## MODELING AND CONTROL OF WEB VELOCITY AND TENSION: AN INDUSTRY/UNIVERSITY PERSPECTIVE ON ISSUES OF IMPORTANCE

By

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#### ABSTRACT

The goal of this paper is to provide an overview of current industrial practice incontrol of web velocity and tension and discuss some critical issues that require futureresearch from the community which is pertinent to problems faced in the industry. It is well known that there is a considerable gap (and time lag) between what is currently practiced in various industries and what is being researched in academia and research laboratories. This gap appears to be much more significant in the web handling industry when compared to some other established industries such as aerospace, automobile, semiconductor manufacturing, and robotics, to name a few. There are a few plausible reasons for this. First, the number of products made from materials manufactured in rolled form is very large. Second, since a wide spectrum of materials is manufactured and processed in rolled form, machines that handle different materials are diverse and so is their operation. Yet, there is a substantial amount of commonality between various web process lines that handle different materials. Two key process variables that need to be monitored and controlled in almost all web process lines are web tension and velocity. The discussions in the paper will highlight and focus on issues related to modeling and control of these two key process variables.

The paper will first give discussions on the "typical" webline, potential performance enhancements and commissioning improvements that modern control methods can provide, and advances in drive and microprocessor technology that allow implementation of modern controllers. A brief description of the models for web tension and velocity is given. Many modern control algorithms have been suggested for controlling tension and velocity in the recent years. But very few, if any, have been transferred to current industrial practice. One of the reasons is that the development and presentation of these new control strategies is often too abstract for controls engineers who are trained in implementing and tuning PID-type algorithms. The paper will examine some of these new control strategies whose implementation is relatively simple and present a reasonable level of complexity while providing superior performance to currently used PID techniques.

# NOMENCLATURE

A	area of cross section of the web [m <sup>2</sup> ]
b	viscosity parameter in the viscoelastic model [N sec/m]
Ε	modulus of elasticity (Young's modulus) $\left[\frac{N}{m^2}\right]$
$E_{v}$	parameter in the visco-elastic model $\left[\frac{N}{m^2}\right]$
е	error between system state vector and reference model state vector $(x - x_m)$
F	force [N]
G	system matrix in state-space representation
H	input matrix in state-space representation
h	web thickness [m]
J L	roller inertia
L k	web span length [m] controller gain parameter vector
n K	controller gain parameter matrix
P	Solution of the Algebraic Riccati Equation
$q_t, q_v$	weights in the performance index
R	roller radius [m]
r	reference [m]
S	state-space representation
t	time [sec]
Т	variation in web tension from reference [N]
$t_i$	web tension in the <i>i</i> -th span [N]
U	input torque in the linearized dynamics [N-m]
и	input torque [N-m]
V	variation in web velocity from reference $[m/s]$
V	web velocity $[m/s]$
w	web width [m]
x	state vector
3	strain
ρ	control input weight in the performance index
Σ	performance index
τ	time constant
ω	roller angular velocity [rad/sec]
Subscripts	
i	span or roller number or <i>i</i> -th subsystem
т	pertaining to the reference model
r	pertaining to the roller

rpertaining to the rollerwpertaining to the web span

#### THE "TYPICAL" WEBLINE

The web handling industry spans a wide variety of products and materials. Materials range from centimeters thick metals to microns thick plastics, with widths ranging from single thin strands to 10 plus meters, and line speeds ranging from millimeters per minute to in excess of 2500 meters per minute. The products themselves have a wide diversity, ranging from a single homogeneous sheet to multiple layers of non-uniform materials. Many web handling machines run numerous products, and must be rapidly reconfigured to run a variety of products efficiently and optimally.

Despite this wide range of materials and construction, a fair amount of commonalty exists among machines. Each machine section is required to transport the web, and generally control tension or strain in that section to meet each product's requirements. Each section's control system is typically implemented as a series of cascaded loops, with an outer tension regulator trimming an inner velocity regulator, which further commands an inner torque loop. Often the unwind and rewind sections of a machine will have an outer loop position regulator (dancer) with a coordinated diameter calculator driving the inner loops. The vast majority of these regulators are PID controllers, with additional provisions for performance enhancements such as feed forward, filtering, and gain adaption.

There has been a dramatic improvement in the performance of drive systems over the past decade. Digital drives have almost universally displaced analog drives. The dramatic improvements in microprocessors have been applied to drive systems in general, and has resulted in faster loop updates, increased resolution, fast communication links allowing high speed section coordination, and more complex and universal web handling algorithms embedded within the drive.

However, there still exists a large unmet need. Many industries, such as aerospace and robotics, have widely applied advanced modern control algorithms with great benefits to their industries. These algorithms are for the most part lacking in commercially available web handling products.

There are two main areas where modern control algorithms could dramatically improve web handling tension control systems: the first is in achieving higher performance, and the second is in greatly reducing the commissioning time and cost.

## MODERN CONTROL PERFORMANCE ENHANCEMENTS

The primary objective of tension control is to maintain the web tension as closely as possible to the tension setpoint at all times. This is a very demanding task, as tension control of webs is extremely sensitive to many parameters (web modulus, thickness, runouts, friction, traction/flotation etc.).

Modern control algorithms have been proven to provide greater performance over the traditional PID controller. These algorithms provide higher tension regulator bandwidth, higher dynamic stiffness (disturbance rejection) and increased accuracy, as well as provide increased disturbance rejection. This translates into lower product variability, improved yields and an opportunity to increase production speed. An additional benefit of these modern algorithms is their increased adaptability. Web lines have many parameters that change during operation. Diameters and inertias change continuously during operation. Load changes can change depending on process conditions. Few web lines run a single product configuration; the majority need to run products of varying web thickness, width, and modulus. While current methods allow for these conditions, they often rely on the operator or a sensor to provide accurate information required for operation. In many cases, the machine is tuned to accommodate a wide variety of

materials, compromising the performance for some materials. Modern algorithms can identify these changes during operation, adapting the controller and thus reducing product variability.

Control systems rely on sensors to provide feedback to control the process. Modern algorithms are capable of estimating or observing a process variable. This can avoid the initial cost of a sensor, as well as the expense required to maintain and calibrate it. Often it is difficult to properly locate a sensor for the best measurement. The observer can often make the best estimate of a desired process variable. Therefore, modern control system observers and state estimators can be used to refine (greater resolution), filter and access process states that are difficult and, in some cases, impossible to measure otherwise. The modern observers typically rely on a few reliable measurements of key process variables and knowledge of the model structure to estimate other process variables that are either difficult to measure directly using existing sensors or the difficulty to locate a sensor at a specific location in the line where a process variable needs to be measured.

The control system is often called upon to correct many mechanical shortcomings and limitations. The ideal machine would have infinite stiffness, with all loads rigidly coupled to zero inertia motors. Non-ideal components, such as gearboxes, keyways, belts, pulleys, power transmission shafts, and so on, would not be used. Load cells would be mounted under large diameter, zero inertia rollers; dancers would be massless and frictionless. In the real world all mechanical systems have inertia (mass), and typically exhibit non-ideal characteristics such as friction, stiction, compliance and lost motion (backlash). Though mechanical systems can always be improved, cost, space and time constraints will drive the mechanical system design. Modern control methods can compensate for mechanical system non-idealities providing improved performance.

## **COMMISIONING IMPROVEMENTS**

Modern control algorithms could greatly ease the task of commissioning a web handling machine. Commissioning a web handling machine is a detailed and demanding procedure and can be difficult. Below is a simplified overview of a typical commissioning sequence for a web line:

- 1. Commission & identify motor models. Setup drive torque regulators.
- 2. Implement velocity regulators. Tune with drive motor only (decoupled). Verify performance.
- 3. Couple drive motor to machine (roll). Identify inertia and friction model of the machine. Set up torque load sharing through mechanical coupling. Compensate for poor mechanics and tune velocity controllers for maximum dynamic stiffness.
- 4. Calibrate tension feedback devices. Implement tension controller. Tune statically.
- 5. Repeat for each section.
- 6. Implement special features for winders, unwinders, coaters, etc.
- 7. Optimize machine speed performance, no web.
- 8. Web up, tune statically for interconnected sections.
- 9. Tune dynamically at various speeds, verify stability and performance.

Steps 4-9 are highly interactive, making optimization difficult. Ideally at each stage, the system would be modeled in great detail, model information would be correlated with system identification parameters found during commissioning, discrepancies would be noted and investigated, and assumptions questioned. Based on the identified system model with issues resolved, the control algorithm and parameters would be designed, and the control system implemented and tested. On high volume systems, such as disk drives

where millions of units are manufactured, this is indeed the standard procedure. For web line systems, in which each system is unique, with highly variable product specifications, there is simply not enough time and budget to apply the ideal commissioning procedure. Predominately "rules of thumb" type procedures are used that allow a skilled control engineer to rapidly achieve a reasonably optimized system.

It is difficult to obtain high fidelity models for web handling machines and web processes because of the possibility of process and parameter changes. The PID algorithm strongly dominates the web handling industry today since it can be in most instances designed and tuned independently of the model. PID loops are widely applied and understood, easily tuned, and are reasonably robust to moderate changes in the system. Indeed the PI control law is used almost exclusively in the inner regulator loops of the drive control structure. At the drive velocity/position regulator level the PI control law (sometimes) applied in a state controller, with command feedforward, velocity observers, acceleration observers, filters, and disturbance estimators represent the state of the art. Why is PID still the preferred control law for drive regulators? Because rigid mechanical systems are well understood, a properly modeled and compensated drive will require very little output from velocity/position loop PI regulator to stiffly hold setpoint. The drive PI regulators are often automatically tuned by merely selecting a desired bandwidth. Indeed it is the drive's task to provide a stiff and linear interface to the rollers that ultimately control the web. A well-tuned drive using PI control should appear as a unity gain, zero lag (at least with a wide bandwidth first order response) web actuator, taking velocity/position/torque setpoints from the outer loop controller.

Given the success of PID algorithms in inner loop control (velocity/position/torque), drive and web handling control engineers naturally apply the same robust and easily understood PID control law to the system outer loop web tension regulators. It turns out that this is a less than optimal solution for the outer loop tension control problem. Modern control algorithms integrated into drive system controllers can and will provide more robust, stiff, and repeatable results than the traditional PID approach. However, modern control methods require a substantially more detailed and accurate model of the plant and the product. This will require increased time, greater effort, more skill and a deep process understanding from the web handling control engineer.

There are two critical requirements that would make modern algorithms more successful in commercial drive platforms. The first is the algorithm must capture the expertise of the implementer, and yet present it to the less sophisticated user in a very understandable manner. While the machine is often commissioned by a controls engineer, daily maintenance and support operations are typically provided by a plant technician It is critical that this less skilled user is able to readily understand and apply the advanced algorithms. The second requirement is that the tools required for systemidentification and parameter optimization be provided in a form that the typical drive user can understand and apply. These tools should allow rapid identification of the plant and product, and then provide optimization of the control system parameters based on this identification.

A web handling machine is a large capital expense, and often the products manufactured on these machines are price sensitive commodity products. There are enormous cost and schedule pressures to place a new machine in operation as quickly and inexpensively as possible. Uptime and rapid time to repair are also very important to economical operation. Any tools that can decrease commissioning time and expense, reduce product variability, increase runnability and increase process availability will be well received in the web handling industry.

Current state of the art tools rely heavily on 3rd party analysis and design tools, such as Matlab & Simulink. The user must design an experiment to provide the proper plant stimulus for identification, and then import this data into Matlab for the plant

identification and control system design and simulation. These tools work well for the skilled user, but even then it is still a substantial effort to fully model and optimize a system. Typically, due to the previously mentioned time and expense pressures, only the more demanding or problematic sections are commissioned with these advanced tools. These tools are costly, and generally cannot be justified for machine commissioning or machine design.

Ideally, the web handling control system processor would incorporate these tools into the control platform. By properly encapsulating the expertise of the designer, the less skilled user is led through the complex process of identifying that plant, designing or configuring the controller, and optimizing and testing its performance. These "wizards" would bring great benefit to the web processor by reducing design and commissioning cost and time, as well as allowing the user to achieve much the higher machine/process performance that was discussed above.

## ADVANCES IN DRIVE TECHNOLOGY

Modern AC drives are capable of operating at torque bandwidths well beyond any conceivable disturbance generated in web handling systems. Recent developments show that deadbeat control of drive motor torque, flux and losses is possible at high fixed modulation frequencies. The switching frequency limit resides in the capability of power semiconductors to switch current at motor voltages - in excess of 10 kHz. Within the period of a single PWM switching event, the commanded torque is available at the motor shaft. Essentially the ideal of the unity gain torque source is now actually available and has been functionally available for several years.

The system drives on the market today have a torque response in the range of 2000 - 4000 rad/sec. This is accomplished by use of discrete state control algorithms making use of Luenberger and Gopinath observer software structures for current, speed, acceleration, position, etc. The drive outer loop regulators are cascaded position to velocity to torque (see Fig. 1). The (optional) process regulator can be cascaded into any summing junction including torque. The outermost regulators (process and position) are processed at a rate of 1 ms to 500  $\mu$ sec whereas the velocity regulator is processed at updates of 500 – 250  $\mu$ sec. The accuracy of the torque developed by drives using these algorithms - independently measured with calibrated torque instrumentation - is in the range of 1% at zero speed. Greater accuracy can be achieved by careful and repeated motor identification. Velocity controlled drives can be implemented for non-zero speed operation using an open loop flux observer - eliminating a sensor (shaft position encoder) - with reasonable speed accuracy of less than 0.5%.

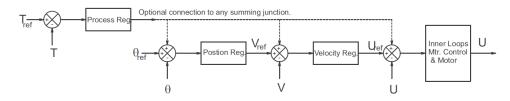


Figure 1 - Cascade structure of drive regulators

Drives employ observer strategy to develop excellent estimates of (zero lag) acceleration feedback and slightly lagged estimates of disturbance torques. These estimates are employed at the torque regulator level to compensate for poor mechanics, e.g. backlash, compliance. The effect of this is that the velocity regulator does not act to compensate in the event of a torque disturbance - disturbance rejection takes place at the torque regulator bandwidth.

System level drives are complex and are not meant for casual application. These drives provide external access to process, position, velocity and torque references. The process, position and velocity regulators installed in system drives are configured for the Proportional-Integral (PI) control law with programmable proportional and integral gains, e.g. gains can be programmed to change with speed. In addition, commands are fed forward through an identified load model to the torque summing junction ahead of the torque regulator. Command feedforward provides a response to operator commands at the wide torque regulator bandwidth. In addition to the advanced regulator strategies, described above, modern drives perform motor and load identification routines to simplify commissioning. The drive software will perform all the measurements required to parameterize a high fidelity model of the drive motor and the driven load - including the load mechanics and losses. Commissioning a single drive out-of-the-box will typically be completed in less than an hour and will be greatly dependent upon the time to couple and uncouple the load.

In addition to the regulatory and commissioning software features, modern drives provide high speed trend buffering capability where events internal or external to the drive can trigger the capture of information that would not otherwise be accessible. Drives have modern digital communication features and can communicate with higher order control systems using high speed TCP/IP and several other protocols. Most drives have a high speed fiber optic communication channel, so coordinated torque control with mechanically coupled drives can occur. These and numerous other features make modern AC drive a quantum leap in capability over the older generation DC drives where the limitation in the DC drive semiconductors (SCR's) limited torque bandwidth to the range of 100 -300 radians/sec.

Drives are evolving to include self-sensing features where the temperature limit of the semiconductor junction can be observed and controlled -the efficiency of these devices, already high, will get higher. Advanced liquid cooling designs allow a large power capability in a small package - present state of the art contains 1500 hp in a volume of 12  $\text{ft}^3$  - making it possible to use present size limited drive control rooms to manage greater and greater power level.

## DYNAMIC MODELS OF LONGITUDINAL WEB BEHAVIOR

A discussion of the derivation of the dynamic models for web tension and velocity are given in Appendices 1 and 2. These models are derived based on first principles, which result in nonlinear differential equations for web tension. The nonlinear equations are linearized around operating or reference values to obtain a linearized set of equations which can be used for model analysis and/or controller design. The linearized equations can be expressed either in terms of transfer functions between inputs and outputs (web velocity and tension) or state-space models. Although these models are different representations of the same dynamics, separate study of them by the control engineer provides flexibility in using tools available for analysis and controller design for different representations.

There are several key assumptions that are made in the derivation of the nonlinear model for web tension. First, uniaxial behavior is assumed in the transport (longitudinal or machine) direction. Second, web strain is assumed to be very small in comparison to other process variables. Third, web tension/strain within a span is assumed to be uniform, that is, it does not vary spatially. Fourth, a linear constitutive relation between web strain and tension is assumed; typically, the web is assumed to be either elastic (Hooke's law is

often used to relate stress and strain) or viscoelastic (a first-order dynamics is often used to relate stress and strain). Relaxing one or more of these assumptions often results in complicated dynamic models of web tension that are not generally amenable for analysis or controller design. Years of experience in the industry has shown that such assumptions are reasonable for many webs.

The dynamics of web velocity are obtained indirectly via roller dynamics and setting up a constitutive relation between roller peripheral velocity and web velocity. The roller angular velocity dynamics are obtained by torque balance and the peripheral velocity is obtained as a product of the roller angular velocity and the radius of the roller. If it is assumed that the web is not slipping on the roller, then the web velocity is same as the roller peripheral velocity. There are slip and friction models available in existing web handling literature that relate roller velocity to web velocity but these have not been successfully validated. Further, these models require knowledge of a number of model parameters whose selection is not trivial. The span tension dynamics is a coupled differential equation including as variables the web velocities of adjacent rollers and transport of tension from the upstream span. The transfer function models for the web velocity on the *i*-th roller and web tension in the *i*-th span are shown in block diagram form in Figures 2 and 3.

Two time constants are involved-one related to the web span and the other to the roller. Further, if a viscoelastic model is used, then the viscosity parameter b is non-zero and must be appropriately chosen to reflect the amount of viscosity present in the web material. Once the dynamics of web tension in each span of the web line and web velocity on each roller are determined, the governing equations that represent the longitudinal dynamics of the web line can be formed.

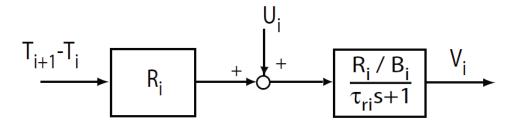


Figure 2 – Web Velocity Dynamics (on the *i*-th roller)

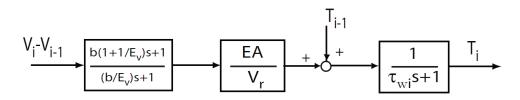


Figure 3 – Web Tension Dynamics (*i*-th span)

In many process lines, the number of idler rolls is large when compared to the number of driven rollers. Therefore, there is a large number of governing equations which represent the free dynamics of the web rollers and the spans in between them. The effect of these idler rolls and free spans on web transport is most significant during acceleration and deceleration of the line. Under the assumption that the line is running at a steady speed, the dynamics of the idler rolls is ignored for steady-state analysis and controller design. This is often done by introducing the notion of a tension zone, which is the web between two driven rollers. This strategy of representing the dynamics via the notion of tension zones considerably reduces the number of governing equations for a web line, and thus, facilitates analysis and controller design.

Another dynamic element that is prevalent in many web process lines is a dancer. As opposed to feedback of web tension that is measured by a tension transducer device, the position of a dancer is used as feedback. The motion of the dancer is a consequence of web tension variations and thus its position reflects web tension variations. Since a dancer is a dynamic element, it must be modeled as well. There are other dynamic elements such as accumulators/festoons which are present in many commercial web lines. Models of these dynamic elements must be appropriately included in the web line dynamics.

Improvement of mathematical models which better reflect dynamic behavior of the web and the machine is an ongoing effort that will continue well into the future. The core dynamic models for web tension and velocity behavior in a web line have been well established. Recent studies have developed models for non-ideal effects such as backlash, dead-zones, etc. Investigations into improving the models to better reflect observed data on real process lines is necessary. It is expected that the process of validation of models will lead to model improvements. It is essential to discover mechanisms that cause certain web behavior and figure out ways to incorporate those mechanisms into models. For example, periodic oscillations are found in all measured tension signals. It is evident that such oscillations arise due to non-ideal rotating components. It was unclear until recently how to incorporate this effect into models so that web tension from model simulations reflects such oscillatory tension behavior. It was found recently that span length variations due to eccentric roller and/or out-of-round rolls are the direct cause for these oscillations, which can be appropriately included into models. Such investigations which can simultaneously analyze theoretical models and experimental data from well-designed experiments on web lines have the potential to result in significant model improvements. These model improvements can be utilized in the design and tuning of controllers which will potentially result in reduction in product variability and increased productivity.

## **CONTROL STRATEGIES**

Many web process lines typically have a large number of idler rolls in comparison to the number of driven rollers. For analysis and controller design purposes a web process line is often divided into tension zones, each tension zone is a section of the web between two driven rollers. Tension in each zone is controlled using a driven roller. A master speed roller which is under pure speed control is often used to set the web line speed. An example of such a strategy with tension feedback from load cells is shown in Figure 3. The decentralized control strategy that is often used in practice for each driven roller is a cascaded two-loop structure as shown in Figure 4. Ignoring the dynamics of idler rolls during controller design leads to two potential issues: (i) energy required to accelerate and decelerate the idler rolls is not taken into consideration during the controller design phase, and, more importantly, (ii) controllers are not designed to account for resonances due to idler roll inertias. The first issue leads to tension variations in the idler roll spans within the tension zone. The second issue leads to overestimation of the performance of the designed controller when in reality the minimum resonant frequency of the idler roll system within a tension zone could very well be smaller than the controller bandwidth. For effective controller design the control engineer should be familiar with the minimum resonant frequency, and possibly the distribution of the resonant frequencies near the minimum resonant frequency.

Another issue that is overlooked is the coupling between different subsystems; the decentralized control design assumes no dynamic coupling between tension zones. This coupling leads to propagation of disturbances from one zone to another. Controllers must be designed in such a way that this coupling is minimized. A natural question to ask is whether there is a way to achieve higher performance from a given tension zone.

Two control strategies are commonly employed in the web handling industry, one uses tension feedback from a load cell (see Fig. 4) and the other uses dancer position feedback (in Fig. 4 replace the web/load cell dynamics block by web/dancer dynamics block; the feedback now will be dancer position and the reference will be regulated dancer position). In the dancer case, the tension variations are reflected as dancer motion and thus the dancer position is expected to provide inferred measurement of web tension. In each strategy, the output of the tension controller acts as a velocity error correction. The velocity controller is designed followed by the design of the tension controller. The most commonly employed controller is the PID controller in both cases. In some situations, a torque controller based on tension measurement is directly used to regulate tension.

Many modern control algorithms have been suggested for controlling tension and velocity. Some design techniques include linear quadratic optimal control design, H-infinity control design, Kalman filter design, and adaptive control design. Many of these techniques have been shown to provide superior performance over existing PID-type controllers. In the following we will discuss some of these techniques and their potential advantages and limitations towards successful application on commercial machines.

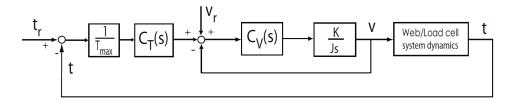


Figure 4 - Load Cell Tension Feedback Control Strategy

#### State Feedback Controllers Via Linear Quadratic Optimal Control Method

In this case the controller is designed in the state-space domain. A quadratic performance index is minimized to obtain a state-feedback optimal controller. In the web handling case, the state variables are tension variations and velocity variations. The performance index consists of two terms, weighted penalty on tension and/or velocity variations and weighted penalty on the input variable. The performance index that is to be minimized is of the form:

$$\Sigma_{i} = \int_{0}^{\infty} q_{ti} T_{i}^{2}(t) + q_{vi} V_{i}^{2}(t) + \rho_{i} U_{i}^{2}(t) dt$$
<sup>{1}</sup>

where  $q_{ii}$ ,  $q_{vi}$  and  $\rho_i$  are weights. Increasing the value of a weight places increased penalty on variations in the corresponding variable. The resulting controller is linear and takes the form,  $U_i(t) = -k_{ii}T_i(t) - k_{vi}V_i(t)$  where  $k_{ii}$  and  $k_{vi}$  are the optimal gains. The optimal control gains for each section of the web line are computed based on a state space model. The implementation of such a controller is shown in Figure 5. The equilibrium control block in the figure is used to provide feedforward actions. Such a state feedback optimal control strategy was successfully implemented on the experimental web platform shown in Fig. 6. Experimental results with a PI controller (same structure as Fig. 4) and the state-feedback optimal controller are shown in Figures 7 and 8, respectively.

The benefits of this approach are: (i) the resulting optimal loop transfer function has guaranteed phase and gain margins; (ii) one can include integral action if needed; (iii) the variables in the performance index can be weighted in the frequency domain; for example, if the actuator has certain frequency response, the control weight can be shaped based on this known frequency response; (iv) if a model of the disturbance is known, then it can incorporated into the controller to reject it; for example, if the disturbance is sinusoidal with a known frequency and unknown amplitude, the effect of this on the output can be attenuated by including an internal model of the disturbance in the controller; and (v) almost all the computations are off-line and can be done prior to implementation and the implementation of the controller is straightforward.

The limitations of the approach are: (i) Selection of weights in the performance index; (ii) off-line computations need computational tools such as MATLAB; (iii) a model is required; and (iv) possible strategies to tune control gains on-line to improve performance are not known.

A natural question to ask is that what additional developments are needed for commercial implementation of this controller in web handling machines. The answer to this question is linked to the ability to address the limitations stated above by systematically constraining some of the design parameters. The main aspect of many of the modern control techniques is that they provide significant flexibility in shaping the algorithms based on the plant or process under consideration. Although this flexibility is welcomed by advanced control engineers, it in fact provides restrictions on commercial implementation and tuning by operators due to the selection of various design parameters. What needs to be done to bring this controller for commercial implementation is a systematic procedure tailored for web tension and roller dynamics in which many of the design parameters are constrained and a limited number of free parameter that can tuned/adjusted by the operator. This would require development of a base algorithm that can be implemented at the machine commission stage and steps to tune/adjust free design parameters that can be performed by the machine operator.

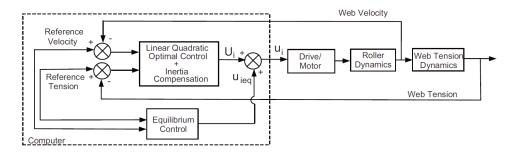


Figure 5 – Linear Quadratic Optimal State Feedback Control

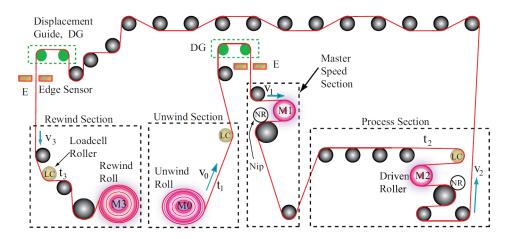


Figure 6 – Sketch of the experimental platform

## H∞ Controllers

The  $H_{\infty}$  control theory involves minimization of the  $H_{\infty}$  norm of a transfer function matrix. This theory can be used to synthesize robust multivariable controllers with disturbance rejection property. For scalar transfer functions (single-input single-input (SISO) systems) the  $H_{\infty}$  norm is simply the peak of the magnitude bode plot. Therefore, if one considers the transfer function from a disturbance to the regulated output, then a controller that minimizes the  $H_{\infty}$  norm of this transfer function is sought. For multiple-input multiple-output (MIMO) systems,  $H_{\infty}$  norm is the maximum singular value of the transfer function matrix. Details of various available algorithms using the  $H_{\infty}$  design can be found in [6,7] and references there-in. The benefits of this approach are: (i) controllers can be designed such that several closed-loop transfer functions can be minimized simultaneously; effect of various process disturbances on outputs can be attenuated; (ii) both structured and unstructured plant uncertainties can be handled. The same limitations as discussed for state feedback controllers above apply. Further, practicing controls engineers would be required to take advanced control courses or training workshops to understand the algorithms.

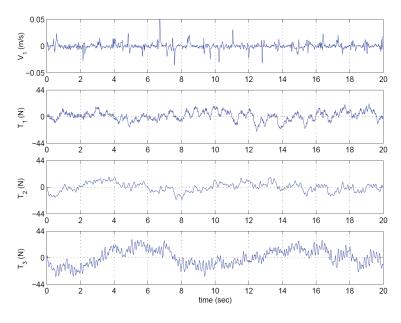


Figure 7 – Experimental results with PI controller: Line speed = 1000

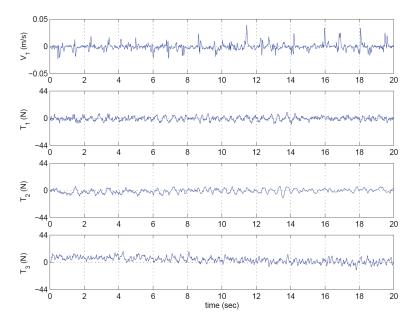


Figure 8 – Experimental results with optimal controller: Line speed = 1000

## Adaptive Controllers

Once tuned fixed gain controllers, such as the ubiquitous PID controller, may not provide adequate performance when the process or plant undergoes changes. Whenever the plant or process changes, the operator or the control engineer has to re-tune the gains to get satisfactory performance. This repeated tuning and re-tuning can be time consuming and often does not provide desired results. A controller that can continuously adapt itself to changes in the process and plant parameters is desirable. The existing adaptive control literature is extensive and a large number of methods exist. So, selection of a particular adaptive method for designing a particular adaptive scheme for web tension and velocity is a considerable task. There has been some work on application of adaptive control to web tension and velocity dynamics; this was reported in [9].

Existing adaptive control strategies generally fall into two broad categories-indirect approach and direct approach. In the indirect approach, the uncertain plant parameters are estimated first and then the controller parameters are derived based on the estimated plant parameters. In the direct approach, the controller parameters are parametrized in terms of the uncertain plant parameters and directly estimated using adaptation algorithms. In both these approaches, the way in which adaptation algorithms are chosen for parameter estimation is important. Two methods are prevalent in estimating unknown parameters: (1) positivity and Lyapunov design and (2) gradient and least-squares methods based on estimation cost criteria. The positivity and Lyapunov design is based on the direct method of Lyapunov and is formulated as a stability problem, that is, the differential equation for the parameter estimation algorithm is chosen such that certain stability conditions based on Lyapunov theory are satisfied. The second approach deals with minimization of a cost function which is specified in terms of an estimation error; the estimation error is explicitly dependent on the estimated and actual parameters.

Application of a direct adaptive algorithm based on Lyapunov design was successfully implemented on the experimental web platform shown in Figure 6. The algorithm was formulated based on the state-space representation of the tension and velocity dynamics of each section of the web line. The outline of the approach is the following. The dynamics of each section of the web line can be represented in the form:

$$\mathbb{S}_{i}: \quad \dot{x}_{i}(t) = G_{i}x_{i}(t) + H_{i}U_{i}(t) + \sum_{j=0, j\neq i}^{3} G_{ij}x_{j}(t)$$
<sup>(2)</sup>

where  $i = 0, 1, 2, 3, U_i$  is the control input,  $G_i, H_i, G_{ii}$  are system matrices, and

$$x_0 := \begin{bmatrix} T_1 & V_0 \end{bmatrix}^{\top}, x_1 := V_1, x_2 := \begin{bmatrix} T_2 & V_2 \end{bmatrix}^{\top}, x_3 := \begin{bmatrix} T_3 & V_3 \end{bmatrix}^{\top}$$

The last term in equation {2}reflects the coupling of the dynamics in the first section with variables in other sections. The dynamics of the entire web platform can be compactly expressed as

$$\mathbb{S}: \dot{x}(t) = Gx(t) + HU(t)$$
<sup>{3</sup>

where  $x^{\top}(t) = [x_0^{\top}(t), x_1^{\top}(t), x_2^{\top}(t), x_3^{\top}(t)], U^{\top}(t) = [U_0(t), U_1(t), U_2(t), U_3(t)], G$  is a matrix composed of block diagonal matrix elements  $G_i$  and off-diagonal matrix elements  $G_{ij}$ , and H is a block diagonal matrix composed of  $H_i$ .

An adaptive control algorithm was designed using Lyapunov method based on the web platform dynamics S mimicking a known reference model which is given by

$$\mathbb{S}_m: \dot{x}_m(t) = G_m x_m(t) + Hr(t) - HK_m^{\dagger} x_m$$
<sup>(4)</sup>

where  $K_m$  is a matrix of gain parameters. The resulting control algorithm and parameter estimation algorithm are given by

$$U_{i}(t) = r_{i}(t) - k_{mi}^{\top} x_{m}(t) - \hat{k}_{i}^{\top} x_{i}(t)$$
(5a)

$$\hat{k}_i(t) = -(e_i^\top(t)P_iH_i)x_i(t)$$
(5b)

where  $\hat{k}_i$  is the estimate of controller parameter  $k_i$  and  $e_i$  is the error given by  $e_i := x_i - x_{mi}$ . Experimental results corresponding to implementation of this adaptive controller are shown in Fig. 9. Complete details of this algorithm can be found in [9].

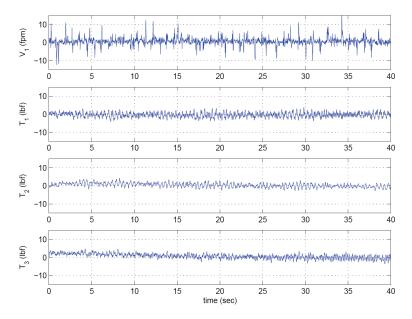


Figure 9 - Decentralized adaptive controller: Reference velocity 1000 ft/min

The benefits of developing controllers using the adaptive approach are the following: (i) ideal for web lines handling different materials; (ii) parametric uncertainties can be addressed; (iii) ability to provide robustness by restricting the parameter estimates to a known range of values; and (iv) significant potential to reduce commissioning time while achieving high performance. The limitations are the following: (i) parameter estimation algorithms must be implemented on-line in real-time; (ii) increase in computational effort when compared to fixed gain controllers; and (iii) structure of the dynamic models are needed. It appears that the first two limitations can be overcome by modern drives and real-time processors, which have the processing capabilities to implement advanced algorithms. The structure of the dynamic models of web tension and velocity are reasonably well developed. So, there are two main additional developments that are needed for the adaptive algorithms to be implemented on commercial web machines: (i) finding the adaptive structures that are well suited for robust commercial implementation on web process lines; and (ii) simplifying these adaptive structures and improving them for implementation by carrying out a series of well developed experimental procedures. From the discussions given in earlier sections, successful development of simple adaptive approaches seems to be within reach and can provide significant benefits.

### SUMMARY

Modern control methods have clearly been proven to achieve higher performance than the existing predominantly PID based methods. However, due to the web handling challenges presented in this paper, the rate of adoption and implementation in commercial hardware has been slow.

The incorporation of advanced control algorithms into commercial web handling drive systems would provide numerous benefits to the industry. However, for these algorithms to be commercially successful, the following features and functionality are required:

- Modern high performance adaptive algorithms provided within commercial web handling controls, allowing the typical control engineer to focus on optimizing web transport as opposed to developing specialized algorithms. This will also provide much more powerful and sophisticated algorithms than a typical engineer could develop on their own.
- The incorporation of adaptive structures with some identification methods to compensate for material and process changes.
- The inclusion of embedded data collection tools with parameter identification "wizards", allowing the typical control engineer to rapidly commission and optimize the application.
- Ideally these algorithms would be implemented to "encapsulate" the expertise of the advanced control expert while hiding the complexities of the underlying algorithms, allowing the typical plant floor maintenance personnel to be comfortable in implementing and supporting these advanced controllers.

The web handling industry requires expensive and complex capital equipment. The products manufactured on these machines are often price sensitive commodity products. In an increasingly global economy, we are often competing against worldwide low cost suppliers. The suggestions provided in this paper could dramatically improve equipment performance, both in the commissioning as well as the operational stages of operation. This expanded performance will enable new more demanding web products, as well as improve quality, increase yields, and reduce costs. The authors are eager to see some of the algorithms and methods presented in this paper appear in commercial equipment.

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### **APPENDIX 1: WEB TENSION DYNAMIC MODEL**

The mathematical model describing the dynamics of web tension in a web span is derived by applying the law of conservation of mass for a control volume encompassing the web span between two rollers as shown in Fig. 10. This is done as a two step procedure: (1) apply law of conservation of mass to obtain the strain dynamics and (2) obtain tension dynamics by choosing a constitutive relation between stress and strain. This two-step procedure allows inclusion of various constitutive relations between stress

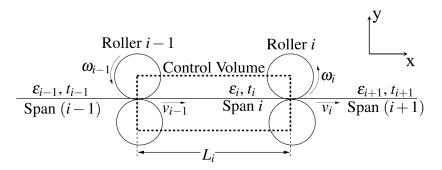


Figure 10 – Web Span and Control Volume

and strain. From step 1 the strain dynamics in a web span is obtained as

$$\frac{d}{dt}\left[\int_0^{L_i} \frac{dx}{1+\varepsilon_i(x,t)}\right] = \frac{v_{i-1}}{1+\varepsilon_{i-1}(t)} - \frac{v_i}{1+\varepsilon_i(t)}.$$
(6)

This is simplified under the assumptions: (i) strain within the length of the span does not vary with x and (ii) strain is small, i.e., the term  $(1/1 + \varepsilon) \approx 1 - \varepsilon$ . Under these assumptions the web strain dynamics can be simplified to

$$L_i \frac{d}{dt} \varepsilon_i(t) = v_i(t)(1 - \varepsilon_i(t)) - v_i(t)(1 - \varepsilon_{i-1}(t))$$
<sup>(7)</sup>

This nonlinear web strain dynamics is linearized around a reference web velocity,  $v_r$ , and reference strain,  $\varepsilon_r$ , by setting  $v_i(t) = v_r + V_i(t)$  and  $\varepsilon_i(t) = \varepsilon_r + E_i(t)$ , and ignoring second-order terms. The resulting linearized dynamics is given by

$$L_i \dot{\varepsilon}_i(t) = V_i(t) - V_{i-1}(t) - v_r(\varepsilon_i(t) - \varepsilon_{i-1}(t))$$

$$(8)$$

In the frequency domain, the linearized strain dynamics can be expressed as

$$E_i(s) = \frac{1}{(\tau_{wi}s+1)} \left(\frac{V_i(s) - V_{i-1}(s)}{v_r}\right) + \frac{1}{(\tau_{wi}s+1)} E_{i-1}(s)$$
<sup>(9)</sup>

where  $\tau_{wi} = L_i/v_r$  is the web span time constant. The second term in the above equation is strain transport from the previous span.

The second step in the modeling procedure is to choose the constitutive relation between stress and strain. The simplest such relation is the Hooke's law for elastic webs, i.e.,  $\sigma_i / \varepsilon_i = E$ . The tension in the span is related to stress as  $t_i = \sigma_i A$ . With this constitutive relation, the tension dynamics is given by

$$T_{i}(s) = \frac{EA/v_{r}}{(\tau_{wi}s+1)} \left( V_{i}(s) - V_{i-1}(s) \right) + \frac{1}{(\tau_{wi}s+1)} T_{i-1}(s)$$

$$\{10\}$$

If the web exhibits viscoelastic behavior, then one has to appropriately choose the constitutive relation between stress and strain. Two well-known models are (i) Maxwell model with a linear spring in parallel and (ii) Kelvin-Voigt model with a linear spring in series. The Maxwell model with a linear spring in parallel is chosen for illustration purpose. The relationship between stress and strain for this model is given by

$$(1/b)\sigma_i + (1/E_v)\dot{\sigma}_i = (E/b)\varepsilon_i + (1+E/E_v)\dot{\varepsilon}_i$$
 {11}

The tension dynamics in the viscoelastic case is given by

$$T_{i}(s) = \left(\frac{b(1+1/E_{v})s+1}{(b/E_{v})s+1}\right)\frac{EA/v_{r}}{(\tau_{wi}s+1)}\left(V_{i}(s) - V_{i-1}(s)\right) + \frac{1}{(\tau_{wi}s+1)}T_{i-1}(s)$$
 (12)

In the derivation of the above equation, for simplicity, it was assumed that the same four viscoelastic constants apply to every span. This may not be the case when the web is being heated or cooled in certain sections of the process line; appropriate modifications to the above equations must be made for this case. Comparison of the above equation {12} with the elastic case {10} shows that by setting the viscosity parameter  $b \equiv 0$  one can retrieve the elastic case. Therefore, this model allows the controls designer to test the robustness of the controllers to webs which exhibit a certain amount of visco-elastic behavior by selecting the two parameters b and  $E_{y}$ .

## **APPENDIX 2: WEB VELOCITY DYNAMIC MODEL**

The web velocity dynamics on each roller is obtained by first finding the roller angular velocity dynamics and then relating the roller peripheral velocity and the web velocity. This is done by either assuming that either the web is not slipping or slipping. For the former case, the web velocity is related to the roller angular velocity through the roller radius,  $v_i = R_i \omega_i$ . When the web is slipping on the roller, one must have a relation between the roller peripheral velocity and web velocity through a friction model; this will not be discussed in this paper. For a roller of fixed radius (such as an idle or driven roller), the roller angular velocity dynamics is given by

$$J_{i}\dot{\omega}_{i} = -B_{i}\omega_{i} + R_{i}(T_{i+1} - T_{i}) + u_{i}$$
<sup>{13}</sup>

where  $u_i$  is the torque applied on the roller. If  $u_i$  is set to zero, then the roller *i* is an idler roll. In the frequency domain, the roller angular velocity dynamics is given by

$$\Omega_i(s) = \frac{1/B_i}{\tau_{ri}s + 1} U_i(s) + \frac{R_i/B_i}{\tau_{ri}s + 1} (T_{i+1} - T_i)$$
<sup>{14</sup>}

where  $\tau_{ri} = J_i/B_i$ . For the no-slip case,  $v_i = R_i\omega_i$ , the following web velocity dynamics is obtained:

$$V_i(s) = \frac{R_i/B_i}{\tau_{ri}s + 1} U_i(s) + \frac{R_i^2/B_i}{\tau_{ri}s + 1} (T_{i+1}(s) - T_i(s))$$
<sup>(15)</sup>

For the material rolls (unwind/rewind), since the radii are time-varying, the inertias are also time-varying. So the dynamics of the unwind and rewind rolls, respectively, is given by

$$\frac{d}{dt}(J_0\omega_0) = -B_0\omega_0 + R_0(t)T_1(t) + u_0$$
<sup>(16)</sup>

$$\frac{d}{dt}(J_N\omega_N) = -B_N\omega_N - R_N(t)T_N(t) + u_N$$
<sup>[17]</sup>

Expanding the derivative on the left side of the above equations leads to the following equations:

$$J_0(t)\dot{\omega_0} + \dot{J}_0(t)\omega_0 = -B_0\omega_0 + R_0(t)T_1(t) + u_0$$
<sup>{18</sup>}

$$J_N(t)\dot{\omega_N} + \dot{J}_N(t)\omega_N = -B_N\omega_N - R_w(t)T_N(t) + u_0$$
<sup>(19)</sup>

Using the relations between linear and angular velocity,  $V_0(t) = R_0(t)\omega_0(t)$  and  $V_N(t) = R_N(t)\omega_N(t)$ , the web velocity dynamics on the unwind and rewind rolls are given by

$$\frac{J_0(t)}{B_0}\dot{V_0} + V_0 + \frac{\left(\dot{J_0}(t) - J_0(t)\dot{R_0}(t)\right)}{B_0}V_0 = \frac{R_0^2(t)}{B_0}T_1(t) + \frac{R_0(t)}{B_0}u_0$$
<sup>(20)</sup>

$$\frac{J_N(t)}{B_N}\dot{V_N} + V_N + \frac{\left(\dot{J_N}(t) - J_N(t)\dot{R_N}(t)\right)}{B_N}V_N = -\frac{R_N^2(t)}{B_N}T_N(t) + \frac{R_N(t)}{B_N}u_N \quad \{21\}$$

By abuse of notation, under the assumption that the radii and inertias are slowly timevarying, the dynamics in the frequency domain is given by

$$V_0(s) = \frac{R_0/B_0}{(\tau_{r0}s + 1 + \beta_0)} U_0(s) + \frac{R_0^2/B_0}{(\tau_{r0}s + 1 + \beta_0)} T_1(s)$$
<sup>(22)</sup>

$$V_N(s) = \frac{R_N/B_N}{(\tau_{rN}s + 1 + \beta_N)} U_N(s) - \frac{R_N^2/B_N}{(\tau_{rN}s + 1 + \beta_N)} T_N(s)$$
<sup>(23)</sup>

where

$$\beta_0 = \frac{\left(\dot{J}_0(t) - J_0(t)\dot{R}_0(t)\right)}{B_0} \text{ and } \beta_N = \frac{\left(\dot{J}_N(t) - J_N(t)\dot{R}_N(t)\right)}{B_N}$$

$$\{24\}$$

## **APPENDIX 3: WEB LINE GOVERNING EQUATIONS**

One can form the governing equations in terms of the transfer functions as given above. But often it may be convenient to express the governing equations in state-space form to use the available tools for analysis and controller design. To obtain state-space representation of the web line, the notion of tension zones will be utilized, i.e., web tension dynamics between two driven rollers is taken into consideration. For example, an experimental web line shown in Figure 6 can be simplified, for control design purpose, to the one shown in Figure 11.

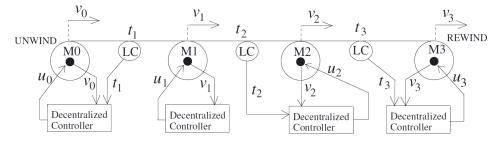


Figure 11 - Sketch of the experimental web line showing tension zones

Modeling and Control of Web Velocity and
Tension: An Industry/University Perspective
on Issues of Importance

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Name & Affiliation Unknown	Question Some of the controllers are model based, right? How do
Name & Affliction	you determine the model?
Name & Affiliation Dan Carlson, 3M Company	Answer Yes. Unfortunately, we didn't have enough time. One of the things that excites me a lot is that these models are not highly accurate. The traditional designs would rely on estimation. You would do elaborate analysis of the system and find very active models. These were pretty coarse models basically by entering inertias, roller diameters, web span distances. They were not input precisely. That is one of the things that gets me excited is because they don't rely on very exact and highly precise methods, such as accurate but high time consuming advanced stimulus system identification methods. Especially because as the web winds up, the model you have now is going to be change later. One thing I see that is crucial in the application of these algorithms being applied to web handling is being
Name & Affiliation	able to adapt and be tolerant of system changes. <b>Question</b>
Tim Walker, T. J. Walker & Associates	In your tension plots, you show $\pm$ offset from the 0. I am assuming that is not the actual tension?
Name & Affiliation	Answer
Dan Carlson, 3M Company	No, deviation from set point (tension error) is shown, in pounds force. So let's say your set point was at 50 lbf, the plot shows the deviation, or actual minus 50.
Name & Affiliation	Comment
Tim Walker, T. J. Walker & Associates	So for PID there was 10 pounds of variation on a 30 pound set point. It looks like <u>E40</u> Newtons. Not quite that high. One of the questions is: When do you need to advance from PID to advanced control? One of the challenges, of course, is that 80% of the applications are not going to need this.
Name & Affiliation Dan Carlson, 3M Company	Answer You're right. You don't need to always use the latest,
	greatest, highest performance algorithms. There are some costs with this. There is nothing wrong with PI, it works very well. Especially when the system changes PI can be a big advantage. With some processes where you do have very stringent performance requirements, or especially where a wide range of materials will be run, it can be very challenging with traditional PID control techniques.