Gravitational Phenomena From Superstrings in Curved Spacetime[*]

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ABSTRACT

The four-dimensional superstring solutions define at low energy effective supergravity theories. A class of them extends successfully the validity of the standard model up to the string scale ($\mathcal{O}(10^{17})$ TeV). We stress the importance of string corrections which are relevant for low energy ($\mathcal{O}(1)$ TeV) predictions of gauge and Yukawa couplings as well as the spectrum of the supersymmetric particles. A class of exact string solutions are also presented, providing non trivial space-time backgrounds, from which we can draw some lessons concerning the regions of space-time where the notion of the effective field theory prescription make sense. We show that the string gravitational phenomena may induce during the cosmological evolution, transitions from one effective field theory prescription to a different one where the geometrical and topological data, as well as the relevant observable states are drastically different.

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1 Introduction

String theory extents the validity of quantum field theory to very short distances and defines consistently quantum gravity. It is thus appropriate to try to investigate the behavior of string dynamics at high energies and in regions of spacetime where the gravitational field is strong. There are several problems in gravity where the classical, and even worse the semiclassical treatment have perplexed physicists for decades. We are referring here to questions concerning the behavior in regions of strong (or infinite) curvature with both astrophysical (black holes) and cosmological (big-bang, wormholes) interest. It is only appropriate to try to elucidate such questions in the context of stringy gravity. There has been progress towards this direction, and by now we have at least some ideas on how different string gravity can be from general relativity in regions of space-time with strong curvatures. We need however exact classical solutions of string theory in order to have more quantitative control on phenomena that are characteristic of stringy gravity.

On the other hand the special characteristics of superstrings do not only reflect in the gravitational sector of the theory. There are also important implications for particle physics, namely, concerning the string low energy predictions at M_Z and at the accessible by the future, energy scale of $\mathcal{O}(1)$ TeV.

The first main property of superstrings is that they are ultraviolet finite theories (at least perturbatively). The second important property is that they unify gravity with all other interactions. This unification does not include only the gauge interactions but also the Yukawa ones as well as the interactions among the scalars. This String Hyper-Unification (SHU) happens at large energy scales $E_t = \mathcal{O}(M_{string}) = 10^{17}$ GeV. At this energy scale however, the first exited string states become important and thus, the whole effective low energy field theory picture breaks down. Indeed, the effective field theory of strings is valid only for $E_t \ll M_{string}$ by means of $\mathcal{O}(E_t/M_{string})^2$ expansion. It is then necessary to evolve the SHU predictions to a lower scale $M_U < M_{string}$ where the effective field theory picture makes sense. Then, at M_U any string solution, provides non-trivial relations among the gauge and Yukawa couplings which can be written as,

$$\frac{k_i}{\alpha_i(M_U)} = \frac{k_j}{\alpha_j(M_U)} + \Delta_{ij}(M_U). \tag{1.1}$$

The above relation looks very similar to the well known unification condition in supersymmetric Grand Unified Theories (SuSy-GUT) where the unification scale is about $M_U = 10^{16}$ GeV and $\Delta_{ij}(M_U) = 0$ in the $\bar{DR}$ renormalization scheme; in SuSy-GUTs the normalization constants k_i are fixed *only* for the gauge couplings ($k_1 = k_2 = k_3 = 1$, $k_{em} = \frac{3}{8}$) but there are no relations among gauge and Yukawa couplings at all. In string effective theories however, the normalization constants (k_i) are known for both gauge and Yukawa interactions. Furthermore, $\Delta_{ij}(M_U)$ are calculable *finite* quantities for any particular string solution. Thus, the predictability of a given string solution is extended for all low energy coupling constants $\alpha_i(M_Z)$ once the string induced corrections $\Delta_{ij}(M_U)$

are determined.

This determination however, requests string computations which at present are not completely known. It turns out that $\Delta_{ij}(M_U)$ are non trivial functions of the vacuum expectation values of some gauge singlet fields[1], $<T_A>=t_A$, the so called moduli (the moduli fields are flat directions at the string classical level and they remain flat in string perturbation theory, in the exact supersymmetric limit) :

$$\Delta_{ij}(M_U) = \delta_{ij} + F_{ij}(t_A). \tag{1.2}$$

$F_{ij}(t_A)$ are modular forms which depend on the particular string solution and are normalized here such that: $F_{ij}(t_A) = 0$ when $t_A = 1/M_{string}^2$. Partial results for F_{ij} exist in the exact supersymmetric limit in many string solutions based on orbifold [2] and fermionic constructions [3]. The finite part δ_{ij} is a function of M_U/M_{string} and at the present time it is only approximately estimated; in principle it is a well defined calculable quantity once we perform our calculations at the string level where all interactions including gravity are consistently defined. The full string corrections to the coupling constant unification, $\Delta_{ij}(M_U)$, as well as the string corrections associated to the soft supersymmetry breaking parameters

$$m_0, \quad m_{1/2}, \quad A, \quad B \text{ and } \mu, \quad \text{at} \quad M_U, \tag{1.3}$$

are of main importance, since they fix the strength of the gauge and Yukawa interactions [4, 5], the full spectrum of the supersymmetric particles as well as the Higgs and the top-quark masses at the low energy range $E_t \sim M_Z$ - $\mathcal{O}(1)$ TeV.

There has been a lot of progress in this direction and some semi-quantitative results are already obtained. A much more detailed study is necessary in order to understand better the SHU-predictions when supersymmetry is spontaneously broken.

In the rest of this talk, however, we will focus mostly on some of the implications of superstring theory for gravitational type phenomena. In particular we will analyze some exact four dimensional string solutions which have been constructed recently and they posses interesting cosmological and astrophysical properties.

Studies on string theory have so far given hints for the presence of several interesting stringy phenomena, like the existence of a minimal distance [6], finiteness at short distances [7], smooth topology change [8, 9], spacetime duality symmetries [11, 12, 13, 14, 15, 16, 9, 10], variable dimensionality of spacetime [17], existence of maximal (Hagedorn) temperature and subsequent phase transitions [18] etc. The lessons we learn from the exact solutions we are going to describe essentially corroborate some items on the list above and we would like to present them in a somewhat general context. Ideas of a similar form have already been presented in [19].

In particular, we will present exact string solutions, in 3+1 dimensions, where one can study a topology and/or geometry changing phenomena which can happen in some regions of the space time where the curvature is strong (of the order of the Planck scale); namely in the regions where the $1/M_{string}^2$ expansion (or α' expansion) breaks down and the notion of the effective field theory is ill defined. In the framework of

exact string solutions however, this expansion is not necessary and thus, we can extend our description at the string level using the powerful techniques of the underlying two dimensional (super-)conformal theory. It is then possible (at least in certain cases) to go through the strong curvature region (where the topology and/or the geometry change occurs) towards another asymptotic region where we have a different low energy field theory. In the exact string solution we have studied, this change is described in terms of a modulus field that varies with time [10, 20] and thus the effective field theory transitions occur dynamically during the cosmological evolution.

Here, we have for the first time the possibility to address an interesting phenomenon such as dynamical topology change in the context of an exact solution to classical string theory.

2 String states and their corresponding effective field theories

We will start from the simple case of a compactification on a flat torus to illustrate some typical stringy phenomena and we will eventually move to discuss non-flat backgrounds. The spectrum of string physical states in a given compactification can be generically separated in three kinds of sub-spectra.

i)The first one consists of Kaluza-Klein-like effective field theory modes (or momentum modes). The masses of such modes are always proportional to the typical compactification scale $M_c = M_{string}/R$ where R is a typical radius in M_{string} units (in more general cases, when there are more than one compact dimensions, one can still define the concept of a scale [21]).

ii) The second set of states are the winding modes which exist here because of two reasons. The first is that the string is an extended object. The second is that the target space has a non-trivial π_1 so the string can wind around in a topologically non-contractible way. In a large volume compact space, these modes are always super-heavy, since their mass is proportional to the compactification volume $1/M_c$, (more precisely, it is proportional to M_{string}^2/M_c).

iii) The third class of states are purely stringy states constructed from the string oscillator operators. Their masses are always proportional to the string scale $M_{string} = 1/\sqrt{\alpha'}$.

This separation of the spectrum is strictly correct in torroidal backgrounds. However, further analysis indicates that one can extend the notions of Kaluza-Klein (KK) type modes and winding modes to at least non-flat backgrounds with some Killing symmetries. This generalization comes with the help of the duality symmetries present in such background fields [12]-[16, 9]. However, the "winding" states are not always associated with non-contractible circles of the manifold. They appear as winding configurations

in a (usually) contractible circle associated with a Killing coordinate [15].

Once we have the picture above concerning different types of string excitations we can state that the notion of a string effective field theory makes sense when the "winding" states as well as the oscillator states are much heavier than the field theory-like KK states. Here, we will assume the presence of a single scale (except M_{string}). If there are more such scales then one has to investigate the different regimes. In any such regime, our discussion below is applicable.

Using the $1/M_{string}^2$-expansion one finds the effective theory which is relevant for the low energy processes ($E_t < M_{string}$) among the massless states, as well as the lowest lying KK states, provided, that the typical mass scale M_c, (which could be a compactification scale, gravitational curvature scale etc) is much below the string scale M_{string}. Otherwise the notion of the effective theory is not applicable.

In the last few years, a lot of activity was devoted in understanding the effective theories of strings at genus zero, [22] and in some cases the genus one corrections were included [1]. The output of this study confirms that the winding and the $\mathcal{O}(M_{string})$ string superheavy modes can be integrated out and one can define consistently the string low energy theory in terms of the massless and lowest massive KK states. Thus, the perturbative string solutions are well described by classical gravity coupled to some gauge and matter fields with unified gauge, Yukawa and self interactions. As long as we stay in the regime where $E_t, M_c < M_{string}$ nothing dangerous is happening and consequently our description of the physics in terms the the effective theory is good with well defined and calculable $\mathcal{O}(E_t/M_{string}), \mathcal{O}(M_c/M_{string})$ corrections.

The situation above changes drastically once the mass of "winding" type states becomes smaller than the string scale and when (usually at the same time) the KK modes have masses above the string scale. This is the case when the typical M_c scale is larger than M_{string}. When this occurs, the relevant modes are not any more the KK ones but rather the winding ones. Thanks to the well known by now generalized string duality [12, 13, 14, 15, 16, 9] (e.g. the generalization of the well known R to 1/R torroidal duality) it is possible to find an alternative effective field theory description by means of $\mathcal{O}((E_t, \tilde{M}_c)/M_{string})$ dual expansion, where one uses the dual background which is characterized by $\tilde{M}_c$ instead of the initial one characterized by a high mass scale. The dual mass scale $\tilde{M}_c = M_{string}^2/M_c$ is small when the M_c becomes big and vice-versa. We observe that in both extreme cases, either (i) $M_c < M_{string} < \tilde{M}_c$ or (ii) $M_c > M_{string} > \tilde{M}_c$, a field theory description exists in terms of the original curved background metric in the first case or in terms of its dual in the second case. This observation is of main importance since it extends the notion of the effective field theories in backgrounds with associated high mass scales (due to size or curvature).

Strictly speaking, there can be many dual backgrounds which correspond to the same string solution and it is an open problem if it is always possible to map all regions of spacetime with high associated scales to ones with small such scales. In fact, this is not always necessary, since, even for regions which are strongly curved or have small

volume, we can have a well defined effective description. Examples could be the region close to special symmetry points in torroidal compactifications, where although the torus has volume of order one, we can easily handle the low energy spontaneously broken gauge theory. Then, a general string solution would give rise to a *set* of *effective field theories* defined in restricted regions of space-time $(x^\mu)_I$, I=1,2,3... with M_I smaller or of the same order as M_{string}; If $T_{I,J}$ is the boundary region among $(x^\mu)_I$ and $(x^\mu)_J$, then on $T_{I,J}$ we have almost degenerate effective characteristic scales $M_I \sim M_{string} \sim M_J$. In such regions, the effective field theory description of regions I, J break down (individually), and the full string theory is needed in order to have a smooth transition between the two.

A goal in that direction would be to establish some simple rules that would provide the extra stringy information that would *glue* the field theoretic regions together. This effectively amounts to a reorganization of the α' expansion. In simple backgrounds, like torroidal ones this *gluing* can be effectively done [23] by constructing an effective action, containing an infinite number of fields.

The models we have studied[10] could very well serve as a laboratory towards answering this question. This is certainly important, since many interesting phenomena happen precisely at such regions in string theory. We can mention, the *gluing* of *dual solutions* in cosmological contexts, [24] and global effective theories for large regions of internal moduli spaces. A related issue here is, that with each region, one has an associated geometry and spatial topology, as dictated by the effective field theory. It turns out that moving from one region to another not only the geometry can change but also the topology. Examples in the context of Calabi-Yau compactifications [8] and more simple models, [9] have been given.

There is another important point about topology change that we would like to stress here. Where topology change happens, depends crucially on the values of some of the parameters in the background. One such parameter is always M_c/M_{string}, but usually, in string backgrounds, there are others, like various different levels for non-simple WZW and their descendant conformal field theories, various radii or related moduli etc. The absolute judge concerning topology change is the effective field theory.

Another application of such solutions could be in considering strings at finite temperature. They would describe a string ensemble with temperature that varies (adiabatically) in space. It might be interesting to entertain such an idea in more detail, in order to investigate temperature gradients in string theory.

3   Stringy dynamical transitions among effective field theories

We will present now a class of exact string cosmological models [10] where the geometry and/or topology of their effective field theories changes dynamically with the time. As a

starting point we will consider first a static four dimensional model based on the following space-time metric:

$$ds^2 = -dt^2 + k[dx^2 + \frac{sin^2x}{cos^2x + R^2sin^2x}d\theta_1^2 + \frac{R^2cos^2x}{cos^2x + R^2sin^2x}d\theta_2^2] \qquad (3.1)$$

The above metric corresponds to an exact string solution for any value of the parameter R. For the special value R=1 the space is a three dimensional sphere (S^3) with constant scalar curvature $\hat{R} = 6/k$. For arbitrary $R \epsilon (0, \infty)$ the above metric, from the conformal theory point of view, corresponds to an exact deformation (by $J_3 \bar{J}_3$) of the $SU(2)_k$ WZW-model with a scalar curvature $\hat{R}$ given by,

$$\hat{R} = -\frac{2}{k}\frac{[2(R^4 - 1)\sin^2 x - 5(R^2 - 1) - 3]}{(1 + (R^2 - 1)\sin^2 x)^2} . \qquad (3.2)$$

The manifold is regular except at the end-points where

$$\hat{R}(R = 0) = -\frac{4}{k\cos^2 x} \quad , \quad \hat{R}(R = \infty) = -\frac{4}{k\sin^2 x} . \qquad (3.3)$$

We have to stress here, that even at the end points the string theory is well defined and thus the curvature singularity of the effective field theory is an artifact. Indeed, at these points the $SU(2)_k$ deformed theory factorizes to the $(SU(2)/U(1))_k$ parafermionic theory and to a $U(1)$ non-compact dimension. Both conformal sub-systems ($SU(2)_k$ and $U(1)$), have well defined and non-singular correlations functions although their effective field theory metric has curvature singularities. It should be noted also that, the geometric data (metric, curvature, etc.) are invariant under the duality transformation $R \rightarrow 1/R$ and $x \rightarrow \pi/2 - x$.

In order to give a more complete description of the above model we must specify the other two non-trivial background fields; The dilaton $\phi(x)$ and the antisymmetric tensor field $B_{\mu,\nu}(x)$:

$$\phi(x, R) = log[\frac{cos^2x + R^2 \; sin^2x}{R^2}] + \phi_0 \quad , \qquad (3.4)$$

$$B_{\theta_1,\theta_2}(x, R) = \frac{cos^2x}{(cos^2x + R^2 \; sin^2x)} \qquad (3.5)$$

The string energy spectrum (in M_{string} units) of the model is given in terms of the deformed $SU(2)_k$ theory quantum numbers; the $SU(2)$ spin j and the left and right $j_3, \bar{j}_3$ charges m and $\bar{m}$

$$E^2(R) = 2\frac{j(j + 1)}{k + 2} - \frac{m^2 + \bar{m}^2}{k} + \frac{1}{2k}\left[(m - \bar{m} + kM)^2R^2 + \frac{(m + \bar{m} + kN)^2}{R^2}\right] + N' + \bar{N}' \qquad (3.6)$$

where $N', \bar{N}'$ denote the string oscillator contributions.

For k relatively large (semi-classical limit) the would be excited states below the string scale (states with $E^2(R) \ll M_{string}^2$) are those which correspond to $N' = \bar{N}' = 0$ and :

6

i) $m - \bar{m} = M = 0$ and $(kN)^2 < R^2$ if $R >> k$,

ii) $N = M = 0$ and $m, \bar{m} << k$ if $R \sim 1$.

iii) $m + \bar{m} = N = 0$ and $(kM)^2 < R^{-2}$ if $R^{-1} >> k$,

In these three R intervals the lower lying states are organized in a different manner; i) $\mathcal{O}(1/k)$ and $\mathcal{O}(k/R)$ expansion when R is large, ii) $\mathcal{O}(1/k)$ when $R \sim 1$ and iii) $\mathcal{O}(1/k)$ and $\mathcal{O}(kR))$ when R is small:

$$E^2(R >> k) = \frac{2}{k}[j(j+1) - m^2 + (m + \frac{kM}{2})^2 \frac{1}{R^2}] \tag{3.7}$$

$$E^2(R \sim 1) = \frac{1}{k}[2j(j+1) - m^2 - \bar{m}^2 + (m - \bar{m})^2 R^2 + (m + \bar{m})^2 \frac{1}{R^2}] \tag{3.8}$$

$$E^2(R << k) = \frac{2}{k}[j(j+1) - m^2 + (m + \frac{kN}{2})^2 R^2] \tag{3.9}$$

The $R >> 1$ effective field theory spectrum is well described by the $[SU(2)/U(1)]_k$ semi-classical spectrum ($\frac{1}{k}[2j(j+1) - m^2]$) and by a large radius, almost decompactified, $U(1)$ spectrum (l^2/R^2); The effective field theory metric in this regime is defined by (with $\tilde{\theta}_1 = \theta_1/R$):

$$ds^2(R >> 1) \sim -dt^2 + k[dx^2 + d\tilde{\theta}_1^2 + \frac{cos^2 x}{sin^2 x} d\theta_2^2] \tag{3.10}$$

Around $R \sim 1$ the semiclassical spectrum is similar to a small deformed S^3 sphere and the effective field theory metric is approximately given by:

$$ds^2(R \sim 1) \sim -dt^2 + k[dx^2 + sin^2 x d\theta_1^2 + cos^2 x d\theta_2^2] \tag{3.11}$$

Finally when $R << 1$ the spectrum is well described by the *dual* $[SU(2)/U(1)]_k$ semi-classical spectrum ($\frac{1}{k}[2j(j+1) - m^2]$) and by a dual $U(1)$ spectrum ($l^2 R^2$). The effective theory metric becomes now: (with $\tilde{\theta}_2 = \theta_2 R$)

$$ds^2(R << 1) \sim -dt^2 + k[dx^2 + \frac{sin^2 x}{cos^2 x} d\theta_1^2 + d\tilde{\theta}_2^2] \tag{3.12}$$

In the above described example the case $R >> 1$ and $R << 1$ give isomorphic effective field theories; they have the same topology and their metrics are diffeomorfic. This property it is not generic for other string examples. For instance, the string example which is based on a deformed H_3^+ hyperboloid, instead of the deformed $SU(2)_k$ described above, defines in $R >> 1$ regime an effective field theory which topologically is different from the effective field theory in the $R << 1$ regime and that which is defined in $R \sim 1$

region. All geometrical data on H_3^+ can be obtain formally from those of $SU(2)_k$ by means of the following analytic continuation:

$$t \rightarrow t, \quad x \rightarrow ix, \quad \theta_1 \rightarrow \theta_1, \quad \theta_2 \rightarrow i\theta_2; \quad k \rightarrow -k, \quad sinx \rightarrow isinhx, \quad cosx \rightarrow coshx. \quad (3.13)$$

The effective theory metric of the deformed H_3^+ model is regular for $R << 1$ and is factorized in a two sub-spaces. The one sub-space is that which is described by the semi-classical metric of the "axial" gauged WZW model $[SU(2)/U(1)]_k$ and has the shape of a "two-dimensional cigar" (x, θ_1), while the second sub-space is described by a non-compact coordinate $(\tilde{\theta}_2)$. The effective theory metric around $R \sim 1$ is that of a static pseudosphere (H_3^+ hyperboloide) with constant negative curvature $\hat{R} = -6/k$. For $R > 1$ the effective theory metric has singularities at $sinh^2 x = 1/(R^2 - 1)$ and for very large R is factorized and is well described by the "vector" gauged WZW model $[SU(2)/U(1)]_k$ together with an extra non-compact $U(1)$ factor. The three dimensional space in this limit is factorized to a two dimensional space with the shape of a "trumpet" (x, θ_2) and to the one which is defined by one non-compact coordinate $\tilde{\theta}_1$.

We would like to make the static picture described above dynamical giving to the parameter R an evolution in time. If that can be done in a stringy way then it will be possible to pass through different effective field theories during the cosmological evolution. This dynamical changing phenomenon cannot be described in terms of a unique effective field theory with finite number of states. We have shown[10] that, in both $SU(2)$ and H_3^+ case, there exists a stringy extension with R now a function of time, which is adiabatic at least is some time intervals provided that $R(t)$ satisfies the following non linear differential equation.

$$\frac{R'''}{R''} = \frac{R''}{R'} + \frac{R'}{R} \quad (3.14)$$

All solutions of this equation correspond to exact string solutions and can be classified in terms of two parameters C_1 and C_2. One has after integration:

$$\frac{R'}{R} = C_1 \ R + C_2 \frac{1}{R} \quad (3.15)$$

The metric and antisymmetric tensor of the static case have the same form in terms of $R(t)$. The dilaton field however gets some corrections:

$$\phi(x, R(t)) = log[\frac{cos^2 x + R(t)^2 \ sin^2 x}{R(t)^2}] - log[\frac{R'(t)}{R(t)}]\phi_0 \ , \quad (3.16)$$

The solutions with non-trivial $R(t)$ are (up to shifts in t):

(ia) $C_1 = 0$:

$$R^2(t) = C_2^2 t^2 \quad (3.17)$$

For $t \in [0, \infty)$, $R^2 \in [0, \infty)$.

8

(ib) $C_2 = 0$:

$$R^2(t) = \frac{1}{C_1^2 t^2} \tag{3.18}$$

For $t \in [0, \infty)$, $R^2 \in [0, \infty)$.

(ii) $C_1 C_2 > 0$:

$$R(t) = \sqrt{\frac{C_2}{C_1}} \tan(\sqrt{C_1 C_2} t) \tag{3.19}$$

For $t \in [0, \pi/2\sqrt{C_1 C_2}]$, $R^2 \in [0, \infty)$.

(iii) $C_1 C_2 < 0$:

$$R(t) = \sqrt{-\frac{C_2}{C_1}} \tanh(\sqrt{|C_1 C_2|} t) \quad \text{and} \quad R(t) = \sqrt{-\frac{C_2}{C_1}} \coth(\sqrt{|C_1 C_2|} t) \tag{3.20}$$

Here, for the *tanh* solution, $R^2 \in [0, 1]$ whereas for the *coth* solution $R^2 \in [1, \infty)$.

All of the above solutions correspond to exact conformal field theories with central charge $c = 4 - 6 \left(1/(k+2) - 4C_1 C_2/(3 - 4C_1 C_2)\right)$. When C_i are such that, $c = 4$, then their supersymmetric extension are nothing but deformations and analytic continuations of the exact $N = 4$, $\hat{c}=4$ supercorfomal systems[25] and which were used[17] to construct exact supersymmetric string solutions in non trivial space-time. The euclidean version of them corresponds to a class of gravitational and axionic instantons[26].

In summary, exact string solutions as those described above, have a twofold interest. First they describe cosmological solutions in which the effective field theory describing the early stages of our universe is completely different from the one describing its later stages. The study of such solutions can provide us with some useful hints for understanding better the string unification at high scales and with quantitative predictions at low energies.

Acknowledgements

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