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The Scattering of Strings in a Black-Hole Background

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Abstract

In this paper, correlation functions of tachyons in the two-dimensional black-hole background are studied. The results are shown to be consistent with the prediction of the deformed matrix model up to external leg factors.

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1 INTRODUCTION

In this letter, I will give some results on tachyon scattering amplitudes in the background of a two-dimensional black hole [1, 2]. For this purpose, I will use the techniques developed in [3] based on Ward identities. The Ward identities will provide powerful constraints on the correlation functions that are otherwise so hard to calculate. The black hole will be constructed as a perturbation around the ordinary linear dilaton vacuum. Indeed, in [4] it was shown that the action of the $SL(2,R)/U(1)$ coset model [2], which describes a two-dimensional black hole, can be written as

$$\frac{1}{2\pi}\int(\partial X\bar{\partial}X+\partial\phi\bar{\partial}\phi-\sqrt{2}R\phi+M\mathcal{B}), \tag{1}$$

where

$$\mathcal{B}\sim(3i\partial X+\partial\phi)(3i\bar{\partial}X+\bar{\partial}\phi)e^{-2\sqrt{2}\phi} \tag{2}$$

is the 'black-hole screener'. For further discussion, see [5]. In the brave spirit of [6], I will consider tachyon correlation functions in the presence of an integer number of such black-hole screeners. The hope is that this will teach us something about the true tachyon correlation functions in the black-hole background. However, this naïve perturbative approach has severe limitations. This will be apparent in the form of the leg factors. I will return to this important issue in the last section.

The main purpose of this letter is to show that the correlation functions of the deformed matrix model [7–13] are, in a highly non-trivial way, consistent with the black-hole Ward identities. More details will be given in a future publication.

2 WARD IDENTITIES IN TWO-DIMENSIONAL STRING THEORY

2.1 Without Black-Hole Screeners

In [3] Ward identities were derived, which related tachyon scattering amplitudes with different numbers of tachyons. These Ward identities were shown to produce recursion relations that determined all the amplitudes. Compared to more standard calculations, see e.g. [14], this was a remarkable simplification. Below I will briefly summarize the results of [3].

The Ward identities are derived by using the charges

$$Q_{m,-m} = \oint \frac{dz}{2\pi i} W_{J,m} - \oint \frac{d\bar{z}}{2\pi i} c\Psi_{m+1,-m}\bar{X}_{m,-m}, \tag{3}$$

where

$$W_{J,m} = \Psi_{J+1,m}(z)\bar{\mathcal{O}}_{J,m}(\bar{z}), \tag{4}$$

first constructed by Witten in [15]. The fields Ψ correspond to the gravitationally dressed primary fields

$$\Psi_{J,m} = \psi_{J,m}(z)e^{\sqrt{2}(-1+J)\phi(z)}. \tag{5}$$

The $\mathcal{O}$'s are elements of the ground ring. The X-fields have ghost number -1, but their precise form need not concern us.

In [3] the action of the currents on tachyons was calculated. In terms of the rescaled tachyons, $\tilde{T}$, where

$$T^{\pm}_p = \pi\Delta(1 \mp \sqrt{2}p)\tilde{T}^{\pm}_p \tag{6}$$

($\Delta(x) = \Gamma(x)/\Gamma(1-x)$), it was found that

$$W_{m,-m}(z)c\bar{c}\tilde{T}^{-}_{-p}(0) = \frac{1}{z}c\bar{c}(\sqrt{2}p + 2m)\tilde{T}^{-}_{-p-m\sqrt{2}}(0) + \ldots \tag{7}$$

$\pm$ refers to chirality. A positive-momentum rightly dressed tachyon is defined to have positive chirality. I will follow [3] and write this, in an obvious notation, as

$$Q_{m,-m}|\tilde{T}^{-}_{-p}\rangle = (\sqrt{2}p + 2m)|\tilde{T}^{-}_{-p-m\sqrt{2}}\rangle. \tag{8}$$

As is clear from above, the first term in (3) has ghost numbers $(0,0)$, while the second term has $(1,-1)$. Since the tachyon (if fixed) has ghost numbers $(1,1)$, and has no $(2,0)$ part, the second term cannot contribute to the right-hand side (apart from a BRST-exact state). It is only for rightly dressed non-tachyonic states that one gets a non-trivial contribution. These are the new moduli discussed in [16].

The same current acting on tachyons of the opposite chirality gives a very different result. In general

$$Q_{m,-m}|\tilde{T}^+_{p_1},...,\tilde{T}^+_{p_{2m+1}}\rangle = (2m+1)!\left(\sqrt{2}\sum_{i=1}^{2m+1}p_i - 2m\right)|\tilde{T}^+_{\sum_{i=1}^{2m+1}p_i-\sqrt{2}m}\rangle. \quad (9)$$

It is hence capable of changing the number of tachyons. This is the reason why it is possible to construct powerful recursion relations. For instance, an insertion of the $W_{1/2,-1/2}$ current leads to the relation

$$(N-2)A(N) = (N-2)(N-3)A(N-1) \quad (10)$$

among the tachyon amplitudes $A(N)$, which have 1 negative and $N-1$ positive chirality tachyons. This, then, implies the well-known result

$$A(N) = (N-3)!, \quad (11)$$

given that $A(3) = 1$. Note that as compared to [3], I have rescaled also the negative chirality tachyon. This would really give zero since it sits on discrete momenta (the bulk amplitudes of these renormalized tachyons are zero), but the zero is compensated by including the volume factor leading to a finite result [17]. This is the convention of collective field theory.

2.2 With Black-Hole Screeners

Let me now generalize the above construction to correlation functions involving black-hole screeners. I will denote such correlation functions by $A(N,m)$, where N is the total (including the negative chirality one) number of tachyons and m is the number of black-hole screeners. Momentum conservation implies

$$p_N = -\frac{N+2m-2}{\sqrt{2}}. \quad (12)$$

I will use the charges $Q_{n/2,-n/2}$, with $n \geq 2m-1$, to derive the Ward identities. From momentum conservation it follows that $Q_{n/2,-n/2}$ transform $n+1-2k$ positive chirality tachyons and k screeners into one tachyon. I will assume, as an ansatz, that the precise form is

$$Q_{n/2,-n/2} \mid \tilde{T}^+_{p_1}...\tilde{T}^+_{p_{n+1-2k}}\mathcal{B}^k\rangle$$

$$= 2^{-k} k!(n+1-2k)! \left(\sqrt{2} a_{n,k} \sum_{i=1}^{n+1-2k} p_i + b_{n,k} \right) \mid \tilde{T}^{+}_{\sum_{i=1}^{n+1-2k} p_i - n/\sqrt{2}} \rangle. \quad (13)$$

The $a_{n,k}$ and $b_{n,k}$ are coefficients depending on n and k only. The prefactor is for latter convenience. This ansatz is based on the assumption of factorization into leg factors. If this assumption is true, it is clear that factors of Δ must appear, in the same way as before when the black-hole screeners are involved. Furthermore, the residual dependence on momenta is clearly symmetric in the different momenta, but must also be at most linear in order for the resulting recursion relations, for all N, not to depend on individual momenta when all contributions are summed up. This is a prerequisite for factorization into leg factors. For each black-hole screener there will appear a regulated zero $\sim 1/\log M$ (see [14] for a discussion on zeroes and poles of correlation functions with dicrete states), which, symbolically, I will write as $\Delta(1)$. The $1/\log M$ will be absorbed into $\mathcal{B}$, or rather M. This is no different from standard $c=1$ where $\Delta/\log\Delta \to \mu$.

Equations (8) and (13) are the only possible non-trivial contributions. This can be seen by using momentum conservation and examining the singularities of the contractions. All other possibilities give at best discrete states at non-discrete momenta. For $n < 2m-1$, however, discrete states at discrete momenta would appear, giving more complicated Ward identities.

It is straightforward to write down the generalization of the $c=1$ Ward identities using the above results. It follows that

$$(N+2m-2)A(N,m) = (N-2)...(N-n-1)(N+2(m-1)(n+1)+n)A(N-n,m)$$

$$+ \sum_{k=1}^{m} m(m-1)...(m-k+1)(N-2)...(N-n-1+2k)$$

$$\times \left(a_{n,k}(n+1-2k)(N+2m-2) + b_{n,k}(N-1)\right)A(N-n+2k, m-k), \quad (14)$$

when all possible non-vanishing cancelled propagators are taken into account. The $a_{n,k}$ and $b_{n,k}$ could in principle be calculated directly using the methods of [3]. I will, however, not attempt this in this paper since a careful study of regularization is first needed.

Let me repeat the logic of the present discussion. *If* the correlation functions factorize as in standard $c=1$, *then* they must satisfy the Ward identities (14) for some values of $a_{n,k}$ and $b_{n,k}$. I stress that even without explicit values for the coefficients $a_{n,k}$ and $b_{n,k}$ these are very powerful Ward identities.

Below I will show that the deformed matrix model results are compatible with (14), while some other proposals are not. But first I need to explain the prediction of the deformed matrix model. Remember that this model has been argued [7–13] to be the matrix model realization of the black hole.

3 SOLUTIONS OF THE WARD IDENTITIES

3.1 The Deformed Matrix Model Prediction

The deformed matrix model is obtained by adding a piece $M/2x^2$ to the matrix model potential. The potential becomes

$$-\frac{1}{2\alpha'}x^2+\frac{M}{2x^2}; \tag{15}$$

M will be positive. The position of the Fermi level is, as usual, measured in terms of its deviation from zero, i.e. μ. However, it is now possible to define a double scaling limit, even when $\mu=0$. One then needs to keep $\hbar/M^{1/2}$ fixed, which will be the string coupling constant.

Special cases of tachyon correlation functions have been calculated in several papers, [7–9,11–13]. In [12] the general formula (in the case with $N-1$ tachyons of the same chirality) was given up to genus one. The genus-zero piece, at $\alpha'=2$, is

$$\langle \tilde{T}^+_{p_1}...\tilde{T}^+_{p_{N-1}}\tilde{T}^+_{-p}\rangle = (N-3)!!(\sqrt{2}p-2)(\sqrt{2}p-4)...(\sqrt{2}p-(N-4))M^{p/\sqrt{2}-N/2+1}, \tag{16}$$

when normalized to collective field theory. This result is valid only for even N: for odd N the matrix model gives zero. Put $p=\frac{N+2m-2}{\sqrt{2}}$ and take m derivatives of (16) to get

$$A(N,m)=2^{-m}(N-3)!!(N+2m-4)!!. \tag{17}$$

This, then, is the deformed matrix model suggestion for the correlation functions with black hole screeners.

3.2 A Check of the Ward Identities

Let us now check whether the matrix model prediction can be made to satisfy the Ward identity (14). As I will discuss in the last section, the vanishing of the matrix model odd-point functions is a consequence of a difference in the leg factors coming from a different choice of vacuum. It is natural to assume that when the perturbative vacuum is used, the odd-point functions are given by a direct 'analytic continuation' of the even-point result. This I will use below. In the next subsection I will provide some evidence for this by showing that it is true for the three-point function.

It is straightforward to substitute (17) into the Ward identities and determine the necessary $a_{n,k}$ and $b_{n,k}$ one by one in k; $m = 1$ provides $k = 1$, $m = 2$ provides $k = 2$ (given the $k = 1$), etc. The answer is

$$a_{n,k} = \frac{n}{k!}(n-2)(n-4)...(n-2k+2) \tag{18}$$

and

$$b_{n,k} = -n a_{n,k}. \tag{19}$$

I stress that *it is a highly non-trivial property of the deformed matrix model correlation functions that coefficients can be found such that the Ward identities are obeyed.* For instance, it can be checked that other proposals, such as $A(N,m) = (N+2m-3)!\frac{m!}{(2m)!}$ (which would correspond to $c = 1$ correlation functions with $\mu^2 \to M$) fail the test at $m = 2$.

3.3 Some Explicit Examples

Even if a direct calculation of the scattering amplitudes is in general very difficult (which is one reason to consider Ward identities), there are some cases where the calculation can be done and hence used as a check of the above results. This is so when $m = 1$. A convenient representation of the black-hole screener is

$$\frac{1}{ia}\lim_{y\to z}\partial_y : e^{-iaX(z)+b\phi(z)+iaX(y)+c\phi(y)} : \tag{20}$$

where $b + c = -2\sqrt{2}$, and the same for the anti-holomorphic piece. With $a = 6\sqrt{2}$ and $b = 0$ eq. (2) is reproduced. I will, for the moment, keep the polarization tensor, i.e. a and c, arbitrary. This can be taken as a useful check on the calculation. As explained in [19], one of the graviton polarizations is pure gauge, i.e. it can be written as L_1 of something. In general, by using the gauge freedom, any special state can be written in a form where it contains no $\partial^n \phi$. Hence, the final answer should not depend on the polarization. However, the resulting integrals simplify only for certain peculiar values of the momenta. Let me pick $p = \frac{N}{\sqrt{2}}$, $p_1 = ... = p_{N-2} = \frac{\sqrt{2}c}{c-a}$, and consequently $p_{N-1} = \frac{N}{\sqrt{2}} - (N-2)\frac{\sqrt{2}c}{c-a}$. The integral to be performed, with the black-hole screener at ∞, is

$$-\frac{1}{a^2}\left((a+c)\frac{N}{\sqrt{2}} - \sqrt{2}c\right)^2 \prod_{i=1}^{N-2} \int d^2 z_i \mid z_i \mid^{2\alpha} \mid 1 - z_i \mid^{2\beta} \prod_{i>j}^{N-2} \mid z_i - z_j \mid^{2\gamma}, \quad (21)$$

where

$$\alpha = \left(1 - \frac{2c}{c-a}\right) N + \frac{2c}{c-a} - 2, \quad (22)$$

$$\beta = N - 2 - (N-3)\frac{2c}{c-a} \quad (23)$$

and

$$\gamma = \frac{4c}{c-a} - 2. \quad (24)$$

The integral can be evaluated by using the results of [20] and the result is, with the volume factor,

$$2(N-2)!\Delta(1-\sqrt{2}p)^{N-2}\Delta(1-\sqrt{2}p_{N-1})\Delta(1-N)\Delta(1), \quad (25)$$

with no dependence left on the polarization tensor as promised. We recognize the familiar leg factors and see that the remaining non-factorizable piece agrees with (17) at $m = 1$ up to the normalization of $\mathcal{B}$.

Let me now consider the three-point in a little more detail. According to (17) it is given by

$$2^{-m}\frac{(2m-1)!!}{m!}M^m. \quad (26)$$

It is now necessary to comment on [21], where the three point is calculated for general m using continuum methods. It was found that the three-point is the same as for $c = 1$ up to a factor

$$\frac{1}{\Gamma(\frac{x+4}{8})}\frac{\Gamma(m+\frac{x+4}{8})}{\Gamma(m+1)}, \quad (27)$$

where x is an unknown parameter introduced by the regularization. In [21] x was fixed so that the factor (27) was one. However, one can argue in favour of another prescription. Let me demand that m and the Liouville momentum of the negative chirality tachyon are untouched by the regularization. After all, it is only for integer m that we can perform the calculation. As is clear from [21], this means that $x = 0$. The three-point then becomes precisely (26). This is the only way to remain consistent with (25), where this regularization prescription has been used implicitly. One can also check that the deformed matrix model expression for the two-point, $\frac{1}{2m} M^m$, is in agreement with the continuum calculation.

4 CONCLUSIONS

In this paper we have seen how the black-hole tachyon correlation functions compare with those of the deformed matrix model. We have seen that the latter obey highly non-trivial Ward identities derived for the black-hole background. I have also provided some examples of explicit calculations, where the agreement can be verified. There are therefore reasons to believe that the deformed matrix model really *is* a black hole. It should be possible to complete a rigorous proof along the lines of this paper.

A difference between the results is the form of the external leg factors or rather the position of the poles in these factors. After all, it is only the latter that are universal and can be confidently predicted by the matrix model without further physical input. In the continuum calculation we are implicitly using the same vacuum as in the linear dilaton theory and we are bound to obtain the same external leg factors in our perturbative treatment. It is easy to see, however, that this vacuum is an unfortunate choice. In the black hole we often demand periodicity in Euclidean time. This is just a choice, but a sensible one. It means that we have chosen a vacuum corresponding to an equilibrium eternal black hole at the Hawking temperature. The particular value of the Euclidean compactification radius R, i.e. the inverse temperature, is obtained if one insists on no conical singularity at the origin of Euclidean space, i.e. the horizon. In the two-dimensional black hole, the compactification radius differs between the semiclassical case [1, 2] and the 'exact' metric of [18], being $1/\sqrt{2}$ and $3/\sqrt{2}$, respectively. Recall that I use conventions such that $\alpha' = 2$. For comparison, the self dual radius is $R_S = \sqrt{2}$ and the Kosterlitz–Thouless phase transition takes place at $R_T = 2\sqrt{2}$. (Since the above values are below R_T, this might be a problem.) In standard $c = 1$ the poles occur at $p = \frac{n}{\sqrt{2}}$ for all integers n. If Euclidean time is compactified the allowed momenta must be of the form $\frac{n}{R}$. Clearly all states are allowed at $R = R_S$. The situation is different when we pick one of the black-hole radia suggested above. In both cases the odd poles are not allowed! This is the reason why we get double spacing of correlation function poles. The matrix model is clever enough to spot this problem. In [8] it was shown that matrix eigenvalue wave functions that might give rise to poles other than the ones above were in general non-normalizable. The perturbative continuum calculation, however, is too crude to take this into account.

To conclude, there is strong evidence that the deformed matrix model is describing a two-dimensional black hole. If this is indeed true, we have a powerful tool at our disposal to explore stringy quantum black-hole phenomena.

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