Attitude Control of an Underactuated Planar Multibody System Using Momentum Preserving Internal Torques

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Abstract

In the last few years interest in versatile reconfigurable arrays for space applications has been growing and several concepts tailored for different mission needs have been proposed. Nevertheless, a compelling application that justifies their higher cost and complexity with respect to conventional systems has not yet been found. Here a novel approach to the design of an Attitude Control System (ACS) for small reconfigurable spacecraft is proposed. It shall exploit momentum-preserving internal torques generated by the modules of the multibody array rotating relative to each other. The goal is to achieve better performance in efficiency, accuracy and robustness with respect to state-of-the-art ACSs, which is a bottleneck of small spacecraft technology. This paper investigates the characteristic behaviour of a planar multibody array whose attitude is controlled using internal joint torques. To do this, relevant reorientation trajectories are shown and discussed. With respect to previous work in the field, optimal attitude control trajectories that take into account module impingement are discussed and the dynamics of momentum-preserving manoeuvres is explained in detail from both physical and mathematical points of view. The results demonstrate that further development of the concept is desirable.

Nomenclature

$\mathbf{A}$	=	rotation matrix [-]
$\mathbf{B}_{di}$	=	virtual displacement transformation matrix [varies]
C	=	constraint equation [varies]
$\mathbf{C}$	=	constraint vector [varies]
$\mathbf{C_q}$	=	constraint Jacobian matrix [varies]
h	=	manoeuvre segment index [-]
I	=	inertial reference frame [-]
I	=	moment of inertia [kgm^2]
$\mathbf{I}$	=	identity matrix [-]
j	=	joint index [-]
k	=	rod index [-]
l	=	rod half-length [m]
$\mathbf{l}$	=	position vector in body frame [m]
m	=	rod mass [kg]
$\mathbf{M}$	=	mass inertia matrix [varies]
n	=	generalised coordinate vector size [-]
N	=	number of modules [-]
q	=	generalised coordinate [varies]
$\mathbf{q}$	=	generalised coordinates vector [varies]
$\mathbf{Q}$	=	generalised forces vector [N]
$\mathbf{R}$	=	position vector in inertial frame [m]

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t	=	time [s]
u	=	internal joint torque [Nm]
$\mathbf{u}$	=	control input vector [Nm]
x, y, z	=	cartesian position components [m]
X, Y, Z	=	cartesian axes [-]
$\Delta\theta$	=	net change in orientation angle [deg]
θ	=	orientation angle [deg]
ψ	=	shape angle [deg]

I. Introduction

A system is said to be reconfigurable if it can adapt its configuration to different situations and comply with the requirements of a certain number of operational needs. Reconfigurable arrays represent the state of the art in robotics technology, closely related to advanced nonlinear control and artificial intelligence. According to the definition of Yim et al. [1], *"self-reconfiguring robots are able to deliberately change their own shape by rearranging the connectivity of their parts, in order to adapt to new circumstances, perform new tasks, or recover from damage"*, and so can potentially be employed for more than one task.

Yim et al. [1] also suggest that space engineering could represent a technological field where it is possible to consider a number of applications that would benefit from the advantages that autonomous reconfigurable arrays offer. Spacecraft whose solar panels remain folded during the launch phase and are deployed to maximise the area exposed to the Sun once released into orbit and robotic manipulators are examples of already existing reconfigurable space systems. In the last few years the research effort for the development of new classes of reconfigurable arrays, such as formation-flying systems, has been growing. An interesting concept, presented by Shoer and Peck [2] and further developed by Nisser et al. [3], is a batch of cubic modules linked by means of temporary electromagnetically-actuated hinges. The shape of the array can be modified by sequentially creating and breaking these interconnections between the cubes. Other examples are self-assembling [4] and self-folding origami structures [5].

Despite the significant advantages reconfigurable arrays offer over fixed-configuration systems and the impressive technological growth they have been recently experiencing, many challenges on the way to their commercial exploitation still have to be overcame. One obstacle is clearly to find specific compelling applications that justify the increase in overall system complexity and theoretical lower performance in single tasks compared to the gain in versatility they could guarantee. Also, as again suggested by Shoer and Peck [2], *"spacecraft reconfiguration technologies have not matured yet to the point in which they can be considered robust"*, this being a driving requirement for every space system.

Another research field that has been increasingly drawing attention and investments over recent years is that of small satellites. These systems, that today are mostly used for Earth Observation and LEO communications, are cheaper and have lower development time with respect to large satellites [6]. They are used by universities for educational purposes, commercial stakeholders seeking for low-risk missions and technology demonstration opportunities or other applications requiring prompt deployment. Also, small satellites are more suitable for serving as modules of distributed space systems such as formations, which may be a crucial technology for the future.

As suggested by Nann and Abbondanza [7], however, *"cheap and light, implies a lot of compromises not only on the operational side but also on the design, the reliability and the lifetime"*. The reason is that today small spacecraft can be equipped with the latest technologies but these are not always space qualified. Also, small spacecraft *"will always be limited by their capacity to carry a payload and to supply it with the required power"* and are still outperformed by large spacecraft in terms of ACS performance. As a result, small satellite missions are often considered complimentary and not competitive to large satellite missions. The latter represents indeed the most appropriate choice when high reliability, functional flexibility, pointing accuracy or high payload power supply are key requirements.

According to Bouwmeester and Guo [8], ACS for small satellites are still in an early development phase and represent a bottleneck for their technology, especially from the points of view of dynamic control and accuracy. The result is that current systems cannot satisfy stringent requirements for tasks including precise remote sensing. It is clear then that consistent improvements of the ACS subsystem would allow for their employment in high performance missions and

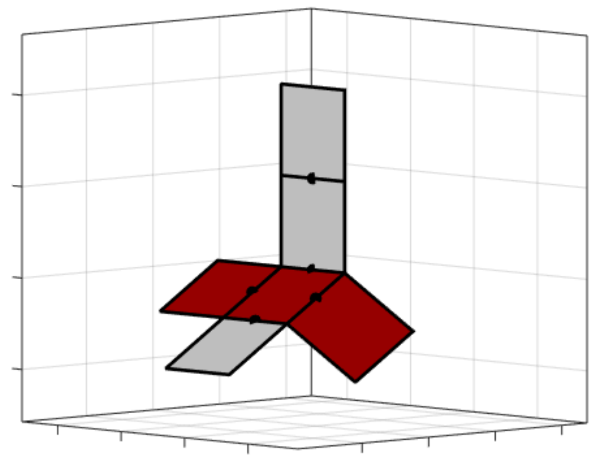


Fig. 1 Multipanel cubic spacecraft unfolded layout.

pave the way for the commercial development of a novel class of space arrays that can compete with large satellites in a wider variety of market sectors.

An ingenious solution to the issues introduced by this analysis could be the redefinition of the architecture of small spacecraft so that the reconfiguration capability is used not only for complying with the requirements of different mission phases but also actively for attitude control purposes. In particular, the idea is to exploit the effect of momentum-preserving internal torques that are generated by the elements of a modular spacecraft rotating with respect to each other. These dynamic contributions in fact have an indirect influence on the absolute orientation of the array and of the single modules themselves. The reference spacecraft we wish to control, whose layout is given in Fig. 1, consists of rigid panels to which instrumentation, solar cells or other subsystems can be attached, interconnected using robust revolute joints. The research objective is to evaluate the potential of an innovative approach to the design of ACS for small spacecraft in which the configuration of the array, i.e., the relative position and relative motion of its modular components, become active players in the control of the attitude of the system in space.

The well-known *falling-cat problem* [9] represents the base of the idea introduced. Its physical explanation demonstrates the possibility, under certain conditions, to obtain absolute reorientation by exclusively using momentum-preserving internal torque contributions. Examples of this concept relevant for our case include the study of Qiao et al. [10], that determine optimal 3-axis reorientation trajectories for a three panel array in an L-shape, and the work of Kawaguchi et al. [11, 12], which instead focus on the design of a planning algorithm that is able to build a reorientation trajectory by patching together a certain number of motion primitives taken from a database obtained by numerical simulation. This algorithm also is developed for the L-shape three panel array but then it is adapted and tested on a system with a larger number of interconnected panels.

A long-term development of this novel approach could justify the increase in cost and complexity of the reconfigurable array not only with advantages in the added versatility of the entire system that is able to fold, unfold and in general tune the orientation of its panels singularly according to pointing, thermal, structural and other subsystem requirements. It could additionally allow for better performance in ACS efficiency, accuracy, stability and even robustness, if appropriate failure mitigation and recovery strategies based on module reconfiguration are developed.

This paper represents the first building block to reach the final goal of an autonomous, intelligent reconfigurable spacecraft. Here the focus is to understand what is the characteristic behaviour of a multibody system rotating in free space when it is subjected to momentum-preserving internal torques and how these can be exploited for attitude control. As explained in Sec. II, the analysis will be conducted on a planar array that is an approximation of the multipanel spacecraft of Fig. 1 and recalls the dynamical properties that are relevant for the purpose of the paper. As shown in Fig. 2, the simplified system is a chain of three rods and its three rotational Degrees Of Freedom (DOF) are to be controlled with only two internal joint torques, which makes its full attitude stabilization fall in the class of *underactuated* control

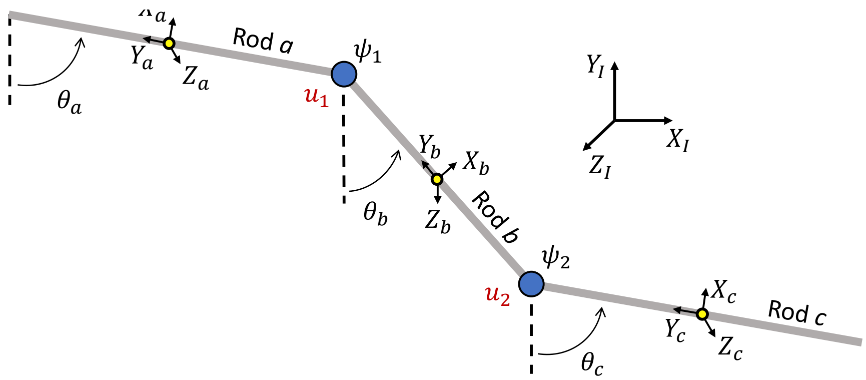


Fig. 2 Planar array of 3 interconnected rods.

problems. These problems are characterised by complex dynamics due to dominating nonlinearities and are most often treated by means of numerical and artificial intelligence techniques, especially in the field of advanced robotics.

The determination of a control scheme for the orientation of a planar system of three rods only using two internal torques is a representative problem in its field that has already been analysed by several authors. The results obtained for it are indeed also valid for analogous chains which count $N > 3$ modules since these can always be reduced to $N = 3$ by locking some hinges. Also, as stated by Reyhanoglu and McClamroch [13], a simpler planar system with $N = 2$ does not satisfy the mathematical conditions of *accessibility* and *small time local controllability* necessary for the internal torque reorientation problem to be solved. Interesting data were derived from the analytical study of *rest-to-rest* manoeuvres. Reyhanoglu and McClamroch [13–15] prove that smooth feedback control of the three rod planar arrays is impossible and derive an explicit nonsmooth mathematical scheme for the system to reach a chosen equilibrium configuration. Walsh and Sastry [16] tested an analogous law on a lab prototype of the three rod system and adapted it for controlling the attitude of a satellite with two rotors. Finally, Sreenath [17] and Krishnaprasad [18] analyse in general how to control the configuration and attitude of classes of planar multibobody systems with N modules.

With respect to previous work on the topic, the present paper adds the analysis of optimal reorientation trajectories for which impingement constraint avoidance is enforced and, importantly, explains in detail the mechanics of momentum-preserving reorientation manoeuvres from both physical and mathematical points of view. Studying the behaviour of a simpler array, whose mechanics can be related to that of system in Fig. 1, is the most efficient choice for reaching this purpose. Section II introduces the planar multibody array used for gathering the results of Section III, where relevant attitude control trajectories are shown and discussed. The focus here will be on the reorientation of the spacecraft as a whole and not on relative pointing of the single modules. Section IV concludes the discussion and presents the next steps.

II. Multibody Mechanics

The most simple subset of the array in Fig. 1, to which internal torques can be successfully applied for attitude control purposes and that can yield relevant information, is the chain of three panels highlighted in red interconnected by two revolute joints. The array in vacuum around its Centre Of Mass (COM), in absence of gravity or any other external disturbance. It is assumed that panels have constant density and that their COMs always lie on the same common plane, perpendicular to the rotation axes of the frictionless revolute joints. As a result the rotation of the system is constrained to this plane. In consideration of this, without loss of generality with respect to the objective of the paper, we can study the equivalent planar system of three interconnected rods given in Fig. 2 rotating around its COM.

Internal joint torques only redistribute angular momentum among the modules and cannot be used for manoeuvres such as detumbling. The attitude control trajectories that will be analysed in the following describe *rest-to-rest* reorientation procedures that start and end in a condition where all the rods are stationary in space. The total angular momentum is always null. The objective is to obtain a variation in the net absolute orientation of the modules at the end of the manoeuvre by exclusively using internal joint torques which cause the shape of the array to change.

The equations of motion for the three rod array are determined and propagated using the approach of Wehage and Haug [19] which is based on Lagrangian dynamics [20]. Each of its rigid elements has one rotational and two translational DOF for a total of 9 DOF for the whole unconstrained multibody. The first constraint to take into account fixes the COM of the multibody to the origin of the inertial frame I and removes two translational DOF. We have:

$$C_1 : \quad \frac{m_a \mathbf{R}_a + m_b \mathbf{R}_b + m_c \mathbf{R}_c}{m_a + m_b + m_c} = \mathbf{0} \tag{1}$$

where $\mathbf{R}_k = [x_k, y_k]$ is the position of the COM of rod $k = a, b, c$ in the I frame and m_k is the corresponding inertial mass. Two other constraints model in 2D the revolute joints connecting the rods:

$$C_2 : \quad \mathbf{R}_a + \mathbf{A}_a \boldsymbol{l}_{1a} - \mathbf{R}_b - \mathbf{A}_b \boldsymbol{l}_{1b} = 0 \tag{2}$$

$$C_3 : \quad \mathbf{R}_b + \mathbf{A}_b \boldsymbol{l}_{2b} - \mathbf{R}_c - \mathbf{A}_c \boldsymbol{l}_{2c} = 0 \tag{3}$$

in which $\mathbf{A}_i$ are the rotation matrices expressing the orientation of the rods and $\boldsymbol{l}_{jk}$, where $k = 1, 2$ is the position of hinge j with respect to the COM of rod k. C_2 and C_3 also remove two translational DOF each. The multibody system, as modelled, is left with only three rotational DOF that, as already mentioned, can be controlled with only two inputs, the joint torques u_1 and u_2. The state of the multibody system is described by a set of generalised coordinates $\mathbf{q}$. These can be subdivided in a group of independent coordinates $\mathbf{q}_i$, whose value can be chosen freely, and a group of dependent coordinates $\mathbf{q}_d$, which are a function of the independent coordinates through the constraint equations. The number of independent coordinates for the three rod array is $n_i = 3$ and corresponds to the number of DOF of the system. The number of dependent coordinates n_d instead is equal to the amount of available constraints. One can add an arbitrary number of dependent coordinates as long as these can be linked to the independent coordinates by means of some mathematical relationship.

The independent coordinates that will be used for our case are the orientation of rod a, θ_a, defined as the angle between the inertial frame reference axis Y_I and the direction of rod a (see Fig. 2), and the relative angles ψ_1 and ψ_2 determining the shape of the array given by[1]:

$$C_4 : \quad \psi_1 = \theta_b - \theta_a \tag{4}$$

$$C_5 : \quad \psi_2 = \theta_c - \theta_b \tag{5}$$

Both Eq. (4) and Eq. (5) have to be included together with Eqs. (1) to (3) in the constraint vector $\mathbf{C}$. The sets of generalised coordinates then are $\mathbf{q}_i = [\theta_a, \psi_1, \psi_2]$ and $\mathbf{q}_d = [\theta_b, \theta_c, x_a, y_a, x_b, y_b, x_c, y_c]$. When using the multibody methodology of Wehage and Haug [19] only the generalised independent accelerations $\ddot{\mathbf{q}}_i$ are directly defined as a function of the inertia of the array and the forces and torques acting on it and then integrated twice in time to get $\mathbf{q}_i$. In the classical Newton form we have:

$$\ddot{\mathbf{q}}_i = \mathbf{M}_i^{-1} \mathbf{Q}_i \tag{6}$$

where $\mathbf{Q}_i$ and $\mathbf{M}_i$ are respectively the generalised force vector and mass inertia matrix with respect to the independent coordinates. The former can be written as [20]:

$$\mathbf{Q}_i = \mathbf{B}_{di}^T \left(\mathbf{Q} + \mathbf{Q}_v \right) - \mathbf{B}_{di}^T \mathbf{M} \bar{\mathbf{Q}}_c \tag{7}$$

in which $\mathbf{B}_{di}$ is by definition a matrix that expresses the virtual displacement of the dependent coordinates with respect to the independent ones and is defined as:

$$\mathbf{B}_{di} = \begin{bmatrix} \mathbf{I}_{n_i} \\ -\mathbf{C}_{\mathbf{q}_d}^{-1} \mathbf{C}_{\mathbf{q}_i} \end{bmatrix} \tag{8}$$

In Eq. (8) $\mathbf{C}_{\mathbf{q}_d}$ and $\mathbf{C}_{\mathbf{q}_i}$ are derived from the Jacobian constraint matrix $\mathbf{C}_{\mathbf{q}}$:

$$\mathbf{C}_{\mathbf{q}} = \frac{\partial \mathbf{C}}{\partial \mathbf{q}}, \qquad \mathbf{q} = \left[\mathbf{q}_i \ \mathbf{q}_d \right] \tag{9}$$

[1]Note that the physical meaning of the results that will follow does not depend on which of the angles θ_a, θ_b or θ_c is chosen as a reference for the orientation of the array.

In particular, $\mathbf{C}_{\mathbf{q}_i}$ is the left submatrix of $\mathbf{C}_\mathbf{q}$ having its same number of rows and taking the first n_i columns while $\mathbf{C}_{\mathbf{q}_d}$ is the remaining squared submatrix with size n_d. The generalised dependent coordinates $\mathbf{q}_d$ can instead be propagated indirectly by integrating the generalised dependent velocities $\dot{\mathbf{q}}_d$ which are expressed as a function of the generalised independent velocities $\dot{\mathbf{q}}_i$ through the constraint equations [20]:

$$\dot{\mathbf{q}}_d = -\mathbf{C}_{\mathbf{q}_d}^{-1}\mathbf{C}_{\mathbf{q}_i}\dot{\mathbf{q}}_i \tag{10}$$

The external dynamic contributions acting on the system are taken into account by means of the vector $\mathbf{Q}$. These forces and torques are decomposed with respect to the generalised coordinates q. The order for listing them in $\mathbf{Q}$ is fundamental and has to be the same specified by vector $\mathbf{q}$. This is valid in general also for the other vectors and matrices that are being defined. For the three rod planar system in absence of external disturbances we have:

$$\mathbf{Q} = [0\ u_1\ u_2\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T \tag{11}$$

The apparent forces and torques are included in vector $\mathbf{Q}_v$ but are null for our planar array. Finally, $\bar{\mathbf{Q}}_c$ is a vector that is used for taking into account the dynamic contributions due to the variation of the constraint matrix $\mathbf{C}$ with respect to time [20]:

$$\bar{\mathbf{Q}}_c = \begin{bmatrix} \mathbf{0}_{[n_i\times 1]} \\ \mathbf{C}_{\mathbf{q}_d}^{-1}\mathbf{Q}_c \end{bmatrix}, \qquad \mathbf{Q}_c = -\frac{\partial^2 \mathbf{C}}{\partial t^2} - \frac{\partial\left(\mathbf{C}_\mathbf{q}\dot{\mathbf{q}}\right)}{\partial \mathbf{q}} - 2\frac{\partial \mathbf{C}_\mathbf{q}}{\partial t} \tag{12}$$

where in $\mathbf{Q}_c$ the first and last terms are null for the three rod array. The generalised mass inertia matrix with respect to the independent coordinates $\mathbf{M}_i$ in Eq. (6) is given by:

$$\mathbf{M}_i = \mathbf{B}_{di}^T\mathbf{M}\mathbf{B}_{di} \tag{13}$$

in which $\mathbf{M}$, assuming that the system moves in a plane and that the origins of the body frames of the single rigid elements are placed at their respective COM, is a diagonal matrix. Similarly to vector $\mathbf{Q}$, it contains the inertia terms of the body elements themselves that directly relate to the generalised coordinates, again arranged according to the order defined by $\mathbf{q}$. Since the mass/inertia terms corresponding to the relative shape angles are null, we have:

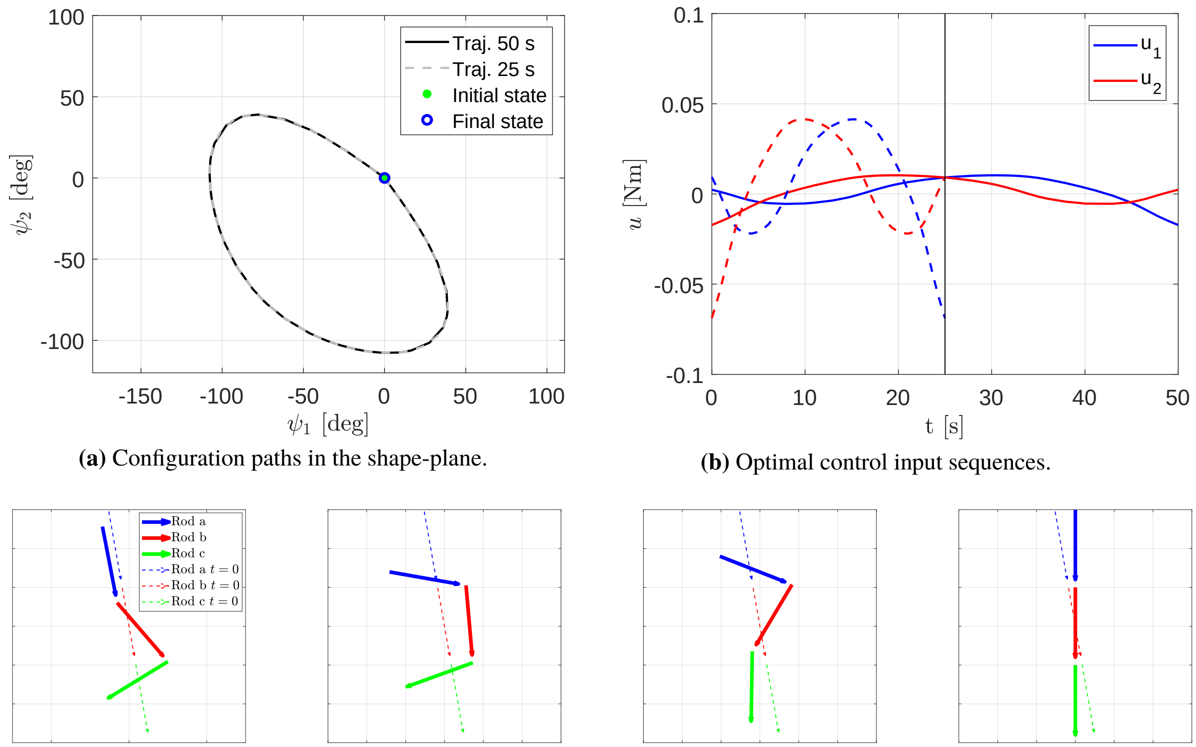
$$\mathbf{M} = \mathtt{diag}(I_a, 0, 0, I_b, I_c, m_a, m_a, m_b, m_b, m_c, m_c) \tag{14}$$

with I_k being the moment of inertia of rod k around body axis Z_k. The rods, if not explicitly mentioned otherwise, are assumed to have a mass m_k of 1 kg and a half-length l_k of 1 m from which the corresponding moment of inertia can easily be calculated.

For the purpose of testing the its validity, this multibody design approach was used for building a triple pendulum oscillating under the effect of Earth gravity. This model features minor modifications with respect to the three rod model presented earlier. It was verified that its total energy remains constant. Also, it was checked that the total angular momentum for every attitude control manoeuvre not involving external dynamic contributions always remains constant. In both cases an acceptable inaccuracy due to numerical propagation error was present.

III. Planar Reorientation Trajectories

In here the three rod multibody model presented earlier is used to generate attitude reorientation trajectories that are significant for the objectives of the paper presented in Sec. I. The manoeuvres taken into account are not only *rest-to-rest* but also *flat-to-flat*, which means that both at the beginning and at the end of the trajectory the COM of the rods are aligned and the chain is flat. These assumptions allow for easier comparison while not affecting the physical meaning of the results. In fact, what is relevant is the $\Delta\theta = \theta_{a,end} - \theta_{a,start}$ obtained and the fact that the system has the same shape and total angular momentum at the beginning and at the end of the manoeuvre. Initial orientation can be chosen arbitrarily because there is no external dynamic contribution that depends on the state of the system. Section III.A will discussed optimal control trajectories and then in Sec. III.B the focus will be moved on rectangular reorientation manoeuvres which, amongst others, are particularly useful to understand the dynamical behaviour of the system.



(a) Configuration paths in the shape-plane. **(b)** Optimal control input sequences.

(c) $t = 0.25t_M$. **(d)** $t = 0.5t_M$. **(e)** $t = 0.75t_M$. **(f)** $t = t_M$.

Fig. 3 Comparison of optimal control effort trajectories correcting $\Delta\theta$=10° in t_M=25 s (dashed) and t_M=50 s (solid).

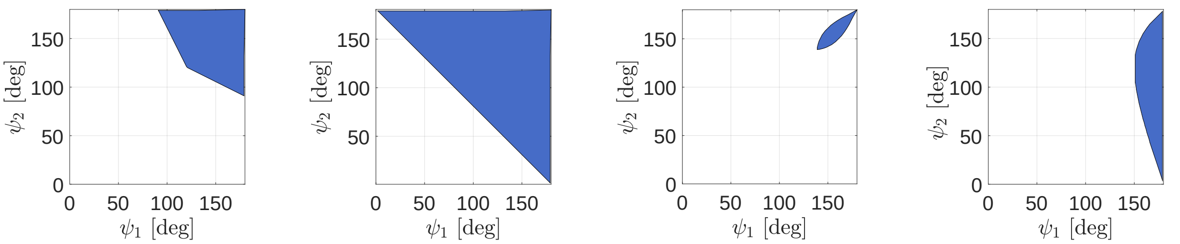
A. Optimal Control Trajectories

The first exploratory results needed for understanding how to exploit internal joint torques for spacecraft attitude control have been gathered using a local pseudospectral optimization tool, GPOPS-II [21], that solves the two point boundary value problem under desired conditions. The objective of the manoeuvres here is to achieve a net desired reorientation $\Delta\theta$ starting from an arbitrary initial attitude. Two types of reorientation trajectories for obtaining $\Delta\theta$ in the range [-180°,180°] have been tested, optimising either manoeuvre time or control effort using cost functions, respectively:

$$J = \int_{\mathbf{t_0}}^{\mathbf{t_f}} 1dt; \qquad J = \int_{\mathbf{t_0}}^{\mathbf{t_f}} \mathbf{u}^T \mathbf{I} \mathbf{u} \, dt; \tag{15}$$

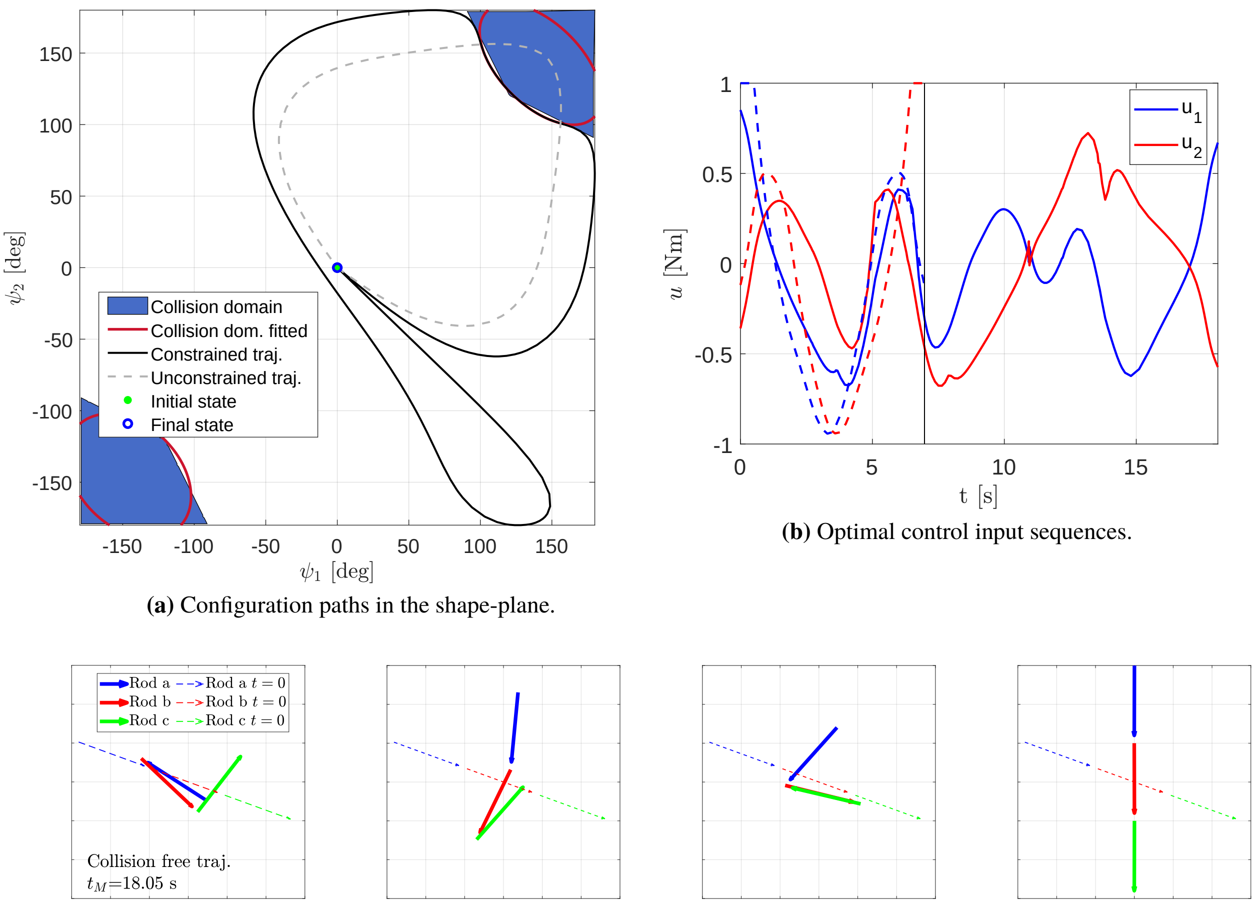
The control input sequences generated for minimum time manoeuvres, not unexpectedly, are *bang-bang* sequences. However, this is the case only when the control input range is small in relation to the inertia of the rods. The projection of *flat-to-flat* manoeuvres in the $\psi_1\psi_2$-plane, that will be defined as shape-plane or configuration space, are closed curves. Figure 3 shows that two minimum control effort manoeuvres with different durations and control input sequences that achieve the same net orientation change $\Delta\theta$ follow the same path in the configuration space. This demonstrates that net orientation variation does not directly depend on the internal joint torques $\mathbf{u}$ but on the trajectory travelled in the shape-plane. Analogous results were obtained using an analytical approach by Walsh and Sastry [16] who find a procedure to obtain a desired $\Delta\theta$ as a function of the area enclosed by circular paths or other simple curves in the $\psi_1\psi_2$-plane. The problem of controlling the attitude of the three rod array then can be separated in two parts. One consists of defining the configuration path that generates the desired net change in orientation and the other part, determining the control effort and manoeuvre time, is to get the path using the available control inputs.

A relevant problem for multibody arrays is module impingement. The collision of two adjacent rods of our three rod chain can be avoided by simply constraining the range of the shape angles ψ_1 and ψ_2 to $[-180°, 180°]$. When the two external rods a and c are concerned, however, the collision area in the shape-plane depends on the relative lengths of all rods. Collision domains in 2D can be obtained with an algorithm that establishes whether two segments intersect



(a) l_a=1, l_b=1, l_c=1 m **(b)** l_a=2, l_b=0.5, l_c=1 m **(c)** l_a=1, l_b=1.5, l_c=1 m **(d)** l_a=2, l_b=1, l_c=0.5 m

Fig. 4 Collision domains (in blue) for different geometric configurations of the three rods array.



(a) Configuration paths in the shape-plane.

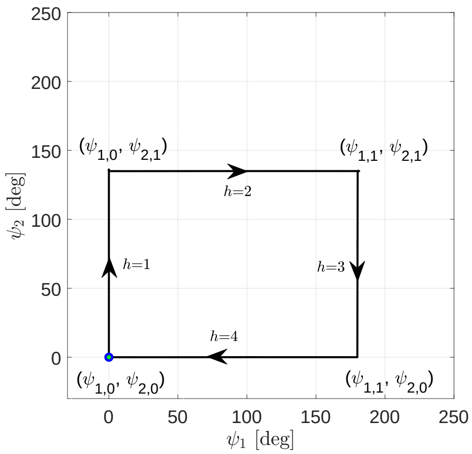
(b) Optimal control input sequences.

(c) t = 0.25t_M. **(d)** t = 0.5t_M. **(e)** t = 0.75t_M. **(f)** t = t_M.

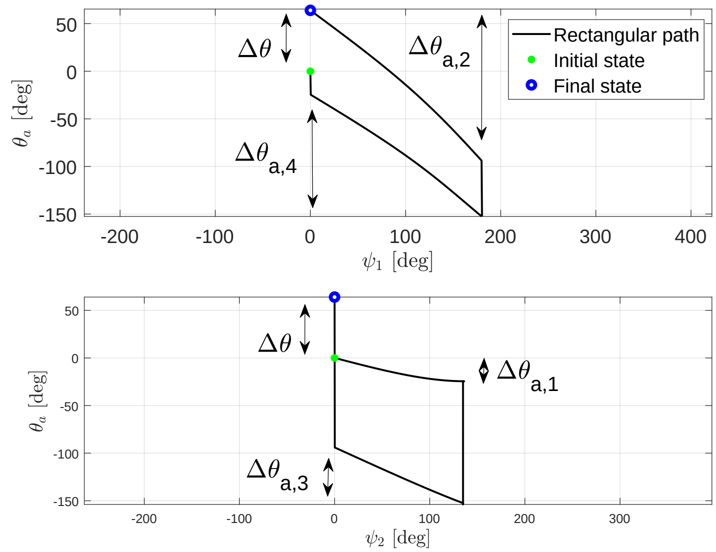
Fig. 5 Comparison of minimum time trajectories correcting $\Delta\theta$=70° with (solid) and without (dashed) impingement avoidance constraint.

using the concept of orientation (clockwise, counter-clockwise or collinear) of triplets of extreme points [22]. Figure 4 shows half of the whole collision domain (the other half is symmetric with respect to the origin) for different geometric configurations of the array. The domain disappears when $l_b > l_a + l_c$.

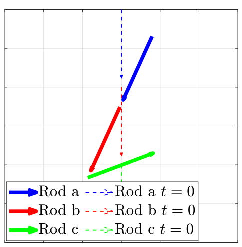
Local pseudospectral optimization algorithms can only handle properly path constraints expressed by continuous functions so that the collision domains have to be defined accordingly if an optimal trajectory enforcing impingement avoidance has to be generated. A straightforward solution to this problems, that works satisfactorily only for some



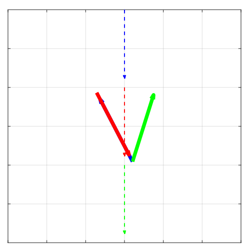
(a) Trajectory in the shape-plane.



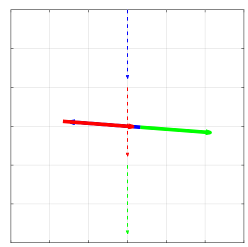
(b) Trajectory in the shape-orientation-plane.



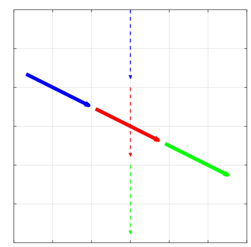
(c) h = 1, end.



(d) h = 2, end.



(e) h = 3, end.



(f) h = 4, end.

Fig. 6 Clockwise rectangular *flat-to-flat* reorientation manoeuvre.

convex collision domains such as the one for our standard case in Fig. 4a, is to approximate the forbidden area with an ellipse. Figure 5 shows the same minimum time manoeuvre reorientation trajectory with and without the collision avoidance constraint. The system circumnavigates the obstacle but the path becomes significantly more complex.

The optimal reorientation trajectories introduced show the peculiar dynamics of the array but obtaining such trajectories is an intensive task from the computational point of view. Using a standard Intel Core i7-7700 CPU @ 3.60 GHz Windows machine the optimization time for a *flat-to-flat* manoeuvre is in the order of minutes, and increases to tens of minutes when the collision avoidance path constraint is enforced.

B. Rectangular Trajectories

Assume now that only one shape angle ψ_v at a time varies due to the application of the corresponding internal joint torque while the other angle ψ_w is locked in position $\psi_{w,0}$. In the configuration space the trajectory going from $\psi_{v,0}$ to $\psi_{v,1}$ is a segment parallel to either one of the ψ_j-axes. This motion, in agreement with the law of conservation of angular momentum, results in a variation of the orientation angle $\Delta\theta_{a,1}$. At the end of this first segment ψ_v gets locked in position $\psi_{v,1}$ and ψ_w is unlocked and brought from $\psi_{w,0}$ to $\psi_{w,1}$, where again it gets locked. Then, θ_a is subjected to a variation $\Delta\theta_{a,2}$. The third and fourth segments are respectively given by ψ_v going from $\psi_{v,1}$ back to $\psi_{v,0}$, with ψ_w locked in position $\psi_{w,1}$ and ψ_w going from $\psi_{w,1}$ back to $\psi_{w,0}$, with ψ_v fixed to $\psi_{v,0}$. While the array configuration travels along these two segments the orientation angle of rod a changes by $\Delta\theta_{a,3}$ and $\Delta\theta_{a,4}$. The net change in absolute orientation of the system at the end of the path is:

$$\Delta\theta = \Delta\theta_{a,1} + \Delta\theta_{a,2} + \Delta\theta_{a,3} + \Delta\theta_{a,4} \tag{16}$$

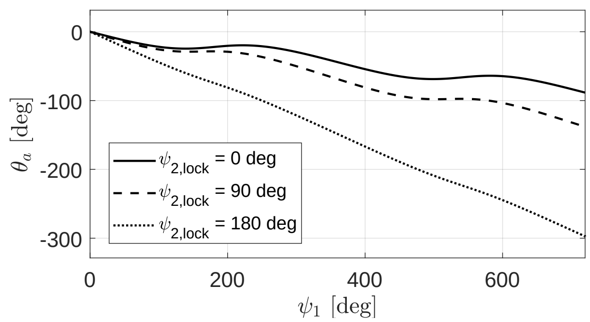


Fig. 7 Variation of θ_a as a function of ψ_1 for different lock angle positions $\psi_{2,lock}$ and $b_5 = 0$.

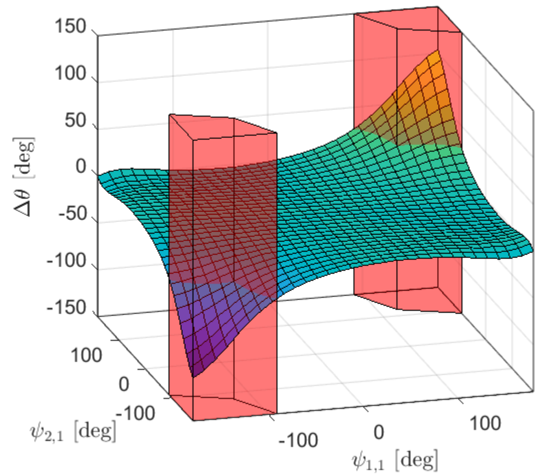


Fig. 8 Net reorientation for different *flat-to-flat* clockwise rectangular manoeuvres.

that, assuming *flat-to-flat* manoeuvres ($\psi_{v,0} = \psi_{w,0} = 0$), exclusively depends on $\psi_{v,1}$ and $\psi_{w,1}$ and on the direction the path is travelled, clockwise ($v = 1$, $w = 2$) or anticlockwise ($v = 2$, $w = 1$). In particular, the same rectangular path travelled in opposite directions yields opposite $\Delta\theta$, which is not null as long as the path encloses a non-null area. Figure 6 shows the rectangular reorientation trajectory described travelling clockwise.

The problem of determining the input joint torque that allows the chosen reorientation trajectory to be followed, that as noted earlier can be treated separately, can be solved easily for the case of rectangular manoeuvres. A proportional-derivative control law drives the array along each segment:

$$u_j = -k_p(\psi_j - \psi_j^*) - k_d\dot{\psi}_j \tag{17}$$

where ψ_j^* is the shape angle to reach at the end of the segment. The gains k_p and k_d have been optimised by means of a step response analysis for the default system mass and geometry to achieve minimum settling time with no overshoot. Each side of the rectangle takes the same time to be travelled.

To obtain a better understanding of the dynamics of the array consider that an arbitrary segment h between $\psi_{v,start}$ and $\psi_{v,end}$, with ψ_w locked in position $\psi_{w,lock}$ describes the motion of a $N = 2$ multibody array. One of the two rigid-bodies is the two rods interconnected by joint w while the other is the remaining rod. The two rods rigidly linked have a specific moment of inertia that depends on the lock angle position $\psi_{w,lock}$ and determines how the orientation angle θ_a varies as a function of ψ_v. The following relationship has been verified to hold true:

$$\Delta\theta_h = f_h\left(\psi_{v,end}\right) - f_h\left(\psi_{v,start}\right), \qquad f_h(\psi_v) = b_1\psi_v + b_2\sin(b_3\psi_v + b_4) + b_5 \tag{18}$$

in which the average slope b_1 and the other parameters b_{2-4} depend on the mass and geometric properties of the array and on $\psi_{w,lock}$ while b_5 is determined by the initial conditions of the segment. The function f_h has been estimated numerically for different shape angle lock positions of the default array configuration and is given in Fig. 7 for $v = 1$, $w = 2$. In agreement with the law of conservation of angular momentum, when the two rods linked rigidly are folded in on themselves ($\psi_{w,lock} = 180°$) the moment of inertia of the rigid element they make is lower and the same $\Delta\psi_v = \psi_{v,end}$ - $\psi_{v,start}$ yields a larger $\Delta\theta_{a,h}$.

If just after reaching $\psi_{v,1}$ angle ψ_v is brought back to initial position $\psi_{v,0}$, while always keeping $\psi_{w,0}$ locked, the full path encloses no area and the net change in θ_a is null. The moment of inertia of the rigid body of two rigidly linked rods is the same for both segments so that they have the same f_h and achieve exactly the opposite $\Delta\theta_{a,h}$ going back and forth between the extremes. This is the reason why there is no *rest-to-rest* manoeuvre that achieves reorientation for a planar multibody system with $N = 2$.

The situation is different when we have $N = 3$ modules. In fact, before ψ_v is brought back to its initial condition $\psi_{v,0}$ as a result of the third segment of the rectangular trajectory, the angle ψ_w has gone from $\psi_{w,0}$ to $\psi_{w,1}$ with the result that the moment of inertia of the block of two rods interconnected by hinge w has changed. The parameters of f_h, b_1 in particular, are not the same for the two parallel segments travelling in opposite directions. In summary, the net change in absolute orientation obtained at the end of the manoeuvre described is due to a nonlinear dynamic effect caused by the variable inertia of the array. This effect is particularly evident for the rectangular path analysed but the same physical concept applies in general to all *rest-to-rest* manoeuvres such as those in Fig. 3 and 5.

Finally, Fig. 8 shows how $\Delta\theta$ for a clockwise rectangular *flat-to-flat* manoeuvre varies as a function of the two shape angles positions characterising it, $\psi_{1,1}$ and $\psi_{2,1}$. The red prisms contain those trajectories that are not feasible due to rod collision. As expected, the larger the difference in the moment of inertia of the array when parallel opposite segments are travelled, the larger the $\Delta\theta$ that can be obtained. Also, given a desired $\Delta\theta$, the function in Fig. 8 can be interpolated to obtain the corresponding set of $\psi_{1,1}$ and $\psi_{2,1}$ that allow us to achieve this change in orientation. The same function can additionally be used for building trajectories that patch together multiple rectangular paths for reaching any planar attitude condition.

IV. Conclusions

The objective of the paper, in addition to justifying a novel approach to small spacecraft ACS design, was to understand how momentum-preserving internal joint torques can be used for spacecraft attitude control. The focus was on *flat-to-flat* reorientation trajectories for a three rod planar array. After giving an overview of existing literature in the field, relevant optimal control trajectories with and without impingement avoidance constraint were shown and the nonlinear behaviour of the system was explained from both physical and mathematical points of view by analysing in detail the rectangular manoeuvre. These results demonstrated that many different strategies can be used to achieve net reorientation and that it is possible to design an innovative ACS that relies on this dynamic effect. The next steps for reaching this final goal consist of studying out-of-plane manoeuvres applicable to the full three-dimensional array, designing an algorithm that can plan a reorientation trajectory and determining how to tune the orientation of the single panels, that can eventually have different mass-inertia properties, using internal joint torques with a feedback logic. Further work can also investigate the use of variable shape to exploit external torques including solar radiation pressure and gravity gradient.

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