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A New Duality Symmetry in String Theory

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ABSTRACT

We consider the conformal gauging of non-abelian groups. In such cases there are inequivalent ways of gauging (generalizing the axial and vector cases for abelian groups) corresponding to external automorphisms of the group. Different σ-models obtained this way correspond to the same conformal field theory. We use the method of quotients to formulate this equivalence as a new duality symmetry.

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Duality symmetries are special to string theory and seem to be very useful in understanding stringy physics. Although discovered in flat torroidal backgrounds [1], they were shown to persist semiclassically in curved backgrounds with abelian [2] or non-abelian [3] symmetries. In [4, 5] it was realized that axial and vector gauging of an abelian chiral symmetry provides with two dual versions of the same σ-model. One on the other hand can use CFT arguments in order to show that axial and vector abelian cosets correspond to the same CFT, [6]. Axial-vector duality was employed in [7] to generate, using the method of quotients, the general abelian duality transformations, [2], and used to generalize the $O(d, d, Z)$ symmetry to curved backgrounds [8]. The exactness of abelian duality symmetries in the compact case is well understood [6, 9]. In the non-compact case, we know that axial vector duality is exact, [10], only for abelian cosets possessing appropriate Weyl symmetries. Concerning non-abelian duality [11, 12, 13] the situation is certainely not clear. It is not obvious if one has an exact symmetry and tools like axial-vector duality are lacking so far.

In this note we will study a non-abelian analog of axial-vector duality which is generated by gauging non-abelian groups in a σ-model, not only using the standard vector gauging but also other possible gaugings related to vector gauging by external automorphisms, [14, 15]. As we will see, a look at the CFT construction of these gaugings indicates that the various cosets obtainable that way, are equivalent CFTs. This will provide us then with some new duality transformations in the σ-model picture. It is plausible that this duality (which from now on we will label *quasi-axial-vector duality*) is related to the standard non-abelian duality. Moreover in general it acts on σ-model backgrounds with no isometries. An easy example of that is G/H with H a maximal subgroup of G.

This quasi-axial-vector duality realized partly a conjecture in [16]. One expects all the underlying exact symmetries of current algebra to generate duality symmetries for the σ-model description. Affine external automorphisms however are not obvious to implement in the σ-model. Here we will deal with usual external automorphisms and provide their implications for quasi-axial-vector duality.

The first explicit example of such duality was given in [15], where two different gaugings were considered for the $(E_2^c \times E_2^c)/E_2^c$ coset model. The respective σ-model backgrounds were related by a series of abelian duality transformations however. The reason is that if one constructs the E_2^c groups by appropriately contracting $SU(2) \times U(1)$ then the quasi-axial-vector duality is generated by standard axial-vector duality on the U(1) before contraction.

We will start by investigating the CFT point of view. Consider a WZW theory for some (simple) group G. The standard spectrum of representations for the left and right current algebras has the form $(R, \bar{R})$, modulo the affine truncation. The operator product fusion rules follow again group theory modulo truncations. If we act on the spectrum by an external automorphism of the right (finite) Lie algebra, then we obtain an equivalent theory*, in the sense that there is a reorganization of the spectrum such that the OPEs are the same.

A similar remark applies to coset models. Consider a non-abelian subgroup H of G (such that it has non-trivial external automorphisms). The standard vector gauging implies the following constraint on the spectrum

$$J_0^a + \bar{J}_0^a = 0 \tag{1}$$

This determines the modular invariant of the G/H model once the spectrum of the original G model is known. If we now gauge the left H subgroup twisted by an automorphism S we will have a different zero mode constraint:

$$J_0^a + S^a{}_b \bar{J}_0^b = 0 \quad . \tag{2}$$

The fact that the two gaugings give equivalent models is obvious once we perform the transformation S in the G theory.

Once we have seen that the two gauged models correspond to the same CFT we can move to the σ-model in order to implement this equivalence as a duality. The strategy we use is that of quotients [7] which is tuned to utilize axial-vector type dualities, [4].

The duality transformations for an abelian isometry, may be derived by starting from a general $(d+1)$-dimensional σ-model with $U(1)_l \times U(1)_r$ symmetry. One then has the choice of either gauging the vector or axial $U(1)$ subgroup, leading to two σ-models which are related by the abelian duality transformations. In particular, since one can argue that both gaugings lead to the same CFT, this proves that the corresponding duality transformation relates two different backgrounds of the same CFT.

Here we will generalize this result using the fact [14] that when we have a world-sheet action, with an $(H_l \times H_r)$ symmetry, we have a distinct anomaly-free world-sheet gauging of the H-symmetry for each external automorphism S of H,

$$S_a{}^d S_b{}^e f_{de}{}^g (S^{-1})_g{}^c = f_{ab}{}^c \quad , \quad S_a{}^c S_b{}^d \eta_{cd} = \eta_{ab} \quad , \ a = 1, \ldots, dim\, H \tag{3}$$

where $f_{ab}{}^c$ and η_{ab} are the structure constants and Killing metric of H respectively. The statement is that the world-sheet gauge group can be chosen to be

$$\mathcal{J}_a^H = J_a^H + S_a{}^b \bar{J}_b^H \tag{4}$$

where J_a^H and $\bar{J}_a^H$ are the left- and right-moving world sheet currents of H respectively. In particular, the usual vector-gauging corresponds to $S = 1$, while the validity of the axial gauging for abelian subgroups follows since $S = -1$ is always an external automorphism in that case. More generally, when a non-abelian group H has a non-trivial automorphism, the result implies that we have other gaugings beyond the vector gauging, which are

*Sometimes the diagonal Lie algebra may have a physical meaning. In such a case one can distinguish between a WZW model and its transformed version. This happens for example in type II_A or II_B strings.

typically of a mixed vector-axial type. For example, when H is a simple compact non-abelian group such automorphisms occur in $SU(n)$ and $SO(2n)$ with $n \geq 3$, and E_6.

We start with the most general σ-model in $d + 2dim\, H$ dimensions that has a chiral $H_l \times H_r$ symmetry,

$$I(h_1, h_2, x) = I(h_1) + I(h_2) + \int \frac{d^2z}{\pi}[B_{ab}(x)\bar{J}_2^a J_1^b + G^1_{ia}(x)\bar{\partial}x^i J_1^a + G^2_{ai}(x)\bar{J}_2^a \partial x^i] + I(x) \quad (5)$$

where $I(h_I),\ I = 1, 2$ are two copies of the WZW action

$$I(h) = \int_\Sigma \frac{d^2z}{2\pi} Tr[h^{-1}\partial h h^{-1}\bar{\partial}h] - i\int_B \frac{d^3z}{6\pi} Tr(h^{-1}dh)^3 \quad (6)$$

satisfying the Polyakov-Wiegmann formula

$$I(hh_0) = I(h) + I(h_0) + \int \frac{d^2z}{\pi} Tr[h^{-1}\partial h \bar{\partial} h_0 h_0^{-1}] \quad . \quad (7)$$

The currents in the action (5) are defined as

$$J_I = h_I^{-1}\partial h_I = J_I^a T_a \quad , \quad \bar{J}_I = \bar{\partial} h_I h_I^{-1} = \bar{J}_I^a T_a \quad , \quad I = 1, 2 \quad (8)$$

where T_a are matrices of some H-representation normalized as $Tr(T_a T_b) = \eta_{ab}$. Finally, the matrices $B_{ab}(x)$, $G^1_{ia}(x)$ and $G^2_{ai}(x)$ are arbitrary functions of some set of target space coordinates $x^i,\ i = 1, \ldots, d$ with an arbitrary σ-model action

$$I(x) = \int \frac{d^2z}{2\pi}[E_{ij}(x)\bar{\partial}x^i \partial x^j + \alpha' R^{(2)}\phi(x)] \quad , \quad E_{ij}(x) = g_{ij}(x) + b_{ij}(x) \quad (9)$$

where g_{ij} is the target space metric on this manifold, b_{ij} the anti-symmetric tensor and ϕ the dilaton field.

We remark that the form (5) can be obtained by decomposing the WZW action $I(h_1(\theta_L)g(x)h_2(\theta_R))$ using eq.(7), in which case a special form of the action (5) is found†, with

$$B_{ab}(x) = Tr[g(x)T_a g^{-1}(x)T_b] \quad (10a)$$

$$\bar{\partial}x^i G^1_{ia}(x) = Tr[T_a \bar{\partial}g(x)g^{-1}(x)] \quad , \quad G^2_{ai}(x)\partial x^i = Tr[T_a g^{-1}(x)\partial g(x)] \quad (10b)$$

$$I(x) = I(g(x)) \quad . \quad (10c)$$

It is clear from the properties of the WZW action, that in this special case we have a chiral $H_l \times H_r$ symmetry. However, this symmetry persists when we drop the conditions in eq.(10), as we will show below.

The action (5) has chiral $H_l \times H_r$ symmetry, generated by the separately conserved left- and right-handed currents

$$\bar{J}_l^a = \bar{J}_1^a + [\bar{J}_2^c B_{cb}(x) + \bar{\partial}x^i G^1_{ib}(x)](\omega_1)^{ba} \quad (11a)$$

†this we can do if the group G inside which H is embedded is large enough.

$$J_r^a = J_2^a + (\omega_2)^{ab}[B_{bc}(x)J_1^c + G_{bi}^2(x)\partial x^i] \tag{11b}$$

$$\partial \bar{J}_l^a = \bar{\partial} J_r^a = 0 \tag{11c}$$

where we have defined the matrices

$$(\omega_I)_{ab} = Tr[h_I T_a h_I^{-1} T_b] \quad , \quad I = 1,2 \tag{12}$$

and indices are raised with the inverse Killing metric η^{ab}.

In particular, these conserved currents follow from the chiral transformations

$$h_1 \to h_l h_1 \quad , \quad h_l = e^{\epsilon_l^a T_a} \tag{13a}$$

$$h_2 \to h_2 h_r^{-1} \quad , \quad h_r = e^{\epsilon_r^a T_a} \tag{13b}$$

which, for infinitesimal ϵ, yield a variation of the action given by

$$\delta S = \int \frac{d^2z}{\pi}[\partial \epsilon_l^a \bar{J}_a^l - \bar{\partial}\epsilon_r^a J_a^r] \quad . \tag{14}$$

To obtain these results from (5), one uses the Polyakov-Wiegmann property (7), along with the definition of the currents in (8) and ω_I in (12).

In the following we will assume that H is such that it has at least one non-trivial external automorphism S. We can then choose to gauge the vector subgroup of the $H_l \times H_r$ symmetry or a mixed vector-axial gauging corresponding to a non-trivial S in (4). Hence, we consider the gauge transformations in (13), with

$$\epsilon_r^a = \epsilon_l^b S_b{}^a \tag{15}$$

where $S = 1$ is the usual vector-gauging.

The resulting gauge-invariant action is given by

$$I_{gauge} = I(h_1, h_2, x) - \int \frac{d^2z}{\pi}[A_l^a \bar{J}_a^l - \bar{A}_r^a S_a{}^b J_b^r - \bar{A}_r^a(1 - S\omega_2 B\omega_1)_{ab} A_l^b] \tag{16}$$

where $I(h_1, h_2, x)$ as given in (5). The gauged action is invariant under the left and right H-transformations in (13), combined with the transformation of the gauge fields,

$$A_l \to h_l(A_l - \partial)h_l^{-1} \quad , \quad \bar{A}_r \to h_l(\bar{A}_r - \bar{\partial})h_l^{-1} \tag{17}$$

where the gauge field components are as usual defined by $A_l^a = \eta^{ab}Tr[T_b A_l]$ and $\bar{A}_r^a = \eta^{ab}Tr[T_b \bar{A}_r]$.

We will choose to gauge fix $h_2 = 1$[‡], so that $J_2 = \bar{J}_2 = 0$ and $\omega_2 = 1$, and drop the subscript 1 on the remaining quantities h_1, J_1, $\bar{J}_1$ and ω_1. Then, after integrating out the gauge fields in the gauged action (17), we obtain after some algebra a $(d + dim\, H)$-dimensional σ-model, whose form is given by

$$I_{gauge}(h,x) = I(h) - \int \frac{d^2z}{\pi}[G_{ab}(h,x)\bar{J}^a J^b + \Gamma_{ia}^1(h,x)\bar{\partial}x^i J^a + \Gamma_{ai}^2(h,x)\bar{J}^a \partial x^i + Q_{ij}(h,x)\bar{\partial}x^i \partial x^j]$$

[‡]Notice that this is a different gauge fixing than the one used in [7].

$$+\int \frac{d^2z}{2\pi}\alpha' R^{(2)}\Phi(h,x) \tag{18a}$$

$$J_a = Tr[T_a h^{-1}\partial h] \quad , \quad \bar{J}_a = Tr[T_a \bar{\partial} h h^{-1}] \quad , \quad \omega_{ab} = Tr[hT_a h^{-1}T_b] \quad . \tag{18b}$$

Here we have defined

$$M \equiv 1 - SB\omega \quad , \quad G = (1-M^{-1})\omega^{-1} \quad , \quad \Gamma^1 = -G^1\omega M^{-1}\omega^{-1} \tag{19a}$$

$$\Gamma^2 = -M^{-1}SG^2 \qquad Q = -\frac{E}{2} - G^1\omega M^{-1}SG^2 \quad , \quad \Phi = \phi - \frac{1}{2}\ln det\, M \tag{19b}$$

where, for brevity, we have dropped the explicit indices of the relevant matrices and matrix multiplication is employed. The shift in the dilaton follows from the jacobian that arises when integrating out the gauge fields.

We remark here that due to the the h-dependence of the couplings, the model (18) cannot be viewed as a non-abelian compactification. Moreover, the h-dependence of the matrices G, $\Gamma^{1,2}$ and Q arises through ω (and hence also M), and the x dependence is encoded in the arbitrary matrices B, $G^{1,2}$ and E of the parent action (5). So although the latter are arbitrary, the couplings in (18) do have a certain form dictaded by eq.(19).

To summarize, starting from the parent action (5) with some group H, we have derived for each external automorphism S of H, a distinct gauged model in $d + dim\, H$ target space dimensions. If the parent action is conformally invariant then so are the gauged versions of it (and vice versa). This implies that when the gauged theories (18) are conformal, they correspond to the same conformal field theory for any S.

We will now use the above to obtain quasi-axial-vector duality transformations. Taking the viewpoint that we are given an action of the form (18) for $S=1$, we need that the quantities

$$B(x) = G(\omega G - 1)^{-1} \quad , \quad G^1(x) = \Gamma^1(\omega G-1)^{-1} \quad , \quad G^2(x) = (G\omega - 1)^{-1}\Gamma^2 \tag{20a}$$

$$E(x) = -2Q + 2\Gamma^1\omega(G\omega-1)^{-1}\Gamma^2 \quad , \quad \phi(x) = \Phi + \frac{1}{2}\ln det\,(1-G\omega) \tag{20b}$$

depend on x only, where ω is defined in (18b). These relations are simply obtained by solving for B, $G^{1,2}$ and E in eq.(19) at $S=1$.

We can now substitute the expressions (20) into (19) to obtain the corresponding dual model, given the non-trivial external automorphism S of H. After some manipulations we find the quasi-axial-vector duality transformation,

$$\tilde{G} = SG[1+\omega(S-1)G]^{-1} \quad , \quad \tilde{\Gamma}^1 = \Gamma^1[1+\omega(S-1)G]^{-1} \quad , \quad \tilde{\Gamma}^2 = S[1+G\omega(S-1)]^{-1}\Gamma^2 \tag{21a}$$

$$\tilde{Q} = Q - \Gamma^1\omega(S-1)[1+G\omega(S-1)]^{-1}\Gamma^2 \quad , \quad \tilde{\Phi} = \Phi - \frac{1}{2}\ln det[1+(S-1)G\omega] \tag{21b}$$

relating the original background to another one, both corresponding to the same CFT.

As a first check on these results we note that performing the transformations (21) twice results in $S \to S^2$, as desired by consistency. In particular, for Z_2 external automorphisms (which is the generic case when an external automorphism is present) we obtain the initial background, while for a Z_3 external automorphism (which occurs for $SO(8)$) we get back to the orginal background after applying the generalized duality three times.

As another check, we choose H to be abelian and evaluate (21) for $S = -1$. As our method here is the non-abelian generalization of the one employed in [7], yielding the abelian duality transformations, we should recover those in this way. Indeed using (21) in (18) along with

$$I(h) = \int \frac{d^2 z}{2\pi} \eta_{ab} \bar{\partial}\theta^a \partial\theta^b \quad , \quad J^a = \partial\theta^a \quad , \quad \bar{J}^a = \bar{\partial}\theta^a \quad , \quad \omega_{ab} = \delta_{ab} \tag{22}$$

it is not difficult to establish that we obtain the correct abelian duality transformations

$$\tilde{G}_t = G_t^{-1} \quad , \quad \tilde{\Gamma}_t^1 = \Gamma_t^1 G_t^{-1} \quad , \quad \tilde{\Gamma}_t^2 = -G_t^{-1}\Gamma_t^2 \tag{23a}$$

$$\tilde{Q}_t = Q_t - \Gamma_t^1 G_t^{-1} \Gamma_t^2 \quad , \quad \tilde{\Phi} = \Phi - \frac{1}{2} \ln det\, G_t \quad . \tag{23b}$$

Here, the total action is written as

$$I(\theta, x) = \int \frac{d^2 z}{2\pi} [\bar{\partial}\theta G_t(x) \partial\theta + \bar{\partial}x \Gamma_t^1(x) \partial\theta + \bar{\partial}\theta \Gamma_t^2(x) \partial x + \bar{\partial}x Q_t(x) \partial x] \tag{24a}$$

$$G_t = 1 - 2G \quad , \quad \Gamma_t^1 = -2\Gamma^1 \quad , \quad \Gamma_t^2 = -2\Gamma^2 \quad , \quad Q_t = -2Q \tag{24b}$$

in terms of the quantities in (21).

There are several questions that remain open. The quasi-axial-vector duality symmetry presented above seems to be in some respects different from the standard abelian axial-vector duality. For one thing, the backgrounds where it applies do not generically have Killing symmetries. Moreover a coordinate independent characterization of the backgrounds which admit such a duality symmetry is not obvious. It would be interesting to study examples of dual pairs of σ-models. However, since groups with non-trivial automorphisms are rather large the calculations are involved. Most important, the possible connection with non-abelian duality must be elucidated. This will help in shedding some light onto the real differences between abelian and non-abelian duality. It seems plausible in that respect that quasi-axial-vector dulaity is related to non-abelian duality upon linearizing the non-abelian symmetry. We hope to address some of these issues in a future publication.

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References

[1] K. Kikkawa, M Yamazaki, Phys. Lett. **B149** (1984) 357;
N. Sakai, I. Senda, Prog. Theor. Phys. **75** (1986) 692;
V. P. Nair, A. Shapere, A. Strominger, F. Wilczek, Nucl. Phys. **B287** (1987) 402;
A. Giveon, E. Rabinovici, G. Veneziano, Nucl. Phys. **B322** (1989) 167.

[2] T. H. Buscher, Phys. Lett. **B194** (1987) 59; ibid **B201** (1988) 466.

[3] X. De La Ossa and F. Quevedo Nucl.Phys. **B403** (1993) 377.

[4] E. Kiritsis, Mod. Phys. Lett. **A6** (1991) 2871.

[5] R. Dijkgraaf, E. Verlinde, H. Verlinde, Nucl. Phys. **B371** (1992) 269.

[6] E. Kiritsis, Nucl. Phys. **B405** (1993) 109.

[7] E. Verlinde, M. Roçek, Nucl. Phys. **B373** (1992) 630.

[8] A. Giveon, M. Rocek, Nucl. Phys. **B380** (1992) 128.

[9] E. Alvarez, L. Alvarez-Gaumé, J. Barbon and Y. Lozano, Nucl. Phys. **B415** (1994) 71.

[10] A. Giveon and E. Kiritsis, Nucl. Phys. **B411** (1994) 487.

[11] A. Giveon and M. Roček, preprint, hep-th/9308154.

[12] M. Gasperini, R. Ricci and G. Veneziano, Phys.Lett. **B319** (1993) 438.

[13] E. Alvarez, L. Alvarez-Gaume and Y. Lozano, preprint, hep-th/9403155.

[14] I. Bars and K. Sfetsos, Mod. Phys. Lett. **A7** (1992) 1091.

[15] I. Antoniadis and N. Obers, preprint, hep-th/9403191. To appear in Nucl. Phys. B

[16] E. Kiritsis, talks given at 'Strings 93' Berkeley and 'EPS 93' Marseille, hep-th/9309064, hep-th/9309101.