

CERN-TH.7269/94
ENSLAPP-A-474/94
CTP-TAMU-26/94
ACT-08/94

Some Physical Aspects of Liouville String Dynamics

John Ellis^{a,◇}, N.E. Mavromatos^{b,◇} and D.V. Nanopoulos^{a,c}

Abstract

We discuss some physical aspects of our Liouville approach to non-critical strings, including the emergence of a microscopic arrow of time, effective field theories as classical “pointer” states in theory space, *CPT* violation and the possible apparent non-conservation of angular momentum. We also review the application of a phenomenological parametrization of this formalism to the neutral kaon system.

^a Theory Division, CERN, CH-1211, Geneva, Switzerland,

^b Theoretical Physics Laboratory, ENSLAPP, Chemin de Bellevue, Annecy-le-Vieux, France, and

On leave from S.E.R.C. Advanced Fellowship, University of Oxford, Dept. of Physics (Theoretical Physics), 1 Keble Road, Oxford OX1 3NP, United Kingdom,

^c Center for Theoretical Physics, Dept. of Physics, Texas A & M University, College Station, TX 77843-4242, USA and Astroparticle Physics Group, Houston Advanced Research Center (HARC), The Mitchell Campus, Woodlands, TX 77381, USA.

[◇] **Invited contributions** to *the First International Conference on ‘Phenomenology of Unification from Present to Future’*, Rome, ‘La Sapienza’ University, March 22-26 1994

CERN-TH.7269/94
ENSLAPP-A-474/94
CTP-TAMU-26/94
ACT-08/94
May 1994

1 Introduction and Summary

One of the most profound issues in microphysics is a consistent formulation of gravity. The only candidate we have for resolving this issue is string theory, which is known [1] to be free of the “trivial” perturbative divergences that beset quantum calculations in a fixed smooth space-time background. Potentially far more profound problems arise when one considers curved backgrounds with horizons or singularities. Semiclassical calculations in such backgrounds indicate that a pure quantum-mechanical description cannot be maintained, as reflected in the thermodynamic description of macroscopic black holes, with non-zero temperature and entropy [2].

The question arises whether such a mixed quantum description is also necessary at the microscopic level, once quantum fluctuations in the space-time background and back-reaction of particles on the metric are taken into account. Hawking [3] has argued that asymptotic scattering must be described in terms of a linear operator \mathcal{S} relating “in” and “out” density matrices :

$$\rho_{out} = \mathcal{S}\rho_{in} \tag{1}$$

that does not factorize as a product of S and S^\dagger matrix elements. It has been argued [4] that, if this is the case, there should be a corresponding modification of the quantum Liouville equation describing the time-evolution of the density matrix:

$$\dot{\rho} = i[\rho, H] + \mathcal{S}H\rho \tag{2}$$

This would cause pure states to evolve into mixed states, yielding a loss of coherence associated with the loss of information accross microscopic event horizons, that would be enhanced for macroscopic systems [5].

Ordinary local quantum field theory is an incomplete guide to these issues, but string theory appears to resolve them. The scattering of particles off a black hole in string theory is described by a well-defined S matrix, which reflects the existence of an infinite set of local symmetries (and associated conserved charges) that interrelate (and characterize) different string states in the presence of a black hole, as long as quantum fluctuations in the space-time background and back-reaction of particles on the metric are ignored. However, we have argued [6, 7] that non-trivial modifications (1) of the effective field theory description of scattering and (2) of the quantum Liouville equation appear once these effects are taken into account. We describe such effects using non-critical string theory [8], with time introduced as a renormalization group flow variable [6], associated with a covariant world-sheet scale introduced via the Liouville field [9]. We do not review here the basics of this approach, which are described elsewhere [7, 10, 11]. However, we do discuss some physical aspects of this approach that are particularly relevant to the focus of this meeting.

One is a possible microscopic arrow of time [6, 7]. As we discuss in sections 2 and 3, string theory satisfies the general condition for the existence of an irreversible “ageing” variable, as a corollary of the existence of the Zamolodchikov metric [12] in the space of couplings of two-dimensional field theories on the world sheet. The arrow of time emerges as a result of the stringy symmetry-induced couplings of light particles to massive, non-propagating solitonic states of the string in a black-hole background [13]. In this respect, we see some similarity with ideas advocated by Penrose in the context of local field theories in highly-curved space-times [14].

A second issue is the emergence of semiclassical “pointer” states which describe specific low-energy effective quantum field theories obtained from string theory. As we discuss in section 4, these also appear as a result of the inevitable couplings to unobservable non-propagating solitonic string states.

Then we discuss the violation of CPT [15] and other conservation laws in section 5. CPT is expected to be violated in any theory, such as ours, which allows pure states to evolve into mixed states [16]. As we have discussed elsewhere, energy [15] and probability [6] are conserved in our string approach to density matrix mechanics. However, the renormalizability of the theory, which guarantees energy conservation, does not guarantee the conservation of angular momentum. Unlike string contributions to the increase in entropy, which cannot cancel, the apparent non-conservation of angular momentum may vanish in some backgrounds, though not in one cosmological background that we study.

Finally, in section 6 we review the formalism [4, 15, 17] for describing phenomenologically possible modifications of quantum mechanics and violations of CPT in the neutral kaon system, which is also discussed here by Huet [18, 19].

2 A Primer in Non-Equilibrium Quantum Statistical Mechanics

One of the open problems of any local field theory is how to incorporate an irreversible time variable. In the way formulated so far, at a ‘microscopic’ or ‘fundamental constituents of matter’ level, time reversal symmetry is unbroken. The arrow of time comes later, when one makes a ‘reduction’ of the degrees of freedom in order to describe the observable world. In cosmology, for instance, the arrow of time is believed to be induced by ‘integrating out’- in a path-integral sense - unobserved states hidden behind the ‘particle horizons’. In a similar spirit, in thermodynamics the arrow of time, or the ‘second law’ as it is commonly called, is induced by the ‘open-ness’ of the subsystem under consideration. Boltzmann’s second law of

entropy increase implies the existence of a Lyapounov function (entropy) for the subsystem, which increases with time, whilst the entropy of the total system remains constant. In the modern formulation of Boltzmann's law [2], where entropy increase is associated with 'information leakage' from the open subsystem to the 'environment', this entropy increase implies an irreversible arrow of 'time' [16].

From the above discussion it becomes evident that the concept of an irreversible time variable in statistical mechanics is different from that of time in Einstein's General Relativity Theory, where it is a coordinate of the space-time manifold that is completely reversible. Misra and Prigogine [20] have discussed this issue formally from a quantum statistical mechanics point of view by introducing the concept of 'measurable entities' into this framework. They assumed the existence of a system whose distribution function in phase-space $\rho(p, q, t)$ evolves reversibly under the flow of a time co-ordinate, t . Such a flow is described by a unitary transformation U_t :

$$\rho(p, q, t) = U_t \rho(p, q, 0) U_t^\dagger \quad (3)$$

Reversible time translations are generated by the Hamiltonian \mathcal{H} of the system, and the time evolution equation for ρ has the familiar Liouville form

$$\partial_t \rho = [\rho, L]_{PB} \quad (4)$$

where $L = e^{-i\mathcal{H}t}$ is the Liouvillean, and $[\cdot]_{PB}$ denotes Poisson brackets or commutators multiplied by a factor i/\hbar , depending whether one considers classical or quantum systems.

To introduce a time arrow in the above framework, Misra and Prigogine [20] assume that the physical states, i.e. the ones that can be measured in an experiment, have a phase-space distribution function $\tilde{\rho}$ which is related to ρ via a *non-unitary* transformation : Λ

$$\tilde{\rho} = \Lambda \rho \quad (5)$$

The transformation Λ may be an invertible map or a projection P . The evolution of $\tilde{\rho}$ is governed by a strongly irreversible Markov process (s.i.m.p.), defined by the adjoint W_t^* of a positive semigroup W_t defined for $t \geq 0$:

$$\tilde{\rho}(t) = W_t^* \tilde{\rho}(0) \quad (6)$$

with W_t^* satisfying the intertwining condition [20]

$$\Lambda^{-1} U_t \Lambda = W_t^* \quad \text{or} \quad P U_t P = W_t^* \quad (7)$$

and

$$\partial_t \tilde{\rho} = \Phi(L) \tilde{\rho} \quad : \quad \Phi(L) = \Lambda L \Lambda^{-1} \text{ or } P L P \quad (8)$$

It can be shown that (8) implies the existence of a Lyapounov function, if and only if the following condition of star hermiticity is satisfied [20]

$$i\Phi(L) = (i\Phi(L))^* \equiv \Phi^\dagger(-L) \quad (9)$$

A particular case of an invertible Λ transformation is provided under certain conditions by a non-conservative force in the statistical mechanics of open systems [21]. If F_i is such a non-conservative force for a system whose hamiltonian \mathcal{H} is such that $\partial\mathcal{H}/\partial p_i \neq 0$ as in a model of dissipation, then the evolution of the density matrix is described by an equation of the form [22]

$$\partial_t \tilde{\rho} = -\{\tilde{\rho}, \mathcal{H}\}_{PB} + \frac{\partial \mathcal{H}}{\partial p_i} \mathcal{G}_{ij} \frac{\partial \tilde{\rho}}{\partial p_j} \quad (10)$$

with $\mathcal{G}_{ij}[F_i]$ a matrix depending on the non-conservative forces. It has been shown in ref. [21] that (10) satisfies the star hermiticity condition (9) if and only if

$$\mathcal{G}_{ij} = \mathcal{G}_{ji} \quad : \quad \mathcal{G}_{ij} \in \mathbf{R} \quad (11)$$

In this way the theory of such an open statistical system is connected to the Λ -transformation theory of reduction to a physical subspace.

The issue of demonstrating that time has an arrow is a bit more subtle and requires more careful consideration. In the Misra-Prigogine [20] approach, the starting point is a unitary transformation U_t , which governs the time t flow of the system ρ . This implies straightforwardly that if one has a s.i.m.p. in the positive time direction $t \geq 0$, described by a transformation Λ_+ , then one will also have a s.i.m.p. in the $t < 0$ direction governed by a transformation Λ_- . The time arrow is introduced into the system only if the dynamics implies that [20] $\Lambda_+ \neq \Lambda_-$.

If the above conditions are met, the construction of an internal time or age operator, T , is possible according to a theorem of Misra and Prigogine [20]. The operator T is defined in terms of its eigenvalues λ as follows:

$$U_t T U_t^\dagger = I + \lambda T \quad (12)$$

and its action on physical states leads to a non-decreasing sequence of the eigenvalues λ .

What we shall argue in this talk is that a similar situation occurs in non-critical string theory [8] when microscopic quantum fluctuations of space-time into non-trivial, highly-curved (singular) backgrounds are taken into account [6] via non-critical string theory. The existence of such singularities in the structure of space-time implies the existence of non-decoupling, delocalized topological string modes which are non-propagating and can be thought of as remnants of a highly-symmetric (topological) phase of string theory [23, 24]. Such modes cannot be measured in local scattering experiments conducted by a conventional observer, and therefore define a sort of ‘environment’ for the propagating string modes. Time is then introduced [6] into this framework as a dynamical local renormalization group scale on the world-sheet [25, 26] (Liouville field), and its arrow is established as a result of the

unitarity of the effective string σ -model describing string propagation in the singular background under consideration [27]. We shall be very brief in this description due to lack of space and time (!). The interested reader may find detailed presentation of this work in ref. [10] and references therein.

3 The Two-dimensional Stringy Black Hole and the Arrow of Time in String Theory

In the first-quantized σ -model string formalism the propagation of string particles in non-trivial backgrounds is described by deforming the σ -model Lagrangian by the appropriate vertex operators. The couplings of such operators are the background fields of the target space-time. Conformal invariance on the world sheet implies the vanishing of the pertinent renormalization-group β functions, which are interpreted as equations of motion for the backgrounds. In this way the dynamics of any consistent background is described by marginal (in a renormalization-group sense) deformations of the world-sheet action, that preserve by construction the conformal symmetry. The construction of the appropriate vertex operators for the various background fields relies on the completeness of the set of operators with zero anomalous dimensions, or $(1, 1)$ operators. As a result of this completeness property, higher-level operators appear in the operator product expansion (OPE) of lower-level operators. For instance, starting from the lowest-level bosonic string states, the so-called tachyons, one finds the following OPE between the respective vertex operators [28]

$$V_T(z, \bar{z}) \otimes V_T(z', \bar{z}') \propto \frac{1}{|z - z'|^4} V_T\left(\frac{z + z'}{2}, \frac{\bar{z} + \bar{z}'}{2}\right) + \sum_{N \geq 1} V_N(\text{less singular}) \quad (13)$$

where the sum is over higher-level string states $N \geq 1$, and V_N denote the associated vertex operators (the corresponding less-singular world-sheet factors are not exhibited for convenience).

The physical spectrum of the theory is defined in flat space time, and is assumed background-independent, for general covariance reasons. Thus the concept of higher-level string states is defined unambiguously in any background. These remarks will be crucial for our subsequent discussion of the spectrum of the two-dimensional stringy black hole.

If the theory is formulated on a world sheet with fixed topology, e.g. a sphere, then the corresponding background field theory will be ‘classical’ in target space. Quantum background corrections are described by higher-genus effects on the world sheet [29]. However, such effects can be effectively projected onto the world-sheet sphere [30], by including corrections to the conformal invariance conditions which supplement those obtained by the perturbative renormalization-group treatment on fixed-genus Riemann surfaces.

In this context, target-space quantum gravity effects, such as black hole creation and evaporation, Hawking radiation, etc., can be studied by conformal field theory methods. The only explicit example which has been solved exactly from a conformal field theory point of view is the two-dimensional (target-space) black hole of ref. [27]. The model is described by a gauged Wess-Zumino conformal field theory on a non-compact group $SL(2, R)/U(1)$. This theory possesses one propagating degree of freedom, a massless scalar field called a ‘tachyon’ for historic reasons, and an infinity of non-propagating higher-level string modes with discrete energies and momenta [31]. The target-space metric for this two-dimensional string theory assumes the following form in the Minkowski $SL(2, R)/O(1, 1)$ case [27]:

$$ds^2 = dr^2 - \tanh^2 r dt^2 \quad (14)$$

where r is space-like and t is time-like. It has been shown in ref. [6, 11] that, as a result of the static nature of the background (14), one may consider it as a fixed point of a renormalization group transformation on the world sheet, with t interpreted as a local renormalization group scale. This formalism is known as Liouville string dynamics. The central charge of the model is $c = \frac{3k}{k-2} - 1$, with k the Wess-Zumino level parameter. In this formalism, due to the $c = 26$ critical string character in the case $k = 9/4$, one obtains dynamically a Minkowski signature [8, 9, 32, 33] in target space-time, and the Euclidean $SL(2, R)/U(1)$ counterpart can be obtained by analytic continuation [11].

An important element in the stringy black hole (14) is the existence of a dilaton field, whose presence is necessitated by the non-critical dimensionality of the target space-time [8]. From our point of view, the existence of a non-trivial dilaton is important because it implies that a two-dimensional black hole can have a non-trivial entropy S despite the absence of an horizon area [34]

$$S = e^{\Phi_H} \quad (15)$$

where Φ_H is the value of the dilaton field at the horizon point [27]. This helps justify the use of a two-dimensional toy model as a prototype for realistic string computation relevant to the definition of time in string theory.

In the flat target space-time case of two-dimensional strings, known as the $c = 1$ string model, the tachyon deformation, which coincides with the world-sheet cosmological constant operator, is exactly marginal, i.e. its renormalization group β -function vanishes identically. This implies the vanishing of the respective OPE coefficients in (13). However, this is not so in the black hole case. From the structure of the pertinent OPE of two tachyon deformations [35], one can immediately conclude that the exactly marginal deformation that turns on a non-trivial tachyon background turns on an infinity of higher-level (topological) modes as well. In the two-dimensional string theory one may use $SL(2, R)$ -isospin quantum numbers (j, m, \bar{m}) , to classify the states, where m (\bar{m}) are third components of the isospin in

left (right) sectors of the closed string, characterized by a common $SL(2, R)$ isospin j . In this framework, the exactly marginal deformation of a Euclidean black hole¹ is represented [35] by an infinite sum of vertex operators $W_{j,m,\bar{m}}$

$$L_0^1 \bar{L}_0^1 = \mathcal{F}_{-\frac{1}{2},0,0}^{c,c} + W_{-1,0,0} - \bar{W}_{-1,0,0} + \dots \quad (16)$$

where

$$\mathcal{F}_{-\frac{1}{2},0,0}^{c,c}(r) = \frac{1}{\cosh r} F\left(\frac{1}{2}, \frac{1}{2}; 1, \tanh^2 r\right) \quad (17)$$

with

$$\begin{aligned} F\left(\frac{1}{2}, \frac{1}{2}, 1; \tanh^2 r\right) &\simeq \frac{1}{\Gamma^2\left(\frac{1}{2}\right)} \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n \left(\frac{1}{2}\right)_n}{(n!)^2} [2\psi(n+1) - 2\psi\left(n + \frac{1}{2}\right) + \\ &+ \ln(1 + |w|^2)] (\sqrt{1 + |w|^2})^{-n} \end{aligned} \quad (18)$$

is the tachyon operator, transforming according to a continuous $SL(2, R)$ representation, denoted by the superscript (c, c) [35].

In a low-energy world, all measurements are made by local scattering experiments employing only the $\mathcal{F}_{-\frac{1}{2},0,0}^{c,c}$ deformation. The global modes $W_{-1,0,0}, \dots$ can be measured by Aharonov-Bohm experiments [37] in higher-dimensional theories (in which case the two-dimensional string prototype considered above is viewed as describing appropriate spherically-symmetric s -wave field configurations [38]). Even in that case, the infinite number of the associated quantum numbers makes a complete measurement impossible in practice, and hence the mere existence of these topological modes implies a ‘quasiparticle’ [22] structure for the propagating string modes, in the sense of modifying their energy-momentum dispersion relations. This is a generic feature of a non-critical string [8]. Any deviation from conformal invariance in the sub-system of the propagating string modes will result in Liouville-dependent dilaton terms. Such terms are responsible for the existence of screening charges in Liouville correlation functions. Their presence affects the energy-momentum dispersion relation of a propagating string mode. For instance, for the tachyon mode one has, in the simplest case of a non-critical bosonic string, of matter central charge C_m , propagating in a linear dilaton background, [8]

$$-\frac{1}{2}(E^2 + Q^2 - \mathbf{p}^2) = 1 \quad ; \quad Q^2 = C_m - 25 \quad (19)$$

In our stringy space-time foam case the modification is much more complicated, due to the non-trivial structure of the background target space-time.

¹At present, exactly-marginal deformations of Wess-Zumino black hole models have been constructed only for the Euclidean case. There are technical subtleties in the Minkowski case, which is assumed to be obtained by analytic continuation. See ref. [36] for a discussion of this procedure.

As we have discussed above, the coupling between propagating and non-propagating string modes is a consequence of the world-sheet conformal symmetry. There is a deeper reason for this, which appears to be generic in string theory. This is provided by the infinite-dimensional gauge symmetries that characterize strings and are responsible for level mixing. Such symmetries can be elevated from the world-sheet to target space-time [39, 40]. Techniques for such an elevation have been described in the literature [41], where we refer the interested reader. For our purposes, we note that in the two-dimensional Wess-Zumino strings the world-sheet ancestors of the target-space gauge symmetries are W_∞ Lie-algebraic structures pertaining to spin 2 and higher world-sheet operators [13, 42]. The sub-algebra generated by the spin 2 (stress-tensor) operator is the conformal algebra, which is part of W_∞ . It is a straightforward computation to show that the world-sheet W_∞ charges do not commute with the string level number operator [37]. Hence the ‘innocent’ requirement of conformal invariance in the deformation (16), which mixes the various string levels, finds a natural explanation within the more general context of stringy gauge symmetries that have no analogue in any local field theory. In higher-dimensional string theories, such symmetries are spontaneously broken by the graviton background field [43], as one expects for a mass-level mixing symmetry in the presence of a non-zero mass gap characterizing a propagating massive mode. However, in the case of a topological mode the notion of a mass gap is not defined in the same way. Hence symmetries mixing propagating with topological modes are not necessarily broken by non-trivial backgrounds. Moreover, in two space-time dimensions spontaneous breaking cannot occur [44], so in the case of the two-dimensional black holes the W_∞ symmetries should be considered as unbroken, provided one takes the infinite set of higher-level discrete modes into account. Their presence is essential for the maintenance of quantum coherence [13], as a result of the phase-space area preserving nature of such algebraic structures [45]. A low-energy measurement process effectively breaks such symmetries, since it effectively integrates out the discrete modes in (16). This implies in turn the non-conservation of the phase-space volume for the propagating modes under target-time evolution, and hence a modification of the quantum mechanics for the low-energy modes, which thereby behave as an ‘open’ system [20, 22].

From this point of view, one can define a reduction to a physically-measurable subspace by a sort of Λ transformation of the type discussed by Misra and Prigogine [20] as follows [10] :

$$\tilde{\rho}(X, P, g^i, p_i, t) \equiv e^{F[g^i, p_i, t] - \mathcal{H}[g^i, p_i, t]} \quad (20)$$

where t is introduced as a covariant renormalization-group scale via a Liouville field, X and P denote the phase space of the string particle, and g^i, p_i the background field theory phase-space over which the string propagation takes place. In equation (20), \mathcal{H} is the corresponding 1st-quantized string Hamiltonian [29] and F is the σ -model effective action that generates string amplitudes for the propagating single-particle string modes g^i . In the two-dimensional case these modes are just the tachyon

deformations $\mathcal{F}_{-\frac{1}{2},0,0}^{c,c}$.

On the other hand, the ‘full’ string theory, including the global modes, which corresponds to the unitary initial system ρ of Misra and Prigogine [20], is defined through a σ -model deformed by the exactly-marginal deformation (16):

$$\rho(X, P, g^i, p_i, t) \equiv e^{F[L_0^1 \bar{L}_0^1, t] - \mathcal{H}[L_0^1 \bar{L}_0^1, t]} \quad (21)$$

The (abstract) analogue of a Λ transformation in this case is essentially the process of a string path integration of the topological modes in (21). Formally such a procedure is still not rigorously known, given the absence of a satisfactory string field theory so far. In practice however, the first quantized approach, based on the σ -model description (20), proves sufficient. Using the previously-mentioned theorem of Misra and Prigogine [20], one expects to be able to define an internal time, that flows irreversibly under the dynamics of the subsystem of the low-energy string modes. Indeed, in our case we can see how to define an internal time by looking at the important difference between (20) and (21). Equation (21) is mathematically consistent as it stands, given the exactly-marginal nature of the deformation (16). In this case, the local renormalization group scale decouples from the background σ -model couplings, which thus become static [27, 11], and it only appears as a coordinate of the target space of the σ -model. In contrast, equation (20) requires renormalization, since the tachyon deformation (17) breaks conformal symmetry, and hence induces ultraviolet divergences in the world-sheet model. Consistency is restored by appropriate Liouville dressing using the formalism of curved-space renormalization in field theory [25, 26]. In our case, to leading order in the inverse Wess-Zumino level parameter $\frac{1}{k}$, and hence to first order in the Regge slope α' , only the OPE coefficients in the β function are important, and hence the appropriate Liouville dressing is given by [11]

$$\int d^2 z g V_g(r) \rightarrow \int d^2 z g e^{\alpha_g \phi} V_g(r) \equiv \int d^2 z g V_g(r) - \int d^2 z g^2 C_{ggg} V_g(r) \phi + \dots \quad (22)$$

where r is a σ -model spatial coordinate, and g is a generic deformation coupling (for simplicity we have assumed a single deformation). It is conjectured that a similar consistent result is obtained for finite k , and hence to all orders in α' .

The dressing (22) corresponds to an effective anomalous dimension for the coupling of the tachyon deformation, which depends on the zero mode of the local renormalization scale/Liouville field ϕ_0 . Technically, one employs the so-called ‘fixed-area constraint’ [32] in correlation functions by inserting the identity

$$\int dA \delta(A - \int d^2 z e^\phi \sqrt{\hat{\gamma}}) = 1 \quad (23)$$

where A is a covariant world-sheet area, and $\hat{\gamma}$ denotes a fiducial world-sheet metric. On the one hand, this formalism allows an integration over the Liouville mode ϕ in

the σ -model path-integral, as appropriate for its target-time interpretation, whilst on the other it preserves an explicit global renormalization-scale A dependence in the integrand of the A -integral corresponding to the ‘physically’ measurable part of the correlation functions, which thus become target-time dependent. The global scale corresponds to the zero mode of the Liouville field, and this will always be understood in the following. It will be essential for determining the physical sense of the time flow to be discussed later. It should be remarked that in this formalism the renormalized deformations of the σ -model Lagrangian are conveniently written as

$$\int d^2 z g^i [X(z, \bar{z}), \phi(z, \bar{z})] V_i [X(z, \bar{z}), \phi(z, \bar{z})] = \int d^2 z \int d^D y g^i(y) \delta^{(D-1)}(\underline{y} - \underline{X}(z, \bar{z})) \delta^{(0)}(y^0 - \phi(z, \bar{z})) V_i [X, \phi] \quad (24)$$

where D is the dimension of target space-time, including the Liouville mode as a coordinate, and appropriate normal ordering is understood. In this way summation over the index i can include the integration over space-time coordinates y . Equation (24) encapsulates the relationship between the coordinate time y^0 and the irreversible Liouville mode. Thus, the above procedure amounts to the advertized ‘temporal’ dependence of the σ -model background couplings, where the time appears as an ‘external’ irreversible evolution parameter in target space-time and is at the same time related to a non-trivial quantum world-sheet field (the Liouville mode) in the σ -model. In the two-dimensional black-hole example this effect describes the collective effects of massive global string modes (16).

The induced arrow of time follows nicely from the properties of correlation functions for tachyon (matter) deformations in the Liouville-dressed theory. The crucial assumption is that the effective theory after integrating out massive global string modes is unitary on the world sheet. This implies that the induced renormalization group flow as a result of the dressing (22) will be irreversible, as a consequence of the Zamolodchikov C-theorem [12]. The associated Lyapounov function is constructed out of components of the world-sheet stress tensor

$$C = 2z^4 \langle T(\sigma)T(0) \rangle - 3z^3 \bar{z} \langle T_{z\bar{z}}(\sigma)T(0) \rangle - 6z^2 \bar{z}^2 \langle T_{z\bar{z}}(\sigma)T_{z\bar{z}}(0) \rangle \quad (25)$$

where $T \equiv T_{zz}$, $T_{z\bar{z}}$ is the trace of the stress tensor, and $\langle \dots \rangle$ denotes a σ -model v.e.v. with respect to the Liouville renormalized deformation $[\mathcal{F}_{-\frac{1}{2},0,0}^{c,c}]$ in the sense of equation (22). The C-theorem states that the function C flows irreversibly under the flow of the scale t , the zero mode of the Liouville field :

$$\partial_t C = -12 \langle T_{z\bar{z}}(\sigma)T_{z\bar{z}}(0) \rangle = -12\beta^i \langle V_i V_j \rangle \beta^j \leq 0 \quad (26)$$

where the metric in coupling constant space

$$G_{ij} \equiv 2|z|^4 \langle V_i(z, \bar{z})V_j(0,0) \rangle \quad (27)$$

is positive for unitary theories on the world-sheet.

A rather straightforward analysis yields the following string analogue of the evolution equation (10) [6]

$$\partial_t \tilde{\rho} = -\{\tilde{\rho}, H\}_{PB} + \beta^j G_{ij} \frac{\partial}{\partial p_i} \tilde{\rho} \quad (28)$$

where in the two-dimensional black-hole example the couplings g^i denote the propagating ‘tachyon’ modes. The key point to notice here is that this equation has the form (10),(11) which guarantees the existence of an ‘ageing’ operator in non-equilibrium quantum statistical mechanics, i.e., a microscopic arrow of time. This is a consequence of the symmetry and reality properties of the Zamolodchikov metric in coupling constant space for unitary world-sheet theories [12].

It may be useful to comment here on the precise meaning of the partial derivative with respect to the zero mode of the renormalization scale. As stated above, this denotes dependence on the covariant world-sheet area A , which occurs in the fixed world-sheet area expectation values $\langle \dots \rangle$. The integration over the Liouville field in the fixed-area-constraint formalism (23) implies that C is a target space-time action functional. For stringy σ -models it is known [46] that any form of the C -theorem which is local in target space will yield anomalous-dimension-like terms on the right-hand-side of (26), which are due to world-sheet infrared divergences. For graviton G_{MN} and dilaton Φ backgrounds they assume the form

$$\gamma(G, \Phi) = -\frac{3}{16\pi^2} \frac{\alpha'}{2} \nabla^2 (\beta_\Phi - \beta_M^M + \dots) \quad (29)$$

where the \dots indicate higher-order α' -corrections. Such anomalous dimension terms spoil the monotonicity properties of the local C -function, even for unitary theories. Fortunately, due to the fact that they are total target space-time derivatives, they are absent[47] in the space-time integrated form of the C -theorem. Thus, the positivity of the action functional (25) is guaranteed for unitary theories, in which case the action C counts correctly the physical degrees of freedom of the system. Moreover, this integrated form guarantees the existence in the associated flow equations of second order derivatives with respect to the renormalization-group scale, which arise from the usual functional derivatives of a string effective action with respect to the background fields/couplings [6, 10]. However, the existence of (dissipative) friction terms, as a result of the non-critical nature of the underlying conformal field-theory [6], implies the irreversibility of such a flow. This property applies in a fixed-genus (sphere) computation. In higher genera there will, in general, be mixing of standard matter states with ghosts circulating along the handles of the Riemann surface. However, measurement in our effective theory cannot detect such effects directly. Following the analysis of ref. [30], it is possible to project higher-genus effects onto the lowest-genus Riemann surface (sphere), in such a way that the string loop effects appear as extra renormalization counterterms on the world-sheet theory, not included in a perturbative fixed-genus renormalization. Thus, in such a formalism the lowest-genus effective theory can always be assumed unitary, and the effective C -theorem is applicable [48].

When one resums over higher genera, the classical couplings g^i become ‘quantum operators’ \hat{g}^i , and the associated Poisson brackets become commutators. The quantum version of (28) is [6]

$$\partial_t \tilde{\rho} = i[\tilde{\rho}, \mathcal{H}] + i\beta^i G_{ij} [g^j, \tilde{\rho}] \quad (30)$$

From equation (26) one can construct an associated statistical entropy which is essentially the exponential of C (25). It has been shown [6] that this quantity can be expressed in terms of the reduced density matrix $\tilde{\rho}$ of the low energy subsystem according to Boltzmann’s prescription

$$S = e^{-C} = -Tr \tilde{\rho} \ln \tilde{\rho} \quad (31)$$

which for unitary theories varies monotonically along the renormalization group flow

$$\dot{S} = \beta^i G_{ij} \beta^j S \geq 0 \quad (32)$$

We have argued in ref. [7, 11] that world-sheet instanton effects can be used to describe qualitatively both higher-genus and global mode effects in the Wess-Zumino black hole model. A rigorous treatment of this issue has not yet become available, and it probably has to wait for a satisfactory matrix model formalism [49] of black holes in string theory, where a summation over higher genera is performed exactly². The association of world-sheet instanton with higher-genus effects is, however, supported by a study of the $N = 2$ topological Wess-Zumino theory on a black-hole space-time background. Such models are believed [24] to describe correctly the topological phase of the two-dimensional string, which is also reflected in the black hole singularity itself [27, 51]. The connection with higher genera comes from the conjecture of Mukhi and Vafa [52] that a $c = 1$ string theory resummed over genera is expressed as a topological world-sheet Wess-Zumino model formulated on a $SL(2, R)/U(1)$ group manifold.

We are concerned next with the sense of the flow of the subsystem of the propagating string modes. In ref. [7] we have established a flow of time which is opposite to the conventional renormalization group flow. There is an unambiguous way of determining the correct sense of the flow, associated with the fact that, according to Zamolodchikov, there is an ‘thinning’ of physical degrees of freedom of the system along the renormalization-group flow. In unitary theories, where the original analysis of Zamolodchikov took place [12], the effective central charge counts correctly such degrees of freedom. This notion can probably be extended [53] even to non-unitary theories. In our case, as we shall discuss below, the ‘bounce’ picture of Liouville flow [54, 7] clearly provides a mathematically rigorous construction which selects the direction of flow along which there is a ‘thinning’ of degrees of freedom in

²For some attempts towards this direction see ref. [50], where a matrix model scenario for string propagation on a fixed black hole geometry is presented.

the observable world: close to the starting point of the flow, in the topological string phase, the delocalized modes are strongly coupled to the propagating string modes. On the other hand, at the end of the flow, where flat space-time is approached asymptotically, the topological modes decouple. The associated entropy production (31) in such a picture, is interpreted as pertaining to the amount of information carried by the ‘environment’ of the topological modes.

We now develop briefly the ‘bounce’ picture. Consider an N -point correlator of ‘tachyons’ in the above Liouville string. Its expression is [55]

$$\langle V_{i_1} \dots V_{i_N} \rangle_{\mu} \propto \left(\int dA e^{-A} A^{-s-1} \right) \langle V_{i_1} \dots V_{i_N} \rangle_{\mu=0} \quad (33)$$

where the subscript μ denotes world-sheet cosmological constant deformations, appropriately modified in the black hole where μ is related to the black hole mass [56]. The quantity s is a kinematic factor involving Liouville energies of the various operators. The quantity A is the covariant area of the world-sheet that sets a renormalization scale [53], and has been introduced via the fixed-area constraint (23). The A -integral is ultraviolet divergent if s is a positive integer, which is the case in a two-dimensional black hole where scattering of tachyons off the black hole results in an excitation of the latter to discrete string states [37]. The regularization of the integral can be made by analytic continuation, representing it by the contour integral depicted in fig. 1 [7, 10]. The contour of fig. 1 implies a ‘bounce’ [54, 7] of the world-sheet (c.f. figure 2) area at the infrared fixed point ($A \rightarrow \infty$) towards the ultraviolet one ($A \rightarrow 0$).

The bounce interpretation of the Liouville renormalization group is different from the ordinary representation describing transitions among string vacua. The bounce picture is in perfect agreement with the time reversal symmetry breaking Λ -transformation approach of Misra and Prigogine [20], if one identifies the two opposite directions of time in fig. 1 with the Λ_{\pm} branches. The bounce picture is supported by explicit instanton computations in the dilute-gas approximation on the world sheet, as discussed in [7, 11].

4 Quantum-Classical Correspondence within the Renormalization Group Framework

In the previous section we have developed a deterministic flow of time, which stems from working within an effectively fixed-genus σ -model. In this picture, higher genus effects are collectively and qualitatively represented by world-sheet instanton effects, that produce extra logarithmic scale dependences not appearing within the conventional fixed-genus renormalization group analysis. However, from a formal

point of view, summation over higher genera leads to a natural quantization of the target-space fields g^i , and one loses the concept of a classical point in coupling constant phase space that evolves deterministically. To include this feature we seek states in this quantum-mechanical system that evolve ‘almost reversibly’ in time, and therefore are the closest quantum counterparts to the classical points in the σ -model background phase space. Given that the degree of irreversibility of an open system is measured by the entropy production, the above states should correspond to minimum entropy production [57, 58]. Their time evolution follows classical phase-space trajectories, which in our case express the usual renormalization-group flow. In conventional quantum-mechanical treatments these states are called ‘pointer states’. For instance, in the case of the harmonic oscillator, the pointer states are identified with the conventional ‘coherent states’ [58]. The existence, as well as the nature, of pointer states depends on the form of the interaction of the open subsystem with the environment [59]. For instance, in a toy two-state spin system, it can be shown that oscillatory pointer states appear at weak coupling with the environment, while constant pointer states appear at very strong coupling. For intermediate couplings there are no pointer states, but only a noisy background [59].

It is the purpose of this section to address these issues for our system. As already mentioned, a difference from the ordinary quantum-mechanical case appears because the ‘phase space’ refers to background fields g^i , and their conjugate momenta p_i in the target space of the string, where the index i includes the target-space coordinates. As a preliminary to finding our pointer states we follow [58] and compute the linear entropy production:

$$\partial_t s^l \equiv \partial_t (\text{Tr} \tilde{\rho} - \text{Tr} \tilde{\rho}(\mathbf{g}, \mathbf{p}, t)^2) = -\partial_t \text{Tr} \tilde{\rho}^2 \quad (34)$$

in our framework. Using equation (30) it is straightforward to derive

$$\begin{aligned} \partial_t s^l &= i \text{Tr}([\beta^i G_{ij}, \tilde{\rho}] g^j) = \\ &2 \ i \ \text{Tr} \tilde{\rho}^2 g^i \frac{d}{dt} G_{ij} g^j - 2i \text{Tr} \tilde{\rho} g^i \frac{d}{dt} G_{ij} \tilde{\rho} g^j - 2i \text{Tr} \tilde{\rho} g^i [G_{ij}, \tilde{\rho}] \beta^j \end{aligned} \quad (35)$$

Using equation (20), we can express the last term in terms of the commutator $[G_{ij}, e^{-H}]$, which is non-zero as a result of the renormalization-group invariance of $\beta^i G_{ij} \beta^j$, and the fact that $\frac{d}{dt} \beta^i \neq 0$ in any scheme. Choosing a scheme such that [12] $G_{ij} = \delta_{ij} + O(g^2)$, we observe that, for the case of (1, 1) deformations that we are interested in, the last term is necessarily of order $O[g^6]$ and it is not dominant in the weak-field $g^i \ll 1$ approximation, which we assume for convenience. The remaining terms can then be time-integrated to yield

$$s^l(t) = i \int_{t_0}^t dt (\text{Tr} \tilde{\rho}^2 g^i \frac{d}{dt} G_{ij} g^j - i \text{Tr} \tilde{\rho} g^i \frac{d}{dt} G_{ij} \tilde{\rho} g^j) \quad (36)$$

Writing $i \frac{d}{dt} G_{ij}$ as $-[G_{ij}, H]$, and taking into account the fact that in the weak-field limit the renormalization of G_{ij} requires linear subtractions in the logarithmic scale

$t = \ln \mu$, one may make the replacement

$$[G_{ij}, H] \propto -\delta_{ij} + \dots \quad (37)$$

where the \dots indicate higher order corrections. Thus, the final result for the linear entropy produced in the time interval $2\Delta t$ is

$$s^l(2\Delta t) = \int_{t-\Delta t}^{t+\Delta t} dt (Tr \tilde{\rho}^2 g^i \delta_{ij} g^j - Tr \tilde{\rho} g^i \delta_{ij} Tr \tilde{\rho} g^j) (\Delta g)^2 \quad (38)$$

Were it not for the self-interaction Hamiltonian of the low-energy string-mode system, the natural candidates for the pointer states would be the position eigenstates in coupling-constant space, corresponding to background fields in the target space of the string. The non-triviality of the Hamiltonian modifies this result by introducing ‘momentum’ uncertainties in coupling constant space. To see this, we may rewrite the right-hand-side of (38) in the twin limit of weak field and weak coupling with the environment, as

$$s^l \simeq \int_{2\Delta t} dt (\langle g^i \delta_{ij} g^j \rangle - \langle g^i \rangle \delta_{ij} \langle g^j \rangle) = \int_{2\Delta t} dt \langle (g^i - \langle g^i \rangle)^2 \rangle \quad (39)$$

For the interaction Hamiltonian we use the matrix-model inverted harmonic oscillator approximate Hamiltonian [49] which is supposed to describe in a closed form the result of the resummation over world-sheet genera for a two-dimensional string,

$$H = \frac{1}{2}(p^2 - q^2) \quad (40)$$

where q, p are related by a canonical collective coordinate transformation to the matter tachyon field of the two-dimensional string theory and its conjugate momentum respectively [49]. From our point of view, this change of variables corresponds to a renormalization group choice, and hence our previous analysis applies. In particular, we can use the known solution of the inverted harmonic oscillator problem to write (39) in the form

$$s^l(2\Delta t) = \int_{2\Delta t} \langle \psi | [(q - \langle q \rangle) \cosh t + (p - \langle p \rangle) \sinh t]^2 | \psi \rangle \quad (41)$$

where $|\psi\rangle$ denote the states of the inverted harmonic oscillator. It is understood that any modification of the matrix model potential, e.g., of the form $-\frac{m^2}{q^2}$ which describes [50] tachyon propagation in the background of a string black hole of mass m , has been omitted here, since such terms have been integrated out by the measurement process which ‘sees’ only the propagating matter. The effect of the black hole is the entropy production due to the non-zero coupling with the topological modes of the string (of which one is the two-dimensional space-time graviton itself). In this specific model one can explicitly verify (37) as follows: as already commented, we can interpret the ‘coordinate’ q and the conjugate ‘momentum’ p as a specific

scheme choice. Unitarity of the σ model theory requires the following generic form for the metric G_{qq} :

$$G_{qq} = (\text{const})^2 + (\alpha)^2 q^2 + \dots \quad ; \quad \alpha \in \mathbf{R} \quad (42)$$

Using equation (40), the canonical commutation relation $[q, p] = i\hbar$, and the representation $p = -i\hbar \frac{\partial}{\partial q}$, it is straightforward to arrive at

$$[G_{qq}, H] = -\alpha^2 + O[q \frac{\partial}{\partial q}] \quad (43)$$

thus verifying equation (37).

We now remark that, for the inverted harmonic oscillator case, $2\Delta t$ can be taken to be the infinite interval $[-\Lambda, \Lambda]$, with $\Lambda \rightarrow \infty$. The result of the time integration then is proportional to

$$s^l \propto [(\Delta g)^2 + (\Delta p)^2] \quad (44)$$

which is our final result that can be compared with the conventional harmonic oscillator case [58]. The pointer states are found by minimizing the entropy s^l . Viewing the latter as a functional of $(\Delta g)^2$ and of the product $\Delta g^i \Delta p_i \geq \hbar/2$ (no sum over i), and minimizing (44) with respect to the Δg^i , we find that the states of minimum entropy are characterized by

$$\Delta g^i \Delta p_i = \frac{\hbar}{2} \quad (45)$$

which is similar to the result of ref. [58], showing that the minimum-entropy-producing initial states are minimum-uncertainty Gaussian wave-packets of the inverted harmonic oscillator that describes the dynamics of the (quantum) two-dimensional string.

In the case of the conventional harmonic oscillator, such Gaussian wave-packets coincide with the Wigner coherent states. However, in the case of the inverted harmonic oscillator, a thorough analysis [60] shows that Gaussian wave-packets are different from the Wigner coherent states. The latter are not square-integrable, though still localizable because they fall like q^{-1} for $q \rightarrow \infty$, and can be obtained by the action of the Weyl operators on energy eigenstates [60]. On the contrary, the Gaussian wave-packet assumes the usual form

$$\psi(q, t = 0) = (b\sqrt{\pi})^{-\frac{1}{2}} \exp(-q^2/2b^2) \quad (46)$$

An interesting property of equation (46) is that the corresponding time-dependent probability distribution $|\psi(q, t)|^2$ retains its Gaussian shape under time evolution. This result can be derived from the action of the Green function of the inverted

harmonic oscillator [60] on equation (46). The time evolution of the probability distribution yields a time-dependent width

$$\tilde{b}^2(t) = b^2 \cosh^2 t + b^{-2} \sinh^2 t \quad (47)$$

We can also consider the scattering of a Gaussian wave-packet incident on the inverted oscillator potential, i.e., a Gaussian distribution in both q and p . As the initial state at $t = 0$ we take [60]

$$\psi(q, 0) = (b\sqrt{\pi})^{-\frac{1}{2}} \exp(-(q - q_0)^2/2b^2 + ip_0x) \quad (48)$$

Its time evolution is found again with the help of the appropriate Green function, and the result for the probability density is

$$|\psi(q, t)|^2 = \frac{1}{\sqrt{\pi\tilde{b}(t)}} \exp(-(q - q(t))^2/\tilde{b}(t)^2) \quad (49)$$

where $\tilde{b}(t)$ is given by (47), and $q(t) = q_0 \cosh t + p_0 \sinh t$ is the classical trajectory of a particle having energy $\mathcal{E} = \frac{1}{2}(p_0^2 - q_0^2)$. Notice that the mean energy $\langle \psi | H | \psi \rangle = \mathcal{E} - (b^2 - 1/b^2)/4$. It coincides with the energy \mathcal{E} for minimum-uncertainty wavepackets with $b = 1$ (in units of $\hbar = 1$), which is the case of the pointer states under consideration. It is straightforward to see that the peak of the Gaussian distribution (49) follows the classical trajectory exactly [60].

A more interesting feature that is directly relevant to our Markovian approach to Liouville string is the effect of the environment, simulated by oscillators à la Caldeira and Leggett [61], on the shape of the Gaussian distribution (46), viewed as an initial pure state of the sub-system at $t = 0$. Assuming thermal equilibrium with the heat bath of the environmental oscillators, the authors of ref. [61] simulated the coupling to the environment by a temperature-dependent fluctuating non-conservative force $F(t, T)$

$$\langle\langle F(t, T) F(t', T) \rangle\rangle = 2\eta k_B T \delta(t - t') \quad (50)$$

where $\langle\langle \dots \rangle\rangle$ denotes a statistical/thermal average, t denotes the time, k_B is Boltzmann's constant, and η is a friction coefficient. The result of ref. [60] is that under the influence of the heat-bath a pure initial state (46) evolves at time t to a probability distribution for the particle, described by the diagonal element of the density matrix, which is given exactly by the Gaussian [60]

$$\rho(q, q, t) = \frac{1}{\sqrt{\pi\tilde{b}(t, T)}} \exp(-\frac{q^2}{\tilde{b}(t, T)^2}) \quad (51)$$

where for a unit-mass and -frequency oscillator

$$\begin{aligned} \tilde{b}^2(t, T) = & \left(\frac{2\sinh\Omega t}{\Omega e^{\frac{\eta}{2}t}}\right)^2 \left\{ \left(\frac{b}{2}\right)^2 (\Omega \coth\Omega t + \frac{\eta}{2}) + \left(\frac{\hbar}{2b}\right)^2 + \right. \\ & \left. \frac{\hbar\eta}{2\pi} \int_0^{\nu_{max}} d\nu \nu \coth\left(\frac{\hbar\nu}{2k_B T}\right) \left| \frac{e^{\frac{\eta}{2}t}}{\sinh\Omega t} \int_0^t d\tau e^{-\frac{\eta}{2}\tau + i\nu\tau} \sinh\Omega\tau \right|^2 \right\} = 2 \langle\langle q^2 \rangle\rangle \end{aligned} \quad (52)$$

with $\Omega \equiv \sqrt{1 + \eta^2/4}$.

What is the meaning of this result in our framework? To answer this question it is necessary to define a notion of “fine” thermodynamics in the sense of Fronteau [22], which allows the definition of a phase-space-dependent ‘temperature’ $T^f(q, p, t)$ even for systems outside thermal equilibrium. In the example considered by Fronteau [22] the work done by a non-conservative force F acting on the system is given infinitesimally by

$$dQ^f = Fdq \quad (53)$$

and is related to the “fine” entropy $S^f(p, q, t)$ and the “fine” temperature $T^f(q, p, t)$ by the conventional thermodynamic relation, in infinitesimal form

$$dS^f(q, p, t) = \frac{dQ^f(q, p, t)}{T^f(q, p, t)} \quad (54)$$

From section 2 and ref. [10] we recall that in our case the non-conservative forces are given by

$$F_i(q) = G_{ij}\beta^j \quad (55)$$

where G_{ij} is the Zamolodchikov metric (27) in σ -model coupling constant space. Using equation (32), and writing (54) in terms of differential rates, we can define a “fine” temperature in our case

$$T^f(\mathbf{g}, \mathbf{p}, t) \equiv S^{-1} = e^{C_{eff}(\mathbf{g}, t) - 25} \quad (56)$$

with $C(\mathbf{g}, t)$ the Zamolodchikov C -function (25), and the critical value 25 arises from an appropriate choice of normalization conventions. In this way, a notion of “fine” temperature is defined in non-critical strings, which measures the deviation from conformal fixed points. Near the infrared fixed point on the world-sheet, where $C_{eff} \rightarrow \infty$ [10, 11], the topological phase of the string is approached [24], and the “fine” temperature is infinite. Adapting the formalism of ref. [62], we find that the uncertainty in σ -model coupling constant/field space diverges in this limit. This implies the breakdown of a low-energy point-like field theory, which should be expected in the topological phase. On the other hand, in the ultraviolet limit $C_{eff} \rightarrow 25$ and the theory approaches that of a conventional critical string. In this case the unit of “fine” temperature is reached, and ‘thermal equilibrium’ is achieved. This is in agreement with the law (54), because in this case, both δQ^f and δS^f vanish, since they are related to temporal changes in momentum and position in phase-space, whilst T^f is still finite for a free ‘particle’ with momentum p , being related to its kinetic energy [22] $\frac{3}{2}T^f = p^2/2m$. We note at this stage that the above definition of temperature as a deviation from conformal equilibrium in non-critical strings is not new. It has appeared previously in the literature [63], when topological defects on the world-sheet were considered. Our “fine” temperature, which is a quantity defined in coupling-constant space, is related to the world-sheet temperature of ref. [63] by a simple exponentiation,

$$T^f = e^{C_{eff} - 25} \equiv e^{T_{ws}} \quad (57)$$

The physical reason behind this relation is the requirement of vanishing fine entropy and heat-transfer in the critical string, as explained above.

Having defined a fine temperature, we can now interpret the result (51,52) in our framework, by first replacing the equilibrium temperature T by the fine quantity T^f (56). Secondly, we take into account the results of ref. [50], according to which the spatial coordinate q of the inverted harmonic oscillator potential can be related by an appropriate canonical collective coordinate transformation to the massless propagating ‘tachyon’ mode of the two-dimensional string theory. From our point of view, such a transformation corresponds to a renormalization-scheme change, i.e., an appropriate coordinate change in the coupling constant space of the two-dimensional string. Thirdly, we recall that tachyon propagation in a two-dimensional black-hole geometry is believed to be described by a modification of the inverted-oscillator potential by anharmonic terms [50]

$$\delta V = -\frac{m^2}{2q^2} + \dots \quad (58)$$

describing the topological string modes, where the \dots denote (an infinity of) possible terms. Integrating out such terms provides a low-energy ‘observer’ measuring only the localizable modes with the analogue of an ‘environment’, where the above decoherence results emerge. Given that the minimum-entropy-producing initial states are Gaussian, whose nature is not affected by the environment, one can safely interpret the result (51,52) as implying that the concept of a low-energy (observable) Gaussian field theory mode survives the procedure of integrating out the topological string modes in the two-dimensional black-hole string theory. The Gaussian wave-packet (51) maintains its minimum uncertainty in coupling-constant space. This should not be confused with the time-dependent modified uncertainty in position and momentum in target-space of a test string found in [11] as a result of a time-varying minimum string length [64]. In the field space of the stringy σ -model, the uncertainty in measurements of a field and its conjugate momentum, both related to pointer states in coupling-constant-space quantum mechanics, retains its conventional form as in local field theory, unrenormalized by the Liouville mode.

It should be stressed that the above result, i.e., the emergence of a quantum field theory as the low-energy limit of a matrix model (or, in more general terms, of a resummed world-sheet σ -model theory), finds a consistent explanation in the formalism of coupling-constant density-matrix mechanics. In the simple Drude-model analogue example of ref. [6], which is argued to capture the essential physical features of the realistic string situation, it was been found that the off-diagonal terms of the density matrix in coupling space of the σ -model, $\rho(g^i, g^j, t)$, behave like

$$\rho(g^i, g^j) \propto e^{-Dt(g^i - g^j)^2 + \dots} \quad (59)$$

where D is a small coefficient, depending on the squares of the anomalous dimensions α_g (22) of the Liouville-dressed deformations g_R^i . In our framework, the topological string modes, which have been integrated out by the low-energy observer, play the rôle of a continuously measuring ‘apparatus’, and the observed time flow is a result of

such a ‘measurement’ process. The ‘collapse of the wave-function’, i.e. the vanishing of the off-diagonal terms (59) of the density matrix in coupling-constant space, occurs at $t \rightarrow \infty$, which in our framework is the ultraviolet fixed point. At that point critical string theory is recovered and a fixed string background is achieved, in the sense of a non-vanishing entry of only one of the diagonal elements of the density matrix³. Away from this equilibrium point there are non-trivial interference terms (59), expressing the quantum nature in the coupling-constant space of the non-equilibrium physics. In this picture, critical string theory is identified with the final result ($t \rightarrow \infty$) of the ‘measurement process’ induced by the topological modes of the string⁴.

The minimum-entropy-production nature of the Gaussian wave-packet of the inverted harmonic oscillator implies an almost reversible, deterministic trajectory in coupling constant space for the resulting states/fields away from equilibrium. This combines the classical deterministic nature of the renormalization group flow with the quantum nature of the string backgrounds in the matrix model formalism, that resums Riemann surfaces. In the above example, as well as in that of ref. [58], such states arise naturally through decoherence effects associated with the interaction of the quantum system with an ‘environment’. In the case of the black hole model, as we discussed above, the environment is provided by the topological modes of the string, which do not decouple, and whose interaction with the propagating low-energy modes results in an irreversible flow of time. The pointer states that arise through decoherence effects in the coupling constant space of the stringy σ -model move almost reversibly with the renormalization group time, and in this way one obtains a conventional quantum field theory (g^i 's and p_i 's) in the target space of the string.

This demonstration in the inverted harmonic oscillator approach appears to possess a deeper interpretation. We remind the reader that it was this model that the geometric interpretation of the W_∞ string symmetry as a coherence-preserving target-space symmetry was given [13, 45]. The linear entropy is related to the area

³It should be understood that this picture would be modified by the action of exactly-marginal deformations that induce uncertainties in a specific background choice. This is related to the well-known string-vacuum degeneracy problem. It is hoped that truly non-perturbative string effects will eventually lift this degeneracy, leading to a unique critical string vacuum, and hence to a single non-zero entry in the string density matrix.

⁴This measurement process and the associated ‘collapse of the wavefunction’ phenomenon in string coupling-constant space should not be confused with their analogues in the target space of the string, discussed in ref. [6]. In that case one considers a density matrix for a test string propagating in the non-trivial background g^i . Its off-diagonal elements refer to spatially-separated points in target space of the string, related to the above formalism in the way explained in ref. [7]. In that case, when many string particles are present, the exponent of the off-diagonal elements (59) acquires a multiplicative factor N , the number of test strings, and the collapse time is diminished significantly [5].

of the p and q phase space by [58]

$$s^l = 1 - 1/A \tag{60}$$

for Gaussian distributions, like the ones corresponding to our pointer states. In the absence of black holes in space-time, the W_∞ symmetries of the matrix model guarantee the invariance of the two-dimensional phase-space area A under (target) time evolution[13, 45]. Its time dependence (35) indicates the breaking of coherence as expected from the fact that the W_∞ symmetries mix the propagating low-energy string modes with the higher-level string states which are delocalized and do not decouple in the presence of a black hole [13, 6]. For the pointer states found above this breaking of coherence is the softest possible one. This justifies the use of the flat space-time matrix model in writing down the temporal evolution of the fields g^i in the weak coupling approximation.

The pointer states are the closest approximations to classical points in the string coupling-constant phase space. Due to their minimal irreversibility, they are the best approximations to the effective quantum field theories usually taken as the low-energy limits of string theories. We note that the statistical entropy (31) is not minimized by the pointer states, as can be easily seen by the fact that the linear entropy s^l (34) provides only a *lower bound* on the statistical entropy (31)[65]. This implies an overall cosmological time arrow, whilst in parallel allowing for the emergence of almost-time-reversible local field theory structures, associated with decoherence-induced pointer states in the coupling-constant space. In this way, we have a rather elegant way of understanding how quantum field theory in target space arises in Liouville strings. An interesting question is whether this is merely an elegant formalism, or has some observable microscopic consequences. This may be possible, as we argue in the following section.

5 Generic CPT Violation and Non-Conservation of Angular Momentum

The above definition of time in string quantum gravity made it necessary to introduce non-perturbative effects on the world sheet, such as instantons. In their presence, certain charges cease to be conserved, as a result of logarithmic renormalization scale dependences. Such a situation implies the non-commutativity of the Hamiltonian operator with the respective charge operators on the world sheet. If one defines a generalized CPT symmetry in such a way that this symmetry leaves the mass of a string state invariant, but changes the sign of the charge, then it is straightforward to argue that in our case the elevation of CPT symmetry to target space fails in general. This is a heuristic argument, and one should really construct a rigorous proof of such a situation, which is not yet available.

However, the evolution of pure states into mixed ones, as a result of the entropy increase (31) along the positive direction of time, implies the violation of CPT symmetry as we now review in the context of a general analysis [16]. Let us make the assumption that a CPT operator $\hat{\Theta}$ exists in target space, such that

$$\begin{aligned}\rho'_{in} &= \hat{\Theta}^{-1} \rho_{out} \\ \rho'_{out} &= \hat{\Theta} \rho_{in}\end{aligned}\tag{61}$$

and the *in* and *out* density matrices are related through the superscattering operator \mathcal{S} [3]

$$\begin{aligned}\rho_{out} &= \mathcal{S} \rho_{in} \\ \rho'_{out} &= \mathcal{S} \rho'_{in}\end{aligned}\tag{62}$$

The following relation is a trivial consequence of the above equations

$$\hat{\Theta} = \mathcal{S} \hat{\Theta}^{-1} \mathcal{S}\tag{63}$$

which implies that \mathcal{S} has an inverse. Clearly this cannot happen if there is evolution of pure states into mixed ones and not vice versa, as implied by the monotonic increase of the entropy (31). This proves the breaking of CPT symmetry in the above framework.

In ref. [7] we have described non-factorisable (i.e. $\mathcal{S} \neq SS^\dagger$, where S is the conventional S -matrix operator) contributions to the string \mathcal{S} -matrix, coming from valleys between topological defects on the world sheet ⁵. This provides an explicit demonstration of the non-existence of an inverse \mathcal{S}^{-1} , and hence of induced CPT violation in the target space of this effective string theory. It should be stressed, however, that the above considerations cannot exclude the possibility of a some weaker form of CPT invariance [16] which might cause violations of CPT symmetry to be unobservable in an experimental apparatus. Such a situation falls beyond the scope of the present talk, and in what follows we simply explore the possibility that a detectable violation of CPT occurs, which we parametrize in a way suitable for present experiments with neutral kaons and at future ϕ factories [68, 69], that constitute the most sensitive probes in a search for violations of quantum mechanics at the microscopic level. For more details we refer the reader to the literature [15, 17, 19] and also to Huet's talk at this meeting [18].

⁵Notice that, in that picture, the creation and annihilation of a target-space black hole is represented as a world-sheet monopole-anti-monopole pair [66]. Instantons induce transitions among such configurations of different charge, the latter being proportional to the black hole mass. We note that there is a formal analogy [7, 10] with the Quantum Hall fluids: in that case, instantons in the respective Wess-Zumino models, describing the effective theories in 'conductivity space', induce transitions among the transverse-conductivity plateaux [67].

Before proceeding with the parametrization of such possible phenomenological effects, it is worth pointing out two important properties of our modification of quantum mechanics due to stringy quantum gravity effects. The first is energy conservation on the average, which follows from renormalizability of the world-sheet theory, and the second is a generic violation of angular momentum conservation. Both properties can easily be understood formally as follows. The renormalized background couplings g^i are assumed to be quantum operators, as a result of a higher-genus resummation. This implies that the modified density matrix equation (30) that describes the time evolution will be used [6].

Consider an operator \mathcal{K} in this framework whose average is given by

$$\langle\langle \mathcal{K} \rangle\rangle \equiv Tr(\tilde{\rho}\mathcal{K}) \quad (64)$$

Its time evolution is given by

$$\partial_t \langle\langle \mathcal{K} \rangle\rangle = \langle\langle \beta^i G_{ij}[g^j, \mathcal{K}] \rangle\rangle + \langle\langle \partial_t \mathcal{K} \rangle\rangle \quad (65)$$

where we have used the fact that $\beta^i G_{ij}$ is a functional of g^i only, and not of $p_i = \frac{\delta}{\delta g^i}$. If the operator \mathcal{K} is the σ -model Hamiltonian (energy) then, using $[g^i, \mathcal{H}] = \beta^i$ as well as the fact that the C -function is related to the string effective action that generates string amplitudes, it is straightforward to derive [15, 11]

$$\partial_t \langle\langle \mathcal{H} \rangle\rangle = \partial_t \left(\frac{\delta \beta^i}{\delta g^i} \right) = 0 \quad (66)$$

as a result of the renormalizability of the σ -model, which implies that the couplings g^i and the associated β functions do not have any explicit scale dependence. This property of energy conservation can be extended straightforwardly to many-particle states.

The same is not true for the angular momentum operator, in target space dimensions higher than two. In the context of the target-space effective field theory this operator is defined as

$$J^{\alpha\beta} = X^\alpha P_{\beta'} G^{\beta\beta'} \quad (67)$$

where Greek indices denote target spatial components, and $G_{\alpha\beta}$ is the metric tensor in target space. The stress tensor is derived from the effective lagrangian

$$\begin{aligned} T_{\alpha\beta} &= \frac{\delta \mathcal{L}^{eff}}{\delta G_{\alpha\beta}} \\ T_{0\beta} &= \frac{\delta \mathcal{L}^{eff}}{\delta G_{0\beta}} \end{aligned} \quad (68)$$

and the target momenta can be defined from the effective theory stress tensor by differentiating with respect to the $G_{0\beta}$ component of the metric (here the “0” component refers to the Liouville time). The effective action is identified with the

Zamolodchikov C -function. By construction, the latter is renormalization-group invariant, hence

$$(\partial_t + \beta^i \partial_i)C = 0 \quad (69)$$

Using the off-shell corollary of the C -theorem $\partial_i C = G_{ij} \beta^j$ [70, 47], and taking into account the fact that the angular momentum operator is a functional of g^i only, as seen in equation (68), we observe that the g -commutator term in (65) vanishes, leaving us with the following non-trivial result for the temporal dependence of the angular momentum operator in this framework :

$$\partial_t J^{\alpha\beta} = X^{[\alpha} G^{\beta]\gamma} \frac{\delta[\beta^i G_{ij} \beta^j]}{\delta G_{0\gamma}} \neq 0 \quad (70)$$

with $[\cdot]$ denoting antisymmetrization. This expression is non-zero in general.

We can evaluate the above expression (70) in an explicit bosonic string background. Consider for instance the case of a maximally-symmetric space-time with a constant dilaton:

$$R_{MN} = G_{MN} R \quad ; \quad \Phi = const \quad (71)$$

To lowest non-trivial order in α' the graviton β -functions are just given by the Ricci tensor

$$\beta_{MN}^G = R_{MN} + \dots \quad (72)$$

whilst the quantity $\beta^i G_{ij} \beta^j$ is given by

$$\beta^i G_{ij} \beta^j = \frac{3}{16\pi^2} [R + \dots] \quad (73)$$

It should be stressed that in the above formulae the target manifold includes the Liouville/local renormalization scale ϕ as a time component [11]. We have ignored for simplicity explicit matter fields, and concentrated on the gravitational sector. Our formulae are easily adapted to the more general case with matter deformations.

It is easy to see that for a maximally-symmetric non-static universe the result (70) becomes

$$\partial_t J^{\alpha\beta} = \frac{3}{8\pi^2} X^{[\alpha} \nabla^{\beta]} \partial_t R \quad (74)$$

We can see from (74) that the average $Tr \tilde{\rho} \partial_t J^{\alpha\beta} \neq 0$ for time-varying curvatures, e.g., expanding universes [11] with $\partial_t R(t) \equiv H(t)R(t)$, where $H(t)$ is a Hubble parameter. For such maximally-symmetric spaces the following operator relation holds

$$\partial_t \langle\langle J^{\alpha\beta} \rangle\rangle = -\frac{3}{8\pi^2} H(t)R \langle\langle J^{\alpha\beta} \rangle\rangle \quad (75)$$

showing a decrease of the average angular momentum in an expanding universe. This amounts to a derivation of Mach's principle, analogous to that [71] in conventional inflationary cosmology.

We can give a physical interpretation of the above results by making a direct comparison with the expanding universe solution in non-critical string theory of ref. [8]. It has been shown [9, 33] that this model can be directly put in the above framework of Liouville strings by the identification of the Liouville field with the target time coordinate X^0 . The relevant point here is that in the model of ref. [8] in $D > 3$ target-space dimensions there is a local antisymmetric tensor field $B_{MN} = -B_{NM}$, which is assumed to depend non-trivially on the ‘cosmic’ time $t \equiv e^{X^0}$, whilst the dilaton is linear in X^0 . In string theory there is an Abelian symmetry that forces B_{MN} to appear only through its field strength, $H_{MNP} = \nabla_{[M}B_{NP]}$. For $D = 4$ one can define an axion (pseudoscalar) field b by

$$H_{MNP} = e^\Phi \epsilon_{MNP\Sigma} \nabla_\Sigma b \quad (76)$$

where the dilaton factor is due to scale invariance, and the time derivative is with respect to t . The axion b may be viewed as the Goldstone boson of the target space symmetry $b \rightarrow b + \text{const}$ [8]. A linear t dependence in b implies a time-independent factor in H_{MNP} . This has been interpreted in ref. [8] as a signal for spontaneous breaking of Lorentz invariance (and hence of angular momentum as well). On the other hand, time-translation invariance is not broken by b and therefore energy is conserved in physical amplitudes derived from the non-critical string model of ref. [8]. In contrast, a constant shift in the dilaton $\Phi \rightarrow \Phi + \text{const}$. scales the overall target-space Lagrangian, and hence the Planck constant, and so the dilaton cannot be viewed as a Goldstone boson of a symmetry at a quantum level. Time-translation invariance is formally restored if the correct string vacuum is a superposition of various ground states corresponding to different constant values Φ_0 of the dilaton field. This is the situation indicated by our two-dimensional toy example considered above. In this case, since only s -wave four-dimensional configurations are described by this model, the antisymmetric-tensor field strength vanishes trivially, and one cannot see explicitly the breaking of Lorentz invariance. However, even in this case one can see certain generic features of the above approach. For instance, as we discussed in ref. [10], near the infrared fixed point of the world-sheet the σ -model action is described by a ‘topological version’ of the Wess-Zumino black hole model of ref. [27], which contains a θ term that is nothing other than a discrete (topological) antisymmetric tensor background. This term is essential in yielding instanton deformations with finite world-sheet action [72], and thus capable of breaking conformal invariance and thereby inducing time flow. Moreover, the coupling of this θ term is proportional to the instanton-renormalized level parameter $k_R(t)$ of the topological Wess-Zumino model [7, 10], related to the integration over topological modes. The latter contains logarithmic scale (Liouville time) dependences, which are assumed to exponentiate beyond the dilute gas approximation, thereby leading to an exponential Liouville-dependence (i.e. linear in the ‘cosmic time’ t) of the antisymmetric tensor coupling, exactly as required for a spontaneous breaking of Lorentz invariance, but not of time-translation invariance. Moreover, in the two-dimensional black-hole example, a constant shift in the dilaton field corresponds to a change in the black-hole mass. Our ground state is assumed to be a foamy superposition of various microscopic

black holes, corresponding to various constant shifts in the dilaton field. From a world-sheet point of view, it corresponds to a Kosterlitz-Thouless plasma (Coulomb gas) of various monopole charges [66]. When averaging over such states in a quantum theory of gravity, as we do in (66), time-translation invariance is restored and energy is conserved on the average, in agreement with our more general result stemming from the renormalizability of the underlying σ -model.

It should be stressed that the above physical picture appears specific to four dimensions. If it is correct in a full four-dimensional stringy model of space-time foam, it leads to severe restrictions on the Liouville scale-dependence of the antisymmetric tensor backgrounds. It would be nice to prove that this feature is a result of ‘integrating out’ topological (global) degrees of freedom of the string that cannot be measured by local scattering experiments, in much the same way as in the two-dimensional (s -wave four-dimensional) example. At this stage this is only a conjecture, given that a realistic four-dimensional space-time foamy configuration is not known, at least to the same level of precision and mathematical exactness as the two-dimensional Wess-Zumino black hole case [27].

Before proceeding, we mention another important property, namely the conservation of total probability:

$$\partial_t \int dp_i dg^i \text{Tr} \tilde{\rho} = \int dg^i dp_i \frac{\partial}{\partial p_i} \tilde{\rho} G_{ji} \beta^j = 0 \quad (77)$$

if one assumes that the momentum-boundary terms vanish. This does not imply that the coupling constant space has no boundary. On the contrary, it is believed that the latter is a non-simply connected manifold, if a topological interpretation in terms of Morse theory is to be given [73].

6 Application to the Neutral Kaon System

We are now well equipped to discuss the phenomenology of violations of quantum mechanics in the above framework. The formalism of [4] will be adopted. Below, we describe briefly the formalism [4, 15] for a discussion of the possible modification of quantum mechanics and violation of CPT in the neutral kaon system, which is among the most sensitive microscopic laboratories for studying these possibilities. In the normal quantum-mechanical formalism, the time-evolution of a neutral kaon density matrix is given by

$$\partial_t \rho = -i(H\rho - \rho H^\dagger) \quad (78)$$

where the Hamiltonian takes the following form in the (K^0, \bar{K}^0) basis:

$$H = \begin{pmatrix} (M + \frac{1}{2}\Delta M) - \frac{1}{2}i(\Gamma + \frac{1}{2}\Delta\Gamma) & M_{12}^* - \frac{1}{2}i\Gamma_{12}^* \\ M_{12} - \frac{1}{2}i\Gamma_{12} & (M - \frac{1}{2}\Delta M) - \frac{1}{2}i(\Gamma - \frac{1}{2}\Delta\Gamma) \end{pmatrix} \quad (79)$$

The non-hermiticity of H reflects the process of K decay: an initially-pure state evolving according to (78) and (79) remains pure.

In order to discuss the possible modification of this normal quantum-mechanical evolution, and allow for the possibility of CPT violation, it is convenient to rewrite [15] (78) and (79) in a Pauli σ -matrix basis [4], introducing components ρ_α of the density matrix:

$$\rho = 1/2\rho_\alpha\sigma_\alpha \quad (80)$$

which evolves according to

$$\partial_t\rho_\alpha = h_{\alpha\beta}\rho_\beta \quad (81)$$

with

$$h_{\alpha\beta} \equiv \begin{pmatrix} Imh_0 & Imh_1 & Imh_2 & Imh_3 \\ Imh_1 & Imh_0 & -Reh_3 & Reh_2 \\ Imh_2 & Reh_3 & Imh_0 & -Reh_1 \\ Imh_3 & -Reh_2 & Reh_1 & Imh_0 \end{pmatrix} \quad (82)$$

It is easy to check that at large times ρ takes the form

$$\rho \simeq e^{-\Gamma_L t} \begin{pmatrix} 1 & \epsilon^* \\ \epsilon & |\epsilon|^2 \end{pmatrix} \quad (83)$$

where ϵ is given by

$$\epsilon = \frac{\frac{1}{2}iIm\Gamma_{12} - ImM_{12}}{\frac{1}{2}\Delta\Gamma - i\Delta M} \quad (84)$$

in the usual way.

A modification of quantum mechanics of the form discussed in section 3 can be introduced by modifying equation (81) to become

$$\partial_t\rho_\alpha = h_{\alpha\beta}\rho_\beta + \mathcal{H}_{\alpha\beta}\rho_\beta \quad (85)$$

The form of $\mathcal{H}_{\alpha\beta}$ is determined if we assume probability and energy conservation, as proved in the string context in section 3, and that the leading modification conserves strangeness:

$$\mathcal{H}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\alpha & -2\beta \\ 0 & 0 & -2\beta & -2\gamma \end{pmatrix} \quad (86)$$

It is easy to solve the 4×4 linear matrix equation (85) in the limits of large time:

$$\rho_L \propto \begin{pmatrix} 1 & \frac{-\frac{1}{2}i(Im\Gamma_{12}+2\beta)-ImM_{12}}{\frac{1}{2}\Delta\Gamma+i\Delta M} \\ \frac{\frac{1}{2}i(Im\Gamma_{12}+2\beta)-ImM_{12}}{\frac{1}{2}\Delta\Gamma-i\Delta M} & |\epsilon|^2 + \frac{\gamma}{\Delta\Gamma} - \frac{4\beta ImM_{12}(\Delta M/\Delta\Gamma)+\beta^2}{\frac{1}{4}\Delta\Gamma^2+\Delta M^2} \end{pmatrix} \quad (87)$$

and of short time:

$$\rho_S \propto \begin{pmatrix} |\epsilon|^2 + \frac{\gamma}{|\Delta\Gamma|} - \frac{-4\beta ImM_{12}(\Delta M/\Delta\Gamma) + \beta^2}{\frac{1}{4}\Delta\Gamma^2 + \Delta M^2} & \epsilon - \frac{i\beta}{\frac{\Delta\Gamma}{2} - i\Delta M} \\ \epsilon^* + \frac{i\beta}{\frac{\Delta\Gamma}{2} + i\Delta M} & 1 \end{pmatrix} \quad (88)$$

We note that the density matrix (87) for K_L is mixed to the extent that the parameters β and γ are non-zero. It is also easy to check [15] that the parameters α , β and γ all violate CPT , in accord with the general argument of [16], and consistent with the string analysis mentioned earlier in this section.

Experimental observables O can be introduced [4, 15] into this framework as matrices, with their measured values being given by

$$\langle O \rangle = Tr(O\rho) \quad (89)$$

Examples are the K to 2π and 3π decay observables

$$O_{2\pi} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad ; \quad O_{3\pi} = (0.22) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (90)$$

and the semileptonic decay observables

$$O_{\pi^- l^+ \nu} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \\ O_{\pi^+ l^- \bar{\nu}} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \quad (91)$$

A quantity of interest is the difference between the K_L to 2π and K_S to 3π decay rates [15]:

$$\delta R \equiv R_{2\pi} - R_{3\pi} = \frac{8\beta}{|\Delta\Gamma|} |\epsilon| \sin\phi_\epsilon \quad (92)$$

where $R_{2\pi}^L \equiv Tr(O_{2\pi}\rho_L)$, and $R_{3\pi}^S \equiv Tr(O_{3\pi}\rho_S)/0.22$, and the prefactors are determined by the measured [74] branching ratio for $K_L \rightarrow 3\pi^0$. (Strictly speaking, there should be a corresponding prefactor of 0.998 in the formula (90) for the $O_{2\pi}$ observable.)

Using (91), one can calculate the semileptonic decay asymmetry [15]

$$\delta \equiv \frac{\Gamma(\pi^- l^+ \nu) - \Gamma(\pi^+ l^- \bar{\nu})}{\Gamma(\pi^- l^+ \nu) + \Gamma(\pi^+ l^- \bar{\nu})} \quad (93)$$

in the long- and short-lifetime limits:

$$\delta_L = 2Re\left[\epsilon\left(1 - \frac{i\beta}{ImM_{12}}\right)\right] \\ \delta_S = 2Re\left[\epsilon\left(1 + \frac{i\beta}{ImM_{12}}\right)\right] \quad (94)$$

The difference between these two values

$$\delta\delta \equiv \delta_L - \delta_S = -\frac{8\beta}{|\Delta\Gamma|} \frac{\sin\phi_\epsilon}{\sqrt{1 + \tan^2\phi_\epsilon}} = -\frac{8\beta}{|\Delta\Gamma|} \sin\phi_\epsilon \cos\phi_\epsilon \quad (95)$$

with $\tan\phi_\epsilon = (2\Delta M)/\Delta\Gamma$, is a signature of CPT violation that can be explored at the CPLEAR and DAΦNE facilities [68, 69].

We have used [15] the latest experimental values of $R_{2\pi}$ and $R_{3\pi}$ to bound δR , and the latest experimental values of $\delta_{L,S}$ to bound $\delta\delta$, expressing the results as contours in the (β, γ) plane [15]. In our formalism, the usual CP -violating parameter ϵ is given by [15]

$$|\epsilon| = -\frac{2\beta}{|\Delta\Gamma|} \sin\phi_\epsilon + \sqrt{\frac{4\beta^2}{|\Delta\Gamma|^2} - \frac{\gamma}{|\Delta\Gamma|} + R_{2\pi}^L} \quad (96)$$

On the basis of this preliminary analysis, it is safe to conclude that

$$\left|\frac{\beta}{\Delta\Gamma}\right| \lesssim 10^{-4} \text{ to } 10^{-3} \quad ; \quad \left|\frac{\gamma}{\Delta\Gamma}\right| \lesssim 10^{-6} \text{ to } 10^{-5} \quad (97)$$

In addition to more precise experimental data, what is also needed is a more complete global fit to all the available experimental data, including those at intermediate times, which are essential for bounding α , and may improve our bounds (97) on β and γ [15, 17, 19]. We now give a brief account of the intermediate-time formalism. As a first step, we consider a perturbative ansatz for the density matrix elements ρ_{ij} , $i, j = 1, 2$ that appear in the system of equations (85) after a (convenient) change of basis to $K_{1,2} \equiv \sqrt{1}\sqrt{2}(K^0 \mp \bar{K}^0)$ [4, 15]. We write [17]

$$\rho_{ij}(t) = \rho_{ij}^{(0)} + \rho_{ij}^{(1)} + \dots \quad (98)$$

where $\rho_{ij}^{(k)}$ are polynomials in α, β, γ and $|\epsilon|$ of degree k :

$$\rho_{ij}^{(k)} \equiv \alpha^{P_\alpha} \beta^{P_\beta} \gamma^{P_\gamma} |\epsilon|^{P_\epsilon} \quad ; \quad P_\alpha + P_\beta + P_\gamma + P_\epsilon = k \quad (99)$$

with the initial condition of having a pure K^0 state, i.e., $\rho_{ij}^0(0) = \frac{1}{2}$, $\rho_{ij}^k(0) = 0$, $k \geq 1$. The ansatz (98) leads to the following iterative system of differential equations, describing the time evolution of the density matrix of the neutral-kaon system at arbitrary time intervals [17] :

$$\frac{d}{dt}[e^{At} \rho_{ij}^{(k)}(t)] = e^{At} \sum_{kl \neq ij} \rho_{kl}^{(k-1)}(t) \quad (100)$$

where A is a generic factor that can be expressed in terms of known data of the neutral-kaon system [17]. In the long and short time limits one recovers the bounds (97) of β and γ . On the other hand, a fit to presently available intermediate time data from two-pion decays [75] can place more stringent bounds on these quantities

[17], confirming that the standard CP -violation (96), observed so far, is mainly quantum mechanical in origin. Moreover, an upper bound on the quantity α can also be placed by such fits,

$$\left|\frac{\alpha}{\Delta\Gamma}\right| \lesssim 2 \times 10^{-3} \quad (101)$$

although more stringent bounds can be placed by a study of ϕ -decays at a ϕ -factory [69, 18, 19].

A concrete phenomenological consequence of the CPT -violation will be a shift $\delta\phi$ in the minimum of the time-dependent semileptonic decay asymmetry $\delta(t)$ (93) as a function of time t . A preliminary estimate of this shift, using the bound (101) yields [17]

$$\delta\phi \lesssim 6 \times 10^{-3} \quad (102)$$

and we expect this range to be probed in the foreseeable future.

We cannot resist pointing out that the bounds (97) are quite close to

$$O(\Lambda_{QCD}/M_P)m_K \simeq 10^{-19} GeV \quad (103)$$

which is perhaps the largest magnitude that any such CPT - and quantum-mechanics-violating parameters could conceivably have. Since any such effects are associated with topological string states that have masses of order M_P , we expect them to be suppressed by some power of $1/M_P$. This expectation is supported by the analogy with the Feynman-Vernon model of quantum friction [76], in which coherence is suppressed by some power of the unobserved oscillator mass or frequency. If the CPT - and quantum-mechanics-violating parameters discussed in this section are suppressed by just one power of M_P , they may be accessible to the next round of experiments with CPLEAR and/or DAΦNE [68, 69].

Acknowledgements

The work of N.E.M. is supported by a EC Research Fellowship, Proposal Nr. ERB4001GT922259 . That of D.V.N. is partially supported by DOE grant DE-FG05-91-GR-40633.

References

- [1] S. Mandelstam, Phys. Lett. B277 (1992), 82.
- [2] S. Hawking, Comm. Math. Phys. 43 (1975), 199 ;
J. Bekenstein, Phys. Rev. D12 (1975), 3077.
- [3] S. Hawking, Comm. Math. Phys. 87 (1982), 395.
- [4] J. Ellis, J.S. Hagelin, D.V. Nanopoulos and M. Srednicki, Nucl. Phys. B241 (1984), 381.
- [5] J. Ellis, S. Mohanty and D.V. Nanopoulos, Phys. Lett. B221 (1989), 113.
- [6] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B293 (1992), 37.
- [7] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, CERN, ENS-LAPP and Texas A & M Univ. preprint, CERN-TH.6896/93, ENS-LAPP-A426-93, CTP-TAMU-29/93; ACT-09/93 (1993); hep-th/9305116 ;
J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, CERN, ENS-LAPP and Texas A & M Univ. preprint, CERN-TH.6897/93, ENS-LAPP-A427-93, CTP-TAMU-30/93; ACT-10/93 (1993); hep-th/9305117.
- [8] I. Antoniadis, C. Bachas, J. Ellis and D.V. Nanopoulos, Phys. Lett. B211 (1988), 393; Nucl. Phys. B328 (1989), 117; Phys. Lett. B257 (1991), 278.
- [9] J. Polchinski, Nucl. Phys. B324 (1989), 123;
D.V. Nanopoulos, in *Proc. Int. School of Astroparticle Physics*, HARC, Houston (World Scientific, Singapore, 1991), p. 183.
- [10] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, preprint CERN-TH.7195/94, ENS-LAPP-A-463/94, ACT-5/94, CTP-TAMU-13/94, *lectures presented at the Erice Summer School, 31st Course: From Supersymmetry to the Origin of Space-Time*, Ettore Majorana Centre, Erice, July 4-12 1993.
- [11] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, *Proc. HARC workshop on "Recent Advances in the Superworld"*, The Woodlands, Texas (USA), April 14-16 1993, eds. J. Lopez and D.V. Nanopoulos, (World Sci., Singapore, 1994), p. 3; preprint CERN-TH.7000/93, CTP-TAMU 66/93, ENSLAPP-A-445/93, OUTF-93-26P, hep-th/9311148,
- [12] A.B. Zamolodchikov, JETP Lett. 43 (1986), 730; Sov. J. Nucl. Phys. 46 (1987), 1090.
- [13] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B267 (1991), 465; *ibid* B272 (1991), 261.
- [14] R. Penrose, *The Emperor's New Mind* (Oxford Univ. Press, 1989).

- [15] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B293 (1992), 142; CERN and Texas A & M Univ. preprint CERN-TH. 6755/92; ACT-24/92;CTP-TAMU-83/92; hep-th/9212057 ;
- [16] R. Wald, Phys. Rev. D21 (1980), 2742.
- [17] J. Ellis, J. Lopez, N.E. Mavromatos and D.V. Nanopoulos, in preparation ;
J. Lopez, preprint CPT-TAMU 38/93 (1993), Proc. at *HARC workshop on "Recent Advances in the Superworld"*, The Woodlands, Texas (USA), April 14-16 1993, eds. J. Lopez and D.V. Nanopoulos, (World Sci., Singapore, 1994), p. 272.
- [18] P. Huet, these proceedings.
- [19] P. Huet and M. Peskin, SLAC (Stanford) preprint SLAC-PUB-6454 (1994), hep-ph/9403257.
- [20] B. Misra, I. Prigogine and M. Courbage, *Physica* A98 (1979), 1;
I. Prigogine, *Entropy, Time, and Kinetic Description*, in *Order and Fluctuations in Equilibrium and Non-Equilibrium Statistical Mechanics*, ed G. Nicolis et al. (Wiley, New York, 1981);
B. Misra and I. Prigogine, *Time, Probability and Dynamics*, in *Long-Time Prediction in Dynamics*, ed G. W. Horton, L. E. Reichl and A.G. Szebehely (Wiley, New York, 1983) ;
B. Misra, *Proc. Nat. Acad. Sci. U.S.A.* 75 (1978), 1627.
- [21] J.P. Constantopoulos and C.N. Ktorides, J. Phys. A17 (1984), L29.
- [22] J. Fronteau, A. Tellez-Arenas and R.M. Santilli, Hadronic J. 3 (1979), 130;
J. Fronteau, Hadronic J. 4 (1981), 742.
- [23] D. Gross, Phys. Rev. Lett. 60 (1988), 1229;
E. Witten, Trans. Roy. Soc. A329 (1989), 345.
- [24] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B288 (1992), 23.
- [25] G. Shore, Nucl. Phys. B286 (1987), 349.
- [26] H. Osborn, Nucl. Phys. B294 (1987), 595; *ibid* B308 (1988), 629; Phys. Lett. B222 (1989), 97.
- [27] E. Witten, Phys. Rev. D44 (1991), 314.
- [28] A.M. Polyakov, *Gauge Fields and Strings* (Harwood, New York, 1987).
- [29] M.B. Green, J.H. Schwarz and E. Witten, *String Theory*, Vol. I and II (Cambridge Univ. Press, 1986).

- [30] W. Fischler and L. Susskind, Phys. Lett. B171 (1986), 262; *ibid* B171 (1986), 383.
- [31] A.M. Polyakov, Mod. Phys. Lett. A6 (1991), 635.
- [32] F. David, Mod. Phys. Lett. A3 (1988), 1651;
J. Distler and H. Kawai, Nucl. Phys. B321 (1989), 509.
- [33] N.E. Mavromatos and J.L. Miramontes, Mod. Phys. Lett. A4 (1989), 1847.
- [34] V. Frolov, *Physics of 2-D Black Holes*, Proc. Workshop on Strings and Quantum Gravity, Ettore Majorana Centre, Erice, June 1992, Eds. N. Sanchez and A. Zichichi, (World Sci., Singapore, 1993)
- [35] S. Chaudhuri and J. Lykken, Nucl. Phys B396 (1993), 270.
- [36] J. Distler and P. Nelson, Nucl. Phys. B374 (1992), 123.
- [37] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B284 (1992), 43.
- [38] J. Ellis, N. Mavromatos and D.V. Nanopoulos, Phys. Lett. B278 (1992), 246.
- [39] G. Veneziano, Phys. Lett. B167 (1985), 388;
J. Maharana and G. Veneziano, Nucl. Phys. B283 (1987), 126;
T. Kubota, Osaka Univ. preprint OU-HET-146 (1990), Proc. *Symposium on Quarks, Symmetries and Strings, in honour of the 60th birthday of B. Sakita, (New York, October 1-2, 1990)*.
- [40] M. Evans and B. Ovrut, Phys. Rev. D39 (1989), 3016.
- [41] M. Evans and B. Ovrut, Phys. Rev. D41 (1990), 3149.
- [42] I. Bakas and E. Kiritsis, Phys. Lett. B301 (1993), 49.
- [43] M. Evans and B. Ovrut, Phys. Lett. B231 (1989), 80.
- [44] S. Coleman, Comm. Math. Phys. 31 (1973), 253.
- [45] E. Witten, Nucl. Phys. B373 (1992), 187.
- [46] G. Curci and G. Paffuti, Nucl. Phys. B312 (1989), 227.
- [47] N.E. Mavromatos and J.L. Miramontes, Phys. Lett. B212 (1988), 33;
N.E. Mavromatos, Phys. Rev. D39 (1989), 1659.
- [48] D. Espriu and N.E. Mavromatos, Phys. Lett. B237 (1990), 370.

- [49] E. Brézin and V.A. Kazakov, Phys. Lett. B236 (1990), 144;
M. Douglas and A. Shenker, Nucl. Phys. B335 (1990), 635;
D. Gross and A.A. Migdal, Phys. Rev. Lett. 64 (1990), 127;
For a recent review see, e.g., I. Klebanov, in *String Theory and Quantum Gravity*, Proc. Trieste Spring School 1991, ed. by J. Harvey et al. (World Scientific, Singapore, 1991), and references therein.
- [50] A. Jevicki and T. Yoneya, Nucl. Phys. B411 (1994), 64.
- [51] T. Eguchi, Mod. Phys. Lett. A7 (1992), 85.
- [52] S. Mukhi and C. Vafa, Harvard Univ. and Tata Inst. preprint HUTP-93/A002; TIFR/TH/93-01 (1993).
- [53] D. Kutasov, Mod. Phys. Lett. A7 (1992), 2943.
- [54] I. Kogan, Phys. Lett. B265 (1991), 269.
- [55] M. Goulian and M. Li, Phys. Rev. Lett. 66 (1991), 2051.
- [56] M. Bershadsky and D. Kutasov, Phys. Lett. B266 (1991), 345.
- [57] H. Partovi, Phys. Rev. Lett. 50 (1983), 1885;
R. Blankenbecler and H. Partovi, Phys. Rev. Lett. 57 (1986), 2887.
- [58] W. Zurek, S. Habib and J.P. Paz, Phys. Rev. Lett. 70 (1992), 1187.
- [59] A. Albrecht, Phys. Rev. D46 (1992), 5504.
- [60] G. Barton, Ann. Phys. 166 (1986), 322.
- [61] A.O. Caldeira and A.J. Leggett, Ann. Phys. 149 (1983), 374.
- [62] B.L. Hu and Y.H. Zhang, Univ. of Maryland prperint, UMD-PP-93-161 (1993)/gr-qc bulletin 930811, and references therein.
- [63] B.A. Ovrut and S. Thomas, Phys. Rev. D43 (1991), 1314.
- [64] D. Amati, M. Ciafaloni and G. Veneziano, Phys. Lett. B197 (1987), 81; *ibid* B216 (1989), 41.
- [65] T Elze, preprint CERN-TH.7131/93 (1994); hep-ph/9404215.
- [66] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B289 (1992), 25; *ibid* B296 (1992), 40.
- [67] A. Pruisken, Nucl. Phys. B290[FS20] (1987), 61.

- [68] R. Adler *et al.*, CPLEAR-Collaboration, *A study of T violation via the semileptonic Decays of Neutral Kaons in CPLEAR*, paper submitted to *Second Biennial Conference on Low-Energy Antiproton Physics (LEAP), Courmayeur 1992*;
- [69] *Daφne Physics Handbook*, edited by L. Maiani, L. Pancheri and N. Paver (INFN, Frascati, 1992).
- [70] H. Osborn, *Phys. Lett.* B222 (1989), 97.
- [71] J. Ellis and K.A. Olive, *Nature* **303** (1983) 679.
- [72] A.V. Yung, *Int. J. Mod. Phys. A*9 (1994), 591 ;
A.V. Yung, Swansea preprint SWAT 94/22 (1994).
- [73] C. Vafa, *Phys. Lett.* B212 (1988), 28.
- [74] *Review of Particle Properties*, Particle Data Group, *Phys. Rev.* D45 (No 11, part II) (1992), 1.
- [75] L.K. Gibbons, *et al.*, E731-Collaboration, *Phys. Rev. Lett.* 70 (1993), 1199 and references therein.
- [76] R.P. Feynman and F.L. Vernon Jr., *Ann. Phys. (NY)* 94 (1963), 118.

Figure Captions

Figure 1 - Contour of integration in the analytically-continued (regularized) version of $\Gamma(-s)$ for $s \in \mathbb{Z}^+$. This is known in the literature as the Saalschutz contour, and has been used in conventional quantum field theory to relate dimensional regularization to the Bogoliubov-Parasiuk-Hepp-Zimmermann renormalization method.

Figure 2 - Schematic representation of the evolution of the world-sheet area as the renormalization group scale moves along the contour of fig. 1.

This figure "fig1-1.png" is available in "png" format from:

<http://arxiv.org/ps/hep-th/9405196v1>