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# THE CHAOTIC SYSTEM AND NEW PERSPECTIVES FOR ECONOMICS METHODOLOGY. A NOTE

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**Indice**

**Abstract**

*Historically, economists have, whenever possible, used linear equations to model economic phenomena, because they are easy to manipulate and usually yield unique solutions. However, it now has become impossible to ignore the fact that many important and interesting phenomena are not amenable to such treatment. With the chaos theory it is possible to take into account those aspects of the phenomena. The theory of chaos is challenging many of the fundamental presuppositions of the traditional older Newtonian world view of science. The implications of the new science vision will be explored starting from physics to arrive at economics in terms of their challenges to the traditional methodological views. In particular the implications of chaos control theory for the economics will be highlighted. The purpose of this paper is to show why the economists can no longer ignore that economics is a complex system and how the application of chaos control methods could improve the system's economic performance.*

*Keywords: economic modeling, chaos theory, chaos control*

## Introduction

Since the beginning, economists have tried to model economy using concepts and tools from so-called exact sciences. They have used linear equations to model economic phenomena, because they are easy to manipulate and usually yield unique solutions deriving this approach from classical physics. However, as progress has been made in field of exact sciences and new mathematical and statistical tools have been available to economists, it has became impossible to ignore that many important and interesting phenomena cannot be understood using this treatment.

In particular, linear models are not appropriate to describe economic phenomena like depressions and recessionary periods, stock market bubbles and crashes, and the occurrence of business cycles.

Market economies seem to resemble dynamically unstable systems rather than deterministic systems in which episodes of instability could be attributed to external shocks. They thus tend to involve a series of complicated decision at each price, each agent must decide what to supply and what to demand, how much to work and how much to play, whether to invest or whether to spend.

In this scenario it is very difficult to think about economic linear relationships of cause-effect, so that it is usually closer to reality to consider that relationships among the economic agents and variables are non-linear[1].

In recent years, the advances in nonlinear dynamical systems theory has determined significant changes also in the mainstream of economic theory. Interest in nonlinear systems and deterministic chaos has increased tremendously and the literature is still growing.

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[1] "[…] relationships between prices and quantities are nonlinear; thus to confine a model to be linear would place very severe restrictions on the forms of economic relationship which it could use. Similarly to confine the social function to be quadratic would be impose very restrictive assumptions". Livesey D. A., (1971), p. 526

Nonlinearity and chaos could mean the reconciliation of economics with a realistic representation of its phenomena. In fact, economic theorists are turning to the study of non-linear dynamics and chaos theory as possible tools to model economic phenomena that traditional economic theory considers not relevant to be explained because exogenous or apparently random.

There are at least two reasons why chaos is so interesting for explaining economic phenomena.

1) Realistic modeling. The proponents of the chaos theory are clearly attempting to articulate a new, more realistic, scientific world-view. The most salient characteristics of chaos theory are challenging and clearly contradictory to the fundamental notions of the Newtonian world view on which traditional economic theory is based. The Newtonian vision portrays the universe as a mechanism susceptible to precise measurement, prediction, and control. Chaos is characterised by complex behaviours represented by interactions among elements and whose behaviours appear random. In this sense it could represent a change of perspective in the explanation of economic pehnomena such as fluctuations, instability, crashes, crisis, and depressions.

2) Controllability of systems. Although chaos is unpredictable, the fact that it is deterministic makes it exploitable and therefore controllable. This means a change in the approach to control economic systems. The current opinion among scientists was that chaotic motion was neither predictable nor controllable because of the sensitive dependence on initial conditions. Small disturbances lead only to other chaotic motions and not to any stable and predictable alternative. Paradoxically sensitivity to initial conditions can be used to control it[2]. This observation opened possibilities for changing behaviour of natural systems without interfering with their inherent properties.

This idea was quickly appreciated in physics and in other natural sciences. Applied to economics the chaotic models are very new and for policy analysis could have different consequences from those associated with more conventional models. It has been shown that one can control deterministic

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[2] Ott E., Grebogi C., Yorke J. A.(1990)

chaos and simultaneously improve efficiency of the economic system. In fact when complex dynamics lead to inferior performance applying control by chaos methods the decision makers could obtain considerable improvement in the system's economical properties in terms of profits and welfare.

Therefore economists can no longer ignore that economy is a complex system and that the application of chaos control methods could improve the control of economic systems in terms of performance.

In this paper the implications of the new science vision will be explored starting from physics to arrive at economics in terms of its challenges to the traditional methodological views. In particular the implications of chaos control theory will be highlighted as a new perspective for modeling economic phenomena.

Therefore we will show which implications for policy implementation could be derived by using a chaotic approach to model and to control economic systems.

## *Looking for a more realistic way to model economic phenomena*

It is generally accepted that economy belongs to complex systems because it contains various and different interconnections that act synergistically upon each other. The global behavior of an economy may be difficult to predict due to nonlinear linkages between its elements and perfect knowledge of the various elements in isolation and is not sufficient to understand and predict the overall behavior of the economy.

So, it is very difficult to think about economic linear relationships of cause-effect. On the contrary it is usually closer to reality to consider that relationships among the economic agents and variables are non-linear[3]. Thus economies involve a series of complicated decisions at each price, each agent must decide what

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[3] "[…] relationships between prices and quantities are nonlinear; thus to confine a model to be linear would place very severe restrictions on the forms of economic relationship which it could use. Similarly to confine the social function to be quadratic would be impose very restrictive assumptions". Livesey D. A., (1971), p. 526

to supply and what to demand, how much to work and how much to play, whether to invest or whether to spend. Market economies resemble dynamically unstable systems rather than deterministic systems in which episodes of instability could be attributed to external shocks.

The reductionist approach, applied by traditional economic theory overlooks these interconnections among elements and their influence upon economic behavior, so both deterministic and stochastic descriptions are used to define main features of economic dynamics[4].

The proponents of chaos and complexity theory are attempting to articulate a new, more realistic, scientific world-view. In particular the characteristics of chaos theory are challenging and clearly contradictory to the fundamental notions of the Newtonian world view dominant in physics and traditional economics. Nonlinearity and chaos could represent the reconciliation of economics with a realistic representation of its phenomena.

This awareness and consequently the requirement of more realistic models have lead to powerful new concepts and tools to detect, analyze and control apparently random phenomena that with a deeper analysis could show chaotic behaviours.

Chaos theory has attracted particular attention because of its ability to produce sequences whose characteristics resemble the fluctuations observed in the market place. Most of economic variables both micro, prices and quantities, and macro, consumption, investment and employment, oscillate. For these oscillations at the level of macro and micro variables it is not possible to find a specific pattern because they are not cyclic[5] and not due to external schocks.

The pioneering studies by Benhabib and Day (1980, 1981, 1982), have been important for making the profession aware of the potential usefulness of chaos theory and its tools for analysing economic phenomena. In the early 1980s Benhabib and Day (1982), Grandmont (1985) and Boldrin and Montrucchio (1986) derived chaotic business cycle models from utility and profit

[4] Holyst J. A, Urbanowicx K. (2000)
[5] Day R. H., (1992)

maximization principles within the general equilibrium paradigm of perfectly competitive markets and rational expectations. Business cycles have several puzzling features such as a continuing wave-like movement, behaviour partially erratic and at the same time serially correlated; expansion phases which are recurring and not periodical and differ by their amplitude and their length. All these phenomena can be represented not by linear but by non-linear and chaotic models.

Day (1983) attracted considerable attention to the possibility of chaos in two quite familiar contexts: a classical growth model and a Solow growth model. Hommes (1991) showed how easy it is to produce chaos in Hichsian type models with lags in investement and consumption. Bala, Maiumdar and Mitra (1998) located sufficient conditions for robust ergodic chaos to appear in growth models.

Chaos has also been analysed in the context of a multiplier accelerator type model by Dana and Malagrange (1984). Benhabib and Nishimura (1979) employ the Hopf bifurcation in their study of how the properties of an optimal growth model are affected by the discount rate. Deneckere and Pelikan (1986) discuss some necessary conditions for chaos. Mitra (2001) shows the existence of chaotic equilibrium growth paths a model of endogenous growth with externalities.

Chaotic dynamics have played an important role in identifying the main sources of economic fluctuations. In this way there has been a change of perspective in the explanation of these fluctuations that are considered internally generated as part of the deterministic process. The books by Day, 1994; Dechert, 1996; Goodwin, 1990; Hommes, 1991; Lorenz, 1993; Medio, 1992; Puu, 1997, and others represent the basic references on this focus.

Grandmont (1986) is concerned with the effects of various government policies while Grandmont and Laroque demonstrate the importance of the expectations formation mechanism for the stability of economy.

Farmer (1986) and Reichlin (1986) both consider production economies and both make use of Hop bifurcation which is often thought to be more robust than the flip bifurcation. In Farmer (1986) chaos depends upon the government's debt policy. In Reichlin (1986) it is shown that fiscal policy can cure chaos.

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The application chaos theory does not only concern the definition of models but also empirical analysis. The introduction of the assumption of non-linearity and chaos has it made possible to reveal in time series behaviours much closer to series observed in macroeconomics and finance.

Nevertheless controversies exist about the existence of nonlinearity or chaos with economic data and with financial data, although regarding the smaller sample sizes available with economic data they have greated deeper differences in the macroeconomic literature than in the financial one. This controversy has led to some misunderstandings on the importance of research of chaos in economic time series. In fact some testing for the presence of chaos in financial and in particular macroeconomic time series data has not been very supportive of the chaos hypothesis (Ramsey, Sayers and Rothman 1988).

Investigators have found substantial evidence for nonlinearity but relatively weak evidence for chaos per se. In general, it has been stressed that an accurate empirical testing of chaos requires the availability of high quality, high frequency data, which makes financial time-series a more promising field of research than macroeconomic data. Macro-economic series are characterised by limitation of economic data, short, noisy, and possibly nonstationary time series.

Brock and Sayers (1988) test various American macroeconomic time series concluding that they are not supportive of low-order deterministic chaos. Sayers (1986) found evidence of nonlinearity in some American work stoppage data. Scheinkman and Lebaron (1989) examined data on American stock prices but their conclusions are not consistent with chaos. Frank and Stengos (1988), and Frank, Gencay, and Stengos (1988) find no evidence of chaos in the U.S., Canadian, and international macroeconomic time series. On the other hand, Barnett and Chen (1988a,b) detected chaos in the U.S. Divisia monetary aggregates, a conclusion confirmed by DeCoster and Mitchell (1991, 1992). Similar results have been obtained by Frank and Stengos (1989). Barnett and Serletis (2000) list seven studies that have used various economic time series to test for nonlinearity or chaos. Those studies highlighted the presence of nonlinear dependence, but there is no consensus at all on whether there is chaos in

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economic time series.

Concerning financial data DeCoster (1992) found evidence of chaos in the futures prices regarding twenty years from 1989. Bajo-Rubio O., et al., (1992) using the Grassberger-Procaccia test and Lypunov exponent found evidence of chaotic behaviour in Spanish Peseta-Dollar spot and forward exchange rates at different periods. Brorsen and Yang (1994) didn't refuse chaos determitic hypothesis. Serletis and Gogas (1997) tested for chaos in seven East European black market exchange rates. They used three inference methods, the BDS test, the NEGM test, and the Lyapunov exponent estimator concluding that there was evidence consistent with a chaotic nonlinear generation process in two out of the seven series: the Russian ruble and East German mark. Abhyankar, Copeland, and Wong (1995) examined the behavior of the U.K. Financial Times Stock Exchange 100 (FTSE 100) index over the first six months of 1993. Applying the Hinich (1982) bispectral linearity test, the BDS test, and the NEGM test, they found evidence of nonlinearity, but no evidence of chaos. Abhyankar, Copeland, and Wong (1997) test for nonlinear dependence and chaos in real-time returns on the world's four most important stock-market indices. They found a rejection of the hypothesis of independence in favor of a nonlinear structure for all data series, but no evidence of low-dimensional chaotic processes.

### Control of chaotic systems. What suggestions for the economic analysis?

"A chaotic motion is generally neither predictable nor controllable. It is unpredictable because a small disturbance will produce exponentially growing perturbation of the motion. It is uncontrollable because small disturbances lead only to other chaotic motions and not to any stable and predictable alternative"[6]. This was the general opinion among the scientists but as Ditto (1995) suggests paradoxically the cause of despair is also the reason to hope. The sensitivity to small changes and the numerous unstable, periodic orbits (UPOs) of various periodicities

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[6] Dyson F., (1988), pp. 182-184

can be used to control the chaotic systems.

Chaos control can be divided into two categories[7]:.

1)      to suppress chaotic dynamics. This control is based on the principle that by relatively small perturbations, unstable periodic orbits (UPOs) could be stabilized, that is, induced to behave periodically, suppressing chaotic behaviour

2)      to generate or enhance chaos in nonlinear systems (known as chaotification or anti-control of chaos). Although the topic of enhancing chaos has attracted some attention in the scientific literature there are indeed very few theoretical publications and even fewer experimental works. In this context we will address methods to suppress chaos by stabilization of chaotic trajectory on stable orbit.

Sensitive dependence to initial conditions is the first cause of instability of chaotic systems but in a control context, this proprety implies that only small adjustments to the system can produce large changes to perform the desidered dynamics.

Starting from these considerations Ott, Grebogi and Yorke[8] (1991) have proposed a method (OGY) for the control of chaos. Moving the orbit in the stable manifold of a designed unstable periodic orbit allows to implement the control with greater success only after the system moves into the region of the phase space that we wish to control. Achieving the chosen UPO is guaranteed by ergodic property of chaos: if we wait long enough a chaotic trajectory will approach arbitrarily close to any point selected within the attractor. Therefore it has to wait until the dynamics approach the desired region before control can be initiated; when we arrive close enough to the target point we use linear control with respect to a parameter in order to stay in that neighborhood[9]. In this way it

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[7] Lu J., Zhou T., Zhang S., (2002)

[8] Control methods inspired by the OGY method tend to be ''closedloop'' control systems in which the applied perturbation is determined by the state of the system. There also exist ''open-loop'' control systems in which the applied perturbation is independent of the system's state; that is, there is no feedback loop. Such control schemes involved modulating the chaotic systems with random or periodic signals.

[9] Chanfreau P., Lyyjynen H., (1999).

is possible to change behaviour of natural systems without interferring with their inherent properties and to extract many types of behaviors from a single system with minimal intervention.

The OGY method and its various versions[10] require precise information about the targed UPO[11] and is typically used when the target point is an unstable fixed point or a point in an unstable periodic cycle[12] embedded in the strange attractor of the chaotic system under consideration.

Nevertheless, often waiting[13] to approach a desired region could take a long time. In fact the perturbations to control the parameter are applied only when the system is close to the chosen UPO. We need a net that can be thrown over the phase space in order to capture the trajectory.

To avoid waiting long it could be very useful to have some techniques to persuade the dynamics to quickly approach the control region. A method devised for this is the targeting[14] which maintains the basic assumptions of the OGY method regarding the lack of prior knowledge about the system dynamics and small a perturbation on control parameters, but differently from OGY control which is local. In the targeting the control activity is extended to regions further away from the chosen target. In this case we will implement a global control[15].

### *Economics and control of chaotic systems*

In the field of control systems the main criticism moved to the

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[10] For survey see Kantz H., Schreiber T., (1997), Chen G., Dong X., (1993), Pyragas K., (1992), Jackson E. A., Grosu I., (1995), Breeden J. L., (1994), Mirus K. A., Sprott J. C., (1999), Tian Y.-C. Tadé M. O., (2000), Boccaletti S. et al. (2000).

[11] Tian Y. P., Chen G., (2001).

[12] Chanfreau P., Lyyjynen H., (1999).

[13] "[…] In general, one expects that the average waiting time before a typical orbit approaches a given saddle periodic point is proportional to the dimension of the attractor", Kostelic E. J., Barreto E., (1997), p.160.

[14] Shinbrot T., et al. (1990).

[15] Kopel M., (1997).

linear models arises from the fact that they mislead a real understanding of the economic phenomenon and can induce inadequate and erroneous economic policies[16]. An incorrect policy advice based on the wrong theory produces effects that will be fundamentally different from those predicted by the theory. An alternative to performing adequate policies could be the use of non-linear models and in particular chaotic ones.

In economics there have been some applications of chaos control methods. Examples are: Holyst et al. (1996) applied the Ott-Grebogi-Yorke method to a model of two competing firms; Kopel (1997) using a simple model of evolutionary market dynamics showed how chaotic behaviour can be controlled by making small changes in a parameter that is accessible to the decision makers and how firms can improve their performance measures by use of the targeting method. Xu et al. (2001) introduced an approach to detect UPOs pattern from chaotic time series from the Kaldor business cycle model. Kaas (1998) proved that within a macroeconomic disequilibrium model stationary and simple adaptive policies are not capable of stabilizing efficient steady states and lead to periodic or irregular fluctuations for large sets of policy parameters. The application of control methods for chaotic dynamical systems shows that the government can, in principle, stabilize an unstable Walrasian equilibrium in a short time by varying income tax rates or government expenditures.

These applications have shown that it is possible to have consequences for policy analysis that are different from those associated with more conventional models[17]. One can control the deterministic chaos in a simple model and improve simultaneously the system efficiency. When complex dynamics lead to inferior performance applying the control by chaos methods, the decision makers could obtain considerable improvement in the system's economical properties in terms of profits and welfare (Day 1994).

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[16] Bullard and Butler A (1991).

[17] Since economic systems are complex non-equilibrium systems, [....], the existing knowledge in complex systems research should provide the right tool for an efficient control of turbulence. Huebler, (1992)

Considering both stabilization[18] and targeting[19] procedures or the combined[20] use of them have highlighted the possibility of modifying the behaviour of a system to achieve its best performance[21].

Therefore we have a change of perspective in the control strategies[22] and this point seems particularly interesting for the insights of economic policies.

1)      Moving from given orbits to other ones on the attractor means choosing different behaviour of the systems, that is, different trade-offs of economic policy.

2)      Employment of an instrument of control in terms of resources in order to achieve a specific goal of economic policy will be smaller if compared to the use of traditional techniques of control.

3)      Using the sensitivity to initial conditions could mean strong "energy saving", i.e. resources, to perform economic policy goals. If the system is non chaotic, the effect of an input on the output is proportional to the latter. Vice versa when the system is chaotic, the relation between input and output are made exponential by the sensitivity to initial conditions. We can obtain a relatively large improvement in system performance by use of small controls. Therefore if the system is chaotic limited resources don't reduce the possibility by policy-makers to catch up prefixed goals of economic policies.

Therefore small parameter changes and the presence of many aperiodic orbits are attractive characteristics for economic policy insights because they could mean resource saving and choosing among different trade-offs of economic policies.

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[18] Holyst (1996).

[19] Kopel (1997).

[20] Kopel (1997).

[21] Ott et al., (1990).

[22] Boccaletti et al. (2000).

### Why Economics didn't accomplish what Physics has accomplished?

Economists traditionally have cared about whether economics is, or could be, a science. In order to become a true science economics has tried since its beginnings to model itself using concepts from so-called exact sciences. "They have taken their mathematics and their deductive techniques from physics, their statistics from genetics and agronomy, their systems of classification from taxonomy and chemistry, their model-construction techniques from astronomy and mechanics, and their methods of analysis of the consequences of actions from engineering"[23].

At first economics has had to adapt the pattern pointed out by the "hard" sciences emulating the success reached by the Newtonian thought in the explanation and prediction of natural phenomena. This success was founded on a belief in the existence of some universal, unalterable and deterministic laws that were supposed to govern the observed behavior in nature.

As Allais highlighted (1992) "the essential condition of any science is the existence of regularities which can be analysed and forecast. This is the case of celestial mechanics but it is true for many economic phenomena whose analysis displays the existence of regularities which are similar to those found in the physical sciences. This consideration is the basis of why economics is a science, and why this science can rest on the same general principles and methods of classical thermodynamics and in general as Physics".

The possibility that there should be similarities of structure or interpretation in the mathematical modeling of economic and physical systems has been an important focus in the economic speculation that produced the neoclassical theory. These topics have been a matter of considerable importance to Walras and many of his contemporaries. To treat the state of an economy as an equilibrium analogous to the equilibrium of a mechanical system has had enormous influence on the development of

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[23] Schoeffler S., (1955), p. 40.

traditional economic theory.

On one hand Jevons (1924) said that "Economics, if it is to be a science at all, must be a mathematical science […]mechanics of utility and self-interest," on the other ~~from other one~~ Walras maintained "that economics, like astronomy and mechanics is both an empirical and a rational science[24]". In fact its explanation of the existence of an "auctioneer", whose only purpose was to generate equilibrium prices remembers Maxwell's imaginary demon who controls an opening in an otherwise impermeable membrane separating two volumes of a fluid[25]. Fisher[26], developed a mechanical analogy between economics and physics, invoking force and distance to be analogous to price and number of goods, respectively.

Instead the description of economic systems in the sense of thermodynamics have pioneering contributions by Keynes, von Neumann (1963), Samuelson (1955), Georgescu-Roegen (1971).

Neoclassical economics and classical thermodynamics seek to describe phenomena in terms of solutions to constrained optimization problems; supply and demand vectors in economics resemble the generalized energies and volumes of physical systems; vice versa prices resemble generalized temperatures and pressures. Following this path the relations among economic phenomena are considered actual laws and thus constant and ubiquitous like the laws of gravity and expansion of material.

By the 1980s Physics had offered new suggestions for a more realistic understanding and modeling of economic phenomena. The spread of non-equilibrium thermodynamic ideas in the natural sciences pushed the economists to follow this approach also in the economic analysis. One of the reasons was widespread recognition that neoclassical economics was an inadequate "forma mentis" to understand real economic processes. The marginalist revolution created three fundamental misunderstandings:

1)        The Representative Agent who is a scale model of the whole society with ~~e~~ extraordinary capacities, particularly

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[24] Jaffè W., (1977), pag. 47

[25] Foley D., (2002)

[26] Thoben H.,(1982).

concerning the area of information processing and computation. The real world is made by different and bounded agents.

2)      Equilibrium as a natural end of economic systems. This concept is not adequate to describe and to support phenomena in perpetual motion. The equilibrium paths of even very standard economic models are much richer than the saddle-point literature suggested by neoclassical theory. In fact equilibria might not approach a steady-state, but could end in limit cycles, in which variables endlessly repeat cyclical movements, or even in chaotic paths of a highly irregular kind.

3)      Linear models or at least the linearization of models in the region of a solution have been tradionally preferred by economists. This is why the linear models with one solution or one equilibrium position can be solved explicitly without using numerical procedures. The linear models, in fact are not appropriate to include phenomena like depressions and recessionary periods, stock market price bubbles and corresponding crashes, persistent exchange rate movements, the occurrence of regular and irregular business cycles.

The new perspective opened by thermodynamics of non-equilibrium and the advances in nonlinear dynamic systems theory has also determined significant changes in the mainstream of economic theory. Chaos theory has allowed to achieve good results in terms of modelling of phenomena and their empirical analysis.

Two problems could be outlined in the application of chaos theory and its tools to economics:

1)      Concerning economic models that display chaos, many of them are classical models, ad hoc models in the sense that the special structures of the underlying difference or differential equations are postulated and are not derived from basic axioms[27]. To demonstrate that simple and known economic models could display chaotic behaviours was a good strategy to push the interest of economists towards the chaos theory. Probably we now need to build economic models that are not derived from classical ones but characterised by structure and dynamics considering the

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[27] Lorenz H. W., (1993)

real features of economy.

2)        Concernig data analysis, application of chaotic tools like the Lyapunov exponent, dimension correlation, BDS test, provoked controversies about the presence of chaotic behaviours. The cause is indicated in the features of economic and financial data. This problem could be overcome by the combined use of tools to analyse those data and considering new tools like VRA (visual recurrence analysis[28]) to study short time series like economic ones.

Along with the chaos theory we took from physics and applied each time to economics methodologies and tools, we forgot an important point: a basic difference exists between physics, in general an exact sciences, and economics that doesn't allow a mechanical application of the  methodologies of physics to economics.

An economic system is composed of many people, whose psychology and relationships are constantly changing. The economic agents are heterogeneous and are endowed with subjectivity; they have different choice capabilities, tastes, information processing, they can be contemporarily both consumers and producers. These elements affect the economics structure that, unlike the physical one, depends upon human behaviours and is the result of human actions. This is a problem that physics has never coped with, and it has caused the mathematical techniques and modeling philosophy in economics to diverge from those in physics

The chaos theory has pushed economists to deal with the following basic questions: economics is not a linear phenomenon, its dynamics does not converge to a unique solution, its actors are not representative agents, but also in this case we had just a mechanical application of this theory without taking into account the specific features of economic systems. Surely, compared with models of neoclassical theory the application of the chaos theory to economics allowed to interpret phenomena considered as ininfluent, exogenous, stochastic and so on. The techniques of chaos control allow to control these systems and we have

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[28] Faggini M., (2004)

indicated above what insights there could be by applying those techniques to economic phenomena.

A way to take into account the typical features of economics could be to use heterogeneous agent based models. In this way we will pass from equations based models (EBM) to agent based models (AMB). In literature some kind of heterogenity has been demonstrated to produce chaotic behaviours[29]. The benefits[30] of ABM can be thus defined: (i) ABM captures emergent phenomena such as chaos; (ii) ABM provides a natural description of a system; and (iii) ABM is flexible.

## *Conclusions*

The subject of economics is not that of an exact science, so economics by its nature cannot conform rigorously to the models of the exact sciences. If we want to consider economics as a science this does not imply that one can be reduced to the other. This, of course, does not mean that a social science is impossible but just a different way of doing science because human actions are conditioned by social mechanisms just as the phenomena of nature are governed by mechanisms of their own[31]. The problems of economic agents are of a different order than the problems of physical objects. In this changeable context we have to face a difficult task: of finding whether or not there are durable patterns for constructing explicit models.

In policy formulation, building an appropriate model is a crucial focus. In fact in order to influence economic outcomes or processes, policy.makers must rely on a model that is, a description of economic systems that could be little more than a rough picture of reality.

We should have a description of all complex

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[29] Some examples: Lux T., (1995, 1998), Brock W. A., Hommes C., (1997, 1998), Gaunersdorfer, A., (2000), Kyrtsou C., Labys W., Terraza M., (2001)

[30] Bonabeau E., (2002)

[31] "In both spheres the aim of scientific work is to reveal the mechanisms involved and there is every reason to expect such work to be as meaningful and illuminating in the social sphere as it is in the natural domain" (Lawson 1998, pag. 170)

interrelationships and feedback loops that exist among the economic variables. We cannot draw conclusions from individual relationships between particular variables of interest, excluding variables or relationships that have a secondary or indirect effect. This can be dangerous, not so much because it aggregates or simplifies the structure of the economy, but because it may ignore or misrepresent some aspect of the economy's structure that play an important part in its dynamic behaviour.

Recognizing the existence of deterministic chaos in economics is important from both a theoretical and practical points of view. From the theoretical point of view, if a system is chaotic we may construct mathematical models, which would provide a deeper understanding of its dynamics. More recently, many studies concern the arising of chaotic behaviours from agent based models. Differently from previously chaotic models these can be considered more realistic because they are not based on assumptions of representative agents but of heterogenous interacting agents. Chaos is demonstrated to manifest because there is a some kind of heterogeneity (income, expectations, beliefs preferences, bounded rationality)

From the practical point of view, the discovery of chaotic behaviours makes it possible to control them. Small parameter changes and the presence of many aperiodic orbits are a characteristic which is attractive for economic policy insights.

Using sensitivity for initial conditions to move from given orbits to other ones of attractors means to choose different behaviour of the systems, that is, different trade-off of economic policy. Moreover the employment of an instrument of control in terms of resources in order to achieve a specific goal of economic policy will be smaller if compared to the use of traditional techniques of control.

The salient feature of applying of chaotic control is the strong "energy saving", that is resources, to perform economic policy goals. If the system is non chaotic the effect of an input on the output is proportional to the latter. Vice versa when the system is chaotic, the relation between input and output are made exponential by the sensitivity to initial conditions. We can obtain a relatively large improvement in system performance by the use of small controls.

Therefore if the system is chaotic, limited resources don't reduce the possibility of policy-makers to catch up prefixed goals of economic policies. Resource saving and choosing among different trade-offs of economic policies (many orbits) could be significant motivations to use chaotic models in ~~the~~ economic analysis.

Considering the opportunity offered by chaotic approach and agent-based models in economics we have to combine the two methodologies. We have to proceed in this way: Testing economic data with chaotic methodology and tools, proved to show chaotic behavior, we can use agent based models to describe that phenomenon and finally to apply chaos control to push it to the desidered performance.

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