# A RETRIAL DISCRETE-TIME QUEUEING SYSTEM WITH ACTIONS IN THE SERVER

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This paper considers a discrete-time retrial queueing system with movements. The arriving customers can opt to go directly to the server obtaining immediately their service or to join the orbit. In the first case, if the server is busy, the customer that is in the server is displaced to the orbit. The arrivals follow a geometrical law and the service times are general.

We study the Markov chain underlying the considered queueing system obtaining the generating function of the number of customers in the orbit and in the system as well as the stationary distribution of the time that a customer spends in the server. We derive the stochastic decomposition law and as an application we give bounds for the proximity between the steady-state distributions for our queueing system and its corresponding standard system.

At time $m^+$ the system can be described by the process $(C_m,\ \xi_m,\ N_m)$ where $C_m$ denotes the state of the server, 0 or 1 according to whether the server is free or busy and $N_m$ the number of repeated customers. If $C_m = 1$, then $\xi_m$ represents the remaining service time of the customer currently being served.

It can be shown that $\{(C_m,\ \xi_m,\ N_m) :\ m \in \mathbb{N}\}$ provides a Markovian description of the queueing system under study, whose states space is

$$\{(0,\ k) : k \geq 0;\ (1,\ i,\ k) : i \geq 1,\ k \geq 0\}.$$

The main objective of this work is to find the stationary distribution

$$\begin{aligned}
\pi_{0,k} &= \lim_{m \to \infty} P[C_m = 0;\ N_m = k],\ k \geq 0 \\
\pi_{1,i,k} &= \lim_{m \to \infty} P[C_m = 1; \xi_m = i; N_m = k],\ i \geq 1, k \geq 0
\end{aligned}$$

of the Markov chain $\{(C_m, \xi_m, N_m) : m \in \mathbb{N}\}$.

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