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# Cognitive, Neural, and Educational Contributions to Mathematics Performance: A Closer Look at the Roles of Numerical and Spatial Skills

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Graduate Program in Psychology  
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## Abstract

The principal aims of this thesis were to (1) provide new insights into the cognitive and neural associations between spatial and mathematical abilities, and (2) translate and apply findings from the field of numerical cognition to the teaching and learning of early mathematics.

Study 1 investigated the structure and interrelations amongst cognitive constructs related to numerical, spatial, and executive function (EF) skills and mathematics achievement in 4- to 11-year old children (N=316). Results revealed evidence of highly related, yet separable, cognitive constructs. Together, numerical, spatial, and EF skills explained 84% of the variance in mathematics achievement (controlling for chronological age). Only numerical and spatial skills, but not EF, were unique predictors of mathematics performance. Spatial visualization was an especially strong predictor of mathematics.

Study 2 examined where and under what conditions spatial and numerical skills converge and diverge in the brain. An fMRI meta-analysis was performed to identify brain regions associated with basic symbolic number processing, mental arithmetic, and mental rotation. All three cognitive processes were associated with activity in and around the bilateral intraparietal sulcus (IPS). There was also evidence of overlap between symbolic number and arithmetic in the left IPS and overlap between mental rotation and arithmetic in the middle frontal gyri. Together, these findings provide a process-based account of common and unique relations between spatial and numerical cognition.

Study 3 addressed the research-to-practice gap in the areas of numerical cognition research and mathematics education. A 25-hour Professional Development (PD) model for teachers of Kindergarten–3<sup>rd</sup> Grade was designed, implemented, and tested. Results indicated that the PD was effective at increasing teachers' self-perceived numerical cognition knowledge and students' general numeracy skills. However, there were notable differences in the effects of the PD across the two sites studied, with much stronger effects at one site than the other. Thus, critical questions remain as to when and why the model may be effective in some school contexts but not others.

Together, these studies contribute to an improved understanding of the underlying relations amongst spatial, numerical, and mathematical skills and a viable new approach to better integrate research and practice.

## Keywords

Numerical cognition, spatial cognition, mathematical cognition, mathematics education, spatial skills, numerical skills, spatial visualization, fMRI, teacher Professional Development (PD)

## Summary for Lay Audience

In the last two decades, research has revealed just how important mathematics is for school and occupational success, but also one's opportunities to live a healthy and happy life. Indeed, there is a growing need to better understand factors that influence and contribute to mathematical thinking and development. The current thesis addresses this objective by focusing on how cognitive competencies, namely numerical and spatial skills, contribute to mathematical learning and performance.

Study 1 examines how numerical, spatial, and executive functioning (i.e., working memory, attention, and inhibitory control) skills relate to one another and predict children's (4-11 year olds) mathematics achievement. Results indicated strong connections between all cognitive skills. Mathematics performance was predicted by both numerical and spatial skills, but not executive function skills. Spatial visualization skill (i.e., the ability to form and manipulate mental images) was found to be an especially strong predictor of mathematics achievement.

Study 2 investigates which brain regions underlie numerical and spatial reasoning. An fMRI meta-analysis was performed to identify brain regions associated with basic symbolic number processing (e.g., comparing the larger of two numbers), mental arithmetic, and mental rotation (e.g., judging objects as the same or different despite being presented at different orientations). Results revealed large areas of overlap in and around the bilateral intraparietal sulcus (IPS), as well as regions in the left IPS potentially more sensitive to numerical processes and regions in the prefrontal cortex potentially more sensitive to domain-general manipulation (mental manipulation of numbers and/or objects).

Study 3 concerns the design, implementation, and effectiveness of a new model of Professional Development (PD) for Kindergarten—3<sup>rd</sup> Grade teachers. Central to the model is the goal of better integrating numerical cognition research with the teaching and learning of early mathematics. The results revealed evidence that the model was effective at improving teachers' self-perceived knowledge of numerical cognition research and students' general numeracy skills. However, there was also evidence that model worked better at one school compared to another, indicating the need for further research.

Together, the current PhD provides new insights into the ways in which cognitive skills and educational experiences influence mathematical thought.

## Co-Authorship Statement

The work presented in this doctoral thesis was designed and written in collaboration with my advisor, Dr. Daniel Ansari. For each of the studies within this thesis, Dr. Ansari contributed to the design, analysis, and interpretation of the findings. Although this thesis is my own work, it should be acknowledged that Dr. Ansari contributed to the revising and editing of each of the chapters.

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I am extremely grateful for the opportunity to complete my PhD in Dr. Daniel Ansari's Numerical Cognition Lab. I could not have imagined a better supervisor. My experiences in the lab have been nothing but positive and have shaped me into a more critical thinker and disseminator of research. You have given me many things; mentorship, guidance, a strong foundation in how to conduct, carryout, and consume research, and privileged access to and opportunity in the world of numerical cognition research. Thank you.

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# Chapter 1

## 1 General Introduction

The overarching goal of this thesis is to contribute to an improved understanding of mathematical thinking and learning. To approach this goal, I carried out three studies. In the first two studies, I aim to provide new insights into the cognitive and neural associations between spatial and mathematical abilities. In my third and final study, I focus on the issue of knowledge translation. I describe a study designed to bridge the gap between research in numerical cognition and the teaching and learning of early years mathematics.

The central problem addressed in Studies 1 and 2 concerns the question of why and under what conditions spatial and mathematical thinking are linked. Over a century of empirical research has demonstrated close relations between spatial and mathematical reasoning (Galton, 1881; Mix & Cheng, 2012). According to a recent review on the topic, “*the connection between space and math may be one of the most robust and well-established findings in cognitive psychology*” (Mix & Cheng, 2012, p. 198). Yet, only recently have researchers begun to ask the question how and why numerical, spatial, and mathematical abilities tend to be highly correlated. The mechanisms that link spatial and mathematical reasoning remain poorly understood. Studies 1 and 2 aims to address this gap in the literature. As described further below, each study was designed to test a set of novel hypotheses aimed to further reveal ways in which spatial, numerical, and mathematical thinking may be linked in both behavior and in the brain. The following questions remain poorly understood and are the subject of Studies 1 and 2:

- (i) To what extent is the relation between spatial and mathematical abilities explained by another variable, such as working memory or general intelligence? For example, it is possible that spatial abilities are essentially a proxy for other high-order cognitive skills.



- (ii) If spatial skills are a unique contributor to mathematics abilities, why might this be? What is it about spatial processing that facilitates mathematics learning and performance?
- (iii) Are spatial skills, namely spatial visualization, more strongly related to novel mathematical content compared to highly familiar content?
- (iv) How do spatial, numerical, and mathematical abilities relate to one another at the neural level? Do they rely on similar mechanisms?
- (v) To what extent are spatial and mathematical processes associated in the brain as a function of the mental operations that are shared and/or distinct between them? For example, might we expect to see overlap between mental rotation (a measure of spatial ability) and mental arithmetic, but not basic numerical representations, in regions sensitive to object mental manipulation?

A better understanding of the space-math link is important for two main reasons. First, by better understanding the underlying nature of the space-math link, we may be afforded new insights and a richer understanding of the cognitive underpinnings of mathematical thought. More specially, by uncovering when and under what specific conditions spatial and mathematical cognition are linked, we gain further knowledge into the potential role(s) that spatial abilities play in mathematics learning and development. For example, as hypothesized and addressed in Study 1, there is reason to believe that spatial abilities, spatial visualization skills in particular, play an especially important role in the learning of unfamiliar mathematical content. With mastery and automaticity (e.g., arithmetic fact retrieval), spatial skills are predicted to play less of a role. This point directly relates to the second reason why uncovering the space-math link is an important endeavor; the findings have the potential to inform mathematics education. It is not enough to say that spatial abilities are highly correlated with mathematics achievement. To fully leverage this relation, educators need to know when, why, and how spatial skills

are related to mathematics performance and, more specifically, the ways in which spatial instruction may stand to benefit mathematics learning. Although progress towards this goal will undoubtedly be a slow process, it is also a critically important one in the effort to improve our understanding of mathematical learning and instruction.

The central problem addressed in Study 3 concerns the research-to-practice gap in the area of numerical cognition and mathematics education. That is, how can we take research findings from the field of numerical cognition and translate and apply them to the classroom? As a discipline, numerical cognition involves the interdisciplinary study of the cognitive, developmental, and neural bases of numerical and mathematical thought. And while the knowledge generated from this field of study has the potential to inform mathematics education, this is seldom the case. Instead, numerical cognition research and mathematics education are siloed from one another. Why is this? One reason is that there is currently no infrastructure or mechanism in place that supports and facilitates opportunities for collaborative, productive, and iterative exchange between the disciplines of numerical cognition and mathematics education. To date, there has been an impressive body of literature espousing the need for *why* stronger connections should exist between disciplines (e.g., see De Smedt, Ansari, Grabner, Hannula-Sormunen, Schneider, & Verschaffel, 2011). However, as of yet, there is little indication as to *how* to forge better connections between numerical cognition and mathematics education.

Study 3 presents a model of teacher Professional Development specifically designed to address this problem. In brief, the model involves a structured approach to bringing researchers and educators together to work towards applying research from numerical cognition to classroom practice. Educators are presented with key ideas from the numerical cognition literature (e.g., foundations of number, number-space mappings, arithmetic strategies) and provided with time and resources to transform the ideas into actual lessons and activities for their own students (Kindergarten – Grade 3). This study represents a first of its kind and presents an important case study on the feasibility and findings associated with integrating numerical cognition research and mathematics education.

## 1.1 Overview of Upcoming Sections

With the above goals in mind, the remainder of the Introduction provides a more detailed literature review of the primary objectives of the current dissertation. In reviewing the literature, I identify where gaps in knowledge exist and briefly describe how my studies attempt to address these gaps. I begin by operationalizing numerical and spatial skills and then propose four candidate mechanisms for how, why, and when spatial and numerical/mathematical reasoning may be related. Moreover, I identify where our knowledge of spatial-mathematical relations falls short and briefly describe how Studies 1 and 2 address these gaps. Moving from the theoretical to the more practical, I then shift focus and discuss the need for knowledge translation in the area of numerical cognition. Lastly, I end the Introduction by providing an overview of the main questions addressed across all three of my empirical studies (Studies 1-3).

## 1.2 Relations between Spatial and Mathematical Skills

The mapping of numbers to space is central to how we operationalize, learn, and do mathematics (Lakoff & Núñez, 2000). From a historical perspective, it is difficult, if not impossible, to sift through the major discoveries in mathematics without acknowledging the central importance placed on the mapping of numbers to space. For example, the Pythagorean Theorem, the Cartesian coordinate system (mapping in general), Euclid's Elements, the real number line, and Cavalieri's principle are but a few famous examples of numerical-spatial mappings (Davis, 2015). More ubiquitous examples include the measurement of time and various other everyday quantities (e.g., cooking ingredients). Mathematical instruments as well as measurement devices are in themselves a testament to the widespread application of mapping numbers to space. These examples include the abacus, number line, clock, and ruler. To flip through any mathematical textbook is to further reveal the intimate relations between numbers and space. Diagrams, graphs, and various other visual-spatial illustrations fill the pages and serve to communicate and improve mathematical understanding.

From these examples, it is clear that numbers and space interact in important ways. But how is it that these spatial-numerical associations come to be in the first place?

What are the cognitive processes that underlie our uniquely human ability to derive the Pythagorean Theorem or to invent concepts and tools to measure the world around us? In both Study 1 and 2, I ask what role spatial abilities might play in numerical and mathematical reasoning. More specifically, I focus on the ways in which spatial visualization is related to numerical and mathematical competencies.

## 1.2.1 Spatial Visualization Skills

Although many spatial skills have been identified (e.g., navigation skills, memory for location), spatial visualization skills appear to be especially related to mathematical thinking (Mix & Cheng, 2012). For example, there is little evidence (to date) to suggest that spatial navigation skills relate to mathematics abilities. In contrast, there is well over a century of research linking spatial visualization and mathematics (Davis, 2015; Galton, 1880; Mix & Cheng, 2012). Broadly defined here as the ability to generate, recall, maintain, and manipulate visual-spatial images and solutions (Lohman, 1996; see Figure 1.1), spatial visualization has been reported to play a critical role in mathematical and scientific discovery and innovation. For example, the discovery of the structure of DNA, the Theory of Relativity, the Periodic table, and the invention of the induction motor are all said to have been borne out of spatial visualization (Davis, 2015; Moss, Bruce, Caswell, Flynn, & Hawes, 2016; Newcombe, 2010). According to famed mathematician Jacques Hadamard (1945), mathematical discoveries first present themselves in the form of intuitions and visual-spatial imagery. Only then does one engage in the more arduous and time-consuming work of testing the veracity of one's imaginings through formal and symbolic logic. This theory is perhaps best articulated by Albert Einstein, who in a letter to Hadamard, wrote:

*The words or language, as they are written or spoken, do not seem to play any role in my mechanism of thought. The physical entities which seem to serve as elements in thought are certain signs and more or less clear images which can be "voluntarily" reproduced and combined. ...Conventional words or other signs have to be sought for laboriously only in a secondary stage, when the mentioned associative play is sufficiently established and can be reproduced at will (Einstein, quoted in Hadamard, 1945, p. 142–143).*

Critically, Einstein is not alone in describing his thought process in this way. Many other mathematicians and scientists, including Poincaré, van't Hoff, and Pasteur, have offered similar introspective accounts (Hadamard, 1945; Root-Bernstein, 1985). These anecdotal accounts provide important, but far from conclusive, accounts of the role(s) that spatial visualization might play in mathematical discovery. But what does the empirical evidence suggest? Further, and more to the point, what role do spatial visualization skills play in the learning and performance of school-based mathematics?

In terms of mathematical and scientific discovery and innovations, there is longitudinal support for strong predictive relations (Wai, Lubinski, & Benbow, 2009). For example, in a nationally representative sample of U.S. high school students ( $N = 400,000$ ), it was found that spatial visualization abilities predicted which students enjoyed, entered, and succeeded in STEM disciplines (science, technology, engineering, and mathematics), even after taking verbal and quantitative competencies into account (Wai, Lubinski, & Benbow, 2009). Follow-up studies of this same sample further demonstrated that spatial visualization skills predicted creativity and innovation in the workplace, suggesting that there may be some truth to the anecdotal reports noted above (Kell, Lubinski, Benbow, & Steiger, 2013).

Consistent and robust correlations have been reported between spatial visualization skills and a breadth of mathematical tasks (Mix & Cheng, 2012). For example, spatial visualization skills have been linked to performance in geometry (Delgado & Prieto, 2004), algebra (Tolar, Lederberg, & Fletcher, 2009), numerical estimation (Tam, Wong, & Chan, 2019), word problems (Hegarty & Kozhevnikov, 1999), mental arithmetic (Kyttälä & Lehto, 2008), and advanced mathematics (e.g., function theory, mathematical logic, computational mathematics; Wei, Yuan, Chen, & Zhou, 2012). Figure 1.1 presents a few examples of the types of measures that are typically used to capture individual differences in spatial visualization skills.

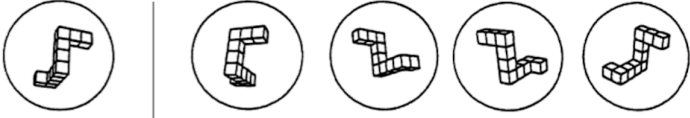

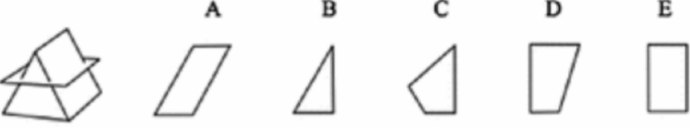
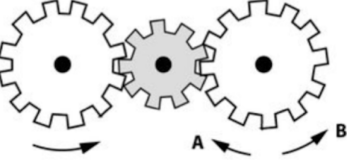
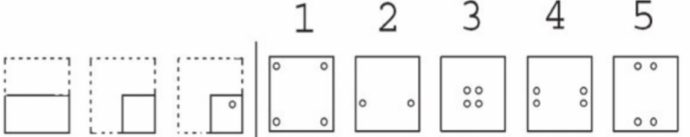
Task Description	Example Item
<p><b>3D Mental Rotation</b></p> <p><i>“Which two figures are identical to the target figure on the left (just seen from different angles)?”</i></p>	
<p><b>Mental Composition</b></p> <p><i>“Which three shapes can be combined to compose the target shape on the far left?”</i></p>	
<p><b>Mental Cutting</b></p> <p><i>“Choose the cross-section that matches the image when cut by a given plane.”</i></p>	
<p><b>Mechanical Visualization</b></p> <p><i>“If the left gear rotates in the direction indicated by the arrow, in which direction will the right gear rotate?”</i></p>	
<p><b>Mental Paper Folding</b></p> <p><i>“A piece of paper has been folded and hole-punched (left). Which image on the right corresponds to the instructions on the left?”</i></p>	

Figure 1.1 Examples of measures used to capture individual differences in spatial visualization skills.

## 1.2.2 Basic Numerical Skills

In addition to spatial visualization, basic numerical skills represent another key construct explored in detail throughout this dissertation. Like spatial visualization, basic numerical skill is often thought of as constellation of related subskills, including the ability to compare and order numbers, perform arithmetic, and answer numerical word problems (see Figure 1.2 for examples). In brief, numerical skills typically refer to an understanding

of number symbols and their various relations and applications. As further discussed in detail below, the relation between spatial visualization and basic numerical skills is an interesting one in that the relations are not overtly obvious. While many branches of mathematics are inherently spatial, including geometry and measurement, the same cannot so easily be said of the most basic of mathematical entities and operations: numbers and arithmetic. Indeed, the question of why spatial visualization skills are linked to basic numerical competencies remains poorly understood.


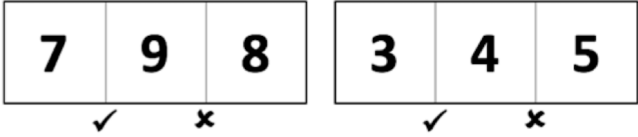
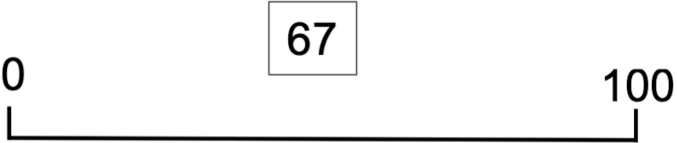
Task Description	Example Item
<p><b>Number Comparison</b></p> <p><i>“As quickly but accurately as possible indicate the larger of two numbers”</i></p>	
<p><b>Number Ordering</b></p> <p><i>“As quickly but as accurately as possible indicate whether or not the sequence of numbers is in correct (ascending) order or not.”</i></p>	
<p><b>Number Line Estimation</b></p> <p><i>“Indicate where the target number belongs on the number line.”</i></p>	
<p><b>Word Problems</b></p>	<p><i>A balloon first rose 200 meters from the ground, then moved 100 meters to the east, then dropped 100 meters. It then traveled 50 meters to the east, and finally dropped straight to the ground. How far was the balloon from its original starting place?</i></p>
<p><b>Arithmetical Operations</b></p>	<p><math>6 + 7, 8 + \_ = 12, 8 - 3 + 5, \text{ etc.}</math></p>

Figure 1.2 Examples of measures used to capture individual differences in numerical reasoning.

### 1.2.3 Mathematics Performance

Definitions of mathematics vary and continue to change as new branches of mathematics and its applications are invented. For this reason, mathematics is difficult, if not impossible, to define. However, in very general terms, definitions of mathematics range from “*the study of objects and their relations*” (retrieved from Wikipedia, December 2017) to “*the science of structure, order, and relation that has evolved from elemental practices of counting, measuring, and describing the shapes of objects*” (retrieved from Encyclopaedia Britannica, June 2019). Given the difficulty in defining mathematics, it is even more important that when studying mathematics performance that we operationalize the specific domain or feature of mathematics under study. Moreover, because mathematics is not a unitary construct, the relation between spatial thinking and basic numerical skills have with mathematics performance and achievement is complex and likely changes as function of the mathematical task under investigation. One’s experiences with mathematics is also likely to change the contributions of spatial and numerical skills to performance. In the present PhD, mathematics is operationalized according to school-based mathematics descriptions, including arithmetic, numeration, and geometry. I also make distinctions between novel vs. familiar mathematics, in the effort to elucidate the ways in which spatial and numerical skills may differentially relate to mathematics dependent on experience with the maths task at hand.

## 1.3 Explanations for Relations between Spatial and Mathematical Skills: A Review of Four Candidate Mechanisms

As already mentioned above, there is clear evidence for strong relations between spatial and mathematical skills. However, the underlying nature of this relationship remains elusive. Questions remain as to how, why, and under what conditions spatial skills and mathematics are linked. The following quote not only speaks to this point, but also makes it clear why we should care about this area of study:



*The relation between spatial ability and mathematics is so well established that it no longer makes sense to ask whether they are related. Rather, we need to know why the two are connected—the causal mechanisms and shared processes—for this relation to be fully leveraged by educators and clinicians (Mix & Cheng, 2012, p. 206).*

In response to this need, the remainder of this section is directed at reviewing the ways in which spatial and mathematical thinking may be linked. Through reviewing and synthesizing research across psychology, neuroscience, and education, I identify and examine four mechanistic accounts for the oft-reported close and potentially causal relations between spatial and mathematical thought. These four accounts include the: 1) *Spatial representation of numbers account*, 2) *shared neural processing account*, 3) *spatial modelling account*, and 4) *working memory account*. They are not mutually exclusive. For example, there is considerable overlap between the spatial representation of numbers account and the shared neural processing account. However, for ease of communication and in an attempt to best represent the research traditions from which these accounts originate, I present them as separate mechanisms. In describing these accounts, I identify outstanding questions and current knowledge gaps. Indeed, it is these knowledge gaps which form the basis of many of the questions addressed in Studies 1 and 2. In other words, these four accounts of the space-math link provide the theoretical basis and motivations for carrying out my first two studies. I also return to these four accounts in the General Discussion section, referring to the four accounts to contextualize the findings from Study 1 and 2. Moreover, I suggest ways in which future research might further test and extend various aspects of the four accounts.

### 1.3.1 Spatial Representations of Number Account

Numbers are the building blocks of mathematics. For this reason, any association between spatial processing and numbers is of potential critical importance in the effort to better understand the robust link between spatial skills and mathematics performance. As reviewed next, there is a substantial body of research indicating that numbers may be represented spatially. According to a recent study on the subject, “*spatial processing of numbers has emerged as one of the basic properties of humans’ mathematical thinking*”

(Patro, Fischer, Nuerk, & Cress, 2016, pp. 126). However, it remains unclear whether and to what extent spatial representations of number may confer any advantages to learning and doing mathematics. Moreover, and most germane to the purposes of the current thesis, it is not well understood what role higher-level spatial skills, namely spatial visualization skills, may play in the spatial representation of numbers.

The idea that numbers might be represented spatially has origins in Sir Francis Galton's mental imagery studies of the late 1800s (Galton, 1881). Galton provided the first evidence to suggest that numbers may be conceived as objects corresponding to specific positions in space:

*Those who are able to visualize a numeral with a distinctness comparable to reality, and to behold it as if it were before their eyes, and not in some sort of dreamland, will define the direction in which it seems to lie, and the distance at which it appears to be. If they were looking at a ship on the horizon at the moment that the figure 6 happened to present itself to their minds, they could say whether the image lay to the left or right of the ship, and whether it was above or below the line of the horizon; they could always point to a definite spot in space, and say with more or less precision that that was the direction in which the image of the figure they were thinking of first appeared. (1881, p. 86)*

Galton referred to such visualizations as number forms, noting that people's descriptions of such visualizations varied according to their visual-spatial properties, including differences in orientation, color, brightness, and perceived weight (e.g., see Figure 1.3). Despite such differences, number forms were said to represent a reliable and stable trait within individuals.

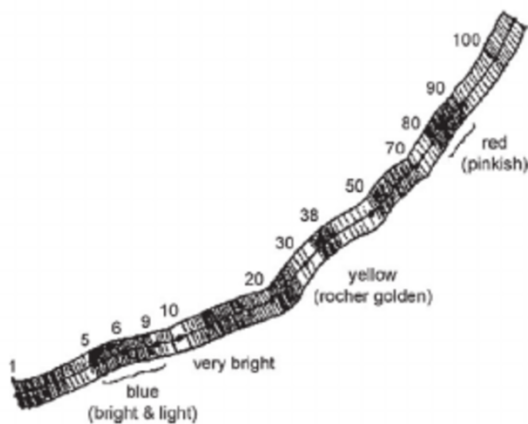


Figure 1.3 An example of how one of the participants in Galton's study described their visualization of numbers.

Galton's studies on number forms is important because it provided the first evidence that people may represent numbers in a spatial format; most typically from left-to-right, akin to an actual number line. During the last several decades, considerable research efforts have followed-up on this possibility through a wide assortment of empirical investigation. Perhaps the most influential study in this regard is Dehaene and colleagues' (1993) original findings on the SNARC effect (Spatial-Numerical Associations of Response Codes). In brief, the SNARC effect refers to the finding that people tend to automatically associate small number (e.g., 1, 2, 3) to the left side of space and larger numbers (e.g., 7, 8, 9) to the right side of space. People are faster and make fewer errors when making parity judgments (i.e., determine whether a number is even or odd) when using the left hand to make judgements about small numbers and use the right hand to make judgements about larger numbers. This finding has been interpreted as evidence for the existence of a 'mental number line': A metaphor used to describe the tendency for individuals from Western cultures to conceive numbers as ordered magnitudes along a left-to-right axis. Indeed, the 'mental number line' has been theorized to underlie a whole host of studies examining spatial-numerical associations (SNAs; e.g., see Toomarian & Hubbard, 2018). For example, line bisection tasks (Calabria & Rossetti, 2005), spatial attention tasks (Fischer & Fias, 2005) and even random number generation are but a few examples of tasks said to reveal spatial-numerical biases, interpreted as support for the presence of a 'mental number line' (Loetscher, Bockisch, Nicholls, & Brugger, 2010). Arithmetic processing has also been suggested to induce automatic spatial-numerical biases (Knops, Viarouge, & Dehaene, 2009). For example, the operation-momentum effect refers to findings of left-right biases associated with addition and subtraction. Adult participants tend to overestimate answers to addition problems and underestimate answers to subtraction problems (McCrink, Dehaene, & Dehaene-Lambertz, 2007). Even when no calculation is required the mere presence of the operators themselves (i.e., + and -) has been found to influence left-right spatial biases (Mathieu et al., 2017). Importantly, evidence suggests that SNAs are mediated through cultural and educational practices. For example, the SNARC effect is reversed in cultures that read from right-to-left (Shaki, Fischer, & Petrusic, 2009). Taken together, there is considerable

evidence to suggest that numerical thinking is related to spatial biases. These biases, in turn, have been taken as evidence of the ‘mental number line.’

Critically, the mental number line has been posited to underlie both automatic/unconscious processing of numbers as well as more effortful/conscious processing of numbers (Fischer & Fias, 2005; Schneider et al., 2018; Toomarian & Hubbard, 2018). As I will now demonstrate, this distinction has important implications in addressing the question of when and why spatial skills and numerical reasoning are related. While Galton’s inquiries centred around conscious visualizations of number, the vast majority of studies on SNAs have examined the automatic numerical-spatial biases. Research on the latter has revealed little evidence that SNAs are related to individual differences in numerical reasoning skills (Cipora, Patro, & Nuerk, 2015). Although a systematic review is needed to more fully investigate these relations, it is reasonable to conclude that automatic spatial-biases (as measured with the SNARC effect for example) have little influence on higher level numerical and mathematical processing. There is even some evidence to suggest that a negative association may exist. Practicing mathematicians, for example, have been found to demonstrate weaker numerical-spatial biases compared to control subjects (Cipora et al., 2016). These findings stand in stark contrast to the research literature on intentional spatial-numerical associations (e.g., see Schneider et al., 2018).

For example, research on the number line estimation task reveals a consistent and reliable association (Schneider et al., 2018). People who are more accurate at estimating where a given number belongs on a horizontal line flanked by two end points (e.g., 0 – 100; see Figure 1.2), tend to also demonstrate better numerical and mathematical reasoning skills. Results of recent meta-analysis revealed an average correlation of .44 between number line task performance and mathematics (counting, arithmetic, school mathematics achievement) across the ages of 4-14 (N=10,576; Schneider et al., 2018). This effect size is considerably larger than the correlations that have been reported between other foundational numerical skills and mathematics achievement. For example, measures of symbolic number comparison – a widely used measure of numerical fluency – is estimated to share a .30 correlation with mathematics achievement (e.g., see Schneider et al., 2017). Moreover, to date, the most effective mathematics interventions

have used the number line as the instructional tool used to enhance students' numerical reasoning (Fischer, Moeller, Bientzle, Cress, & Nuerk, 2011; Link, Moeller, Huber, Fischer, & Nuerk, 2013; Ramani & Siegler, 2008). Interestingly, number line training studies are theorized to be effective because they lead to a more refined 'mental number line' (Fischer et al., 2011; Siegler & Ramani, 2009).

Thus, in considering the above findings, we are left with an interesting paradox. Automatic/unconscious spatial-numerical associations do not appear to be related to individual differences in mathematics. On the contrary, intentional spatial-numerical associations appear to be strongly related to individual differences in mathematics. Moreover, both types of spatial-numerical associations – the unconscious and the conscious – are said to reflect the 'mental number line.' What might explain this disconnect?

To gain insight into this question, I turn to the role that spatial visualization may play in first forming spatial-numerical associations. Several studies have now provided evidence that spatial visualization skills relate to improved number line performance, which in turn is related to improved arithmetic and mathematics performance (Gunderson, Ramirez, Beilock, & Levine, 2012; LeFevre, Jimenez Lira, Sowinski, Cankaya, Kamawar, & Skwarchuk, 2013; Tam, Wong, & Chan, 2019). In other words, linear numerical representations have been found to mediate relations between spatial and numerical reasoning. Other researchers have found that spatial visualization skills are positively correlated to automatic SNAs, including the SNARC effect (Viarouge, Hubbard, & McCandliss, 2014). It has been hypothesized that strong spatial visualization skills underlie a greater ease and fluency in which one can move up and down and carryout arithmetical operations along the mental number line (Viarouge, Hubbard, & McCandliss, 2014). However, this finding is somewhat at odds with the evidence viewed above. That is, if spatial visualization skills are linked to automatic SNAs, might we also expect automatic SNAs to relate to mathematics? Currently, it remains unclear whether, how, and why automatic SNAs mediate relations between spatial visualization and mathematics.

While it is easy to imagine the role that spatial visualization skills play in tasks that explicitly call upon the need to map numbers to space (e.g., the empty number line

task), it is more difficult to imagine why spatial visualization skills are associated with automatic SNAs. One possibility is that automatic SNAs are an artefact of numerical-spatial relations formed earlier in development. That is, early in development spatial visualization skills may help children to construct relations between space and number. Over time, children may internalize these spatial-numerical relations, a process which eventually gives rise to automatic numerical-spatial biases. An important question is whether spatial visualization skills are still related to automatic SNAs, once the ‘building process’ is complete. While the study by Viarouge et al. (2014), discussed above, suggests that the answer to this question is yes, this is, to the best of my knowledge, the one and only study to directly address this question. Moreover, even if follow-up research confirms relations between spatial visualization skills and automatic SNAs, we are still left with the question of why conscious SNAs but not automatic SNAs relate to mathematics. As discussed in the next section, it is also possible that automatic SNAs are not as automatic as they appear, but rather constructed on the fly, within the confines of working memory and dependent on the specific task demands.

Critically, the mapping of numbers to space – by way of a ‘mental number line’ – might be but *one* example in which spatial visualization skills are used to map and make sense of numerical-spatial relations (e.g., see Lakoff & Núñez, 2000; Marghetis, Núñez, & Bergen, 2014). As pointed out earlier, mathematics is full of examples in which numbers are mapped to space (e.g., geometric proofs, measurement, topology, etc.). Might spatial visualization skills play a more general role in mapping numbers, but also other mathematical entities and concepts, onto space? Indeed, as discussed earlier, the relationship between spatial visualization skills extends to a wide variety of mathematical tasks (Mix & Cheng, 2012). Moreover, numbers do not appear to be unique in their automatic association of left-right biases. For example, the SNARC effect has been extended and replicated with other ordered stimuli such as the days of week, months of the year, and letters of the alphabet (Gevers, Reynvoet, & Fias, 2003; 2004). Relatedly, the SNARC effect appears to be flexible and prone to priming effects. For example, Fischer et al., (2010) trained participants to view large numbers on the left and small numbers on the right and found evidence of a reversed SNARC effect (Fischer, Mills, & Shaki, 2010). Together, these findings suggest that the SNARC effect is a) not limited to

numbers, and b) easily modulated by context. These findings have led to the hypothesis that the SNARC effect is indicative of context-dependent mappings between ordered stimuli (numbers, months, letters) and space. Importantly, these findings challenge the long-held belief that numbers are inherently spatial and automatically associated with space. Instead, an alternative viewpoint has emerged, positing that numerical-spatial associations are constructed in working memory during task execution (van Dijck & Fias, 2011). Whether or not spatial visualization plays a role in this online constructive process remains to be studied. However, given the close link between spatial visualization skills and explicit numerical-spatial mappings (i.e., number line estimation tasks), spatial visualization skills may also facilitate more covert numerical-spatial mappings.

Taken together, questions remain regarding the extent to which numbers are automatically associated with space versus actively constructed on a moment-to-moment basis. Moreover, the role of spatial visualization in mapping numbers to space remains largely unknown. In the next section, we continue to expand on the central idea presented in this section; that is, spatial and numerical skills may be linked because numbers are represented spatially. While this section has revealed behavioural evidence in favor of a close coupling of numbers and space, the next section addresses questions about the neural mechanisms that underlie these relations (the explicit focus of Study 2).

### 1.3.2 Shared Neural Processing Account

According to the shared neural processing account, spatial and numerical processing may be related because they rely on the same brain regions and utilize similar neural computations. The first indication that this may be the case came from neurological case studies. Individuals with damage to the parietal lobes were sometimes observed to demonstrate joint deficits in both spatial and numerical processing (Gerstmann, 1940; Holmes, 1918; Stengel, 1944). In fact, Gerstmann's Syndrome, presents a rare but specific example of how damage to the parietal lobes (i.e., the left angular gyrus) is associated with impaired spatial and numerical reasoning. People with Gerstmann's Syndrome typically display a tetrad of symptoms including acalculia, left-right confusion, finger agnosia (difficulty identifying one's fingers), and dysgraphia (difficulty with

writing) (Gerstmann, 1940). It has been suggested that these difficulties may be due to a more general deficit in the mental manipulation of visual-spatial images, including impaired mental rotation skills (e.g., see Mayer et al., 1999).

Research on patients with hemi-spatial neglect provides further evidence that space and number may depend on intact parietal lobes. Individuals with hemi-spatial neglect demonstrate an inability to attend to the contralesional portion of space (e.g., inability to attend to the left side of space when the lesion is in the right parietal lobe). This condition is associated with a skewed ability to indicate the mid-point of both real and imagined objects, but also the mid-point of numerical intervals (Bisiach & Luzatti, 1978; Zorzi et al., 2002). For example, Zorzi et al. (2002) asked right-lateralized neglect patients to indicate the mid-point of two spoken numbers, such as “two” and “six.” Presumably, due to an impaired ‘mental number line,’ patients were found to bias their estimates to the right and erroneously state “five” as the mid-point. Taken together, neuropsychological case studies provide the earliest evidence that spatial and numerical processing may rely on common parietal cortex.

More recently, the advent of fMRI has given way to a host of follow-up investigations into the neural correlates of numerical and spatial thinking. This body of research points to the intraparietal sulcus (IPS) as the critical juncture in which numbers and space may interact (e.g., see Study 2). Indeed, it is now well-established that the IPS and its neighboring regions play a critical role in reasoning about a variety of magnitudes, including non-symbolic quantities (e.g., arrays of dots), space (size and shape), luminance, and even abstract notions such as number and time (see Kadosh, Lammertyn, & Izard, 2008; Sokolowski et al., 2017; Walsh, 2003). Thus, there is evidence to suggest that *basic* spatial and numerical processes rely on common regions in and around the IPS.

There is also evidence that higher-level spatial skills, such as mental rotation, may also draw on these same parietal regions. For example, it has long been recognized that a central function of the parietal lobes is the performance of spatial transformations. Support for this can be seen in the results of a meta-analysis by Zacks (2008) on the neural correlates of mental rotation. He found evidence to suggest that the IPS was the most robust and consistently activated brain region associated with mental rotation. Other spatial visualization processes, such as being able to compose/decompose and translate



geometric shapes, have also been associated with activity in this region (Jordan, Heinze, Lutz, Kanowski, & Jäncke, 2001; Seydell-Greenwald, Ferrara, Chambers, Newport, & Landau, 2017). One reason that spatial and numerical reasoning may be linked is through shared processes related to mental transformations. According to Hubbard et al. (2009): *parietal mechanisms that are thought to support spatial transformation might be ideally suited to support arithmetic transformations as well*” (2009, p. 238). Indeed, this is an intriguing possibility and one that supports the neuronal re-cycling hypothesis.

According to the neuronal recycling hypothesis, numbers as well as other mathematical symbols and concepts, may re-use the brain’s neural resources that were originally specialized for interacting with the physical world (e.g., see Anderson, 2010; 2015; Dehaene & Cohen, 2007; Lakoff & Núñez, 2000; Marghetis, Núñez, & Bergen, 2014). In other words, numerical processing may co-opt or re-use the brain’s more ancient and evolutionary adaptive spatial and sensorimotor systems, which originally served our abilities to interact with tools, objects, and locations in space (Dehaene et al., 2003; Johnson-Frey, 2003; Lakoff & Núñez, 2000). Marghetis et al. (2014) offer this summary of the neuronal re-cycling account: “*we may recycle the brain’s spatial prowess to navigate the abstract mathematical world*” (p. 1580). The neuronal recycling hypothesis has been used by many as explanation for numerical-spatial biases observed through both behavioral as well as neuroimaging studies.

Taken together, there is compelling evidence that spatial and numerical processing are associated with overlapping regions of parietal cortex, namely in and around the IPS. However, there are also some notable gaps in the literature. One such gap is the emphasis placed on uncovering how *basic* spatial processes (e.g., comparing line lengths) relate to *basic* numerical processes (e.g., comparing Arabic digits; e.g., see Sokolowski, Fias, Mousa, & Ansari, 2017). To date, research on higher-level spatial skills (i.e., those that require spatial transformation, such as mental rotation) have been studied in isolation from neuroimaging studies of numerical cognition. So, although there is good evidence to suggest that higher-level spatial skills also rely on processes associated with the IPS, we do not yet have any direct evidence (i.e., from a single study) for this correlation. However, this is a critical gap in the literature for reasons discussed earlier. While there is robust evidence for relations between spatial visualization skills and numerical and

mathematical performance, there is little evidence that spatial representations of number relate to individual differences in numerical and mathematical performance. Thus, when it comes to better understanding individual differences in mathematics performance, much can be gained by studying the neural relations between spatial skills proper and numerical and mathematical reasoning. In Study 2, I address the question of spatial and numerical cognition are related in the brain. More specifically, I report the results of an fMRI meta-analysis that was designed to uncover brain regions associated with symbolic number, arithmetic, and mental rotation.

### 1.3.3 Spatial Modelling Account

According to the spatial modelling account, spatial visualization is related to mathematical reasoning because it provides a “mental blackboard” of which mathematical relations and operations can be modeled and visualized. More specifically, spatial visualization has been posited to play a critical role in how one organizes, models, and ultimately conceptualizes *novel* mathematical problems (Ackerman, 1988; Mix et al., 2016; Uttal & Cohen, 2012). Although there may be little to no need to model familiar mathematical content, such as memorized arithmetic facts, the visualization process may prove beneficial when confronted with novel mathematical content, such as arithmetic questions that require multi-step calculations. Moreover, the spatial modelling account functions to bridge past, present, and future knowledge states. For example, to solve the question,  $58 + 63$ , one might use their prior knowledge of arithmetic facts to arrive at a previously unknown arithmetic fact (e.g., reason that  $50 + 60 = 110$  and  $8 + 3 = 11$ ; therefore, the solution is  $110 + 11 = 121$ ). To do this – bridge prior knowledge with newly created knowledge – one must also maintain the problem and interim solutions in mind. Whether or not these same functions might just as easily be ascribed to a working memory account is an important question and one we further address below (and address in Study 1).

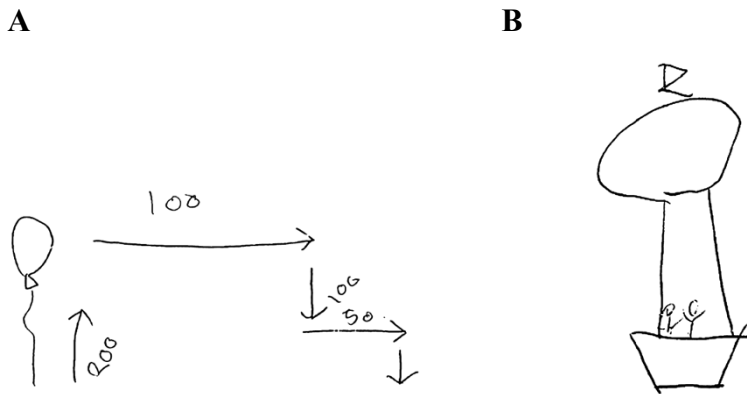
Arguably, the most impressive feature of the spatial-modelling account, but also perhaps its Achilles heel when it comes to empirical study, is that there are few, if any, limitations on the type of mathematical content that can be modeled by way of spatial

visualization. Indeed, spatial visualization processes provide a space in which one can move back and forth between a multitude of representations; between the concrete (e.g., an array of 5 objects) and the abstract (e.g., the number word ‘five’), the symbolic (e.g., Arabic numerals) and nonsymbolic (e.g., collections of objects), real and the imagined, and static and dynamic representations (Antonietti, 1999). In short, nothing is off limits when it comes to what mathematical relations can be modeled through visualizations. It is for this reason that it can be difficult to empirically investigate the spatial-modeling account. How does one reveal the specific type of spatial modelling that occurs in the ‘mind’s eye’ of any given individual? Are some types of spatial modelling more conducive to effective mathematical reasoning than others?

One promising approach to these questions comes from studying how children model solutions to mathematical word problems. For example, Hegarty and Kozhevnikov (1999) presented children with the following word problem:

*“A balloon first rose 200 meters from the ground, then moved 100 meters to the east, then dropped 100 meters. It then traveled 50 meters to the east, and finally dropped straight to the ground. How far was the balloon from its original starting place?”*

Children’s accompanying drawings to the problem revealed key insights and differences into how children modeled/visualized the problem. While some children’s drawings were literal representations of the problem, others were more abstract and contained only the relevant mathematical details needed to answer the question. Based on these differences, children’s drawings were categorized as either pictorial (more literal in representation) or visual-schematic (more abstraction in representation; emphasis on relevant numerical-spatial relations; See Figure 1.4 for an example). Children who produced visual-schematic representations were more likely to arrive at the correct solution. Moreover, children who produced visual-schematic representations were also found to demonstrate significantly higher spatial visualization skills. Several studies have since replicated this finding (see Boonen, van der Schoot, van Wesel, de Vries, & Jolles, 2013; Boonen, van Wesel, Jolles, & van der Schoot, 2014). Taken together, these studies suggest that spatial visualization skills may indeed help learners to better model mathematical relations, which in turn, may lead to improved mathematical performance.



*Figure 1.4* An example of a visual-schematic representation (A) vs. a pictorial representation (B).

In the above studies on word problems, it appears best to create mental models of only the relevant mathematical details. However, the question of what to model is likely task/question specific. For some maths problems, it is not so much about ‘doing away’ with irrelevant details, but about retaining, manipulating, and forming new relations with the information given. For example, take missing term problems, such as  $5 + \_ = 7$ . It has been suggested that one of the ways in which children come to develop fluency with such questions, is through the ability to re-structure (re-model) the problem. So, instead of  $5 + \_ = 7$ , the learner might transform the question into the more familiar form,  $\_ = 7 - 5$ . What role might spatial visualization skills play in this process? To investigate this question, Cheng and Mix (2014) carried out a randomized controlled trial with 6- to 8-year-olds. Half the children were assigned to mental rotation training condition and the other half were assigned to an active control group. Compared to the control group, children in the mental rotation group demonstrated significant gains on the missing term problems. Consistent with the spatial-modelling account, the authors suggested that gains on the missing term problems may have resulted from an improved ability to re-model the problems into an easier format. This study provided the first causal evidence that spatial visualization training may transfer to mathematics. However, a recent follow-up study by Hawes et al. (2015), failed to replicate this finding. It is clear that more research is needed before causal claims can be made about the generalizability of spatial training

to mathematics. In moving forward, such efforts should also try to more specifically address the mechanism of transfer. For example, what evidence is there that the changes in mathematics occur because of the effect that spatial training has on the way the problems are modelled? Insights into this question are needed in order to test the validity and make causal claims about the spatial-modelling account.

As mentioned earlier, one of the predictions of the spatial-modelling account is that spatial-modelling is most used when dealing with novel versus familiar mathematical content. There is some evidence that this may be the case. To test this possibility, Mix et al. (2016) examined the relation between spatial skills, including spatial visualization, and novel and familiar mathematical content. Their results suggested that spatial skills were most closely related to novel mathematical problems. In Study 1, I provide additional insights into this issue. Using a latent-variable analysis, I examine relations between basic numerical skills, more advanced mathematical skills (e.g., applied number problems, number operations), and spatial visualization skills. Based on the spatial-modelling account stronger relations should be observed between spatial visualization skills and more advanced mathematics compared to more basic numerical skills. However, based on the ‘spatial representation of numbers account’ we might also predict that the link between spatial visualization and higher-level mathematics is mediated through basic numerical competencies. I test both of these possibilities in Study 1.

It is important to note that the spatial-modelling account overlaps with other theories of numerical and mathematical cognition. In particular, it bears close resemblance with grounded and embodied accounts of mathematical cognition. According to these perspectives, mathematical thought is grounded in our everyday sensory and bodily experiences (Anderson, 2010; 2015; Lakoff & Núñez, 2000; Marghetis, Núñez, & Bergen, 2014). It is through engaging with metaphors, mental imagery, and simulated actions that mathematics becomes meaningful, and ironically, ‘groundless’ (e.g., see Lakoff & Núñez, 2000). This view contrasts with the perspective that mathematics is largely independent of sensorimotor experiences and instead is a function of symbolic amodal thought (e.g., see Barsalou, 2008). Most relevant to the spatial-modelling account is the role that mental simulation has been hypothesized to play in cognition in general, and in mathematics, in particular (Anderson, 2015; Barsalou, 2008). Indeed, mental

simulation and mental modelling are alike in that they describe mental processes related to the reenactment of sensorimotor experiences (e.g., mental imagery) in the service of a future goal (e.g., arriving at the correct solution to a word problem). The following provides an apt summary of the grounded cognition account, including clear parallels with mental simulation and the spatial modelling account:

*Operations with some of the objects in mental models are like operations with physical objects. In reasoning about these objects, the person mentally moves about on them or in them, combines them, changes their sizes and shapes, and performs other operations like those that can be formed on objects in the physical world (Greeno, 1991, p. 178).*

To be clear, the spatial-modelling account is a more specific instantiation of mental simulation; one that is confined to the discipline of mathematics and deals explicitly with spatial relations. The above quote speaks to the ‘neuronal recycling’ hypothesis mentioned earlier, offering additional insights into why space and number might both heavily recruit bilateral regions in and around the IPS. It is possible that numbers and various other mathematical concepts are processed in ways similar to the planning and actions associated with our handling of everyday objects. It is not unusual, for example, to hear of mathematicians speak of numbers as objects, as entities to be manipulated and acted on. In fact, common definitions of mathematics include “the study of objects and their relations” (retrieved from Wikipedia, December 2017). An interesting question moving forward is the extent to which certain mathematical operations (e.g., division) are (in)distinguishable from our embodied as well imagined experiences of dividing/decomposing quantities (e.g., see Lakoff & Núñez, 2000). Functional MRI studies are ideally suited to examine such questions.

### 1.3.4 Working Memory Account

Another way in which spatial visualization and mathematical skills may be related is through another variable which shares relations with performance in both of these areas. For example, it is possible that spatial visualization skills are essentially a proxy for other cognitively demanding skills, such as executive function and working memory skills. Visual-spatial working memory (VSWM), in particular (i.e., the capacity to temporarily

store, maintain, and manipulate visual-spatial information), may explain the relations between spatial visualization and numerical skills. In this section, I review the evidence for and against this proposal and outline how Study 1 further contributes to this possibility.

Research to date suggest that both spatial visualization skills and VSWM are strongly related to numerical reasoning. As discussed above, performance on spatial visualization tasks, such as mental rotation, have been linked to basic measures of numerical competencies, including arithmetic, number comparison, and number line estimation. Similarly, VSWM has also been found to explain similar amounts of variance in these same measures. Furthermore, there is evidence of close relations between all three of these variables – VSWM, spatial visualization, and numerical reasoning – when measured concurrently in the same studies (Alloway & Passolunghi, 2011; DeStefano & LeFevre, 2004; Kaufman, 2007; Kyllonen & Christal, 1990; Kyttälä et al., 2003; Li & Geary, 2017; Mix et al., 2016). Together, these findings question the extent to which spatial visualization and VSWM skills make unique contributions to numerical abilities.

It has been suggested that poor spatial abilities are a result of low VSWM. For example, several researchers have demonstrated notable differences in people of low- versus high-spatial abilities in their abilities to form, maintain, and transform visual-spatial representations (Carpenter & Just, 1986; Just & Carpenter, 1985; Lohman, 1988). Carpenter and Just (1986) concluded that “*a general characterization...is that low spatial subjects have difficulty maintaining a spatial representation while performing transformations*” (p. 236). That is, individuals with low-spatial abilities tend to “lose” information as they engage in the act of spatial transformation. For example, when mentally rotating cube figures, individuals with low-spatial abilities often lose “sight” of the mental image and require multiple attempts at rotation (Carpenter & Just, 1986; Lohman, 1988). Against this background, researchers have attributed individual differences in spatial visualization as primarily due to differences in working memory (e.g., see Hegarty & Waller, 2005).

Evidence to suggest that spatial visualization skills and VSWM are not as related as suggested above comes from three separate bodies of research: factor analyses, research on sex differences, and training studies. Studies from factor analytic studies

suggest that VSWM, spatial visualization, and executive functions (EFs) represent distinct cognitive constructs (i.e., latent variables; Miyake et al., 2001). In Study 1, I examine the extent to which numerical, spatial, and EF skills (including measures VSWM) and mathematics achievement represent distinct cognitive constructs. Moreover, I test whether relations between spatial visualization and mathematics achievement can be explained by general intelligence (*g*-factor), EFs, VSWM, or basic numerical skills. In doing so, I provide the most stringent test to date on whether the relation(s) between spatial visualization and mathematics is attributable to third party variables, including primary candidates VSWM and general intelligence.

Further evidence that spatial visualization and VSWM are separable constructs can be gleaned from findings of reliable sex differences on measures of spatial visualization but not VSWM (Halpern et al., 2007). Beginning by about the age of ten males tend to outperform females on measures of mental rotation, with estimated effects sizes ranging from .9 – 1.0 (Halpern et al. 2007; Titze, Jansen, & Heil, 2010). Importantly, sex differences are not confined to mental rotation tasks but also emerge on other spatial visualization tasks, including mental paper folding tasks (Halpern et al. 2007). Findings of sex differences in spatial visualization skills, but not VSWM, further suggests that these two aspects of visual-spatial processing may represent distinct constructs.

Training studies provide further evidence that VSWM and spatial skills behave and operate in unique ways. Although the effects of VSWM training are hotly debated and there is little evidence that training generalizes to other related tasks (e.g., mathematics; Redick, Shipstead, Wiemers, Melby-Lervåg, & Hulme, 2015), a different picture has emerged with respect to spatial training. A recent meta-analysis of 217 spatial training studies by Uttal and colleagues (2013) indicates that spatial thinking can be improved in people of all ages and through a wide assortment of training approaches (e.g., course work, task-based training, video games). Furthermore, the researchers concluded that although further evidence is still required, it appears as though the effects of spatial training transfer to a variety of novel and untrained spatial tasks. In subsequent sections, I return to the topic of spatial training and the extent to which spatial training transfers to numerical reasoning. The take away point in this section, however, is that



compared to VSWM, spatial visualization skill appears to represent a more flexible and adaptive cognitive system, providing further insight into the separability of VSWM and spatial skills. I return to the idea that spatial skills may be more malleable and transferrable to mathematics in the General Discussion.

At this point, it is worth returning to the question at hand: Does VSWM explain the relationship between spatial visualization skills and numerical/mathematical abilities? Based on the available evidence, there are reasons to suspect that 1) spatial visualization and VSWM are separable constructs, and 2) that each share independent pathways with numerical skills. An important follow-up question is why VSWM and spatial visualization skills may differentially contribute to numerical and mathematical learning and performance.

One proposal is that VSWM and spatial visualization differ according to the cognitive demands placed on the need to “recall” versus “generate” visual-spatial information. For example, at a measurement level, most VSWM measures primarily require participants to recall, maintain, and (sometimes) manipulate visual-spatial information. Most spatial visualization measures, on the other hand, require participants to perceive, maintain, manipulate, and ‘generate’ visual-spatial solutions. Thus, the shared need to maintain and manipulate visual-spatial information may explain the previously reported correlations between VSWM and spatial visualization. However, the differences in task requirements might be one reason to predict differential relations with numerical performance. While VSWM skills may play a greater role in numerical tasks that emphasize the need to recall and maintain information (e.g., basic arithmetic), spatial visualization skills may play a greater role in numerical tasks that emphasize the need to generate novel solutions (e.g., word problems, applied problems). Notably, this prediction supports the spatial-modelling account discussed earlier. Spatial visualization skills are predicted to be especially useful, even more so than VSWM, on problems that require the modelling and generation of problem solutions. In Study 1, I examine this possibility by testing, for the first time, whether ‘recall’ and ‘generative’ cognitive tasks load on separate factors. I then address whether there is any evidence that these two factors differentially relate to mathematics achievement.

## 1.4 Knowledge Translation – Integrating the Science of Learning and the Practice of Teaching

In this section, I take a step back from the goal of elucidating more basic mechanisms linking spatial, numerical, and mathematical thinking. Instead, I ask whether any of this knowledge might be leveraged in ways to improve mathematics teaching and learning. This is the focus of my third and final study. I address the question of how to better integrate research in numerical cognition and mathematics education to strengthen the teaching and learning of early number. More specifically, I report on the design, implementation, and effects of an in-service mathematics Professional Development (PD) model for teachers of Kindergarten–3<sup>rd</sup> Grade. This study was designed to address the research-to-practice gap.

### 1.4.1 A Brief History of the Research-Practice Gap in Psychology and Education

One of the draws of psychological research, in particular studies concerned with human development, cognition, and learning, is the promise that the findings from such research can be used to inform our knowledge of how people learn. This information in turn has the potential to bring about improved learning outcomes across a variety of contexts, including, most notably, the classroom. However, the application of psychological research into principles and practices of teaching and learning has proven to be an extremely difficult endeavor.

Indeed, questions of how to apply research to practice (i.e., the research-to-practice gap) has plagued the discipline of psychology since its beginnings. William James, the father of American psychology, wrote about the difficulties surrounding the research-to-practice gap, as did other early influential thinkers including John Dewey and Edward J. Thorndike (James, 1899; Dewey, 1897/1998; Thorndike, 1917;1921). However, instead of merely acknowledging the problem, these psychologists, as well as many others of this time period (e.g., Pyle, 1928; Ragsdale, 1932), actively sought

solutions to the research-practice gap. It was not uncommon for researchers to meet with practicing teachers to discuss methods of applying psychological research to classroom learning (e.g., see Chase, 1998). For example, William James regularly met with teachers and spoke of evidence-informed instructional tactics for teachers (Chase, 1998; James, 1899) and Thorndike made connections between his basic research on connectionism to the teaching and learning of arithmetic (Thorndike, 1917;1921).

However, this trend towards efforts to close research-to-practice gaps did not continue into the mid 20<sup>th</sup> Century. Prominent researchers including E. C. Tolman (1932), E. R. Guthrie (1935), Kurt Lewin (1936), B. F. Skinner (1938), Kenneth Spence (1942), and Clark Hull (1943), proposed learning theories predominantly derived from carefully controlled laboratory experiments, often involving animals as subjects. Learning was described with highly technical terms, such as drive reduction, schedules of reinforcement, inhibition, extinction, and cognitive maps. According to Chase (1998), these new terms “*were nonsense syllables to most front line educators*” and “*The vocabulary necessary for communication with other educational specialists was disappearing*” (Chase, 1998, p. 242). Taken together, the 3<sup>rd</sup> and 4<sup>th</sup> decades of the 20<sup>th</sup> Century psychology were marked by a widening of the research-practice gap.

Behaviorist models of learning eventually gave way to cognitive and constructivist models of learning. Influential theories of learning were proposed by Jerome Bruner, Lev Vygotsky, and Jean Piaget. Interestingly, these theories continue to be frequently discussed in teacher preparation and Educational Psychology courses and text books. However, a criticism of this work is that it has remained overly theoretical and not immediately useful for actual classroom practice (Berliner, 1992). So, although this work has had some influence on approaches to educational practice (e.g., Vygotsky’s ideas around the Zone of Proximal Development), many questions remain about how and whether these theories can directly be applied to classroom practice.

The question of how to better integrate research and practice has resurfaced as a critical issue in contemporary psychology. Beginning at around the turn of the 21<sup>st</sup> Century and continuing to present day, researchers in a variety of subfields of psychology, including cognitive science and neuroscience, have become increasingly interested in addressing the research-practice gap.

Indeed, over the last 15 years there has been a steep rise in the number of journals, societies, and research labs dedicated to the mission of knowledge translation and application across the disciplines of psychology, neuroscience, and education. This new field of study is often referred to as Mind, Brain, and Education or Educational Neuroscience. The goals of the Learning Sciences are also closely aligned with the development of these new disciplines. The development of journals associated with this movement include, *Mind, Brain, and Education*, *Trends in Neuroscience and Education*, *npj Science of Learning*. Taken together, we are in the midst of a new wave of efforts to bring research and practice into closer contact.

I provide the above history of the research-practice gap because it helps contextualize the goals of my third study. More broadly, it also helps contextualize the aims and scope of my research program as a whole. As evidenced in the present PhD, I seek to better understand how people learn and what this might mean for education and intervention through combining methodological approaches and disciplinary perspectives across the psychological, neural, and educational sciences. For reasons discussed next, I am hopeful that the effort to better integrate research and practice will fare better than efforts of the past.

#### 1.4.2 Moving Forward – A better Integration of Research and Practice Across Psychology, Neuroscience, and Education

One proposed reason for the lack of successful research-practice integration in the past was due to the lack of infrastructure or mechanisms needed to facilitate more optimal translation and application of research to practice (Ansari & Coch, 2006). As mentioned above, progress has been made in this regard; the development of new journals, societies, and fields of study have contributed to an improved infrastructure and means of communication across disciplines interested in addressing the research-practice gap. However, it is also clear that this is not enough. Indeed, the future of successful research-practice integration may require a return to the earliest days of psychology. A return to the

efforts of James and Thorndike who not only recognized the importance of both basic research and applied practice, but the need to build intermediary links between them. That is, rather than continually justify *why* efforts should be directed at ‘bridging the gap,’ actions and concrete examples of *how* to bridge the gap are needed. My third study addresses this need. I propose the need for researchers and educators to directly interact with one another and work collaboratively towards the goal of integrating research and practice.

## 1.5 Summary of Background, Rationale, and Study Objectives

The literature reviewed above reveals several gaps in our understanding of mathematics learning and performance. This thesis was designed and carried out to address these gaps. I will now provide a brief review of each study, summarizing the background and rationale for each study, as well as the more specific goals of each study. I will also make it clear how and why each study contributes to an improved understanding of mathematical cognition and learning.

### 1.5.1 Overview of Study 1 – Relations between Numerical, Spatial, and Executive Function Skills and Mathematics Achievement: A Latent-Variable Approach

Current evidence suggests that numerical, spatial, and executive function (EF) skills each play critical and independent roles in the learning and performance of mathematics (e.g., see De Smedt, Noël, Gilmore, & Ansari, 2013; Mix & Cheng, 2012; Cragg & Gilmore, 2014). However, these conclusions are largely based on isolated bodies of research and without measurement at the latent variable level. While prior research has examined latent relations between two of these constructs (e.g., spatial and mathematical abilities; Mix et

al., 2016; 2017; spatial and EF abilities; Miyake, Friedman, Rettinger, Shah, & Hegarty, 2001), relations between all four constructs has yet to be examined. Thus, questions remain regarding the extent to which these skills represent distinct constructs and whether numerical, spatial, and EF skills afford differentiated pathways to mathematics achievement. This study aims to address this gap by examining the latent structure and interrelations between numerical, spatial, EF, and mathematics abilities in a sample of 4- to 11-year-olds.

In addition to providing a cognitive model of children's mathematics achievement, this study was designed to more closely reveal insight into the underlying nature of the oft reported space-math association. Although there is extensive correlational evidence linking spatial and mathematical cognition, including decades of behavioral and neural research (Mix & Cheng, 2012; Hubbard, Piazza, Pinel, & Dehaene, 2005), relatively few efforts have been made to reveal potential mechanisms linking space and math. Here, we test the hypothesis that spatial visualization plays a critical role in mathematical problem solving and achievement. By testing a model of mathematical achievement that includes numerical, spatial, and EF factors, we were able to test and control for specific pathways connecting spatial visualization skills and mathematics achievement. This allowed us to examine the extent to which the space-math link is best explained by direct relations between spatial visualization and mathematics or whether the space-math link might be better explained by alternative mechanisms. Specifically, we test whether general intelligence and/or EF skills (including visual-spatial working memory) might better explain the space-math link. In addition, we test the hypothesis that spatial visualization skills are indirectly related to mathematics through basic numerical skills (e.g., see Gunderson, Ramirez, Beilock, & Levine, 2012; LeFevre et al., 2013). The findings related to these pathways are crucial in order to advance current theories of spatial and mathematical associations.

To summarize, this study was designed to:

- (i) Test whether numerical, spatial, and EF skills and mathematics achievement represent distinct cognitive constructs

- (ii) If they do represent distinct constructs, how do numerical, spatial, and EF skills relate to mathematics achievement
- (iii) Test whether numerical and EF skills mediate relations between spatial skills and mathematics achievement

### 1.5.2 Overview of Study 2 – Neural Underpinnings of Numerical and Spatial Cognition: An fMRI Meta-Analysis of Brain Regions Associated with Symbolic Number, Arithmetic, and Mental Rotation

Study 2 explores the extent to which spatial and numerical skills – key foundations of mathematical thinking – rely on shared and distinct neural mechanisms. Although there is extensive behavioral evidence for strong relations between spatial and numerical thinking, questions remain regarding the underlying neural relations between these two cognitive constructs. To date, research on the neural correlates of visual-spatial skills, such as mental rotation, and numerical reasoning have been studied in isolation from one another (e.g., Zacks, 2008). Thus, it remains unclear whether and to what extent spatial and numerical cognition rely on similar neural networks.

To address this gap in the literature, we report on a meta-analysis of brain regions associated with neural activity in three key aspects of mathematical thinking: basic symbolic number processing, arithmetic, and mental rotation (a widely accepted archetype of visual-spatial reasoning). We targeted these three cognitive skills in an effort to tease apart brain regions potentially related to symbolic number processing, including arithmetic, as well as regions more attuned to mental manipulation. Taken together, we see the current study as an important step in providing a comprehensive overview of the neural correlates of mathematical thinking. More specifically, this study delves into the age-old question of why spatial and numerical abilities are related by mapping – for the first time – the common and distinct brain regions associated with spatial and numerical cognition.

To summarize, this study was designed to:

- (i) Reveal the neural correlates of three key cognitive processes found to underlie mathematical thought - basic symbolic number, arithmetic, and spatial reasoning (mental rotation)
- (ii) Test a theoretical model that makes predictions about when, where, and why numerical and spatial cognition may converge and diverge in the human brain
- (iii) Relatedly, tease apart regions of activation subserving mental manipulation versus symbolic number representation.

### 1.5.3 Overview of Study 3 – Integrating Numerical Cognition Research and Mathematics Education to Strengthen the Teaching and Learning of Early Number

This study addresses the question of how to translate and apply the science of learning with the practice of teaching. More specifically, it addresses the research-to-practice gap in the area of numerical cognition and mathematics education. Because the implications for classroom instruction do not immediately follow from the science of learning, there is a need to build infrastructure that supports and facilitates opportunities for collaborative, productive, and iterative exchange between the disciplines of education and developmental cognitive science.

In response to this need, the current study reports on the design, implementation, and effects of a 3-month (~25 hour) teacher intervention for teachers of kindergarten – 3<sup>rd</sup> grade. To test the replicability of the model, we carried out a two-year pre-post controlled study across two different intervention sites. We report on the effects of the intervention model at both the teacher and student level. The intervention is grounded in research in numerical cognition and uses this knowledge to inform teachers' assessment and instructional practice. In brief, this study describes and tests a new model of teacher PD designed to: 1) Enrich teachers' awareness of and understanding of research on children's



numerical thinking, and 2) uses this knowledge to inform teachers' assessment and instructional practice.

To summarize, this study was designed to:

- (i) Design and implement a new teacher PD model aimed at creating stronger connections between developmental cognitive science and early mathematics education
- (ii) Test the effects of the intervention model on both teachers and their students
- (iii) Test the replicability of the model across two different school sites

## 1.6 Summary Statement

To summarize, the current thesis is focused on better understanding how humans are able to learn and perform abstract mathematics. To approach this task, I consider the influence of cognitive, neural, and educational factors in mathematics performance and achievement. More specifically, my research aims to more closely reveal the specific roles that numerical and spatial skills play in mathematics as well as the ways in which numerical cognition research can be leveraged to improve mathematics teaching and learning. Ultimately, my hope is that this work will make a small contribution to the much larger goal of advancing our understanding of what underlies mathematical cognition and learning.

## 1.7 References

- Ackerman, P. L. (1988). Determinants of individual differences during skill acquisition: Cognitive abilities and information processing. *Journal of Experimental Psychology: General*, *117*, 288–318.
- Alloway, T. P., & Passolunghi, M. C. (2011). The relationship between working memory, IQ, and mathematical skills in children. *Learning and Individual Differences*, *21*(1), 133–137.

- Anderson, M. L. (2010). Neural reuse: A fundamental organizational principle of the brain. *Behavioral and Brain Sciences*, *33*(4), 245-266.
- Ansari, D., & Coch, D. (2006). Bridges over troubled waters: Education and cognitive neuroscience. *Trends in Cognitive Sciences*, *10*(4), 146-151.
- Antonietti, A. (1999). Can students predict when imagery will allow them to discover the problem solution?. *European Journal of Cognitive Psychology*, *11*(3), 407-428.
- Barsalou, L. W. (2008). Grounded cognition. *Annual Review of Psychology*, *59*, 617-645.
- Berliner, D. C. (1992). Telling the stories of educational psychology. *Educational Psychologist*, *27*(2), 143-161.
- Bisiach, E., & Luzatti, C. (1978). Unilateral neglect of representational space. *Cortex*, *14*, 129-133.
- Boonen, A. J., van der Schoot, M., van Wesel, F., de Vries, M. H., & Jolles, J. (2013). What underlies successful word problem solving? A path analysis in sixth grade students. *Contemporary Educational Psychology*, *38*(3), 271-279.
- Boonen, A. J., van Wesel, F., Jolles, J., & van der Schoot, M. (2014). The role of visual representation type, spatial ability, and reading comprehension in word problem solving: An item-level analysis in elementary school children. *International Journal of Educational Research*, *68*, 15-26.
- Calabria, M., & Rossetti, Y. (2005). Interference between number processing and line bisection: a methodology. *Neuropsychologia*, *43*(5), 779-783.
- Carpenter, P. A., & Just, M. A. (1986). Spatial ability: An information processing approach to psychometrics. In R. J. Stenberg (Ed.), *Advances in the psychology of human intelligence* (Vol. 3, pp. 221-252). Hillsdale, NJ: Erlbaum.
- Chase, C. I. (1998). A chat room—Educational psychologists but no curriculum methodologists?. *Educational Psychology Review*, *10*(2), 239-248.
- Cheng, Y. L., & Mix, K. S. (2014). Spatial training improves children's mathematics ability. *Journal of Cognition and Development*, *15*(1), 2-11.
- Cipora, K., Hohol, M., Nuerk, H. C., Willmes, K., Brożek, B., Kucharzyk, B., & Nęcka, E. (2016). Professional mathematicians differ from controls in their spatial-numerical associations. *Psychological Research*, *80*(4), 710-726.
- Cipora, K., Patro, K., & Nuerk, H. C. (2015). Are spatial-numerical associations a

- cornerstone for arithmetic learning? The lack of genuine correlations suggests no. *Mind, Brain, and Education*, 9(4), 190-206.
- Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education*, 3(2), 63-68.
- Davis, B and the Spatial Reasoning Study Group (2015). *Spatial reasoning in the early years: Principles, assertions, and speculations*. New York, NY: Routledge.
- Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General*, 122(3), 371-396.
- Dehaene, S., & Cohen, L. (2007). Cultural recycling of cortical maps. *Neuron*, 56(2), 384-398.
- Delgado, A. R., & Prieto, G. (2004). Cognitive mediators and sex-related differences in mathematics. *Intelligence*, 32(1), 25–32.
- De Smedt, B., Noël, M. P., Gilmore, C., & Ansari, D. (2013). The relationship between symbolic and non-symbolic numerical magnitude processing skills and the typical and atypical development of mathematics: a review of evidence from brain and behavior. *Trends in Neuroscience & Education*, 2(2), 48-55.
- De Smedt, B., Ansari, D., Grabner, R. H., Hannula-Sormunen, M., Schneider, M., & Verschaffel, L. (2011). Cognitive neuroscience meets mathematics education: It takes two to tango. *Educational Research Review*, 6(3), 232-237.
- DeStefano, D., & LeFevre, J. A. (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology*, 16(3), 353–386.
- Dewey, J. (1998/1897). My pedagogic creed. In L. A. Hickman & T. M. Alexander (Eds.), *The essential Dewey: Vol. 1. Pragmatism, Education, Democracy* (pp. 229–235). Bloomington: Indiana University Press. (Original work published 1897).
- Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., ... & Sexton, H. (2007). School readiness and later achievement. *Developmental Psychology*, 43(6), 1428-1446.
- Fischer, M. H., & Fias, M. H. (2005). Spatial representation of numbers. *Handbook of*

*Mathematical Cognition*, 43-54.

- Fischer, U., Moeller, K., Bientzle, M., Cress, U., & Nuerk, H. C. (2011). Sensori-motor spatial training of number magnitude representation. *Psychonomic Bulletin & Review*, 18(1), 177-183.
- Galton, F. (1880). Visualised numerals. *Nature*, 21(533), 252-256.
- Gerstmann, J. (1940). Syndrome of finger agnosia, disorientation for right and left, agraphia and acalculia: local diagnostic value. *Archives of Neurology & Psychiatry*, 44(2), 398-408.
- Gevers, W., Reynvoet, B., & Fias, W. (2003). The mental representation of ordinal sequences is spatially organized. *Cognition*, 87(3), B87-B95.
- Gevers, W., Reynvoet, B., & Fias, W. (2004). The mental representation of ordinal sequences is spatially organized: Evidence from days of the week. *Cortex*, 40, 171-172.
- Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education*, 22(3), 170-218.
- Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skill and early number knowledge: the role of the linear number line. *Developmental Psychology*, 48(5), 1229-1241.
- Guthrie, E. R. (1935). *The Psychology of Learning*. New York, NY: Harper.
- Hadamard, J. (1945). *The psychology of invention in the mathematical field*. Princeton, NJ: Princeton University Press
- Halpern, D. F., Benbow, C. P., Geary, D. C., Gur, R. C., Hyde, J. S., & Gernsbacher, M. A. (2007). The science of sex differences in science and mathematics. *Psychological Science in the Public Interest*, 8(1), 1-51.
- Hawes, Z., Moss, J., Caswell, B., & Poliszczuk, D. (2015). Effects of mental rotation training on children's spatial and mathematics performance: A randomized controlled study. *Trends in Neuroscience and Education*, 4(3), 60-68.
- Hegarty, M., & Kozhevnikov, M. (1999). Types of visual-spatial representations and mathematical problem solving. *Journal of Educational Psychology*, 91(4), 684-689
- Hegarty, M., & Waller, D. (2005). Individual differences in spatial abilities. *The*

- Cambridge Handbook of Visuospatial Thinking*, 121-169.
- Holmes, G. (1918). Disturbances of visual orientation. *The British Journal of Ophthalmology*, 2(9), 449-468.
- Hubbard, E. M., M. Piazza, P. Pinel, & Dehaene (2009). Numerical and spatial intuitions: a role for posterior parietal cortex? In L. Tommasi, L. Nadel, and M. A. Peterson (Eds.), *Cognitive biology: Evolutionary and developmental perspectives on mind, brain and behavior* (pp. 221–246). Cambridge, MA: MIT Press.
- Hull, C. L. (1932). *Principles of Behavior*. New York, NY: Appleton-Century-Crofts.
- James, W. (1899/2001). *Talks to teachers on psychology and to students on some of life's ideals*. New York, NY: Dover Publications.
- Johnson-Frey, S. H. (2004). The neural bases of complex tool use in humans. *Trends in Cognitive Sciences*, 8(2), 71-78.
- Jordan, K., Heinze, H. J., Lutz, K., Kanowski, M., & Jäncke, L. (2001). Cortical Activations during the mental rotation of different visual objects. *NeuroImage*, 13(1), 143-152.
- Just, M. A., & Carpenter, P. A. (1985). Cognitive coordinate systems: accounts of mental rotation and individual differences in spatial ability. *Psychological Review*, 92(2), 137.
- Kadosh, R. C., Lammertyn, J., & Izard, V. (2008). Are numbers special? An overview of chronometric, neuroimaging, developmental and comparative studies of magnitude representation. *Progress in Neurobiology*, 84(2), 132-147.
- Kaufman, S. B. (2007). Sex differences in mental rotation and spatial visualization ability: Can they be accounted for by differences in working memory capacity? *Intelligence*, 35, 211–223.
- Kell, H. J., Lubinski, D., Benbow, C. P., & Steiger, J. H. (2013). Creativity and technical innovation: Spatial ability's unique role. *Psychological Science*, 24(9), 1831-1836.
- Knops, A., Viarouge, A., & Dehaene, S. (2009). Dynamic representations underlying symbolic and nonsymbolic calculation: Evidence from the operational momentum effect. *Attention, Perception, & Psychophysics*, 71(4), 803-821.
- Kyllonen, P. C., & Christal, R. E. (1990). Reasoning ability is (little more than) working-memory capacity. *Intelligence*, 14(4), 389–433.

- Kyttälä, M., Aunio, P., Lehto, J. E., Van Luit, J., & Hautamaki, J. (2003). Visuospatial working memory and early numeracy. *Educational and Child Psychology, 20*, 65–76.
- Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York, NY: Basic Books.
- LeFevre, J. A., Jimenez Lira, C., Sowinski, C., Cankaya, O., Kamawar, D., & Skwarchuk, S. L. (2013). Charting the role of the number line in mathematical development. *Frontiers in Psychology, 4*, 641.
- Lewin, K. (1936). *Principles of Topological Psychology*. New York, NY: McGraw Hill Book Co.
- Li, Y., & Geary, D. C. (2017). Children’s visuospatial memory predicts mathematics achievement through early adolescence. *PLoS One, 12*(2), e0172046.
- Link, T., Moeller, K., Huber, S., Fischer, U., & Nuerk, H. C. (2013). Walk the number line—An embodied training of numerical concepts. *Trends in Neuroscience and Education, 2*(2), 74-84.
- Loetscher, T., Bockisch, C., Nicholls, M. E. R., & Brugger, P. (2010). Eye position predicts what number you have in mind. *Current Biology, 20*, R264–R265
- Lohman, D. F. (1988). Spatial abilities as traits, processes, and knowledge. In R. J. Sternberg (Ed.), *Advances in the psychology of human intelligence* (Vol. 4, pp. 181-248). Hillsdale, NJ: Erlbaum.
- Lowrie, T., Logan, T., & Ramful, A. (2017). Visuospatial training improves elementary students’ mathematics performance. *British Journal of Educational Psychology, 87*(2), 170-186.
- Marghetis, T., Núñez, R., & Bergen, B. K. (2014). Doing arithmetic by hand: Hand movements during exact arithmetic reveal systematic, dynamic spatial processing. *The Quarterly Journal of Experimental Psychology, 67*(8), 1579-1596.
- Mathieu, R., Epinat-Duclos, J., Sigovan, M., Breton, A., Cheylus, A., Fayol, M., ... & Prado, J. (2017). What's Behind a “+” Sign? Perceiving an Arithmetic Operator Recruits Brain Circuits for Spatial Orienting. *Cerebral Cortex, 28*(5), 1673-1684.
- Mayer, E., Martory, M. D., Pegna, A. J., Landis, T., Delavelle, J., & Annoni, J. M. (1999).

- A pure case of Gerstmann syndrome with a subangular lesion. *Brain*, 122(6), 1107-1120.
- McCrink, K., Dehaene, S., & Dehaene-Lambertz, G. (2007). Moving along the number line: Operational momentum in nonsymbolic arithmetic. *Perception & Psychophysics*, 69(8), 1324-1333.
- Mix, K. S., & Cheng, Y. L. (2012). The relation between space and math: developmental and educational implications. *Advances in Child Development and Behavior*, 42, 197-243.
- Mix, K. S., Levine, S. C., Cheng, Y. L., Young, C., Hambrick, D. Z., Ping, R., & Konstantopoulos, S. (2016). Separate but correlated: The latent structure of space and mathematics across development. *Journal of Experimental Psychology: General*, 145(9), 1206-1227.
- Mix, K. S., Levine, S. C., Cheng, Y. L., Young, C. J., Hambrick, D. Z., & Konstantopoulos, S. (2017). The latent structure of spatial skills and mathematics: A replication of the two-factor model. *Journal of Cognition and Development*, 18(4), 465-492
- Miyake, A., Friedman, N. P., Rettinger, D. A., Shah, P., & Hegarty, M. (2001). How are visuospatial working memory, executive functioning, and spatial abilities related? A latent-variable analysis. *Journal of Experimental Psychology: General*, 130(4), 621-640.
- Newcombe, N. S. (2010). Picture this: Increasing math and science learning by improving spatial thinking. *American Educator*, 34(2), 29-35.
- Parsons, S., & Bynner, J. (2005) *Does numeracy matter more?* (National Research and Development Centre for Adult Literacy and Numeracy, London)
- Patro, K., Fischer, U., Nuerk, H. C., & Cress, U. (2016). How to rapidly construct a spatial–numerical representation in preliterate children (at least temporarily). *Developmental Science*, 19(1), 126-144.
- Pyle, H. W. (1928). *The psychology of learning*. Baltimore, MD: Warwick and York.
- Ragsdale, C. E. (1932). *Modern psychology and education*. New York, NY: MacMillan.
- Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in

- low-income children's numerical knowledge through playing number board games. *Child Development*, 79(2), 375-394.
- Redick, T. S., Shipstead, Z., Wiemers, E. A., Melby-Lervåg, M., & Hulme, C. (2015). What's working in working memory training? An educational perspective. *Educational Psychology Review*, 27(4), 617-633.
- Root-Bernstein, R. S. (1985). Visual thinking: The art of imagining reality. *Transactions of the American Philosophical Society*, 75(6), 50-67.
- Schneider, M., Beeres, K., Coban, L., Merz, S., Susan Schmidt, S., Stricker, J., & De Smedt, B. (2017). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: A meta-analysis. *Developmental Science*, 20(3), e12372.
- Schneider, M., Merz, S., Stricker, J., De Smedt, B., Torbeyns, J., Verschaffel, L., & Luwel, K. (2018). Associations of number line estimation with mathematical competence: A meta-analysis. *Child Development*, 89(5), 1467-1485.
- Seydell-Greenwald, A., Ferrara, K., Chambers, C. E., Newport, E. L., & Landau, B. (2017). Bilateral parietal activations for complex visual-spatial functions: Evidence from a visual-spatial construction task. *Neuropsychologia*, 106, 194-206.
- Shaki, S., Fischer, M. H., & Petrusic, W. M. (2009). Reading habits for both words and numbers contribute to the SNARC effect. *Psychonomic Bulletin & Review*, 16(2), 328-331.
- Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games—but not circular ones—improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology*, 101(3), 545-560.
- Skinner, B. F. (1938). *The Behavior of Organisms: An Experimental Analysis*. New York, NY: Appleton-Century.
- Sokolowski, H. M., Fias, W., Mousa, A., & Ansari, D. (2017). Common and distinct brain regions in both parietal and frontal cortex support symbolic and nonsymbolic number processing in humans: A functional neuroimaging meta-analysis. *NeuroImage*, 146, 376-394.
- Spence, K. W. (1942). Theoretical interpretations. In Moss, F. A. (ed.), *Comparative*



- Psychology*, New York, NY: Prentice Hall.
- Stengel, E. (1944). Loss of spatial orientation, constructional apraxia and Gerstmann's syndrome. *Journal of Mental Science*, *90*(380), 753-760.
- Tam, Y. P., Wong, T. T. Y., & Chan, W. W. L. (2019). The relation between spatial skills and mathematical abilities: The mediating role of mental number line representation. *Contemporary Educational Psychology*, *56*, 14-24.
- Thorndike, E. L. (1917). *Thorndike arithmetic; book two*. New York, NY: Rand McNally & Co. Thorndike, E. L. (1921). *The new methods in arithmetic*. New York, NY: Rand McNally & Co.
- Titze, C., Jansen, P., & Heil, M. (2010). Mental rotation performance and the effect of gender in fourth graders and adults. *European Journal of Developmental Psychology*, *7*(4), 432-444.
- Tolar, T. D., Lederberg, A. R., & Fletcher, J. M. (2009). A structural model of algebra achievement: computational fluency and spatial visualisation as mediators of the effect of working memory on algebra achievement. *Educational Psychology*, *29*(2), 239–266.
- Tolman, E. C. (1932). *Purposive Behavior in Animals and Men*. New York, NY: Appleton-Century.
- Toomarian, E. Y., & Hubbard, E. M. (2018). On the genesis of spatial-numerical associations: Evolutionary and cultural factors co-construct the mental number line. *Neuroscience & Biobehavioral Reviews*, *90*, 184-199.
- Uttal, D. H., & Cohen, C. A. (2012). Spatial thinking and STEM education: When, why and how. *Psychology of Learning and Motivation*, *57*, 147–181.
- Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., & Newcombe, N. S. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological Bulletin*, *139*(2), 352.
- van Dijck, J. P., & Fias, W. (2011). A working memory account for spatial–numerical associations. *Cognition*, *119*(1), 114-119.
- Viarouge, A., Hubbard, E. M., & McCandliss, B. D. (2014). The cognitive mechanisms of the SNARC effect: An individual differences approach. *PloS one*, *9*(4), e95756.
- Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial ability for STEM domains:

- Aligning over fifty years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology*, 101, 817–835.
- Walsh, V. (2003). A theory of magnitude: common cortical metrics of time, space and quantity. *Trends in Cognitive Sciences*, 7(11), 483-488.
- Wei, W., Yuan, H., Chen, C., & Zhou, X. (2012). Cognitive correlates of performance in advanced mathematics. *British Journal of Educational Psychology*, 82(1), 157–181.
- Zacks, J. M. (2008). Neuroimaging studies of mental rotation: a meta-analysis and review. *Journal of Cognitive Neuroscience*, 20(1), 1-19.
- Zorzi, M., Priftis, K., & Umiltà, C. (2002). Brain damage: neglect disrupts the mental number line. *Nature*, 417(6885), 138-139.

## Chapter 2

# 2 Relations between Numerical, Spatial, and Executive Function Skills and Mathematics Achievement: A Latent-Variable Approach

## 2.1 Citation

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## 2.2 Introduction

How do humans learn to think mathematically? What role do cognitive skills play in the ability to engage in abstract mathematical thought? During the past two decades, researchers from a wide variety of disciplines, including psychology, cognitive neuroscience, and education, have become increasingly interested in answering these and other related questions. This is due, in part, to the growing recognition of the importance of mathematics for both school and life success (e.g., see Duncan et al., 2007; Parsons & Bynner, 2005). For instance, early mathematics skills strongly predict later mathematics achievement, as well as educational attainment more generally, and contribute to important life outcomes, such SES, health and personal well-being (Duncan et al., 2007; Parsons & Bynner, 2005; Ritchie & Bates, 2013). In short, there is a need to better understand factors that contribute to individual differences in the development of and achievement in mathematics.

Current evidence suggests that numerical, spatial, and executive function (EF) skills each play critical and independent roles in the learning and performance of

mathematics (e.g., see De Smedt, Noël, Gilmore, & Ansari, 2013; Mix & Cheng, 2012; Cragg & Gilmore, 2014). However, these conclusions are largely based on isolated bodies of research and without measurement at the latent variable level. While prior research has examined latent relations between two of these constructs (e.g., spatial and mathematical abilities; Mix et al., 2016; 2017; spatial and EF abilities; Miyake, Friedman, Rettinger, Shah, & Hegarty, 2001), relations between all four constructs has yet to be examined. Thus, questions remain regarding the extent to which these skills represent distinct constructs and whether numerical, spatial, and EF skills afford differentiated pathways to mathematics achievement. In this paper, we address this gap in the literature and examine the latent structure and interrelations between numerical, spatial, EF, and mathematics abilities in a sample of 4-to 11-year-olds.

In addition to providing a comprehensive cognitive model of children's mathematics achievement, this study was designed to more closely reveal insight into the underlying nature of the oft reported space-math association. Although there is extensive correlational evidence linking spatial and mathematical cognition, including decades of behavioral and neural research (Mix & Cheng, 2012; Hubbard, Piazza, Pinel, & Dehaene, 2005), relatively few efforts have been made to reveal potential mechanisms linking space and math. Here, we test the hypothesis that spatial visualization plays a critical role in mathematical problem solving and achievement. By testing a model of mathematical achievement that includes numerical, spatial, and EF factors, we were able to test and control for specific pathways connecting spatial visualization skills and mathematics achievement. This allowed us to examine the extent to which the space-math link is best explained by direct relations between spatial visualization and mathematics or whether the space-math link might be better explained by alternative mechanisms. Specifically, we test whether general intelligence and/or EF skills (including visual-spatial working memory) might better explain the space-math link. In addition, we test the hypothesis that spatial visualization skills are indirectly related to mathematics through basic numerical skills (e.g., see Gunderson, Ramirez, Beilock, & Levine, 2012; LeFevre et al., 2013). The findings related to these pathways are crucial in order to advance current theories of spatial and mathematical associations.

In the next section, we provide a more detailed review of space-math associations. We then operationalize basic numerical skills and EFs, as defined in the current study, and review evidence to suggest differentiated pathways from each of the targeted constructs to mathematics achievement. Our main study objectives and hypotheses are then discussed in light of this review.

## 2.2.1 Relations between Spatial Skills and Mathematics

The scientific study of associations between spatial and mathematical thinking has a lengthy history, dating back to Sir Francis Galton's inquiries into the visualization of numerals in the late 1800's (Galton, 1880). Indeed, a large body of research supports the finding that people with strong spatial skills also tend to do well in mathematics (Mix & Cheng, 2012). Of the various spatial skills identified, spatial visualization skills appear to play an especially important role in mathematics learning and achievement (Mix et al., 2016). Defined as the ability to generate, retrieve, maintain, and manipulate visual-spatial information (Lohman, 1996), spatial visualization skills have been linked to performance across a breadth of mathematics tasks, including arithmetic (Kyttälä & Lehto, 2008), word problems (Hegarty & Kozhevnikov, 1999), geometry (Delgado & Prieto, 2004), algebra (Tolar, Lederberg, & Fletcher, 2009), and highly advanced mathematics, including function theory, mathematical logic, and computational mathematics (Wei, Yuan, Chen, & Zhou, 2012). Moreover, the link between spatial visualization and mathematics is not limited to tasks that are inherently spatial, such as geometry or measurement. Research demonstrates that even basic number processing, such as comparing which digit is numerically larger (7 vs. 2), is closely associated with spatial visualization skills, such as mental rotation (Thompson, Nuerk, Moeller, & Cohen Kadosh, 2013; Viarouge, Hubbard, & McCandliss, 2014).

What explains the math-space link? One popular theory posits that numbers are represented spatially (de Hevia, Vallar, & Girelli, 2008). That is, humans come to conceive of and arrange numbers along a "mental number line," with small numbers belonging to the left and larger numbers extending to the right (Dehaene, Bossini, & Giraux, 1993). Interestingly, the left-to-right orientation of the mental number line

appears to be culturally-specific and is reversed in cultures that read and write right-to-left (Göbel, Shaki, & Fischer, 2011). It is hypothesized that one of the ways children make sense of symbolic numbers is to learn to represent numbers according to their spatial relations (e.g., 1 and 2 are “close together,” while 1 and 9 are “far apart.”). Said differently, spatial skills are predicted to play an active role in the development of children’s conceptualization and visualization of the various meanings of number (e.g., see Sella, Berteletti, Lucangeli, & Zorzi, 2017).

Moreover, the same spatial reasoning capacities that help ground various symbolic number relations may also help with the learning and representation of symbolic mathematics more generally (Lakoff & Núñez, 2000). That is, spatial visualization skills are hypothesized to play a critical role in one’s ability to model, simulate, and form mathematical relationships. Accordingly, spatial visualization skills may provide a means to conceptualize numbers as ascending from left to right (akin to a number line), but also allow one to visualize and model various other mathematical transformations, such as the decomposition of 12 into a unit of 10 and 2. In sum, spatial visualization skills might represent one cognitive tool which children draw from to learn and make sense of not only basic numerical relations but novel and higher-level mathematics as well.

In the present study, we targeted spatial visualization skills by including measures of mental rotation and visual-spatial reasoning. These measures were selected as they were hypothesized to involve the recruitment of spatial visualization in the service of solving novel visual-spatial problems. More generally, we operationalized spatial visualization as a construct involving the “generation” or “creation” of visual-spatial solutions to problems (e.g., imagining how a folded and punctured piece of paper might appear when unfolded). We placed special emphasis on this aspect of spatial visualization (i.e., the need to generate mental images) in an effort to examine and better understand the hypothesized link between spatial thinking and mathematics described above. That is, one reason for the consistent relations between spatial and mathematical thinking may be due to the shared task requirements involved in the generation and manipulation of visual-spatial representations and solutions to problems in both respective domains.

## 2.2.2 Relations between Numerical Skills and Mathematics

Numbers – and their various relations with one another – lie at the heart of mathematics. As such, concerted efforts have been directed at studying how humans, and other species for that matter, perceive, represent, manipulate, and make sense of number. To date, the study of basic numerical skills has been approached through various paradigms that target the measurement of an individual’s numerical magnitude representations. For example, the speed and accuracy in which individuals can compare and select the larger of two numerical magnitudes (5 vs. 3 or  $\bullet\bullet\bullet$  vs.  $\bullet\bullet$ ) is a commonly used approach to assess the precision of an individual’s mental representation of number (Siegler, 2016). There is an extensive body of research linking individual differences in magnitude comparison tasks and various measures of mathematics. In general, children and adults who are faster and more accurate at comparing numerical symbols (5 vs. 3) and nonsymbolic number ( $\bullet\bullet\bullet$  vs.  $\bullet\bullet$ ), tend to also do better on higher-level mathematical tasks, such as arithmetic (Schneider et al., 2017).

Another important marker of an individual’s basic number skills relates to their understanding and processing of numbers as ordered sequences (i.e., ordinality). Performance on ordinality tasks, typically assessed by the speed and accuracy in which an individual can recognize ordered numerical sequences (e.g., 4-5-6; 5-7-9), are thought to index the strength of an individual’s associations of numerical relations. Research indicates positive relations between children and adults’ ordinality skills and mathematics performance (Lyons et al., 2014; Lyons, Vogel, & Ansari, 2016).

Taken together, current research in the field of numerical cognition point to magnitude and ordinal processing skills as foundational numerical competencies with strong links to more formal mathematics. For this reason, the current study included measures of children’s magnitude comparison (symbolic and nonsymbolic) and ordinality skills as key indicators of the targeted construct of numerical ability.

The hypothesized causal link between basic numerical skills and higher-level mathematics is rather straightforward: Mathematics is a particularly hierarchical subject,

where earlier learned concepts and skills are needed to give rise to new and more advanced mathematical knowledge, and thus, basic numerical skills represent and serve the role of fundamental building blocks.

## 2.2.2 Relations between Executive Functions and Mathematics

The last decade has seen a sharp rise in research linking executive functioning (EF) and mathematics achievement (e.g., see Cragg & Gilmore, 2014). Although definitions vary, EF is most typically defined as a suite of highly related but separable cognitive control abilities that includes working memory, inhibitory control, and shifting or flexible attention (Friedman & Miyake, 2017; Miyake et al., 2000). Each sub-component has been found to both concurrently and longitudinally predict mathematics achievement (Cragg & Gilmore, 2014). Working memory, in particular, has been found to be a consistent predictor of mathematics (Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013; Fuchs et al., 2010), with especially strong relations between visual-spatial working memory (VSWM) and mathematics performance (Reuhkala, 2001). Despite evidence and theory to suggest that all three of these components are related and represent a unified construct (i.e., EF; Miyake et al., 2000), there is a scarcity of research studying relations between EF and mathematics at the latent variable level. Thus, it remains to be shown how EF – as a unified construct – relates to mathematics achievement.

In the present study, we attempted to measure EF by including measures of VSWM as well as a single behavioral measure of EF (i.e., the Head-Toes-Knees-Shoulders task; Ponitz et al., 2009); a comprehensive measure thought to tap into each sub-component of EF, but most notably inhibitory control (McClelland & Cameron, 2012). The decision to target VSWM also allowed us to test the extent to which spatial ability and VSWM represent potentially distinct constructs.

Given that many mathematics tasks are complex and involve multi-step solutions, EF skills have been theorized to play a critical, if not causal, role in mathematics learning and performance. Moreover, different components of EF have been proposed to play unique roles in the service of different mathematical goals. For example, working



memory is called upon to remember the specifics of a given problem as well as to temporarily hold partially completed solutions in mind while performing other aspects of the problem. Inhibitory control is needed to ignore or suppress certain responses in favor of other more appropriate responses (e.g., inhibiting knowledge of whole number operations when dealing with fractions). Shifting or flexible attention is recruited when switching between different operations, such as problems that involve both addition and subtraction. From these examples, it can be seen how individual differences in EF skills may constrain one's capacity to learn and carry out various mathematical tasks.

## 2.3 Main Questions and Hypotheses

### 2.3.1 Rationale for Testing a Four-Factor Model

The first goal of this study was to carry out a confirmatory factor analysis (CFA) to examine the degree of evidence in favor of a four-factor model, with factors corresponding to numerical, spatial, EF, and mathematical constructs. Based on theory and a small body of empirical research suggesting the existence of highly-related, but *distinct* constructs, we had reason to expect similar findings. For example, Mix et al. (2016) found evidence to suggest that spatial and mathematical abilities represent separate, but highly overlapping, constructs in a cross-sectional sample of children aged 5–13. In adults, Miyake et al. (2001) demonstrated that spatial abilities and EFs are highly related factors and distinguishable to the extent that the spatial abilities measured were theorized to involve high executive demands. For example, latent variables related to spatial visualization and EF skills were highly related to one another (.91) compared to relations between spatial perceptual speed and EFs (.43). The extent to which spatial abilities and EFs represent separate factors in children is currently unknown. The present study aims to shed light on this issue.

To our knowledge, researchers have yet to examine the latent structure of basic numerical skills (e.g., basic understandings of numerical symbols and their associated magnitudes) and its relations with constructs related to spatial, EF, and mathematical abilities. However, prior research using single indicator variables has revealed consistent

relations between basic numerical and spatial skills (e.g., see Newcombe, Levine, & Mix, 2015), as well as basic numerical skills and EFs (e.g., see Cragg et al., 2017). Effect sizes related to these studies are typically in the moderate range and provide reason to suspect that numerical skills will share both overlapping and unique variance with spatial and EF skills. However, the extent to which basic numerical skills and mathematics achievement represent distinct constructs remains an open question. Moreover, if such a distinction does exist, what construct might spatial ability share more variance with? How might EF skills modify these relations?

With these questions in mind, we examined the latent structure of constructs related to numerical, spatial, EF, and more general mathematical skills. Given empirical evidence suggesting overlapping but unique relations between each construct, along with general consensus that each construct does indeed refer to something specific, we predicted that results of a CFA would offer support for a four-factor model. However, it should be noted that this is the first CFA that we are aware of that tests evidence for all four factors in the same model. It is possible, given their high associations with one another, that a single factor (i.e., general intelligence or *g*) might emerge as the best model fit of the data. Thus, to rule out this possibility, we also ran an exploratory analysis of a single-factor (*g*) model for comparison purposes.

### 2.3.2 Which Construct Does Visual-Spatial Working Memory Belong to: Spatial or EF?

Another follow-up objective of testing the four-factor model was to examine the extent to which measures of visual-spatial working memory (VSWM) load more closely on the EF versus spatial factor. Currently, the decision to classify VSWM as a marker of spatial ability or EF is up to individual researchers and it is not always clear whether such a decision is theoretically guided or post hoc. Here, we make the prediction that VSWM is better characterized as an indicator of EF than spatial visualization ability. This prediction was theoretical and made a priori based on the following criteria: Although both constructs are similar in that both are presumed to place heavy demands on cognitive control, there are some important distinctions in task requirements that potentially result

in differential relations with mathematics. The EF measures were selected based on their involvement of working memory and inhibitory control. These measures were selected as “recall-based” measures; they require the storage and recall of information. In contrast, the spatial reasoning measures were selected based on their heavy demands on spatial perception, reasoning, and most notably, spatial visualization. More specifically, each spatial task required participants to reason and visualize solutions to problems involving parts of objects in relation to their whole. Critically, the spatial measures are distinct from the EF measures in that they are “prospective” or “generative” in nature. Thus, the spatial measures place a low demand on recall of information and place a heavy demand on the visualization or modeling of problems and their solutions. As detailed further below, loading VSWM on the EF factor also allowed us to test the important question of whether spatial visualization makes unique contributions to mathematics over and above EF skills.

### 2.3.3 Differentiated Pathways to Mathematics Achievement

Given sufficient evidence for the existence of a four-factor model, our second objective was to test the shared and unique contributions of each predictor variable with mathematics achievement. As reviewed above, separate bodies of research have identified numerical, spatial, and EF skills as robust and consistent predictors of mathematics achievement. For this reason, we expected the combination of factors to explain a large proportion of variance in children’s mathematics performance. To our knowledge, this is the first study to include each construct within the same model and to simultaneously examine the extent to which each cognitive construct uniquely relates to mathematics achievement. Therefore, it is currently unknown how each variable relates to one another and potentially afford differential pathways to mathematics achievement. However, as reviewed above, each cognitive construct in the current study has been posited to differentially explain individual differences in mathematical performance. A metaphor of building a house serves as an example: Whereas basic numerical skills represent the fundamental building blocks (bricks), spatial visualization skills are considered a tool in

which to manipulate and assemble the bricks, and EF skills place certain constraints, such as rules and regulations, on the building process.

### 2.3.4 Does Age Moderate Potential Relations between Constructs?

As a follow-up to the above objective and analyses, we were interested in testing the extent to which age might moderate the observed relations. One reason for targeting the selected age-range (4- to 11-year-olds) was to better understand how each one of these foundational skills develop and potentially interact with one another across the early to middle childhood years. While separate bodies of research suggest that each construct undergoes rapid development during this time frame (e.g., see Mix, Huttenlocher, & Levine, 2002; Newcombe & Huttenlocher, 2003; Zelazo, Carlson, & Kesek, 2008), we currently know very little about the potential influence of age on these relations. Given that children's numerical, spatial, and EF skills have all been posited to play a critical role in children's mathematical development, it is important to better understand the potential impact that age might have on these various relations. This information, in turn, may be useful when designing educational interventions. Thus, an important question concerns the extent to which relations between constructs remain consistent across time or show evidence of change during specific periods of development.

### 2.3.5 Uncovering the Space-Math Association

Our third, and most theoretically-guided objective, involved working towards an improved understanding of the space-math association. Critically, the inclusion of the targeted constructs provided opportunities to test specific hypotheses about the underlying nature of this relationship. Namely, we sought to determine the potentially mediating roles of children's numerical and EF skills in the relation between spatial visualization and mathematics achievement.

Reasons to suspect that numerical skills might mediate the space-math link includes research pointing to the fundamental importance of spatial thinking in the acquisition and development of basic numerical competencies (Dehaene, 2011; Geary, 2004). Indeed, there is evidence that both numerical and spatial cognition rely on highly similar neural networks (e.g., see Hubbard et al., 2005; Toomarian & Hubbard, 2018). This has led some to speculate that reasoning about symbolic number – a relatively recent cultural invention – is rooted in more evolutionarily adaptive neural networks specialized for performing various visual-spatial tasks, such as using and reasoning with objects and tools (Anderson, 2010; Dehaene & Cohen, 2007; Lakoff & Núñez, 2000). Moreover, not only do numerical and spatial processing appear to share biological underpinnings, but they also appear to share close conceptual links (Lakoff & Núñez, 2000; Marghetis, Núñez, & Bergen, 2014). One proposal is that the learning of the number system involves the mapping of numbers to space, a process that has been found to implicate higher-level spatial skills, such as spatial visualization (Gunderson et al., 2012; Marghetis, Núñez, & Bergen, 2014; Sella et al., 2017). For example, children’s ability to estimate the locations of numbers along a physical number line, has been found to mediate relations between spatial skills and mathematics performance (Gunderson et al., 2012; LeFevre et al., 2013; Tam, Wong, & Chan, 2018). These findings dovetail with theoretical claims that the development of number knowledge corresponds to the refinement of one’s ‘mental number line’ (Dehaene, 2011; Siegler & Booth, 2004; Siegler & Ramani, 2008). In the current study, we look to extend this finding by testing whether or not basic numerical skills more generally mediate the space-math link.

However, this is but one pathway in which spatial ability is potentially linked to mathematics. Moving beyond basic numerical-spatial associations, spatial skills may also be recruited and utilized across a breadth of mathematical tasks, including those that are more distally related to basic numerical processing, including geometric reasoning. Moreover, as discussed earlier, reasoning about numbers in novel contexts, as is required in word problems, algebra, or even arithmetic, may be augmented through the mapping and modeling of these various mathematical relations onto space. We suspect that the same spatial system that allows one to both map and conceptualize numbers along a ‘mental number line,’ is the same system that allows one to map, model, and

conceptualize various other abstract mathematical relations (e.g., see Lakoff & Núñez, 2000; Marghetis, Núñez, & Bergen, 2014). Accordingly, we predicted that spatial ability would relate to mathematics indirectly through its relation with basic numerical skills, but also directly, due to spatial processes that are not specific to number.

Another reason why spatial skills and mathematics may be linked is due to the high executive demands of spatial tasks (e.g., see Miyake et al., 2001). It is possible that spatial tasks are essentially a proxy for EF. Indeed, although a large body of research demonstrates close connections between spatial and mathematical thinking (Mix & Cheng, 2012), it remains to be tested whether EF skills might serve as the common and potentially explanatory source for this relationship. Said differently, it could be the case that spatial thinking is only related to mathematics inasmuch as the spatial tasks also recruit and rely on executive functions, such as working memory and inhibitory control. By testing the mediating role of EF in the space-math association, we were able to test the extent to which the space-math link might be explained by individual differences in children's EFs. If the space-math link is fully attributable to children's EF skills then we should expect full mediation. If the space-math link is best explained by children's spatial skills, over and above EF skills, then we should not expect strong evidence of mediation. However, if spatial and EF skills represent distinct constructs with differentiated relations to mathematics achievement – as current theory suggests – we should expect to find evidence of both direct and indirect relations between spatial skills and mathematics achievement. This finding would provide evidence of both shared and unique relations with mathematics. Indeed, we predicted that EF skills would explain some of the shared variance between spatial ability and mathematics performance, but would not fully account for the space-math relation. As outlined above, we hypothesized that differences in the “generative” versus “recall” requirements of the spatial and EF tasks, respectively, should result in separate factors but also differential relations with mathematics performance.

### 2.3.6 Different Pathways for Different Mathematical Reasoning

Our final objective dealt with issues around the multidimensionality of mathematics. Mathematics is not a unified construct and represents multiple components and skills sets (Mix & Cheng, 2012). Yet, most researchers use arithmetic or calculation-based tasks as mathematics outcome measures. Although arithmetic represents a foundational mathematics skill, more comprehensive mathematics outcome measures are needed to capture the type of mathematics that is more representative of the subject as a whole. Furthermore, multiple measures of the different branches of mathematics are needed to better capture specific relations amongst cognitive skills and different aspects of mathematics. In the current study, numeration and geometry were selected as the two outcome measures of mathematics and used in combination to form the mathematics achievement factor. However, we were also interested in how numerical, spatial, and EF skills might differentially relate to numeration and geometry as separate outcome measures of mathematics. It was predicted that spatial skills will best predict geometry performance, numerical skills will best predict numeration performance, and EF skills will equally predict both. Although these predictions are relatively straightforward, they are a necessary first step in moving towards a more nuanced picture of the cognitive foundations of mathematics performance.

## 2.4 Methods

### 2.4.1 Participants

Three-hundred and sixteen 4- to 11-year-olds (kindergarten – 4<sup>th</sup> grade) participated in the study ( $M_{\text{age}}=6.68$  years,  $SD=1.40$ ; Females=165). The mean age was the same for males ( $M_{\text{age}}=6.74$  years) and females ( $M_{\text{age}}=6.62$  years),  $t(314) = -0.81, p = .42$ . Table 2.4 provides a summary of the number of children and mean ages for each grade level. The sample was drawn from eight schools located in both rural ( $n=6$ ) and urban communities ( $n=2$ ) in northwestern and southwestern regions of Ontario, Canada. Based on 2016 Canadian census data, all participating schools serve communities with family income levels below the Canadian median (\$70,336), ranging from \$55,936 to \$68,062. The schools represent a range of low-to-moderately high performing schools in mathematics

based on available standardized provincial test scores. Exactly half of the sample identified as Indigenous peoples of Canada; 94% identified as Anishinaabe and 6% identified as Métis.<sup>1</sup> Based on available 2016 census data, the vast majority (>95%) of the remaining population identified as Caucasian. Note that although all 316 participants were included in the analyses, data were incomplete or missing for performance on individual measures due to time restrictions (109 cases) or the child's inability to understand task requirements (46 cases). Missing data accounted for 4% of all cases. Written consent was provided by a parent/guardian for all participants and research was carried out in agreement with the ethics boards of the University of Toronto and University of Western Ontario.

## 2.4.2 Measures and Testing Procedures

Participants completed a cognitive test battery involving eleven separate measures (see Table 2.1). All measures were selected from previously published research. Participants completed the measures in pseudo-random order and in two approximately 30-minute sessions (1-5 days apart). Due to the nature of the tests, the following measures were presented within ordered blocks: Symbolic number comparison, nonsymbolic number comparison, and ordering; path span forward and path span reverse; and KeyMath numeration and KeyMath geometry. All tests were carried out in a quiet location of the child's school (e.g., empty classrooms or private testing rooms) and were administered one-to-one by trained experimenters. The details of each test are provided below.

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<sup>1</sup> Note that information on Indigenous status was not collected at two of the participating schools due to prior knowledge that these schools predominantly serve Caucasian populations and an extremely low number of Indigenous students; e.g., the 2016 census listed the number of Indigenous families in these communities at zero. For this reason, we did not see a need to inquire about Indigenous status at these schools.



### 2.4.2.1 Description of Numerical Indicators

The following three measures were adopted from Lyons et al. (2018) and presented to participants in a paper booklet (12 items per page). Participants marked their responses using a pencil. For all three tasks, children were provided with 1 minute to complete as many items as possible. The tasks were presented to children in the order in which they are described below. Both the symbolic and nonsymbolic comparison tasks consisted of 72 items and the ordering task consisted of 48 items. For all three measures, the same scoring procedures were used: To adjust for potential speed-accuracy trade-offs or guessing behavior, adjusted raw scores were computed by subtracting the total number of incorrect items from the total number of correct items (see Lyons et al., 2018).

*Symbolic Number Comparison:* Participants were presented with pairs of Hindu-Arabic numerals (e.g., 4 | 9) and asked to indicate the larger of two numerals as quickly as possible. Numerals ranged from 1–9, with absolute numerical distances ( $N1 - N2$ ) of 1 to 3. All 15 combinations of 1–9 with distances of 1 or 2 were included as well as three combinations with distance 3 (1|4; 3|6; 6|9). This resulted in 18 possible combinations. Trials were counterbalanced to ensure that the larger number appeared on the left and right side of the page an equal number of times.

*Nonsymbolic Number Comparison:* Participants were presented with pairs of dot arrays (e.g., : | ::) and asked to select the array with the most dots as quickly as possible. Dot arrays ranged from 1–9 dots and included the same numerical distances as those used in the symbolic task. That is, 18 combinations of dot arrays were used and were counterbalanced in the exact same order as the symbolic task. This was done to allow for direct comparison between the symbolic and non-symbolic versions of the task (e.g., see Lyons et al., 2018). Children were instructed not to count the dots. In an effort to control for the influence of the continuous properties of the dot stimuli on performance, both area and contour length were manipulated and controlled for across trials. More specifically, on half the trials dot area was positively correlated with numerosity and overall contour length was negatively correlated with numerosity. On the other half of the trials the

opposite was true. Thus, relying on either area or contour length to would result in chance performance (Gebuis & Reynvoet, 2012).

*Ordering Task:* Participants were presented with a sequence of numerals (e.g., 2 – 3 – 4) and asked to indicate whether or not the sequence was in numerical order (i.e., are the numerals in an ascending sequence?). Numerals ranged from 1–9, with absolute numerical distances of 1 (e.g., 2 – 3 – 4) or 2 (e.g., 2 – 4 – 6). There were an equal number of correct and incorrect sequences of distances 1 and 2. For half of the items, the sequences were in correct ‘ascending order’ (e.g., 2 – 3 – 4 or 3 – 5 – 7) and for the other half, the sequences were in incorrect order (e.g., 2 – 4 – 3 or 5 – 3 – 7). Participants put a line through a checkmark to indicate when the sequence was believed to be in order and a line through an ‘X’ when the order was not believed to be in order.

#### 2.4.2.2 Description of Spatial Indicators

*2D Mental Rotation:* This measure was adapted from Levine et al.’s (1999) Children’s Mental Transformation Task (CMTT); a widely used measure of young children’s spatial visualization skills, namely mental rotation (Ehrlich, Levine, & Goldin-Meadow, 2006; Gunderson et al., 2012; Hawes, LeFevre, Xu, & Bruce, 2015). Children were presented with two halves of a shape, bisected either along the horizontal or vertical line of symmetry (e.g., a diamond that has been divided into two triangles) and separated and rotated 60° from one another on either the same plane (direct rotation items) or diagonal plane (diagonal rotation items). Four response items (2D shapes) were presented in a 2 x 2 array below the bisected shape. For each item, children were asked to point to the shape that could be made by putting the two pieces together (e.g., a diamond can be made by rotating and translating two triangles). There were 16 items in total; half of which required direct rotations and half of which required diagonal rotations. Note that we modified the original measure by only including items that involved mental rotation (we eliminated items that required translations only). This modification has been shown to make the task more difficult and more appropriate for our targeted grade range (K-3; e.g.,

see Casey et al., 2018; Hawes et al., 2015). Each item had one correct response. Children were awarded one point for each correct response.

*Visual-Spatial Reasoning:* This measure was adapted from Hawes et al. (2017) and was designed as a comprehensive measure of children's spatial visualization skills. The test consists of 20 items divided into four different problem types: missing puzzle pieces (two variations), mental paper folding, and composition/decomposition of 2D shapes. For each problem, children were asked to identify the correct answer among four options. One point was awarded for each correct response.

*Raven's Progressive Matrices:* This is a widely used measure of children's visual-spatial analogical reasoning (Raven, 2008). Previous research has shown that performance on the task can be linked to a latent spatial visualization factor (Lynn, Backhoff, & Contreras-Niño, 2004; also see Kunda, McGregor, & Goel, 2010). For each item, participants are presented with a partially completed visual-spatial pattern and must select from amongst six alternatives the puzzle piece that will complete the pattern. The test consists of 36 items. One point was awarded for each correct response.

### 2.4.2.3 Description of Executive Function Indicators

*Head-Toes-Knees-Shoulders Task (HTKS):* This measure was adapted from Ponitz et al. (2009). The task requires children to engage in flexible attention, working memory, and inhibitory control (McClelland & Cameron, 2012) and closely aligns with Miyake et al.'s (2000) model of executive functioning. For each item, children listen to an instruction to touch a body part (e.g., "Touch your head") and then must touch a paired "opposite" body part (e.g., toes). Head and toes represented one pair and shoulders and knees represented the other pair. The test was divided into two sections. In the first section, participants were only asked to deal with one pair of body parts (head and toe pairings or shoulder and knee pairings). These pairings were counterbalanced across tests and participants were randomly administered a test version that started with the head and toe pairings or one that started with shoulder and knee pairings. The second section included both pairings.

Both sections included 10 items. For each item participants were given a score of 0, 1, or 2; a score of 0 corresponded to incorrect body movements (touching one's head when asked to touch their head), a score of 1 corresponded to a self-corrected body movements (initiating movement towards the wrong body part and then making a correction), and a score of 2 corresponded to correct body movements (touching one's toes when asked to touch their head). Children were given a total score out of 40.

*Forward Path Span:* This task was completed on an iPad and used to measure children's working memory, a key component of executive functioning (Miyake et al., 2000). Participants were presented with a set of nine randomly arranged green circles and instructed to watch as the circles lit up one at a time. Each circle was presented for .6 seconds, with .5 seconds of wait time between presentation. Participants then attempted to recall the sequence by touching/tapping the circles in the same order in which they were presented. Following a practice trial, participants began by attempting two trials at a sequence length of two. Upon successful recall of one or two sequences the child progressed to the next level. The task was discontinued when the child failed to recall both sequences at any given level. Children were assigned a score based on the total number of correct sequences recalled.

*Reverse Path Span:* This task was identical to the one above but required participants to recall the given sequence in reverse order. For this reason, this task is considered to place even more demands on executive control. For more information and to access both path span tasks see: <http://hume.ca/ix/pathspan.html>

Note that we did not include a manifest measure of shifting ability. This decision was based on research indicating that the working memory and inhibitory components of EF are stronger predictors of mathematics than shifting (e.g., see Cragg & Gilmore, 2014). Moreover, shifting skills are presumably implicated in the HTKS task (McClelland & Cameron, 2012).

## 2.4.2.4 Description of Mathematics Achievement Indicators

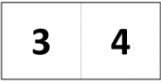
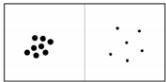
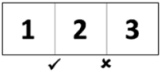

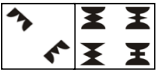



To assess children's mathematics achievement, we used the Numeration and Geometry subtests of KeyMath (Connolly, 2007). We selected KeyMath as our mathematics outcome measure because it is a standardized Canadian normed test and the items represent a broad range of content knowledge closely aligned with the Ontario mathematics curriculum. The test is administered with an easel booklet and each problem refers to information presented in the form of an image and/or writing. The test is adaptive in that it begins by establishing baseline performance and continues with questions of increasing difficulty. The test is discontinued when the child answers four questions incorrectly in a row. Thus, the test captures a range of children's mathematics skills and not all children are administered the exact same questions. Moreover, because the test is adaptive and continues until a ceiling level of performance is established children are almost inevitably presented with novel mathematical content. The majority of questions are also novel in that they require children to apply their knowledge of mathematical concepts, facts, and procedures within contexts likely unfamiliar to the students (e.g., rather than solving a standard arithmetic problem children must apply their knowledge of arithmetic to solve a problem dealing with combinations of block structures). The majority of items in both subtests require knowledge of the symbolic number system. Children were awarded a total raw score by subtracting the total number of incorrect responses from the maximum item number reached.


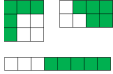
*Numeration Test:* This measure includes a total of 49 questions related to counting, comparing quantities, recognizing and ordering number symbols, operations, place value, and proportions/fractions/decimals.

*Geometry Test:* This measure includes a total of 36 questions related to shape recognition, positional language, geometrical transformations (e.g., rotations), measurement, grid coordinates, angles, geometric proofs.

Table 2.1

*Summary of measures used in the study*

Measures	Task Description	Example Items
<b>Numerical Measures</b>		
Symbolic Number Comparison	<ul style="list-style-type: none"> <li>Participants select the numerically larger of two Hindu-Arabic Numerals</li> <li>1 minute to complete as many items as possible</li> </ul>	
Nonsymbolic Number Comparison	<ul style="list-style-type: none"> <li>Participants select the numerically larger of two dot arrays</li> <li>1 minute to complete as many items as possible</li> </ul>	
Ordering	<ul style="list-style-type: none"> <li>Participants indicate whether or not a sequence of numerals are in numerical order</li> <li>1 minute to complete as many items as possible</li> </ul>	
<b>Spatial Measures</b>		
Visual-Spatial Reasoning	<ul style="list-style-type: none"> <li>Participants are presented with 4 different types of 'spatial puzzles' requiring participants to visualize solutions to partially completed puzzles, composition/decomposition tasks, and mental paper folding challenges</li> </ul>	 <p><i>"Which three pieces will go together to make the shape above?"</i></p>
2D Mental Rotation	<ul style="list-style-type: none"> <li>Participants select amongst four options a given shape that can be made by mentally rotating and translating two separated shapes</li> </ul>	
Raven's Matrices	<ul style="list-style-type: none"> <li>Participants are presented with a partially completed image or visual-spatial pattern and must select amongst 6 options the piece that best completes the image/pattern</li> </ul>	
<b>Executive Function Measures</b>		
Head-Toes-Knees-Shoulders	<ul style="list-style-type: none"> <li>Participants touch the opposite body part of the one instructed</li> </ul>	<i>"When I say touch your head, I really want you to touch your toes"</i>
VSWM - Forward Path Span	<ul style="list-style-type: none"> <li>Participants are presented with a random sequence of green dots on an iPad screen and watch as individual dots light up one at a time</li> <li>Participants recall the exact sequence</li> </ul>	
VSWM - Reverse Path Span	<ul style="list-style-type: none"> <li>Participants are presented with a random sequence of green dots on an iPad screen and watch as individual dots light up one at a time</li> <li>Participants recall the exact sequence but in reverse order in which they occurred</li> </ul>	

Mathematics Measures		
Numeration	<ul style="list-style-type: none"> <li>Comprehensive suite of questions targeting numeration, including questions related to counting, ordering, operations, place value, fractions/proportions/decimals</li> </ul>	 <p><i>“How many more dots are needed to make ten?”</i></p>
Geometry	<ul style="list-style-type: none"> <li>Comprehensive suite of questions targeting geometry, including questions related to shape recognition, positional language, transformations, measurement, angles, proofs and formulas</li> </ul>	 <p><i>“Here are three shapes. Which shape will have the most green squares when it’s filled completely?”</i></p>

Note that for copyright reasons the example items for Raven’s matrices, numeration, and geometry measures were reproduced and do not constitute direct replicas of the actual items. Also note that the for the actual 2D mental rotation task, the bisected shape was presented above the four response items. VSWM = visual-spatial working memory.

### 2.4.3 Analytical Approach

Analyses were carried out using the recommended two-step approach to structural equation modeling (SEM; see Kline, 2015). The first step involved testing the measurement model using confirmatory factor analyses (CFA). The purpose of the measurement model is to test and observe the relations between the observed variables (aka indicator or manifest variables) and the relations these variables have with the hypothesized construct or constructs (aka factors or latent variables). Failure to obtain adequate fit statistics at this stage may indicate the need to reconsider the model and/or make modifications to the model. The second step involved analyses of the full structural equation model(s). The purpose of this step is to test hypothesized interrelations between constructs/factors and is similar in some ways to general linear regression models. However, a major advantage of SEM over general linear models is that SEM takes error variances into account (regression analyses assume variables are measured without error: see Weston & Gore, 2006) and allows one to model both variability common to a latent variable (i.e., error-free scores) as well as the variability not explained by the latent variable (i.e., error). Moreover, SEM allows for the creation of weighted aggregate variables of targeted constructs. That is, latent variables are not merely an average of scores obtained across different measures but a composite score that has been weighted

according to the various contributions that each indicator variable makes to the construct of interest.

Analyses were performed with Mplus Version 7.4 (Muthén and Muthén, 1998-2015) using the default maximum likelihood estimation (MLE) procedures. All analyses were conducted on raw (continuous) scores. Modification indices were requested for chi-squared values equal to or greater than 10. Missing data (4% of all cases) were treated with full information likelihood (FIML) estimation procedures (the default option in Mplus). Confidence intervals were computed using Mplus' bias corrected bootstrapped confidence interval procedure. Note that the following link provides an annotated copy of the Mplus scripts used for each of our analyses along with the corresponding output/results (<https://osf.io/2y7xu/>).

We used three goodness-of-fit statistics to compare our CFA models and determine model fit: (1) Root Mean Square Error of Approximation (RMSEA), (2) Comparative Fit Index (CFI), and (3) Standardized Root Mean Residual (SRMR). Decisions about what constitutes acceptable or 'good' model fit were based on the following recommendations: RMSEA values of less than .10, and CFI values  $> .95$ , and SRMR values  $< .08$  (Kline, 2015). Note that we also report chi-squared ( $\chi^2$ ) values for comparison purposes but due to the large sample size ( $>200$ ) did not interpret statistically significant results in any meaningful way (see Kline, 2015).

Power analyses were conducted to determine the minimum sample size needed to detect a medium effect size with an alpha of  $= .05$  and power  $= 0.95$  (Soper, 2018). Using a SEM with four latent variables and 11 indicator variables, the results indicated a recommended sample size of 241 participants.

## 2.5 Results

### 2.5.1 Part I: Measurement Model

Table 2.2 shows the descriptive statistics for each measure. As can be seen, the kurtosis and skewness values of each indicator variable fall within the acceptable limits of  $\pm 2$  (Field, 2009). Table 2.3 shows the bivariate zero-order correlations between all variables.



As can be seen, there were moderate to high correlations between all measures included in the measurement model (.42 – .83): Note that age was included as covariate in the structural models, but not the measurement model, as we had little reason to suspect measurement invariance across age (see section 7.2.2. for analyses related to the moderating effects of age). Scatter plots were used to visualize the data distributions between all variables and no concerns were noted (e.g., nonnormality, lack of homoscedastic, outliers). Furthermore, data were screened to ensure normal distributions of performance for each task across each grade level. These analyses indicated relative normal distributions across tasks and grades. Table 2.4 provides a summary of the number of children and mean scores for each grade level.

In total, five different CFA models were run on the data and associated covariance matrix (see Table 2.5 for summary of each model run). Of primary interest was to test the hypothesized four-factor measurement model. The results of this model indicated good fit statistics, with the RMSEA value (.057) below the recommended cut-off of .10 and the CFI value (.983) above the recommended threshold of .95 (Kline, 2015). The high CFI value suggests that the model is superior to a “null” model or one that assumes zero correlations between the variables. Importantly, the recommended modification indices were relatively low (MIs < 17) and inconsequential to the overall model fit. These recommendations ran contrary to the theoretical model (later to be tested with SEM), as they implicated cross-loadings between the mathematics outcome variables and individual indicator variables. Overall, the results provide support for a four-factor model.

Although the four-factor model demonstrated good fit statistics, we tested four alternative models. These modifications served the purpose of hypothesis testing as well as attempts to improve the overall model fit. The first of these modifications included the removal of the path span reverse indicator (a measure of VSWM) from the Executive Function Factor and including it as an indicator of the Spatial Factor. This modification was justified based on the grounds that the task demands share some features with the other spatial ability measures (i.e., require storage and manipulation of visual-spatial information) and has been used in prior investigations as a measure of spatial ability. The modification did not improve the model. As can be seen in Table 2.5, the change resulted in a slight increase in the chi-square value and a small increase in the RMSEA values. As

a follow-up to this analysis, and based on a similar rationale, another modification was made in which both VSWM measures were made to load on the Spatial Factor. The HTKS variable was converted into a single indicator latent variable. This was achieved by multiplying the mean/variance associated with the HTKS measure (120.307) with a reasonable estimate of assumed error variance (.20; see Kline (2015) for more details on this approach). This modification did not improve the model fit. Thus, there appears to be little difference between a model that includes a well-defined Spatial Factor with three indicator variables and a more comprehensive, yet less defined, Spatial Factor with five indicator variables. To further test the relative separability of the Spatial Factor from the EF factor, a three-factor model was carried out in which the spatial and EF measures were made to load on the same factor. Although this model demonstrated good fit (see Table 2.5), it failed to achieve the same quality of fit statistics of the four-factor model. Results of a nested chi-square difference test revealed statistically significant differences between the four-factor (Model 1) and the three-factor model (Model 4);  $\chi^2(3) = 32.56, p < .001$ . These results suggest that spatial and EF skills – as measured in the present study – represent distinct constructs.

Finally, a measurement model was evaluated in which all predictor variables were made to load on a single general factor (i.e., *g*). The rationale for such a modification was based on recent research suggesting that general intelligence, or a *g*-factor, might be responsible for previously observed relations between cognitive variables and academic achievement (e.g., see Ritchie, Bates, & Deary, 2015). Moreover, this modification was justified based on the relatively high correlations between all predictor variables. The results of this model indicated marginally acceptable fit (RMSEA = .101, CFI = .94), but worse fit statistics compared to the hypothesized four-factor model. Results of a nested chi-square difference test revealed statistically significant differences between the four-factor (Model 1) and the single-factor model (Model 5);  $\chi^2(5) = 105.03, p < .001$ . This suggests that the four-factor model fits the data significantly better than the one-factor model.

### 2.5.1.1 Summary of Results

Overall, the five measurement models evaluated demonstrated acceptable fit statistics and any one of them could technically be retained and considered decent representations of the data. However, based on a priori theoretical decisions and the finding that models 1-4 were all comparable in fit, the four-factor model was retained and used in all subsequent analyses.

Figure 2.1 shows the final measurement model and the relations between factors as well as the relations between indicators and their residuals in relation to each factor. As can be seen, the correlations between factors are extremely strong ( $> .84$ ), with a range of correlation values between .84 (spatial with numerical) and .94 (spatial and mathematics achievement). Despite high correlations between factors, multicollinearity analyses at the latent variable level revealed acceptable tolerance and VIF statistics (Spatial factor; tolerance = .402, VIF = 2.488; Numerical factor; tolerance = .370, VIF = 2.699; EF factor; tolerance = .423, VIF = 2.364). Note that mathematics was entered as the dependent variable in the model that was used to derive these statistics. Concerns of multicollinearity occur when the VIF statistic exceeds 10 and the tolerance statistic is below .10 (e.g., see O'Brien, 2007). Accordingly, multicollinearity of factors does not appear to present a problem in the present study and further structural analyses were planned to potentially reveal the unique relations between factors.

The indicator variables also appear to adequately reflect the factors of interest as can be seen by the relatively low residuals. All indicators explain at least 50% of the variance with their associated factor. This is a desirable outcome and provides further evidence that the factors are adequately represented by their hypothesized indicator variables (Kline, 2015).

In sum, the four-factor model provides a good fit of the data, despite extremely high correlations between each factor. Thus, there is evidence to suggest that numerical skills, spatial ability, EF, and mathematics achievement are highly correlated but separable constructs.

Table 2.2

*Descriptive statistics for all measures and reliability estimates.*

	N	Mean Score (SD)	Range (min - max)	Kurtosis	Skewness	Reliability
<b>Demographic Measures</b>						
Age (years)	316	6.68 (1.40)	4.08 – 10.92	-.398	.331	N/A
Gender	316	52% female	0 – 1	N/A	N/A	N/A
<b>Numerical Measures</b>						
Symbolic Number Comparison	304	21.91 (13.31)	-6 – 64	-.398	.143	.84
Nonsymbolic Number Comparison	305	15.52 (7.61)	-5 – 35	-.272	-.203	.71
Ordering	285	7.24 (5.44)	-10 – 21	-.096	-.118	.72
<b>Spatial Measures</b>						
Visual-Spatial Reasoning	313	9.75 (3.66)	2 – 19	-.642	.294	.70
2D Mental Rotation	309	8.83 (3.35)	1 – 16	-.762	-.175	.66
Raven's Matrices	309	20.31 (6.05)	4 – 33	-.523	-.084	.68
<b>EF Measures</b>						
Head-Toes-Knees-Shoulders	292	27.52 (10.99)	0 – 40	.423	-1.21	.73
VSWM - Forward Path Span	286	4.06 (2.25)	0 – 12	-.005	.244	.66
VSWM - Reverse Path Span	285	3.03 (2.45)	0 – 9	-.692	.581	.60
<b>Mathematics Measures</b>						
Numeration	314	9.63 (5.08)	0 – 22	-.736	.436	.91
Geometry	311	10.75 (4.78)	0 – 24	-.089	.205	.60

Note that reliability estimates represent test-retest coefficients and were based on a subsample of participants ( $N=106$ ) who were administered the measures at two time points (~4 months apart).

Table 2.3

*Zero-order correlations between variables.*

Measures	1	2	3	4	5	6	7	8	9	10	11	12	13
<b>Demographic Measures</b>													
1. Age	-												
2. Gender	.05	-											
<b>Numerical Measures</b>													
3. Symbolic Number Comparison	.76	.02	-										
4. Nonsymbolic Number Comparison	.72	-.03	.83	-									
5. Ordering	.51	.03	.66	.64	-								
<b>Spatial Measures</b>													
6. Visual-Spatial Reasoning	.59	.02	.58	.56	.47	-							
7. 2D Mental Rotation	.64	-.03	.63	.61	.48	.64	-						
8. Raven's Matrices	.67	.00	.66	.65	.51	.65	.67	-					
<b>EF Measures</b>													
9. Head-Toes-Knees-Shoulders	.49	-.07	.61	.61	.47	.48	.51	.52	-				
10. VSWM – Forward Path Span	.56	-.03	.57	.56	.46	.42	.48	.52	.50	-			
11. VSWM – Reverse Path Span	.56	.07	.61	.56	.52	.54	.54	.59	.51	.57	-		
<b>Mathematics Measures</b>													
12. Numeration	.70	.06	.76	.72	.69	.69	.67	.72	.58	.57	.63	-	
13. Geometry	.56	-.01	.62	.55	.54	.68	.61	.65	.54	.46	.55	.75	-

Note: With the exception of the correlations between gender and the other variables ( $ps > .05$ ), all other correlations were statistically significant at  $p < .001$ . Sex was dummy coded: 0 = females and 1 = males.

Table 2.4

*Descriptive statistics showing the number of children and mean scores for each grade level.*

	Grade	VisSpat (max 20)	MR2D (max 16)	Raven's (max 36)	SymNum (max 72)	NonSym (max 72)	Ordering (max 48)	HTKS (max 40)	PathFor (max 18)	PathRev (max 18)	Numeration (max 49)	Geometry (max 36)
<u>N</u>	JK	32	32	32	32	32	32	32	31	31	32	32
	SK	71	69	70	67	66	54	59	65	65	72	72
	1	77	76	76	75	76	73	73	72	72	76	76
	2	74	73	74	73	74	69	72	67	66	74	73
	3	46	46	45	45	45	45	44	39	39	47	45
	4	13	13	12	12	12	12	12	12	12	13	13
<u>Mean</u>	JK	6.06	4.75	13	4.84	5.16	2.81	17.9	1.71	0.71	4.41	7.09
	SK	7.48	6.8	16.1	11.6	10	4.39	21.6	2.71	1.57	5.75	7.38
	1	9.55	8.82	20.6	22	16.2	6.88	28.5	4.35	3.32	9.63	10.9
	2	11.4	10.2	22.6	26.1	18.1	8.3	30	4.81	3.55	10.9	12.3
	3	12.7	11.4	26.1	37	23	11.3	35.1	6.08	5.05	15.5	15.1
	4	12.7	12.7	26.8	41.8	25.5	12.8	34.1	5.08	5.67	15.4	13.9
<u>SD</u>	JK	1.58	1.68	3.66	5.07	4.11	3.2	12.3	1.27	0.864	1.7	3.32
	SK	2.52	2.42	4.37	8.88	5.56	4.2	12.5	1.8	1.51	2.79	3.72
	1	3.16	2.84	4.4	7.97	5.06	4.41	9.34	1.91	2.26	3.66	3.64
	2	3.36	2.84	5.02	9.02	5.48	5.52	9.02	1.87	2.16	4.66	4.48
	3	3.12	2.35	4.06	9.71	4.99	5.27	4.01	2.08	2.55	3.49	4.22
	4	3.12	2.59	4	4.59	4.1	4.13	2.43	1.44	2.23	3.73	3.33

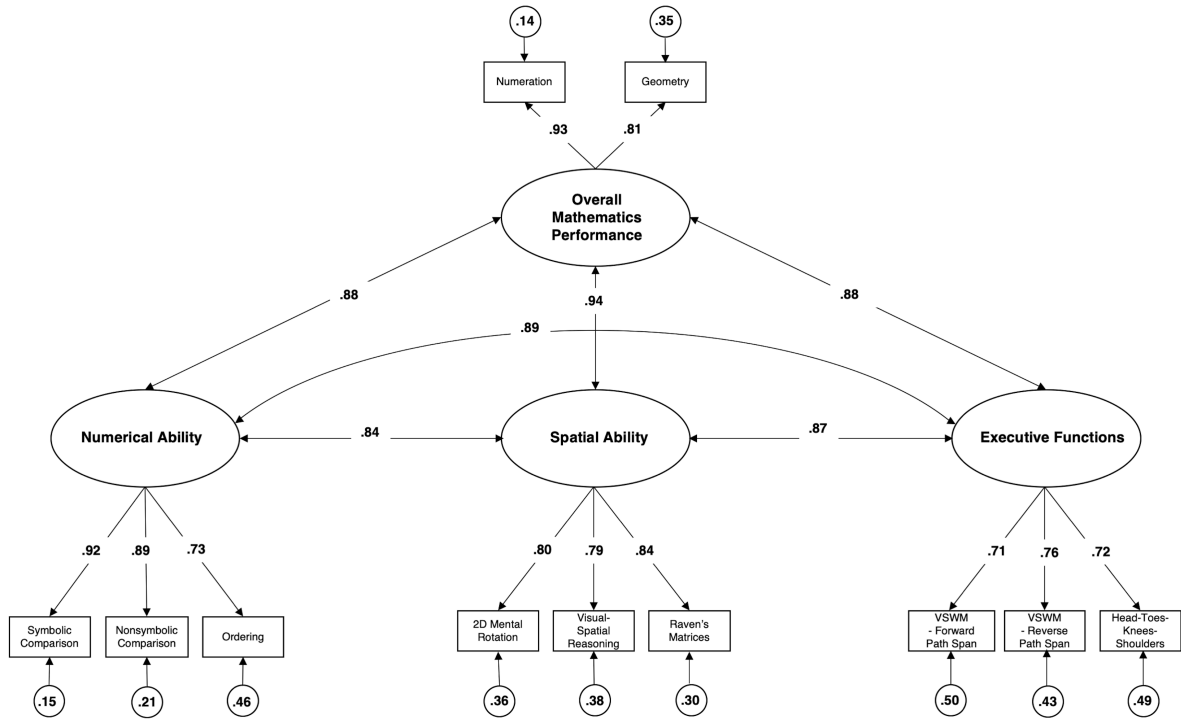
Note: JK = Junior Kindergarten (equivalent of pre-school in the U.S.); SK = Senior Kindergarten (equivalent of kindergarten in the U.S. and other countries). Max = maximum possible score on measure. SD = standard deviation. VisSpat = Visual-Spatial Reasoning; MR2D = 2D Mental Rotation; NumComp = Symbolic Number Comparison; DotComp = Non-Symbolic Number Comparison; Ordering = Ordering Task; HTKS = Head-Toes-Knees-Shoulders Task; PathFor = Path span forward; PathRev = Path span reverse.

Table 2.5

*CFA (measurement model) goodness-of-fit statistics for original hypothesized model and four modified alternative models.*

Model	$\chi^2$ (df)	RMSEA (90% CIs)	CFI	SRMR	AIC
1. Hypothesized four-factor model	77.31(38) $p < .001$	.057 (.039-.075)	0.983	0.024	17959.95
2. Loading VSWM (path span reverse) with Spatial Factor	89.19(38) $p < .001$	.065 (.048-.083)	0.978	0.029	17971.83
3. Loading both VSWM measures with Spatial Factor and HTKS as single indicator	98.56(39) $p < .001$	.070 (.053 -.087)	0.975	0.031	17979.19
4. Three-factor model where spatial and EF measures load on same factor	109.87(41) $p < .001$	.073 (.057-.090)	0.971	0.033	17986.51
5. Single, g-factor, model	182.34(43) $p < .001$	.101 (.086-.117)	0.941	0.038	18054.97

Note: VSWM = visual spatial working memory; HTKS = head-toes-knees-shoulders.



*Figure 2.1* The four-factor measurement model and the one retained for further structural analyses. Double-headed arrows represent correlations between factors and single-head (unidirectional) arrows indicate factor loadings (interpreted as regression coefficients). The smaller circles with arrows leading to each indicator variable represent unexplained variance or residual/error terms (interpreted as proportions of unexplained variance). Note that correlations can be squared to determine the proportion of shared variance between variables (e.g., the proportion of shared variance between spatial ability and mathematics is  $.94^2 = .88$ ).

## 2.5.2 Part II: Structural Models

### 2.5.2.1 Cognitive Predictors of Mathematics Achievement

Our first set of structural analyses tested the unique and shared relations between the numerical, spatial, and EF factors and their predictive relations with mathematics

achievement. Figure 2.2 shows the various relations with one another after controlling for age<sup>2</sup>. Overall, the model explained a large proportion, .84, of the variance in mathematics achievement even after controlling for age. Both the numerical and spatial factors were unique predictors of mathematics achievement,  $\beta = .289$ ,  $SE = .12$ ,  $p = .013$ , 95%  $CI$  [.06, .52], 99%  $CI$  [-.01, .59] and  $\beta = .673$ ,  $SE = .11$ ,  $p < .001$ , 95%  $CI$  [.46, .88], 99%  $CI$  [.40, .95], respectively. Accordingly, a 1-unit increase on the numerical factor was associated with a .29 standard deviation unit increase on the mathematics factor, controlling for the effects of age, spatial ability, and EF skills. A 1-unit increase on the spatial factor was associated with a .67 standard deviation unit increase on the mathematics factor, controlling for the effects of age, numerical, and EF skills. The relation between the spatial and mathematics factors remains robust even at 99% CIs, whereas the relation between the numerical and mathematics factors is no longer statistically significant at 99% CIs. There were no unique relations between the EF and mathematics factor once the numerical and spatial factors were taken into account,  $\beta = .056$ ,  $SE = .17$ ,  $p = .734$ , 95%  $CI$  [-.27, .38], 99%  $CI$  [-.37, .48].

A follow-up test was carried out to examine the possibility that the above results may have been driven by the inclusion of Raven's Matrices as an indicator of spatial ability. Given that matrix reasoning is typically considered a measure of nonverbal intelligence and not necessarily a measure of spatial ability proper, it was important to determine whether the above results remained once this measure and its contributions were removed. Moreover, this allowed us to further isolate and more narrowly examine the effects of spatial visualization on mathematics achievement. The results were highly consistent with those reported above. Both the numerical and spatial factors remained independent predictors of mathematics,  $\beta = .302$ ,  $SE = .12$ ,  $p = .014$ , 95%  $CI$  [.06, .54], 99%  $CI$  [-.01, .62] and  $\beta = .665$ ,  $SE = .12$ ,  $p < .001$ , 95%  $CI$  [.44, .89], 99%  $CI$  [.37, .96], respectively. There was no unique relation between EF and mathematics,  $\beta = .08$ ,  $SE = .17$ ,  $p = .630$ , 95%  $CI$  [-.25, .42], 99%  $CI$  [-.36, .53]. Notably, highly similar results were

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<sup>2</sup> Hereafter, all analyses were carried out with age as a covariate at the individual variable level (i.e., scores on each manifest variable was regressed on each child's age in months). Figure 2.2 represents the retained four-factor model; all subsequent analyses involve a deconstruction of this model in an attempt to better understand the potentially explanatory pathways that give rise to these observed relations.



obtained when Raven's Matrices was used as a control measure. Both numerical and spatial performance were strongly related to mathematics performance,  $\beta = .38$ ,  $SE = .13$ ,  $p = .004$ , 95%  $CI$  [.12, .64], 99%  $CI$  [.04, .72], and ,  $\beta = .60$ ,  $SE = .12$ ,  $p < .001$ , 95%  $CI$  [.37, .83], 99%  $CI$  [.30, .90]. Again, there was no unique relation between EF and mathematics,  $\beta = .10$ ,  $SE = .17$ ,  $p = .556$ , 95%  $CI$  [-.23, .42], 99%  $CI$  [-.33, .52]. These results demonstrate that the relation between spatial and mathematics performance remains strong when Raven's Matrices is excluded from the analyses altogether, but also when it is included as general covariate in the model. Therefore, the relation between spatial ability and mathematics appears to be related to spatial visualization skills. Taken together, the results indicate that in combination, numerical, spatial, and EF skills explain a large proportion of the variance in mathematics performance. More specifically, the results reveal significant unique relations between numerical and spatial skills with mathematics achievement and no unique relations between EF and mathematics. Spatial ability appears to be an especially strong contributor to mathematics achievement, over and above contributions from EF and numerical skills.

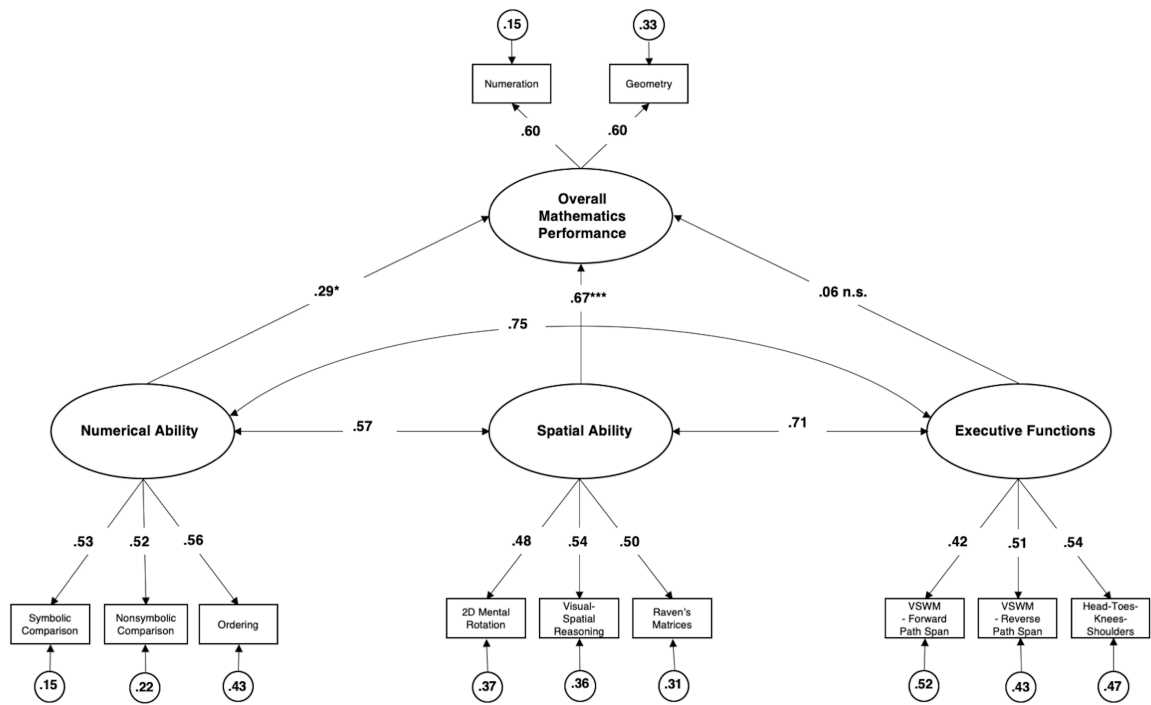
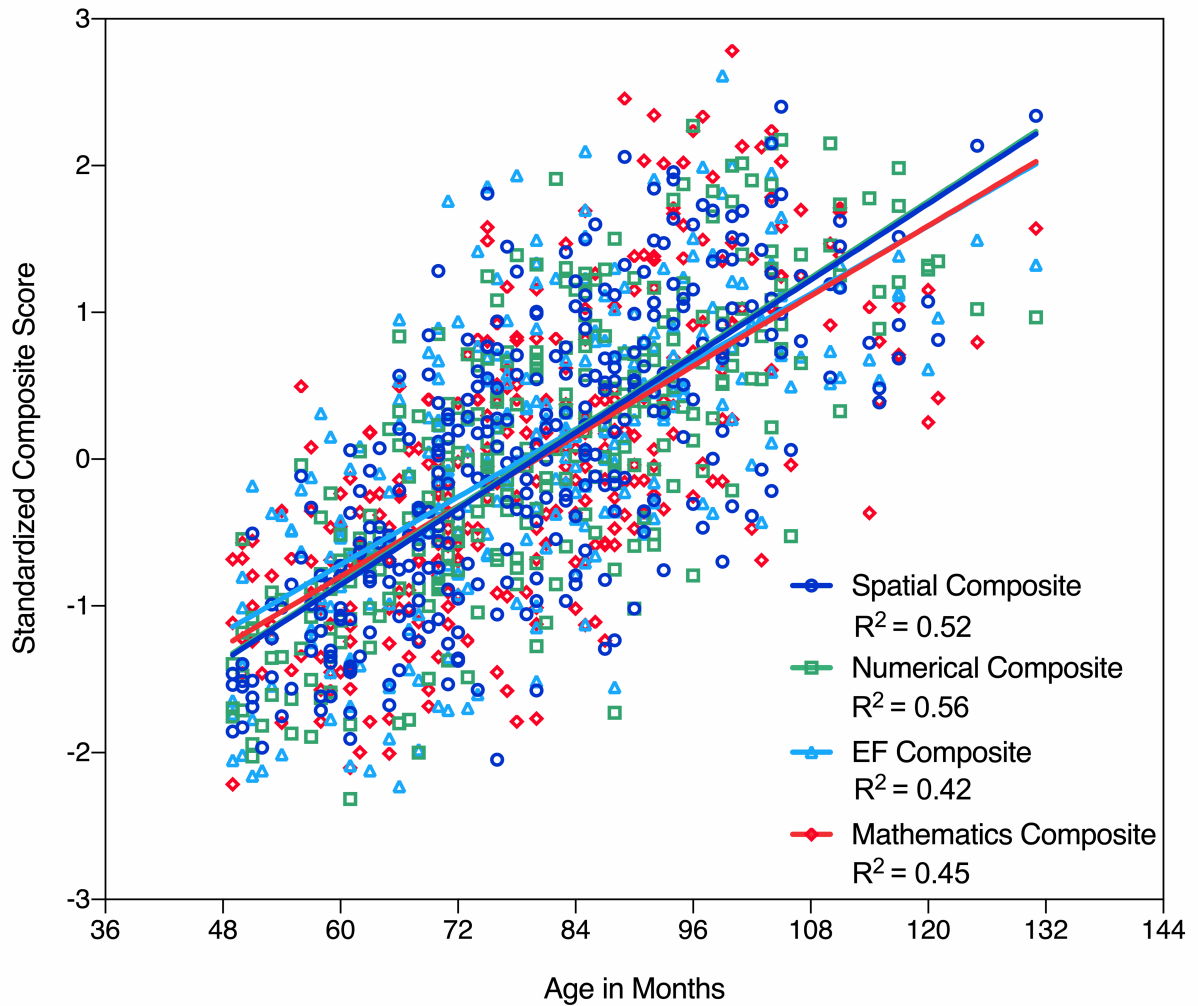


Figure 2.2 Relations between cognitive predictors and overall mathematics achievement controlling for age. All pathways are significant ( $p < .05$ ) except for the path between EF and mathematics. All values represent standardized estimates.

### 2.5.2.2 Stability of Performance Across Age

To examine the extent to which scores on the various factors were stable across age, composite ‘factor’ scores were computed for each individual. Importantly, these scores are weighted according to each indicator’s contributions (i.e., performance on each test) to the latent construct of interest. In this way, composite scores are not merely an average score on a given number of measures. To test whether factor scores vary as a function of development, a grade (6) by factor (4) repeated measures ANOVA was carried out. Results revealed that mean factor scores differed significantly between grades,  $F(13.849, 667.544) = 1.830, p = .032, \eta_p^2 = .037$ . Follow-up Bonferroni corrected comparisons revealed statistically significant differences between factor scores only amongst the youngest grade tested (i.e., junior kindergarten); scores differed significantly from one another on the spatial versus mathematics factor ( $p = .007$ ) and numerical versus

mathematics factor ( $p=.037$ ). There were no other significant differences in factor scores in kindergarten through 4<sup>th</sup> grade. A Bayesian repeated measures ANOVA was carried out to further examine the strength of evidence for the presence of grade by factor interactions. These results revealed a Bayes Factor of .003, indicating extremely weak support for the hypothesis of grade by factor interactions. Overall, despite a statistically significant grade by factor interaction, a closer look at the data reveals highly similar developmental trajectories of each construct across age. Note that Mix et al. (2016) also reported consistent relations between space and mathematics across grades K, 3, and 6. Figure 2.3 provides an illustration of the relation between age and children's individual factor scores. An analysis of potential differences in the slopes of each factor revealed statistically insignificant results,  $F(3, 1161) = .980, p = .402$ . There was also no statistically significant differences in the intercepts,  $F(3, 1164) = .294, p = .830$ . Overall, the results suggest fairly consistent relations between factors over developmental time.



*Figure 2.3* Scatterplot of individual participants' age and standardized composite score for each factor. Each column represents one year.

### 2.5.2.3 Mediation Analyses: Numerical and EF Skills as Mediators of the Space-Math Link

Next, mediation analyses were carried out based on theory to suggest that numerical and EF skills potentially mediate the shared relations between spatial and mathematical processing. That is, the analyses reported above were further decomposed to test whether EF and numerical skills independently mediate the relation between spatial thinking and mathematics achievement. For example, to test the mediating role of numerical skills in

the space-math relationship, we removed the EF factor from the model presented in Figure 2.2; conversely, to test the mediating role of EF skills we removed the numerical factor from the model. Thus, mediation models were achieved by removing irrelevant pathways from the four-factor model and retaining only the pathways of specified interest. Note that the rationale and interpretation of each mediation analysis differed somewhat according to construct and question of interest (numerical vs EF). Numerical skills were targeted as a potential mediator for reasons that are best described by the *causal steps approach* to mediation (Judd & Kenny, 1981; Baron & Kenny, 1986), while EF skills were entered into the model for reasons that align with the *confounding variables approach* (e.g., see MacKinnon, Fairchild, & Fritz, 2007). While the causal steps approach involves testing a causal chain of events and typically assumes temporal precedence (e.g., spatial skills → numerical skills → mathematics achievement), the confounding variables approach – although a mathematically equivalent model – is used to test the influence of a potentially confounding or third variable in a given bi-variate relationship (e.g., testing the extent to which EF skills might explain the space-math link; MacKinnon, Fairchild, & Fritz, 2007; also see Fiedler, Harris, Schott, 2018).

As shown in Figure 2.4, numerical skills were found to partially mediate the relation between spatial skills and mathematics achievement,  $\beta = .182$ ,  $SE = .04$ ,  $p < .001$ , 95%  $CI [.10, .27]$ , 99%  $CI [-.07, .29]$ . The direct effect between spatial skills and mathematics remained robust,  $\beta = .697$ ,  $SE = .08$ ,  $p < .001$ , 95%  $CI [.55, .85]$ , 99%  $CI [.50, .89]$ . Figure 2.5 shows the results of EF as a mediator between spatial ability and mathematics. As can be seen, EF failed to mediate these relations,  $\beta = .159$ ,  $SE = .09$ ,  $p = .064$ , 95%  $CI [-.01, .33]$ , 99%  $CI [-.06, .38]$ . The direct effect between spatial skills and mathematics remained robust,  $\beta = .740$ ,  $SE = .11$ ,  $p < .001$ , 95%  $CI [.52, .96]$ , 99%  $CI [.45, 1.03]$ <sup>3</sup>. In sum, numerical skills, but not EF, were found to partially mediate the relation between spatial ability and mathematics.

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<sup>3</sup> As a follow-up, we also conducted an analysis in which we partialled out the effects of EF skills from *both* spatial and mathematics skills. (The mediation model is more akin to a partial correlation in which the influence of the mediator is partialled out from the outcome variable only). These results further confirmed strong relations between spatial and mathematics performance even after the influence of EF on both variables was taken into account. More specifically, spatial skills explained 81% of the variance in

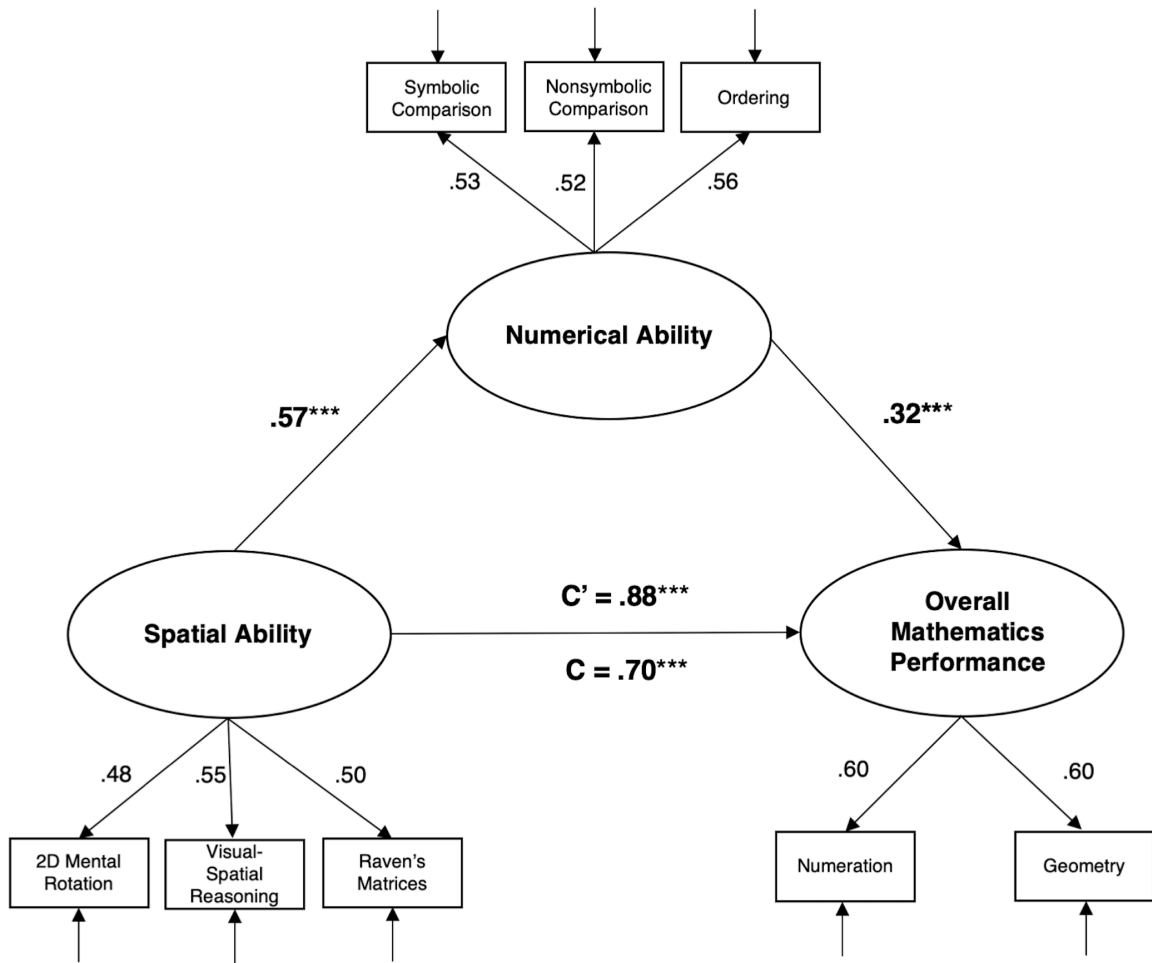


Figure 2.4 Mediation model of numerical ability in the relation between spatial ability and overall mathematics performance. \*\*\*  $p < .001$ . Values represent standardized coefficients.

mathematics before taking EF into account and 62% of variance after EF was taken into account. Therefore, EF skills explained approximately 23% (i.e., 19/81) of the variance in the space-math link. In short, regardless of analytical approach, the space-math link does not appear to be explained by children's EF skills.

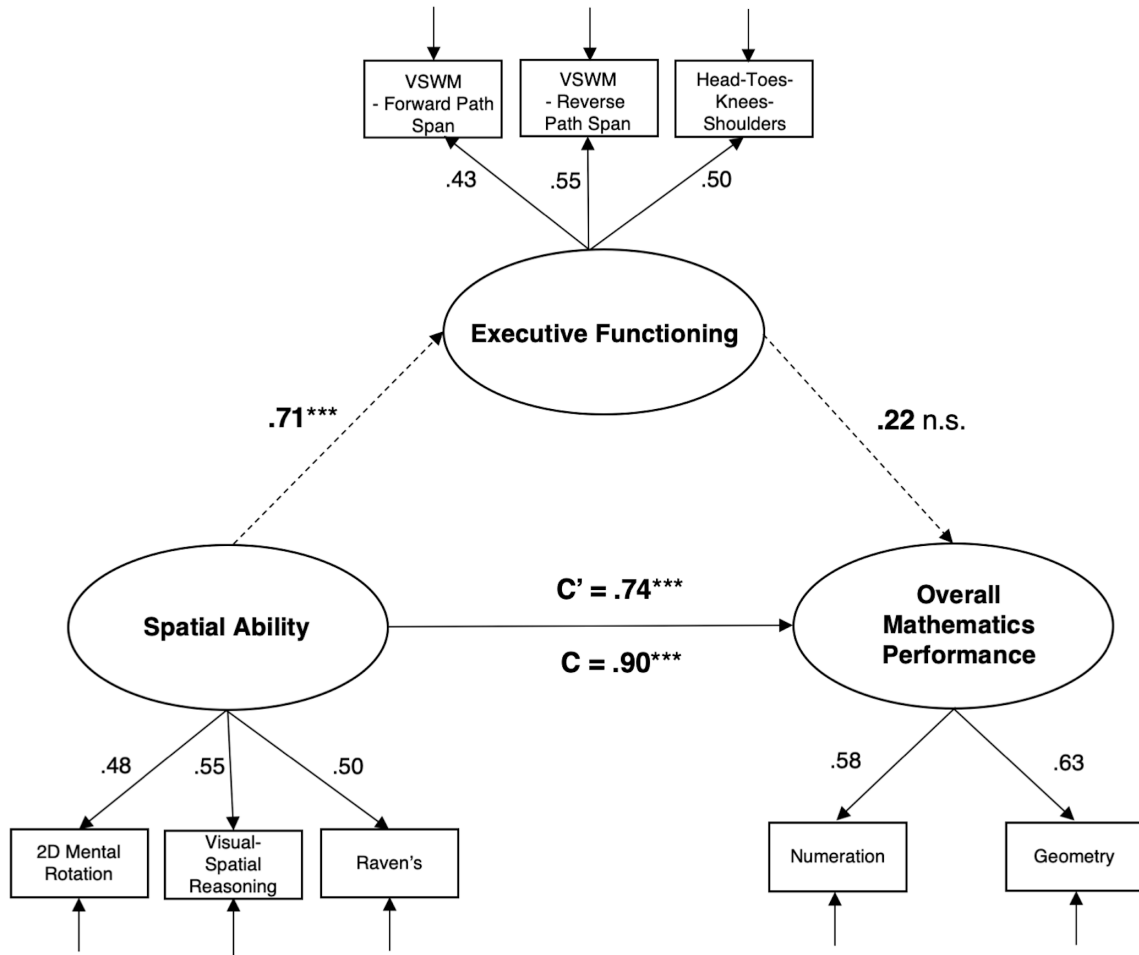


Figure 2.5 Mediation model of EF in the relation between spatial ability and overall mathematics performance. \*\*\*  $p < .001$ . n.s. = non-significant. Values represent standardized coefficients.

### 2.5.2.4 Predictive Relations with Different Components of Mathematics

The final set of analyses examined the relations between the cognitive predictors and each mathematics outcome measure. That is, numeration and geometry were entered as single indicator outcome variables and two separate structural models were run. These analyses were carried out to determine how the various relations previously observed potentially vary as a function of the mathematics activity in question. Figure 2.6 shows a summary of

the results when numeration was used as a single outcome variable. Similar to the results of the full model, scores on the numerical and spatial factors were both uniquely related to numeration performance,  $\beta = .252$ ,  $SE = .07$ ,  $p = .001$ , 95%  $CI [.11, .40]$ , 99%  $CI [.06, .44]$  and,  $\beta = .342$ ,  $SE = .07$ ,  $p < .001$ , 95%  $CI [.21, .48]$ , 99%  $CI [.16, .52]$  respectively. Scores on the EF factor were not statistically predictive of performance on the numeration test,  $\beta = .024$ ,  $SE = .11$ ,  $p = .821$ , 95%  $CI [-.18, .23]$ , 99%  $CI [-.25, .30]$ .

Figure 2.7 shows the results when geometry was used as a single outcome variable. Scores on the spatial factor were strongly related to performance in geometry,  $\beta = .532$ ,  $SE = .09$ ,  $p < .001$ , 95%  $CI [.36, .71]$ , 99%  $CI [.30, .77]$ . Scores on the numeration and EF factor did not predict performance on the geometry test,  $\beta = -.003$ ,  $SE = .10$ ,  $p = .979$ , 95%  $CI [-.19, .19]$ , 99%  $CI [-.25, .25]$  and,  $\beta = .064$ ,  $SE = .14$ ,  $p = .648$ , 95%  $CI [-.21, .34]$ , 99%  $CI [-.29, .43]$ .

Taken together, both spatial and numerical skills predicted performance on the numeration test, but only spatial skills predicted performance on the geometry test. Executive functioning skills did not explain any unique variance on either measure.

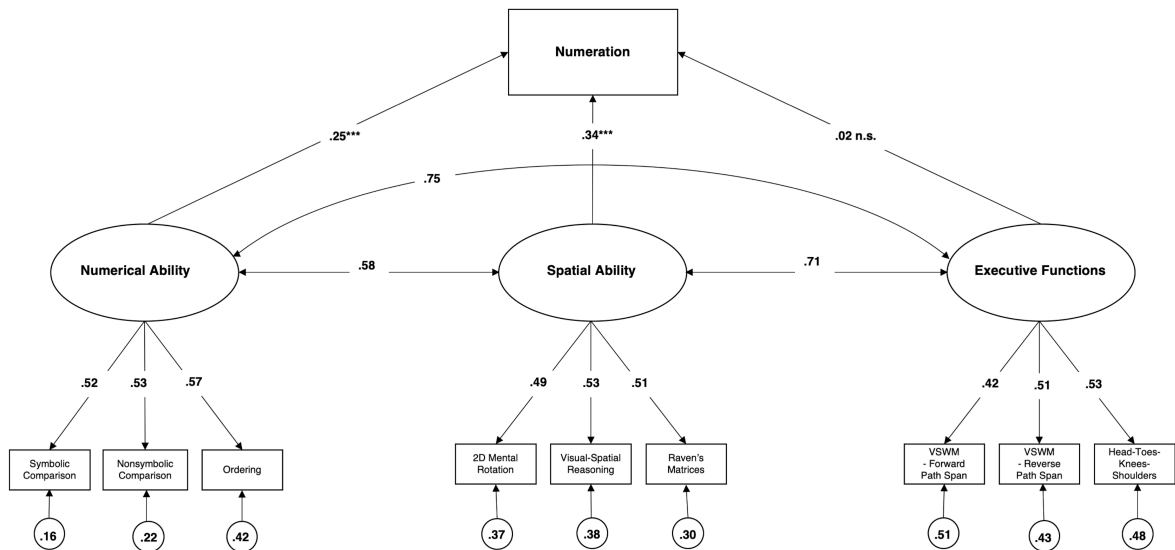


Figure 2.6 Cognitive predictors of numeration as a single test outcome measure. Values represent standardized coefficients. \*\*\*  $p \leq .001$



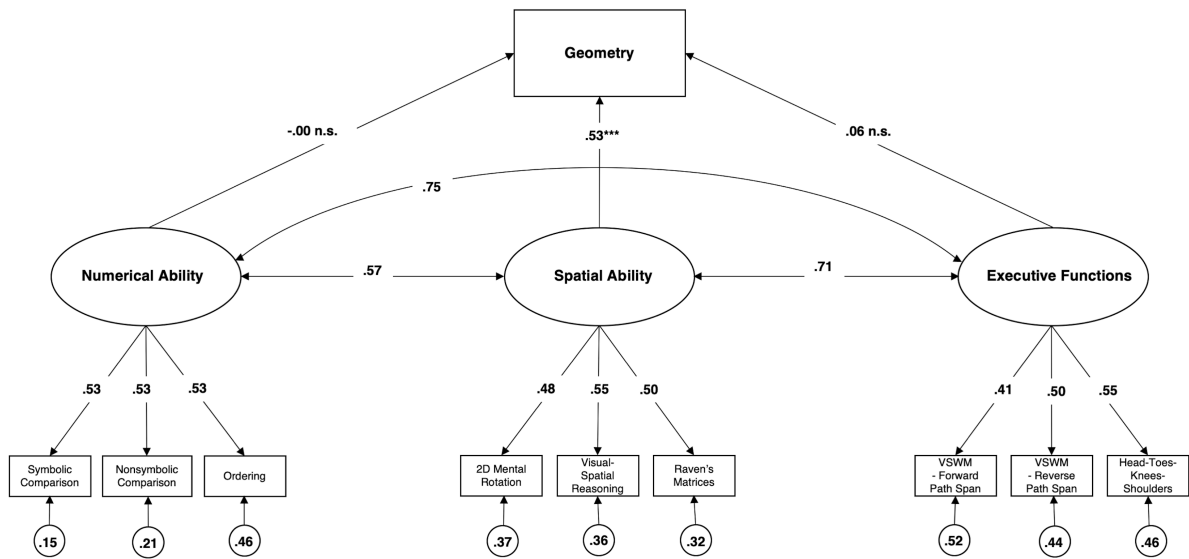


Figure 2.7 Cognitive predictors of geometry as a single test outcome measure. Values represent standardized coefficients. \*\*\*  $p < .001$

## 2.6 Discussion

The current study examined the cognitive foundations of early mathematics achievement in a sample of 4- to 11-year-olds. Analyses were first carried out to test the psychometric properties associated with a hypothesized four-factor model, with cognitive constructs related to numerical, spatial, and executive function skills and mathematics achievement. The four-factor model revealed robust correlations between each factor while also demonstrating good fit statistics; a finding that suggests that numerical, spatial, EF, and mathematics abilities are highly related but separable constructs. Importantly, the original four-factor model achieved better fit statistics than several alternative models, including a model in which a general ( $g$ ) factor was used to link each individual predictor variable with mathematics achievement.

Given evidence of a four-factor model, our primary analyses aimed to more closely reveal the structure and underlying relations between numerical, spatial, EF, and mathematics skills. To this regard, we had several goals: (i) to examine the shared and unique contributions of children's numerical, spatial, and EF skills to mathematics

achievement, (ii) to determine the relative stability of these relations across childhood, (iii) to test the potentially mediating roles of numerical and EF skills in the oft reported space-math link, and, lastly, (iv) to examine the extent to which relations between the predictor variables and mathematics vary as a function of the mathematics task in question (i.e., numeration vs. geometry).

Results revealed that children's numerical, spatial, and EF skills collectively explained 84% of the variance in mathematics achievement, even after controlling for the effects of age. These results provide evidence of a fairly comprehensive model of children's mathematics achievement. However, only the numerical and spatial factors were uniquely predictive of mathematics achievement. The observed relations between factors remained stable across age and grade, appearing to undergo highly parallel growth trajectories. Follow-up mediation analyses revealed that numerical skills, but not EF skills, partially mediated the relation between spatial skills and mathematics achievement. Our last set of analyses examined how the predictive utility of the model potentially varies as a function of the mathematics task being assessed, that is, numeration vs. geometry. Scores on the numerical and spatial factors were uniquely related to numeration performance, while only spatial ability was a unique predictor of geometry performance.

In the following sections, we provide a more detailed review of the main findings just described. We begin by discussing the results of the CFA analysis and then review and offer interpretations of the findings related to the structural models employed. We focus much of our attention on the space-math link and more carefully consider the role of spatial visualization in children's mathematics performance.

### 2.6.1 Evidence of a Four-Factor Model

We found evidence to suggest that numerical, spatial, EF, and more general mathematics skills are highly related but separable constructs. The correlations amongst these factors were strikingly high and similar in strength ( $r_s$  .84 – .94) and indicate higher relations at the latent variable level than what would be predicted by examining the relations amongst the single indicator variables alone. This finding in itself demonstrates the potential utility of forming and testing the relations between latent variables, as they offer a more

comprehensive model of the targeted constructs; one not defined by a single measure – but rather a combination of measures – and less influenced by measurement error.

Subsequent analyses revealed that the four-factor model achieved better fit statistics than a single-factor (*g*) model. While the four-factor model demonstrated good fit, the single-factor model straddled the boundary of what is considered acceptable fit statistics. Overall, our results suggest the need to be cautious in interpreting each factor as fully independent constructs. Instead, numerical, spatial, EF, and general mathematics achievement appear to strongly overlap with one another and yet are distinct enough to represent separable constructs. This result adds further support to the results of Mix et al. (2016), who found evidence of highly related but separable factors associated with spatial and mathematical domains in a large sample (N=854) of 5-to 13-year-olds. Moreover, these authors found evidence of strong cross-domain loadings for certain spatial and mathematical tasks, suggesting that a common cognitive network might underlie certain spatial and mathematical tasks. Notably, Mix et al. (2016) also presented evidence showing that spatial and mathematical tasks loaded on to a single factor in an orthogonal EFA model. Thus, our findings, like those reported by Mix et al. (2016), suggest a tight coupling of spatial and mathematical thinking. The current findings suggest that numerical skills and EFs might also be implicated in this same cognitive network.

Several follow-up analyses were carried out to further confirm evidence of a four-factor model as well as to test specific theoretical distinctions in measurement. Of primary interest was whether measures of VSWM would more strongly load on the EF or spatial factor. Our results indicated better model fit when the VSWM measures were made to load on the EF factor. Moreover, the four-factor model fit the data better than a three-factor model in which the spatial and EF measures were made to load on the same construct. This suggest that spatial and EF skills – as measured in the current study – represent distinct constructs.

Taken together, our results suggest that VSWM is better defined as a measure of EF than spatial ability. This finding has important implications as VSWM and spatial visualization skills appear to represent different constructs and, as further discussed below, share different relations with measures of mathematics achievement. Follow-up research is needed to further test the extent to which differences in constructs are

potentially due to the amount that the respective tasks emphasize the need to ‘recall’ visual-spatial information as opposed to self-generate and manipulate visual-spatial information. Do these differences in recall- versus generative-based tasks represent shared or distinct underlying cognitive mechanisms? Moreover, assuming VSWM and spatial visualization do represent distinct mechanisms, and our data suggest that they might, how do individual differences in these areas relate to mathematics achievement? Interestingly, while our data point to spatial visualization (i.e., generative spatial reasoning) as a more important contributor to mathematics achievement, it is possible that VSWM (i.e., recall-based spatial reasoning) may play a more important role in mathematics tasks that emphasize fluency, such as the retrieval of arithmetic facts. Answering questions such as these will contribute to a more nuanced understanding of when and how spatial and mathematical thinking interact.

In summary, the results of the CFA provided support for the hypothesized four-factor model in which performance on numerical, spatial, EF, and mathematics tasks emerged as separate but highly related factors. Although we retained this model for all subsequent pathway analyses, some caution is warranted as our results also suggest strong cross-loadings between factors. Future research is needed to replicate the current findings and to further test the extent to which each factor is indeed independent from the other. Furthermore, research is needed that seeks to better explain the underlying mechanisms that give rise to similarities and differences across constructs.

## 2.6.2 Predictors of Mathematics Achievement

Our results indicated that only the numerical and spatial factors explained unique variance in children’s mathematics achievement. Children’s scores on the EF factor failed to explain performance in mathematics once the other two factors and age were taken into account. Spatial ability was an especially strong predictor of children’s mathematics achievement.

Given that the majority of the mathematics test items required the understanding and/or manipulation of symbolic mathematics (e.g.,  $34 = 30 + \_$ ), it is somewhat surprising that the spatial factor, and not the numerical factor, was the best predictor of mathematics achievement. Critically, the relations between spatial ability and

mathematics could not be explained by the inclusion of matrix reasoning as an indicator of spatial ability. The same pattern of results was obtained when matrix reasoning was eliminated from, as well as controlled for, in the analyses. These findings provide support for specific relations between spatial visualization and mathematics.

One explanation for this finding, and one not unique to our original hypothesis, has to do with the role of spatial visualization in mathematical problem solving. Indeed, it has been hypothesized that spatial visualization plays a critical role in how one mentally organizes, models, and ultimately makes sense of novel mathematical problems (Ackerman, 1988; Mix et al., 2016; Uttal & Cohen, 2012). Accordingly, the ‘spatial modelling hypothesis,’ as we have come to refer to it, predicts especially strong relations between spatial visualization and performance on novel mathematical tasks compared to highly familiar tasks. For example, spatial visualization would be expected to play a more important role when one is first learning arithmetic compared to when one has mastered their arithmetic facts. Interestingly, recent findings of Mix et al., (2016) provide support for the spatial modelling hypothesis, in which it was found that spatial skills were more related to novel mathematics problems than familiar ones. In the current study, the mathematics tests predominantly featured applied problems, lending further support for the role of spatial visualization in solving novel problems. This hypothesis dovetails nicely with the metaphor of spatial visualization as a cognitive tool used to construct spatial-numerical/mathematical relations.

Interestingly, our findings also provide evidence of relations between basic numerical skills and spatial visualization (for similar findings see Thompson, Nuerk, Moeller, & Cohen Kadosh, 2013; Viarouge, Hubbard, & McCandliss, 2014). More specifically, our results indicate considerable overlap at the latent variable level ( $r=.84$ ) as well as evidence that basic numerical skills partially mediate relations between spatial visualization and overall mathematics achievement. These results suggest that spatial visualization might also be involved in processing familiar and well-learned mathematical content, such as making rapid judgments about numerical symbols. Thus, our results implicate spatial visualization in numerical tasks that are solved both quickly and with seemingly little effort as well as tasks that require deliberate and effortful reasoning.

Although these results appear to run counter to the spatial modeling hypothesis (i.e., spatial visualization plays a greater role in novel mathematics), one possibility is that the association between basic number skills and spatial visualization is an artefact of numerical-spatial relations formed earlier in development. Spatial visualization may play an important role in early number learning as children actively construct spatial-numerical associations. Eventually, over development, these early conceptual groundings become increasingly more automatic and give rise to procedural fluency. Findings from our mediational analysis offer preliminary – albeit far from causal – support for this possibility and suggest that spatial visualization skills may facilitate numerical development. However, the relation between spatial visualization and mathematics achievement appears to be much stronger than the one shared between spatial visualization and basic numerical skills. Thus, the relation between spatial visualization and numerical skills cannot explain the robust relationship between spatial visualization and mathematics achievement more broadly and lends support to the spatial modelling hypothesis. This suggest that although numbers and spatial processes are linked at a relatively basic level, the association is even stronger at higher levels of numerical and mathematical processing.

Taken together, our findings suggest that spatial visualization skills play an important role in both basic numerical skills as well as more advanced numerical and mathematical reasoning. However, there appears to be an asymmetry in these relations, as spatial visualization was found to be more strongly related to novel or much less practiced mathematical tasks compared to tasks assessing numerical fluency. Future research efforts are needed to further disentangle when, why, and how spatial visualization is implicated in both basic and advanced mathematical reasoning.

### 2.6.3 Effects of Age and Grade on Observed Relations

Our findings suggest that the relations between numerical, spatial, and EF skills, and mathematical achievement develop in parallel and maintain relatively stable relations during early childhood (4-to-10 years of age). On the one hand, these findings are to be expected based on prior research showing strong and consistent relations between these

variables in isolated studies of both children and adults (e.g., see Miyake et al., 2001; Mix & Cheng, 2012). On the other hand, these findings run counter to the idea that certain cognitive skills, such as spatial visualization, share stronger relations during initial learning of academic content, such as symbolic number, as compared to when the content has become more procedural and automatic (e.g., see Holmes & Adams, 2006; Huttenlocher, Jordan, & Levine, 1994; Mix et al., 2016; Rasmussen & Bisanz, 2005). For example, prior research has demonstrated that the learning of new mathematical content relies more on VSWM and less on verbal working memory (Rasmussen & Bisanz, 2005). However, with learning and development, the role of verbal working memory becomes increasingly more important for representing the learned material and the role of VSWM appears to become less important (Huttenlocher, Jordan, & Levine, 1994; Rasmussen & Bisanz, 2005). Indeed, this ‘spatial’ to ‘verbal’ shift is thought to correspond to changes in how the content is conceptualized; that is, as information that is initially grounded and understood in terms of concrete, spatial, and embodied experiences, but over time and experience, becomes increasingly more abstract and verbal in its representation (Bruner, 1966; Lakoff & Núñez, 2000). Notably, this shift also corresponds to a decrease in the need to exhibit effortful top-down executive control, suggesting that the role of EFs is dampened with mastery of content in a given area. Paradoxically, when one is confronted with the learning of new mathematics material, the role of EFs, most notably inhibitory control, is needed to inhibit prior learning experiences (e.g., overcoming the ‘whole number bias’ when introduced to fractions;  $2/3 > 4/7$ ; Gómez, Jiménez, Bobadilla, Reyes, & Dartnell, 2015).

Taken together, the research above helps to shed light on why we may have observed consistent relations between spatial ability, EF skills, and mathematics achievement across such a wide age range of children. So long as the mathematics tasks are adaptive and requires the use of spatial skills and EFs to make sense of new or rarely encountered problems, relatively stable correlations between constructs are predicted to emerge. According to this view, successful performance on novel or difficult mathematical problems requires both independent and integrated contributions from both spatial and EF skills. Interestingly, our data offer only partial support for this hypothesis as only spatial skills were found to uniquely predict mathematics performance. Future

research efforts are needed to further investigate this hypothesis using a different suite of EF measures. It is possible that the EF measures enlisted in the current study were too closely related to the spatial ability measures and any remaining variance was simply not enough to detect individual contributions of EF skills to mathematics achievement.

Based on the hypothesis stated above, we should expect to see tighter relations between spatial, EF, and numerical skills earlier in development and a gradual divergence of relations between spatial and EF skills and their relations with basic numerical skills over development. As symbolic number skills become more automatic the roles of higher cognitive skills should be minimized. However, this does not mean that the correlations between these various skills should necessarily become minimized. On the contrary, and to use the relation between EFs and mathematics as an example, if a child enters the learning of new mathematics material with strong EF skills, he/she should be able to harness these skills to better learn the new task(s). There is little reason to suspect that the relations would weaken over time, despite fundamental changes in the recruitment and reliance on EFs as the learner progresses from novice to ‘expert.’ This same explanation might underlie the relative stable relations in the current study between spatial visualization and basic numerical skills as well as more sophisticated mathematics tasks.

Longitudinal research is needed to further test the stability of the factors at the individual level. This approach will provide further insight into the relative stability and change that occurs in performance over development. For example, is it the case that children who start low on any given factor are likely to remain low throughout development? Moreover, how are improvements on any one factor associated with improvements across the other factors, perhaps most notably, mathematics achievement? In short, longitudinal research provides a means to better understand directional relations between the various factors. This information, in turn, has implications for educational design and intervention.

## 2.6.4 Mediating Roles of EFs and Numerical Skills

Our results indicated that basic numerical skills, but not EFs, partially mediated the relation between spatial visualization and mathematics achievement. As noted above, the



finding that numerical skills mediated the relation is in line with prior theoretical and empirical support that spatial skills facilitate numerical development (e.g., see Gunderson et al., 2012; Tam, Wong, & Chan, 2018).

Our failure to reveal a mediating role of EF skills in the space-math link is a novel and surprising finding. Recall that the decision to test EF as a mediator in the relation between space and maths was based on the proposal that EF skills may be driving the space-math link due to the shared recruitment and reliance on top-down effortful control mechanisms. However, our findings indicate that although the two constructs were highly related, they were found to differentially relate to mathematics achievement. Spatial visualization appears to share a more direct link to our measures of mathematics than children's EF skills. This finding supports the longitudinal findings of Verdine et al. (2014), who found that children's spatial skills at the age of three uniquely predicted children's mathematics performance one year later, explaining an additional 27% of the variance over and above children's EF skills.

However, it is also worth considering an alternative explanation for why EF skills were not uniquely related to mathematics achievement and similarly failed to mediate the space-math link. Rather than assuming that spatial visualization and EFs represent distinct constructs, as we have done, it is possible that the spatial measures enlisted may in fact better represent indicators of EF than the measures enlisted to represent EF. Our attempt to separate spatial ability from EFs based on distinctions, in part, between the need to 'generate' versus 'recall' information may have resulted in a misrepresentation of EF. For example, it could be argued that the best measures of EF used in the current study were those used to measure spatial visualization, as these measures required a greater degree of manipulation of information in the service of a task. Future studies are needed to further investigate this possibility. It is possible that EF tasks that require greater amounts of planning and manipulation, as opposed to more recall-based tasks, would potentially result in stronger relations with both spatial visualization tasks but also mathematics achievement. In order to further test our claims made about the 'spatial modeling hypothesis' this is a critical next step: Is it the ability to generate and model *visual-spatial* solutions to problems that is most important to mathematical problem solving? Or is it a more general ability to generate solutions to problems, including verbally mediated

processes, that matters most? As it stands, our data suggest that mathematics performance is best explained by an underlying construct related to the ability to generate and reason about visual-spatial images compared to a construct related to visual-spatial recall and inhibitory control.

## 2.6.5 Predicting Numeration vs. Geometry Performance

As just alluded to, mathematics is not a unitary construct but rather a varied and complex one. For this reason, it has been suggested that any attempt to predict mathematical behaviour should first consider the task requirements of the particular mathematics in question (see Mix & Cheng, 2012). In the present study, we used separate tests of numeration and geometry as examples of outcome measures that were expected to call upon different cognitive resources. More specifically, we predicted that numeration would best be predicted by basic number skills and geometry would best be predicted by spatial ability. These predictions were only partially supported. Although basic numerical skills did predict performance on the numeration test, spatial ability was found to be an even stronger predictor. Only spatial ability was a unique predictor of geometry.

Why might spatial ability better explain performance in both these areas of mathematics? One possibility is that basic numerical skills are necessary but not sufficient in order to perform well on both tests of mathematics. To do well requires not only a familiarity and fluency with numbers, but perhaps more importantly, knowledge and skills in the *use* and *application* of numbers within broader mathematical contexts. As hypothesized above, spatial visualization skills might serve as an important cognitive tool in this regard. To further illustrate this point, we return to the building metaphor in which numbers might be seen as the building blocks and spatial visualization as a tool used to manipulate and assemble the building blocks. Our findings suggest that key differences in mathematics performance are explained by both one's fluency with basic numerical skills but also – and perhaps to a greater extent – one's ability to operate on, use, and apply numbers within and across various mathematical problems. Ultimately, mathematics performance likely rests on one's ability to coordinate multiple representations and uses of number and various other mathematical symbols. Future research efforts are needed to

better understand how different cognitive skills not only differentially relate to different branches of mathematics but also potentially different numerical and mathematical concepts, procedures, and facts within each branch.

## 2.6.6 Limitations

There are several limitations worth pointing out. First, this study was carried out in low SES populations, living in mostly rural areas. For this reason, one must be careful not to generalize the current findings to the general population. It is possible that children of lower SES backgrounds may rely more heavily on informal approaches to mathematics problem solving compared to their higher SES peers who may rely more heavily on formal learning experiences (e.g., see Jordan, Huttenlocher, & Levine, 1994). Accordingly, children in higher SES populations may rely less on spatial visualization skills and more on symbolic numerical skills (e.g., see Butterworth, Reeve, & Reynolds, 2011). However, recent evidence challenges this prediction. Reeve and colleagues (2018) demonstrated that the predictors of arithmetic in indigenous and nonindigenous children in rural and urban Australia were highly comparable and driven by similar visual-spatial factors. This finding highlights the importance of visual-spatial abilities for early numerical cognition regardless of SES and cultural divides. Given the mixed results to date, future work of this sort should strive to use a more economically diverse sample and seek to better understand the potentially moderating effects of SES on the observed relations.

Second, another concern with the current study has to do with the issue of common method variance (Kline, 2015); when variables are measured in highly similar ways. One reason we may have found separate factors for each construct might be partially explained by the common measurement approaches used to test each construct. For example, the numerical measures were all timed tests and involved highly similar task demands (e.g., crossing out the correct response). The spatial and mathematical tasks were all untimed and involved pointing to the correct response. Consequently, the spatial and mathematics measures may have been more closely related because individuals who were careful, took their time, and double-checked their work in the spatial measures may have also been more likely to do so in the mathematics measures. Issues of common

measurement variance should be carefully considered in any future efforts to replicate the current findings.

Third, our results should be interpreted with acknowledging that all four constructs were highly correlated. Although a four-factor model was found to best fit the data, other models also fit the data to a satisfactory degree as well. So, although we can be confident that in combination, numerical, spatial, and EF skills provide a robust model of mathematics achievement, we are less confident of the more specific relations observed. The current model of mathematics, including the individual pathways, needs to be replicated.

Finally, our data were cross-sectional and limit any conclusions we can make about the directionality of the mediation analyses. Future research is needed to test longitudinal relations between numerical, spatial, and EF skills and their relations with mathematics achievement. Given their high correlations with one another, it seems germane to study the extent to which these variables interact with one another over time and potentially develop in part due to synergistic effects of one domain on the other. Said differently, does growth or improvement in one domain predict growth in the other domains? Intervention studies that target each construction in isolation but also in combination with one another will be critical in order to arrive at a better understanding of causal pathways between variables. Moreover, these efforts have the potential to eventually inform educational practice.

## 2.6.7 Conclusion

Results of a CFA demonstrated that numerical, spatial, EF, and mathematics skills are highly related, yet separable, constructs. Follow-up structural analyses revealed that numerical, spatial, and EF latent variables explained 84% of children's mathematics achievement scores, even after controlling for age. These results further highlight the potential importance of numerical, spatial, and EF skills in the learning and performance of foundational mathematics competencies, such as numeration and geometry. Further analyses revealed spatial reasoning as a particularly strong contributor to mathematics achievement. It is hypothesized that this relation rests on the critical role that spatial visualization plays in forming the problem and potential solutions to novel mathematics

tasks. This study contributes to the growing need to further understand the dynamic interplay of basic cognitive skills and performance in various branches of mathematics.

## 2.7 References

- Ackerman, P. L. (1988). Determinants of individual differences during skill acquisition: Cognitive abilities and information processing. *Journal of Experimental Psychology: General*, *117*, 288–318.
- Barron, R. M., & Kenny, D. A. (1986). The moderator-mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. *Journal of Personality and Social Psychology*, *51*(6), 1173-1182.
- Bruner, Jerome. 1966. *Toward a theory of instruction*. Cambridge, MA: Belknap Press
- Butterworth, B., Reeve, R., & Reynolds, F. (2011). Using mental representations of space when words are unavailable: studies of enumeration and arithmetic in indigenous Australia. *Journal of Cross-Cultural Psychology*, *42*(4), 630-638.
- Casey, B. M., Lombardi, C. M., Thomson, D., Nguyen, H. N., Paz, M., Theriault, C. A., & Dearing, E. (2018). Maternal support of children's early numerical concept learning predicts preschool and first-grade math achievement. *Child Development*, *89*(1), 156-173.
- Claessens, A., Duncan, G., & Engel, M. (2009). Kindergarten skills and fifth-grade achievement: Evidence from the ECLS-K. *Economics of Education Review*, *28*(4), 415-427.
- Connolly, A. J. (2007). *KeyMath diagnostic assessment (3rd ed.)*. Minneapolis, MN: Pearson
- Cragg, L., Keeble, S., Richardson, S., Roome, H. E., & Gilmore, C. (2017). Direct and indirect influences of executive functions on mathematics achievement. *Cognition*, *162*, 12-26.
- Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education*, *3*(2), 63-68.
- Dehaene, S. (2011). *The number sense: How the mind creates mathematics*. New York,

NY: Oxford University Press.

- Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General*, *122*(3), 371-396.
- de Hevia, M. D., Vallar, G., & Girelli, L. (2008). Visualizing numbers in the mind's eye: The role of visuo-spatial processes in numerical abilities. *Neuroscience & Biobehavioral Reviews*, *32*(8), 1361-1372.
- Delgado, A. R., & Prieto, G. (2004). Cognitive mediators and sex-related differences in mathematics. *Intelligence*, *32*(1), 25-32.
- De Smedt, B., Noël, M. P., Gilmore, C., & Ansari, D. (2013). How do symbolic and non-symbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior. *Trends in Neuroscience and Education*, *2*(2), 48-55.
- Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., ... & Sexton, H. (2007). School readiness and later achievement. *Developmental Psychology*, *43*(6), 1428-1446.
- Ehrlich, S. B., Levine, S. C., & Goldin-Meadow, S. (2006). The importance of gesture in children's spatial reasoning. *Developmental Psychology*, *42*(6), 1259-1268.
- Fiedler, K., Harris, C., & Schott, M. (2018). Unwarranted inferences from statistical mediation tests—An analysis of articles published in 2015. *Journal of Experimental Social Psychology*, *75*, 95-102.
- Field, A. (2009). *Discovering statistics using SPSS*. London, UK: Sage publications.
- Friedman, N. P., & Miyake, A. (2017). Unity and diversity of executive functions: Individual differences as a window on cognitive structure. *Cortex*, *86*, 186-204.
- Galton, F. (1880). Visualised numerals. *Nature*, *21*(533), 252-256.
- Geary, D. C. (1993). Mathematical disabilities: cognitive, neuropsychological, and genetic components. *Psychological Bulletin*, *114*(2), 345-362.
- Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities*, *37*(1), 4-15.
- Göbel, S. M., Shaki, S., & Fischer, M. H. (2011). The cultural number line: a review of

- cultural and linguistic influences on the development of number processing. *Journal of Cross-Cultural Psychology*, 42(4), 543-565.
- Gebuis, T., & Reynvoet, B. (2012). The interplay between nonsymbolic number and its continuous visual properties. *Journal of Experimental Psychology: General*, 141(4), 642-648.
- Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skill and early number knowledge: the role of the linear number line. *Developmental Psychology*, 48(5), 1229-1241.
- Hegarty, M., & Kozhevnikov, M. (1999). Types of visual-spatial representations and mathematical problem solving. *Journal of Educational Psychology*, 91(4), 684-689.
- Holmes, J., & Adams, J. W. (2006). Working memory and children's mathematical skills: Implications for mathematical development and mathematics curricula. *Educational Psychology*, 26(3), 339-366.
- Hu, L. T., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling: A Multidisciplinary Journal*, 6(1), 1-55.
- Hubbard, E. M., Piazza, M., Pinel, P., & Dehaene, S. (2005). Interactions between number and space in parietal cortex. *Nature Reviews Neuroscience*, 6(6), 435-448.
- Huttenlocher, J., Jordan, N. C., & Levine, S. C. (1994). A mental model for early arithmetic. *Journal of Experimental Psychology: General*, 123(3), 284-296.
- Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1994). Assessing early arithmetic abilities: Effects of verbal and nonverbal response types on the calculation performance of middle-and low-income children. *Learning and Individual Differences*, 6(4), 413-432.
- Judd, C. M., & Kenny, D. A. (1981). Process analysis: Estimating mediation in treatment evaluations. *Evaluation Review*, 5(5), 602-619.
- Kline, R. B. (2015). *Principles and practice of structural equation modeling*. New York, NY: Guilford publications.
- Kyttälä, M., & Lehto, J. E. (2008). Some factors underlying mathematical performance:

- The role of visuospatial working memory and non-verbal intelligence. *European Journal of Psychology of Education*, 23(1), 77-94.
- Kunda, M., McGregor, K., & Goel, A. (2010). Taking a look (literally!) at the Raven's intelligence test: Two visual solution strategies. *Proceedings of the Cognitive Science Society*, 32 (32), 1690-1696.
- Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York, NY: Basic Books.
- LeFevre, J. A., Jimenez Lira, C., Sowinski, C., Cankaya, O., Kamawar, D., & Skwarchuk, S. L. (2013). Charting the role of the number line in mathematical development. *Frontiers in Psychology*, 4, 641. doi: 10.3389/fpsyg.2013.00641
- Levine, S. C., Huttenlocher, J., Taylor, A., & Langrock, A. (1999). Early sex differences in spatial skill. *Developmental Psychology*, 35(4), 940-949.
- Lohman, D. F. (1996). Spatial ability and G. In I. Dennis & P. Tapsfield (Eds.), *Human abilities: Their nature and assessment* (pp. 97–116). Hillsdale, NJ: Lawrence Erlbaum.
- Lynn, R., Backhoff, E., & Contreras-Niño, L. A. (2004). Sex differences on g, reasoning and visualisation tested by the progressive matrices among 7–10 year olds: some normative data for Mexico. *Personality and Individual Differences*, 36(4), 779-787.
- Lyons, I. M., Bugden, S., Zheng, S., De Jesus, S., & Ansari, D. (2018). Symbolic number skills predict growth in nonsymbolic number skills in kindergarteners. *Developmental Psychology*, 54(3), 440-457.
- Lyons, I.M., Hutchison, J. E., Bugden, S., Goffin, C., & Ansari, D. (2018). *Kindergarteners reliability mis-classify ordered sequences of non-adjacent numbers*. Paper presented at the Mathematical Cognition and Learning Society conference, Oxford University, UK.
- Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014). Numerical predictors of arithmetic success in grades 1–6. *Developmental Science*, 17(5), 714-726.
- Lyons, I. M., Vogel, S. E., & Ansari, D. (2016). On the ordinality of numbers: a review of neural and behavioral studies. *Progress in Brain Research*, 227, 187-221.



- MacKinnon, D. P., Fairchild, A. J., & Fritz, M. S. (2007). Mediation analysis. *Annual Review of Psychology*, *58*, 593-614.
- Marghetis, T., Núñez, R., & Bergen, B. K. (2014). Doing arithmetic by hand: Hand movements during exact arithmetic reveal systematic, dynamic spatial processing. *The Quarterly Journal of Experimental Psychology*, *67*(8), 1579-1596.
- McClelland, M. M., & Cameron, C. E. (2012). Self-regulation in early childhood: Improving conceptual clarity and developing ecologically valid measures. *Child Development Perspectives*, *6*(2), 136-142.
- Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex “frontal lobe” tasks: A latent variable analysis. *Cognitive Psychology*, *41*(1), 49-100.
- Miyake, A., Friedman, N. P., Rettinger, D. A., Shah, P., & Hegarty, M. (2001). How are visuospatial working memory, executive functioning, and spatial abilities related? A latent-variable analysis. *Journal of Experimental Psychology: General*, *130*(4), 621-640.
- Mix, K. S., & Cheng, Y. L. (2012). The relation between space and math: Developmental and educational implications. *Advances in Child Development and Behavior*, *42*, 197-243.
- Mix, K. S., Huttenlocher, J., & Levine, S. C. (2002). *Quantitative development in infancy and early childhood*. New York, NY: Oxford University Press.
- Mix, K. S., Levine, S. C., Cheng, Y. L., Young, C. J., Hambrick, D. Z., & Konstantopoulos, S. (2017). The latent structure of spatial skills and mathematics: a replication of the two-factor model. *Journal of Cognition and Development*, *18*(4), 465-492.
- Mix, K. S., Levine, S. C., Cheng, Y. L., Young, C., Hambrick, D. Z., Ping, R., & Konstantopoulos, S. (2016). Separate but correlated: The latent structure of space and mathematics across development. *Journal of Experimental Psychology: General*, *145*(9), 1206-1227.
- Muthén, L. K., & Muthén, B. O. (1998-2015). Mplus (Version 7.4). *Los Angeles, CA: Muthén & Muthén*.

- Newcombe, N. S., & Huttenlocher, J. (2003). *Making space: The development of spatial representation and reasoning*. Cambridge, MA,US: MIT Press.
- Newcombe, N. S., Levine, S. C., & Mix, K. S. (2015). Thinking about quantity: The intertwined development of spatial and numerical cognition. *Wiley Interdisciplinary Reviews: Cognitive Science*, 6(6), 491-505.
- O'Brien, R. M. (2007). A caution regarding rules of thumb for variance inflation factors. *Quality & Quantity*, 41(5), 673-690.
- Parsons, S., & Bynner, J. (2005). *Does numeracy matter more?* National Research and Development Centre for Adult Literacy and Numeracy, London, UK: Institute of Education.
- Ponitz, C. C., McClelland, M. M., Matthews, J. S., & Morrison, F. J. (2009). A structured observation of behavioral self-regulation and its contribution to kindergarten outcomes. *Developmental Psychology*, 45(3), 605.
- Rasmussen, C., & Bisanz, J. (2005). Representation and working memory in early arithmetic. *Journal of Experimental Child Psychology*, 91(2), 137-157.
- Raven, J (2008). *Coloured progressive matrices and Chrichton vocabulary scale*. London, UK: Pearson.
- Reuhkala, M. (2001). Mathematical skills in ninth-graders: Relationship with visuo-spatial abilities and working memory. *Educational Psychology*, 21(4), 387-399.
- Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science*, 24(7), 1301-1308.
- Ritchie, S. J., Bates, T. C., & Deary, I. J. (2015). Is education associated with improvements in general cognitive ability, or in specific skills?. *Developmental Psychology*, 51(5), 573-582.
- Sella, F., Berteletti, I., Lucangeli, D., & Zorzi, M. (2017). Preschool children use space, rather than counting, to infer the numerical magnitude of digits: Evidence for a spatial mapping principle. *Cognition*, 158, 56-67.
- Schneider, M., Beeres, K., Coban, L., Merz, S., Susan Schmidt, S., Stricker, J., & De

- Smedt, B. (2017). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: A meta-analysis. *Developmental Science*, *20*(3).DOI: 10.1111/desc.12372
- Siegler, R. S. (2016). Magnitude knowledge: the common core of numerical development. *Developmental Science*, *19*(3), 341-361.
- Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development*, *75*(2), 428-444.
- Siegler, R. S., & Ramani, G. B. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental Science*, *11*(5), 655-661.
- Soper, D.S. (2018). A-priori Sample Size Calculator for Structural Equation Models [Software]. Available from <http://www.danielsoper.com/statcalc>
- Tam, Y. P., Wong, T. T. Y., & Chan, W. W. L. (2018). The relation between spatial skills and mathematical abilities: The mediating role of mental number line representation. *Contemporary Educational Psychology*, *56*, 14-24.
- Thompson, J. M., Nuerk, H. C., Moeller, K., & Kadosh, R. C. (2013). The link between mental rotation ability and basic numerical representations. *Acta Psychologica*, *144*(2), 324-331.
- Tolar, T. D., Lederberg, A. R., & Fletcher, J. M. (2009). A structural model of algebra achievement: computational fluency and spatial visualization as mediators of the effect of working memory on algebra achievement. *Educational Psychology*, *29*(2), 239-266.
- Toomarian, E. Y., & Hubbard, E. M. (2018). On the genesis of spatial-numerical associations: Evolutionary and cultural factors co-construct the mental number line. *Neuroscience & Biobehavioral Reviews*.
- Tosto, M. G., Hanscombe, K. B., Haworth, C., Davis, O. S., Petrill, S. A., Dale, P. S., ... & Kovas, Y. (2014). Why do spatial abilities predict mathematical performance? *Developmental Science*, *17*(3), 462-470.
- Uttal, D. H., & Cohen, C. A. (2012). Spatial thinking and STEM education: When, why and how. *Psychology of Learning and Motivation*, *57*, 147–181.  
<http://dx.doi.org/10.1016/B978-0-12-394293-7.00004-2>

- Verdine, B. N., Irwin, C. M., Golinkoff, R. M., & Hirsh-Pasek, K. (2014). Contributions of executive function and spatial skills to preschool mathematics achievement. *Journal of Experimental Child Psychology, 126*, 37-51.
- Viarouge, A., Hubbard, E. M., & McCandliss, B. D. (2014). The cognitive mechanisms of the SNARC effect: an individual differences approach. *PloS ONE, 9*(4), e95756.
- Wei, W., Yuan, H., Chen, C., & Zhou, X. (2012). Cognitive correlates of performance in advanced mathematics. *British Journal of Educational Psychology, 82*(1), 157-181.
- Weston, R., & Gore Jr, P. A. (2006). A brief guide to structural equation modeling. *The Counseling Psychologist, 34*(5), 719-751.
- Zelazo, P. D., Carlson, S. M., & Kesek, A. (2008). The development of executive function in childhood. In C. A. Nelson & M. Luciana (Eds.), *Developmental cognitive neuroscience. Handbook of developmental cognitive neuroscience* (pp. 553-574). Cambridge, MA, US: MIT Press.

## Chapter 3

### 3 Neural Underpinnings of Numerical and Spatial Cognition: An fMRI Meta-Analysis of Brain Regions Associated with Symbolic Number, Arithmetic, and Mental Rotation

#### 3.1 Citation

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#### 3.2 Introduction

Mathematics is frequently conceived of and expressed in terms of spatial relations. Historically, many mathematical discoveries have made use of the human capacity to think and reason about space (Davis et al., 2015; Dehaene, 2011; Hubbard, Piazza, Pinel, & Dehaene, 2005). For example, famous mathematical discoveries, such as Pythagoras's Theorem, the Real Number Line, Cavalieri's principle, and the Cartesian coordinate system all speak to the intricate and intimate connections between space and mathematics. Moreover, ancient tools such as the abacus and knotted arithmetic rope, and more recently the number line, are but a few examples of cultural inventions that directly map numbers and their relations onto space.

Critically, the link between numbers and space is not limited to inherently spatial aspects of mathematics, such as geometry and measurement, but appears to extend down to the most fundamental of mathematical entities and operations: numbers and arithmetic.

Although there is extensive behavioral evidence for strong relations between spatial and numerical thinking (e.g., see Mix & Cheng, 2012; Hawes, Moss, Caswell, Seo, & Ansari, 2019), questions remain regarding the underlying neural relations between these two cognitive constructs. To date, research on the neural correlates of spatial skills, such as mental rotation, and numerical reasoning have been studied in complete isolation from one another (e.g., see Zacks, 2008). While it has been well established that *basic* spatial processes (e.g., comparing line lengths) are related to basic numerical processes (e.g., comparing Arabic digits; e.g., see Sokolowki, Fias, Mousa, & Ansari, 2017), it is not yet known whether higher-level spatial skills (e.g., mental rotation) relate to numerical and mathematical processing in the brain. Thus far, investigations into the neural correlates of spatial and numerical processes has been limited to studies examining Spatial-Numerical Associations (SNAs; e.g., see Toomarian and Hubbard, 2018). This body of research is based largely on experimental paradigms that do not require intentional and effortful spatial processing, such as mental rotation. Instead, this body of research is interested in uncovering the unconscious links between space and number. Crucially, in this paper, we aim to do the opposite. We address the conscious and intentional processing of numbers, space, and the operations that link them.

The decision to focus on high-level spatial skills (of which mental rotation is but one of many), rather than lower-level spatial skills, was informed by the literature on individual differences. While consistent and robust relations exist between spatial visualization abilities<sup>4</sup>, including mental rotation skills, and numerical and arithmetical reasoning, relations between low-level spatial and numerical processing (e.g., automatic SNAs) has failed to reveal reliable associations with higher level mathematics, including arithmetic (Cipora, Patro, & Nuerk, 2015; Hawes et al., 2019; Mix & Cheng, 2012). Thus,

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<sup>4</sup> Note that mental rotation is but one example of what we refer to more generally as spatial visualization, which is defined here as the ability to generate, maintain, and transform visual-spatial images in mind (Lohman, 1996). In addition to mental rotation, other measures of spatial visualization include mental paper folding, composition/decomposition of 2D/3D shapes, and block design (Carroll, 1993; Hawes et al., 2019; Hegarty & Waller, 2005). We targeted mental rotation as our construct of interest to constrain our search criteria, but also because it is a well-established measure of spatial ability, has been found to correlate strongly with a variety of mathematical tasks, and has been subject to numerous fMRI investigations (Mix & Cheng, 2012; Zacks, 2008).

by revealing the neural relations between mental rotation and numerical and arithmetical reasoning, we may be afforded new insights into the relations between high-level spatial skills (mental rotation) with both basic and more advanced numerical reasoning processes (i.e., basic symbolic number processes and arithmetic, respectively). To summarize, we have a good understanding of where, and to a lesser extent, how low-level spatial and numerical processes are associated in the brain (Dehaene, Piazza, Pinel, & Cohen, 2003; Sokolowski et al., 2017; Sokolowski, Fias, Ononye, & Ansari, 2017). We do not, however, have a good understanding of where or how spatial visualization abilities are related to numerical and arithmetical processes in the brain.

To address this gap in the literature, we report the results of a meta-analysis of brain regions associated with neural activity in three key aspects of mathematical thinking: basic symbolic number processing, mental arithmetic, and mental rotation (a widely used measure of spatial ability). We targeted these three cognitive processes because they provided opportunities to test theoretically informed predictions as to when, why, and where we should expect to see common and distinct neural activity. As outlined in Figure 3.1 – and described in detail in the following literature review – these three cognitive processes are hypothesized to be related to the extent that task performance involves common and distinct operations. For example, common to mental arithmetic and symbolic number, but not mental rotation, is the need for symbolic number processing. Accordingly, we hypothesized that we should see overlap in brain regions that are associated with symbolic number processing, shared by both arithmetic and symbolic number processes, but not mental rotation. Using this same logic, we should expect to see overlap between mental rotation and mental arithmetic, but not symbolic number, in regions that are more closely associated with mental manipulation. While mental arithmetic and mental rotation involve domain-general mental manipulation, symbolic number processing presumably does not (or at least to a much lesser degree)<sup>5</sup>. Lastly, we

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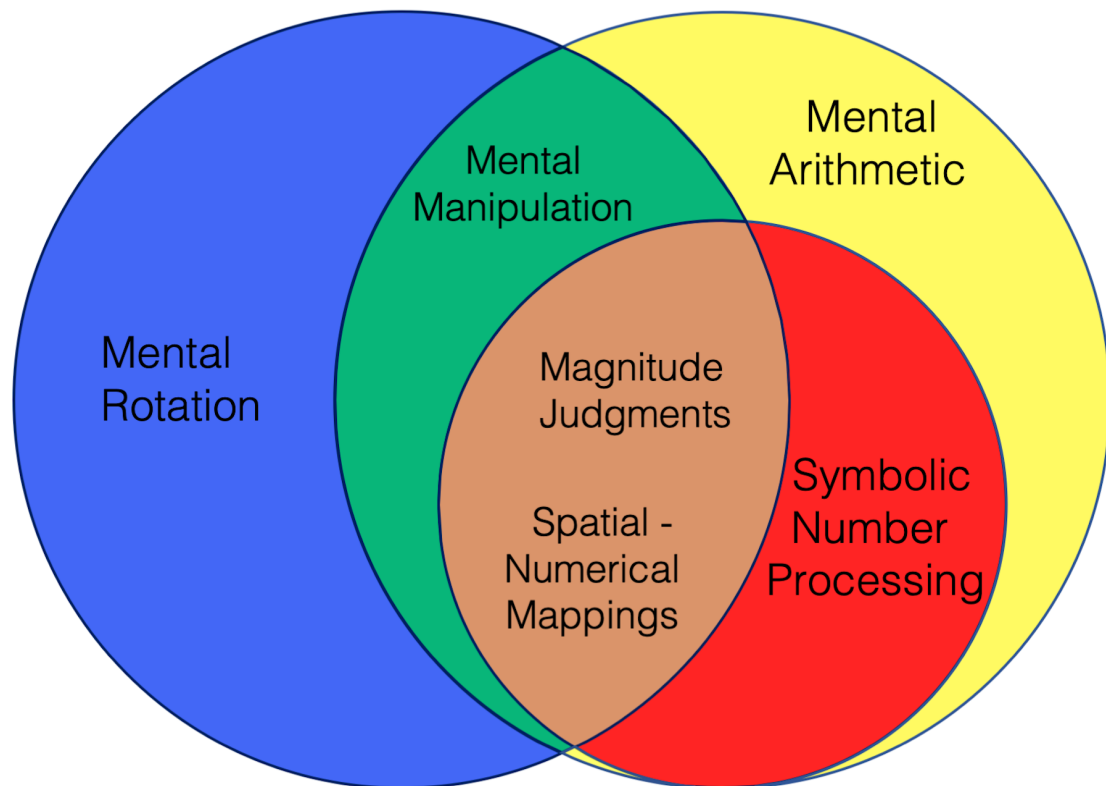
<sup>5</sup> We acknowledge that not all types of arithmetic require mental manipulation (e.g., memorized arithmetic facts). However, as revealed in the Methods section, many of the fMRI studies on mental arithmetic were explicitly designed to elicit effortful calculation and mental manipulation processes. We deliberately made no distinction between low-effort (recall-based) vs. high-effort (calculation-based) problems in creating our mental arithmetic ALE map. As discussed later, this decision was based on our intent to reveal brain

should expect to see overlap between all three processes based on the common need to represent and reason about magnitudes (e.g., see Walsh, 2003). Additionally, we hypothesize that these processes may also be linked through the role that spatial visualization (measured here with mental rotation) plays in mapping numbers onto space. By examining the representation versus manipulation of numerical information and the associated overlap with mental rotation, we aimed to better pinpoint the specific relationships between spatial and numerical processing. Taken together, the goals of this study were 1) to provide a meta-analysis of brain regions associated with three key aspects of mathematical thinking, and 2) provide a more nuanced and theoretically driven approach to understanding when and why spatial and numerical thinking may or may not recruit common neural mechanisms.

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regions associated with both basic symbol processing but also higher-level spatial reasoning (i.e., mental rotation). Note that domain-general manipulation refers to the manipulation of unspecified and amodal stimuli and forms of information (e.g., cube structures or numbers; verbally or visually coded information).





*Figure 3.1* Process-based account of common and distinct operations associated with symbolic number, mental arithmetic, and mental rotation.

### 3.2.1 Behavioral Evidence of Connections between Spatial and Numerical Cognition

The scientific study of relations between numbers and space has a lengthy history, beginning with studies by Sir Francis Galton in the late 1800's and continuing to the present day (Galton, 1980; Toomarian & Hubbard, 2018). The majority of research in this area posits the 'mental number line' as the source of various empirical accounts of 'numerical-spatial associations.' According to this theory, humans conceptualize numbers and their various relations along a mental number line in which numbers are ordered in ascending magnitude from left-to-right. Empirical support for the theory comes from a number of behavioral findings, including the SNARC effect, (spatial-numerical

association of response codes; Dehaene, Bossini, & Giraux, 1993), line bisection effects (Calabria & Rossetti, 2005), and the operation momentum effect (Knops, Viarouge, & Dehaene, 2009). In brief, the SNARC effect refers to the automatic association of small numbers (e.g., 1, 2, 3) to the left side of space and larger numbers (e.g., 7, 8, 9) to the right side of space. For example, people are faster to make parity judgments (i.e., determine whether or not a number is even or odd) when the left hand is used to make judgments about small numbers and the right hand is used to make judgments about larger numbers. This effect is said to be automatic because the task itself does not actually involve intentional judgments about the magnitude of the numbers. The line bisection effect is much less studied than the SNARC effect but similarly demonstrates automatic biases of associating small numbers with the left side of space and large numbers to the right side of space. For example, in one version of the line bisection task, individuals are asked to use a pencil to mark the midpoint of a string of numerals of small single-digit numerals (e.g., 2222222) compared large single-digit numerals (e.g., 9999999). Results of these studies indicate that adult participants bias their estimates to the left when bisecting small single-digit numerals and bias their estimates to the right when bisecting large single-digit numbers (Calabria & Rossetti, 2005). Finally, operation momentum effects refer to the oft-reported finding that left-right response biases are associated with addition and subtraction, and even the operators themselves (i.e., + and -). For example, individuals tend to overestimate answers to addition problems and underestimate answers to subtraction problems (McCrink, Dehaene, & Dehaene-Lambertz, 2007). Importantly, these associations appear to be culturally mediated and indicate the roles of learning, development, and cultural influences (left-to-right written notation) in forming these spatial-numerical associations. For example, the SNARC effect is reversed in cultures that read from right-to-left (Shaki, Fischer, & Petrusic, 2009). Taken together, a large body of research supports the presence of spatial-numerical associations and the tendency to map numbers and their various relations to space.

### 3.2.1.1 Contributions of Spatial Skills in Mapping Numbers to Space

What are the cognitive bases for the ability to map numbers and mathematical objects onto space? Recent research suggests that spatial abilities play a key role in this process. For example, individual differences in the ability to map numbers to space (e.g., estimating where a number belongs on an empty number line) has been found to mediate relations between spatial ability and mathematics performance (Gunderson, Ramirez, Beilock, & Levine, 2012; Tam, Wong, & Chan, 2019). One explanation for these findings is that stronger spatial abilities, such as being able to mentally rotate objects and visualize various visual-spatial relations, underlies a greater ease and fluency in which one can move up and down and carry out various operations along the ‘mental number line’ (Viarouge, Hubbard, & McCandliss, 2014). Thus, spatial ability represents one potential cognitive mechanism that underlies numerical-spatial mappings.

Critically, the mapping of numbers to space might represent but *one* instantiation of the role spatial skills plays in conceptualizing mathematical relations. Individual differences in spatial skills, such as mental rotation, have been linked to performance across a variety of mathematical tasks, including geometry (Delgado & Prieto, 2004), algebra (Tolar, Lederberg, & Fletcher, 2009), word problems (Hegarty & Kozhevnikov, 1999), mental arithmetic (Kyttälä & Lehto, 2008), and advanced mathematics (e.g., function theory, mathematical logic, computational mathematics; Wei, Yuan, Chen, & Zhou, 2012). According to a recent review, “*the connection between space and math may be one of the most robust and well-established findings in cognitive psychology*” (Mix & Cheng, 2012, p. 198). Taken together, an emerging body of research suggests that spatial skills, such as mental rotation, may play an important role in forming spatial-numerical associations, specifically, and spatial-mathematical associations, more generally (Marghetis, Núñez & Bergen, 2014; Hubbard, Piazza, Pinel, & Dehaene, 2009).

### 3.2.2 Neural Evidence for Links between Spatial and Numerical Cognition

### 3.2.2.1 Neuropsychological Studies and the Role of the Left Angular Gyrus

Given the close coupling of number and space in behavioral studies, might we also see a close coupling of underlying neural mechanisms? Evidence to date suggests that this indeed may be the case. Some of the earliest studies that indicate that there is a link between numerical and spatial processing at the neural level came from neuropsychological case studies. It has long been recognized that lesions to the parietal lobe result in joint impairments in numerical and spatial processing (Gerstmann, 1940; Holmes, 1918; Stengel, 1944). For example, Gerstmann's Syndrome, a rare condition associated with lesions to the left angular gyrus, is marked by deficits in numerical and spatial thinking and more specifically by a tetrad of symptoms that include deficits in carrying out basic calculations, left-right confusion, finger agnosia (trouble identifying one's fingers), and dysgraphia (difficulty with writing) (Gerstmann, 1940). There is some evidence to suggest that the core deficit associated with Gerstmann's Syndrome is due to difficulties in the mental manipulation of images, including impaired mental rotation skills (Mayer et al., 1999). These case studies suggest a potential interaction of number and space in the left angular gyrus. Recent support for this possibility has been demonstrated across several studies using transcranial magnetic stimulation (TMS); a methodology used to temporarily induce 'lesion-like' effects through altering electrical current in targeted areas of the brain. Studies have shown that disruptions to the left angular gyrus appear to impair one's spatial representation of number, also referred to as the 'mental number line' (Cattaneo, Silvanto, Pascual-Leone, & Battelli, 2009; Göbel, Calabria, Farne, & Rossetti, 2006; Göbel, Walsh, & Rushworth, 2001).

Another line of neuropsychological research that supports the interaction of space and number in the parietal lobes comes from studies on patients with hemi-spatial neglect; a condition marked by the inability to attend to the contralesional portion of space (e.g., ignoring left side of space when the lesion is in the right parietal lobe). This results in a skewed ability to indicate the mid-point of both imagined and actual objects, including the mid-point of a physical line, but also the mid-point of numerical intervals (Bisiach & Luzatti, 1978; Zorzi et al., 2002). For example, Zorzi and colleagues (2002)

found evidence to suggest that right-lateralized neglect patients tended to overestimate the mid-points of two spoken numbers, such as “two” and “six”; that is, rather than state that “four” falls in between “two” and “six,” patients were more likely to bias their estimates to the right and erroneously state “five” as the mid-point.

In sum, lesion studies as well as temporarily altered brain activity via TMS, suggests that the parietal lobe and specifically the left angular gyrus subserve both numerical and spatial processing. However, more recent research findings challenge these claims. For example, accumulating evidence suggests that the left angular gyrus may be the source of verbally stored symbolic number understanding and associated number facts, including arithmetic facts (Polspoel, Peters, Vandermosten, & De Smedt, 2017). This shift away from the left angular gyrus as a neural region associated with both numerical and spatial processes is perhaps best represented in Dehaene et al.’s (1992; 2003) ‘Triple Code Model’ of numerical cognition. This model posits that the left angular gyrus is specific to verbally mediated symbolic number processes and the bilateral intraparietal sulci (IPS) supports the processing of abstract numerical magnitudes, including the spatial and semantic representation and manipulation of numbers (Dehaene & Cohen, 1997; Dehaene et al., 2003). A recent fMRI meta-analysis further suggests that the left angular gyrus might play a role in verbally mediated symbolic number knowledge (Sokolowki, Fias, Mousa, & Ansari, 2017). More specifically, while both symbolic and non-symbolic numbers (e.g., dot arrays) were processed by shared frontal and parietal regions, only symbolic number uniquely activated the left angular gyrus. Additionally, a meta-analysis of functional brain activity related to mental rotation failed to reveal regions specific to the left angular gyrus and instead pointed to activity in bilateral frontal and parietal regions (Zacks, 2008).

Taken together, while there is some evidence that the left angular gyrus might be implicated in both numerical and spatial processing, there is a growing body of evidence to suggest that the left angular gyrus is more specifically related to verbally mediated numerical knowledge. By directly contrasting brain regions associated with activity in basic symbolic number processing, arithmetic, and mental rotation, we aim to further shed light on the specificity of this region as one potentially more attuned to numerical and/or spatial processing. Furthermore, by contrasting regions specific to basic symbolic

number processes and more complex symbolic number processes, i.e., arithmetic, we may be able to offer additional insight into whether this region is more active for basic vs. higher-level numerical tasks.

### 3.2.2.2 fMRI Studies and the Role of the Intraparietal Sulcus

The intraparietal sulcus (IPS) has been targeted as a central region of interest to researchers of numerical and spatial cognition alike. However, the conclusions and claims about the importance of the IPS for numerical and spatial cognition differ according to each field. Research on numerical cognition has described the IPS as the locus of the putative “number module,” “core quantity system,” and the “number-essential” region (Butterworth, 1999; Dehaene et al., 2003). Research on spatial cognition has described the IPS as a region underlying visual-spatial transformations (Jordan, Heinze, Lutz, Kanowski, & Jäncke, 2001; Zacks, 2008). Presumably, these differences are because studies on the role of the IPS for numerical and spatial processes have been carried out in isolation from one another. Moreover, this lack of ‘cross-talk’ between fields may underlie differences in the ways in which domain-specific functions are ascribed to the IPS. These differences are especially apparent within the domain of numerical cognition.

For over two decades, the IPS has been theorized to house domain-specific processes related to number. Indeed, there is a large body of evidence showing that the IPS – the horizontal segment of the IPS in particular – is consistently activated during both symbolic (“3” or “three”) and non-symbolic (••) number tasks. The fact that the meaning of number is processed and retained across formats (e.g., hearing the number “three” and seeing three objects) has been taken as evidence that the IPS represents number in the abstract. According to Dehaene’s influential ‘Triple Code Model,’ the IPS plays a critical role in the semantic manipulation of numbers and is the most plausible candidate for domain-specificity.

Critically, other perspectives on the role of the IPS in number processing espouse far less ‘domain-specific’ views. Instead, the IPS may represent an area that underlies a far more general magnitude system; one that is sensitive to a variety of magnitudes,

including space, luminance, and even time (e.g., see Kadosh, Lammertyn, & Izard, 2008; Sokolowski et al., 2017). For example, the IPS and other parietal regions are similarly activated when participants make number comparisons but also when comparing various line lengths (Pinel, Piazza, Le Bihan, & Dehaene, 2004). There is strong evidence that basic spatial properties of objects are processed in the parietal cortex, including the IPS. In fact, a central challenge in the attempt to isolate number-specific regions of cortex is controlling for confounds related to basic spatial properties of objects. As is the case in natural world, continuous quantity and numerosity appear to be highly correlated in the brain (Newcombe, Levine, & Mix, 2015; Walsh, 2003). The most influential model in this regard is Vincent Walsh's (2003), 'A Theory of Magnitude', aka, ATOM. Walsh posits evolutionary reasons for widespread overlap for between the magnitudes of time, space, and quantity.

Given that the processing of basic spatial properties, such as size and shape, have been implicated in a general magnitude system, might higher-level spatial skills, such as mental rotation, also recruit some of the same neural resources? Although the neural foundations of mental rotation have been studied in isolation from studies of numerical reasoning, a review of the literature suggests highly overlapping areas of activation in the parietal lobes, including the IPS. In fact, a meta-analysis by Zacks (2008) demonstrated that the IPS was the most consistent and robust brain region associated with mental rotation performance. This finding has led to speculation that this brain region is responsible for visual-spatial transformations, including mental rotation but other visual-spatial transformations as well, such as geometric translations (Jordan, Heinze, Lutz, Kanowski, & Jäncke, 2001; Seydell-Greenwald, Ferrara, Chambers, Newport, & Landau, 2017; Zacks, 2008). According to this view, the IPS is representative of a more general network that is involved in a variety of visual-spatial transformations.

Taken together, current evidence suggests that the IPS and closely surrounding parietal regions play a foundational role in numerical and spatial processes. However, the functions ascribed to the IPS vary and represent a range of possibilities, including number-specific processes, more general magnitude processes, and visual-spatial transformations. One of the aims of this study is examine the common and distinct regions in and around the IPS as they relate to numerical and spatial processes. If it is

found that a high degree of overlap exists between symbolic number processing, arithmetic, and mental rotation, there may be reason to revisit current theories related to the functions of the IPS. The presence of distinct regions associated with each task might further provide guidance for future studies, as these regions might be particularly suited to specific processes related to each task.

### 3.2.2.3 Mathematical Cognition and the General Role of the Fronto-Parietal Network

In addition to the parietal lobes, the frontal lobes are also consistently active during numerical, mathematical, and visual-spatial reasoning tasks (Desco et al., 2011; Matejko & Ansari, 2015; O'Boyle et al., 2005). However, in comparison to the parietal lobes, the frontal cortex has received less attention as a region of targeted interest. This may be due in part to more general functions ascribed to the frontal regions compared to the parietal lobes. It is well-recognized that the prefrontal cortex is commonly associated with top-down attentional and executive control processes (Fincham et al., 2002; Owen, McMillan, Laird, R., & Bullmore, 2005; Smith & Jonides, 1999). Thus, task-related activity in frontal regions is often taken as evidence of increased top-down control requirements. For example, increases in task difficulty are associated with increased activation of the dorsolateral prefrontal cortex (e.g., Kroger et al., 2002).

Neuroimaging studies of numerical reasoning demonstrate consistent activation in frontal regions (e.g., see Sokolowski et al., 2017). However, the amount of frontal activity appears to be somewhat dependent on development and task difficulty. Early in development children tend to rely heavily on frontal regions but over time a general shift occurs and parietal regions become more actively engaged (Ansari et al., 2005; Cantlon et al., 2006; Zamarian, Ischebeck, & Delazer, 2009). Relatedly, rote number processing, including memorized arithmetic facts, appears to rely less on frontal regions and more on parietal regions; calculation-based numerical reasoning, however, appears to more broadly recruit the fronto-parietal network. In short, fluency with number symbols and



arithmetic facts is associated with less frontal activity and more parietal activity. Mental rotation also appears to rely on frontal regions, including regions thought to reflect general cognitive effort, but also regions thought to underlie motor planning and control (e.g., premotor cortex; Zacks, 2008).

Overall, the fronto-parietal network is implicated in both numerical and spatial reasoning and collectively represents the neural underpinnings of mathematical cognition (Desco et al., 2011; Matejko & Ansari, 2015). However, activity in the frontal regions appears to vary somewhat depending on task difficulty. In the current study, we expected to find more diffuse frontal activity for mental rotation and arithmetic compared to basic symbolic number processes.

### 3.2.3 The Present Study

The purpose of the current study was to identify underlying neuroanatomical structures that converge across multiple empirical neuroimaging studies to support numerical, arithmetical, and spatial reasoning at the meta-analytic level. We targeted these three cognitive functions because they represent some of the most well-established building blocks of mathematics (e.g., see Mix & Cheng, 2012; LeFevre et al., 2010). Relatedly, a better understanding of the neural correlates of these skills might provide additional evidence and insights into the historically tight relationship between spatial and mathematical thinking (Smith, 1964; Mix & Cheng, 2012). Another motivating factor behind this study was the intent to merge two traditionally separate bodies of neuroimaging research; one devoted to numerical processes and the other devoted to mental rotation. Critically, each body of literature suggests that numerical reasoning and mental rotation are sub-served by a highly overlapping fronto-parietal network; the IPS being of particular interest within each distinct body of literature. Thus, one of the aims of this study was to examine the common and distinct regions in and around the IPS as they relate to numerical and spatial processes. Identifying brain regions that converge and diverge across the targeted constructs is an important step in working towards a better operational understanding of the brain (e.g., see Price & Friston, 2005). That is, rather than assign disciplinary specific terminology to different brain structures based on the

findings from independent studies (e.g., the “number module”), a more fruitful approach may be to evaluate and define functional brain regions across studies and according to the operations that different areas perform (Price & Friston, 2005). Quantitative fMRI meta-analytic techniques, such as coordinate based Activation Likelihood Estimation (ALE), are ideally suited for this purpose (Eickhoff et al., 2009). By pooling data from different studies, which examine the same construct (e.g., mental arithmetic) but may employ variations of the experimental approach, one is better able to identify consistent responses across experiments (Laird et al., 2009a; Laird et al., 2009b). In addition, this approach may help combat common problems associated with individual fMRI studies, including small sample sizes (low power), low reliability, and the problems inherent to the subtraction logic used to differentiate between two conditions (Price, Devlin, Moore, Morton, Laird, 2005).

Against the background of the literature reviewed above, we entered this study with several predictions (see Figure 3.1 and Table 3.1). Broadly speaking, we predicted the fronto-parietal network would be implicated in all three cognitive tasks. However, we predicted more frontal activation for arithmetic and mental rotation compared to basic symbolic number processing due to the higher cognitive demands of the former tasks. That is, from an operational perspective, we expected to see overlap between mental arithmetic and mental rotation due to the shared need to mental manipulate information (be they objects or numbers). We also reasoned that there may be regions of overlap specific to symbolic number and arithmetic processes, but not mental rotation. The presence of these regions, potentially in and around the left angular gyrus, might suggest areas that deal more exclusively with the representation of symbolic number compared to magnitudes more generally (e.g., angles of rotation). Finally, we predicted that we might identify regions that are specific to mental rotation that correspond to mental imagery and motor control.

In sum, by revealing the neural correlates of all three cognitive processes we aimed to systematically test the ways in which spatial and numerical cognition may converge and diverge in the brain. Specifically, we sought out to tease apart regions of activation subserving mental manipulation versus symbolic number representation.

Table 3.1

*Names of contrasts carried out in the meta-analysis and main mental process remaining after the contrast has been performed.*

<b>Name of contrasts</b>	<b>Predicted remaining mental process</b>	<b>Potential corresponding brain region(s)</b>
Arithmetic > mental rotation	Symbolic number processing	Left angular gyrus
Symbolic number > mental rotation	Symbolic number processing	Left angular gyrus
Mental rotation > symbolic number	Mental manipulation	Frontal regions/prefrontal cortex
Arithmetic > symbolic number	Mental manipulation	Frontal regions/prefrontal cortex
Symbolic number > arithmetic	None	None
Mental rotation > arithmetic	Motor/object simulation	Motor cortex

### 3.3 Methods

#### 3.3.1 Literature Search and Article Selection

Three separate literature searches were conducted; one for each cognitive construct of interest. Each literature search involved the same two-step process: (1) a search of the PUBMED and PsychInfo databases, and (2) a review of the reference sections for any other relevant papers that may not have shown up in the initial search. Although the inclusion/exclusion criteria differed somewhat across constructs (detailed below), we adhered to the following general guidelines when deciding whether or not a study was relevant for inclusion: (1) Studies had to use and report whole-brain group analyses with stereotactic coordinates in Talairach/Tournoux or Montreal Neurological Institute (MNI) space. Contrasts that used region of interest (ROI) or multivariate statistical approaches were excluded; (2) Studies had to include a sample of healthy adults; (3) Only fMRI or PET imaging methods were accepted as these methods have comparable spatial uncertainty; (4) Studies had to have contrasts with active control conditions; studies that included contrasts against baseline, rest, or fixation were excluded. Note that all studies involved button/computer responses; (5) Studies had to be published in English. Our literature search includes papers published prior to August 9<sup>th</sup> 2018.

### 3.3.1.1 Mental Rotation

Combinations of the key terms “mental rotation,” “mental imagery,” “spatial,” “visual-spatial,” “visuospatial,” “object rotation,” “mental transformation,” “PET,” “positron emission topography,” “fMRI,” “functional magnetic resonance imaging,” “neuroimaging,” and “imaging” were entered into the search databases. Studies that included the mental rotation of 2D or 3D task stimuli, including depictions of real world objects or abstract shapes, were included. As a result, the mental rotation ALE map is largely made up of studies that involved the mental rotation of 2D or 3D task stimuli contrasted against an active control condition. As is typical in mental rotation tasks, the control condition involved presenting participants with the same stimulus type and required the same response as the other mental rotation trials (e.g., ‘same’ or ‘different’ response) but the angle of disparity between the objects being compared was categorically smaller (e.g.,  $< 30^\circ$ ) or 0. Studies were excluded if they, 1) involved the mental rotation of body parts (e.g., hands), 2) included contrasts that included mental rotation of number symbols, and 3) were designed to isolate stimulus-dependent mental rotation neural activation (e.g., contrasts mentally rotate tools>non-tools). We excluded studies that included mental rotation of body parts because prior research has found that mental rotation of body parts is distinguishable from mental rotation of objects (e.g., see Tomasino & Gremese, 2016). Moreover, research on relations between mental rotation and mathematics is almost exclusively based on paradigms that involve the mental rotation of objects (and not body parts). Thus, in an attempt to better reveal neural correlates of the well-established behavioral relations between mental rotation and mathematics (Mix & Cheng, 2012), we deliberately excluded studies that included rotation of body parts.

Table 3.2 provides a detailed summary of each study included in the mental rotation meta-analysis, including details on the number of participants per study, type of contrasts run, and the number of foci reported. In total, 28 studies (papers) met the inclusion criteria, providing data on 363 healthy adult participants. These studies included 276 activation foci obtained from 45 contrasts.

### 3.3.1.2 Symbolic Number

The symbolic number map was initially created in a prior study by Sokolowski et al. (2017). Using the two-step literature search process as outlined above, the authors conducted a meta search for studies on numerical and non-numerical magnitude processing. The key terms used in this search included: “number,” “numeral,” “symbol” “nonsymbolic,” “magnitude,” “fMRI,” “PET,” “functional magnetic resonance imaging,” “positron emission topography,” “neuroimage,” “imaging,” “congruent,” “incongruent,” “stroop,” “quantity,” “amount,” “physical size,” “numerical size,” “object size,” “size,” “size interference,” “length,” “duration,” “distance,” and “area”. For the purpose of the current study, we only included studies from the meta-analysis by Sokolowski et al. (2017a; 2017b) that included active and intentional symbolic number processing. Additionally, only studies that included whole numbers were included. As shown in Table 3.3, the majority of symbolic number studies used a number comparison paradigm where participants were asked to make within category comparisons (large vs. small numbers) or between category comparisons (number vs. size comparison). Studies were excluded if they contained 1) only nonsymbolic number processing or non-numerical magnitude processing, 2) only passive viewing or automatic processing. Notably, the current study (unlike previous basic number processing meta-analyses; Sokolowski et al., 2017a, 2017b) excluded passive viewing tasks in an attempt to more closely align the symbolic number processing map to the novel arithmetic and mental rotation maps.

Table 3.3 provides a detailed summary of each study included in the symbolic number meta-analysis, including details on the number of participants per study, type of contrasts run, and the number of foci reported. In total, 24 studies (papers) met the inclusion criteria, providing data on 396 healthy adult participants. These studies included 229 activation foci obtained from 42 contrasts.

### 3.3.1.3 Mental Arithmetic

Combinations of the key terms “arithmetic”, “mental arithmetic”, “problem-solving”, “math”, “arithmetic operations,” “addition”, “subtraction”, “multiplication”, “division,” “mental math” “PET,” “positron emission topography,” “fMRI,” “functional magnetic resonance imaging,” “neuroimaging,” and “imaging” were entered into the search databases. Studies were included if they involved arithmetic with integers and visually presented problem stimuli requiring active responses done on a computer/button press. In an effort to create a general map of mental arithmetic all problem types were included (e.g., easy/automatically recalled facts vs. difficult problems involving overt calculation). Moreover, because prior research has revealed distinct brain regions dependent on the operation being performed (e.g., multiplication vs. addition; see Table 3.4), we included contrasts between operation types. Studies were excluded if they 1) involved arithmetic with fractions and decimals 2) reported only effects relating to arithmetic training. We excluded studies that involved arithmetic with fractions or decimals in an effort to best align the arithmetic and symbolic number maps.

Table 3.4 provides a detailed summary of each study included in the mental arithmetic meta-analysis, including details on the number of participants per study, type of contrasts run, and the number of foci reported. In total, 31 studies (papers) met the inclusion criteria, providing data on 527 healthy adult participants. These studies included 710 activation foci obtained from 80 contrasts.

### 3.3.2 Analysis Procedure

All analyses were done using GingerALE version 2.3.6, a freely available application by BrainMap (<http://www.brainmap.org>; Eickoff et al., 2017, 2012, 2009; Turkeltaub et al., 2012). Preparation of the data to be analyzed in GingerALE was conducted with two programs developed by BrainMap: Scribe (version 3.3) and Sleuth (version 2.4). Scribe was used to code specific study details and input the coordinates (i.e. foci) from all relevant papers that were not already available in BrainMap database. Sleuth was used to select relevant experimental contrasts from papers in the BrainMap database, as well as those we entered into scribe, and create a text-file with foci included in the meta-analyses. Foci were grouped by subject group. Prior to analyses, all foci (coordinates) were

converted into a common Talairach space; a process that involved transforming MNI coordinates into Talairach space. This was computed in Sleuth using the Lancaster transformation *icbm2tal* (Laird et al., 2010; Lancaster et al., 2007). Finally, GingerALE was used to carry out single dataset meta-analyses for each construct. That is, a 3D map was created for each construct. These single dataset analyses were then used to carry out conjunction and contrast (subtraction) analyses.

### 3.3.2.1 Single Dataset Analyses

The present meta-analysis used activation likelihood estimation (ALE) to examine patterns of brain activity related to basic symbolic number processes, arithmetic, and mental rotation. ALE is used to quantitatively synthesize peak activation locations across many empirical neuroimaging studies in stereotactic coordinates ( $x, y, z$ ) on normalized and ‘standard’ brain templates (Talairach or MNI). The input for ALE meta-analyses is 3D coordinates of peak activation within an empirical study that are referred to as foci. An ALE analysis involves modeling the foci from contrasts within each study as centers of 3D Gaussian probability distributions (Eickhoff et al., 2009). This is done to model the spatial uncertainty associated with coordinate-based point estimates. The ALE algorithm then generates 3D activation maps by finding the maximum of each foci group’s Gaussian (Research Imaging Institute UTHSCSA [RII], 2013). This approach of using the maximum is a non-additive method and was created to deal with problems of within-experiment effects (e.g., see Turkeltaub et al., 2012). More specifically, the ALE algorithm was modified in an effort to prevent the influence of between study differences in the number of within study contrasts; a limitation of earlier ALE meta-analyses (Eickhoff et al., 2009; Turkeltaub et al., 2012). On a related note, ALE accounts for differences in sample sizes between studies by adjusting the shape of the Gaussian distribution; larger sample sizes are weighted to have a tighter and taller Gaussian. The 3D activation maps are referred to as pre-ALE Modeled Activation (MA) maps and are generated for each contrast coded for and entered into GingerALE. It is through combining each MA map that a single dataset ALE map is created (RII, 2013). More

specifically, the ALE maps are computed as the voxel-wise union of the MA maps across all studies.

GingerALE then creates a null-distribution by randomly redistributing the ALE scores and probability statistics from the activation maps. This procedure results in an analog brain space that shares the same properties as the original data, such as number of foci and sample sizes, but assumes no preferences for the spatial arrangement of the data. The null-distribution is then used to calculate the probability of obtaining statistically meaningful clusters present in the actual data. More specifically, the ALE algorithm performs a random-effects significance test and determines whether the clustering of converging areas of activity across contrasts is greater than chance. This process results in a parametric 3D map of the data along with the associated  $p$ -values.

Once the  $p$ -value image has been obtained, it is then used to set a significance threshold on the ALE scores (Rii, 2013). In the present study, we used the recommended cluster-forming uncorrected threshold of  $p < .001$  and the cluster-level corrected threshold of  $p < .05$ , obtained from running 1000 threshold permutations (Eickhoff et al., 2012; Rii, 2013). This approach addresses the issue of multiple-comparisons through family-wise error (FEW) correction and has been found to provide optimal compromise between sensitivity and specificity (Eickhoff et al., 2017).

Lastly, GingerALE generates a list of anatomical regions (clusters) that have passed the selected thresholds. GingerALE also provides the following statistics for each cluster identified: volume ( $\text{mm}^3$ ), bounds, weighted center, and the locations and values at peaks within the region. Anatomical labels are also provided for each cluster using Talairach Daemon (talairach.org). In order to visualize the results (i.e., each cluster), we used a combination of Mango (Rii, 2015) and the BrainNet toolbox for MATLAB (Xia, Wang, & He, 2013). To supplement the anatomical labels provided by Talairach Daemon, we also report on the MNI labels provided in Anatomy Toolbox v2.2c (Eickhoff et al., 2005). This allowed us to more narrowly define certain anatomical regions, such as gyri, sulci, and even sulci subdivisions.

### 3.3.2.2 Conjunction and Contrast Analyses



Conjunction and contrast analyses were conducted in GingerALE and used to identify overlapping and distinct brain regions associated with symbolic number, arithmetic, and mental rotation. The single dataset ALE maps described above provided the bases for these analyses. We used an uncorrected threshold of  $p < .01$  with 5000 threshold permutations and a minimum cluster volume of  $50 \text{ mm}^3$ . Note that the cluster-level correction used to produce the single dataset ALE maps (reported above), is not available for conjunction and contrast analyses. The choice to use a threshold of  $p < .01$  was based on its use in prior meta-analyses (e.g., see Pollack & Ashby, 2017; Sokolowski et al., 2017a and 2017b). Moreover, the use of  $p < .01$  is appropriate given that the clusters used for conjunction and contrast analyses have already passed the strict cluster thresholds used to make the single data ALE maps.

Conjunction analyses were conducted in a pairwise fashion to compare regions of overlap amongst all three cognitive constructs. For each conjunction analysis, ALE uses the single dataset ALE maps for each construct of interest (e.g., symbolic number and mental rotation) and looks for voxels that are significantly active across both datasets. A conjunction or overlapping region is identified if it passes the statistical thresholds noted above and reaches a minimum size of  $50 \text{ mm}^3$ . The following three conjunctions were performed: symbolic  $\cap$  mental rotation; symbolic  $\cap$  arithmetic; mental rotation  $\cap$  arithmetic.

Contrast analyses were conducted in order to determine regions of distinct activation between the three constructs. These analyses involved subtracting one single dataset ALE map from another. To conduct the subtraction analyses, ALE first pools the data from across the two studies and then randomly distributes the data into two groupings that are equal in size to the original datasets. One null dataset is then subtracted from the other. The remaining image is then compared to the true data. After a set number of permutations have been performed, a  $p$ -value image is created indicating where the true data's values sit on the distribution of values for any given voxel. In the current study, we ran 5000 permutations with an uncorrected threshold of  $p < .01$ . The following six contrasts were performed: symbolic  $>$  mental rotation; symbolic  $>$  arithmetic; mental rotation  $>$  symbolic; mental rotation  $>$  arithmetic; arithmetic  $>$  symbolic; arithmetic  $>$

mental rotation. To simplify the interpretation of ALE contrast images, they are converted into *z*-scores.

Table 3.2

*Summary of studies included in the mental rotation meta-analysis.*

1st Author	Year	Journal	N	Imaging method	Mean age	Gender	Tasks	Contrast name	Location
Barnes J	2000	Neuropsychologia	6	fMRI	34	4M 2F	Rotation of 3D objects	Rotational transformation > Rotational reference	6
Ecker C	2006	NeuroImage	10	fMRI	25	10F	Rotation of 3D objects	100 degree angular disparity > 20 degree angular disparity	1
Gauthier I	2002	Neuron	15	fMRI		8M 7F	Rotation of 3D objects	60 degree angular disparity > 0 degree angular disparity	2
Halari R	2006	Experimental Brain Research	19	fMRI	25.8	9M 10F	Rotation of 3D objects	Large > Small rotations	3
Hugdahl K	2006	Neuropsychologia	11	fMRI	31	5M 7F	Rotation of 3D objects	Rotation > Control (men)	6
Johnston S	2004	Neuroscience Letters	9	fMRI	25.8	5M 4F	Rotation of 2D abstract shapes	Rotation > Control (women)	6
Jordan K	2001	NeuroImage	9	fMRI	21	1M 8F	Rotation of 2D shapes and 3D objects	3D experimental > 2D control	4
							3D > Control	2	
							Abstract > Control	4	
							Letter > Control	2	
							Mental rotation > Control	2	
Jordan K	2002	Neuropsychologia	24	fMRI	23.55	10M 14F	Rotation of 2D shapes and 3D objects	Women (rotation > control)	13
							Men (rotation > control)	6	
Kawamichi H	2007	Brain Research	12	fMRI	25.5	12M	Rotation of 3D objects in 2D and 3D space	3D Large > Small rotations	5
							2D Large > Small rotations	9	
Keehner M	2006	NeuroImage	14	fMRI		7M 7F	Rotation of 3D objects	Mental rotation > 0	6
Kosslyn S M	1998	Psychophysiology	12	PET	20.1	12F	Rotation of 3D objects	Cubes > Cubes baseline	8
Kosslyn S M	2001	NeuroReport	8	PET	20	8F	Rotation of 3D objects	External action > Baseline	7
							Internal action > Baseline	9	
Lamm C	2007	NeuroImage	13	fMRI	23-31	13M	Rotation of 2D shapes	Orientation > Location	11
Levin SL	2005	Evolutionary Psychology	12	fMRI	20.67	6M 6F	Rotation of 3D objects	Spatial > Same (collapsing across sex)	4
							Spatial > Different (collapsed across sex)	2	
Ng V W K	2001	Journal of Cognitive Neuroscience	12	fMRI	20.25	12F	Rotation of letters	Activation > Baseline	9
Podzebenko K	2005	Neuroscience	16	fMRI	31.5	8M 8F	Rotation of 2D abstract shapes	Rotation > Control	5
Schendan H E	2007	NeuroImage	13	fMRI	21.6	6M 7F	Rotation of 3D objects	Rotation > Control	26
Seurinck R	2005	NeuroImage	24	fMRI	23	24M	Rotation of 3D tool images	Rotation > Control (fixed-paced)	10
							Rotation > Control (self-paced)	6	
Suchan B	2002	Behavioural Brain Research	10	PET	28.9	4M 6F	Rotation of 2D matrices	Main effect rotation	14
							Simultaneous matrix rotation > Simultaneous matrix comparison	8	
							Successive matrix rotation > Successive matrix comparison	5	
Thomsen T	2000	Medical Science Monitor	11	fMRI	30	5M 6F	Rotation of 3D objects	Main effect of stimulus	4
							3D drawings > 2D control (males)	2	
							3D drawings > 2D control (females)	4	
Logie R	2011	Neuropsychologia	21	fMRI	20-35	7M 14F	Rotation of 3D objects	Mental rotation > Control	2
Vanrie J	2002	Neuropsychologia	6	fMRI	25.5	3M 3F	Rotation of 3D objects	Invariance > Control	6
							Rotation > Control	8	
Vingerhoets G	2001	NeuroImage	10	PET	26	5M 5F	Rotation of 2D abstract shapes	Figures rotation > Figures control	2
Vingerhoets G	2002	NeuroImage	12	fMRI	29	12M	Rotation of 3D tool images	Rotated tools > Non-rotated tools	9
Weiss E M	2003	Neuroscience Letters	20	fMRI		10M 10F	Rotation of 3D objects	Rotated images > Control	7
Wilson K D	2006	Perception	7	fMRI	18-23	3M 4F	Rotation of 3D objects and 2D letters	Rotation letters > Control	4
							Rotation objects > Control	7	
Wraga M	2003	Brain and Cognition	16	PET	18-39	16M	Rotation of 3D objects	Rotation > Control	6
							Rotation > Control	4	
Wraga M	2005	Neuropsychologia	11	fMRI	25	7M 4F	Rotation of 3D objects	Rotation > Control	7

Table 3.3

*Summary of studies included in the symbolic number meta-analysis.*

1st Author	Year	Journal	N	Imaging Method	Mean Age	Gender	Tasks (s)	Contrast Name	Location
Ansari D	2005	NeuroReport	12	fMRI	19.8		Comparison	Distance effect (small > large) adults	12
Ansari D	2006	NeuroImage	14	fMRI	21	8F 6M	Comparison Size Congruity	Main effect of distance (small > large)	10
								Main effect of distance in the neutral condition (small>large)	7
Ansari D	2007	Journal of Cognitive Neuroscience	13	fMRI	21.5		Comparison	Conjunction of small and large symbolic number	8
Attout L	2014	PLoS ONE	26	fMRI	21	15F 11M	Order Judgment	Distance effect of numerical order judgment	7
Chen C	2007	NeuroReport	20	fMRI	22.7	10F 10M	Delayed-number-matching	Unmatched numbers > Matched numbers	8
Chochon F	1999	Journal of Cognitive Neuroscience	8	fMRI	22.3	4F 4M	Naming Comparison	Digit naming > Control	2
								Comparison > Control	13
								Comparison > Digit naming	1
Eger E	2003	Neuron	9	fMRI	27.9	5F 4M	Target detection	Number > Letter and colour (fixed-effect)	4
								Number > Letter and colour (random-effect)	2
								Number > Letter (fixed-effect)	2
								Number > Letter (random-effect)	2
								Number > Colour (fixed-effect)	4
								Number > Colour (random-effect)	3
Fias W	2003	Journal of Cognitive Neuroscience	18	PET	23	18M	Comparison	Number comparison vs Nonsymbolic stimuli comparison	13
Fias W	2007	Journal of Neuroscience	17	fMRI	20-37	9F 8M	Comparison	(Number comparison-number dimming) - (letter comparison-letter dimming)	3
Franklin M S	2009	Journal of Cognitive Neuroscience	17	fMRI	21.8	10F 7M	Ordering Task	Magnitude near > Far (common regions with order near > far)	1
								Order far > Near (common regions with magnitude near > far)	1
								Magnitude near > Far (unique regions)	3
								Order far > near (unique regions)	1
Fulbright R K	2003	American Journal of Neuroradiology	19	fMRI	24	8F 11M	Order Identification	Number > Shapes	0
								Far order number vs. Near order number	0
He L	2013	Cerebral Cortex	20	fMRI	21	8F 12M	Comparison	Symbolic > Nonsymbolic	2
								Digit-digit > Cross notation trials	1
								Overlap between (symbolic>nonsymbolic) and (small > large)	2
Holloway I D	2010	NeuroImage	19	fMRI	23.5	10F 9M	Comparison	(symbolic - control) - (non-symbolic - control)	2
Kadosh R C	2005	Neuropsychologia	15	fMRI	28	7F 8M	Comparison	Numerical vs. Size	7
								Numerical vs. Luminance	8
								Numerical distance	3
Kadosh R C	2007	Journal of Cognitive Neuroscience	14	fMRI	25.6	9F 5M	Comparison	Numerical comparison task: Incongruent vs. Congruent	2
Kaufmann L	2005	NeuroImage	17	fMRI	31	7F 10M	Stroop	Numerical comparison > physical comparison	5
								Numerical comparison (Distance 1 > Distance 4, only neutral trials)	5
								Physical comparison (incongruent trials > congruent trials)	0
Le Clec'H G	2000	NeuroImage	5	fMRI	37	5M	Compare to 12	Numbers > Body parts (Block)	4
			6	fMRI	27	3F 3M	Compare to 12	Numbers > Body parts (ER)	3
Liu X	2006	Journal of Cognitive Neuroscience	12	fMRI	18-45	7F 5M	Stroop	Distance of 18 vs. Distance of 27	6
Lyons IM	2013	Journal of Neuroscience	33	fMRI	18-22	16F 17M	Comparison	Symbolic: Number ordinal > Luminance symbolic ordinal	3
								Symbolic ordinal > Luminance ordinal (symbolic) and Symbolic cardinal > Luminance cardinal (symbolic)	10
Park J	2012	Journal of Cognitive Neuroscience	20	fMRI	23.4	11F 9M	Visual matching task	Number > Letter	1
Pesenti M	2000	Journal of Cognitive Neuroscience	8	PET	21-29	8M	Comparison	Number comparison > Number control (orientation judgement)	7
Pinel P	2004	Neuron	15	fMRI	24	18F 6M	Stroop	Number comparison vs. Size comparison	5
								Number comparison small distance vs. Number Comparison large distance	3
Robertson B D	2015	NeuroImage	16	fMRI	23	8F 8M	Comparison	Incongruent - Congruent contrast	34
Tang J	2006	Journal of Cognitive Neuroscience	18	fMRI	25	7F 11M	Comparison	Numerical > Physical	10
								Physical task conflict trials > Physical task non-conflict trials	1
Vogel S E	2013	Neuropsychologia	14	fMRI	25	7F 7M	Number estimation	Number > Control	10
								Number specific activation	5

Table 3.4

*Summary of studies included in the mental arithmetic meta-analysis.*

Author	Year	Journal	N	Imaging Method	Mean Age	Gender	Tasks	Contrast Name	Location
Andres M	2011	NeuroImage	10	fMRI	21	10M	Subtraction and multiplication of Hindu-Arabic digits	Multiply and subtract	8
								Multiply > Subtract	7
Andres M	2012	NeuroImage	18	fMRI	21.3	18M	Subtraction and multiplication of Hindu-Arabic digits	Arithmetic > Letter reading	5
								Multiplication > Subtraction	2
Chochon F	1999	Journal of Cognitive Neuroscience	8	fMRI	20-30	4M 4F	Subtraction and multiplication of Hindu-Arabic digits	Multiplication > Control	12
								Subtraction > Control	14
								Multiplication > Digit naming	4
								Multiplication > Comparison	1
								Subtraction > Digit naming	11
								Subtraction > Comparison	13
								Subtraction > Multiplication	4
De Visscher A	2015	NeuroImage	20	fMRI	29	10M 10F	Multiplication of Hindu-Arabic digits	Retrieval > Non-retrieval	7
								Non-retrieval > Retrieval	7
Delazer M	2003	Cognitive Brain Research	13	fMRI	30.5	7M 6F	Complex multiplication (2 digit times 1 digit) and fact retrieval multiplication (1 digit times 1 digit)	Retrieval > Number matching	14
								Untrained complex multiplication > Number matching	13
Fehr T	2007	Brain Research	11	fMRI	26.8	5M 6F	1 and 2-digit addition, subtraction and multiplication	Addition complex > Addition simple (A)	17
								Subtraction complex > Subtraction simple (B)	18
								Multiplication complex > Multiplication simple (C)	9
								Division complex > Division simple (D)	15
								Conjunction (A+B+C+D)	8
Fehr T	2010	Neuropsychologia	11	fMRI	26.8	5M 6F	1 and 2-digit addition, subtraction and multiplication	Control Group: Complex > Simple (B)	18
Grabner R H	2007	NeuroImage	25	fMRI	25.7	25M	1 and 2 digit multiplication	Multi-Digit > Single-Digit	15
								Single-Digit > Multi-Digit	1
Grabner R H	2009	Human Brain Mapping	28	fMRI	26.9	28M	Multiplication of Hindu-Arabic digits	Multiplication > Figural-spatial, untrained	2
Grabner R H	2009	Neuropsychologia	28	fMRI	26.9	28M	Addition, subtraction, multiplication and division of Hindu-Arabic digits	Retrieval > Procedural	1
							Procedural > Retrieval	9	
Gruber O	2001	Cerebral Cortex	6	fMRI	25.8	6M	Multiplication of Hindu-Arabic digits	Compound number calculation > Result matching	7
								Simple number calculation > Result matching	8
Gullick M M	2014	Human Brain Mapping	24	fMRI	19	12M 12F	Addition and subtraction of positive and negative integers	Positive operand problems > Negative-operand problems	7
								Negative operand problems > Positive operand problems	8
								Addition problems > Subtraction problems	1
								Subtraction problems > Addition problems	11
Hayashi N	2000	Journal of the Neurological Sciences	10	PET	26.2	10M	Serial number subtraction and multiplication	Subtra-task > Count-task	5
								Multi-task > Count-task	4
Hugdahl K	2004	American Journal of Psychiatry	12	fMRI	31	5M 7F	Addition of Hindu-Arabic digits	Mental Arithmetic - Vigilance, healthy subjects	4
Ischebeck A	2006	NeuroImage	12	fMRI	26.8	4M 8F	Subtraction and multiplication of Arabic digits	Multiplication untrained > Number matching	13
								Subtraction untrained > Number matching	21
Jost K	2009	NeuroImage	16	fMRI	24.5	6M 10F	Single digit multiplication	Small multiplication > Storage	16
								Large multiplication > Small multiplication	8
								Small multiplication > Large multiplication	1
								Small multiplication > Zero multiplication	1
								Small multiplication > storage, Large multiplication > storage, Zero multiplication > storage	12
Keller K	2009	NeuroImage	49	fMRI	23.99	24M 25F	Addition and subtraction in 3 operand equations	Conjunction (males and females) (Calculation - identification)	9
Knops A	2017	NeuroImage	17	fMRI	24.9	8M 9F	Addition and subtraction of Hindu-Arabic digits	Subtraction > Addition	10
Kong J	2005	Cognitive Brain Research	16	fMRI	28	7M 9F	Addition and subtraction of double digits	Main effect of arithmetic type (subtraction > addition)	5
								Main effect of procedure complexity (with carrying > without carrying)	4
Kuo B C	2008	Brain Research	11	fMRI	21-29	6M 6F	Single or dual addition and subtraction	Single addition > Baseline	13
								Single subtraction > Baseline	17
								Dual addition > Baseline	15
								Dual subtraction > Baseline	21
								Dual operation > Baseline	26
Lee K M	2000	Annals of Neurology	11	fMRI	25-35	6M 5F	Subtraction and multiplication of Hindu-Arabic digits	Multiplication > Subtraction	6
							Subtraction > Multiplication	8	
Menon V	2000	Neuroimaging	16	fMRI	20.28	8M 8F	2 and 3 operand equations	3s 3-operand equations > Control equations	16
								3s 2-operand > Control equations	6
								6s 3-operand equations > Control equations	5
								6s 2-operand equations > Control equations	0
								Main effect, operands	3
Pesenti M	2000	Journal of Cognitive	8	PET	21-29	8M	Single digit addition	Addition > Comparison	2

Neuroscience								
							Addition > Orientation, characters (masked with addition > rest)	5
Rosenberg Lee M	2011	Neuropsychologia	20	fMRI	23.9	11M 9F	Addition, subtraction, multiplication and division of Hindu-Arabic digits	5
							Subtraction: Calculation > Identification	6
							Multiplication: Calculation > Identification	6
Soylu F	2017	Cognitive Processing	13	fMRI	24.67	7M 6F	1 and 2-digit addition	6
							Division: Calculation > Identification	4
							Main effect of addition difficulty (two digit > single digit)	6
							Main effect of addition difficulty (One digit > Two digit)	6
Stanescu-Cosson R	2000	Brain	7	fMRI	22-26	3M 4F	Single digit addition and approximation	16
							All calculation > Letter matching	7
							Small numbers > Letter matching	7
Tschentscher N	2014	NeuroImage	26	fMRI		13M 13F	1 and 2-digit addition and multiplication	6
							Arithmetic fact retrieval > Procedural strategies	7
							Procedural strategies > Arithmetic fact retrieval	7
van der Ven F	2016	Brain Research	23	fMRI	21.04	8M 15F	Addition and subtraction of Hindu-Arabic digits	24
							[(Arabic addition > Arabic memory) > (Dot addition > Dot memory)]	24
Venkatraman V	2005	Neuropsychologia	10	fMRI	20-25	7M 3F	Single digit addition and approximation	5
							Symbolic exact addition > Control	7
							Symbolic approximate addition > Control	7
							Conjunction (symbolic and nonsymbolic addition, exact and approximate)	5
Zago L	2001	NeuroImage	6	PET	21	6M	1 and 2-digit multiplication	7
							Retrieve > Read	14
							Compute > Read	14
							Compute and retrieve > Read conjunction	15
							Compute > Retrieve masked by compute > Read	15
							Retrieve > Compute masked by retrieve > Read	2
Zarnhofer S	2012	Behavioral and Brain Functions	42	fMRI	23	20M 21F	Subtraction and multiplication of Hindu-Arabic digits	13
							Subtraction > Multiplication	8
							Multiplication > Subtraction	8

## 3.4 Results

### 3.4.1 Single Dataset Meta-Analyses

#### 3.4.1.1 Mental Rotation ALE Map

The ALE map for mental rotation included 28 individual studies (Table 3.5) and revealed six clusters of convergent brain regions associated with mental rotation performance. From largest to smallest, these regions included the right precuneus (hIP3), left superior parietal lobe, left inferior parietal lobe, left middle frontal gyrus, right middle frontal gyrus, and left middle frontal gyrus (Figure 3.2; see Table 3.5 for details). In sum, mental rotation was associated with neural activity in the bilateral parietal and frontal regions, with the largest regions of convergence in the right IPS.

#### 3.4.1.2 Symbolic Number ALE Map

The ALE map for basic symbolic number skills included 24 individual studies (Table 3.6) and revealed four clusters of convergent brain regions associated with symbolic number processing. From largest to smallest, these regions included the left superior parietal lobule, right inferior parietal lobe (IPS), right superior frontal gyrus, and right insula

(Figure 3.2; see Table 3.6 for details). In sum, symbolic number processing was associated with bilateral parietal activity and right frontal activity.

### 3.4.1.3 Mental Arithmetic ALE Map

The ALE map for mental arithmetic included 31 individual studies (Table 3.7) and revealed nine clusters of convergent brain regions associated with mental arithmetic. From largest to smallest, these regions included the left inferior parietal lobule (hIP3), right precuneus, left inferior frontal gyrus, left superior frontal gyrus, left insula, right insula, right middle frontal gyrus, left middle frontal gyrus, and right sub-gyral. (Figure 3.2; see Table 3.7 for details). In sum, mental arithmetic was associated with neural activity in the left IPS and a host of bilateral parietal and frontal regions.

### 3.4.1.4 Summary of Single Dataset Meta-Analyses

All three cognitive tasks were associated with brain activity in fronto-parietal cortex (see Figure 3.3). More specifically, for all three tasks the largest region of convergence was found in the IPS as well as neighboring regions including the inferior and superior parietal lobes. Additionally, all three tasks were associated with frontal activity.

Table 3.5

*Mental rotation single dataset analyses.*

Cluster #	Anatomical Labels		BA	X	Y	Z	ALE	Vol/mm3
	Talairach Daemon	Anatomy Toolbox						
1	R. Precuneus	hIP3 (IPS)	7	24	-60	48	0.029	12528
	R. Inferior Parietal Lobule	hIP2 (IPS)	40	36	-42	40	0.026	
	R. Precuneus	Area 7 (SPL)	7	20	-66	44	0.026	
	R. Superior Parietal Lobule	Area 5L (SPL)	7	16	-60	58	0.021	
	R. Precuneus	R. Middle Occipital	19	28	-74	30	0.020	
	R. Superior Occipital Gyrus	R. Middle Occipital	19	32	-70	22	0.016	
	R. Superior Parietal Lobule	Area 7PC (SPL)	7	24	-56	60	0.016	
	R. Inferior Parietal Lobule	hIP3 (IPS)	40	36	-48	50	0.016	
2	L. Superior Parietal Lobule	Area 7PC (SPL)	7	-28	-60	54	0.025	6568
	L. Precuneus	Area 7A (SPL)	7	-18	-64	46	0.020	
	L. Precuneus	L. Inferior Parietal	19	-28	-72	38	0.019	
	L. Superior Parietal Lobule	Area 7A (SPL)	7	-14	-62	56	0.018	
	L. Precuneus	L. Superior Parietal	7	-18	-76	42	0.014	
	L. Precuneus	L. Middle Occipital	31	-24	-76	26	0.013	
3	L. Inferior Parietal Lobule	Area 2	40	-40	-40	50	0.027	2408
4	L. Middle Frontal Gyrus	L. Middle Frontal	6	-26	-2	60	0.024	2216
	L. Sub-Gyral	L. Middle Frontal	6	-24	-2	54	0.021	
	L. Middle Frontal Gyrus	L. Middle Frontal	6	-34	2	50	0.013	
5	R. Middle Frontal Gyrus	R. Middle Frontal	6	26	-8	52	0.024	2064
	R. Middle Frontal Gyrus	R. Superior Frontal	6	26	0	62	0.016	
6	L. Middle Frontal Gyrus	L. IFG	46	-42	18	22	0.016	928
	L. Middle Frontal Gyrus	Area 44	9	-52	4	32	0.014	

Table 3.6

*Symbolic number single dataset analyses.*

Cluster #	Anatomical Labels		BA	X	Y	Z	ALE	Vol/mm3
	Talairach Daemon	Anatomy Toolbox						
1	L. Superior Parietal Lobule	L. Superior Parietal	7	-26	-58	42	0.025	7208
	L. Inferior Parietal Lobule	hIP1 (IPS)	40	-34	-50	36	0.020	
	L. Superior Parietal Lobule	Area 7A (SPL)	7	-28	-66	46	0.019	
	L. Supramarginal Gyrus	hIP2 (IPS)	40	-42	-44	36	0.019	
	L. Inferior Parietal Lobule	Area 7PC (SPL)	40	-38	-50	50	0.017	
2	R. Inferior Parietal Lobule	hIP1 (IPS)	40	34	-44	38	0.031	5576
	R. Precuneus	R. Angular Gyrus	7	28	-64	38	0.025	
	R. Precuneus	R. Middle Occipital	31	26	-72	26	0.020	
3	R. Superior Frontal Gyrus	R. Posterior-Medial	6	2	10	48	0.021	936
4	R. Insula	R. Insula		30	20	2	0.021	880

Table 3.7

*Mental arithmetic single dataset analyses.*

Cluster #	Anatomical Labels		BA	X	Y	Z	ALE	Vol/mm3
	Talairach Daemon	Anatomy Toolbox						
1	L. Inferior Parietal Lobule	hIP3 (IPS)	7	-30	-56	42	0.051	12144
	L. Superior Parietal Lobule	hIP3 (IPS)	7	-28	-62	44	0.048	
	L. Inferior Parietal Lobule	Area PFt (IPL)	40	-44	-38	40	0.038	
	L. Inferior Parietal Lobule	L. Inferior Parietal	40	-40	-44	42	0.031	
	L. Precuneus	Area 7A (SPL)	7	-12	-64	50	0.023	
2	L. Superior Parietal Lobule	Area 7A (SPL)	7	-12	-70	54	0.022	9680
	R. Precuneus	R. Angular Gyrus	19	30	-66	40	0.046	
	R. Inferior Parietal Lobule	hIP2 (IPS)	40	42	-42	44	0.041	
3	R. Inferior Parietal Lobule	hIP3 (IPS)	40	34	-50	42	0.036	7024
	L. Inferior Frontal Gyrus	L. IFG	9	-44	6	28	0.069	
4	L. Middle Frontal Gyrus	L. IFG	9	-46	26	30	0.027	5952
	L. Superior Frontal Gyrus	L. Posterior-Medial	6	-2	14	50	0.033	
5	L. Superior Frontal Gyrus	R. Superior Medial	8	0	18	48	0.032	2440
	L. Insula	L. IFG	47	-32	24	2	0.036	
6	R. Insula	R. IFG	47	32	24	0	0.038	2328
	R. Middle Frontal Gyrus	Area 45	9	48	14	26	0.032	
7	L. Middle Frontal Gyrus	L. Middle Frontal	6	-26	-6	52	0.025	1936
	L. Middle Frontal Gyrus	L. Middle Frontal	6	-26	6	60	0.023	
8	R. Sub-Gyral	R. Middle Frontal	6	28	2	52	0.021	912



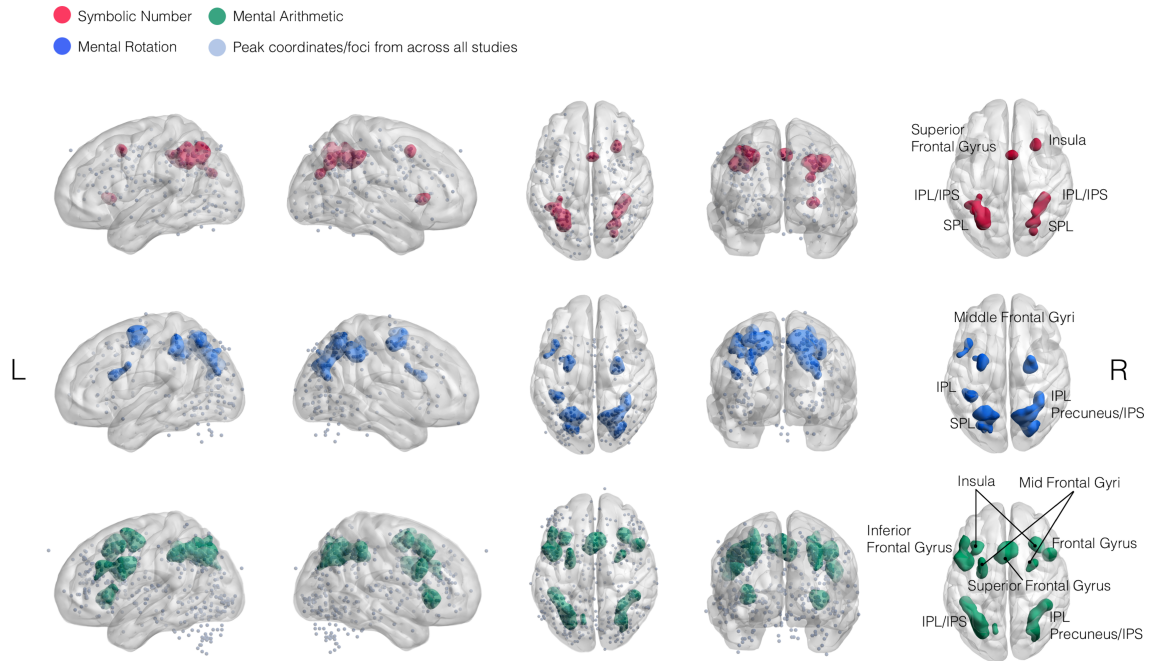


Figure 3.2 Single dataset ALE maps for each cognitive construct of interest.

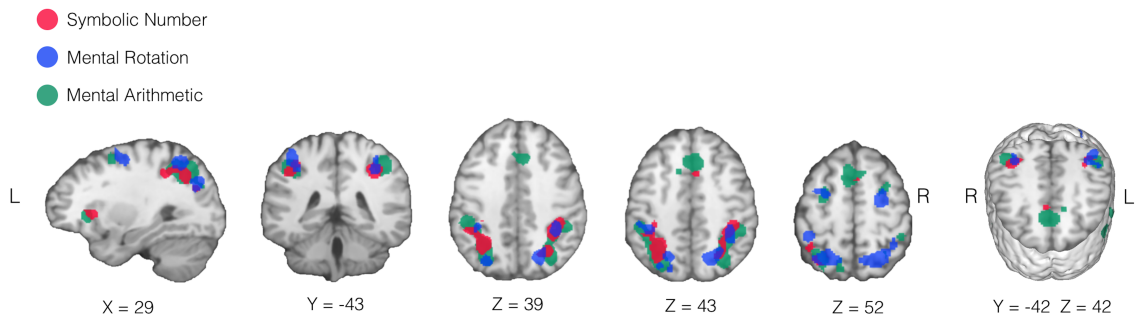


Figure 3.3 Qualitative map of overlapping ALE maps for each cognitive process.

### 3.4.2 Conjunction and Contrast Analyses

Conjunction and contrast analyses were computed to identify regions of brain activation that were overlapping and distinct for mental rotation, arithmetic, and symbolic number processing. Each conjunction and contrast analysis was carried out through a series of

pairwise comparisons. All reported results were statistically significant at an uncorrected threshold of  $p < .01$ .

### 3.4.2.1 Conjunction and Contrast ALE Maps: Mental Rotation and Symbolic Number

The conjunction analysis for mental rotation and symbolic number revealed five brain regions that were activated by both cognitive processes, including the right inferior parietal lobule (IPS), the left superior parietal lobule, the left inferior parietal lobule (IPS), and two separate regions of the precuneus (Figure 3.4; Table 3.8).

Contrast analyses revealed several brain regions that were specific to mental rotation (i.e., mental rotation > number), including the right precuneus, left middle frontal gyrus, left precuneus, right precuneus, right superior frontal gyrus, and the left cuneus (Figure 3.4; Table 3.8). Regions that were specific to number (i.e., number > mental rotation) included the left inferior parietal lobule (hIP3) and right claustrum/insula (Figure 3.4; Table 3.8).

These analyses highlight that both mental rotation and symbolic number processing were associated with overlapping brain activity in around the parietal lobe. However, each construct was also sub-served by specific distinct regions within the parietal lobe. Additionally, mental rotation was associated with frontal activation in the superior and middle frontal gyri.

### 3.4.2.2 Conjunction and Contrast ALE Maps: Mental Rotation and Mental Arithmetic

The conjunction analysis for mental rotation and mental arithmetic revealed six brain regions that were activated by both cognitive processes. From largest to smallest, these regions included the right precuneus (hIP3), left superior parietal lobule, left inferior parietal lobule, left sub-gyral, left middle frontal gyrus, and right sub-gyral (Figure 3.4; Table 3.9).

Contrast analyses identified brain regions that were specifically related to mental rotation (i.e., mental rotation > mental arithmetic) including, the right superior parietal lobule, two separated regions of the right precuneus, and the left postcentral gyrus (Figure 3.4; Table 3.9). Contrast analyses also identified brain regions that were specifically related to mental arithmetic (i.e., mental arithmetic > mental rotation) including the left inferior frontal gyrus, left precuneus/angular gyrus, right precuneus, right inferior parietal lobule, right insula, left claustrum, right medial frontal gyrus, left medial frontal gyrus, right middle frontal gyrus, left inferior parietal lobe (hIP2), left inferior frontal gyrus, and left middle frontal gyrus (Figure 3.4; Table 3.9).

Together, these conjunction and contrast analyses revealed that mental rotation and mental arithmetic were associated with overlapping brain activity in regions associated with the fronto-parietal network. However, each task was also associated with distinct activity in the parietal lobe and in the case of mental arithmetic, regions in the frontal lobe as well.

### 3.4.2.3 Conjunction and Contrast ALE Maps: Mental Arithmetic and Symbolic Number

Results of the conjunction analysis for mental arithmetic and symbolic number revealed five brain regions that were activated by both tasks. These regions included large bilateral regions of the superior and inferior parietal lobes, including the IPS, right insula, and the left superior frontal gyrus (Figure 3.4; Table 3.10).

Contrast analyses identified brain regions specifically related to mental arithmetic ((i.e., mental arithmetic > symbolic number), including the left inferior frontal gyrus, left medial frontal gyrus, right precuneus, right inferior parietal lobule, left sub-gyral regions, left precuneus, left claustrum, left inferior parietal lobule, right inferior frontal gyrus, right middle frontal gyrus, left middle frontal gyrus, left precuneus, and another region of the right middle frontal gyrus. No brain regions were specifically activated during symbolic number processing that were not also activated during arithmetic (i.e., number > mental arithmetic).

Therefore, mental arithmetic and symbolic number were associated with large overlapping regions in the bilateral parietal lobes, including all embankments of the IPS (i.e., hIP1-3). Mental arithmetic was also associated with distinct brain activity in a number of regions in the fronto-parietal network. There was no distinct brain associated with symbolic number.

Table 3.8

*Conjunction and contrast analyses (mental rotation, number)*

Cluster #	Anatomical Labels		BA	X	Y	Z	ALE	Vol/mm3
	Talairach Daemon	Anatomy Toolbox						
<i>Mental rotation and Number</i>								
<b>1</b>	R. Inferior Parietal Lobule	hIP2 (IPS)	40	36	-42	40	0.026	2624
	R. Superior Parietal Lobule	hIP3 (IPS)	7	26	-62	42	0.019	
	R. Precuneus	R. Middle Occipital	31	26	-74	28	0.017	
	R. Superior Parietal Lobule	hIP3 (IPS)	7	30	-54	44	0.013	
	R. Precuneus	R. Superior Occipital	7	26	-66	34	0.013	
	R. Precuneus	R. Superior Occipital	19	30	-68	34	0.012	
<b>2</b>	L. Superior Parietal Lobule	Area 7A (SPL)	7	-22	-64	46	0.016	1016
	L. Superior Parietal Lobule	Area 7A (SPL)	7	-26	-62	48	0.015	
	L. Superior Parietal Lobule	Area 7A (SPL)	7	-32	-62	50	0.014	
	L. Precuneus	L. Superior Parietal	7	-22	-56	46	0.013	
	L. Precuneus	L. Superior Parietal	19	-28	-68	42	0.013	
<b>3</b>	L. Inferior Parietal Lobule	Area 2	40	-38	-44	46	0.014	656
	L. Inferior Parietal Lobule	hIP3 (IPS)	40	-38	-44	40	0.013	
<i>Mental rotation &gt; Number</i>								
<b>1</b>	R. Precuneus	Area 7A (SPL)	7	13.6	-64	51.7	3.72	2096
	R. Precuneus	R. Precuneus	7	7.3	-62.4	51.1	3.54	
	R. Superior Parietal Lobule	Area 7A (SPL)	7	12	-63	60	3.24	
<b>2</b>	L. Middle Frontal Gyrus	unknown area	6	-27.7	0.8	63.4	3.72	1264
	L. Middle Frontal Gyrus	L. Superior Frontal	6	-22	-6	61	2.82	
<b>3</b>	L. Precuneus	Area 7A (SPL)	7	-13	-60	52	3.35	1176
	L. Superior Parietal Lobule	Area 7A (SPL)	7	-10	-64	58	3.24	
<b>4</b>	R. Precuneus	R. Superior Occipital	19	24	-78	38	3.09	368
	R. Cuneus	R. Superior Occipital	19	26	-82	36	2.85	
<b>5</b>	R. Superior Frontal Gyrus	R. Superior Frontal	6	26	-2	64	3.04	144
	R. Middle Frontal Gyrus	R. Middle Frontal	6	28	-6	58	2.38	
<b>6</b>	R. Inferior Parietal Lobule	Area 7PC (SPL)	40	40	-48	52	2.56	56
<i>Number &gt; Mental Rotation</i>								
<b>1</b>	L. Inferior Parietal Lobule	hIP3 (IPS)	40	-32	-53	44	3.04	560
	L. Superior Parietal Lobule	hIP3 (IPS)	7	-30	-58	42	2.99	
<b>2</b>	R. Claustrum	unknown area		28	18	6	2.40	136
	R. Insula	R. Insula	13	32	20	6	2.38	
	R. Claustrum	unknown area		27	24	5	2.33	

Note. Bolded numbers represent clusters that passed the uncorrected threshold of  $p < .001$  whereas unbolded number indicate cluster regions significant at  $p < .01$ .

Table 3.9

*Conjunction and contrast analyses (mental rotation, mental arithmetic)*

Cluster #	Anatomical Labels		BA	X	Y	Z	ALE	Vol/mm3
	Talairach Daemon	Anatomy Toolbox						
<i>Mental rotation and Arithmetic</i>								
<b>1</b>	R. Precuneus	hIP3 (IPS)	7	28	-56	48	0.025	3936
	R. Inferior Parietal Lobule	hIP2 (IPS)	40	36	-44	42	0.024	
	R. Superior Parietal Lobule	hIP3 (IPS)	7	26	-62	44	0.022	
	R. Cuneus	R. Middle Occipital	19	30	-76	30	0.019	
<b>2</b>	L. Superior Parietal Lobule	Area 7PC (SPL)	7	-28	-60	52	0.024	2968
	L. Precuneus	L. Inferior Parietal	19	-28	-72	38	0.019	
	L. Precuneus	Area 7A (SPL)	7	-14	-62	50	0.017	
	L. Precuneus	Area 7A (SPL)	7	-18	-74	42	0.013	
<b>3</b>	R. Inferior Parietal Lobule	Area 2	40	-42	-42	48	0.021	1056
<b>4</b>	L. Sub-Gyral	L. Middle Frontal	6	-24	-2	54	0.021	920
	L. Middle Frontal Gyrus	L. Middle Frontal	6	-26	0	58	0.020	
<b>5</b>	L. Middle Frontal Gyrus	L. IFG	9	-42	14	24	0.014	464
	L. Inferior Frontal Gyrus	Area 44	9	-52	4	32	0.014	
<b>6</b>	R. Sub-Gyral	R. Superior Frontal	6	24	-4	54	0.017	248
	R. Sub-Gyral	R. Middle Frontal	6	28	-2	52	0.016	
<i>Mental rotation &gt; Arithmetic</i>								
<b>1</b>	R. Superior Parietal Lobule	Area 7PC (SPL)	7	22	-54	58	2.748	240
<b>2</b>	R. Precuneus	R. Precuneus	7	14	-64	41	2.706	176
<b>3</b>	R. Precuneus	R. Superior Occipital	19	22	-74	34	2.447	112
	R. Cuneus	R. Superior Occipital	19	20	-80	32	2.374	
<b>4</b>	L. Postcentral Gyrus	L. Postcentral Gyrus	40	-36	-36	54	2.536	112
<i>Arithmetic &gt; Mental Rotation</i>								
<b>1</b>	L. Inferior Frontal Gyrus	L. Precentral Gyrus	9	-42.6	6.1	31	3.719	4688
	L. Inferior Frontal Gyrus	Area 44	9	-47.2	1.2	26.8	3.540	
	L. Precentral Gyrus	L. Middle Frontal	9	-44	22	34	3.090	
	L. Middle Frontal Gyrus	L. Middle Frontal	9	-48	22	34	3.036	
	L. Inferior Frontal Gyrus	Area 44	44	-52	8	16	2.989	
	L. Middle Frontal Gyrus	L. Middle Frontal	9	-43	28	34	2.878	
<b>2</b>	L. Precuneus/Angular Gyrus	hIP3 (IPS)	39	-33	-57.5	42.3	3.719	3080
	L. Superior Parietal Lobule	Area 7A (SPL)	7	-30	-68	46	3.156	
<b>3</b>	R. Precuneus	R. Angular Gyrus	19	33.3	-67.1	41	3.719	1528
<b>4</b>	R. Inferior Parietal Lobule	R. Area PF (IPL)	40	47.1	-39.5	43.8	3.719	1400
<b>5</b>	R. Insula	R. Insula	13	33.9	20.9	5.4	3.719	1216
	R. Insula	unknown area	13	28	26	8	3.353	
<b>6</b>	L. Claustrum	L. Insula		-30.7	17.3	6	3.719	1208
	L. Insula	L. Insula	13	-30	20	4	3.540	
<b>7</b>	R. Medial Frontal Gyrus	R. Superior Medial	8	8	20	44	3.036	1040
	R. Cingulate Gyrus	R. MCC	32	10	24	38	2.911	
<b>8</b>	L. Medial Frontal Gyrus	L. Posterior Medial	8	-8	17	50	3.156	744
	L. Medial Frontal Gyrus	L. Superior Medial	8	-8	22	48	3.036	
<b>9</b>	R. Middle Frontal Gyrus	Area 44	9	46.7	18.7	31.3	3.239	680
<b>10</b>	L. Inferior Parietal Lobe	hIP2 (IPS)	40	-44	-42	38	3.353	680
	L. Supramarginal Gyrus	hIP2 (IPS)	40	-48	-40	34	3.239	
	L. Supramarginal Gyrus	hIP2 (IPS)	40	-44	-38	34	3.090	
<b>11</b>	L. Inferior Frontal Gyrus	L. IFG	47	-38	26	0	2.489	64
<b>12</b>	L. Middle Frontal Gyrus	L. IFG	46	-44	36	16	2.948	64

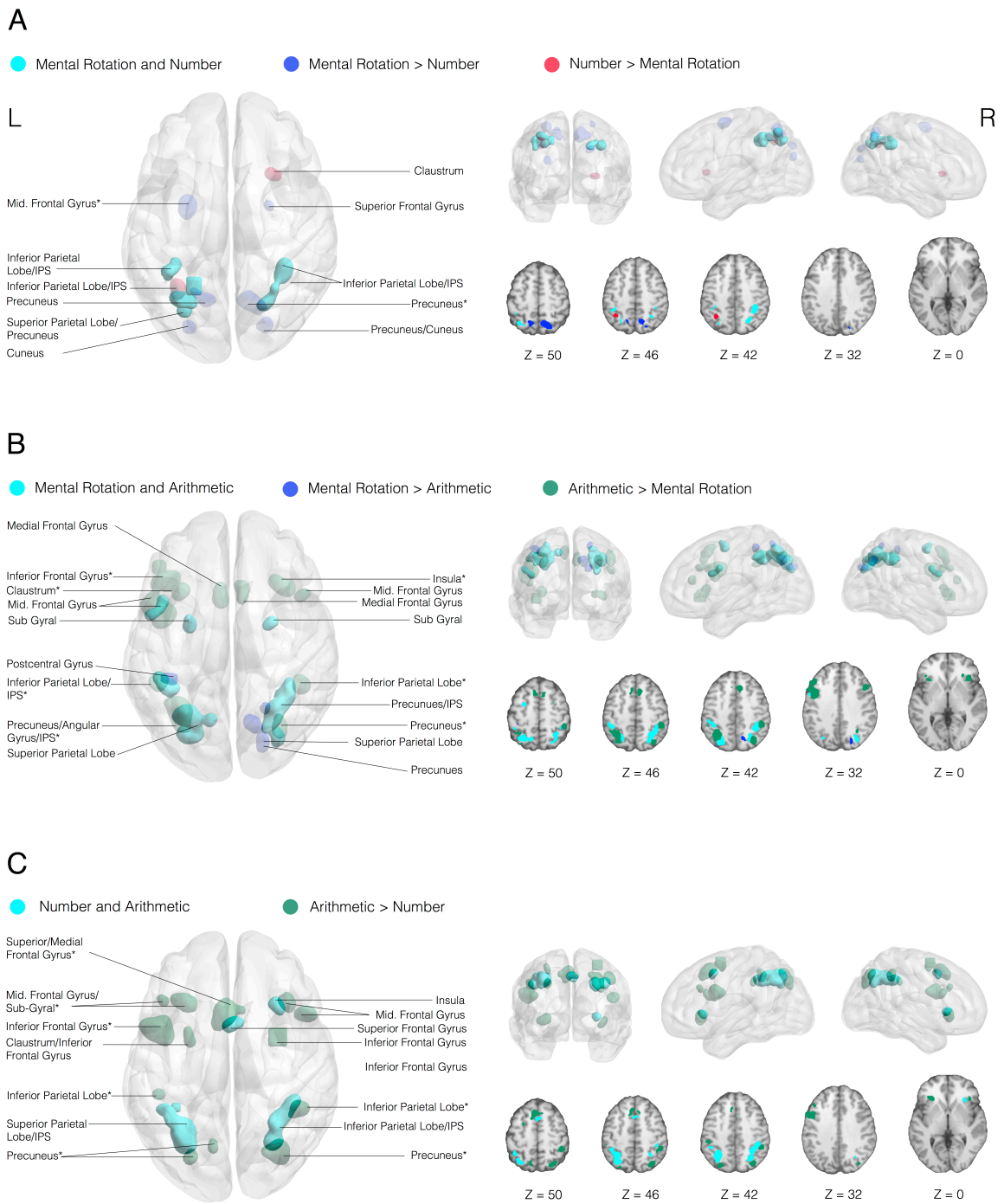
Note. Bolded numbers represent clusters that passed the uncorrected threshold of  $p < .001$  whereas unbolded number indicate cluster regions significant at  $p < .01$ .

Table 3.10

*Conjunction and contrast analyses (mental arithmetic, number)*

Cluster #	Anatomical Labels		BA	X	Y	Z	ALE	Vol/mm3
	Talairach Daemon	Anatomy Toolbox						
<i>Arithmetic and Number</i>								
<b>1</b>	L. Superior Parietal Lobule	L. Superior Parietal	7	-26	-58	42	0.025	5272
	L. Superior Parietal Lobule	Area 7A (SPL)	7	-28	-66	46	0.019	
	L. Supramarginal Gyrus	hIP2 (IPS)	40	-42	-44	36	0.019	
	L. Inferior Parietal Lobe	Area 7PC (SPL)	40	-38	-50	50	0.017	
	L. Inferior Parietal Lobe	hIP1 (IPS)	40	-36	-48	36	0.017	
	L. Inferior Parietal Lobe	hIP1 (IPS)	40	-34	-52	38	0.017	
<b>2</b>	R. Inferior Parietal Lobule	hIP2 (IPS)	40	38	-40	40	0.026	3344
	R. Inferior Parietal Lobule	hIP3 (IPS)	40	34	-46	40	0.025	
	R. Precuneus	R. Angular Gyrus	7	28	-64	38	0.025	
<b>3</b>	R. Insula	R. Insula		30	22	2	0.021	512
<b>4</b>	L. Superior Frontal Gyrus	R. Posterior-Medial	6	0	10	48	0.020	496
<b>5</b>	R. Cuneus	R. Middle Occipital	19	30	-76	26	0.012	8
<b>6</b>	R. Cuneus	R. Middle Occipital	19	28	-76	28	0.013	8
<b>7</b>	R. Cuneus	R. Middle Occipital	19	30	-74	28	0.012	8
<b>8</b>	R. Precuneus	R. Middle Occipital	19	28	-72	30	0.011	8
<i>Arithmetic &gt; Number</i>								
<b>1</b>	L. Inferior Frontal Gyrus	Area 44 (IFG)	44	-46.3	9.6	27	3.72	3320
	L. Precentral Gyrus	L. IFG	6	-39	0	25	3.24	
<b>2</b>	L. Medial Frontal Gyrus	L. Superior Medial	8	-6.7	19.3	51.9	3.72	1976
	L. Superior Frontal Gyrus	L. Posterior-Medial	6	-8	14	50	3.54	
	R. Medial Frontal Gyrus	R. Superior Medial	8	6	22	48	3.09	
	L. Cingulate Gyrus	L. Superior Medial	32	-6	24	40	2.50	
	R. Precuneus	Area PGp (IPL)	19	31.3	-74.2	42.8	3.54	
<b>4</b>	R. Inferior Parietal Lobule	Area PFm (IPL)	40	43.3	-46	49.3	3.72	1072
	R. Inferior Parietal Lobule	Area PFm (IPL)	40	48	-46	46	3.54	
<b>5</b>	L. Sub-Gyral	L. Middle Frontal	6	-24	5.4	58.6	3.35	880
	L. Middle Frontal Gyrus	L. Middle Frontal	6	-25.3	0	60.7	3.16	
<b>6</b>	L. Inferior Parietal Lobe	Area 2	40	-46	-34	44	2.77	672
	L. Inferior Parietal Lobe	Area PFt (IPL)	40	-48	-34	34	2.55	
<b>7</b>	L. Precuneus	Area PFt (IPL)	19	-25	-78	41	3.09	624
	L. Precuneus	L. Superior Parietal	19	-26	-74	39	3.04	
<b>8</b>	R. Inferior Frontal Gyrus	R. IFG	47	40	24	-4	3.35	552
	R. Inferior Frontal Gyrus	R. IFG	47	38	28	-6	2.95	
<b>9</b>	L. Inferior Frontal Gyrus	L. IFG	47	-32	28	-2	2.99	520
	L. Claustrum	unknown area		-24	26	2	2.95	
<b>10</b>	R. Inferior Frontal Gyrus	Area 45	9	52	18	22	2.91	520
	R. Middle Frontal Gyrus	Area 45	46	46	20	26	2.82	
	R. Middle Frontal Gyrus	R. IFG	46	46	18	22	2.77	
<b>11</b>	L. Middle Frontal Gyrus	L. IFG	9	-42	24	32	2.69	232
	L. Middle Frontal Gyrus	L. Middle Frontal Gyrus	9	-44	28	34	2.54	
<b>12</b>	L. Precuneus	Area 7A (SPL)	7	-10	-66	48	2.73	224
	L. Precuneus	Area 7A (SPL)	7	-8	-70	50	2.56	
<b>13</b>	R. Middle Frontal Gyrus	R. Middle Frontal Gyrus	6	30	2	62	2.69	56
<i>Number &gt; Arithmetic</i>								

Note. Bolded numbers represent clusters that passed the uncorrected threshold of  $p < .001$  whereas unbolded number indicate cluster regions significant at  $p < .01$ .



*Figure 3.4* Brain regions associated with the conjunction and contrast analyses. Note: \* indicates regions that passed the uncorrected threshold of  $p < .001$ .

## 3.5 Discussion

This study was designed to achieve two goals. First, we aimed to reveal the locations of brain regions associated with neural activity across three key aspects of mathematical thinking: Basic symbolic number processing, mental arithmetic, and spatial reasoning (mental rotation). Second, we aimed to go beyond identifying the locations of these processes, by also testing theoretically-informed predictions as to when, why, and where we should expect to see cognitively-defined associations and dissociations between numerical and spatial processing (see Figure 3.1 and Table 3.1). Specifically, given the common need to engage in mental manipulation, we predicted overlap in brain regions subserving this shared process between mental arithmetic and mental rotation. Using similar logic, we aimed to reveal regions more sensitive to symbolic number processing by comparing neural activity common to symbolic number and arithmetic processes, but not mental rotation. Examining these three processes provided a means to examine the representation versus manipulation of numerical information in the brain. Moreover, by also studying the neural correlates of mental rotation, we were able to better pinpoint specific points of convergence and divergence between spatial and numerical processing.

Overall, results of the current quantitative meta-analyses revealed considerable overlap across mental rotation, arithmetic, and symbolic number processing in bilateral regions along the parietal lobe. This was apparent through a qualitative comparison of the meta-analytic ALE maps for each cognitive task (i.e., single dataset meta-analyses), but critically, further revealed through quantitative conjunction analyses. More specifically, the IPS was found to be the largest and most consistent region of overlap across all three cognitive tasks. Whereas the left IPS was the largest region of activation for symbolic number and arithmetic, the right IPS was the largest region of activation for mental rotation. The neighboring regions of the inferior and superior parietal lobules were also common to all three tasks. In addition, mental rotation and mental arithmetic were also associated with overlapping frontal regions, namely the middle frontal gyrus.

The results of the contrast analyses revealed several distinct regions of activity associated with each task. Despite widespread regions of overlap in the bilateral parietal lobes, all three tasks were also found to activate distinct activity in nearby parietal



regions. Bilateral regions of the inferior parietal lobes, including the left IPS, were more active for symbolic number processing, including arithmetic, compared to mental rotation. Compared to symbolic number and arithmetic processes, mental rotation was associated with greater activity in the right precuneus. Regions common to both mental manipulation tasks (i.e., mental arithmetic and mental rotation), but not basic symbolic number processes, included the middle frontal gyrus. Lastly, compared to basic numerical processes and mental rotation, mental arithmetic was associated with a host of unique regions in both frontal and parietal regions.

In summary, our findings indicate that the performance of symbolic number processing, mental arithmetic, and mental rotation are all associated with widespread activity in the bilateral parietal lobes. Mental rotation and mental arithmetic were also associated with common frontal activity in the left middle frontal gyrus. Mental arithmetic and symbolic number were associated with common frontal activity in the right insula/clastrum. These findings provide important insight into the neural regions that support mathematical thinking more generally and the neural underpinnings of numerical and spatial reasoning more specifically. In the following sections, we discuss these key findings and offer several theoretical accounts for why spatial and numerical cognition recruit a common bilateral parietal network. We then turn our attention to brain regions found to be more uniquely active for some cognitive operations (e.g., mental manipulation) compared to others (symbolic processing).

### 3.5.1 Brain Regions Common to All Three Cognitive Tasks

In line with prior research and theory, our findings suggest the parietal lobe is actively engaged during various mathematical tasks (Desco et al., 2011; Matejko & Ansari, 2015). More specifically, the neural activity associated with all three mathematical reasoning domains – symbolic number processing, arithmetic, and mental rotation – were all found to recruit the bilateral IPS and the closely neighboring regions of the inferior and superior parietal lobes. These results challenge domain-specific accounts of the IPS, suggesting instead that the IPS may play a more general role in mathematical cognition.

What explains the observed neural overlap between number, arithmetic, and mental rotation? One explanation is that all three processes are part of a general magnitude system (Walsh, 2003; Leibovitz, Naama, Maayan, & Henik, 2017). That is, all three tasks involve making comparisons and judgments about magnitudes. In the case of number and arithmetic, participants are required to reason about discrete and symbolic quantities (numerals 0-9). Mental rotation, however, involves reasoning about continuous relations and degrees of magnitude between objects (e.g., angles of rotation). The common need to reason about quantitative relations between objects (be they symbolic numbers or meaningless objects) may indeed be one reason for the observed overlap. That time and luminance judgments have also been found to consistently activate bilateral parietal regions (e.g., see Walsh, 2003), provides further evidence that a general magnitude system might be at work.

Another way in which number, arithmetic, and mental rotation might be linked is through a common action-based neural network dedicated to perceiving and acting on objects. Critically, this view is not at odds with the general magnitude theory, but aims to extend it through incorporating goal-directed behavior into the account (Walsh, 2003). For example, according to Walsh's 'a theory of magnitude,' space, quantity, and time are all linked through a common metric for action (Walsh, 2003). In this view, numbers and mental rotation stimuli (e.g., 3D cube figures) are alike in that they both represent objects to be acted on. Indeed, there is both theoretical as well as empirical support for the embodied perspective that numbers – although abstract – rely on neural resources specialized for interacting with the physical world (e.g., see Anderson, 2010; 2015; Lakoff & Núñez, 2000; Marghetis, Núñez, & Bergen, 2014). According to the 'neuronal re-cycling hypothesis' (Dehaene & Cohen, 2007), numbers as well as other mathematical symbols, may co-opt or re-use the brain's more ancient and evolutionarily adaptive spatial and sensorimotor systems; systems that originally served the purpose of interacting with tools, objects, and locations in space (Johnson-Frey, 2003; Lakoff & Núñez, 2000; Dehaene et al., 2003). In short, "*we may recycle the brain's spatial prowess to navigate the abstract mathematical world*" (Marghetis, Núñez, & Bergen, 2014, p. 1580).

Taken together, both the ‘general magnitude theory’ and ‘neuronal re-cycling hypothesis’ present plausible explanations for the common neural activity observed between all three processes. More specifically, the ‘neuronal re-cycling hypothesis’ offers a more pointed explanation of why spatial and numerical thinking may recruit common neural substrate.

### **3.5.2 Spatial Visualization as a Key Contributor to Spatial-Numerical Relations**

The present findings offer an extended possibility for the involvement of spatial processing in performing numerical and mathematical tasks. Although prior research efforts have examined neural relations between lower-level spatial processes, such as making simple comparative judgments involving a variety of spatial magnitudes (e.g., line lengths), the relations between more cognitively demanding visual-spatial reasoning tasks, such as mental rotation, and numerical cognition has yet to be examined. Our findings demonstrate that brain regions associated with mental rotation – a widely accepted proxy for higher-level visual-spatial reasoning – are also activated during numerical and arithmetical reasoning. This finding suggests that the relation between space and number is not limited to lower-level spatial processes, namely magnitude judgements. Instead, our findings hint at the possibility that higher-level spatial skills may be implicated in the formation of numerical-spatial associations. Consistent with prior behavioral findings, including the ‘mental number line’ hypothesis, spatial visualization skills may play a critical role in mapping number as well as other mathematical entities to space. In other words, one of the ways humans might conceptualize the meaning of numbers and various other mathematical concepts is by visualizing and, through practice, internalizing their inherent visual-spatial relations and structure. Further research is needed to more fully examine this possibility.

### **3.5.3 Distinct Brain Regions Associated with Each Task**

### 3.5.3.1 Brain Regions More Attuned to Symbolic Number Processing

To gain insight into brain regions potentially underlying symbolic number processing, we carried out conjunction analyses between the symbolic number and arithmetic maps and then contrasted each individual map with the mental rotation map. Based on this logic, we hypothesized that if a symbolic number region exists it should be present in both the symbolic number and arithmetic maps and either absent or present to a much lesser extent in the mental rotation map. This approach yielded evidence that compared to mental rotation, symbolic numerical reasoning, including arithmetic, may be associated with larger regions of activity in the inferior parietal lobes, including the left IPS and regions that appear to overlap with the left angular gyrus. One explanation for this finding might have to do with the relatively ease and automaticity in which individuals are able to access the meaning of numerical symbols and basic operations (e.g.,  $2 + 1$ ). Prior research indicates that fluency and automatic processing of numbers and arithmetic facts is associated with activity in left lateralized ‘language based’ regions, namely the left angular gyrus and supramarginal gyrus (Dehaene et al., 2003; Polspoel, Peters, Vandermosten, & De Smedt, 2017). The current findings might reflect the neural consequences of learning the symbolic number system and associated arithmetic facts. Compared to mental rotation, symbolic number and arithmetic facts are more likely to be stored as verbally mediated knowledge. This view is in general agreement with Dehaene’s triple code model (2003), in which the left angular gyrus is posited as the location where number names and arithmetical facts are stored.

It is worth mentioning that the above findings are based on an uncorrected p-value of .01. When the more stringent cut-off is used ( $p < .001$ ), a different pattern of findings emerges. Instead, the data fail to support the presence of regions unique to symbolic number compared to mental rotation. Thus, the above finding of regions more attuned to symbolic number compared to mental rotation should be interpreted with caution. A more parsimonious interpretation of the current meta-analysis is that both numerical and spatial reasoning engage highly similar bilateral regions of the parietal lobe. Evidently, more research is needed to further disentangle whether, when, and how symbolic number

processes and visual-spatial reasoning engage distinct neural regions. The findings from these studies may prove useful in advancing theories of symbolic specific regions (triple code model) versus more general multi-purpose theories of cognitive processing (e.g., neuronal recycling and redeployment).

### 3.5.3.2 Brain Regions More Attuned to Mental Manipulation

Using the same logic as above, we also aimed to reveal brain regions potentially underlying mental manipulation. That is, we carried out conjunction analyses between the mental arithmetic and mental rotation maps and then contrasted each individual map with the symbolic number map. We reasoned that regions common to mental arithmetic and mental rotation but not symbolic number processing might be indicative of regions related to the general ability to engage in mental manipulation. Results revealed the left middle frontal gyrus as a potential site for mental manipulation. Note that this region survived the stricter threshold of  $p < .001$ . As outlined earlier, the dorsolateral prefrontal cortex, which is situated in the middle of the middle frontal gyri, is an important region for carrying out top-down executive tasks, such as planning, working memory, inhibition, and abstract reasoning (Owen et al., 2005; Miller & Cummings, 2017; Smith & Jonides, 1999). The current findings provide further evidence that the left middle frontal gyrus may indeed play a role in mental manipulation of information. However, some caution is warranted, as this region has also been associated with a variety of other cognitive tasks including the identification of sound sources (Giordano et al., 2014), imagined grasping (Grafton, Arbib, Fadiga, & Rizzolatti, 1996), and emotional prosody in speech (Mitchell, Elliott, Barry, Crittenden, Woodruff, 2003). Thus, as is the case with the IPS, more research is needed to further operationalize the functions associated with this region.

The parietal lobes may also play an important role in the mental manipulation of information. Mental rotation has been found to consistently activate bilateral regions in and around the IPS (Zacks, 2008); a finding that has led some to conclude the IPS plays a critical role in performing visual-spatial transformations (e.g., see Jordan, Heinze, Lutz, Kanowski, & Jäncke, 2001; Seydell-Greenwald, Ferrara, Chambers, Newport, & Landau,

2017; Zacks, 2008). The current study shows that mental arithmetic is associated with activation in some of these same regions. These findings provide preliminary support for the hypothesis put forward by Hubbard et al.: “*parietal mechanisms that are thought to support spatial transformation might be ideally suited to support arithmetic transformations as well*” (2009, pp. 238). An important question moving forward is the extent to which the common overlap in the parietal regions for spatial and arithmetical transformations (as well as other mathematical computations) are due to shared reliance on visual-spatial representations. Is it a coincidence that arithmetic relies on cerebral cortex most strongly associated with visual-spatial reasoning and not the traditional language regions, namely structures in and around the left sylvan fissure (e.g., inferior frontal lobe and temporal regions; Monti, Parsons, & Osherson, 2009)? On the one hand, evidence to date suggests not. There is emerging consensus that arithmetical and mathematical thinking do not appear to be rooted in the neural mechanisms of natural language (Amalric & Dehaene, 2016; Monti & Osherson, 2011). However, the extent to which arithmetic operations are dependent on visual-spatial representations and not some other form of mental representation remains an important research question (e.g., see Marghetis, Núñez, & Bergen, 2014). For example, it is possible that arithmetic is carried out through purely symbolic or propositional processes independent from visual-spatial representations and also distinct from natural language mechanisms. Future research efforts are needed to test the extent to which the parietal regions that subserve visual-spatial transformations also subserve mental operations devoid of visual-spatial referents.

### 3.5.3.3 Brain Regions Associated with Mental Arithmetic

Mental arithmetic was associated with widespread frontal activity. Compared to mental rotation and symbolic number, mental arithmetic was associated with significantly more activation in the following frontal regions: left inferior frontal gyrus, left medial frontal gyrus, and right middle frontal gyrus. Based on prior research and as noted above, these regions are likely representative of activity associated with executive control processes (e.g., see Miller & Cummings, 2017). Given that mental rotation is commonly thought to be a highly cognitively demanding task, it is somewhat surprising that mental arithmetic

was associated with more widespread frontal activity. In fact, mental rotation was not associated with any frontal activity that was not also engaged by mental arithmetic. This finding is deserving of more attention and perhaps points to differentiated frontal activity more attuned to the manipulation of symbols compared to less culturally defined visual-spatial objects (e.g., 3D cube figures).

The findings of widespread frontal and parietal activity associated with mental arithmetic may be due in part to the decision to include all types of arithmetic problem solving. That is, the arithmetic map includes arithmetical reasoning associated with relatively easy problem types (e.g.,  $2 + 1$ ) but also difficult problem types ( $37 + 68$  or  $3 + 8 - 4$ ). Thus, the arithmetic map includes questions requiring little cognitive effort as well as questions requiring concerted cognitive effort. These differences in the need to recall arithmetic facts compared to need to carry out novel calculations have been found to be associated with common and distinct neural networks (Zamarian, Ischebeck, & Delazer, 2009). The decision to include all types of arithmetic problems was motivated by our aim to reveal regions associated with both basic symbol processing but also higher-level spatial reasoning (i.e., mental rotation). Although not directly tested, we reasoned that recall-based arithmetic would have more in common with basic symbolic processing and calculation-based arithmetic would have more in common with mental rotation. Thus, in an attempt to avoid such biases, we decided to include all studies on arithmetic processing. A logical next step is to formally test the hypothesis that low-effort arithmetic (recall-based) will share more neural regions associated with basic symbolic processes, while high-effort arithmetic (calculation-based) will share more neural regions associated with higher-level spatial reasoning, such as mental rotation. Such relations would provide additional evidence in favour of the grounded or embodied theories of the space-math link (as mentioned above; also see Mix et al., 2016 for further details). An absence of such relations would require reconsideration of such theories.

### 3.5.3.4 Brain Regions Associated with Mental Rotation

In comparison to both numerical reasoning tasks, mental rotation was more associated with activity in the right precuneus/superior parietal lobe. One interpretation of this

finding is that the precuneus may play a role in visual-spatial imagery. Indeed, one of the primary functions ascribed to the precuneus is visual-spatial imagery (Cavanna & Trimble, 2006; Fletcher et al., 1995; Oshio et al., 2010). More specifically, the precuneus has been suggested to play a role in directing attention in space and planning and imagining goal-directed movements (Cavanna & Trimble, 2006; Kawashima, Roland, & O’Sullivan, 1995). However, as evidenced in the present study, the precuneus has been found to be involved in a variety of cognitive tasks, including a pivotal role in the default mode network (Fransson & Marrelec, 2008). Thus, it appears that the precuneus serves a variety of functions, with visual-spatial (motor) imagery potentially being one of them.

Based on prior research, we had expected that we might see the activation of canonical motor regions (e.g., premotor cortex). Instead, we found very little evidence for activation of primary motor cortices. Like symbolic number and mental arithmetic, convergent activation of mental rotation was largely confined to activation in the bilateral parietal lobes. Although prior research has reported that mental rotation is associated with brain activation in motor regions (e.g., see Zacks, 2008), more recent research paints a more complicated picture. A recent meta-analysis suggests that the activation of motor cortex is dependent on experimental stimuli (Tomasino & Gremese, 2016). Mental rotation was found to be correlated with motor activity when the task involved imagining the rotation of body parts but not when it involved the rotation of objects. Thus, our decision to focus on the rotation of objects and to exclude studies that included rotation of body parts is the most probable reason for the absence of observed motor activity.

### 3.5.4 Limitations and Future Directions

Both a strength and a limitation of fMRI meta-analyses is that they provide a broad overview of the neural correlates of cognitive functions. However, by using this technique to ‘see the forest through the trees’ one runs the risks of obscuring important methodological details and findings. The very nature of the meta-analytic method employed – an averaging of peak activation across multiple studies – limits the ability to make specific claims about the findings. Indeed, this process may overestimate the amount of overlap between tasks by averaging across studies and minimizes potentially



small, but important, differences across paradigms. For example, our decision to include all types of arithmetic problems, ranging from easy to difficult, may have resulted in an arithmetic map that is in fact an average of two relatively distinct maps – one associated with solving simple problems and the other for solving complex problems. While this was a desirable outcome for the current study, it stands as an example of what might be happening more generally across and within fMRI studies. One approach to reduce problems associated with averaging across individuals and studies is the use of within-subject designs. By having the same individual perform multiple tasks (e.g., mental rotation and number comparison), it is possible to examine whether the same voxels are co-activated for different tasks.

At the same time, it is important to recognize that co-activation does not necessarily indicate functional equivalence. To this point, the shared neuronal account has been used as evidence and a potential causal explanation for the widely observed behavioural links between spatial and numerical cognition (e.g., see Cheng & Mix, 2014; Hawes, Tepylo, & Moss, 2015). For example, even though mental rotation, basic numerical competencies, and arithmetic appear to recruit common parietal regions, this does not mean that these regions perform the same functions across all three tasks. Moreover, neither does it indicate that the same region is for all three tasks within individuals. Thus, going back to the point above, the present study is only able to provide a general overview of common and distinct regions associated with the three targeted cognitive tasks. Whether or not the overlap observed is functionally meaningful remains an open question; ALE meta-analyses do not permit one to evaluate patterns of activation within overlapping regions. Moving forward, more sensitive methods of analyzing fMRI data, including multivariate pattern analyses (MVPA), are needed to better understand ways in which the same brain region(s) performs multiple cognitive functions. To this aim, we see the present meta-analyses as an important first step in demonstrating the engagement of a common parietal network underlying numerical and spatial cognition. We hope the present findings prove a source of motivation to carry out more sensitive studies and analyses in an effort to better understand the complex neural underpinnings of spatial and numerical cognition.

In interpreting the present findings it is worth considering how our decision to include within-category contrasts (e.g., two-digit addition > single-digit addition) may have influenced the results. On the one hand, within-category contrasts provide a stringent control condition, allowing one to optimally control for perceptual features (e.g., visual processing of numerals). On the other hand, our decision to include within-category contrasts may have resulted in the removal of regions more typically associated with other processes, including visual and language processing. For example, with respect to our symbolic number and arithmetic maps, the inclusion of within-category contrasts may have resulted in the removal of lower-level numerical processes (e.g., numeral identification); processes which have recently been shown to correlate with neural activity in the ventral visual pathway, namely the inferior temporal gyrus (ITG; Baek, Daitch, Pinheiro-Chagas, & Parvizi, 2018; Grotheer, Jeska, & Grill-Spector, 2018; Pinheiro-Chagas, Daitch, Parvizi, & Dehaene, 2018; Yeo, Wilkey, & Price, 2017). However, the presence of this region has not been consistently detected across studies to date (e.g., see Sokolowski et al., 2017) and appears highly sensitive to task demands and the specificity of the contrasts employed (e.g., see Yeo, Wilkey, & Price, 2017). Together, these reasons may help explain why we did not see evidence of a “number form area” in the ITG or more wide spread activity in regions typically associated with language processing for arithmetic.

Lastly, we acknowledge that the current study represents but one of many ways in which spatial and mathematical thinking may converge/diverge in the brain. Both spatial and mathematical abilities are not unitary constructs, but skills made up of many different sub-skills (Mix & Cheng, 2012). Thus, in moving forward, it will be of value to study the neural correlations of spatial-mathematical relations beyond the one studied here. For example, an emerging body of research indicates strong relations between spatial scaling abilities (i.e., the ability to relate distances in one space to distances in another space) and mathematical performance across a variety of tasks, including proportional reasoning, number line estimation, and comprehensive tests of school-based mathematics (Frick, 2018; Gilligan, Hodgkiss, Thomas, & Farran, 2018; Jirout, Holmes, Ramsook, & Newcombe, 2018; Möhring, Frick, & Newcombe, 2018). In short, we have only just begun to scratch the surface of the neural underpinnings of the space-math link.

Opportunities to further probe the space-math link are many and varied and represent a promising area for future research.

### 3.5.5 Conclusions

Decades of behavioral, neuropsychological, and neuroimaging studies have demonstrated consistent and reliable associations between spatial and numerical processing (Hubbard et al., 2005; Mix & Cheng, 2012; Toomarian & Hubbard, 2018). However, much less is known about why and under what conditions spatial and numerical processes converge and/or diverge from one another (Mix & Cheng, 2012). The present study aimed to narrow this gap in understanding by carrying out the first systematic ALE meta-analysis on brain regions associated with spatial and numerical cognition. Consistent with a shared processing account, we revealed that symbolic number, arithmetic, and mental rotation processes were all associated with bilateral parietal activity. We also found evidence that numerical and arithmetic processing were associated with overlap in the left IPS, whereas mental rotation and arithmetic both showed activity in the middle frontal gyri. These patterns suggest regions of cortex potentially more specialized for symbolic number representation and domain-general mental manipulation, respectively. Additionally, arithmetic was associated with unique activity throughout the fronto-parietal network and mental rotation was associated with unique activity in the superior parietal lobe. Taken together, these findings contribute new insights into the neurocognitive mechanisms supporting spatial and numerical thought specifically, and mathematical thought more generally.

## 3.6 References

- Amalric, M., & Dehaene, S. (2016). Origins of the brain networks for advanced mathematics in expert mathematicians. *Proceedings of the National Academy of Sciences*, *113*(18), 4909-4917.
- Ansari, Garcia, Lucas, Hamon, & Dhital (2005). Neural correlates of symbolic number processing in children and adults. *NeuroReport*, *16*, 1769-1773.

- Baek, S., Daitch, A. L., Pinheiro-Chagas, P., & Parvizi, J. (2018). Neuronal population responses in the human ventral temporal and lateral parietal cortex during arithmetic processing with digits and number words. *Journal of Cognitive Neuroscience*, *30*(9), 1315-1322.
- Bisiach, E., & Luzatti, C. (1978). Unilateral neglect of representational space. *Cortex*, *14*, 129-133.
- Bogen, J. E., & Gazzaniga, M. S. (1965). Cerebral commissurotomy in man: Minor hemisphere dominance for certain visuospatial functions. *Journal of Neurosurgery*, *23*(4), 394-399.
- Broca, P. (1861). Remarks on the seat of the faculty of articulated language, following an observation of aphemia (loss of speech). *Bulletin de la Société Anatomique*, *6*, 330-57.
- Butterworth, B. (1999). *The mathematical brain*. London: Macmillan Calabria, M., & Rossetti, Y. (2005). Interference between number processing and line bisection: a methodology. *Neuropsychologia*, *43*(5), 779-783.
- Cantlon, J. F., Brannon, E. M., Carter, E. J., & Pelphrey, K. A. (2006). Functional imaging of numerical processing in adults and 4-y-old children. *PLoS Biology*, *4*(5), e125.
- Carroll, J. (1993). *Human cognitive abilities: A survey of the factor-analytic studies*. New York: Cambridge University Press.
- Cattaneo, Z., Silvanto, J., Pascual-Leone, A., & Battelli, L. (2009). The role of the angular gyrus in the modulation of visuospatial attention by the mental number line. *NeuroImage*, *44*(2), 563-568.
- Cavanna, A. E., & Trimble, M. R. (2006). The precuneus: a review of its functional anatomy and behavioural correlates. *Brain*, *129*(3), 564-583.
- Cipora, K., Patro, K., & Nuerk, H. C. (2015). Are spatial-numerical associations a cornerstone for arithmetic learning? The lack of genuine correlations suggests no. *Mind, Brain, and Education*, *9*(4), 190-206.
- Davis, B., & Spatial Reasoning Study Group. (2015). *Spatial reasoning in the early years: Principles, assertions, and speculations*. Chicago, IL: Routledge.
- de Hevia, M. D., Vallar, G., & Girelli, L. (2008). Visualizing numbers in the mind's eye:

- The role of visuo-spatial processes in numerical abilities. *Neuroscience & Biobehavioral Reviews*, 32(8), 1361-1372.
- Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, 44(1-2), 1-42.
- Dehaene, S. (2011). *The number sense: How the mind creates mathematics*. Oxford, UK: Oxford University Press.
- Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General*, 122(3), 371-396.
- Dehaene, S., & Cohen, L. (1997). Cerebral pathways for calculation: Double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex*, 33(2), 219-250.
- Dehaene, S., & Cohen, L. (2007). Cultural recycling of cortical maps. *Neuron*, 56(2), 384-398.
- Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, 20(3-6), 487-506.
- Delgado, A. R., & Prieto, G. (2004). Cognitive mediators and sex-related differences in mathematics. *Intelligence*, 32(1), 25-32.
- Desco, M., Navas-Sanchez, F. J., Sanchez-González, J., Reig, S., Robles, O., Franco, C., ... & Arango, C. (2011). Mathematically gifted adolescents use more extensive and more bilateral areas of the fronto-parietal network than controls during executive functioning and fluid reasoning tasks. *NeuroImage*, 57(1), 281-292.
- Eickhoff, S. B., Bzdok, D., Laird, A. R., Kurth, F., & Fox, P. T. (2012). Activation likelihood estimation meta-analysis revisited. *NeuroImage*, 59(3), 2349-2361.
- Eickhoff, S. B., Laird, A. R., Fox, P. M., Lancaster, J. L., & Fox, P. T. (2017). Implementation errors in the GingerALE Software: Description and recommendations. *Human Brain Mapping*, 38(1), 7-11.
- Eickhoff, S. B., Laird, A. R., Grefkes, C., Wang, L. E., Zilles, K., & Fox, P. T. (2009). Coordinate-based activation likelihood estimation meta-analysis of neuroimaging data: A random-effects approach based on empirical estimates of spatial uncertainty. *Human Brain Mapping*, 30(9), 2907-2926.
- Fincham, J. M., Carter, C. S., Van Veen, V., Stenger, V. A., & Anderson, J. R. (2002).

- Neural mechanisms of planning: a computational analysis using event-related fMRI. *Proceedings of the National Academy of Sciences*, 99(5), 3346-3351.
- Fletcher, P. C., Frith, C. D., Baker, S. C., Shallice, T., Frackowiak, R. S., & Dolan, R. J. (1995). The mind's eye—precuneus activation in memory-related imagery. *NeuroImage*, 2(3), 195-200.
- Fransson, P., & Marrelec, G. (2008). The precuneus/posterior cingulate cortex plays a pivotal role in the default mode network: Evidence from a partial correlation network analysis. *NeuroImage*, 42(3), 1178-1184.
- Frick, A. (2018). Spatial transformation abilities and their relation to later mathematics performance. *Psychological Research*, 1-20.
- Galton, F. (1880). Visualised numerals. *Nature*, 21(533), 252-256.
- Gerstmann, J. (1940). Syndrome of finger agnosia, disorientation for right and left, agraphia and acalculia: local diagnostic value. *Archives of Neurology & Psychiatry*, 44(2), 398-408.
- Gilligan, K. A., Hodgkiss, A., Thomas, M. S., & Farran, E. K. (2018). The developmental relations between spatial cognition and mathematics in primary school children. *Developmental Science*, e12786.
- Giordano, B. L., Pernet, C., Charest, I., Belizaire, G., Zatorre, R. J., & Belin, P. (2014). Automatic domain-general processing of sound source identity in the left posterior middle frontal gyrus. *Cortex*, 58, 170-185.
- Göbel, S. M., Calabria, M., Farne, A., & Rossetti, Y. (2006). Parietal rTMS distorts the mental number line: simulating 'spatial' neglect in healthy subjects. *Neuropsychologia*, 44(6), 860-868.
- Göbel, S., Walsh, V., & Rushworth, M. F. (2001). The mental number line and the human angular gyrus. *NeuroImage*, 14(6), 1278-1289.
- Grafton, S. T., Arbib, M. A., Fadiga, L., & Rizzolatti, G. (1996). Localization of grasp representations in humans by positron emission tomography. *Experimental Brain Research*, 112(1), 103-111.
- Grotheer, M., Jeska, B., & Grill-Spector, K. (2018). A preference for mathematical processing outweighs the selectivity for Arabic numbers in the inferior temporal gyrus. *Neuroimage*, 175, 188-200.

- Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skill and early number knowledge: the role of the linear number line. *Developmental Psychology, 48*(5), 1229-1241.
- Hawes, Z., Moss, J., Caswell, B., Seo, J., & Ansari, D. (2019). Relations between numerical, spatial, and executive function skills and mathematics achievement: A latent-variable approach. *Cognitive Psychology, 109*, 68-90.
- Hawes, Z., Tepylo, D., & Moss, J. (2015). Developing spatial thinking: Implications for early mathematics education. In B. Davis and Spatial Reasoning Study Group (Eds.), *Spatial reasoning in the early years: Principles, assertions and speculations* (pp. 29-44). New York, NY: Routledge.
- Hegarty, M., & Kozhevnikov, M. (1999). Types of visual-spatial representations and mathematical problem solving. *Journal of Educational Psychology, 91*, 684–689.
- Holmes, G. (1918). Disturbances of visual orientation. *The British Journal of Ophthalmology, 2*(9), 449.
- Hubbard, E. M., Piazza, M., Pinel, P., & Dehaene, S. (2005). Interactions between number and space in parietal cortex. *Nature Reviews Neuroscience, 6*(6), 435-448.
- Hubbard, E. M., M. Piazza, P. Pinel, & Dehaene (2009). Numerical and spatial intuitions: a role for posterior parietal cortex? In L. Tommasi, L. Nadel, and M. A. Peterson (Eds.), *Cognitive biology: Evolutionary and developmental perspectives on mind, brain and behavior* (pp. 221–246). Cambridge, MA: MIT Press
- Jeannerod, M. (1994). The representing brain: Neural correlates of motor intention and imagery. *Behavioral and Brain sciences, 17*(2), 187-202.
- Johnson-Frey, S. H. (2004). The neural bases of complex tool use in humans. *Trends in Cognitive Sciences, 8*(2), 71-78.
- Jordan, K., Heinze, H. J., Lutz, K., Kanowski, M., & Jäncke, L. (2001). Cortical activations during the mental rotation of different visual objects. *NeuroImage, 13*(1), 143-152.
- Jirout, J. J., Holmes, C. A., Ramsook, K. A., & Newcombe, N. S. (2018). Scaling up spatial development: A closer look at children's scaling ability and its relation to number knowledge. *Mind, Brain, and Education, 12*(3), 110-119.
- Kadosh, R. C., Lammertyn, J., & Izard, V. (2008). Are numbers special? An overview of

- chronometric, neuroimaging, developmental and comparative studies of magnitude representation. *Progress in Neurobiology*, 84(2), 132-147.
- Knops, A., Viarouge, A., & Dehaene, S. (2009). Dynamic representations underlying symbolic and nonsymbolic calculation: Evidence from the operational momentum effect. *Attention, Perception, & Psychophysics*, 71(4), 803-821.
- Kosslyn, S. M., Ganis, G., & Thompson, W. L. (2001). Neural foundations of imagery. *Nature Reviews Neuroscience*, 2(9), 635-642.
- Kroger, J. K., Sabb, F. W., Fales, C. L., Bookheimer, S. Y., Cohen, M. S., & Holyoak, K. J. (2002). Recruitment of anterior dorsolateral prefrontal cortex in human reasoning: a parametric study of relational complexity. *Cerebral Cortex*, 12(5), 477-485.
- Kyttälä, M., & Lehto, J. E. (2008). Some factors underlying mathematical performance: The role of visuospatial working memory and nonverbal intelligence. *European Journal of Psychology of Education*, 22(1), 77-94.
- Laird, A.R., Eickhoff, S.B., Kurth, F., Fox, P.M., Uecker, A.M., Turner, J.A., Robinson, J.L., Lancaster, J.L., Fox, P.T., (2009a). ALE meta-analysis workflows via the brainmap database: progress towards a probabilistic functional brain atlas. *Frontiers in Neuroinformatics*, 3, 23.
- Laird, A.R., Eickhoff, S.B., Li, K., Robin, D.A., Glahn, D.C., Fox, P.T., (2009b). Investigating the functional heterogeneity of the default mode network using coordinate based meta-analytic modeling. *Journal of Neuroscience*, 29, 14496-14505.
- Laird, A.R., Robinson, J.L., McMillan, K.M., Tordesillas-Gutiérrez, D., Moran, S.T., Gonzales, S.M., et al., (2010). Comparison of the disparity between Talairach and MNI coordinates in functional neuroimaging data: validation of the Lancaster transform. *NeuroImage*, 51 (2), 677-683.
- Lancaster, J. L., Tordesillas-Gutiérrez, D., Martinez, M., Salinas, F., Evans, A., Zilles, K., ... & Fox, P. T. (2007). Bias between MNI and Talairach coordinates analyzed using the ICBM-152 brain template. *Human Brain Mapping*, 28(11), 1194-1205.
- LeFevre, J. A., Fast, L., Skwarchuk, S. L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of



- performance. *Child Development*, 81(6), 1753-1767.
- Leibovich, T., Katzin, N., Harel, M., & Henik, A. (2017). From “sense of number” to “sense of magnitude”: The role of continuous magnitudes in numerical cognition. *Behavioral and Brain Sciences*, 40. e164
- Lohman, D. F. (1996). Spatial ability and G. In I. Dennis & P. Tapsfield (Eds.), *Human abilities: Their nature and assessment* (pp. 97–116). Hillsdale, NJ: Lawrence Erlbaum.
- Marghetis, T., Núñez, R., & Bergen, B. K. (2014). Doing arithmetic by hand: Hand movements during exact arithmetic reveal systematic, dynamic spatial processing. *The Quarterly Journal of Experimental Psychology*, 67(8), 1579-1596.
- Matejko, A. A., & Ansari, D. (2015). Drawing connections between white matter and numerical and mathematical cognition: a literature review. *Neuroscience & Biobehavioral Reviews*, 48, 35-52.
- Mayer, E., Martory, M. D., Pegna, A. J., Landis, T., Delavelle, J., & Annoni, J. M. (1999). A pure case of Gerstmann syndrome with a subangular lesion. *Brain*, 122(6), 1107-1120.
- McCrink, K., Dehaene, S., & Dehaene-Lambertz, G. (2007). Moving along the number line: Operational momentum in nonsymbolic arithmetic. *Perception & Psychophysics*, 69(8), 1324-1333.
- Miller, B. L., & Cummings, J. L. (2017). *The human frontal lobes: Functions and disorders*. New York, NY: Guilford Press.
- Milner, B. (1971). Interhemispheric differences in the localization of psychological processes in man. *British Medical Bulletin*, 27(3), 272-277.
- Mitchell, R. L., Elliott, R., Barry, M., Cruttenden, A., & Woodruff, P. W. (2003). The neural response to emotional prosody, as revealed by functional magnetic resonance imaging. *Neuropsychologia*, 41(10), 1410-1421.
- Mix, K. S., & Cheng, Y. L. (2012). The relation between space and math: Developmental and educational implications. *Advances in Child Development and Behavior*, 42, 197–243.
- Möhring, W., Frick, A., & Newcombe, N. S. (2018). Spatial scaling, proportional

- thinking, and numerical understanding in 5-to 7-year-old children. *Cognitive Development*, 45, 57-67.
- Monti, M. M., Parsons, L. M., & Osherson, D. N. (2009). The boundaries of language and thought in deductive inference. *Proceedings of the National Academy of Sciences*, 106(30), 12554-12559.
- Newcombe, N. S., Levine, S. C., & Mix, K. S. (2015). Thinking about quantity: The intertwined development of spatial and numerical cognition. *Wiley Interdisciplinary Reviews: Cognitive Science*, 6(6), 491-505.
- O'Boyle, M. W., Cunnington, R., Silk, T. J., Vaughan, D., Jackson, G., Syngeniotis, A., & Egan, G. F. (2005). Mathematically gifted male adolescents activate a unique brain network during mental rotation. *Cognitive Brain Research*, 25(2), 583-587.
- Oshio, R., Tanaka, S., Sadato, N., Sokabe, M., Hanakawa, T., & Honda, M. (2010). Differential effect of double-pulse TMS applied to dorsal premotor cortex and precuneus during internal operation of visuospatial information. *NeuroImage*, 49(1), 1108-1115.
- Owen, A. M., McMillan, K. M., Laird, A. R., & Bullmore, E. (2005). N-back working memory paradigm: A meta-analysis of normative functional neuroimaging studies. *Human Brain Mapping*, 25(1), 46-59.
- Parsons, S., & Bynner, J. (2005). Does numeracy matter more? National Research and Development Centre for Adult Literacy and Numeracy, Institute of Education.
- Pinel, P., Piazza, M., Le Bihan, D., & Dehaene, S. (2004). Distributed and overlapping cerebral representations of number, size, and luminance during comparative judgments. *Neuron*, 41(6), 983-993.
- Pinheiro-Chagas, P., Daitch, A., Parvizi, J., & Dehaene, S. (2018). Brain mechanisms of arithmetic: A crucial role for ventral temporal cortex. *Journal of Cognitive Neuroscience*, 30(12), 1757-1772.
- Price, C.J., Devlin, J.T., Moore, C.J., Morton, C., Laird, A.R., (2005). Meta-analyses of object naming: effect of baseline. *Human Brain Mapping*, 25, 70–82.
- Price, C. J., & Friston, K. J. (2005). Functional ontologies for cognition: The systematic definition of structure and function. *Cognitive Neuropsychology*, 22(3-4), 262-275.

- Raemaekers, M., Vink, M., Zandbelt, B., van Wezel, R.J., Kahn, R.S., Ramsey, N.F. (2007). Test-retest reliability of fMRI activation during prosaccades and antisaccades. *NeuroImage*, *36*, 532–542
- Seydell-Greenwald, A., Ferrara, K., Chambers, C. E., Newport, E. L., & Landau, B. (2017). Bilateral parietal activations for complex visual-spatial functions: Evidence from a visual-spatial construction task. *Neuropsychologia*, *106*, 194-206.
- Shaki, S., Fischer, M. H., & Petrusic, W. M. (2009). Reading habits for both words and numbers contribute to the SNARC effect. *Psychonomic Bulletin & Review*, *16*(2), 328-331.
- Smith, I. M. (1964). *Spatial ability: Its educational and social significance*. London, UK: University of London Press
- Smith, E. E., & Jonides, J. (1999). Storage and executive processes in the frontal lobes. *Science*, *283*(5408), 1657-1661.
- Sokolowski, H. M., Fias, W., Mousa, A., & Ansari, D. (2017). Common and distinct brain regions in both parietal and frontal cortex support symbolic and nonsymbolic number processing in humans: A functional neuroimaging meta-analysis. *NeuroImage*, *146*, 376-394.
- Sokolowski, H. M., Fias, W., Ononye, C. B., & Ansari, D. (2017). Are numbers grounded in a general magnitude processing system? A functional neuroimaging meta-analysis. *Neuropsychologia*, *105*, 50-69.
- Stengel, E. (1944). Loss of spatial orientation, constructional apraxia and Gerstmann's syndrome. *Journal of Mental Science*, *90*(380), 753-760.
- Tam, Y. P., Wong, T. T. Y., & Chan, W. W. L. (2019). The relation between spatial skills and mathematical abilities: The mediating role of mental number line representation. *Contemporary Educational Psychology*, *56*, 14-24.
- Tolar, T. D., Lederberg, A. R., & Fletcher, J. M. (2009). A structural model of algebra achievement: computational fluency and spatial visualisation as mediators of the effect of working memory on algebra achievement. *Educational Psychology*, *29*, 239–266.
- Tomasino, B., & Gremese, M. (2016). Effects of stimulus type and strategy on mental

- rotation network: an activation likelihood estimation meta-analysis. *Frontiers in Human Neuroscience*, 9(693), 1-24.
- Toomarian, E. Y., & Hubbard, E. M. (2018). On the genesis of spatial-numerical associations: Evolutionary and cultural factors co-construct the mental number line. *Neuroscience & Biobehavioral Reviews*, 90, 184-199.
- Turkeltaub, P. E., Eickhoff, S. B., Laird, A. R., Fox, M., Wiener, M., & Fox, P. (2012). Minimizing within-experiment and within-group effects in activation likelihood estimation meta-analyses. *Human Brain Mapping*, 33(1), 1-13.
- Viarouge, A., Hubbard, E. M., & McCandliss, B. D. (2014). The cognitive mechanisms of the SNARC effect: An individual differences approach. *PloS one*, 9(4), e95756.
- Walsh, V. (2003). A theory of magnitude: common cortical metrics of time, space and quantity. *Trends in Cognitive Sciences*, 7(11), 483-488.
- Wei, W., Yuan, H., Chen, C., & Zhou, X. (2012). Cognitive correlates of performance in advanced mathematics. *British Journal of Educational Psychology*, 82(1), 157–18.
- Yeo, D. J., Wilkey, E. D., & Price, G. R. (2017). The search for the number form area: A functional neuroimaging meta-analysis. *Neuroscience & Biobehavioral Reviews*, 78, 145-160.
- Zacks, J. M. (2008). Neuroimaging studies of mental rotation: a meta-analysis and review. *Journal of Cognitive Neuroscience*, 20(1), 1-19.
- Zamarian, L., Ischebeck, A., & Delazer, M. (2009). Neuroscience of learning arithmetic—evidence from brain imaging studies. *Neuroscience & Biobehavioral Reviews*, 33(6), 909-925.

## Chapter 4

### 4 Integrating Numerical Cognition Research and Mathematics Education to Strengthen the Teaching and Learning of Early Number

#### 4.1 Citation

This chapter is currently under review and involved the following co-authors.

Zachary Hawes, Rebecca Merkley, Christine Stager, and Daniel Ansari.

#### 4.2 Introduction

*“I say moreover that you make a great, a very great mistake, if you think psychology, being the science of the mind’s laws, is something from which you can deduce definite programmes and schemes and methods of instruction for immediate schoolroom use. Psychology is a science, and teaching, is an art; and sciences never generate arts directly out of themselves. An intermediary inventive mind must make the application, by using its originality” (William James, 1899, p. 23).*

The above quote points to a central problem facing both educators and psychologists alike: How can we translate and apply the science of how children learn to the classroom? As this quote also reminds us, the implications for classroom instruction do not immediately follow from the science of learning. Instead, intermediary actions are needed to most optimally merge the science of learning and the practice of teaching.

The current study was designed to address the research-to-practice gap. Of primary interest was whether and to what extent both teachers and their students benefit from a model of teacher professional development (PD) explicitly aimed to better integrate research in numerical cognition with mathematics instruction. More specifically, we report on the design, implementation, and effects of an in-service mathematics Professional Development (PD) model for teachers of Kindergarten–3<sup>rd</sup> Grade. The PD

model centres around the translation and application of key findings from the field of numerical cognition – a branch of cognitive science that involves the interdisciplinary study of the cognitive, developmental, and neural bases of numerical and mathematical thought. Throughout the PD model (25 hours over a 3-month period), numerical cognition research serves as both a base and point of return to better understand children’s numerical thinking. Indeed, central to our model is the hypothesis that by better understanding children’s numerical thinking, teachers may be better equipped to assess, plan, and deliver mathematics instruction. To summarize, the present study describes and tests a new model of teacher PD designed to: (1) Enrich teachers’ awareness of and understanding of research on children’s numerical thinking, and (2) use this knowledge to inform teachers’ assessment and instructional practice.

#### 4.2.1 Background and Foundations on which the Current Teacher PD Model was Built

If research-to-practice gaps are the problem, what are some potential solutions? In this section we briefly review three bodies of work that have each achieved some levels of success in better integrating research and practice. These research programs were instrumental in forming the theoretical underpinnings and design of the current intervention.

One approach to narrowing the research-to-practice gap is represented by the invention of a methodological approach to educational interventions known as design-research (Brown, 1992; Collins, 1992). In a nut shell, design-research involves an iterative cycle of intervention design, implementation, and evaluation in real-world learning environments (e.g., classrooms). Importantly, this occurs in partnership with the various stakeholders involved (e.g., teachers). Design-research was borne in response to the difficulties of taking lab-based learning interventions and implementing them in classroom and school contexts (Brown, 1992). These difficulties include the emergent properties of real-world learning environments (classrooms) that are the products of multifaceted and largely uncontrollable variables (e.g., social dynamics of individual students across different classrooms). As the name suggests, design-research has its basis

in the scientifically-informed ‘trial and error’ approaches of the design sciences, including engineering, artificial intelligence, and aeronautics (Collins, Joseph, & Bielaczyc, 2004; Nathan & Sawyer, 2014; Simon, 1969). This approach is akin to beta testing. A product is first designed and then released to actual users who then provide feedback, report bugs, etc. This feedback is then used to create a more optimally functioning and user-informed product. Educational design-research functions similarly. Learning interventions are not viewed as static, prescriptive ‘how-to-teach x’ recipes but are implemented with built-in feedback mechanisms. For example, teachers might be encouraged to adapt the intervention where they see fit based on the feedback they receive from their students. In the present intervention, we borrowed this particular feature of design-research. In designing our intervention, we built certain degrees of freedom into the intervention model – specifying beforehand where and what aspects of the intervention we allow and *want* to vary. Specifically, we aimed to utilize teacher expertise in the delivery of the student intervention activities. Teachers were encouraged to take the activities (designed and presented to the group by the research team) and adapt them where they saw need for revision. In line with design-research, we did this in an effort to a) build teacher agency and incorporate professional feedback into the model, and b) to gradually refine and ultimately build better student intervention activities (e.g., see Moss, Bruce, Caswell, Flynn, & Hawes, 2016).

Another approach to narrowing the research-practice gap, and one specific to early years mathematics instruction, is a form of teacher PD known as *Cognitively Guided Instruction* (CGI; Carpenter, Fennema, Franke, Levi, & Empson, 2014; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996). At its core, CGI is an approach to working with teachers that involves sharing research on children’s mathematical thinking and then using this knowledge as a basis for assessment and instruction (Carpenter et al., 1989; Franke, Carpenter, Levi, & Fennema, 2001). For example, as described in various iterations of this model, teachers are introduced to research-based frameworks for understanding children’s arithmetic development and strategy use (e.g., the counting on strategy). This knowledge is then more readily accessible and utilized by teachers’ during their assessment and instruction of children’s arithmetic (Carpenter et al., 1989). While our approach to teacher PD differs

from CGI in some important ways, it resembles CGI in that it puts student thinking at its center. Likewise, we echo the hypotheses put forth by the authors of CGI for why introducing teachers' to cognitive developmental research on children's mathematical thinking can be used to leverage teacher practice and student learning. That is, research on children's mathematical learning can assist teachers by providing a more organized and structured 'mental model' of the learner (Willingham, 2017), providing teachers with an improved reference for what to look for in terms of student thinking and what this means for moving forward with instruction (Carpenter et al., 1989). Unfortunately, despite the widespread support for CGI, the evidence that this approach leads to changes in student thinking is limited. For example, to our knowledge, this approach has not yet been subjected to a pre-post research design involving both an experimental and control group and, critically, using the same pre- and post-test measures across two time points and through comparing the effects across conditions (experimental vs. control). Using a more rigorous study design, the present study aims to further reveal the extent to which students benefit from a model of PD where the central focus is on using research on children's numerical thinking to inform instruction.

Lastly, the current intervention builds on a model of teacher PD known as the *Math for Young Children* Project (Hawes, Moss, Caswell, Naqvi, & MacKinnon, 2017; Moss et al., 2016). While various iterations of this approach exist (e.g., see Bruce, Flynn, & Bennett, 2016; Moss, Hawes, Naqvi, & Caswell, 2015), the present study is most closely related to the model described in Hawes et al. 2017. In this study, the authors describe a 32-week teacher-led intervention aimed at improving children's spatial and geometrical thinking. Similar to the current study, teachers and researchers engaged in semi-regular meetings to share and discuss activities and strategies for improving young children's (Kindergarten – Grade 2) mathematical thinking. Teachers were provided with a series of intervention activities to implement in their classrooms. Critically, just as we do in the present study, teachers were encouraged to take the activities and 'make them their own,' adapting and revising the activities based on their own professional judgment and assessment of their own students' learning needs. Compared to a control group, children in the intervention classrooms demonstrated widespread gains on assessments of spatial and geometric thinking, as well as some evidence of far transfer to a measure of



numerical reasoning. Other iterations of this approach to teacher PD and the associated classroom-based intervention have been linked to quantitative gains in children's geometric and spatial reasoning, as well as qualitative evidence of change in teachers' content knowledge and self-confidence (see Moss et al., 2015; Bruce & Hawes, 2015).

In addition to incorporating features of design-research, this model of PD also includes two other features hypothesized to facilitate teacher change, and in turn, student change. These features include, 1) the design and implementation of clinical interviews with students, and 2) teacher engagement in mathematical tasks designed for the eventual implementation with students. Originally pioneered by Jean Piaget, clinical interviews provide an adaptive method of questioning children as a means to reveal children's conceptual understandings of a given phenomenon (e.g., conservation of number; see Ginsburg, 1997). Prior research suggests that clinical interviews contribute to improvements in teachers' capacity to assess, understand, and further promote children's mathematical thinking, while also providing teachers with important insight into their own mathematical thinking (Clarke, Clarke, & Roche, 2011; Mast & Ginsburg, 2010; Moss et al., 2015). It was for these reasons, that we included the practice of teacher-student clinical interviews in the present model. These same reasons underlay our decision to have teachers engage in mathematical tasks eventually intended for their students. In addition, we hypothesized that having teachers approach mathematical tasks as both learners, but also through the perspective of their own students, may lend itself to improved teacher content knowledge as well as comfort level (lowered anxiety) with the teaching and learning of mathematics.

The present study builds on the design and approach to PD described by Hawes et al., (2017), but aims to extend it in some key ways. First, in the current study, we focus our PD on improving key facets of children's numerical reasoning (i.e., arithmetic, number line estimation, and applied number problems). Given the widely recognized importance of young children's numerical reasoning for later mathematical and academic success (e.g., see Duncan et al., 2007), it is crucial to target this area of instruction in the early years. Second, our model places much more emphasis on the integration of cognitive science and mathematics instruction. More specifically, we focus more time and effort sharing and discussing relations between cognitive processes and strategies and

their relations to mathematical learning and performance. Additionally, in accordance with the emerging disciplines of Mind, Brain, and Education (aka Educational Neuroscience), we share and discuss with teachers some of the recent insights from cognitive developmental neuroscience hypothesized to be relevant to the improvement of classroom instruction (e.g., brain plasticity, neuromyths, brain-related responses during arithmetic; see Dubinsky, Roehrig, & Varma, 2013). Lastly, empirical studies of the model have been limited to measuring the effects of the intervention at the student level. This study is the first to measure the effectiveness of the model at both the student and teacher level. Specifically, we examine the extent to which the intervention influences teachers' content knowledge, self-perceived content knowledge, and math anxiety/comfort level.

## 4.2.2 The Present Study

The purpose of this study was to address the research-to-practice gap in the teaching and learning of early number. Building on the teacher PD models described above, we designed a 25-hour in-service PD intervention that aimed to better integrate research in numerical cognition with the instruction of early years mathematics. Borrowing from these different approaches, our model incorporates features of design-research (i.e., built-in teacher feedback mechanisms) and uses research on children's numerical thinking as the basis for facilitating both teacher and student change. For reasons provided above, as well as discussed in further details below (Methods), we predicted that our model of teacher PD would be an effective means for increasing both teacher and student learning. More specifically, we predicted that our intervention would lead to gains in teachers' awareness and knowledge of numerical cognition research and work towards alleviating teacher math anxiety. In addition, our model also provided a platform to share and discuss research related to developmental cognitive neuroscience (i.e., research not limited to numerical cognition). For this reason, we predicted that teachers might also report increased knowledge of terminology and content related to developmental cognitive neuroscience more broadly. It was through engaging teachers in research and its application to classroom learning that we also expected to see evidence of increased

student learning. Given that the teacher PD was aimed at the translation and application of key topics within the numerical cognition literature (e.g., research related to cardinality, ordinality, number lines, and arithmetic strategies), we predicted that these would be the aspects of children’s mathematical thinking where the largest gains would occur.

## 4.3 Methods

### 4.3.1 Study Design and Procedure

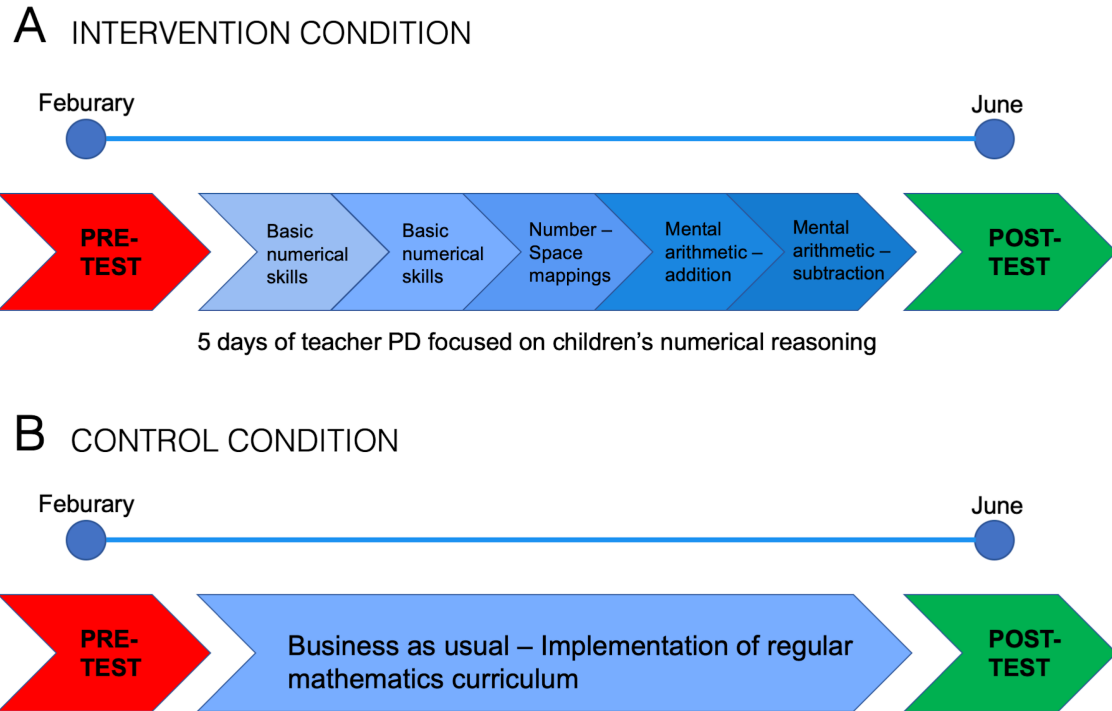
This study occurred over two consecutive school years (2016/2017 and 2017/2018) and involved a combination of two different study designs: a quasi-experimental pre-post research design and a within-group cross-over intervention design. The cross-over design was possible because the control school in the first year of the study (Year 1) participated as the intervention group in the second year of the study (Year 2). In total, three elementary schools participated across the two-year study. These schools were not selected at random but were based on consultation with the district school board and the explicit need to work with schools serving families of similar demographics (income and neighborhood characteristics) and highly comparable academic performance levels. With these constraints in mind, the school board selected three schools to participate in the research project. All three schools were located within the same general neighborhood, serving families of similar demographics and with comparable student performance on the standardized provincial achievement tests in mathematics and language (reading and writing). All three schools consistently perform below the provincial average in mathematics. Taken together, the three participating schools were well-matched in sociodemographic characteristics and mathematics performance, fulfilling our need as researchers to conduct research with highly comparable schools and the school board’s need to provide additional mathematics instructional support in these particular schools.

In the first year of the study, two of the three schools were randomly assigned to either the experimental or the ‘waitlist’ control condition (see Figure 4.1). Prior to the collection of pre- and post-test data, the school principal and Kindergarten to Grade 2

teachers gave their consent to participate. Information letters and consent forms were then sent home by the participating classroom teachers to the parents of children in their classrooms. Due to time constraints, we were unable to test all children for whom we had consent. For this reason, children were randomly selected to participate in the pre-post assessments.

In the second year of the study, the control group from the previous year participated as the experimental group. In that same year, the third school, introduced above, participated as a control group. The same teacher, principal, and parent/child consent procedure described above was employed. Likewise, children whose parent/guardian provided consent for them to participate were randomly selected to participate in the pre- and post-testing assessments. The study design and procedure were approved by the University of Western's Non-Medical Research Ethics Board (NMREB) as well as the participating school board's ethics committee.

In both years of the study, the intervention occurred over the same 3-month period (1<sup>st</sup> week of March to 1<sup>st</sup> week of June). Within this time frame, teachers received 5 full-days (9am-3:30pm) of paid teacher release to participate in the intervention. Each one of these days were spaced out 3-4 weeks apart from one another. As outlined further below, each day of the intervention followed the same general structure, but varied in the specific content addressed (see Figure 4.1). In total, the in-school teacher intervention was approximately 25 hours in duration (excluding lunch and mid-morning/day breaks). All pre-and-post testing also occurred during the same time frame in each year of the study. Moreover, because some children (n=48) participated in both the control and experimental conditions (in different years), we tested these children at near identical pre and postdates across both years. This allowed us to accurately compare within-participant growth across both conditions (experimental vs. control).



*Figure 4.1* A schematic of the research design/time frame and structure of the teacher professional development intervention.

## 4.3.2 Participants

### 4.3.2.1 Year 1

#### 4.3.2.1.1 Teacher Participants

In the first year of the study, a total of 24 educators participated. Fifteen educators participated in the intervention condition and 9 educators participated in the control condition. The two groups were well-matched in terms of years of teaching experiences (Mean intervention group = 10.57 years,  $SD = 5.88$ ; Mean control group = 11.00 years,  $SD = 8.43$ ). Note that one teacher in the intervention group did not provide years of teaching experience. Teachers in both groups completed identical pre- and post-test measures prior to and immediately following the 3-month intervention period.

### 4.3.2.1.2 Child Participants

A total of 107 children participated ( $M_{age} = 5.95$  years,  $SD = 1.37$ ; Females = 58) in the pre-post testing. Fifty-two children were randomly selected for pre-post testing from the intervention classrooms ( $M_{age} = 6.09$ ,  $SD = 1.17$ ; Females = 27) and fifty-five were randomly selected from the control classrooms ( $M_{age} = 5.81$ ,  $SD = 1.22$ ; Females = 31). Note that random selection was done for each grade level in an effort to balance the number of children from each grade across both conditions. Pre- and post-testing took part during a two-week period before and immediately following the intervention.

### 4.3.2.2 Year 2

#### 4.3.2.2.1 Teacher Participants

A total of 27 educators participated in Study 2. Fifteen educators participated in the intervention condition and 12 educators participated in the control condition. The two groups were well-matched in terms of years of teaching experiences (Mean intervention group = 11.83 years,  $SD = 8.68$ ; Mean control group = 10.92 years,  $SD = 6.35$ ). Teachers in both groups completed identical pre- and post-test measures prior to and immediately following the 3-month intervention period.

#### 4.3.2.2.1 Child Participants

A total of 121 children participated ( $M_{age} = 6.72$  years,  $SD = 1.42$ ; Females = 66) in the pre-post testing. The intervention group consisted primarily of children who had participated as control participants in the previous year ( $n=48$ ). That is, 48 students from the Study 1 control group were available to take part in testing one year later (Study 2): This time as part of the intervention group. In order to increase the sample size and better match the intervention group with the Study 2 control group, an additional 9 children were selected to participate. In total, 57 children were randomly selected to participate in

the intervention group ( $M_{\text{age}} = 6.57$  years,  $SD = 1.36$ ; Females = 32). Sixty-four children were randomly selected to participate in the control group ( $M_{\text{age}} = 6.86$  years,  $SD = 1.47$ ; Females = 34). Pre- and post-testing took part during a two-week period before and immediately following the intervention.

### 4.3.3 Overview of the Teacher Intervention and Rationale for Including Each Component

The teacher intervention occurred over 5 days spread out over a 3-month period. All meetings were held in the school's library and facilitated by authors Hawes, Merkley, and Ansari. As shown in Figure 4.1, the focus of the first two sessions was on the foundations of number, the third session focused on number-space associations, and the fourth and fifth sessions focused on arithmetic (addition and subtraction) strategies. Table 4.1 provides a summary of the main mathematical content/concepts addressed across sessions. Although each day had its own focus, the general structure of each session was the same. As reviewed next, each day included the same five components: 1) A researcher-led presentation of numerical cognition research (e.g., arithmetic strategies), 2) a group discussion of one or two research articles, 3) assessments of students' mathematical thinking via clinical interviews, 4) teacher engagement with mathematics, and 5) design and implementation of student activities/lessons. For complete details and the scheduling of each session visit: (<https://osf.io/tqs7e/>)

*Researcher-Led Presentation of Numerical Cognition Research.* During the morning of each session, researchers Hawes, Merkley, and/or Ansari prepared and presented a brief presentation on the day's given theme (e.g., numerical foundations). Examples of topics from numerical cognition research included research on the counting principles, dyscalculia, number line training studies, and arithmetic strategies (a more detailed description of the specific topics is addressed further below). Examples of topics on developmental cognitive (neuro)science included sharing and discussing research related to distributed/spaced practice effects, neuromyths, conceptual vs. procedural knowledge, brain plasticity, and effects of home and environment on early academic achievement. Moreover, discussing research in these various areas naturally led to sharing

and discussing various other terms frequently used in cognitive science research, including inhibitory control, executive functions, and working memory.

The purpose of these presentations was to introduce and share research findings from the field of numerical cognition as well as developmental cognitive (neuro)science more generally. More specifically, by sharing, translating, and discussing research it was our intent to provide a springboard from which to focus our collective thinking and theorizing about children's numerical thinking and the types of classroom activities that relate to such research findings. We saw these presentations as an opportunity to initiate a group discussion on whether and how research in numerical and cognitive science is or can be applied to the classroom. The central topic of these presentations (e.g., arithmetic strategies) also served as the focal point and unifying feature of all other aspects of the professional learning across each session.

This specific component of the intervention was hypothesized to facilitate teachers' understanding of research knowledge and terminology related to numerical cognition and, to a lesser degree, developmental cognitive neuroscience more generally. For this reason, we expected to see gains in teachers' actual and self-perceived numerical cognition knowledge, as well as potentially increases in self-perceived general cognition terminology (see measures below).

*Whole Group Discussion of Research Articles.* In between sessions, group members were expected to read one or two research articles related to the session's main topic. Table 4.2 provides a list of the articles read and discussed. Group members prepared questions based on the reading(s), providing a catalyst for the group discussion of the readings. This component of the intervention served the same purpose of the researcher-led research presentation. It was our intention that reading and discussing research in numerical cognition would help familiarize group members with key concepts and terminology from the field of numerical cognition. We also viewed this component as an extension of the research presentations and an opportunity for group members to further consolidate and question their understanding of the targeted topics. This component was hypothesized to further facilitate teachers' content knowledge in the area of numerical cognition as well as issues related to bridging the gap between research and practice.



*Assessments of Students' Mathematical Thinking.* As a follow-up to research on children's numerical thinking, as well as means to bridge between research and practice, we carried out brief assessments of children's mathematical thinking (i.e., clinical interviews). These assessments were based on established measures within the numerical cognition literature and targeted the session's given focus. During our session on the foundations of numerical thinking, team members were provided with a copy of Okamoto and Case's Number Knowledge Test (1996) and administered the assessment with a minimum of three of their own students. During the session where we investigated numerical-spatial associations, teachers were introduced to the number line task (i.e., a task involving the placement of a given number on a horizontal line marked with bounded end points, e.g., 0 and 100). During the sessions on arithmetic, teachers were introduced to methods of observing and recording children's arithmetic strategy use. With the exception of the Number Knowledge Test, which occurred in between sessions, the other assessments occurred as part of the professional learning. After introducing the group to a particular assessment (e.g., number line estimation task), teachers were asked to select three students from their classrooms that they were interested in assessing. Teachers then worked with their same-grade teacher partners to adapt the measure to their own needs and prepare a series of questions for the students they would be interviewing/assessing. Teachers then took turns sharing their questions with the whole group, offering a rationale for the creation of their questions as well as their predictions for how children would perform on the task. Teachers then went to their own classrooms and retrieved the student(s) they were interested in carrying out the assessment with and brought the student(s) to our common meeting place (i.e., the library). Teachers then conducted the interviews/assessments, typically with one student at a time, but sometimes with two or three children at once. Following the assessments, we would come back together as a group and take turns sharing our observations of student thinking. Teachers were also encouraged, whenever possible, to record their interviews and assessments with students and later upload them to our group's shared Google Drive. Teachers were given opportunities to show a brief video clip of their students' thinking and discuss it with the group.

The primary purpose of this component of the intervention was to make students' mathematical thinking visible, providing teachers with new insights into students' mathematical thinking (Ginsburg, 1997). Relatedly, it was our hope that these observations/insights would help inform subsequent teacher planning and instruction in the given areas of focus. For these reasons, teachers were encouraged to carry out the assessments as clinical interviews as opposed to standard test administration. In other words, we encouraged teachers to focus less on test administration and more on what the child's response to the question might reveal about their current mathematical understanding. We encouraged the group to 'go off script' and improvise new questions and extensions in direct response to the child's responses.

By orienting attention towards student thinking (and what this might mean for instruction), we predicted that teacher-student interviews/assessments may confer a number of benefits. In line with previous research, we predicted that teacher-student interviews/assessments may enhance teachers' mathematical content knowledge as well as pedagogical content knowledge (Ball, Thames, & Phelps, 2008; Clarke et al., 2011; Mast & Ginsburg, 2010; Moss et al., 2015). Moreover, the use of teacher-student interviews has also been associated with increased teacher confidence in teaching mathematics (Clarke et al., 2011). For these reasons, we had reason to believe that the inclusion of teacher-student assessments was an important potential agent of teacher change.

*Teacher Engagement with Mathematics.* During each meeting, teachers engaged in a variety of mathematical activities related to each session's targeted theme (e.g., number-space associations). While some of these activities were specifically intended for adults, the majority of the activities were intended to be implemented in the teachers' classrooms with their own students. In other words, with few exceptions, the activities that we asked teachers to engage in were the same as those that were to be implemented with students in the teachers' own classrooms (for details on the student intervention implementation see the next section). This component of the intervention was designed to achieve several related aims. First, as a means of focusing attention on student thinking – a guiding principle of approach to PD. Teachers were asked to engage in the activities with the perspective of their students' in mind (e.g., "*How might you approach this task if you*

*were a student? If you were a 5-year-old what might you find difficult? What questions might you have?*). A second purpose of having teachers engage in mathematics was to increase content knowledge and to further raise the group's familiarity with the concepts discussed previously in the context of research. For example, by engaging in an activity targeting various arithmetic strategies (e.g., counting on from the largest of two addends), it was hoped that teachers would become better acquainted with concepts related to arithmetic strategies, and in turn, would be better able to recognize their students' arithmetic strategies. A third purpose of doing mathematics as a group was an attempt to lower teacher's mathematics anxiety. It is well documented that early years teachers demonstrate high levels of mathematics anxiety; that is, feelings of fear or apprehension of mathematics or the prospect of doing math (Maloney & Beilock, 2013). We intentionally selected activities that we thought would give teacher's a new appreciation for mathematics and that they would be excited to share with their students. Moreover, by having teachers engage in mathematics through the mind of a child, we aimed to make it clear that we were not evaluating the teachers' mathematical performance, but rather, we were interested in learning more about how children think about and learn mathematics.

To summarize, this component of the intervention was intended as a means to a) orient teachers' attention towards students' mathematical thinking, b) to increase mathematical content knowledge, and c) foster positive attitudes towards mathematics (i.e., lower levels of math anxiety). In addition, we anticipated that teachers would be more likely to implement the activities in their classrooms if they were more familiar with them and what they might expect from students.

*Design and Implementation of Student Intervention.* The last component of the teacher intervention centered around the implementation of classroom-based activities. Each session the research team presented the teacher team with a series of activities targeting the specific focus of each session (e.g., number-space associations). As noted above, these activities were first presented to and tried by the teacher team. Then, as a team, we discussed how the activities might be implemented, and if necessary, adapted, in the teachers' own classrooms. To access the activities for each session visit:

[\(https://osf.io/tqs7e/\)](https://osf.io/tqs7e/)

The activities were referred to as Quick Challenge activities and, as the name suggests, were designed as brief activities (5-20 minute) that could easily be implemented and continually adapted over multiple iterations. That is, the Quick Challenge activities were not designed to be stand-alone lessons, but activities that could be used and continually adapted to meet the learning needs of children in different grades (K-3) and abilities. The selected activities were intended to build-up children's numerical reasoning gradually and in accordance with the principle of distributed/spaced practice (Kang, 2016; Rohrer, 2015). In fact, we presented and discussed research on distributed/spaced practice as a means to first introduce the group to Quick Challenge activities and the rationale for their design and implementation.

On two different occasions, on the first and third session together, the first author modeled the implementation of two separate Quick Challenge activities for the group. Children from different teacher's classrooms were brought to the library and participated in the activities in front of the group. This was done to model how the activities were intended to be implemented; that is, in a playful yet mathematically rigorous approach, using careful observation of student reasoning as a basis to adapt and expand the specific questions asked of students.

In terms of implementation, teachers were encouraged to try all of the shared activities as part of their regular mathematics instructional time. Teachers were provided with log sheets to record notes and the name and duration of the Quick Challenge activities implemented. During each meeting, with the exception of the first one, teachers shared the successes and challenges they faced with implementation.

We predicted that having students participate in these activities throughout the intervention would a) provide a context in which teachers could further observe the concepts discussed as part of the professional learning, and 2) provide opportunities for students' to further strengthen their numerical reasoning. More specifically, given the content to the Quick Challenge activities, we expected to see the largest evidence of student gains in their basic numerical reasoning (number comparison and ordering), mental arithmetic, number-line estimation, and abilities to apply their numerical knowledge across a variety of number-based contexts.

Table 4.1

*Summary of main mathematical content addressed across each session.*

Session Number	Main Numerical Content Addressed
1/2	<ul style="list-style-type: none"> <li>• Overview of counting principles</li> <li>• Main focus on cardinality and ordinality</li> <li>• Place value</li> <li>• Overview of arithmetic strategies</li> </ul>
3	<ul style="list-style-type: none"> <li>• Number-space mappings</li> <li>• Main focus on number lines as a tool for assessment, understanding, and representing numerical relations</li> <li>• Grids/Coordinates</li> </ul>
4/5	<ul style="list-style-type: none"> <li>• Arithmetic strategies (counting up, counting on, composing/decomposing, automatic retrieval)</li> <li>• Composing/decomposing number beyond context of arithmetic</li> </ul>

Table 4.2

*List of articles read and discussed as part of teacher professional learning intervention.*

Session Number	Title of Research Article	Year	Author(s)
1	<i>Numerical Symbols Count for Mathematical Success</i>	2017	Merkley & Ansari
	<i>Laying the Foundation for Computational Fluency</i>	2003	Griffin
2	<i>Improving the Numerical Understanding of Children from Low-Income Families</i>	2009	Siegler
3	<i>Early Number Competencies and Mathematical Learning: Individual Variation, Screening, and Intervention</i>	2014	Jordan, Fuchs, & Dyson
4	<i>Bridges Over Troubled Waters: Education and Cognitive Neuroscience</i>	2006	Ansari & Coch

## 4.3.4 Pre- and Post-Test Measures

### 4.3.4.1 Teacher Measures and Testing Procedure

*Math Anxiety.* Teacher mathematics anxiety was measured using the short Mathematics Anxiety Rating Scale (sMARS; Alexander & Martray, 1989). The questionnaire includes 25 items. For each item, participants are asked to indicate the degree to which a given math-related situation (e.g., receiving a math textbook, being given a set of subtraction problems to solve on paper) would make them feel anxious on a 5-point scale, from “not at all” to “very much.” Each teacher received a total score across all 25 items. To keep the total scores meaningful and within the 5-point rating scale, we divided each teacher’s total score by 25. Thus, each teacher was given a score out of 5, with lower score indicating lower math anxiety and higher scores indicating higher math anxiety. In Year 1, data were missing for 4 teachers in the intervention group and 2 teachers in the control group. Data were missing due to absenteeism on the day of testing (1 intervention: 1 control) or incomplete surveys. In Year 2, data were missing for 3 teachers in the intervention group and 2 teachers in the control group. Data were missing due to absenteeism on the day of testing (2 intervention: 2 control) or incomplete surveys.

*Math Comfort Level.* As an additional means of measuring teacher mathematics comfort/anxiety, teachers completed the Math for Young Children Survey (see Hawes et al., 2017). The survey includes 9 items in which teachers are asked to indicate their comfort level teaching and learning math on a 5-point scale, from “not at all comfortable” to “very comfortable” (e.g., How are comfortable are you teaching math? How comfortable are you as a math learner?). Each teacher received a total score across all 9 items. To keep the total scores meaningful and within the 5-point rating scale, we divided each teacher’s total score by 9. Thus, each teacher was given a score out of 5, with lower scores indicating lower comfort levels with math and higher scores indicating higher levels of comfort with math. In Year 1, data were missing for 4 teachers in the intervention group and 2 teachers in the control group. Data were missing due to absenteeism on the day of testing (1 intervention:1 control) or incomplete surveys. In

Year 2, data were missing for 3 teachers in the intervention group and 3 teachers in the control group. Data were missing due to absenteeism on the day of testing (2 intervention: 2 control) or incomplete surveys.

*Numerical Cognition Test.* This test was specifically designed for the purposes of this study. The test includes 12 multiple choice questions on key topics and concepts discussed within the numerical cognition literature and addressed within the current intervention. For example, the measure assesses knowledge of concepts and terms related to arithmetic strategies, numerical distance effects, the ‘mental number line,’ the counting principles, and dyscalculia. See Appendix A for a copy of the test. One point was awarded for each correct response on the test and teachers were given a total score out of 12. In Year 1, data were missing for 1 teacher in the intervention group and 1 teacher in the control group. Data were missing due to absenteeism on the day of testing (1 control) or incomplete surveys. In Year 2, data were missing for 3 teachers in the intervention group and 3 teachers in the control group. Data were missing due to absenteeism on the day of testing (2 intervention: 2 control) or incomplete surveys.

*Self-Perceived Numerical Cognition Knowledge.* This measure consisted of 5 items from the Mind, Brain, and Education Questionnaire (Goffin, Sokolowski, Matejko, Bugden, Lyons, & Ansari, 2018). Participants were presented with terms related to numerical cognition, such as dyscalculia, cardinality, mental number line, and asked to indicate their level of knowledge on a 6-point scale: ““None” means you have never heard of the term and “Excellent” mean you could explain the term to a peer.” Each teacher received a total score out of 30. To keep the total scores meaningful and within the 6-point rating scale, we divided each teacher’s total score by 5. Thus, each teacher was given a total score of 6 to indicate their self-perceived numerical cognition knowledge, with lower scores indicating lower self-perceived knowledge and higher scores indicating higher self-perceived knowledge. In Year 1, data were missing for 1 teacher in the control group due to an incomplete survey. In Year 2, data were missing for 2 teachers in the intervention group and 3 teachers in the control group. Data were missing due to absenteeism on the day of testing (2 intervention: 2 control) or incomplete surveys.

*Self-Perceived General Cognition Knowledge.* This measure consisted of the remaining 9 items from a subsection of the Mind, Brain, Education Questionnaire noted above. Participants were presented with terms related to cognitive science research more broadly, including brain plasticity, working memory, dyslexia, executive functions, and the scientific method. The same 6-point scale and scoring procedures described above were used. Thus, each teacher was given a total score of 6 to indicate their self-perceived general cognition knowledge, with lower scores indicating lower self-perceived knowledge and higher scores indicating higher self-perceived knowledge. In Year 1, data were missing for 1 teacher in the control group due to an incomplete survey. In Year 2, data were missing for 2 teachers in the intervention group and 3 teachers in the control group. Data were missing due to absenteeism on the day of testing (2 intervention: 2 control) or incomplete surveys.

#### 4.3.4.2 Child Measures and Testing Procedure

Participating children completed 13 measures over two approximately 30 minutes testing sessions (1-5 days a part) 2-3 weeks prior to the intervention and within a 2-week period following the intervention. With the exception of the Mental Arithmetic measure, which was designed specifically for this study, all measures were selected from published research. Participants completed the measures in pseudo-random due to the blocked nature of some of the tests. Symbolic Number Comparison, Non-symbolic Number Comparison, and Ordering were administered to children in this order. Children also always completed the Path Span Forward prior to Path Span Reverse and Numeration prior to Geometry. All testing occurred in a quiet location of the school (i.e., empty classrooms or private testing rooms) and was administered one-to-one by trained experimenters.

##### 4.3.4.2.1 Measures of Numerical and Mathematical Reasoning



The following three measures were adopted from Lyons, Bugden, De Jesus, and Ansari (2018) and Lyons, Hutchison, Bugden, Goffin, and Ansari (2018) and part of the same paper-and-pencil measure. As noted above, the three separate measures were presented in fixed order. Both the symbolic and nonsymbolic number comparison tasks consisted of 72 items and the ordering task included 48 items. Children were provided with 1 minute to complete as many items as possible. For all three measures, the same scoring procedures were used. To adjust for potential speed-accuracy trade-offs/guessing behavior, adjusted raw scores were computed by subtracting the total number of incorrect items from the total number of correct items (see Lyons, Bugden, et al., 2018; Lyons, Hutchison, et al., 2018).

*Symbolic Number Comparison.* Children were presented with pairs of Hindu-Arabic numerals (e.g., 2 | 5) and asked to indicate the larger number as quickly and accurately as possible. Comparisons were confined to single-digit numerals (1-9) and the absolute distances between numerals ranged from 1 to 3. Trials were counterbalanced so that the larger number appeared an equal number of times on the left side of the page as the right.

*Nonsymbolic number comparison.* Children were presented with pairs of dot arrays (e.g., : | ::) and asked to indicate the array with the most dots as quickly and accurately as possible. Dot arrays ranged from 1 to 9 dots and included the same numerical distances as those used in the symbolic number comparison task. Children were instructed not to count the dots. To control for the influence of the continuous properties of the dot stimuli on performance, both the area and contour length were manipulated and controlled for across trials. On half the trials, dot area was positively correlated with numerosity and overall contour length was negatively correlated. The reverse was true on the other half of the trials. In Year 2, data were incomplete/missing for 2 students (1 intervention: 1 control).

*Ordering Task.* Children were presented with a sequence of numerals (e.g., 1 – 2 – 3) and asked to indicate whether or not the sequence was in numerical order. Numerals ranged from 1 to 9 and included absolute numerical distances of 1 (e.g., 1 – 2 – 3) or 2 (e.g., 1 –

3 – 5). There were an equal number of correct and incorrect sequences of distances 1 and 2. For half of the items, the sequences were in the correct ‘ascending order’ and for the other half, the sequences were in incorrect order. In Year 2, data were incomplete/missing for 1 student in the control group.

*Mental Arithmetic.* Children were orally administered 12 addition problems of increasing difficulty. The first 4 problems were considered ‘easy’ and involved solutions with sums of 5 or less. The next 4 problems were considered ‘medium’ difficulty and involved solutions between 6 and 10. The last 4 problems were considered ‘difficult’ and involved solutions between 11 and 15. Questions were counterbalanced so that on half of the questions the smaller addend was presented first (e.g.,  $1 + 2$ ) and on the other half the larger addend was presented first (e.g.,  $2 + 1$ ). All questions were solved without paper-and-pencil or concrete materials. Children were awarded 1 point for each correct response and given a total score out of 12. In Year 1, data were incomplete/missing for 1 student in the control group. In Year 2, data were incomplete/missing for 1 student in each group.

*Number Line Estimation.* This measure was administered on an iPad (to access the application see: <https://hume.ca/ix/estimationline.html>). Children were presented with a horizontal line marked with “0” at the far left end of the line and either “10” or “100” at the far right end of the line. Kindergarten children were administered the 0-10 number line and children in grades 1-3 were administered the 0-100 number line. The goal of the task was to indicate where on the line a given target number belongs (e.g., “*Where does the number six belong on the line?*”). To familiarize children with the task, children were first presented with a practice trial: For kindergarten children, the practice trial involved the placement of “5” and for children in grades 1-3 the practice trial involved the placement of “50.” The test trials for kindergarten children included numbers 1-9 (with the exception of 5). For children in Grades 1-3, test trials included the following target numbers adopted from Laski and Siegler (2007): 2, 3, 5, 8, 12, 17, 21, 26, 34, 39, 42, 46, 54, 58, 61, 67, 73, 78, 82, 89, 92, and 97. All trials were randomly presented to children. The accuracy of each trial was recorded by the computer. We then used this information

to calculate each child's overall accuracy across all estimates. To do this, we calculated each child's percent absolute error (PAE) using the following formula:

$$\text{PAE} = \left| \frac{\text{Estimate} - \text{Estimated Quantity}}{\text{Scale of Estimates}} \right| \times 100$$

To put this into context, if a child was asked to estimate the location of 3 on the 0-10 number line and placed his/her response at the location that corresponded to 5, the percent absolute error (PAE) would be 20%:  $[(5 - 3)/10] \times 100$ . A lower PAE is associated with greater accuracy (less error). In Year 1, data were incomplete/missing for 6 students (4 intervention: 2 control). In Year 2, data were incomplete/missing for 4 students (3 intervention: 1 control).

*Numeration Test.* Children's overall numeracy performance was assessed with the Numeration subtest from KeyMath (Connolly, 2007); a standardized Canadian normed test designed for students in kindergarten to 12<sup>th</sup> grade. This test provides a comprehensive and curriculum-aligned assessment of children's numeration skills, including knowledge and concepts related to counting, comparing quantities, recognizing and ordering number symbols, operations, place value, and proportions/fractions/decimals. The test is administered with an easel booklet and each problem refers to information presented in the form of an image and/or writing. The test is adaptive in that it begins by establishing baseline performance and continues with questions of increasing difficulty. The test is discontinued when the child answers four questions incorrectly in a row. Thus, children are presented with problems that vary from the familiar to the unfamiliar/novel. The test includes 49 items in total. Children were given a total raw score by subtracting the total number of incorrect responses from the maximum item number reached. In Year 1, data were incomplete/missing for 1 student (control). In Year 2, data were incomplete/missing for 1 student (control).

*Geometry Test.* To assess children's geometry performance, we used the Geometry subtest from the KeyMath assessment described above (Connolly, 2007). This test proves a comprehensive and curriculum-aligned assessment of children's geometry skills, including knowledge and concepts related to shape recognition, positional language, geometrical transformations (e.g., rotations), measurement, grid coordinates, angles, geometric proofs. The same scoring procedures described above were used for this measure. The test included a total of 36 items. In Year 1, data were incomplete/missing for 3 students (2 intervention: 1 control). In Year 2, data were incomplete/missing for 4 students (2 intervention: 2 control).

#### 4.3.4.2.2 Measures of Spatial Ability

*2D Mental Rotation.* Children's mental rotation was measured with an adapted version of the Children's Mental Transformation Task (Levine, Huttenlocher, Taylor, & Langrock, 1999); a widely used measure of children's mental rotation skills (e.g., see Ehrlich, Levine, & Goldin-Meadow, 2006; Hawes, LeFevre, Xu, & Bruce, 2015). In this task, children are presented with two halves of 2D shape (printed on cardstock), which have been separated and rotated 60° from one another on either the same plane (direct rotation items) or diagonal plane (diagonal rotation items). Children are then asked to identify which shape (amongst four options) can be made by putting two halves together; a process that presumably relies on the ability to mentally rotate the puzzle pieces and visualize the correct solution. There were 16 items and children were awarded one point for each correct response. In Year 1, data were incomplete/missing for 1 student (control). In Year 2, data were incomplete/missing for 3 students (1 intervention: 2 control).

*Visual-Spatial Reasoning.* This measure was adopted from Hawes, Moss, Caswell, Naqvi, and MacKinnon (2017) and provides a comprehensive measure of children's spatial visualization skills. The test consists of 20 items divided into four different problem types: missing puzzle pieces (two variations), composition/decomposition of 2D shapes, and mental paper folding. For each problem, children were asked to identify the correct answer among four options. One point was awarded for each correct response. In Year 1,

data were incomplete/missing for 1 student (control). In Year 2, data were incomplete/missing for 2 students (1 intervention: 1 control).

*Raven's Coloured Progressive Matrices.* This is a widely used measure of children's visual-spatial analogical reasoning (Raven, 2008). Children are presented with partially completed visual-spatial patterns and must select from amongst six alternatives the puzzle piece that will complete the pattern. The test consists of 36 items. One point was awarded for each correct response. In Year 1, data were incomplete/missing for 1 student (control). In Year 2, data were incomplete/missing for 2 students (1 intervention: 1 control).

#### 4.3.4.2.3 Measures of Executive Functioning

*Head-Toes-Knees-Shoulders task (HTKS).* This measure was adapted from Ponitz et al. (2009) and was designed to measure children's ability to engage in flexible attention, working memory, and inhibitory control (McClelland & Cameron, 2012). For each item, children listen to an instruction to touch a body part (e.g., "Touch your toes") and then must touch a paired "opposite" body part (e.g., head). The task uses 'head' and 'toes' as one pairing and 'knees' and 'shoulders' as the other pairing. There are 20 items in total. For each item, children were given a score of 0, 1, or 2; a score of 0 corresponded to incorrect body movements (touching one's head when asked to touch their head), a score of 1 corresponded to a self-corrected body movements (initiating movement towards the wrong body part and then making a correction), and a score of 2 corresponded to correct body movements (touching one's toes when asked to touch their head). Children were given a total score out of 40. In Year 1, data were incomplete/missing for 1 student (control). In Year 2, data were incomplete/missing for 3 students (2 intervention: 1 control).

*Visual-Spatial Working Memory - Forward Path Span.* This measure was administered on an iPad and was used as a measure of children's visual-spatial working memory (to access the application see: <https://hume.ca/ix/pathspan.html>). Children were presented with a set of nine green circles randomly arranged on the screen and watched as the

circles lit up one at time. Children were then instructed to recall the sequence in the same order in which they were presented. After a practice trial, children were first presented with two trials at a sequence length of two. Upon successful recall of one or both of the sequences, the child progressed to the next level (i.e., two trials with sequence lengths of three). The task was discontinued when the child failed to recall both sequences at any given level. Children were assigned a score based on the total number of correct sequences recalled. In Year 1, data were incomplete/missing for 7 students (4 intervention: 3 control). In Year 2, data were incomplete/missing for 3 students (2 intervention: 1 control).

*Visual-Spatial Working Memory - Reverse Path Span.* This task was identical to the one above but required children to recall the given sequence in reverse order. In Year 1, data were incomplete/missing for 7 students (4 intervention: 3 control). In Year 2, data were incomplete/missing for 4 students (2 intervention: 2 control).

#### **4.3.5 Measurement of Time Spent Implementing Teacher-Led Student Intervention Activities**

Teachers in the intervention groups were provided with tracking sheets where they recorded the date, duration, name of activity, and a brief description/notable observations of the implementation of all activities conducted. It is worth noting that in each participating school (including the control group), teachers of grades 1-3 reported adhering to the Ontario Ministry of Education policy of teaching mathematics for 60 minutes per day. More specifically, in each school, the class schedule was structured to ensure one 60-minute block of mathematics per day. While there is no mandate or guidelines for how much time should be devoted to mathematics instruction in Kindergarten, all participating schools reported between 30-45 minutes of mathematics instruction per day. Thus, we can be fairly certain that the participating schools engaged in equivalent amounts of mathematics instruction. This information is useful in helping to rule out explanations that any potential changes in mathematics of one group over another was due more time spent in mathematics.

As noted above, teachers were also encouraged to contribute to the group's shared Google Drive. Specially, teachers were encouraged to upload any videos of teacher-led assessments, pictures/videos of student work based on the teacher-led student activities, and any adapted versions of the activities tried by teachers in their own classrooms. Both intervention groups were provided with a total score based on the number of unique items uploaded. We then used this score as an exploratory means of measuring and comparing the intervention groups' engagement and/or commitment to the project.

### 4.3.6 Analytical Approach

Analyses were based on the analytical approaches outlined in the pre-registration of the Year 1 (<https://osf.io/efyqy/register/5771ca429ad5a1020de2872e>) and Year 2 studies (<https://osf.io/kpr9g/>). Data were analyzed using Bayesian statistics and conducted with JASP (Version 0.9.0.1). Findings from both the preliminary and main analyses are reported using Bayes factors: A statistic that provides a means of directly comparing and evaluating the strength of evidence for one statistical model (e.g., there is a group difference) over another (e.g., there is no group difference). One of the benefits of using Bayes factors is that they provide a means to quantify the amount of support both for and against the alternative hypothesis over the null. Moreover, Bayes factors can be used to indicate when there is insufficient evidence in support of the alternative hypothesis *or* the null. Knowing whether there is support for the null and/or whether more data are needed before claiming support for the null (i.e., "there is no effect") is especially important when analyzing and reporting intervention-based research. Another advantage of using Bayesian statistics, compared to traditional frequentist statistics, is that smaller sample sizes are needed to reach conclusions about the presence of a given effect, while having the same or lower long-term error rate (Schönbrodt, Wagenmakers, Zehetleitner, & Perugini, 2017). Given the small sample size of teacher participants in the present study, Bayesian analyzes were ideally suited for this purpose.

For all preliminary analyses, we report on Bayes factors as they correspond to evidence in favor of the alternative hypothesis (i.e., that there are differences between groups at pre-test) compared to the null hypothesis (i.e., there are no differences between

groups at pre-test). For these analyses, the symbol  $BF_{10}$  is used to signify the strength of evidence for the alternative hypothesis ( $H_1$ ) over the null ( $H_0$ ). As detailed further below, we considered Bayes factors of 3 and above as evidence for the alternative (i.e., the presence of group differences at pre).

To address our main questions of whether or not the intervention had any positive effects on both teacher and student outcomes, we used mixed-design Bayesian repeated measures ANOVA. In both Studies 1 and 2, we analyzed the extent to which teacher and student change from pre- to post-test was dependent on group assignment (i.e., experimental vs. control). In addition, in Experiment 2 we also evaluated the effects of the intervention by carrying out within-group Bayesian repeated measures ANOVAs. In all cases, we report on the Bayes factors from a model with the interaction term (group x time) from models without the interaction term. More specifically, we report on the statistic referred to as Bayes factor inclusion (hereafter  $BF_{incl}$ ). The  $BF_{incl}$  provides a means to quantify the amount by which the prior odds of including an effect term in the model (in this case a group x time interaction) is updated after observing the data. For example, a  $BF_{incl}$  of 5 indicates that the observed data have increased the odds of an interaction by a factor of 5. Said differently, a model which includes the interaction term is 5 times more likely than all other models of the data that do not contain an interaction.

Given that Bayes factors are open to subjective interpretation (e.g., should an effect that is 5 times more likely than the null be considered as strong evidence? Is that convincing enough?), the following guidelines for interpreting the strength of Bayes factors have been recommended (e.g., see Jarosz & Wiley, 2014): Bayes factors between 1 and 3 = weak/anecdotal support (not enough evidence to make any substantial claims either for or against the predicted relationship); Bayes factors between 3 and 10 = substantial support (enough evidence to make moderate claims about effect); Bayes factors between 10-100 = strong evidence (enough evidence to be make moderate/strong claims about effect); Bayes factors greater than 100 = very strong/decisive evidence (enough evidence to make strong claims about effect). As mentioned above, in the present study, we report on the Bayes factors associated with a model that includes an interaction compared to all other models that do not include the interaction term. In cases where the reported Bayes factors are below 1, this is an indication that there is more support for a



model that does not include an interaction factor. In cases for the Bayes factor is 3 or above, this is considered evidence in support of an interaction. In short, the higher the Bayes factor, the higher the odds of there being a group difference from pre-to-post.

Note that for all analyses we used the default settings in JASP for repeated measure ANOVA (Version 0.9.0.1). These settings include an  $r$  scale for fixed effects of .5 (i.e.,  $h = .5$ ). We used the default prior because we had no prior information about which size effects to expect. Moreover, the default prior contains a reasonable range of data coverage without being committed to any one point (Rouder, Morey, Verhagen, Swagman, & Wagenmakers, 2017).

## 4.4 Results

### 4.4.1 Year 1 – Teacher Results

#### 4.4.1.1 Preliminary Analyses

To assess the presence of any group differences at pre-test on any of the measures, a series of Bayesian independent samples  $t$ -tests were conducted (see Table 4.3 for a comparison of mean scores by each group and across both time points). Note that for these analyses we excluded the pre-test data available from the school principal in the intervention group. We did this because we were unable to collect her post-test data and we also did not have any pre- or post-test data from the school principal in the control group. Results revealed no evidence of group differences on any of the five pre-test measures: Math Comfort Level (16),  $BF_{10} = 0.48$ ; Math Anxiety (16),  $BF_{10} = 0.72$ ; Numerical Cognition Knowledge (20),  $BF_{10} = 0.59$ ; Self-Perceived Numerical Cognition Knowledge (21),  $BF_{10} = 0.41$ ; Self-Perceived General Cognition Knowledge (21),  $BF_{10} = 0.40$ . Note that the numbers in brackets refer to the degrees of freedom for each particular  $t$ -test conducted. These findings suggest that the groups were well-matched on all measures of interest.

Table 4.3

*Mean scores and standard deviations by teacher group at pre- and post-test (Year 1)*

		Math Comfort	Math Anxiety	Num. Cog. Knowledge Test	Self-Perceived Num. Cog. Knowledge	Self-Perceived Gen. Cog. Knowledge
Pre-Test	Experimental	25.82 (5.08)	82.64 (15.27)	7.43 (1.45)	14.67 (6.20)	28.53 (9.16)
	Control	27.57 (6.32)	92.14 (15.60)	6.75 (1.39)	15.63 (4.27)	27.75 (6.45)
Post-Test	Experimental	25.33 (5.25)	84.27 (19.52)	8.93 (1.83)	19.00 (3.51)	44.33 (8.17)
	Control	28.14 (6.57)	87.43 (12.87)	6.25 (1.83)	12.50 (4.07)	33.88 (9.02)

#### 4.4.1.2 Main Analyses

Bayesian repeated measures ANOVA were used to analyze the extent to which the intervention and control groups changed in relation to one another from pre- to post-test (see Table 4.3; Figure 4.2). More specifically, we conducted a group (intervention vs. control) by time (pre vs post) analysis for each dependent variable. On both the Math Comfort Level and Math Anxiety surveys there was evidence in favor of the null (i.e., support against a model that includes a time x group interaction); Math Comfort Level (16),  $BF_{incl} = 0.25$ ; Math Anxiety (16),  $BF_{incl} = 0.19$ ; Support for the presence of a group by time interaction in favor of the intervention group was observed on three of the measures: Numerical Cognition Knowledge (20),  $BF_{incl} = 9.24$ ; Self-Perceived Numerical Cognition Knowledge (21),  $BF_{incl} = 7.47$ ; Self-Perceived General Cognition Knowledge (21),  $BF_{incl} = 11.64$ . Figure 4.2 displays each educator's individual scores on each measure and across each time point. Overall, our analyses indicated that the intervention group demonstrated greater improvements than the control group on a test of Numerical Cognition and questionnaires examining Self-Perceived Numerical Cognition Knowledge and Self-Perceived General Cognition Knowledge.

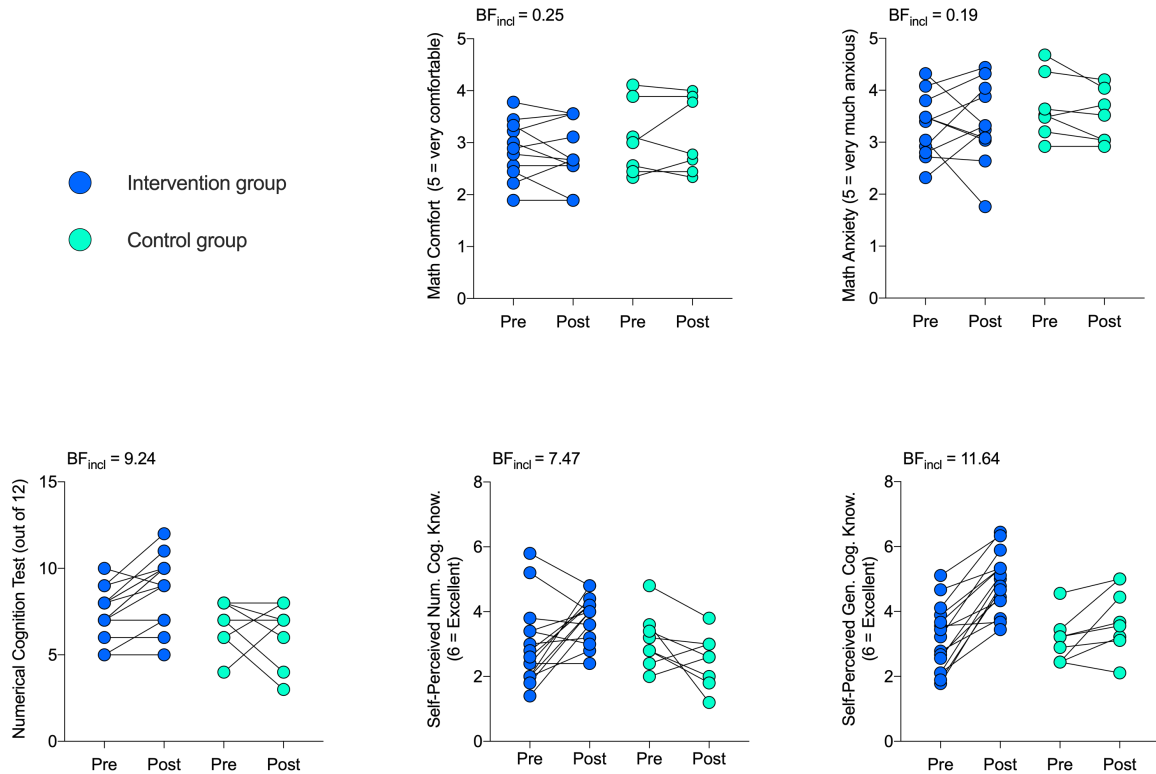


Figure 4.2 Comparison of pre-post performance by teachers in the intervention and control group (Year 1). Each circle and the lines that connect them represents the pre-post scores for an individual teacher.

## 4.4.2 Year 1 – Student Results

### 4.4.2.1 Preliminary Analyses

Table 4.4 shows the mean scores and standard deviations by group at pre and post. To assess the presence of any group differences at pre-test on any of the measures (as well as age), a series of Bayesian independent samples  $t$ -tests were conducted. As preregistered, group differences were determined by Bayes factors greater than three. Based on this criteria, no group differences were observed on any of the measures: Age(105),  $BF_{10} = 0.40$ ; Numeration(105),  $BF_{10} = 0.27$ ; Geometry(104),  $BF_{10} = 0.22$ ; Non-Symbolic Number Comparison(105),  $BF_{10} = 2.015$ ; Symbolic Number Comparison(105),  $BF_{10} =$

0.52; Ordering(105),  $BF_{10} = 1.36$ ; Arithmetic(105),  $BF_{10} = 0.25$ ; Number Line(PAE)(99),  $BF_{10} = 0.42$ ; Visual-Spatial Working Memory – Forward Path Span(100),  $BF_{10} = 0.24$ ; Visual-Spatial Working Memory – Reverse Path Span(100),  $BF_{10} = 0.25$ ; Head-Toes-Knees-Shoulders (105),  $BF_{10} = 0.21$ ; Raven’s Matrices(105),  $BF_{10} = 0.27$ ; 2D Mental Rotation(105),  $BF_{10} = 0.24$ ; Visual-Spatial Reasoning(105),  $BF_{10} = 0.30$ . Note that the numbers in brackets refer to the degrees of freedom for each particular *t*-test conducted. These findings suggest that the groups were well-matched in terms of age and performance at pre-test.

Table 4.4

*Mean scores and standard deviations by group at pre- and post-test (Year 1).*

	Mathematics						Executive Functions				Spatial Reasoning		
	Numeration	Geometry	Non-Sym Comp	Sym Comp	Order	Arithmetic	Number Line (PAE)	VSWM-F	VSWM-R	HTKS	Raven's	2D MR	Visual-Spatial
Pre-Test	9.56 (5.10)	10.94 (5.44)	14.12 (6.29)	18.02 (10.85)	7.14 (5.00)	6.44 (4.60)	.15 (.09)	3.74 (2.17)	2.55 (1.77)	26.72 (11.62)	18.31 (5.94)	7.98 (3.40)	8.42 (3.610)
Control	8.80 (4.97)	10.64 (3.45)	11.05 (7.52)	14.93 (11.18)	5.15 (4.94)	5.84 (4.62)	.17 (.10)	3.49 (2.27)	2.30 (2.28)	26.11 (10.80)	17.42 (5.86)	7.62 (3.08)	9.00 (2.99)
Post-Test	10.85 (5.11)	11.73 (4.27)	20.71 (10.70)	14.37 (6.84)	7.14 (5.42)	8.35 (3.68)	.10 (.04)	4.53 (1.90)	3.28 (2.06)	32.25 (9.73)	20.67 (5.60)	8.73 (3.67)	9.29 (3.59)
Control	9.06 (5.23)	10.96 (4.77)	17.29 (12.43)	14.62 (7.55)	5.51 (6.08)	6.37 (4.50)	.16 (.10)	4.06 (2.57)	2.85 (2.12)	32.06 (7.49)	19.20 (5.86)	8.72 (3.36)	9.82 (3.97)

## 4.4.2.2 Main Analyses

Bayesian repeated measures ANOVA were used to analyze the extent to which the intervention and control groups changed in relation to one another from pre- to post-test (see Table 4.4; Figure 4.3). More specifically, we conducted a group (intervention vs. control) by time (pre vs post) analysis for each dependent variable. Bayes factors for the inclusion of the interaction term (group x time) were used to determine the strength of the intervention. As noted above, our minimum a priori criteria for evidence of positive intervention effects was associated with a Bayes factor of 3. Based on this criteria, our analyses indicated evidence of pre-post gains by the intervention group compared to the control group on three measures: Numeration(104),  $BF_{incl} = 9.65$ ; Arithmetic(104),  $BF_{incl} = 8.50$ ; Number Line(PAE)(97),  $BF_{incl} = 4.53$ . There was evidence of pre-post gains in favor of the control group on the Non-Symbolic Number task (105),  $BF_{incl} = 28.40$ . There was no evidence of group differences from pre-to-post on any of the other measures: Geometry(102),  $BF_{incl} = 0.06$ ; Symbolic Number Comparison(105),  $BF_{incl} = 0.44$ ; Ordering(105),  $BF_{incl} = 0.07$ ; Visual-Spatial Working Memory – Forward Path Span(98),  $BF_{incl} = 0.22$ ; Visual-Spatial Working Memory – Reverse Path Span(98),  $BF_{incl} = 0.31$ ; Head-Toes-Knees-Shoulders (104),  $BF_{incl} = 0.21$ ; Raven’s Matrices(104),  $BF_{incl} = 0.34$ ; 2D Mental Rotation(104),  $BF_{incl} = 0.23$ ; Visual-Spatial Reasoning(104),  $BF_{incl} = 0.23$ . Figure 4.3 shows all children’s pre-post scores by group and across all the mathematics measures (see Supplementary Figure 1 for pre-post scores by group for performance on the spatial and EF measures; <https://osf.io/tqs7e/files/>).

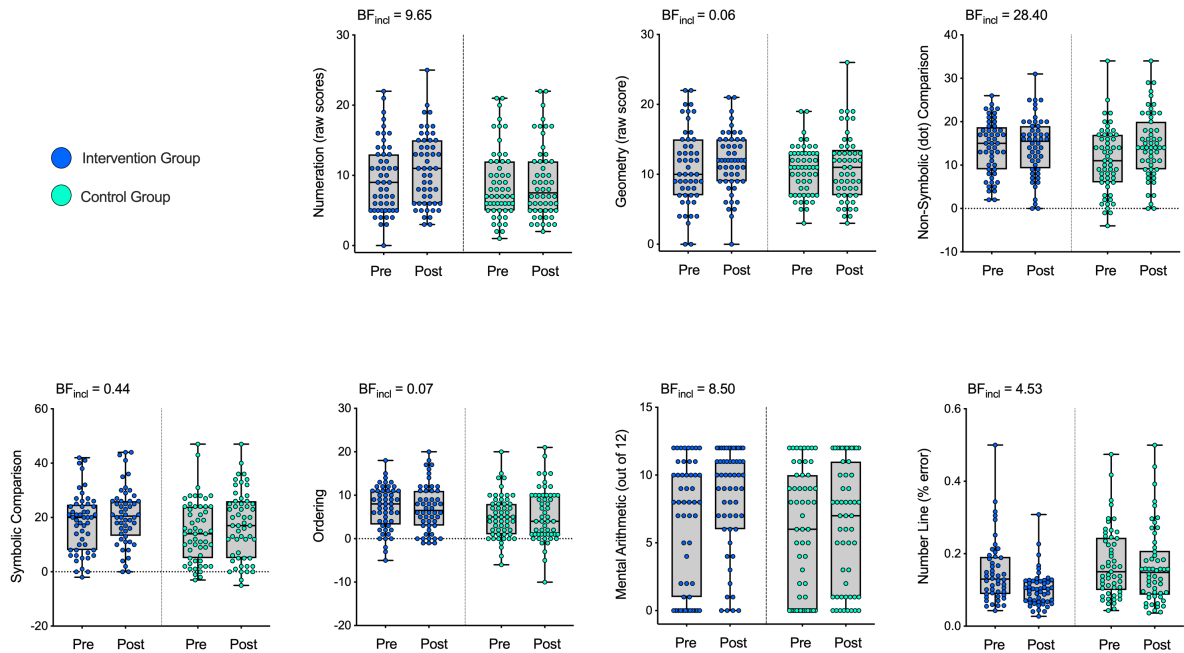


Figure 4.3 Comparison of pre-post performance by students in the intervention and control group (Year 1).

### 4.4.3 Summary of Year 1 Results

Teachers in the intervention group demonstrated greater gains than the control group on measures of numerical cognition knowledge, self-perceived numerical cognition knowledge, and self-perceived general cognition knowledge. There was support in favor of an absence of gains (i.e., support for the null) on measures of math anxiety and comfort in the teaching and learning of mathematics. Children in the intervention classrooms demonstrated greater gains compared to the control group on measures of number line estimation, mental arithmetic (addition), and overall numeration performance. Both groups of children made highly similar gains on measures of spatial and EF skills, which were not targeted during PD. Thus, the gains made by the intervention group were highly specific to content and activities covered as part of the teacher PD.

### 4.4.4 Year 2 – Teacher Results

#### 4.4.4.1 Preliminary Analyses

To assess the presence of any group differences at pre-test on any of the measures, a series of Bayesian independent samples *t*-tests were conducted (see Table 4.5). Results revealed no evidence of group differences on any of the five pre-test measures: Math Comfort Level (19),  $BF_{10} = 0.62$ ; Math Anxiety (20),  $BF_{10} = 0.44$ ; Numerical Cognition Knowledge (20),  $BF_{10} = 0.39$ ; Self-Perceived Numerical Cognition Knowledge (20),  $BF_{10} = 0.45$ ; Self-Perceived General Cognition Knowledge (20),  $BF_{10} = 0.41$ . Note that the numbers in brackets refer to the degrees of freedom for each particular *t*-test conducted. These findings suggest that the groups were well-matched on all measures of interest.

Table 4.5

*Mean scores and standard deviations by teacher group at pre- and post-test (Year 2)*

		Math Comfort	Math Anxiety	Num. Cog. Knowledge Test	Self-Perceived Num. Cog. Knowledge	Self-Perceived Gen. Cog. Knowledge
Pre-Test	Experimental	26.83 (8.58)	86.00 (23.60)	6.46 (1.81)	15.92 (4.15)	28.77 (7.44)
	Control	30.56 (5.46)	91.44 (19.62)	6.33 (1.23)	17.11 (4.46)	30.22 (10.39)
Post-Test	Experimental	26.42 (7.86)	88.92 (23.46)	7.77 (1.92)	20.92 (3.77)	36.00 (5.51)
	Control	29.56 (6.06)	95.22 (20.75)	6.78 (1.86)	15.44 (4.50)	29.44 (10.17)

#### 4.4.4.2 Main Analyses

Bayesian repeated measures ANOVA were used to analyze the extent to which the intervention and control groups changed in relation to one another from pre- to post-test (see Table 4.5; Figure 4.4). On both the Math Comfort Level and Math Anxiety surveys there was evidence in favor of the null (i.e., support against models that included the time x group interaction); Math Comfort Level (19),  $BF_{incl} = 0.21$ ; Math Anxiety (20),  $BF_{incl} = 0.23$ . Support for the presence of a group by time interaction in favor of the intervention group was observed on the measure of Self-Perceived Numerical Cognition Knowledge (20),  $BF_{incl} = 9.23$ . There was insufficient evidence for or against a group x time interaction on the remaining two measures: Numerical Cognition Knowledge (20),  $BF_{incl}$



= 0.76; Self-Perceived General Cognition Knowledge (20),  $BF_{incl} = 1.97$ . Figure 4.4 displays each educator's individual scores on each measure and across each time point. Overall, our analyses indicated that the intervention group demonstrated greater improvements than the control group on the measure of Self-Perceived Numerical Cognition Knowledge.

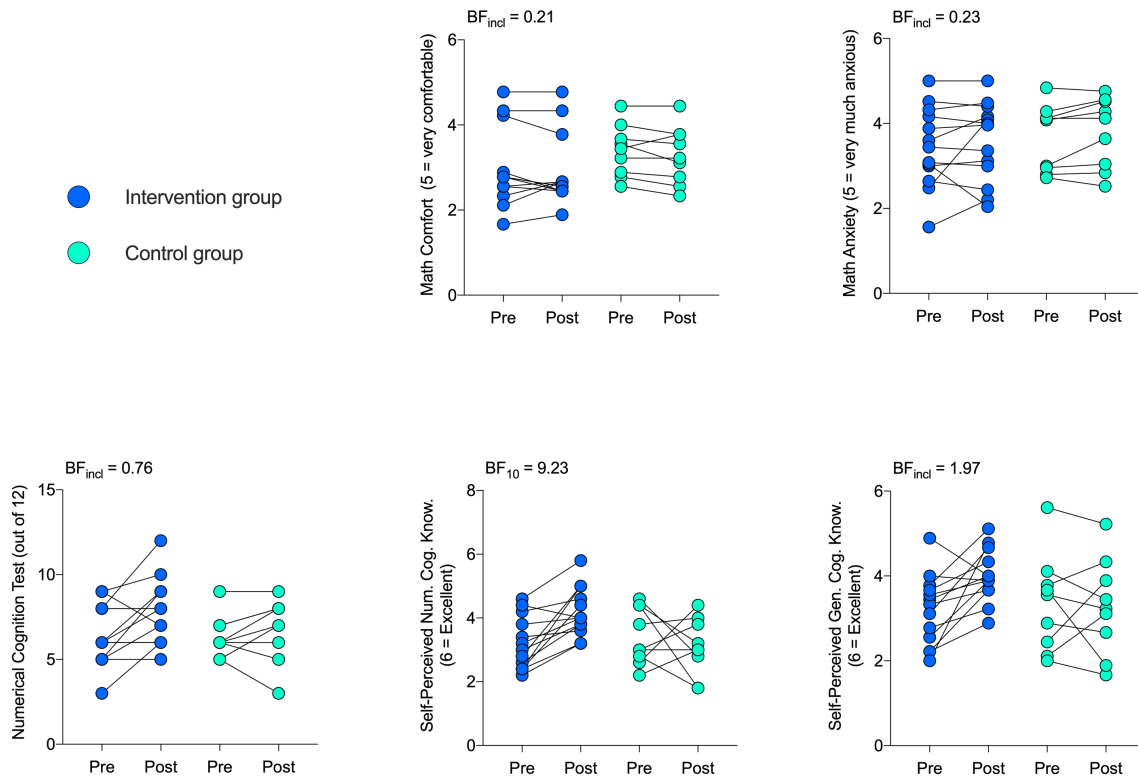


Figure 4.4 Comparison of pre-post performance by teachers in the intervention and control group (Year 2). Each circle and the lines that connect them represents the pre-post scores for an individual teacher.

## 4.4.5 Year 2 – Student Results

### 4.4.5.1 Preliminary Analyses

Table 4.6 shows the mean scores and standard deviations by group at pre and post. To assess the presence of any group differences at pre-test on any of the measures (as well as

age), a series of Bayesian independent samples *t*-tests were conducted. No group differences were observed on any of the measures: Age(119),  $BF_{10} = 0.35$ ; Numeration(119),  $BF_{10} = 0.22$ ; Geometry(118),  $BF_{10} = 0.20$ ; Non-Symbolic Number Comparison(119),  $BF_{10} = 0.22$ ; Symbolic Number Comparison(119),  $BF_{10} = 0.21$ ; Ordering(118),  $BF_{10} = 0.20$ ; Arithmetic(119),  $BF_{10} = 0.31$ ; Number Line(PAE)(118),  $BF_{10} = 0.20$ ; Visual-Spatial Working Memory – Forward Path Span(118),  $BF_{10} = 0.20$ ; Visual-Spatial Working Memory – Reverse Path Span(117),  $BF_{10} = 0.22$ ; Head-Toes-Knees-Shoulders (119),  $BF_{10} = 0.25$ ; Raven’s Matrices(119),  $BF_{10} = 0.20$ ; 2D Mental Rotation(119),  $BF_{10} = 0.19$ ; Visual-Spatial Reasoning(119),  $BF_{10} = 0.21$ . Note that the numbers in brackets refer to the degrees of freedom for each particular *t*-test conducted. That there were no differences between groups on any of the measures suggests that the groups were well-matched in age and performance.

Table 4.6

*Mean scores and standard deviations by group at pre- and post-test (Year 1).*

	Mathematics										Executive Functions				Spatial Reasoning	
	Numeration	Geometry	Non-Sym Comp	Sym Comp	Order	Arithmetic	Number Line (PAE)	VSWM-F	VSWM-R	HTKS	Raven's	2D MR	Visual-Spatial			
Pre-Test	Experimental	10.37 (5.99)	11.66 (5.73)	15.75 (9.07)	20.91 (14.01)	7.05 (6.35)	7.16 (4.17)	.16 (.10)	3.96 (2.52)	2.91 (2.43)	31.05 (9.28)	19.72 (6.69)	8.70 (3.71)	10.54 (3.79)		
	Control	10.92 (5.51)	11.92 (5.44)	14.95 (7.46)	21.95 (11.95)	7.40 (6.35)	7.92 (4.16)	.16 (.11)	3.94 (2.38)	3.13 (2.38)	29.77 (10.18)	19.41 (6.40)	8.94 (3.53)	10.23 (4.09)		
Post-Test	Experimental	12.12 (6.19)	12.68 (5.64)	16.70 (8.38)	22.14 (13.80)	8.75 (7.80)	8.00 (3.83)	.13 (.06)	4.36 (2.54)	3.11 (2.57)	33.05 (10.08)	21.59 (6.63)	9.73 (3.86)	10.82 (4.37)		
	Control	11.98 (5.84)	12.82 (5.11)	16.70 (7.82)	24.39 (12.42)	7.92 (6.34)	8.56 (3.90)	.13 (.08)	4.54 (2.47)	3.21 (2.60)	32.70 (6.36)	21.90 (6.50)	9.52 (3.86)	11.32 (3.96)		

#### 4.4.5.2 Main Analyses

Bayesian repeated measures ANOVA were used to analyze the extent to which the intervention and control groups changed in relation to one another from pre- to post-test (see Table 4.6; Figure 4.5). We found no evidence of pre-post gains by the intervention group compared to the control group on any of the measures: Numeration(118),  $BF_{incl} = 0.48$ ; Geometry(115),  $BF_{incl} = 0.18$ ; Non-Symbolic Number Comparison(117),  $BF_{incl} = 0.30$ ; Symbolic Number Comparison(119),  $BF_{incl} = 0.33$ ; Ordering(118),  $BF_{incl} = 0.14$ ; Arithmetic(117),  $BF_{incl} = 0.27$ ; Number Line(PAE)(115),  $BF_{incl} = 0.16$ ; Visual-Spatial Working Memory – Forward Path Span(116),  $BF_{incl} = 0.22$ ; Visual-Spatial Working Memory – Reverse Path Span(115),  $BF_{incl} = 0.03$ ; Head-Toes-Knees-Shoulders (116),  $BF_{incl} = 0.41$ ; Raven’s Matrices(117),  $BF_{incl} = 0.39$ ; 2D Mental Rotation(116),  $BF_{incl} = 0.14$ ; Visual-Spatial Reasoning(117),  $BF_{incl} = 1.35$ . Figure 4.5 shows all children’s pre-post scores by group and across all the mathematics measures (see Supplementary Figure 2 for pre-post scores by group for performance on the spatial and EF measures; <https://osf.io/tqs7e/files/>).

As a follow-up to the above analysis, we also carried out a series of within-group Bayesian repeated measures ANOVAs. Because the intervention group had previously participated as the control group, we were able to test for differences in their growth across the two conditions (control vs. intervention; see Table 4.7). As outlined in our pre-registration, we considered this analysis as a more robust and reliable measure of the effectiveness of the intervention. These analyses revealed three condition x time interactions with a Bayes factor greater than three. Children demonstrated greater gains on the Numeration test when part of the intervention condition compared to the control condition (i.e., business as usual); Numeration(46),  $BF_{incl} = 9.42$ . Unexpectedly, children demonstrated greater gains on the Non-Symbolic Number Comparison task and HTKS task when part of the control group compared to the intervention group; Non-Symbolic Number Comparison(47),  $BF_{incl} = 7.30$ ; Head-Toes-Knees-Shoulders (45),  $BF_{incl} = 4.21$ . On all of the remaining measures, there was no evidence of greater gains when children were members of the intervention compared to the control condition: Geometry(43),  $BF_{incl} = 0.35$ ; Symbolic Number Comparison(47),  $BF_{incl} = 0.78$ ; Ordering(47),  $BF_{incl} =$

0.56; Arithmetic(47),  $BF_{incl} = 0.34$ ; Number Line(PAE)(43),  $BF_{incl} = 0.70$ ; Visual-Spatial Working Memory – Forward Path Span(43),  $BF_{incl} = 0.75$ ; Visual-Spatial Working Memory – Reverse Path Span(43),  $BF_{incl} = 0.39$ ; Raven’s Matrices(45),  $BF_{incl} = 0.87$ ; 2D Mental Rotation(45),  $BF_{incl} = 2.10$ ; Visual-Spatial Reasoning(45),  $BF_{incl} = 0.24$ . Figure 4.6 shows children’s gain scores across all four time points and under both conditions (intervention vs. control) for all the mathematics measures (see Table 4.7 and Supplementary Figure 4.3 for gains scores by time and condition on the remaining spatial and EF measures; <https://osf.io/tqs7e/files/>).

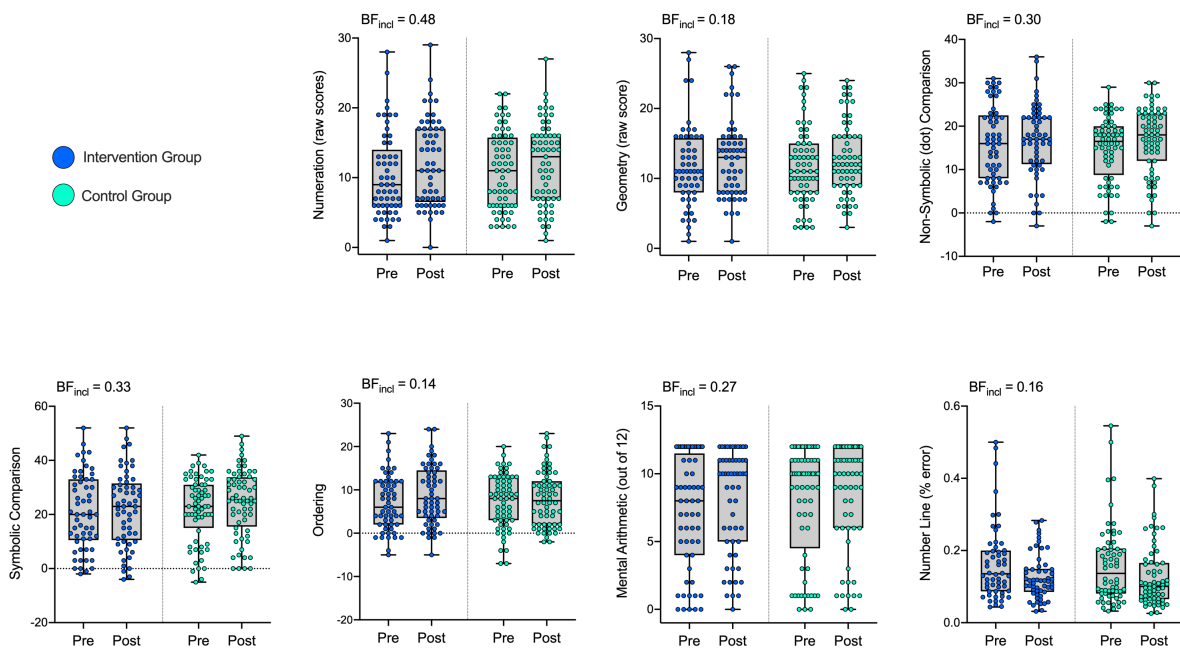


Figure 4.5 Comparison of pre-post performance by students in the intervention and control group (Year 2).

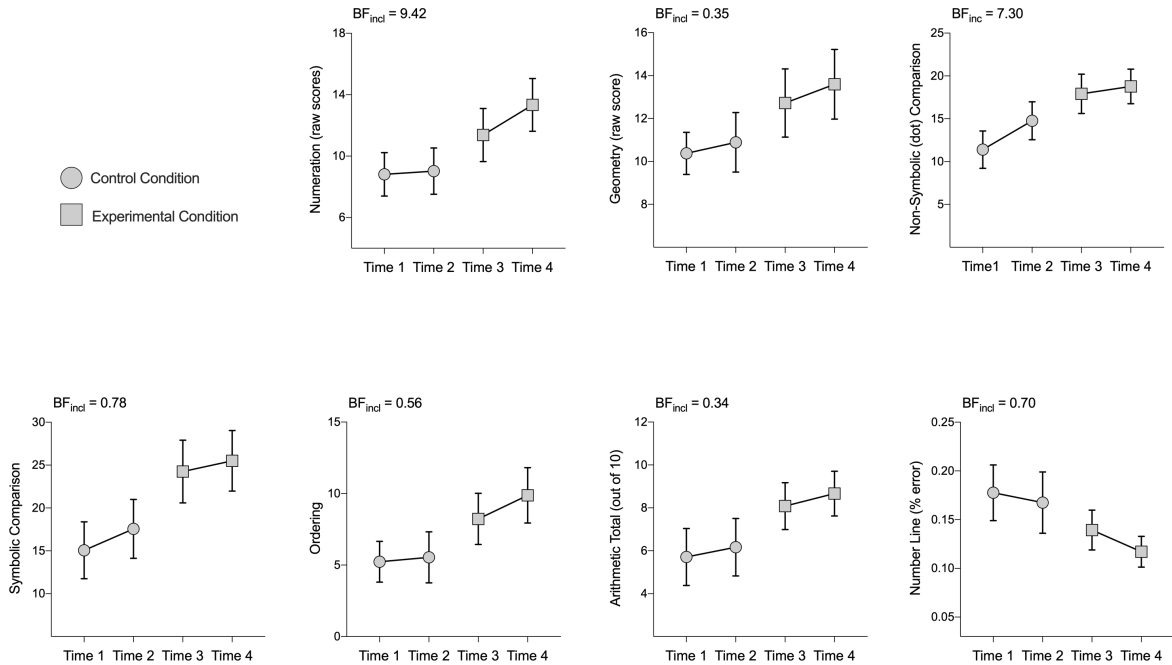


Figure 4.6 Within group change across multiple time points and under both conditions. Bars represent 95% Confidence Intervals around the mean.

#### 4.4.6 Summary of Year 2 Results

Relative to the control group, teachers in the intervention group demonstrated gains on the measure of self-perceived numerical cognition. Bayesian analyses indicated insufficient evidence to claim support for or against an effect on measures of numerical cognition knowledge and self-perceived general cognition knowledge. Thus, whether or not the intervention had an effect on these aspects of teacher knowledge remains ambiguous. Replicating the Year 1 results, there was support for the null on both the measure of math anxiety as well as comfort in the teaching and learning of mathematics. As per the student results, there was no evidence of gains by the intervention group compared to the control group on any of the measures. In fact, on the mathematics measures, except for numeration, there was support in favor of the null. The within-group analyses revealed a somewhat different picture, indicating greater improvements in

children's numeration performance when they were participated in the intervention compared to the control condition.

#### 4.4.7 Implementation of Teacher-Led Student Intervention Activities

On average, the teachers in Year 1 engaged their students in the intervention activities for a total of nearly 12 hours ( $M=11.80$ ,  $SD = 6.97$ , range = 3.67–22.42 hrs). In Year 2, teachers engaged their students in the intervention activities for an average of approximately 3 hours ( $M=3.37$ ,  $SD = 1.46$ , range = 1.67–5.67 hrs). A Bayesian independent t-test was conducted to assess whether and to what extent the two groups varied in the total time spent implemented the student intervention activities. Results revealed  $B_{10} = 6.74$ , indicating a group difference in favor of the Year 1 teachers. Indeed, the Year 1 teachers engaged their students in the activities for approximately 3 and  $\frac{1}{2}$  times longer than the Year 2 teachers. Also note that while all participating teachers in Year 1 returned their log sheets, one teacher in Year 2 failed to return theirs and another teacher's log sheet was incomplete and unusable. It is clear that teachers in Year 1 engaged their students in the intervention activities to a much greater extent than the teachers in Year 2.

There was also a clear difference between groups in the number of items uploaded to each group's shared Google Drive. The Year 1 teachers uploaded 53 items compared to the 11 items uploaded by the Study 2 teachers.

Table 4.7

*Mean scores and standard deviations across all four time points and under both conditions (intervention vs. control)*

	Mathematics										Executive Functions			Spatial Reasoning	
	Numeration	Geometry	Non-Sym Comp	Sym Comp	Order	Arithmetic	Number Line (PAE)	VSWM-F	VSWM-R	HTKS	Raven's	2D MR	Visual-Spatial		
Control	Time 1 (pre)	8.92 (4.88)	10.39 (3.42)	11.40 (7.49)	15.06 (11.42)	5.23 (4.91)	5.71 (4.58)	.17 (.10)	3.64 (2.35)	2.39 (2.40)	26.22 (11.35)	17.13 (5.79)	7.54 (3.11)	9.30 (2.96)	
	Time 2 (post)	0.02 (5.13)	10.93 (4.73)	14.77 (7.63)	17.56 (11.84)	5.54 (6.14)	6.17 (4.62)	.15 (.09)	4.39 (2.63)	2.91 (2.25)	31.85 (7.98)	18.93 (5.81)	8.87 (3.36)	9.78 (3.83)	
Intervention	Time 3 (pre)	11.40 (6.02)	12.91 (5.47)	17.92 (7.91)	24.25 (12.58)	8.23 (6.15)	8.08 (3.76)	.14 (.07)	4.27 (2.53)	3.39 (2.51)	33.11 (6.43)	21.33 (6.06)	9.87 (3.43)	11.20 (3.83)	
	Time 4 (post)	13.38 (5.97)	13.73 (5.66)	18.77 (6.94)	25.50 (12.16)	9.88 (6.67)	8.67 (3.59)	.11 (.05)	4.75 (2.52)	3.57 (2.64)	35.57 (6.23)	23.15 (6.13)	10.37 (3.22)	11.54 (4.29)	



## 4.5 Discussion

In this study, we designed, implemented, and tested the effects of a novel approach to teacher PD. At its core, the model was designed to achieve two major objectives: 1) Enrich teachers' awareness and understanding of research on children's numerical thinking, and 2) use this knowledge to inform teachers' assessment and implementation of a teacher-led classroom intervention. We predicted that this model would provide an effective means for improving both teacher and student learning in the area of early number. In an effort to provide a more stringent test of the model, we carried out a two-year replication study. Year 1 results indicated that relative to a control group, teachers who participated in the PD intervention demonstrated gains in their numerical cognition knowledge, self-perceived numerical cognition knowledge, and self-perceived general cognition knowledge. Compared to a control group, children in the intervention classrooms demonstrated gains in number line estimation, mental arithmetic (addition), and a comprehensive test of numeration. Together, these results provide evidence to suggest that the intervention was effective at increasing both teacher and student knowledge in the areas most specifically targeted throughout the intervention. However, our attempt to replicate these effects (Year 2) paints a somewhat different picture. In Year 2, teachers in the intervention group demonstrated greater improvements than the control group on a measure of self-perceived knowledge of numerical cognition. Thus, this result was consistent across both years. Unlike Year 1, however, we failed to replicate evidence of teacher gains in their actual content knowledge of numerical cognition. At the student level, there was no evidence that the intervention group outperformed the control group on any of the measures in Year 2. However, the within-group analyses revealed greater improvements in children's numeration performance when they participated in the intervention compared to the control condition. This finding, coupled with the Year 1 results, provides some evidence that the intervention may have had positive effective on children's numeration performance. As discussed in greater detail below, one reason we may have obtained mostly discrepant results between years of study may have been due to group differences in teacher uptake and implementation of the intervention. For example, compared to the Year 2, the Year 1 teachers spent, on average, considerably

more time (3 x) implementing the student intervention, were more likely to identify with goals of the PD, and had the support of their school principal. Overall, although our findings are far from conclusive, a careful weighing of the evidence across years of study, suggest the current PD model is a viable approach to better integrate research and practice. In what follows is a more detailed summary and interpretation of the effects of the PD, as well as explanations for the inconsistencies in findings between years of study. We conclude our discussion by considering limitations and next steps.

### 4.5.1 Teacher Results

In both Year 1 and Year 2, teachers in the intervention reported higher-levels of perceived numerical cognition knowledge compared to the control group. More specifically, at the end of the intervention, teachers who participated in the PD reported experiencing increased levels of knowledge on the following terms: numerical cognition, dyscalculia, mental number line, cardinality, and ordinality. Given that these terms were central to and used throughout the PD intervention, these findings were expected. As a follow-up to this measure, we also included an actual test of numerical cognition knowledge; a multiple choice test that was designed to assess the understanding of these and other terms in classroom-based contexts. For example, for one of the questions, teachers were asked to identify the name of the property associated with a scenario in which a child recognizes that  $3 + 2$  results in the same answer as  $2 + 3$  (i.e., the commutative property). To our surprise, improvements on this measure were present in Year 1 but not Year 2. Thus, although gains in self-perceived numerical cognition were consistent across years of study, improvements in actual numerical cognition knowledge was restricted to the Year 1 group.

This finding lends itself to some interesting questions regarding the influence of teachers' numerical cognition knowledge on student learning. If teachers' numerical cognition knowledge is more related to student learning than self-perceived knowledge, then we should expect to see greater evidence of student gains in Year 1 than Year 2. Indeed, at first glance, this is what the results seem to suggest. As further discussed below, there was stronger evidence for improvements in numerical reasoning by students

in Year 1 compared to Year 2. This finding is consistent with prior research, in which teachers' understanding of different facets of children's numerical thinking (e.g., arithmetic strategies) has been associated with gains in students' numerical thinking (e.g., see Carpenter et al., 1989; Fennema et al., 1996; Franke et al., 2001). However, upon closer reflection, it is clear that this trend in the current data needs to be interpreted with caution. Due to the small sample sizes and uneven distribution of students across grades, we were unable to directly address the question of whether teacher change was associated with student change and furthermore dissociable by group. Thus, statistically speaking, we were unable to state whether the gains in Year 1 were a result of greater gains in teachers' numerical cognition knowledge in Year 1 compared to Year 2. Moreover, the Bayes factors associated with the Year 2 teacher gains on the numerical cognition knowledge test failed to provide evidence for or against the presence of an interaction effect. Taken together, the results provide some hints that teachers' numerical cognition knowledge may be linked to student growth in numerical reasoning. However, due to the small sample sizes (at the teacher level) and ambiguous Year 2 teacher results, more research is needed to examine the effects of the intervention on teachers' numerical cognition knowledge, and in turn, the effects that this knowledge has on student learning outcomes.

The absence of intervention effects on teachers' math anxiety/comfort was far less ambiguous. Across both years of the study, on both a measure of teacher math anxiety and a separate measure of teacher comfort level teaching and learning math, there was support in favor of the null: that is, there was enough evidence to suggest that the intervention did not have an effect in these areas. These results run counter to our original predictions. Entering this study, we were cognizant of the widespread math anxiety amongst early years teachers (Maloney & Beilock, 2013) and the potential negative effects that such feelings might have on student learning (Beilock, Gunderson, Ramirez, & Levine, 2010). To combat teacher math anxiety, we had teachers engage in mathematical tasks in non-threatening, playful contexts and within all teachers' capabilities. Moreover, in preparation for implementation with their students, we had teachers engage in these various math activities through the mind/perspective of their students. Although previous research suggests that this approach is effective at lowering teachers' math anxiety (e.g.,

see Tooke and Lindstrom, 1998), we found no such evidence. And yet, despite evidence to suggest our intervention was not effective at lowering teacher math anxiety, we still obtained some evidence of intervention-related improvements in student learning. Our findings thus question whether and to what extent effective teaching and consequently, student learning is dependent on teachers' math anxiety/comfort level. However, we must also question the extent to which our inability to lower teachers' math anxiety was associated with our limited evidence of student change. Moving forward, it is clear that much more research is needed to uncover when, why, and how teachers' math anxiety is linked to student learning. Moreover, concerted efforts are needed to study the malleability of teacher math anxiety and the effect that reductions in math anxiety have on student learning.

## 4.5.2 Student Results

The present intervention targeted both teachers and their students. While primary efforts were directed at intervening at the teacher level, our primary outcomes of interest were directed at the student level. To this aim, the implementation of the student intervention consisted of teacher-led activities targeting the three major foci of the teacher intervention (basic numerical relationships, number-space mappings, and arithmetic). Based on the success of the teacher intervention by Hawes et al. (2017), we provided teachers with a curated bank of numerical reasoning activities to draw from and implement in their own classrooms. These activities were aligned with the specific foci of the teacher PD, providing opportunities for teachers to make links between the PD and their practice. Furthermore, in line with design-research practices, teachers were encouraged and given opportunities to adapt the activities based on their own professional judgment (Brown, 1992).

As noted above, the relative effectiveness of the teacher-led student intervention varied across Year 1 and 2. In Year 1, children in the experimental classrooms made larger improvements than children in the control classrooms on measures of mental arithmetic, number line estimation, and a comprehensive test of numeration. Critically, both groups of children made highly similar gains on measures of spatial and EF skills, which were

not targeted during PD. Thus, the gains made by the experimental group were highly specific to activities designed and implemented as part of the PD. These results, coupled with the evidence of teacher change observed in Year 1, were in line with our original hypotheses, as well as the results of Hawes et al. (2017), and provided reasons to be confident in the current model of teacher PD. However, promising as these results appeared, it was important to us to see whether the results would replicate.

Despite employing the same methodologies as Year 1, only one of the teacher results replicated and none of the student-level results replicated. In fact, across all student-level measures, Bayesian analyses suggested more support for the null than the alternative hypothesis. However, slightly different results emerged when analyzing the data with what we preregistered as a more stringent approach involving within-group comparisons. That is, we compared the same students' growth across the two different conditions, intervention vs. control. We considered this analysis to be a more robust analysis as it allowed us to better control for school, teacher, and individual effects. These analyses indicated that students demonstrated larger gains in their numeration performance when they were part of the intervention group compared to the control. These results are promising in so much as the numeration test is a psychometrically reliable and robust measure of children's overall numerical reasoning (Connolly, 2007). The test requires the integration and application of a wide range of both procedural skills and conceptual understanding of number and operations. For these reasons, it was our primary outcome measure.

The comprehensiveness of this test may also help explain why we obtained some evidence of change on this measure, but not others, across both years of study. Change on this measure may simply be a reflection of our approach to intervention; that is, as broad in scope, targeting and aiming to integrate key facets of numerical reasoning. While this particular measure may have afforded varied and multiple opportunities for students to demonstrate what they have learned through the intervention, the other measures may have been too narrowly focused (e.g., symbolic number comparison). Interestingly, this is possibly at odds with our original predictions. Entering the study, we assumed we would see the largest gains on the measures more directly aligned to the content addressed in our PD (basic number relations, number-space mappings, and arithmetic). While we obtained

some evidence of this in Year 1, our overall results suggest the greatest improvements occurred on the most advanced measure of numerical reasoning. One plausible account for why change might occur on a comparatively more higher-level numerical reasoning tasks compared to an absence of change on more basic number tasks is due to change at the level of student strategy use. While the current intervention targeted both basic skill development as well as strategy use, it is possible that teachers were keen to focus on strategy use in their students. In future iterations of this model, we aim to gain further insights into this issue by directly observing teacher instructional practices.

### 4.5.3 Explanations for the Inconsistencies in Findings Across Years

There are many potential reasons for why we observed inconsistent findings between the two years of study. In discussing these reasons, we will limit ourselves to explanations that we see as most probable, based on the data as well as our own observations. First, as discussed above, the null results from the students in Year 2 may have been due to the null results obtained from the teacher measures in Year 2. Indeed, our Year 2 findings, do not contradict our original hypotheses, but in some ways, support it. That is, the success of the current intervention is dependent on there being a relationship between the gains in children's numerical thinking and those obtained at the teacher level. Put differently, given our design within which teacher gains are expected to translate into student gains, it is hardly surprising that if teachers did not benefit from the intervention that student gains were not observed. This leads to the critical question as to why one group of teachers appeared to gain from the teacher intervention while another group did not.

A second reason for the discrepant findings between Year 1 and Year 2 has to do with group differences in uptake and implementation of the student focused intervention activities. Compared to Year 2, teachers in Year 1 spent 3 ½ more amounts of time engaging their students in the intervention activities. For this reason alone, we may not have observed clear evidence of student gains in Year 2. Moving forward, it will be important to also examine factors related to the quality of teacher-led activity implementation and associations between quality of implementation and student learning.

Further evidence that the two groups differed in the uptake and commitment to the project can be observed by comparing the shared Google Drives between groups. Recall that as part of the intervention teachers were encouraged to upload video/picture or paper-and-pencil examples of student reasoning, including student assessments and student work samples, as well as adapted versions of the student intervention activities. The Year 1 teachers uploaded 53 items compared to the 11 items uploaded by the Study 2 teachers. These data support the greater amounts of engagement we observed with teachers in Year 1 compared to Year 2.

A third reason for the discrepant findings may be related to the involvement of the school principal. Indeed, prior research on what makes for effective teacher PD points to principal involvement as an important factor in increasing the likelihood of instructional improvement (McLaughlin, 1990; Santagata, Kersting, Givvin, & Stigler, 2011; Wanless, Patton, Rimm-Kaufman, & Deutsch, 2013; Wilson, 2013). In the present study, the school principal was actively involved and a regular participant of the PD sessions in Year 1 (attending all 5 sessions) but not in Year 2 (attending no sessions). In line with the research literature cited above, the Year 1 principal not only participated, but appeared to play a critical role as a leader in encouraging teacher uptake and commitment to the project. Prior to our first meeting, the principal had taken the time to explain to the group of participating teachers the purpose of the project. During the actual PD sessions, the principal asked questions, made connections between research and practice, and perhaps most importantly, demonstrated a keen interest in learning from the project. In between sessions, the principal visited the teachers' classrooms to observe the implementation of the student activities and shared her observations of student learning in our subsequent meetings together. Taken together, we have some evidence to suggest that the school principal plays an important role in liaising teacher-researcher collaborations.

A fourth and final reason for the difference in success between years of study may have been related to the degree of (mis)alignment between researcher and teacher goals. The overall goal of this project was to improve children's numerical thinking. However, the extent to which this was a priority amongst the two groups of teachers appeared to vary. This was clear throughout the PD, but was especially apparent during our concluding focus group interviews, held during the last 45 minutes of the final session.

Teachers were asked to reflect on and share their thoughts about the PD process. In Year 1, not only was there widespread support for the approach to teacher PD but there was also clear alignment between teachers' perceptions of the PD and our researcher-designed rationale and purpose behind each component of the PD. In other words, teachers in Year 1 were easily able to identify and appreciate the purpose of the PD and its various components. For example, in the following quote by a Year 1 2<sup>nd</sup> grade teacher, we see evidence of appreciation for this approach to PD, but also some evidence of teacher-researcher goal alignment:

*“I think this is the best PD I’ve ever had – like ever – and it’s obvious I’ve been doing this for a while. It was more of an in-depth understanding of how really children learn math and mathematical concepts, and things like that. And then what I did personally, I took that and looked at the curriculum and it really helped me blend the two together. I absolutely didn’t discount the curriculum because that’s where our direction is, and I really incorporated a lot of what you guys offered to us...and I just think it’s a really good way to offer PD for teachers. It was wonderful, I really enjoyed it”* (Grade 2 teacher, 25 years of experience).

This teacher's mention of taking what she has learned about how children learn mathematics and applying it to the mathematics curriculum speaks to one of the ultimate goals of this approach to PD. In line with the principles of *Cognitively Guided Instruction* (e.g., see Carpenter 1989; 2014; Fennema et al., 1996), we aimed to equip teachers with a better understanding of children's numerical reasoning and in turn a better 'mental model' of the learner (Willingham, 2017). In this way, teacher learning is not bound to the delivery of specific lessons/activities, but has the potential to be applied across a number of contexts, including various aspects of the curriculum. Other teachers also referred to the PD process as an effective means to bridge research and practice, making explicit mention of the importance of going beyond giving 'lip service' to research and instead highlighted the need to go one step further; that is, use research to inform the design and actual implementation of student focused activities. Moreover, it is clear from the quotes below that teachers appreciated working with their students in an effort to bridge research and practice:



*“I liked how the research translated into activities. So, if the research says children need to be able to do these things, then let’s build some activities that will actually get students to do these things. But I thought that was really powerful. That’s kind of that marriage of research with professional practice that seems to not happen a lot.”* (Instructional coach)

*“The fact that we would hear it [research] and we went back and did it. Because you go to an outside PD and you sit there all day, and they tell you this and this and this, and they give you the research behind it... and if you’re skeptical at all, you’re going yeah right. And you come back to class and you don’t necessarily do it because you are skeptical about it, but here, we did it, we tried it. We went, just like Ian said, ‘woah, yeah, I would have never thought to do that and look what happened.’”* (Kindergarten teacher)

*“Well, I guess I might be interpreting research a little bit bigger than this, but I think when you bring those students in [to the shared meeting space in the library] and those teachers are working with their own students and making those observations that are so powerful saying ‘I never thought about that, I forgot to think about that.’ I think our teachers become researchers and that becomes very powerful... I think that makes a huge difference. This part of the PD where you’re bringing your students in is the most powerful, I think.”* (Principal)

Collectively, these quotes speak to what William James referred as the necessity of intermediary actions in order to bridge the research-to-practice gap (James, 1899). These teachers were able to identify the purpose of conducting one-to-one assessments with students and piloting activities with students as a whole group. They saw these components of the PD as effective mechanisms in making the translation from research to practice.

This same level of enthusiasm and ability to provide mechanistic accounts of the various components of the PD was not as apparent in Year 2. Although teachers spoke of the PD in positive terms, there was far less indication that teachers, both as individuals but also as a collective, identified with the purpose of the PD. There was little talk about the specific components of the PD model and at no point any explicit mention of how this model may better afford the application of research to practice. Instead, much of the conversation was centered around topics tangential to the actual PD experience. For example, the majority of our conversation centred around questions and concerns about their students’ home lives and “emotional availability to learn.”

*“I would really like to get some more insight, I guess, understanding of children coming to school that aren’t prepared to learn, that aren’t able to learn.”* (Kindergarten teacher)

*“Because they’re [the students] kind of in that flight response all the time. So, they’re not available to learn cause they’re there, right?”* (Instructional coach)

Indeed, the amount of time spent discussing issues related to their students’ home lives is a potentially indicative of poor teacher-researcher goal alignment. Simply put, the teachers in Year 2 may not have been interested in the PD we had to offer because they saw the need for PD of a different sort; for example, PD that places greater emphasis on understanding the emotional and behavioral well-being of their students. However, it should also be mentioned that the Year 1 teachers also identified students’ behavioral and emotional challenges as key obstacles in their ability to carry out effective instruction. And yet, compared to the Year 2 cohort, it was apparent that the Year 1 teachers were better able to juggle what some teachers identified as the competing goals of delivering academic content while also attending to their students’ emotional readiness to learn. It is unclear to us why one school was better able to do this than the other. We must also be careful not to assume the needs of both schools were the same, despite serving students of the same neighborhood and their almost identical performance on both cognitive and academic measures of achievement. It is possible that the particular cohort of students in the Year 2 school presented a unique set of problems; more severe than what was experienced in the Year 1 school. In returning to the idea of teacher-researcher goal alignment, it is plausible that the goals of our intervention were at odds with the school’s identified need to prioritize the emotional and behavioral well-being of their students. In future iterations of the model, we aim to further investigate the potential moderating influence that teacher-researcher goal alignment has on the implementation and overall success of the intervention.

#### 4.5.4 Limitations and Next Steps

There are several limitations of this study worth pointing out. First, the teacher sample sizes were small. This prevented us from directly assessing how teacher change related to student change as a function of the intervention. Moving forward, it will be important to demonstrate whether, to what extent, and what particular aspects of teacher learning are related to student growth. For example, our findings provide some hints that teachers' numerical cognition content knowledge may be more strongly related to students' numerical thinking than teachers' self-perceived numerical cognition knowledge. However, larger sample sizes, at the teacher level, are needed to directly address this line of inquiry.

Another limitation of the present study was our inability to randomly assign teachers to the intervention. Difficult and impractical as this may be to achieve, such an approach would ultimately provide a more robust test of the intervention, allowing for better control of various school-level effects (e.g., principal, school philosophy, student makeup, etc.). For example, it is possible that by randomly assigning teachers to the intervention, the group differences in balancing the delivery of academic content and attending to students' emotional needs, as noted above, may have been made equivalent across groups. However, because randomization at the teacher level is not always possible or may, arguably, not be the best option for reasons to do with ecological validity, one way of maintaining high scientific rigor is to use a within-group repeated measure design. We did so in the current study in an effort to better control for and examine the effects of the intervention at the individual level as opposed to the group level. This also allowed us to be more confident in the null results but also provided some evidence of potential gains in students' numeration performance that were not detected through between-group analyses.

In moving forward, it will be important to more thoroughly examine the specific ways in which the intervention may have influenced teachers' assessment and instructional practices. For example, although the quotes from the Year 1 teachers above suggest that teachers were better able to apply research-to-practice, it remains unclear how exactly this manifested itself in practice. We have hypothesized that a better understanding of research on children's thinking provides teachers with a better basis on which to observe (assess) and extend children's thinking during instruction. For example,

one must know what cardinality is in order to look for it in student reasoning, identify it as an area of strength/concern, and then use these observations to plan for appropriate instruction. Given that teachers are likely to differ on how they perceive and use research to inform assessment and instruction, it is critical to capture these differences and ultimately relate them to student thinking. Fennema et al., (1996), for example, were able to show that their approach to teacher PD (i.e., CGI) was related to increases in teachers' attention to and instructional focus on mathematical problem solving. This change, in turn, was related to student gains in problem solving. It is this sort of detail that will be important to document in future research of the current model.

Lastly, it is worth asking whether the PD model itself may be a limitation in the pursuit of establishing an effective intervention. In other words, should we consider abandoning the model altogether, making changes to the model, or keep the model entirely intact? At this point, we side with keeping the model intact and instead urge the need for more research. Although we did not obtain unambiguous support for the model, we did see evidence of teacher and student gains in Year 1. More importantly, it seems that the gains observed in Year 1 and the mostly absent gains in Year 2 could be attributed to poor uptake and implementation of the PD. Moreover, the results of Year 1 align with the success of the model in the study by Hawes et al., (2017). For these reasons, we remain hopeful that the present model has the potential to be an effective agent of both teacher and student change. However, it has also become clear that this *potential* rests on variety of factors that, at the moment, remain poorly understood. As others have shown, it may not be enough to build a model of teacher PD based on established features of effective PD (Hill, Corey, & Jacob, 2018). Indeed, even when teacher PD models do incorporate effective features of PD, including sustained focus on student's mathematical thinking, studies of these models yield mixed results (Hill et al., 2018; Jacob, Hill, Corey, 2017). By including two studies of the same approach to teacher PD, but with differing results across the two contexts, we were able to further examine why this might be. While we have suggested these differences reside in uptake and implementation, future efforts are needed to follow-up on these possibilities and examine their influence with finer grained analyses and measurement.

## 4.5.5 Conclusion

This study provides new insights into an old problem: How to address the research-to-practice gap? We demonstrate ‘proof of concept’ for the design and implementation of a 5-day teacher PD model that aims to better integrate numerical cognition research and the teaching of early years mathematics. Our approach is interdisciplinary in design, built to foster improved communication and understanding of children’s learning among both researchers and practitioners alike. For this reason, we see the model as one not limited to bridging numerical cognition research and practice, but as one that has the potential to be applied to other research-practice gaps (e.g., literacy). Although the current findings provide some indication that the model is effective at bringing about change at both the level of teacher and student, the inconsistent findings between Year 1 and 2 make it clear that more research is needed. More specifically, in contrasting the results from Years 1 and 2, it may not be strictly a question of *whether* the model is effective but also a question as to *when* and under *what* conditions the model is effective. We obtained evidence to suggest widespread buy-in and uptake in Year 1, and much less evidence of this in the Year 2 group. This is but one plausible reason for the discrepancies in results. Moving forward, it will be important to more systematically examine why the same approach and model of teacher PD might be taken up differently in different contexts.

## 4.6 References

- Alexander, L., & Martray, C. (1989). The development of an abbreviated version of the Mathematics Anxiety Rating Scale. *Measurement and Evaluation in Counseling and Development*, 22(3), 143-150.
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407.
- Beilock, S. L., Gunderson, E. A., Ramirez, G., & Levine, S. C. (2010). Female teachers’ math anxiety affects girls’ math achievement. *Proceedings of the National Academy of Sciences*, 107(5), 1860-1863.
- Brown, A. L. (1992). Design experiments: Theoretical and methodological challenges in

- creating complex interventions in classroom settings. *The Journal of the Learning Sciences*, 2(2), 141-178.
- Bruce, C. D., Flynn, T. C., & Bennett, S. (2016). A focus on exploratory tasks in lesson study: The Canadian 'Math for Young Children' project. *ZDM–The International Journal on Mathematics*, 48(4), 541-554.
- Bruce, C. D., & Hawes, Z. (2015). The role of 2D and 3D mental rotation in mathematics or young children: what is it? Why does it matter? And what can we do about it?. *ZDM–The International Journal on Mathematics*, 47(3), 331-343.
- Carpenter, T., Fennema, E., Franke, M., L. Levi, & S. Empson (2014). *Children's Mathematics, Second Edition: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann
- Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C. P., & Loeff, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, 26(4), 499-531.
- Clarke, D., Clarke, B., & Roche, A. (2011). Building teachers' expertise in understanding, assessing and developing children's mathematical thinking: the power of task-based, one-to-one assessment interviews. *ZDM–The International Journal on Mathematics Education*, 43(6–7), 901–913.
- Collins A. (1992) Toward a Design Science of Education. In Scanlon E., O'Shea T. (Eds.) *New Directions in Educational Technology*. NATO ASI Series (Series F: Computer and Systems Sciences), Vol. 96. Berlin: Springer
- Collins, A., Joseph, D., & Bielaczyc, K. (2004). Design research: Theoretical and methodological issues. *The Journal of the Learning Sciences*, 13(1), 15-42.
- Connolly, A. J. (2007). *KeyMath diagnostic assessment* (3rd ed.). Minneapolis, MN: Pearson.
- Dubinsky, J. M., Roehrig, G., & Varma, S. (2013). Infusing neuroscience into teacher professional development. *Educational Researcher*, 42(6), 317-329.
- Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., ... & Sexton, H. (2007). School readiness and later achievement. *Developmental Psychology*, 43(6), 1428-1446.
- Ehrlich, S. B., Levine, S. C., & Goldin-Meadow, S. (2006). The importance of gesture in

- children's spatial reasoning. *Developmental Psychology*, 42(6), 1259-1268.
- Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, 27(4), 403-434.
- Franke, M. L., Carpenter, T. P., Levi, L., & Fennema, E. (2001). Capturing teachers' generative change: A follow-up study of professional development in mathematics. *American Educational Research Journal*, 38(3), 653-689.
- Ginsburg, H. P. (1997). *Entering the child's mind: the clinical interview in psychological research and practice*. New York: Cambridge University Press.
- Goffin, C., Sokolowski, H. M., Matejko, A. A., Bugden, S., Lyons, I. M. & Ansari, D. (2018). Assessing knowledge translation in the field of mind, brain and education in pre-service teachers. International Mind, Brain, and Education Society Conference, Los Angeles, U.S.
- Hawes, Z., LeFevre, J. A., Xu, C., & Bruce, C. D. (2015). Mental rotation with tangible three-dimensional objects: A new measure sensitive to developmental differences in 4-to 8-year-old children. *Mind, Brain, and Education*, 9(1), 10-18.
- Hawes, Z., Moss, J., Caswell, B., Naqvi, S., & MacKinnon, S. (2017). Enhancing children's spatial and numerical skills through a dynamic spatial approach to early geometry instruction: Effects of a 32-week intervention. *Cognition and Instruction*, 35(3), 236-264.
- James, W. (1899/2001). *Talks to teachers on psychology and to students on some of life's ideals*. New York, NY: Dover Publications
- Jarosz, A. F., & Wiley, J. (2014). What are the odds? A practical guide to computing and reporting Bayes factors. *The Journal of Problem Solving*, 7(1), 2-9.
- Kang, S. H. (2016). Spaced repetition promotes efficient and effective learning: Policy implications for instruction. *Policy Insights from the Behavioral and Brain Sciences*, 3, 12-19.
- Laski, E. V., & Siegler, R. S. (2007). Is 27 a big number? Correlational and causal connections among numerical categorization, number line estimation, and numerical magnitude comparison. *Child Development*, 78(6), 1723-1743.
- Levine, S. C., Huttenlocher, J., Taylor, A., & Langrock, A. (1999). Early sex differences

- in spatial skill. *Developmental Psychology*, 35(4), 940-949.
- Lyons, I. M., Bugden, S., Zheng, S., De Jesus, S., & Ansari, D. (2018). Symbolic number skills predict growth in no symbolic number skills in kindergarteners. *Developmental Psychology*, 54(3), 440–457.
- Lyons, I. M., Hutchison, J. E., Bugden, S., Goffin, C., & Ansari, D. (2018). Kindergarteners reliability mis-classify ordered sequences of non-adjacent numbers. Paper presented at the Mathematical Cognition and Learning Society conference. UK: Oxford University.
- Maloney, E. A., & Beilock, S. L. (2012). Math anxiety: Who has it, why it develops, and how to guard against it. *Trends in Cognitive Sciences*, 16(8), 404-406.
- Mast, J. V., & Ginsburg, H. P. (2010). Child study/lesson study: Developing minds to understand and teach children. In N. Lyons (Ed.), *Handbook of reflection and reflective inquiry: Mapping a way of knowing for professional reflective inquiry* (pp. 257–271). New York: Springer Publishing Co.
- McClelland, M. M., & Cameron, C. E. (2012). Self-regulation in early childhood: Improving conceptual clarity and developing ecologically valid measures. *Child Development Perspectives*, 6(2), 136-142.
- McLaughlin, M. W. (1987). Learning from experience: Lessons from policy implementation. *Educational Evaluation and Policy Analysis*, 9(2), 171–178.
- Moss, J., Bruce, C. D., Caswell, B., Flynn, T., & Hawes, Z. (2016). *Taking shape: Activities to develop geometric and spatial thinking. Grades K-2*. Toronto, CA: Pearson Canada Incorporated.
- Moss, J., Hawes, Z., Naqvi, S., & Caswell, B. (2015). Adapting Japanese Lesson Study to enhance the teaching and learning of geometry and spatial reasoning in early years classrooms: a case study. *ZDM—The International Journal on Mathematics*, 47(3), 377-390.
- Nathan, M. J., & Sawyer, K. (2014). Foundations of Learning Sciences. In K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (2nd ed., pp. 21–43). Cambridge, England: Cambridge University Press.
- Okamoto, Y., & Case, R. (1996). Exploring the microstructure of children’s central



- conceptual structures in the domain of number. *Monographs of the Society for Research in Child Development*, 61(1–2), 27–58.
- Ponitz, C. C., McClelland, M. M., Matthews, J. S., & Morrison, F. J. (2009). A structured observation of behavioral self-regulation and its contribution to kindergarten outcomes. *Developmental Psychology*, 45(3), 605.
- Raven, J. (2008). *Coloured progressive matrices and Chrichton vocabulary scale*. London, UK: Pearson.
- Rohrer, D. (2015). Student instruction should be distributed over long time periods. *Educational Psychology Review*, 27(4), 635-643.
- Rouder, J. N., Morey, R. D., Verhagen, J., Swagman, A. R., & Wagenmakers, E. J. (2017). Bayesian analysis of factorial designs. *Psychological Methods*, 22(2), 304-321.
- Santagata, R., Kersting, N., Givvin, K. B., & Stigler, J. W. (2011). Problem implementation as a lever for change: An experimental study of the effects of a professional development program on students' mathematics learning. *Journal of Research on Educational Effectiveness*, 4(1), 1–24.
- Schönbrodt, F. D., Wagenmakers, E. J., Zehetleitner, M., & Perugini, M. (2017). Sequential hypothesis testing with Bayes factors: Efficiently testing mean differences. *Psychological Methods*, 22(2), 322-339.
- Simon, H. A. (1969). *The Sciences of the Artificial*. Cambridge, MA: MIT Press
- Tooke, D. J., & Lindstrom, L. C. (1998). Effectiveness of a mathematics methods course in reducing math anxiety of preservice elementary teachers. *School Science and Mathematics*, 98, 136-139.
- Wanless, S. B., Patton, C. L., Rimm-Kaufman, S. E., & Deutsch, N. L. (2013). Setting-level influences on implementation of the Responsive Classroom approach. *Prevention Science*, 14(1), 40–51.
- Wilson, S. M. (2013). Professional development for science teachers. *Science*, 340(6130), 310–313.
- Willingham, D. T. (2017). A mental model of the learner: Teaching the basic science of educational psychology to future teachers. *Mind, Brain, and Education*, 11(4), 166-175.

## Chapter 5

### 5 General Discussion

#### 5.1 Overview

This thesis was carried out to better understand how humans are able to learn and perform mathematics. To approach this goal, I studied the ways in which cognitive, neural, and educational factors influence mathematical thinking and learning. While the first two studies focused on revealing the cognitive and neural underpinnings of spatial, numerical, and mathematical cognition, the third and final study investigated how research in numerical cognition can be used to inform the teaching and learning of early years mathematics (Kindergarten – Grade 3).

In light of the findings and common themes examined in Studies 1 and 2, I begin this Discussion by revisiting explanations of why, when, and how spatial and mathematical cognition may be linked. I discuss how the current thesis (Studies 1 and 2) contributes to an improved understanding of the four candidate mechanisms linking space and math outlined in the Introduction. I then turn my attention to the role that spatial training studies can play in further elucidating the causal mechanisms hypothesized to underlie spatial-mathematical relations. The second major section of the Discussion focuses on the research-practice gap in numerical cognition research and mathematics education. Lastly, I end by considering the implications of the current thesis and how it adds to the larger goal of an improved understanding of mathematical thinking and learning.

#### 5.2 What Explains the Relations between Spatial and Mathematical Performance?

As outlined in the Introduction, at least four candidate mechanisms have been put forward to explain the reliably robust relations between spatial and mathematical cognition. These mechanistic accounts include the: (1) *Spatial representation of numbers account*, (2)

*shared neural processing account*, (3) *working memory account*, and (4) *spatial modelling account*. I will now discuss how the current findings contribute to an improved understanding of each account and speculate what this might mean moving forward.

### 5.2.1 Spatial Representation of Numbers Account

According to the *spatial representation of numbers account*, numbers and their various relations are represented along a ‘mental number line’ (Fischer & Fias, 2005). In turn, the precision of one’s mental number line has been posited to play an important role in performing a host of numerical reasoning tasks, including comparing, ordering, and operating on numbers (Fischer et al., 2011; Siegler & Ramani, 2009). Although the current thesis did not directly test this account of the space-math link (and is least informed by the present thesis), the findings point to spatial visualization as a potential variable of interest in the development of spatial-numerical associations (a relation potentially best explained by the spatial-modelling account further discussed below).

To date, the majority of research in this area has focused on the automatic mappings of numbers to space (aka numerical-spatial biases; see Toomarian, & Hubbard, 2018). However, the precise mechanisms underlying the mental processes related to the automatic mapping of numbers to space remains unclear. Recently, an alternative view has emerged which argues that the mapping of numbers to space is not automatic but an active process. Accordingly, numerical-spatial associations/biases may reflect learned associations of number-space relations (e.g., internalizing the structure of physical number lines) and/or numerical-space relations constructed in working memory during task execution (van Dijck & Fias, 2011). Although working memory has been posited as the cognitive resource underlying the active construction of numerical-spatial relations, the current thesis indicates that spatial visualization might also play an important role in the mapping of numbers to space. Findings from Study 1 indicated strong behavioral relations between spatial visualization and basic numerical competencies. Study 2 demonstrated that basic symbolic number processes and spatial visualization (defined as mental rotation) activated large areas of overlapping neural cortex in and around the IPS.

Overall, the findings from Study 1 and 2 suggest a close coupling of spatial visualization and basic numerical processes. These findings suggest the need to more closely consider the role of spatial visualization processes in forming spatial-numerical associations. Moving forward, it will necessary to further test whether spatial visualization underlies both so-called automatic spatial-numerical mappings (e.g., as measured with the SNARC paradigm) compared to more deliberate mappings of numbers to space (e.g., number line estimation tasks). Moreover, research is needed to further disentangle whether and to what extent working memory and spatial visualization processes are differentially related to spatial representations of number.

### 5.2.2 Shared Neural Processing Account

The *shared neuronal processing account* suggests that numbers and space are linked through shared underlying neuroanatomical substrate, typically taken as evidence of the ‘mental number line.’ To this point, neural relations between spatial and numerical thought have been limited to the relation between lower-level spatial skills and basic numerical competencies (e.g., see Hubbard, Piazza, Pinel, & Dehaene, 2005). Study 2 aimed to offer additional explanations for the ways in which spatial and numerical cognition may be linked in the brain; thus, shedding new light on the *shared neuronal processing account*.

Findings from Study 2 revealed that cognitive processes related to basic numerical skills, mental rotation, and mental arithmetic were all associated with large areas of overlapping activity in and around the bilateral IPS. This study is significant in that it demonstrates that the neural relations between spatial and numerical processing (including arithmetic) extend beyond lower-level associations (Cf. Toomarian, & Hubbard, 2018). Instead, the neural correlates of higher-level spatial processing (mental rotation) also appear to relate to numerical processing, including arithmetic. This finding aligns with the findings from Study 1, suggesting that spatial visualization skills may play an important role in forming number-space relations. Though because spatial-numerical mappings were not explicitly probed, the role of spatial visualization in forming space-number relations remains speculative.

Although the ‘mental number line’ account may be one explanation for space-number associations in the brain, the findings from Study 2 suggest other ways in which spatial and numerical cognition may be linked. While numerical and arithmetic processing were associated with overlap in the left IPS, mental rotation and arithmetic were associated with overlap in the middle frontal gyri. These findings are significant in that they suggest that spatial and numerical thinking may be linked through task dependent operations. For instance, mental rotation and mental arithmetic both share the need to mentally manipulate information. This common operation might be one reason for the observed overlap in frontal regions typically associated with executive functions (Owen, McMillan, Laird, & Bullmore, 2005; Smith & Jonides, 1999). Basic symbolic number processing arguably requires far less top-down executive control mechanisms, which may explain why symbolic number was not associated with activity in this same region. As argued in Study 2, this process-based approach to understanding convergence and divergence in cognitive functions may prove useful in future research aiming to further reveal the ways in which spatial and mathematical cognition are linked.

### 5.2.3 Working Memory Account

The *working memory account* calls into question unique relations between spatial and mathematical skills. Instead, the link may have its roots in individual differences in other cognitively demanding skills, including executive function skills and working memory capacity (e.g., see Lourenco, Cheung, & Aulet, 2018). To test this possibility, Study 1 examined whether relations between spatial visualization skills and mathematics achievement could be explained by third party variables, including visual-spatial working memory (VSWM), EF skills, and general intelligence (*g*-factor). Results indicated that relations between spatial visualization and mathematical skills could not be explained by any of these other variables. This finding is significant in that it is the first source of evidence, at least for the time being, to rule out the working memory account as a potential explanation for the space-math link. The question of why spatial visualization might be a better predictor of mathematics is an important one and is further discussed in the next section.

## 5.2.4 Spatial Modelling Account

The *spatial modelling account* places emphasis on spatial visualization as a general mechanism used to model, organize, and simulate a wide variety of numerical and mathematical concepts. According to this account, spatial visualization is predicted to play an especially important role when the mathematical problem is unfamiliar to the individual. As I will argue next, of all the accounts, the spatial modelling account offers the best explanation for the data and findings revealed in the present thesis. Moreover, the spatial modelling account gives meaningful context to the understanding of the other accounts.

The spatial modelling account might explain why we observed stronger relations between novel mathematical content compared to familiar mathematical content. That is, although Study 1 revealed strong latent relations between spatial visualization and basic numerical skills, the relation was considerably stronger between spatial visualization and mathematics achievement. Critically, the mathematics achievement measures used in this study focused on applied problem solving and, because it was adaptive, included at least a portion of questions that were novel to the participant. Thus, one way in which children may have made sense of these novel problems was to mentally generate and model various solutions to the problems – mental operations typically associated with spatial visualization processes (Lohman, 1996).

The emphasis placed on the need to generate solutions to mathematics problems may also help explain why spatial visualization was a stronger predictor of mathematics than VSWM or EF skills. As noted in the Introduction, VSWM and spatial visualization may differ according to cognitive demands placed on the need to “recall” versus “generate” visual-spatial information. While most VSWM measures primarily emphasize the need to recall information, most spatial visualization measures primarily emphasize the need to generate and mentally manipulate mental models of stimuli. These differences, at least at the measurement level, may be one reason to predict stronger relations between spatial visualization skills and novel mathematical content. Moreover, VSWM may play a greater role in mathematical tasks that emphasize the need to recall

and maintain information (e.g., basic arithmetic). Thus, relations between spatial and mathematical performance may be dependent on the mathematical task in question. Accordingly, the space-math link may best be explained by the spatial modelling account under some conditions (novel mathematical content) and the working memory account under other conditions (coordination of familiar mathematical content).

According to this proposal, the space-math link may differ across individuals as a function of their experience and familiarity with the mathematical task in question. For example, a child who is first learning basic arithmetic may find it useful to model the solution, whereas a child fluent in basic arithmetic may have no need to pause, reflect, and model the problem and solution. This suggests the need to more carefully consider the learner's familiarity with the mathematical content under investigation when examining mechanisms underlying the space-math link. Said differently, experience may moderate relations between space and math. To my knowledge, this represents a major gap in the literature and represents a promising area of future study.

As revealed in Study 2, neuroimaging may prove to be a useful tool in further understanding when and under what conditions spatial visualization may correlate with different components of mathematics. Moreover, neuroimaging may prove useful in testing whether spatial visualization is more associated with novel mathematical content vs. familiar content. If spatial visualization does indeed play a role in helping novice learners model mathematical relations, then we might expect to see increased neural activity in regions associated with spatial visualization processes compared to regions associated with mastery of the content. For advanced learners (those that have mastered the content under question), we might expect to see increased neural activity in regions associated with mastery of the content and less activity in regions associated with spatial visualization processes. Indeed, the fronto-parietal shift is thought to reflect a shift in effortful to more automatic numerical processing (Ansari, Garcia, Lucas, Hamon, & Dhital, 2005). It is possible that a similar type of shift might occur from regions more attuned to spatial processing to regions more attuned to verbal storage as a function of going from a novice to mastery learner. Future research is needed in this regard as it has the potential to shed new light on the spatial modelling account.

Lastly, it is worth considering how the spatial modelling account may help to explain the spatial representation of numbers account (aka the mental number line account). Numbers and their various relations may be conceptualized and visualized in a variety of ways. The ‘mental number line’ might be but *one* demonstration of the ways in which numbers and their relations are represented spatially. Indeed, the spatial modelling account might also explain why other entities, such as days and months of the week (Gevers, Reynvoet, & Fias, 2003; 2004) and even emotions (Holmes, Alcat, & Lourenco, 2019), have been found to be mapped to space. Accordingly, the capacity to mentally organize and model concepts according to spatial metrics is not limited to numbers and other magnitudes, but might extend to other mathematical domains and even other non-mathematical domains as well. However, given that mathematics is frequently expressed and conceived in terms of numerical-spatial relations (e.g., Pythagorean Theorem), it seems reasonable to predict that spatial modelling may play an especially important role in mathematical thought.

## 5.2.5 An Integrated Description of the Four Accounts

The spatial modelling account provides the best description of the current thesis. However, it is possible that all four accounts interact with one another and at certain points in time and under different testing conditions present themselves as the most likely explanation for a particular space-math link. Moreover, the extent to which these various accounts are descriptions of the same underlying mechanism but in different forms and at different levels of analysis is an important question. For example, it is possible that one of the ways in which numbers become represented spatially is through the active processes of spatial modelling (e.g., visualizing a number line to reason about numerical relations). From a biological perspective, it could be that the IPS and closely associated regions provide the necessary neuronal networks to carry out these modelling and transformational processes. Moreover, even when the spatial modelling of numerical concepts no longer serves the individual (i.e., the concepts at hand have become automatized more or less), these same neural substrates may continue to underlie both



numerical and spatial processes. This may occur despite an independence in function. If we assume that spatial visualization is a relatively stable trait, then we should expect to see lasting correlations between spatial visualization and numerical skills even when spatial visualization no longer serves a purpose in one's semantic understanding of number. In other words, spatial and numerical processes may continue to be correlated, both neurally and behaviorally, long after they have become conceptually divorced from one another. This relation may remain because of individual differences in spatial visualization skills that once helped give rise to conceptual mappings between numbers and space. This integrated account may explain why we continue to see correlations between spatial visualization skills and basic numerical competencies into adulthood. It might also explain why we see relations between intentional numerical-spatial mappings (e.g., as measured with the number line task) and mathematics (Schneider et al., 2018), but mixed evidence for relations between automatic numerical-spatial mappings (i.e., SNARC) and mathematics (Cipora, Patro, & Nuerk, 2015). Moving forward, it will be important to continue to theorize and test how and when the four accounts are both related and distinct from one another.

### 5.3 Next Steps – Establishing Causal Relations between Spatial and Mathematical Thinking

The current thesis adds to a growing body of research suggesting close relations between spatial and mathematical thought (e.g., see Mix & Cheng, 2012). And while the findings from Study 1 and 2 offer new insights into ways in which spatial and mathematical thinking may be linked, follow-up studies are needed to test for causal relations between spatial and mathematical cognition. To this end, spatial training studies offer an ideal methodological approach. A recent meta-analysis suggests that spatial training is an effective means for improving spatial thinking in people of all ages and through a wide assortment of training approaches (e.g., in-class training, video games, spatial task training; Uttal et al., 2013). Moreover, the effects of spatial training appear to generalize to intermediate transfer measures; that is, other spatial measures not part of the training. Overall, current evidence suggests that spatial reasoning is a highly malleable construct.

Spatial training studies have the potential to further reveal the ways in which spatial and mathematical thinking may be linked. Indeed, spatial training studies offer the means to more thoroughly investigate the hypothesized relations between spatial and mathematical thought discussed in Study 1 and 2. More specifically, spatial training studies offer opportunities to test the space-math link as they relate to the four candidate mechanisms reviewed above. For example, different predictions can be made depending on the different accounts reviewed. According to the *spatial representation of numbers account*, one might predict that spatial training is related to improvements in one's internal representation of numbers according to a more spatially precise mental number line. This refinement in one's 'mental number line,' in turn, is predicted to facilitate greater numerical reasoning. Critically, in order to test this hypothesis, future training studies will need to include measures of spatial-numerical mappings (e.g., intentional number line estimation tasks, automatic SNA tasks, including SNARC effects). Any gains in more general measures of numerical reasoning should theoretically be mediated by change on these measures. One way of testing the *spatial-modelling account* would be to gain insights into the strategies that participants use while engaging in the numerical and mathematical tasks. What evidence is there that the spatial visualization training actually leads to an improved ability to mentally model the problem at hand? For example, collecting process data of the sort used in Hegarty and Kozhevnikov's (1999) word problem studies (see Introduction), could be used to demonstrate the extent to which spatial training results in improved schematic representations of the problems. Evidence of this sort would lend support for the spatial modelling account. In terms of the *shared neural processing account*, researchers have yet to examine the neural correlates of spatial training. However, a rather straightforward prediction would be that training-induced changes in neural activity (or the underlying neuroanatomical structures) should be correlated with improvements in numerical/mathematical reasoning. Lastly, according to the *working memory account*, changes in spatial visualization should more broadly be encapsulated by changes in working memory. It is possible that spatial visualization training is akin to working memory training. Future training studies thus need to also include measures of working memory to provide evidence for or against this possibility.

Moving forward, it will also be important to establish whether mathematical training in itself is a form of spatial training. That is, is there any evidence that mathematics training is related to improvements in spatial cognition? The results from Study 3 suggest not, as there was no indication that gains in children's numerical/mathematical thinking was associated with gains in children's spatial thinking of EF skills. However, given the rather short time frame of this study, as well as the limited evidence of gains in children's numerical skills, more research is needed to more fully address this question. It seems plausible that bidirectional relations exist, but that the amount of transfer may depend on the degree of overlap in the mental operations that is shared between the two domains. Presently, the question of whether mathematics learning generalizes to spatial learning remains an open question.

To conclude, future training studies have the potential to provide new insights into the theorized mechanisms underlying the space-math link. This approach is critical in revealing why and under what conditions training might be effective for some individuals but not others. Moreover, as mentioned in the Introduction, the better understanding we have of why spatial and mathematical thinking are linked, the more likely it is that this information can be used in educational and clinical practice.

## 5.4 Bringing Numerical Cognition Research into the Classroom

Research into numerical cognition, including the research discussed above, has the potential to inform educational practice. However, all too often the findings and insights revealed in peer-reviewed journal articles fail to have any bearing whatsoever on educational practice. In many regards, this may be a good thing. For example, the implementation of unreplicable findings into practice may actually lead to misguided teaching efforts. Other times, however, research findings provide a firm base on which teachers and the teaching profession as whole can use as a guide, framework, or inspiration to effective instruction. For example, to borrow from the language cognition literature, years of research have provided strong evidence that deliberate phonics instruction is an essential component of effective reading programs (e.g., see Castles,

Rastle, & Nation, 2018). Although numerical cognition as a field is relatively new compared to the field of literacy research, there is still a large body of research on which to stand on and use to inform early mathematics instruction. The primary goal of Study 3 was to take some of the well-established research findings from the numerical cognition literature and work alongside practising teachers (Kindergarten to Grade 3) to integrate it into their own practice.

The results of this two-year study were interesting but ultimately difficult to interpret. While the Year 1 results demonstrated program success at both the teacher and student level, the results of Year 2 indicated minimal evidence of success at both the teacher and student level. The Discussion section in Study 3 offers several explanations for why we may have observed differential effects across both years of study, including differences in principal involvement, implementation of the student-focused intervention, and overall teacher buy-in. However, these explanations are, at present, mostly speculative and it is clear that more research efforts are needed to further evaluate what makes the teacher intervention effective in one context but not another.

What was not explicitly discussed in Study 3 is how these findings fit into the broader literature on the effectiveness of in-service teacher PD interventions. As it turns out, our mixed findings are reflective of the field as whole. Interestingly, mixed findings appear to be the norm, even when teacher PD programs adhere to widely regarded effective features of PD (e.g., see Hill, Corey, & Jacob, 2018). Indeed, there is general consensus amongst educational researchers that effective teacher PD consists of several key features: a focus on subject matter content; teachers as active participants in the PD process; coherence with school goals and local policies; and includes collaborative participation (Desimone, 2009; Penuel, Fishman, Yamaguchi, & Gallagher, 2007). Arguably, the teacher PD model described in Study 3 adheres to all of these principles. Yet, our study, like many others that have come before, indicate that the inclusion of these features are not a guarantee of program success (e.g., see Garet et al., 2011; Hill, Corey, & Jacob, 2018; Jacob, Hill, & Corey, 2017; Santagata, Kersting, Givvin, & Stigler, 2010). According to Hill et al. (2018) the field of PD research has reached a crossroad. Now, more than ever, there is a need to better understand what makes some teacher PD programs – along with the individual features that make-up a program – more effective

than others. However, as I argue next, there is also a need to consider more than just the PD program itself.

Study 3 offers some insight into this need, suggesting that even the same program may have differential effects from one school to another. This points to the importance of replicating previously identified successful teacher interventions. Our findings also suggest that more research efforts need to be directed at identifying how features of the local school context influence teacher uptake and implementation. This includes the individual and collective characteristics of the teachers involved. In the end, it is likely not enough to evaluate the effectiveness of PD programs based on the specific features it entails. Even the best designed PD programs are likely to vary in degrees of effectiveness as a function of the school climate and teachers involved. Thus, in order to advance from the current crossroad, it will be important to not only evaluate the programs themselves, but the context in which the program is carried out, and ultimately the ways in which the program and context interact with one another.

While this may seem like an ambitious endeavour (and it is), understanding what makes for effective PD has major implications for the improvement of teaching and learning. For example, from an economics standpoint, the financial cost of teacher PD in the United States is estimated at \$8 billion per year or an average annual spending by school districts of \$18,000 per teacher (Layton, 2015). While this figure varies from year to year and from study to study, it is clear that school boards, including those here in Ontario, spend an enormous amount of money and resources on teacher PD. At least some of this money may be better spent on investigating what makes for effective PD, including an increased research focus on how the local context influences PD uptake and implementation. It is only through a better understanding of what works and what does not work when it comes to teacher PD that we can hope to reliably meet the goal of improved teaching and student learning.

#### 5.4.1 Contributions to the Discipline of Mind, Brain, and Education

The teacher PD intervention described in Study 3 is unique in that it places much more emphasis on the role of developmental cognitive neuroscience in mathematics education. Indeed, in bringing these two disciplines together, Study 3 addresses some of the central aims of Mind, Brain, and Education (aka Educational Neuroscience). Chief amongst these aims is the creation of an infrastructure that creates productive bidirectional exchange between researchers and practitioners. As discussed in the Introduction, the relatively new discipline of Mind, Brain, and Education remains at a standstill, fully acknowledging the potential benefits of bridging cognitive science and education, but falling short in providing a means to do so. As William James noted over 100 years ago, an “intermediary inventive mind” is needed to bridge the science of the mind and art of teaching (James, 1899). In the model presented in Study 3, we see evidence of both teachers and researchers working together to fulfill this role. Through reading, discussing, and engaging in cognitive, developmental, and educational research, teachers are brought into closer contact with the ‘science of the mind.’ Theoretically, this knowledge can then be used to positively inform and influence the ‘art of teaching.’ Researchers, on the other hand, are brought into closer contact with the art of teaching and everyday classroom practice. In addition, this knowledge may further serve to inform and influence research and understanding into the science of the mind. Thus, in its ideal state, all members involved in the teacher-research PD model contribute and come away from the process with an improved and more integrated understanding of mind, brain, and education. While it is clear that more research is needed to further test the realization of these goals, the results of Study 3 do indicate some promising results. Moving forward it will be necessary to better quantify the extent to which these bidirectional goals of the PD model are being met.

Future goals aside, Study 3 marks an important advance, providing proof of concept that it is possible to bring learning scientists and practitioners together to work towards evidence-informed mathematics instruction. This is but one small advance in the goal of building better connections between research and practice.

## 5.5 Concluding Remarks

The present thesis contributes to an improved understanding of mathematics thinking and learning in several ways. First, the results of Study 1 and 2 provide new perspectives on the ways in which spatial, numerical, and mathematical thinking may be linked at both the behavioral as well as neural levels of analysis. The results from these studies demonstrate close behavioral as well as neural associations between spatial visualization processes and various numerical and mathematical processes. These findings have led to the hypothesis that spatial visualization may play an important role in how people come to mentally organize, model, and simulate numerical and mathematical relations. The spatial modelling account, as it has been named in this thesis, is particularly appealing because it offers new ways of thinking about and contextualizing other alternative accounts of the space-math link (e.g., the ‘mental number line’ account). Research is now needed to test the predictions associated with this account (e.g., that spatial visualization processes are especially important during the learning of novel mathematical content). A second contribution of this thesis concerns the progress made towards bridging the research-practice gap between numerical cognition and mathematics education. Study 3 describes the design, implementation, and results of a new model of teacher Professional Development aimed to better integrate numerical cognition research with the teaching and learning of early number. Findings from this study indicated that the PD may have been effective at increasing teachers’ self-perceived numerical cognition knowledge and students’ general numeracy skills. However, there were notable differences in the effects of the PD across the two sites studied, with much stronger effects at one site than the other. Thus, critical questions remain as to when and why the model may be effective in some school contexts but not others. Although more research is needed, the PD model presents a promising new approach in the effort to apply research findings from numerical cognition the teaching and learning of early years mathematics. Together, the present thesis provides new insights into the cognitive and neural underpinnings of mathematical thought and a viable approach to the translation and application of numerical cognition research to authentic classroom settings.

## 5.6 References

- Ansari, D., Garcia, N., Lucas, E., Hamon, K., & Dhital, B. (2005). Neural correlates of symbolic number processing in children and adults. *NeuroReport*, *16*(16), 1769-1773.
- Bulf, H., de Hevia, M. D., & Macchi Cassia, V. (2016). Small on the left, large on the right: Numbers orient visual attention onto space in preverbal infants. *Developmental Science*, *19*(3), 394-401.
- Castles, A., Rastle, K., & Nation, K. (2018). Ending the reading wars: Reading acquisition from novice to expert. *Psychological Science in the Public Interest*, *19*(1), 5-51.
- Cipora, K., Patro, K., & Nuerk, H. C. (2015). Are spatial-numerical associations a cornerstone for arithmetic learning? The lack of genuine correlations suggests no. *Mind, Brain, and Education*, *9*(4), 190-206.
- De Hevia, M. D., & Spelke, E. S. (2010). Number-space mapping in human infants. *Psychological Science*, *21*(5), 653-660.
- Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher*, *38*(3), 181-199.
- Fischer, M. H., & Fias, M. H. (2005). Spatial representation of numbers. *Handbook of Mathematical Cognition*, 43-54.
- Fischer, U., Moeller, K., Bientzle, M., Cress, U., & Nuerk, H. C. (2011). Sensori-motor spatial training of number magnitude representation. *Psychonomic Bulletin & Review*, *18*(1), 177-183.
- Garet, M. S., Wayne, A. J., Stancavage, F., Taylor, J., Eaton, M., Walters, K., & Doolittle, F. (2011). *Middle School Mathematics Professional Development Impact Study: Findings after the second year of implementation* (NCEE 2011-4024). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.
- Gevers, W., Reynvoet, B., & Fias, W. (2003). The mental representation of ordinal sequences is spatially organized. *Cognition*, *87*(3), B87-B95.



- Gevers, W., Reynvoet, B., & Fias, W. (2004). The mental representation of ordinal sequences is spatially organised: Evidence from days of the week. *Cortex*, 40, 171-172.
- Hegarty, M., & Kozhevnikov, M. (1999). Types of visual-spatial representations and mathematical problem solving. *Journal of Educational Psychology*, 91(4), 684–689
- Hill, H. C., Corey, D. L., & Jacob, R. T. (2018). Dividing by zero: Exploring null results in a mathematics professional development program. *Teachers College Record*, 120(6), n6.
- Holmes, K. J., Alcat, C., & Lourenco, S. F. (2019). Is Emotional Magnitude Spatialized? A Further Investigation. *Cognitive Science*, 43(4), e12727.
- Hubbard, E. M., Piazza, M., Pinel, P., & Dehaene, S. (2005). Interactions between number and space in parietal cortex. *Nature Reviews Neuroscience*, 6(6), 435.
- Jacob, R., Hill, H., & Corey, D. (2017). The impact of a professional development program on teachers' mathematical knowledge for teaching, instruction, and student achievement. *Journal of Research on Educational Effectiveness*, 10(2), 379-407.
- James, W. (1899/2001). *Talks to teachers on psychology and to students on some of life's ideals*. New York, NY: Dover Publications
- Layton, L. (2015, August 4). Study: Billions of dollars in annual teacher training is largely a waste. The Washington Post. Retrieved from [https://www.washingtonpost.com/local/education/study-billions-of-dollars-in-annual-teacher-training-is-largely-a-waste/2015/08/03/c4e1f322-39ff-11e5-9c2d-ed991d848c48\\_story.html?utm\\_term=.934f1df95a68](https://www.washingtonpost.com/local/education/study-billions-of-dollars-in-annual-teacher-training-is-largely-a-waste/2015/08/03/c4e1f322-39ff-11e5-9c2d-ed991d848c48_story.html?utm_term=.934f1df95a68)
- Lohman, D. F. (1996). Spatial ability and G. In I. Dennis & P. Tapsfield (Eds.), *Human abilities: Their nature and assessment* (pp. 97–116). Hillsdale, NJ: Lawrence Erlbaum.
- Lourenco, S. F., Cheung, C-N., & Aulet, L. S. (2018). Is visuospatial reasoning related to early mathematical development? A critical review. In A. Henik & W. Fias (Eds.), *Heterogeneity of function in numerical cognition* (pp. 177-210). Elsevier: Academic Press

- Mix, K. S., & Cheng, Y. L. (2012). The relation between space and math: developmental and educational implications. *Advances in Child Development and Behavior*, 42, 197-243.
- Owen, A.M., McMillan, K.M., Laird, A.R., Bullmore, E. (2005). N-back working memory paradigm: a meta-analysis of normative functional neuroimaging studies. *Human Brain Mapping*, 25 (1), 46–59.
- Penuel, W. R., Fishman, B. J., Yamaguchi, R., & Gallagher, L. P. (2007). What makes professional development effective? Strategies that foster curriculum implementation. *American Educational Research Journal*, 44(4), 921-958.
- Popescu, T., Sader, E., Schaer, M., Thomas, A., Terhune, D. B., Dowker, A., ... & Kadosh, R. C. (2019). The brain-structural correlates of mathematical expertise. *Cortex*, 114, 140-150.
- Santagata, R., Kersting, N., Givvin, K. B., & Stigler, J. W. (2010). Problem implementation as a lever for change: An experimental study of the effects of a professional development program on students' mathematics learning. *Journal of Research on Educational Effectiveness*, 4, 1–24.
- Schneider, M., Merz, S., Stricker, J., De Smedt, B., Torbeyns, J., Verschaffel, L., & Luwel, K. (2018). Associations of number line estimation with mathematical competence: A meta-analysis. *Child Development*, 89(5), 1467-1485.
- Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games—but not circular ones—improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology*, 101(3), 545-560.
- Smith, E.E., & Jonides, J. (1999). Storage and executive processes in the frontal lobes. *Science*, 283 (5408), 1657–1661.
- Toomarian, E. Y., & Hubbard, E. M. (2018). On the genesis of spatial-numerical associations: Evolutionary and cultural factors co-construct the mental number line. *Neuroscience & Biobehavioral Reviews*.
- Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., & Newcombe, N. S. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological Bulletin*, 139(2), 352.
- van Dijck, J. P., & Fias, W. (2011). A working memory account for spatial–numerical

associations. *Cognition*, 119(1), 114-119.

# Appendices

## Appendix A: Documentation of Ethics Approval



Research Ethics

### Western University Non-Medical Research Ethics Board NMREB Full Board Initial Approval Notice

**Principal Investigator:** Prof. Daniel Ansari  
**Department & Institution:** Social Science\Psychology, Western University

**NMREB File Number:** 108327  
**Study Title:** Mathematics teaching and learning in the early years: Experimenting with a new model of teacher Professional Development

**NMREB Initial Approval Date:** November 17, 2016  
**NMREB Expiry Date:** November 17, 2017

#### Documents Approved and/or Received for Information:

Document Name	Comments	Version Date
Assent	Individual Child Assent Script and Form - also includes written consent for children aged 7 and over	2016/07/23
Western University Protocol	Received October 31, 2016	
Other	Support letter to conduct study from TVDSB - Received for Information	2016/07/20
Letter of Information & Consent	Principal	2016/10/27
Letter of Information & Consent	Teacher	2016/10/27
Letter of Information & Consent	Parent	2016/10/27
Assent	Whole Class Assent Script	2016/07/22
Instruments	Description and Examples of Child Assessment Items - Received July 26, 2016	
Other	Quality of Lessons and Activities Examples - Received for Information	2016/10/27
Instruments	Teacher Assessment Measures - Received July 26, 2016	

The Western University Non-Medical Research Ethics Board (NMREB) has reviewed and approved the above named study, as of the NMREB Initial Approval Date noted above.

NMREB approval for this study remains valid until the NMREB Expiry Date noted above, conditional to timely submission and acceptance of NMREB Continuing Ethics Review.

The Western University NMREB operates in compliance with the Tri-Council Policy Statement Ethical Conduct for Research Involving Humans (TCPS2), the Ontario Personal Health Information Protection Act (PHIPA, 2004), and the applicable laws and regulations of Ontario.

Members of the NMREB who are named as Investigators in research studies do not participate in discussions related to, nor vote on such studies when they are presented to the REB.

The NMREB is registered with the U.S. Department of Health & Human Services under the IRB registration number IRB 0000094.



Western University, Research, Support Services Bldg., Rm. 5150  
London, ON, Canada N6G 1G9 t. 519.661.3036 f. 519.850.2466 www.uwo.ca/research/ethics



# Western Research

**Date:** 25 October 2017

**To:** Prof. Daniel Ansari

**Project ID:** 108327

**Study Title:** Mathematics teaching and learning in the early years: Experimenting with a new model of teacher Professional Development

**Application Type:** Continuing Ethics Review (CER) Form

**Review Type:** Delegated

**Meeting Date:** November 3, 2017

**Date Approval Issued:** 25/Oct/2017 09:56

**REB Approval Expiry Date:** 17/Nov/2018

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Dear Prof. Daniel Ansari ,

The Western University Research Ethics Board has reviewed the application. This study, including all currently approved documents, has been re-approved until the expiry date noted above.

REB members involved in the research project do not participate in the review, discussion or decision.

The Western University NMREB operates in compliance with the Tri-Council Policy Statement Ethical Conduct for Research Involving Humans (TCPS2), the Ontario Personal Health Information Protection Act (PHIPA, 2004), and the applicable laws and regulations of Ontario. Members of the NMREB who are named as Investigators in research studies do not participate in discussions related to, nor vote on such studies when they are presented to the REB. The NMREB is registered with the U.S. Department of Health & Human Services under the IRB registration number IRB 00000941.

Please do not hesitate to contact us if you have any questions.

Sincerely,



9 Feb 2017

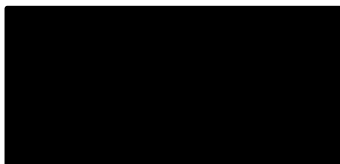
Dear Mr. Hawes:

Your project, entitled "Mathematics teaching and learning in the early years: Experimenting with a new model of teacher Professional Development" has been approved by Research and Assessment Services at the Thames Valley District School Board. Please ensure that all members of your research team who will be assisting with data collection involving students have an up-to-date criminal record check. You are welcome to begin data collection for your study.

The continued willingness of our families and staff to participate in research studies is greatly enhanced by pertinent feedback of findings. It is suggested that direct feedback be provided to the school(s), staff, students, and/or families involved in the study. Please find attached the Thames Valley District School Board Study Completion Form. Once you have completed your research in our board, please complete this form and submit it to Research and Assessment Services. This form should be submitted within two years of receiving approval. If the study is not completed within two years of the date on this letter, please submit a study extension request to Dr. Sarah Folino.

All the best with your research. Please feel free to contact me if I can be of further assistance.

Sincerely,



Research and Assessment Services  
Thames Valley District School Board  
Email: [Redacted]

/sd

cc: [Redacted] Superintendent of Student Achievement

---

**Thames Valley District School Board - Research and Assessment**  
1250 Dundas Street, P.O. Box 5888, London, Ontario N6A 5L1 Tel: 519-452-2000 Fax: 519-452-1499 website: www.tvdsb.ca

*We build each student's tomorrow, every day.*

# Curriculum Vitae

## Zachary Hawes

Assistant Professor  
Department of Applied Psychology & Human Development  
University of Toronto

## Address

University of Toronto, OISE  
Human Development and Applied Psychology  
Dr. Eric Jackman Institute of Child Study  
45 Walmer Road  
Toronto, ON, Canada, M5R 2X2

## Education

- |             |  |
|-------------|--|
| 2015 – 2019 | PhD Cognitive, Developmental, and Brain Sciences<br>University of Western University<br>Advisor: Daniel Ansari   |
| 2009 – 2011 | MA Child Studies and Education<br>Applied Psychology & Human Development<br>Ontario Institute for Studies in Education (OISE)<br>University of Toronto |
| 2008        | Post-degree studies in Psychology<br>Wilfrid Laurier University  |
| 2002 – 2006 | BKin Honours Kinesiology<br>Faculty of Sciences<br>Wilfrid Laurier University  |

## Academic Positions

- |                |   |
|----------------|---|
| 2019 – present | Assistant Professor<br>Applied Psychology & Human Development<br>University of Toronto/Ontario Institute for Studies in Education   |
| 2011 – 2015    | Research Officer<br>The Robertson Program for Inquiry-Based Teaching in<br>Mathematics and Science<br>Applied Psychology & Human Development<br>University of Toronto/Dr. Eric Jackman Institute of Child Study |

## Refereed Journal Publications

**Hawes, Z., & Ansari, D.** (accepted with minor revision). Why are spatial and mathematical skills related? A review of evidence from brain and behavior. *Psychonomic Bulletin & Review*

**Hawes, Z., Moss, J., Caswell, B., Seo, J., Thomson, M., Thomson, N., Bailey, C.** (2019). Effects of a teacher-designed and teacher-led numerical board game intervention: A randomized controlled study with 4- to 6- year-olds. *Mind, Brain, and Education*

**Hawes, Z., Sokolowski, H. M., Ononye, C. B., & Ansari, D.** (in press). Neural underpinnings of spatial and numerical cognition: An *f*MRI meta-analysis of brain regions associated with symbolic number, arithmetic, and mental rotation. *Neuroscience & Biobehavioral Reviews*, *103*, 316-336.

**Hawes, Z., Moss, J., Caswell, B., Seo, J., & Ansari.** (2019). Relations between numerical, spatial, and executive function skills and mathematics achievement: A latent-variable approach. *Cognitive Psychology*, *109*, 68-80.

Sokolowski, H. M., **Hawes, Z.**, & Lyons, I. M. (2019) What explains sex differences in math anxiety? A closer look at the role of spatial processing. *Cognition*, *182*, 193-212. <https://osf.io/hxsrd/>

**Hawes, Z., Nosworthy, N., Archibald, L., & Ansari, D.** (2019). Kindergarten children's symbolic number comparison skills predict 1<sup>st</sup> grade mathematics achievement: Evidence from a two-minute paper-and-pencil test. *Learning and Instruction*, *59*, 21-33.

**Hawes, Z., Moss, J., Caswell, B., Naqvi, S., & MacKinnon, S.** (2017). Enhancing children's spatial and numerical skills through a dynamic spatial approach to early geometry instruction: Effects of a 32-week intervention. *Cognition and Instruction*, *35*(3), 236-264.

Bruce, C.D., Davis, B., Sinclair, N., McGarvey, L., Hallowell, D., Drefs, M., Francis, K., **Hawes, Z.**, Moss, J., Mulligan, J. & Okamoto, Y. (2017). Understanding gaps in research networks: Using "spatial reasoning" as a window into the importance of networked educational research. *Educational Studies in Mathematics*, *95*(2), 143-161.

Francis, K., Bruce, C., Davis, B., Drefs, M., Hallowell, D., **Hawes, Z.**, McGarvey, L., Moss, J., Mulligan, J., Okamoto, Y. & Sinclair, N. (2017). Multidisciplinary perspectives on a video case of children designing and coding for robotics. *Canadian Journal of Science, Mathematics and Technology Education*, *17*(3), 165-178.

**Hawes, Z., Moss, J., Caswell, B., & Poliszczuk, D.** (2015). Effects of mental rotation training on children's spatial and mathematics performance: A randomized controlled study. *Trends in Neuroscience and Education*, *4*(3), 60-68.



**Hawes, Z.**, LeFevre, J., Xu, C., & Bruce, C. (2015). Mental rotation with tangible three-dimensional objects: A new measure sensitive to developmental differences in 4- to 8-year-old children. *Mind, Brain, and Education*, 9(1), 10-18.

Moss, J., **Hawes, Z.**, Naqvi, S., & Caswell, B. (2015). Adapting Japanese Lesson Study to enhance the teaching and learning of geometry and spatial reasoning in early years classrooms: A case study. *ZDM – The International Journal on Mathematics Education*, 47(3), 377-390.

Bruce, C. D., & **Hawes, Z.** (2015). The role of 2D and 3D mental rotation in mathematics for young children: What is it? Why does it matter? And what can we do about it? *ZDM – The International Journal on Mathematics Education*, 47(3), 331-343.

**Hawes, Z.**, Moss, J., Finch, H., & Katz, J. (2013). Choreographing patterns and functions. *Teaching Children Mathematics*, 19(5), 302-309.

## Book Chapters

Sinclair, N., Moss, J., **Hawes, Z.**, & Stephenson, C. (2019). Learning through and from drawing in early years geometry. In K. Mix and M. Battista (Eds.), *Visualizing mathematics: The role of spatial reasoning in mathematical thought* (pp. 229-252). New York, NY: Springer Nature Publishing.

McGarvey, L., Luo, L., **Hawes, Z.**, & Spatial Reasoning Study Group. (2018). Spatial skills framework for young engineers. In L. English and T. Moore (Eds.), *Early engineering learning* (pp. 53-81). Singapore: Springer Nature Publishing

**Hawes, Z.**, Tepylo, D., & Moss, J. (2015). Developing spatial thinking: Implications for early mathematics education. In B. Davis and Spatial Reasoning Study Group (Eds.), *Spatial reasoning in the early years: Principles, assertions and speculations* (pp. 29-44). New York, NY: Routledge.

Bruce, C., Sinclair, N., Moss, J., **Hawes, Z.**, & Caswell, B. (2015). Spatializing the mathematics curricula. In B. Davis and Spatial Reasoning Study Group. (Eds.), *Spatial reasoning in the early years: Principles, assertions and speculations* (pp. 85-106). New York, NY: Routledge

## Books

Moss, J., Bruce, C.D., Caswell, B., Flynn, T. & **Hawes, Z.** (2016). *Taking shape: Classroom activities to improve young children's geometric and spatial thinking*. Toronto, ON: Pearson

Davis, B., & Spatial Reasoning Study Group (2015). *Spatial reasoning in the early years: Principles, assertions and speculations*. New York, NY: Routledge

## Educational Policy and Government Publications

**Hawes, Z.**, Flynn, T., (co-lead authors) & Ontario Ministry of Education (2014). *Paying attention to spatial thinking: Support document for paying attention to mathematical education; K-12*. Toronto, ON: Queen's Printer for Ontario.

**Hawes, Z.** (2013). *Capacity Building Series: Inquiry-based learning*. Toronto, ON: Queen's Printer for Ontario.

**Hawes, Z.**, Gibson, A., Mir, S., & Pelletier, J. (2012). Children's experiences in full-day programs for 4-and 5-year-olds: Play and self-regulation. In Corter, C., Janmohamed, Z., & Pelletier, J. (Eds.), *Toronto First Duty Phase 3 Report* (pp. 31-55). Toronto, ON: Atkinson Centre for Society and Child Development, OISE/University of Toronto.

## Refereed Conferences and Proceedings

**Hawes, Z.**, Sokolowski, H. M., Ononye, C., & Ansari, D. (2019). *Where and under what conditions do spatial and numerical cognition converge and diverge in the brain? An fMRI meta-analysis*. Poster presented at the Mathematical Cognition and Learning Society conference, Carleton University, ON, Canada.

Sokolowski, H. M., **Hawes, Z.**, Peters, L., & Ansari, D. (2019). *Symbols are special: An fMRI adaptation study of symbolic, nonsymbolic, and non-numerical magnitude processing in the human brain*. Poster presented at the Mathematical Cognition and Learning Society conference, Carleton University, ON, Canada.

**Hawes, Z.**, Moss, J., Caswell, B., Seo, J., & Ansari. (2018). *Relations between Numerical, Spatial, and Executive Function Skills and Mathematics Achievement: A Latent-Variable Approach*. Poster presented at the Mathematical Cognition and Learning Society conference, Oxford University, UK.

Lyons, I.M., Daker, R.J., Sokolowski, H.M., **Hawes, Z.**, Ramirez, G., Maloney, E.A., Rendina D.N., Levine, S.C., & Beilock, S.L. (2018). *Spatial Anxiety – A novel tool with applications for STEM education*. Paper presentation at the Mathematical Cognition and Learning Society conference, Oxford University, UK.

Merkley, R., **Hawes, Z.**, & Ansari, D. (2018). What are the barriers to MBE? Insights from conversations and collaborations with early maths educators and researchers. Paper symposium at *International Mind, Brain, and Education Society* conference. Los Angeles, California.

**Hawes, Z.**, Merkle, R., Stager, C., & Ansari, D. (2018). Bridging the gap: Integrating developmental cognitive research and educational practice to strengthen students' number knowledge. Presentation at *7<sup>th</sup> Latin American School for Education, Cognitive, and Neural Sciences* (LA School) (June), Aconcagua Valley, Chile, 2018.

Ansari, D., & **Hawes, Z.** (2017). Do students benefit when teachers learn about the science of learning? Evidence from an early math professional development program. Talk presented at *ResearchEd* Toronto 2017, Toronto, ON, Canada.

Moss, J., Caswell, B., **Hawes, Z.**, & Jones, J. (2017). Prioritizing visual spatial mathematical approaches in first nation early years classrooms. In A. Chronaki's (Ed.), *Mathematics Education and Life at Times of Crisis; Proceedings of the 9<sup>th</sup> International Mathematics Education and Society Conference* (pp. 728-738), Volos, Greece.

Flynn, T., Bruce, C., **Hawes, Z.**, Moss, J., & Caswell, B. (2017). Math for Young Children: A classroom-based spatial reasoning intervention. Paper presented at the *Biennial Meeting of the Society for Research in Child Development*. Austin, TX.

Sokolowski, H. M., **Hawes, Z.**, Leibovich, T., & Ansari, D. (2017). The interference of symbolic and nonsymbolic numbers in a novel enumeration Stroop task. Poster presented at the 29<sup>th</sup> annual *American Psychological Association* convention. Boston, MA.

**Hawes, Z.**, Moss, J., Caswell, B., Cain, M., Thomson, N., Bruce, C., Flynn, T., & Le Duc, P. (2016). Integrating Mind, Brain, and Education through teacher-researcher collaborations. Paper symposium at the *International Mind, Brain, and Education Society* conference. Toronto, ON.

**Hawes, Z.**, Moss, J., Caswell, B., & MacKinnon, S. (2016). Enhancing children's spatial and numerical skills through a dynamic spatial approach to early geometry instruction: Effects of a 32-week intervention. Poster presented at the *6<sup>th</sup> Latin American School for Education, Cognitive and Neural Sciences* (LA School) (March), Buenos Aires, Argentina, March, 2016.

Moss, J., **Hawes, Z.**, & Caswell, B. (2016). Infusing spatial reasoning into early years mathematics: A year-long K-2 intervention. Paper presented at the *National Council for Teachers of Mathematics Conference*, San Francisco, CA, April, 2016.

Moss, J., **Hawes, Z.**, & Caswell, B. (2016). A spatial approach to early years geometry: Effects of a year-long teacher-led intervention. Part of symposium entitled: Lessons learned about increasing mathematics achievement through spatial instruction. Paper presented at the *American Educational Research Association* Conference, Washington, DC.

**Hawes, Z.**, Moss, J., Caswell, B., Naqvi, S., Bruce, C., & Flynn, T. (2015). A 'spatial' approach to early Mathematics instruction: Effects of a yearlong intervention. Paper presented at the *Biennial Meeting of the Society for Research in Child Development*. Philadelphia, PA.

**Hawes, Z.,** Moss, J., Caswell, B., Naqvi, S., & Poliszczuk, D. (2015). Effects of computerized spatial training on children's spatial and mathematical thinking: A randomized, controlled study. Poster presented at the *Biennial Meeting of the Society for Research in Child Development*. Philadelphia, PA.

**Hawes, Z.,** Moss, J., Caswell, B., & Poliszczuk, D. (2015). To what extent is spatial thinking malleable: Examining near and far transfer effects of computerized mental rotation training. Paper presented at the *45<sup>th</sup> Annual Jean Piaget Society Conference*. Toronto, Canada.

Tookenay, B., Caswell, B., Jones, J., Moss, J., MacKinnon, S., Snowball, R., Jones, S., **Hawes, Z.,** & Kabatay, T. (2015). Equitable access to quality mathematics for First Nation children. Paper presented at the World Indigenous Research Alliance Symposium, *World Indigenous Nations Higher Education Consortium*, Nanicoste, ON, August, 2015.

Moss, J., **Hawes, Z.,** Naqvi, S., Caswell, B., Bruce, C., Flynn, T. (2014). Changing perceptions of young children's geometry and spatial reasoning competencies: Lessons from the 'Math for Young Children' (M4YC) project. In Liljedahl, P., Nicol, C., Oesterle, S., & Allan, D. (Eds.). *Proceedings of the 38<sup>th</sup> Conference of the International Group for the Psychology of Mathematics Education and the 36<sup>th</sup> Conference of the North American Chapter of the Psychology of Mathematics Education* (Vol. 1; pp. 79-81). Vancouver, Canada: PME.

Moss, J., Tepylo, D., & **Hawes, Z.** (2014). The malleability of spatial reasoning. In Liljedahl, P., Nicol, C., Oesterle, S., & Allan, D. (Eds.). *Proceedings of the 38<sup>th</sup> Conference of the International Group for the Psychology of Mathematics Education and the 36<sup>th</sup> Conference of the North American Chapter of the Psychology of Mathematics Education* (Vol. 1; pp. 87-89). Vancouver, Canada: PME.

**Hawes, Z.,** & Bruce, C. (2014). A misnomer no more: Using tangible cube-figures to measure 3D mental rotation in young children. In Liljedahl, P., Nicol, C., Oesterle, S., & Allan, D. (Eds.). *Proceedings of the 38<sup>th</sup> Conference of the International Group for the Psychology of Mathematics Education and the 36<sup>th</sup> Conference of the North American Chapter of the Psychology of Mathematics Education* (Vol. 1; pp. 95-97). Vancouver, Canada: PME.

**Hawes, Z.,** Chang, D., Naqvi, S., Olver, A., & Moss, J. (2013). Uncovering the processes of young children's 3D mental rotation abilities: Implications for lesson design. In Martinez, M. & Castro Superfine, A (Eds.), *Proceedings of the 35<sup>th</sup> annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Chicago, IL: University of Illinois at Chicago.

**Hawes, Z.,** Chang, D., Olver, A., & Moss, J. (2013). Investigating spatial reasoning in young children: Developing a new measure to inform curriculum design. Paper presented at *Fields MathEd Forum Meeting*, Toronto, ON: The Fields Institute for Research in Mathematical Sciences, University of Toronto.

**Hawes, Z.**, Mancini, M., Morris, J., Olver, A., & Moss, J. (2013). Mental and physical rotations of 3D cube figures in 4-11 year old's: Differences in gender, age, and mathematical ability. Paper presented at the *Biennial Meeting of the Society for Research in Child Development*. Seattle, WA.

**Hawes, Z.**, McManus J., Naqvi S., Martinussen R., Pelletier J. (2013). Improving children's ability to self-regulate through computerized cognitive training: a randomized controlled study. Poster presented at the *Biennial Meeting of the Society for Research in Child Development*. Seattle, WA.

**Hawes, Z.**, Moss, J., Chang, D., & Naqvi, S. (2013). Math for Young Children: A professional development model that bridges cognitive science and mathematics education. In Martinez, M. & Castro Superfine, A (Eds.), *Proceedings of the 35<sup>th</sup> annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Chicago, IL: University of Illinois at Chicago.

Naqvi, S., **Hawes, Z.**, Chang, D., & Moss, J. (2013) Exploring pentominoes in 7 diverse Pre-k/K classrooms. In Martinez, M. & Castro Superfine, A (Eds.), *Proceedings of the 35<sup>th</sup> annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Chicago, IL: University of Illinois at Chicago.

Moss, J., **Hawes, Z.**, Chang, D., Tepylo, D. (2013). From mental rotation task design to lesson planning: The case of the polyomino challenge. Paper presented at *Fields MathEd Forum Meeting*, Toronto, ON: The Fields Institute for Research in Mathematical Sciences, University of Toronto.

## Invited Papers and Talks

Moss, J., & **Hawes, Z.** (2015). Learning to visualize through “quick image” activities: Building spatial and numerical reasoning for students of all ages. Presentation at *Ontario Association for Mathematics Educators (OAME)*. Toronto, ON.

Caswell, B., Moss, J., & **Hawes, Z.**, & Naqvi, S. (2013). Math for Young Children (M4YC) project: A no ceiling approach to math learning in an urban school. *Centre for Urban Schooling*, 1(1),1-4.

**Hawes, Z.**, Rohel, M., Chang, D., & Naqvi (2013). Tapping into symmetry: Young children's understanding of geometry and spatial reasoning. Presentation at *Ontario Literacy and Numeracy math forum: Creativity and innovation in mathematics*. Toronto, ON.

Bruce, C., Moss, J., Flynn, T., & **Hawes, Z.** (2013). A model of teacher professional development: Math for young children (M4YC). Presentation at *Ontario Ministry of Education Far and Northern Boards Meeting*. Toronto, ON.

Flynn, T., & **Hawes, Z.** (2013). Young children's spatial reasoning. *Ontario Association for Mathematics Education (OAME)*. Toronto, ON.

McLaughlin, J., **Hawes, Z.**, & Ellingson, P. (2012). Research-based educational games: Good for the brain? Presentation at *Brainpower*. Toronto, ON: University of Toronto.

## Papers Under Review or in Preparation

Sokolowski, H. M., **Hawes, Z.**, & Ansari, D. (under review). Neural correlates of symbolic, nonsymbolic, and non-numerical magnitude processing: An fMRI adaptation Study. <https://osf.io/2549a/>

**Hawes, Z.**, Merkley, R., Stager, C., & Ansari, D. (under review). Integrating numerical cognition research and mathematics education to strengthen the teaching and learning of early number. <https://osf.io/tqs7e/>

Sokolowski, H. M., **Hawes, Z.**, Leibovich, T., & Ansari, D. (in preparation). Number symbols are processed more automatically than the quantities they represent: Findings from an enumeration Stroop task.

Muñoz, F., Will, R., **Hawes, Z.**, & James, K (in preparation). Enhancing spatial thinking through mechanical problem solving.

Sokolowski, H. M.\*, **Hawes, Z.\***, & Ansari, D. (in preparation) The neural correlates of simple versus complex mental arithmetic: A functional neuroimaging meta-analysis. <https://osf.io/vqnt6/>

\*These authors contributed equally to the work

**Hawes, Z.**, Moss, J., Caswell, B., Seo, J., Thomson, M., Thomson, N., Bailey, C. & Sinclair, N. (in preparation). Learning to see geometrical relationships: Effects of a teacher-led drawing intervention for 4- to 6- year-olds.

**Hawes, Z.**, Gilligan, K., & Mix, K. (in preparation). Does spatial training generalize to mathematics performance? A meta-analysis and critical review

Lau, T.T., **Hawes, Z.**, & Ansari, D. (in preparation). Fear of math: Using the TIMSS international database to disentangle sources of math anxiety around the world

## Scholarships and Awards

2019 - 2021                      Chilean Commission for Scientific and Technological Research (CONICYT). Fund for Promotion of Scientific and Technological Development (FONDEF IDeA I+D): CAD \$367, 678.66.  
Relations between spatial skills and mathematics performance in

	elementary school children. Role: Co-Investigator/International Collaborator
2019 - 2021	Chilean Commission for Scientific and Technological Research (CONICYT). Scientific and Technological Development Fund for Early Career Researchers (FONDECYT de Iniciación): CAD \$105,100.96 An investigation of causal relations between mechanical reasoning and spatial skills. Role: Co-Investigator/International Collaborator
2016 - 2019	University of Western Ontario Doctoral Excellence Research Award (\$5000/year)
2015 - 2019	Western Graduate Research Scholarship (\$ ~ 5,900/year)
2015 - 2019	Social Sciences and Humanities Research Council of Canada (SSHRC) Doctoral Fellowship (\$80,000 over 48 months)
2018	Latin American School for Education, Cognition, and Neural Sciences Fellowship., Aconcagua Valley, Chile. June 17 <sup>th</sup> – 23 <sup>rd</sup> .
2016	Latin American School for Education, Cognition, and Neural Sciences Fellowship. Buenos Aires, Argentina. March 7 <sup>th</sup> – 18 <sup>th</sup> .
2015 - 2016	Ontario Graduate Scholarship (declined)
2014 - 2015	TVOntario (TVO) technology grant: \$9, 004.80 Examining the effects of computerized spatial training Role: Co-Principal Investigator with Joan Moss
2011 - 2012	TVOntario (TVO) technology grant: \$31, 280 Computerized learning Role: Co-Principal Investigator with Janette Pelletier
2011	Dr. Eric Jackman Graduate Studies Researcher Award, University of Toronto (\$500)

## Teaching Experience

2019 – 2020	Assistant Professor; Master of Arts Program Mathematics Theory and Curriculum (APD2210 & APD 2212) OISE/University of Toronto Department of Applied Psychology & Human Development
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2018 - 2019	Part-time lecturer Master of Arts program, Theory & Curriculum II: Mathematics (APD2212) OISE/University of Toronto Department of Applied Psychology & Human Development
2019	Teaching Assistant for the Psychological Trauma (PSYCH3316G) University of Western Ontario Department of Psychology
2018	Teaching Assistant for Cognitive Developmental Neuroscience (PSYCH3440) University of Western Ontario Department of Psychology
2017	Teaching Assistant for the Maladjusted Mind (PSYCH2030) University of Western Ontario Department of Psychology
2016	Teaching Assistant for Child Development (PSYCH2410) University of Western Ontario Department of Psychology
2007	Elementary School Teacher Gumi Public School, South Korea

## Ad Hoc Reviews

Developmental Science  
 Child Development  
 Journal of Experimental Child Psychology  
 Frontiers in Psychology  
 Learning and Individual Differences  
 Learning and Instruction  
 Cognition and Instruction  
 Developmental Psychology  
 Mind, Brain, and Education  
 Mathematical Thinking and Learning  
 Journal of the Learning Sciences  
 Journal of Cognition and Development



