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# Dominance of the light-quark condensate in the heavy-to-light exclusive decays

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## Abstract

Using the QCD *hybrid* (moments–Laplace) sum rule, we show *semi-analytically* that, in the limit  $M_b \rightarrow \infty$ , the  $q^2$  and  $M_b$  behaviours of the heavy-to-light exclusive ( $\bar{B} \rightarrow \rho$  ( $\pi$ ) semileptonic as well as the  $B \rightarrow \rho\gamma$  rare) decay–form factors are *universally* dominated by the contribution of the soft light–quark condensate rather than that of the hard perturbative diagram. The QCD–analytic  $q^2$  behaviour of the form factors is a polynomial in  $q^2/M_b^2$ , which mimics quite well the usual pole parametrization, except in the case of the  $A_1^B$  form factor, where there is a significant deviation from this polar form. The  $M_b$ –dependence of the form factors expected from HQET and lattice results is recovered. We extract with a good accuracy the ratios:  $V^B(0)/A_1^B(0) \simeq A_2^B(0)/A_1^B(0) \simeq 1.18 \pm 0.04$ ,  $A_1^B(0)/f_+^B(0) \simeq 1.64 \pm 0.06$ , and  $A_1^B(0)/F_1^B(0) \simeq 1.19 \pm 0.03$ ; combined with the “world average” value of  $f_+^B(0)$  or/and  $F_1^B(0)$ , these ratios lead to the decay rates:  $\Gamma_{\bar{B} \rightarrow \pi e \bar{\nu}} \simeq (3.6 \pm 0.6) \times |V_{ub}|^2 \times 10^{12} \text{ s}^{-1}$ ,  $\Gamma_{\bar{B} \rightarrow \rho e \bar{\nu}}/\Gamma_{\bar{B} \rightarrow \pi e \bar{\nu}} \simeq 1.4 \pm 0.2$ , and to the ratios of the  $\rho$ –polarised rates:  $\Gamma_+/\Gamma_- \simeq 0.10 \pm 0.04$ ,  $\alpha \equiv 2\Gamma_L/\Gamma_T - 1 \simeq -(0.75 \pm 0.15)$ .

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# 1 Introduction

In previous papers [1, 2], we have introduced the *hybrid* (moments–Laplace) sum rule (HSR), which is more appropriate than the *popular* double exponential Laplace (Borel) sum rule (DLSR) for studying the form factors of a heavy–to–light quark transition; indeed, the *hybrid* sum rule has a well-defined behaviour when the heavy quark mass tends to infinity. In [2], we studied analytically with the HSR the  $M_b$ –dependence of the  $B \rightarrow K^* \gamma$  form factor and found that it is dominated by the light–quark condensate and behaves like  $\sqrt{M_b}$  at  $q^2 = 0$ . We have also noticed in [1] that the light–quark condensate effect is important in the numerical evaluation of the  $\bar{B} \rightarrow \rho$  ( $\pi$ ) semileptonic form factors, while it has been noticed numerically in [3] using the DLSR that for the  $\bar{B} \rightarrow \rho$  semi–leptonic decays, the  $q^2$  behaviour of the  $A_1^B$  form factor in the time–like region is very different from the one expected from the *standard* pole representation. In this paper, we shall study analytically the  $M_b$ –behaviour of the different form factors for a better understanding of the previous numerical observations. As a consequence, we shall re–examine with our analytic expression the validity of the  $q^2$ –dependence obtained numerically in [3], although we shall mainly concentrate our analysis in the Euclidian region ( $q^2 \leq 0$ ). There, the QCD calculations of the three–point function are reliable; also the lattice results have more data points. For this purpose, we shall analyse the form factors of the  $\bar{B} \rightarrow \pi(\rho)$  semileptonic and  $B \rightarrow \rho \gamma$  rare processes defined in a standard way as:

$$\begin{aligned}
\langle \rho(p') | \bar{u} \gamma_\mu (1 - \gamma_5) b | B(p) \rangle &= (M_B + M_\rho) A_1 \epsilon_\mu^* - \frac{A_2}{M_B + M_\rho} \epsilon^* p' (p + p')_\mu \\
&\quad + \frac{2V}{M_B + M_\rho} \epsilon_{\mu\nu\rho\sigma} p^\rho p'^\sigma, \\
\langle \pi(p') | \bar{u} \gamma_\mu b | B(p) \rangle &= f_+(p + p')_\mu + f_-(p - p')_\mu, \\
\langle \rho(p') | \bar{s} \sigma_{\mu\nu} \left( \frac{1 + \gamma_5}{2} \right) q^\nu b | B(p) \rangle &= i \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p^\rho p'^\sigma F_1^{B \rightarrow \rho} \\
&\quad + \left\{ \epsilon_\mu^* (M_B^2 - M_\rho^2) - \epsilon^* q (p + p')_\mu \right\} \frac{F_1^{B \rightarrow \rho}}{2}. \quad (1)
\end{aligned}$$

In the QCD spectral sum rules (QSSR) evaluation of the form factors, we shall consider the generic three–point function:

$$V(p, p', q) = - \int d^4x \int d^4y \exp(ip'x - ipy) \langle 0 | T J_L(x) O(0) J_b(y) | 0 \rangle, \quad (2)$$

whose Lorentz decompositions are analogous to the previous hadronic amplitudes. Here  $J_L \equiv \bar{u} \gamma_\mu d$  ( $J_L \equiv (m_u + m_d) \bar{u} i \gamma_5 d$ ) is the bilinear quark current having the quantum numbers of the  $\rho$  ( $\pi$ ) mesons;  $J_b \equiv (M_b + m_d) \bar{d} i \gamma_5 b$  is the quark current associated to the  $B$ -meson;  $O \equiv \bar{b} \gamma_\mu u$  is the charged weak current for the semileptonic transition, while  $O \equiv \bar{b} \frac{1}{2} \sigma^{\mu\nu} q_\nu$  is the penguin operator for the rare decay. The vertex function obeys the double dispersion relation:

$$V(p^2, p'^2, q^2) = \frac{1}{4\pi^2} \int_{M_b^2}^\infty \frac{ds}{s - p^2} \int_0^\infty \frac{ds'}{s' - p'^2} \text{Im} V(s, s', q^2) + \dots \quad (3)$$

As already emphasized in [2], we shall work with the HSR:

$$\mathcal{H}(n, \tau) \equiv \frac{1}{n!} \left( \frac{\partial}{\partial p^2} \right)_{p^2=0}^n \mathcal{L} \left( V(p^2, p'^2, q^2) \right)$$

$$= \frac{1}{\pi^2} \int_{M_b^2}^{\infty} \frac{ds}{s^{n+1}} \int_0^{\infty} ds' \exp(-\tau' s') \text{Im} V(s, s', q^2), \quad (4)$$

rather than with the DLSR ( $\mathcal{L}$  is the Laplace transform operator). This sum rule guarantees that terms of the type:

$$\frac{M_b^{2l}}{(M_b^2 - p^2)^k p'^{2k'}}, \quad (5)$$

which appear in the successive evaluation of the Wilson coefficients of high-dimension operators, will not spoil the OPE for  $M_b \rightarrow \infty$  unlike the case of the double Laplace transform sum rule, which blows up in this limit for some of its applications in the heavy-to-light transitions.

In order to come to observables, we insert intermediate states between the charged weak and hadronic currents in (2), while we smear the higher-states effects with the discontinuity of the QCD graphs from a threshold  $t_c$  ( $t'_c$ ) for the heavy (light) mesons. Therefore, we have the sum rule:

$$\begin{aligned} \mathcal{H}_{res} &\simeq 2C_L f_B \frac{F(q^2)}{M_B^{2n}} \exp(-M_L^2 \tau) \\ &\simeq \frac{1}{4\pi^2} \int_{M_b^2}^{t_c} \frac{ds}{s^{n+1}} \int_0^{t'_c} ds' \exp(-\tau s') \text{Im} V_{PT}(s, s', q^2) + NPT. \end{aligned} \quad (6)$$

$PT$  ( $NPT$ ) refers to perturbative (non-perturbative) contributions;  $C_L \equiv f_P M_P^2$  for light pseudoscalar mesons, while  $C_L \equiv M_V^2 / (2\gamma_V)$  for light vector mesons;  $M_L$  is the light meson mass. The decay constants are normalized as:

$$\begin{aligned} (m_q + M_Q) \langle 0 | \bar{q}(i\gamma_5) Q | P \rangle &= \sqrt{2} M_P^2 f_P \\ \langle 0 | \bar{q} \gamma_\mu Q | V \rangle &= \epsilon_\mu^* \sqrt{2} \frac{M_V}{2\gamma_V^2}. \end{aligned} \quad (7)$$

$F(q^2)$  is the form factor of interest. For our purpose, we shall consider the expression of the decay constant  $f_B$  from moments sum rule at the same order (i.e. to leading order) [4]:

$$\frac{2f_B^2}{(M_B^2)^{n_2-1}} \simeq \frac{3}{8\pi^2} M_b^2 \int_{M_b^2}^{t_c} \frac{ds}{s^{n_2+1}} \frac{(s - M_b^2)^2}{s} - \frac{\langle \bar{q}q \rangle}{M_b^{2n_2-1}} \left\{ 1 - \frac{n_2(n_2+1)}{4} \left( \frac{M_0^2}{M_b^2} \right) \right\}. \quad (8)$$

For convenience, we shall work with the non-relativistic energy parameters  $E$  and  $\delta M_{(b)}$ :

$$s \equiv (M_b + E)^2 \quad \text{and} \quad \delta M_{(b)} \equiv M_B - M_b, \quad (9)$$

where, as we saw in the analysis of the two-point correlator, the continuum energy  $E_c$  is [4]:

$$\begin{aligned} E_c^D &\simeq (1.08 \pm 0.26) \text{ GeV} \\ E_c^B &\simeq (1.30 \pm 0.10) \text{ GeV} \\ E_c^\infty &\simeq (1.5 \sim 1.7) \text{ GeV}. \end{aligned} \quad (10)$$

In terms of these continuum energies, and at large values of  $M_b$ , the decay constant reads [4]:

$$f_B^2 \simeq \frac{1}{\pi^2} \frac{(E_c^B)^3}{M_b} \left(\frac{M_B}{M_b}\right)^{2n_2-1} \left\{ 1 - \frac{3}{2}(n_2+1) \left(\frac{E_c^B}{M_b}\right) + \frac{3}{5} \left( (2n_2+3)(n_2+1) + \frac{1}{4} \right) \left(\frac{E_c^B}{M_b}\right)^2 - \frac{\pi^2}{2} \frac{\langle \bar{q}q \rangle}{(E_c^B)^3} \left( 1 - \frac{n_2(n_2+1)M_0^2}{4M_b^2} \right) \right\}, \quad (11)$$

## 2 The $\bar{B} \rightarrow \rho$ semileptonic decay

The corresponding form factors defined in (1) have been estimated with the HSR [1] and the DLSR [1], [3]. Instead of taking the average values from the two methods as was done in [1], we shall only consider the HSR estimates, because of the drawbacks previously found in the DLSR approach:

$$A_1^B(0) \simeq 0.16 - 0.41, \quad A_2^B(0) \simeq 0.26 - 0.58, \quad V^B(0) \simeq 0.28 - 0.61. \quad (12)$$

The errors in these numbers are large, as the HSR has no  $n$ -stability. In the following, we derive semi-analytic formulae for the form factors. Using the leading order in  $\alpha_s$  QCD results of the three-point function, and including the effect of the dimension-6 operators as given in [5], one deduces the sum rule ( $q^2 \leq 0$ ):

$$A_1^B(0) \simeq -\frac{1}{2} \langle \bar{q}q \rangle \frac{\rho_1}{f_B} \left(\frac{M_B}{M_b}\right)^{2n} \left\{ 1 - \frac{q^2}{M_b^2} + \delta^{(5)} + \delta^{(6)} + \frac{\mathcal{I}_1}{M_b^2} \right\}, \quad (13)$$

with:

$$\begin{aligned} \rho_1 &\equiv \left(\frac{\gamma_\rho}{M_\rho^2}\right) \frac{M_b}{(M_B + M_\rho)} \exp(M_\rho^2 \tau') \\ \delta^{(5)} &\equiv \frac{\tau' M_0^2}{6} \left\{ n - \frac{1}{\tau' M_b^2} \left( \frac{3}{2} n^2 + 6n + \frac{5}{2} \right) - \frac{q^2}{M_b^2} \left( (n+1) \left( \frac{3}{2} n - 1 \right) + 2\tau' M_b^2 (1+2n) - 2(n+1)q^2 \tau' \right) \right\} \\ \delta^{(6)} &\equiv -\frac{1}{M_b^3} \left(\frac{8\pi}{81}\right) \frac{\langle O_6 \rangle}{\langle \bar{q}q \rangle} \left\{ 4(n+1) - \tau' M_b^2 (10 - 3n + n^2) + \frac{15}{2} (\tau' M_b^2)^2 - \frac{1}{2} (\tau' M_b^2)^3 \right\} \end{aligned} \quad (14)$$

where  $\mathcal{I}_1$  is the integral from the perturbative expression of the spectral function. It is constant for  $M_b \rightarrow \infty$ . Its value and behaviour at finite values of  $M_b$  and for  $q^2 = 0$  is given in Fig. 1. At  $M_b = 4.6$  GeV, it reads:  $\mathcal{I}_1 \simeq (3.6 \pm 1.2)$  GeV<sup>2</sup> and behaves to leading order in  $1/M_b$  as  $t_c'^2 E_c / \langle \bar{q}q \rangle$ , which is reassuring as it gives a better meaning of the expansion in (13). The other values of the QCD parameters are [6]:  $\langle \bar{q}q \rangle = -(189 \text{ MeV})^3 (\log M_b / \Lambda)^{12/23}$ ,  $M_0^2 = (0.80 \pm 0.01)$  GeV<sup>2</sup> from the analysis of the  $B, B^*$  sum rules and  $\langle O_6 \rangle \equiv \rho \alpha_s \langle \bar{q}q \rangle^2 \simeq (3.8 \pm 2.0) \times 10^{-4}$  GeV<sup>6</sup>. The  $\rho$ -meson coupling is  $\gamma_\rho \simeq 2.55$ .

One can deduce from the previous expression that  $A_1^B$  is dominated by the light-quark condensate in the  $1/M_b$ -expansion counting rule. Moreover, the perturbative contribution is also numerically small at the  $b$ -mass. The absence of the  $n$ -stability is explicit from our formula, due to the meson-quark mass difference entering the overall factor. This effect could be however minimized by using the expression of  $f_B$  in (11) and by imposing that the effects due to the meson-quark mass differences from the three- and two-point functions compensate each other to leading order. This is realized by choosing:

$$2n = n_2 - \frac{1}{2}, \quad (15)$$

which, fixes  $n$  to be about 2, in view of the fact that the two-point function stabilizes for  $n_2 \simeq 4-5$ . In this way, one would obtain the leading-order result in  $\alpha_s$ :

$$A_1^B \simeq 0.3 - 0.6, \quad (16)$$

where we have used the leading-order value  $f_B^{L,O} \simeq 1.24f_\pi$ . However, although this result is consistent with previous numerical fits in (12) and in [3], we only consider it as an indication of a consistency rather than a safe estimate because of the previous drawbacks for the  $n$ -stability. One should also keep it in mind that the values given in (12) correspond to the value of  $f_B \simeq 1.6f_\pi$ , which includes the radiative corrections of the two-point correlator and which corresponds to smaller values of  $n$ . Improvements of the result in (16) need (of course) an evaluation of the radiative corrections for the three-point function. The  $q^2$ -dependence of  $A_1^B$  can be obtained with good accuracy, without imposing the previous constraint. We obtain the numerical result in Fig. 2, which is well approximated by the effect from the light-quark condensate alone:

$$R_1^B(q^2) \equiv \frac{A_1^B(q^2)}{A_1^B(0)} \simeq 1 - \frac{q^2}{M_b^2}. \quad (17)$$

Performing an analytic continuation of this result in the time-like region, we reproduce the numerical result from the DLSR [3](see Fig. 2), which indicates that the result is independent of the form of the sum rule used, while in the time-like region the perturbative contribution still remains a small correction of the light-quark condensate one. This result is clearly in contradiction with the *standard* pole-dominance parametrization, as, indeed, the form factor decreases for increasing  $q^2$ -values. A test of this result needs improved lattice measurements over the ones available in [7]. From the previous expressions, and using the fact that  $f_B$  behaves as  $1/\sqrt{M_b}$ , one can also predict the  $M_b$ -behaviour of the form factor at  $q_{max}^2 \simeq M_b^2 + 2M_\rho M_b$ :

$$A_1^B(q_{max}^2) \sim \frac{1}{\sqrt{M_b}}, \quad (18)$$

in accordance with the expectations from HQET [8] and the lattice results [7]. The analysis of the  $V^B$  and  $A_2^B$  form factors will be done in the same way. Here, one can realize that the inclusion of the higher dimension-5 and -6 condensates tends to destabilize the results, although these still remain small corrections to the leading-order results. Then, neglecting these destabilizing terms, one has:

$$\begin{aligned} V^B(q^2) &\simeq -\frac{1}{2} \langle \bar{q}q \rangle \frac{\rho_V}{f_B} \left( \frac{M_B}{M_b} \right)^{2n} \left\{ 1 + \frac{\mathcal{I}_V}{M_b^2} + \dots \right\} \\ A_2^B(q^2) &\simeq -\frac{1}{2} \langle \bar{q}q \rangle \frac{\rho_2}{f_B} \left( \frac{M_B}{M_b} \right)^{2n} \left\{ 1 + \frac{\mathcal{I}_2}{M_b^2} + \dots \right\} \end{aligned} \quad (19)$$

with:

$$\begin{aligned}\rho_V &\equiv \left(\frac{\gamma_\rho}{M_\rho^2}\right) \frac{M_b(M_B + M_\rho)}{M_B^2} \exp(M_\rho^2 \tau') \\ \rho_2 &\equiv \left(\frac{\gamma_\rho}{M_\rho^2}\right) \frac{(M_B + M_\rho)}{M_b} \exp(M_\rho^2 \tau').\end{aligned}\quad (20)$$

$\mathcal{I}_{V,2}$  are integrals from the perturbative spectral functions, which also behave like  $\mathcal{I}_1$  to leading order in  $1/M_b$ . They are given in Fig. 1 for  $q^2 = 0$  and for different values of  $M_b$ . As expected, they are constant when  $M_b \rightarrow \infty$ , although, as in the previous case, the asymptotic limit is reached very slowly. Here, the  $n$ -stability of the analysis is also destroyed by the overall  $(M_B/M_b)^{2n}$  factor, which hopefully disappears when we work with the ratios of form factors. We show in Fig. 2 the  $q^2$ -dependence of the normalized  $V^B$  and  $A_2^B$ , which is very weak since the dominant light-quark condensate contribution has no  $q^2$ -dependence. The small increase with  $q^2$  is due to the  $q^2$ -dependence of the small and non-leading contribution from the perturbative graph. Lattice points in the Euclidian  $q^2$ -region [7] agree with our results. An analytic continuation of our results at time-like  $q^2$  agrees qualitatively with the one in [3]. The numerical difference in this region is due to the relative increase of the perturbative contribution in the time-like region due to the effect of the additional Landau-type singularities. However, this effect does not influence the  $M_b$  behaviour of the form factors at  $q_{max}^2$ , which can be safely obtained from the leading-order expression given by the light-quark condensate. One can deduce:

$$V^B(q_{max}^2) \sim \sqrt{M_b}, \quad A_2^B(q_{max}^2) \sim \sqrt{M_b}. \quad (21)$$

This result is in agreement with HQET and lattice data points. Finally, we can also extract the ratios of form factors. At the  $\tau'$ -maxima and at the  $n$ -maxima or inflexion point, we obtain from Fig. 3:

$$r_2 \equiv \frac{A_2^B(0)}{A_1^B(0)} \simeq r_V \equiv \frac{V^B(0)}{A_1^B(0)} \simeq 1.18 \pm 0.04, \quad (22)$$

where the accuracy is obviously due to the cancellation of systematics in the ratios. This result is again consistent with the lattice results [7], but more accurate.

### 3 The $\bar{B} \rightarrow \pi$ semileptonic decay

The relevant form factor defined in (1) has been numerically estimated within the HSR with the value [1]:

$$f_+^B(0) \simeq 0.20 \pm 0.05, \quad (23)$$

where the contribution of the  $\pi'(1.3)$  meson has been included for improving the sum rule variable stability of the result. In this paper, we propose to explain the meaning of this numerical result from an analytic expression of the sum rule. Using the QCD expression given in [5], we obtain for a pseudoscalar current describing the pion:

$$f_+^B(q^2) \simeq -\frac{(m_u + m_d)\langle \bar{q}q \rangle}{4f_\pi m_\pi^2} \frac{1}{f_B} \left(\frac{M_B}{M_b}\right)^{2n} \left\{ 1 + \delta^{(5)} + \frac{\mathcal{I}_\pi}{M_b^2} \right\}, \quad (24)$$

where  $\mathcal{I}_\pi$  is the spectral integral coming from the perturbative graph. Its value at  $q^2 = 0$  for different values of  $M_b$  is shown in Fig. 1. It indicates that at  $M_b = 4.6$  GeV, the perturbative contribution, although large, still remains a correction compared with the light-quark condensate term;  $\delta^{(5)}$  is the correction due to the dimension-5 condensate and reads:

$$\delta^{(5)} \simeq -\tau' M_0^2 \left\{ \frac{(n+1)}{3} + \frac{\tau'^{-1}}{4M_b^2} (n^2 + 3n + 4) \right\}. \quad (25)$$

One can use the well-known PCAC relation

$$(m_u + m_d) \langle \bar{q}q \rangle = -m_\pi^2 f_\pi^2, \quad f_\pi = 93.3 \text{ MeV} \quad (26)$$

into the previous sum rule in order to express  $f_+^B$  in terms of the meson couplings. Unlike the case of the  $B \rightarrow \rho$  form factors where the scale dependence is contained in  $\langle \bar{q}q \rangle$ ,  $f_+^B$  is manifestly renormalization-group-invariant. It should be noted, as in the case of the sum rule determination of the  $\omega\rho\pi$  coupling [6], that the  $f_\pi$ -dependence appears indirectly via (26) in a correlator evaluated in the deep Euclidian region, while the pion is off shell, which is quite different from soft-pion techniques with an on-shell Goldstone boson. One can also deduce from (24) that for large  $M_b$ ,  $f_+^B$  behaves like  $\sqrt{M_b}$ . In this limit the  $q^2$ -dependence is rather weak, as it comes only from the non-leading  $1/M_b$  contributions; we therefore have, to a good accuracy:

$$f_+^B(q_{max}^2) \simeq f_+^B(0) \sim \sqrt{M_b}. \quad (27)$$

As in the previous case, the slight difference between the  $q^2$ -behaviour in the time-like region and the one from that obtained in [3], at a finite value of  $M_b (=4.6 \text{ GeV})$ , is only due to a numerical enhancement caused by the Landau singularities of the perturbative contribution in this region, but does not disturb the  $M_b$ -behaviour of the form factor. Finally, we extract the ratio of the form factor in Fig. 3, from which we deduce:

$$r_\pi \equiv \frac{A_1^B(0)}{f_+^B(0)} \simeq 1.64 \pm 0.06. \quad (28)$$

## 4 The $B \rightarrow \rho\gamma$ rare decay

We can use the previous results into the HQET [8] relation among the different form factors of the rare  $B \rightarrow \rho\gamma$  decay ( $F_1^B \equiv F_1^{B \rightarrow \rho}$ ) and the semileptonic ones. This relation reads around  $q_{max}^2$ :

$$F_1^B(q^2) = \frac{q^2 + M_B^2 - M_\rho^2}{2M_B} \frac{V^B(q^2)}{M_B + M_\rho} + \frac{M_B + M_\rho}{2M_B} A_1^B(q^2), \quad (29)$$

from which we deduce:

$$F_1^B(q_{max}^2) \sim \sqrt{M_b}. \quad (30)$$

However, we can also study, directly from the sum rule, the  $q^2$ -dependence of  $F_1^B$ . Using the fact that the corresponding sum rule is also dominated by the light-quark condensate for  $M_b \rightarrow \infty$  [2], an evaluation of this contribution, at  $q^2 \neq 0$ , shows that the light-quark

condensate effect has no  $q^2$ -dependence to leading order. Then, we can deduce, to a good accuracy:

$$F_1^B(q_{max}^2) \simeq F_1^B(0) \sim \sqrt{M_b}. \quad (31)$$

Let us now come back to the parametrization of the form factor at  $q^2 = 0$ . We have given in [2] an expanded interpolating formula that involves  $1/M_b$  and  $1/M_b^2$  corrections due to the meson-quark mass difference, to  $f_B$  and to higher-dimension condensates. Here, we present a slightly modified expression, which is:

$$F_1^B(0) \simeq -\frac{1}{2} \langle \bar{q}q \rangle \frac{\rho_\gamma}{f_B} \left( \frac{M_B}{M_b} \right)^{2n} \left\{ 1 + \frac{\mathcal{I}_\gamma}{M_b^2} + \dots \right\}, \quad (32)$$

with:

$$\begin{aligned} \rho_\gamma &\equiv \left( \frac{\gamma_\rho}{M_\rho^2} \right) \exp(M_\rho^2 \tau'), \\ \mathcal{I}_\gamma &\simeq (20 \pm 4) \text{ GeV}^2 \quad \text{for } M_b \geq 4.6 \text{ GeV}, \end{aligned} \quad (33)$$

where we have neglected the effects of higher-dimension condensates;  $\mathcal{I}_\gamma$  is the perturbative spectral integral. One should notice that unlike the other spectral integrals in Fig. 1,  $\mathcal{I}_\gamma$  reaches quickly the asymptotic limit when  $M_b \rightarrow \infty$ . Using the estimated value of  $F_1^B(0)$  in [2], we can have, in units of GeV:

$$F_1^B = \frac{1.6 \times 10^{-2}}{f_B} \left( 1 + \frac{20 \pm 4}{M_b^2} \right), \quad (34)$$

which leads of course to the same formula at large  $M_b$  as in [2]. However, due to the large coefficient of the perturbative contribution, it indicates that an extrapolation of the result obtained at low values of  $M_c$  is quite dangerous, as it may lead to a wrong  $M_b$ -behaviour of the form factor at large mass. One should notice that (34) and the one in [2] lead to the same numerical value of  $F_1^D(0)$ . Proceeding as for the former cases, we can also extract the ratio:

$$r_\gamma \equiv \frac{A_1^B(0)}{F_1^B(0)} \simeq 1.19 \pm 0.03, \quad (35)$$

from the analysis of the  $\tau'$ - and  $n$ -stability shown in Fig. 3.

## 5 Values of the $B$ -form factors

The safest prediction of the absolute value of the form factors available at present, where different versions of the sum rules and lattice calculations have a consensus, is the one for  $f_+^B(0)$ :

$f_+^B(0) \simeq$	$0.26 \pm 0.12 \pm 0.04$	Lattice [7]
	$0.26 \pm 0.03$	DLSR [3](see also[9])
	$0.23 \pm 0.02$	HSR+DLSR [1]
	$0.27 \pm 0.03$	Light-cone [10],

(36)



from which one can deduce the “world average”:

$$f_+^B(0) \simeq 0.25 \pm 0.02. \quad (37)$$

Using this value in the previous ratios of the form factors, we obtain:

$$A_1^B(0) \simeq 0.40 \pm 0.03 \quad A_2^B(0) \simeq V^B(0) \simeq 0.47 \pm 0.04 \quad F_1^B(0) \simeq 0.34 \pm 0.03. \quad (38)$$

Alternatively, one can use the present most reliable estimate of  $F_1^B$  [2], [11]:

$$F_1^B(0) \simeq 0.27 \pm 0.03, \quad (39)$$

where we have used the strength of the  $SU(3)$ -breakings obtained in [2] in order to convert the result for  $B \rightarrow K^* \gamma$  of [11] into the  $B \rightarrow \rho \gamma$  of interest here. Then, we deduce:

$$A_1^B(0) \simeq 0.36 \pm 0.03, \quad A_2^B(0) \simeq V^B(0) \simeq 0.43 \pm 0.04, \quad f_+^B(0) \simeq 0.22 \pm 0.02. \quad (40)$$

We consider as a final *best* estimate the average of the two results:

$$\begin{aligned} A_1^B(0) &\simeq 0.38 \pm 0.04, & A_2^B(0) &\simeq V^B(0) \simeq 0.45 \pm 0.05, \\ f_+^B(0) &\simeq 0.23 \pm 0.02, & F_1^B(0) &\simeq 0.30 \pm 0.04. \end{aligned} \quad (41)$$

## 6 $B$ -semileptonic-decay rates

We are now in a good position to predict the different decay rates. In so doing, we shall use the pole parametrization, except for the  $A_1^B$  form factor. For the  $B \rightarrow \pi$ , we shall use the experimental value 5.32 GeV of the  $B^*$  mass. For the  $B \rightarrow \rho$ , we shall use the fitted value  $(6.6 \pm 0.6)$  GeV [3] for the pole mass associated to  $A_2^B$  and  $V^B$ . For  $A_1^B$ , we use the linear form suggested by (13), with an effective mass of  $(5.3 \pm 0.7)$  GeV, which we have adjusted from the numerical behaviour given in [3] (we have not tried to reproduce the change of the behaviours for  $t \simeq (0.76 - 0.95)M_b^2$  obtained in [3], which is a minor effect). Using the standard definitions and notations, we obtain:

$$\Gamma_{\bar{B} \rightarrow \pi e \bar{\nu}} \simeq (3.6 \pm 0.6) \times |V_{ub}|^2 \times 10^{12} \text{ s}^{-1} \quad (42)$$

We also obtain the following ratios:

$$\frac{\Gamma_{\bar{B} \rightarrow \rho e \bar{\nu}}}{\Gamma_{\bar{B} \rightarrow \pi e \bar{\nu}}} \simeq 1.4 \pm 0.2, \quad \frac{\Gamma_+}{\Gamma_-} \simeq 0.10 \pm 0.04, \quad \alpha \equiv 2 \frac{\Gamma_L}{\Gamma_T} - 1 \simeq -(0.75 \pm 0.15). \quad (43)$$

Thanks to a better control of the ratios of form factors, the ratio of the  $\bar{B}$  decays into  $\pi$  over the  $\rho$  can be predicted, to a good accuracy. It becomes compatible with the prediction obtained by only retaining the contribution of the vector component of the form factors. Our predictions are compatible with the ones in [3] except for  $\Gamma_+/\Gamma_-$ , where the one in [3] is about one order of magnitude smaller. The difference of two of these three quantities with ones in [1] ( the large branching ratio into  $\rho$  over  $\pi$  and the positive value of the asymmetry  $\alpha$  in [1] and in most other pole dominance models for  $A_1^B$ ) is mainly due to the different  $q^2$ -behaviour of  $A_1^B$  used here.

## 7 Conclusions

We have studied, using the QCD *hybrid* sum rule, the  $M_b$ - and  $q^2$ -behaviours of the heavy-to-light transition form factors. We find that these quantities are dominated in a *universal* way by the light-quark condensate contribution.

The  $M_b$ -dependence obtained here is in perfect agreement with the HQET and lattice expectations.

The  $q^2$ -dependence of the  $A_1^B$  form factor, which is mainly due to the one from the light-quark condensate contribution, is in clear contradiction with the one expected from a pole parametrization. The other form factors can mimic *numerically* this pole parametrization. Our QCD-analytic  $q^2$ -behaviours confirm the previous numerical results given in [3].

We have also shown that it can be incorrect to derive the  $M_b$ -behaviour of the form factors at  $q^2 = 0$  by combining the HQET result at  $q_{max}^2$  with the pole parametrization.

We have also shown that the unusual  $q^2$ -behaviour of the  $A_1^B$  form factor affects strongly the branching ratio of  $B \rightarrow \rho$  over  $B \rightarrow \pi$  and the  $\rho$ -polarisation parameter  $\alpha$ . A measurement of these quantities complemented by the one of the  $q^2$ -behaviour of the form factor should provide a good test of the sum rules approach.

We want also to stress that the extrapolation of the results obtained in this paper to the case of the  $D$ -meson would be too audacious: the uses of the HSR in that case cannot be *rigorously* justified since the value of the  $c$ -quark mass is smaller, although it may lead to acceptable phenomenological results. We are investigating this point at present.

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## Figure captions

**Fig. 1**  $M_b$ -dependence of the perturbative spectral integrals at  $q^2 = 0$ .

**Fig. 2**  $q^2$ -behaviour of the normalized form factors:  $R_1 \equiv A_1^B(q^2)/A_1^B(0)$ ,  $R_2 \equiv A_2^B(q^2)/A_2^B(0)$ ,  $R_V \equiv V^B(q^2)/V^B(0)$  and  $R_\pi \equiv f_+^B(q^2)/f_+^B(0)$ . The squared points in the timelike region are from [3].

**Fig. 3**  $\tau'$ - and  $n$ -dependences of the ratios of form factors at  $q^2 = 0$ :  $r_2 \equiv A_2^B(0)/A_1^B(0)$ ,  $r_V \equiv V^B(0)/A_1^B(0)$ ,  $r_\pi \equiv A_1^B(0)/f_+^B(0)$  and  $r_\gamma \equiv A_1^B(0)/F_1^B(0)$ .

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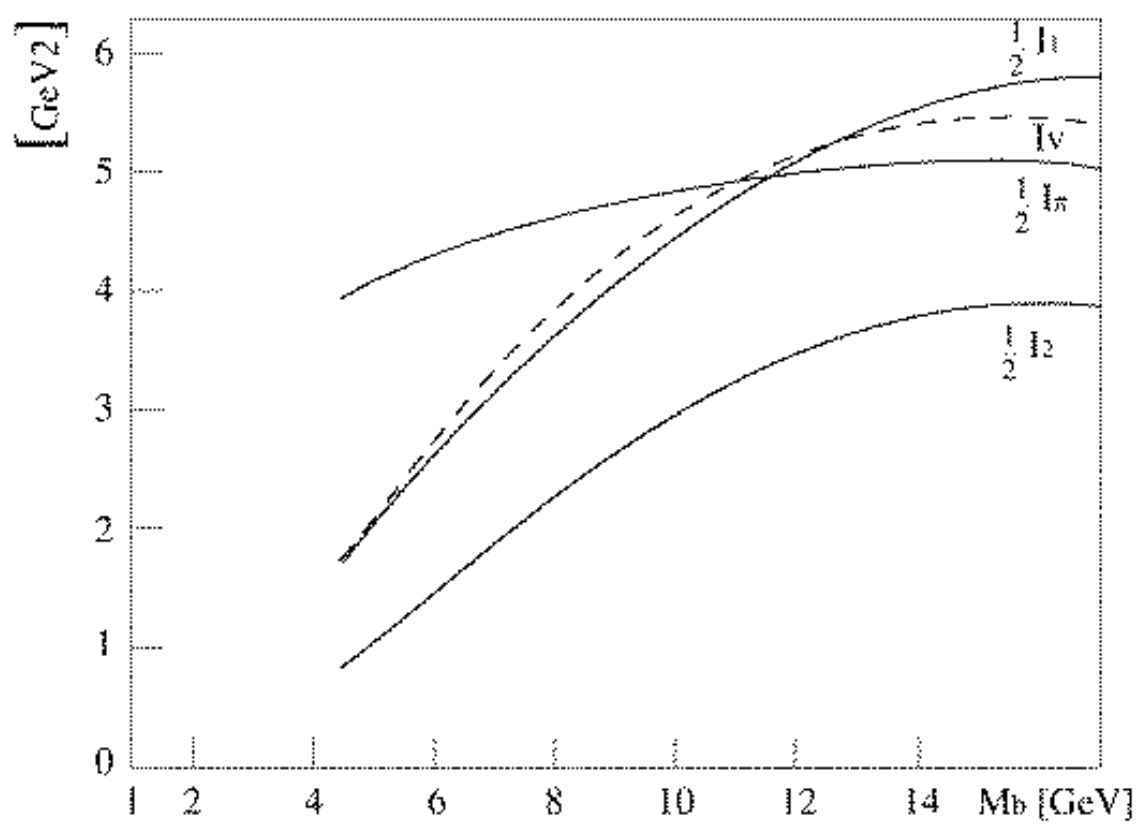


Fig. 1

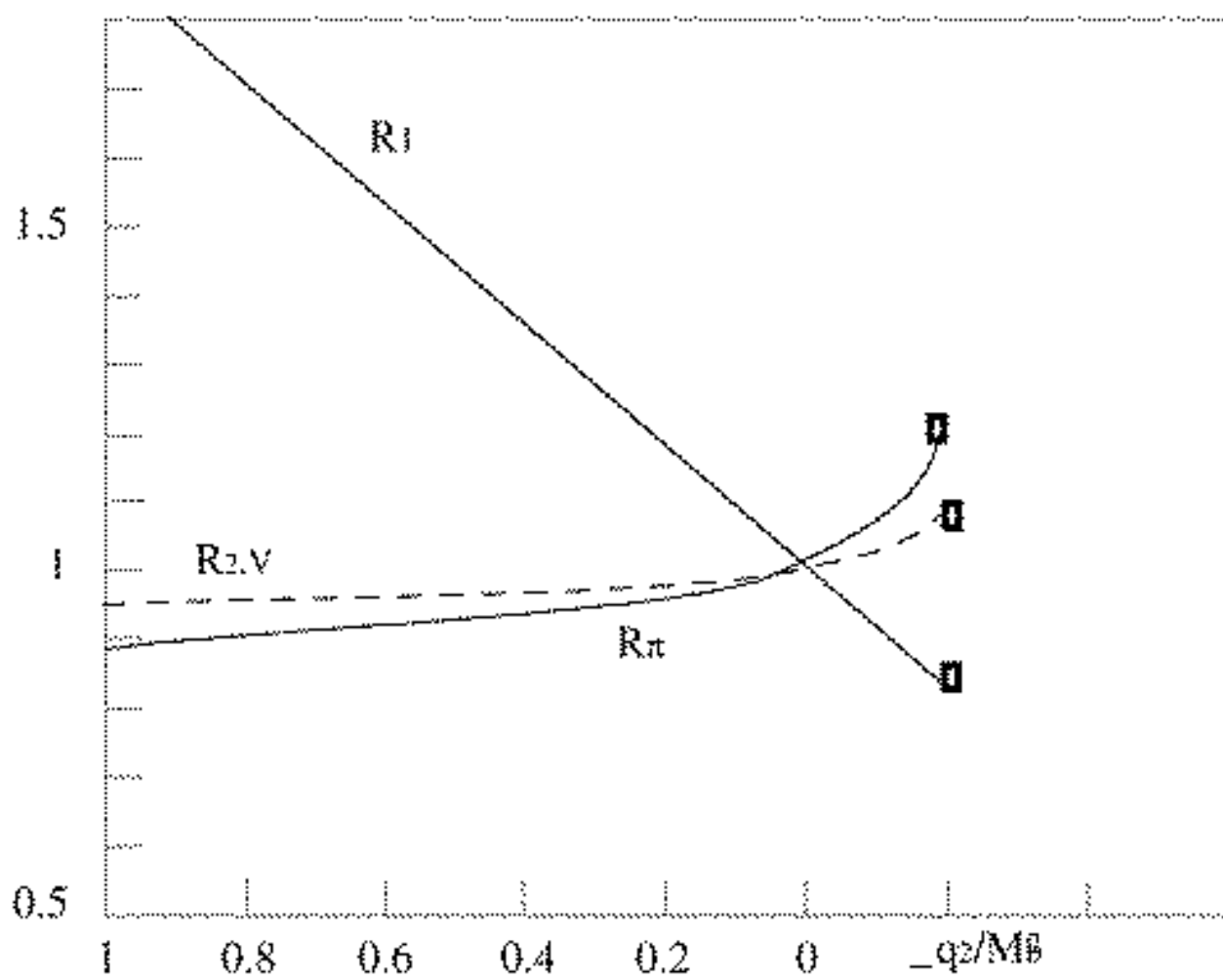


Fig. 2

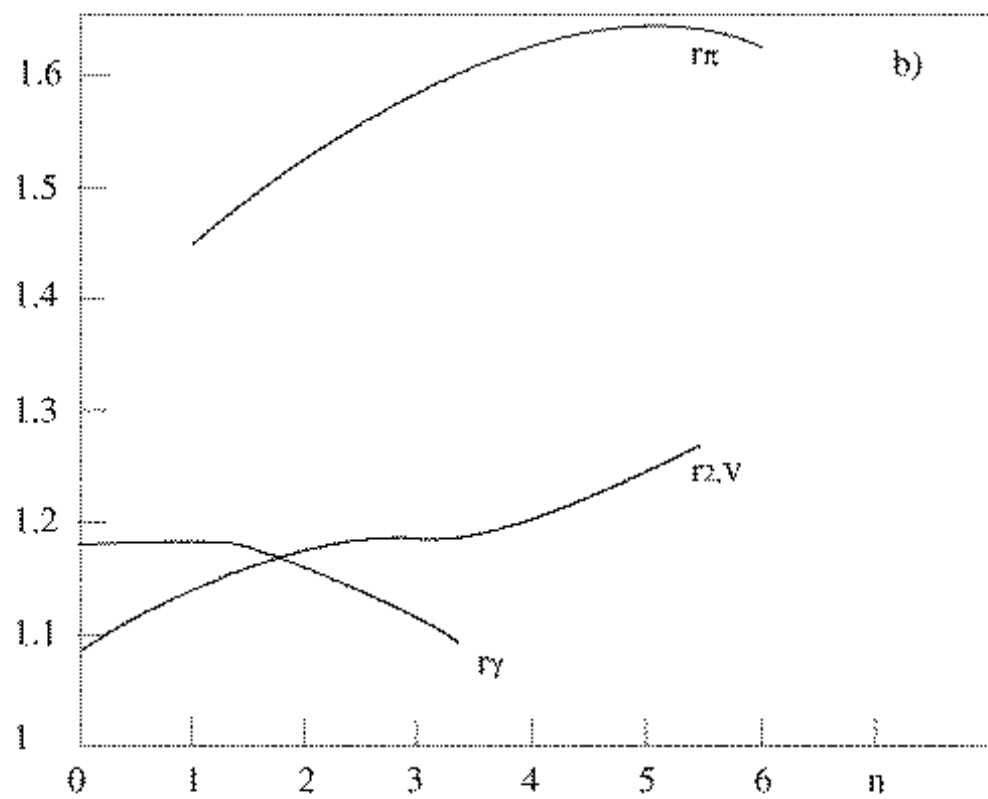
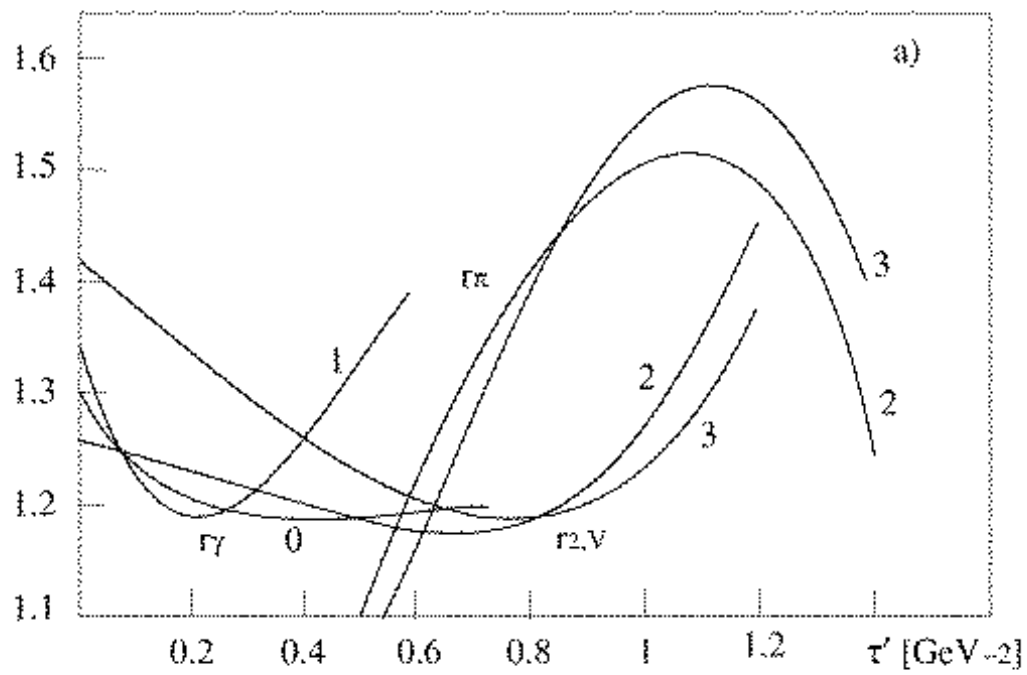


Fig. 3