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Implications of the Top Quark Mass Measurement for the CKM Parameters, x_s and CP Asymmetries

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Abstract

Motivated by the recent determination of the top quark mass by the CDF collaboration, $m_t = 174 \pm 10^{+13}_{-12}$ GeV, we review and update constraints on the parameters of the quark flavour mixing matrix V_{CKM} in the standard model. In performing these fits, we use inputs from the measurements of $|\epsilon|$, the CP-violating parameter in K decays, $x_d = (\Delta M)/\Gamma$, the mixing parameter in B_d^0-$\overline{B_d^0}$ mixing, the present measurements of the matrix elements $|V_{cb}|$ and $|V_{ub}|$, and the B-hadron lifetimes. The CDF value for m_t considerably reduces the CKM-parameter space previously allowed. An interesting result of our analysis is that the present data can be used to restrict the coupling constant product ratio $f_{B_d}\sqrt{B_{B_d}}$ to the range 110-270 MeV – in comfortable agreement with existing theoretical estimates of this quantity. We use the updated CKM matrix to predict the B_s^0-$\overline{B_s^0}$ mixing ratio x_s, as well as the quantities $\sin 2\alpha$, $\sin 2\beta$ and $\sin^2\gamma$, which characterize CP-violating asymmetries in B-decays.

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1. Introduction

The CDF collaboration at Fermilab has recently published evidence for top quark production in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV. The search is based on the final states expected in the decays of the top quark in the standard model (SM). Based on this analysis a top quark mass $m_t = 174 \pm 10^{+13}_{-12}$ GeV and a production cross section $\sigma(p\bar{p} \to t\bar{t} + X) = 13.9^{+6.1}_{-4.8}$ pb have been reported [1]. The CDF value for the top quark mass is in very comfortable agreement with the prediction based on the SM electroweak fits of the LEP and SLC data, $m_t = 177 \pm 11^{+18}_{-19}$ GeV [2]. The top quark production cross section measured by CDF is roughly a factor ~ 2 larger than the expected theoretical value in QCD [1] but is consistent with the upper limit presented by the D0 collaboration: $\sigma(p\bar{p} \to t\bar{t} + X) < 13$ pb (95% C.L.) for a top quark mass of 180 GeV [3]. The neat overlap between the estimates of m_t based on the SM-electroweak analysis and its direct measurement, together with the implied dominance of the decay mode $t \to W^+ b$, is a resounding success of the standard model [4].

It is well appreciated that the top quark plays a crucial role in the phenomenology of the electroweak interactions, flavour mixing, rare decay rates and CP violation. Therefore the new experimental input for m_t, while still not very precise, should help in reducing the present uncertainties on the parameters of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [5]. Conversely, the knowledge of m_t can be used to restrict the range of the relevant hadronic matrix elements, which in turn should help in firming up SM-based predictions for rare decays and CP asymmetries in a number of K- and B-hadron decays. The aim of this article is to update the profile of the CKM matrix elements, in particular the CKM unitarity triangle, taking into account all present measurements and theoretical estimates of hadronic matrix elements, along with their uncertainties. In doing this update, we also include the improvements reported in a number of measurements of the lifetime, mixing ratio, and the CKM matrix elements $|V_{cb}|$ and $|V_{ub}/V_{cb}|$ from B decays, measured by the ARGUS, CLEO, CDF and LEP experiments. The allowed ranges for the CP-violating phases that will be measured in B decays, characterized by $\sin 2\beta$, $\sin 2\alpha$ and $\sin^2\gamma$, are also presented. They can be measured directly through asymmetries in the decays $\overset{(-)}{B_d} \to J/\psi K_S$, $\overset{(-)}{B_d} \to \pi^+\pi^-$, and in $\overset{(-)}{B_s} \to D_s^{\pm} K^{\mp}$, respectively. We also give the allowed domains for two of the angles, $(\sin 2\alpha, \sin 2\beta)$ and the SM estimates for the B_s^0-$\overline{B_s^0}$ mixing parameter, x_s.

2. An Update of the CKM Matrix

In updating the CKM matrix elements, we make use of the Wolfenstein parametrization [6], which follows from the observation that the elements of this matrix exhibit a hierarchy in terms of λ, the Cabibbo angle. In this parametrization the CKM matrix can be written approximately as

$$V_{CKM} \simeq \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 - iA^2\lambda^4\eta & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} . \quad (1)$$

We shall restrict ourselves to specifying the main input, pointing out the significant changes in the determination of the CKM parameters λ, A, ρ, and η, since we presented our earlier fits [7]-[9].

We recall that $|V_{us}|$ has been extracted with good accuracy from $K \to \pi e \nu$ and hyperon decays [10] to be

$$|V_{us}| = \lambda = 0.2205 \pm 0.0018 \ . \tag{2}$$

This agrees quite well with the determination of $V_{ud} \simeq 1 - \frac{1}{2}\lambda^2$ from β-decay:

$$|V_{ud}| = 0.9744 \pm 0.0010 \ . \tag{3}$$

The parameter A is related to the CKM matrix element V_{cb}, which can be obtained from semileptonic decays of B mesons. We shall restrict ourselves to the methods based on the heavy-quark effective theory (HQET) to calculate the exclusive and inclusive semileptonic decay rates. In the heavy quark limit it has been observed that all hadronic form factors in semileptonic decays can be expressed in terms of a single function, the Isgur-Wise function [11]. It has been shown that the HQET-based method works best for $B \to D^* l \nu$ decays, since these decays are unaffected by $1/m_b$ corrections [12, 13, 14]. Furthermore, the perturbative corrections calculated in HQET turn out to be small [14]. This method has been used by both the ARGUS and CLEO collaborations to determine $|V_{cb}|$.

Using HQET, the differential decay rate in $B \to D^* \ell \nu_\ell$ is

$$\begin{aligned}\frac{d\Gamma(B \to D^* \ell \bar{\nu})}{d\omega} &= \frac{G_F^2}{48\pi^3}(m_B - m_{D^*})^2 m_{D^*}^3 \eta_A^2 \sqrt{\omega^2 - 1}(\omega + 1)^2 \\ &\quad \times [1 + \frac{4\omega}{\omega + 1}\frac{1 - 2\omega r + r^2}{(1-r)^2}]|V_{cb}|^2 \xi^2(\omega),\end{aligned} \tag{4}$$

where $r = m_{D^*}/m_B$, $\omega = v \cdot v'$ (v and v' are the four-velocities of the B and D^* meson, respectively), and η_A is the short-distance correction to the axial vector form factor estimated to be $\eta_A = 0.99$ [14]. In the absence of any power corrections $\xi(\omega = 1) = 1$. The size of the $O(1/m_b^2)$ and $O(1/m_c^2)$ corrections to the Isgur-Wise function $\xi(\omega)$ has recently become a matter of some discussion [15, 16]. We recall that the effects of such power corrections were previously estimated as [15]:

$$\xi(1) = 1 + \delta(1/m^2) = 0.98 \pm 0.04 \tag{5}$$

In a recent paper Shifman, Uraltsev and Vainshtain [16] have argued that the deviation of $\xi(1)$ from unity is larger than the estimate given in eq. (5). Following [16], this deviation can be expressed as:

$$1 - \xi^2(1) = \frac{1}{3}\frac{\mu_G^2}{m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{4}(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}) + \sum_{i=1,2,\ldots} \xi_{B \to excit}^2, \tag{6}$$

where the contribution to the higher excited states is indicated by the last term, and μ_G^2 and μ_π^2 parametrize the matrix elements of the chromomagnetic and kinetic energy operators, respectively. These have been estimated to be:

$$\begin{aligned}\mu_G^2 &= \frac{3}{4}(M_{B^*}^2 - M_B^2) \simeq 0.35 \ \mathrm{GeV}^2, \\ \mu_\pi^2 &= (0.54 \pm 0.12) \ \mathrm{GeV}^2 \ ,\end{aligned} \tag{7}$$

where the numbers for μ_π^2 are based on QCD sum rules [17]. Using the central value for this quantity and ignoring the contribution of the excited states, one gets

$$\eta_A \xi(1) = 0.92 \ . \tag{8}$$

The contribution of the higher states is positive definite. However, its actual value can only be guessed at present. Shifman et. al. estimate [16]:

$$\eta_A \xi(1) = 0.89 \pm 0.3 \ . \tag{9}$$

The values of $\eta_A \xi(1)$ given in eqs. (8) and (9) are substantially smaller than the estimates given in eq. (5) (with $\eta_A = 0.99$) and used in previous theoretical and experimental analyses. If the estimates by Shifman et. al. are valid then one must conclude that the $O(1/m_Q^2)$ corrections to $\xi(1)$ are not as innocuous as claimed previously! In the analysis for $|V_{cb}|$ presented here, we shall use the maximum allowed value for $\eta_A \xi(1)$ in the estimate of eq. (9), i.e. $\eta_A \xi(1) = 0.92$. Clearly, a better theoretical calculation for the excited states is needed which might be forthcoming as the contribution of the inelastic channels in semileptonic B decays is measured more accurately.

Not only are theoretical estimates for $\xi(1)$ in a state of flux, so are experimental numbers! The previously reported value for $|V_{cb}|$ by the ARGUS collaboration from the decays $B \to D^* + \ell \bar{\nu}$ using the HQET formalism yielded a value $|V_{cb}| = 0.047 \pm 0.007$ with a considerably higher value for the slope of the Isgur-Wise function, $\xi'(1) \equiv -\rho^2$, in the range $1.9 < \rho^2 < 2.3$ [18]. In a recent analysis by ARGUS, significantly lower values for both $|V_{cb}|$ and ρ^2 have been obtained, yielding [19]:

$$\begin{aligned} |V_{cb}|\left(\frac{\tau(B_d^0)}{1.53 \text{ ps}}\right)^{1/2} &= 0.039 \pm 0.005 \ , \\ \rho^2 &= 1.08 \pm 0.12 \ , \end{aligned} \tag{10}$$

where the value of $|V_{cb}|$ corresponds to a linear extrapolation of the Isgur-Wise function $\xi(\omega) = 1 - \rho^2(\omega - 1)$ and the error quoted includes also that from the B_d^0 lifetime. The slope parameter in eq. (10) is now in agreement with the theoretical bounds, which suggest $\rho^2 \leq 1$ [20, 21].

The numbers obtained by the CLEO collaboration from a similar method are [22]:

$$\begin{aligned} |V_{cb}|\left(\frac{\tau(B_d^0)}{1.5 \text{ ps}}\right)^{1/2} &= 0.039 \pm 0.006, \\ & 1.0 \pm 0.04 < \rho^2 < 1.2 \pm 0.7. \end{aligned} \tag{11}$$

where no constraints on the slope at zero recoil are assumed. The two results are in remarkable agreement!

Using the ARGUS and CLEO values of $|V_{cb}|$ given in eqs. (10) and (11), renormalizing the Isgur-Wise function to $\eta_A \xi(1) = 0.92$, and using the LEP value for the mean B lifetime, $\langle \tau_B \rangle = 1.54 \pm 0.03 \ ps$, the updated values for $|V_{cb}|$ can be expressed as†:

$$|V_{cb}|\left(\frac{\tau(B_d^0)}{1.54\text{ps}}\right)^{1/2}\left(\frac{\eta_A \xi(1)}{0.92}\right) = 0.041 \pm 0.006 \tag{12}$$

†If the B_d^0 lifetime, $\tau(B_d^0) = 1.52 \pm 0.11 \ ps$ is used instead, this does not change the result significantly.

The corresponding analysis for the inclusive semileptonic B decays incorporating the power corrections given in eq. (8) above has also been undertaken by Shifman et. al. [16]. Updating the B-lifetime, their estimate of $|V_{cb}|$ can be expressed as:

$$|V_{cb}| = 0.041(\frac{1.54 \text{ ps}}{\tau(B_d^0)})^{1/2}(\frac{\mathcal{B}_{SL}(B)}{0.106})^{1/2}(\frac{4.8 \text{ GeV}}{m_b})^5 \qquad (13)$$

in which an error of $\pm 3\%$, mostly due to the uncertainty in the b quark pole mass, or, equivalently, on $(m_b - m_c)$, has been anticipated [16]. The error estimate notwithstanding, the HQET-based analyses of the exclusive and inclusive semileptonic B decays are in better quantitative agreement than they have a right to be!

For the purposes of the fits which follow, we shall use the value of the CKM parameter A obtained from the above HQET-based methods:

$$A = 0.84 \pm 0.12 \ . \qquad (14)$$

The other two CKM parameters ρ and η are constrained by the measurements of $|V_{ub}/V_{cb}|$, $|\epsilon|$ (the CP-violating parameter in the kaon system), x_d (B_d^0-$\overline{B_d^0}$ mixing) and (in principle) ϵ'/ϵ ($\Delta S = 1$ CP-violation in the kaon system). We shall not discuss the constraints from ϵ'/ϵ, due to the various experimental and theoretical uncertainties surrounding it at present, but take up the rest in turn and present fits in which the allowed region of ρ and η is shown.

First of all, $|V_{ub}/V_{cb}|$ can be obtained by looking at the endpoint of the inclusive lepton spectrum in semileptonic B decays. The present average of this ratio, based on the recent analysis of the ARGUS [23] and CLEO [24, 25] data, is [9, 26]:

$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.08 \pm 0.02 \ . \qquad (15)$$

This gives

$$\sqrt{\rho^2 + \eta^2} = 0.36 \pm 0.09 \ . \qquad (16)$$

The experimental value of $|\epsilon|$ is [10]

$$|\epsilon| = (2.26 \pm 0.02) \times 10^{-3} \ . \qquad (17)$$

Theoretically, $|\epsilon|$ is essentially proportional to the imaginary part of the box diagram for K^0-$\overline{K^0}$ mixing and is given by [27]

$$\begin{aligned} |\epsilon| \ &= \ \frac{G_F^2 f_K^2 M_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K} B_K \left(A^2\lambda^6\eta\right) \left(y_c \left\{\eta_{ct} f_3(y_c, y_t) - \eta_{cc}\right\}\right. \\ &\qquad \left. + \eta_{tt} y_t f_2(y_t) A^2 \lambda^4 (1-\rho)\right). \qquad (18) \end{aligned}$$

Here, the η_i are QCD correction factors, $\eta_{cc} \simeq 0.82$, $\eta_{tt} \simeq 0.62$, $\eta_{ct} \simeq 0.35$ for $\Lambda_{QCD} = 200$ MeV [28], $y_i \equiv m_i^2/M_W^2$, and the functions f_2 and f_3 are given by

$$\begin{aligned} f_2(x) \ &= \ \frac{1}{4} + \frac{9}{4}\frac{1}{(1-x)} - \frac{3}{2}\frac{1}{(1-x)^2} - \frac{3}{2}\frac{x^2 \ln x}{(1-x)^3} \ , \\ f_3(x,y) \ &= \ \ln\frac{y}{x} - \frac{3y}{4(1-y)}\left(1 + \frac{y}{1-y}\ln y\right) . \qquad (19) \end{aligned}$$

(The above form for $f_3(x,y)$ is an approximation, obtained in the limit $x \ll y$. For the exact expression, see ref. [29].)

The final parameter in the expression for $|\epsilon|$ is B_K, which represents our ignorance of the matrix element $\langle K^0|(\bar{d}\gamma^\mu(1-\gamma_5)s)^2|\overline{K^0}\rangle$. The evaluation of this matrix element has been the subject of much work. The results are summarized in ref. [7]. Considering all the various calculational techniques, one is led to the range $1/3 \le B_K \le 1$. However, the $1/N$ and lattice approaches are generally considered the most reliable. They yield:

$$B_K = 0.8 \pm 0.2. \tag{20}$$

We now turn to B_d^0-$\overline{B_d^0}$ mixing. The latest value of x_d, which is a measure of this mixing, is [30]

$$x_d = 0.71 \pm 0.07 \ . \tag{21}$$

The mixing parameter x_d is calculated from the B_d^0-$\overline{B_d^0}$ box diagram. Unlike the kaon system, where the contributions of both the c- and the t-quarks in the loop were important, this diagram is dominated by t-quark exchange:

$$x_d \equiv \frac{(\Delta M)_B}{\Gamma} = \tau_B \frac{G_F^2}{6\pi^2} M_W^2 M_B \left(f_{B_d}^2 B_{B_d}\right) \eta_B y_t f_2(y_t) |V_{td}^* V_{tb}|^2 \ , \tag{22}$$

where, using eq. 1, $|V_{td}^* V_{tb}|^2 = A^2\lambda^6\left[(1-\rho)^2+\eta^2\right]$. Here, η_B is the QCD correction. In ref. [31], this correction is analyzed including the effects of a heavy t-quark. It is found that η_B depends sensitively on the definition of the t-quark mass, and that, strictly speaking, only the product $\eta_B(y_t)f_2(y_t)$ is free of this dependence. In the fits presented here we use the value $\eta_B = 0.55$, following ref. [31].

For the B system, the hadronic uncertainty is given by $f_{B_d}^2 B_{B_d}$, analogous to B_K in the kaon system, except that in this case, also f_{B_d} is not measured. Most lattice-QCD based estimates, as well as those from the QCD sum rules, are compatible with the following ranges for $f_{B_d}^2 B_{B_d}$ and B_{B_d} [32, 33]:

$$\begin{aligned} f_{B_d} &= 180 \pm 50 \text{ MeV}, \\ B_{B_d} &= 1.0 \pm 0.2 \end{aligned} \tag{23}$$

3. The unitarity triangle

The allowed region in ρ-η space can be displayed quite elegantly using the so-called unitarity triangle. The unitarity of the CKM matrix leads to the following relation:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \ . \tag{24}$$

Using the form of the CKM matrix in eq. 1, this can be recast as

$$\frac{V_{ub}^*}{\lambda V_{cb}} + \frac{V_{td}}{\lambda V_{cb}} = 1 \ , \tag{25}$$

which is a triangle relation in the complex plane (i.e. ρ-η space), illustrated in Fig. 1. Thus, allowed values of ρ and η translate into allowed shapes of the unitarity triangle.

Figure 1: The unitarity triangle. The angles α, β and γ can be measured via CP violation in the B system.

In order to find the allowed unitarity triangles, the computer program MINUIT is used to fit the CKM parameters A, ρ and η to the experimental values of $|V_{cb}|$, $|V_{ub}/V_{cb}|$, $|\epsilon|$ and x_d. Since λ is very well measured, we have fixed it to its central value given above. The new ingredient in these fits is the value of the top quark, for which we use the value $m_t = (174 \pm 16)$ GeV, measured by CDF [1]. We present here two types of fits:

- Fit 1: the "experimental fit." Here, only the experimentally measured numbers are used as inputs to the fit with Gaussian errors; the coupling constant $f_{B_d}\sqrt{B_{B_d}}$ and B_K are given fixed values.

- Fit 2: the "combined fit." Here, both the experimental and theoretical numbers are used as inputs assuming Gaussian errors for the theoretical quantities.

We first discuss the "experimental fit" (Fit 1). The goal here is to restrict the allowed ranges of the CKM parameters (ρ, η) for given values of the coupling constants, $f_{B_d}\sqrt{B_{B_d}}$ and B_K. We first fix the bag factor to the value $B_K = 0.8$ and vary the value of the coupling constant $f_{B_d}\sqrt{B_{B_d}}$. The effect of varying the bag factor B_K in the range $B_K = 0.8 \pm 0.2$ is not crucial and will be discussed later. The resulting fits for fixed values of $f_{B_d}\sqrt{B_{B_d}}$ are shown in Fig. 2. In all these graphs, the solid line has $\chi^2 = \chi^2_{min} + 1$. Note that this corresponds to only a 39% confidence level region [10]! For comparison, we include the dashed line, which is the 90% C.L. region ($\chi^2 = \chi^2_{min} + 4.6$). As we pass from Fig. 2(a) to Fig. 2(e), the most likely unitarity triangles become more and more obtuse. This behaviour has already been anticipated [7, 34, 35]. However, unlike the previous such analyses, now that the top quark mass has been measured, the dependence of the CKM triangle on the coupling constant product $f_{B_d}\sqrt{B_{B_d}}$ can be disentangled from that on m_t. As shown in these fits, the allowed region in (ρ, η)-space is now quite restricted for a given value of the coupling constant. This underscores the importance of measuring this quantity, for example through the decays $B^\pm \to \tau^\pm \nu$.

The most probable values of the parameters (ρ, η) are given in Table 1, together with their χ^2. We remark that the values $f_{B_d}\sqrt{B_{B_d}} \leq 110$ MeV and $f_{B_d}\sqrt{B_{B_d}} \geq 250$ MeV give very poor fits of the existing data. The minimum and maximum allowed values of $f_{B_d}\sqrt{B_{B_d}}$ are somewhat correlated with the value of B_K, which is the only

$f_{B_d}\sqrt{B_{B_d}}$ (MeV)	(ρ, η)	χ^2_{min}
130	$(-0.33,\ 0.18)$	0.10
155	$(-0.27,\ 0.27)$	0.14
180	$(-0.05,\ 0.33)$	0.33
205	$(0.15,\ 0.32)$	0.03
230	$(0.28,\ 0.30)$	0.39

Table 1: The "best values" of the CKM parameters (ρ, η) as a function of the coupling constant $f_{B_d}\sqrt{B_{B_d}}$, obtained by a minimum χ^2 fit to the experimental data discussed in the text including the CDF value $m_t = 174 \pm 16$ GeV. We fix $B_K = 0.8$. The resulting minimum χ^2 values from the MINUIT fits are also given.

remaining theoretical parameter of the fit. For lower values of B_K, higher values of $f_{B_d}\sqrt{B_{B_d}}$ are disfavoured, while for higher values of B_K, somewhat higher values of the coupling constant are allowed. Specifically, for $B_K = 0.6$, the allowed range is 110 MeV $\leq f_{B_d}\sqrt{B_{B_d}} \leq$ 220 MeV, whereas for $B_K = 1.0$, the corresponding range is 110 MeV $\leq f_{B_d}\sqrt{B_{B_d}} \leq$ 270 MeV. For the lower value $B_K = 0.4$, which is not favoured by the lattice and QCD sum rules, the allowed range of $f_{B_d}\sqrt{B_{B_d}}$ is restricted to the range 110-180 MeV, with generally higher values of χ^2 than for the case B_K in the range 0.6-1.0. This suggests that present data disfavour (though do not exclude) $B_K \leq 0.4$ solutions. Summing up, present data exclude all values of $f_{B_d}\sqrt{f_{B_d}}$ which lie below 110 MeV and above 270 MeV for the entire B_K range.

One notices in Table 1 that certain values of the coupling constant $f_{B_d}\sqrt{B_{B_d}}$ give smaller χ^2 than others. Indeed, as one scans through the allowed parameter space in the coupling constants, one obtains a double-valleyed solution corresponding to two minima in χ^2. For example, for $B_K = 0.8$, there are minima in the χ^2 distribution at $f_{B_d}\sqrt{B_{B_d}} = 140$ and 210 MeV. We do not believe, however, that any exciting conclusions can be drawn from this observation. The other values of $f_{B_d}\sqrt{B_{B_d}}$ in the range 110-250 MeV also give quite good fits to the data, so that the presence of the minima at 140 and 210 MeV is not statistically significant.

We now discuss the "combined fit" (Fit 2). Strictly speaking, this fit is not on the same footing as the "experimental fit" presented above, since theoretical "errors" are not Gaussian. On the other hand, experimental systematic errors are also not Gaussian, but it is common practice to treat them as such, and to add them in quadrature with statistical errors. In this sense, the method used in this fit is not unreasonable. Since the coupling constants are not known and the best we have are estimates given in the ranges in eqs. (20) and (23), which are allowed by data, a reasonable profile of the unitarity triangle at present can be obtained by letting the coupling constants vary in this range. The resulting CKM triangle is shown in Fig. 3, which still leaves quite a bit of uncertainty in the (ρ, η)-space, though it is much reduced compared to the previous such analyses, due to the knowledge of m_t. The preferred values obtained from the "combined fit" are

$$(\rho, \eta) = (0.14, 0.32) \quad (\text{with } \chi^2 = 0.17). \tag{26}$$

Figure 2: Allowed region in ρ-η space, from a fit to the experimental values given in the text, including $m_t = 174 \pm 16$ GeV. We have fixed $B_K = 0.8$ and vary the coupling constant product $f_{B_d}\sqrt{B_{B_d}}$ as indicated on the figures. The solid line represents the region with $\chi^2 = \chi^2_{min} + 1$; the dashed line denotes the 90% C.L. region. The triangles show the best fit.

Figure 3: Allowed region in ρ-η space from a simultaneous fit to both the experimental values given in the text (including $m_t = 174 \pm 16$ GeV) and the theoretical quantities B_K and $f_{B_d}\sqrt{B_{B_d}}$. The theoretical errors are treated as Gaussian for this fit. The solid line represents the region with $\chi^2 = \chi^2_{min} + 1$; the dashed line denotes the 90% C.L. region. The triangles show the best fit.

The resulting unitarity triangle is almost identical to the one for the "experimental fit" (Fit 1) with $f_{B_d}\sqrt{B_{B_d}} = 205$ MeV, shown in Fig. 2(d), which is suggestive but not compelling.

4. x_s and the unitarity triangle

Mixing in the B_s^0-$\overline{B_s^0}$ system is quite similar to that in the B_d^0-$\overline{B_d^0}$ system. The B_s^0-$\overline{B_s^0}$ box diagram is again dominated by t-quark exchange, and the mixing parameter x_s is given by a formula analogous to that of eq. (22):

$$x_s \equiv \frac{(\Delta M)_{B_s}}{\Gamma_{B_s}} = \tau_{B_s} \frac{G_F^2}{6\pi^2} M_W^2 M_{B_s} \left(f_{B_s}^2 B_{B_s} \right) \eta_{B_s} y_t f_2(y_t) |V_{ts}^* V_{tb}|^2 \ . \tag{27}$$

Using the fact that $|V_{cb}| = |V_{ts}|$ (eq. 1), it is clear that one of the sides of the unitarity triangle, $|V_{td}/\lambda V_{cb}|$, can be obtained from the ratio of x_d and x_s:

$$\frac{x_s}{x_d} = \frac{\tau_{B_s} \eta_{B_s} M_{B_s} \left(f_{B_s}^2 B_{B_s} \right)}{\tau_{B_d} \eta_{B_d} M_{B_d} \left(f_{B_d}^2 B_{B_d} \right)} \left| \frac{V_{ts}}{V_{td}} \right|^2 . \tag{28}$$

All dependence on the t-quark mass drops out, leaving the square of the ratio of CKM matrix elements, multiplied by a factor which reflects $SU(3)_{flavour}$ breaking effects. The only real uncertainty in this factor is the ratio of hadronic matrix elements. Whether or not x_s can be used to help constrain the unitarity triangle will depend crucially on the theoretical status of the ratio $f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d}$.

The lifetime of the B_s meson has now been measured at LEP and Tevatron. The present average for the B_s^0 lifetimes is [36]:

$$\tau_{B_s^0} = (1.50 \pm 0.18) \times 10^{-12} \text{ s} \ . \tag{29}$$

Within the experimental errors, this value is consistent with the averaged value of τ_B used in the previous section. The mass of the B_s^0 meson has also now been measured and its present best measurement is from CDF: $\langle M_{B_s}\rangle = 5367.7 \pm 2.4 \pm 4.8$ MeV [37], very close to the ALEPH measurement $\langle M_{B_s}\rangle = 5368.6 \pm 5.6 \pm 1.5$ MeV [38], with the DELPHI result $\langle M_{B_s}\rangle = 5374.6 \pm 16 \pm 2$ MeV [39] quite compatible with the other two. We expect the QCD correction η_{B_s} to be equal to its B_d counterpart, i.e. $\eta_{B_s} = 0.55$. The main uncertainty in x_s is now $f_{B_s}^2 B_{B_s}$. Using the determination of A above, τ_{B_s} from eq. (29) and $m_t = 174 \pm 16$ GeV, we obtain

$$x_s = (428 \pm 147)\,\frac{f_{B_s}^2 B_{B_s}}{(1\ \mathrm{GeV})^2}. \tag{30}$$

With $f_{B_s}\sqrt{B_{B_s}} = 170 - 300$ MeV, this gives at 90% C.L. (defined as 1.64σ),

$$5.4 \le x_s \le 60.2. \tag{31}$$

The standard model therefore predicts very large values for x_s.

Another estimate can be obtained by using the relation in eq. (28). Two quantities are required. First, we need the CKM ratio $|V_{ts}/V_{td}|$. From our "experimental fit," we have obtained the 90% C.L. bound on the inverse of this ratio as a function of $f_{B_d}\sqrt{B_{B_d}}$. This is shown in Fig. 4. From this we find

$$2.91 \le \left|\frac{V_{ts}}{V_{td}}\right| \le 8.06 \tag{32}$$

The second ingredient is the SU(3)-breaking ratio $f_{B_s}^2 B_{B_s}/f_{B_d}^2 B_{B_d}$. Estimating this to be 1.2 – 1.5, we determine at 90% C.L.:

$$7.2 \le x_s \le 52. \tag{33}$$

for $x_d = 0.71$ [‡]. Thus, the two ranges of x_s are quite comparable and a value of $x_s \le 5$ is disfavoured in SM.

5. CP Violation in the B System

It is expected that the B system will exhibit large CP-violating effects, characterized by nonzero values of the angles α, β and γ in the unitarity triangle (Fig. 1) [40]. These angles can be measured via CP-violating asymmetries in hadronic B decays. In the decays $\overset{(-)}{B_d} \to \pi^+\pi^-$, for example, one measures the quantity $\sin 2\alpha$, and in $\overset{(-)}{B_d} \to J/\psi K_S$, $\sin 2\beta$ is obtained. The CP asymmetry in the decay $\overset{(-)}{B_s} \to D_s^{\pm} K^{\mp}$ is slightly different, yielding $\sin^2\gamma$.

These CP-violating asymmetries can be expressed straightforwardly in terms of the CKM parameters ρ and η. The 90% C.L. constraints on ρ and η found previously can be used to predict the ranges of $\sin 2\alpha$, $\sin 2\beta$ and $\sin^2\gamma$ allowed in the standard model. The allowed ranges which correspond to each of the figures in Fig. 2, obtained from the "experimental fit" (Fit 1), are found in Table 2. In this Table we have assumed that the angle β is measured in $\overset{(-)}{B_d} \to J/\Psi K_S$, and have therefore included the extra minus sign due to the CP of the final state.

‡ Folding in also the present experimental error on x_d, the corresponding range is $6.1 \le x_s \le 79.9$.

Figure 4: Allowed values of the CKM matrix element ratio $|V_{td}/V_{ts}|$ as a function of the coupling constant product $f_{B_d}\sqrt{B_{B_d}}$, from the "experimental fit" shown in Fig. 2. The solid line corresponds to the best fit values and the dashed curves correspond to the maximum and minimum allowed values at 90 % C.L.

$f_{B_d}\sqrt{B_{B_d}}$ (MeV)	$\sin 2\alpha$	$\sin 2\beta$	$\sin^2\gamma$
130	0.43 - 0.98	0.18 - 0.41	0.1 - 0.56
155	0.42 - 1.0	0.28 - 0.62	0.29 - 1.0
180	-1.0 - 1.0	0.33 - 0.78	0.2 - 1.0
205	-1.0 - 0.91	0.38 - 0.87	0.14 - 1.0
230	-1.0 - 0.53	0.46 - 0.92	0.12 - 0.97

Table 2: The allowed ranges for the CP asymmetries $\sin 2\alpha$, $\sin 2\beta$ and $\sin^2\gamma$, corresponding to the constraints on ρ and η shown in Figs. 2. Values of the coupling constant $f_{B_d}\sqrt{B_{B_d}}$ are stated. The range for $\sin 2\beta$ includes an additional minus sign due to the CP of the final state $J/\Psi K_S$.

Since the CP asymmetries all depend on ρ and η, the ranges for $\sin 2\alpha$, $\sin 2\beta$ and $\sin^2 \gamma$ shown in Table 2 are correlated. That is, not all values in the ranges are allowed simultaneously. We illustrate this in Fig. 5, corresponding to the "experimental fit" (Fit 1), by showing the region in $\sin 2\alpha$-$\sin 2\beta$ space allowed by the data, for various values of $f_{B_d}\sqrt{B_{B_d}}$. Given a value for $f_{B_d}\sqrt{B_{B_d}}$, the CP asymmetries are fairly constrained. However, since there is still considerable uncertainty in the values of the coupling constants, a more reliable profile of the CP asymmetries at present is given by our "combined fit" (Fit 2), where we convolute the present theoretical and experimental values in their allowed ranges. The resulting correlation is shown in Fig. 6. From this figure one sees that the smallest value of $\sin 2\beta$ occurs in a small region of parameter space around $\sin 2\alpha \simeq 0.4$-0.6. Excluding this small tail, one expects the CP-asymmetry in $\overset{(-)}{B_d} \to J/\Psi K_S$ to be at least 30%.

6. Summary and Outlook

We summarize our results:

(i) We have presented an update of the CKM unitarity triangle following from the additional experimental input of $m_t = 174 \pm 16$ GeV [1]. The fits can be used to exclude extreme values of the pseudoscalar coupling constants, with the range 110 MeV $\leq f_{B_d}\sqrt{B_{B_d}} \leq 270$ MeV, still allowed for all values of B_K. The solutions for $B_K = 0.8 \pm 0.2$ are favoured by the data as compared to the lower values. These numbers are in very comfortable agreement with QCD-based estimates from sum rules and lattice techniques. The statistical significance of the fit is, however, not good enough to determine the coupling constant more precisely.

(ii) The allowed CKM unitarity triangle in the (ρ, η)-space is more restricted than obtained previously without the top quark mass input. However, the present uncertainties are still large unless the pseudoscalar coupling constant could be determined independently. It may be possible to measure the parameter f_{B_d}, using isospin symmetry, via the charged-current decay $B_u^\pm \to \tau^\pm \nu_\tau$. With $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ and $f_{B_d} = 180 \pm 50\ MeV$, one gets a branching ratio $BR(B_u^\pm \to \tau^\pm \nu_\tau) = (0.3\text{-}2.4) \times 10^{-4}$, which lies in the range of the future LEP and asymmetric B-factory experiments. Along the same lines, the prospects for measuring (f_{B_d}, f_{B_s}) in the FCNC leptonic and photonic decays of B_d^0 and B_s^0 hadrons, $(B_d^0, B_s^0) \to \mu^+\mu^-, (B_d^0, B_s^0) \to \gamma\gamma$ in future B physics facilities are not entirely dismal [9].

(iii) We have determined bounds on the ratio $|V_{td}/V_{ts}|$ from our fits. For 110 MeV $\leq f_{B_d}\sqrt{B_{B_d}} \leq 270$ MeV, i.e. in the entire allowed domain, we find at 90 % C.L.:

$$0.12 \leq \left|\frac{V_{td}}{V_{ts}}\right| \leq 0.34 \ . \tag{34}$$

The upper bound from our analysis is more restrictive than the current experimental upper limit following from the CKM-suppressed radiative penguin decays $\mathcal{B}(B \to \omega + \gamma)$ and $\mathcal{B}(B \to \rho + \gamma)$, which at present yield at 90% C.L. [41]:

$$\left|\frac{V_{td}}{V_{ts}}\right| \leq \frac{1}{1.8} \ . \tag{35}$$

Figure 5: Allowed values of the CP asymmetries $\sin 2\alpha$ and $\sin 2\beta$ resulting from the "experimental fit" of the data for different values of the coupling constant $f_{B_d}\sqrt{B_{B_d}}$ indicated on the figures a) – e).

Figure 6: Allowed values of the CP asymmetries $\sin 2\alpha$ and $\sin 2\beta$ resulting from the "combined fit" of the data for the ranges for $f_{B_d}\sqrt{B_{B_d}}$ and B_K given in the text.

Furthermore, both the upper and lower bounds are better than those obtained from unitarity, which gives $0.06 \leq |V_{td}/V_{ts}| \leq 0.59$ [10].

(iv) Using the measured value of m_t, we find

$$x_s = (428 \pm 147)\,\frac{f_{B_s}^2 B_{B_s}}{(1\ \mathrm{GeV})^2}. \tag{36}$$

For $f_{B_s}\sqrt{B_{B_s}} = 170 - 300$ MeV, this gives

$$5.4 \leq x_s \leq 60.2 \tag{37}$$

at 90% C.L.

(v) The ranges for the CP-violating asymmetries $\sin 2\alpha$, $\sin 2\beta$ and and $\sin^2\gamma$ are determined to be at 90% C.L.:

$$\begin{aligned} -1.0 &\leq \sin 2\alpha \leq 1.0\ , \\ 0.16 &\leq \sin 2\beta \leq 0.91\ , \\ 0.09 &\leq \sin^2\gamma \leq 1.0\ . \end{aligned} \tag{38}$$

(For $\sin 2\alpha < 0.4$, we find $\sin 2\beta \geq 0.3$.)

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References

[1] F. Abe et al. (CDF Collaboration) FERMILAB-PUB-94/097-E; CDF/PUB/TOP/PUBLIC/2561 (1994).

[2] The LEP Collaborations ALEPH, DELPHI, L3 and OPAL and the LEP Electroweak Working Group, Preprint CERN/PPE/93-157 (1993) and updates presented at the Moriond Workshop, March, 1994;
M. Swartz, in *Proc. XVI Int. Symp. on Lepton-Photon Interactions*, Ithaca, NY (1993).

[3] F. Abachi et al. (D0 Collaboration), Phys. Rev. Lett. **72** (1994) 2138; P. Grannis, CERN Particle Physics Seminar, 3 May 1994.

[4] S.L. Glashow, Nucl. Phys. **22** (1961) 579; A. Salam in Elementary Particle Theory, ed. N. Svartholm (Almqvist and Wiksell, Stockholm, 1968); S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264.

[5] N. Cabibbo, Phys. Rev. Lett. **10** (1963) 531; M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652.

[6] L. Wolfenstein, Phys. Rev. Lett. **51** (1983) 1945.

[7] A. Ali and D. London, J. Phys. G: Nucl. Part. Phys. **19** (1993) 1069.

[8] A. Ali and D. London, DESY Report 93-016(1993) and in B-Physics Working Group Report, ECFA Workshop on a European B Meson factory; eds. R. Aleksan and A. Ali, ECFA 93/151, DESY 93-053 (1993).

[9] A. Ali, CERN-TH.7168/94 (1994), to appear in B Decays (2. Edition), ed. S. Stone (World Scientific, Singapore, 1994).

[10] K. Hikasa et al. (Particle Data Group), Phys. Rev. **D45** (1992) 1.

[11] N. Isgur and M. Wise, Phys. Lett. **B232** (1989) 113; **B237** (1990) 527.

[12] M.E. Luke, Phys. Lett. **B252** (1990) 447.

[13] C.G. Boyd and D.E. Brahm, Phys. Lett. **B257** (1991) 393.

[14] M. Neubert and V. Rieckert, Nucl. Phys. **B382** (1992) 97; M. Neubert, Phys. Lett. **B264** (1991) 455, Phys. Rev. **D46** (1992) 2212.

[15] M. Neubert, CERN-TH.7255/94 (1994); to appear in the Proceedings of TASI-93, Boulder, Colorado, 1993.

[16] M. Shifman, N.G. Uraltsev and A. Vainshtain, Preprint TPI-MINN-94/13-T (1994).

[17] P. Ball and V. Braun, Phys. Rev. **D49** (1994) 2472.

[18] H. Albrecht et al. (ARGUS Collaboration), Phys. Lett. **B275** (1992) 195; Z. Phys. **C57** (1993) 533.

[19] H. Albrecht et al. (ARGUS Collaboration), Phys. Lett. **B324** (1994) 249; H. Schröder (private communication).

[20] J.D. Bjorken, in Gauge Bosons and Heavy Quarks, Proceedings of 18th SLAC Summer Institute on Particle Physics, Stanford, California, 1990, ed. J.F. Hawthorne (SLAC Report No. 378, Stanford, 1991).

[21] M.B. Voloshin, Phys. Rev. **D46** (1992) 3062.

[22] G. Crawford et al. (CLEO Collaboration), CLEO CONF 93-30, submitted to *XVI Int. Symp. on Lepton-Photon Interactions*, Ithaca, NY (1993).

[23] H. Albrecht et al. (ARGUS Collaboration), Phys. Lett. **B234** (1991) 297.

[24] R. Fulton et al. (CLEO Collaboration), Phys. Rev. Lett. **64** (1990) 16.

[25] J. Bartelt et al. (CLEO Collaboration), Phys. Rev. Lett. **71** (1993) 4111.

[26] S. Stone, to appear in B Decays (2. Edition), ed. S. Stone (World Scientific, Singapore, 1994).

[27] A.J. Buras, W. Słominski and H. Steger, Nucl. Phys. **B238** (1984) 529; **B245** (1984) 369.

[28] J. Flynn, Mod. Phys. Lett. **A5** (1990) 877.

[29] T. Inami and C.S. Lim, Prog. Theor. Phys. **65** (1981) 297.

[30] M. Danilov, in *Proc. of the Europhysics Conference on High Energy Physics*, Marseille (1993), and private communication.

[31] A.J. Buras, M. Jamin and P.H. Weisz, Nucl. Phys. **B347** (1990) 491.

[32] R. Sommer, DESY Report 94-011 (1994).

[33] S. Narison, Preprint CERN-TH.7091/93 (1993);
S. Narison and A. Pivovarov, Preprint CERN-TH.7140/94 (1994).

[34] G. Martinelli, in B-Physics Working Group Report, ECFA Workshop on a European B Meson factory; eds. R. Aleksan and A. Ali, ECFA 93/151, DESY 93-053 (1993).

[35] M. Schmidtler and K.R. Schubert, Z. Phys. **C53** (1992) 347; A. J. Buras and M.K. Harlander, MPI-PAE/PTh 1/92, TUM-T31-25/92, and in *Heavy Flavors* eds. A.J. Buras and M. Lindner (World Scientific, Singapore, 1992).

[36] V. Sharma and F. Weber, to appear in B Decays (2. Edition), ed. S. Stone (World Scientific, Singapore, 1994) and update.

[37] F. Abe et al. (CDF Collaboration), Phys. Rev. Lett. **71** (1993) 165; J. Mueller (CDF Collaboration), Invited talk at the Beauty '94 Workshop, Le Mont St. Michel, April 1994.

[38] D. Buskulic et al. (ALEPH Collaboration), Phys. Lett. **B311** (1993) 425.

[39] P. Abreu et al. (DELPHI Collaboration), Preprint CERN-PPE/94-22 (1994).

[40] For reviews, see, for example, Y. Nir and H.R. Quinn in "B Decays," ed. S. Stone (World Scientific, Singapore, 1992), p. 362; I. Dunietz, *ibid* p. 393.

[41] D. Payne (CLEO Collaboration), Invited talk at the Beauty '94 Workshop, Le Mont St. Michel, April 1994.