PROSPECTIVE ELEMENTARY TEACHERS' MATHEMATICS LEARNING EXPERIENCES AND BELIEFS ABOUT THE NATURE OF MATHEMATICS

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Abstract: In the current era of mathematics education reform, researchers have sought to show how beliefs about mathematics change due to experiences learning mathematics (Hannula, 2006) and how these beliefs can influence mathematics teaching and learning (Leinwand, Brahier, & Huniker, 2014). A convergent parallel mixed methods study was conducted to determine the past mathematics learning experiences and the nature of mathematics beliefs of a group of 38 elementary preservice teachers (ePSTs). Additionally, a goal of the study was to determine if there was a relationship between mathematics learning experiences and nature of mathematics (NOM) beliefs. Analysis of the data revealed that mathematics learning experiences of the ePSTs were more aligned with traditional mathematics instruction, and not the reformed based instruction advocated by the National Council of Teachers of Mathematics (NCTM, 2000). Mathematics learning experiences were characterized by attitudes toward learning mathematics, perceptions of instruction, perceptions of mathematics understanding, perceptions of success and a focus on speed and memorization. The analysis revealed that the NOM beliefs of the ePSTs were characterized by views that mathematics is a subject we learn, rules and procedures, the study of the world around us, a type of puzzle, and problem solving through critical thinking. A Pearson correlation analysis was conducted and showed a statistically significant correlation between positive or negative experiences and NOM beliefs. Findings also indicated that in general ePSTs regarded their mathematics learning experiences in a positive way which leads to concerns for their future instructional practice in light of the research that suggests their NOM beliefs evolve from these experiences and beliefs have been shown to influence instructional practice. Based on these results, implications for future research and practice are shared.

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CHAPTER I

INTRODUCTION

In 2014, the National Council of Teachers of Mathematics (NCTM) published a companion book to their *Principles and Standards* (NCTM, 2000) called *Principles to Action: Ensuring Mathematical Success for ALL* that outlined a series of essential components of mathematics teaching and learning as well as "specific actions that teachers and stakeholders need to take to realize our shared goal of ensuring mathematical success for all" (Leinwand, Brahier, & Huinker, 2014, p. vii). This book along with other bodies of literature intended to communicate decades of research-proven best practices in mathematics instruction have formed a road map for what is needed to help solve the problem of low mathematics achievement and avoidance (Bay-Williams, McGatha, Kobett, & Wray, 2014; Boaler, 2016; Dacey, Lynch, & Salemi, 2013; Smith & Stein, 2011).

While the content of this research highlights a number of elements that are necessary for effective mathematics instruction, one foundational point is the idea that a teacher must possess a specific set of beliefs to be an effective mathematics educator. According to Leinwand et al. (2014), effective mathematics educators hold many different sets of beliefs including those about the teaching and learning of mathematics, access and equity, mathematics curriculum, tools and technology, assessment, and professionalism in mathematics education. These beliefs can be productive or unproductive toward the goal of producing mathematically proficient students (Leinwand et al., 2014). While consideration of these beliefs is important for a mathematics teacher to be effective, beliefs about what mathematics is--the Nature of Mathematics (NOM)—are particularly important for directly influencing the mathematics achievement of students.

In fact, multiple researchers have shown that decisions made in the classroom are filtered through a teacher's belief system including his or her NOM beliefs (Bay-Williams et al., 2014; Goos, 2006). More specifically, the research literature presents a clear picture of how NOM beliefs can dictate the flow of instruction from the goals written (or not written) for a lesson, to the task chosen for students, to the questions the teacher poses during the lesson, to the way the teacher conducts assessment (Leinwand et al., 2014). Studies show how some teachers take the stance that mathematics is one dimensional, a set of rules and procedures that should be handed down to the student from the teacher who is the sole authority on what mathematical concepts should and should not be learned (Beswick, 2012; Cross, 2009; Dossey, 1992). The research literature defines teachers with such a stance as those with a static NOM belief (Dossey, 1992; Grigutsch & Torner, 1998). With instruction that aligns with the belief that mathematics is a static set of knowledge, most students are taught mathematics content in a way that encourages them to memorize the material for the exam and then subsequently dislodge the information because they believe they will not use it again (Bay-Williams et al., 2014; Marchionda, Bateiha, & Autin, 2014).

In contrast, teachers can believe that mathematics is a human activity ripe with all creative power necessary for a student to construct mathematical knowledge in their own mind according to their own interests (Dossey, 1992; Ernest, 1989; Hersh, 1979). This view, often termed the dynamic view of mathematics, aligns with Boaler's (2016) claim that mathematics is "about creativity and making sense" (p. 273) and "making connections and communicating" (p. 274). Additionally, a dynamic NOM belief defines mathematics as a way of thinking about and interpreting the world (Dossey, 1992; Grigutsch & Torner, 1998). Mathematics instruction that is influenced by a dynamic NOM belief aligns more closely with an inquiry-based approach to learning mathematics (Dossey, 1992; Marchionda et al., 2014; Yusof & Tall, 1999) and the reformed teaching practices recommended by NCTM in *Principles to Action* (Leinwand, et. al., 2014). Research has shown that students who experience mathematics teaching with this approach are more successful at developing mathematical proficiency and reach higher levels of achievement in mathematics (Boaler, 2016; Leinwand et al., 2014; Marchionda et al., 2014).

While many studies have investigated what beliefs teachers hold about mathematics (Dossey, 1992), a few researchers have sought to understand how beliefs form in the first place. The general consensus in the research community is that beliefs "are the consequence of an evolutionary process that involves all of an individual's experiences with mathematics throughout their entire life" (Maab & Schloglmann, 2009, p. vii). In other words, beliefs about mathematics form from experiences with or learning about mathematics (Goos, 2006; Presmeg, 2002). In particular, teachers who have memories of success in their experiences learning mathematics, tend to teach the way they were taught (Wilkins, 2008). In the current era of reform, and with the recommendations put forth by NCTM regarding effective mathematics instruction, there is a need to understand what mathematics learning experiences teachers have had (Dossey, 1992; Leinwand, et. al., 2014; NCSEAD, 2018; Skemp, 1976) in order to more clearly understand the beliefs about mathematics they have developed.

Statement of the Problem

With the implication that teachers should possess a specific set of beliefs to promote effective instructional decisions when teaching mathematics (Leinwand et al., 2014), understanding the beliefs a teacher holds about mathematics is imperative. This understanding can only come if teachers are given opportunities to reflect and articulate their beliefs. In particular, according to Yang and Leung (2015), it is important for researchers to identify preservice teachers' beliefs during their teacher preparation programs so that efforts can be made by mathematics teacher educators to provide opportunities for preservice teachers to modify their beliefs to be more supportive of effective mathematical instruction (Bay-Williams et al., 2014; Yang & Leung, 2015). Beliefs that align with effective mathematics teaching practices (Leinwand et al., 2014) should be promoted in teacher preparation programs because upon graduation, preservice teachers enter the classroom and bring their "vision of mathematics" to their students (Goos, 2006, p. 8). Therefore, asking preservice teachers to reflect upon and articulate their beliefs about the nature of mathematics before completing their teacher preparation programs acts as early detection of experiences learning mathematics that are inconsistent with reform-based experiences. These experiences need to be identified, studied, and corrected before the teacher preparation program ends.

In order to form a more complete understanding of a teachers' beliefs about mathematics, it is necessary to understand how those beliefs were formed. To this end, some researchers (e.g. Beswick, 2012; Mapolelo, 2009) have investigated teachers' past learning experiences with mathematics. In these two studies, teachers were asked to reflect on and articulate their past mathematical learning experiences—data that could be compared to previously collected data on NOM beliefs (Beswick, 2012; Mapolelo, 2009). The goal was to determine if past experiences learning mathematics influenced the NOM beliefs of the teachers and both studies found this was the case. While some research has been conducted on the topic of past mathematical experiences, there is still a need for more research to better understand the apparent relationship that might exist between the learning experiences of teachers and their developed NOM beliefs.

Theoretical Framework

In 1955, George Kelly defined personal construct (PC) theory as a way to explain how individuals use their constructions of reality to anticipate the outcomes of future events (Kelly, 1955). PC theory provides a foundational concept from which multiple models can be developed according to the particular content and context being investigated to explain the way people gather and evaluate information and then act according to the perceptions they form (Kelly, 1955; Snow, Corno, & Jackson, 1996). In short, PC theory maintains that people develop personal constructs to interpret their experiences and understand how the world works (Kelly, 1955). PC theory is useful for describing the individual differences or constructs—both within a personality and between a person and their environment—in human thought and how those thoughts relate to subsequent actions (Middleton & Spanias, 1999; Snow et al., 1996) Additionally, a personal construct model helps to describe the system used by an individual that is "constantly evolving" (Owens, 1987, p. 194) while also being influenced by past experiences and currently formed perceptions about future experiences. PC theory works well for this study in light of the fact that experiences and beliefs can be quite individualized since not every person can have the exact same perceptions develop from the same experiences. Indeed, two students learning mathematics in the same classroom with the same teacher will come into the experience with their own individual differences (i.e. past learning experiences, emotions and thoughts) and could leave the experience with completely different perceptions of what mathematics is.

The theoretical framework for this study serves to combine the ideas of George Kelly (1955), American philosopher John Dewey (1938), and mathematics education researcher Markku Hannula (2006) to generate a hypothesized personal construct that models the perceived relationship between experiences with mathematics and formation of beliefs about mathematics. Figure 1.1 below presents a pictorial representation of the model where 'mathematics learning experience' is represented by Dewey (1938) and 'nature of mathematics beliefs' and 'actions in the classroom' are represented by Hannula (2006).

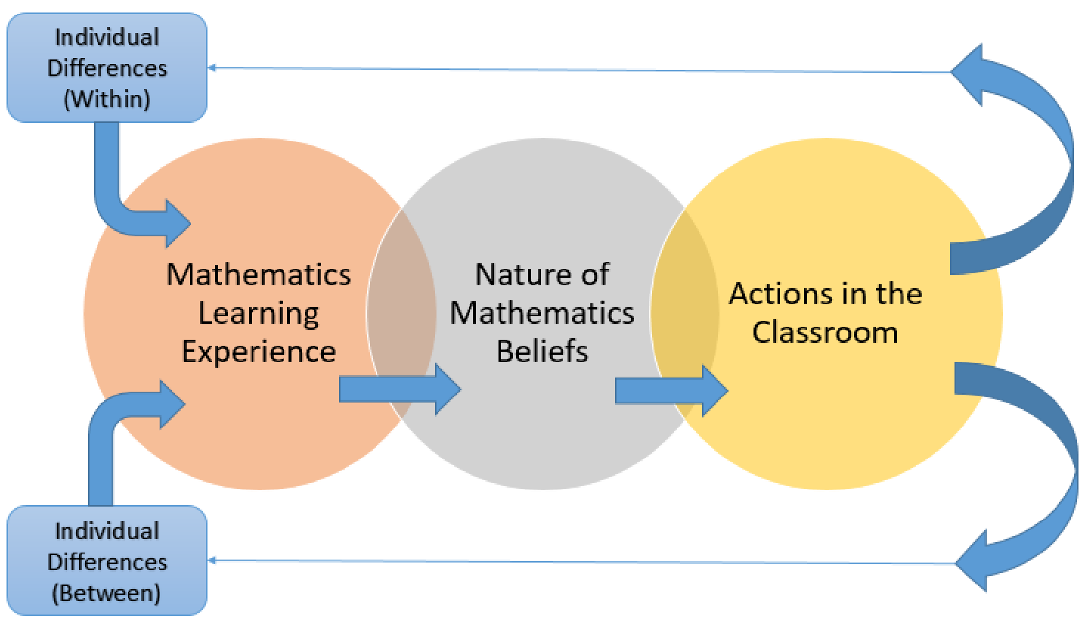


Figure 1.1. Hypothesized personal construct model

Dewey (1938) was a proponent of the influence that experience plays in education and believed it was possible to miseducate with respect to experiences in learning. His idea was that the traditional model of education stripped away any real experience for a learner to create their own knowledge of a subject. Often the traditional model robbed students of the opportunities to experience true learning while it filled the void with experiences that had the "effect of arresting or distorting the growth of further experience" (Dewey, 1938, p. 25). In other words, Dewey maintained that experiences that left learners with the idea that they should just sit and wait for the teacher to fill them up with knowledge were the type that restricted the gains made from future experiences. Dewey (1938) developed the "principle of continuity of experience" (p. 35) or the principle that a current experience will be influenced by past experiences and in turn will influence future experiences. More importantly, Dewey (1938) said that "if an experience arouses curiosity, strengthens initiative and sets up desires and purposes that are sufficiently intense to carry a person over dead places in the future, continuity works" (p. 38) to provide true learning or growth that can be applied to subsequent experiences.

According to Markku Hannula (2006), beliefs encompass needs and goals which, like Dewey's experiences, have the potential to dictate future actions or changes to needs and goals. Hannula (2006) conducted a study that attempted to explain the connections between the beliefs individuals hold about the needs and goals they set during a mathematical task, and how the satisfaction (or dissatisfaction) of those needs and goals influences their motivation to complete the task. Hannula (2006) outlined a model—a personal construct model—that showed how experiences with a mathematical task determine beliefs about that mathematical task, and those beliefs then determine future actions. The degree to which success on the task is measured (e.g. the belief an individual has that they were successful) has much to do with the degree to which needs were met and goals achieved (Hannula, 2006). A productive experience will yield growth in what is learned and will propel an individual to future experiences (Dewey, 1938; Hannula, 2006). Additionally, Hannula (2006) maintains that what an individual believes they need or what they aspire to will be influenced by multiple factors including self-efficacy beliefs, context, and the social norms of the environment. All of these elements feed into the collective experience—as both within and between individual differences—and serve to produce effective learning or arrest learning completely.

This study will focus on the relationship that mathematical learning experiences play in building nature of mathematics beliefs. Combining the ideas of Kelly, Dewey and Hannula into one framework helps to explain how an individual's experiences affects what they believe about the experiences they may have in the future. Specifically, this study will use this framework as a lens through which to view the qualitative and quantitative data gathered with regard to the mathematics learning experiences and NOM beliefs of preservice elementary teachers. Specifically, the framework will serve to illuminate the influence experience plays in the construction of NOM beliefs. Later, the model will be employed again to interpret the findings of this study and develop implications and pathways for further research.

Purpose Statement and Research Questions

Considering current research from NCTM (Leinwand et al., 2014), it is likely that the students who aspire to be the educators of tomorrow have experienced mathematics learning while studying under a teacher who holds a specific set of NOM beliefs. This mixed methods study will address the NOM beliefs of preservice teachers and how past mathematics learning experiences may be related to those beliefs. Thus, this study seeks to answer the following research questions using both qualitative and quantitative methods:

1. What are elementary preservice teachers' past experiences with mathematics?
2. What are elementary preservice teachers' nature of mathematics beliefs?
3. In what ways, if any, are the mathematics experiences of elementary preservice teachers related to their beliefs about the nature of mathematics?

While this study has multiple goals, one main objective is to explore in greater depth the possibility of a connection between mathematics learning experiences and the beliefs about the nature of mathematics that an individual forms with respect to these experiences. This research will add to the literature on this subject and raise awareness of the influence of experience on the formation of beliefs and consider the potential impact that NOM beliefs can make on choices made in the mathematics classroom.

Assumptions

The following section details the assumptions inherent in this research study. Assumptions stem from the choices made by the researcher but are not necessarily controlled by the researcher (Simon, 2011). If the researcher makes different choices, an entirely different study might result but a set of assumptions exists just the same. In particular, this study is governed by a set of basic assumptions and paradigmatic assumptions. The basic assumptions are listed below:

1. Participants will respond honestly and thoughtfully to Likert-type response items and to open-ended questions. Justification of this assumption lies in the procedures for confidentiality and anonymity for each participant. Additionally, each participant will have the option of withdrawing from the study at any time without penalty.
2. Participant's instructors will provide data on the participant's mathematical experiences and NOM beliefs.
3. The experiences of the participants will be highly detailed in the data that is collected. Mathematical learning experiences will be captured in different forms of medium, the first is a mathematical autobiography, the second is a set of open-ended written prompts, the third is a drawing instrument.
4. The sample of participants is representative of the population of elementary preservice teachers. Even though the sample is purposeful and convenient, the sample will represent elementary preservice teachers as they are being recruited out of the education department at a large mid-western university.

Delimitations

Delimitations are the characteristics of a study that set the boundaries and limits of the study and are in the researcher's control (Simon, 2011). Every choice from the topic of the study, the research questions to be answered, to the population chosen for the investigation represents a delimitation. In particular, for this study, elementary preservice teachers were chosen as the population of interest because of the unique timing and the potential improvements to teacher education programs that are possible from the results of this study. The phenomenon of the potential relationship between experiences learning mathematics and nature of mathematics beliefs was selected and therefore eliminated any other potential problems for investigation. Research shows that beliefs of all kinds influence the instructional choices that teachers make in the classroom (Leinwand et al., 2014) and often nature of mathematics beliefs are lumped in with other beliefs. However, beliefs about the nature of mathematics are the less researched and analyzed set of beliefs. The phenomenon of mathematical learning experience plays a role in the formation of beliefs but few studies exist that provide evidence or support of empirical links between NOM beliefs and experience. All of the aforementioned choices necessarily delimit the boundaries of the study at hand and dictate the depth and breadth of the applicability of the findings.

Organization of the Study

This study and the results will be organized in a five-chapter format. Chapter 1 provides an introduction to the study including the foundation of the problem under investigation, statement of the problem, purpose of the study, research questions that are being addressed, and assumptions. Chapter 2 includes an extensive review of the literature related to the study in order to provide the reader with examples of other studies that have been done with respect to the phenomenon under investigation. The literature review helps to extend previous research and situates this study within the current conversation about the topic. Chapter 3 presents the details of the methodology of the study to facilitate replication of the study. More specifically, chapter 3 addresses the study participants, research design, data collection procedures, instruments used for data collection, and data analysis procedures. Chapter 4 will present the findings of the analysis of the data that was collected while Chapter 5 will outline the findings of the study, conclusions, implications, limitations, and calls for additional research.

CHAPTER II

LITERATURE REVIEW

The purpose of chapter two is to outline the research literature that pertains to the mathematics learning experiences and nature of mathematics (NOM) beliefs that preservice teachers hold. The research questions that are guiding this review are as follows:

1. *What are elementary preservice teachers' past experiences with mathematics?*
2. *What are elementary preservice teachers' nature of mathematics beliefs?*
3. *In what ways, if any, are the mathematics experiences of elementary preservice teachers related to their beliefs about the nature of mathematics?*

There are many areas of research that serve to extend the conversation in the mathematics education community with regard to experiences and beliefs about mathematics. The sections included in this chapter outline the various types of experiences that help to build beliefs about mathematics. Additionally, the sections in this chapter describe what the research community specifically defines as nature of mathematics beliefs. While the first research question deals with the past mathematics learning experiences of the participants in this study, the definitions used for nature of mathematics beliefs are defined for the reader first. Furthermore, this chapter outlines the affective components (i.e. emotion, values, and attitude) that are powerful in a mathematics learning experience. Finally, the research literature that outlines the connections that exist between experiences learning mathematics and nature of mathematics beliefs with respect to teachers' actions in the math classroom will be presented.

Defining Nature of Mathematics Beliefs

An effective mathematics educator should hold many beliefs about the type of mathematics teaching and learning that produces mathematically proficient students (Leinwand, Brahier, & Huinker, 2014). In particular, to the nature of mathematics there is some uncertainty in the mathematics education community on a common and accurate view. The act of defining nature of mathematics beliefs has confounded the mathematics community for decades, in fact. The quest for a singular definition has resulted in a dichotomous relationship between those who would seem to have opposing views about the nature of mathematics. The opposition only illuminates the reality that mathematics can be defined differently depending on the individual submitting the definition. The following section will highlight the various definitions that have been proposed through history in order to provide context for the significance of this study.

Opposing Definitions

Defining the nature of mathematics is a centuries old debate first articulated by Plato and his student Aristotle. Plato defined mathematics as "an abstract mental activity on externally existing objects that have only representations in the sensual world…an external, independent, unobservable body of knowledge" (Dossey, 1992, p. 40). However, Aristotle disagreed. Aristotle believed that mathematics was a human endeavor in a state of constant flux with human beings thinking logically and creating a whole new world of mathematics (Dossey, 1992). In Aristotle's view, mathematics was created through "experimentation, observation, and abstraction" (Dossey, 1992, p. 40) in a given mathematical situation. Across time, it seems as though the debate centered on two contrasting views of the nature of mathematics. Early mathematicians and philosophers, like Descartes and Immanuel Kant, all took sides offering their own idea of what mathematics is (Dossey, 1992).

Fast forward to the late 1970s when mathematicians began to offer less dichotomous views. In 1979, mathematician Reuben Hersh put out a call that the discussion of 'what is mathematics' needed a revival of sorts. In his analysis of the state of NOM beliefs, Hersh presents his own answer that seems to acknowledge all critical aspects from both sides of the NOM debate. According to Hersh "mathematical work is work with ideas" (Hersh, 1979, p. 40) created by humans interacting with daily life. Hersh's definition is the result of the quest to abandon the search for one single foundation for the nature of mathematics. He maintains that it is this search for certainty within mathematics that effectively installs "intellectual blinders" (Hersh, 1979, p. 34) and prevents the mathematics education community from moving forward to a new understanding of NOM. Hersh chides his fellow mathematicians for not showing more leadership to resolve the "contradictory views on the nature and meaning of their work" (Hersh, 1979, p. 32).

Ten years later, Ernest (1989) published his definition of the nature of mathematics. Like Hersh before him, Ernest attempted to present a unified definition of NOM simultaneously representing the various views proffered by the mathematics community up through the late 1980s. Ernest's definition of mathematics includes three separate but equally important views. These three views—the problem-solving view, the Platonist view, and the Instrumentalist view—resulted from years of debate among mathematicians, educators and researchers responding to the criticism that what mathematicians believe about NOM is not transmitted in their lesson delivery when they are asked to teach mathematics. According to Ernest, the person who ascribes to the problem-solving view believes that mathematics is not a static discipline but a "dynamic problem-driven" product that "represents a continually expanding field of human inquiry" (Ernest, 1989, p. 21). The Platonist view—named after Plato—holds that mathematics is a "static but unified body of knowledge, consisting of…truths…which [are] discovered, not created" (Ernest, 1989, p. 21). Ernest adds a third piece to the definition by declaring another view—the Instrumentalist view. The instrumentalist sees mathematics as a "useful but unrelated collection of facts, rules and skills" (Ernest, 1989, p. 21). Useful because the rules and skills help a student arrive at a solution but unrelated because the symbols being manipulated have no meaning for the student—they do not understand the 'why' behind the rules and facts they are working with.

Modern Definitions

Approximately ten years after Ernest outlined his definition, Keith Devlin (1997), a mathematician, offered his version of the definition of the nature of mathematics. And in 1999, Rueben Hersh clarified his original attempt from 1979. Both definitions are more comprehensive than those offered by their predecessors. Indeed, Devlin's book, *Mathematics: The Science of Patterns*, is devoted to helping individuals who read it understand the nature of mathematics. Devlin compares mathematics to a science and he claims there is no feature of our lives not influenced by math in some way. He maintains that abstract patterns compose our thoughts, the way we communicate, the way we interact with each other, and the calculations we do. Abstract patterns govern our lives (Devlin, 1997). Hersh continued refining his definition too. Originally published in 1979, Hersh expanded his definition and modernized it to include his critique of how mathematics is represented in schools. Namely, Hersh (1999) argued that school math is not real math because students who only do math looking for the answer are missing the idea that math is driven by questions. (Hersh, 1999). Hersh maintains that mathematics is first solving a problem—a question that was posed—then asking more questions. Questions separate from the life dilemmas that created them are essentially lifeless (Hersh, 1999).

In 2010, Conrad Wolfram, director of Wolfram-Alpha, proposed that working with mathematics involves four stages. The first stage is "posing the right question" (Wolfram, 2010) or even just posing a question to start, which involves actively and critically thinking about a situation or a set of data. After posing a question about some real world situation, the second stage is to create a mathematical model that represents the problem. "Real world [to] math formulation" (Wolfram, 2010) involves deep understanding of how mathematics applies to the problem situation. In stage three, the mathematical model is used as a tool to calculate a solution. Wolfram (2010) calls it "[c]omputation" and indicates that 80% of mathematics taught in school is centered on this stage. The irony is employers do not need human calculators—calculations are done with computers running coded programs that do the calculations with given input parameters and the touch or click of the single button. The fourth stage involves moving back to the real world from the mathematical model to generate an interpretation and to determine if the question posed in the beginning was answered (Wolfram, 2010). Wolfram's definition placed emphasis on the action of mathematics—the doing of math as it were. Unfortunately, many students think that doing math is simply performing calculations (Wolfram, 2010).

Between 2010 and 2016 multiple researchers offered definitions of the nature of mathematics (e.g. Azram, 2014; Tosmur-Bayazit & Ubuz, 2013); however, the most recent definition has been proposed by Jo Boaler, a mathematics educator and researcher at Stanford University. In her book, *Mathematical Mindsets*, Boaler (2016) outlines that mathematics is a "subject full of uncertainty; it is about explorations, conjectures, and interpretations, not definitive answers" (Boaler, 2016, p. 22). Additionally, Boaler claims that it is through exploring, conjecturing, and interpreting that students can experience the beauty and visual creativity and make math a "living subject" (Boaler, 2016, p. 31). Boaler's definition exemplifies the multidimensional nature of mathematics. The fact that mathematics can represent so many different phenomena from natural patterns to scientific man-made ones implies that the traditional way the subject is taught leaves an enormously vast field relatively unexplored by today's student body.

Multiple Definitions

Multiple definitions of the nature of mathematics serve to highlight the individual characteristics that mathematics can possess depending on the person doing the defining. From the definitions presented here, we can see the various dimensions mathematics can take.

At a minimum these definitions represent the true nature of mathematics, one that is characterized as a

> cultural phenomenon; a set of ideas, connections, and relationships that we can use to make sense of the world. At its core, mathematics is about patterns. We can lay a mathematical lens upon the world, and when we do, we see patterns everywhere; and it is through our understanding of the patterns, developed through mathematical study, that new and powerful knowledge is created. (Boaler, 2016, p. 23)

However, there does not seem to be acceptance of any common and accurate NOM definition. Indeed, there are several studies that have shown evidence that individuals can possess multiple beliefs about mathematics. Grigutsch and Torner asked mathematicians what they believed mathematics was and the results of the study showed that the participants believed that mathematics was a set of "axioms, terms and relations between these terms" where the procedure involving these axioms was executed in a precise "deductive method" (Grigutsch & Torner, 1998, p. 5). Additionally, the results also showed that the mathematicians believed that mathematics is "an activity of contemplating about problems, acquiring resolutions and creating knowledge" (Grigutsch & Torner, 1998, p. 6). Both views are distinct and complementary and both were beliefs held at the same time. In 2003, Op't Eynde and De Corte studied 365 Flemish students in junior high and found evidence that seems to support the students holding multiple beliefs about NOM. Some students believed mathematics was constructed through social life while also believing that math was a precise domain for rules and procedures. In 2015, Yang and Leung studied a group of preservice mathematics teachers and found that, like the mathematicians and students in the previous studies, the preservice teachers believed math was rules and procedures and a process of inquiry.

Definitions Used in This Study

Throughout this study, reference will be made to the terms ‘static’ NOM beliefs and ‘dynamic’ NOM beliefs. The terms were first used by Dossey and his colleagues in 1986; however, each term is defined in Dossey’s article (1992) titled *The Nature of Mathematics: Its Role and Its Influence*. Namely, static NOM beliefs are defined as those views of mathematics as a “static discipline, with a known set of concepts, principles, and skills” (Dossey, 1992, p. 39). Dynamic NOM beliefs are those beliefs where mathematics is seen as a “constantly changing [discipline] as a result of new discoveries from experimentation and application” (Dossey, 1992, p. 39). In several ways, these terms are used to represent opposing ends of the nature of mathematics beliefs continuum that includes multiple definitions (Boaler, 2016; Weldeana & Abraham, 2014). However, some researchers see them as complimentary views (Grigutsch & Torner, 1998; Op’t Eynde & De Corte, 2003; Yang and Leung, 2015). In particular, for this study, static NOM beliefs refer to mathematics as rules and procedures while dynamic NOM beliefs refer to mathematics as a process of inquiry (Tatto, 2012).

Mathematics Learning Experiences

There is a large amount of research evidence to suggest that the type of mathematical learning experience matters a great deal when considering the formation of static NOM beliefs versus dynamic NOM beliefs (Leinwand et al., 2014; NCTM, 2018). The section that follows will outline the types of experiences that research says creates both types of NOM beliefs. Even more important are the mathematics learning experiences of teachers (both preservice and inservice) in light of the research that supports a direct influence of NOM beliefs on instructional choices in the math classroom (Leinwand et al., 2014). More specifically, this section will outline the research on mathematics learning experiences of preservice teachers where studies have focused on understanding the influence of experiences on beliefs. Finally, this section will outline how mathematics learning experiences can be characterized by the emotions and attitudes generated during the study of math (Maab & Scholgmann, 2008; Tobias, 1993).

Experiences That Build Static NOM Beliefs

Over half of graduating high school seniors are making decisions about taking mathematics in college without having fully developed ideas about the true nature of mathematics (Leinwand et al.,2014). In the U.S., adolescents are not mathematically proficient when comparing scores from the Program for International Student Assessment (PISA), which implies they lack a complete understanding of concepts, problems solving strategies, and adaptive reasoning (Leinwand et al., 2014). Additionally, they are avoiding mathematics classes in college because they do not see mathematics as “useful and worthwhile” for their lives in college or career (Leinwand et al., 2014; NCTM, 2018).

Many studies have been conducted over the past several decades that illuminate strong evidence to support the conclusion that traditional math instruction is a main contributor to spawning students who can only perform calculations and see those calculations as something to avoid in the future (Boaler, 2016; Leinwand et al., 2014).

Traditional mathematics instruction can be characterized by the idea that the teacher is the sole authority of knowledge and the students are receptacles that must be filled with that knowledge (Freire, 2000). Typically, teachers using this pedagogical approach spend a great amount of time showing students how to do a math problem then require the students to regurgitate the procedures, facts and formulas on assignments and exams. The teaching as telling model is not inherently wrong. Indeed, Boaler (2016) presents research that reveals an optimal time to show students how to do a procedure or method but that moment is after they have been exposed to a problem and been given relatively little instruction on how to solve it. Repeated exposure to the show and tell form of instruction produces students who have only practiced and experienced one characteristic of mathematics, not the full multidimensional nature of math as defined by the research community (Boaler, 2016; Devlin, 1997; Dossey, 1992; Ernest, 1989; Hersh, 1999;).

John Dewey (1938) classifies the traditional type of educational experiences as "miseducative" in light of the fact that it produces students with incomplete knowledge and in the case of mathematics education, the wrong ideas of the true nature of mathematics. Countless studies completed more recently have indicated the connection between past experiences learning math in school and beliefs held about the nature of mathematics. Ethnomathematics educator, Norma Presmeg (2002), studied her students (high school and graduate level) and found that most of her students believed that mathematics was solving problems by calculations. Presmeg collected qualitative data and was able to conclude that her students held beliefs consistent with their past experiences and these beliefs inhibited them from making connections to the mathematics that exists in culture and life (Presmeg, 2002). In a study by Amirali and Halai (2010), quantitative data were analyzed and the researchers concluded that teachers believed that mathematical knowledge was “static and truth” (p. 57), and also that this belief was a result of teacher experiences with schooling and instruction. Most recently, Jo Boaler reported on some interviews she did with a group of young people in their mid-twenties. All of these individuals had been educated with traditional mathematics instruction and expressed dissatisfaction with the knowledge they gained and how it applied to their lives and careers. Specifically, they indicated that their “school experiences of math had not given them any sense of the real nature of math and its importance to their future” (Boaler, 2016, p. 194). In other words, traditional mathematics instruction breeds a singular view of what mathematics is, but there are experiences that do paint a more complete picture of the nature of mathematics.

Experiences That Build Dynamic NOM Beliefs

In contrast to the traditional form of mathematics instruction that tends to create a learning experience that serves to focus on only one characteristic of mathematics (procedural formulaic calculation), a learning experience that includes mathematics instruction that teaches students to model the real world with mathematical language to interpret the meaning of a solution reveals a more accurate picture of the nature of mathematics. Boaler (2016) shares her own experience as a student learning mathematics indicating that she was not forced to memorize math facts, formulas or procedures. Now a prominent mathematics professor at Stanford University, Boaler advocates for “learning of math facts along with deep understanding of numbers and the ways they relate to each other” (Boaler, 2016, p. 38).

Research shows that mathematics learning experiences that build dynamic NOM beliefs typically yield higher mathematics achievement as well. In a study he conducted on mathematical problem posing, Silver (1994) showed that students who experience the essence of real mathematics in situations that require them to pose questions and think deeply, are more successful and achieve to higher levels. Silver's research illuminates the connection between experiences designed around the true nature of mathematics producing students who engage in activities that are associated with depth of understanding and greater mathematical proficiency. However more than just increased mathematics achievement takes place—students begin to develop dynamic NOM beliefs for themselves.

Other mathematics education research has provided evidence that learning experiences that involve problem-solving activity, including real world career situations, build dynamic NOM beliefs. In one qualitative study done by Tosmur-Bayazit (2013), a set of five practicing engineers were interviewed in an effort to determine their NOM beliefs and their beliefs about mathematics education at the university level. Tosmur-Bayazit found the engineers NOM beliefs to be dualistic in that their beliefs developed from both experiences in traditional mathematics instruction and experiences in the working world. She reported that the engineers' experience influenced their beliefs that mathematics is more of a problem-solving activity while they also felt their university mathematics education was adequate to prepare them for their careers (Tosmur-Bayazit & Ubuz, 2013). Additionally, Tosmur-Bayazit (2013) detailed how the engineers believed mathematics to be "a language, a tool to model and design, to construct a logical way of thinking, for predictions and explanations of reality, and…for describing and analyzing engineering processes and systems" (p. 34).

Decades of research on the types of experiences that produce dynamic NOM beliefs is represented in *Principles to Action: Ensuring Mathematical Success for ALL* (Leinwand, et. al., 2014). In this report, Leinwand and colleagues present a list of the components of the types of mathematics learning experiences that students should be having in school. Namely, the mathematics learning experiences should include challenging tasks that actively involve students in making meaning of mathematics (Leinwand, et. al., 2014). In addition, these mathematics learning experiences should help students develop conceptual and procedural knowledge, as well as the skills on how to apply this knowledge to their own personal experiences (Leinwand, et. al., 2014). A goal in providing experiences such as this is to graduate students from high school who have been able to "experience the joy, wonder and beauty of mathematics" (NCTM, 2018, p. 9).

Types of Mathematics Learning Experiences of Preservice Teachers

Preservice teachers are featured in several studies that the research community has conducted to gain a deeper understanding of their mathematics learning experiences and how these experiences influence or change beliefs, attitudes, and emotions. Indeed, there is a consensus in the research community that beliefs, attitudes and emotions "are the consequence of an evolutionary process that involves all of an individual's experiences with mathematics throughout their entire life" (Maab & Schloglmann, 2009, p. vii). This section will outline the research completed to understand the mathematics learning experiences of preservice teachers and the research that seeks to understand the components of emotion and attitude within mathematics learning experiences.

Understanding preservice teachers' experiences learning math. While preservice teachers are a popular sample for research studies, there are few investigations that seek to understand the cumulative set of mathematics learning experiences a preservice teacher might have engaged in. Qualitative studies are typically oriented to provide a "thick description" (Erlandson, 1993) focusing on the deeper dive necessary to understand how, in this case, mathematics learning experiences have influenced an individual. In 1987, Nespor published a theoretical study she conducted based in qualitative data to develop a conceptualization of beliefs and suggest "several key functions of beliefs in teachers' thinking" (p. 317). While the participants in her study were not preservice teachers, Nespor defines several ways in which beliefs are formed in the mental structure, including the "episodic memory" that is "organized in terms of personal experiences" (Nespor, 1987, p. 320). Nespor's study showcases how the teacher participants accessed their own experiences of being a student learning mathematics as far back as primary, secondary and early college. These experiences, she maintains, are the driving force behind the "subjective power, authority, and legitimacy" of the beliefs teachers have (Nespor, 1987, p. 320).

Similar to Nespor's study, Cooney, Shealy, and Arvold (1998) conducted a study of the beliefs of four preservice teachers who were in the last year of their teacher preparation program and would continue on to teach secondary mathematics. Based on the idea that "teachers' beliefs about mathematics and how to teach mathematics are influenced in significant ways by their experiences with mathematics and schooling long before they enter the formal world of mathematics education" (Cooney, et. al., 1998, p. 306), Cooney and colleagues sought to gather the reflective data provided by the participants in their study. The four participants selected for analysis in the study were selected based on the wide range of initial beliefs about mathematics and mathematics teaching. In each of the four cases, details are provided that showcase the participants' reliance on multiple past experiences learning mathematics when developing their own ideas of what mathematics is and how it should be taught. In one case in particular, the participant named Sally shares about her experiences beginning in 2^{nd} grade and Cooney et. al. (1998) assess the reflection saying in part,

> Early successful experiences in doing essentially computational mathematics contributed to her limited notions of teaching and learning…[h]er mathematics consisted of facts, algorithms, and computation. Authority for mathematical truths was attributed to teachers and textbooks. (p. 319)

In Sally's example, her belief that mathematics is 'facts, algorithms, and computation' aligns with the static NOM ideal, and her experience supports this belief formation. In fact, in every case presented by Cooney, et. al. (1998), the beliefs about mathematics were derived from past experiences learning mathematics that included various episodes from the participants' educational past.

In other studies (Goos, 2006; McDuffie & Slavit, 2003), researchers have homed in on the reflective component for analyzing preservice teachers past experiences learning mathematics and challenging the beliefs they hold about mathematics. In a study conducted by McDuffie and Salvit (2003), preservice teachers were exposed to an online discussion board where they engaged in reflective practices that resulted in the "self-examination of beliefs about mathematics and the teaching and learning of mathematics" (p. 448). The goal of the study was to present the results of the reflective aspects of the online discussion board at inducing the preservice teachers to assess their static NOM beliefs through challenges they experienced via communicating with their peers, and the instructors (McDuffie & Slavit, 2003). Like the study presented by Cooney, et. al. (1998), the preservice teachers in this study reflected about past experiences learning mathematics but indicated through their responses that they hoped to avoid the experiences they had when they begin teaching in their own classrooms. The challenges presented to the preservice teachers to call to question their initial static NOM beliefs seemed successful in moving the preservice teachers toward more dynamic NOM beliefs. Evidence of this comes from the participants sharing how their mathematics learning experiences were non-reformed experiences and expressing that they wished they had been encouraged to be creative rather than being told to follow the rules of mathematics (McDuffie & Slavit, 2003). The study by McDuffie and Slavit (2003) illustrates the realization that teaching mathematics according to a static NOM belief, will not "encourage students to be creative" (p. 460) in their mathematics learning. Indeed, this was the goal of the study, to ensure preservice teachers understand how the experiences they had learning mathematics can influence the beliefs they hold about what mathematics is and how it should be taught.

Reflection by preservice teachers on their past mathematics learning experiences is an important practice, and one in which Goos (2006) showcases in her article *Why Teachers Matter*. Goos indicates that in her effort to understand the past experiences of the preservice teachers she trains, she recognizes that they need to provide her with "their personal mathematical life history, in which they describe their experiences of learning mathematics at school and at university and recall the influence of good and bad teachers they may have encountered" (Goos, 2006, p. 9). Goos describes how the preservice teachers recall "fondly" the teachers of their past who inspired in them a love for mathematics, but there are also preservice teachers who remember "ridicule or harsh words" that caused more negative feelings (p. 9). In her attempt to understand the mathematics learning experiences of the preservice teachers, Goos describes how she sets out to create an opportunity for the preservice teachers to provide details about their "daily experiences in classrooms, [as] students [who] develop long lasting perceptions about mathematics and mathematics teachers" (Goos, 2006, p. 9).

Mathematics learning experiences that change beliefs. A large portion of the research on the mathematics learning experiences of preservice teachers (PSTs) focuses on experiences designed to change the beliefs of the PSTs in order to align their beliefs with what research says effective mathematics educators should believe (Leinwand, et. al., 2014). These studies involve both qualitative and quantitative methods where they seek to show statistically significant change in beliefs due to experiences while also diving deeper into the reasons why the change occurred (or did not occur). Contrary to the studies showcased in the previous section, the research outlined here does not seek to investigate the past school-life experiences learning mathematics of preservice teachers but does seek to understand how a particular experience learning mathematics changes beliefs about mathematics.

While not specifically related to the experiences of preservice teachers, a study by Yusof and Tall (1999) investigated the change in beliefs about mathematics of university students learning mathematics through problem solving. The university students' beliefs about mathematics were measured before the course began and the same questionnaire was administered at the end of the course. The pre-test and post-test scores were analyzed for a statistically significant change and the qualitative data gathered via observation in the classroom and semi-structured interviews between was analyzed to determine why beliefs about mathematics changed or did not change. During the problem solving mathematics course, the students were "encouraged to experience all aspects of mathematical thinking—formulating, modifying, refining, reviewing problems and their solutions, specializing to simple cases, generalizing through systematic specialization, seeking patterns, conjecturing, testing and justifying…and [reflecting] on their mathematical experiences [to discuss] their attempts to solve problems" (Yusof & Tall, 1999, p. 2). The results of this study show that through this experience learning mathematics, the university students' beliefs about mathematics changed from a static to a more dynamic view and the change was statistically significant (Yusof & Tall, 1999). Participants in the study declared before the learning experiences that they believed mathematics was "facts and procedures to be memorized" (Yusof & Tall, 1999, p. 1); however, after the experiences in the course they reported they believed mathematics to be "a process of thinking" (Yusof & Tall, 1999, p. 8).

Another study focused directly on preservice teachers measured changes in beliefs about mathematics with respect to the preservice teachers' experience of "a university mathematics course that aimed, among other things, to promote a problem solving view about mathematics" (Shilling & Stylianides, 2013, p. 393). The problem solving view about mathematics denoted by Shilling and Stylianides is that defined by Paul Ernest in 1989 and previously described. Like the previous study by Yusof and Tall, a goal of this study was to collect data on the reasons why a change in beliefs did or did not occur as a result of the mathematics learning experience. When describing the details about the type of mathematics learning experience designed for the preservice teachers, Shilling and Stylianides note that the "environment allowed prospective teachers to experience active learning that may have been absent when they were students and reflects an environment that [encourages] the exploration of and potential change in held beliefs" (p. 395). The results of this study indeed show a change in beliefs about mathematics toward a more problem solving view (i.e. more dynamic view). The study also showed that the preservice teachers did not change their initial views of mathematics (Shilling & Stylianides, 2013). For some of the preservice teachers this finding means they continued to hold dynamic NOM beliefs, however, other preservice teachers continued to hold static NOM beliefs (Shilling & Stylianides, 2013). This study therefore raises questions as to the effectiveness of the designed mathematics learning experience to change the NOM beliefs of the preservice teachers and how beliefs can be resistant to change.

The most recent study conducted by Weldeana and Abraham (2014) focused on preservice teachers' mathematics learning experience in a "history-based intervention program" (p. 310) designed to elicit cognitive conflict and help preservice teachers move from static NOM beliefs to more dynamic ones. The experience in this course was grounded in problem solving and the preservice teachers were required to "reflect, invent, imagine, and play with ideas; create and solve problems; theorize, generalize and describe concepts; express meanings of concepts; and explore, explain, criticize, and justify their reasoning" (Weldeana & Abraham, 2014, p. 311). These activities all took place under the umbrella of historically based mathematics content where the preservice teachers check the "historical development of mathematical topics" and see that mathematics is created by humans to interpret and understand the world around them (Weldeana & Abraham, 2014, p. 308). The results showed favorable change in the NOM beliefs of the preservice teachers, away from the static view and toward a more dynamic view.

Inhibited experiences learning mathematics. There were a few investigations that, like the studies mentioned above, focused on the change in beliefs due to a particular experience but in ways not anticipated were able to indicate that static NOM beliefs tended to inhibit some participants from a positive experience learning mathematics that encourages the development of dynamic NOM beliefs. In other words, in an attempt to change static NOM beliefs through a specially designed mathematics learning experience, researchers discovered how some participants who held static NOM beliefs did not agree with the instructional methods of courses designed around inquiry-based mathematics instruction (McGinnis et. al., 2002; Presmeg, 2002).

In the study conducted by McGinnis and colleagues (2002), the change in attitudes and beliefs about mathematics was investigated for elementary and middle school preservice teachers in the "Maryland Collaborative for Teacher Preparation (MCTP), a statewide, standards-based project in the National Science Foundation's Collaborative in Excellence in Teaching Preparation (CETP) Program" (McGinnis et. al., 2002, p. 713). The courses in this program were focused on mathematics and science content and were aligned with reform-based recommendations that included inquiry-based instructional methods (McGinnis et. al., 2002). The courses offered in the MCTP program were open to all preservice teachers therefore there were participants designated as “MCTP teacher candidates” and “non-MCTP teacher candidates (McGinnis et. al., 2002). According to McGinnis et al., the difference between MCTP and non-MCTP teacher candidates are the hours required for coursework in the program. Specifically, MCTP candidates must take 18 hours each of math and science (36 total hours) while non-MCTP preservice teachers only need 26 hours total for math specialty and 29 hours for science (McGinnis et al., 2002). Results of the study showed that the MCTP teacher candidates did indeed experience a change in the beliefs about mathematics with the majority of them staying true and growing stronger in their dynamic NOM beliefs. However, the non-MCTP teacher candidates did not show such improvement and even showed a decline in subscales scores used to measure beliefs about the nature of mathematics (McGinnis et. al., 2002). Upon consideration of these results, McGinnis et. al. report that:

> In a debriefing meeting for all MCTP course instructors, a dominate discourse referent was that a noticeable number of the non-MCTP students in the courses struggled to earn the high grades they were accustomed to getting in the past. Those students singled out the change in instruction and assessment from an emphasis on rote memorization to an emphasis on problem posing and conceptual change as the cause of their difficulties. (p. 725)

The non-MCTP teacher candidates consequently experienced an affirmation of their static NOM beliefs, and a negative impact on their attitude toward mathematics therefore inhibiting the positive experience learning mathematics characteristic of previous studies.

Another study that exhibited a similar trend was one conducted to investigate the influence of an ethnomathematics course on the beliefs about mathematics of high school students and graduate students (Presmeg, 2002). The ethnomathematics course was designed to connect everyday life and cultural experiences with the mathematics represented but often not seen in those experiences. Initial assessments of the high school students' beliefs about mathematics indicated they were very static. In fact, Presmeg reports that it was due to these beliefs (and similar beliefs held by the high school students' teacher) that she was unable to "implement successfully the cultural mathematical activities we had planned for this class" (Presmeg, 2002, p. 297). While unable to change NOM beliefs of the high school students via her ethnomathematics content, Presmeg was able to use reflective question at multiple junctures to help the students' NOM beliefs expand beyond where they were initially (Presmeg, 2002). The evidence shows, however, that due to their highly static NOM beliefs, the high school students' experience learning mathematics—at least the kind of mathematics intended by Presmeg—was inhibited from growing their beliefs toward a more dynamic view of mathematics.

Mathematics Learning Experiences Characterized by Emotion and Attitude

The research presented thus far has described the various types of mathematics learning experiences in which preservice teachers (and university students) have engaged.

Acknowledgement of the role emotions and attitude play in these mathematics learning experiences cannot go unaddressed. Specifically, there is much research on the influence and attachment of emotions with beliefs and attitudes inside mathematics learning experiences (Di Martino & Zan, 2011; Op't Eynde, De Corte, & Verschaffel, 2002; Quinn, 1997; Wilkins, 2008).

In their research review, Op't Eynde and colleagues sought to "develop a theoretical framework that coherently integrates the major components of prevalent models of students' beliefs" (Op't Eynde, De Corte, & Verschaffel, 2002, p. 13). The framework presented showcased the intertwining complexity that beliefs have with emotions and value components. More specifically, they claimed that beliefs about mathematics evolve through experiences in the social context (i.e. the classroom), emotions are an expression of beliefs, and the importance or value that a student places on the experience will determine if they engage in learning (Op't Eynde, De Corte, & Verschaffel, 2002).

For teachers, emotions can influence what they choose to learn (i.e. what they chose to experience) and the value they place on content can influence what they choose to teach (Nespor, 1987). Additionally, the beliefs a teacher holds are "bound up with the personal, episodic, and emotional experiences" (Nespor, 1987, p. 321) of the teacher and in order to understand the teacher's perspective on teaching, "we have to understand the beliefs with which they define their work" (Nespor, 1987, p. 323).

These types of research claims lead to studies that seek to investigate the relationships that beliefs, attitudes and emotions have with mathematics learning experiences. Quinn (1997) set out to measure the change in attitude toward mathematics of preservice teachers under the idea that teachers who have "poor attitudes toward the subject often exacerbate the problems that students experience in learning mathematics" (Quinn, 1997, p. 108). Attitude toward mathematics was defined by Quinn as "the level of like or dislike felt by an individual toward mathematics" (Quinn, 1997, p. 108). Here, emotion is viewed as a component of attitude and to change the attitudes of the preservice teachers, a mathematics learning experience aligned to reformed teaching practices was implemented. Results showed a statistically significant change in attitude toward mathematics was seen for some preservice teachers in the study but not for all. The secondary preservice teachers' attitudes toward mathematics improved but the change was not statistically significant (Quinn, 1997). This study showed that the experience learning mathematics via reformed instructional practices serves to influence attitudes—and consequently emotions—toward mathematics although it is unclear if these attitudes are related to the preservice teachers' beliefs about mathematics.

Another study that investigated the relationships between beliefs about inquiry-based instruction and attitudes toward mathematics of inservice teachers and sought to determine the level of mathematics content knowledge, beliefs and attitudes under the premise they all influence instructional practice (Wilkins, 2008). This study was not specifically measuring beliefs about the nature of mathematics but did ask teachers about their level of enjoyment and liking of the subject, the value they placed on the content, and their feelings of success with regard to their experiences with mathematics. In a quantitative study, Wilkins (2008) showed statistically significant relationships between components of content knowledge, beliefs and attitude indicating that attitude did have a positive effect on instructional practice with teachers expressing positive attitudes toward mathematics would be more likely to use inquiry-based instructional methods. Further, Wilkins highlights the idea that positive attitudes relate to emotions that are established based on enjoyment of the past experience of learning mathematics.

A study conducted by Di Martino and Zan investigated students' autobiographical descriptions of their relationship with mathematics and found three main themes related to 1) emotional disposition, 2) students' self-efficacy, and 3) students' beliefs about mathematics (Di Martino & Zan, 2011). According to Di Martino and Zan, the autobiographical accounts provided by the students act as a "bridge between the writer's beliefs and emotions, and highlight the psychologically central role [the beliefs and emotions] play for the narrator" (Di Martino & Zan, 2011, p. 476). Specifically, Di Martino and Zan found that beliefs about mathematics played a "crucial role" in establishing a "negative emotional disposition" and beliefs about the inability to do mathematics (p. 480). The beliefs about mathematics expressed by the students in this study were highly static in nature indicating singular ideas of mathematics as rules and mathematics is not creative (Di Martino & Zan, 2011). These static NOM beliefs in turn motivated negative emotions and attitudes toward mathematics when the students with such beliefs experienced learning mathematics. Additionally, Di Martino and Zan maintain that the apparent links between beliefs, attitudes and emotions were too complex to resolve in a singular study and that more investigations needed to be undertaken.

Beliefs Influence Instructional Practice

Up to this point, this chapter has presented how the mathematics community has defined the nature of mathematics as a multidimensional subject and how particular types of experiences help to generate beliefs aligned with that definition. The ultimate reason why we must understand how these beliefs are formed is due to the significant influence NOM beliefs can have on the instructional actions and choices of a mathematics educator. Simply stated, mathematics educators filter everything they do and say through their beliefs system (Bay-Williams et al., 2014; Hannula, 2006). Additionally, when every other component of mathematics education is put into perspective, teachers are the single most influential force for learning in the classroom (Balka, Hull, & Miles, 2010). In response to these factors and the persistent lack of mathematically proficient students in the U.S., the NCTM published a detailed recipe of all of the characteristics of an effective mathematics educator (Leinwand et al., 2014). Specifically, the NCTM maintains that effective mathematics educators are those who hold dynamic NOM beliefs—dynamic beliefs that support instructional choices made to build mathematical proficiency in students. It is important to consider some of the direct pathways through which NOM beliefs influence math instruction and consequently mathematics learning. The following sections will outline how NOM beliefs influence classroom norms the teacher may establish, and the instructional goals and tasks a teacher chooses to use.

Beliefs Influence Classroom Norms

Over the past several decades, many research studies have been conducted that seek to understand the NOM beliefs teachers hold and how those beliefs influence their instructional practices (Amirali & Halai, 2010; Beswick, 2012; Cross, 2009; Lerman, 1990; Speer, 2008). While teachers must make many instructional decisions, the classroom norms they choose to establish are the first point of influence for their NOM beliefs. In her book, *Mathematical Mindsets*, Jo Boaler (2016) encourages mathematics teachers to establish a set of expectations or positive classroom norms. Boaler lists the seven norms she uses, several of which directly express some primary characteristics of dynamic NOM beliefs. Her students are encouraged to believe that math is creative, involves sense-making, connections and communicating and that as their instructor she will facilitate experiences in learning according to these norms (Boaler, 2016).

Setting norms allows the teacher to explicitly state their dominant beliefs about mathematics and the learning that will happen in the classroom. These explicit statements (usually posted around the classroom for all to see) are quite powerful in light of recent research that claims a direct and profound relationship between positive beliefs about math and higher achievement, especially for females. Beilock, Gunderson, Ramirez, and Levine (2009) found that female teachers' negative beliefs about mathematics, transmitted to students via comments about being bad at math in school, resulted in lower mathematics achievement for female students. By associating school math with negative feelings, the teacher solidifies the idea that mathematics is one-dimensional and any attempt to do math will breed failure. Consequently, students who meet with failure in the math classroom come away with the belief that mathematics is school math and since their attempts were unsuccessful (i.e. they couldn't remember how to do the problem or forgot the formula for solving the problem) they ought not to have any further interaction with it. When the messages that students receive from their teachers paint a one-dimensional picture of mathematics, students believe in that image and this belief acts as a filter influencing future decisions to engage in mathematics.

Beliefs Influence Instructional Goals

Similar to setting classroom norms, math teachers also set goals and objectives for each math lesson. Ideally, these goals and objectives reflect the classroom norms but in the absence of norms, the goals can communicate to students the beliefs a teacher holds about the nature of the math they are learning. For a given mathematical lesson, goals and objectives typically come from an adopted curriculum or textbook mandated by a teacher's district or department. Additionally, goals and objectives are written to be measureable and applicable to a wide range of students. Consequently, textbooks are written with the "idea of isolating methods, reducing them to their simplest form" (Boaler, 2016) for students to practice. Teachers who believe that mathematics is one-dimensional often use the textbook as the only source of goals, objectives and practice. In fact, Stephen Lerman (1990) showed that teachers in his study who ascribed to static NOM beliefs indicated that mathematics learning involved certain knowledge from the textbook and a high degree of direction from the teacher (i.e. the traditional form of instruction). However, the teachers in Lerman's study who held dynamic NOM beliefs concentrated on context and meaning for the individual student, focusing instead on the problem-solving experience rather than the rigidity in the textbook (Lerman, 1990).

According to Jo Boaler (2016), "[r]eal problems often require the choice and adaptation of methods that students have often never learned to use or even think about" (p. 45). Teachers who offer experiences with real problems are typically those who believe that mathematics is a creative and visual subject of patterns and connections. Therefore, these teachers supplement or supplant mandated curriculum to offer their students opportunities to experience the true nature of mathematics. In order for a teacher to give the experiences he or she must set goals appropriate to the task.

Beliefs Influence Instructional Tasks

After goals are set in a mathematics learning experience, it's time for students to practice their math. When it comes to the formation of dynamic NOM beliefs in students, it matters what they practice. When students spend most of their time working on mathematical questions that are focused on a particular procedure and require mostly calculation it is difficult for them to believe that mathematics is anything but one-dimensional (Boaler, 2016). Experiences of this type are typical of a classroom where the teacher works from the textbook and sets goals according to her one-dimensional view of mathematics.

On the contrary, a teacher who believes the nature of mathematics is more than just facts, procedures and calculations will design and administer mathematical tasks that reflect her beliefs. A significant amount of research has been done to define a quality mathematical task and all of its components, including the beliefs an effective mathematics educator must have to design and facilitate one (Bay-Williams et al., 2014; Boaler, 2016; Leinwand et al., 2014; Smith & Stein, 2011). In general, most researchers agree on five characteristics of every quality mathematical experience (Boaler, 2016; Smith & Stein, 2011):

1) The task should be inquiry based and open-ended with no suggested pathway to a solution.
2) The task should contain a visual component or an opportunity for students to visualize a pattern or portion of the problem.
3) The task should allow for multiple methods and representations to find a solution.
4) The task should be classified as a high cognitive demand task inducing productive struggle but allowing any student to tackle the problem at their current level of ability. Boaler (2016) calls this characteristic “low floor, high ceiling” (p. 84)
5) The task should require students to critique the reasoning and methods of their classmates in an effort to convince.

Each element described above represents at least one or multiple dimensions of the nature of mathematics. Tasks designed with these elements in mind will not only build dynamic NOM beliefs in the students who experience them, but those students will also begin to build mathematical proficiency (Leinwand, et. al., 2014). The key idea here is that if a teacher does not hold dynamic NOM beliefs she will not offer her students tasks like the ones described above that help her students develop dynamic NOM beliefs.

Definition of Terms

Now that the problem statement, research questions and background of this study have been established, the following section outlines terms used throughout the rest of this study.

- *Nature of Mathematics (NOM) beliefs* – a collection of the beliefs a person has in regard to what mathematics is in terms of the discipline or subject of mathematics; For instance, a belief that mathematics is the study of mathematical ideas and connections, or a belief that one can use mathematics as a tool to solve problems, or the belief in the creative power of mathematics in daily life (Boaler, 2016; Ernest, 1989; Hersh, 1979); beliefs can be singular or exhibit multiple views.
- *Dynamic NOM Beliefs* – multidimensional beliefs consistent with the NOM definition that says that mathematics is a creative activity involving the study of patterns, connections and ideas that can be applied to solve problems in the world (Boaler, 2016; Ernest, 1989; Hersh, 1979; Leinwand et al., 2014); a belief that can grow as more perceptions of NOM are experienced.
- *Static NOM Beliefs* – singular beliefs consistent with the NOM definition that says that mathematics is a useful set of unrelated rules, formulas and procedures that need to be memorized but do not apply to any situation outside of the school learning environment (Ernest, 1989; Hersh, 1979; Leinwand et al., 2014).
- NOM beliefs continuum – the range of static NOM to dynamic NOM where static beliefs represent singular views and dynamic beliefs represent multiplistic views; movement along the NOM continuum corresponds to movement from static NOM beliefs to dynamic NOM beliefs or vice versa.
- *Mathematics Experiences* – any interaction with mathematics in a learning environment consistent with typical in-school environments including math

learned in school and during school projects; can be characterized by traditional or non-traditional learning elements.

- *Non-reformed experience* – a general term for an experiences that involves an instructional approach that includes many elements but mainly teacher-centered activities characterized by direct instruction either at the front of the class or with individual students, show and tell sessions; teacher acts to give the impression there is only one correct answer and problems can only be solved one particular way; teacher considered the sole authority on mathematics content being taught.
- *Reformed experience* – typically characterized as an experience that involves reformed instructional practice; problem-centered learning or inquiry-based learning; a general term for an instructional approach that includes many elements but mainly whole class discussion, small group discussion and activities, student-centered tasks that have multiple levels of access and are cognitively demanding; the teacher acts as a facilitator helping to build new knowledge based on existing knowledge by using thought provoking questions and constantly assessing student cognition (Leinwand et al., 2014).

Chapter Summary

This chapter has thus far presented the background on the definitions of nature of mathematics beliefs, mathematics learning experiences, and how beliefs have been shown to influence teaching practice. Connecting all of the existing research together leaves some areas still in need of attention.

While there seems to be a consensus that static and dynamic NOM beliefs exist (Dossey, 1992), there is still a question about what should fill the space between these purported extremes. Much of the research outlined in this chapter (Boaler, 2016; Devlin, 1997; Ernest, 1989; Hersh, 1999;) showcases multiple dimensions of NOM beliefs (i.e. mathematics is a creative endeavor or mathematics is a science of patterns) and these views were held concurrently by some individuals (Grigutsch & Torner, 1998; Yang & Leung, 2015). Future research needs to look at the NOM beliefs of individuals—especially preservice teachers—to investigate how these beliefs are formed and where they fit with respect to the static-dynamic NOM continuum.

Understanding more specifically how NOM beliefs are formed is also an area of research study still in need of attention. According to existing research, certain types of experiences learning mathematics help to build certain types of NOM beliefs (Boaler, 2016; Leinwand, et. al., 2014; NCTM, 2018). Much of the research to understand these mathematics learning experiences showcases the change in NOM beliefs associated with an intervention experience learning mathematics (Shilling & Stylianides, 2013; Weldeana & Abraham, 2015; Yusof & Tall, 1999). Only a few case studies investigated the episodic memory (Nespor, 1987) where PSTs reflect on a variety of past experiences learning mathematics (Cooney, et. al.,1998; McDuffie & Slavit, 2003; Nespor, 1987). Case studies are not generalizable across the larger population of PSTs, therefore, more mixed methods research needs to be completed to illuminate relationships that exist between past experiences learning mathematics and NOM beliefs while also seeking to understand why the relationship might exist.

There were only a few studies outlined in this chapter that found some unanticipated results when investigating the change in NOM beliefs due to a particular experience. Namely, the results showed that participants in these studies held NOM beliefs that inhibited future experiences and helped to keep NOM beliefs unchanged (McGinnis, et. al., 2002; Presmeg, 2002; Shilling & Stylianides, 2013). In effect, the studies showed that for some individuals who hold static NOM beliefs, experiences aligned with reformed instruction equate to negative experiences for them. Future research needs to investigate this phenomenon particularly as it relates to preservice teachers and the NOM beliefs they hold. If preservice teachers have positive experiences that align with static NOM beliefs, these positive experiences will carry over to their instructional choices (Cooney, et. al., 1998; Nespor, 1987; Wilkins, 2008).

The existing research literature on beliefs about mathematics illuminates how beliefs are complexly intertwined with emotions, attitudes and values (Nespor, 1987; Op't Eynde, De Corte, & Verschaffel, 2002) and several studies show empirical evidence of the changes in attitude toward mathematics based on specific experiences learning mathematics (DiMartino & Zan, 2011; Quinn, 1997; Wilkins, 2008). Due to the complex nature of the system of beliefs, emotion, attitude and values, future studies should seek to gain a deeper understanding of this complexity by providing opportunities for participants to reflect and provide details about past experience and beliefs not ordinarily present in quests for empirical evidence.

In Chapter 3, the Methodology used in this study to answer the research questions will be discussed. Participant sampling, data collection (instrument and procedures) and data analysis will be described in detail.

CHAPTER III

METHODOLOGY

This convergent parallel mixed methods research study uses both qualitative and quantitative data to ascertain the beliefs about the nature of mathematics that prospective elementary teachers hold and how their mathematics learning experiences may be related to these beliefs. Chapter three describes the participants and sampling strategy used in the study, the survey instruments used for quantitative data collection, qualitative data collection procedures, and the methods for analysis of the data collected. In sum, the goal of the study is to answer the following research questions:

1. *What are elementary preservice teachers' past experiences with mathematics?*
2. *What are elementary preservice teachers' nature of mathematics beliefs?*
3. *In what ways, if any, are the mathematics experiences of elementary preservice teachers related to their beliefs about the nature of mathematics?*

Research Design

Mixed methods research can be defined as a methodological orientation that combines "philosophical positions, inferences, and the interpretations of results" (Creswell & Plano Clark, 2011, p. 3) from both qualitative and quantitative approaches to a study. According to Patton (2015), use of multiple data collection and analysis techniques provides "more grist for the analytical mill" (p. 661). The researcher in this study chose a mixed methods approach to gain a deep understanding of the mathematics learning experiences and nature of mathematics beliefs held by a group of preservice elementary teachers. Comparison and integration of both qualitative and quantitative data through analysis will illuminate any inconsistencies or corroborations and provide a fuller picture of the data.

This mixed methods study can be classified as a convergent parallel design that will address and describe the nature of mathematics (NOM) beliefs of elementary preservice teachers (ePSTs) and their past experiences learning math, while also trying to discern if a relationship exists between the two. While measuring individual beliefs can be accomplished using a quantitative Likert-style instrument, experiences are best described through individual narrative. Thus, this study incorporates both quantitative and qualitative data and uses both quantitative and qualitative data analysis (e.g. univariate/multivariate statistical methods and analytical triangulation and thematic analysis). After analysis of all qualitative and quantitative data was complete, the results of the analyses were integrated for interpretation and explanation. Figure 3.1 shows a diagram depicting the research design of this study.

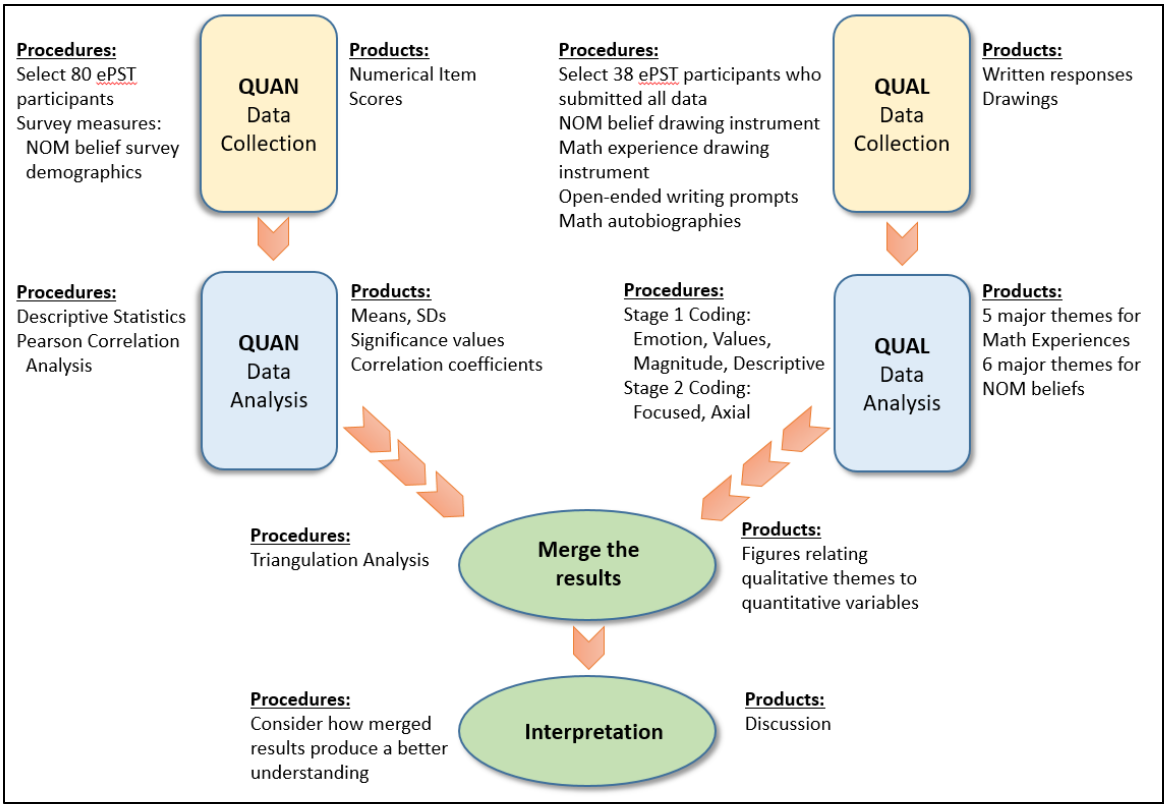


Figure 3.1. Study diagram for convergent parallel design

As Figure 3.1 indicates, the quantitative and qualitative data were collected concurrently. The qualitative data were analyzed using first and second cycle coding procedures (Saldana, 2013) through multiple stages to describe and categorize the data. The qualitative data were also triangulated to uncover any inconsistencies and corroborations in the data. According to Patton (2015), triangulation of qualitative data is intended to ascertain the consistency of the various data sources. In this study, there were three separate qualitative data sources for both mathematics learning experiences and NOM beliefs and these sources were compared for consistency within experiences and within beliefs. The quantitative data were analyzed using standard statistical analysis methods (e.g. descriptive statistics, and Pearson correlation). To add more strength of credibility to this study and after each distinct analysis was completed, the qualitative data were compared with the quantitative data to check consistency of results generated by different data collections methods (Patton, 2015). Finally, both sets of analyses were integrated and interpreted to answer the research questions posed. Utilizing multiple methods strengthens a study in order to illuminate the "different aspects of empirical reality and social perceptions" (Patton, 2015, p. 661). Integration of both qualitative and quantitative data collection, and analysis methods has the distinct benefit of "offering opportunities for deeper insight into the relationship" (Patton, 2015, p. 661) that may exist between mathematics learning experiences and NOM beliefs of the prospective elementary teachers in this study. Consequently, the interpretations drawn from this effort will serve to boost the understanding of inconsistences or corroborations between experiences and beliefs.

Sampling Strategy

The sample for this study was chosen both due to convenience and purpose as defined by Creswell and Plano-Clark (2011) for mixed methods research. A convenience sample is one in which the researcher has access to with respect to location and time allotted for the study. The sample for this study is a convenience sample as it consisted of individuals from the group of elementary preservice teachers at a large Midwestern land-grant university to which the researcher is also affiliated. Purposeful sampling "means that researchers intentionally select participants who have experienced the central phenomenon or the key concept being explored in the study" (Creswell & Plano Clark, 2011, p. 173). All participants in this study have developed beliefs about the nature of mathematics and have had personal experiences learning mathematics.

During Fall 2017, there were approximately 80 elementary preservice teachers enrolled in the intermediate mathematics methods course. Since a large sample size is desired—greater than 30 participants if possible—to increase statistical power for the quantitative analysis of the study (Lomax, 2012), the researcher personally solicited participation from all students enrolled in this course at that time. All participants were asked to complete the quantitative and qualitative instruments.

Initially, the researcher met participants during their intermediate mathematics methods course meeting time. During the last fifteen minutes of class time, participants were asked to complete the demographic survey, the NOM beliefs questionnaire as well as the NOM beliefs drawing instrument. Participants were instructed to complete the mathematics autobiographies, the open-ended writing prompts, and the mathematics experiences drawing outside of class time and return their responses to their instructor. The NOM beliefs questionnaire, demographic survey, open-ended writing prompts and drawings were voluntary components, only the mathematics autobiographies were required. Because the autobiographies were an assignment required by the instructors of the intermediate math methods course, each instructor agreed to collect the voluntary components (i.e. the writing prompt responses and the experience drawings) and a submitted copy of the mathematics autobiographies. The researcher procured these items two weeks after collecting the demographic and NOM beliefs data during the intermediate math methods class time.

During data collection of the demographic survey, NOM beliefs questionnaire, and the NOM beliefs drawing, participants were encouraged to be as detail oriented as possible, but were limited by time—they had approximately ten minutes to complete all items. Additionally, the participants were encouraged to include as much information as possible when they completed the mathematics experiences drawings and open-ended writing prompts during their time outside of class. The mathematics autobiographies were governed by specific instructions due to the fact that it was a required assignment for the intermediate mathematics methods course.

While approximately 80 ePSTs were surveyed with the demographic survey, NOM Beliefs Questionnaire, and the NOM Beliefs drawing, only 38 ePSTs completed and returned their open-ended writing prompts and mathematics experiences drawings. All ePSTs were required to complete the mathematics autobiographies; however, this study was focused on the ePSTs for which a complete data set was collected.

Participants

The participants in this study were 38 prospective elementary teachers (ePSTs) enrolled in a teacher education program at a large Midwestern land-grant university. Participants were in their last semester of coursework prior to student teaching and were enrolled in 15 credit hours including an intermediate mathematics methods course. Participants were also required to spend several hours engaging with elementary students in a classroom setting and tutoring elementary students in mathematics.

By this point in their program, the ePSTs had already completed about 97 hours of course work. Included in the 97 hours of course work are 12 hours of college-level mathematics courses, six hours (two courses) of which are courses specifically designed for prospective elementary teachers. The first course, Geometric Structures, covers the fundamentals of plane geometry, transformations, polyhedra, and applications to measurement. The second course, Mathematical Structures, covers the foundations of number, algebraic systems, functions, applications of functions, and probability. To account for the other six hours of the 12 required for college-level mathematics, the ePSTs are able to choose from a variety of course work including courses in College Algebra to Calculus. The demographic data for the 38 participants is presented in Table 3.1.

Table 3.1 *Participant Demographic Characteristics as a Percentage of Sample Size, n=38*

Characteristic	n (%)	Characteristic	n (%)
Gender		*College-level Mathematics Courses Taken*	
Female	36 (95)	Functions	25 (66)
Male	2 (5)	Applications of Modern Math	12 (32)
Ethnicity		College Algebra	33 (87)
White	32 (84)	Mathematical Structures	38 (100)
Native American	4 (11)	Geometric Structures	38 (100)
Hispanic	2 (5)	Other [a]	12 (32)
Age Range		*High School Mathematics Courses Taken*	
20 to 23	29 (76)	Algebra 1	36 (95)
24+	7 (18)	Algebra 2	34 (89)
Not Indicated	2 (5)	Geometry	34 (89)
High School Classification		Trigonometry	13 (34)
Rural	11 (29)	Pre-Calculus	20 (53)
Suburban	20 (53)	Calculus	3 (8)
Urban	5 (13)	Statistics	3 (8)
Other	2 (5)	Other [b]	2 (5)

[a]Other college-level mathematics courses taken included remedial math, statistics, trigonometry, mathematical proofs and business calculus. [b]Other high school mathematics courses taken included pre-college algebra and AP Calculus.

Instrumentation

The data collection instruments used in this study include both quantitative and qualitative measures. The quantitative instruments include a NOM questionnaire with two subscales that measure NOM beliefs and a demographic survey. The qualitative instruments used in this study include open-ended writing prompts, mathematics autobiographies, visual metaphor drawings about NOM beliefs, and mathematics experience drawings.

Quantitative Measures

Demographic survey. The demographic survey (see Appendix A) collected demographic information from the participants such as gender, ethnicity, age, and mathematics courses taken in college and high school. This demographic information was used to calculate and report descriptive statistics for the sample population and aid in data analysis.

Nature of mathematics beliefs questionnaire. The nature of mathematics beliefs questionnaire (see Appendix B) used in this study contains two subscales designed to measure beliefs about mathematics as rules and procedures and mathematics as a process of inquiry. The NOM belief subscales were developed for the Teacher Education and Development Study in Mathematics (TEDS-M) (Tatto, 2013) and were adopted for this study to assess elementary preservice teachers' beliefs about the nature of mathematics. The TEDS-M study had a primary goal to assess the characteristics of the mathematics and teaching knowledge that prospective elementary teachers had attained by the conclusion of their teacher education program (Tatto, Schwille, Senk, Ingvarson, Rowley, Peck & Reckase, 2012). Specifically, the TEDS-M study sought to investigate the level and depth of prospective teachers' content knowledge, pedagogical knowledge, and the beliefs about the nature of mathematics they had acquired by the end of their teacher preparation programs (Tatto et al., 2012).

While content and pedagogical knowledge are intertwined with beliefs (Ernest, 1989; Leinwand, Brahier, & Huinker, 2014; Liljedahl, 2009; Nespor, 1987), these topics were not the focus of this study. Consequently, the TEDS-M subscales that measure the depth and level of content and pedagogical knowledge were not employed for this study. However, like the TEDS-M study, this study is focused on the NOM beliefs of preservice teachers. Therefore, the NOM belief subscales used in the TEDS-M study were employed for this investigation. The NOM belief subscale that measures mathematics as rules and procedures (NOM-RP) corresponds to the research-based claim that mathematics is a static subject full of facts that must be memorized (Grigutsch & Torner, 1998; Op't Eynde & De Corte, 2003; Tatto et al., 2012; Tatto, 2013). The development of the NOM-RP subscale for the TEDS-M study came from the word of several studies that show evidence that "suggests that the beliefs about mathematics and mathematics learning that beginning teachers carry with them may influence how they teach, and subsequently may influence how their students learn" (Tatto et al., 2012, p. 154). As a consequence, the NOM-RP subscale was created to measure how much preservice teachers agree with the view that mathematics is rules and procedures. The subscale contains a set of six statements about mathematics as rules and procedures to which participants must indicate their level of agreement based on a 6-point Likert-scale (e.g. 1 – strongly disagree to 6 – strongly agree). Higher scores indicate that respondents align with the belief that mathematics is more static in nature. The Cronbach's alpha for this subscale was reported as 0.94 (Tatto, 2013, p. 268). Using the sample size for this study, $n = 38$, the Cronbach's alpha for this subscale was 0.86.

The NOM belief subscale that measures mathematics as a process of inquiry (NOM-PI) corresponds to the research-based idea that mathematics is a creative endeavor and dynamic process where new ideas lead to the creation of new connections (Grigutsch & Torner, 1998; Op't Eynde & De Corte, 2003; Tatto et al., 2012; Tatto, 2013). Like the NOM-RP subscale, the NOM-PI subscale was created to measure how much preservice teachers agree with the view that mathematics is a process of inquiry. In the TEDS-M study, the NOM-RP and NOM-PI subscales were negatively correlated (Tatto et al., 2012) meaning agreement with one view would position a preservice teacher at one end of the NOM continuum while agreement with the other view would result in placement at the opposite end. The NOM-PI subscale contains a set of six statements about mathematics as a process of inquiry to which participants must indicate their level of agreement based on a 6-point Likert-scale (e.g. 1 – strongly disagree to 6 – strongly agree). Higher scores indicate that respondents align with the belief that mathematics is more process-oriented and open to multiple definitions of what math is. The Cronbach's alpha for this subscale was reported as 0.91 (Tatto, 2013, p. 268). Using the sample size for this study, n = 38, the Cronbach's alpha for this subscale was 0.86. The overall Cronbach alpha for all twelve scale items on the NOM belief questionnaire was 0.84.

Qualitative Measures

Open-ended writing prompts. Each participant provided responses to three open-ended writing prompts as shown in Appendix C. The researcher-designed writing prompts were designed to collect qualitative data with regard to past mathematics experiences and beliefs about mathematics. The prompts were designed to serve as an initial opportunity for the study participants to reflect on their past mathematics learning experiences and articulate the NOM beliefs they hold. Participants were asked to respond to each prompt with as much detail as possible and to complete the prompts before the mathematics autobiography assignment. The main goal of the prompts was to record the initial thoughts and memories that participants hold with respect to their NOM beliefs and past math experiences. Question one asked “what is mathematics?”, the second question asked participants to describe their experiences learning mathematics before college, and the third question asked for descriptions of mathematics learning experiences during college. The first prompt was designed to record the first idea participants have when asked what mathematics is. This initial record can be compared to other records obtained by subsequent methods like the biography and drawings. The second prompt was designed to focus participants’ attention on memories related to learning mathematics before college. The third prompt was designed to focus participants to their college experiences while learning math.

Mathematics autobiographies. The elementary preservice teachers completed mathematics autobiographies as an assignment for their intermediate mathematics methods course. The mathematics autobiography assignment is assigned during the first week of the semester by each instructor of the intermediate math methods course. Developed by the methods course instructors, the assignment is intended to gain a detailed picture of the students taking the course and provide the students an opportunity to reflect on the math learners they have become. The purpose of the autobiographies for this study was to gain detailed information about past experiences learning mathematics and associated beliefs about mathematics. Participants were told in the instructions for the assignment that their mathematics autobiography should paint a detailed picture of themselves as learners of mathematics. Participants were asked to share about their mathematics experiences—both the positive and the negative types—and reflect on how the good/bad experiences have affected them. Participants were also asked to share about how they would describe mathematics to a friend. Finally, participants were asked to describe their mathematics dispositions and beliefs about learning and teaching mathematics. For a complete description of the mathematics autobiography assignment see Appendix D.

Mathematics experience drawings. A primary goal of this study is to explore the past mathematics learning experiences of elementary preservice teachers. Drawings, also known as visual metaphors, can be a powerful way to get a glimpse of an individual's past or their current mindset with regard to a particular phenomenon. Drawings have been used before to investigate preservice teachers' beliefs about mathematics teaching and learning (e.g. Reeder, Utley & Cassel, 2009). In their study, Reeder and colleagues (2009) asked a cohort of elementary preservice teachers to draw a visual metaphor for mathematics teaching and learning and subsequently explain their metaphor with words. Similarly, participants in this study were asked to draw a singular image or a collage of images depicting their mathematics learning experiences. They were provided colored pencils and a sheet of paper with a space marked off for the drawing and instructions on what to draw and where to write the description of the drawing. See Appendix E for an example of the instrument.

NOM beliefs drawings. Similar in method to the mathematics experience drawings (Reeder, Utley & Cassel, 2009), the NOM beliefs drawings were used to investigate preservice teachers' beliefs about the nature of mathematics. At the top of the drawing instrument, participants were asked to complete a metaphor/simile—'mathematics is like'—that conveys their NOM beliefs. In addition, the participants were asked to draw a picture of their metaphor and explain their drawing with a few sentences as to why they chose their particular metaphor. See Appendix F for an example of the instrument. Table 3.2 provides a summary description of each of the instruments plus the research questions addressed by each measure. Each research question is associated with a set of instruments, a description of each instrument, and the data analysis that is intended to answer the research question.

Table 3.2 *Research Questions and Related Measures Employed*

Research Question	Instrument	Description	Data Analysis
1) What are elementary preservice teachers' past experiences with mathematics?	-Open-ended Writing Prompts -Mathematics Autobiographies -Math Experience Drawings	-Participant generated written/drawing data	-Code for mathematics experiences -Triangulation -Generate magnitude scores
2) What are elementary preservice teachers' nature of mathematics beliefs?	-Open-ended Writing Prompts -Mathematics Autobiographies -NOM Beliefs Drawings	-Participant generated written/drawing data	-Code for nature of mathematics beliefs -Triangulation
	-NOM Beliefs Questionnaire	-Likert-type scale	-Univariate/Bivariate statistical methods
3) In what ways, if any, are the mathematics experiences of elementary preservice teachers related to their beliefs about the nature of mathematics?	-Open-ended Writing Prompts -Mathematics Autobiographies -Math Experience Drawings -NOM Beliefs Drawings	-Participant generated written/drawing data	-Integration and interpretation -Triangulation
	-NOM Beliefs Questionnaire -Math experience scores	-Likert-type scale -Quantified qualitative Codes	-Correlation analysis

Data Collection Procedures

Prior to data collection, the researcher sought approval by the university Institutional Review Board (IRB). Upon securing IRB approval, the researcher began soliciting participation from the elementary preservice teachers enrolled in the Fall 2017 sections of the intermediate mathematics methods course at a Midwestern land-grant university. Several instruments were administered during the data collection process and each instrument has a unique identifying number so that the data can be paired and tracked during analysis. Informed consent was garnered from 40 of the 80 students during the first week of classes—at which time they also completed three instruments—the demographic survey, NOM beliefs questionnaire, and NOM beliefs drawings. Two more instruments—the open-ended writing prompts and mathematics experience drawings—were voluntary and collected two weeks after initial consent was obtained. The final instrument—the mathematics autobiography—was a class assignment completed during the first week and then sent to the researcher two weeks after initial consent. For this study, one case is defined for a single participant who completed all instruments in full—the demographic survey, NOM beliefs questionnaire, open-ended writing prompts, mathematics autobiography, mathematics experience drawing, and a NOM beliefs drawing. A total of 38 cases were analyzed.

Data Analysis

Data analysis was separated into several distinct stages—some purely qualitative and others purely quantitative. During the analysis, each research question was considered separately; however, some coding procedures were shared. Additionally, the qualitative analysis was separated into several sub-stages on the pathway to integration and interpretation. Each stage of data analysis is detailed below.

Qualitative Analysis for RQ1 and RQ2

A complete set of qualitative data in this study is classified as a single case where the participant provided a mathematics autobiography, open-ended writing prompt, and drawings—one each of the mathematics experience drawing and nature of mathematics (NOM) belief drawing. The first research question (RQ1) was in reference to the elementary preservice teachers' experiences learning mathematics. The qualitative data analyzed for RQ1 were the mathematics autobiography, the open-ended writing prompt, and the mathematics experience drawing. The second research question (RQ2) sought to reveal the NOM beliefs that the participants held. Similar to RQ1, the qualitative data analyzed for RQ2 was again the mathematics autobiography and open-ended writing prompt, but the NOM beliefs drawings replaced the experience drawings.

Eclectic coding procedure. The analysis for both RQ1 and RQ2 was conducted using an Eclectic Coding procedure (Saldana, 2013). Eclectic Coding is defined by Saldana as "using a repertoire of methods simultaneously" (Saldana, 2013, p. 188) to code a piece of qualitative data. To provide a thick description (Erlandson, 1993) of the mathematics learning experiences and the NOM beliefs of the elementary preservice teachers in this study, it was necessary to codify all elements described by the participants. As a consequence of this objective, using a multitude of relevant coding processes seemed to be the best fit.

The Eclectic Coding process was split between two stages. In stage one, the coding procedures that were used for RQ1 were Emotion, Values and Descriptive Coding. Stage one for RQ2 involved the Values and Descriptive Coding procedures. Stage two for both RQ1 and RQ2 involved Focused and Axial Coding procedures to define themes emerging from the data. Table 3.3 outlines each research question, the coding procedures used, and the stage in which they were employed. In the section that follows, both stages and the coding procedures employed are described in detail.

Table 3.3 *Research Questions and Stages of Eclectic Coding Procedure*

Research Question	Stage One	Stage Two	Complete Data Set
1) What are elementary preservice teachers' past experiences with mathematics?	-Emotion Coding -Values Coding -Descriptive Coding	-Focused Coding -Axial Coding	-Mathematics Autobiography -Open-ended Writing Prompt -Mathematics Experience Drawing
2) What are elementary preservice teachers' nature of mathematics beliefs?	-Values Coding -Descriptive Coding	-Focused Coding -Axial Coding	-Mathematics Autobiography -Open-ended Writing Prompt -NOM Beliefs Drawing

Stage one. Stage one of the Eclectic Coding process at first involved the removal of identifying information and assigning a case number for each of the 38 participants that provided completed set of qualitative data. Due to the fact that the data for RQ1 was focused on experiences learning mathematics, the affective elements—particularly emotions and attitude—were of primary concern. According to Saldana, emotions "are a universal human experience [and] our acknowledgement of them in our research provides deep insight into the participants' perspectives, worldviews, and life conditions"

(Saldana, 2013, p. 106). Consequently, Emotion Coding is appropriate for this study due to the qualitative data that reveals the emotive component residing within participants' experiences learning mathematics. Initially, the qualitative data set for RQ1 was emotion coded, where emotions that were described by the ePSTs or emotions that were inferred by the researcher from the context of the data were labeled with the appropriate emotion word. For example, a statement like 'doing math makes me happy' would be coded with the emotion code 'HAPPY'. Each piece in the completed qualitative data set for RQ1 was emotion coded.

First for RQ2 but subsequent to the Emotion Coding procedure for RQ1, the qualitative data was coded using Values Coding which assigns codes based on the participant's described or inferred values, beliefs, and attitudes (Saldana, 2013). Saldana (2013) defines a value as "the importance we attribute to oneself, another person, thing, or idea", attitude as "the way we think and feel about ourselves, another person, thing, or idea", and belief as "part of a system that includes our values and attitudes, plus our personal knowledge, experiences, opinions, prejudices, morals, and other interpretive perceptions of the social world" (p. 111). Values Coding was appropriate for this study due to the qualitative data for RQ1 describing attitudes of the participants surrounding their experiences learning mathematics. Comparatively, the qualitative data for RQ2 revealed the NOM beliefs that participants described in their responses. In several ways, the Emotion Coding and Values coding for RQ1 intertwined and seemed to occur one on top of the other. However, RQ2 dealt specifically with beliefs about mathematics and therefore the Values Coding procedure generated a set of unique codes for the analysis of RQ2.

Along with Emotion Coding (for RQ1) and Values Coding, Descriptive Coding was employed to illuminate *what* the elementary preservice teachers were describing in their mathematics learning experiences and in their NOM beliefs. Due to the fact that a main goal of both RQ1 and RQ2 is to describe what participants' experiences learning mathematics were and what their NOM beliefs are, respectively, Descriptive Coding was employed to capture the structural elements of the experiences and beliefs described by the participants in their responses. The structural elements are those parts of the descriptions that define *what* the experience or belief was about. In fact, Saldana (2013) indicates that Descriptive Coding "summarizes in a word or short phrase—most often as a noun—the basic topic of a passage of qualitative data" (p. 88). Consequently, short phrases were taken directly from participant's responses provided in the qualitative data. These phrases were coded according to the structural elements of the mathematics learning experience or the NOM beliefs being described by the participants. For example, when describing their beliefs about mathematics, a participant might say, "mathematics is formulas, rules, and procedures". The description code assigned for this case would be 'rules and procedures' as this is the topic of the data.

Stage two. Stage two of the Eclectic Coding process encompasses two additional coding procedures and the subsequent generation of the codebook for mathematics learning experiences and the codebook for nature of mathematics (NOM) beliefs. In particular, for both RQ1 and RQ2 the processes of Focused Coding and Axial Coding were employed in order to further filter and focus the salient features of the qualitative data for generating categories and themes and building meaning from the data (Saldana, 2013). Focused Coding was employed to illuminate the most significant codes to develop categories for the data. Once categories were developed, Axial Coding was used to extend the analytic work from the processes used in stage one and the Focused Coding process (Saldana, 2013). Axial Coding specifies the elements of a category, how big the category is and where it fits on a continuum, and what properties belong to the category. Both Focused Coding and Axial Coding are appropriate for this study due to the qualitative data occurring in many forms (e.g. mathematics autobiographies, writing prompts, and drawings) and the presence of dominant codes that lend well to the development of categories. The code books for mathematics learning experience and NOM beliefs are presented in Appendix G.

Qualitative Analysis for RQ3

The third research question (RQ3) was aimed at determining if the past experiences learning mathematics and the nature of mathematics (NOM) beliefs held by elementary preservice teachers were related. While RQ3 is a quantitative question, the qualitative data needed to be quantified before the quantitative analysis could happen. As previously discussed, affective coding procedures (e.g. Emotion Coding and Values Coding) were employed to analyze the affective components running throughout the qualitative data. During the coding process, a clear difference between cases became apparent. Some of the participants' descriptions of their experiences were more intensely negative or positive in terms of emotion or attitude. Likewise, descriptions of NOM beliefs seemed to align with a clear direction on a continuum from mathematics as a static subject to mathematics as a dynamic process. In order to capture this characteristic of the descriptions and to enable comparisons between data sets, a Magnitude Coding process was employed. According to Saldana (2013), Magnitude Coding "consists of and adds a supplemental alphanumeric or symbolic code or subcode to an existing coded datum or category to indicate its intensity, frequency, direction, presence or evaluative content" (p. 72-73). In the sections that follow, the Magnitude Coding process will be outlined for both mathematics learning experiences and NOM beliefs.

Magnitude coding for mathematics learning experiences. During the first stage of the Eclectic Coding process, emotion codes were generated from the mathematics autobiographies, open-ended writing prompts and experience drawings. In each case, the data were scoured and words describing emotions were identified and coded. Next, emotion codes were used to determine a magnitude score of the intensity of the positivity or negativity present in the mathematics experiences being described by the elementary preservice teachers (ePSTs). Use of emotional words such as "love" and "enjoyed" bring a more positive connotation; while, emotional words like "hate" and "afraid" connote a very negative state of mind. The emotion codes were considered with respect to this positive-negative scale and were used to determine a score to represent the relative magnitude of positivity or negativity present in each data as a whole. As indicated in Table 3.4 below, Magnitude Coding was used to assign numeric scores that indicated the intensity of the emotions present in the mathematics learning experiences of ePSTs based on a scale of 1 (strongly negative) to 7 (strongly positive).

Table 3.4 *Magnitude Coding Scale with Descriptions and Examples for Mathematics Learning Experiences*

Score	Magnitude Code	Description and Example
1	Strongly Negative	Description: Several (more than two) strongly negative emotions are mentioned, drawn, labeled; emotional wording is highlighted, capitalized, underlined or mentioned more than one time; no positive emotions are mentioned Example: "Mathematics has always been hard for me…The teacher…made me feel like I was just a burden to her…That was a really bad experience for me to go through especially when I was already struggling so much, I just felt like she didn't care…I have always struggled with math no matter how easy or hard it is, all my memories with mathematics are not nice ones." (ePST23)
2	Negative	Description: Minimal (one or two) negative emotions are mentioned, drawn, labeled; emotional wording is not highlighted, capitalized, underlined Example: "I have never considered myself a numbers person and I remember I would get so frustrated that I couldn't solve problems as fast as other kids in my class for example, taking timed tests." (ePST21)
3	Slightly Negative	Description: Both positive and negative emotions are mentioned, drawn, labeled; emotional wording is more negative than positive in number or emphasis Example: "I was that student who really struggled in math class… Math was one of those things that never really clicked with me… In seventh grade I had an outstanding pre-algebra teacher who made math a lot of fun for me" (ePST1)
4	Neutral	Description: No emotional wording is used, drawn, labeled; only a description of the elements of the mathematics learning experiences is offered Example: "In elementary, middle, and high school, I learned math best by memorizing one way of solving a specific type of problem and practicing it until I felt that I had memorized the steps." (ePST14)
5	Slightly Positive	Description: Both positive and negative emotions are mentioned, drawn, labeled; emotional wording is more positive than negative in number or emphasis Example: "My Algebra 1 teacher and my geometry teacher were not very helpful to me and I struggled in those classes because I

		was not able to get much help from them…However, my Algebra 2 and pre-calculus teachers were great. They were very hands on and interactive and I loved taking their classes…In my college math classes, I have had mainly positive experiences and I have enjoyed math." (ePST22)
6	Positive	Description: Minimal (one or two) positive emotions are mentioned, drawn, labeled; emotional wording is not highlighted, capitalized, underlined; no negative emotions are mentioned Example: "All through elementary school we are forced to remember a string of facts to make a foundation. This was actually a very good time for me I thrived with the memorization did very well. I learned all the formulas and flourished in the assignments" (ePST18)
7	Strongly Positive	Description: Several (more than two) strongly positive emotions are mentioned, drawn, labeled; emotional wording is highlighted, capitalized, underlined or mentioned more than one time; no negative emotions are mentioned Example: "Throughout my academic career, I have always had a love for math. I love the fact that there are so many different ways that one can discover an answer to a problem, but I also like how there is just one answer…I had wonderful math teachers all through middle school and high school…I think that one positive experience helped make every other math class I took a positive experience as well… My teachers made math fun and made us enjoy the challenge of finding the answers" (ePST13)

For clarity, Table 3.4 shows the magnitude scores (i.e. 1 to 7), each level (i.e. strongly negative to strongly positive), the criteria that must be met for each score to be assigned, and an example from the data. For each mathematics autobiography, open-ended writing prompt and experience drawing, the emotion codes associated with participants' responses were evaluated according to the criteria set forth in Table 3.4. The criteria for each magnitude score level were not predetermined but were apparent in the data as Emotion Coding progressed in stage one of the Eclectic coding process. It should be noted however that these criteria are based on researcher interpretation of the intensity of the emotions present in the elementary preservice teachers' experiences learning mathematics.

Mathematics experience magnitude sum scores. Each case was analyzed according to the emotion codes and magnitude scale in Table 3.4. Correspondingly, each case earned a set of three magnitude scores—one score for each piece of data in the case (e.g. mathematics autobiography, writing prompt or drawing)—based on the researcher's interpretation of the positive or negative intensity of the emotion within the experiences described. A total of 114 magnitude scores were generated. The three magnitude scores were summed per participant to generate a score representing the positive or negative intensity across all data for each participant. For example, ePST20 earned a score of 1 on their mathematics autobiography, and a score of 2 for their writing prompt and drawing. Adding these scores gives a sum score of 5. The magnitude sum scores ranged from 3 to 21, where a strongly negative experience would equal 3 and a strongly positive experience would equal 21. Scores of 12 represent the neutral point where neither positive or negative intensity was assigned. A score of 5 indicates that for ePST20, their mathematics learning experiences were negative. Fifteen of the 38 participants indicated that they had negative experiences with mathematics. There were two participants whose magnitude sum score totaled 12, which is half way between positive and negative. There were 21 participants who indicated that their mathematics experiences were positive.

Magnitude coding for nature of mathematics beliefs. During the Eclectic Coding cycle, each mathematics autobiography, writing prompt, and experience drawing was analyzed to assign a values code to identify the beliefs associated with the participants in this study. According to Saldana (2013), values coding highlights the values, beliefs and attitudes within qualitative data. Due to the goals of this study, beliefs about mathematics were of specific interest. Each case was coded for values and beliefs about mathematics and then a magnitude score that aligns with a static-dynamic scale was assigned. In Table 3.5 each magnitude score and level of the static-dynamic mathematics beliefs continuum is defined. In a similar way to the aforementioned Emotion Coding for mathematics experiences, the data for NOM beliefs seemed to align naturally. A magnitude coding scale was employed to assign numeric scores that indicate the direction of the NOM beliefs along a continuum from static beliefs to dynamic beliefs base on a scale of 1 (strongly static) to 5 (strongly dynamic). A strongly static belief about mathematics relates to how closely participants believe that mathematics is only rules and procedures. A strongly dynamic belief about mathematics relates to how closely participants believe that mathematics is a process of inquiry or a subject that involved multiple forms of process and rules to describe naturally occurring phenomena. Therefore, magnitude codes on the dynamic end of the continuum indicate that beliefs include several dimensions to answer the question of what mathematics is.

Table 3.5 *Magnitude Coding Scale with Descriptions and Examples for Nature of Mathematics Beliefs*

Score	Magnitude Code	Description and Example
1	Static	Description: Only one single NOM belief is stated, drawn or labeled; description has a distinctly fixed minded approach to math, expressed belief has not changed over time Example: "When I think of mathematics I think of equations and word problems" (ePST3)
2	Less Static	Description: Two NOM beliefs are mentioned, drawn, labeled; less fixed minded approach Example: "Personally, I believe mathematics is being able to understand values and what they mean" (ePST27)

3	Middle	<u>Description:</u> Three NOM beliefs are mentioned, drawn, labeled; more growth minded approach <u>Example:</u> "…a subject in school that we use in our everyday lives to answer questions about things like architecture, accounting, and many more… It deals greatly with numbers and finding out how they work together to solve different problems" (ePST1)
4	More Dynamic	<u>Description:</u> Four NOM beliefs are mentioned, drawn, labeled; more growth minded approach; expressed belief is one that has changed over time <u>Example:</u> "If I were to describe mathematics to someone, I would describe is as sets of real life problems that involve rigorous thinking and problem solving skills…Now that I am on my way to being a teacher, I have realized more and more how useful and essential math is" (ePST5)
5	Dynamic	<u>Description:</u> More than four NOM beliefs are stated; description has a distinctly growth minded approach to math; expressed belief that belief has changed over time <u>Example:</u> "To me, mathematics is problem-solving, reasoning, questioning, making connections, and using various strategies and processes to learn more about the world around us…It is the building block of our world and one can see it all around them" (ePST6)

The criteria outlined in Table 3.5 were strictly enforced across the qualitative data. For example, there were many times that a participant shared in one writing prompt response a multifaceted NOM belief. The example for magnitude score 5 highlights this. However, there were several other examples of participants sharing about a singular NOM belief across a complete data set. For cases such as these, the criteria in Table 3.5 were used to compare each component of the data to verify that only one NOM belief was shared. Additionally, for a single data set (e.g. one mathematics autobiography, one open-ended writing prompt, and one NOM beliefs drawing), each component was considered in comparison to the criteria in Table 3.5 separately. Accordingly, a mathematics autobiography could earn a magnitude score of 1 while the corresponding writing prompt could earn a 3 and the drawing a 2. The idea here is that each piece is contributing a small portion to the whole system of NOM beliefs for that particular participant. Triangulation of the data in this way helps to illuminate these inconsistences and corroborations.

NOM beliefs magnitude sum scores. Each case was analyzed according to values codes and the static-dynamic magnitude scale in Table 3.5. Correspondingly, each case earned a set of three magnitude scores—one score for each piece of data in the case (e.g. mathematics autobiography, writing prompt or drawing). A total of 114 magnitude scores were generated. The three magnitude scores were summed per participant to generate a score representing the most accurate position on the static-dynamic continuum. The magnitude sum scores range from 3 to 15, where a static NOM belief would equal 3 and a dynamic NOM belief would equal 15. As an example, a participant could earn a score of 3 for their mathematics autobiography, a score of 2 for their writing prompt response, and a score of 2 for their NOM beliefs drawing. Adding these scores gives a sum score of 7. This score indicates that the NOM beliefs for this participant are just shy of the middle of the NOM continuum. While it seems appropriate to issue a definition of what a NOM magnitude sum score of 7 might mean the only conclusion that can be drawn as this point is that the score of 7 clearly positions this participant between the ends of the continuum. Later in chapter five the meaning of such a finding with be addressed.

Quantitative Analysis for RQ3

Due to the fact that research question three (RQ3) asked about a potential relationship between past experiences learning mathematics and nature of mathematics beliefs of elementary preservice teachers, a comparison of the quantitative data was necessary. Analysis of the scores from the NOM beliefs questionnaire, and the magnitude scores for mathematics learning experiences and NOM beliefs involved both descriptive and inferential statistical methods. Descriptive statistics were used to characterize the mathematics learning experiences and NOM beliefs of the elementary preservice teachers involved in this study. Pearson product-moment correlation analysis (Freund, Wilson, & Mohr, 2010) was used to determine if the magnitude scores for mathematics learning experiences are linearly related to either the magnitude scores for NOM beliefs or the scores from the NOM beliefs questionnaire. A level of statistical significance of *alpha* = .05 was used to test the hypothesis that the correlation coefficient is not zero (no relationship). A significant linear relationship was found, therefore, the correlation analysis explains the amount of variation in NOM beliefs due to mathematics learning experiences. For the sample size in the analysis of research question three (RQ3), three cases were thrown out due to the fact that some NOM beliefs questionnaire scores were lower than what they should have been. Those scores were lower because the ePSTs left responses off of the questionnaire. This created artificially low sum scores that needed to be removed from the analysis only for RQ3. A total of 35 cases were analyzed.

Validity and Trustworthiness

In the quantitative strand of this study, a main concern was for the quality of the conclusions drawn from the results of the quantitative analysis. Standard and robust quantitative analysis methods were employed to generate interpretations with regard to any potential relationship between experiences and NOM beliefs of the elementary preservice teachers. These tried and true methods have been long accepted by the research community as a powerful and valid approaches to making inferences based on numerical data that might represent a relationship (Creswell & Plano Clark, 2011; Lomax, 2012). In particular, internal and external validity represent concerns for this study. Internal validity (conclusions of cause and effect relationships among variables) is supported because threats such as participant attrition, selection bias and maturation of participants are accounted for in the design. External validity (conclusions that the results apply to a larger population) is also supported because the sample selected is representative of the population experiencing the phenomenon in the study.

With regard to the qualitative strand of the study, building trustworthiness is a primary issue. According to Erlandson (1993), a study exhibits trustworthiness when it guarantees "some measure of credibility about what it has inquired…[communicates] in a manner that will enable application by its intended audience and…[enables] its audience to check on its findings and the inquiry process by which the findings were obtained" (p. 28). In particular, methods to establish trustworthiness are: prolonged engagement, persistent observation, triangulation, referential adequacy materials, peer debriefing, member checks, thick description, purposive sampling, and dependability and confirmability audits (Erlandson, 1993). For this study, trustworthiness is established based on the use of the all of the following methods:

1. Triangulation - data in this study is drawn from several sources the credibility of which is checked during analysis (Patton, 2015)
2. Mixed qualitative-quantitative methods triangulation - verification of consistency of findings generated by both quantitative and qualitative data collection methods (Patton, 2015).
3. Referential adequacy materials/"slice of life" materials – mathematics autobiographies
4. Thick description – via thick data collected in various ways
5. Purposive sampling – participants chosen specifically because they experienced the phenomenon being investigated

Ethical Considerations

To protect the identity of the research participants, all data were coded. Pseudonyms were used for all of the ePSTs to protect their identity, privacy, and confidentiality. The participants were made aware of the risks and benefits of participation in advance and also signed consent forms that described in writing how confidentiality will be handled. All participants were given the opportunity to withdraw from the study at any time.

It should be noted that research participants in this study were offered an opportunity to enter a drawing for one of three $10 Amazon gift cards after they complete the NOM beliefs questionnaire, writing prompts, and drawings and submit their mathematics autobiographies to their instructor. The use of the payment scheme is to ensure a large sample of participants and increase the power and generalizability of the quantitative strand.

Chapter Summary

A summary of the research questions that were studied and the related measurement instruments and data analysis is presented below:

1. What are elementary preservice teachers' past experiences with mathematics? This was measured by the two writing prompts (describe your experiences learning mathematics before/during college), mathematics autobiographies, and mathematics experience drawings. This question was analyzed using eclectic coding methods.
2. What are elementary preservice teachers' beliefs about the nature of mathematics? This was measured by the writing prompt (What is mathematics?), NOM beliefs metaphors and drawings, and the mathematics autobiographies. This question was analyzed using eclectic coding methods.
3. In what ways, if any, are the mathematics experiences of elementary preservice teachers related to their beliefs about the nature of mathematics? This was measured by the NOM belief questionnaire scores, mathematics experience magnitude scores, and NOM beliefs magnitude scores. This question was analyzed by both descriptive and inferential statistical methods.

The results of the data analysis will be presented in Chapter IV and a discussion of those results will follow in Chapter V.

CHAPTER IV

RESULTS

This convergent parallel mixed methods study set out to examine the data related to 38 elementary preservice teachers' experiences learning math, their nature of mathematics (NOM) beliefs and the potential relationship between the two phenomena. The research questions guiding this study were:

1. *What are elementary preservice teachers' past experiences with mathematics?*
2. *What are elementary preservice teachers' nature of mathematics beliefs?*
3. *In what ways, if any, are the mathematics experiences of elementary preservice teachers related to their beliefs about the nature of mathematics?*

This chapter will present the findings by research question. For the first research question, themes that emerged from analytical triangulation of the mathematics autobiographies, open-ended writing prompts, and mathematics experience drawings will be discussed. Similarly, the second section will present the results of analytical triangulation of the mathematics autobiographies, open-ended writing prompts, and NOM beliefs drawings for the second research question. Finally, the results of emotion and magnitude coding analysis, and the means and standard deviations as well as the results of a correlation analysis to answer the third research question will be presented. All results will be summarized to conclude the chapter.

RQ1: Elementary Preservice Teachers' Past Experiences with Mathematics

This section presents the results from the data analysis completed to answer the first research question guiding this study and gain a deeper understanding of the past mathematics experiences of the elementary preservice teacher participants (ePSTs). The emergent themes that describe the mathematics learning experiences for the participants are presented.

Intertwining of Themes Within Each Experience

The mathematics learning experiences of the preservice teachers were quite complex with emergent themes intertwining within each case (e.g. mathematics autobiography, open-ended writing prompt, or drawing). For some cases, the ePSTs describe experiences that illustrate multiple themes, while in other cases participants might describe categories that make up a single theme. Throughout the next section, each theme that emerged from the data for mathematics learning experiences will be outlined in detail. However, the reader should note that only the subject theme or category in the current section will be discussed. For instance, in some cases the drawings show a split scene and two different experiences. Each side of the scene illustrates different themes but the drawing may be used to showcase one theme in particular. Throughout, the cases typically illustrate a comparison between two or more time frames in the mathematics learning experiences of the participants. Each of these time frames represents a singular experience, and each single experience contributes to the depth of understanding of the entirety of the experience.

Mathematics Learning Experience Themes

The mathematics learning experiences of the preservice teachers fell into five themes including: (1) attitudes toward learning mathematics; (2) perceptions of instruction; (3) perceptions of mathematical understanding; (4) perceptions of success and (5) focus on speed and memorization. Each theme emerged from the data during the triangulation analysis of the mathematics autobiographies, open-ended writing prompts, and mathematics experience drawings. While each theme emerged in a unique way from the data analysis, several themes were intertwined within the data. Table 4.1 below reveals the number of participants who described each theme as well as the total number of instances for each theme. For example, within one mathematics autobiography, a participant might describe their attitude while learning math in more than one way. Each description would count as one instance of that theme.

Table 4.1 offers an overall picture of how participants responded across the three data sources. Examination of this data indicates that more than two-thirds of the preservice teachers tended to reflect upon mathematics experiences that were tied to their attitudes toward learning mathematics or how they perceived the mathematics instruction that occurred. Additionally, depending on the data sources between one-third and two-thirds of the preservice elementary teachers mentioned experiences that related to their mathematical understanding or their success (or lack thereof) in mathematics. About half of the ePSTs, shared instances where speed and memorization stood out in their experiences learning mathematics.

Table 4.1 *Mathematics Learning Experiences: Emergent Themes in ePST Autobiographies, Open-ended Writing Prompts and Experience Drawings (n=38)*

Theme	Description	Autobiographies		Writing Prompt		Drawings	
		n (%)	Total # of Instances	n (%)	Total # of Instances	n (%)	Total # of Instances
Attitudes toward Learning Mathematics	Confidence; Anxiety; Enjoyment; Perseverance; Frustration	35 (92)	142	26 (68)	60	29 (76)	85
Perceptions of Instruction	Teaching style/learning style; Commitment to teaching; Connection with teacher; Types of math work	31 (82)	103	33 (87)	62	26 (68)	43
Perceptions of Mathematical Understanding	Understanding comes easily with/without teacher influence; Understanding as innate ability; Understanding for specific content; Deeper understanding;	29 (76)	49	18 (47)	25	15 (39)	25
Perceptions of Success	Good grades with little struggle; Good/bad grades with hard work; Success journey; Success on quizzes & exams	21 (55)	36	28 (74)	60	13 (34)	22
Focus on Speed & Memorization	Timed tests or worksheets; Repetition drills/flash cards; Slow down/need more time; Speed is smart; Repeated practice; Difficulty remembering	19 (50)	39	8 (21)	8	8 (21)	9

Attitudes toward learning mathematics. The theme of attitudes toward learning mathematics arose from the large percentage of participants who described their mathematics learning experiences in terms of the emotions they felt. According to Saldana (2013), "[a]n attitude is the way we think and feel about ourselves, another person, thing, or idea" (p. 111). For the purpose of this study, an attitude toward learning mathematics encompasses how the elementary preservice teachers think and feel about learning mathematics. While emotions are inseparable from everyday lived experiences (Saldana, 2013), there are many cases where the participants described their experiences exclusively in terms of their attitude toward learning mathematics. Table 4.1 shows that 92% of ePSTs described their attitudes toward learning mathematics in their autobiographies, 68% address attitude in their writing prompts, and 76% describe their attitudes in the experience drawings. Throughout the data sets, ePSTs described various aspects of their attitude toward mathematics but the main elements were their anxiety, their frustration caused from confusion, their level of enjoyment, their confidence level with doing math, and their level of perseverance in learning mathematics. Each of these components of attitude are discussed below.

Anxiety. Many of the participants shared about their feelings of anxiety when they were learning mathematics in school. For the most part, the type of mathematics work that was assigned during the ePSTs experiences was the reason behind the anxiousness. One participant mentions how "working on [her]…math problems or with more advanced math" made her feel "scared and insecure" (ePST5). Others more specifically targeted the timed tests, or the word problems they were assigned. In her descriptions of her mathematics experiences, ePST6 mentioned both of these elements, saying,

> For myself and others like me, these timed tests caused an overwhelming amount of anxiety and the inability to perform to the best of our abilities…I did not, and still don't, do well with any type of word problem. Anytime I am faced with math, I begin to feel a sense of anxiety because I fear I will not be able to understand the problem or what I am supposed to be doing. (ePST6)

This same sentiment is echoed in Figure 4.1, where there is a clear division of the drawing space into two areas. The image on the left shows the face of an individual that is not happy and the thought bubble is surrounded by question marks as if to indicate a state of confusion or questioning—both of which could lead to anxiety. Contrast the left side of Figure 4.1 with the right side image where the participant is not alone, and everyone shows smiles while working with math shapes—the opposite of an anxious scene.

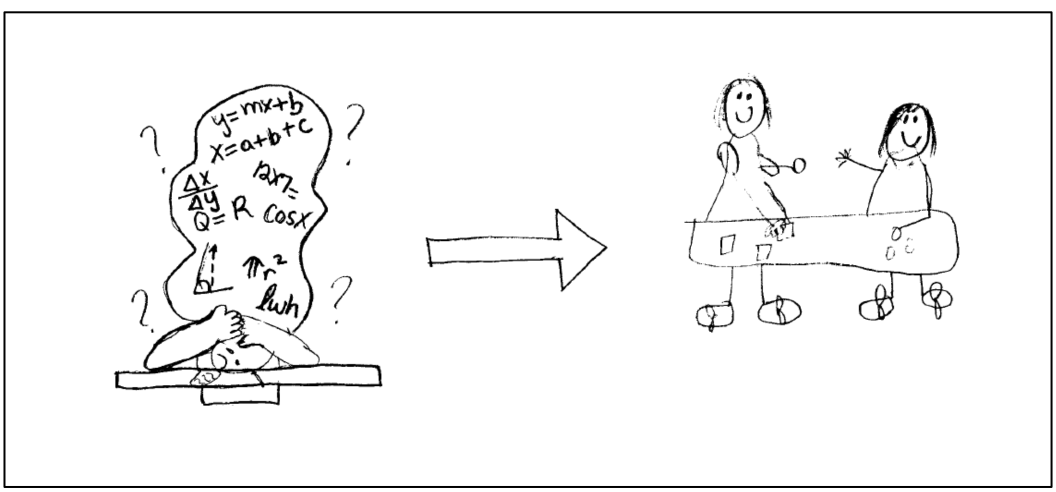


Figure 4.1. Mathematics experience drawing for ePST6

Several participants mentioned feeling scared (ePST12) or having emotions of fear (ePST11) during their mathematics learning experiences. According to the data, these fearful emotions originate from the realization made by the ePSTs that they did not understand or know how to do the mathematics they were learning. In fact, ePST8 said, "...math became increasingly difficult for me. One of the worst feelings that I remember from this time was receiving a packet of math homework and knowing that I had no idea how to complete it." In the same way another participant shared this feeling of 'looking stupid' and how it created feelings of anxiety in her, saying, "[m]ath made me feel anxious and I was always worried I would end up looking 'dumb' if I wasn't able to provide the correct answer on the board or when called on in class" (ePST35). Similarly, ePST21 said that if she had to "go up to the board and solve a problem [her] heart feels like it's about to beat out of [her] chest". She goes on to say that she would "get so nervous" that she would make a mistake and her "peers [would] secretly judge" her.

Frustration. In addition to feelings of anxiety, many participants mentioned emotions associated with frustration or the state of being frustrated within their mathematics learning experiences. For most cases, the frustration seemed to stem from a state of confusion where the ePST might be trying to understand a procedure or concept but is not comprehending the information. For example, ePST8 said, "[w]hen I cannot find a solution after multiple attempts, I tend to get frustrated." And another participant said,

> I recall many long nights up at the kitchen table with my parents trying to help me understand what we were learning in class, ending in tears — even though my dad

was a CPA and excelled in mathematics, and I was frustrated with myself. (ePST11)

Still other participants mention “hours and hours of confusion” (ePST15) and a desire for the answer to a math problem to “just…be there” (ePST5) as if the answer could be read off of the page. Some ePSTs wrote that they were “extremely frustrated” (ePST28) and used their experience drawings to show what that frustration might look like. In Figure 4.2, ePST28 drew an image of a female individual, holding her head and looking as if she was worried in her thoughts. The thought bubble shows a variety of elements that the girl is thinking about. We see words (e.g. “CONFUSED!”, “SO HARD”, and “SO LOST”) to indicate this individual’s state of mind and emotions. The word “FRUSTRATED” can be seen as well as questions like “What does that even mean?” and “Why don’t I ever understand any of this?”—evidence for the emotive state of frustration.

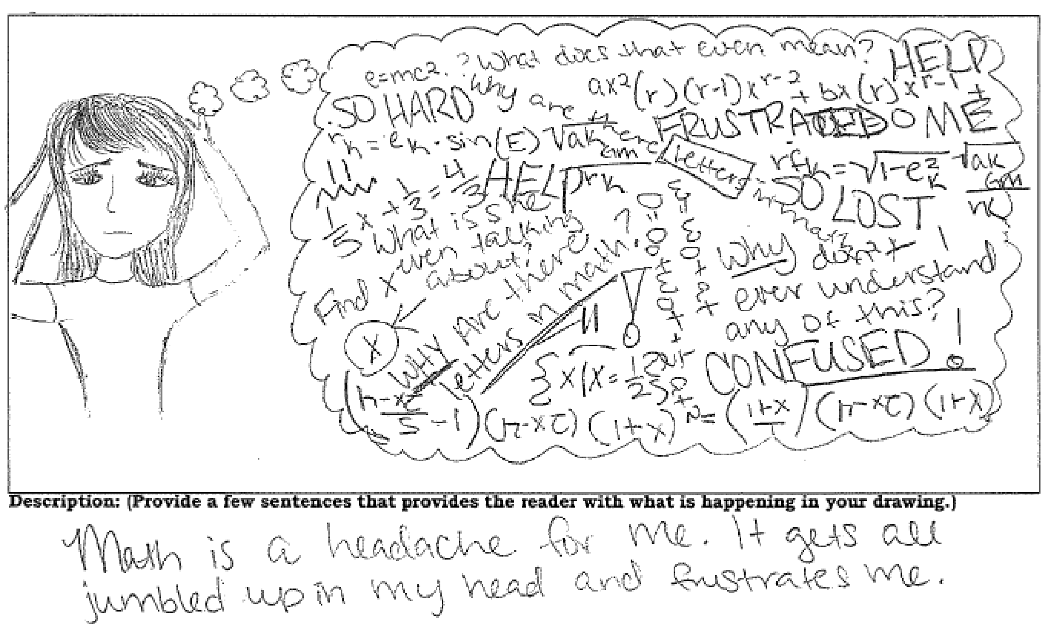


Figure 4.2. Mathematics experience drawing for ePST28

Enjoyment. In other cases, the elementary preservice teachers shared about their level of enjoyment during their mathematics learning experiences. For the most part, when they shared about their enjoyment of mathematics, the participants spoke about how they liked math, developed a love for math or thought math was fun. For example, ePST2 said, “I like the challenge of dissecting the problem to find out exactly what it is that I need to do in order to solve. It is great fun and when I accomplish this, I feel like I can do anything”. In a similar way, ePST13 spoke about an ever-present love for math, saying,

> Throughout my academic career, I have always had a love for math. I love the fact that there are so many different ways that one can discover an answer to a problem, but I also like how there is just one answer. (ePST13)

While these particular responses give an indication of what specifically the participants enjoyed about learning mathematics, other responses simply stated “I have always enjoyed math” (ePST33). Likewise, for ePST18 the only element in her drawing (not shown) is a smiling face, which can be associated with a positive and pleasant experience, while the writing that describes the drawing indicates that for ePST18 learning mathematics was “always…really enjoyable”.

Several responses outlined a comparison between mathematics learning experiences before college and those during college. One case in particular, ePST8, includes many details about the enjoyment level of her mathematics experiences in both pre-college and college, saying,

> I never really struggled with math in elementary school but it became more difficult in junior high and high school. I continued to succeed in math classes but I began to dislike the subject. Most of the work was boring to me and I didn't enjoy struggling with problems...I didn't enjoy my non-elementary math classes math functions and college algebra I thought they were boring and tedious however I LOVED all of my elementary math classes geometric structures math structures primary math these classes helped me look at math differently as a creative and exploratory subject at this point math is one of my favorite subjects. (ePST8)

The mathematics experience for ePST8 included varied levels of enjoyment matched with levels of struggle for her. The mathematics content early on was not difficult, so her enjoyment while learning math was higher than in her mid-high school years when content became more challenging. There seems to be a difference in enjoyment for ePST8 between courses taken in college within her major and general education course work. Indeed, ePST8 enjoys math due to her experiences in her major course work that allowed her to "look at math differently as a creative and exploratory subject".

A few ePSTs mentioned being successful in math in elementary school but growing to dislike the subject in subsequent years, however no other details are shared by the participants to indicate why the dislike for math came about (ex. ePST 11 and 2). On the other hand, some participants shared about how they "had awful experiences" in their pre-college days where teachers made them cry and induced in them a "negative attitude about math" to the point they thought they would never enjoy math again (ePST15).

Confidence. Similar to the element of enjoyment in the mathematics experiences of the participants, many ePSTs shared about their level of confidence with respect to learning mathematics content, doing mathematics work, or solving math problems. Several participants simply stated they were or were not confident in their mathematics abilities. For example, ePST5 said "I am not very confident in the subject" while ePST13 said, "I have strong confidence in most of my math skills". Low levels of confidence were more common, however, some ePSTs mentioned building confidence from certain types of mathematical activities and work. For instance, ePST15 said that getting "an A in trig…boosted" her confidence. In a similar way, ePST17 said, "[f]inding patterns, solving equations, and discovering the correct answer is appealing to me and I feel confident in myself when I accomplish these things". The clearest example of experiences that built confidence is from the response of ePST2,

> As I said earlier I am not extremely confident about my mathematical abilities, but the more classes I take, the more I work with and try to understand mathematic[s] the more confident I get. I have to repeatedly work with a math concept to fully understand it. (ePST21)

The idea that confidence levels can rise and fall with the type of mathematics content that the ePSTs engaged in is evident in the drawing in Figure 4.3. The image shows columns of "confidence levels" that correspond to specific time periods within ePST12's mathematics learning experience. No wording is below the drawing area, therefore, no further details can be gleaned as to what might be happening in the drawing. However, the drawing does seem to show how confidence in learning mathematics can fluctuate with time.

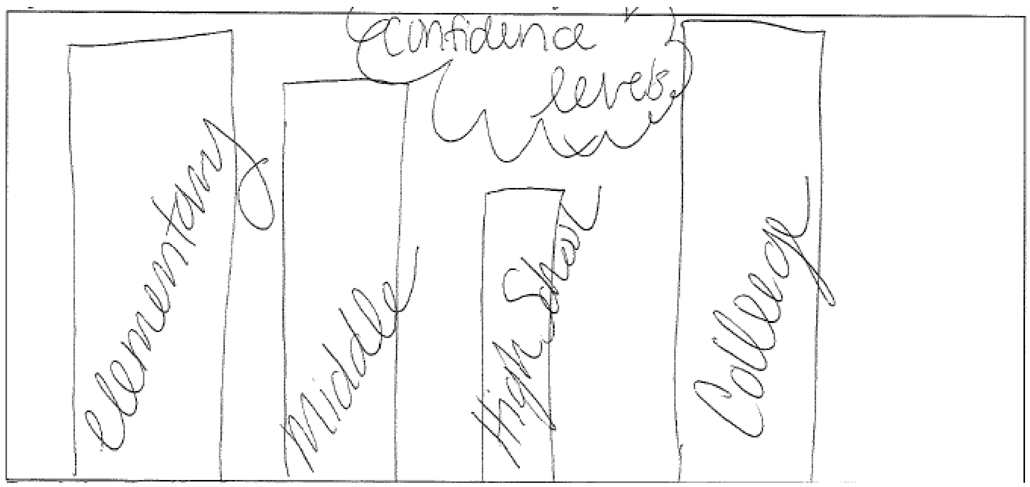


Figure 4.3. Mathematics experience drawing for ePST12

Perseverance. The last element of attitude toward learning mathematics is perseverance. More specifically, participants spoke of either having or showing perseverance during math work or giving up easily when a challenge was presented. For example, ePST21 said, "[as] a child when I would have to do math if I didn't understand it immediately I pretty much gave up". Another participant lamented about trying "to persevere when solving math problems" but admitted that she is "easily discouraged and feel[s] the urge to give up from time to time" (ePST6). A major point made by the ePSTs was that perseverance was the activity of pressing forward through the challenging task in front of them. In most cases, the participants talked about being frustrated or challenged in the mathematics work they were doing but also shared about how they knew they needed to push through those feelings to reach a solution. For example, ePST22 said,

> As a learner, I know that I sometimes get frustrated when I do not understand something and it makes me feel slightly discouraged, but when I do finally

> understand the concept I am working on it feels very rewarding and this is what encourages me to go on. (ePST22)

Another ePST mentions how she gives her brain a break and then she "revisit[s] the problem with fresh eyes and new energy…[which] helps me to persevere through difficult problems" (ePST8). The drawing data did not contain images of perseverance as was described by the participants in their written responses.

Perceptions of Instruction. Subsequent to the theme of attitude while learning mathematics, many participants described mathematics experiences that focused on their perception of the ways they were taught mathematics. With respect to the participants' mathematics autobiographies, 82% described their experiences in terms of their perceptions of instruction. Eighty-seven percent of the open-ended writing prompt responses detailed perceptions of instruction while 68% of the experience drawings described perceptions of instruction. Several components of the theme of perceptions of instruction are detailed in the following section including teaching style and learning style, teacher commitment to teaching, connection with the teacher, and types of math work.

Teaching style/Learning style. Participants frequently compared the teaching style they experienced with their learning style. For example, one participant mentioned that they needed to have content explicitly shown to them before they felt they could complete a problem or assignment, saying, "I have to see the teacher physically work out example problems on the board" (ePST1). Similarly, others described various moments where they perceived differences between the way they learned math and the methods of instruction used by their teachers. Another participant outlined the reason for their struggles with math was due to instructional techniques not matching the way they learn indicating that she "struggled so much because I needed a visual approach to the problems and he was not delivering" (ePST3). The 'he' refers to the teacher and while the term 'visual approach' is not specific, the perception of a mismatch in learning style and teaching style is still evident. Several ePSTs mentioned being flexible thinkers who develop multiple strategies to solve a problem over time which is contrary to the traditional worksheet practice and drills for math facts. For example, one participant shared how they learn through problem solving, but the instruction received in math class was focused on practice of a single method for solving problems. She said:

> The word I would use to describe myself as a learner of mathematics is adaptable, working to make sure I do not give up on a problem until I have exhausted all my resources and thinking. Personally, I like to work problems in different ways to ensure I understand how a problem can be solved…In relation to this, I dislike that many of my teachers taught me that there was only one correct way to do a problem. This was discouraging for me, and many times led for me to do problems "incorrectly". (ePST15)

Another participant discussed the mismatch between learning style and the instructional style of a few of the mathematics teachers encountered:

> My Algebra 1 teacher and my geometry teacher were not very helpful to me and I struggled in those classes because I was not able to get much help from them. Their teaching style, lecture style, just mainly did not work for me…However, my Algebra 2 and pre-calculus teachers were great. They were very hands on and interactive and I loved taking their classes. (ePST22)

Commitment to teaching. In addition to learning style and teaching style mismatches, ePSTs also shared specifics regarding their perceptions of instruction with respect to how much their teacher was committed to the job of teaching. Statements like "my teacher did not want to take the extra time to help me in mathematics" and "[my teacher] was a coach before he was a teacher, that meaning coaching was his priority" (ePST31) suggest the perception of poor instruction due to commitments outside of mathematics teaching. In line with this, several ePSTs recalled that they did not believe their mathematics teacher cared or were concerned for their learning, making statements like "[a]s I got older it seemed like the less my teachers cared about math so likewise I cared less about math" (ePST19), and:

> The teacher I had for this course made me feel like I was just a burden to her and she would always sigh if I had a question to ask her. That was a really bad experience for me to go through especially when I was already struggling so much, I just felt like she didn't care. (ePST23)

Figure 4.4 below is an example of what one participant drew to describe elements of teacher commitment (e.g. a teacher modeling and a teacher not engaged). The image on the left shows happy faces and indicates the teacher is "modeling" and "making sure the student is successful" (ePST5). This description suggests the teacher is committed to an instructional approach that includes directly explaining or modeling to the student the math content on board. On the right side, the image shows a very different experience. A student with a sad face is drawn with their hand in the air and the teacher is unengaged and unwilling to help the student resolve her confusion and stress.

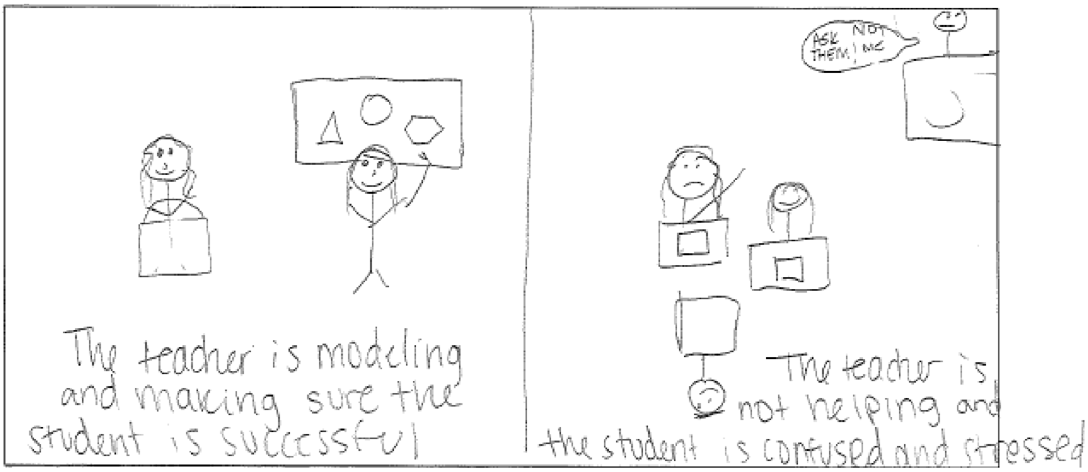


Figure 4.4. Mathematics experience drawing for ePST5.

Connection with teacher. Participants also shared about their best teachers indicating that they felt connected to their math teacher and received effective mathematics instruction. In some cases, ePSTs said they had "*great*", "*awesome*", or "*wonderful*" mathematics teachers but no other details were shared to indicate exactly what characteristics, instructional style, or procedures the teachers employed to make them so effective. In other cases, the teacher's passion for mathematics content or for teaching was designated as one of the elements of a more positive and effective learning experience. For example, ePST13 says:

> I had wonderful math teachers all through middle school and high school. My eighth grade math teacher is the one that always comes to mind. He had such a passion for math and I think that one positive experience helped make every other math class I took a positive experience as well. (ePST13)

Additionally, relatability between the participant and their teacher was cited as a main factor that attributed to the participants' positive experiences learning mathematics.

In the following statement by ePST23 the connection with the teacher is described in terms of relational characteristics:

> I did have this really great geometry teacher my freshman year of high school. Her class was the only class in my high school math career that I made a B in math. She was one of the most caring, thoughtful and helpful teachers I had ever had. (ePST23)

Similarly, ePST2 describes an experience where she perceived both a positive connection with an instructor in college but not before feeling as though she was degraded by past teachers. In the drawing in Figure 4.5, ePST2 represents her experiences with a clear designation between elementary and post-secondary time frames. The left side of the drawing shows a 3rd grade teacher presumably announcing that special instructional accommodations were "made" specifically for ePST2 because she was "still [not] understanding". The type of math work shown on the board is multiplication facts and mathematical calculations. On the right side of the figure we see a professor drawn with a happy face and the words written under the board indicate the professor used instructional techniques that presumably build understanding of "the reasoning behind" the math. In the space below the drawing, ePST2 indicates that her 3rd grade teacher "belittled" her while her professor was "amazing" and "healed the hurt [she] had for math". Each of the scenarios in Figure 4.5 showcases different connections made with math teachers—each contributing in a unique way to the mathematics learning experiences for ePST2.

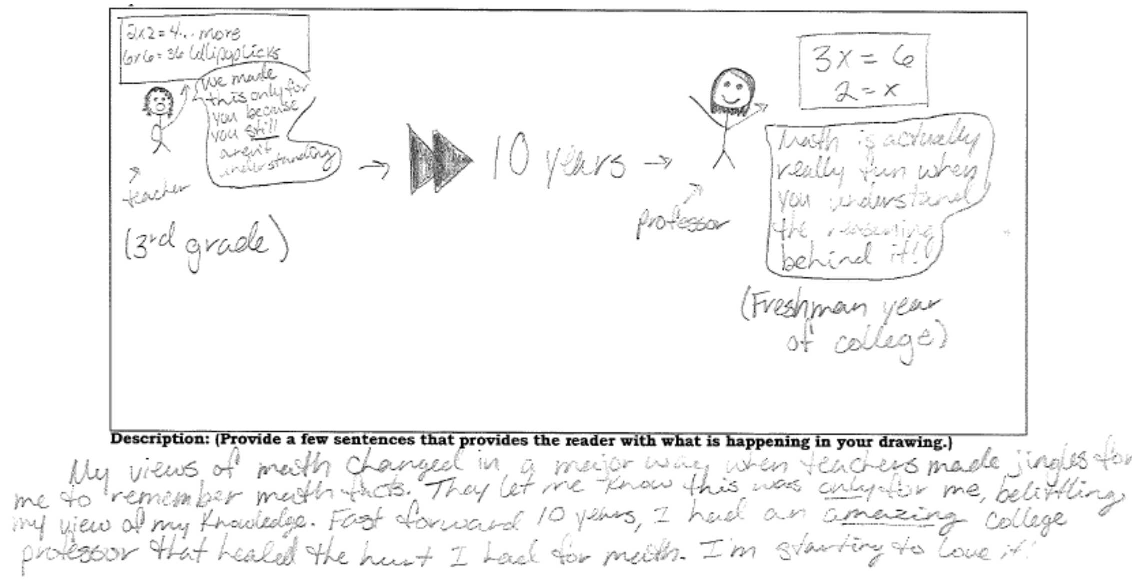


Figure 4.5. Mathematics experience drawing for ePST2.

Types of math work. Assigning mathematics work is a primary instructional task of all math teachers, therefore, the types of math work mentioned by the ePSTs in this study were categorized under perceptions of instruction. Specifically, ePSTs mentioned engaging in particular types of mathematics work or practice during their mathematics learning experiences. Many ePSTs mentioned that their mathematics work consisted of worksheets, timed tests or math fact drills, repeated practice and exams. For example, one participant said, “I went to elementary school doing the usual timed multiplication papers (which I liked because I always tried to be first) and the typical worksheets” (ePST32). In some cases, ePSTs mentioned hands-on inquiry-based mathematics work as the norm. One participant detailed how her kindergarten teacher “did an excellent job” incorporating mathematics into “daily conversation” and used “art projects” to help “deepen [her] knowledge of numbers” (ePST4). Many of the ePSTs indicated that their experiences included quite a bit of worksheet practice and not much hands-on learning.

The drawing represented in Figure 4.6 showcases types of mathematics work present in ePST14's mathematics experiences. The drawing indicates two different time periods of experience. On the left side, a student is drawn sleeping on the desk because "[m]ath was boring and routine before college". The math work on the left side of the drawing is labeled with the word 'MATH' but it is unclear what the mathematics content would be. The right side features several people around a table looking happy and using manipulatives (shapes, etc.) and indicates that college math was "interactive" and "accessible to more people".

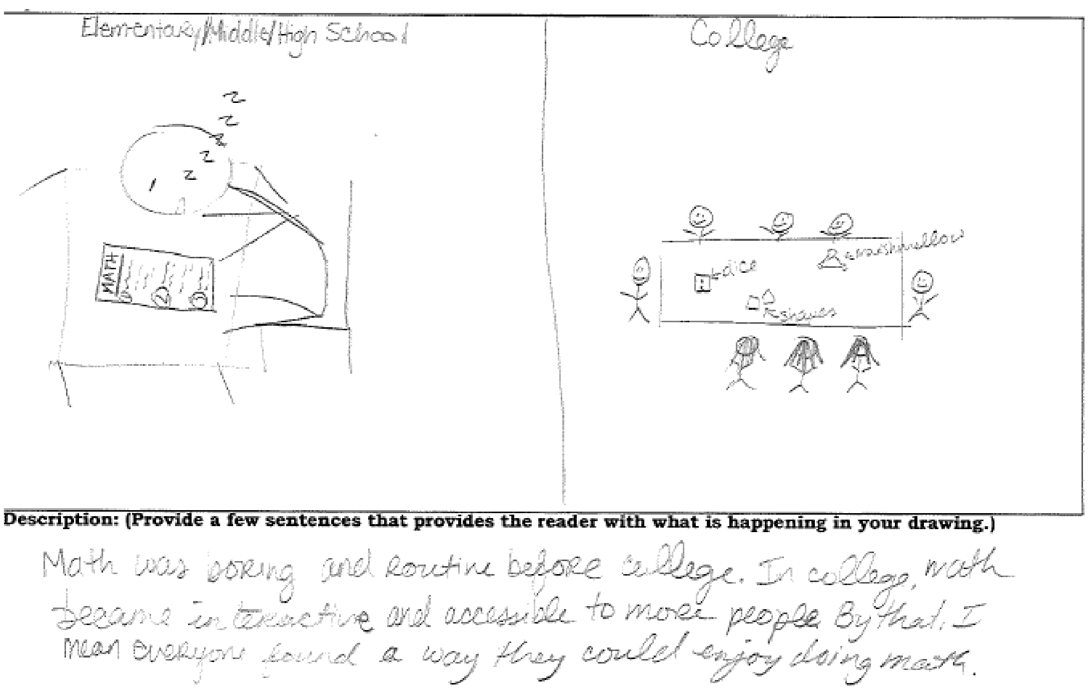


Figure 4.6. Mathematics experience drawing for ePST14

Types of math work seemed to be present in all of the data as it is a main component of instruction. Therefore, the thematic category as part of the theme of perceptions of instruction intertwined heavily with other categories for this theme and for other themes. As an example in the Figure 4.6 above, the wording below the drawing mentions that "everyone found a way they could enjoy doing math" which overlaps with the attitudes in learning mathematics theme previously described.

Perceptions of mathematical understanding. The theme of perceptions of mathematical understanding arose from the emphasis that study participants placed on understanding or not understanding the mathematics they were learning during their educational experiences. Several participants described the degree to which understanding the mathematics they were learning either came easy or did not. Participants described how they understood some types of mathematics content like, geometry, but were unable to understand other types of content, like algebra. Some indicate that they achieved understanding but only with extra help from teachers or parents. Seventy-six percent of the mathematics autobiographies described participant experiences in terms of their perceptions of mathematical understanding. Additionally, 47% of the writing prompt responses and 39% of the experience drawings indicated that descriptions of experiences were based upon perceptions of mathematical understanding. Each of the components of the theme of perceptions of mathematical understanding are presented in the section that follows.

Understanding achieved. Elementary preservice teachers described their perceptions of mathematical understanding in many ways, and this section describes the component of understanding achieved in terms of three types of experiences. The common thread of mathematical understanding gained is evident but the experiences in which understanding was achieved is unique in each case.

Understanding came easy without teacher influence. For many elementary preservice teachers, understanding the mathematics content they were learning came easily for them, but they do not indicate that their understanding was achieved with influence by their teacher. Statements such as, "[m]oving on to middle and high school, math continued to come easy to me" (ePST27) and "math has always come pretty easily to me" (ePST30) show a perception that mathematical understanding was achieved via the efforts of only the participant and no other individual. In responses such as these, the participant describes how understanding mathematics came easily without influence from the teacher. For example, ePST25 said, "[n]o matter the grade or area of math, it always clicked very easily and I was fast at completing my work" (ePST25). Here, ePST25 credits herself for completing her work quickly and gaining mathematical understanding at any grade-level. Likewise, one description is particularly descriptive of how the moment of understanding mathematics manifests, and how mathematics can be like a puzzle to solve but makes no mention of teacher influence—ePST30 says,

> One of the most influential experiences in mathematics, for me, was in my high school Trigonometry class. It was the first class to really challenge me, and make me push myself. One day, I was trying to decipher the Unit Circle, and I was getting beyond frustrated. I kept trying to memorize the different values, but none of it made sense. It was a puzzle with pieces that didn't fit together. I stared and stared at it**.** Finally something clicked -- I saw the pattern. There was reason behind the madness! Once I understood the Unit Circle, everything else fell into place, and I realized that Trigonometry was beautiful. I loved its curves, how they would rise and fall. To this day, Trigonometry was one of my favorite subjects. (ePST30)

The drawing in Figure 4.7 corroborates the assessment by ePST30. The picture shows the face of an individual who understands mathematics as connections or puzzle pieces fitting together and 'lightbulb' moments of clarity. The drawing does not show a teacher figure anywhere, and the comments below the drawing do not mention any teacher influence contributing to the "moment when everything clicks".

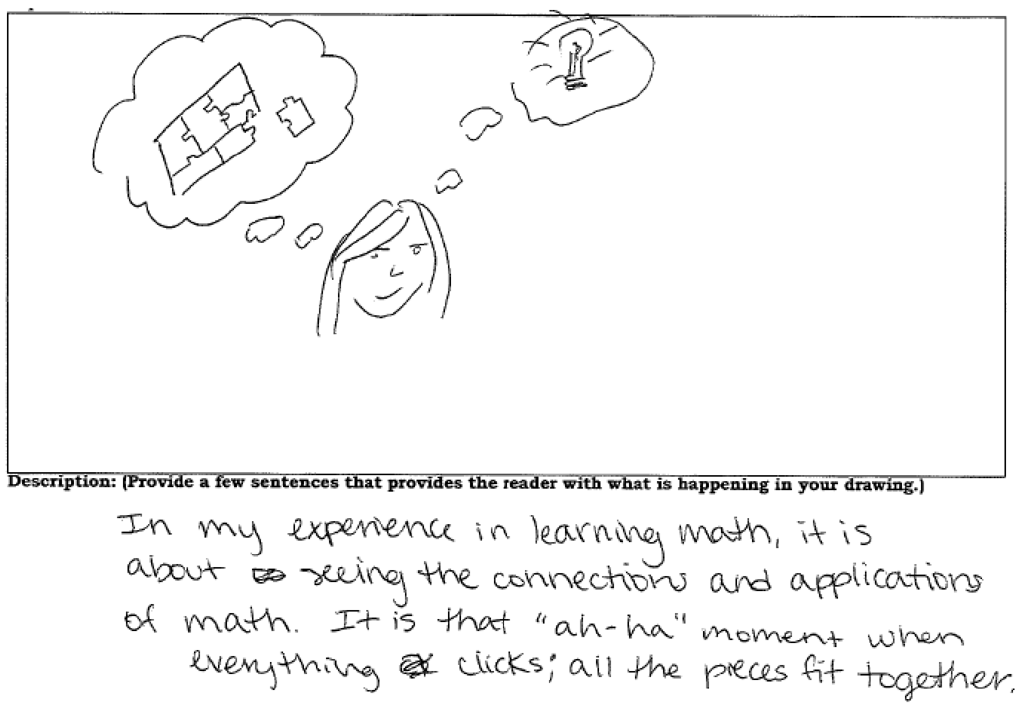


Figure 4.7. Mathematics experience drawing for ePST30

For each of the participants highlighted above, understanding of the mathematics they were learning came easily for a variety of reasons. In one case it was because they completed their work quickly, and another it was the visualization of the patterns in mathematics that helped them understand. However, all of the examples indicate that mathematical understanding was achieved without teacher influence.

Understanding came easy with teacher influence. In contrast to those participants to claim mathematical understanding without teacher influence, several other participants claim understanding came because of the influence of their teachers or instructors. In particular, ePST3 describes her experiences in geometry, stating

> My most positive math experience was in tenth grade. I was in geometry class...the way she presented material was so easy to understand and it clicked with me. I think the way she presented geometry was very effective because I could see, visualize, and apply it to the real world. (ePST3)

Here ePST3 credits teacher influence in gaining mathematical understanding. In a similar way, several participants reported achieving mathematical understanding with the influence provided by their college instructors. For one participant mathematical understanding came easy because "instructors in college were more helpful and open to helping [her] understand the material that [they] were going over in class" (ePST21). Another participant recalls how the instructor in college algebra "showed [her] that one must think out of the box in order to understand the fun challenge of completing a problem" (ePST2). Still another participant spoke of a professor in college who "truly helped [her] understand the purpose" (ePST16) of the mathematics work she was doing.

Understanding achieved with help outside the teacher. For some participants, their mathematical understanding was achieved only when they sought help outside of the main teacher in their math class. A common thread in experiences in this category is the struggle to reach understanding even after the teacher has delivered the lesson. Participants describe experiences seeking outside help from parents, tutors and other teachers. In her mathematics experience, ePST3 describes how she needed extra help to understand how to solve a mathematics problem, saying "I had to ask for help from many different teachers…before I started to get the hang of the steps I needed to take to…solve a problem". For ePST3, mathematical understanding came via the influence of multiple teachers outside of her main teacher delivering the lesson. Similarly, ePST2 reports needing help to reach deeper understanding, saying, "[m]y parents enrolled me in tutoring at [a] Learning Center with an excellent tutor who used real-life examples of mathematics to help me more deeply understand what I was doing". In this experience, ePST2 describes the help she received from a professional tutoring service to reach a deeper understanding of the mathematics she was learning. In fact, ePST2 goes on to credit her parents with providing assistance in her developing mathematical understanding. She says, "[m]y parents did everything in their power to help me understand math…[t]hey bought flash-cards, [and] special boards that helped with my retention of math facts" (ePST2). And still ePST11 recalls the help she received from her parents stating there were "many long nights up at the kitchen table with my parents trying to help me understand what we were learning in class, ending in tears" (ePST11).

Understanding as innate ability. While several of the preservice teacher participants described how mathematics understanding came easily or 'clicked' for them, many more ePSTs describe very different experiences in terms of understanding mathematics. Many ePSTs found mathematical understanding to be virtually unachievable because they indicated that understanding mathematics was not an innate ability for them. For example, ePST8 says, "I certainly didn't consider myself a 'math person'…I assumed that my brain was lacking in that area because, unlike most other subjects, math understanding did not come naturally to me" (ePST8). In line with this assessment, other participants mention how their perceived inability to reach a level of mathematical understanding indicated for them that they were not "a math person" (ePST35) or a “numbers person” (ePST21) and that mathematics is “not natural” (ePST20; ePST29) for them. These ideas merged in the case of ePST10, who said that since mathematics understanding “did not come easy” to her, that meant that she was “born without the ability to understand certain aspects of math”.

Understanding of specific content. In many cases, the participants described mathematics experiences that included understanding some mathematics content but not others. More specifically, the experiences described in this section fit into two distinct sub-categories—fundamental mathematics content and geometric or algebraic reasoning.

Fundamental mathematics content. For some participants in this study, experiences learning mathematics were described in terms of an understanding (or lack of) fundamental mathematics content. Many ePSTs commented about understanding basic math operations like addition, subtraction, multiplication and division; however, some participants admitted not understanding why certain operations were used. For example, one participant lamented over her lack of understanding with respect to multiplication, saying, “I did not connect with multiplication for I did not understand the purpose or exactly what I was doing” (ePST2). The mathematics experience drawing shown in Figure 4.8 below places a focus on mathematical facts, procedures and calculation where memorization was frequently encouraged. The participant here indicated that memorization was a common action but she “never really understood why” (ePST3).

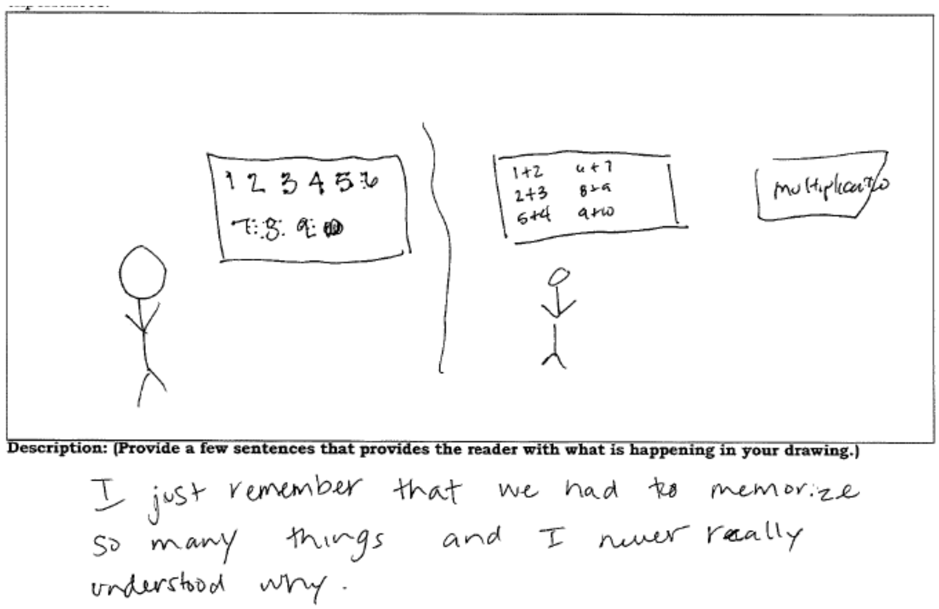


Figure 4.8. Mathematics experience drawing for ePST3

Geometric or algebraic reasoning. While some of the elementary preservice teachers described their understanding of fundamental mathematics content, other participants focused their descriptions on a lack of understanding for geometric or algebraic reasoning and facts. For example, ePST17 described doing well in her mathematics learning until her 10^{th} grade year in school when her “struggles began in geometry” ePST17). In a similar way, ePST23 said that she “truly didn’t understand” the algebra course she chose to take in college. In another case, the lack of understanding of algebraic reasoning is showcased when ePST28 says, “I don’t understand why there are letters in math, I don’t get why it matters and that $a^2+b^2=c^2$. I don’t know what it means that $E=mc^2$” (ePST28). Each of the cases described previously indicate that at least in some ways, understanding of certain types of mathematics content (e.g. math fundamentals, or geometry/algebra) proved to be elusive for some participants.

Deeper understanding of mathematics. Contrary to situations that resolve to a lack of mathematical understanding, several participants reported experiencing deeper understanding of the mathematics they were learning. When reflecting about her college experience, ePST30 reported, “I have thoroughly enjoyed my college level math classes they go deeper into concepts and understanding of math” (ePST30). Another ePST reported a better understanding of the many pathways to a mathematical solution and the creative side of mathematics saying, “[n]ow I know that approaching problems with a lens of creativity and analyzing the many ways to view a problem and find the solution enables me to understand mathematics at a level that sticks with me” (ePST10).

While some participants reported attaining a deeper understanding, others described realizations that they did not reach a deeper level. In particular, ePST14 describes how singular and rigid methods taught to her for solving mathematics problems actually prevented her from achieving a deeper understanding of mathematics. She stated,

> Throughout school, I always learned that there were specific ways for doing math and that I should not stray from those ways… This, I think, helped me to find structure in math when I sometimes felt lost. However, it kept me from having a deeper understanding of math. (ePST14)

Some participants compare their mathematics learning to other subjects and lament that they “could not connect with it as deeply as [they] did other subjects” (ePST2).

Perceptions of success. The theme of perceptions of success arose from the multitude of participants describing their mathematics experiences in terms of success by high grades or failure by low grades. Many participants mention how they made good grades in math class with little effort, while other participants describe more difficult experiences in terms of the effort required to get good grades. Several describe their success in terms of a journey through time where they experienced failure first then success. For some participants, success in mathematics was determined by the scores they earned on quizzes and exams, including higher stakes exams like the sort of which determine entrance to college-level coursework or end of instruction exams. In total, 55% of the mathematics autobiographies fit into the theme of perceptions of success. Likewise, 74% of writing prompt response and 34% of experience drawings showcased this theme.

Good grades with little effort. For some of the elementary preservice teachers, their mathematics experiences were characterized by getting good grades in their mathematics classes with little to no struggle or effort. Statements like "I have always excelled in math" (ePST17), and "I never really struggled with math" (ePST8) indicate the absence of struggle and an element of success inside the mathematics experiences of these participants. Other ePSTs specifically mention that they remember receiving high marks on mathematics work throughout their mathematics learning experience. For example, ePST1 says:

> In seventh grade I had an outstanding pre-algebra teacher who made math a lot of fun for me, and I actually got a lot better at it… By the time I was in high school, I was wanting to take trigonometry and pre-calculus just for the fun of it. I passed both of those classes with A's. (ePST1)

While ePST1 credits her "outstanding pre-algebra teacher"—an intertwining of the current theme with the perceptions of instruction theme—she also indicates that she took other math classes "just for the fun of it" all while earning high marks in the process.

Another participant describes success in college-level mathematics courses stating that "I have made an A in every college math course I've taken" (ePST38).

Good grades with hard work. Several participants described their mathematics learning in terms of the hard work required to achieve success or high grades. For instance, one ePST says that "[o]ver the years I have taken many math classes, all of which I had to work extremely hard for to receive a good grade" (ePST20). Similar to that response, another ePST states, "I have always had to work harder in math…I never got less than an A or B in my math classes" (ePST5). While participants share about the hard work required to be successful in their mathematics experiences, not all experiences where characterized by hard work equating to good grades. Many ePSTs lamented that they worked hard but still did not achieve high marks on their mathematics work. Early mathematics experiences for ePST1 were characterized by "constantly having to miss recess to come in to make up assignments that [she] had scored a low grade on…" (ePST1). In another case, despite the instructional methods used by a college instructor, ePST5 passed the class through hard work and extra time spent with a tutor. In this example, she shares about her college mathematics experience stating:

> One really negative experience I have had was in college… I had a D in this [college] class most of the semester even though I went to tutoring and was working really hard… I ended up passing the class but I have forever been scarred by the teaching methods of this certain teacher. (ePST5)

Note the intertwining of the current theme of perceptions of success with the aforementioned theme of perceptions of instruction. This example showcases how the participant's perception that the teaching methods were ineffective is inseparable from the perception of success with hard work.

The journey of success. In many cases, the participants' descriptions resembled a journey through their success or failure in their mathematics learning. While it is expected that the participants would provide information about their experiences at various moments in their educational history, there were several that spoke of successes and failures at specific points in their education. For example, ePST26 shared that she "did not believe [she] did well in math" in elementary school, while in junior high she was "okay at math" (ePST26). Upon reaching high school, ePST26 lamented that she "did not do well in geometry…but did very well in algebra 2" (ePST26). For this type of mathematics experience, success is not directly tied to high or low grades but a more general description of success or failure. In Figure 4.9 below the drawing shows two time periods in the experience with grades as a central feature of success. In the left picture, a girl is drawn with a frown, holding a paper with a large letter F at the top, and writing indicating that middle and high school math "actually made me cry". On the other hand, the right side of the image shows a vastly different type of experience in terms of success. The girl on the right side is drawn with a happy face, holding papers with a "You Rock!" and an "A+" in college courses, and describe beginning to excel and "seeing how much fun it could be".

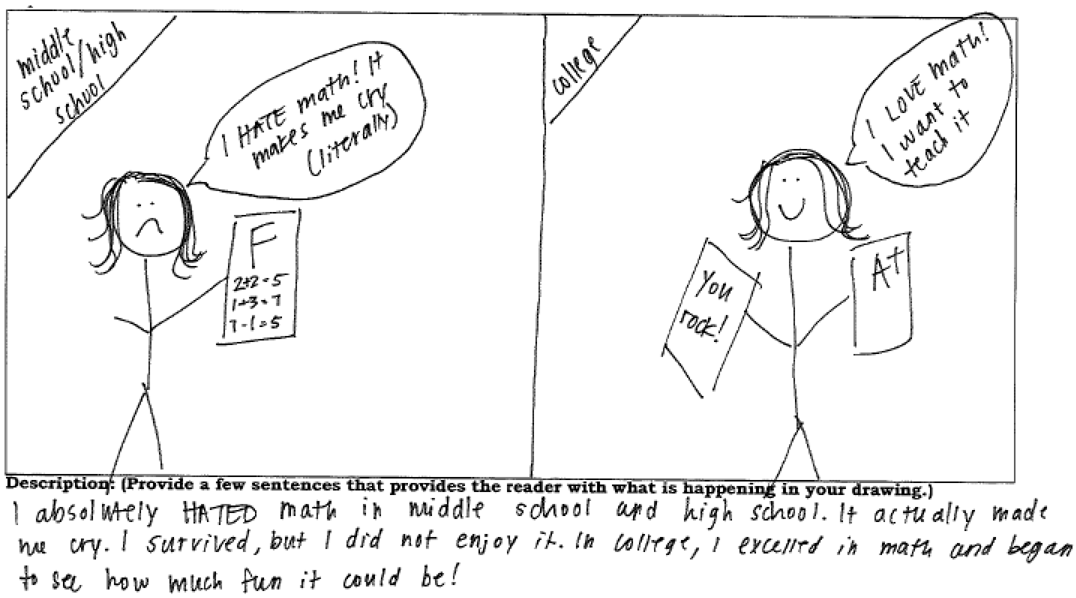
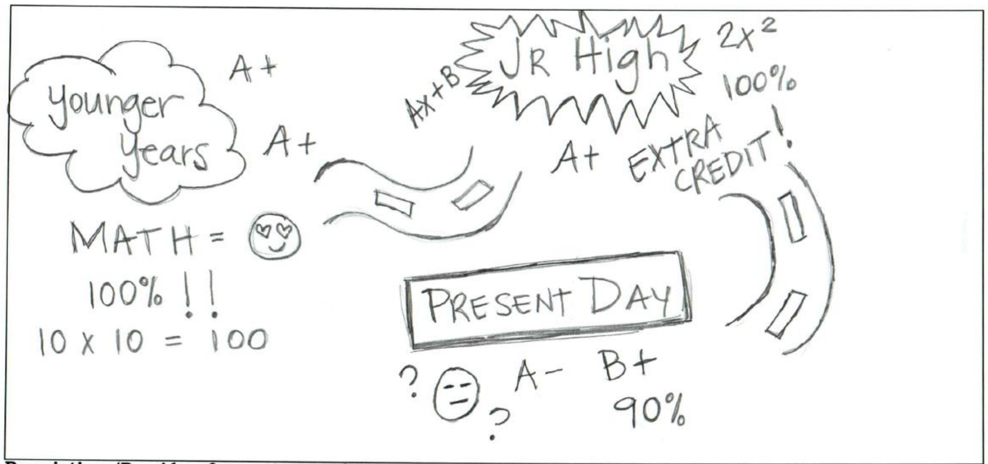


Figure 4.9. Mathematics experience drawing for ePST15

In the same way, Figure 4.10 below shows how grades for another participant are an important indicator of success. The drawing shows a journey on a road from the "younger years" to "JR High" and on to the "Present Day". At each stage, the drawing shows a series of symbols meant to indicate good grades (e.g. A+, B- and 100%), and correct mathematical answers (e.g. "10 x 10 = 100", and Ax + B) to describe the successes at each point along the journey, though her enjoyment of the subject has declined.



Description: (Provide a few sentences that provides the reader with what is happening in your drawing.)

The picture depicts my initial interest in math and feeling excited about the subject and my success. It gradually declined as I got older and I dont enjoy it like I used to.

Figure 4.10. Mathematics experience drawing for ePST29

Success on quizzes and exams. For many of the participants, success in their mathematics learning experiences was defined by their performance on quizzes and exams. One participant described how her success was related to the attention her teacher gave her but in her memory the good grades on tests and quizzes was the evidence of her success. She stated:

> In 7th grade, I had a particularly talented teacher who really took attention to me and was determined to help me, this was a more positive experience in math and I remember making several 100s on tests and quizzes. (ePST11)

Here the intertwining of the current theme of perceptions of success with the aforementioned theme of perceptions of instruction shows how the participant's perception that the teacher's commitment and connection with her contributed to her success.

While high marks on tests and quizzes is a characteristic of success for some, more ePSTs highlighted times when their performance on high stakes exams—like those for college readiness and state mandated exams that assess for knowledge and skills gained in school—was less than perfect and even prevented the ePSTs from moving forward in their mathematics education. For example, ePST1 says:

> [I]n college...I had to try and test out of an intermediate algebra class because my ACT score was not high enough on the math portion to put me into a regular algebra class… I ended up completely bombing the placement test and ended up in intermediate algebra. (ePST1)

Another ePST expressed the same performance issue on a state mandated exam in Texas but shares how the failure influenced her mindset about algebra. She said:

> Because I struggled so much that year in math I failed the Texas Assessment of Knowledge and Skills (TAKS). I was devastated and this put me in a mindset that I would never be good at algebra. (ePST3)

Focus on speed and memorization. The elementary preservice teachers' descriptions of their mathematics experiences include several references to speed and memorization throughout their time learning mathematics. Several participants describe a combination of speed drills and timed tests used to improve or assess memorization of math facts. Fifty percent of the participants described a focus on speed and memorization in their mathematics autobiographies. The writing prompts and experience drawings each accounted for 21% of the descriptions of a focus on speed and memorization.

Timed tests and speed drills. Solving a math problem quickly and remembering math facts were actions commonly mentioned in participant descriptions. For example, one participant remembers her experience in elementary school saying, "[in] my third grade year...I recall timed tests with one hundred problems that we had to solve in under one minute…each day with addition and subtraction" (ePST2). Mentions of "timed tests" especially for multiplication (ePST6 and ePST21) or "timed multiplication papers" (ePST32) were quite common throughout the mathematics biographies, and open-ended writing prompts. The mathematics experience drawings also indicated that timed tests and speed drills were commonplace in mathematics learning. Figure 4.11 shows the experience drawing of ePST11. On the left side of the drawing a student is sitting at a desk taking a timed multiplication test. Additionally, a speech bubble above the student's head indicates she is certain she does not have time to take the test and that it will be "too hard".

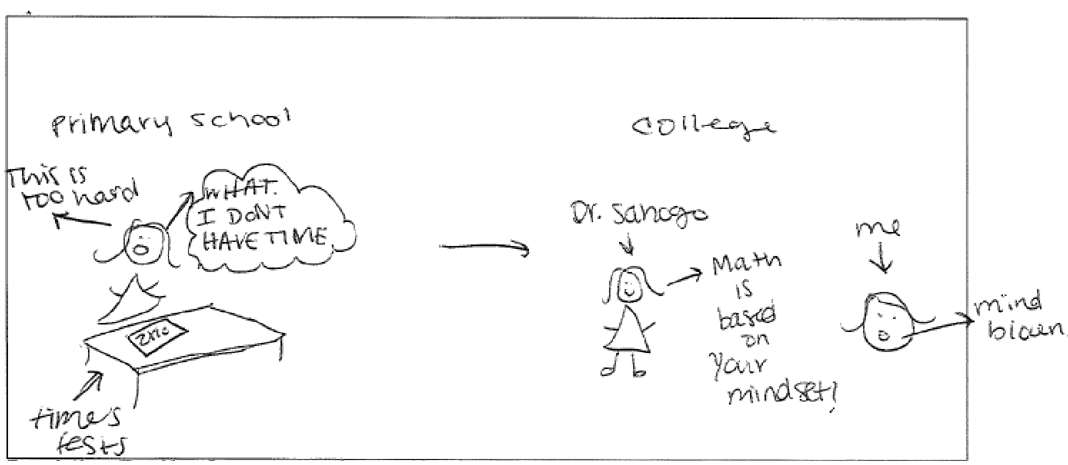


Figure 4.11. Mathematics experience drawing for ePST11

Slow down/need more time. For some of the participants, the time and attention spent on timed tests and worksheets did not produce good memories of mathematics learning. Many ePSTs recall moments of struggle when quizzed on math facts because they needed more time to process the lesson, remember the necessary facts, or understand the concept. One ePST lamented, “I just remember getting drilled with math facts and I was one of those students who needs just a bit more time to think through a problem so I struggled a bit with those” (ePST3). For some ePSTs who enjoyed learning math and doing math, needing more time was not negotiable. Despite loving math when she was younger, ePST9 describes how it took her “a little longer to understand certain math subjects” and her grade was lower because “they still had to go on with the lesson” while she had to catch up. Another participant said that she would “get so frustrated that [she] couldn’t solve problems as fast as other kids in [her] class” (ePST21). Statements such as these indicate that the ePSTs felt less intelligent than their peers if they could not work as quickly on mathematics problems as their peers. Similarly, ePST16 said that she saw “math as intimidating and something that takes up a huge chunk of [her] time because it is difficult for [her] to complete a math task quickly” (ePST16).

Not all participants focused on the negative memories of speed drills and timed tests. The experience drawing in Figure 4.12 shows what seems to be a more positive experience with the same worksheets and speed drills. The drawing shows a math worksheet or exam and whiteboard for speed games along with a horn and some balloons which indicate the celebration of an accomplishment for the aforementioned math work. This figure indicates that worksheets and speed drills were more enjoyable for this participant.

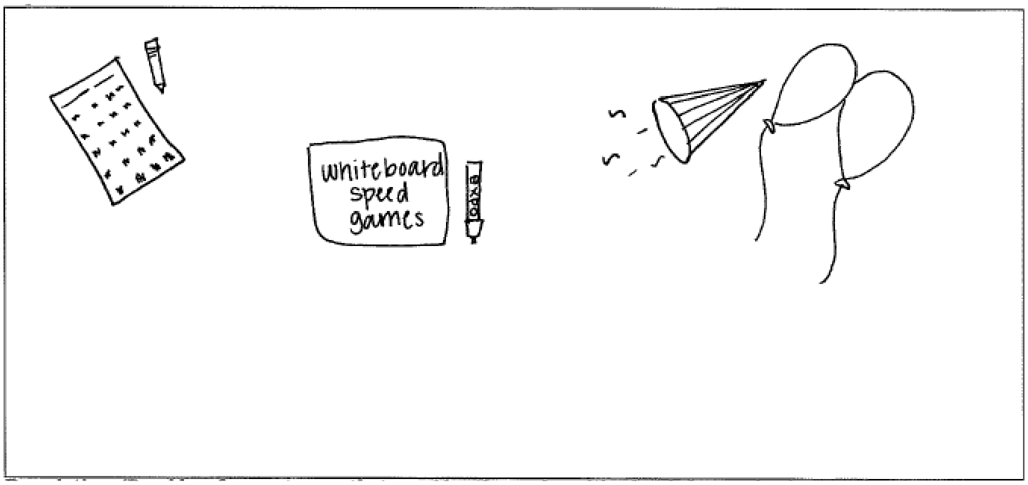


Figure 4.12. Mathematics experience drawing for ePST34

Memorization. A focus on memorization runs throughout the data describing mathematics experiences. For example, ePST14 describes her experience saying, "[in] elementary, middle, and high school, I learned math best by memorizing one way of solving a specific type of problem and practicing it until I felt that I had memorized the steps" (ePST14). Mathematics learning was governed by "procedural memorization" for ePST7 and ePST8 describes how she developed "a strong memory and followed formulas and procedures" in her experience. There were several drawings where the participants described either in drawing or in words that they learned to remember math facts or learned to "memorize" (ePST7). Some ePSTs remembered lessons focused on memorization as positive moments. For instance, ePST18 stated,

> All through elementary school we are forced to remember a string of facts to make a foundation. This was actually a very good time for me […] I thrived with the memorization [and] did very well. I learned all the formulas and flourished in the assignments. (ePST18)

Most participants, however, describe memorization of mathematics in a less positive way. Math work is described as "repetitive and monotonous" (ePST2), and "drill and kill worksheets" (ePST25). The experience drawing in Figure 4.13 shows a more negative image. The details in the student's expression show wide surprised eyes, an open mouth, and sweat droplets coming off the student's head. The words written in the space below the drawing indicate frustration with timed tests and flash cards.

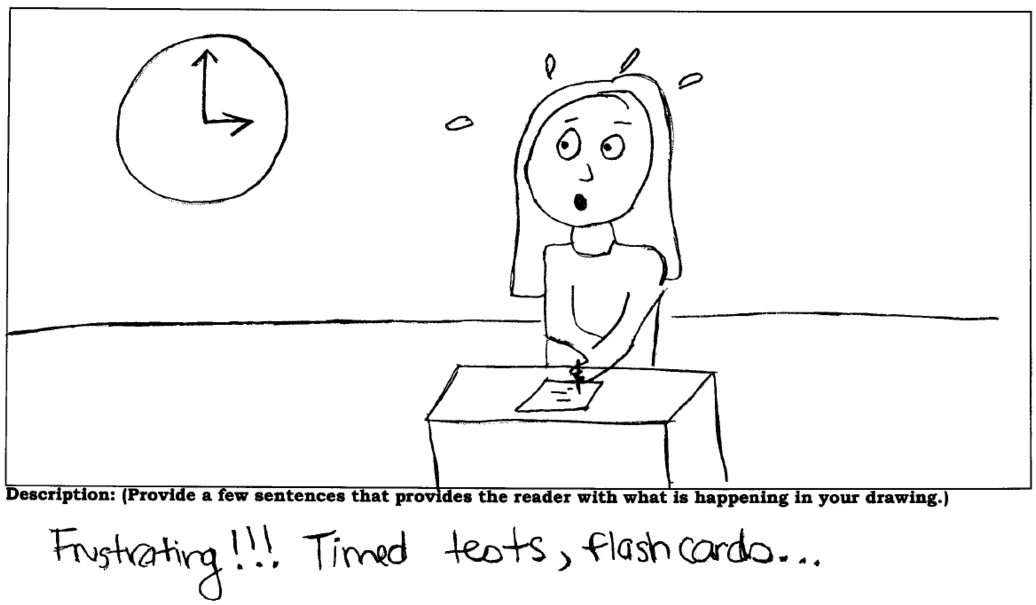


Figure 4.13. Mathematics experience drawing for ePST35

While several of the participants shared about specific elements of speed or memorization in mathematics learning, other descriptions combined several elements to provide a richer image of their experience. In one case, ePST36 said,

> These unfortunate timed assessments were given to students to fill out in a given amount of time. The tests were broken up into multiplication, addition, subtraction, and division facts. The incentive for getting all of the answers correct and within the time limit was an ice cream party. As a lower elementary grade

> student, I always wanted to go to the ice cream parties and if I did not get all of the answers correct I felt defeated. (ePST36)

Timed tests or “assessments” were a large part of the elementary school experience for ePST36 and it is clear that these moments sometimes resulted in a “defeated” state of mind. In a similar way, ePST38 describes her experience,

> In elementary school, math concepts were not exceptionally difficult, but they were taught using only one method through repetition and rote memorization. I remember sitting at my desk, facing the chalkboard, and churning out my timed fact tables in a frenzy. When we weren’t battling the clock, we were battling basal worksheets and book work. (ePST38)

Here the description includes all of the elements for a focus on speed and memorization while also intertwining with elements from the theme of perceptions of instruction in the mathematics learning experience. This example showcases how the perception of instruction gives rise to the idea that the mathematics learning experience for ePST38 was dominated by speed and memorization.

Summary of RQ1 Results

This section presented the results from the data analysis completed to answer the first research question guiding this study and gain a deeper understanding of the past mathematics experiences of the elementary preservice teacher participants. Five emergent themes were discussed including: (1) attitudes toward learning mathematics, (2) perceptions of instruction, (3) perceptions of mathematical understanding, (4) perceptions of success, and (5) focus on speed and memorization. Each theme carried with it specific examples from the data including excerpts from mathematics autobiographies, open-ended writing prompts, and drawings. Table 4.2 outlines the categories that reside in each theme.

Table 4.2 *Themes in Elementary Preservice Teacher Descriptions of Their Mathematics Learning Experiences*

Theme	Description
Attitude toward Learning Mathematics	Confidence; Anxiety; Emotion; Perseverance; Frustration
Perceptions of Instruction	Teaching style/learning style; Good/bad instruction; Commitment to teaching; Connection with teacher; Types of math work
Perceptions of Mathematical Understanding	Understanding comes easily with/without teacher influence; Understanding as innate ability; Understanding for specific content; Deeper understanding;
Perceptions of Success	Good grades with little struggle; Good/bad grades with hard work; High/low scores on tests/quizzes; Performance on high stakes exams
Focus on Speed & Memorization	Timed tests/worksheets; Repetition drills/flash cards; Slow down/need more time; Speed is smart; Repeated practice; Difficulty remembering
Focus on Speed & Memorization	Timed tests/worksheets; Repetition drills/flash cards; Slow down/need more time; Speed is smart; Repeated practice; Difficulty remembering

During analysis, each theme proved to intertwine with other themes and categories within themes would overlap providing multiple instances of the category within one case. The examples from participant responses were presented to illustrate the dominant themes present in the data. Table 4.1 at the beginning of this section highlights the number of instances that each theme occurred as well as the number of participant responses that fit each theme.

RQ2: Elementary Preservice Teachers' Nature of Mathematics Beliefs

This section presents the results from the data analysis completed to answer the second research question guiding this study and gain a deeper understanding of the nature of mathematics beliefs of the elementary preservice teacher participants. The emergent themes that describe the NOM beliefs for the participants are presented in the sections that follow.

Intertwining of Themes Within Each Case

For the results presented in this section, themes that describe nature of mathematics beliefs intertwine in multiple ways. For example, one participant might reflect on their NOM beliefs within their autobiography and mention one or more themes. This same participant might reiterate the themes in their writing prompt response or their drawing. Similarly, one participant might mention a single theme in their autobiography, a different theme in their writing prompt response, and still another theme (or more) in their drawing.

Another way the themes of nature of mathematics beliefs intertwined was via the categories or components of a single theme. In some cases, a participant might share about multiple categories of a single theme. This mixture can happen across one single data type (e.g. mathematics autobiography, writing prompts or drawing) or across the whole case where each data type is contributing multiple categories for a theme. However, the reader should note that only the subject theme or category in the current section will be discussed. For the purpose of this study, intertwining of themes was expected and reveals the multiple dimensions beliefs can take. Research supports the notion that a dynamic view of the nature of mathematics tends to be multiplistic and creative (Boaler, 2016; Tosmur-Bayazit & Ubuz, 2013), where mathematics is defined by many different but supporting roles. Consequently, the static view of the nature of mathematics represents singularity—mathematics is defined as only rules and procedures or only as a subject we learn (Dossey, 1992). Therefore, a case that showcases multiple themes intertwining might suggest the participant holds a NOM belief that is more dynamic than static.

Emergent Themes for Nature of Mathematics Beliefs

The nature of mathematics beliefs of the preservice teachers fell into five themes including: (1) mathematics is a subject we learn; (2) mathematics is rules and procedures; (3) mathematics is the study of the world around us; (4) mathematics is a type of puzzle, and (5) mathematics is problem solving through critical thinking. Each theme emerged from the data during the triangulation analysis of the mathematics autobiographies, open-ended writing prompts, and mathematics experience drawings. While each theme emerged in a unique way from the data analysis, several themes were intertwined within the data. Table 4.3 reveals the number of participants who described each theme as well as the total number of instances for each theme. For example, within one mathematics autobiography, a participant might describe their belief that mathematics is a subject we learn in more than one way. Each description would count as one instance of that theme. Table 4.3 offers an overall picture of how participants responded across the three data sources. Examination of the data for mathematics autobiographies indicates that more than three-quarters of the preservice teachers tended to describe their beliefs that mathematics is a subject we learn, and that mathematics is rules and procedures.

Approximately half of the participants tended to say that they believed mathematics was the study of the world around us, a type of puzzle or problem solving through critical thinking. The data for writing prompts shows three themes that were represented by more than one-third of the preservice teachers. Examination of the data for the NOM beliefs drawings indicates that almost two-thirds of the participants believe mathematics is a subject we learn, almost half believe mathematics is a type of puzzle and more than one-quarter believe mathematics is rules and procedures or the study of the world around us.

Table 4.3 *Nature of Mathematics Beliefs: Emergent Themes in ePST Mathematics Autobiographies, Open-ended Writing Prompt and NOM Beliefs Drawings (n=38)*

Theme	Description	Autobiographies		Writing Prompt		Drawings	
		n (%)	Total # of Instances	n (%)	Total # of Instances	n (%)	Total # of Instances
Mathematics is a subject we learn	School subject; Textbook subject; Conceptual subject; Hard/difficult subject; Tedious, memorizing, time intensive subject; a negative experience; a science of number	33 (87)	78	5 (13)	9	23 (61)	41
Mathematics as rules and procedures	A useful tool; rules, formulas, expressions, variables, functions; equations & word problems; algorithms, processes, procedures; solving problems with rules & procedures; numbers & shape	29 (76)	68	17 (45)	20	13 (34)	28
Mathematics is the study of the world around us	Adaptable, ever changing; explains how and why things happen; sets of real life problems; is all around us; is essential for interpreting the world; a language to understand and communicate about the world	22 (58)	51	15 (39)	26	10 (26)	15

Mathematics is a type of puzzle	Math is a puzzle; connections & patterns; emotions associated with a puzzle (rewarding); whole picture/solved puzzle; a challenging problem	20 (53)	38	5 (13)	6	17 (45)	28
Mathematics is problem solving through critical thinking	requires rigorous thinking; requires multiple methods; requires logic, reasoning; a deep and flexible subject; requires creative thinking	19 (50)	42	14 (37)	14	6 (16)	10

Mathematics is a subject we learn. The theme of mathematics is a subject we learn arose from the large percentage of participants who described their nature of mathematics beliefs this way. Table 4.3 above indicates that in 87% of mathematics autobiographies, 13% of writing prompt responses and 61% of NOM beliefs drawings participants indicated that they believed mathematics is a subject we learn primarily in school. Across several cases, preservice teachers described various aspects of their beliefs stating that mathematics is a school subject that is studied at various content levels (e.g. addition/subtraction, multiplication/division, algebra, or geometry) usually with the aid of a textbook and a teacher. For some participants mathematics is a tedious subject—one that involves large amounts time dedicated to repetition and memorization. Likewise, mathematics is described as a hard or difficult subject where participants describe the challenge or frustration associated with learning the subject of math. A few of the preservice teachers noted that they believe mathematics is a conceptual subject dealing with abstract theories and relationships between numbers. Each of these components of the theme that mathematics is a subject we learn are discussed below.

School subject. One of the main ideas communicated among the preservice teachers in this study is that mathematics is a subject we learn in school. For example, ePST9 said, "I believe that it is one of the most important subjects in school" (ePST9). For many of the participants, the mathematics they learned in school is compartmentalized according to content areas so when they speak about math being a subject to learn, they typically reference school subjects like algebra or geometry. They also describe school mathematics in terms of content from early education such as addition, subtraction, multiplication, and division. The drawing in Figure 4.14 showcases the idea that mathematics learned in school early on builds toward more "complex math" later in school. Figure 4.14 illustrates the belief that mathematics is like "starting at the bottom [and] working up" (ePST1) much like the idea that mathematics taught in elementary school represents steps toward mathematics learned in secondary school. In a similar way, ePST6 believes that mathematics is "a priority in schools" (ePST6) while another participant compared mathematics to other subjects taught in school—"history, [and] language arts" (ePST27)—to further solidify the idea that they believe mathematics is a school subject we learn.

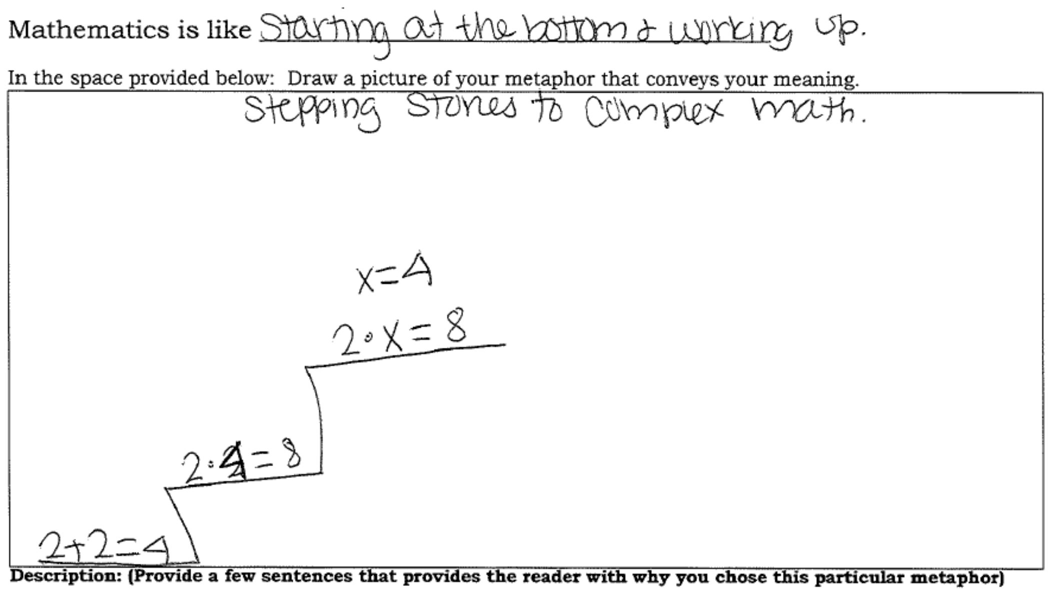


Figure 4.14. Nature of mathematics belief drawing for ePST1

Difficult subject. Several participants described mathematics as a difficult subject, although the reasons they give for why it was a difficult subject vary across responses. For some preservice teachers math as a subject was difficult because of the frustration it caused when 1) they were confused and did not understand the content or 2) they were tired of the repetitious nature of the subject and the memorization they had to do. Some participants just stated they believed “math is hard” (ePST3) while offering few details as to why they believe this way. Still others used the NOM belief drawing to visually represent their belief that math is a difficult subject. In Figure 4.15, ePST32 communicates the she believes math is hard because it is frustrating.

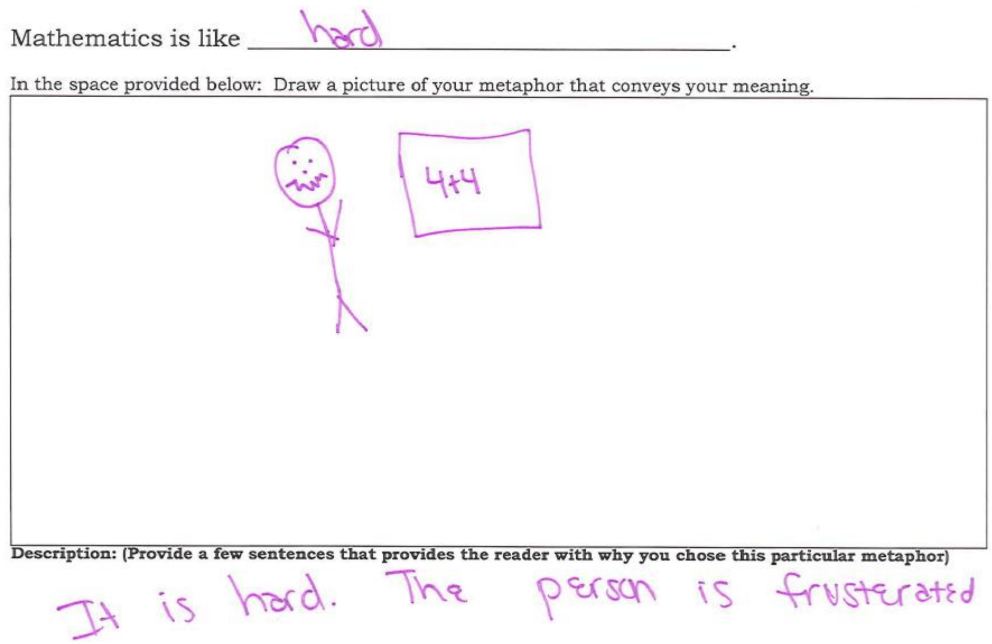


Figure 4.15. Nature of mathematics belief drawing for ePST32

Some participants provided more detail in their responses. For example, ePST15 elaborated stating, "[m]athematics is a confusing subject that is commonly misunderstood…I find that math can be interesting and intriguing while also being extremely confusing to me" (ePST15). Here ePST15 seems to indicate that for some people mathematics is hard because it is confusing and "misunderstood". However, for ePST15, while she might be confused, she sees the difficulty in mathematics as more of an "interesting and intriguing" challenge. Like ePST15, confusion is one of the main reasons why ePST28 believes mathematics is a difficult subject but she also cites frustration as a culprit. She says,

> So many of the stupid formulas I used in school have not been seen in my adult life so far…it has never made sense to me…I immediately think of crying and

> tears, frustration and feeling like I'm stupid, and just feeling like I will never understand anything. (ePST28)

In summary, these participants indicate that the subject of mathematics is difficult due to distinct reasons. For some participants mathematics is a difficult subject because of the frustration induced while learning either due to misunderstanding or confusion.

Tedious subject. While some of the participants mentioned that mathematics is a hard because it is a frustrating or confusing subject, other participants talk about how mathematics is a tedious subject. For example, ePST18 says, "math is usually the subject that students loath in school…[t]he process is tedious, formula based, and most people find that stressful." As if to elaborate on why people find math stressful she adds, "[a]ll through elementary school we are forced to remember a string of facts" (ePST18). According to ePST18, the tedium of mathematics learning that involves time-intensive memorization can bring about negative emotions such as dislike, stress, and frustration for some students. In fact, ePST21 and ePST32 use virtually the same language to describe this idea. In her mathematics autobiography, ePST21 described the words that came to her mind when she thought about mathematics, namely, "hard, time consuming, challenging, numbers and equations". In an almost identical fashion, ePST32 said that in thinking about mathematics the words that came to her mind were "hard, time consuming, stress, frustrating, and annoying". The examples in this section showcase the idea that mathematics is a difficult subject for some participants because it is a tedious, annoying and frustrating endeavor involving numbers and equations.

Painful subject. A few participants used their drawings to show they believed that mathematics was a difficult subject because of the intensity of the negative emotions they felt while learning the subject. Some describe their beliefs in terms of physical pain, or more appropriately they would metaphorically compare the study of mathematics to a situation that is detrimental or painful. For example, ePST23 drew a scene of a person stepping on a Lego® block in the dark of night. The statement at the top of the drawing in Figure 4.16 illustrates a comparison—that the subject of mathematics brings pain similar to stepping on the corner of a hard object when you least expect it.

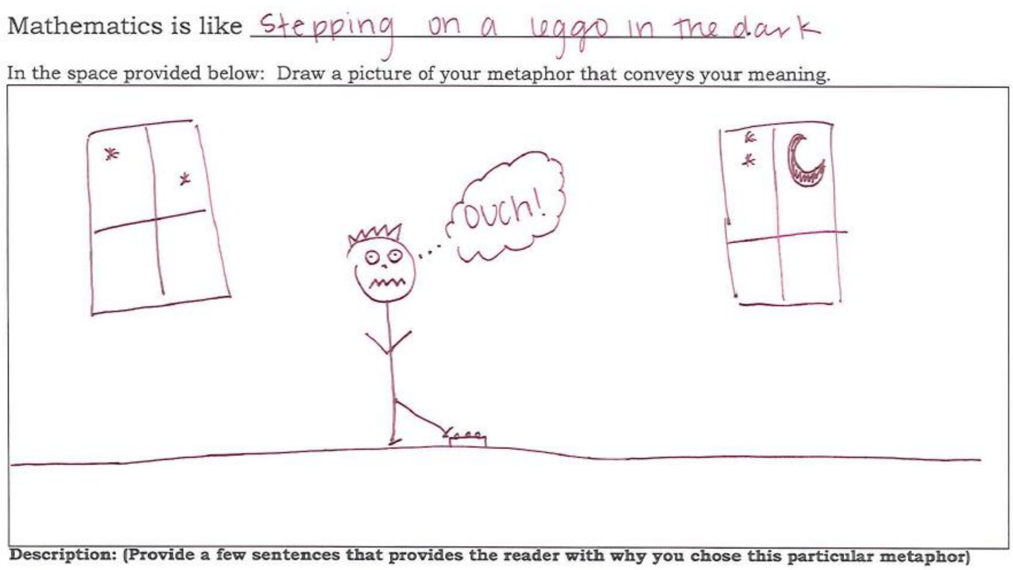


Figure 4.16. Nature of mathematics belief drawing for ePST23

This preservice teacher (ePST23) goes on to share in her mathematics autobiography about her belief that mathematics as a subject can be "your worst nightmare" and how, for her, studying mathematics in school encompassed "some of the worst experiences that [she] had" (ePST23). In a similar way, ePST2 states that she,

> …viewed math as detrimental or something that set me back in my education…math was gray and monotonous, not enjoyable…math was scary and created bad feelings in me when it was time to start our math lesson. (ePST2)

While both responses are intertwined with the experiences that each preservice teacher had in their mathematics education, each participant communicated how they believe the subject of mathematics was difficult for them because of the "bad feelings" they endured during their schooling.

Mathematics as rules and procedures. In 76% of mathematics autobiographies, 45% of writing prompt responses, and 34% of NOM beliefs drawings, participants indicated that they believed mathematics is a set of rules and procedures. Several participants indicated the rules and procedures were useful tools for solving problems like word problems or calculating a solution. Other participants shared about the various elements of rules (e.g. numbers, symbols, formulas and equations) and types of procedures (e.g. algorithms, processes, and systems). Each of these components of the theme are discussed below.

A useful tool. Many of the preservice teachers in this study commented that they believed mathematics is a collection of rules. In mathematics, a rule can be defined depending on the type of problem that is being solved. There are algebraic rules for defining the relationship between two variables; there are operational rules that govern the order in which we calculate a string of numbers being combined through addition, subtraction, multiplication or division; there are formulas that represent conceptual rules for naturally occurring phenomena, like the gravitational force on the surface of the

Earth. For this study, the majority of participants defined mathematics as rules in a general way to mean a collection of anything needed to find a solution. Multiple responses given in the mathematics autobiographies, writing prompt responses and NOM belief drawings illustrate the idea that mathematics is a collection of rules. For example, ePST3 says that when she thinks of mathematics, she thinks of "equations and word problems". However, ePST12 believes that mathematics is a larger collection including not only equations but "systems, numbers, and…, components, and solutions." In a similar way, ePST24 defines mathematics as "equations, formulas, shapes, and dimensions."

For some of the participants, their NOM belief drawings indicated that mathematics is a collection of rules. For instance, the drawing shown in Figure 4.17 compares mathematics as a collection of rules to "a box of chocolates", and the words below the drawing suggest that the collection represents "different opportunities and possibilities available" for working on a math problem.

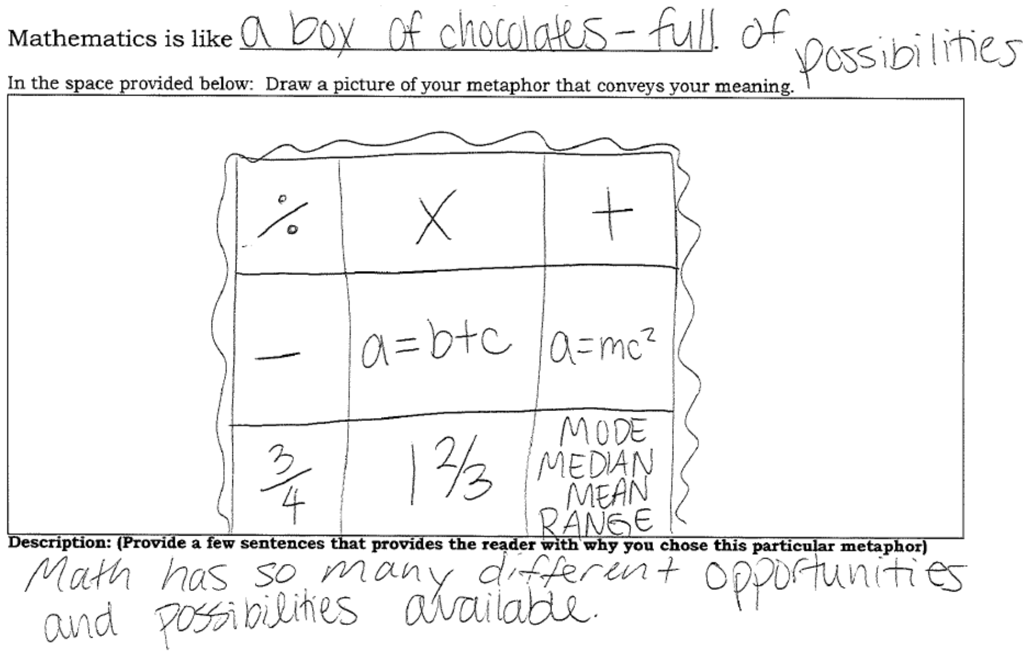


Figure 4.17. Nature of mathematics belief drawing for ePST12

In a similar way, Figure 4.18 shows the NOM beliefs drawing created by ePST13 where the words at the top liken mathematics to a "never ending problem". It is possible that the problem has many different solution pathways but Figure 4.18 seems to suggest the math problem itself is a "never ending" collection of symbols and variables in an equation to solve. Similarly, ePST 31 said she believes mathematics is a "long, exasperating, abstract, complex, [set of] written formulas."

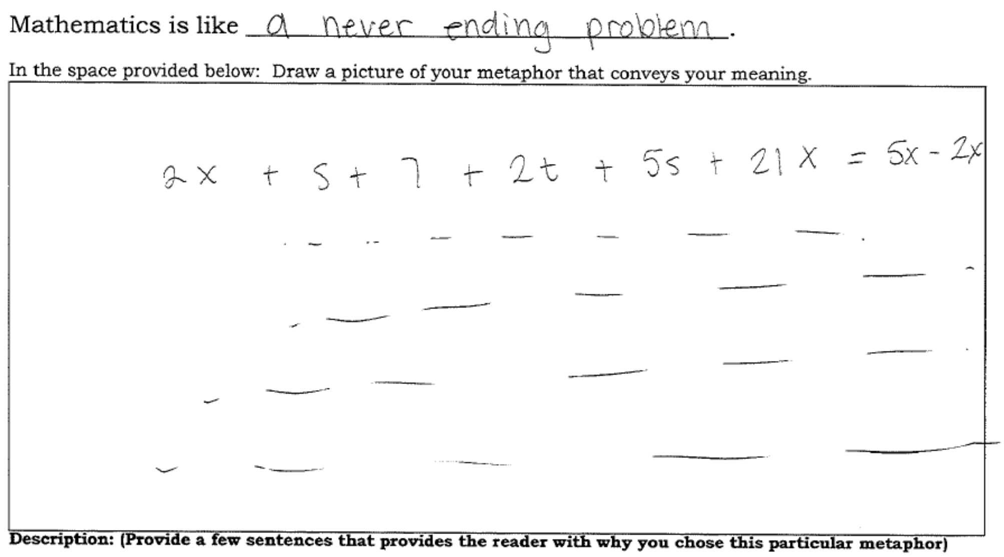


Figure 4.18. Nature of mathematics belief drawing for ePST13

Use the tool to solve. While, many participants indicated that they believed that mathematics is a collection of rules they also suggested in their responses that rules needed to be followed or used to reach a solution or solve a problem. In fact, ePST28 said exactly that "[m]athematics is a set of rules to follow". While mathematics is defined by several participants as a collection of rules, equations and formulas, the actionable use of these elements is also acknowledged by the preservice teachers across much of the data. Indeed, there are several examples that showcase the idea that mathematics is not just a collection of rules but also involved an action, where to solve a problem means to use the rules. One participant supports this notion with her statement that "[m]ath isn't just 2+2=4, it is measuring, calculating, and finding…the solution" (ePST9). For ePST9 mathematics is a set of rules (e.g. addition rule of two plus two) and procedures (e.g. measuring, calculating). Similarly, ePST11 states,

> “Math is not simply a list of functions, formulas, and diagrams to memorized and drilled into your head… Rather, mathematics includes a wide range of problems that can be solved through a multitude of strategies” (ePST11)

Here again is the idea that there exists a multitude of procedures or “strategies” in which rules like functions and formulas can been employed to solve a math problem. Some participants stated their beliefs more simply like the following examples show:

> “Math is the use of number and or symbols to explain a problem and provide a solution” (ePST25)
>
> “Strategically working through an equation to solve a problem” (ePST20)
>
> “Working out problems using equations and following steps and rules” (ePST19)
>
> “Math involves solving problems through steps and approaches” (ePST16)
>
> “Mathematics is the manipulation and use of numbers shapes and patterns” (ePST14)

The NOM beliefs drawing of ePST17 also showcases the idea that rules and procedures can be used to reach a solution. The drawing in Figure 4.19 seems to suggest that using rules and procedures to find a solution in mathematics is like “struggling to climb a mountain and finally reaching the top” (ePST17). Mathematical operation rules, formulas and calculation procedures are shown as ‘tools’ used to find the summit or solution.

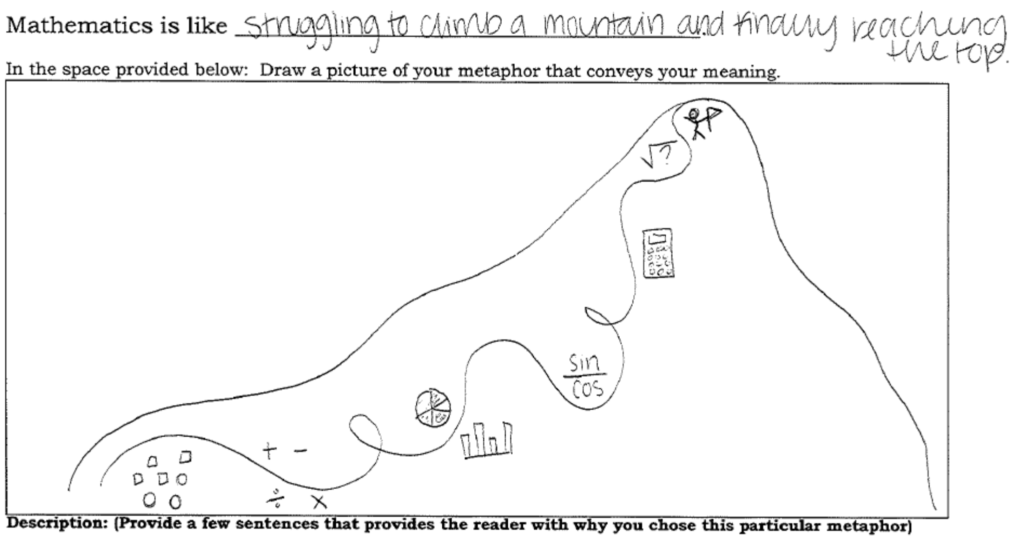


Figure 4.19. Nature of mathematics belief drawing for ePST17

Mathematics is the study of the world around us. Just over half (58%) of the participants in their mathematics autobiographies discussed how they believed that mathematics is the study of the world around us. Additionally, 39% of the writing prompt responses and 26% of the NOM belief drawings showcased this theme. In some cases, participants shared that they believed mathematics is all around us in nature and in sets of real-life problems that must be solved. These responses highlighted the belief that mathematics is adaptable to the ever-changing character of nature and the changing needs of the human race. Many participants described how they believed that mathematics was a language that humans used to explain how the world works and why things happen. Some said that mathematics is essential for interpreting the world. In the section that follows, each of these categories for the theme that mathematics is the study of the world around us will be discussed.

All around us. Across many cases, preservice teachers shared about their belief that mathematics is the study of the world around us. In one example, ePST6 states,

> To me, mathematics is problem-solving, reasoning, questioning, making connections, and using various strategies and processes to learn more about the world around us…It is the building block of our world and one can see it all around them. (ePST6)

Here ePST6 describes her belief that while mathematics is the action of studying the world to learn about it, she also believes that mathematics is a thing found in the world around us. The idea that mathematics is in the world around us is echoed by ePST35 in her NOM belief drawing shown in Figure 4.20.

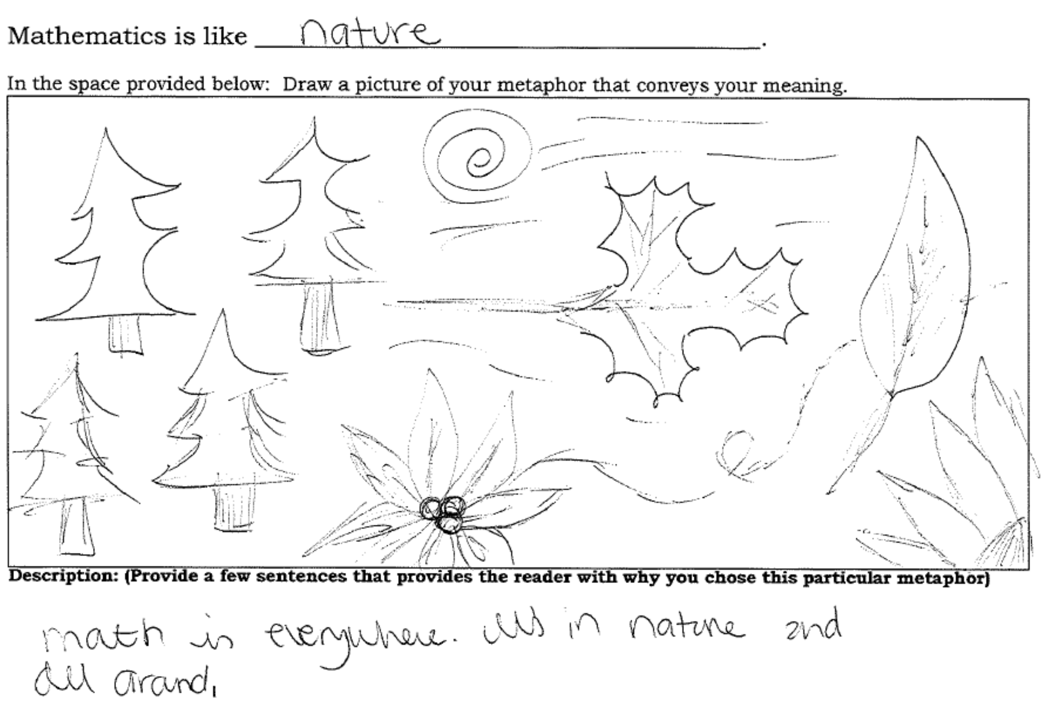


Figure 4.20. Nature of mathematics belief drawing for ePST35

From Figure 4.20, ePST35 indicated that "math is everywhere…[it's] in nature and all around" therefore signaling the belief that while mathematics can be viewed as an actionable sequence of studying the world around us, it is also possible to see mathematics in nature. In a similar way, ePST4 reiterates the idea that mathematics is in nature. She says,

> I think that math can be many things and is found in many places. When I hear the word 'mathematics' I think about money and decimals, but I also think about flower petals, and the depth of the ocean, and splitting a candy bar with my siblings… Math is addition and subtraction, but it is also the world around us… I find that math is a useful tool in organization and I love to see math in nature. (ePST4)

The intertwining of themes is quite evident with the responses of ePST6, ePST35 and ePST4. In each response, evidence is present of the belief that mathematics is a useful tool that can be employed to study phenomena in the world. At the same time, each response showcases the belief that mathematics is in the world, in nature, that it exists as a thing and we can use rules and procedures to study it.

Real-life problems. While the idea that mathematics is all around us is a prominent belief among the participants in this study, so too is the belief that mathematics is a collection of real life problems. Across multiple mathematics autobiographies and writing prompt responses, preservice teachers define "sets of real life problems" (ePST5) in two ways; 1) the real life problems that represent the mundane, daily events for most people and 2) the real life problems that are the type professional engineers and mathematicians tackle— "like architecture, accounting, and many more" (ePST1). This way of defining real life problems is evident in a statement made by ePST25 when she says,

> "Math is very logical, one must use reasoning- whether it be inductive, or deductive, to solve what is being asked of them. It can range from a real world problem such as, "how much change do I receive back from a purchase?" to the most complex theorems still being tested by extremely advanced mathematicians today" (ePST25)

In a more general way, this same belief is echoed by ePST38,

> Mathematics is many things mathematics quantifies everything in our lives it offers explanations for how and why things happen mathematics is concrete but also flexible. (ePST38)

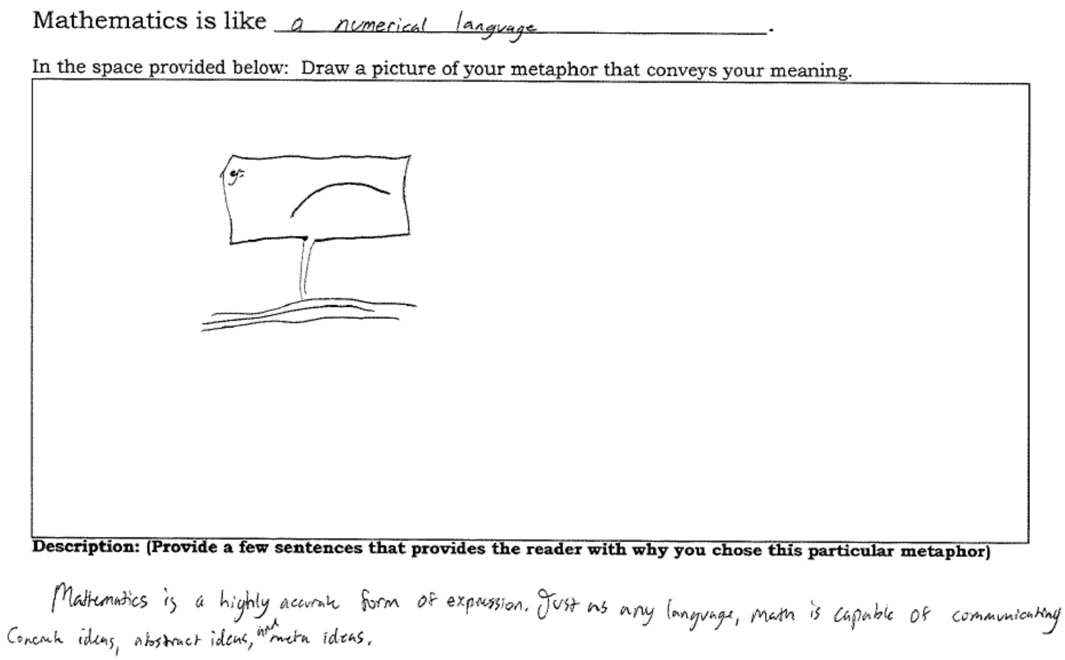
While the response by ePST38 highlights the idea that "mathematics quantifies everything in our lives" it also illuminates the adaptable characteristic of mathematics to be useful and meaningful *for* and *in* many aspects of the human existence. Indeed, ePST16 believes that mathematics is crucial to her daily life, she says,

> I believe that math is vital. I know that without math, I would not be able to make it through the day. I use math daily. I tell time, I estimate how long it will take me to get from one destination to another, I spend money, I budget money, I pay bills. (ePST16)

Essential for interpreting the world. Similar to the idea that mathematics is a useful tool for studying the world and solving real life problems, several participants indicated that they believed that mathematics is essential for interpreting the world. Understanding how and why things work the way they do was a prominent idea in many of the responses and some participants even likened mathematics to being a language that is used to communicate and understand relationships between numbers. For example, ePST1 says "[b]ut that is exactly what mathematics is, it helps us solve everyday issues". In a similar way, ePST17 says that "[m]ath is a way of understanding the world and figuring out a way for humans to comprehend the complex systems surrounding us". The idea that our understanding of the world can be communicated via mathematics as a tool is showcased in the mathematics autobiography response of ePST6. She states that mathematics is "the use of numbers and processes to solve [,] communicate [,] reason [,]and make connections" (ePST6). For ePST7, the notion that mathematics is a tool to communicate is paramount. In fact, he indicates this belief in his mathematics autobiography, writing prompt response and NOM belief drawing. In his mathematics autobiography he states,

> I firmly believe math is a language…It is a set of functions, numerals, procedures, rules, and concepts, which combine in multiple ways to prove new ideas; thus, mathematics is language and expression. (ePST7)

The shorter writing prompt response echoes the idea stating that "[m]athematics is a numerical form of communication [,] it communicates facts about the world…and the relationships between things" (ePST7). And again in the NOM belief drawing, ePST7 indicates that he believes mathematics is a language used to communicate mathematical ideas and concepts (see Figure 4.21).


Mathematics is like a numerical language.

In the space provided below: Draw a picture of your metaphor that conveys your meaning.

Description: (Provide a few sentences that provides the reader with why you chose this particular metaphor)

Mathematics is a highly accurate form of expression. Just as any language, math is capable of communicating concrete ideas, abstract ideas, and meta ideas.

Figure 4.21. Nature of mathematics belief drawing for ePST7

In the words below his drawing, ePST7 compares mathematics to other languages and highlights previously mentioned ideas that mathematics is a "form of expression" which is "capable of communicating concrete ideas (e.g. number and shape), abstract ideas (e.g. algebraic reasoning), and meta ideas". For clarification, concrete ideas in mathematics refer to elements such as number and shapes, while abstract ideas in mathematics are related to algebraic reasoning or conceptual ideas. It is unclear what ePST7 means by "meta ideas", however, the term 'meta' can be a noun that means "an abstract, high-level analysis" (www.m-w.com/meta, 2019). Using this definition with the rest of the response, it seems as though ePST7 is referencing the higher level mathematical ideas that are the work of professional mathematicians.

Like a language is essential for interpreting the ideas of people, mathematics is essential because “it is a way for people to understand the world through numbers” (ePST9). It is a way to communicate, but more importantly—as one participant points out—“[w]ithout math, people could not make skyscrapers and bridges, they could not get a man to the moon” (ePST9). As several responses have shown, the belief that mathematics is essential for interpreting the world is common to the theme that mathematics is the study of the world around us.

Mathematics is a type of puzzle. The theme math is a type of puzzle represents a collection of the descriptions the ePSTs provided with regard to the beliefs they hold that mathematics is a type of puzzle. Just over half of the mathematics autobiographies, about 13% of the writing prompts and 45% of the NOM beliefs drawings showcased this belief. Many ePSTs described mathematics as being a challenging problem, like a puzzle, that when completed revealed the whole picture or concept being studied. Some of the ePSTs indicated that they thought mathematics was a puzzle that involved thinking and engagement. The participants likened the puzzle pieces to connections and patterns and some ePSTs used the term “jigsaw puzzle” when describing their idea of what math is. These categories for this theme are described in the next section.

Math is a jigsaw puzzle. When describing their beliefs about what mathematics is, many ePSTs shared that they believed mathematics was a “jigsaw puzzle” (ePST21). A few participants were not as specific and simply compared mathematics to a type of puzzle. One participant said mathematics is “much like solving a puzzle” (ePST25) and drew a picture of puzzle pieces fitting together (Figure 4.22). The words below the drawing in Figure 4.22 add a bit more detail as ePST25 indicates what she means by stating that "solving a math equation is much like solving a puzzle." Two participants believed that mathematics is like a "maze" (ePST16 and ePST29) with a "reward" or solution at the end while another participant compared mathematics to being like puzzle pieces where each piece contributes to the whole (ePST18). In other ways, some participants see mathematics as a problem that must be solved, like a puzzle that will not see completion until it is assembled. Indeed, ePST30 says that "[e]ach problem is a puzzle, a challenge, that I am compelled to solve".

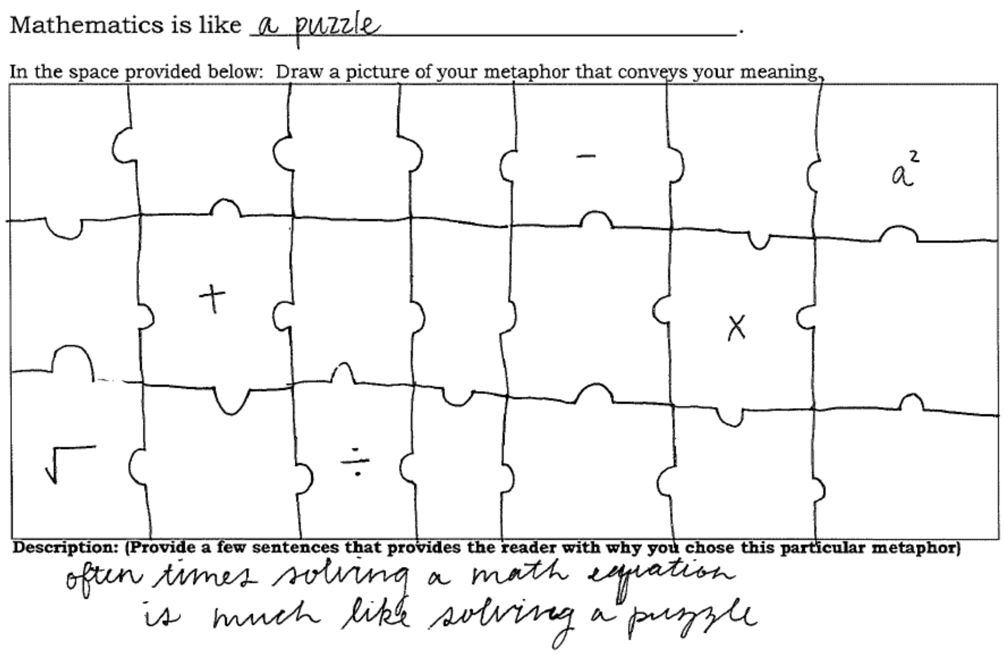


Figure 4.22. Nature of mathematics belief drawing for ePST25

Connections and patterns. This section is related to the descriptions provided by the participants that mathematics involves connections and patterns similar to the way a puzzle involves seeing connections and patterns to assemble the puzzle together. One participant said the reason she believes that mathematics is a type of puzzle is because for "every piece you learn[,] the more you begin to understand the whole subject" (ePST18).

Similarly, another participant referenced the different areas of math and said "it all fits together" (ePST36). According to ePST35, connections and patterns are found when "doing math puzzles" but she noted that after a certain point in her education, her NOM beliefs changed. She said,

> But once I saw how wonderful and interesting Math is, my attitude toward math completely changed. Math became more like puzzles to solve and connections/patterns to look for as opposed to one-approach, generic math problems (ePST35)

And similar to the way a puzzle works, ePST17 says that "[m]ath can illustrate how patterns occur, give meaning to the quantities and order of things." In a puzzle, the individual pieces fill a spot in the big picture or pattern but the order of the picture is not revealed until all of the pieces are connected.

Whole picture/concept. The whole picture/concept element arose from the participants' descriptions of their beliefs that mathematics was a puzzle that once it was solved revealed the whole concept being studied. Indeed, several ePSTs indicted that mathematics was a puzzle— "a puzzle to be solved" (ePST24). The words they used indicate an understanding that once the puzzle pieces fit together, the puzzle reveals the whole picture or solution. Some participants talked about how the whole picture is the concept while the pieces of the puzzle represent the mathematical operations or skills involved with that concept. Figure 4.23 shows the NOM beliefs drawing of ePST18. In the drawing we see what looks to be a puzzle that is nearly complete but still lacks a couple of pieces. The words above the drawing say that mathematics is like "puzzle pieces" and the words below the drawing indicate that the whole picture cannot be understood until all of the pieces are placed.

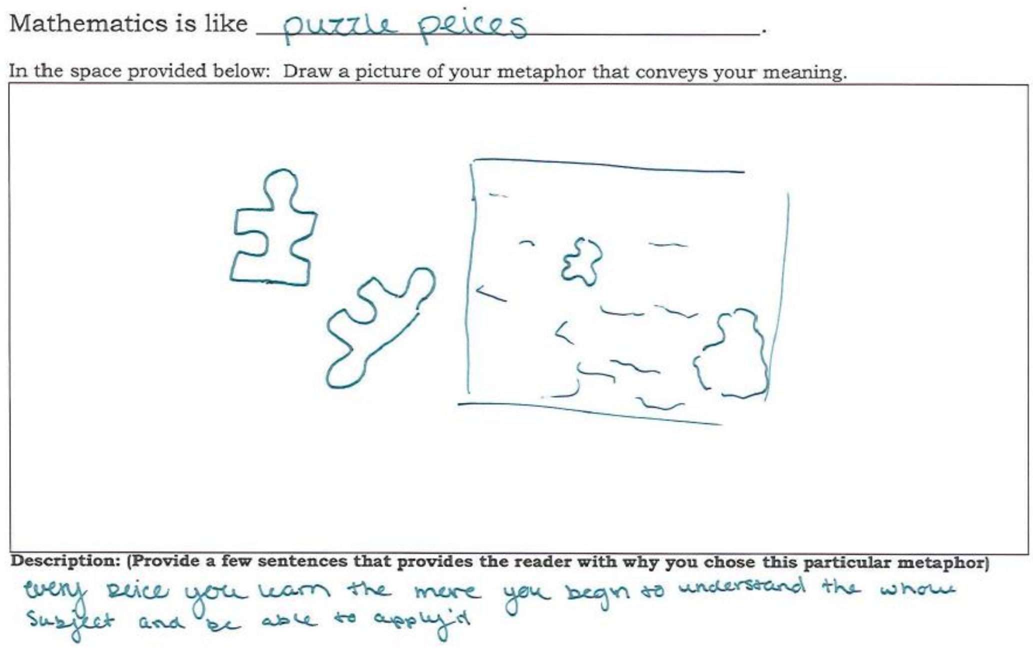


Figure 4.23. Nature of mathematics belief drawing for ePST18

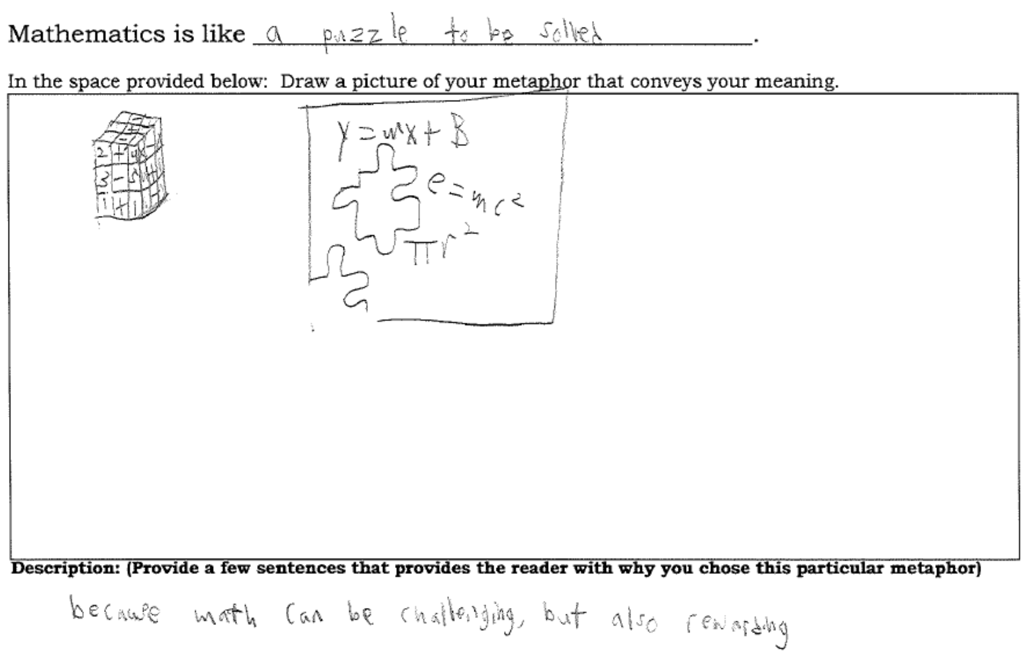
Requires thinking and engagement. Some participants described mathematics as a puzzle requiring thinking and engagement to solve. Specifically, one ePST said that solving a mathematics puzzle involves "reasoning, questioning, making connections" (ePST6) similar to the connections made between puzzle pieces and the thinking involved in organizing the different types of pieces. Another participant used words like "rigorous thinking" to describe what it takes to solve a puzzle or mathematics problem (ePST5). Like making connections is necessary to solve a puzzle, mathematics involves making connections between mathematics concepts. In the mathematics autobiography for ePST25, this idea is evident. She says,

> When I think of mathematics, I think of a puzzle. First off, you have to figure out what is missing- or the problem at hand. Then you start examining the pieces to see what goes where, or how one piece fits into another. Eventually, you realize how the problem connects and interacts with a whole series of other problems that surround it. (ePST25)

Emotions associated with a puzzle. Several participants described their belief that mathematics is a puzzle but focused their responses on words to describe the feelings associated with doing and completing a puzzle, which for them, are similar to the emotions they experience when doing math. For example, one participant said that doing math was like a puzzle because it is "rewarding" (ePST24) to complete the puzzle. ePST21 elaborates on this idea saying,

> If I had to describe what comes to mind when I think about what mathematics is I would say it's a jigsaw puzzle. It is a giant and hard jigsaw puzzle that will drive you crazy until you finally put it together, but once you put it together nothing is a more satisfying feeling. (ePST21)

The NOM belief drawing by ePST24 (Figure 4.24) illustrates the same idea.


Mathematics is like a puzzle to be solved.

In the space provided below: Draw a picture of your metaphor that conveys your meaning.

Description: (Provide a few sentences that provides the reader with why you chose this particular metaphor)

because math can be challenging, but also rewarding

Figure 4.24. Nature of mathematics belief drawing for ePST24

According the ePST24, mathematics is like a “puzzle to be solved” because it is “challenging” and “rewarding”. Other participants describe their belief that math is a type of puzzle because both are “exciting” (ePST17) and “interesting” (ePST35).

Challenging problem. Many participants compared the challenge of completing a puzzle with the challenge of doing a mathematics problem. A puzzle can be challenging in various ways which is similar the challenge offered in a mathematics problem. In particular, one ePST said “mathematics is like a puzzle to be solved because math can be challenging” (ePST24). Another participant said a jigsaw puzzle is “challenging” (ePST21) in the same way mathematics is challenging. And yet another participant said “[e]ach problem is a puzzle, a challenge, that I am compelled to solve” (ePST30). Not all visual images were provided by the NOM belief drawings. One participant in particular used words to create a visual of the challenge of climbing a mountain,

> Math is a lot like climbing a mountain… sometimes the path is smooth and obstacles are far and few between, but sometimes the path seems steep and rocky with a new challenge at every turn. Either way, when you reach the top, no matter how difficult or easy the journey felt, there is always a feeling of accomplishment and success. (ePST17)

Intertwining of the categories in this theme is highly evident. The belief that mathematics is a type of puzzle has several elements that are related. Some math problems are challenging as are some puzzles. Each situation involves understanding how the pieces fit together and in both cases there is a rewarding feeling when you finally see the big picture or understand how all the connections fit.

Mathematics is problem solving through critical thinking. The theme that mathematics is problem solving through critical thinking arose from preservice teachers describing their belief that mathematics is the action of problem solving that requires more than just memorization and practice. Several participants noted that mathematics is problem solving that requires rigorous thinking and multiple methods to reach a solution. For the purpose of this study, rigorous thinking is defined as critical thinking that which involves various types of thinking processes like logic, reasoning and creativity. Related to this idea, many ePSTs described how mathematics is problem solving through critical thinking because it is a deep and flexible subject. Fifty percent of the mathematics autobiographies, 37% of the writing prompt responses, and 16% of the NOM beliefs drawings fit into this theme. Each of the components of the theme are discussed below.

Requires rigorous thinking. Many of the preservice teachers responded that they believed mathematics is problem solving through critical thinking by describing the type of thinking that is necessary in the problem solving process. Some noted that mathematics is the ability to use a higher level of thinking to solve a problem" (ePST4) but others said that mathematics "is a way in which we solve problems using creative ways to solve them" (ePST3). Likewise, ePST11 reiterates that "math is…to use high order thinking, not just simply do the process and not understand how it works" (ePST11). Additionally, ePST5 defines mathematics as "problem-solving in a unique and creative way…math is reasoning[,] comparing[,] and making connections…I would describe it as sets of real life problems that involve rigorous thinking…" (ePST5). In a similar way, ePST8 explains that she believes mathematics is a subject of exploration, creativity, and conceptual understanding". In this case, ePST8 adds the notion that knowing rules and procedures is not enough—critical thinking means understanding why the rules work to solve the problem. The same is true for ePST27, she says, "I believe mathematics is being able to understand values and what they mean". This idea is echoed by ePST37 when she says that mathematics is "understanding numbers and number relationships …understanding procedures and formulas". Lastly, the creative element of critical thinking is supported by ePST13 when she states math is a problem solving process that "helps us look at situations in new ways". All of these responses combine to form evidence that mathematics is problem solving through critical thinking because it requires rigorous thinking to find a solution—not just knowing how to do a problem.

Requires multiple methods. The belief that mathematics is problem solving through critical thinking requires the knowledge and understanding of specific rules and procedures and then application of some or all of those rules and procedures to solve a problem. According to ePST4, mathematics "is critical thinking and problem-solving to come up with an answer [and] we use schema to understand and build on the knowledge". It seems as though ePST4 is saying that existing knowledge is built upon with new knowledge as multiple methods are used when solving a problem. The primary point here is that when mathematics is problem solving through critical thinking, multiple methods are not only employed but are necessary. For ePST11, "mathematics includes a wide range of problems that can be solved through a multitude of strategies". In a similar way, ePST6 defines mathematics as "problem-solving, reasoning, questioning, making connections, and using various strategies and processes to learn more about the world around us". To further the point, ePST13 makes the statement,

> If I were to describe what mathematics was to someone, I would explain how it is the process of solving problems through different methods. I would tell them how there isn't one right way to get an answer, and that there are many different ways to look at a problem. (ePST13)

And ePST27 agrees, indicating that mathematics "is understanding that there are multiple ways to find a solution to a certain problem". The NOM belief drawings for some participants illustrated this idea as well, that mathematics is problem solving through critical thinking because of the necessity to employ multiple methods to find a solution. Figure 4.25 below shows the drawing that ePST4 completed. While the words that the top compare mathematics to a pizza with many toppings, the toppings are indented to represent the “different ways to do math problems that all come together to get similar results” (ePST4).

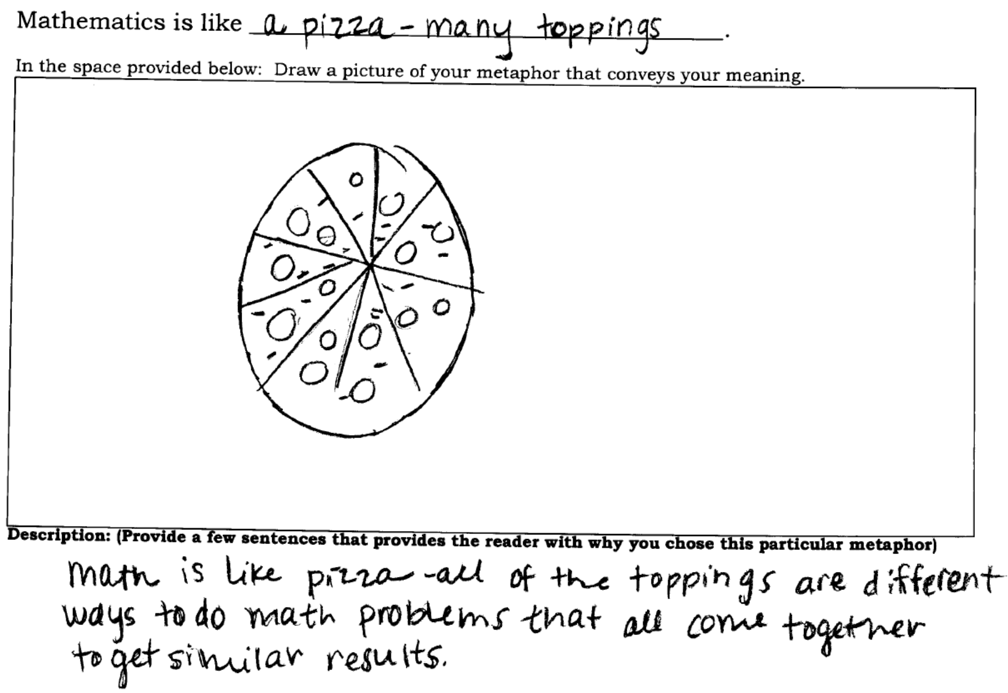


Figure 4.25. Nature of mathematics belief drawing for ePST4

A deep and flexible subject. Several participants in this study defined mathematics as a “deep and flexible” (ePST14) subject. The mathematics autobiography response from ePST14 describes this idea, saying,

> Mathematics is an incredibly flexible and deep subject, but it is often not taught that way… When I think of the mathematics, I often think of the different types of math classes that I have taken. I also think of numbers, shapes, and specific formulas that I have memorized… Before coming to college and taking education-based math courses, I believed that math was a static subject that never changed. I was convinced that math was stoic and that there was only one way to

> do math… After taking education-based math courses, I have changed my way of thinking about math. I have discovered that math is a constantly changing and flexible subject. There is a lot more to math than I had ever believed. (ePST14)

The belief that mathematics is problem solving through critical thinking highlights the characteristic of mathematics to be both deep and flexible. The notion that mathematics is "deep" indicates that it is more than just memorizing a procedure; it is the meaning behind the numbers we use, procedures we do and it is the nature we see; math is a thing, and an action. Related to the depth of mathematics is the idea that mathematics is "flexible" which refers to the fact that math can be a process, a tool, a number, shape or pattern. Depending on the context and the need, mathematics can be useful in multiple forms. These ideas are echoed in the response by ePST38:

> Mathematics is many things…mathematics quantifies everything in our lives…it offers explanations for how and why things happen…mathematics is concrete but also flexible" (ePST38)

Similarly, a few NOM beliefs drawings add a visual representation that seem to show how mathematics can be deep and flexible. The drawing in Figure 4.26 shows compares mathematics to "a painting" and then below the drawing ePST8 describes what her image conveys.

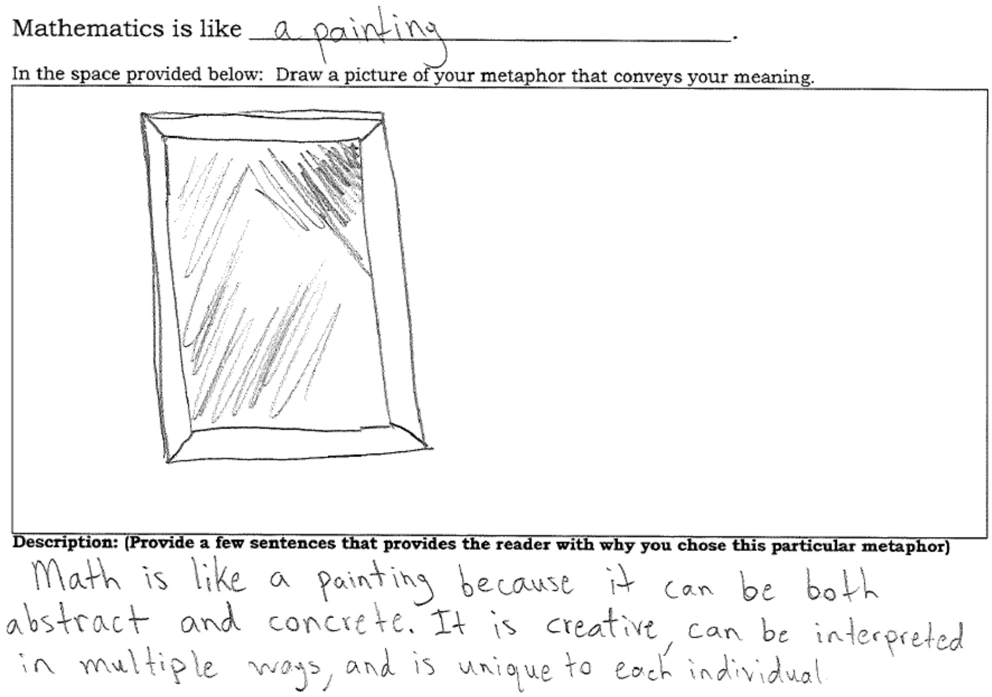


Figure 4.26. Nature of mathematics belief drawing for ePST8

For ePST8, mathematics can take multiple forms and can even be "unique to each individual". The idea that math is flexible is evident here when ePST8 says that "it can be both abstract and concrete". Indeed, this assessment intertwines with characteristics of NOM beliefs described in previous sections. Like the painting she compares it to, mathematics is "creative" and "interpreted in multiple ways"—two ideas that have been discussed before.

NOM Beliefs That Change

The results presented in this section showcase how the participants described their nature of mathematics beliefs changing in some way. In the process of describing what their NOM beliefs were, several participants shared about how their beliefs changed over time. While some of the elementary preservice teacher participants shared few details regarding their NOM beliefs before and after a specific time period, many ePSTs would interweave their description of the nature of mathematics with an experience they had. The section below presents the examples of the ePSTs who expressed a change in their NOM beliefs.

NOM beliefs change at some point. The examples in this section showcase the language used by ePSTs that indicate a change in NOM beliefs. However, within the descriptions there are no other identifying details that would create a richer image of the reason or impetus behind the change. For example, ePST5 said, "[n]ow that I am on my way to being a teacher, I have realized more and more how useful and essential math is". This statement does not provide any information as to the NOM beliefs of ePST5 before she pursued her teaching career, however; the language does seem to indicate that a new realization has been made and ePST5 believes mathematics is useful and essential. In a similar way, ePST7 provides a description of the change in her NOM belief when she said, "[n]ow, with a solid understanding [of the] relationship between elements of math, I firmly believe math is a language" (ePST7). Presumably, ePST7 did not believe that mathematics was a language before developing an understanding of the "elements of math" but ePST7 does not provide any other details about her NOM beliefs before this moment in time. Lastly, ePST15 showcases the change in her NOM belief in terms of her mathematics learning experiences. She does not provide details as to what her NOM belief was before these moments but her statement below indicates there was a change based in her different experiences learning mathematics. She said,

> One thing I like about math, that I mostly learned in my college mathematics courses, is that math is adaptable, and can be done in many different ways…In

> relation to this, I dislike that many of my teachers taught me that there was only one correct way to do a problem. (ePST15)

For this participant, mathematics was defined as something other than "adaptable" before her college experiences, and she recalls that it was the experience of her teachers teaching her "only one correct way to do a problem" which contributed to this comparison.

NOM beliefs change due to an experience. This section shows the examples of the ePSTs responses in which several details are given with respect to the change in NOM beliefs experienced by the participants. In some cases, the participants shared a variety of detail which provides a very robust account of what NOM beliefs were held prior to and after a specific experience learning mathematics. The description provided by ePST14 is a prime example of this type of detail, she stated,

> Before coming to college and taking education-based math courses, I believed that math was a static subject that never changed. I was convinced that math was stoic and that there was only one way to do math…After taking education-based math courses, I have changed my way of thinking about math. I have discovered that math is a constantly changing and flexible subject. There is a lot more to math than I had ever believed. (ePST14)

Like ePST14, other participants shared details about their change in NOM beliefs and the experiences they feel contributed to the change. In her mathematics autobiography, ePST8 shared about two separate mathematics experiences which contributed to her change in NOM belief. She stated,

> I didn't enjoy my non-elementary math classes math functions and college algebra I thought they were boring and tedious however I LOVED all of my elementary math classes [like] geometric structures, math structures, [and] primary math…these classes helped me look at math differently as a creative and exploratory subject. (ePST8)

While it is unclear what NOM belief ePST8 held before her experiences in the teacher education program, her response does show how she holds a different NOM belief now having taken the courses. In most cases, the participants shared about their experiences and NOM beliefs in their responses via a comparison of pre-college and college level experiences. A particular case, ePST11, showcases this comparison and the resulting change in NOM beliefs that occurred for this participant. She wrote,

> Conversely, when I reached college and started taking methods classes…, my perspective on mathematics drastically changed and reading books like Mathematical Mindsets [Boaler, 2016] really resonated with me. I learned so much about myself as a student and how to approach math as future teacher. That was one of the few positive and uplifting experiences that I have had with math…[and]…If I were to describe to someone what I think mathematics is now* I would start by telling them what it is not. Math is not simply a list of functions, formulas, and diagrams to memorized and drilled into your head…Rather, mathematics includes a wide range of problems that can be solved through a multitude of strategies. (ePST11)

In each of the cases presented in this section, intertwining of mathematics learning experiences and NOM beliefs is evident. More specifically, the examples showcase the resulting change in NOM beliefs for certain participants based on their experiences learning mathematics. The reader should note that the discovery of the theme of change in NOM beliefs was not a specific goal of this study. This theme arose from the descriptions provided by the participants in the study, in a natural way.

Summary of RQ2 Results

This section presented the results from the data analysis completed to answer the second research question guiding this study. A goal was to gain a deeper understanding of the nature of mathematics beliefs that elementary preservice teachers hold. Five emergent themes were discussed including: (1) mathematics is a subject we learn, (2) mathematics is rules and procedures, (3) mathematics is the study of the world around us, (4) mathematics is a type of puzzle, and (5) mathematics is problem solving through critical thinking. Each theme involved specific examples from the data including excerpts from mathematics autobiographies, open-ended writing prompts, and NOM beliefs drawings. Table 4.4 below outlines the themes and the categories that they are composed of.

Table 4.4 *Themes in Elementary Preservice Teacher Descriptions of Their Nature of Mathematics Beliefs*

Theme	Description
Mathematics is a subject we learn	School subject; Textbook subject; Conceptual subject; Hard/difficult subject; Tedious, memorizing, time intensive subject; a negative experience; a science of number
Mathematics as rules and procedures	A useful tool; rules, formulas, expressions, variables, functions; equations & word problems; algorithms, processes, procedures; solving problems with rules & procedures; number & shape
Mathematics is the study of the world around us	Adaptable, ever changing; explains how and why things happen; sets of real life problems; is all around us; is essential for interpreting the world; a language to understand and communicate about the world
Mathematics is a type of puzzle	Math is a puzzle; connections & patterns; emotions associated with a puzzle (rewarding); whole picture/solved puzzle; a challenging problem
Mathematics is problem solving through critical thinking	requires rigorous thinking; requires multiple methods; requires logic, reasoning; a deep and flexible subject; requires creative thinking

During the analysis, each theme proved to intertwine with other themes and categories. Additionally, categories within themes would overlap providing multiple instances of the category within one case. The examples of mathematics autobiography, and writing prompt excerpts as well as the NOM beliefs drawings were presented to illustrate the dominant themes in the data. Table 4.3 at the beginning of this section highlighted the number of instances that each theme occurred as well as the number of participant responses that fit each theme.

RQ3: Relationship Between Mathematics Experiences and NOM Beliefs

This section presents the details that relate to the third research question of this study. While the last two sections dealt with themes present within the elementary preservice teachers' descriptions of their mathematics experiences and nature of mathematics beliefs, the goal of this section is to present the findings of a correlation analysis completed to determine if a relationship between mathematics experiences and NOM beliefs exists. Recall from chapter three the method for generating magnitude sum scores from the mathematics experience and NOM belief data in the ePSTs mathematics autobiographies, open-ended writing prompts and drawings. The magnitude scores were added together to create a sum score. In a similar way, the scores on the twelve scale items on the NOM beliefs questionnaire were added to create a sum score. The descriptive statistics for each of these sets of sum scores are presented in the following section. Second, a discussion of the assumptions that may have been violated is shared. Finally, the results of a parametric correlation analysis are presented.

Descriptive Statistics

This section presents the descriptive statistics for the mathematics experiences magnitude sum scores, the NOM belief magnitude sum scores, and the NOM belief questionnaire sum scores.

Table 4.5 *Descriptive Statistics for Mathematics Experience and NOM Beliefs Sum Scores*

Descriptive Statistic	Experience Magnitude	NOM Magnitude	NOM Beliefs Questionnaire
Mean	12.6	8.057	46.6
Standard Error	0.909	0.500	1.495
Median	14	8	47
Mode	17	9	52
Standard Deviation	5.375	2.960	8.842
Sample Variance	28.894	8.761	78.188
Kurtosis	-0.973	-0.660	3.878
Skewness	-0.327	-0.001	-1.150
Range	18	11	48
Minimum	3	3	15
Maximum	21	14	63
Sum	441	282	1631
Count	35	35	35

As shown in Table 4.5, Figures 4.27 and Figure 4.28, the mathematics experience magnitude sum scores ranged from 3 to 21. The sum score that occurred most often was 17 according to Figure 4.28. Therefore, the distribution of scores is negatively skewed as confirmed by Table 4.5. Skewness (-0.327) is also evident from the box-plot (Figure 4.27) as the quartiles are not equally spaced. The distribution overall shows that the ePSTs tended to have more positive experiences learning mathematics than negative ones. Table 4.5 also shows the mean (*mean* = 12.6) and standard deviation (*S.D.* = 5.375) for the distribution.

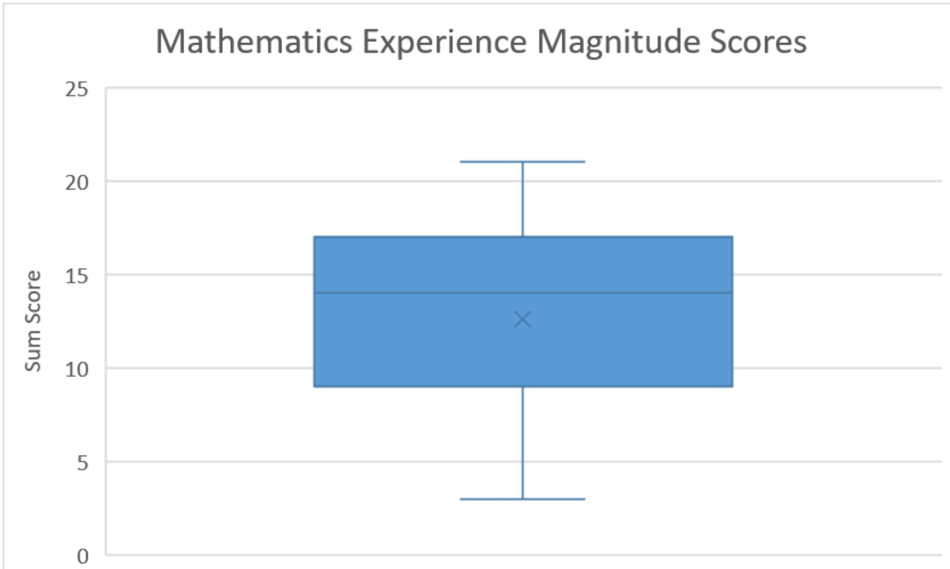


Figure 4.27. Mathematics experience magnitude sum scores box plot

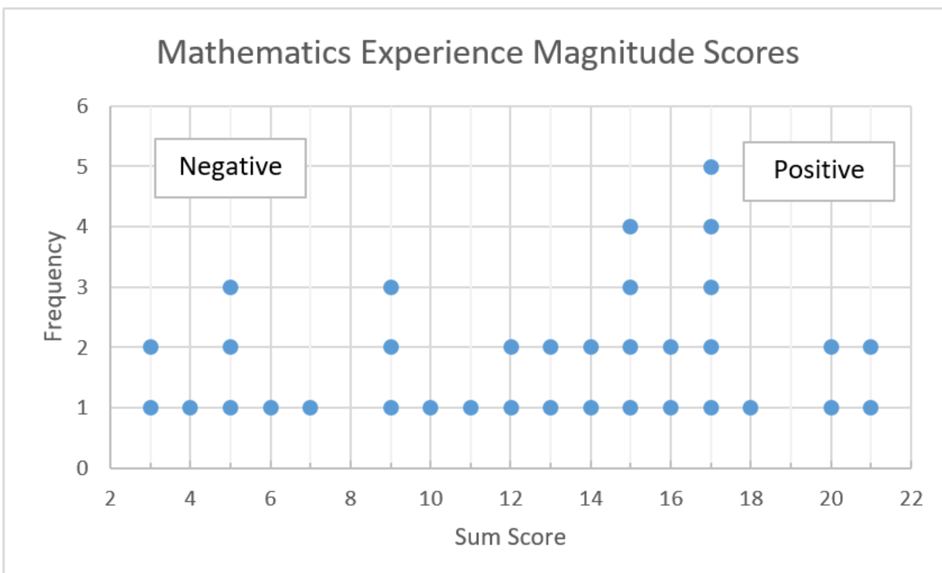


Figure 4.28. Mathematics experience magnitude sum scores frequency distribution

As shown in Table 4.5, Figures 4.29 and Figure 4.30, the NOM belief magnitude sum scores ranged from 3 to 14. The sum score that occurred most often was 9 according to Figure 4.30. The distribution of scores is only slightly negatively skewed as confirmed by Table 4.5. Skewness (-0.001) is hardly evident from the box-plot (Figure 4.29) as the quartiles are nearly equally spaced. The distribution shows that the ePSTs tended to be evenly divided between static beliefs about mathematics and dynamic beliefs. Table 4.5 also shows the mean (*mean* = 8.057) and standard deviation (*S.D.* = 2.960) for the distribution.

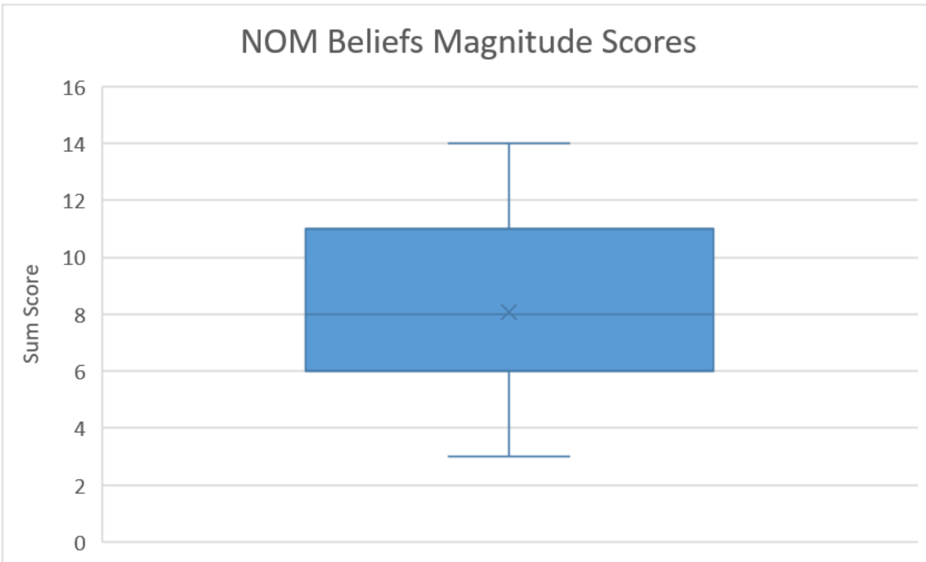


Figure 4.29. NOM beliefs magnitude sum scores box plot

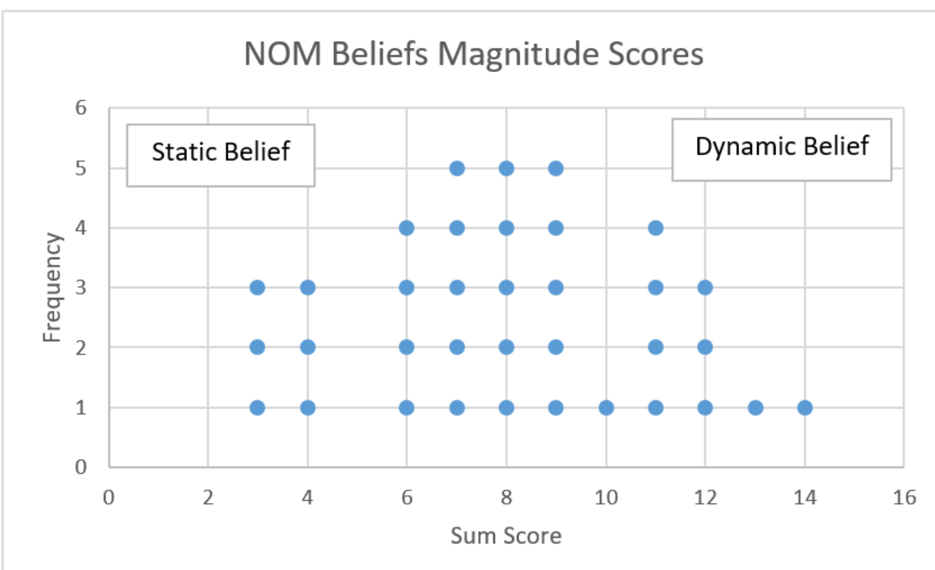


Figure 4.30. NOM beliefs magnitude sum scores frequency distribution

Lastly, shown in Table 4.5, Figures 4.31 and Figure 4.32, the sum scores for the NOM belief questionnaire ranged from 15 to 63. The score that occurred most often was 52, while the median of the distribution occurred at 47 according to Figure 4.32. The distribution of scores is negatively skewed as confirmed by Table 4.5. Skewness (-1.150) is also evident from the box-plot (Figure 4.31) as an outlier is shown past the outer boundary of first quartile. The outlier is a true outlier (Lomax, 2012) and represents a single ePST who expressed more disagreement in general with the subscale items on the NOM beliefs questionnaire. Therefore, the sum score for all of the subscale items for this participant was lower than the rest of the sample. Overall, the distribution shows that ePSTs tended to rate their NOM beliefs more toward the middle of the static-dynamic scale. Table 4.5 also shows the mean (*mean* = 46.6) and standard deviation (*S.D.* = 8.842) for the distribution.

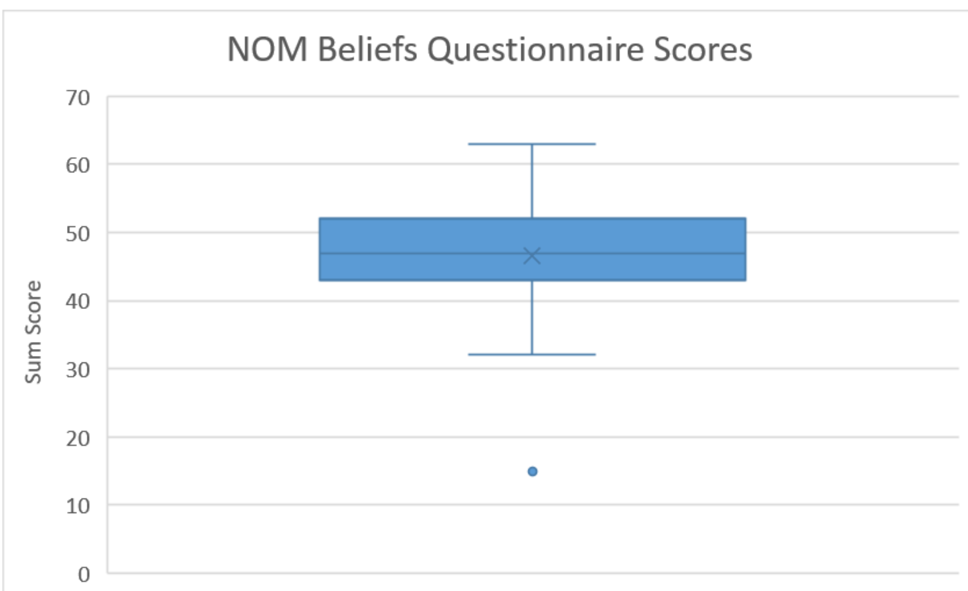


Figure 4.31. NOM beliefs questionnaire sum scores box plot

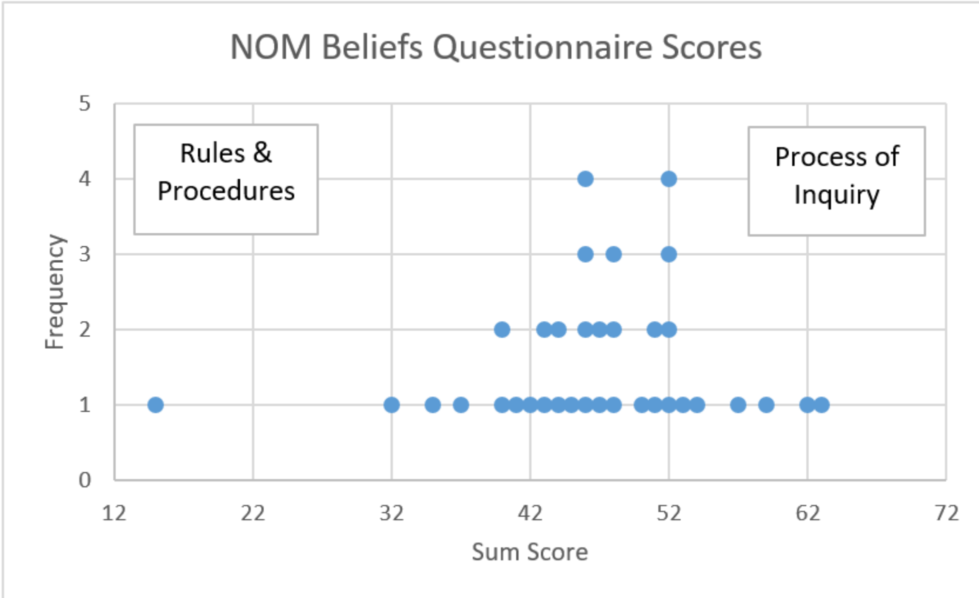


Figure 4.32. NOM beliefs questionnaire sum scores frequency distribution

Assumptions Governing Correlation Analysis

This section presents a discussion of the assumptions that may have been violated in this study. Due to the nature of the third research question, a correlation analysis is the best route to determine if a relationship exists between the mathematics learning experiences and NOM beliefs of the elementary preservice teacher participants. A parametric correlation procedure such as Pearson's product moment correlation is governed by several assumptions (Lomax, 2012). Each of the assumptions is detailed below.

Linearity. The scatter plots of the independent variable (experience magnitude sum score) and the dependent variables, NOM magnitude sum score (Figure 4.33), and NOM beliefs questionnaire sum score (Figure 4.34), indicate that the assumption of linearity is reasonable. Namely, as experience magnitude sum score increases, NOM magnitude and NOM beliefs questionnaire scores increase as well. With Figures 4.33 and 4.34 the linearity assumption is shown to be met.

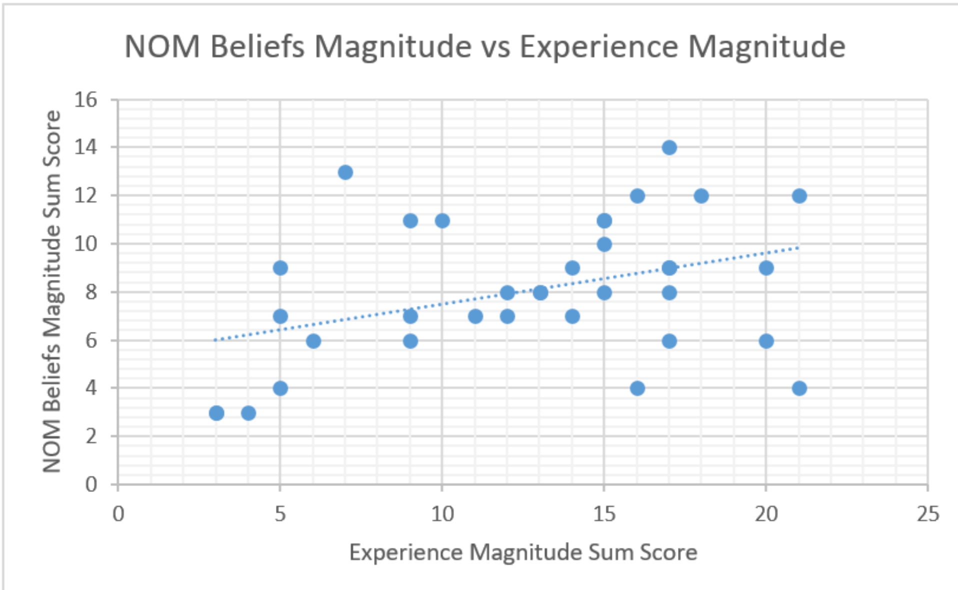


Figure 4.33. Scatterplot of NOM magnitude sum scores versus mathematics experience magnitude sum scores

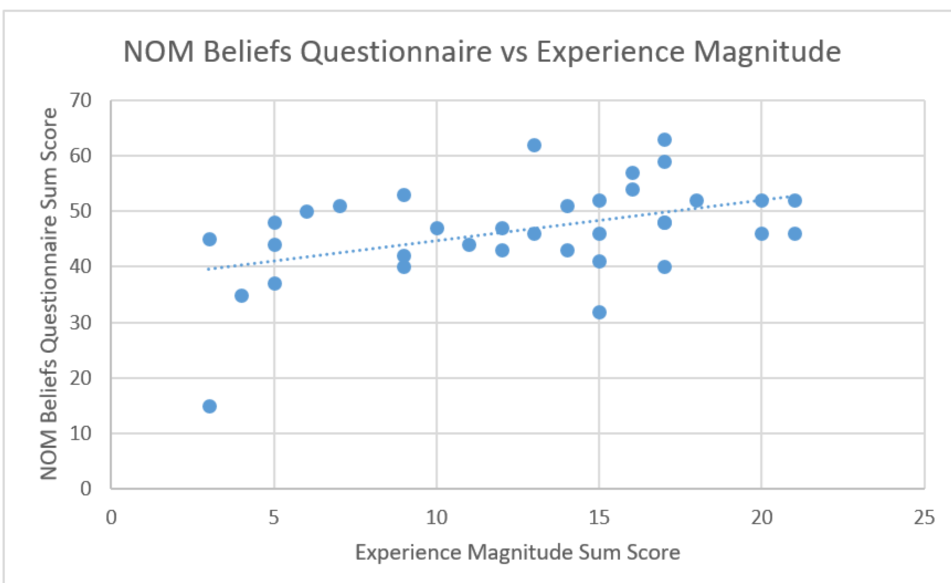


Figure 4.34. Scatterplot of NOM belief questionnaire sum scores versus mathematics experience magnitude sum scores

Normality. The assumption of normality has been tested via examination of the unstandardized residuals and the frequency distributions of the mathematics experience magnitude sum scores, the NOM magnitude sum scores, and the NOM belief questionnaire sum scores. Frequency distributions were previously discussed and skewness and kurtosis values were reported with descriptive statistics in Table 4.5. The unstandardized residuals also showed a small amount of skewness (.108) for NOM belief magnitude residuals and a negative skewness (-0.745) for NOM belief questionnaire residuals. The histograms of the residuals as shown in Figures 4.35 and Figure 4.36 also confirm that the assumption of normality has been met.

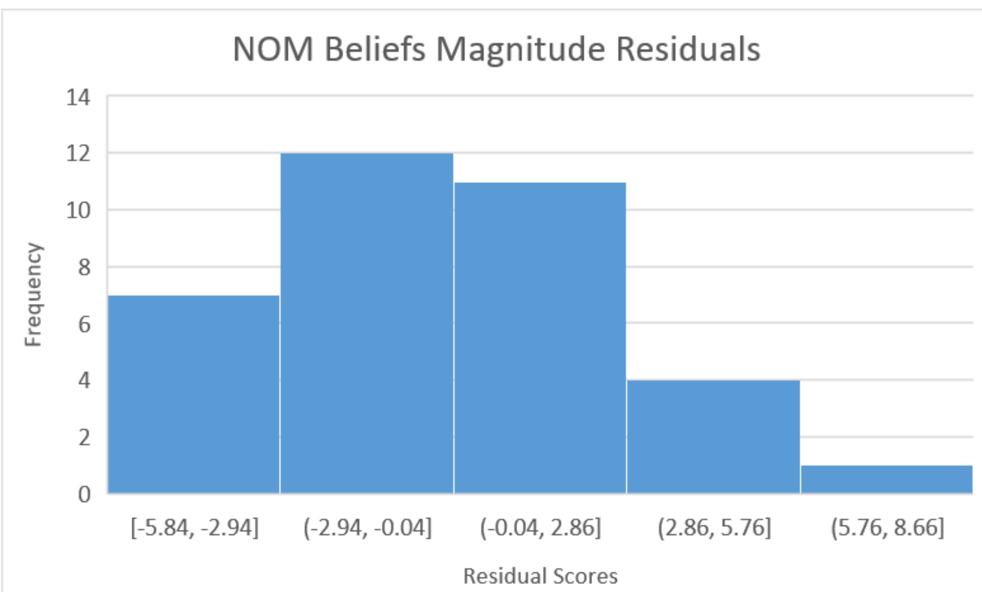


Figure 4.35. Histogram of NOM beliefs magnitude score residuals

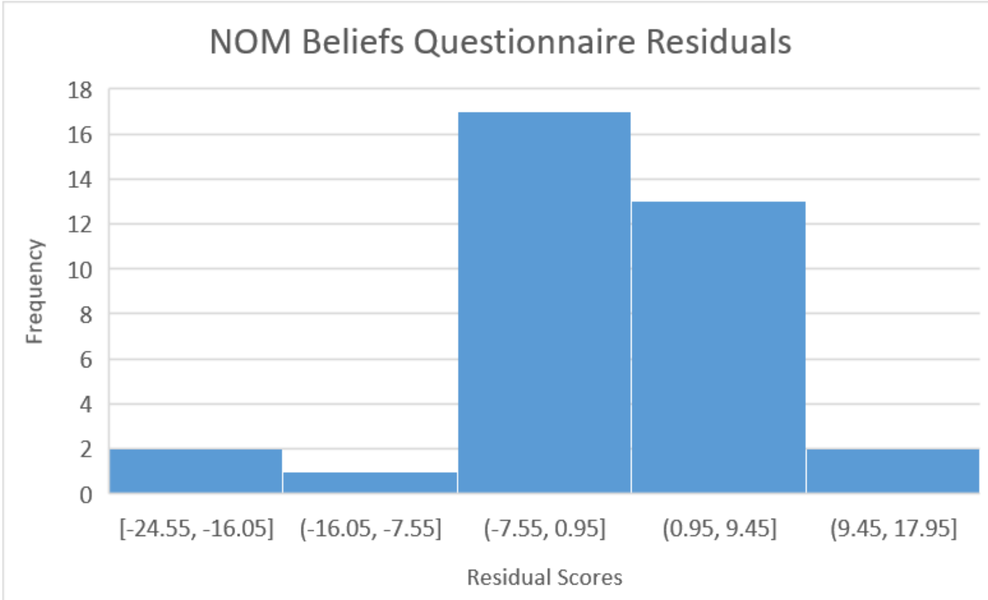


Figure 4.36. Histogram NOM beliefs questionnaire score residuals

Independence. The assumption of normality has been tested via examination of the unstandardized residuals and the random display of points in the scatterplots shown in Figure 4.37 and Figure 4.38. The unstandardized residuals are plotted against values of the independent variable (mathematics experience magnitude sum score). Each of the figures show that the independence assumption has been met for both NOM beliefs magnitude and NOM beliefs questionnaire sum scores.

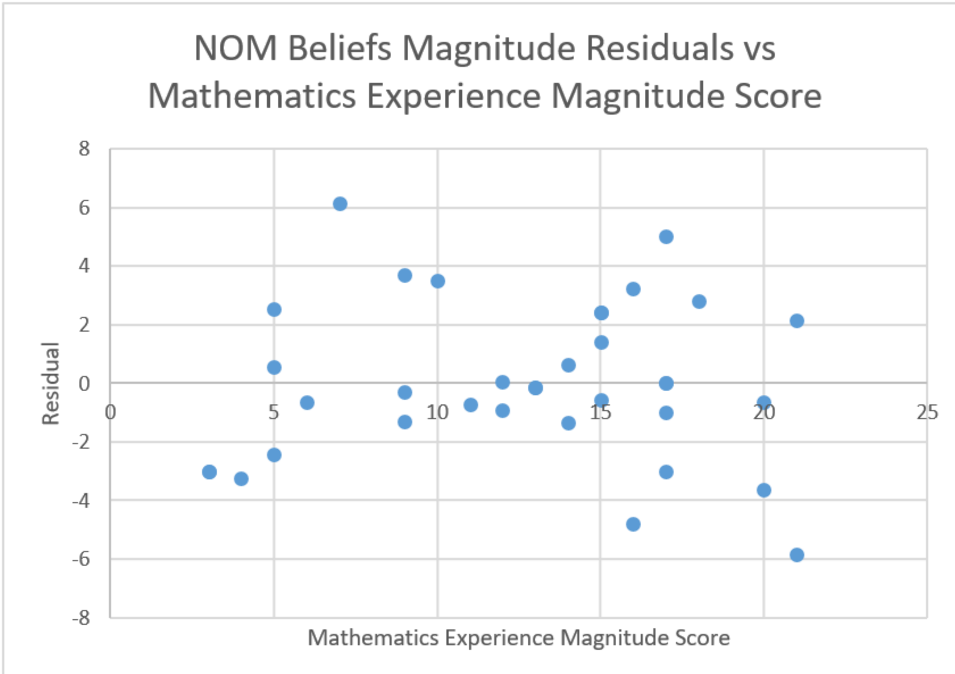


Figure 4.37. Residual Scatterplot: NOM beliefs magnitude residuals versus experience magnitude sum scores

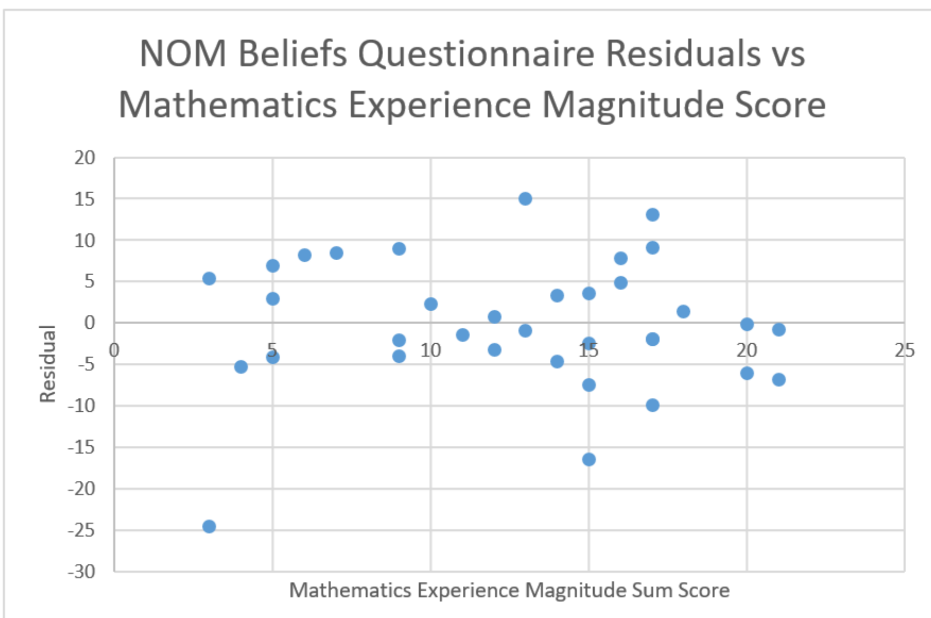


Figure 4.38. Residual Scatterplot: NOM beliefs questionnaire residuals versus experience magnitude sum scores

Homogeneity of variance. The assumption of homogeneity of variance has been met based on the inspection of the scatterplots in the figures above. A relatively random display of points, where the spread of the residuals appears fairly constant over the range of values of the independent variable provides evidence of homogeneity of variance.

Correlation Analysis

This section presents the results of a Pearson correlation analysis completed to determine if there is a relationship between the mathematics learning experiences of elementary preservice teachers and their nature of mathematics beliefs. Recall from chapter three, that while 38 cases were analyzed for RQ1 and RQ2, three cases were removed for the analysis of RQ3 because the participants' NOM belief questionnaire sum scores were arbitrarily low. The scores were low because the participants failed to show their level of agreement with every subscale item on the questionnaire. The total number of cases analyzed for RQ3 was 35.

Experience magnitude vs. NOM questionnaire. The Pearson correlation between mathematics experiences and NOM belief questionnaire scores is .446, which is a weak positive correlation and is statistically different from zero (r = .446, n = 35, p = .007). Thus, the null hypothesis that the correlation is zero was rejected at the .05 level of significance. There is a weak positive correlation between the mathematics experience magnitude sum scores and the NOM beliefs questionnaire sum scores among elementary preservice teachers in this study. The outlier point represented in the NOM beliefs questionnaire data was a true outlier, therefore the Pearson correlation analysis was rerun without the outlier to understand the effect it exerted on the analysis results (Lomax, 2012). The Pearson correlation coefficient was calculated to be 0.34 with a *p-value* = .049. Without the outlier, the positive weak correlation (Cohen, 1988) still exists and is just barely significant at an alpha level of .05.

Experience magnitude vs. NOM beliefs magnitude. Similarly, the Pearson correlation between mathematics experiences and NOM magnitude scores is .39, which is a weak positive correlation, and is statistically different from zero (r = .386, n = 35, p = .022). Thus, the null hypothesis that the correlation is zero was rejected at the .05 level of significance. There is a weak positive correlation (Cohen, 1988) between the mathematics experience magnitude sum scores and the NOM magnitude sum scores among elementary preservice teachers in this study.

NOM beliefs magnitude vs. NOM beliefs questionnaire. Note that the correlation between NOM magnitude scores and NOM beliefs questionnaire scores is .396, which is a weak positive correlation and is statistically significant from zero (r = .396, n = 35, p = .018). While not a relationship that is specific to answering RQ3, this significant correlation shows that NOM belief magnitude scale is linearly related to the NOM beliefs questionnaire scale meaning both scales represent the same direction. Namely, the static end of the NOM beliefs magnitude scale can be related to the 'rules and procedures' (NOM-RP) end of the NOM beliefs questionnaire. Likewise, the dynamic end of the NOM beliefs magnitude scale can be related to the 'process of inquiry' (NOM-PI) end of the NOM beliefs questionnaire.

Summary of RQ3 Results

A Pearson correlation coefficient was computed to determine if there is a relationship between the mathematics learning experiences of ePSTs and their beliefs about the nature of mathematics. The test was conducted using an alpha of .05. The null hypothesis was that the relationship would be zero. The assumption of independence was met via random selection and via an inspection of the unstandardized residuals in Figure 4.37 and Figure 4.38 above. The assumption of linearity was reasonable given a review of a scatterplot of the variables (see Figures 4.33 and 4.34 above). Results of the correlation analysis are summarized in Table 4.6.

Table 4.6 *RQ3 Correlation Analysis Summary of Results (N = 35)*

		Mathematics Experience Magnitude	NOM Beliefs Magnitude	NOM Beliefs Questionnaire
Mathematics Experience Magnitude	Pearson correlation	1.00	0.386*	0.446*
	Sig. (two-tailed)		.022	.007
NOM Beliefs Magnitude	Pearson correlation	0.386*	1.00	0.396*
	Sig. (two-tailed)	.022		.018
NOM Beliefs Questionnaire	Pearson correlation	0.446*	.396*	1.00
	Sig. (two-tailed)	.007	.018	

*Correlation is significant at the 0.05 level (two-tailed)

Chapter Summary

This chapter presented the findings of the analysis by research question. The research questions investigated in this study were:

1. *What are elementary preservice teachers' past experiences with mathematics?*
2. *What are elementary preservice teachers' nature of mathematics beliefs?*
3. *In what ways, if any, are the mathematics experiences of elementary preservice teachers related to their beliefs about the nature of mathematics?*

To answer the first research question, themes that emerged from analytical triangulation of the mathematics autobiographies, open-ended writing prompts, and mathematics experience drawings were discussed. In a similar way, the second section presented the results of analytical triangulation of the datasets for research question two. Finally, the means and standard deviations as well as the results of the correlation analysis to answer the third research question was presented in the third section of this chapter. The results showed that there was a significant relationship between past experiences learning mathematics and the beliefs held about the nature of mathematics among the participants of this study. The next chapter will outline the integration and interpretation of these results.

CHAPTER V

DISCUSSION

This convergent parallel mixed methods research study used both qualitative and quantitative data to ascertain the beliefs about the nature of mathematics (NOM) that prospective elementary teachers hold and how their mathematical learning experiences may be related to these beliefs. Chapter five describes connections that can be made with the results of this study. Specifically, this section outlines how this research can be applied practically within the current era of mathematics education reform. In sum, the goal of the study was to answer the following research questions:

1. *What are elementary preservice teachers' past experiences with mathematics?*
2. *What are elementary preservice teachers' nature of mathematics beliefs?*
3. *In what ways, if any, are the mathematics experiences of elementary preservice teachers related to their beliefs about the nature of mathematics?*

One goal of this chapter is to connect the answers to these three research questions to the existing literature on the topics of mathematics learning experiences for preservice teachers, and their beliefs about the nature of mathematics. Another goal of this chapter is to interpret the findings in light of existing research and to situate the findings inside the hypothesized personal construct (PC) theory described in chapter one.

Elementary Preservice Teachers' Mathematics Learning Experiences

This section draws connections between the mathematics learning experiences shared by the participants in this study and what the existing research literature says about mathematics learning experiences. Recall that the mathematics learning experiences of the elementary preservice teachers (ePSTs) in this study were characterized by five themes as discussed in chapter four: (1) attitudes toward learning mathematics; (2) perceptions of instruction; (3) perceptions of mathematical understanding; (4) perceptions of success; and (5) focus on speed and memorization.

The Prevalence of Emotion

The most prominent theme characterizing the mathematics learning experiences of the elementary preservice teachers in this study was the theme of attitudes toward learning mathematics. The majority of participants (92%) shared about how they felt or the attitude they developed at various moments during their mathematics education. This emotional connection to experiences learning mathematics is supported by research (Di Martino & Zan, 2011; Nespor, 1987; Op't Eynde, De Corte, & Verschaffel, 2002; Quinn, 1997; Wilkins, 2008) and provides a way to understand the emotional intensity to which the experiences learning mathematics affected the participants in this study. In many cases, words chosen by the participants to elaborate on the emotions or attitudes they experienced served to communicate the intensity of a negative experience. Several participants expressed hate toward particular content courses. Emotional wording such as this is strongly negative in intensity and reveals negative elements of the experience. Other ePSTs described feeling frustrated during their learning due to confusion, mismatch of learning style with teaching style, being pressed for time, focus on memorization, or a lack of appreciation of how math was useful for them. Frustration and confusion together reveal additional negative emotional components of the experience. Sheila Tobias (1993), author of *Overcoming Math Anxiety*, showcases this frustration and confusion and how it can bring about severe anxiety about mathematics—a very negative state of emotion. Other researchers (e.g. Di Martino & Zan, 2011; McDuffie & Slavit, 2003) indicate how participants in their studies also feel negatively because their mathematics experience was characterized by rules and procedures.

In the same way, emotional intensity is indicated in the positive emotional wording used to describe a mathematics learning experience. Participants chose words to indicate the intensity of the positivity in their mathematics learning experiences when some described their love for a particular math teacher, content, or form of instruction. Emotional wording such as this is strongly positive in intensity and reveals positive elements of the experience. They expressed experiencing enjoyment for particular types of math work, like memorizing math facts, or the successful feeling that came with good grades on a math test. These positive emotional associations are supported by research. Several studies showcase how some preservice teachers associate positive emotions with mathematics learning experiences that they had which included getting good grades (McGinnis, et. al., 2003), loving a teacher or the way a teacher taught (Goos, 2006) or the positive feeling that math always came easy for them (Cooney, et. al., 1998).

While the extremes of the emotional intensity were easily deciphered, each mathematics learning experience held within it a certain level of positive *and* negative emotional connotation. For some participants, frustration from a challenging mathematics problem was described with positive emotional wording (e.g. they enjoyed the challenge even though the problem brought frustration). However, in the same description, the participant might also describe the feelings of disappointment when a good grade was not achieved on a particular mathematics assignment. In this way, several participants fell into the middle range of emotional intensity with 'somewhat' negative or positive interpretations. This complexity and intertwining of contradictory emotions within the same mathematics experience is also evident in the existing research literature. Goos (2006) reports that her preservice teachers often shared both positive and negative emotions when describing their past mathematics learning experiences.

In virtually every way, emotional descriptors were used to showcase the variety of mathematics learning experiences that governed the past mathematics education for the ePSTs. This result supports the prominent nature with which emotions can characterize a mathematics learning experience (DiMartino & Zan, 2011; Tobias, 1993). In addition, the results show how emotions are connected to mathematics learning (DiMartino & Zan, 2011; Maab & Scholgmann, 2008; Nespor, 1987). In fact, the learning of mathematics cannot be completely understood and analyzed as a "purely cognitive process" (Maab & Schloglmann, 2009, p. vii).

Mathematics Learning Experiences: Red Flags

In conjunction with the significance of the emotional component of the mathematics learning experiences of the participants in this study, there are some real concerns for the participants illuminated by the other themes that characterize their experiences. These themes deal with ePSTs perceptions of instruction, perceptions of mathematics understanding, perceptions of success and a focus on speed and memorization. The section below showcases how the experiences described by the participants are not aligned with recent efforts to reform mathematics education and how positive emotional association with these experiences raises concerns for the instructional practice of the preservice teachers in this study.

Inconsistency with reformed experiences. Aside from the theme of attitude toward learning mathematics, the mathematics learning experiences of the participants in this study were characterized by four other themes that seem to be inconsistent with recommendations for reformed mathematics instruction (Boaler, 2014; Leinwand et. al., 2014; NCSEAD, 2018; NCTM, 2000; NCTM, 2018). The theme of perceptions of instruction describes components of the mathematics instruction the participants believed they experienced. In the cases where ePSTs describe their perceptions of the instruction, many write about or draw teacher-directed classrooms. The teachers stand in front of the class and make assignments or give instructions, and show how to do mathematics problems rather than engage the ePSTs in active learning (Boaler, 2014; Leinwand et. al., 2014; NCTM, 2000). A reformed mathematics learning experience should be governed by inquiry-based teaching where students are encouraged to discover mathematics within a real-world context (Boaler, 2014; Leinwand et. al., 2014; NCTM, 2018). However, preservice teachers who have not had such experiences learning math are still motivated by the experiences they did have (Nespor, 1987). Whether they classify the experiences as positive or negative, they will continue to make decisions about what they learn and teach based on these experiences (Nespor, 1987; Wilkins, 2008).

The theme of perceptions of mathematical understanding deals with elements of the experience that relates to understanding (or not) the mathematics content being taught or learned. Many participants described their mathematics learning experiences in terms of their degrees of confusion. Several indicated that they were confused because they did not understand the concepts, while others indicated their confusion stemmed from the type of instruction they received (e.g. the teacher would not show them how to do the problem, or the teacher would not help them understand). Reformed mathematics instruction serves to build mathematically proficient students that can demonstrate conceptual understanding of the mathematics concepts they learn (Leinwand et. al., 2014) and when preservice teachers have experiences that involve misunderstanding math, the experiences are characterized in a negative way and could still influence the value PSTs place on the math they teach (Op't Eynde, De Corte, & Verschaffel, 2002).

The theme of perceptions of success deals with the ideas participants had of how successful they were in their mathematics learning. Many participants described their mathematics learning experiences in terms of their grades. Several wrote about or drew pictures of "A's" or "F's" on their math papers and some shared about how they never made good grades in math class. Motivation to do well in math class should not be measured by grades alone. Motivation to do the work of learning should be present in the learning environment and visualized in the active engagement of the students doing the work (Boaler, 2014; NCSEAD, 2018). However, some individuals are motivated by external rewards, like grades, and consider experiences such as earning good grades with positive emotions. Research shows that positive and negative experiences influence beliefs (McDuffie & Slavit, 2003, McGinnis, et. al., 2002). Namely, a positive experience equates to good grades and mathematics is easy, while a negative experience equates to bad grades.

Finally, the theme of focus on speed and memorization serves to characterize direct references to elements of mathematics learning experiences that involved speed and memorization. While some of the ePSTs shared that the speed and memorization drills were fun, several participants shared about how the drills made them equate speed at math with being smart. Reformed mathematics instruction moves away from the dependence on speed in remembering math facts. Built on the idea that mathematics learning occurs at different rates for different students, reformed mathematics instruction focuses on building mathematics proficiency which includes procedural fluency and adaptive reasoning rather than speed and memorization (Boaler, 2014; Leinwand et. al., 2014). More importantly, mathematics learning experiences that focus on speed and memorization do not allow students to see the "joy, wonder and beauty of mathematics" (NCTM, 2018, p. 9) because the singular idea that math is facts and rules to be remembered inhibits this growth (McGinnis et. al., 2002; Presmeg, 2002). Secondly, while some ePSTs indicated they did not have a positive experience when asked to be fast or to remember, several ePSTs shared that these sorts of experiences were positive episodes for them. The importance of this finding for the participants in this study is concerning due to the research that supports how these experiences will lead to the formation of beliefs consistent with static NOM beliefs and those static NOM beliefs could potentially influence instructional choices. Additionally, the findings of this study with respect to the themes that characterize the mathematics learning experiences do not seem to align with reformed teaching and learning as defined by the research community (Boaler, 2016; Leinwand, et. al., 2014, NCTM, 2018).

A concern for preservice teachers' practice. Despite the contrasting nature of the types of mathematics learning experiences described in this study with those aligned with reformed mathematics teaching, the general trend among the participants was more positive in regard to the intensity of the emotions associated with these experiences. While both negative and positive emotions were used to describe experiences, more participants regarded their mathematics learning experiences as positive experiences. As shown in Figure 5.1, there are more participants who report a more positive emotional association with mathematics learning experiences. This finding agrees with research that when they reflect, prospective teachers share both positive and negative emotions when considering their past experiences (Goos, 2006; Cooney, et. al. 1998; Nespor, 1987) however the positive experiences are more likely to influence beliefs and future actions (Cooney, et. al. 1998; Nespor, 1987). Recall from chapter three that each case consisting of one mathematics autobiography, one open-ended writing prompt, and one experience drawing was scored using a magnitude scale from 1 to 7 with 1 indicating a strongly negative experience and 7 a strongly positive experience. The three scores were summed and the sum score is reported below in Figure 5.1. Each dot represents the experience magnitude sum score for one elementary preservice teacher participant. The general trend shows more ePSTs on the positive end of the scale and in comparison, 12 ePSTs are in the positive to strongly positive range (16 to 21), while 8 are in the negative to strongly negative range (3 to 8).

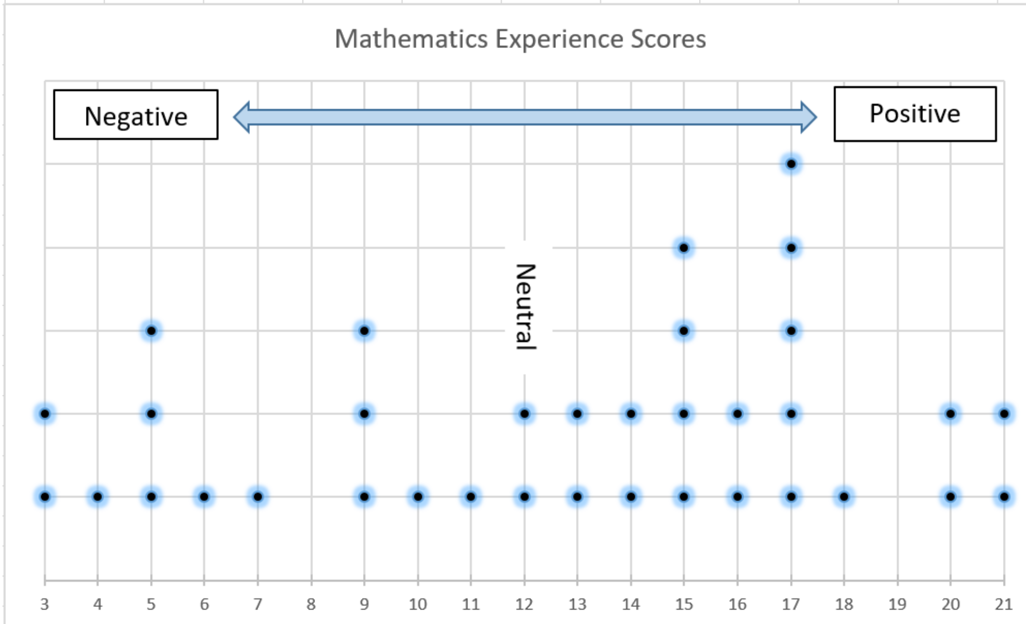


Figure 5.1. Mathematics experience magnitude sum scores for every participant

When considering the trend in Figure 5.1 and the participants' descriptions of their mathematics learning experiences, what is concerning for this sample of preservice teachers is the positive emotional association with non-reform oriented mathematics learning experiences. Namely, the participants in this study are elementary preservice teachers which means that they will or have just recently entered the classroom as full-time professional educators. The data reported here indicate the they will be taking the aforementioned mathematics learning experiences into the classroom with them (Goos, 2006). Research indicates that, preservice teachers use their prior experiences, both positive and negative, to make instructional decisions (Cooney, et. al., 1998; Nespor, 1987; Wilkins, 2008). In effect, these preservice teachers will teach the way they have been taught. While there is no argument that positive experiences learning mathematics are desired, the types of experiences described by the participants in this study are inconsistent with the mathematics reform effort and therefore raise concerns for the potential learning opportunities designed by these new educators. If indeed, these participants pull from their past experiences learning mathematics based off of the positive emotional association they have, the learning opportunities they design will consequently be inconsistent with reformed mathematics instruction and potentially harmful to many students. These learning opportunities, while positive in the minds of the participants, will serve to further remove students of the future from the goal of mathematical proficiency (Leinwand et. al., 2014).

To answer the first research question, this study has revealed the mathematics learning experiences of a group of elementary preservice teachers. While their experiences were associated with more positive emotions than negative, the experiences that were described do not align with reformed mathematics experiences.

Elementary Preservice Teachers' Beliefs about the Nature of Mathematics

This section draws connections between multidimensional NOM beliefs shared by the participants in this study, and what existing research says about how these beliefs are derived from experiences with mathematics. In chapter four the themes that characterized the NOM beliefs of the participants in this study were presented. The themes were (1) mathematics is a subject we learn, (2) mathematics is rules and procedures, (3) mathematics is the study of the world around us, (4) mathematics is a type of puzzle, and (5) mathematics is problem solving through critical thinking. In this section, each theme represents a distinct dimension of the NOM beliefs held by the sample of participants in this study.

NOM Beliefs Are Multidimensional

The elementary preservice teachers in this study were asked in four distinct ways to describe what mathematics is to them. The mathematics autobiography asked them to describe what they would say to a friend if they were trying to describe what mathematics was. The open-ended writing prompt asked the question ‘what is mathematics’, and the drawing instrument asked them to draw a visual metaphor and explain in words how the drawing represented what mathematics is. The fourth measurement occurred in the form of the survey instrument where participants had to self-report their agreement with statements made about the nature of mathematics. Generally, the themes show a multidimensional characteristic that can align with different and even complimentary NOM beliefs held by the participants. The majority (87%) believe that mathematics is a subject we learn in school that requires tedious memorization. Likewise, just over 75% of participants believe that mathematics is rules and procedures or solving problems with these tools. NOM beliefs such as these have been characterized in the research literature as static beliefs (Grigutsch & Torner, 1998). Often these rules and procedures are what is learned in mathematics classrooms in school so that the tools might be employed to solve problems from a textbook (Dossey, 1992; Grigutsch & Torner, 1998). These static NOM beliefs are just two dimensions that were shared by the ePSTs in this study. A third dimension is evident when several participants (58%) shared about how they believe that mathematics is the study of the world around them where it acts as a language to allow them to communicate about how and why things work. The fourth dimension comes to light when half of the preservice teachers shared that they believe mathematics is problem solving through critical thinking, a deep and flexible subject that requires multiple methods and creative thinking. In the existing research literature, NOM beliefs such as these are classified as dynamic because they present mathematics as an ever-changing creative discipline, a process of inquiry that begins with a problem and creates a new pathway to the solution (Grigutsch & Torner, 1998).

In effect, the participants in this study hold NOM beliefs that are complementary viewpoints of what mathematics is (Grigutsch & Torner, 1998; Op't Eynde & De Corte, 2003; Yang & Leung, 2015;). These four dimensions of NOM beliefs expressed by the participants showcase particular characteristics of what mathematics is. The interesting thing is that these distinct beliefs about math are complementary and held at the same time. In other words, an individual elementary preservice teacher could express that they believe math is rules and procedures and also a process of inquiry. This sort of multiplicity inside NOM beliefs is supported by existing research studies (Grigutsch & Torner, 1998; Op't Eynde and De Corte & Vershaffel, 2003; Yang & Leung, 2015). Figure 5.2 below illustrates the dual dimensions of participants' NOM beliefs when they had to self-report their NOM beliefs on the NOM beliefs questionnaire. Participants were asked to show agreement or disagreement with statements that characterized math as rules and procedures or as a process of inquiry (Tatto, 2013). Most of the scores fell in the range of 42 to 52, and to earn a score in this range, a participant would have to indicate they agree with both views that math is rules and procedures and a process of inquiry. The number of participants (on the vertical axis) shows how many ePSTs scored in the range of 42 to 52. This finding is important because it shows how when ePSTs have to choose between two ends, the participants in this study placed themselves in the middle of the continuum. Research supports this finding for other groups (i.e. university students) and preservice teachers (Yang & Leung, 2015; Yusof & Tall, 1999) where both views were indicated by study participants as valid in their minds.

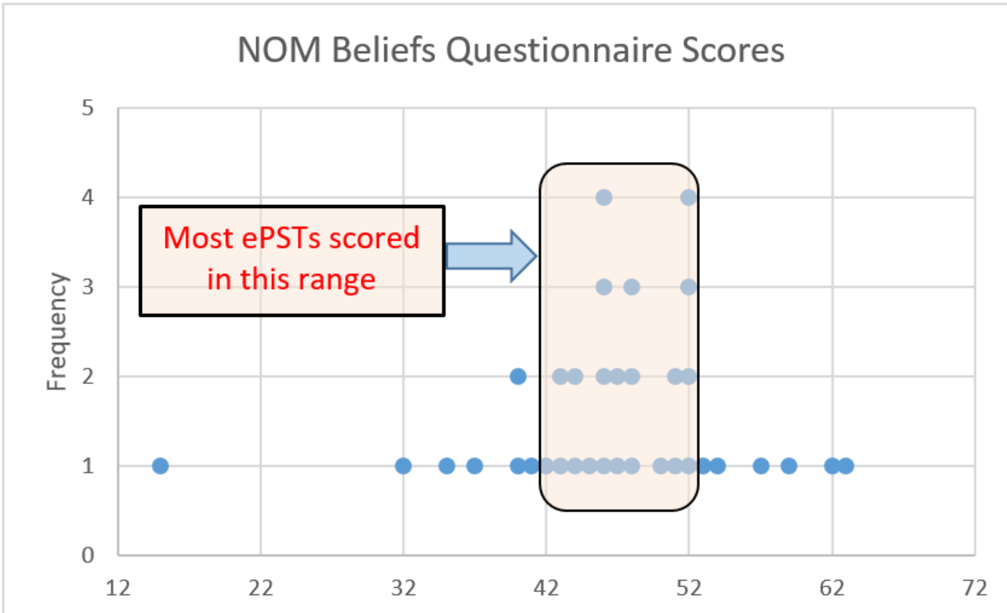


Figure 5.2. Frequency plot of NOM beliefs questionnaire sum scores

While static and dynamic NOM beliefs reside on opposite ends of the NOM beliefs continuum, the space between these complementary ends includes various other NOM beliefs. For example, NOM beliefs that align with mathematics is a creative discipline (Boaler, 2016), mathematics is posing questions (Hersh, 1999), or mathematics is the science of patterns (Devlin, 1997) are all views that can be represented on the continuum. But no one view can win out, they all have to coexist for an individual to have a truly dynamic NOM belief. Believing one (i.e. math is only the science of patterns or only rules and procedures) is an indicator of a position on the NOM continuum closer to the static end, dynamic NOM beliefs are multidimensional including many distinct characteristics of mathematics while leaving room to grow and add more.

Adding dimensions to one's NOM belief means moving more toward the dynamic end of the continuum. Much of the existing research looks that the duality of NOM beliefs (Tatto, 2013; Yang & Leung, 2015; Yusof & Tall, 1999) asking if NOM beliefs are static or dynamic or both. One study investigated NOM beliefs in terms of the three views defined by Ernest in 1989 (Shilling & Stylianides, 2013). Another study investigated twelve different characteristics or themes of mathematics to analyze NOM beliefs of preservice teachers (Weldeana & Abraham, 2014). However, based on the results of this study, evidence exists to support the fact that NOM beliefs are multidimensional and some beliefs fit better in the space between the static and dynamic ends of the NOM continuum.

The notion that the middle of the NOM beliefs continuum has room for other distinct beliefs is supported by one theme in this study that 'mathematics is a type of puzzle'. This theme in effect adds a fifth dimension to the others previously discussed. Many elementary preservice teachers shared that they believed that mathematics is a type of puzzle. Indeed, over half of the mathematics autobiography responses and approximately 45% of the NOM beliefs drawings indicated this idea. The participants shared that because mathematics involves connections and patterns, it is like a puzzle that requires thinking and engagement to solve. The participants indicated that when solved, a puzzle reveals the whole picture in much the same way solving a mathematics reveals the solution. Other studies have shown this type of wording used to describe NOM beliefs (Goos, 2006), however, no other details are shared with regard to how this NOM belief (i.e. math is a type of puzzle) might fit on the continuum between static and dynamic. Due to this lack of existing research it seems appropriate to suggest placement of this NOM belief between the static and dynamic end of the continuum somewhere near the middle or just shy of the middle. More research is necessary to adequately place this NOM belief as it seems more than only static but not as multidimensional as a dynamic NOM belief can be. However, the prevalence of this theme in the data adds to the complex nature of the preservice teachers' beliefs about mathematics (Speer, 2008; Weldeana & Abraham, 2014).

To answer the second research question, this study has defined the NOM beliefs for a group of elementary preservice teachers. The themes that characterize the ePSTs NOM beliefs showcase multiple dimensions where both static and more dynamic views are held concurrently.

The Relationship of Experiences to Beliefs

One of the goals of this study was to determine if a relationship existed between the mathematics learning experiences and the nature of mathematics beliefs of a sample of elementary preservice teachers. Considering the themes previously discussed, this study found that the mathematics learning experiences for the ePSTs were non-reform oriented, focusing on mathematics that is rules and procedures that required time for memorization and performance measured by grades. In consideration of the themes which characterize the ePSTs' NOM beliefs, the most prevalent theme—mathematics is a subject we learn—was mentioned or described by 87% of the participants in their mathematics autobiographies, and 61% of the participants' drawings. The second most prevalent theme with respect to NOM beliefs was the theme of mathematics as rules and procedures. Over three-quarters of the participants described aspects of this theme in their mathematics autobiographies, while 34% drew a representation of the theme. The experiences outlined by the preservice teachers were non-reform aligned which means the experiences were the type described by Leinwand, et. al. (2014) as unproductive toward building dynamic NOM beliefs. Indeed, the prevalent dimensions of the NOM beliefs represented in this study are aligned to the more static end of the continuum. Additionally, a Pearson correlation analysis yielded a statistically significant weak positive correlation ($r = .39$, $n = 35$, $p < .05$) between the emotional intensity of the mathematics learning experiences and the NOM beliefs of the ePSTs. Similarly, a statistically significant weak positive correlation ($r = .45$, $n = 35$, $p = .007$) was found between emotional intensity of experiences and NOM beliefs questionnaire scores. These findings indicate that in a small way emotionally positive mathematics learning experiences relate with more dynamic NOM beliefs, while emotionally negative experiences relate to more static NOM beliefs. Adding this empirical link to the qualitative results seems to indicate that for the sample of elementary preservice teachers who participated in this study, their NOM beliefs are related to their experiences learning mathematics. Further support for this link is provided by a plethora of research studies that have helped to develop a consensus that beliefs evolve through a process that includes all experiences with mathematics during a person's life (Leinwand, et. al., 2014; Maab & Schloglmann, 2009; Wilkins, 2008).

Further support for the relationship between mathematics learning experiences and NOM beliefs is evidenced by the change in NOM beliefs that can happen with a given type of experience. At multiple points in the qualitative data, several participants described their beliefs about mathematics changing as a result of their experience in a particular class in their teacher education program. The instructor for this class incorporated reading literature that defined what NOM beliefs were and how to cultivate these NOM beliefs in students. In addition, the instructor required the ePSTs to reflect on their mathematics life history via the mathematics autobiography assignment. The ePSTs described that before taking this class, they used to believe that mathematics was only rules and procedures to memorize and know for a test. They reported a change to more dynamic NOM beliefs after their exposure to course content and reflective opportunities that illuminated the importance of NOM beliefs and how experiences can be designed for students to build dynamic NOM beliefs. Research supports this change in NOM beliefs based on experiences designed to build dynamic NOM beliefs (Boaler, 2016; Leinwand, et. al., 2014; NCTM, 2018). In several studies that involve preservice teachers, interventions were designed as the experience necessary to change the static NOM beliefs of preservice teachers (PSTs) to more dynamic NOM beliefs and the results of these studies indicate that the interventions were successful (Shilling & Stylianides, 2013; Weldeana & Abraham, 2014; Yusof & Tall, 1999). Other studies show that giving PSTs and inservice teachers opportunities or reflect and relate their beliefs to their practice can bring about changes in NOM beliefs (Boaler, 2016; Cooney, et. al, 1998; Goos, 2006; McDuffie & Salvit, 2003; Mewborn & Cross, 2007).

In conclusion, the findings reported here along with the corresponding interpretations seem to work well with the personal construct (PC) theory that was used in this study. PC theory acted as the theoretical framework and combined the ideas of John Dewey (1938) and Markku Hannula (2006). PC theory models the perceived relationship between mathematics learning experiences and the formation of NOM beliefs but also leaves room for the input of individual differences from within (i.e. beliefs about self) and those between (i.e. beliefs about social context). In several ways, the findings of this study have verified the relationship between mathematics learning experiences and nature of mathematics beliefs. By showcasing the mathematics learning experiences and NOM beliefs of the ePSTs in this study, a connection between experiences and beliefs could be illuminated as in the hypothesized model. However, the limitations of this study will serve to mitigate the applicability of the PC theory in terms of the complexity and power that other factors, like individual differences, might exert on the relationship between experiences and beliefs.

Limitations

Every research study has a set of limitations or potential weaknesses that are beyond the control of the researcher after a plan of study is formed (Simon, 2011). Initial choices regarding the study are made with respect to the research topic, worldview and physical limitations (e.g. available time, funding, and location etc.). These choices dictate a set of limitations that must be clearly outlined. This section will outline the limitations of this study.

1. The sample of elementary preservice teachers in this study was purposefully and conveniently chosen even though random selection for correlation analysis was employed. This selection strategy effectively limits the generalizability to the entire population of elementary preservice teachers.
2. Another limitation of this study is the researcher bias with respect to positive and negative mathematics experiences. This researcher views positive mathematics

experiences as those aligned with reformed mathematics teaching (Leinwand, et. al., 2014). Additionally, negative experiences were viewed as those not aligned with reformed teaching. There is a possibility this bias influenced the data analysis in this study, however, data analysis was checked by a second reader in order to mitigate this.

3. The researcher relied on participants' use of detailed language to describe past experiences and the memory recall necessary to outline the details.
4. The researcher relied on participants' use of detail language to describe beliefs about mathematics which is a subject not discussed often and articulated even less often.
5. While the Pearson correlation coefficient was statistically significant (in all cases that were run), the weak positive relationship indicates there are likely other factors at play in the development of NOM Beliefs with respect to mathematics learning experiences. Indeed, beliefs are highly complex, interwoven with attitudes, values and emotions (Nespor, 1987; Op't Eynde, De Corte, & Verschaffel, 2002).
6. There was a certain amount of participation bias due to the use of financial incentive as payment for time spent participating in the study. Preservice teachers who respond to the questionnaire were entered into a drawing to win a ten-dollar gift card.

Implications for Research

This study has shown that the mathematics experiences of preservice teachers are not consistent with those they need to develop a dynamic NOM belief. Indeed, the weak positive correlation illuminates the fact that their experiences have at least in some significant way served to build the NOM beliefs they do have. In light of the fact that in-service teachers have to make so many decisions in the classroom (Francis, 2015; Mewborn & Cross, 2007; Wilkins, 2008), a strongly dynamic NOM belief is necessary to cut across instructional choices that might be hindered by administrative drag or classroom disruptions. In fact, Francis (2015) was able to show how beliefs about classroom and behavior management, and relationships with parents and administration served to mitigate the dynamic NOM beliefs and reformed aligned teaching practices that inservice teachers subscribed to. While the ePSTs in this study did show some dynamic NOM beliefs, it is unlikely that these beliefs are strong enough to override the positive emotional association the ePSTs showed for their non-reformed experiences. Therefore, the future lessons they plan may not be orchestrated based on the foundation of a dynamic NOM belief to create an environment where students experience the joy and beauty of mathematics rather than the rules and procedures that need to be memorized (Leinwand, et. al., 2014; NCTM, 2018). Essentially, a dynamic NOM belief needs to influence every aspect of planning from goals to tasks to discussions about mathematics

Implications for Practice

To encourage the development of dynamic NOM beliefs in teachers, further research needs to focus attention on developing effective curriculum for teacher preparation and professional development programs. The National Commission on Social, Emotional and Academic Development recently reported (2018) that teacher preparation should focus on the whole teacher. Teachers need a strong understanding of their beliefs, attitudes and emotions with regard to the content they teach and how those factors influence their lesson planning and classroom community (NCSEAD, 2018). Effective programs need to be designed and developed so that teachers—particularly those who teach mathematics—can learn and develop their own dynamic beliefs about the nature of mathematics, then they will be more equipped to develop those beliefs in their students. Indeed, some of the participants in this study were able to engage in a mathematics learning experiences that allowed them to reflect on and understand the connections their NOM beliefs had to potential instructional choices. According to their descriptions, these participants were given opportunities to see mathematics as a creative, process of inquiry and in effect move along the continuum toward more dynamic NOM beliefs. This type of experience should be incorporated into mathematics teacher education programs so that future mathematics teachers can learn how to design experiences and learning opportunities for students that build dynamic NOM beliefs.

Significance of the Study

The significance of a study can be measured in terms of the number of contributions the investigation makes to the existing research literature. This study adds to the existing research in several ways. First, this study adds a deeper understanding of the types of mathematics learning experiences that current elementary preservice teachers have encountered. In general, the experiences are non-reform oriented focused on mathematics for memorization, speed, and performance. Second, this study adds to the existing literature a more fine-grained analysis and understanding of the nature of mathematics (NOM) beliefs that current elementary preservice teachers hold. These beliefs seem to be consistent with their experiences although there is a multi-dimensional characteristic that exists. Finally, this study adds to the research base an empirical link between experiences learning mathematics and NOM beliefs providing further evidence that beliefs do derive from experiences.

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APPENDICES

APPENDIX A

Demographic Information Survey

Directions: Please indicate your response to the following questions by clearly circling your answer below.

Gender Female Male Prefer not to respond

Ethnicity African American Asian White Hispanic Native American Other

Please indicate your age____

Please indicate which of the following **college-level** mathematics courses you have completed by placing a mark in the blank (indicate all that apply):

______MATH 1483 (Functions)

______MATH 1493 (Applications of Modern Math)

______MATH 1513 (College Algebra)

______MATH 3603 (Mathematical Structures)

______MATH 3403 (Geometric Structures)

Other - Please list all additional mathematics or statistics courses you have completed to-date:

____

Indicate which of the following courses you successfully completed in **high school** by placing a circle around all that apply:

Algebra 1 Algebra 2 Geometry Trigonometry Pre-Calculus

Calculus Statistics Other (list)_________________________________

APPENDIX B

NOM Beliefs Questionnaire

Please respond by indicating your level of agreement on the following items. Indicating 1 (one) means you strongly disagree with the statement while indicating 6 (six) means you strongly agree with the statement.

	NOM-RP SCALE	1 Strongly Disagree	2 Disagree	3 Slightly Disagree	4 Slightly Agree	5 Agree	6 Strongly Agree
A	Mathematics is a collection of rules and procedures that prescribe how to solve a problem.						
B	Mathematics involves remembering and application of definitions, formulas, mathematical facts and procedures.						
E	When solving mathematical tasks you need to know the correct procedure else you would be lost.						
G	Fundamental to mathematics is its logical rigor and preciseness.						
K	To do mathematics requires much practice, correct application of routines, and problem solving strategies.						
L	Mathematics means learning, remembering and applying.						

Note: This nature of mathematics beliefs subscale was taken from (Tatto, 2012) the TEDS-M report

	NOM-PI SCALE	1 Strongly Disagree	2 Disagree	3 Slightly Disagree	4 Slightly Agree	5 Agree	6 Strongly Agree
C	Mathematics involves creativity and new ideas.						
D	In mathematics, many things can be discovered and tried out by oneself.						
F	If you engage in mathematical tasks, you can discover new things (e.g. connections, rules, concepts)						
H	Mathematical problems can be solved correctly in many ways.						
I	Many aspects of mathematics have practical relevance.						
J	Mathematics helps solve everyday problems and tasks.						

Note: This nature of mathematics beliefs subscale was taken from (Tatto, 2012) the TEDS-M report

APPENDIX C

Writing Prompts

Directions: Please respond to the following prompts with as much detail as possible. If not enough space is provided on the front of this page, feel free to use the back of the page.

1) What is mathematics?

2) Describe your experiences learning mathematics before college.

3) Describe your experiences learning mathematics during college.

APPENDIX D

Mathematical Autobiography

Description:

Elementary preservice teachers enrolled in an intermediate mathematics methods course will be required to complete the assignment outlined below. The set of instructions that follow provide guidelines for the type of information that should be included in the mathematical autobiography.

Assignment Instructions:

Math Autobiography in the dropbox in D2L. Be sure to put your name and section number at the top of your mathematical autobiography before your post. Be mindful of sentence and paragraph structure as well as grammar and punctuation.

Your math autobiography should paint a *detailed picture of yourself as a learner of mathematics*. Be sure to address **each** of the following within your autobiography:

- In one paragraph, introduce us to who you are (e.g. where are you from, and anything else that you would like to share with us about who You are).
- Mathematical Experiences: Please describe mathematical experiences that have affected you – both positively and negatively; provide specific examples (do not use specific school/ teacher names); reflect upon why it was a good/bad experience
- Mathematical Dispositions: Please paint a picture of you and your interaction with mathematics (eg. likes, dislikes, feelings, emotions, confidence, anxiety, usefulness of mathematics, etc.) (are you curious, flexible in your math thinking, do you persevere while solving problems, etc.)
- Mathematical Beliefs about Learning: Describe who you are as a learner of mathematics (e.g. your beliefs about how to learn mathematics, feelings when you encounter a mathematics problem, feelings/beliefs about your ability to do mathematics, etc.)
- Mathematical Beliefs about Teaching: Describe what you believe are important qualities of the Ideal mathematics teacher.
- Beliefs about Mathematics: Share how you would describe to someone what comes to mind when you think about *what is mathematics.* The last sentence, please list any word/phrases that come to your mind when you hear the word mathematics.

In your last paragraph, describe anything else that you helps you reflect upon your experiences with learning and teaching mathematics.

APPENDIX E

Mathematical Experiences Drawing Instrument

Name___________________________________ Date:________________

In the space provided below: Create a drawing or multiple drawings that depict your mathematical learning experiences.

Description: (Provide a few sentences that provides the reader with what is happening in your drawing.)

APPENDIX F

Nature of Mathematics Beliefs Drawing Instrument

Name___________________________________ Date:________________

Complete the following statement in such a way that it conveys to me your **beliefs about the nature of mathematics**.

Mathematics is like

____.

In the space provided below: Draw a picture of your metaphor that conveys your meaning.

Description: (Provide a few sentences that provides the reader with why you chose this particular metaphor)

APPENDIX G

Codebook for Mathematics Learning Experiences

Order	Code	Description
1	Teaching/Learning Style	Teaching matches/does not match learning style; T teaches only one method/way
2	Distracted Teacher	Teacher's personal life enters the classroom
3	Type of Instruction- just Good	Teacher described as being "good" but no other description is offered
4	Type of Instruction- good but T-C "fixed"	Teacher described as being "good" but teacher-centered, fixed, lecture, lots of practice problems
5	Type of Instruction- good but S-C "growth"	Teacher described as being "good" but student-centered/growth/real-life application
6	Type of Instruction- just bad	Teacher described as being "bad" but no other description is offered
7	Type of Instruction- bad but T-C "fixed	Teacher described as being "bad" but teacher-centered, fixed, lecture lots of practice problems
8	Type of Instruction- bad but S-C "growth"	Teacher described as being "bad" but student-centered/growth/real-life application
9	Teacher Commitment	Teacher attitude is described as "don't care"/care a great deal or passionate
10	Teacher Connection	Connection with teacher, S - T, T relates to S
11	Homework (problems from a book)	Describes homework as a main component of experience
12	Worksheets In Class (practice problems)	Describes worksheets as a main component of experience
13	tests/Quizzes	Describes tests/quizzes as a big part
14	projects	Describes projects/hands-on as a big part
15	Everyday Math	Describes using math everyday; real life (calculation of tips, discounts, etc)

APPENDIX G

Codebook for Mathematics Learning Experiences (continued)

Order	Code	Description
16	Add/Sub to Mult/Div	Compares pos/neg experience with these
17	Geometry to Algebra	Compares pos/neg experience with these (Geo preferred)
18	Algebra to Geometry	Compares pos/neg experience with these (Algebra preferred)
19	Perseverance	Describes lack of perseverance/giving up or developing perseverance
20	Fear of not knowing/understanding	Describes fear of not knowing how/the answer/or understanding
21	I am confident in my math ability	Describes confidence with doing math
22	Confidence Early (K-12)	Confident with K-12 but not confident in college
23	Confidence Later (college - career)	Confident with college/career but not confident K-12
24	I am not confident in my math ability	General descriptions of low/no confidence
25	Confidence Destroyed/Built	Describes confidence being destroyed or built up, gained
26	Confidence in specific areas	Describes confidence with certain subjects/tasks/problems
27	Ability to Understand	Describes belief in ability to understand or not
28	Emotional Response/Negative	Overwhelmed; disengaged; anxious; “tears”; defeated; frustration; devastated; confusion/lost; “thrown for a loop”; discouraged; mortified; fear/scared; shunned; insecure; unprepared; intimidated; not enjoy; aggravated; nervous; struggle; forced
29	Emotional Response/Positive	Accomplished; curiosity; look forward to; intrigued; engaged; proud; enthusiastic; love; happy; enjoyed; uplifting; determined/persistent

APPENDIX G

Codebook for Mathematics Learning Experiences (continued)

Order	Code	Description
30	Comes Easy to me	Describes math coming easy or not coming easy
31	Not a math person/comes naturally	Describes being a math person, numbers person
32	It "clicked" or "never clicked"	Describes understanding in terms of "clicking" or connecting
33	Understand math better later in life	Compares elements of experience at life stages
34	Understand math through experiences	Describes gaining understanding through stages or experiences
35	Deep Understanding	Describes the presences or absence of deep understanding
36	Needed extra help	Needed help from others not teacher to understand
37	Always made good grades with no struggle	Made good grades but never struggled/excelled
38	made all A's/still successful some struggle	Made all A's but struggled/did well but had to work really hard
39	made B's and C' (grade type)	Discussion of grade types/passed despite struggling through
40	Bring up or make up bad/low grades	Doing extra to make up or bring up a low grade
41	High grades/Best Abilities	Describes making high grades/never making high grades
42	Bad grades but working hard	Describes working really hard but still making poor grades
43	Bad grades too slow	Describes being too slow/behind
44	Going blank/freezing up	Describes difficulty remembering on exams
45	Failed/Bombed a test	Describes scoring a low grade on a test
46	not a good test taker/over-thinker	Describes struggle with taking tests
47	ACT scores not good enough in college	Describes standardized testing struggles
48	Success	Describes doing well/good grades on

APPENDIX G

Codebook for Mathematics Learning Experiences (continued)

Order	Code	Description
49	timed tests/worksheets	Describes completing timed tests/worksheets
50	drills	Describes completing drills for speed
51	Slow Down/More time	Describes needing to slow down or needing more time
52	Speed is Smart	Describes belief that speed = smart
53	rhymes to help	Describes learning rhymes for memory
54	flash cards	Describes using flash cards for memory
55	Repetition/drills	Describes completing drills for memory
56	practice one way	Describes learning one method for memory
57	lots of problems	Describes doing many of the same problems to build memory
58	PSTs memory (I can't remember…had a good memory)	Describes difficulty with remembering

APPENDIX H

Codebook for Nature of Mathematics Beliefs

Order	Code	Description
1	Math is a puzzle	math is a jigsaw puzzle; a fun struggle; actively working toward an answer; understandable by anyone; accomplished by everyone; easy
2	Connections & patterns	math became more like puzzles to solve and connections and patterns to look for; illustrate how patterns occur; give meaning to the order of things
3	emotions associated with a puzzle	rewarding; interesting; exciting; compelling; cool & fascinating
4	Whole picture/concept;	Puzzle is solved; creates clear picture of how pieces are connected
5	solution/solved puzzle	
	Challenge/challenging problem	runs the spectrum of being easy to challenging; more challenge requires more thinking
6	thinking/requires thinking; engagement	comparing, questioning, making connections
7	A useful tool	something that can help solve simple equations; useful; using tools to solve real-world problems
8	Rules, formulas, expressions, variables; functions	rules; order of operations; sets of rules; formulas; variables; expressions; functions
9	equations and word problems	require procedures to solve
10	Algorithms, processes, procedures	how; algorithms; processes; procedures; step-by-step
11	solving problems with rules & procedures	calculating, working out problems, manipulation of symbols; use of symbols to get a solution
12	numbers, numerals	Math is numbers, numerals, values, quantity
13	shapes	Math is shapes, dimensions, components
14	systems	things that use numbers or are composed of numbers

APPENDIX H

Codebook for Nature of Mathematics Beliefs (continued)

Order	Code	Description
15	a science of	A science of numbers, quantities, space, shapes and change (dictionary definition)
16	school subject	a subject in school; subject students loath; priority in schools
17	textbook subject	Algebra, Geometry, Calculus; addition, subtraction; multiplication, division
18	conceptual subject	a subject of conceptual understanding; abstract; theories; looks at relationships between numbers; being able to understand values and what they mean
19	hard/difficult subject	math is hard or difficult; challenging; complex; confusing; frustrating
20	tedious, memorizing, time intensive subject	math is tedious, requires memorizing, requires a lot of time
21	a negative experience	(frustrating); torture; detrimental; painful; not enjoyable; not useful; not valuable
22	requires rigorous thinking	critical thinking, understanding and knowing how to critically think about numbers
23	requires multiple methods	using multiple methods, strategies, process to solve; looking at things in new ways
24	requires logic, reasoning	using reasoning, logic, logical thinking
25	a deep and flexible subject	deep and flexible; engaging; developing
26	requires creative thinking	thinking creatively; ingenuity; exploration and discovery

APPENDIX H

Codebook for Nature of Mathematics Beliefs (continued)

Order	Code	Description
27	Adaptable, ever changing	math is adaptable to everyday problems; understanding there are multiple ways to use it
28	Explains how and why things happen	use in our everyday; understand how the world works
29	sets of real life problems	real world applications, engineering, physics, apply procedures to real world problems
30	is all around us	the building block of our world; all around us; found in many places; many things
31	is essential for interpreting the world	essential; vital; necessity; important
32	a language to understand and communicate about the world	a language; form of expression; communicating ideas; numerical form of communication; a collection of symbols

VITA

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