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MODELLING ASYMMETRIC VOLATILITY DYNAMICS BY MULTIVARIATE BL-GARCH MODELS

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Contents

Abstract

The class of Multivariate BiLinear GARCH (MBL-GARCH) models is proposed and its statistical properties are investigated. The model can be regarded as a generalization to a multivariate setting of the univariate BL-GARCH model proposed by Storti and Vitale (2003a; 2003b). It is shown how MBL-GARCH models allow to account for asymmetric effects in both conditional variances and correlations. An EM algorithm for the maximum likelihood estimation of the model parameters is provided. Furthermore, in order to test for the appropriateness of the conditional variance and covariance specifications, a set of robust conditional moments test statistics are defined. Finally, the effectiveness of MBL-GARCH models in a risk management setting is assessed by means of an application to the estimation of the optimal hedge ratio in futures hedging.

Keywords

Multivariate GARCH, Asymmetry, Conditional Correlation, EM algorithm, Robust Conditional Moment Tests, Futures Hedging.

1 Introduction

In the past two decades massive empirical evidence has confirmed the presence of asymmetric effects in the conditional variance of financial assets. For stocks these effects are usually related to the so called *leverage* effect (Black, 1976). The empirical findings have motivated a remarkably long series of papers proposing modifications of the standard GARCH model in order to allow for asymmetric volatility dynamics (for a review see Hagerud, 1997).

Recently, evidence has been provided suggesting the presence of similar effects in the dynamics of conditional correlations. Similarly to the conditional variance, the asymmetry is due to the presence of a significant relationship between the level of the conditional correlation between two assets and the signs of past returns on both assets. Multivariate conditional heteroskedastic models naturally provide an useful framework for dealing with asymmetries in the dynamics of conditional variances and correlations. However, although multivariate conditional heteroskedastic models are currently applied in many fields, in both macroeconomics and finance, and rely on a consolidated set of theoretical results (for a survey see Bauwens et al., 2006), just a few contributions have been dedicated to propose models able to deal with asymmetric conditional correlations. The first contribution is due to Kroner and Ng (1998). Their General Dynamic Covariance (GDC) model is able to simultaneously capture asymmetric effects in both conditional variances and covariances and nests several multivariate GARCH specifications such as the DVEC and BEKK models. More recently, Sheppard (2002) has proposed a generalization of the Dynamic Conditional Correlation (DCC) model

(Engle, 2002) which allows for asymmetric dynamics in the conditional variances as well as in the conditional correlations. A different approach is followed by Audrino and Barone-Adesi (2003) who suggest a semi-parametric model allowing for both time varying correlations and asymmetric response to past shocks. Further evidence of asymmetric dynamics in the time varying conditional correlations of asset returns is given by Beckaert and Wu (2000) and Errunza and Hung (1999). A discussion on the topic and an extensive survey of the presence of asymmetric effects in the conditional correlations of global equity and bond returns can be found in the paper by Cappiello et al. (2003)

All these papers provide empirical evidence supporting the asymmetric nature of the relationship between conditional correlation and past returns. From an economic standpoint, a plausible explanation for asymmetries in conditional correlations, based on the assumption of time-varying risk premia, is provided by Cappiello et al. (2003). They argue that, in a CAPM setting, after having observed negative returns for both assets, if asymmetric effects in volatility are present, their conditional variances will increase more than after positive returns of the same magnitude. Investors will then require higher returns in order to compensate the larger risk implied by the increased volatilities, leading to a decrease in the prices of both assets and, hence, to an increase in their correlation.

In this case, the estimates of conditional correlations generated by a symmetric model could be biased. More precisely, in the presence of asymmetric dynamics, in a down market, the symmetric model could underestimate conditional correlations leading to under-conservative risk management strategies. In hedging applications, for example, the underestimation of conditional correlations could result in a substantial underestimation of the optimal hedge ratio while, in a similar fashion, if the model is to be used for Value at Risk (VaR) estimation, underestimation of conditional correlations could result in minimum risk capital requirements lower than appropriate.

This paper proposes a multivariate conditional heteroskedastic model, the Multivariate BiLinear GARCH (MBL-GARCH) model, able to capture asymmetries in both conditional variances and correlations. In section 2 the general formulation and the probabilistic properties of the MBL-GARCH model are illustrated while section 3 focuses on the way in which asymmetric effects are introduced into the MBL-GARCH model formulation. The estimation problem is dealt with in section 4 where an EM algorithm for Maximum Likelihood Estimation (MLE) of model parameters is described. The finite sample properties of the estimates are assessed by means of a simulation study. Section 5 discusses a strategy for model checking based on robust conditional moment test statistics developed under the general framework proposed by Wooldridge (1991). In section 6, the MBL-GARCH model is applied to two sets of real data. First, we present an illustrative example on a trivariate time series of U.S. stock returns. Second, the MBL-GARCH model is applied to the problem of estimating the optimal hedge ratio in futures market hedging. Section 7 concludes.

2 The MBL-GARCH model

Let $y_t = [y_{1t} \dots y_{kt}]'$, for $t = 1, \dots, T$, be a $(k \times 1)$ time series of returns such that $(y_t | I^{t-1}) \sim MVN(0, H_t)$ with $I^{t-1} = [y_0, y_1, \dots, y_{t-1}]$. The $(i,j)th$ element of H_t denotes the conditional covariance between y_{it} and y_{jt}. A MBL-GARCH model of orders (p, q) is defined as:

$$y_t = C_t x_t \tag{2.1}$$

with

$$C_t = [I_k \quad B_t^{(1)} \dots B_t^{(p^*)}]$$

where $p^* = max(p, q)$ and I_k is an identity matrix of order k. For $h = 1, \dots, r$, with $r = min(p, q)$, $B_t^{(h)}$ is a $(k \times 2k)$ matrix such that

$$B_t^{(h)} = \begin{bmatrix} S_{1,t}^{(h)'} & 0_{1,2} & \dots & 0_{1,2} & 0_{1,2} \\ 0_{1,2} & S_{2,t}^{(h)'} & \dots & 0_{1,2} & 0_{1,2} \\ \vdots & \dots & \dots & \dots & \vdots \\ 0_{1,2} & \dots & \dots & S_{k-1,t}^{(h)'} & 0_{1,2} \\ 0_{1,2} & \dots & \ddots & 0_{1,2} & S_{k,t}^{(h)'} \end{bmatrix}$$

with $0_{1,2}$ being a (1×2) vector of zeros and

$$S_{it}^{(h)'} = [y_{i,t-h} \qquad \sqrt{h_{ii}(t-h)}] \qquad \text{for } i = 1, \ldots, k; h = 1, \ldots, r \quad (2.2)$$

with $h_{ij}(t) = cov(y_{it}, y_{jt}|I^{t-1})$, for $i, j = 1, \ldots, k$. For $h = r+1, \ldots, p^*$, $B_t^{(h)}$ is a ($k \times k$) diagonal matrix whose non-zero elements are given by $y_{i,t-h}$, if $r = q$, or $\sqrt{h_{ii}(t-h)}$, if $r = p$. The vector x_t has dimension ($K \times 1$) with $K = k + (2k)r + k(p^* - r)$. From a probabilistic point of view, x_t is a random coefficients vector whose distribution has been here assumed to be multivariate normal $x_t \sim MVN(0, V)$. By definition, x_t is independent of I^{t-1}. Furthermore, it is assumed that x_t can be partitioned as:

$$x_t = \{x'_{R,t} \quad x'_{Q,t}\}', \quad (2.3)$$

with $x_{R,t}$ being of dimension ($k \times 1$), and that the covariance matrix V is block diagonal:

$$V = \begin{bmatrix} R & 0_{k,K-k} \\ 0_{K-k,k} & Q \end{bmatrix}$$

where $R = var(x_{R,t})$ and $Q = var(x_{Q,t})$. The matrix Q can be written as

$$Q = \begin{bmatrix} Q_r & 0_{2rk,k(p^*-r)} \\ 0_{k(p^*-r),2rk} & Q_{p^*} \end{bmatrix}$$

where Q_r is a $(2rk \times 2rk)$ block diagonal matrix, with each block being ($2k \times 2k$), and Q_{p^*} is a $(k(p^*-r) \times k(p^*-r))$ block diagonal matrix, with each block being ($k \times k$).
The block diagonal structure of V implies that the conditional covariance matrix of y_t is given by:

$$H_t = C_t V C_t' = R + \sum_{h=1}^{r} B_t^{(h)} Q_r^{(h)} B_t^{(h)'} + \sum_{m=1}^{(p^*-r)} B_t^{(m+r)} Q_{p^*}^{(m)} B_t^{(m+r)'} \quad (2.4)$$

where $Q_r^{(h)}$, $h = 1, \ldots, r$, is the h-th diagonal block of Q_r and $Q_{p^*}^{(m)}$, $m = 1, \ldots, (p^* - r)$, is the m-th diagonal block of Q_{p^*}. Each of the quadratic forms in (2.4) measures the contribution of a specific lag to the conditional covariance matrix H_t. It can be easily noted how equation (2.4) is the multivariate counterpart of an analogous relationship holding for the conditional variance in univariate GARCH (and BL-GARCH) models. If each of the matrices R, $Q_r^{(h)}$, $Q_{p^*}^{(m)}$ is positive definite, H_t is positive semi-definite at each time point t. However, this condition is sufficient but not necessary since, paralleling a similar result for univariate GARCH models, H_t can still be positive semi-definite even if one or more of the above matrices are not positive-definite.

For a k-dimensional process, the total number of parameters is equal to

$$n_p = r\frac{2k(2k+1)}{2} + (p^* + 1 - r)\frac{k(k+1)}{2} \tag{2.5}$$

For a MBL-GARCH model of order (1,1) the expression in (2.5) simplifies to $n_p = (5k^2 + 3k)/2$. Application of this formula for k=2,3,4 gives n_p=13,27,46, respectively. In this case, the matrix Q can be written as a $(2k \times 2k)$ block matrix with $Q = Q_1$. In order to better understand the meaning of each of the elements of Q, it is useful to represent it as a block matrix with each block having dimension (2×2). The block of place $\{i, j\}$ (i,j=1,...,k) is denoted by $[Q]_{ij}$ and is given by the (2×2) submatrix containing the elements of the matrix Q situated at the intersection of the rows $\{2i - 1, 2i\}$ with the columns $\{2j - 1, 2j\}$. It can be easily shown that $[Q]_{ij} = [Q]^{'}_{ji}$.

The i-th component of the vector process y_t under study follows a univariate BL-GARCH(1,1) process (Storti and Vitale, 2003a) with conditional variance equation given by

$$h_{ii}(t) = r_{ii} + S^{'}_{it}[Q]_{ii}S_{it} \tag{2.6}$$

where r_{ij} is the element of place (i, j) in R, for $i, j = 1, \ldots, k$. The conditional covariances between the components of y_t are parameterized by the off-diagonal blocks of Q and the off-diagonal elements of R:

$$h_{ij}(t) = r_{ij} + S^{'}_{it}[Q]_{ij}S_{jt} \tag{2.7}$$

with r_{ij} being the element of place (i, j) in R, for $i, j = 1, \ldots, k$.

This scheme can be easily generalized to higher order models by simply augmenting equations (2.6) and (2.7) with the contributions of lags greater than 1. For the general case of a MBL-GARCH(p,q) model, we obtain the following updating equations for the conditional variances and covariances:

$$h_{ii}(t) = r_{ii} + \sum_{h=1}^{r} S_{it}^{(h)'}[Q_r^{(h)}]_{ii} S_{it}^{(h)} + \sum_{m=1}^{p^*-r} S_{it}^{(m+r)'}[Q_{p^*}^{(m)}]_{ii} S_{it}^{(m+r)}$$

$$h_{ij}(t) = r_{ij} + \sum_{h=1}^{r} S_{it}^{(h)'}[Q_r^{(h)}]_{ij} S_{jt}^{(h)} + \sum_{m=1}^{p^*-r} S_{it}^{(m+r)'}[Q_{p^*}^{(m)}]_{ij} S_{jt}^{(m+r)}.$$

3 Asymmetric dynamics in MBL-GARCH models

This section focuses on the ability of MBL-GARCH models to reproduce asymmetric conditional variance and covariance dynamics. For ease of exposition we focus on the case of a model of order (1,1). However, the analysis can be easily extended to higher order models by simple but tedious algebra.

First of all, we must note that asymmetric effects enter the conditional variance model in (2.6) by means of the interaction terms

$$2q_{ii}(1,2)y_{i,t-1}\sqrt{h_{ii}(t-1)},$$

with $q_{ij}(h,l)$ being the element of place (h,l) in $[Q]_{ij}$, for $i,j = 1,\dots,k$, $h,l = 1,2$. If $q_{ii}(1,2) < 0$ and $y_{i,t-1} < 0$, a positive penalty term is added to $h_{ii}(t)$ while, if $y_{i,t-1} > 0$, the same quantity is subtracted from $h_{ii}(t)$. The asymmetric response of the conditional covariance $h_{ij}(t)$ is guaranteed in a similar fashion by the sum of cross products:

$$AS_t(i,j) = q_{ij}(2,1)y_{j,t-1}\sqrt{h_{ii}(t-1)} + q_{ij}(1,2)y_{i,t-1}\sqrt{h_{jj}(t-1)}$$

MBL-GARCH models also allow for time varying conditional correlations between financial assets. These can be calculated as:

$$\rho_{ij}(t) = \frac{h_{ij}(t)}{\sqrt{h_{ii}(t)h_{jj}(t)}} \qquad (3.1)$$

The conditional correlations in (3.1) can potentially allow for different response patterns to positive and negative shocks as well. More precisely, for a given $\epsilon > 0$, let $h_{ii}^{+}(t)$ and $h_{ii}^{-}(t)$ be the conditional variances at time t derived from having observed $y_{i,t-1} = \epsilon$ and $y_{i,t-1} = -\epsilon$, respectively. The impact of asymmetry at time t can be measured as the difference between $h_{ii}^{+}(t)$ and $h_{ii}^{-}(t)$. Furthermore, assuming $q_{hh}(1,2) \leq 0$, for $h = i, j$, define:

$$\alpha_{i,j}(t) = \frac{\sqrt{h_{ii}^{-}(t)h_{jj}^{-}(t)} - \sqrt{h_{ii}^{+}(t)h_{jj}^{+}(t)}}{\sqrt{h_{ii}^{-}(t)h_{jj}^{-}(t)} + \sqrt{h_{ii}^{+}(t)h_{jj}^{+}(t)}},$$

with $0 \leq \alpha_{i,j}(t) \leq 1$, and the *asymmetry free* conditional covariance:

$$h_{ij}^{*}(t) = h_{ij}(t) - AS_t(i,j)$$

It can then be shown that if:

$$AS_t(i,j) > \alpha_{i,j}(t)h_{ij}^{*}(t)$$

negative returns at time (t-1), i.e. $y_{i,t-1} = -\epsilon$ and $y_{j,t-1} = -\eta$, for ϵ and η both positive, will generate an increase of the conditional correlation coefficient $\rho_{ij}(t)$ greater than that due to having observed positive returns of the same magnitude, which is $y_{i,t-1} = \epsilon$ and $y_{j,t-1} = \eta$. This is an important issue in risk estimation and, more specifically, in portfolio allocation problems where, in a down market, inadequate modelling of asymmetries in the conditional correlation dynamics could lead to an insufficient degree of diversification. This would result in a portfolio allocation which could be quite far from the optimum.

4 Maximum Likelihood Estimation

The unknown parameters in the Multivariate BL-GARCH model described in section 2 are collected in the matrix V. Under the assumption of conditional normality, $(y_t|I^{t-1}) \sim MVN(0, H_t)$, up to a constant, the log-likelihood function of the observed data for the MBL-GARCH model can be expressed in the prediction error decomposition form as

$$\log L(y;\boldsymbol{\theta}) = -\frac{1}{2}\sum_{t=1}^{T}\log|H_t| - \frac{1}{2}\sum_{t=1}^{T} y_t' H_t^{-1} y_t \tag{4.1}$$

with $\boldsymbol{\theta} = [vech(R)', vech(Q)']'$ where $vech(.)$ denotes the operator that stacks the lower triangular portion of a $(N \times N)$ matrix as a $N(N+1)/2 \times 1$ vector. The problem can be reformulated as that of estimating a covariance matrix. Hence, in order to obtain well-defined estimates, the numerical maximization of the above log-likelihood function by standard numerical algorithms requires the solution of a constrained optimization problem. This can be avoided by resorting to a multivariate generalization of the EM type algorithm proposed by Storti and Vitale (2003b) for univariate BL-GARCH models. The algorithm exploits the random coefficient representation of the MBL-GARCH model and is based on the EM algorithm proposed by Shumway and Stoffer (1982) for the estimation of linear state-space models. The proposed algorithm allows to obtain estimates of the model parameters which, by construction, guarantee the positive definiteness of the estimated conditional covariance matrix. It is based on a sequence of simple regression calculations and, hence, it is very easy to implement. Furthermore, a key point is that the EM algorithm does not require the calculation of numerical derivatives. Differently, for a conditionally Gaussian model, the implementation of gradient based methods requires the calculation of the score vector. Since, in most cases analytical expressions for the score of multivariate GARCH models are not easily available, it is usually necessary to resort to the computation of numerical derivatives. It is worth noting that, even for medium-sized models, the use of numerical derivatives can lead to numerically unstable estimates and to a substantial increase of the computational burden, slowing down the estimation process. At the same time, closed form expressions for the score are available only in a few cases. For the BEKK model (Engle and Kroner, 1995), an analytic expression for the score is provided by Lucchetti (2002) while Hafner and Herwartz (2003) show that, for the same model, maximum likelihood estimates based on numerical derivatives are clearly outperformed by their counterparts computed using analytical derivatives. Also, the numerical instability of maximum likelihood estimates of multivariate GARCH models is addressed by Jerez et al. (2001) who show that the log-likelihood can be ill-conditioned due to presence of high inter-asset correlations and to the fact that the log-likelihood surface is very flat in a neighbourhood of the optimum.

Finally, it has been documented (Watson and Engle, 1983) that the EM algorithm, differently from standard gradient based procedures, like the scoring algorithm, is robust to poor initial estimates.

To this purpose note that, in the random coefficient representation of the MBL-GARCH model, the observation equation (2.1) can be reformulated as

$$y_t = B_t x_{Q,t} + x_{R,t}$$

with $B_t = [B_t^{(1)}, \ldots, B_t^{(p^*)}]$ and where $x_{Q,t}$ and $x_{R,t}$ can be interpreted as a state vector and an observation error, respectively. The transition equation for $x_{Q,t}$ does not incorporate any dynamic structure since $x_{Q,t}$ is a sequence of i.i.d. random variables. This fits into the standard linear state-space framework by assuming the transition matrix to be given by a matrix of zeros. In practice the transition equation can be represented by the identity

$$x_{Q,t} = \mathbf{0} x_{Q,t-1} + x_{Q,t}$$

where $\mathbf{0}$ is a $(2k \times 2k)$ matrix of zeros. A similar state-space representation is adopted by Sentana et al. (2004) for the estimation of conditionally heteroskedastic factor models.

Following the terminology adopted by Dempster et al. (1977), the log-likelihood of the *complete* data ($x_{Q,t}$,y_t) is given by

$$\begin{aligned} \log L^c(x_{Q,t}, y_t; \boldsymbol{\theta}) = & - \frac{T}{2}\log|R| - \frac{1}{2}\sum_{t=1}^{T} x_{R,t}^{'} R^{-1} x_{R,t} \\ & - \frac{T}{2}\log|Q| - \frac{1}{2}\sum_{t=1}^{T} x_{Q,t}^{'} Q^{-1} x_{Q,t} \end{aligned}$$

The *Expectation Step* of the algorithm is performed using the Kalman filter to calculate

$$
\begin{aligned}
E(\log L^c(x_{Q,t}, y_t; \boldsymbol{\theta})|I^T) = & -\frac{T}{2}\log|R| - \frac{1}{2}\text{tr}(R^{-1}\Gamma_R) \\
& -\frac{T}{2}\log|Q| - \frac{1}{2}\text{tr}(Q^{-1}\Gamma_Q)
\end{aligned} \tag{4.2}
$$

where $\Gamma_A = \sum_{t=1}^{T}[P_{At}^T + x_{At}^T(x_{At}^T)']$ with $P_{At}^T = var(x_{At}|I^T)$ and $x_{At}^T = E(x_{At}|I^T)$, to be calculated by a simplified version of the ordinary Kalman Filter recursions. Since, in the linear state space representation of the MBL-GARCH model, no dynamic structure is implied by the transition equation, only a filtering step, i.e. only a forward pass through the data, instead of a smoothing step, i.e. a forward and backward pass[1], is necessary in order to calculate these quantities.

In the case of a model of order (1,1), the *Maximization Step* is performed by simply differentiating the expected log-likelihood in (4.2) with respect to R and Q. The values of R and Q which maximize (4.2) at the $(i+1)$th iteration are given by

$$
\begin{aligned}
\hat{R}^{(i+1)} &= T^{-1}\Gamma_R \\
\hat{Q}^{(i+1)} &= T^{-1}\Gamma_Q
\end{aligned}
$$

It follows that the estimate of V at iteration $i+1$ is

$$
\hat{V}^{i+1} = \begin{pmatrix} \hat{R}^{(i+1)} & 0_{k,2k} \\ 0_{2k,k} & \hat{Q}^{(i+1)} \end{pmatrix}
$$

The above steps must be iteratively repeated until convergence is reached. In the general case of a model of order (p,q), the log-likelihood of the com-

[1] In the state-space models terminology, *filtering* is the operation of recursively estimating the current state vector given past and present information. In this sense, the *filtering* step implies only a *forward* pass through the data. Differently, *smoothing* implies the estimation of the current state vector given the whole stretch of data. In Gaussian linear state-space models, this can be achieved by means of the Kalman smoother combining a *filtering* step with a *back-filtering* step performed on data in reversed order. In this sense, the *smoothing* step implies a *forward* and a *backward* pass through the data.

plete data can be written as

$$\log L^c(x_{Q,t}, y_t; \boldsymbol{\theta}) = -\frac{T}{2}\log|R| - \frac{1}{2}\sum_{t=1}^{T} x_{R,t}^{'} R^{-1} x_{R,t}$$
$$- \sum_{h=1}^{r}\left[\frac{T}{2}\log|Q_r^{(h)}| + \frac{1}{2}\sum_{t=1}^{T} x_{Q,th}^{'}(Q_r^{(h)})^{-1} x_{Q,th}\right]$$
$$- \sum_{m=1}^{p^*-r}\left[\frac{T}{2}\log|Q_{p^*}^{(m)}| + \frac{1}{2}\sum_{t=1}^{T} x_{Q,t(m+r)}^{'}(Q_{p^*}^{(m)})^{-1} x_{Q,t(m+r)}\right]$$

where $x_{Q,ti}$ $(i = 1, \ldots, p^*)$ is the sub-vector of $x_{Q,t}$ associated to the i-th lag. Hence, the expected likelihood can be further decomposed as

$$E(\log L^c(x_{Q,t}, y_t; \boldsymbol{\theta})|I^T) = -\frac{T}{2}\log|R| - \frac{1}{2}\mathrm{tr}(R^{-1}\Gamma_R)$$
$$- \sum_{h=1}^{r}\left\{\frac{T}{2}\log|Q_r^{(h)}| + \frac{1}{2}\mathrm{tr}[(Q_r^{(h)})^{-1}\Gamma_{Q_r^{(h)}}]\right\}$$
$$- \sum_{m=1}^{p^*-r}\left\{\frac{T}{2}\log|Q_{p^*}^{(m)}| + \frac{1}{2}\mathrm{tr}[(Q_{p^*}^{(m)})^{-1}\Gamma_{Q_{p^*}^{(m)}}]\right\}. \quad (4.3)$$

Each of the components in (4.3) depends on a specific sub-matrix of model coefficients, R, $Q_r^{(h)}$, $Q_{p^*}^{(m)}$ $(h = 1, \ldots, r;\ m = 1, \ldots, p^* - r)$. In order to obtain an estimate of these matrices, the M-step can then be performed separately for each of the expected log-likelihood components in (4.3) leading to formulas similar to those reported for the (1,1) case.
The EM algorithm does not allow to directly calculate the standard errors associated to the estimated parameters. However, their asymptotic value can be approximated by evaluating the observed Information Matrix $\tilde{I}(\boldsymbol{\theta})$ at the maximum likelihood estimate i.e. for $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$. The ij-th element of $\tilde{I}(\boldsymbol{\theta})$ can be shown to be given by

$$\tilde{I}_{ij}(\boldsymbol{\theta}) = \frac{1}{2}\sum_{t=1}^{T}\left\{\mathrm{tr}\left[H_t^{-1}\frac{\partial H_t}{\partial\theta_i}H_t^{-1}\frac{\partial H_t}{\partial\theta_j}\right]\right\} \quad (4.4)$$

for $i, j = 1, \ldots, n_p$. The derivatives in (4.4) can be recursively calculated as

$$\frac{\partial H_t}{\partial \theta_i} = \frac{\partial C_t}{\partial \theta_i} V C_t' + C_t \frac{\partial V}{\partial \theta_i} C_t' + C_t V \frac{\partial C_t'}{\partial \theta_i} \qquad i = 1, \ldots, n_p$$

The above recursions follow from standard linear state-space theory (Harvey, 1989).
Recently much research has been dedicated to the estimation of multivariate conditional heteroskedastic models under deviations from conditional normality (see Bauwens and Laurent, 2005, among the others). Under this respect, it is worth noting that, if the assumption of conditional normality is not satisfied, the log-likelihood in (4.1) can be considered only as a Gaussian approximation to the underlying (unknown) log-likelihood function. In this case the estimates obtained by the procedure described in this section can be considered as Quasi Maximum Likelihood estimates and the expressions for the asymptotic standard errors must be modified accordingly (Bollerslev and Wooldridge, 1992).
In order to give further insight on the performance of the proposed algorithm in finite samples, a simulation study has been performed. The Data Generating Process (DGP) has been assumed to be given by a conditionally Gaussian bivariate MBL-GARCH(1,1) model with

$$Q = \begin{bmatrix} 0.1 & -0.1 & 0.1 & -0.075 \\ -0.1 & 0.7 & -0.1 & 0.7 \\ 0.1 & -0.1 & 0.1 & -0.08 \\ -0.075 & 0.7 & -0.08 & 0.7 \end{bmatrix} \qquad R = \begin{bmatrix} 0.5 & 0.3 \\ 0.3 & 0.5 \end{bmatrix} \times 10^{-3} \tag{4.5}$$

Namely, 500 time series have been generated from the DGP in (4.5) considering two different sample sizes, $T_1 = 1500$ and $T_2 = 5000$.

The results of the simulation experiment have been reported in tables 4.1, for $\hat{R}$, and 4.2, for $\hat{Q}$. Although, for $T = 1500$, the estimates appear to be characterized by a slight bias component, this tends to disappear as we move to the case $T = 5000$. Also, the simulated variance decreases quickly as the sample size increases.

$T = 1500$			
$\hat{E}(\hat{r}_{11})$ = 0.550^{*}		$v\hat{a}r(\hat{r}_{11})$= $0.012^{\dagger}$	
$\hat{E}(\hat{r}_{21})$ = 0.389^{*}	$\hat{E}(\hat{r}_{22})$ =0.541^{*}	$v\hat{a}r(\hat{r}_{21}) = 0.011^{\dagger}$	$v\hat{a}r(\hat{r}_{22}) = 0.014^{\dagger}$
$T = 5000$			
$\hat{E}(\hat{r}_{11})$ = 0.523^{*}		$v\hat{a}r(\hat{r}_{11})$ = $0.003^{\dagger}$	
$\hat{E}(\hat{r}_{21})$ = 0.357^{*}	$\hat{E}(\hat{r}_{22})$=0.509^{*}	$v\hat{a}r(\hat{r}_{21})$=$0.003^{\dagger}$	$v\hat{a}r(\hat{r}_{22})$= $0.004^{\dagger}$

Table 4.1: Simulated expectations ($\hat{E}(.)$) and variances ($v\hat{a}r(.)$) of the estimated elements of R ($^{(*)} \times 10^{3}$, $^{(\dagger)} \times 10^{6}$).

5 Diagnostic Tests

In order to assess the ability of the estimated model to explain the conditional covariance dynamics, we derive a set of test statistics based on the strategy illustrated in procedure 3.1 in the paper by Wooldridge (1991). This procedure has been shown to yield test statistics which are robust under conditional and unconditional variance misspecification (Wooldridge, 1990). The null assumes that the conditional variance or covariance model is correctly specified while, under the alternative, some sort of misspecification is present. Namely the test detects if appropriately defined misspecification indicators are useful in predicting a generalized residual (in the sense defined by Wooldridge, 1991) which, under the null, should have conditional expectation equal to zero.
The conditional covariance between $y_{i,t}$ and $y_{j,t}$ can be expressed as the conditional expectation $E(w_{ij,t}|I^{t-1})$, where $w_{ij,t} = y_{i,t}y_{j,t}$. In order to implement the test it is necessary to define an appropriate weighting function. To this purpose we use the conditional variance of $w_{ij,t}$. Note that it is possible to express y_t as $y_t = G_t z_t$, where z_t is an i.i.d. $(k \times 1)$ random vector such that $E(z_t) = 0$, $Var(z_t) = I_k$ and $G_t = H_t^{1/2}$ with $H_t^{1/2}$ being any positive definite matrix such that$(H_t^{1/2})(H_t^{1/2})' = H_t$. Also assume that $z_{i,t}$ and $z_{j,s}$ are independent random variables for all t and for any $i \neq j$. The i-th element of y_t can be expressed as $y_{i,t} = G_{i,t}z_t$, where $G_{i,t}$ is the

$T = 1500$			
$\hat{E}(\hat{q}_{11})$ = 0.104		$v\hat{a}r(\hat{q}_{11})$= 0.411*	
$\hat{E}(\hat{q}_{21})$ = -0.101	$\hat{E}(\hat{q}_{22})$ =0.674	$v\hat{a}r(\hat{q}_{21}) = 0.353^*$	$v\hat{a}r(\hat{q}_{22}) = 2.632^*$
$\hat{E}(\hat{q}_{33})$ = 0.103		$v\hat{a}r(\hat{q}_{33})$= 0.464*	
$\hat{E}(\hat{q}_{43})$ = -0.080	$\hat{E}(\hat{q}_{44})$ =0.679	$v\hat{a}r(\hat{q}_{43}) = 0.341^*$	$v\hat{a}r(\hat{q}_{44}) = 2.972^*$
$\hat{E}(\hat{q}_{31})$ = 0.100	$\hat{E}(\hat{q}_{32})$ = -0.099	$v\hat{a}r(\hat{q}_{31})$= 0.413*	$v\hat{a}r(\hat{q}_{32})$= 0.364*
$\hat{E}(\hat{q}_{41})$ = -0.080	$\hat{E}(\hat{q}_{42})$ =0.661	$v\hat{a}r(\hat{q}_{41}) = 0.412^*$	$v\hat{a}r(\hat{q}_{42}) = 2.386^*$
$T = 5000$			
$\hat{E}(\hat{q}_{11})$ = 0.103		$v\hat{a}r(\hat{q}_{11})$= 0.116*	
$\hat{E}(\hat{q}_{21})$ = -0.101	$\hat{E}(\hat{R}_{22})$ =0.688	$v\hat{a}r(\hat{q}_{21}) = 0.080^*$	$v\hat{a}r(\hat{q}_{22}) = 0.661^*$
$\hat{E}(\hat{q}_{33})$ = 0.102		$v\hat{a}r(\hat{q}_{33})$= 0.131*	
$\hat{E}(\hat{q}_{43})$ = -0.079	$\hat{E}(\hat{q}_{44})$ =0.695	$v\hat{a}r(\hat{q}_{43}) = 0.079^*$	$v\hat{a}r(\hat{q}_{44}) = 0.996^*$
$\hat{E}(\hat{q}_{31})$ = 0.100	$\hat{E}(\hat{q}_{32})$ = -0.097	$v\hat{a}r(\hat{q}_{31})$= 0.117*	$v\hat{a}r(\hat{q}_{32})$= 0.091*
$\hat{E}(\hat{q}_{41})$ = -0.080	$\hat{E}(\hat{q}_{42})$ =0.678	$v\hat{a}r(\hat{q}_{41}) = 0.088^*$	$v\hat{a}r(\hat{q}_{42}) = 0.706^*$

Table 4.2: Simulated expectations ($\hat{E}(.)$) and variances ($v\hat{a}r(.)$) of the estimated elements of Q ($^{(*)} \times 10^3$).

i-th row of G_t. The conditional variance of $w_{ij,t}$ is given by

$$\begin{aligned} v_{ij}(t) = Var(y_{i,t}y_{j,t}|I^{t-1}) &= E(y_{i,t}^2 y_{j,t}^2|I^{t-1}) - \{E(y_{i,t}y_{j,t}|I^{t-1})\}^2 = \\ &= E(y_{i,t}^2 y_{j,t}^2|I^{t-1}) - h_{ij}^2(t) \end{aligned}$$

Letting $G_{ih,t}$ be the h-th element of $G_{i,t}$, it can be shown (see Appendix) that

$$\begin{aligned} E(y_{i,t}^2 y_{j,t}^2|I^{t-1}) &= \sum_{h=1}^{k} G_{ih,t}^2 G_{jh,t}^2 \nu_4 + \sum\sum_{h \neq k} G_{ih,t}^2 G_{iw,t}^2 \\ &+ \sum\sum_{h \neq k} G_{ih,t} G_{iw,t} G_{jh,t} G_{jw,t} \end{aligned} \tag{5.1}$$

where $\nu_4 = E(z_{h,t}^4)$. Under the assumption of conditional multivariate normality of y_t, $\nu_4 = 3$. Alternatively, an estimate of ν_4 can be obtained by

calculating its sample equivalent from the estimated standardized residuals $\hat{z}_t = \hat{H}_t^{-1/2} y_t$.
The test statistic can then be computed by means of two auxiliary regressions. First, define a scalar misspecification indicator λ_t and divide it by $\sqrt{\hat{v}_{ij}(t)}$, which is the estimated conditional standard deviation of $w_{ij,t}$, obtaining the weighted misspecification indicator

$$\tilde{\lambda}_t = \frac{\lambda_t}{\sqrt{\hat{v}_{ij}(t)}}$$

Then regress it on

$$\nabla_{\theta_{ij}} \tilde{h}_{ij}(t) = \frac{\nabla_{\theta_{ij}} h_{ij}(t)}{\sqrt{\hat{v}_{ij}(t)}}$$

where θ_{ij} is the vector of parameters involved in the calculation of the conditional covariance $h_{ij}(t)$, $\hat{v}_{ij}(t)$ is the estimated conditional variance of $w_{ij,t}$ and $\nabla_{\theta_{ij}} h_{ij}(t)$ is the gradient of $h_{ij}(t)$ with respect to θ_{ij} evaluated at the MLE $\hat{\theta}_{ij}$. In order to compute this quantity, it is possible to use the recursions described in the previous section. At the second stage, define a generalized residual as $\hat{e}_{ij,t} = w_{ij,t} - \hat{h}_{ij}(t)$ and regress 1 on $\tilde{e}_{ij,t}\tilde{\eta}_{ij,t}$ where

$$\tilde{e}_{ij,t} = \frac{\hat{e}_{ij,t}}{\sqrt{\hat{v}_{ij}(t)}}$$

and $\tilde{\eta}_{ij,t}$ is the residual from the previous regression. Finally, the test statistic is given by $TR_u^2 = T - SSR$ where SSR is the ordinary sum of squared residuals from the second regression. It can be shown (Wooldridge, 1990) that TR_u^2 is asymptotically distributed as a χ_1^2.
Similarly, a robust test statistic for detecting misspecifications of the conditional variance models can be obtained following procedure 5.1 in Wooldridge (1991). As potential misspecification indicators the following variables have

been considered

$$
\begin{aligned}
\lambda_{1,t-1} &= I(y_{i,t-1} < 0, y_{j,t-1} < 0) \\
\lambda_{2,t-1} &= I(y_{i,t-1} < 0, y_{j,t-1} > 0) \\
\lambda_{3,t-1} &= I(y_{i,t-1} > 0, y_{j,t-1} < 0) \\
\lambda_{4,t-1} &= I(y_{i,t-1} > 0, y_{j,t-1} > 0) \\
\lambda_{5,t-1} &= y_{i,t-1}y_{j,t-1}I(y_{i,t-1} < 0, y_{j,t-1} < 0) \\
\lambda_{6,t-1} &= y_{i,t-1}y_{j,t-1}I(y_{i,t-1} < 0, y_{j,t-1} > 0) \\
\lambda_{7,t-1} &= y_{i,t-1}y_{j,t-1}I(y_{i,t-1} > 0, y_{j,t-1} < 0) \\
\lambda_{8,t-1} &= y_{i,t-1}y_{j,t-1}I(y_{i,t-1} > 0, y_{j,t-1} > 0)
\end{aligned}
$$

where $I(.)$ is the indicator function which assumes value 1 when the argument is true and 0 otherwise. The first set of misspecification indicators

$$\{\lambda_{1,t-1}, \lambda_{2,t-1}, \lambda_{3,t-1}, \lambda_{4,t-1}\}$$

allow to test for fixed effects of the sign of shocks on the conditional covariance while the second set

$$\{\lambda_{5,t-1}, \lambda_{6,t-1}\lambda_{7,t-1}\lambda_{8,t-1}\}$$

gives further insight taking into account the magnitude of shocks. Furthermore we have considered the usual univariate sign indicators

$$
\begin{aligned}
\lambda_{9,t-1} &= I(y_{i,t-1} < 0) \\
\lambda_{10,t-1} &= I(y_{j,t-1} < 0) \\
\lambda_{11,t-1} &= I(y_{i,t-1} > 0) \\
\lambda_{12,t-1} &= I(y_{j,t-1} > 0)
\end{aligned}
$$

Since the effect of the size of a shock on the conditional variance could depend on its sign or on the sign of the shocks observed for other assets, we also consider the following transformations of the above indicators:

$$
\begin{aligned}
\lambda_{13,t-1} &= y^2_{i,t-1}\lambda_{9,t-1} \\
\lambda_{14,t-1} &= y^2_{i,t-1}\lambda_{11,t-1}
\end{aligned}
$$

In order to test the conditional covariance specification, the first twelve indicators $\{\lambda_{i,t-1}, i = 1, \ldots, 12\}$ are considered while, for testing the specification of the conditional variance of the i-th asset, the set of indicators $\{\lambda_{9,t-1}, \lambda_{11,t-1}, \lambda_{13,t-1}, \lambda_{14,t-1}\}$ is used.

6 Applications to real data

6.1 An illustrative example on U.S. stock returns

The class of MBL-GARCH models is here used to model the dynamics of the conditional covariance matrix of a multivariate time series of daily returns on three U.S. stock market indexes: Dow Jones Industrials (J) S&P 500 (S) NASDAQ (N). The observation period goes from January 3rd, 1995 to February, 9th, 2001 (1543 observations). Prior to the estimation of the volatility model, the returns series have been mean corrected. Furthermore, a MA model was fitted to the NASDAQ series in order to remove the serial correlation structure of the conditional mean [2].

The next step has been to fit a MBL-GARCH model of order (1,1) to the returns series by means of the procedure described in section 4. The estimated parameters and the associated z-statistics (table 6.1) provide evidence in favour of the hypothesis that asymmetric effects are present in the dynamics of the conditional variances and covariances: the parameters driving asymmetry in the conditional variance and covariance models (see the discussion in section 3) are all negative and significantly different from zero at any reasonable level.

The impact of asymmetry on the models for the conditional variances and correlations, respectively, has been graphically depicted in figure 6.1. The News Impact Curves (NIC) associated to the estimated conditional variance models have been represented on the main diagonal while the off-diagonal contour plots can be intepreted as Correlation News Impact Surfaces (CNIS)

[2] The fitted model for the NASDAQ returns series r_t^N was

$$r_t^N = a_t - \underset{(0.0254)}{0.0582} a_{t-2} - \underset{(0.0255)}{0.0963} a_{t-8}$$

where a_t is a serially uncorrelated white noise process.

	coeff.	z-stat.		coeff.	z-stat.		coeff.	z-stat.
r_{JJ}	3.791 *	5.879	$q_{JJ}(1,1)$	0.081	5.268	$q_{JS}(1,1)$	0.059	4.744
r_{JS}	3.071*	6.029	$q_{JJ}(2,2)$	0.505	6.692	$q_{JS}(2,2)$	0.505	8.149
r_{SS}	2.899 *	6.677	$q_{JJ}(1,2)$	-0.067	-4.712	$q_{JS}(1,2)$	-0.063	-4.710
r_{JN}	1.649 *	3.262	$q_{SS}(1,1)$	0.075	6.346	$q_{JS}(2,1)$	-0.066	-4.181
r_{SN}	1.980 *	4.903	$q_{SS}(2,2)$	0.572	10.073	$q_{SN}(1,1)$	0.059	4.932
r_{NN}	3.344 *	8.141	$q_{SS}(1,2)$	-0.069	-5.799	$q_{SN}(2,2)$	0.519	12.417
			$q_{NN}(1,1)$	0.129	8.012	$q_{SN}(1,2)$	-0.061	-3.619
			$q_{NN}(2,2)$	0.596	14.638	$q_{SN}(2,1)$	-0.060	-4.006
			$q_{NN}(1,2)$	-0.066	-5.292	$q_{JN}(1,1)$	0.039	2.941
						$q_{JN}(2,2)$	0.413	8.579
						$q_{JN}(1,2)$	-0.054	-3.044
						$q_{JN}(2,1)$	-0.048	-2.721

Table 6.1: ML estimates and associated z-statistics for MBL-GARCH model parameters. Legend: (J) Dow Jones (S) S&P500 (N) NASDAQ ($^{(*)} \times 10^5$).

in the sense of Kroner and Ng (1998). The CNIS are clearly asymmetric revealing how correlations tend to be higher in the negative (- -) news quadrant than in the positive (+ +) one.

The ability of the estimated model to reproduce the conditional variance and covariance dynamics has been assessed by means of the robust conditional moment tests described in section 5. The results of the diagnostic tests performed for both the conditional variance and covariance models have been reported in table 6.2. The estimated MBL-GARCH model performs well in capturing the asymmetric nature of the conditional covariances between the three stock market indexes. In particular, this is true for situations in which both returns are negative at time $(t-1)$. At the same time, the model partially fails to fully capture the asymmetry in the covariance between Dow-Jones and Nasdaq when both indexes are increasing at time $(t-1)$. In a similar fashion, the robust conditional moment tests also suggest that the model does not completely explain the asymmetric response of the conditional covariance between S&P 500 and NASDAQ to past returns on both indexes when these are positive at time $(t-1)$.
For the conditional variance model, the robust conditional moment test turns out to be significant only in one case detecting some residual dependence of the NASDAQ standardized returns on past positive returns.

	DJ-SP	DJ-NAS	SP-NAS	DJ	SP	NAS
$\lambda_{1,t-1}$	0.179	0.252	0.270			
$\lambda_{2,t-1}$	0.315	0.166	0.214			
$\lambda_{3,t-1}$	0.251	0.273	0.072			
$\lambda_{4,t-1}$	0.249	0.290	0.101			
$\lambda_{5,t-1}$	0.285	0.085	0.268			
$\lambda_{6,t-1}$	0.191	0.300	0.127			
$\lambda_{7,t-1}$	0.307	0.161	0.054			
$\lambda_{8,t-1}$	0.186	0.029*	0.034*			
$\lambda_{9,t-1}$	0.226	0.315	0.304	0.252	0.219	0.276
$\lambda_{10,t-1}$	0.108	0.283	0.250			
$\lambda_{11,t-1}$	0.301	0.261	0.232	0.260	0.258	0.316
$\lambda_{12,t-1}$	0.209	0.186	0.151			
$\lambda_{13,t-1}$				0.176	0.294	0.108
$\lambda_{14,t-1}$				0.171	0.272	0.022^*

Table 6.2: Robust conditional moment test statistics for the conditional covariance (left) and variance (right) specifications. (*) denotes values significant at the 0.05 level.

6.2 An application to futures hedging

Hedging can be defined as a strategy for reducing risk exposure. One of the most popular hedging strategies makes use of futures contracts to compensate the risk associated to an investment on the spot market. In this case hedging takes place whenever an investor, trading a given asset or commodity, tries to reduce risk exposure by simultaneously assuming opposite positions on the spot and future markets. The hedge ratio is given by the ratio between the number of units which is traded on the futures markets and the number of units which is traded on the spot market. A survey on the estimation of the hedge ratio can be found in the paper by Lien and Tse (2002).

If we denote by $r_{p,t}$ and $r_{f,t}$ the spot and futures market returns, respectively, the optimal hedge ratio can be shown to be given by[3]

$$OHR_t = h_{pf,t}/h^2_{f,t} \tag{6.1}$$

where $h^2_{f,t} = var(r_{f,t}|I^{t-1})$ and $h_{pf,t} = cov(r_{p,t}, r_{f,t}|I^{t-1})$. Since the optimal hedge ratio depends on the value of the conditional variance of the futures returns and the conditional covariance between these and the spot returns, its value is likely to react asymmetrically to bad and good news. Also, the presence of asymmetries in the response of the optimal hedge ratio to past news has been recently documented by Brooks et al. (2002).

In this section the proposed MBL-GARCH model is applied to the estimation of the optimal hedge ratio for the S&P500 stock market index. The data are daily time series of spot and futures closing prices covering the period from January 2nd, 1997 to October 1st, 2002. For the S&P500 futures contracts, there are four delivery months: March, June, September and December. The contracts are cash settled on the third friday of the delivery month. In order to obtain a continuous time series of futures prices, following the usual practice, at each time point we consider the contract closest to maturity. Also, in order to focus on the most actively traded contracts, each contract is replaced by the next closest on the Thursday preceeding the second Friday of the contract month i.e. one week before maturity. The first choice is motivated by the consideration that volume in the "far" contracts is usually very small while the latter is due to the fact the traded volume of futures contracts tends to fall sharply in the last week before the settlement date. Returns are calculated as the first difference of the log-transformed price series

$$r_{p,t} = log(P_{p,t}) - log(P_{p,t-1}) \qquad r_{f,t} = log(P_{f,t}) - log(P_{f,t-1})$$

where by $P_{p,t}$ and $P_{f,t}$ we have denoted the spot and futures prices, respectively.

The results of the ADF test suggest that both future and cash log-transformed prices are I(1). The following step is to further investigate the nonstationary features of the bivariate price process by means of Johansen's cointegration

[3]See Baillie and Myers (1991) for a discussion of this result.

SP		FUT	
ADF statistic	p-value	ADF statistic	p-value
-1.95625	0.3066	-1.960307	0.3047

Johansen's Coint. Test		
No. of Coint. Eq.	Test	p-value
0	271.0518	0.0001
1	3.889239	0.4288

Table 6.3: Augmented Dickey-Fuller (top) and Johansen's cointegration (bottom) tests for the log-transformed spot (SP) and futures (FUT) price series.

test (Johansen, 1991). The null hypothesis of no cointegrating relationships is rejected at any reasonable level while the hypothesis that one cointegrating relationship exists cannot be rejected (table 6.3). Hence, the following Vector Error Correction (VEC) model for the conditional mean of the log-transformed prices was tentatively specified and estimated:

$$\begin{bmatrix} r_{f,t} \\ r_{p,t} \end{bmatrix} = \begin{bmatrix} \underset{(2.9275)}{0.2389} \\ \underset{(-0.9663)}{-0.0825} \end{bmatrix} \eta_{t-1} + \begin{bmatrix} u_{f,t} \\ u_{p,t} \end{bmatrix}$$

with

$$\eta_{t-1} = \log(P_{f,t-1}) \underset{(-735.4670)}{-1.0078} \log(P_{p,t-1}) + \underset{(5.1702)}{0.0499}$$

with the values in brackets being z-statistics. In order to estimate the conditional variance matrix of $r_t = [r_{p,t} \quad r_{f,t}]'$ a MBL-GARCH (1,1) model is then fitted to the residuals of the model in (6.2), u_t, using the procedure described in section 4. The maximum likelihood estimates and associated z-statistics have been reported in table 6.4. All the coefficients are signicantly different from zero and, as expected, the estimates of $q_{11}(21)$, $q_{22}(21)$, $q_{21}(21)$ and $q_{21}(12)$ are negative suggesting the presence of asymmetries in the conditional variance and covariance dynamics. Again, a graphical appraisal of the impact of asymmetry has been offered in figure 6.2 by means of the NICs and the CNIS associated to estimated model. Despite the asymmetric nature of the conditional variances and covariance models, the CNIS appears

	coeff.	z-stat.		coeff.	z-stat.		coeff.	z-stat.
r_{pp}	0.338*	5.601	$q_{pp}(1,1)$	0.093	2.283	$q_{pf}(1,1)$	0.0870	2.014
r_{pf}	0.352*	5.376	$q_{pp}(2,1)$	-0.091	-6.716	$q_{pf}(2,1)$	-0.077	-4.883
r_{ff}	0.411*	5.681	$q_{pp}(2,2)$	0.739	18.422	$q_{pf}(1,2)$	-0.104	-6.268
			$q_{ff}(1,1)$	0.084	5.348	$q_{pf}(2,2)$	0.720	50.769
			$q_{ff}(2,1)$	-0.094	-6.813			
			$q_{ff}(2,2)$	0.721	49.170			

Table 6.4: ML estimates and associated z-statistics for the MBL-GARCH (1,1) model parameters. Legend: (p) S&P500 spot returns (f) S&P500 futures returns($^{(*)} \times 10^{-4}$).

fairly symmetric. Dynamic asymmetries in the estimated optimal hedge ratios (OHR) can be graphically detected in a similar fashion by means of analogous news impact surface for the OHR (Brooks et al., 2002). It is interesting to note how, differently from what observed for the conditional correlations, the asymmetry of the conditional covariance model is directly transmitted to the estimate of the OHR (figure 6.2) which reacts asymmetrically to bad and good news, being higher in the negative quadrant (- -) than in the positive one (+ +). The average OHR value is equal to 0.9255 while its minimum and maximum value are equal to 0.8019 and 1.1489, respectively. Looking at the time plot of the estimated OHR (figure 6.2) it is easily seen that the hedge ratio starts decreasing from the beginning of 2000 while it appears to be increasing from the beginning of 2001 to the end of the sample period.
The hedging performance of the estimated MBL-GARCH model has been compared to those of two alternative multivariate GARCH models: the DCC and the Constant Conditional Correlation (CCC) model proposed by Bollerslev (1990). For both models, the conditional variance has been assumed to follow a GARCH (1,1) model and the order of the correlation model in the DCC specification has also been set equal to (1,1). For the sake of brevity, the parameters estimates and associated standard errors for the DCC and CCC model parameters have not been reported but are available upon request. As a benchmark we have also considered a *naive* hedging strategy which assumes a constant OHR equal to one.
The effectiveness of the hedging strategy implied by each model can be

readily evaluated in terms of the variance of the returns on the hedged portfolio

$$r_{H,t} = r_{p,t} - OHR_t^{(A)} r_{f,t}$$

where $OHR_t^{(A)}$ denotes the OHR implied by model A. Obviously, better models are expected to yield lower variances. In table 7 we have reported the values of the implied variances for each of the hedged portfolios (σ_H^2), the rate of variance reduction with respect to the unhedged portfolio

$$VR = 1 - \frac{\sigma_H^2}{\sigma_U^2}$$

where σ_U^2 is the variance of the spot returns series, and, finally, the rate of variance reduction with respect to the benchmark which is the naive hedging strategy

$$VR_n = 1 - \frac{\sigma_H^2}{\sigma_{H,n}^2}$$

where, $\sigma_{H,n}^2$ is the variance of the hedged portfolio implied by the naive hedging. It can be easily seen that the MBL-GARCH model performs better than the other approaches since the hedged variance implied by this model is lower than that implied by the other hedging strategies. Namely, the relative variance reduction due to the hedging strategy is equal to the 94.78% of the variance of the unhedged portfolio while the improvement over the benchmark is 7.46%. Furthermore, compared to the other alternatives, the MBL-GARCH yields a hedged portfolio characterized by lighter tails implying that less probability mass is associated to extreme returns. At the same time the negative skewness of the hedged returns is less pronounced than in other cases.
Finally, the ability of the estimated model to reproduce the conditional variance and covariance dynamics has been assessed by means of the robust conditional moment tests described in section 5. The results of the diagnostic tests performed for both the conditional variance and covariance models have been reported in table 6.6. The estimated MBL-GARCH model performs well in reproducing the volatility dynamics of the observed dataset

	$naive$	$MBL-G$	DCC	CCC
$mean(\times 10^4)$	-0.05	-0.19	-0.01	0.02
$\sigma_H^{2\dagger}$	9.81	9.08	9.38	9.45
$VR(\%)$	94.36	94.78	94.61	94.57
VR_n (%)	-	7.46	4.38	3.72
$skew.$	-0.48	-0.37	-0.58	-0.67
$kur.$	6.24	5.62	7.31	8.31

Table 6.5: Hedging performance of different models: mean returns on the hedged portfolio, hedged variance (σ_H^2), relative variance reduction over the unhedged portfolio (VR), relative variance reduction over the *naive* hedging (VR_n) ($\dagger \times 10^6$), skewness and kurtosis coefficients.

being able to capture the asymmetric impact of past news on conditional variances as well as on conditional covariances. The null hypothesis of a correctly specified model is rejected only once. In particular, some residual dependence of the S&P500 futures volatility on past positive shocks is detected.

	SP-FUT	SP	FUT	SP-FUT	SP	FUT
$\lambda_{1,t-1}$	0.234		$\lambda_{8,t-1}$	0.187		
$\lambda_{2,t-1}$	0.317		$\lambda_{9,t-1}$	0.239	0.307	0.289
$\lambda_{3,t-1}$	0.218		$\lambda_{10,t-1}$	0.304		
$\lambda_{4,t-1}$	0.312		$\lambda_{11,t-1}$	0.246	0.257	0.316
$\lambda_{5,t-1}$	0.179		$\lambda_{12,t-1}$	0.310		
$\lambda_{6,t-1}$	0.250		$\lambda_{13,t-1}$	-	0.317	0.303
$\lambda_{7,t-1}$	0.289		$\lambda_{14,t-1}$	-	0.096	0.006 †

Table 6.6: Robust conditional moment test statistics for the conditional covariance (left) and variance (right) specifications: p-values. (†) denotes values significant at the 0.01 level.

7 Concluding remarks

The multivariate conditional heteroskedastic model proposed in this paper has been found to be able to adequately explain the sensitivity of conditional variances and covariances to the signs of past shocks. Furthermore, the parameterization implied by MBL-GARCH models, together with the EM algorithm used for the maximum likelihood estimation of the model parameters, allow to guarantee the positive definiteness of the estimated conditional covariance matrix at each time point, with no need for arbitrary untested constraints. Due to these features, MBL-GARCH models can be successfully applied to several problems in financial as well as in macroeconomic modelling. Namely, in my opinion their application to hedging problems can be particularly profitable as explored in section 6. Further investigation of this point is left for future research.

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Appendix

Proof of equation (5.1). By long and tedious algebra it follows that:

$$
\begin{aligned}
& E(y_{i,t}^2 y_{j,t}^2 | I^{t-1}) = E\{(G_{i,t} z_t)^2 (G_{j,t} z_t)^2 | I^{t-1}\} = \\
= \quad & E\{(\sum_{h=1}^{k} G_{ih,t} z_{h,t})^2 (\sum_{h=1}^{k} G_{jh,t} z_{h,t})^2 | I^{t-1}\} = \\
= \quad & E\{(\sum_{h=1}^{k} G_{ih,t}^2 z_{h,t}^2 + \sum_{h \neq k}\sum G_{ih,t} G_{iw,t} z_{h,t} z_{w,t})(\sum_{h=1}^{k} G_{jh,t}^2 z_{h,t}^2 + \\
+ \quad & \sum_{h \neq k}\sum G_{jh,t} G_{jw,t} z_{h,t} z_{w,t}) | I^{t-1}\} = \\
= \quad & E\{(\sum_{h=1}^{k} G_{ih,t}^2 z_{h,t}^2)(\sum_{h=1}^{k} G_{jh,t}^2 z_{h,t}^2) | I^{t-1}\} + E\{(\sum_{h=1}^{k} G_{ih,t}^2 z_{h,t}^2) \times \\
\times \quad & (\sum_{h \neq k}\sum G_{jh,t} G_{jw,t} z_{h,t} z_{w,t}) | I^{t-1}\} + \\
+ \quad & E\{(\sum_{h=1}^{k} G_{jh,t}^2 z_{h,t}^2)(\sum_{h \neq k}\sum G_{ih,t} G_{iw,t} z_{h,t} z_{w,t}) | I^{t-1}\} + \\
+ \quad & E\{(\sum_{h \neq k}\sum G_{ih,t} G_{iw,t} z_{h,t} z_{w,t})(\sum_{h \neq k}\sum G_{jh,t} G_{jw,t} z_{h,t} z_{w,t}) | I^{t-1}\}.
\end{aligned}
$$

Under the assumption $E(z_t^3) = 0$, the above expression can be simplified to give

$$\begin{aligned} & E(y_{i,t}^2 y_{j,t}^2 | I^{t-1}) = \\ & \sum_{h=1}^{k} G_{ih,t}^2 G_{jh,t}^2 E(z_t^4 | I^{t-1}) + \sum_{h \neq k}\sum G_{ih,t}^2 G_{iw,t}^2 E(z_{h,t}^2 z_{w,t}^2 | I^{t-1}) + \\ + & \sum_{h \neq k}\sum G_{ih,t} G_{iw,t} G_{jh,t} G_{jw,t} E(z_{h,t}^2 z_{w,t}^2 | I^{t-1}) = \\ = & \sum_{h=1}^{k} G_{ih,t}^2 G_{jh,t}^2 \nu_4 + \sum_{h \neq k}\sum G_{ih,t}^2 G_{iw,t}^2 + \sum_{h \neq k}\sum G_{ih,t} G_{iw,t} G_{jh,t} G_{jw,t} \end{aligned}$$

which is the desired result.

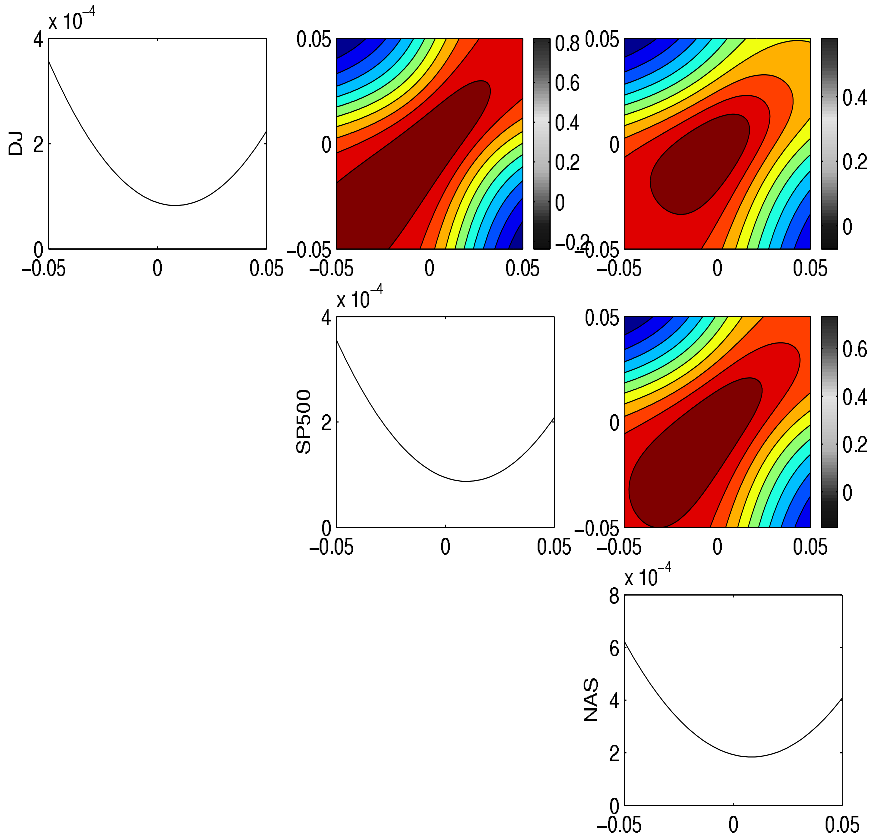

Figure 6.1: Conditional variance (main diagonal) *news impact curves* and correlation (off-diagonal) *news impact surfaces* for the DJ-SP500-NASDAQ dataset.

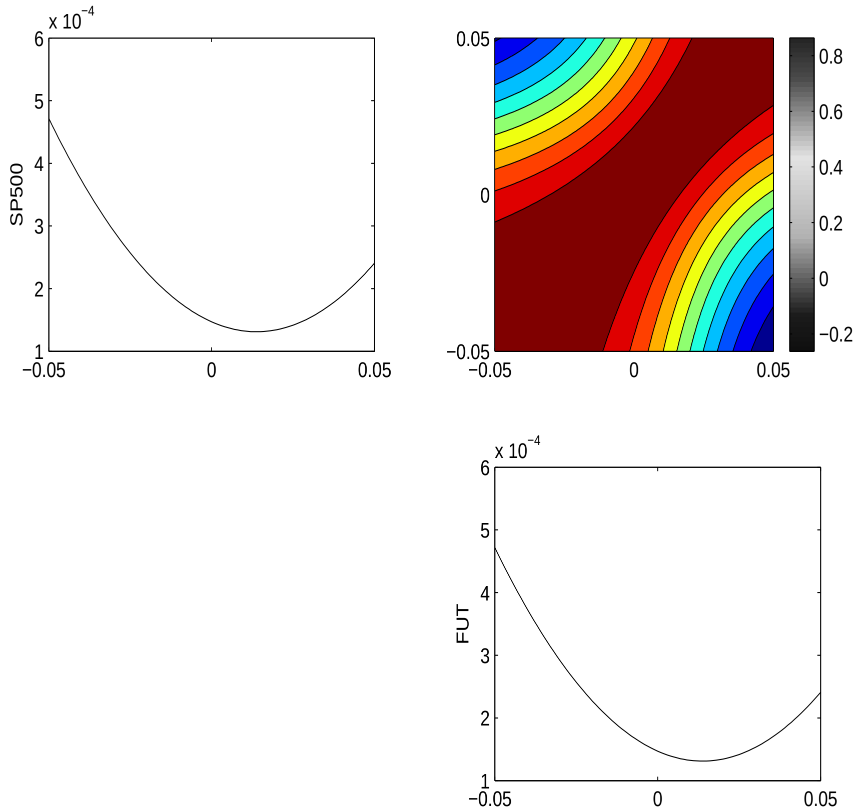

Figure 6.2: Conditional variance (main diagonal) *news impact curves* and correlation (off-diagonal) *news impact surface* for the model in table 6.4.

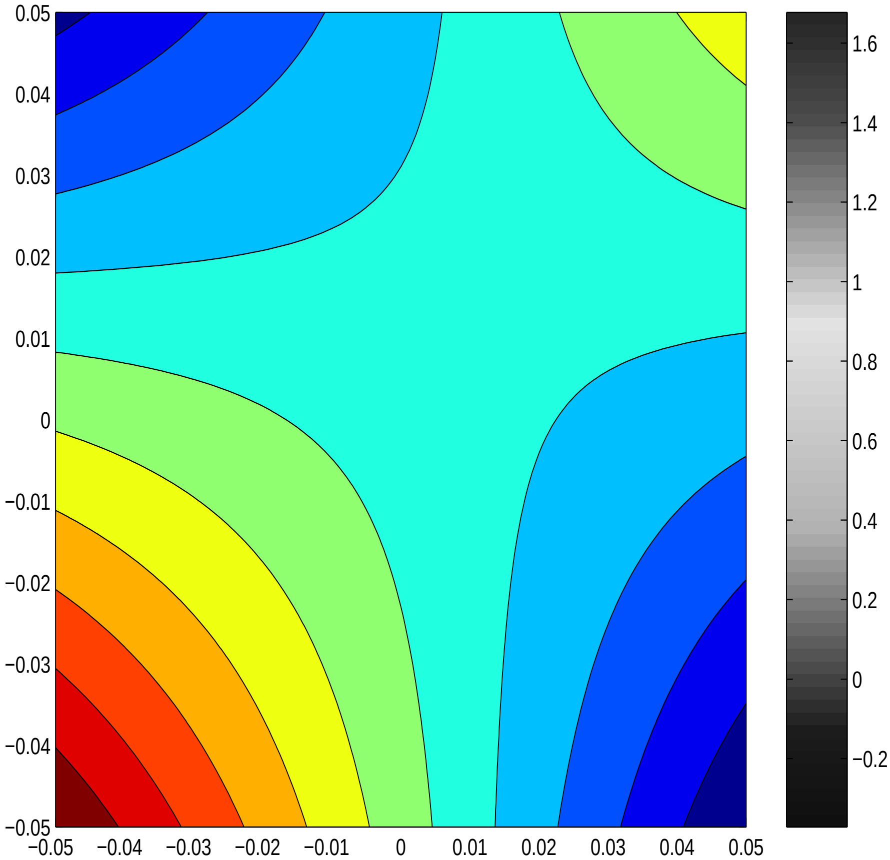

Figure 6.3: OHR news impact surface for the model in table 6.4.

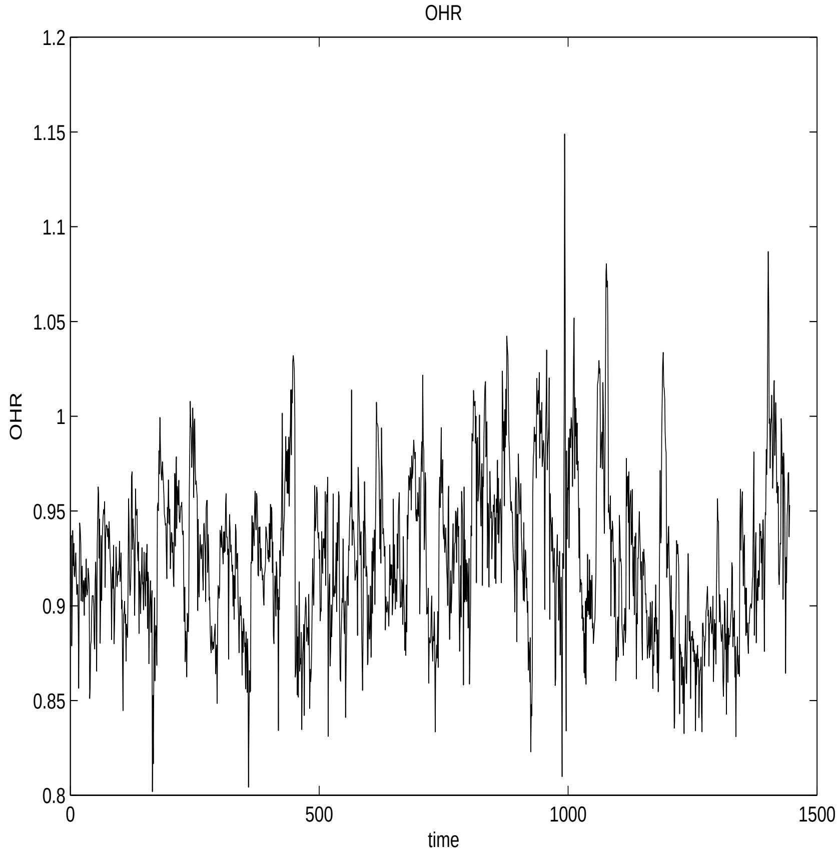

Figure 6.4: Time plot of the estimated OHR for the model in table 6.4.

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