DESIGN OF FIXED STRUCTURE CONTROLLERS FOR WEB TENSION CONTROL

by

P.R. Pagilla[1], M. Cimino[1], and D. Knittel[2]
[1]Oklahoma State University
USA
[2]Louis Pasteur University and INSA, Strasbourg
FRANCE

ABSTRACT

It is a common practice in industry to design a Proportional-Integral (PI) velocity feedback controller cascaded with a PI outer tension loop to regulate web velocity and tension to their desired values. The controller gain tuning is often heuristic and does not explicitly account for process variations, and often fails to provide adequate performance in the presence of uncertainties. To address these issues, one can pose two key questions: (1) How does one systematically obtain controller gains for a given controller structure and a set of operating conditions? (2) Is it possible to systematically choose controller gains to satisfy some pre-defined performance specifications for the closed-loop system when the operating conditions and web material properties have variations?

The goal of this paper is to investigate methods to address the above two questions. Methods from robust control theory are used to investigate and develop techniques to systematically design fixed structure controllers that satisfy pre-specified performance criterion. Although the design procedure allows for choosing controller structures with different number of gain parameters, emphasis will be given to controller structures that contain two or three gain parameters (The PI controller structure has two gain parameters). The objective is to use parametric methods whose end result is a region of controller gains which will satisfy the specified performance criteria. Although emphasis is given to tension control, the proposed techniques can be used for other control loops such as velocity or dancer or lateral position control systems. Since the methods used are an outgrowth of classical time and frequency response methods, it is expected that a control engineer with an understanding of classical techniques will be able to comprehend the design procedures discussed in the paper.

INTRODUCTION

Design methods for developing controllers to satisfy pre-defined performance criteria have been investigated since the 1960's. A number of techniques exist for the design of fixed structure controllers; in this design the designer specifies the controller structure and desired performance criteria for the closed-loop system and seeks to find the controller gains that result in a stable closed-loop system and satisfies the desired performance specifications. Fixed structure controller design can be either parametric or non-parametric. In the parametric method, a region of controller gains for the specified controller is obtained, whereas, in the non-parametric case a single controller is obtained. Details about parametric techniques can be found in [1]. Non-parametric techniques to design fixed structure controllers for web winding systems can be found in [2]. The goal in this paper is to specify a simple controller for tension regulation with two or three tunable gains such as a PI controller, and use a parametric approach to obtain regions for the controller gains which will satisfy the desired performance specifications. Since the number of gains are three or less, the D-decomposition technique [3] is utilized for obtaining the controller gain regions. To incorporate robustness to plant parameter variations into the controller design, the Kharitonov theorem [1] in conjunction with the D-decomposition technique are utilized. For web tension control, it is common practice to divide a process line into several tension zones by denoting the span between two successive driven rollers as a tension zone. Figure 1 shows a web line with three tension zones; the line consists of unwind/rewind rolls and two intermediate driven rollers. In the figure, LC denotes the load cell roller and the driven rollers are represented by M_i for $i = 0, \ldots, 3$, u_i represents input torque from the i-th motor, v_i represents the transport velocity of the web on the i-th roller, and t_i represents web tension in the span between $(i-1)$-th and i-th driven rollers. Decentralized controllers are often preferred, and mostly used, by the web handling industry due to the ease of tuning individual stations; they also provide

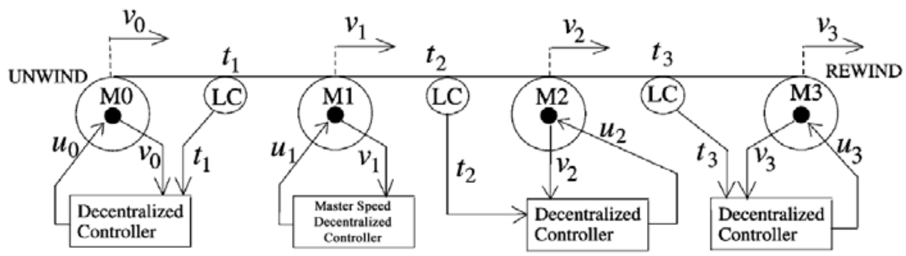

Figure 1: Sketch of a web line with three tension zones.

certain degree of isolation between subsystems in the event of actuator and sensor malfunctions. In many industrial web process lines, the decentralized control scheme for each section has two cascaded PI control loops, as shown in Fig. 2; the output of the tension loop becomes reference velocity error correction for the velocity loop. Also, note that there is one master speed driven roller (roller M1 in Fig. 1) which primarily sets the web line speed and has only the velocity loop. Further, many process lines can also have an outer dancer feedback loop based on dancer position measurement instead of the tension feedback based on measurement of web tension using load cells.

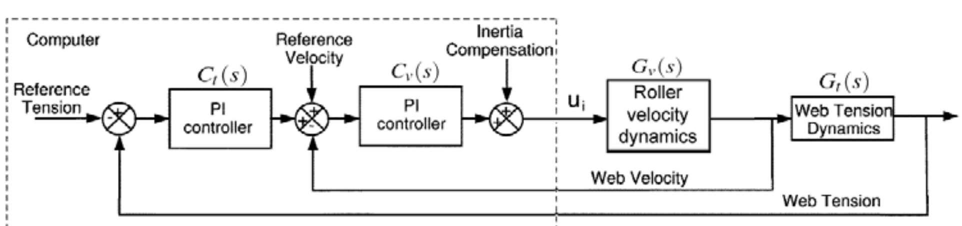

Figure 2: Control strategy with two cascaded PI controllers.

Two problems are usually encountered in the design and tuning of controllers for tension control: (1) How does one systematically obtain the controller gains for a given set of conditions? The conditions include web material properties. (2) Is it possible to choose the controller gains to preserve the desired closed-loop performance criteria in the presence of uncertainty in the operating conditions and web material properties? There is a continuous effort by practicing control engineers to seek methods to systematically tune controller gains for regulating web tension, web transport velocity, and web lateral position on the rollers. The off-line methods that are employed for such tuning are often heuristic and fail to provide the required performance, especially in the presence of uncertainties. Some examples of these tuning techniques are the Ziegler-Nichols (ZN) [4] and Iterative Feedback Tuning (IFT) [5] methods. The ZN method is a heuristic tuning procedure for PID-type controllers and the gains are selected only based on the critical gain of the system; this method does not account for variations in plant parameters. In the IFT method the controller parameters are selected to minimize a particular objective function based on data collected from previous experiments. This off-line model-free tuning procedure is not heuristic, but is highly sensitive to measurement noise or changes in the operating conditions of the system. Any change in the operating conditions may require re-tuning of the controller. Many other off-line tuning procedures are derived from the IFT method and require the minimization of particular objective functions. Besides not being robust with respect to different operating conditions or model parameter uncertainties, these methods do not provide any insights into how the controller parameters could be varied to account for different performance criteria.

With a view towards tuning of controllers in the presence of uncertainties in the plant parameters, a class of fixed structure controllers obtained by varying three free parameters is considered (one can simply look at varying just two or one gain parameter by freezing the others at some design values). The controllers in this class are denoted by

$$C(s,K)=\frac{N_c(s,K)}{D_c(s,K)}, \tag{1}$$

where $K = [k_1, k_2, k_3]$ is the vector of the three free parameters to be tuned. Considering the velocity loop controller of Fig. 2 to be known, the control system structure is shown in Fig. 3, where $u(t)$ is the controller output, $G(s)$ denotes the plant transfer function, and $e(t)$ is the error between the reference tension and web tension.

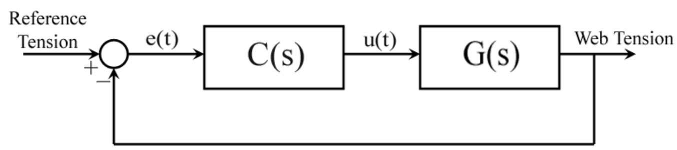

Figure 3: Control system block diagram

In the following the D-decomposition technique and Kharitonov type results on robust stability of polynomials is applied to the tension control problem to design a three free parameter controller for the outer tension loop of Fig. 2. The aim is to generate an entire region of suitable controllers with a given fixed structure for which all the desired performance criteria are satisfied. Moreover, the approach accounts for uncertainty in the operating conditions and model parameters, and provides a starting point to search for appropriate controller gains. Further, the approach also gives flexibility to the control engineer for narrowing the region of suitable controllers by incorporating additional performance criteria. Hardware limitations, such as the control saturation level, can also be taken into account to make the design problem more practical. Theoretical results and experimental corroboration show that the pre-specified performance criteria are met for various web materials and under different operating conditions. The addressed performance criteria are specific to the considered application, but apply to other control systems. A number of performance criteria such as the α-stability, gain margin, phase margin, and H_∞ norm are considered, and details are provided on incorporation of these performance specifications into the synthesis of the controller regions.

ROBUST FIXED STRUCTURE CONTROLLER DESIGN

Consider the plant transfer function in the following form:

$$G(s, A_G) = \frac{N(s, A_G)}{D(s, A_G)} := \frac{c_0 + c_1 s + \cdots + c_m s^m}{c_{m+1} + c_{m+l} s + \cdots + c_{m+l} s^{l-1} + s^l}, \qquad \{2\}$$

where $AG := [c_0, c_1, \ldots, c_{m+l}]$ is the vector of all coefficients of the the polynomials N and D. Let $[c_i^-, c_i^+]$ be the interval modeling the uncertainty on the plant coefficients c_i, $i = 0, \ldots, m + l$. The bounds c_i^- and c_i^+ for each plant parameter c_i are assumed to be known. Moreover, we will say that A_G is admissible if $c_i \in [c_i^-, c_i^+], \forall i = 0, \ldots, m + l$.

Let

$$C_t(s, K) = \frac{N_c(s, K)}{D_c(s, K)}, \qquad \{3\}$$

be the desired controller structure for web tension regulation ($N_c(s,K)$ and $D_c(s,K)$ are polynomials of fixed degree). Given the plant transfer function {2} and the controller structure {3}, the problem of achieving common performance criteria, such as α-stability (the requirement that the poles of the closed-loop system be to the left of the $-\alpha$ line in the complex plane; this is stronger than just requiring the closed-loop system to be stable), damping ratio of the closed-loop poles, gain margin, phase margin, and H_∞ norm, can be reduced to that of placing the roots of some particular polynomials in determined regions of the complex plane [1]. For example, achieving the α-stability performance corresponds to locating the roots of the characteristic polynomial (denominator of the closed-loop transfer function from tension reference to tension output)

$$P_1(s,A_G,K) \equiv N(s,A_G)Nc(s,K) + D(s,A_G)D_c(s,K) \tag{4}$$

to the left of the vertical line $s = -\alpha$ in the complex plane. Instead, achieving the damping ratio requirement for the closed-loop system poles corresponds to placing the roots of equation {4} in between two oblique lines symmetric with respect to the real axis and passing through the origin of the complex plane. Since the gain margin (GM) corresponds to the amount of gain that can be added to the system without destabilizing it, achieving GM $\geq \gamma*$ corresponds to requiring the roots of the characteristic polynomial

$$P_2(s,A_G,K) \equiv \gamma N(s,A_G)N_c(s,K) + D(s,A_G)D_c(s,K) \tag{5}$$

of the control system shown in Fig. 4 to lie in the left-half of the complex plane for every value of γ in the interval [1, γ∗]. Similarly, since the phase margin (PM) corresponds to

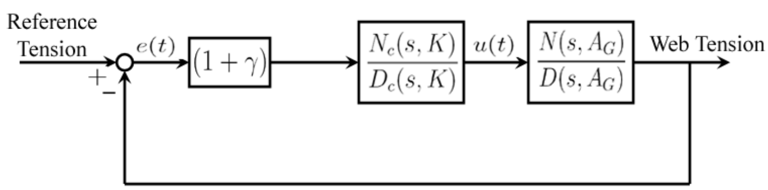

Figure 4: Control system to evaluate the gain margin.

the amount of phase lag that can be added to the system without destabilizing it, achieving PM ≥ q∗ corresponds to requiring the roots of the characteristic polynomial

$$P_3(s,A_G,K) \equiv e^{j\theta}N(s,A_G)N_c(s,K) + D(s,A_G)D_c(s,K) \tag{6}$$

of the control system shown in Fig. 5 to lie in the left-half of the complex plane for every value of θ in the interval [0,θ∗]. Using this procedure one can also minimize the H_∞ norm

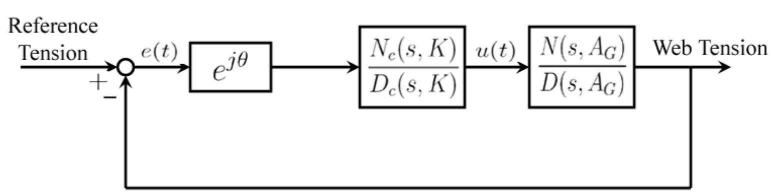

Figure 5: Control system to evaluate the phase margin.

of a transfer function from an input to any target output or a weighted output. The H_∞ norm of a transfer function matrix $Gzr(s)$ from an input $r(t)$ to an output $z(t)$ is defined as

$$\|G_{zr}\|_\infty = \sup_\omega \sigma_{\max}\left(G_{zr}\left(j\omega\right)\right) \tag{7}$$

where σ_{max} denotes the maximum singular value. In the single-input single-output case ($G(s)$ is a scalar transfer function), the H_∞ norm is simply the peak value of the Bode magnitude plot of $G(s)$. The minimization of the H_∞ norm of a transfer function is usually required when it is desired to reduce the effect of a exogenous disturbance on the system output, as in the case depicted in Fig. 6. The transfer function from the

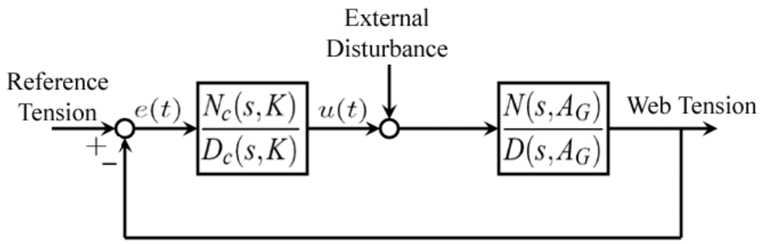

Figure 6: Control system with external disturbance

disturbance to web tension is given by

$$W_{dt}\left(s\right) := \frac{N_{dt}\left(s, A_G, K\right)}{D_{dt}\left(s, A_G, K\right)} = \frac{N\left(s, A_g\right)D_c\left(s.K\right)}{N_c\left(s, K\right)N\left(s, A_G\right) + D_c\left(s, K\right)D\left(s, A_G\right)} \tag{8}$$

To minimize the disturbance effect on web tension, it is required that

$$\left\|W_{dt}\left(s\right)\right\|_\infty \leq \gamma \tag{9}$$

for some real value of γ. It can be shown that this requirement is equivalent to the polynomial

$$P_4\left(s, A_G, K\right) \equiv \gamma e^{j\theta} N_{dt}\left(s, A_G, K\right) + D_{dt}\left(s, A_G, K\right) \tag{10}$$

being Hurwitz for all θ in the interval $[0,2\pi)$.

Performance	Polynomial $P(s, A_G,K)$
Stability α-stability Damping factor	$N(s,A_G)N_c(s,K) + D(s,A_G)D_c(s,K)$
Gain Margin γ*	$\gamma N(s,A_G)N_c(s,K) + D(s,A_G)D_c(s,K)$
Phase Margin θ*	$e^{j\theta}N(s,A_G)N_c(s,K) + D(s,A_G)D_c(s,K)$
$\mathcal{H}_\infty$ norm < γ	$\gamma e^{j\theta}N_{dt}(s,A_G,K) + D_{dt}(s,A_G,K)$

Table 1: Summary of the performance criteria and the respective polynomials used to evaluate them.

Table 1 gives a summary of the performance criteria and the corresponding polynomials. Therefore, the problem of achieving the desired performance criterion can now be posed as that of finding *all* the possible values of the controller gains (K) that place the roots of the corresponding polynomial in the correct region of the complex plane.

For a particular performance criterion, let $\mathcal{H}$ denote the region in the complex plane where the roots of the corresponding polynomial $P(s,AG,K)$ are required to lie. If there is no uncertainty in the model parameters or in the operating conditions (that is, if the model parameters A_G are exactly known), then the set of all the controller gains K can be found by searching the boundary solutions K_b for which $P(s,K)$ has a root (or a complex pair) on the boundary of $\mathcal{H}$. This corresponds to solving the following equation with respect to the unknown gains K:

$$P(s = \delta\mathcal{H}, K) = 0, \tag{11}$$

where $\delta\mathcal{H}$ is the boundary of $\mathcal{H}$.

If there is an uncertainty in the model parameters or operating conditions, the value of A_G is not known exactly. Therefore, the procedure discussed previously has to be extended to account for these uncertainties. Let $P(s,A_G,K)$ be the polynomial corresponding to the desired performance criterion. By fixing reasonable bounds for the entries in A_G (as explained at the beginning of this section), the following family of polynomials can be generated by varying AG among its admissible values:

$$\mathcal{P} = \{P(s,A_G,K) \mid A_G \text{ is admissible}\}. \tag{12}$$

The desired performance can be achieved robustly if there are some values of K for which all the roots of all the polynomials in $\mathcal{P}$ lie in $\mathcal{H}$. Therefore, if the model parameters are uncertain, the problem can be posed as follows: find all possible controller gains K that place the roots of all the polynomials in the family $\mathcal{P}$ in the region $\mathcal{H}$ (corresponding to the selected performance criterion). Notice that, since A_G can assume an infinite number of values, the family $\mathcal{P}$ is made of an infinite number of polynomials. Therefore the application of the previous described procedure to determine the set of solutions K will require simultaneously solving an infinite number of equations (each corresponding to a polynomial in $\mathcal{P}$). It is possible to prove [1] that all the roots of all the polynomials of $\mathcal{P}$ are in $\mathcal{H}$ if and only if a finite number of polynomials have all their roots in $\mathcal{H}$; the primary result on this problem is due to Kharitonov and the finite number of polynomials are called the Kharitonov polynomials. There are four or eight Kharitonov polynomials based on whether the coefficients of the polynomials in the family $\mathcal{P}$ are real or complex, respectively. Therefore, the problem of finding the values of K that place all the roots of all the polynomials of $\mathcal{P}$ in $\mathcal{H}$ reduces to that of finding the values of K that place all the roots of the Kharitonov polynomials in H. Therefore, the set of solutions K can be obtained by solving just a limited number of equations of the type {11} with respect to the unknown K.

The Kharitonov polynomials can be considered as the polynomials representative of the family $\mathcal{P}$. For example, consider the polynomial

$$P(s, A_G) = s^2 + a_1 s + a_0 \tag{13}$$

where $A_G = [a_1, a_0]$, and the bounds on the coefficients are given by

$$a_1 \in \left[a_1^-, a_1^+\right], \quad a_0 \in \left[a_1^-, a_1^+\right] \tag{14}$$

Based on the bounds {14} the following family of polynomials can be generated

$$\mathcal{P}(s) = \left\{P(s, A_G) \mid a_i \in \left[a_i^-, a_i^+\right],\ i = 0,1\right\}. \tag{15}$$

The four Kharitonov polynomials associated with the real family of polynomials {15} are:

$$\begin{aligned} T_1(s) &= a_0^- + a_1^- s + s^2, \\ T_2(s) &= a_0^- + a_1^+ s + s^2, \\ T_3(s) &= a_0^+ + a_1^- s + s^2, \\ T_4(s) &= a_0^+ + a_1^+ s + s^2. \end{aligned} \tag{16}$$

Notice that the Kharitonov polynomials {16} do not depend on the nominal values of the model parameters *AG*, but just on the bounds of the entries of A_G.

The following step-by-step procedure summarizes the design method:

1) Define the bounds for the unknown model parameters A_G;

2) Select the desired performance criterion to be achieved;

3) Choose the controller structure;

4) Based on the desired performance criterion and the selected controller structure, generate the polynomial $P(s,K)$ and define the region H where the roots of $P(s,K)$ must lie in order to achieve the desired performance criterion.

5) Generate the family $\mathcal{P}$ of polynomials $P(s,A_G,K)$, where *AG* is any set of model parameters contained in the bounds defined in step 1.

6) Derive the Kharitonov polynomials $T_i\,(s,A_G,K)$, $i = 1, ..., n$, representative of the family $\mathcal{P}$ (where $n = 4$ or $n = 8$ based on whether $\mathcal{P}$ is a family of real or complex polynomials, respectively).

7) Solve simultaneously the equations

$$T_i\left(s = \delta\mathcal{H}, A_{G,} K\right) = 0,\ i = 1, ..., n \tag{17}$$

with respect to the unknown controller gains K. The solution corresponds to a set of curves, in the controller gain space, delimiting the region of the admissible solutions K for which the desired performance criterion is achieved robustly with respect to the model parameter uncertainty.

8) Plot the curves obtained in the previous step, and shade the region of interest (delimited by the curves). Choice of any controller in the shaded region results in a stable closed-loop system satisfying pre-specified performance criteria, and provides control engineers with a useful starting point to further tune the controller for other performances.

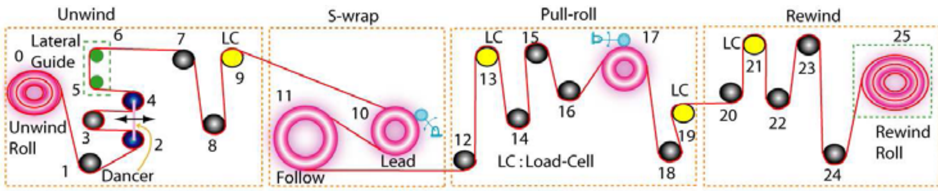

Figure 7: Sketch of the experimental platform.

APPLICATION TO WEB TENSION REGULATION

Fig. 7 shows a sketch of a web line used as an example for this study. The line is divided into different sections and contains four driven rollers. The tension regulation loop is implemented only in the unwind and rewind sections, and only a velocity loop is used in the two intermediate sections to regulate the longitudinal velocity of the web. To measure the tension in the unwind and rewind sections, load cell (LC) rollers are used as shown in Fig. 7.

The control goal is to regulate web tension in the unwind and rewind sections while maintaining the prescribed web transport velocity. The block diagram of Fig. 2 represents the control scheme adopted in the unwind and rewind sections. A simplified model of the motor and roll dynamics of the unwind section is given by

$$G_v(s) = \frac{1}{n_0 J_0 s}, \tag{18}$$

where n_0 and J_0 are the gear ratio and inertia of the roll, motor, and transmission elements reflected to the motor side, respectively. The gear ratio is equal to 8 for both the unwind and rewind rollers. The inertia J_0 of the roller is continuously varying, and is calculated based on online estimation of the radius. The following velocity PI controller is chosen:

$$C_v(s) = \frac{k_p(s + k_v)}{s}. \tag{19}$$

The controller gain k_p is selected to compensate for the gear ratio and time-varying inertia J_0; $k_p = k_r n_0 J_0$ is chosen, where $k_r = 1.5$ is a constant gain. The other controller gain is chosen as $k_v = 3.1$. A web material with Young's modulus $E = 11900$ lbf/in^2 (8.2×10^7 N/m^2) is considered. The following linearized model of the web tension dynamics is used:

$$G_t(s) = \frac{AE - t_0}{Ls + V_r}, \tag{20}$$

where t_0 = 20 lbf (88.96 N) is the wound-in tension of the web in the roll, A = 0.042 in^2 (2.71 × 10^{-5} m^2) is the cross sectional area of the web, V_r = 20.04 in/s (0.51 m/s) is the web reference speed, L = 270 in (6.86 m) is the length of the web span between the unwind and master speed (lead) rollers.

In view of designing the outer loop tension controller, $C_t(s)$, the inner velocity loop together with the web tension dynamics (see Figure 8) can be treated as a composite system whose transfer function is given by

$$G(s) = \frac{\alpha_1 s + \alpha_0}{s^3 + \beta_2 s^2 + \beta_1 s + \beta_0}, \tag{21}$$

where

$$\alpha_1 = \frac{(AE - t_0)k_r}{L}, \ \alpha_0 = \frac{(AE - t_0)k_r k_v}{L}, \tag{22}$$

$$\beta_2 = \left(\frac{V_r}{L} + k_r\right), \ \beta_1 = \left(\frac{V_r k_r}{L} + k_r k_v\right), \ \beta_0 = \frac{V_r k_r k_v}{L}. \tag{23}$$

Therefore, by considering the nominal values of the model parameters, the transfer

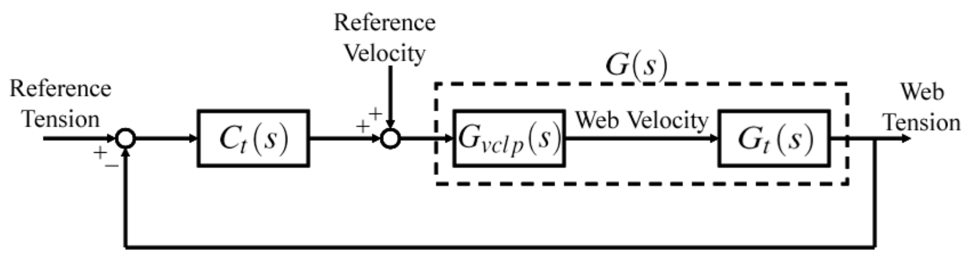

Figure 8: Control system block diagram.

function {21} reduces to

$$G(s) = \frac{31.99s + 99.16}{s^3 + 1.57s^2 + 4.76s + 0.34}. \tag{24}$$

To design the outer loop (tension) controller, the procedure explained in the previous section is applied to this specific example, the details of which are given below.

1. Since the Young's modulus of the web material is seldom known exactly, variations in the modulus over a nominal value must be considered. To design a controller robust with respect to such variations, define an uncertainty of ±5% on the web Young's modulus E about its nominal value ($E \in [11305, 12495]$). The vector A_G contains the coefficients of the model {21}: $A_G = [\alpha_1, \alpha_0, \beta_2, \beta_1, \beta_0]$. By varying E in the defined uncertainty range, the entries in A_G vary in the following ranges:

$$\alpha_1 \in [30.32, 33.65],\ \alpha_0 \in= [93.99, 104.32]$$
$$\beta_2 \in [1.57, 1.57],\ \beta_1 \in [4.76, 4.76],\ \beta_0 \in [0.34, 0.34] \quad \{25\}$$

Since the denominator coefficients b_i ($i = 0,1,2$) do not depend on E, there is no uncertainty on their values.

2. Since PI-type controllers are widely used in industry for tension regulation, the controller structure was chosen as

$$C(s, K) = \frac{k_{it} + k_{pt}s}{s}, \quad \{26\}$$

where $K = [k_{it}, k_{pt}]$ is the controller gain vector.

3. To achieve the a-stability, the solutions K must be such that the roots of the closed-loop characteristic polynomial

$$\begin{aligned} P(s, A_G, K) &= (k_{it} + k_{pt}s)(\alpha_1 s + \alpha_0) + s(s^3 + \beta_2 s^2 + \beta_1 s + \beta_0) \\ &= s^4 + \beta_2 s^3 + (k_{pt}\alpha_1 + \beta_1)s^2 + (k_{pt}\alpha_0 + k_{it}\alpha_1 + \beta_0)s + k_{it}\alpha_0 \end{aligned} \quad \{27\}$$

should be to the left of the line $s = -\alpha$. Notice that each coefficient of the polynomial {27} is affinely dependent on the controller gains K.

4. Let γ_4, γ_3, γ_2, γ_1 and γ_0 be the coefficients of the polynomial {27}:

$$\begin{aligned} &\gamma_4 = 1,\ \gamma_3 = \beta_2,\ \gamma_2 = k_{pt}\alpha_1 + \beta_1 \\ &\gamma_1 = k_{pt}\alpha_0 + k_{it}\alpha_1 + \beta_0,\ \gamma_0 = k_{it}\alpha_0. \end{aligned} \quad \{28\}$$

The uncertainty on the parameters α_i, defined by {25}, is reflected as the following uncertainty ranges for the coefficients γ_i of the polynomial {27}:

$$\begin{aligned} &\gamma_4 \in [\gamma_4^-, \gamma_4^+] \equiv [1,1]\ , \\ &\gamma_3 \in [\gamma_3^-, \gamma_3^+] \equiv [1.57, 1.57], \\ &\gamma_2 \in [\gamma_2^-, \gamma_2^+] \equiv [30.32k_{pt} + 4.76,\ 33.65k_{pt} + 4.76], \\ &\gamma_1 \in [\gamma_1^-, \gamma_1^+] \equiv [93.99k_{pt} + 30.32k_{it} + 0.34,\ 104.32k_{pt} + 33.65k_{it} + 0.34], \\ &\gamma_0 \in [\gamma_0^-, \gamma_0^+] \equiv [93.99k_{it}, 104.32k_{it}]. \end{aligned} \quad \{29\}$$

Therefore, for a given value of the controller gain vector $K = [k_{pt}, k_{it}]$, each coefficient γ_i has to lie in the range $[\gamma_i^-(K), \gamma_i^+(K)]$ given by {29}. Therefore, the following family of real polynomials can be generated:

$$\mathcal{P}(K) = \{P(s, A_G, K) \mid \gamma_i \in [\gamma_i^-(K), \gamma_i^+(K)], i = 0, \ldots, 4\}. \quad \{30\}$$

Notice that the nominal values of the model coefficients are not needed to generate the family {30}. The family of polynomials can be generated just by knowing the bounds {25} of the model coefficients.

5. Generate the four Kharitonov polynomials which represent the entire family {30} of polynomials $\mathcal{P}$. These four polynomials can be thought of as the corners of a region containing all the polynomials in $\mathcal{P}$. Therefore, it is intuitively simple to imagine that the coefficients of the Kharitonov polynomials do not depend on the nominal values of the coefficients {28}, but just on their bounds {29} and the controller gains K. The four Kharitonov polynomials have the form [1]:

$$\begin{aligned} T_1(s,K) &= \gamma_0^- + \gamma_1^- s + \gamma_2^+ s^2 + \gamma_3^+ s^3 + \gamma_4^- s^4, \\ T_2(s,K) &= \gamma_0^- + \gamma_1^- s + \gamma_2^+ s^2 + \gamma_3^+ s^3 + \gamma_4^- s^4, \\ T_3(s,K) &= \gamma_0^- + \gamma_1^- s + \gamma_2^+ s^2 + \gamma_3^+ s^3 + \gamma_4^- s^4, \\ T_4(s,K) &= \gamma_0^- + \gamma_1^- s + \gamma_2^+ s^2 + \gamma_3^+ s^3 + \gamma_4^- s^4. \end{aligned} \qquad \{31\}$$

The Kharitonov polynomials {31} are functions of controller gains only.

6. Consider the first Kharitonov polynomial $T_1(s,K)$. To find the solutions K that place the roots of T_1 to the left of the vertical line $s = -\alpha$ of the complex plane, first find the values of K that place at least one root of T_1 exactly on that line ($s = -\alpha$). This corresponds to solving the equation

$$T_1(-\alpha + j\omega, K) \equiv \mathrm{Re}\{T_1(-\alpha + j\omega, K)\} + j\omega\,\mathrm{Im}\{T_1(-\alpha + j\omega, K)\} = 0. \qquad \{32\}$$

Considering that the coefficients of the Kharitonov polynomials are affine functions of the controller gains k_{pt} and k_{it}, equation {32} can be rewritten as

$$\left[c_1(\omega)k_{pt} + c_2(\omega)k_{it} + c_3(\omega)\right] + j\omega\left[d_1(\omega)k_{pt} + d_2(\omega)k_{it} + d_3(\omega)\right] = 0 \qquad \{33\}$$

where c_i and d_i ($i = 1,2,3$) are functions of the real variable ω. For the equation {33} to hold, both its real and imaginary parts have to vanish. This results in the following two conditions:

$$c_1(\omega)k_{pt} + c_2(\omega)k_{it} + c_3(\omega) = 0, \qquad \{34\}$$

$$j\omega\left[d_1(\omega)k_{pt} + d_2(\omega)k_{it} + d_3(\omega)\right] = 0. \qquad \{35\}$$

Now the problem of finding the values of K that place at least one root of T_1 on the line $s = -a$ can be divided in the following two subproblems:

- *Finding the values of K that place at least one root of T_1 on the real axis, at the point $s = -\alpha$.* Since in this case $\omega = 0$, the equation {35} is trivially satisfied, and the equation {34} can be rewritten as:

$$c_1(0)k_{pt} + c_2(0)k_{it} + c_3(0) = 0, \qquad \{36\}$$

which represents a straight line in the k_{pt} -k_{it} plane.

- *Finding the values of K that place at least one pair of complex conjugate roots of* T_1 *at* $s = -\alpha \pm j\omega$, $(\omega \neq 0)$. In this case, the equations {34}-{35} correspond to a system of two equations in the two unknowns k_{pt} and k_{it}. By grading ω in a reasonable range of values, and by solving that system for each value of w, it is possible to get a curve in the two dimensional plane with the gains *kpt* and *kit* as its axes.

Therefore, from equation {32}, one line and one curve in the k_{pt} -k_{it} plane can be obtained. Each point on them corresponds to a set of controller gains *K* that place at least one root (or a pair of complex conjugate roots) of T_1 on the vertical line $s = -\alpha$ of the complex plane. Similarly, a straight line and a curve can be obtained for each of the four Kharitonov polynomials in {31}, as shown in Figure 9.

7. The lines and curves obtained in the previous step delimit the boundary of the controller gain region. In particular, the curves and straight lines divide the k_{pt} -k_{it} plane in several closed and open regions. But just one of them is the region of interest – containing all the controller gains *K* that place *all* the roots of *all* the four Kharitonov polynomials to the left of the line $s = -a$ of the complex plane. To find this region, the following procedure can be used:

   (i) Select a closed or open region from Figure 9;
   (ii) Select a point $K*$ in that region;

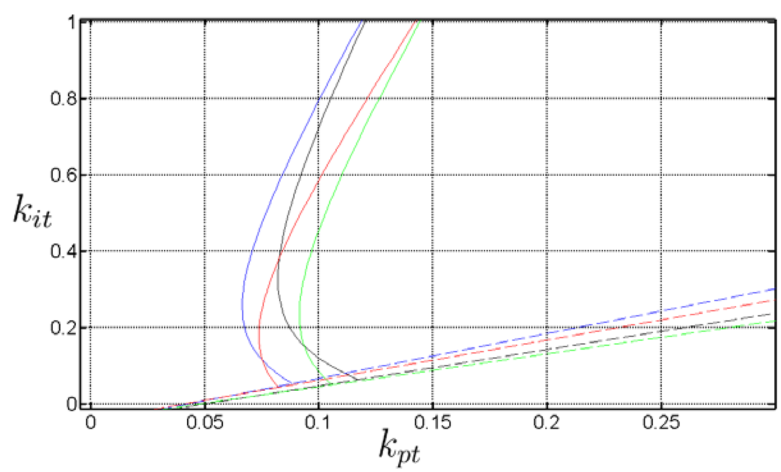

Figure 9: Set of lines and curves in the controller gain space k_{pt} -k_{it}. Each curve (and line) contains the points *K* for which the corresponding Kharitonov polynomial has at least one root on the vertical line $s = -\alpha$ of the complex plane.

   (iii) Check (for that point) if all the roots of all the Kharitonov polynomials lie to the left of the line $s = -\alpha$. If the answer is yes, then the selected region is the one containing the required solution. Otherwise, return to step (i) and select a different region. Note that the designer has to check only a finite number of regions; the check can be stopped once a stable region is found.

The solution for the a-stability problem discussed until now corresponds to the open region of controller gains shaded in Figure 10.

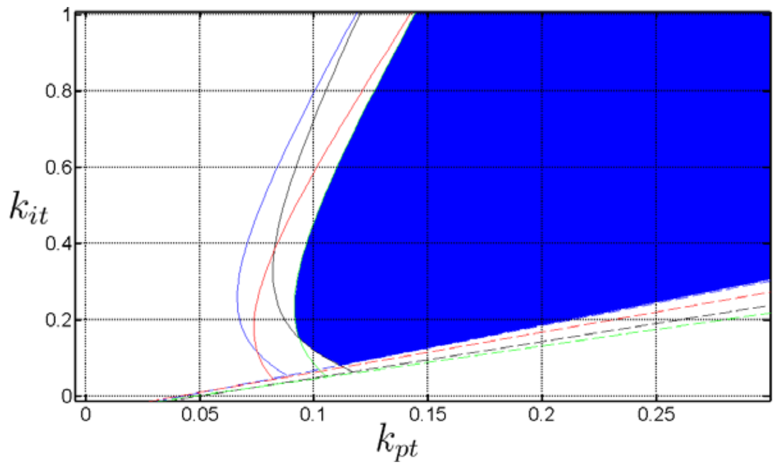

Figure 10: Controller gain region for which the α-stability performance is robustly achieved.

The procedure explained above can be applied for each of the performance criteria given in Table 1. The intersection of all the resulting regions gives the region of controller gains that can achieve all the selected performance criteria. As an example, Figures 11, 12, 13, and 14 show the resulting regions for four different scenarios. The α-stability, with $\alpha = 1$, is considered among the desired performance criteria to guarantee a certain fastness of the system response. Further, a phase margin of at least 10 deg is considered to compensate for delays in the real-time system. Figures 11 and 12 show the region of controller gains suitable to achieve a phase margin of 10 and 15 degrees, respectively. As expected, a larger value of the required phase margin results in a smaller region for the controller gains. Figures 13 and 14 show the results obtained by considering also a ±5% of variation on the Young's modulus E about its nominal value. Since the cardinality of the family of polynomials $\mathcal{P}$ increases as the percentage of uncertainty increases, the region obtained by considering the same performance criteria are visibly smaller than the ones obtained without considering any uncertainty. The controller gain regions serve as a good starting point for a control engineer which can be used to further tune the controller on-line. Moreover, these regions can also be used as a tool to understand the distribution of the admissible controller gains (those for which the performance criteria are met) in the controller gain space.

CONCLUSIONS

A fixed structure controller design technique was proposed for control systems in a web process line. Different performance criteria can be included into the design procedure while considering variations in the physical parameters of the process as well as operating conditions. The parametric design procedure gives a region of controller gains as opposed to a single set of controller gains. The choice of a set of gains from this region will be robustly stable and satisfy all the performance criteria specified for the closed-loop system. The approach provides additional flexibility because once the regions are obtained, the designer can further tune the controller gains based on practical experience and/or observation of the measured signals in real-time.

ACKNOWLEDGEMENTS

The work of the first two authors was supported in part by the National Science Foundation.

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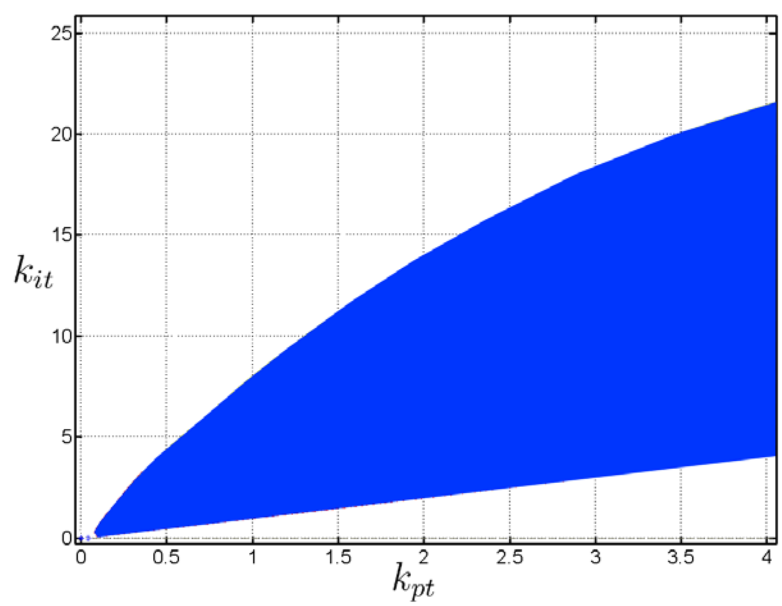

Figure 11: Controller gain region: no uncertainty on the Young’s modulus, phase margin of at least 10 degrees, and α = 1.

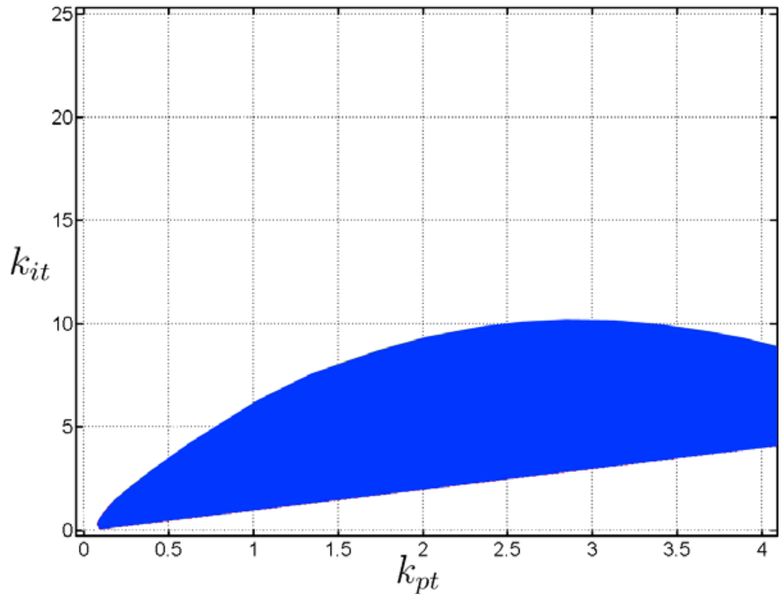

Figure 12: Controller gain region: no uncertainty on the Young's modulus, phase margin of at least 15 degrees, and $\alpha = 1$.

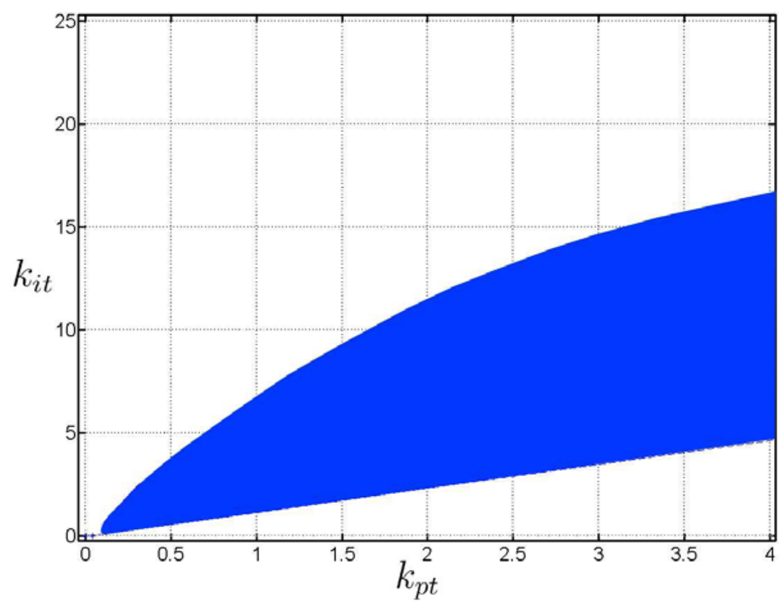

Figure 13: Controller gain region: ±5% of uncertainty on the Young's modulus, phase margin of at least 10 degrees, and $\alpha = 1$.

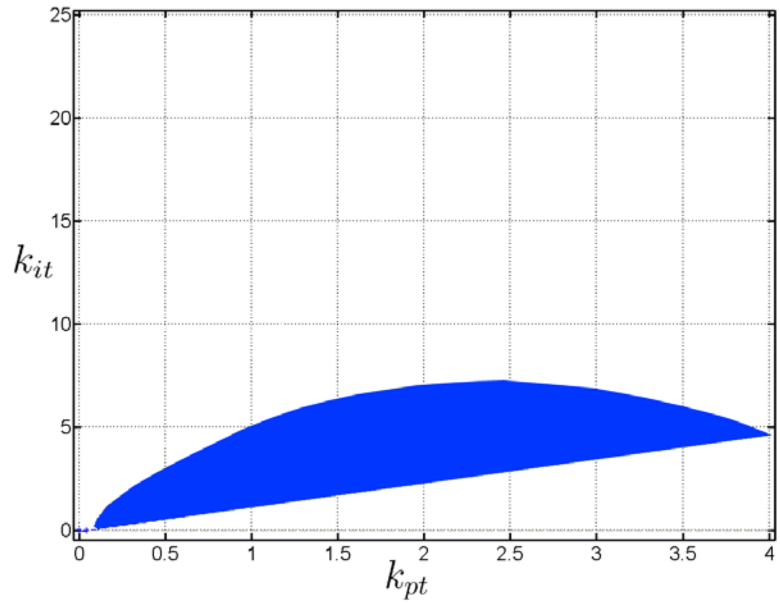

Figure 14: Controller gain region: ±5% of uncertainty on the Young's modulus, phase margin of at least 15 degrees, and $\alpha = 1$.