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Entropy, Quantum Decoherence and Pointer States in Scalar "Parton" Fields [1]

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Abstract

Entropy arises in strong interactions by a dynamical separation of "partons" from unobservable "environment" modes due to confinement. For interacting scalar fields we calculate the statistical entropy of the observable subsystem. Diagonalizing its density matrix yields field pointer states and their probabilities in terms of Wightman functions. It also indicates how to calculate a finite geometric entropy proportional to a surface area.

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The long-standing "entropy puzzle" of high-multiplicity events in strong interactions at high energy has been analysed from a new point of view [1]. The problem dates back to Fermi and Landau and is related to understanding the rapid thermalization of high energy density ($\gg 1$ GeV/fm^3) matter [2]. Why do thermal models work so well? Why do they work at all?

Or, why does high-energy scattering of pure initial states lend itself to a statistical description characterized by large apparent entropy from a mixed-state density matrix describing intermediate stages in a space-time picture of parton evolution? Effectively, *unitary time evolution* of the observable part of the system breaks down in the transition from a quantum mechanically pure initial state to a highly impure (more or less thermal) high-multiplicity final state. In Ref. [1] this was discussed in detail. Based on analogies with studies of the quantum measurement process ("collapse of the wave function") [3] and motivated by related problems in quantum cosmology and by non-unitary non-equilibrium evolution resulting in string theory [4], we argued that *environment-induced quantum decoherence solves the entropy puzzle* of strong interactions.

A complex pure-state quantum system can show quasi-classical behaviour, i.e. an impure density (sub)matrix together with decoherence of associated *pointer states* in the observable subsystem [1]. In particular, there is a *Momentum Space Mode Separation* due to confinement, which is defined in the frame of initial conditions for the time evolution and for the physical (gauge) field degrees of freedom. Thus, almost constant QCD field configurations form an *unobservable environment*, since they neither hadronize nor initiate hard scattering among themselves. It interacts with the *observable subsystem* composed of partons [5].

The induced quantum decoherence and entropy production were studied in a non-relativistic single-particle model resembling an electron coupled to the quantized electromagnetic field, however, with an enhanced oscillator spectral density in the infrared. The Feynman-Vernon influence functional technique for quantum Brownian motion [6] provided the remarkable result that in the *short-time strong-coupling limit* the model parton behaves like a *classical particle* [1]: Gaussian parton wave packets experience *friction* and *localization*, i.e. no quantum mechanical spreading, and their coherent *superpositions decohere.*

Summarizing, partons feel an unobservable (gluonic) environment, which manifests its strong non-perturbative interactions on a short time scale ($\ll 1$ fm/c) through *decoherence* of partonic pointer states, their *quasi-classical behaviour*, and *entropy production.* If confirmed in QCD, this will have important consequences for parton-model applications to complex hadronic or nuclear reactions [7]. The emergence of *structure functions* from initial-state wave functions will be further studied in our approach.

We defined a model of two coupled scalar fields representing partons and their non-perturbative environment. In the functional Schrödinger picture employing Dirac's time-dependent variational principle we derived its Cornwall-Jackiw-Tomboulis (CJT) effective action and the equations of motion for renormalizable interactions [1, 8]. Thus, analysis of the entropy puzzle in strong interactions leads to study an observable field (open subsystem) interacting with a dynamically hidden one (unobservable environment), i.e. *quantum field Brownian motion.*

In the following we derive the *entropy in any system of two interacting real scalar fields.* Their most general normalized *Gaussian wave functional* in the Schrödinger picture can be written as

$$\begin{aligned}\Psi_{12}[\phi_1,\phi_2;t] \;&\equiv\; N_{12}(t)\;\Psi_{G_1}[\phi_1;t]\;\Psi_{G_2}[\phi_2;t] \\ &\cdot\;\exp\left\{-\tfrac{1}{2}[\phi_1-\bar{\phi}_1(t)]\,[G_{12}(t)-i\Sigma_{12}(t)]\,[\phi_2-\bar{\phi}_2(t)]\right\}\;, \qquad (1)\end{aligned}$$

with $(j=1,2)$

$$\Psi_{G_j}[\phi_j;t] \;\equiv \qquad (2)$$

$$N_j(t)\;\exp\left\{-[\phi_j-\bar{\phi}_j(t)]\left[\tfrac{1}{4}G_j^{-1}(t)-i\Sigma_j(t)\right][\phi_j-\bar{\phi}_j(t)]\;+\;i\bar{\pi}_j(t)[\phi_j-\bar{\phi}_j(t)]\right\}\;.$$

We suppress all spatial integrations. The normalization factors are

$$N_j(t)\;=\;\det\{2G_j(t)\}^{-1/4}\;,\quad N_{12}(t)\;=\;\det\{1-G_1(t)G_{12}(t)G_2(t)G_{12}(t)\}^{1/4}\;,\qquad (3)$$

discarding an irrelevant constant factor in N_j. Thus, the time-dependent Hartree-Fock approximation (TDHF) for the quantum field Schrödinger equation [1, 8] is embodied in the variational parameter one-point functions $\bar{\phi}_j(x,t)$, $\bar{\pi}_j(x,t)$ (mean fields) and symmetric two-point functions $G_j(x,y,t)$, $\Sigma_j(x,y,t)$, $G_{12}(x,y,t)$, $\Sigma_{12}(x,y,t)$ (related to Wightman functions). Their meaning was discussed in [1].

All physical quantities of the complex system can be calculated with Ψ_{12}, expressing inner products by functional integrals. The functional density submatrix $\hat{\rho}_{\mathcal{P}}$ for the observable "parton" subsystem (ϕ_1) is obtained by tracing over the unobservable degrees of freedom (ϕ_2),

$$\hat{\rho}_{\mathcal{P}}(t) \;\equiv\; \mathrm{Tr}_2\, |\Psi_{12}(t)\rangle\langle\Psi_{12}(t)| \;\; , \tag{4}$$

as calculated explicitly in [1] (we henceforth omit $\mathcal{P}$). The matrix elements of $\hat{\rho}$ contain *all* the information about the subsystem. Our aim is to obtain the *von Neumann* or *statistical entropy*, $S \equiv -\mathrm{Tr}_1\, \hat{\rho} \ln \hat{\rho}$. Before, we calculated the simpler *linear entropy* directly, which provides a lower bound for the statistical entropy [1],

$$S(t) \;\geq\; -\frac{1}{2}\,\mathrm{Tr}\;\ln\left(\frac{1-G_1(t)G_{12}(t)G_2(t)G_{12}(t)}{1+G_1(t)\Sigma_{12}(t)G_2(t)\Sigma_{12}(t)}\right) \;\; , \tag{5}$$

tracing over coordinates. Equation (5) is also valid for *non-translation invariant* systems, which is relevant for calculating the geometric entropy related to spatial boundaries separating observable and unobservable subsystems.

Geometric entropy is intimately connected to black-hole entropy [9]. Here, one identifies ϕ_1 as the part of a scalar field ϕ with support *outside* a given spatial region and $\phi_2 \equiv \phi - \phi_1$, which has its support *inside* the complement. Our results (5) and (17) below indicate that geometric entropy comes out *finite*, once a renormalization of the equations for the two-point functions, G's and Σ's in (5), is performed or a UV regularization introduced to provide sufficient integrability constraints.

We proceed by *diagonalizing* $\hat{\rho}$. Determining its eigenstates and eigenvalues is equivalent to constructing *field pointer states* [1, 3] within TDHF and their probabilities. The eigenvalue problem $\hat{\rho}|\rho\rangle = \rho|\rho\rangle$ to be solved is of the form

$$\rho\; F[\phi]\;\exp\{-\phi\alpha\phi + \beta\phi\}\;= \tag{6}$$

$$N^2\ \exp\{-\phi a\phi + b\phi\} \int \mathcal{D}\phi'\ F[\phi']\ \exp\{-\phi'[a^* + \alpha]\phi' + [b^* + \beta + \phi c^*]\phi'\}\ \ ,$$

using the ansatz $\langle(\phi + \bar{\phi}_1)|\rho\rangle \equiv F[\phi]\exp\{-\phi\alpha\phi + \beta\phi\}$ with unknown one- and two-point functions β and α and a non-exponential functional F. According to results for $\hat{\rho}_{\mathcal{P}}$ from [1], we define $N \equiv N_1 N_{12}$, $b \equiv i\bar{\pi}_1$ ($\bar{\phi}_1$ does not appear in (6)),

$$\begin{aligned} a &\equiv \tfrac{1}{4}G_1^{-1}A - i[\Sigma_1 - \tfrac{1}{8}(G_{12}G_2\Sigma_{12} + \Sigma_{12}G_2G_{12})] \ = \ a^t\ \ , \\ c &\equiv \tfrac{1}{2}G_1^{-1}B - \tfrac{i}{4}[G_{12}G_2\Sigma_{12} - \Sigma_{12}G_2G_{12}] \ = \ c^\dagger\ \ , \end{aligned}$$

and combinations of two-point functions

$$\begin{aligned} A &\equiv 1 - \tfrac{1}{2}G_1G_{12}G_2G_{12} + \tfrac{1}{2}G_1\Sigma_{12}G_2\Sigma_{12}\ \ , \\ B &\equiv \tfrac{1}{2}G_1G_{12}G_2G_{12} + \tfrac{1}{2}G_1\Sigma_{12}G_2\Sigma_{12}\ \ . \end{aligned}$$

Choosing $\beta \equiv b$ in (6), completing the square, shifting ϕ', and requiring resulting Gaussians in ϕ to cancel yields the eigenvalue problem:

$$\rho\ F[\phi] \ = \ N^2 \int \mathcal{D}\phi'\ F[\phi' + Y\phi]\ \exp\{-\phi' X\phi'\}\ \ , \tag{7}$$

with $X \equiv a^* + \alpha = X^t$, $Y \equiv \frac{1}{2}X^{-1}c$, and where $\alpha = \alpha^t$, by (6), is determined to solve the equation $a - \alpha = \frac{1}{4}c^*[a^* + \alpha]^{-1}c$. Note the similarity to the finite-dimensional oscillator problem of Srednicki [9].

Equivalently, replacing $F[\phi'] \to F[\delta/\delta(\phi c^*) + \delta/\delta(c\phi)]$ and $\phi c^*\phi' \to \frac{1}{2}[\phi c^*\phi' + \phi' c\phi]$ in (6), we obtain by integration

$$\rho\ F[\phi] \ = \ N^2\ \det\{X\}^{-1/2}\ \exp\{-\tfrac{1}{4}\phi c^* X^{-1}c\phi\}\ F[\tfrac{\delta}{\delta(\phi c^*)} + \tfrac{\delta}{\delta(c\phi)}]\ \exp\{+\tfrac{1}{4}\phi c^* X^{-1} c\phi\}\ \ , \tag{8}$$

which is more convenient than (7). Looking for polynomial functional solutions of (8), we find first of all a *constant*,

$$F_0[\phi] \ \equiv \ 1 \quad \Rightarrow \quad \rho_0 \ = \ N^2\ \det\{X\}^{-1/2}\ \ . \tag{9}$$

Secondly, instead of a general *linear* functional, the Fourier transform is sufficient,

$$F_k[\phi] \ \equiv \ \int d^dx\ \mathrm{e}^{-ikx}\phi(x) \ \equiv \ \phi_k\ \ , \tag{10}$$

since the problem is linear in F. Then, from (8) $-$ (10),

$$\rho_1\phi_k \;=\; \tfrac{1}{4}\rho_0[X^{-1}c\phi+\phi c^* X^{-1}]_k \;=\; \rho_0 Y_{kk'}\phi_{-k'} \;\;, \tag{11}$$

summing over indices occurring twice. For a translation-invariant system, (11) could immediately be solved. Generally, however, denoting eigenvalues and eigenvectors of $(Y_{kk'})$ by ξ_k and $\tilde{\phi}_k$, one obtains a set of *linear* eigenvalues $\rho_k = \rho_0\xi_k$. Due to the Gaussian structure in (8), the *higher-order eigenfunctionals* can be built up as linear combinations of products of $\tilde{\phi}_k$'s and lower-order ones. For example, $F_{kk'}[\phi]\equiv \tilde{\phi}_k\tilde{\phi}_{k'}+C_{kk'}$, which yields a set of *quadratic* eigenvalues $\rho_{kk'}=\rho_0\xi_k\xi_{k'}\Theta(k'-k)$. Note the constraint $k'\geq k$; interchange of k and k' does not lead to a new eigenfunctional due to the scalar (bosonic) character of the fields. The *constant* $C_{kk'}$ follows with the help of the matrix diagonalizing $(Y_{kk'})$. We do not construct explicitly the higher-order eigenfunctionals. However, the *n-th order set of eigenvalues*,

$$\rho_{k_1\ldots k_n} \;=\; \rho_0\xi_{k_1}\prod_{i=2}^{n}\,\xi_{k_i}\Theta(k_i-k_{i-1}) \;\;, \tag{12}$$

is easily found, similarly to $\rho_{kk'}$ above. To check the result (12), we calculate

$$\begin{aligned}
\mathrm{Tr}\;\hat{\rho}(t) \;&=\; \sum\;\mathrm{eigenvalues} \;=\; \rho_0+\sum_{n=1}^{\infty}\sum_{k_1\ldots k_n}\;\rho_{k_1\ldots k_n} \\
&=\; \rho_0[1+\sum_{k_1}\;\xi_{k_1}+\sum_{k_1\leq k_2}\;\xi_{k_1}\xi_{k_2}+\sum_{k_1\leq k_2\leq k_3}\;\xi_{k_1}\xi_{k_2}\xi_{k_3}+\;\ldots\,] \\
&=\; \rho_0\prod_k\sum_{n_k=0}^{\infty}\;\xi_k^{\,n_k} \;=\; \rho_0\prod_k\,[1-\xi_k]^{-1} \;=\; \rho_0\det\{1-Y\}^{-1} \;=\; 1 \;\;,
\end{aligned} \tag{13}$$

which resembles the evaluation of a bosonic partition function. In the last step we used $\rho_0=\det\{X^{-1}\mathrm{Re}[2a-c]\}^{1/2}=\det\{[1-\frac{1}{2}X^{-1}c^*][1-\frac{1}{2}X^{-1}c]\}^{1/2}$, which follows from the equation determining α or X.

Similarly, we obtain the *linear entropy*,

$$S^{lin} \;\equiv\; \mathrm{Tr}_1\left\{\hat{\rho}-\hat{\rho}^{\,2}\right\} \;=\; 1-\;\mathrm{Tr}_1\;\hat{\rho}^{\,2} \;=\; 1-\;\det\left\{\frac{1-Y}{1+Y}\right\} \;\;. \tag{14}$$

In order to express Y in terms of A and B, we observe that in a *direct* calculation [1] of $\mathrm{Tr}_1\hat{\rho}^2$ (and in the n-fold functional integral for $\mathrm{Tr}_1\hat{\rho}^n$) *imaginary parts* of a and c cancel. Therefore, we replace a and c by their *real parts*, $a=\frac{1}{4}G_1^{-1}A$ and $c=\frac{1}{2}G_1^{-1}B$, simplifying the equation for α, X, or Y, $a-\alpha = \frac{1}{4}c[a+\alpha]^{-1}c$. The solution (for integrable eigenfunctionals) is

$$Y \;=\; (c/2)^{-1/2}\left[\tilde{a}+(\tilde{a}^2-1)^{1/2}\right]^{-1}(c/2)^{1/2} \quad , \tag{15}$$

with $\tilde{a}\equiv (c/2)^{-1/2}a(c/2)^{-1/2}$. Finally, inserting (15) into (14),

$$S^{lin}(t) \;=\; 1-\;\det\left\{\frac{A(t)-B(t)}{A(t)+B(t)}\right\}^{1/2} \quad , \tag{16}$$

which confirms our earlier result, employed in (5) [1]. Next, we calculate the *statistical entropy* using the "replica trick":

$$\begin{aligned} S(t) \;&\equiv\; -\operatorname{Tr}_1\hat{\rho}(t)\ln\hat{\rho}(t) \;=\; -\frac{d}{dn}\operatorname{Tr}_1\hat{\rho}^{\,n}\,|_{n=1} \;=\; -\frac{d}{dn}\det\left\{\frac{(1-Y)^n}{1-Y^n}\right\}|_{n=1} \\ &=\; -\operatorname{Tr}\left\{\ln(1-Y)+\frac{Y}{1-Y}\ln Y\right\} \quad . \end{aligned} \tag{17}$$

Together with (15), eq. (17) presents our main result. It generalizes eq. (6) of Srednicki [9]. Basically, the TDHF approximation for *interacting* quantum fields preserves a Gaussian structure of the wave functionals, see (1) $-$ (3), which is exact in the non-interacting case and can be reduced to a coupled harmonic oscillator problem.

To evaluate the entropy (17) is still a formidable task for any realistic situation. Before trying, it seems worth while to draw some general conclusions:

I. Neither mean fields $\bar{\phi}_{1,2}$, nor their conjugate momenta $\bar{\pi}_{1,2}$, nor imaginary parts $\Sigma_{1,2}$ of the "parton" and environment two-point functions contribute to S.

II. *Vanishing correlations between "partons" and environment*, i.e. $G_{12}=\Sigma_{12}=0$ (independent subsystems), *imply* $A=1, B=0$, i.e. $Y=0$, *and* $S=0$.

III. *Vanishing widths of "parton" or environment wave functionals*, i.e. $G_{1,2}\to 0$ (one or the other subsystem classical/reversible [1]), *imply* $Y=0$ *and* $S=0$.

This presumably holds for *any* field theory of "partons" coupled to environment modes independently of the interactions in TDHF approximation. The time-evolution, however, follows specific equations of motion for the one- and two-point functions [1].

Our considerations confirm that *quantum decoherence* and *entropy production* in a subsystem is induced by an active environment [1, 3, 4, 6]. The above diagonalization of the "parton" density functional yields time-dependent *field pointer states*, the simplest one of largest probability ρ_0 being

$$\Psi_0[\phi;t] \;=\; \exp\{-[\phi-\bar{\phi}_1(t)]\alpha(t)[\phi-\bar{\phi}_1(t)]+i\bar{\pi}_1(t)[\phi-\bar{\phi}_1(t)]\} \;\; , \tag{18}$$

cf. (6) $-$ (9), with $\alpha=[(c/2)(\tilde{a}^2-1)(c/2)]^{1/2}$. Higher-order eigenfunctionals are less probable, see (12), *and* have higher kinetic energy, since their wave functionals have additional nodes, e.g. (10), analogous to excited oscillator states.

As a first application of (17) we consider the *large-entropy limit*, i.e. $Y\approx 1$ or $A\approx B$. Then, using (14) and (16), we find:

$$\begin{aligned} S(t) \;&\approx\; -\ \mathrm{Tr}\ \ln(1-Y)/(1+Y) \;=\; -\tfrac{1}{2}\ \mathrm{Tr}\ \ln(A-B)/(A+B) \\ &=\; \tfrac{1}{2}\textstyle\sum_{n=1}^{\infty}\frac{1}{n}\ \mathrm{Tr}\ \{[G_1G_{12}G_2G_{12}]^n-[-G_1\Sigma_{12}G_2\Sigma_{12}]^n\} \;\; , \end{aligned} \tag{19}$$

i.e. (5). If we assume a *spatial surface* of area $\mathcal{A}$ dividing the system into two, which is flat on the scale of the *short-ranged* correlations in (19), then $\mathrm{Tr}[\ldots]^n$ can be interpreted as a sum of closed loops of strings of G's or Σ's intersecting the surface $2n$ times: once for each factor G_{12} or Σ_{12} correlating in- and outside fields. The dominant contribution to the trace comes from small loops (let $\Sigma_{12}^2\ll G_{12}^2$). Regularizing their contribution by a short-distance cut-off $\mathcal{L}$, their size transverse to the surface will be $\mathrm{O}(\mathcal{L}^2)$ for $d=3$. Transverse to the surface the system is locally translation-invariant. Therefore, the *geometric entropy* is approximately

$$S(t) \;\propto\; -\frac{\mathcal{A}}{\mathcal{L}^2}\ \mathrm{Tr}_{\mathcal{L}}\ \ln\left(\frac{1-G_1G_{12}G_2G_{12}}{1+G_1\Sigma_{12}G_2\Sigma_{12}}\right) \;\; , \tag{20}$$

where $\mathrm{Tr}_{\mathcal{L}}$ is evaluated locally on the scale of $\mathcal{L}$. A dimensional analysis led Srednicki to propose $S\propto\mathcal{A}$ before [9]. Equation (20) can be applied to the moving mirror model; following Kabat et al. [9], it approximates the *thermal entropy* outside a black hole of radius much larger than $\mathcal{L}$.

Secondly, coming back to partons interacting with their (gluonic) environment, the *rate of entropy production*, which follows from (17), is most interesting. We define a dynamical *decoherence time* τ,

$$\tau^{-1}(t) \;\equiv\; \frac{d}{dt}\ \ln\ S(t) \;\approx\; \frac{\mathrm{Tr}\ \dot{Y}\ln Y}{\mathrm{Tr}\ Y\ln Y} \;=\; \frac{\int\tilde{d}k\ \dot{Y}_k\ln Y_k}{\int\tilde{d}k\ Y_k\ln Y_k} \;\; , \tag{21}$$

with $\tilde{dk} \equiv d^d k/(2\pi)^d$. For simplicity we assumed small Y or S and a translation-invariant system; the Fourier transform is $Y_k = B_k[A_k + (A_k^2 - B_k^2)^{1/2}]^{-1}$, since A, B are convolutions of two-point functions now. Generally, two limits are particularly important: $\tau(t \to 0)$ gives the time scale for the decay of a Gaussian partonic field state, cf. (1) $-$ (3), into an *incoherent superposition* of pointer states, e.g. (18), with impure density matrix and *non-zero entropy*; $\tau(t \gg 0)$ reflects the approach to a stationary state (*thermalization*), if it exists. Using the equations of motion from [1], the decoherence time will be calculated for phenomenologically interesting situations elsewhere.

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