ON THE DURABILITY OF CORES AND REST ROLL DYNAMICS

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ABSTRACT

The printing industry is facing the same challenges as many other industries, in that demand is increasing for its processes to run faster and more reliably. Roll weight, width and web speed are steadily increasing which put increasing demands on unwinding stability and durability of cores.

Durability and especially the unwinding stability of cores is one of the most important topics when designing new wider printing machine unwinders. The majority of paper and printing industry processes run with recyclable paperboard cores. A change in core geometry could lead to significant retrofitting costs for both the paper mill and printer.

Winder and printing press reel stand design, especially with regards to chucks, can have a considerable effect on the maximum tolerated roll weight and unwinding web speed. It is important to use the right combinations of cores and chucks to maximize performance and cost savings.

The risk of rest reel explosions increases rapidly in the vicinity of the resonance frequency of a rest reel. It is important to know the safe web speed ranges in different situations. The range of safe web speed can be estimated according to the presented theory. The maximum tolerated mid span vibration is typically +/- 5 mm.

Cores must also withstand the alternating, cyclic roll supporting stresses during winding and unwinding. The risk of core failure can be minimized by using cores which have sufficiently high dynamic delamination strength (roll weight capacity).

The durability of cores can be estimated by dynamic chuck load capacity tests and by testing cores in simulated winding-unwinding conditions. The simulation test results are reduced to correspond to certain confidence levels by reducing the simulation curve by a certain number of standard deviations.

With a sufficient number of chuck load capacity tests, statistically reduced winding-unwinding simulation results and test data concerning behavior of cores in the vicinity of resonance, core recommendations can be built for core users as shown here.

NOMECLATURE

Ac=area of the core cross section
Ap=area of the paper cross section
b0=coefficient of the least squares fit
b1=slope factor of the least squares fit
C=confidence level
c=viscous damping coefficient
c_{cr}=critical damping
Di=inside diameter of a core
Do=outside diameter of a core
Dp=outside diameter of the rest roll
f=rotation frequency
f(n,h)=probability density of the chi-square distribution
f(n,t)=probability density function of t-distribution
f_1=first natural frequency of a rest roll clamped in an unwinding stand
f2= free-free mode frequency of the rest roll at the free span length L
f_{free}=free hanging frequency of a core or rest roll at full length Lc
f_n= natural frequency
Ic=second moment of inertia of core cross section
Ip=second moment of inertia of rest paper cylinder cross section
L=free span length
Lc=full length of a core
Li=distance from the start of the chuck to the middle of furthest expanding element
$\log(x)_R$= reduced log(x) data points
$\log(y)_R$= reduced log(y) data points
M=mass per unit length of a test core
$mean_C$=population mean with confidence level C
$mean_{reduced}$= population mean can be reduced to the failure probability P
N=number of full oscillations
n=number of data samples
P=failure probability
R_i= residuals representing the deviation of the y_i from the fitted data points $y(x_i)$
S=standard deviation of residuals of curve fit
safe=safety factor
S_C=population standard deviation with confidence level
$S_{Crelative}$=relative population standard deviation with confidence level
S_F= safety factor with regard to population mean
S_{log}=standard deviation of residuals of linearly fitted data points $\log(y_i)$
t_2=upper integration limit for probability density function f(n,t) of t-distribution
v_{safe}=safe web speed of a rest roll
X_0=vibration amplitude in the start
x_i, y_i=random samples
X_N=vibration amplitude after N full oscillations
X_{stat}=static deflection
ζ = relative damping factor
λ=independent variable in the standard normal distribution at failure probability P
π=3.142
ρ_c=core density
ρ_p=paper density

INTRODUCTION

The core is an integral part of a paper roll. Cores work as winding shafts in paper mill slitter-rewinders and as unwinding shafts in printing press unwinders. Cores must be strong enough to withstand the chuck load cycles during winding and unwinding and the unwinding vibrations must stay below a certain safe level. The majority of the paper industry cores are made of paperboard. Paperboard cores possess properties such as good strength, good stiffness to weight ratio, and they are generally easy to handle & recycle.

Roll weight, width and web speed in the industry are steadily increasing, which creates added demands for winding and unwinding processes and poses increasing challenges with regards to the unwinding stability and dynamic strength of cores. Cores and chucks are important factors to consider when designing new, wider and faster printing presses.

In the rotogravure industry, the maximum roll width recently increased from 3.6 m to 4.32 m. Also, in the News and Heatset-Offset printing the current trend is towards 2-2.5 m wide rolls.

WINDING AND UNWINDING

Cores in the paper mill

Figures 1 a) and b) illustrate two ways of supporting rolls of paper in a paper mill slitter-rewinder. In a two-drum surface winder in figure 1 a), the roll weight is supported by the roll surface nip. Figure 1 b) shows a center-surface winder in which the roll weight is supported by a combination of a winding drum and chucks. Approximately 70-80 % of the roll weight is supported by the chucks in the center-surface winder. The maximum center torque in the start of the winding process is typically 100–250 Nm. Some 76x15 mm high strength paper industry cores can withstand torque loads in excess of 2000–2500 Nm.

The most important core properties in paper mill winding processes are straightness, strength, bending stiffness and moisture content. The out of straightness of cores should generally be no more than +/- 0.5 mm/m.

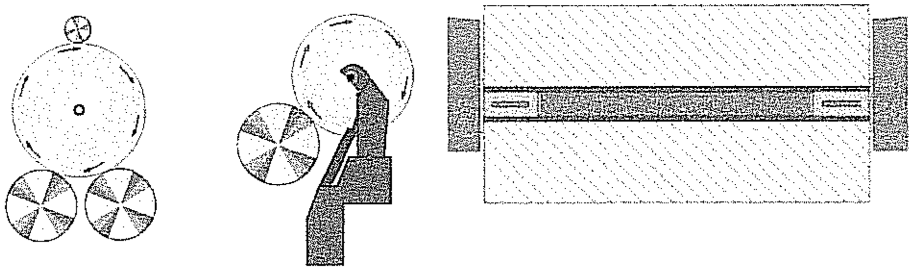

a) b) c)

Figure 1 –Two-drum winder a), center-surface winder b), printing press unwinder c)

Cores in the printing press

In the printing press, rolls are supported from the core ends by chucks as in figure 1 c). At the start of the unwinding process, the Rotogravure roll is accelerated to the appropriate paper web speed by acceleration belts contacting the surface of the rolls. Depending on the design, acceleration belts can contribute 10 kN extra load or support during the acceleration process. In modern constructions, the acceleration belts are located below the roll, which contributes support to the roll during the acceleration process. Typically the chucks on the printing press are equipped with expanding elements to prevent slippage between the core and chuck.

Unwinding paper rolls in printing presses requires core / chuck combinations that support rolls in such a way that excessive vibrations, core delamination and center burst problems do not occur. For safety reasons, cores should not explode too easily if driven accidentally to resonance. If the rotation frequency of rest roll is far enough from the first resonance, the vibrations stay at sufficiently low levels. The natural frequency of a core can be increased by increasing its bending stiffness and decreasing its mass. Furthermore, reducing the free vibration span by proper chuck design will have a positive impact on unwind speed.

Important core properties considering winding and unwindig

Some of the most important core properties, considering winding and unwinding, are:

-Accuracy in outside diameter, especially when using cores in a two-drum winder.
-Accuracy in inside diameter to prevent slippage and chew-out problems.
-Minimal elongation to reduce bending and vibrations in two-drum winding.
-Straight and clean end cut and low friction at core ends to minimize roll bouncing and eccentricity problems in two-drum winding.
-Accuracy in core cut to minimize roll bouncing in two-drum winding.
-Minimal ply gaps, or seamless outside plies in an effort to reduce marking of paper at the start of the wind.
-Low weight (handling, transportation, as high as possible natural frequency).
-Cores must contribute only minimal winding drum wear.
-Cores must tolerate dropping and mishandling to some extent.
-Cores should be able to tolerate good chuck contact to prevent chew out problems.
-The inside and outside diameter of cores must not decrease too much during winding and storing of paper rolls to get the chucks out easily after winding and to minimize the winding pressure drop at roll bottom during storage.
-Not too high radial elasticity to allow the nip generated tension mechanism to work in the start of winding.
-As high as possible damping to decrease vibrations near resonance.
-Straightness of cores is important since it contributes to roll eccentricity, winding pressure distribution and vibrations during winding and unwinding.
-Sufficient flat crush resistance to prevent handling and transportation damage.
-Sufficiently high roll weight capacity.
-Bending stiffness. Higher bending stiffness contribute to higher resonance frequency and stiffer cores bend less in the winding and unwinding process. Some of the author's test results suggest also that better bending stiffness of cores could contribute in reducing paper waste with center burst problem (flagging). There are less paper bursts and they are located closer to the core, which means less waste paper.
-Cores should tolerate dynamic bending deflections of order +/- 50 - 100 mm before explosion (safety, if a core or rest roll is driven too close to resonance).
-Small moisture exchange with paper (keep dimensions and minimize changes in winding tightness during storage).
-Minimal space utilization (transportation and storage).
-Recyclable raw material is advantageous since it allows easy disposal of cores after usage and helps optimize logistics in the paper mill and press room.

UNWINDING VIBRATIONS OF CORES AND REST ROLLS

The effect of damping on vibrations in resonance

Figure 2 shows a sample of damped sine wave vibrations of a core after impact excitation. The amplitudes decrease in geometric series. The smaller the damping in the vibrating system, the more cycles it will take for the vibrations to die down. If the vibrating core is modelled as a single-degree-of-freedom system with viscous damping, the relative damping ζ can be calculated by equation

$$\zeta = \frac{c}{c_{cr}} = \frac{1}{2\cdot\pi\cdot N}\cdot\ln\left(\frac{X_0}{X_N}\right) \qquad \{1\}$$

where c is the viscous damping coefficient, and c_{cr} is the critical damping, N is the number of oscillations, X_0 is vibration amplitude in the start and X_N is the vibration amplitude after N full oscillations.

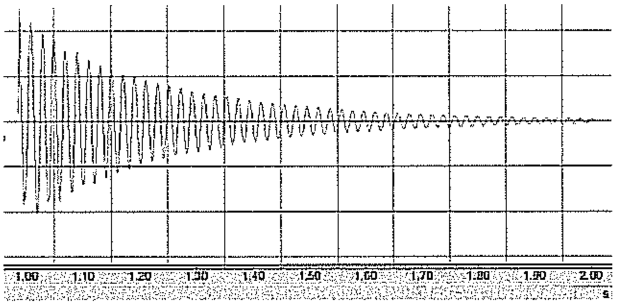


Figure 2 - A sample of core vibrations after impact excitation

If we study forced vibrations of a single-degree-of-freedom system with viscous damping, the amplitude of the dynamic forced vibrations as a function of vibration frequency f can be calculated according to equation [1]

$$X_{dyn}(f) = \frac{X_{stat}}{\sqrt{\left(1 - r(f)^2\right)^2 + \left(2\cdot\zeta\cdot r(f)\right)^2}} \qquad \{2\}$$

where X_{stat} is static deflection and the frequency ratio r(f) is

$$r(f) = \frac{f}{f_n} \qquad \{3\}$$

where f_n is the natural frequency. The amplitude ratio d(f) is

$$d(f) = \frac{X_{dyn}(f)}{X_{stat}} \qquad \{4\}$$

Figure 3 shows examples of amplitude ratios of forced vibrations for various degrees of relative damping ζ. The values of ζ of paper industry board cores (and rest rolls) are of order 0.005-0.02 depending on the raw material, construction and dimensions of the core, core supporting, rest paper cylinder and unwinder construction. The paper web brings also some damping to the system.

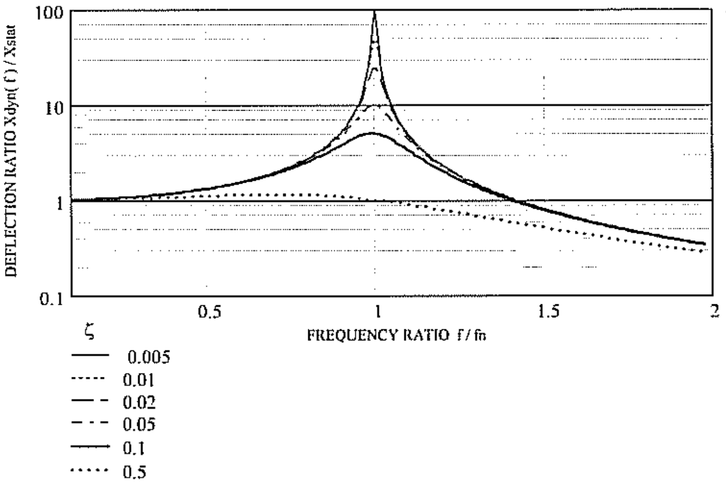
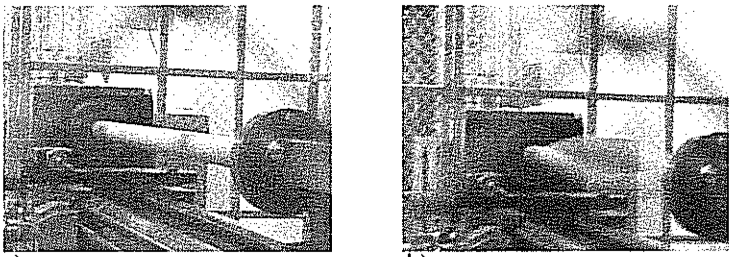


Figure 3 –Amplitude ratios of forced vibrations for various degrees of damping

The theoretical, single degree of freedom results in figure 3 are in accordance with author's practical experiences. The core or rest roll explosion risk increases in the vicinity of resonance frequency. The maximum safe web speed of a rest roll depends on the core and rest paper properties as well as the unwinding conditions. Core stiffness, density, straightness and strength as well as the free vibrating span can impact the safe web speed at roll change. The more the chucks and the machine frame are vibrating with the rest roll and the looser is the contact between the core and chucks, the lower the safe web speed.

There are significant differences in the behaviour of different core types in the vicinity of resonance. Some cores explode almost without warning; some cores can show some +/- 100 mm mid span vibration before explosion and in proper conditions, some cores can even be driven safely over the first resonance. The maximum tolerated limit of rest roll center span vibrations is typically +/- 5 mm. For safety reasons, cores should not explode too easily and should tolerate some +/- 50-100 mm center span vibrations before exploding. Such a high vibration level is a clear warning sign, and will likely cause the web to break, thus stopping the process before explosion.

Figures 5 and 6 show test results of conventional cores and wide ply cores tested in a rotation tester shown in figure 4. The x-scale show the time [s], the upper y-scale shows the mid-span vibrations [mm] and the lower y-scale show the rotation frequency [Hz]. The revolution speed was accelerated from 0 to 4000 rpm in about 3 minutes. The conventional core exploded near the first resonance but the stiffer and stronger wide ply board core was driven safely over the first resonance several times.



a) b)

Figure 4 - An example of a 76x15 mm core resonating in a rotation tester

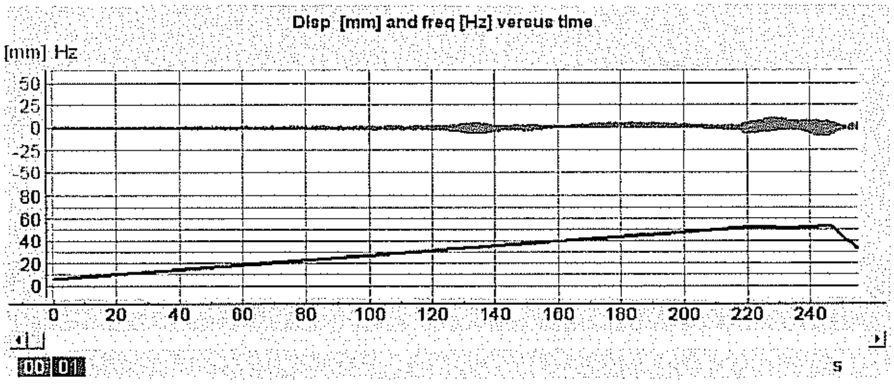


Figure 5 - Rotation test of 76x15 mm wide ply core (E = 6500 MPa) in a rotation tester Core length 2211mm, supporting length 70 mm. Max mid span vibration +/- 5 mm at first resonance and +/- 8,45mm at second resonance (52.8 Hz).

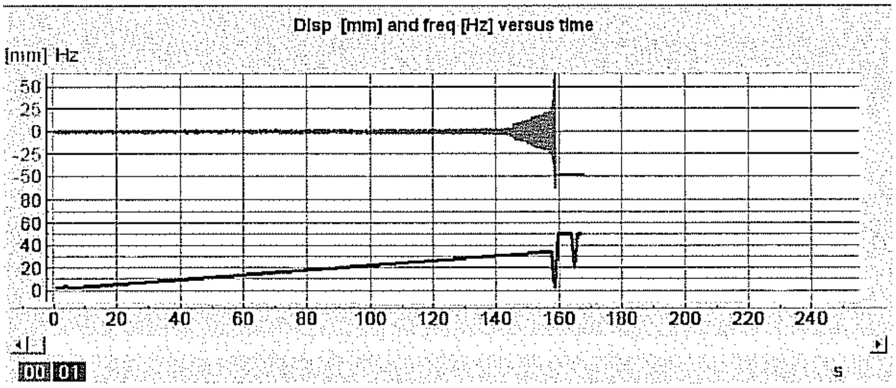


Figure 6 - Rotation of conventional narrow ply 76x15 mm core (E = 4400 MPa) Core length = 2200mm. Supporting length 70mm, core explosion at 33,2Hz.

About safety precautions

Chucks and cores can have a significant impact on the problem free operation of an unwinding stand. This is especially true with regards to the maximum tolerated unwinding speed. A malfunction in chucks or the use of the wrong kind of core can increase the core explosion risk considerably. Core users should understand the potential for core explosion and take necessary safety precautions to minimize the danger of core failure.

It can be very dangerous to run an unwinder without a proper safety cage, since an exploding rest roll can cause death or serious injury. If an unwinder is not equipped with a safety cage, persons should not operate in areas where pieces of an exploding rest roll could fly.

Estimating the safe web speed of a core or rest roll

The safe web speed [m/s] of a rest roll can be estimated by an equation

$$v_{safe} = safe \cdot \pi \cdot Dp \cdot f_1 \qquad \{5\}$$

where f_1 is the first natural frequency [Hz] of a core or rest roll, clamped (the chucks expanded) in an unwinding stand. The first natural frequency f_1 can be measured or estimated by equation {7}. Dp is the outside diameter [m] of the rest roll.

The safety factor, safe, in equation {5} is an empirical coefficient which reduces the resonance web speed to a safe level where the rest roll mid span peak to peak vibration should typically be not more than of order +/- 5 mm. It is a good practice to start unwinding experiments using a safety factor of order 0.7-0.75, and possibly slightly increase it up to 0.8 if there are no signs of excessive vibrations.

Supporting of a rest roll

Figure 7 shows rest rolls supported from the ends. Figure 7 a) shows a theoretical situation with built in ends and infinite stiffness in the end supports. A beam with free or fixed ends has the same modes and natural frequencies. The situation in figure 7 b) is closer to a real unwinding situation. The supporting of the rest roll is not ideally stiff and the free vibration span of the vibrating system does not depend only on the rest roll itself. Examples of free span lengths with different chuck types are shown in figures 7 c) and d). The free span length is assumed as the distance between the middle points of the furthest expanding elements as shown in figure 7 d).

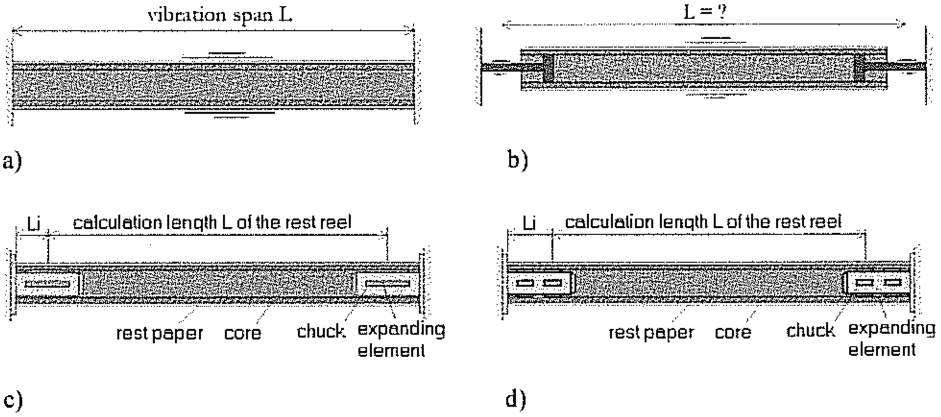


Figure 7 - The free span length L of a rest roll in different situations

The chuck factor

Rigidity of the supports at free span length L is considered by a chuck factor k which is determined experimentally. The chuck factor depends on the rigidity and mass of the unwinding frame, rest roll width, mass, bending stiffness, expansion force and the construction of the expanding chucks. The chuck factor is calculated by equation

$$k = f_1/f2 \qquad \{6\}$$

where f_1 is the first natural frequency of the core or rest roll, clamped in an unwinding stand and f2 is the free-free mode frequency of the core or rest roll at the free span length L. An example of determining the chuck factor is shown in table 1. Li is the distance from the start of the chuck to the middle of furthest expanding element as shown in figures 7 c) and d).

Measured frequency f_l of a core clamped in unwinding stands	37.11 Hz
Free hanging frequency f_{free} of a core, measured according to [2] at full length Lc	39.06 Hz
Full length of rest roll Lc	3455 mm
Distance Li to the middle of expansion element as in figure 7 c) and d)	55 mm
Calculated free hanging frequency f2 of a rest roll at clamped free span length Lc-(2Li) $f2 = f_{free}(Lc^2/(Lc-2Li)^2)$	41.67 Hz
Chuck factor $k = f_l / f2$	0.89

Table 1. - An example of determining the chuck factor.

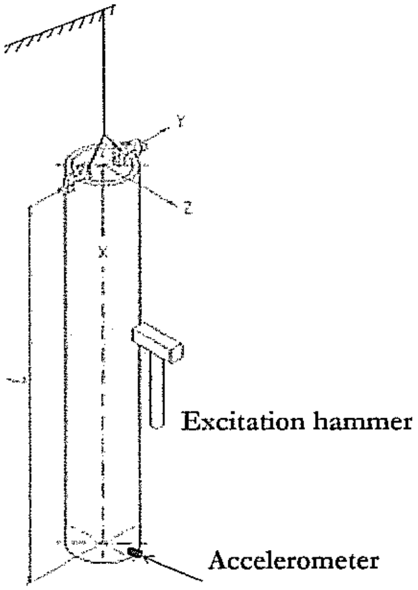


Figure 8 - Measuring free-free mode natural frequency of a core [2]

The author studied the effect of free span width and chuck expansion force on the chuck factor. The test device is shown in figure 9. The 150x13 mm test cores were first measured at 5 m length and shortened finally to 3.25 m length. The test chucks were 340 mm long. The measured chuck factors decreased as the cores were shortened.

With non-expanded chucks, the chuck factor was only 50 % of the value measured with fully expanded chucks. The risk of core explosion increases considerably if one or both of the chucks are not well expanded.

According to the author's experience, the chuck factor of modern unwinders equipped with cylindrical expanding chucks is in good conditions of order 0.85. It can also be more or less depending on the core supporting, the size and type of cores and the rest paper layer. Decreasing the free vibration span by proper chuck design and improving the chuck-core contact, the maximum tolerated web speed can be increased.

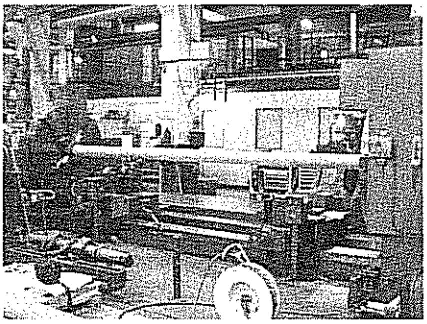


Figure 9 - Measuring the effect of core length and expansion force on chuck factor

Estimating the first natural frequency of a core or a rest roll

Applying the equation of natural frequency of a beam with free or fixed ends, we can calculate the first natural frequency of a core or rest roll by an equation

$$f_1 = k\cdot\frac{22.373}{2\cdot\pi}\cdot\frac{1}{L^2}\cdot\sqrt{\frac{E_c\cdot I_c + E_p\cdot I_p}{\rho_c\cdot A_c + \rho_p\cdot A_p}} \qquad \{7\}$$

where k is the chuck factor (a constant or a function). Core and paper densities are denoted by ρ_c and ρ_p [kg/m^3] respectively. The coefficient 22.373 comes from the fact that we are calculating the first natural frequency. Figure 1 in reference [5] shows data from a freely suspended roll of paper (k=1). For paper thickness up to 45 mm, equation {7} is quite accurate using L as the total roll length. In a chuck supported case, the free span length L is

$$L = Lc - 2\cdot Li \qquad \{8\}$$

where Lc is the full length of the rest roll and Li is the distance from the start of the chuck to the middle of furthest expanding element. The second moment of inertia [mm^4] of the core and rest paper cylinder cross sections Ic and Ip are calculated by equations

$$Ic = \frac{\pi}{64}\cdot\left[(Do)^4 - (Di)^4\right] \qquad \{9\}$$

$$Ip = \frac{\pi}{64}\cdot\left[(Dp)^4 - (Do)^4\right] \qquad \{10\}$$

The area of the core and rest paper cross sections, Ac and Ap [mm^2] are calculated as

$$Ac = \frac{\pi}{4}\cdot\left[(Do)^2 - Di^2\right] \qquad \{11\}$$

$$Ap = \frac{\pi}{4}\cdot\left[(Dp)^2 - Do^2\right] \qquad \{12\}$$

where Do is the outside diameter [mm] and Di is the inside diameter [mm] of the core. Dp is the outside diameter [mm] of the rest roll.

The natural frequency of a core, with certain length L, can be increased by increasing its bending stiffness and decreasing its mass. If the flexural modulus of the core Ec [N/mm^2] is determined by experimental modal analysis according to [2], then

$$\mathrm{Ec} = \frac{7.88 \cdot 10^{-8} \cdot (f_{free})^2 \cdot M \cdot L^4 \cdot Q}{\mathrm{Ic}} \qquad \{13\}$$

where M is mass per unit length of the test core [Kg/m] and f_{free} is the free hanging frequency of a core, measured according to [2]. A dimensionless coefficient Q is determined according to equation

$$Q = \frac{82.66 \cdot \mathrm{Ic}}{\mathrm{Ac} \cdot L^2} + 1 \qquad \{14\}$$

ISO 11093-8 standard [2] says that the minimum core length is 8 times the inside diameter and minimum wall thickness is 0.02 times the inside diameter or not less than 2.0 mm. A good practice is to use at least 1.5 m long samples when measuring 76 mm or 150 mm cores.

Rest paper cylinder E-modulus

The practice has shown that the estimations of paper cylinder properties should be based on results measured from rest rolls, not from single paper layers. These measurements are taken with 5–15 mm rest paper thickness. Solving equation {7} for rest paper E-modulus results

$$\mathrm{Ep} = \frac{-\mathrm{Ec} \cdot \mathrm{Ic}}{\mathrm{Ip}} + \frac{0.0788699}{\mathrm{Ip}} \cdot (f_{free})^2 \cdot L^4 \cdot \delta c \cdot \mathrm{Ac} + \frac{0.0788699}{\mathrm{Ip}} \cdot (f_{free})^2 \cdot L^4 \cdot \delta p \cdot \mathrm{Ap} \qquad \{15\}$$

where f_{free} is the measured first free-free mode frequency of the rest roll.

The effect of paper web on rest roll vibrations

The author did some experiments on the effect of rest paper web tension on the resonance frequency of a rest roll. The size of cores was 76x15 mm, the rest roll width was 2211 mm and the expanding chucks were 106 mm long. The paper web improved the clamped frequency from 48.8 Hz to 51.8 Hz in comparison to the situation when the paper web was cut. Laser sensor measurement of rotating rest rolls support this observation. The mid span vibrations of rest rolls are slightly higher after web cut than before it (see table 2 on next page).

An example of calculated and measured results

Figure 10 shows estimations and measured results of safe web speed of a 4.32 m wide printing press unwinder. The core dimension is 150.5x15.8 mm. Core E-modulus is 6500 MPa and density 920 kg/m^3. The rest paper cylinder density is 1350 kg/m^3 and E-modulus 5000 MPa. Distance Li is 250 mm. The safe web speed means max. +/- 5 mm mid span vibrations. The data points in the calculated curve are evaluated by equation {5}. The chuck factor was measured as a function of rest paper and this function was used in the calculation.

The circles in figure 10 are the measured natural frequencies of rest rolls without the effect of paper web. The calculated and measured natural frequencies were multiplied by the safety factor 0.8, the magnitude of which was determined by rotation tests.

The vibration movement of rotating cores and rest rolls was also measured. The square in figure 10 represents a measured result with 3.2 mm rest paper thickness. The mid span vibration was first +/- 4.4 mm and after 15 seconds 5.7 mm. Results in figure 10 and in table 2 show that the estimated and measured results are well in accordance. The long term practice has also shown that the 4.32 meter wide printing machine runs reliably at 14.5 – 15 m/s with certain special 150 mm recyclable high quality paperboard cores.

rest paper [mm]	paper web tension	web speed [m/s]	mid span amplitude [+/- mm]	
			before web cut	after web cut
4.7	yes	14.5	0.3	0.55
3.2	no	15.5	no paper web	4.4 – 5.7
0	no	15.1	no paper web	3.7
0	no	15.4	no paper web	3.5

Table 2. - An example of rest roll mid span vibrations.

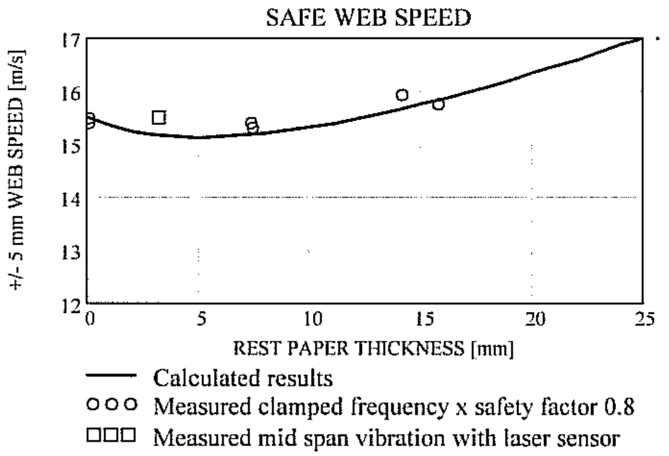


Figure 10 - The effect of rest paper on safe web speed in 4.32 m wide example printing press unwinder.

DYNAMIC DELAMINATION STRENGTH OF CORES

Cores must withstand the alternating, cyclic roll supporting stresses during winding and unwinding. The risk of core failure can be minimized by using cores which have sufficiently high dynamic delamination strength.

Delamination mechanism of cores

Web gaps and gluing defects function as initial cracks and crack growth typically starts from the edges of web gaps. Cracks continue to grow during cyclic winding and unwinding process. Figure 11 shows an example of crack growth in a core cross section as a function of revolutions with constant 15 kN load. The chuck length is 100 mm.

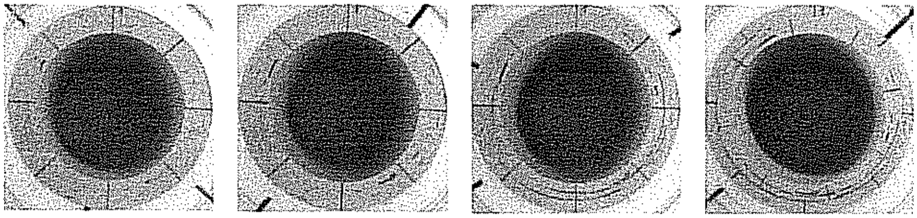

a) b) c) d)

Figure 11- Example of crack propagation on spiral core with 1.5–5 mm initial cracks Cracks after 1000 rev. a), 7000 rev. b), 21000 rev. c) and 21779 rev. d)

Chuck load capacity test

Trying to estimate the dynamic delamination strength of cores from a static flat crush test, in which the sample is compressed to rupture between flat platens, can lead to erroneous conclusions. The cyclic, dynamic chuck load capacity test is an example of considerably more advanced test method. The term chuck load capacity is also referred to as, dynamic load resistance, roll weight capacity and dynamic strength. The chuck load capacity test serves as a simple, short term cyclic production test. It simulates the loading conditions in core supported winding with a linearly increasing load.

Figure 12 b) shows an example of a chuck load capacity tester. Without additional reference information, the test does not directly indicate the maximum roll weight that can be safely wound with a given core. However, test results can be compared to the reference results to draw conclusions about the strength level of a core. The test also reveals manufacturing and raw material defects in a core.

Dynamic chuck load simulator

More accurate information of the durability of cores in winding-unwinding processes could be achieved by tests with full size rolls or by simulating different winding-unwinding conditions. Since, the possibility to arrange full scale winding-unwinding tests with paper rolls are limited the author designed the simulator shown in figure 12 a).

The simulator creates the opportunity to simulate different winding & unwinding conditions (different loading-unloading curves, roll size, chucks, etc.). The construction is optimized according to comprehensive FEM-analysis to match the core stresses in simulated and in real conditions.

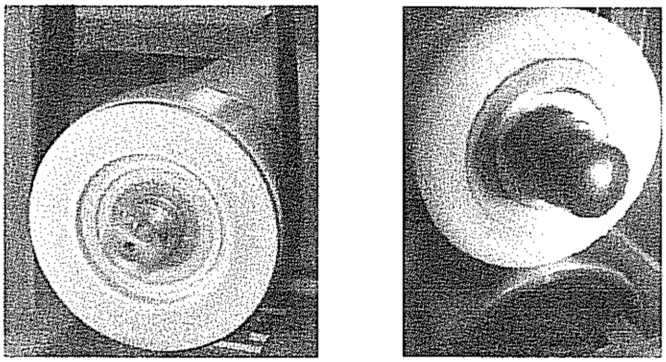
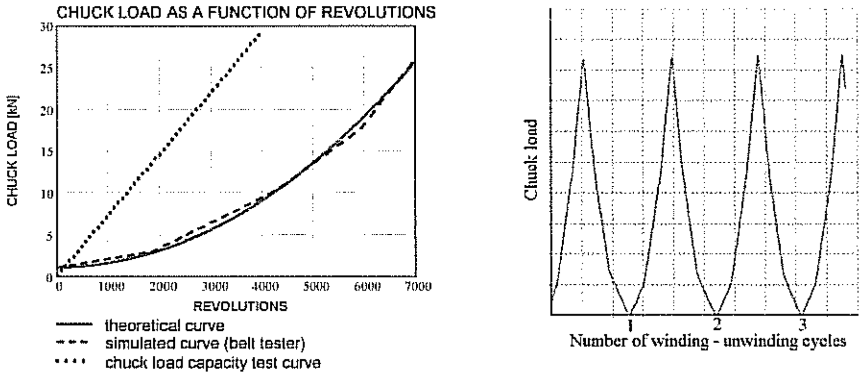

a) b)

Figure 12 – Dynamic chuck load simulator a) and chuck load capacity tester b)

Examples of winding-unwinding simulations

Figure 13 a) show examples of simulation and testing curves. The idea in simulations is to find how many winding-unwinding cycles the core can take before delaminating. Figure 13 b) shows examples of such cycles. One winding cycle is half of the winding-unwinding cycle in figure 13 b). The x-scale starts from 0.5. The durability is 0.5 cycles if the core breaks before the unwinding (the first decreasing part in curve in figure 13 b).



a) b)

Figure 13 – Loading curves a) and winding-unwinding simulation cycles b)

Figure 14 shows an example of simulations with 150x13 mm cores. There is relatively high variation in the results. The simulations have been done with non expanding chuck which heat up during long test periods and affect the durability of test cores. The simulations with cold chuck (blue points) give usually the highest results. After a few tests the chuck warms up and the test results tend to decrease.

The winders are usually equipped with expanding chucks which prevent slippage between the core and chuck. Printing press unwinders have expanding chucks which do not typically get hot if the chuck bearings are working properly. The normal chuck temperature is in the range 35-55 °C but with poor bearings, the temperature can rise considerably.

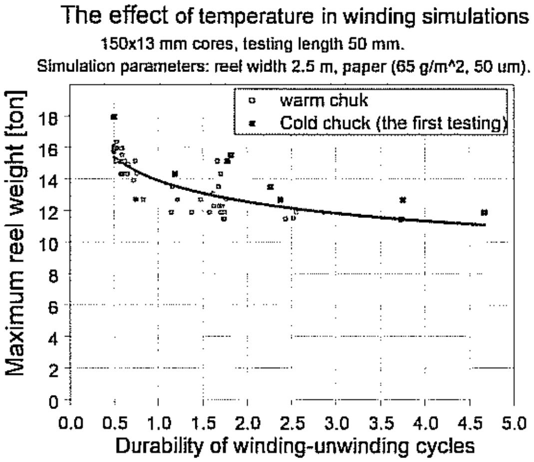


Figure 14 - An example of the effect on temperature in winding simulations

Durability of cores with different failure probabilities

The measured results in figure 15 represent mathematically random samples of unknown population. Let x_i and y_i be these measured random samples. The measured results in winding simulation curves follow the power law

$$y(x) = b0\cdot x^{b1} \tag{16}$$

where x represents the number of winding-unwinding cycles starting from 0.5 and y(x) represents the predicted maximum roll weight results. The curve in equation {16} can be returned to a linear equation by taking the logarithms on both sides as

$$\log(y(x)) = \log(b0) + b1\cdot\log(x) \tag{17}$$

We will study next the linearly fitted data points $\log(x_i)$ and $\log(y_i)$. The coefficients b0 and b1 can be solved by using the least squares estimates by minimizing the vertical distance from points to curve. Considering the lower limit load, this is physically better choice than horizontal minimization. The equations of the unknown coefficients are [3]

$$b1 = \frac{n\cdot\sum_{i=1}^{n}\log(x_i)\cdot\log(y_i) - \sum_{i=1}^{n}\log(x_i)\cdot\sum_{i=1}^{n}\log(y_i)}{n\cdot\sum_{i=1}^{n}(\log(x_i))^2 - \left(\sum_{i=1}^{n}\log(x_i)\right)^2} \tag{18}$$

$$\log(b0) = \left(\sum_{i=1}^{n}\frac{\log(y_i)}{n}\right) - b1\cdot\left(\sum_{i=1}^{n}\frac{\log(x_i)}{n}\right) \tag{19}$$

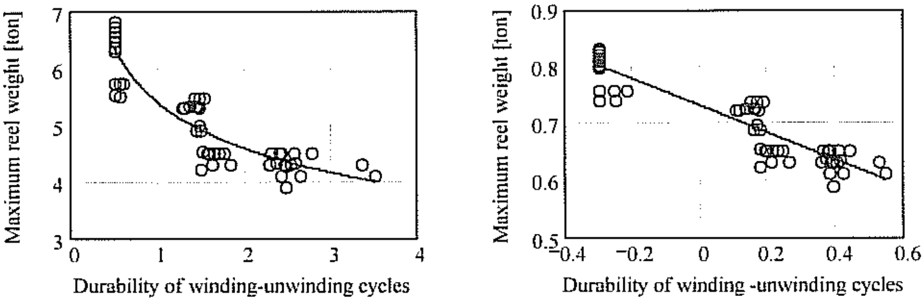
$$b0 = 10^{\log(b0)} \tag{20}$$

where the x_i and y_i are the measured data points of the winding-unwinding simulation and n is the number of the measured (x_i, y_i) data pairs. Figure 15 a) shows 58 measured (x_i, y_i) winding-unwinding simulation test data points with least squares curve fit. Figure 15 b) shows the $(\log(x_i), \log(y_i))$ data points. The residuals R_i represent the deviation of the observed y_i from the fitted data points $y(x_i)$. The residuals are written as

$$R_i = y_i - y(x_i) \tag{21}$$

The mean and variance of the example results are also random variables. The curve fits represent the local sample averages. It can usually be assumed, without making a significant error, that the residuals of winding-unwinding tests are normally distributed.

The sample mean of a normally distributed random variable is characterized by t-distribution. With degrees of freedom 30, the t-distribution is almost identical with normal distribution.



a) b)

Figure 15 – Linear a) and logarithmic b) data points

In durability studies, we are interested in the probability that the local sample average could fall below a certain limit. We can reduce the local population mean with confidence level by equation [4]

$$\text{mean}_C = \text{mean}_{\text{sample}} - t_2 \cdot \frac{S}{\sqrt{n-1}} \qquad \{22\}$$

where mean_C is the population mean with confidence level C, t_2 is the upper integration limit for probability density function f(n,t) of t-distribution, S is the standard deviation of residuals of the curve fit and n is the number of data samples.

If the given confidence level is denoted by C (for example 99 %), then the upper integration limit t_2 can be iterated from equation [4]

$$C = \int_{-\infty}^{t2} f(n-1, t2)\, dt2 \qquad \{23\}$$

The standard deviation of residuals of $\log(y_i)$ data points is [3]

$$S_{\log} = \sqrt{\frac{\sum_{i=1}^{n} \left(\log(y_i) - \log(b0) - b1 \cdot \log(x_i)\right)^2}{n-2}} \qquad \{24\}$$

It is to be noted here that $S_{\log}$ is not equal to log(S) which would denote the logarithm of standard deviation of residuals of y_i data points. The subscript log in $S_{\log}$ denotes that we analyze linearly fitted data points $\log(x_i)$, $\log(y_i)$ using equation for linear data points instead of studying data points (x_i, y_i) which follow the power fit.

If the population mean is replaced by the sample mean, the resulting random variable follows a chi-square distribution with n-1 degrees of freedom [3]. The higher limit of the population standard deviation with confidence level is [4]

$$S_C = \sqrt{\frac{n}{h1}} \cdot S \qquad \{25\}$$

where h1 is iterated from equation

$$C = \int_{hi}^{\infty} f(n-1,h)\,dh \tag{26}$$

where f(n,h) is the probability density of the chi-square distribution and C is the confidence level (for example 99 %).

Core durability can be assumed to be normally distributed with estimated population mean and standard deviation. We can ask how many standard deviations the mean value or core durability curve fit should be reduced to reach certain failure probability. The failure probability is equal to the integral of the density function of standard normal distribution from –infinity to this number. The value of λ is iterated from equation [4]

$$P(x \le \lambda) = \int_{-\infty}^{\lambda} \frac{1}{\sqrt{2\cdot\pi}}\cdot e^{\left(\frac{-x^2}{2}\right)} dx \tag{27}$$

where P is the probability of failure and λ is the independent variable in the standard normal distribution at failure probability P. When the value of λ has been iterated, the population mean can be reduced to the failure probability P by equation [4]

$$\text{mean}_{\text{reduced}} = \text{mean}_C + \lambda\cdot S_C \tag{28}$$

where mean_C is the population mean with needed confidence level {22} and S_C is the population standard deviation with confidence level {25}. It is often more convenient to use the relative standard deviation defined as

$$S_{C\text{relative}} = \frac{S_C}{\text{mean}_C} \tag{29}$$

Using the relative standard deviation, the safety factor with regard to population mean can be calculated by equation [4]

$$S_F = \frac{\text{mean}_C}{\text{mean}_{\text{reduced}}} = \frac{1}{1+\lambda\cdot S_{C\text{relative}}} \tag{30}$$

Table 3 shows the relationship between failure probability, reduction factor and safety factor for relative standard deviations 5 %, 10 % and 15 %.

Failure probability P	0.5	0.01	0.001	0.0001	0.00001	0.000001
Reduction factor λ	0.0	-2.326	-3.090	-3.719	-4.265	-4.753
Safety factor S_f Sc_{relative} 5 %	1.0	1.132	1.183	1.228	1.271	1.312
Safety factor S_f Sc_{relative} 10 %	1.0	1.303	1.447	1.592	1.774	1.906
Safety factor S_f Sc_{relative} 15 %	1.0	1.536	1.864	2.262	2.776	3.484

Table 3. - Relation between failure probability, reduction factor and safety factor

Figure 16 shows an example where the curve fit of the measured results is reduced by one standard deviation. The reduced log(x) and log(y) data points $\log(x)_R$ and $\log(y)_R$ can be constructed using the slope of the fitted curve and $S_{\log}$ as

$$\log(x)_R = \log(x) + \lambda \cdot \frac{S_{\log}}{-b1} \qquad \{31\}$$

$$\log(y)_R = \log(y(x)) + \lambda \cdot S_{\log} \qquad \{32\}$$

where b1 is the slope factor of the least squares fit in equation {18}, λ determines how many standard deviations the durability curve fit should be reduced to reach certain failure probability. The value of λ is iterated from equation {27} and standard deviation $S_{\log}$ is calculated from equation {24}. The logarithmic results can be returned to the linear coordinates by plotting the log(x) and log(y) data points as $10^{\log(x)}$, $10^{\log(y)}$.

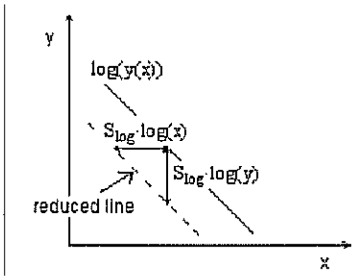


Figure 16 - Reduction of the curve fit to reach certain failure probability

Figure 17 illustrates the application of theory to the results in figure 15. The curves also indicate the confidence and prediction limits. The curve fit has been reduced for different failure probabilities. It can be estimated how many winding-unwinding cycles the 100 mm long test core samples can take with certain failure probability in a winding simulator equipped a with 100 mm long non expanding test chuck.

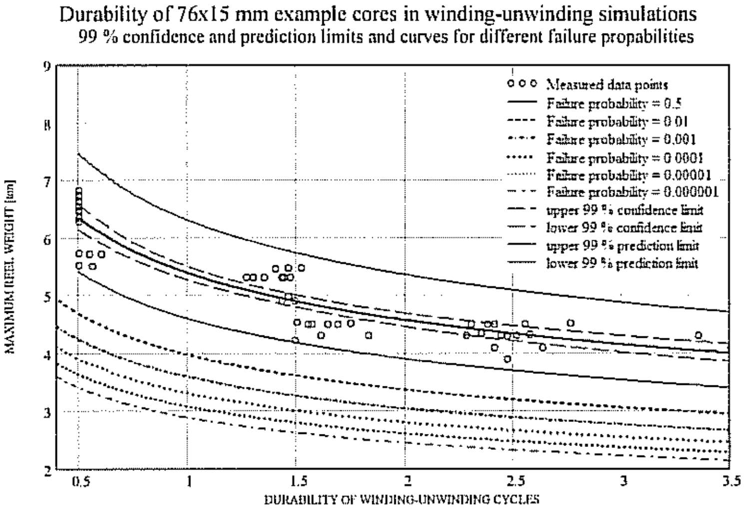


Figure 17 – Durability of example cores in winding simulations

Examples of choosing cores for different applications

Figures 18 and 19 show reel weight and web speed limits for example paperboard cores in different conditions. The examples are based on the presented theories and experimental results.

In figure 18, the assumed unwinder chuck length is 200 mm and the rolls are wound with a center winder where 80 % of the roll weight is supported by 150 mm long chucks (Rotogravure example). In figure 19, the unwinder chucks are 100 mm long and the rolls are wound with a two drum winder (News and Offset example). If the rolls are wound with a center winder instead of a two drum winder, the maximum tolerated roll weight is decreased 20 %. The maximum web speed is in both cases 15 m/s. Under the specified conditions the mid span vibration is not more than +/- 5 mm (even without paper web tension) and the delamination failure probability is of order 0.00005 - 0.00001.

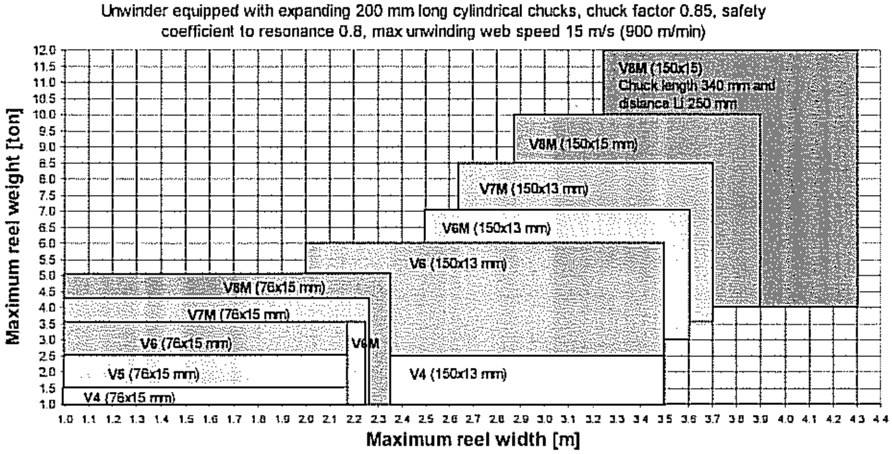

Unwinder equipped with expanding 200 mm long cylindrical chucks, chuck factor 0.85, safety coefficient to resonance 0.8, max unwinding web speed 15 m/s (900 m/min)

Figure 18. The rolls are wound with a center winder and the unwinder chucks are 200 mm long. This represents the typical case with Rotogravure rolls in Europe.

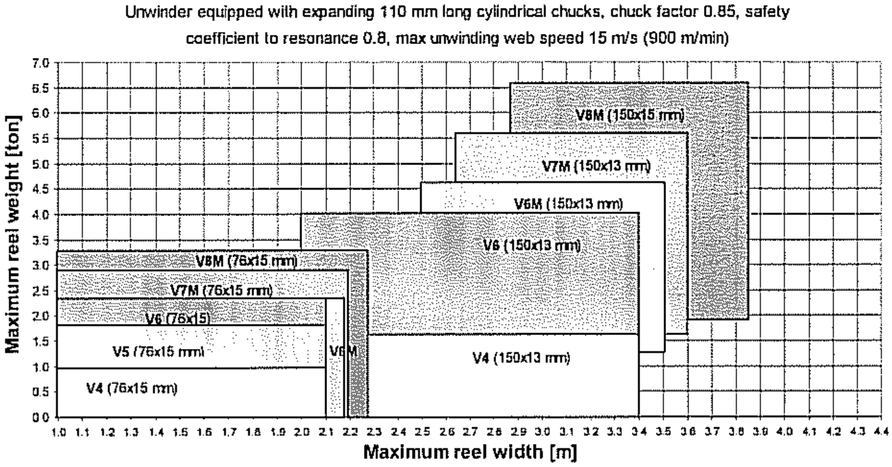

Unwinder equipped with expanding 110 mm long cylindrical chucks, chuck factor 0.85, safety coefficient to resonance 0.8, max unwinding web speed 15 m/s (900 m/min)

Figure 19. The rolls are wound with a two drum winder and the unwinder chucks are 110 mm long. This represents the typical case with News & Offset rolls in Europe.

About choosing cores

The construction of the winder, unwinder, and especially the chucks can have a considerable effect on the maximum tolerated roll weight and unwinding web speed. The supporting length of cores (cylindrical length of chucks) is very important considering both the maximum tolerated roll weight and the maximum tolerated web speed: as an example, see the 150x15 mm core F in figure 18 with 200 and 340 mm chucks. If the cylindrical supporting length of a chuck is doubled, the maximum tolerated roll mass can be doubled if the chuck bending does not increase considerably.

The situation in paper mill winders is typically such that the rolls are lying against the winding drum or drums (the exception is a pure center winder). In such a case, the most important core properties considering vibrations are the straightness and roundness. The web speed limit concerns usually only the unwinding process since the rest roll is supported only by the chucks from the core ends. A good estimation of the maximum safe unwinding web speed can be achieved by measuring the first resonance frequency of a rest roll clamped (the chucks expanded) in an unwinding stand. This result is transformed to a web speed and multiplied by the safety factor which is 0.8 or less. One should start experimenting with a lower value like 0.7 especially if there is no safety cage around the unwinder. The core grades in table 4 represent some existing paper industry cores. It is very important that core manufacturers know well the performance of their products and can assist their customers choosing right cores.

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On the Durability of Cores and Rest Roll Dynamics

M. Ilomäki, Sonoco-Alcore, FINLAND

Name & Affiliation	**Question**
Keith Good Oklahoma State University	Marko, you had some plots of durability in terms of number of winding-unwinding cycles. There were numbers on the x axis that were not positive integers. My question: I'm seeing durability as a function of fractional unwind-rewind cycle. For instance, there may be data points and some of them are at 1.6. How can you have 1.6 winding-unwinding?
Name & Affiliation	**Answer**
Marko Ilomäki Sonoco-Alcore	That's good. Actually, if we would think about the situation when you start winding a reel. If you never stop winding. You just put on more and more paper. What happens is finally the core gives up. This is the point when we have 0.5 durability. But if we are able to wind the paper and unwind the paper and it breaks at the end of the unwinding process. Then we have an unwind-rewind cycle number of 1.
Name & Affiliation	**Question**
Glenn Gentile Hewlett-Packard	I'm really glad that you presented this. I think it's really good to realize the safety involved regarding cores. I'm wondering if there is some type of service available through Sonoco. For example, when I first started working on the winder we were leaving the cores sticking out of the rolls. Maybe a foot sometimes. Then have the core chuck even beyond that. I never really thought about the fact that the roll could break free from the cores. Do you have more details? Is there a service where you can actually go and present or ask the question as an engineer? Where could you ask: If we have a 5000 pound roll and want to have a core that is sticking out 6 inches, will it be able to survive running a linear velocity of 3000 fpm?
Name & Affiliation	**Answer**
Marko Ilokämi Sonoco-Alcore	Yes. We present tools to analyze many things and many winding problems. We visit paper mills, printing presses, and other customers and help them with these problems.