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SUSY PARTICLES[1]

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Abstract

Analysis of the SUSY spectrum in supergravity unified models is given under the naturalness criterion that the universal scalar mass (m_0) and the gluino mass $(m_{\tilde{g}})$ satisfy the constraint $m_0, m_{\tilde{g}} \lesssim 1$ TeV. The SUSY spectrum is analysed in four different scenarios: (1) minimal supergravity models ignoring proton decay from dimension five operators (MSSM), (2) imposing proton stability constraint in supergravity models with SU(5) type embedding which allow proton decay via dimension five operators, (3) with inclusion of dark matter constraints in models of type (1), and (4) with inclusion of dark matter constraint in models of type (2). It is found that there is a very strong upper limit on the light chargino mass in models of type (4), i.e., the light chargino mass $\lesssim 120$ GeV.

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I. Introduction

Supergravity unification[1,2] is currently the leading candidate theory for physics beyond the Standard Model. It allows for a phenomenologically viable breaking of supersymmetry[2,3], and the formalism also generates its own breaking of the electroweak symmetry via renormalization group effects. The search for supersymmetric particles is of great interest at supercolliders, in dark matter experiments and elsewhere. Here we discuss the computation of the SUSY mass spectrum in four specific scenarios. These consist of supergravity models excluding or including proton stability constraint, and supergravity models with and without dark matter constraints. The models where we impose proton stability are those which have an SU(5) type embedding.

The outline of the paper is as follows: in Section 2 we discuss general features of SUSY models. In Section 3 we discuss supergravity models, the parameter space of the theory, and the radiative breaking of the electroweak symmetry and constraints that are imposed in the analysis of the SUSY spectrum. In Section 4 we give the renormalization group analysis of SUSY parameters and analytic formulae for the SUSY spectrum. Section 5 is devoted to analysis and results of the spectrum for four different scenarios. Conclusions are given in Section 6.

II. SUSY models: particles and interactions

SUSY models are built using two types of massless supermultiplets; chiral multiplets with spin $(0,1/2)$ is a left-handed chiral multiplet, and vector multiplets with spin $(1/2,1)$ where spin $1/2$ is a Majorana spinor and spin 1 is a vector boson. For SUSY extension of the standard $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$ model, the chiral multiplets consist of the following:

$$
\begin{array}{cccc}
& J=0 & & J=1 \\
\text{lepton} & \begin{pmatrix} \tilde{\nu}_{iL} \\ \tilde{e}_{iL} \end{pmatrix}, \tilde{e}_{iR} & & \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix}, e_{iR} \\
\text{multiplet} \\
\\
\text{quark} & \begin{pmatrix} \tilde{u}_{iL} \\ \tilde{d}_{iL} \end{pmatrix}, \tilde{u}_{iR}, \tilde{d}_{iR} & & \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}, u_{iR}, d_{iR} & (2.1) \\
\text{multiplet} \\
\\
\text{Higgs} & H_1 = \begin{pmatrix} H_1^0 \\ H_2^- \end{pmatrix}, H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} & \tilde{H}_1 = \begin{pmatrix} \tilde{H}_1^0 \\ \tilde{H}_1^- \end{pmatrix}, \tilde{H}_2 = \begin{pmatrix} \tilde{H}_2^+ \\ \tilde{H}_2^0 \end{pmatrix} \\
\text{multiplets}
\end{array}
$$

The potential of this theory is given by

$$ V = V_W + V_D + V_{SB} \tag{2.2} $$

where $V_W = \sum |\partial W/\partial z_a|^2$ and W is the superpotential given by

$$ W = \mu H_1 H_2 + \lambda_{ij}^{(e)} \ell_i H_1 e_j^c + \lambda_{ij}^{(u)} q_i H_2 u^c + \lambda_{ij}^{(d)} q_i H_1 d_j^c \tag{2.3} $$

In Eq. (2.2) V_D is the D-term given by $V_D = \frac{1}{2} g_A^2 D_A D_A^\dagger$ with $D_A = z_a^+ (T^A)_{ab} z_b$, and V_{SB} is the SUSY breaking terms. It has the general form[4]

$$
\begin{aligned}
V_{SB} = {} & m_{ab}^2 z_a^* z_b + \Big[A_{ij}^{(e)} \lambda_{ij}^{(e)} \ell_i H_1 e_j^c + A_{ij}^{(u)} \lambda_{ij}^{(u)} q_i H_2 u_j^c \\
& + A_{ij}^{(d)} \lambda_{ij}^{(d)} q_i H_1 d_j^c + B\mu H_1 H_2 + \text{h.c.} \Big]
\end{aligned} \tag{2.4}
$$

The SUSY breaking terms in Eq. (2.4) have a large number of arbitrary parameters. Thus this theory is not very predictive. We shall next discuss supergravity unification where there is a sharp reduction in the number of arbitrary parameters, and the theory is very predictive.

III. Supergravity Unification

In supergravity unification supersymmetry can be broken spontaneously via a hidden sector, and the effective low-energy theory has only four arbitrary parameters. In this theory the effective potential below the GUT scale M_G is given by[2,3,5,6]

$$V_{SB} = m_0^2 z_i z_i^+ + \left(A_0 W^{(3)} + B_0 W^{(2)} + \text{h.c.} \right) \tag{3.1}$$

In Eq. (3.1) $W^{(2)}, W^{(3)}$ are the quadratic and the cubic parts of the effective superpotential which in general has the expansion

$$W_{eff} = W^{(2)} + W^{(3)} + \frac{1}{M} W^{(4)} \tag{3.2}$$

where $W^{(2)} = \mu_0 H_1 H_2, W^{(3)}$ contains terms cubic in fields and involves the interactions of quarks, leptons and Higgs, and $W^{(4)}$ contains terms quartic in fields and in general has interactions which violate baryon number. In addition to SUSY breaking terms, in Eq. (3.1) one also has a universal gaugino mass term of the form $m_{1/2} \bar{\lambda}^\alpha \lambda^\alpha$. Thus the effective theory below the GUT scale depends on the following set of parameters:

$$m_0, m_{1/2}, A_0, B_0 \; ; \quad \mu_0 \; ; \quad \alpha_G, \; M_G \tag{3.3}$$

where M_G is the GUT mass and α_G is the GUT gauge coupling constant.

Below the GUT scale one evolves the gauge and Yukawa coupling constants, and the soft SUSY breaking parameters using renormalization group equations. As is well known, an interesting aspect of supergravity unification is that the electroweak symmetry can be broken via renormalization group effects[7]. It is in this framework that we shall discuss the computation of SUSY particle spectrum. The breaking of the electroweak symmetry is controlled by an effective potential which has the form $V = V_0 + \Delta V_1$ where V_0 is the renormalization group improved tree potential and ΔV_1 is the one-loop effective potential[8]. Assuming charge and colour conservation, V_0 is given by

$$\begin{aligned} V_0 &= m_1^2(t)|H_1|^2 + m_2^2(t)|H_2|^2 - m_3^2(t)(H_1 H_2 + \text{H.C.}) \\ &\quad + \frac{1}{8}(g_2^2 + g_Y^2)\left(|H_1|^2 - |H_2|^2\right)^2 \end{aligned} \tag{3.4}$$

where $t = \ln(M_G^2/Q^2)$ and Q is the running scale, and V_1 is given by

$$V_1 = \frac{1}{64\pi^2} \sum_a (-1)^{2s_a} n_a M_a^4 \log \frac{M_a^2}{e^{3/2} Q^2} \tag{3.5}$$

The importance of including V_1 in the analysis has been emphasized recently[9]. The parameters $m_i^2(t), g_2, g_Y$, satisfy the following boundary conditions:

$$m_i^2(0) = m_0^2 + \mu_0^2 \; ; \quad i = 1, 2 \tag{3.6a}$$

$$m_3^2(0) = -B_0 \mu_0 \tag{3.6b}$$

$$\alpha_2(0) = (5/3)\alpha_Y(0) = \alpha_G \tag{3.6c}$$

Electroweak symmetry breaking requires satisfaction of a number of conditions. These include boundedness of the potential from below, i.e., $m_1^2 + m_2^2 - 2|m_3^2| > 0$ and negativeness of the

Higgs (mass)^2 matrix, i.e. $m_1^2 m_2^2 - m_3^4 < 0$. Minimization of the potential then yields the relations

$$\frac{1}{2} M_Z^2 = \frac{\mu_1^2 - \mu_2^2 \tan^2 \beta}{\tan^2 \beta - 1} \; ; \quad \sin^2 \beta = \frac{2m_3^2}{\mu_1^2 + \mu_2^2} \tag{3.7}$$

where $\mu_i^2 = m_i^2 + \Sigma_i, (i = 1, 2)$ and Σ_i is the loop correction arising from ΔV_1: it has the form

$$\Sigma^a = \frac{1}{32\pi^2} \sum_i (-1)^{2s_i} n_i \; M_i^2 \ln \left[M_i^2 / eQ^2 \right] \frac{\partial M_i^2}{\partial v_a^2} \tag{3.8}$$

where $v_a = \langle H_a \rangle$. The particles that make the largest contributions are the stops and the charginos. For the stops one has $\Sigma_{\tilde{t}_i}^a$ $(i = 1, 2)$ where[10]

$$\begin{aligned}
\Sigma_{\tilde{t}_i}^1 &= \frac{3\alpha_2}{8\pi \cos^2 \theta_W} \; M_{\tilde{t}_i}^2 \left[\frac{1}{4} \mp \left\{ \frac{1}{2} \left(m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2 \right) \left(\frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \right) \right. \right. \\
&\left. \left. + \frac{m_t^2 \mu}{M_Z^2 \sin^2 \beta} (A_t m_0 \tan \beta \mu) \right\} \frac{1}{M_{\tilde{t}_2}^2 - M_{\tilde{t}_1}^2} \right] \ln \left(\frac{M_{\tilde{t}_i}^2}{eQ^2} \right)
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
\Sigma_{\tilde{t}_i}^2 &= \frac{3\alpha_2}{8\pi \cos^2 \theta_W} \; M_{\tilde{t}_i}^2 \left[\left(\frac{m_t^2}{M_Z^2 \sin^2 \beta} - \frac{1}{4} \right) \mp \left\{ \frac{1}{2} \left(m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2 \right) \left(-\frac{1}{2} + \frac{4}{3} \sin^2 \theta_W \right) \right. \right. \\
&\left. \left. + \frac{m_t^2 A_t m_0}{M_Z^2 \sin^2 \beta} (A_t m_0 + \mu \; ctn\beta) \right\} \times \frac{1}{M_{\tilde{t}_2}^2 - M_{\tilde{t}_1}^2} \right] \ln \left(\frac{M_{\tilde{t}_i}^2}{eQ^2} \right)
\end{aligned} \tag{3.10}$$

where $m_{\tilde{t}_i}$ are the stop masses and $\tilde{t}_{L,R}$ are defined in Section 4. [Equations (3.9) and (3.10) include a colour factor of three in squark contributions missing in Ref. 10]. The chargino contributions are given by[10]

$$\Sigma_{\tilde{W}_i}^a = -\frac{\alpha_2 s_i}{2\pi} \frac{\lambda_i^3 (\lambda_i - u^a \lambda_j)}{(\lambda_i^2 - \lambda_j^2)} \ln \left(\frac{M_{\tilde{W}_i}^2}{eQ^2} \right) \quad i, j = 1, 2 \; ; \quad i \neq j \tag{3.11}$$

where λ_i are the eigenvalues of the chargino mass matrix (see Section 4) and $u^a = (\tan \beta, \cot \beta)$.

In the analysis of the SUSY spectrum one imposes several constraints both theoretical and experimental. We list the full list of constraints below:

i) charge and colour conservation at the electroweak scale and at the GUT scale;

ii) absence of tachyonic particles;

iii) a lower bound on SUSY particle masses as indicated by CDF, DO and LEP data;

iv) an upper limit on SUSY masses from naturalness criterion which we assume as follows: $m_0, m_{\tilde{g}} < 1$ TeV;

v) proton lifetime satisfies the current experimenta bounds;

vi) the neutralino relic density satisfies a constraint consistent with the COBE data.

We shall discuss analyses of the SUSY spectrum both including and excluding constraints v) and vi) listed above. Before proceeding further we discuss the constraints v) and vi) in some detail.

v) Proton stability constraint

Proton decay is a generic feature of a class of GUT models and string models, and thus the current experimental limits on proton lifetime act as a constraint on the model. In analyses discussed below we shall assume that the SUSY theories we consider have an SU(5) embedding and the Higgs doublets are embedded in $5 + \bar{5}$ representations of SU(5). In this case, there is a model independent proton decay amplitude that arises from the exchange of the Higgs triplet fields. The dominant decvay mode of the proton from this amplitude is the mode[11] $p \to \bar{\nu} K^+$ and the total decay width for this mode is given by

$$\Gamma(p \to \bar{\nu} K^+) = \sum_{i=e,\mu,\tau} \Gamma(p \to \bar{\nu}_i K^+) \tag{3.12}$$

The contribution of the first generation is essentially negligible. For the remaining two generations one gets the following relation[12]

$$\Gamma(p \to \bar{\nu}_i K^+) = C \left(\frac{\beta_p}{M_{H_3}}\right)^2 |A|^2 |B_i|^2 \tag{3.13}$$

where A depends on quark masses and mixings and is given by

$$A = \frac{\alpha_2^2}{2 M_W^2} m_s m_c V_{21}^\dagger V_{21} A_L A_S \tag{3.14}$$

where V_{ij} are the CKM matrix elements and $A_{L,S}$ are the suppression factors with values $A_L = 0.283, A_S = 0.833$. C is a chiral Lagrangian factor given by

$$C = \frac{m_N}{32\pi f_\pi^2} \left| \left(1 + \frac{m_N(D+F)}{m_B}\right) \left(1 - \frac{m_K^2}{m_N^2}\right) \right|^2 \tag{3.15}$$

where $f_\pi = 139$ MeV, $D = 0.76, F = 0.48$ and $m_B = 1154$ MeV. β_p is the three-quark matrix element of the proton defined by

$$\epsilon_{abc} \epsilon_{\alpha\beta} \langle 0 | d_{aL}^\alpha u_{bL}^\beta u_{cL}^\gamma | p \rangle = \beta_p U_L^\gamma \tag{3.16}$$

where U_L^γ is the proton-wave function. Recent lattice gauge calculations give the following evaluation for β_p [13]:

$$\beta_p = (5.6 \pm 0.8) \times 10^{-3} \text{ GeV}^3 \tag{3.17}$$

Finally B_i in Eq. (3.13) is a dressing loop given by[12]

$$B_i = \frac{m_i^d V_{i1}^\dagger}{m_s V_{21}^\dagger} \left[P_2 B_{2i} + \frac{m_t V_{31} V_{32}}{m_c V_{21} V_{22}} P_3 B_{3i} \right] \frac{1}{\sin 2\beta} \tag{3.18}$$

where P_2, P_3 are the intergenerational phases which on CP conserving manifolds have values, ± 1 and B_{ij} is the contribution of the jth generation to B_i and can be written as $B_{ji} = F(\tilde{u}_i, \tilde{d}_j, \tilde{W}) + (\tilde{d}_j \to \tilde{e}_j)$.

The 2nd generation contribution B_{2i} where one neglects L-R mixing is quite straightforward. It is given by $B_{2i} = F_2(\tilde{c}, \tilde{d}_i, \tilde{W}) + (\tilde{d}_i \to \tilde{\ell}_i)$ where

$$F_2(\tilde{c}, \tilde{d}_i, \tilde{W}) = \sin\gamma_+ \cos\gamma_- f(\tilde{c}, \tilde{d}_i, \tilde{W}_1) + \cos\gamma_+ \sin\gamma_- f(\tilde{c}, \tilde{d}_i, \tilde{W}_2) \tag{3.19}$$

and where

$$f(a,b,c) = \frac{m_c}{(m_b^2 - m_c^2)} \left[\left[\frac{m_b^2}{(m_a^2 - m_b^2)} \right] \ln \left(\frac{m_a^2}{m_b^2} \right) - (b \to c) \right] \tag{3.20}$$

In Eq. (3.19) $\gamma_{\mp} = \beta_+ \mp \beta_-$ and $\beta_{\pm}$ are defined by

$$\sin 2\beta_{\pm} = \frac{(\mu \mp \tilde{m}_2)}{[4\nu_{\pm}^2 + (\mu \pm \tilde{m}_2)^2]^{1/2}} \; ; \quad \sqrt{2}\, \nu_{\pm} = M_W(\sin\beta \pm \cos\beta) \; . \tag{3.21}$$

The contributions of the third generation involve L-R mixing due to the top quark mass and are more complex. They are discussed in detail in Ref. 12). For the purpose of analysing the p-stability constraint it is useful to introduce the quantity B defined by[14,15]

$$B \equiv \left[|B_2|^2 + |B_3|^2 \right]^{1/2} \left[\frac{M_S}{10^{2.4}\ \mathrm{GeV}} \right]^{0.33} \times 10^6\ \mathrm{GeV}^{-1} \tag{3.22}$$

where M_S is the effective SUSY mass that appears in Amaldi et al. type analyses in fitting the LEP data to SU(5) type SUSY GUT. The multiplicative factor with the M_S term takes into account the anticorrelation between M_S and M_G in this fit. The current experimental lower limit on $\bar{\nu}K^+$ mode from Kamiokande is[16]

$$\tau(p \to \bar{\nu}K^+) > 1.0 \times 10^{32}\ yr \tag{3.23}$$

Using the above limit one finds[14,15]

$$B \leq 100 \left(\frac{M_{H_3}}{M_G} \right)\ \mathrm{GeV}^{-1} \tag{3.24}$$

A reasonable upper bound on M_{H_3} is $M_{H_3} \leq 10\, M_G$. Thus upper limit keeps the GUT Yukawa couplings perturbative and also keeps M_{H_3} significantly below the Planck scale so that quantum gravity effects will be negligible.

vi) Neutralino Relic Density Constraint

In SUSY theories with R-parity conservation, the lightest supersymmetric particle (LSP) is stable and would contribute to the matter density of the Universe. In supergravity unified theories with radiative breaking one finds[17,18] that for a large part of the parameter space LSP in fact is the lightest neutralino ($\tilde{Z}_1$). Thus in such situations $\tilde{Z}_1$ is a natural candidate for cold dark matter. If one assumes the inflationary scenario with $\Omega = 1$ (where $\Omega = \rho/\rho_c$ with ρ the matter density of the Universe and ρ_c being the critical matter density needed to close the Universe). The COBE data is consistent with a mix of cold and hot dark matter in the ratio of 2:1. Then one finds

$$0.1 < \Omega h^2 < 0.35 \tag{3.25}$$

where h is the Hubble constant in units of 100km/s.Mpc and lies in the range $0.5 < h < 0.75$. The imposition of the constraint of Eq. (3.25) requires considerable care. The reason for this is that the theoretical evaluation of Ωh^2 is a very delicate affair when one is close to thresholds and poles in the annihilation cross-section. We discuss this issue more concretely below.

The standard formula for the computation of the relic density is given by[19,20]

$$\Omega_{\tilde{Z}_1} h^2 \simeq 2.53 \times 10^{-11} \left(\frac{T_{\tilde{Z}_1}}{T_\gamma} \right)^3 \left(\frac{T_\gamma}{2.75} \right)^3 \frac{N_f^{1/2}}{J(x_f)} \tag{3.26}$$

where $(T_{\tilde{z}_1}/T_\gamma)^3$ is a reheating factor, T_γ is the current microwave temperature, N_f is the number of degrees of freedom and $J(x_f)$ is given by

$$J(x_f) = \int_0^{x_f} dx \langle \sigma v \rangle \tag{3.27}$$

where σ is the annihilation cross-section of neutralinos and v is their relative velocity. In usual analyses one uses an expansion on σv of the form[21−23)]

$$\sigma v = a + b v^2 \tag{3.28}$$

However, it is known[24)] that Eq. (3.28) is a poor approximation in the neighbourhood of threshold and poles. Precisely this situation arises for the case of annihilation of neutralinos in supergravity models with masses computed via radiative breaking. Here one finds that in the physically interesting domain of the parameter space annihilation of neutralinos occurs near Higgs and Z-poles[17,18)]. In this circumstance the expansion of Eq. (3.28) no longer holds. However, it turns out that it is fairly straightforward to carry out the correct thermal averaging in the presence of poles. A technique for accomplishing this is discussed in Refs. 17) and 18). A similar analysis is also discussed in Ref. 25).

IV.   SUSY Particle Masses in Supergravity Unification

In renormalization group analyses of the SUSY particle spectrum one begins by extracting the GUT parameters α_G, M_G by using the two-loop renormalization group equations of the gauge coupling constants and fitting to the high precision LEP resulfs for $\alpha_i(M_Z), i = 1, 2, 3$. The two-loop evolution equations are[26]

$$\frac{d}{dt}\alpha_i = -\frac{1}{4\pi}\left[b_i + \frac{1}{4\pi}\sum_j b_{ij}\alpha_j\right]\alpha_i^2 \tag{4.1}$$

where

$$b_i = (0, -6, -9) + (2, 2, 2)N_F + (\frac{3}{10}, \frac{1}{2}, 0)N_H \tag{4.2}$$

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix} + \begin{pmatrix} \frac{38}{15} & \frac{6}{5} & \frac{88}{15} \\ \frac{2}{5} & 14 & 8 \\ \frac{11}{15} & 3 & \frac{68}{3} \end{pmatrix} N_F + \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} N_H \tag{4.3}$$

The α_i of Eq. (4.1) satisfy the boundary conditions $\alpha_i = \alpha_G$ at scale M_G. The α_i computed from Eq. (4.1) are fitted to the LEP data for α_i. The current determinations for these are[27−29)]

$$\alpha_1(M_Z) \equiv \left(\frac{5}{3}\right)\alpha_Y(M_Z) = 0.016985 \pm 0.00002 \tag{4.4}$$

$$\alpha_2(M_Z) = 0.03358 \pm 0.0001 \tag{4.5}$$

$$\alpha_3(M_Z) = 0.118 \pm 0.007 \tag{4.6}$$

A fit to the above data[30)] gives the following values for α_G, M_G and M_S:

$$\alpha_G^{-1} = 25.4 \pm 1.7 , \quad M_G \cong 10^{16.2+5.7(\alpha_3/0.118-1)} \tag{4.7a}$$

$$M_S \cong 10^{2.4+17.4(1-\alpha_3/0.118)} \tag{4.7b}$$

The analysis of the soft SUSY breaking parameters and of the μ-parameter is done using one-loop renormalization group equations[31]. The gaugino masses are assumed to obey the RG equation

$$\frac{d\tilde{m}_i}{dt} = -\frac{b_i}{4\pi}\tilde{\alpha}_i(t)\tilde{m}_i(t) ; \quad \tilde{m}_i(0) = m_{1/2} \tag{4.8}$$

The μ-parameter and the top Yukawa coupling obey the equation[7]

$$\frac{d\mu^2}{dt} = \left(3\tilde{\alpha}_2 + \frac{3}{5}\tilde{\alpha}_1 - 3Y_t\right)\mu^2 \tag{4.9}$$

$$\frac{dY_t}{dt} = \left(\frac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{13}{15}\tilde{\alpha}_1 - 6Y_t\right)Y_t^2 \tag{4.10}$$

where $Y_t = h_t^2/4\pi$ and h_t is the top quark Yukawa coupling. The chargino masses are determined completely in terms of μ and $\tilde{m}_2$ [1]:

$$\lambda_i = \frac{1}{2}\left(|4\nu_+^2 + (\mu - \tilde{m}_2)^2|^{1/2} \mp |4\nu_-^2 + (\mu + \tilde{m}_2)^2|^{1/2}\right) \tag{4.11}$$

where $\sqrt{2}\nu_\pm = M_W(\sin\beta \pm \cos\beta)$ and $m_{\tilde{W}_i} = |\lambda_i|, i = 1, 2$. Similarly the neutralino masses are given by roots of the secular equation $f(\lambda) = 0$ where[1],[31]

$$\begin{aligned}
f(\lambda) &\equiv \lambda^4 - (m_{\tilde{\gamma}} + m_{\tilde{Z}})\lambda^3 - (M_Z^2 + \mu^2 + m_{\tilde{\gamma}\tilde{Z}}^2 - m_{\tilde{\gamma}}m_{\tilde{Z}})\lambda^2 \\
&+ \left[(m_{\tilde{\gamma}} - \mu\sin 2\beta)M_Z^2 + (m_{\tilde{\gamma}} + m_{\tilde{Z}})\mu^2\right]\lambda \\
&+ (\mu^2(m_{\tilde{\gamma}\tilde{Z}}^2 - m_{\tilde{\gamma}}m_{\tilde{Z}}) + \mu m_{\tilde{\gamma}}M_Z^2\sin 2\beta)
\end{aligned} \tag{4.12}$$

where $m_{\tilde{\gamma}}, m_{\tilde{Z}}$ and $m_{\tilde{\gamma}\tilde{z}}$ are as defined in the first paper of Ref. 1.

Sleptons (i = 1,2,3) Masses of the three generation of sleptons are given in a straightforward fashion and are as follows[1]:

$$m_{\tilde{e}_{iL}}^2 = m_0^2 + m_{ei}^2 + \tilde{\alpha}_G\left[\left(\frac{3}{2}\right)f_2 + \left(\frac{3}{10}\right)f_1\right]M_{1/2}^2 + \left(-\frac{1}{2} + \sin^2\theta_W\right)M_Z^2\cos 2\beta \tag{4.13}$$

$$m_{\tilde{e}_{iR}}^2 = m_0^2 + m_{ei}^2 + \tilde{\alpha}_G\left(\frac{6}{5}\right)f_1 m_{1/2}^2 - \sin^2\theta_W M_Z^2\cos 2\beta \tag{4.14}$$

$$m_{\tilde{\nu}_{iL}}^2 = m_0^2 + \tilde{\alpha}_G\left[\left(\frac{3}{2}\right)f_2 + \left(\frac{3}{10}\right)f_1\right]m_{1/2}^2 + \left(\frac{1}{2}\right)M_Z^2\cos 2\beta \tag{4.15}$$

where $\tilde{\alpha}_G = \alpha_G/4\pi, f_a(t) = t(2 - \beta_a t)/(1 + \beta_a t)^2$ and where $\beta_a = (33/5, 1, -3)$

Squarks (i = 1,2): for the first two generation of squarks one can ignore the left-right mixing since the quark masses are small. In this approximation masses of the first two squark generations are given by[1]

$$m_{\tilde{u}_{iL}}^2 = m_0^2 + m_{u_i}^2 + \tilde{\alpha}_G\left[\left(\frac{8}{3}\right)f_3 + \left(\frac{3}{2}\right)f_2 + \left(\frac{1}{30}\right)f_1\right]m_{1/2}^2 + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)M_Z^2\cos 2\beta \tag{4.16}$$

$$m_{\tilde{d}_{iL}}^2 = m_0^2 + m_{d_i}^2 + \tilde{\alpha}_G\left[\left(\frac{8}{3}\right)f_3 + \left(\frac{3}{2}\right)f_2 + \left(\frac{1}{30}\right)f_1\right]m_{1/2}^2 + \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right)M_Z^2\cos 2\beta \tag{4.17}$$

$$m_{\tilde{u}_{iR}}^2 = m_0^2 + m_{u_i}^2 + \tilde{\alpha}_G\left[\left(\frac{8}{3}\right)f_3 + \left(\frac{8}{15}\right)f_1\right]m_{1/2}^2 + \left(\frac{2}{3}\right)\sin^2\theta_W M_Z^2\cos 2\beta \tag{4.18}$$

$$m_{\tilde{d}_{iR}}^2 = m_0^2 + m_{d_i}^2 + \tilde{\alpha}_G \left[\left(\frac{8}{3}\right) f_3 + \left(\frac{2}{15}\right) f_1 \right] m_{1/2}^2 + \left(-\frac{1}{3}\right) \sin^2 \theta_W M_Z^2 \cos 2\beta \tag{4.19}$$

An interesting feature of supergravity masses is that at any scale

$$m_{\tilde{c}}^2 - m_{\tilde{u}}^2 = m_c^2 - m_u^2 \tag{4.20}$$

Eq.(4.20) thus leads to a natural suppression of flavour changing neutral currents[4].

Squarks $(i = 3)$: the t-squark masses are affected significantly due to the top mass. There is a significant amount of L-R mixing and the stop(mass)^2 are given by eigenvalues of the following (mass)^2 matrix[1]:

$$\begin{pmatrix} m_{\tilde{t}_L}^2 & m_t(A_t + \mu \cot \beta) \\ m_t(A_t + \mu \cot \beta) & m_{\tilde{t}_R}^2 \end{pmatrix} \tag{4.21}$$

where[1,7]

$$m_{\tilde{t}_L}^2 = m_Q^2 + m_t^2 + \left[\left(-\frac{1}{2}\right) + \left(\frac{2}{3}\right) \sin^2 \theta_W \right] M_Z^2 \cos 2\beta \tag{4.22}$$

$$m_{\tilde{t}_R}^2 = m_U^2 + m_t^2 + \left[\left(-\frac{2}{3}\right) \sin^2 \theta_W \right] M_Z^2 \cos 2\beta \tag{4.23}$$

$$m_U^2 = \frac{1}{3}m_0^2 + \frac{2}{3}f A_0 m_{1/2} - \frac{2}{3}k A_0^2 + \frac{2}{5}h m_0^2 + \left[\frac{2}{3}e + \tilde{\alpha}_G \left(\frac{8}{3}f_3 - f_2 + \frac{1}{3}f_1\right) \right] m_{1/2}^2 \tag{4.24}$$

$$m_Q^2 = \frac{2}{3}m_0^2 + \frac{1}{3}f A_0 m_{1/2} - \frac{1}{3}k A_0^2 + \frac{1}{3}h m_0^2 + \left[\frac{1}{3}e + \tilde{\alpha}_G \left(\frac{8}{3}f_3 + f_2 - \frac{1}{15}f_1\right) \right] m_{1/2}^2 \tag{4.25}$$

where the functions e, f, h, k are as defined in Ibáñez et al.[32]. The $\tilde{b}_R$ mass is given by Eq. (4.19) as for the two generation case, while the $\tilde{b}_L$ mass is modified by third generation effects as is given by

$$m_{\tilde{b}_L}^2 = \frac{1}{2}m_0^2 + m_b^2 + \frac{1}{2}m_{\tilde{U}}^2 + \tilde{\alpha}_G \left[\left(\frac{4}{3}\right) f_3 + \left(\frac{1}{15}\right) f_1 \right] m_{1/2}^2 + \left(-\frac{1}{2} + \frac{1}{3}\sin^2 \theta_W\right) m_Z^2 \cos 2\beta \tag{4.26}$$

Higgs: The parameters of the Higgs potential are given by the following evolution equations at the one-loop level

$$m_1^2(t) = m_0^2 + \mu^2(t) + g m_{1/2}^2 \tag{4.27}$$

$$m_t^2(t) = \mu^2(t) + e(t)m_{1/2}^2 + A_0 m_{1/2}f + m_0^2 h - A_0^2 k \tag{4.28}$$

$$m_3^2 = -B_0 \mu^2(t) + r\mu_0 m_{1/2} + s A_0 \mu_0 \tag{4.29}$$

where g, r, s are as defined in Ref. 32). The Higgs masses using just the tree potential are given by[1]

$$m_H^2 = m_A^2 + M_W^2 \tag{4.30}$$

$$m_A^2 = m_1^2 + m_2^2 = \frac{2m_3^2}{\sin 2\beta} \tag{4.31}$$

$$m_{h,H}^2 = \frac{1}{2} \left[M_Z^2 + M_A^2 \mp \left\{ (M_Z^2 + m_A^2)^2 - 4m_A^2 M_Z^2 \cos 2\beta \right\}^{1/2} \right] \tag{4.32}$$

However, there can be significant corrections from the loop effects. We discuss here the corrections to the neutral Higgs sector. Retaining only the top Yukawa coupling one has[33,34]

$$m_A^2 = \frac{\Delta}{\sin^2 \beta}$$

$$\Delta = 2m_3^2 - \frac{3\alpha_2}{8\pi} \frac{\mu A_t}{\sin^2 \beta} \left(\frac{m_t}{M_W}\right)^2 \left(\frac{f(m_{\tilde{t}_1}^2) - f(m_{\tilde{t}_2}^2)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \right) \tag{4.33}$$

where $f(m^2) = 2m^2(\log(m^2/Q^2) - 1)$. The values of $m_{h,H}$ are also modified. One has[33]

$$m_{h,H}^2 = \frac{1}{2}\left[M_Z^2 + m_A^2 + \epsilon \mp \left\{(M_Z^2 + m_A^2 + \epsilon)^2 - 4m_A^2 M_Z^2 \cos 2\beta + \epsilon_1\right\}^{1/2}\right] \tag{4.34}$$

where ϵ, ϵ_1 are the loop corrections which are given by

$$\epsilon = Tr\,\Delta\;;\quad \epsilon_1 = -4(Tr\,\nu\Delta + \det\Delta) \tag{4.35}$$

and ν and Δ are defined by

$$
\begin{aligned}
\nu_{11} &= s^2 M_Z^2 + c^2 m_A^2\;;\quad \nu_{22} = c^2 M_Z^2 + s^2 m_A^2\;;\quad \nu_{12} = \nu_{21} = sc(M_Z^2 + m_A^2)\\
\Delta_{11} &= x\mu^2 y^2 z\;;\quad \Delta_{12} = x\mu y(w + A_t yz) = \Delta_{21}\\
\Delta_{22} &= x(v + 2A_t yw + A_t^2 y^2 z)
\end{aligned}
\tag{4.36}
$$

where

$$
\begin{aligned}
x &= \frac{3\alpha^2}{4\pi}\frac{m_t^4}{M_W^2 s^2}\;;\quad y = \frac{A_t + \mu ctn\beta}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\;;\quad z = 2 - w\frac{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\\
w &= \ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right)\;:\quad v = \ln\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right)
\end{aligned}
\tag{4.37}
$$

where $(s, c) = (\sin\beta, \cos\beta)$.

V.   Analysis and Results

We begin by discussing the parameter space of the supergravity unified models. From Eq. (3.3) we find that the low-energy theory is defined by the seven parameters given there. As discussed in Section 4, α_G and M_G are determined using the LEP data. Further using the first of the two equations in Eq. (3.7). Thus the 7-dimensional parameter space reduces to a 4-dimensional parameter space defined by

$$m_0\;,\quad m_{1/2}\;,\quad A_t\;,\quad \tan\beta \tag{5.1}$$

where A_t is the value of the trilinear coupling A_0 at the electroweak scale.

One of the interesting features that emerges from the analysis is that the parameter μ, which as discussed above is determined by the $E - W$ symmetry breaking equation, is typically large[14,15,36] in the sense $|\mu| \gg M_Z$ over much of the parameter space. The largeness of μ leads to certain scaling properties. For the charginos one finds for $|\mu| \gg M_Z$ the result

$$m_{\tilde{W}_1} \simeq \tilde{m}_2 - \frac{M_W^2 \sin 2\beta}{\mu}\;;\quad m_{\tilde{W}_2} \cong \mu + \frac{M_W^2}{\mu} \tag{5.2}$$

Similarly for neutralinos one has

$$m_{\tilde{Z}_1} \cong \tilde{m}_1 - \frac{M_Z^2 \sin 2\beta \sin^2\theta_W}{\mu}\;;\quad m_{\tilde{Z}_2} \cong \tilde{m}_2 - \frac{M_W^2 \sin 2\beta}{\mu} \tag{5.3a}$$

$$m_{\tilde{Z}_3,\tilde{Z}_4} \cong \left|\mu - \frac{1}{2}\frac{M_Z^2}{\mu}(1 \pm \sin^2\beta)\right| \tag{5.3b}$$

From Eqs. (5.2) and (5.3) one finds the following scaling relations[14,15,35)]

$$m_{\tilde{W}_1} \simeq 2m_{\tilde{Z}_1} \simeq m_{\tilde{Z}_2} \tag{5.4a}$$

$$m_{\tilde{W}_2} \simeq m_{\tilde{Z}_3} \simeq m_{\tilde{Z}_4} \tag{5.4b}$$

The numerical analysis exhibits the above scaling relations. Also to a good approximation the following relations between the chargino and gluino masses emerge:

$$m_{\tilde{W}_1} \cong \frac{1}{4}m_{\tilde{g}}(\mu > 0) \; ; \quad m_{\tilde{W}_1} \simeq \frac{1}{3}m_{\tilde{g}} \quad (\mu < 0) \tag{5.5}$$

There is a scaling relation in the Higgs sector also. Here one finds

$$m_{H^0} \simeq m_A \simeq m_{H^\pm} \tag{5.6}$$

Next we discuss the result of the analysis for four specific models. These are (1) models with constraints (i)–(iv), (2) models with constraints (i)–(v), (3) models of type (1) with constraint (vi) and (4) models of type (2) with constraints (vi).

(1) These models (sometimes referred to as MSSM) are a truncated version of the supergravity unification where proton decay via dimension five operators is discarded or suppressed. Thus here only the constraints (i)–(iv) are imposed[36)]. Due to the absence of constraints (v) and (vi) $m_{\tilde{g}}$ here can run up to its naturalness limit. The characteristic spectrum after one integrates over the full parameter space of the theory is shown in Fig. 1. One finds a wide dispersion in the spectrum from a lower limit of $0(25 \text{ GeV})$ for the lightest neutralino to an upper limit of well above a TeV for charged Higgs. An interesting feature of the spectrum is that the lightest neutral Higgs has an upper limit on its mass of about 130 GeV. However, the lightest chargino in this scenario can be as large as 300 GeV. The lower limits on masses of $\tilde{e}_R, \tilde{\nu}_L, \tilde{t}_1, \tilde{Z}_1, \tilde{Z}_2, \tilde{W}_1$ and h^0 are set only by experiment. The lower limits for the remaining particles lie typically in the region 100–200 GeV. Specifically one finds that the mass of the charged Higgs is greater than 100 GeV.

(2) Here one imposes constraints (i)–(v) and the model corresponding to the minimal supergravity grand unification. The characteristic spectrum assuming $M_{H_3} \leq 10M_G$ (and hence from Eq. (3.24), $B \leq 1000 \text{ GeV}^{-1}$) is given in Fig. 2 when one integrates over the full parameter space of the model. An interesting feature of the model is that because of proton stability requirement $m_0 > m_{\tilde{g}}$, and since $m_{\tilde{g}}$ has an experimental lower bound of $\approx$ 150 Gev, it implies that the heavier Higgs $H^0, A, H^\pm$ cannot be too light. In fact the detailed analysis shows that the $H^0, A, H^\pm$ masses are always larger than 200 Gev. In this context we may recall that there is a so-called "hole" in the Higgs mass range of 100–200 GeV $(5 \leq \tan \beta \leq 20)$ which cannot be probed at LEP and LHC[37)]. In the context of supergravity grand unification discused here, such a "hole" is excluded. The light Higgs mass obeys the limit $m_{h^0} \leq 130$ GeV (see Figs. 3 and 4). As for the case of H^0, A and $H^\pm$ masses, the lower limits of the sleptons and squark masses (except for the stop1 mass) and also the lower limits of $\tilde{Z}_3, \tilde{Z}_4$ and $\tilde{W}_2$ masses are pushed up above 200 GeV. These results can be easily understood from the proton stability constraint which requires $m_0 > m_{\tilde{g}}$, and leads to larger lower limits for the slepton and squark masses. Also $m_0 > m_{\tilde{g}}$ implies a larger lower limit on $|\mu|$ in radiative electroweak breaking and thus leads to larger lower limits on $\tilde{Z}_3, \tilde{Z}_4$ and $\tilde{W}_2$ masses.

(3) Supergravity Unification with Dark Matter Constraints: here we impose the constraints (i)–(iv) and (vi). In this case one finds an upper limit on the gluino mass of $m_{\tilde{g}} \lesssim 800$ GeV.

Also one finds an upper limit on the lighter chargino mass, $M_{\tilde{W}_1} \lesssim 250$ GeV. The mass spectrum with integration over the full parameter space under the constraints (i)–(iv) and (vi) is exhibited in Fig. 5. The lower limits on masses of the three generation sleptons and squarks are typically similar, though somewhat larger, as for case (1).

(4) Supergravity Unification with Proton Stability and Relic Density: for this case the full set of constraints (i)–(vi) are imposed. Here we find an upper limit on the gluino mass of $m_{\tilde{g}} \leq 400$ GeV, and an upper limit on the chargino mass of $M_{\tilde{W}_1} \leq 120$ GeV. The mass spectrum after integration over the full parameter space under the constraints (ii)–(vi) is shown in Fig. 6. Here the lower limits on masses of the three generation of sleptons and squarks (except stop 1) and on masses of $\tilde{Z}_3, \tilde{Z}_4, \tilde{W}_2$, and of $H^0, A, H^\pm$ Higgs are substantially higher than those for cases (1) and (3), and are similar to those for case (2). An interesting result of the analysis is that the mass of the lightest chargino is typically smaller than the mass of the lightest higgs, i.e., $m_{\tilde{W}_1} \leq m_{h^0}$.

VI. Conclusion

SUSY spectrum is discussed within the framework of supergravity unified theories. There are 32 SUSY particles in these theories whose masses can be predicted in terms of 4 parameters. Thus there are 28 predictions some of which can be translated also in terms of sum rules[38]. It is found that the SUSY spectrum exhibits certain scaling properties over much of the parameter space of the theory. Computation of the SUSY spectrum is carried out for four different scenarios: supergravity grand unification without proton decay, supergravity grand unification including proton stability, supergravity unification with neutralino relic density constraint but without proton stability constraint, and supergravity unification with neutralino relic density and proton stability constraints. The analysis of supergravity grand unification with proton stability constraint shows that the so-called "hole" in the CP odd Higgs mass between 100–200 GeV which cannot be explored experimentally at LEP2 and LHC, is eliminated.

One also finds some interesting features in the spectrum when the dark matter constraint is included in the analysis. One of these is the observation that $m_{\tilde{g}} \lesssim 800$ GeV, and $m_{\tilde{W}_1} \lesssim 250$ GeV over the allowed region of the parameter space. With inclusion of both p-stability and dark matter constraints one finds the remarkable result that $m_{\tilde{W}_1} \lesssim 120$ GeV and $m_{\tilde{W}_1} \lesssim m_{h^0}$. Also for all the four scenarios we find $m_{h^0} \lesssim 130$ GeV. It should be of interest to pursue signals of supersymmetry in this context using, for example, the trileptonic signal in off-shell W decays[39].

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Figure Captions

Fig. 1: Mass ranges of the SUSY mass spectrum in radiative electroweak symmetry breaking for MSSM under the natural constraint $m_0, m_{\tilde{g}} \lesssim 1$ TeV, for the case $m_t = 160$ GeV and $\mu < 0$. The particles are labelled top to bottom as follows: $\tilde{e}_L, \tilde{e}_R, \tilde{\nu}, \tilde{u}_L, \tilde{u}_R, \tilde{d}_L, \tilde{d}_R, \tilde{b}_L, \tilde{t}_1, \tilde{t}_2,$ $\tilde{Z}_1, \tilde{Z}_2, \tilde{Z}_3, \tilde{Z}_4, \tilde{W}_1, \tilde{W}_2, \tilde{g}, h^0, H^0, A$ and $H^{\pm}$.

Fig. 2: Mass ranges of the SUSY masses in supergravity GUTs including constraint of p-stability. Particles are labelled as in Fig. 1. The parameters are also as in Fig. 1.

Fig. 3: Light Higgs mass as a function of the dressing loop function for $m_t = 160$, and $\mu < 0$ when all other parameters are integrated out as in Fig. 2.

Fig. 4: Same as Fig. 3 except $\mu > 0$.

Fig. 5: Same as Fig. 1 except including dark matter constraint.

Fig. 6: Same as Fig. 5 except also including proton stability constraint. The particles are labelled top to bottm as follows: $\tilde{e}_L, \tilde{e}_R, \tilde{\nu}, \tilde{u}_L, \tilde{u}_R, \tilde{d}_L, \tilde{d}_R, \tilde{b}_L, \tilde{t}_1, \tilde{t}_2, \tilde{Z}_1, \tilde{Z}_2, \tilde{Z}_3/\tilde{Z}_4, \tilde{W}_1, \tilde{W}_2, \tilde{g}, h^0, H^0, A$ and $H^{\pm}$.

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