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Towards a dynamical determination of parameters in the Minimal Supersymmetric Standard Model[1]

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Abstract

In the Minimal Supersymmetric Standard Model (MSSM), the scale M_{SUSY} of soft supersymmetry breaking is usually *assumed* to be of the order of the electroweak scale. We reconsider here the possibility of treating M_{SUSY} as a dynamical variable. Its expectation value should be determined by minimizing the vacuum energy, after including MSSM quantum corrections. We point out the crucial role of the cosmological term for a dynamical generation of the desired hierarchies $m_Z, M_{SUSY} \ll M_P$. Inspired by four-dimensional superstring models, we also consider the Yukawa couplings as dynamical variables. We find that the top Yukawa coupling is attracted close to its effective infrared fixed point, corresponding to a top-quark mass in the experimentally allowed range. As an illustrative example, we present the results of explicit calculations for a special case of the MSSM.

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1. The most plausible solution of the naturalness or hierarchy problem of the Standard Model is low-energy supersymmetry, whose simplest realization is [1] the Minimal Supersymmetric extension of the Standard Model (MSSM). In the MSSM, supersymmetry breaking is parametrized by a collection of explicit, but soft, mass parameters, with a typical scale M_{SUSY} of the order of the electroweak scale, $M_{SUSY} \lesssim 1\ TeV$. In standard notation,

$$\begin{array}{rcl} -\mathcal{L}_{soft} & = & \sum_i m_i^2 |\varphi_i|^2 + \frac{1}{2} \sum_A M_A \overline{\lambda}_A \lambda_A + \Big(h^U A^U Q U^c H_2 \\ & & + h^D A^D Q D^c H_1 + h^E A^E L E^c H_1 + m_3^2 H_1 H_2 + h.c. \Big) \, . \end{array} \quad (1)$$

Such a choice is sufficient to guarantee the quantum stability of the gauge hierarchy $m_Z \ll M$, where M is the appropriate ultraviolet cut-off scale for the MSSM, somewhere[1] near the grand-unification scale $M_U \simeq 2 \times 10^{16}\ GeV$ and the Planck scale $M_P \equiv G_N^{-1/2}/\sqrt{8\pi} \simeq 2.4 \times 10^{18}\ GeV$. Usually, the general form of $\mathcal{L}_{soft}$ is further constrained by assuming[2], as boundary conditions at M_U, a universal scalar mass m_0, a universal gaugino mass $m_{1/2}$, a universal mass parameter A for the cubic scalar couplings, and negligible CP-violating phases apart from the Kobayashi-Maskawa one.

In addition to the stabilization of the hierarchy, another very attractive feature of the MSSM is the possibility of describing the spontaneous breaking of the electroweak gauge symmetry as an effect of radiative corrections [2]. Thanks to the quantum corrections associated with the large top-quark Yukawa coupling, the effective potential of the MSSM gives a phenomenologically acceptable vacuum, with $SU(2)_L \times U(1)_Y$ broken down to $U(1)_{em}$ at a scale that is naturally of order M_{SUSY}. For appropriate numerical assignments of the boundary conditions, one obtains a mass spectrum compatible with present experimental data.

Besides these virtues, the MSSM has a very unsatisfactory feature: the numerical values of its explicit mass parameters must be arbitrarily chosen 'by hand' (the fact that also the gauge and Yukawa couplings must be chosen 'by hand' is as unsatisfactory as in the Standard Model). This means a certain lack of predictivity, and in particular does not provide any dynamical explanation for the origin of the hierarchy $M_{SUSY} \ll M_P$, which is just assumed to be there. To go further, one must have a model for spontaneous supersymmetry breaking in the fundamental theory underlying the MSSM. The only possible candidate for such a theory is $N = 1$ supergravity coupled to gauge and matter fields [3], where (in contrast with the case of global supersymmetry) the spontaneous breaking of local supersymmetry is not incompatible with vanishing vacuum energy. In realistic supergravity models, supersymmetry breaking typically occurs in a 'hidden sector', whose interactions with the MSSM states are of gravitational strength. For spontaneous breaking

[1]In models where supersymmetry is spontaneously broken by non-perturbative phenomena such as gaugino condensation, the appropriate cut-off scale might be different, but it is in any case much larger than the electroweak scale.

[2]One should keep in mind, however, that such assumptions are not based on fundamental symmetry principles, and that different sets of assumptions can also give phenomenologically viable models.

on a flat background, the order parameter is the gravitino mass, $m_{3/2}$, and all the explicit mass parameters of the MSSM are calculable [4] (but model-dependent) functions of $m_{3/2}$.

The general structure of $N = 1$ supergravity is not very constraining, since it allows for a large amount of arbitrariness in the choice of gauge group, matter content, Yukawa couplings, generalized kinetic terms, etc. This structure is too general to provide significant information on the MSSM parameters. However, $N = 1$ supergravity allows us to address some very important questions, which, irrespectively of the specific mechanism that breaks supersymmetry, can considerably restrict the previously mentioned arbitrariness, and hopefully guide us towards the identification of a more fundamental theory. Among the possible questions, we would like to concentrate here on the following ones: 1) how to avoid a large cosmological constant, already at the classical level; 2) how to generate the hierarchy $m_{3/2} \ll M_P$ without explicitly introducing small mass parameters; 3) how to avoid that quadratically divergent loop corrections to the effective potential generate an unacceptably large cosmological constant $\mathcal{O}(m_{3/2}^2 M_P^2)$ and possibly destabilize the hierarchy $m_{3/2} \ll M_P$.

The first two questions were the main motivations of the so-called *no-scale* supergravity models [5, 6]. Their main feature is that the vacuum energy is equal to zero [or at most $\mathcal{O}(m_{3/2}^4)$] at the classical level, even after supersymmetry breaking, thanks to the existence of at least one flat [or almost flat] direction in the scalar potential [5]. The gravitino mass $m_{3/2}$, which sets the scale M_{SUSY} of the soft masses, is not fixed at the classical minimum, but is a function of the scalar field(s) parametrizing the flat [or almost flat] direction(s). If quantum-gravity effects at the Planck scale do not change drastically the approximate flatness of the scalar potential, then $m_{3/2}$ is determined by the quantum corrections associated with the light MSSM fields [6]. Within this class of models, there exists a possibility of explaining the hierarchy by minimizing the effective potential not only with respect to the Higgs fields, but also with respect to M_{SUSY}: as we shall see in detail later, there are situations in which m_Z and M_{SUSY} are both dynamically determined to be exponentially suppressed with respect to M_P.

In a generic $N = 1$ supergravity, the viability of the above scenario (and of other scenarios for the generation of the hierarchy $m_{3/2} \ll M_P$) is plagued by the possible existence of quadratically divergent one-loop contributions to the vacuum energy, proportional to $Str\,\mathcal{M}^2 \equiv \sum_i (-1)^{2J_i}(2J_i+1)m_i^2$, where m_i and J_i are the field-dependent mass-eigenvalue and the spin of the i-th particle, respectively. In the fundamental quantum theory of gravity, these corrections would correspond to finite contributions to the effective potential, $\mathcal{O}(m_{3/2}^2 M_P^2)$. Since $m_{3/2}$ is a dynamical variable in supergravity, via its dependence on the scalar field VEVs, the presence of such contributions may induce, upon minimization of the effective potential, either $m_{3/2} = 0$ or $m_{3/2} \sim M_P$. Moreover, $\mathcal{O}(m_{3/2}^2 M_P^2)$ contributions to the vacuum energy cannot be cancelled by symmetry-breaking phenomena occurring at much lower energy scales. This is the meaning of our third question, which forces us to go beyond classical $N = 1$ supergravity, to a more fundamental theory where quantum gravitational effects can be consistently computed. Today, the most natural candidate for such a theory is the heterotic superstring [7], and more specifically its four-dimensional versions [8], supplemented by some assumptions on the way in which supersymmetry is broken (existing possibilities are non-perturbative phenomena such as gaugino condensation [9] and tree-level string constructions such as coordinate-dependent compactifications [10–12], but there may be others). Even if there is some arbitrariness, connected with the assumptions on the supersymmetry-breaking mechanism, the underlying string structure preserves some useful general properties, which strongly restrict the form of the resulting effective supergravity theories. For example, it is remarkable that no-scale models naturally emerge from the string framework [13, 9, 11]. In addition, an answer to our third question can be found in the restricted class of string-derived no-scale supergravities [14–16,12] that are free from quadratically divergent contributions to the one-loop vacuum energy, proportional to $Str\,\mathcal{M}^2$. In this class of 'large-hierarchy-compatible' (LHC) models, the only corrections to the low-energy effective potential coming from the underlying fundamental theory are, up to logarithms, $\mathcal{O}(m_{3/2}^4)$. Then, since the only scale in the low-energy effective theory is the sliding $m_{3/2}$, the program of inducing the hierarchy by MSSM quantum corrections can be self-consistently carried out.

In this paper, we perform a critical reappraisal of how the hierarchy $M_{SUSY} \ll M_P$ can be generated by MSSM quantum corrections. In particular, we study the influence of the cosmological term of the MSSM effective potential, which was previously neglected but plays a very important role on the dynamical determination of M_{SUSY}. We find that, considering the MSSM as the low-energy limit of a LHC supergravity model, the desired hierarchy $m_Z \sim M_{SUSY} \ll M_P$ can be dynamically realized by the perturbative MSSM quantum corrections.

At the level of the underlying string theory, not only the mass parameters, but also the gauge and Yukawa couplings do indeed depend on the VEVs of some gauge singlet fields with classically flat potentials, called moduli, which parametrize the size and the shape of the six-dimensional compactified space. This suggests an exciting possibility. If the dynamical mechanism that breaks supersymmetry fixes the gauge coupling constant α_U to a given numerical value at M_U, but leaves a residual moduli dependence of the Yukawa couplings, then also the latter should be treated as dynamical variables in the low-energy effective theory. This means that the effective potential of the MSSM should also be minimized with respect to the moduli on which the Yukawa couplings depend. In this paper, we discuss some conceptual issues connected with this minimization procedure. We find that the top-quark Yukawa coupling is dynamically driven, as a result of the minimization with respect to the moduli fields, near its effective infrared fixed point, corresponding to a top-quark mass $m_t \simeq M_t^{IR} \sin\beta$, where $M_t^{IR} \simeq 190\ GeV$ (with roughly a 10% uncertainty due to the error on α_3, threshold and higher-loop effects, etc.), and $(1/\sqrt{2}) < \sin\beta < 1$ (the actual value being determined by the form of the boundary conditions at M_U), thus in the range presently allowed by experimental data [17].

We conclude the paper with a quantitative example, which realizes the above-mentioned ideas. To illustrate more clearly the important features of the problem, we choose to work with a simplified version of the MSSM, leaving a more complete study to a future publication [18].

2. The conventional treatment of radiative symmetry breaking [2] can be briefly summarized as follows. As a starting point, one chooses a set of numerical input values for the independent model parameters at the unification scale $Q = M_U$: the soft masses $(m_0, m_{1/2}, A, m_3^2)$, the superpotential mass μ, the unified gauge coupling α_U and the third-generation[3] Yukawa couplings $(\alpha_t, \alpha_b, \alpha_\tau)$. One then evolves all the running parameters down to a low scale $Q \sim M_{SUSY}$, according to the appropriate [19] renormalization group equations (RGE), and considers the renormalization-group-improved tree-level potential

$$V_0(Q) = m_1^2 v_1^2 + m_2^2 v_2^2 + 2 m_3^2 v_1 v_2 + \frac{g'^2 + g^2}{8} \left(v_2^2 - v_1^2 \right)^2 + \Delta V_{cosm} \, . \tag{2}$$

In eq. (2), only the dependence on the real neutral components of the Higgs fields has been kept, and ΔV_{cosm} stands for a Higgs-field-independent contribution to the vacuum energy (cosmological term). All masses and coupling constants are running parameters, evaluated at the scale Q. The minimization of the potential in eq. (2), with respect to the dynamical variables $v_1 \equiv \langle H_1^0 \rangle$ and $v_2 \equiv \langle H_2^0 \rangle$, is straightforward: to generate a stable minimum with non-vanishing VEVs, one needs $\mathcal{B} \equiv m_1^2 m_2^2 - m_3^4 < 0$ and $\mathcal{S} \equiv m_1^2 + m_2^2 - 2|m_3^2| \geq 0$. In the determination of the vacuum, a crucial role is played by the large top Yukawa coupling, which strongly influences the RGE for $\mathcal{B}$ and $\mathcal{S}$. For appropriate numerical assignments of the boundary conditions, both $\mathcal{S}$ and $\mathcal{B}$ are non-negative at M_U, but the effects of h_t on the RGE drive $\mathcal{B} < 0$ at scales $Q \sim M_{SUSY}$, while keeping $\mathcal{S} \geq 0$, and give a phenomenologically acceptable vacuum, with $SU(2)_L \times U(1)_Y$ broken down to $U(1)_{em}$ and a mass spectrum compatible with present experimental data.

In this paper, we regard the MSSM as the low-energy effective theory of an underlying supergravity model of the LHC type, where the gravitino mass $m_{3/2}$ cannot be determined at the classical level, due to an approximately flat direction in the space of the scalar fields of the hidden sector, and there are no quantum corrections to the effective potential carrying positive powers of the cut-off scale M_P. As can be shown in several examples [14–16], this assumption can be naturally fulfilled in a class of four-dimensional superstring models. With the presently known mechanisms for supersymmetry breaking, in order to allow for $m_{3/2} \ll M_P$ one is led to consider large values of some of the moduli fields [20, 10–12]. In this limit, the discrete target-space duality symmetries leave as remnants some approximate scaling symmetries for the Kähler metric, and one naturally obtains approximately flat directions in moduli space, along which the gravitino mass can slide. Moreover, as was shown in ref. [16], the coefficient of the $\mathcal{O}(m_{3/2}^2 M_P^2)$ contributions to the one-loop effective potential is controlled by the modular weights associated with target-space duality, and there is a non-empty set of LHC models in which such a coefficient is identically vanishing.

[3] For the purposes of the present paper, mixing effects and all other Yukawa couplings can be neglected.

In the present paper, we shall not consider the most general class of LHC models, but restrict our attention to those in which some non-perturbative dynamics fixes the VEVs of the moduli associated with α_U and M_U, and the boundary conditions on the explicit mass parameters of the MSSM can be written as[4]

$$m_{1/2} = \xi_1 \cdot m_{3/2}\,, \quad m_0 = \xi_2 \cdot m_{3/2}\,, \quad A = \xi_3 \cdot m_{3/2}\,, \quad m_3 = \xi_4 \cdot m_{3/2}\,, \quad \mu = \xi_5 \cdot m_{3/2}\,, \tag{3}$$

where the scaling weights with respect to target-space duality fix the ξ parameters to constant numerical values of order one or smaller. Then all the moduli dependence of the MSSM mass parameters is entirely encoded in the gravitino mass $m_{3/2}$, which should thus be considered as an extra dynamical variable, in addition to the Higgs VEVs v_1 and v_2. If we take the low-energy limit and neglect the interactions of gravitational strength, we can formally decouple the hidden sector and recover the MSSM. Quantum effects in the underlying fundamental theory, however, would induce a cosmological term in the resulting MSSM effective potential; for LHC models, this term contains no positive powers of M_P and must therefore be proportional to $m_{3/2}^4$:

$$\Delta V_{cosm} = \eta \cdot m_{3/2}^4\,, \tag{4}$$

obeying certain boundary conditions at M_U, dictated by the structure of the hidden sector (number of states and tree-level mass splittings):

$$\eta(M_U) = \eta_0\,. \tag{5}$$

We stress that, in contrast with conventional treatments, in the present context we are forced to include the cosmological term, since the gravitino mass is not taken as an external parameter, but rather as a dynamical variable.

According to our program, we would like now to minimize the effective potential of the MSSM not only with respect to the Higgs fields, but also with respect to the new dynamical variable $m_{3/2}$, keeping (for the moment) the values of $\alpha_U \simeq 0.04$, $M_U \simeq 2 \times 10^{16}\, GeV$, $\vec{\xi}$, η_0 and $(\alpha_t^U, \alpha_b^U, \alpha_\tau^U)$ as external input data. As in the standard approach, the role of radiative corrections will be crucial in developing a non-zero value for the Higgs VEVs at the minimum. Quantum corrections to the classical potential are summarized, at the one-loop level, by the well-known formula

$$V_1 = V_0(Q) + \Delta V_1(Q)\,, \tag{6}$$

where[5]

$$\Delta V_1(Q) = \frac{1}{64\pi^2} Str\, \mathcal{M}^4 \left(\log \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right) = \sum_i \frac{n_i m_i^4}{64\pi^2} \left(\log \frac{m_i^2}{Q^2} - \frac{3}{2} \right)\,, \tag{7}$$

[4]Even if μ is a globally supersymmetric mass parameter, several examples show that it can be related to the scale of local supersymmetry breaking, i.e. the gravitino mass [21, 11, 22, 16].

[5]We work as usual in the 't Hooft Landau gauge and in the mass-independent $\overline{DR}$ scheme.

and $\mathcal{M}^2$ is the generalized field-dependent mass matrix: the tree-level mass eigenvalues m_i^2 and the integer coefficients n_i, which account for the number of degrees of freedom and for statistics, can be trivially calculated.

In eq. (6), $V_0(Q)$ has the same functional form as the tree-level potential, eq. (2), and is expressed in terms of renormalized fields and parameters at the scale Q, solutions of the appropriate set of RGE. The RGE for the new dimensionless coupling of the theory, the coefficient η of the cosmological term, reads

$$\frac{d\eta}{dt} = \frac{1}{32\pi^2} \left[\frac{Str\,\mathcal{M}^4}{m_{3/2}^4} \right]_{v_{1,2}=0} , \qquad t \equiv \log Q \, . \tag{8}$$

In the MSSM,

$$\begin{aligned} [Str\,\mathcal{M}^4]_{v_{1,2}=0} &= 4(m_1^4 + m_2^4 + 2m_3^4) + 6(m_{U_3^c}^4 + m_{D_3^c}^4 + 2m_{Q_3}^4) + 12(m_{U^c}^4 + m_{D^c}^4 \\ &+2m_Q^4) + 2(m_{E_3^c}^4 + 2m_{L_3}^4) + 4(m_{E^c}^4 + 2m_L^4) - 16M_3^4 - 6M_2^4 - 2M_1^4 - 8\mu^4 \, . \end{aligned} \tag{9}$$

To illustrate the renormalization group evolution of the cosmological constant term, which will play an important role in the determination of the supersymmetry-breaking scale, we plot in fig. 1 the quantity $(\eta - \eta_0)$, as a function of Q, for a number of representative boundary conditions at M_U. We neglect for simplicity the effects of α_b and α_τ, which can play a role only for very large values of $\tan\beta \equiv v_2/v_1$. We can observe that the MSSM particle content is such that $(\eta - \eta_0)$ is always driven towards negative values at sufficiently low scales, and that the dependence on the top Yukawa coupling is not very large within the experimentally interesting region. As we shall see in detail on an example, the gravitino mass dynamically relaxes to a value that is closely related to the scale at which the coupling η turns from positive to negative. We can then expect that the desired hierarchy can be generated for values of η_0 between zero and $\mathcal{O}(100)$, depending on the values of the $\vec{\xi}$ parameters, which are not in contrast with our present ideas on the mass splittings in the hidden sector of the theory.

To illustrate the main point of our approach, it is convenient to choose, as independent variables, the supersymmetry-breaking scale $m_{3/2}^2$ and the dimensionless ratios $\hat{v}_i \equiv (v_i/m_{3/2})$ $(i = 1,2)$. Then the minimization condition of the one-loop effective potential with respect to $m_{3/2}$ can be written in the form

$$m_{3/2}^2 \frac{\partial V_1}{\partial m_{3/2}^2} = 2V_1 + \frac{Str\,\mathcal{M}^4}{64\pi^2} = 0 \, . \tag{10}$$

Eq. (10) can be interpreted as defining an infrared fixed point for the cosmological term, since it corresponds to the vanishing of the associated β-function, which displays a naïve scaling dimension 2 (with respect to $m_{3/2}^2$) and an anomalous dimension determined by the value of $Str\,\mathcal{M}^4$. Such an equation can be interpreted as a non-trivial constraint to be satisfied by the MSSM parameters, and could hopefully discriminate among different superstring models in which these parameters will be eventually calculable. The minimization conditions with respect to the variables $\hat{v}_i$ are completely equivalent to the ones that are usually considered in the MSSM, when the supersymmetry breaking scale is a fixed numerical input.

Using the Q-independence of the effective potential, one can look for a suitable scale $\hat{Q} \sim m_{3/2}$, where the ΔV_1 correction can be neglected, and the previous formula can be rewritten in the simplified form

$$2V_0(\hat{Q}) + \frac{Str\,\mathcal{M}^4(\hat{Q})}{64\pi^2} = 0\,. \qquad (11)$$

A detailed study of these problems shows [23] that the qualitative behaviour of the effective potential, around its minima with $m_{3/2} \sim m_Z$, can still be reproduced by the RGE-improved potential of eq. (2), provided that the renormalization scale $\hat{Q}$ is suitably chosen.

A general study of the MSSM predictions, as functions of the boundary conditions $\vec{\xi}$, η_0, $h^U_{t,b,\tau}$, can be performed numerically, but goes beyond the aim of the present paper. Some illustrative numerical results, for a particularly simple choice of boundary conditions, will be presented in the concluding paragraph.

3. We shall now extend the previous approach by assuming that also $\alpha^U_{t,b,\tau}$ are dynamical variables, in analogy to what we did before for the supersymmetry-breaking scale M_{SUSY}. The main motivation for our proposal comes from superstring theory, where all the parameters of the effective low-energy theory are related to the VEVs of some moduli fields, e.g. gauge-singlet fields in the hidden sector, which after supersymmetry breaking may still correspond to approximately flat directions at the classical level. We assume here, as in the previous section, that non-perturbative phenomena, such as gaugino condensation or others, will fix the VEV of the modulus associated with the unified gauge coupling constant (in conventional language [13], the dilaton-axion modulus field S if there are no threshold corrections depending on the compactification moduli T, a non-trivial combination of the S and T moduli in the opposite case).

It is a well-known fact that, in general four-dimensional string models, tree-level Yukawa couplings are either vanishing or of the order of the unified gauge coupling [13, 24]. One naturally expects the top Yukawa coupling to fall in the latter category, so for the moment we shall concentrate on it, assuming a tree-level relation of the form

$$\alpha^U_t = c_t \alpha_U\,, \qquad (12)$$

where c_t is a model-dependent group-theoretical constant of order unity (for example, in some fermionic constructions $c_t = 2$). At the one-loop level, it is also well known that both gauge [25] and Yukawa [26] couplings receive in general string threshold corrections, induced by the exchange of massive Kaluza-Klein and winding states. Such states have masses that depend on the VEVs of some moduli fields. One can then consider two main possibilities, each of which can be realized in four-dimensional string models, as can be shown on explicit orbifold examples.

The first possibility is that the top Yukawa coupling receives a string threshold correction identical to the one of the gauge coupling, so that the unification condition (12) is preserved. In this case the non-perturbative phenomena, which we have assumed to determine α_U, also fix the value of α_t^U, the latter is no longer an independent parameter, and one can perform the analysis described in the previous paragraph with one parameter less. In particular, the structure of the RGE for α_t is such that its numerical value at the electroweak scale is always very close to its effective infrared fixed point [27], $\alpha_t \simeq 1/(4\pi)$. Phenomenological implications of this fact have been extensively studied in the recent literature [28].

The second possibility is that eq. (12) receives non-trivial moduli-dependent threshold corrections, of the form

$$\frac{c_t}{\alpha_t^U} = \frac{1}{\alpha_U} + F_t(T_i)\,, \tag{13}$$

where $F_t(T_i)$ is a modular function of the singlet moduli, here generically denoted by T_i. We are now in a very interesting situation, in which both the gravitino mass $m_{3/2}$ and the top Yukawa coupling α_t^U depend non-trivially on the moduli fields T_i. To deal with this case, we need to say something about these moduli dependences. As for the gravitino mass, one can always choose a field parametrization such that

$$m_{3/2}^2 = \frac{\alpha_U |k|^2}{V(t_V)}\,, \tag{14}$$

where k is a superpotential term parametrizing local supersymmetry breaking, which does not depend on the moduli corresponding to the approximately flat directions; $V(T_i) = e^{-K}$ is the overall volume of the internal moduli space; and K is the associated Kähler potential. In general, besides the overall combination of moduli associated with the gravitino mass, to be called from now on the 'volume' modulus t_V, there are additional moduli. It is plausible to assume that also some of these 'shape' moduli, to be denoted from now on by t_i, correspond to approximately flat directions, even after the inclusion of the non-perturbative physics, which breaks supersymmetry and fixes the value of the unified gauge coupling constant. We shall assume here that the top Yukawa coupling α_t^U depends on at least one of these additional moduli. Then the minimization conditions with respect to the moduli relevant for the low-energy theory can be written as

$$\frac{\partial V_1}{\partial t_V} = \left(\frac{\partial V_1}{\partial m_{3/2}^2}\right)\left(\frac{\partial m_{3/2}^2}{\partial t_V}\right) + \left(\frac{\partial V_1}{\partial \alpha_t}\right)\left(\frac{\partial \alpha_t}{\partial \alpha_t^U}\right)\left(\frac{\partial \alpha_t^U}{\partial t_V}\right) = 0\,, \tag{15}$$

$$\frac{\partial V_1}{\partial t_i} = \left(\frac{\partial V_1}{\partial \alpha_t}\right)\left(\frac{\partial \alpha_t}{\partial \alpha_t^U}\right)\left(\frac{\partial \alpha_t^U}{\partial t_i}\right) = 0\,. \tag{16}$$

We examine first the possible stationary points that can emerge from the minimization with respect to the shape moduli t_i. There are three generic situations that can fulfil the minimum conditions

$$\frac{\partial \alpha_t^U}{\partial t_i} = 0\,, \tag{17}$$

or

$$\frac{\partial \alpha_t}{\partial \alpha_t^U} = 0\,, \tag{18}$$

or

$$\frac{\partial V_1}{\partial \alpha_t} = 0\,. \tag{19}$$

The first possibility, eq. (17), requires that

$$\frac{\partial F(t_i)}{\partial t_i} = 0\,, \tag{20}$$

and thus involves stringy information about the form of the $F(t_i)$ shape modular functions. There are always such stationary points (or curves) associated with the self-dual points of the theory ($t_i = 1$). This is due to the target-space duality symmetries, $F(t_i, t_j) = F(t_i, 1/t_j)$ which are by now widely studied [29]. Whether these stationary points are minima or maxima depends on the details of the string model. We shall return to this point later.

The second possibility, eq. (18), is the most interesting, since it always gives a universal minimum, which corresponds to the effective infrared fixed point. This stationary point, however, corresponds also to the maximum allowed value for α_t, which strictly speaking is excluded by the requirement of perturbative unification $\alpha_t^U \ll 1$. Our numerical analysis (see the following paragraph) shows that the minimum value of the effective potential corresponds indeed to the case in which α_t has the largest permitted value, namely it is very close to the effective infrared fixed point. On the contrary, the $t_i = 1$ stationary point can correspond either to a minimum or to a maximum, depending on the particular string model under consideration. Notice also that the infrared 'minima' automatically put to zero the second term in the minimization condition with respect to the volume modulus t_V, eq. (15). One then obtains, in the large volume limit for which $\partial m_{3/2}^2/(\partial t_V) \neq 0$ and $m_{3/2} \ll M_P$,

$$m_{3/2}^2 \frac{\partial V_1}{\partial m_{3/2}^2} = 0 \qquad and \qquad \alpha_t \simeq \alpha_t^{IR}\,. \tag{21}$$

This universal minimum is present also if there is a non-trivial dependence of the hidden-sector parameters on the shape moduli t_i: the only additional effect would be to force those parameters to relax to their effective infrared fixed points.

The third possibility, eq. (19), depends on the low-energy structure of the effective potential, which as a function of α_t is sensitive to low-energy threshold phenomena. In general, there exist local stationary points of this kind, but our analysis (see the following section) shows that the infrared fixed point is the deepest minimum that can be obtained, so we shall forget about possible other local minima in what follows. We have therefore been left with two possibilities as far as the determination of α_t is concerned: (i) the infrared fixed point; (ii) the $t_i = 1$ minima.

In fig. 2, we plot $\alpha_t(Q)$ for some representative values of α_t^U. Numerically, for $\alpha_t^U \geq \alpha_U$, α_t at the electroweak scale is always close to its effective infrared fixed point.

In summary, for the minima with respect to t_i, corresponding to eq. (17), satisfied by $t_i = 1$, we can write the residual moduli dependence of α_t^U as

$$\frac{c_t}{\alpha_t^U} = \frac{1}{\alpha_U} + f(t_V)\,, \tag{22}$$

where $f(t_V) \equiv [F(T_i)]_{t_i=1}$. Then α_t is fixed by the minimization with respect to $m_{3/2}$. The range of $f(t_V)$ can be estimated. If $f(t_V)$ happens to be positive and large, then the possible minima associated with the $t_i = 1$ stationary points can be disregarded, since they give small Yukawa couplings at the scale M_{SUSY}, and in this case the minimum corresponding to the infrared fixed point is the deepest one. The relevant stationary points correspond to the situation when $f(t_i)$ takes the smallest possible value, in order to obtain the largest possible value for α_t^U/c_t. However, whatever this value is, due to the infrared structure of the RGE for α_t, the value obtained for α_t at scales $M_{SUSY} \lesssim 1\ TeV$ is very close to the value corresponding to the infrared fixed point. This situation is very interesting, since it gives us the numerical *prediction* that at low energy $m_t \sim M_t^{IR} \sin\beta$. Using further the perturbative constraint $\alpha_t^U < 1$, we find $\alpha_t \sim 1/(4\pi)$ as an absolute upper limit, in the case when the form of the function $f(t_V)$ does not forbid the possibility $\alpha_t^U \gg \alpha_U$.

The reason of this attraction of $\alpha_t(m_{3/2})$ towards the infrared fixed point is the particular structure of the effective potential after the minimization with respect to the volume moduli. Indeed, eq. (11) can be rewritten as

$$V_0(\hat{Q})|_{min} = -\frac{1}{128\pi^2} Str\,\mathcal{M}^4(\hat{Q})\,, \tag{23}$$

or, more explicitly,

$$V_0(\hat{Q})|_{min} = \frac{1}{128\pi^2}(-m_t^2 C_t^2 - \ldots)\,, \tag{24}$$

where

$$C_t^2 = 12\left[m_{Q_3}^2 + m_{U_3}^2 + \frac{m_Z^2}{2}\cos 2\beta + \left(A_t + \frac{\mu}{\tan\beta}\right)^2\right]. \tag{25}$$

Equation (24) looks unbounded from below in the variable α_t. However, the actual bound is set by the effective infrared fixed point, which therefore corresponds to the deepest minimum of the effective potential, if permitted by the structure of the moduli space of the underlying string theory.

Equation (24) can be seen as a quadratic constraint on m_t, and suggests an interesting generalization to the case in which also α_b^U and α_τ^U are considered as dynamical variables:

$$V_0(\hat{Q})|_{min} = \frac{1}{128\pi^2}(-m_t^2 C_t^2 - m_b^2 C_b^2 - m_\tau^2 C_\tau^2 + \ldots)\,, \tag{26}$$

where

$$C_b^2 = 12\left[m_{Q_3}^2 + m_{D_3}^2 - \frac{m_Z^2}{2}\cos 2\beta + (A_b + \mu\,\tan\beta)^2\right], \tag{27}$$

$$C_\tau^2 = 4\left[m_{L_3}^2 + m_{E_3}^2 - \frac{m_Z^2}{2}\cos 2\beta + (A_\tau + \mu\,\tan\beta)^2\right]. \tag{28}$$

A similar constraint was suggested by Nambu [30], in a completely different context, as a possibility for dynamically explaining the m_t/m_b and m_b/m_τ ratios. As remarked also in [31], it might be possible to adapt his approach to the present context, and work along these lines is in progress [18].

4. For simplicity, we consider here a special case of the MSSM defined as follows: 1) all Yukawa couplings are neglected, apart from the top-quark one, h_t; 2) the boundary conditions at the unification scale are chosen to be $10\,m_{1/2} = m_0 = m_{3/2}$, $A = m_3^2 = \mu = 0$. This case does not correspond to a fully realistic model, but is suitable to illustrate in a simple way the main conceptual points of our approach. We would like to stress, however, that our considerations can be easily generalized to the fully-fledged MSSM [18]. Since, in the special case under consideration, $m_3^2 = 0$ and $m_1^2 > 0$ at all scales $Q < M_U$, as can be verified by looking at the general structure of the RGE [19], we can assume $v_1 = 0$ at the minimum, and restrict our attention to the dependence of the RG-improved tree-level potential on $v \equiv v_2$,

$$V_0 = m_2^2 v^2 + \frac{g^2 + g'^{\,2}}{8} v^4 + \eta\, m_{3/2}^4 = \frac{1}{8\pi\alpha_Z}\left(4 m_2^2 m_Z^2 + m_Z^4\right) + \eta\, m_{3/2}^4\,, \tag{29}$$

where to write the second expression we have used the definition $\alpha_Z \equiv (g^2 + g'^{\,2})/(4\pi)$ and the tree-level relation $m_Z^2 = (g^2 + g'^{\,2})v^2/2$. In the following, we shall always choose $Q = 2\,m_{3/2}$, which will give an approximation good enough for the present purposes [23]. It is however clear that, if one wants to extend the present work to the fully-fledged MSSM and give accurate quantitative predictions, threshold effects have to be more accurately modelled [18].

The potential of eq. (29) is easily studied. The stability condition along the $v_1 = v_2$ direction is just $m_1^2 + m_2^2 \geq 0$, and minimization with respect to m_Z (or, equivalently, with respect to v) gives

$$\begin{aligned} m_Z^2 &= -2 m_2^2 \quad if \quad m_2^2 < 0\,, \\ m_Z^2 &= 0 \qquad\;\; if \quad m_2^2 > 0\,. \end{aligned} \tag{30}$$

Substituting this into eq. (29), we get

$$\begin{aligned} V_0 &= \left(\eta - \frac{1}{2\pi\alpha_Z}\frac{m_2^4}{m_{3/2}^4}\right) m_{3/2}^4 \quad if \quad m_2^2 < 0\,, \\ V_0 &= \eta\, m_{3/2}^4 \qquad\qquad\qquad\quad\; if \quad m_2^2 > 0\,. \end{aligned} \tag{31}$$

The two RGEs that control the dynamical determination of m_Z and $m_{3/2}$ are the one for the cosmological parameter η, equation (8), and the one for the mass parameter m_2^2, which reads in our simplified case:

$$\frac{dm_2^2}{dt} = \frac{1}{2\pi}\left[-3\alpha_2 M_2^2 - \alpha' M_1^2 + 3\alpha_t\left(m_{Q_3}^2 + m_{U_3^c}^2 + m_2^2 + A_t^2\right)\right] . \qquad (32)$$

The results of the minimization of the effective potential are illustrated in fig. 3, which shows contours of constant m_Z, $m_{3/2}$ and m_t/m_Z in the (η_0, α_t^U) plane. For $\eta_0 \sim 9$, the transmutation scale at which η turns from positive to negative is close to the electroweak scale, and the experimentally observed value of m_Z can be reproduced. At these minima, the gravitino mass is also of order m_Z, and $m_t \simeq (1.8\text{–}2)\, m_Z$ can be easily reproduced for an appropriate choice of α_t^U.

5. To conclude, we summarize the general features of the dynamical determination of $(m_Z, m_{3/2}, m_t)$, when one regards the MSSM as the low-energy limit of one of the LHC supergravity models considered in ref. [16].

The LHC models were defined as the $N = 1$ supergravities where, in the presence of well-defined quantum gravitational corrections, the contributions to the vacuum energy are not larger than $\mathcal{O}(m_{3/2}^4)$. At least at the one-loop level, examples of LHC models can be obtained from four-dimensional superstring models where supersymmetry is spontaneously broken at the tree level, e.g. by coordinate-dependent compactifications. Other candidate LHC models are certain superstring effective supergravities, where supersymmetry breaking is induced by non-perturbative phenomena such as gaugino condensation. In LHC models, the effective potential is cut-off-independent, up to benign logarithmic corrections, and the scale is set by the gravitino mass $m_{3/2}$, which in turn depends on a singlet modulus field t_V, corresponding to an approximately flat direction. In particular, we concentrated here on the case in which the only moduli dependence of the MSSM mass terms is via $m_{3/2}$. In LHC models, minimization of the effective potential V with respect to the volume modulus t_V sets $m_{3/2}$ to a value defined by the infrared fixed point of the vacuum energy $m_{3/2}^2(\partial V/\partial m_{3/2}^2) = 0$. We found that, for a wide and reasonable range of coupling constants $(\alpha_U, \alpha_{t,b,\tau}^U)$ and other dimensionless parameters $(\vec{\xi}, \eta_0)$, this corresponds to the desired hierarchy $m_Z \sim m_{3/2} \ll M_P$.

Inspired by four-dimensional superstring models, we examined the case in which also the Yukawa couplings are field-dependent dynamical variables, concentrating on the largest one, associated with the top quark. In analogy with the previous result on $m_{3/2}$, we found that also the top Yukawa coupling α_t is driven to its effective infrared fixed point. This left us with a definite prediction for the top-quark mass, $m_t = M_t^{IR} \sin\beta$, where $M_t^{IR} \sim 190\ GeV$ and $\sin\beta$ can be computed in terms of the $(\vec{\xi}, \eta_0)$ parameters, derivable from the fundamental theory at the cut-off scale. A more general analysis of mass hierarchies, including not only $m_{3/2}/M_U$, m_Z/M_U and m_t/M_U, but also some fermion mass ratios such as m_b/m_t and m_τ/m_t, appears to be feasible in the framework of LHC models and is currently under study [18].

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Figure captions

Fig.1: Renormalization of the cosmological term of the MSSM for some representative boundary conditions at M_U: a) $m_{1/2} = m_{3/2}$, $m_0 = A = m_3 = \mu = 0$; b) $m_0 = m_{3/2}$, $m_{1/2} = A = m_3 = \mu = 0$; c) $m_{1/2} = m_0 = m_{3/2}$, $A = m_3 = \mu = 0$; d) $m_{1/2} = m_0 = A = m_3 = \mu = m_{3/2}$. For simplicity, the top Yukawa coupling has been assigned the representative values $\alpha_t^U = 0$ (dashed lines), $\alpha_t^U = \alpha_U$ (solid lines), $\alpha_t^U = 4\alpha_U$ (dash-dotted lines), and all other Yukawa couplings have been neglected.

Fig.2: The running top Yukawa coupling $\alpha_t(Q)$, in the interval $m_Z \le Q \le M_U$, computed at the one-loop level and neglecting threshold effects, for some representative choices of the boundary condition at the unification scale: $\alpha_t^U = \alpha_U$ (solid line); $\alpha_t^U = 2\alpha_U$ (dashed line); $\alpha_t^U = 4\,\alpha_U$ (dash-dotted line).

Fig.3: Contours of constant $m_Z = 60, 90, 120\ GeV$ (solid lines), $m_0 = 10\,m_{1/2} = m_{3/2} = 100, 300, 500\ GeV$ (dashed lines) and $m_t/m_Z = 1.8, 1.9, 2$ (dash-dotted lines), in the (η_0, α_t^U) plane, for the simplified version of the MSSM discussed in the text.