UNIVERSIDADE CATÓLICA PORTUGUESA

The equity risk premium and the low frequency of the term spread

International Evidence

Ana Isabel Oliveira Novais

Católica Porto Business School
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by

Ana Isabel Oliveira Novais

under the supervision of
(PhD) Gonçalo Faria and (PhD) Fabio Verona

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Resumo

Nesta tese analisamos o poder de previsão *out-of-sample* do *term-spread* e dos seus domínios de frequência sobre prémios de risco de mercado. A variável *term spread* representa a diferença entre taxas de juro de longo e curto prazo de obrigações do governo e a sua decomposição em domínio de frequência é feita através do método *Maximum Overlap Discrete Wavelet Transform*.

Foi comprovado pela literatura que no mercado dos Estados Unidos da América a componente de baixa frequência do *term spread* tem uma performance forte e robusta em exercícios *out-of-sample* sobre prémios de risco de mercado. Nesta tese, abordamos a possibilidade de este indicador ter a mesma performance em mercados internacionais (Alemanha, França, Japão, Reino Unido, Canada, África do Sul e Australia). Até então, esta alternativa ainda não foi abordada na literatura, e consideramos muito importante a sua análise para os mais diversos investidores, tanto locais como internacionais.

A principal conclusão desta tese é que a série original em domínio temporal e a componente de baixa frequência do *term spread* tem uma performance *out-of-sample* forte e robusta a prever prémios de risco de mercado para além dos Estados Unidos da América, na Alemanha, na França e no Canada.

Palavras-chave: Prémio de Risco de Mercado, *Term Spread*, Domínio de Frequência

Abstract

In this thesis we analyze the equity risk premium out-of-sample forecasting power of the term spread and its frequency components. The term spread is the difference between long and short term governmental interest rates and its frequency decomposition is done by applying a Maximum Overlap Discrete Wavelet Transform approach.

It has been shown in the literature that, in the United States of America equity market, the low frequency of the term spread is a strong and robust out-of-sample predictor of equity risk premium. In this thesis we address the empirical question if in alternative geographic zones (Germany, France, Japan, United Kingdom, Canada, South Africa and Australia) that continues to be the case.

This question has not been addressed so far in the literature and we foresee it as highly relevant for both local and international diversified equity investors.

Our main conclusion is that the original time series and the low frequency component of the term spread are a strong and robust out-of-sample predictors of the equity risk premium beyond United States of America, namely in Germany, France and Canada.

Keywords: Equity Risk Premium, Term Spread, Frequency Domain

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Introduction

Forecasting Equity Risk Premiums (ERP) is a crucial input for investors in their process of dynamic portfolio adjustments. Additionally, an improved exercise of forecasting ERP plays a crucial rule in finance because it helps to understand some factors that command ERP and improve the ability to use more realistic asset pricing models.

Since a long time ago, the predictability of equity risk premiums has been an active research question for academic researchers (e.g., Dow (1920), Rapach and Zhou (2013)) and a very important topic in empirical finance.

The literature on this topic is very extensive and extremely difficult to embrace. Its majority is unanimous on the fact that ERP are predictable but is rather weak on the consensus about where does come from the forecasting power of the equity risk premium. Several studies in the last decades, have asked whether ERP can be predictable by macroeconomic variables such as dividend-price ratio, interest rate spread, consumption wealth ratio while other researchers focus on technical indicators to test this predictability. These predictors have been tested both in-sample (IS) and out-of-sample exercises (OOS) (e.g., Goyal and Welch 2008), although they have a very poor performance in out-of-sample exercises. Moreover, the extensive research on this topic is mostly dedicated to the United States (US) market.

In this paper we focus our analysis in the out-of-sample predictability of the equity risk premium given that in order to effectively predict ERP in real-time the out-of-sample exercise is the most suitable one.

We focus on one specific predictor: the spread between long and short governmental interest rates (term spread). We consider this variable very attractive once it is closely related with the business cycle[1] and it is very easy to compute using public available data. Furthermore, researchers as Campbell (1987) and Fama and French (1989), state that the term spread has predictability power over equity risk premiums, although the forecasting power of the term spread performs rather poorly when tested out-of-sample[2], as it is the case of many other variables. Being in line with that, we are particularly motivated by the findings of Faria and Verona (2018) which found out that the low frequency (long-term dynamics) of the term spread is a strong and robust predictor of the equity risk premium in out-of-sample exercises for the United States (U.S).

The major novelty of this thesis is that we extend this analysis beyond the U.S market to seven additional markets: Germany, France, Japan, United Kingdom, Canada, South Africa and Australia. This has not been addressed in the literature so far and we foresee it as highly relevant for both local and internationally diversified equity investors.

In line with that, the main research question is: "Is the original time series or any frequency component of the term spread a good out-of-sample predictor of the equity risk premium in international countries?".

We closely follow the method used in Faria and Verona (2018) to test out-of-sample forecasting power of different frequency components of term spread over equity risk premiums. Using a frequency domain approach allow us to extract hidden information from the predictor that can be highly important to forecast

[1] As it is showed by Wheelock and Wohar (2009)
[2] See Goyal and Welch (2008)

ERP. Particularly, Ferreira and Santa Clara (2011) found that different frequency components of some indicators (earnings growth, dividend-price ratio) capture different frequencies of ERP. Similarly, Bandi, Perron, Tamoni, and Tebaldi (2018) and Faria and Verona (2018) succeeded on improving ERP predictability by having in account frequency components dependence between ERP and its predictors. The different frequency components of the term spread are computed using a Maximum Overlap Discrete Wavelet Transform method and are tested to forecast horizons of 1,3,6,12 and 24 months.

In this paper was found that the original time series and the low frequency component of the term spread have significant forecasting power on Germany, France, United States of America and Canada.

The dissertation is divided as follows: In Chapter 1 we provide a review on related stands of literature, while placing our contribution. In Chapter 2 is explained the data and the methodology. In Chapter 3 are presented the main results as well as a robustness analysis.

Chapter 1
Literature Review

Forecasting the equity risk premium has been in the mind of a lot of researchers in the last decades. Despite its extreme importance in Finance and the fact that many economists have tried to forecast ERP using different methods and predictors, the truth is that the predictability of this variable continues to motivate a lot of active research.

1.1 Forecasting Equity Risk Premium

Since the very beginning of the XX century, (e.g. Dow (1920)), the predictability of the equity risk premium has been an active research topic. This happens because the equity risk premium is a very important indicator for equity investors. The ERP tell us on an ex-ante basis how much additional return the investors demand as a premium for taking additional risks carried from stock ownership. Indeed, when we buy a stock its actual price is known, but investors do not know what expected returns are being priced on it. The equity risk premium offers an idea of how much is worth (or not) to invest on a stock against the alternative of investing in risk-free instruments.

As early as 1984, Rozeff (1984) explored the relationship between ERP and dividend yields, pointing out that "Stock market returns are not a random walk and that the current dividend yield provides a clue to future return predictability." According to his studies, as dividend yield increases, the stock market returns tends to move in the same direction.

Fama and French (1989) studied the relationship between expected returns on bonds and stocks. According to their research, the expected excess returns on stocks move together with expected excess returns on corporate bonds, which is in line with the fact that dividend yields, which are commonly used to forecast stock returns, are found to be also good predictors to forecast bond returns. Moreover, Fama and French (1989) share findings that associate other predictors to business cycles conditions. They specifically pointed out that the major movements in dividend yields and default spreads are connected to long term business conditions, whereas the term spread is associated to shorter business cycles. The term spread exhibits lower values across business-cycle peaks and higher values around troughs.

Martin Lettau and Sydney Ludvigson (2001) debated "whether expected returns vary at cyclical frequencies and with macroeconomic variables" arguing that returns are predictable because they represent a rational response of economic agents to economic conditions driven by risk aversion and by time-varying investment opportunities, against the hypothesis of inefficient markets. Following this idea and assuming that markets are efficient, it is reasonable to believe that macroeconomic variables occupy an important position concerning the prediction of stock returns.

Many more studies consider these and other macroeconomic variables to predict ERP, including nominal interest rate and interest rate spread (Keim and Stambaugh 1986, Campbell 1987, Fama and French 1989), inflation rate (Nelson 1976, Fama and Schwert 1977, Campbell and Vuolteenaho 2004), consumption wealth ratio (Lettau and Ludvigson 2001), dividend yield (Rozeff 1984, Campbell and Shiller 1988, Fama and French 1988, Campbell and Yogo 2006,), price-earnings ratio (Campbell and Shiller 1988, …), among others.

Nevertheless, the predictors of the equity risk premium considered in the literature are not limited to macroeconomic variables. Some researchers proposed the inclusion of technical indicators.

As stated by Pring (2002): "The technical approach to investment is essentially a reflection of the idea that prices move in trends that are determined by the changing attitudes of investors toward a variety of economic, monetary, political, and psychological forces." Research on technical analysis in this topic has received less attention, although it is possible to identify the study of trading strategies based on some technical indicators, for example, Fama and Blume (1966) proposed filter rules, Conrad and Kaul (1998) introduced momentum and Brock, Lakonishok and LeBaron (1992) presented moving averages.

Neely, Rapach, Tu and Zhou (2014) analyzed the capability of technical indicators to directly predict equity risk premium and compared their performance with macroeconomic variables' performance. Their findings support the existence of predictability power of both types of predictors, although they capture different information about the ERP. They argue that macroeconomic variables perform better in detecting increases in the equity risk premium near business cycle troughs and technical analysis is dominant when it comes to predicting declines in the equity risk premium near business cycle peaks, sustaining the inclusion of both indicators to better forecast stock returns.

The fact is that macroeconomic variables and technical indicators are considered by part of the literature to have predictive power over the equity risk premium. Following Neely, Rapach, Tu and Zhou (2014) the economic explanation is based on asset pricing models: the changes on macroeconomic trends are tracked by macroeconomic variables and the time varying expected stock returns are mostly driven by the future state of the economy. Concerning technical indicators, the economic explanation is not so clear; four models exist to explain their predictive power and they require market inefficiency.

This dissertation reassess the equity risk premium predictability of the term spread by considering its different frequencies components. We start by running an in-sample analysis and then we move to an out-of-sample exercise which is the principal analysis.

Recently, on this continuous search for a good predictor of the ERP, Faria and Verona (2018) found that in the U.S market, the low frequency of the term spread performs very well on forecasting equity risk premiums and better than that, the predictor has greater levels of predictability both in-sample and out-of-sample exercises. We are particularly surprised by the performance of the term spread on their research and motivated by this conjecture we study the out-of-sample forecasting power of the term spread and its frequency components over ERP in international markets. The term spread represents the difference in interest rates between two sovereign bonds with different maturities. Usually this difference is the difference between long-term and short-term governmental interest rates, and it is a proxy for the slope of the yield curve.

A great deal of effort has been made in the last century in order to test expectation models about the term structure of interest rates, which states that the slope of the yield curve (difference between long-term and short-term rates) reflects the market forecast about the changes in interest rates. When the forward interest rate equals the expected future spot rate and thus the expectation hypothesis holds, it is reasonable to use the yield curve to anticipate market expectations concerning future states of the economy. Under this hypothesis, low current short-term spot rates and high long-term rates are a sign of future economic growth.

However, the expectations theory about the term structure of interest rates has been rejected by some researchers, such as Campbell and Shiller (1991).

In 1987, Campbell showed that the variables which have been used in the expectations model as proxies for risk premium on twenty-year treasury bonds also predict excess stock returns, which is in line with the findings of Fama and French (1989): expected excess returns on corporate bonds and stocks move together.

Regarding the out-of-sample approach which is the main analysis of this thesis, we consider the most reasonable approach to use once it uses information available until the moment of forecasting to forecast, being that way a real-time forecasting.

Goyal and Welch (2008), instead of keep looking for another predictor or another method which could improve the results of forecasting ERP exercise, decided to test on in-sample and out-of-sample exercises all variables considered as good predictors of ERP in the literature. Their findings state that most of the variables perform poorly in out-of-sample exercises and as they say: "OOS, most models not only fail to beat the unconditional benchmark (the prevailing mean) in a statistically or economically significant manner but underperform it outright." Later, Faria and Verona (2018) found out that the low-frequency of the term spread is a good predictor of ERP in out-of-sample exercises. This is very relevant because the indicators performing well out-of-sample are almost inexistent which is the reason why this thesis focus on this predictor, following Faria and Verona (2018) to international markets. In the next subsection is provided a very brief summary of the authors that used the out-of-sample approach to financial purposes

1.1.1 Out-of-Sample Forecast

A considerable number of recent papers use the out-of-sample approach to test the predictive performance of equity risk premiums, using variables such as book-to-market ratios, dividend-to-earnings, consumption-to-wealth ratios and term spread. Lettau and Ludvigson (2001), Campbell and Thomson (2007), and Rapach et. al, (2005) are some examples of authors that make use of these models.

Goyal and Welch (2003, 2004) argue that the variables purposed as predictors of future ERP do not provide predictive gains on in-sample exercises. They stated that "Our paper has systematically investigated the empirical real-world out-of-sample performance of plain linear regressions to predict the equity premium. We find that none of the popular variables has worked – and not only post-1990 … Our profession has yet to find a variable that has had meaningful robust empirical equity premium forecasting power, at least from the perspective of real-world investor." Authors such as Campbell and Thomson (2007) disagree with Goyal and Welch's opinion, arguing that by implementing some restrictions when constructing out-of-sample forecasts, strong evidence emerges in regard to the out-of-sample predictive power in excess stock returns.

1.1.2 International Evidence

The literature about the prediction of the equity risk premium is extensive, although it is mostly dominated by the U.S market. This paper is a contribution for the study of equity risk premium predictability in international markets, i.e., beyond U.S. market.

There are a few papers studying the predictability of equity risk premium in international markets. Asprem (1989) explored the relationship between stock indexes, asset portfolios and macroeconomic variables in ten European countries. The findings of this paper state that inflation, imports, interest rates and employment are inversely related to stock indexes. Campbell and Hamao (1992) explored the U.S and Japan markets and found that variables such as dividend-price ratio and interest rates help to forecast excess monthly returns in both countries.

More recently, Schemeling (2009) explored how consumer confidence affects expected stock returns in 18 industrialized countries, and concluded that when the sentiment is low, future stock returns tend to be higher and when an investor sentiment is high the future stock returns tend to be low. McMillan and Wohar (2011) used sum of parts modeling method to study the predictability of ERP. They found that in Italy, UK, USA and Korea this approach outperforms the alternative models. Later, Kumar Narayan, Seema Narayan and Thuraisamy (2014) tested the predictability of excess stock return and found evidence of in-sample predictability for 15 countries. They used a mean variance investor framework and argued that investors, in most of these emerging markets countries, can make significant profits if they adopt dynamic trading strategies. Moreover, Jordan, Vivian and Wohar (2014) stated that "macro and technical indicators can (statistically) improve forecast accuracy and generate gains to investors; in contrast to the U.S. results, predictability in our sample of European countries exists in recent data."

1.2. Frequency Domain Analysis and Wavelet Methods

Financial analysis is mainly based on time series[3] methods, which can track the movements of any variable that changes over time.

The most popular approach used to study time series is time domain analysis, which represents the analysis of the signals displayed by the times series with respect to time. Moreover, the time domain analysis allows the study of the temporal properties of a given economic variable whose records occur at one determined frequency. The problematic issue here is whenever the occurrences of the variable are not visible in just one frequency (which would be the original time-series), but they occur in several different frequencies (which only frequency domain approach is able to identify). Whenever this happens, the time domain approach is not able to correctly process all the information contained in

[3] A time series is a set of data points that are recorded at specific moments in time.

the time series. Multiscale features is an important item of financial time series which translates the several structures observed in the time series, each one occurring in a different time scale. At this point, it is important to mention the definition of frequency domain analysis as a complementary tool for time domain analysis, so as to surpass the issues that may occur with time domain as described earlier. In opposition to time domain, frequency domain focuses on the analysis of mathematical functions in respect to frequency rather than time and investigates the significance of the different frequency levels on the behavior of the variable. Put simply, the time domain illustrates the extent at which the signal varies over time, whereas the frequency domain represents the extent at which a signal resides within each certain frequency band over an interval of frequencies.

Wavelet analysis takes into account both approaches, as it has the capacity to decompose the time series in a group of sub-time series each one associated to a given time scale. Wavelet methods offer a different view for the researcher, working as a zoom tool on details and offer a larger picture of the features of the series. As pointed by Ranta (2010): "…with wavelet methods we are able to see both the forest and the trees." They are regarded as very attractive since they allow us to break down economic activity in different frequency components and to study them separately. In other words, wavelets methods can be employed to study an indicator's time evolution which depends on the interface of a mix of different frequencies components.

The Fourier transform is a traditional approach in the frequency domain analysis and possesses the capacity to show how much each frequency exists in the signal. However, it does not have the capacity to identify the moment in time when these frequencies exist because it uses constant length windows. This implies a great probability that the fixed time windows contain a large number of high frequencies cycles and a small number of low frequencies, preventing an adequate examination for all frequencies. On the other hand, the wavelet transform supports windows of different size, improving the time resolution of the high frequencies and the frequency resolution of low frequencies. The reason for that is that high frequencies are better located in time, whereas the low frequencies are better located in frequency. Another big disadvantage of the Fourier transform is that it requires the time series to be stationary[4], while wavelets work well when it comes to non-stationary data. This fact is particularly relevant because numerous economic and financial time series are scarcely stationary. These methods allow gathering information about a phase (expansion or regression) or the length of a cycle (e.g. business cycle).

In recent decades, wavelet methods have become more popular in a considerable number of areas, such as geophysics, medicine and engineering.

In finance, some examples using wavelet's methods are: Capobianco (2004) that uses wavelet methods to study Nikkei stock index data and argues that it matches perfectly on the analysis of financial data, Crowly and Lee (2005) which applied wavelet multiresolution analysis to analyze different frequency components of European business cycles and Gençay et al. (2001a) investigating the scaling properties of foreign exchange rates.

[4] A stationary time series means that its joint probability distribution does not change over time, that is, its mean and variance are constant over time.

Chapter 2
Data and Methodology

The aim of this thesis is to analyze the out-of-sample predictability power of the equity risk premium by the term spread and of its different frequencies components in international markets.

A detailed description of the data and the method used will be presented in the following two sub-sections.

2.1 Data

The sample set covers eight countries: Germany, France, Japan, United States of America, United Kingdom, Canada, South Africa and Australia. We use monthly observations from March 1973 to August 2018 and data was gathered from DataStream database, Organization for Economic Cooperation and Development (OECD), Federal Reserve Economic Data (FRED) and International Monetary Fund (IMF). A more detailed description of the variables and their source is provided below.

- **Equity Risk Premium (ERP):** This variable is calculated as the log return on the country's stock index minus the log return on a one-month governmental bond of the corresponded country as follows:

$$ERP_{i,t} = \log(1 + sr_{i,t}) - \log(1 + tr_{i,t}) \quad (1)$$

where $ERP_{i,t}$ corresponds to the equity risk premium of the country i in the month t, the $sr_{i,t}$ corresponds to the stock return of the country i in the month t, and the $tr_{i,t}$ corresponds to one-month government bond return rate of the country i in the month t.

- **Stock Returns ($sr_{i,t}$):** The stock returns are calculated according to the following formula: $sr_{i,t} = \left(\frac{S_{i,t}}{S_{i,t-1}}\right) - 1$, where $S_{i,t}$ represents the Stock Index of the country i in the month t.

- **Stock Index ($S_{i,t}$):** The stock index is from Thomson DataStream, Global Equity Index, which is adjusted for dividends and stock splits. The codes[5] on the database are TOTMK** and the ** denote the code for each country.

- **Return on one-month government bond rate ($tr_{i,t}$):** Like in stock returns, the return on the one-month rate is calculated as follows: $tr_{it} = \left(\frac{rf1_{i,t}}{rf1_{i,t-1}}\right) - 1$, where the $rf1_{i,t}$ is the one-month government bon rate of the country i in the month t.

- **One-month government bond rate ($rf1_{i,t}$):** The one-month rate is calculated as follows: $rf1_{i,t} = \left[\left(1 + rf3_{i,t}\right)^{1/12} - 1\right]$[6], where $rf3_{i,t}$ represents the three-months rate. This variable had to be estimated due to lack of data.

[5] Germany: TOTMKBD; France: TOTMKFR; Japan: TOTMKJP; U.S: TOTMKUS; U.K: TOTMKUK; Canada: TOTMKCN; South Africa: TOTMKSA; Australia: TOTMKAU

[6] As in, Thomadakis (2016)

- **Three-months government bond rate ($rf3_{i,t}$):** The three-months government bond rate corresponds to the governmental rate on three-month bonds of the correspondent country and is gathered from Organization for Economic Cooperation and Development (OECD) and International Monetary Fund (IMF). The reason why two different databases were used[7] in this variable was the range of the period, for which records for the chosen countries did not exist for the entire period in just one database.

- **Term Spread ($TMS_{i,t}$):** This variable is calculated as the difference between long-term and short-term governmental rates of the different countries. The formula is: $TMS_{i.t} = rf10_{i,t} - rf3_{i,t}$, representing the $rf10_{i,t}$ the ten-years rate on governmental bonds, the long-term government bond.

- **Ten-years government bond rate ($rf10_{i,t}$):** The data for this variable was obtained on Federal Reserve Bank of St. Louis, Federal Reserve Economic Data section and is related to the rate on ten-years governmental bonds of the country in question.

[7] However, to reinforce the validity of the data, correlations between the records on common periods for each country were carried out, which presented high levels of correlation (about 98%).

2.2 Method

This paper extends Faria and Verona (2018) to international countries. We therefore use exactly the same forecasting method. As in Faria and Verona (2018), the forecasting power of ERP from the original time series of term spread is studied, as well as three different components of the latter.

The three frequencies of the term spread computed are: High frequency expressed as TMS_{HF}, Business Cycle frequency denoted by TMS_{BCF} and Low frequency symbolized by TMS_{LF}. We used a J=5 level MODWT MRA[8], with a Haar filter reflecting boundary conditions and a J=6 level MODWT MRA analysis to compare and complement the study as a robustness analysis. This is possible because the sample set covers the necessary number of observations to run a J=6 level, as the number of observations limits the level of J[9]. An explanation of these methods is provided in the next sub-section 2.2.1..

2.2.1 Wavelet Methods

The term wavelet means small wave and, as mentioned earlier, the wavelet method has the capacity to decompose an original signal into several sub-series, each one occurring at a different frequency. In the following explanation, in this sub-section, we closely follow Ramsey (2002), Masset (2008), Ranta (2010) and Rua (2011).

There are two types of wavelet transform: The Continuous Wavelet Transform (CWT) and the Discrete Wavelet Transform (DWT). The continuous wavelet transform quantifies the variation in a signal at a given frequency and at a particular point in time. The wavelets are created from a single basic wavelet, the

[8] Maximum Overlap Discrete Wavelet Transform, Multiresolution Analysis

[9] Regarding the choice of J, the number of observations decides the maximum number of frequency bands that are possible to use. In this case, the in-sample period has N=202 observations, so J is such that J $\leq \log_2 N \simeq 7{,}7$.

mother wavelet, which captures high frequency details of the time series which is translated and scaled as:

$$\psi_{us}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right) \tag{2}$$

where u translates the location, s the scale and the term $\frac{1}{\sqrt{s}}$ guarantees that the norm of $\psi_{us}(t)$ equals 1.

To access the features of a signal on a large scale, or a low frequency component, the value of s should be large, whereas the characteristics of a series on a small scale, or a high frequency component, are reached by small values of s.

On the other hand, the Discrete Wavelet Transform differs from the Continuous Wavelet Transform in the sense that it uses limited number of translated and dilated combinations of the mother wavelet to decompose. Here, u and s are chosen in a way that minimizes the number of wavelet coefficients to summarize the signal. This objective is achieved by verifying the following conditions:

$$s = 2^j \quad \text{and} \quad u = k2^j \qquad (3)(4)$$

where k and j are integers that represent the set of discrete dilations and translations. This implies that the Discrete Wavelet Transform of the original form is calculated at dyadic scales, that is, at scales of 2^j. This condition implies that for a time-series with N observations, the bigger number of scales used on the DWT equals the Integer J, such that, $J \leq \frac{\log(T)}{\log(2)}$.

As mentioned before, the discrete wavelet transform decomposes a times series into its constituent multiresolution components. The approximation of the orthogonal wavelet series to a time series is well-defined by the following equation:

$$y(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\,\psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t) \qquad (5)$$

where J represents the number of scales (or multiresolution levels) and k varies from one to the number of coefficients of the corresponding component, defining the length of the filter. Moreover, the terms $s_{J,k}$, $d_{J,k}$, $d_{J-1,k}$, … and $d_{1,k}$ on equation (5) represent the wavelet transform coefficients, which provide a measure of the influence of the given wavelet function to the signal, and are given by:

$$S_{J,k} = \int y(t)\phi_{J,k}(t)\,dt, \qquad (6)$$

$$d_{j,k} = \int y(t)\psi_{j,k}(t)\,dt, \qquad j = 1,2,3,\ldots,J \qquad (7)$$

which are a function of $\phi_{J,k}$ and $\psi_{j,k}$, the father and the mother wavelet respectively. The father wavelet, or scaling function, works like a low-pass filter, capturing the smooth and the low frequency component of the series. On the other hand, the mother wavelet captures the detail and the high frequency components. The mother wavelet ($\psi_{j,k}$) and the father wavelet ($\phi_{J,k}$) are given by the following expressions:

$$\phi_{J.k}(t) = 2^{-J/2}\phi\left(\frac{t-2^J k}{2^J}\right), \qquad (8)$$

$$\psi_{j.k}(t) = 2^{-j/2}\psi\left(\frac{t-2^j k}{2^j}\right), \qquad j = 1,2,3,\ldots,J \qquad (9)$$

Assuming that $S_J(t) = \sum_k s_{J,k}\phi_{J,k}(t)$, $D_J(t) = \sum_k d_{J,k}\psi_{J,k}(t)$, $D_{J-1}(t) = \sum_k d_{J-1,k}\,\psi_{J-1,k}(t)$, and $D_1(t) = \sum_k d_{1,k}\psi_{1,k}(t)$, it is possible to state that equation (5) can be rewritten as:

$$y(t) = S_J(t) + D_J(t) + D_{J-1}(t) + \cdots + D_1(t) \tag{10}$$

The previous expression represents the decomposition of the series y(t) into orthogonal components, $S_J(t)$, the smooth component and $D_j(t)$ for $j = 1,2,2, \dots,$ J, the detail components, at distinctive resolutions constituting the multiresolution analysis decomposition. Given the J multiresolution components, the signal y(t) will have J detail components and a smooth component. The detail components track the high frequency features of y(t) whereas the smooth component captures the low frequencies characteristics of y(t). [10]

When it comes to the levels of frequency, a high J that is compressed by a wavelet function captures slowly changing features, that is, low frequencies, whereas a small J compacted by a wavelet function catches fast changing details, or in other words, high frequencies.

2.2.1.1. Maximal Overlap Discrete Wavelet Transform Multiresolution Analysis (MODWT MRA)

The maximal overlap discrete wavelet transform (MODWT) Multiresolution Analysis (MRA) was the wavelet method used to compute these frequency components. This method was created as a solution to overpass some Discrete Wavelet Transform (DWT) limitations such as (Masset (2008)):

- It requires a dyadic length series ($T = 2^J$)
- Discrete Wavelet Transform troughs or peaks in the original time-series may not be appropriately aligned with similar events in the multiresolution analysis.

[10] Further details about the wavelet decomposition methods can be found in Ramsey (2002), Masset (2008), Vaasa (2010) and Rua (2011).

- Discrete Wavelet Transform is not shift invariant, which means that if the series is shifted one period to the right, the multiresolution coefficients will not be equal.

This method of wavelet transform supports any sample size, is invariant to translation, offers a higher resolution at greater scales and is more efficient in terms of wavelet variance.

The main difference between the two methods is related to the fact that in MODWT we consider every integer translations, i.e., u=k (and not and $u = k2^j$ as in DWT). In other words, this means that the Maximal Overlap Discrete Wavelet Transform achieves a complete resolution of the time series at each different frequency. Moreover, no matter what wavelet scale is considered, the length of the original time-series will always be equal to the length of the wavelet and scaling coefficients.

Considering a signal *s(n)* of length N where $N = 2^J$ for some integer J, a low-pass filter, $h_1(n)$, and a high-pass filter, $g_1(n)$, defined by an orthogonal wavelet, at the first level of MODWT, $h_1(n)$ and $g_1(n)$ are applied to *s(n)* in order to obtain detail and approximation coefficients. $g_1(n)$ is used to obtain detail components, $d_1(n)$, and $h_1(n)$ is used to obtain approximation components, $a_1(n)$, as represented in the following expressions:

$$a_1(n) = h_1(n) * s(n) = \sum_k h_1(n-k)\, s\,(k) \tag{11}$$

$$d_1(n) = g_1(n) * s(n) = \sum_k g_1(n-k)\, s\,(k) \tag{12}$$

If the time-series is not subsampled $a_1(n)$ and $d_1(n)$ have the length N and not N/2 as would be the case of Discrete Wavelet Transform. However, as long as the level of MODWT increases, $a_1(n)$ is filtered using the same system but with different filters: $h_2(n)$ and $g_2(n)$, which are obtained by a dyadic subsampling $h_1(n)$ and $g_1(n)$. This process continues for j=1,2,3,…, J-1 as illustrated in the following figure:

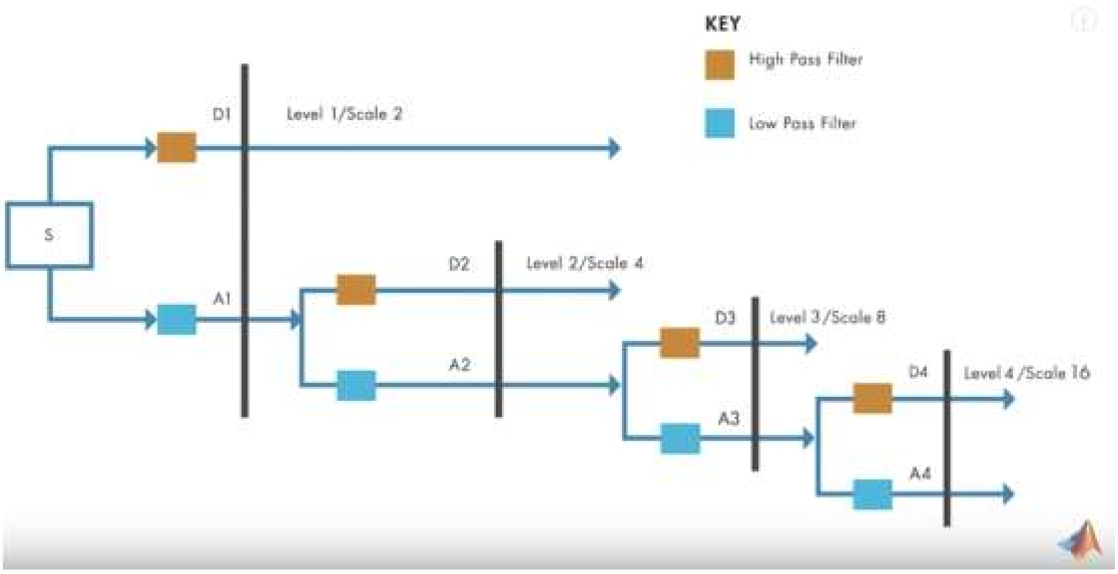

Figure 1: Decomposition of Maximum Overlap Discrete Wavelet Transform (Source: Matlab Tech Talks: Kirthi Devleker)

As mentioned previously, this dissertation tests a J=5 level MODWT MRA to compute the frequency components of the Term Spread. In addition, a J=6 level MODWT MRA is also computed and tested as a robustness analysis. As so, and taking into account that monthly observations are used, the detail and smooth components capture oscillations between:

	J=5	J=6
$TMS_t^{D_1}$	2 and 4 months	2 and 4 months
$TMS_t^{D_2}$	4 and 8 months	4 and 8 months
$TMS_t^{D_3}$	8 and 16 months	8 and 16 months
$TMS_t^{D_4}$	16 and 32 months	16 and 32 months
$TMS_t^{D_5}$	32 and 64 months	32 and 64 months
$TMS_t^{D_6}$	Nonexistent	64-128 months
$TMS_t^{S_5}$	Exceeding 64 months	Nonexistent
$TMS_t^{S_6}$	Nonexistent	Exceeding 128 months

Table 1: Months captured by the oscillations of detail and smooth components

Thenceforth, in the presence of J=5 level, the high frequency component of the term spread corresponds to $TMS_{HF,t} = \sum_{j=1}^{3} TMS_t^{D_j}$, the business cycle frequency is computed as $TMS_{BCF,t} = \sum_{j=4}^{5} TMS_t^{D_j}$ and the smooth or low frequency component represents $TMS_{LF,t} = TMS_t^{S_5}$. In the case of J=6 level, the high frequency component of the term spread corresponds to $TMS_{HF,t} = \sum_{j=1}^{3} TMS_t^{D_j}$, the business cycle frequency is computed as $TMS_{BCF,t} = \sum_{j=4}^{6} TMS_t^{D_j}$ and the smooth or low frequency component represents $TMS_{LF,t} = TMS_t^{S_6}$. Furthermore, by summing the three different components, the original time series is obtained.

The exercise of summing is possible because the different times-series defined in each individual frequency are orthogonal.

Chapter 3
Empirical Results

In this section we present the set of the main results obtained from our empirical analysis. Regarding the U.S. market the results will also be compared with Faria and Verona (2018). This chapter is divided as follows: Frequency components of the term spread, In-Sample Predictability results and Out-of-Sample predictability results including some Robustness Analysis. Details such as summary statistics, correlations and a graph of the detail ($TMS_t^{D_j}$) and the smooth ($TMS_t^{S_T}$) components are provided in the Appendix.

3.1 Frequency components of the term spread

We consider the original time series of the term spread, calculated as the difference between a long and short term governmental rates, and three of its frequency components (high frequency, business cycle frequency and low frequency) as equity risk premium predictors. As explained in section 2.2, the method used to extract these frequency components is the Maximum Overlap Discrete Wavelet Transform Multiresolution Analysis (MODWT MRA), using J=5 level of details. The next set of figures 2-9 illustrate, for each of the eight countries under analysis, the dynamics of the original term spread time series as well as for its three frequencies under analysis. The blue line represents the original time series of the term spread, the orange line denotes the high frequency component of the term spread, the yellow line represents the business cycle frequency and the purple line reflects the low frequency of the term spread.

Germany

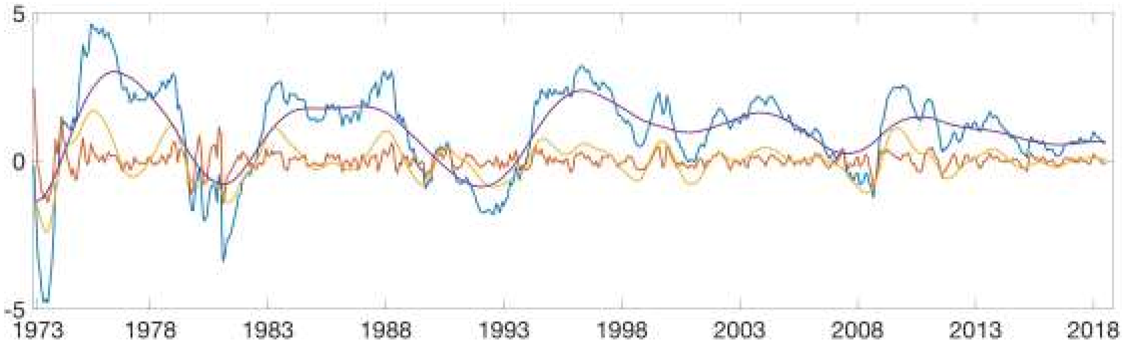

Figure 2: Time series of the term spread and of its different components for Germany

France

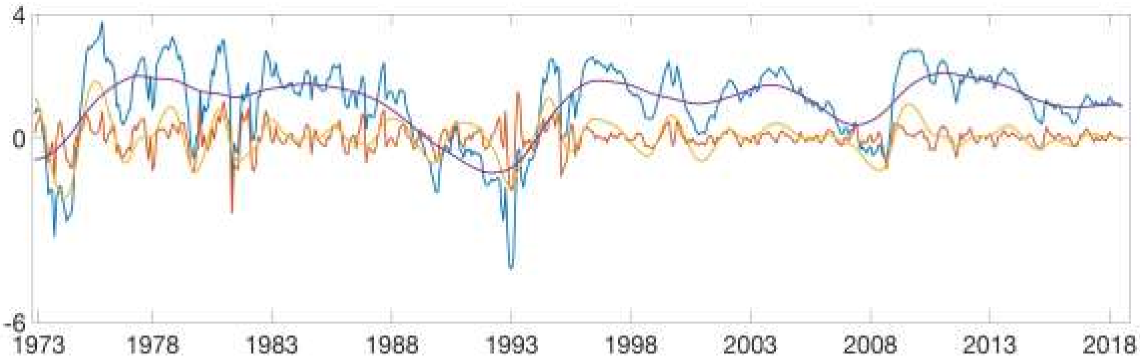

Figure 3: Time series of the term spread and of its different components for France

Japan

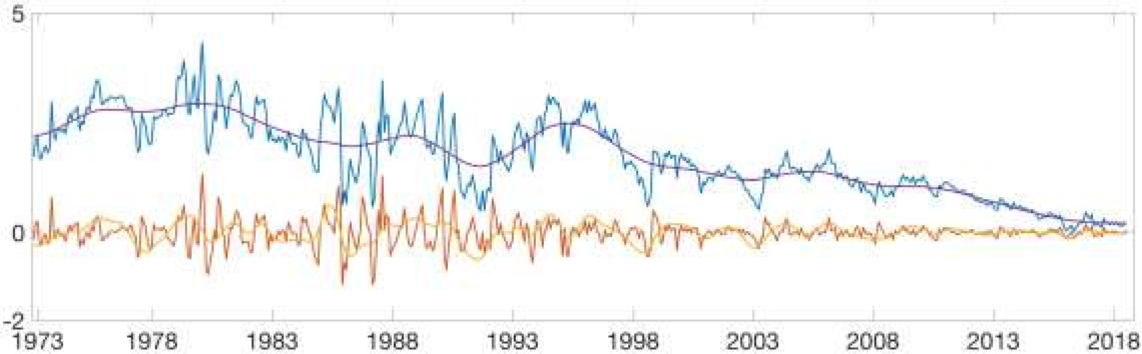

Figure 4: Time series of the term spread and of its different components for Japan

United States of America

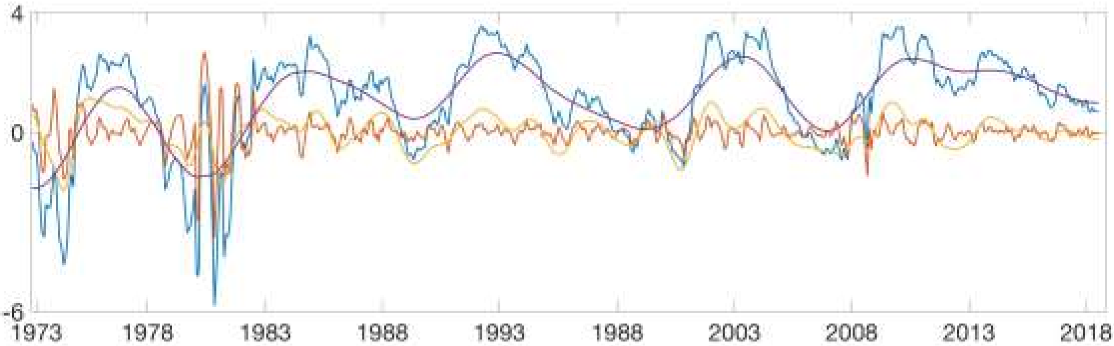

Figure 5: Time series of the term spread and of its different components for U.S.A

United Kingdom

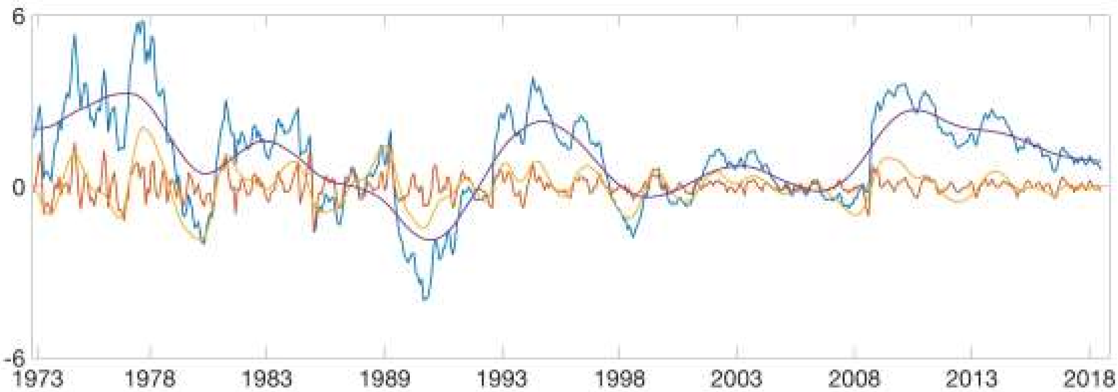

Figure 6: Time series of the term spread and of its different components for U.K

Canada

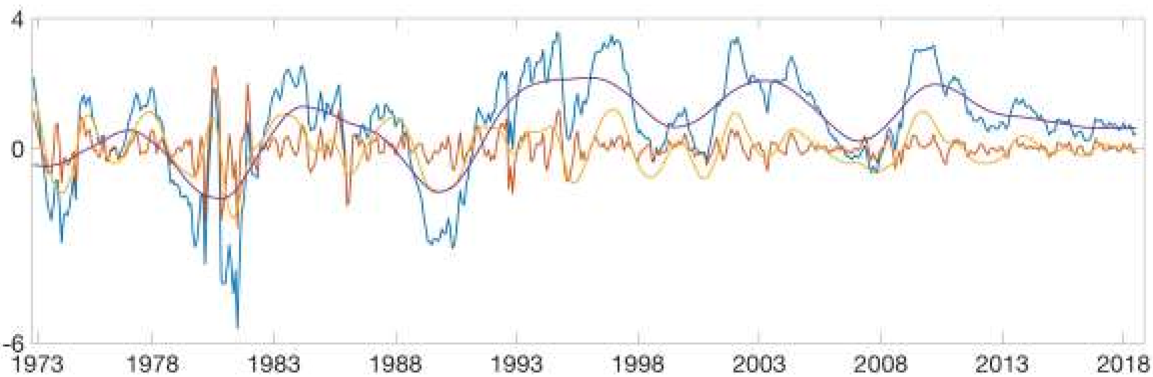

Figure 7: Time series of the term spread and of its different components for Canada

South Africa

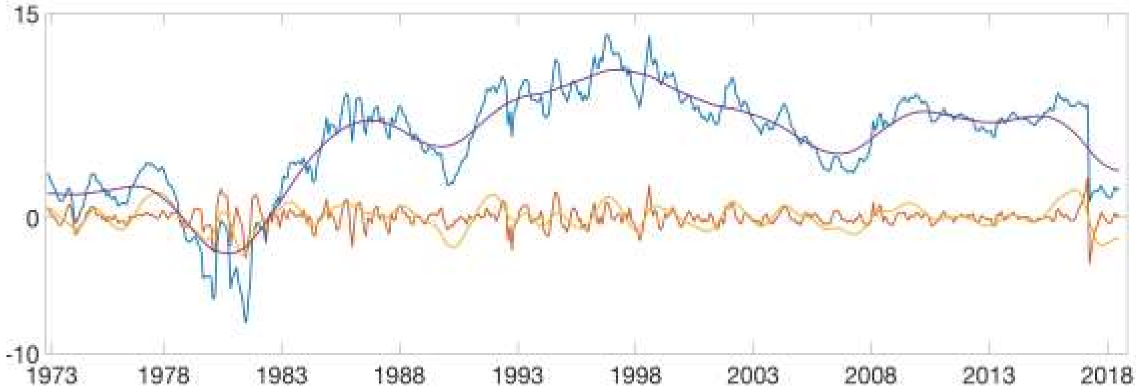

Figure 8: Time series of the term spread and of its different components for South Africa

Australia

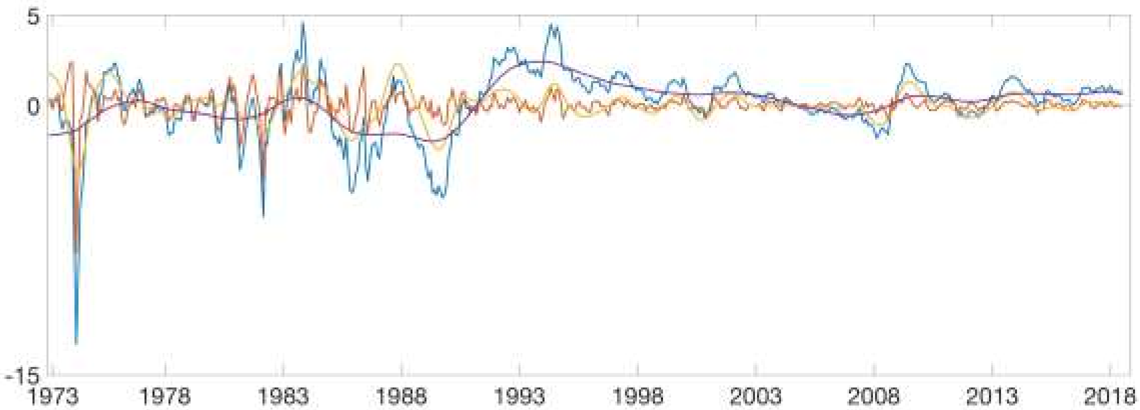

Figure 9: Time series of the term spread and of its different components for Australia

3.2 In-Sample Forecasting

Following Faria and Verona (2018), let $ERP_{i,t}$ represent the equity risk premium of the country i in month t. For each predictor $x_{i,t}$, the predictive regression is:

$$ERP_{i,t:t+h} = \alpha + \beta x_{i,t} + \varepsilon_{i,t:t+h} \qquad \forall t = 1, \dots, T-h \tag{13}$$

where h represents the forecasting horizon and $ERP_{i,t:t+h}$ is also given by:

$$ERP_{i,t:t+h} = \frac{ERP_{i,t+1} + \cdots + ERP_{i,t+h}}{h} \tag{14}$$

In the In-Sample predictability analysis we estimate the predictive regression (13), throughout ordinary least squares (OLS), to obtain the β's and test their significance. Nevertheless, there are few constraints related to Stambaugh (1999) bias and Campbell and Yogo (2006) in the predictive regression (13). In order to avoid this bias, it was used a heteroscedasticity and autocorrelation-robust t-statistic and a wild bootstrapped p-value was calculated to test the null hypothesis $H_0: \beta = 0$ against the hypothesis $H_1 = \beta > 0$ in the predictive regression (13). Additionally, each predictor variable was standardized to have a standard deviation of one.

Therefore, 546 observations exist for one-month ahead (h=1), 544 observations for three months ahead or one quarter forecasting horizon (h=3), 541 observations for six months ahead or one semester ahead (h=6), 535 observations for one year ahead (h=12) and 523 observations for two years ahead (h=24).

In the next subsections are provided the results of each country for the in-sample analysis for each country.

3.2.1 Results

The following tables report, for each forecasting horizon and each country, the β estimates of the predictive model by OLS and the resultant R^2 statistics (given in percentages). Brackets below the β coefficients represent the heteroscedasticity and autocorrelation robust t-statistic for $H_0: \beta = 0$ against $H_1 = \beta > 0$. Moreover, *** denote a significance level of 1%, ** denote a significance level of 5% and * denotes a significance level of 10%, according to bootstrapped p-values. The sample period starts in March 1973 and ends in August 2018.

3.2.1.1 – Germany

Predictor	h = 1		h = 3		h = 6		h = 12		h = 24	
	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2
TMS_{TS}	0,46 [2,27]***	0,78	0,43 [2,51]***	1,89	0,42 [2,56]**	3,27	0,39 [2,47]**	5,67	0,25 [1,87]*	4,97
TMS_{HF}	-0,06 [-0,25]	0,01	-0,20 [-1,07]	0,39	-0,10 [-0,65]	0,19	0,02 [0,22]	0,02	0,01 [0,31]	0,02
TMS_{BCF}	0,48 [2,12]**	0,86	0,47 [2,45]**	2,24	0,43 [2,37]**	3,54	0,38 [2,26]**	5,25	0,24 [1,84]*	4,42
TMS_{LF}	0,43 [2,23]**	0,70	0,45 [2,63]**	2,02	0,41 [2,64]**	3,25	0,37 [2,34]**	5,08	0,24 [1,70]*	4,59

Table 2: In-Sample predictive regression results for Germany

The original time series, the business cycle frequency and the low frequency of the term spread, have predictability power of the equity risk premium in all forecasting horizons, although the levels of significance at which the predictors are significant, are decreasing as the forecasting horizon increases. When it comes to the coefficient of determination (R^2) we can see that they are rather small, although they increase as the forecasting horizon increases. This is in line with the literature.

3.2.1.2 – France

Predictor	h = 1		h = 3		h = 6		h = 12		h = 24	
	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2
TMS_{TS}	0,54 [1,96]**	0,87	0,58 [2,30]**	2,77	0,55 [2,11]**	4,40	0,48 [2,03]*	6,47	0,36 [2,30]*	7,82
TMS_{HF}	-0,22 [-0,75]	0,14	-0,07 [-0,29]	0,04	-0,07 [-0,43]	0,08	-0,01 [-0,10]	0,00	-0,03 [-0,42]	0,04
TMS_{BCF}	0,76 [2,71]***	1,68	0,66 [2,65]**	3,60	0,55 [2,04]**	4,34	0,33 [1,48]	3,02	0,15 [0,93]	1,42
TMS_{LF}	0,42 [1,68]*	0,51	0,47 [2,15]**	1,81	0,51 [2,33]**	3,75	0,53 [2,35]**	7,71	0,47 [2,55]**	13,27

Table 3: In-Sample predictive regression results for France

It is possible to verify the existence of In-Sample predictability power for France of the original time series, the business cycle component and the smooth component, however it is decreasing as the forecasting horizon increases. Again, the coefficient of determination increases as the forecasting horizon increases.

3.2.1.3 – Japan

Predictor	h = 1		h = 3		h = 6		h = 12		h = 24	
	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2
TMS_{TS}	-0,01 [-0,03]	0,00	0,13 [0,64]	0,16	0,21 [0,99]	0,65	0,19 [0,97]	1,06	0,25 [1,48]	3,37
TMS_{HF}	-0,83 [-2,72]	2,40	-0,36 [-2,06]	1,14	-0,08 [-0,64]	0,11	-0,11 [-1,51]	0,37	-0,01 [-0,21]	0,00
TMS_{BCF}	0,01 [0,06]	0,00	-0,05 [-0,25]	0,02	-0,11 [-0,47]	0,19	-0,11 [-0,45]	0,36	-0,10 [-0,74]	0,66
TMS_{LF}	0,29 [1,40]*	0,29	0,30 [1,46]	0,80	0,30 [1,41]	1,40	0,30 [1,45]	2,58	0,35 [1,80]	5,97

Table 4: In-Sample predictive regression results for Japan

For Japan there is no evidence of in-sample predictability power of the equity risk premium from the term spread or any of its frequencies (exception is the low frequency at one-month horizon).

3.2.1.4 – United States of America

Predictor	h = 1		h = 3		h = 6		h = 12		h = 24	
	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2
TMS_{TS}	0,48 [2,36]***	1,11	0,35 [1,85]*	1,75	0,26 [1,50]	1,78	0,25 [1,64]	3,39	0,19 [1,75]	4,46
TMS_{HF}	0,55 [2,60]***	1,49	0,18 [1,05]	0,47	-0,04 [-0,39]	0,05	0,00 [-0,01]	0,00	-0,02 [-0,63]	0,04
TMS_{BCF}	0,40 [1,86]**	0,78	0,35 [1,76]*	1,77	0,27 [1,43]	1,96	0,19 [1,25]	1,86	0,10 [0,75]	1,16
TMS_{LF}	0,25 [1,33]	0,30	0,25 [1,53]*	0,91	0,26 [1,57]	1,85	0,28 [1,75]*	4,18	0,25 [2,41]*	7,26

Table 5: In-Sample predictive regression results for U.S.

This table demonstrates one more country where it is possible to observe the significance of the predictors, yet in a different way. For the United States of America, we can observe the presence of significant predictors mostly for forecasting horizons of one month. The original times and the high frequency component are significant at 1% level, whereas the business cycle frequency and the low frequency component, though significant, are relevant at minor levels of significance. Again, the coefficients of determination are increasing as the forecasting horizon increases.

3.2.1.5 – United Kingdom

Predictor	h = 1		h = 3		h = 6		h = 12		h = 24	
	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2
TMS_{TS}	0,03 [0,10]	0,00	0,12 [0,48]	0,15	0,17 [0,81]	0,55	0,24 [1,55]	2,42	0,21 [1,83]*	5,33
TMS_{HF}	-0,20 [-0,73]	0,14	0,07 [0,24]	0,05	0,07 [0,33]	0,10	0,19 [1,29]*	1,42	0,03 [0,53]	0,08
TMS_{BCF}	0,12 [0,42]	0,05	0,18 [0,74]	0,32	0,25 [1,15]	1,20	0,28 [1,32]	3,35	0,13 [1,27]	2,03
TMS_{LF}	0,03 [0,12]	0,00	0,05 [0,22]	0,03	0,07 [0,36]	0,11	0,12 [0,78]	0,61	0,21 [1,85]*	5,49

Table 6: In-Sample predictive regression results for U.K

The empirical evidence of in-sample predictability in United Kingdom has a similar with that of Japan. There are almost no significant predictors with the exception of forecasting horizons of twelve and twenty-four months. Even considering the significance of these predictors, they are all significant only at 10%. In addition, the $R^2$are increasing as the forecasting horizon increases, but they are slightly lower than the R^2 verified in the other countries.

3.2.1.6 – Canada

Predictor	h = 1		h = 3		h = 6		h = 12		h = 24	
	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2
TMS_{TS}	0,35 [1,82]**	0,63	0,30 [1,73]*	1,25	0,33 [1,98]*	2,81	0,34 [2,18]*	5,88	0,23 [2,93]**	7,23
TMS_{HF}	0,05 [0,27]	0,01	-0,15 [-0,89]	0,31	-0,08 [-0,74]	0,17	0,04 [0,47]	0,10	-0,01 [-0,33]	0,03
TMS_{BCF}	0,45 [2,14]**	1,03	0,46 [2,42]**	2,86	0,46 [2,55]**	5,34	0,33 [2,13]*	5,76	0,12 [1,28]	2,13
TMS_{LF}	0,22 [1,15]	0,25	0,23 [1,41]	0,75	0,25 [1,50]	1,58	0,28 [1,79]*	4,11	0,28 [2,76]**	10,77

Table 7: In-Sample predictive regression results for Canada

For Canada, it is observable that the two significant predictors are the original time series and the business cycle frequency. Furthermore, we can see the significance of lower frequency, for forecasting horizons of 12 and 24 months, of 10% and 5% respectively. The coefficient of determination does not always increase as the forecasting horizon increases, although we can verify that it is rather small when the predictor is not significant.

3.2.1.7 – South Africa

Predictor	h = 1		h = 3		h = 6		h = 12		h = 24	
	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2
TMS_{TS}	0,07 [0,20]	0,01	0,06 [0,20]	0,02	0,05 [0,20]	0,04	0,06 [0,24]	0,12	0,01 [0,04]	0,00
TMS_{HF}	0,29 [0,94]	0,20	0,18 [0,83]	0,23	0,04 [0,20]	0,02	0,12 [1,13]*	0,47	0,01 [0,16]	0,01
TMS_{BCF}	0,15 [0,61]	0,05	0,21 [1,05]	0,30	0,27 [1,31]	0,96	0,30 [1,41]	2,51	0,19 [1,31]	2,13
TMS_{LF}	-0,02 [-0,06]	0,00	-0,03 [-0,08]	0,00	-0,02 [-0,06]	0,00	-0,03 [-0,10]	0,02	-0,04 [-0,20]	0,10

Table 8: In-Sample predictive regression results for South Africa

The only country belonging to the African continent seems to have similar results to Japan and United Kingdom. There is extremely little evidence of in-sample predictability power of the term spread and its frequency components over the equity risk premium.

3.2.1.8 – Australia

Predictor	h = 1		h = 3		h = 6		h = 12		h = 24	
	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2
TMS_{TS}	0,34 [0,87]	0,39	0,32 [0,85]	0,97	0,23 [0,83]	0,96	0,09 [0,52]	0,29	-0,01 [-0,14]	0,01
TMS_{HF}	0,28 [0,52]	0,26	0,25 [0,63]	0,58	0,12 [0,49]	0,25	0,03 [0,37]	0,05	0,00 [0,02]	0,00
TMS_{BCF}	0,36 [0,89]	0,42	0,35 [0,92]	1,14	0,25 [0,86]	1,16	0,04 [0,24]	0,06	-0,11 [-1,33]	1,24
TMS_{LF}	0,10 [0,37]	0,03	0,10 [0,38]	0,09	0,11 [0,48]	0,21	0,10 [0,52]	0,35	0,08 [0,74]	0,57

Table 9: In-Sample predictive regression results for Australia

Obtained results for Australia are similar to those found in the group of Japan, United Kingdom and South Africa. There is no evidence supporting the existence of in-sample predictability power for none of the predictors and for none of the forecasting horizons.

Overall, this set of empirical results support the existence of a subsample of countries (Germany, France, United States of America and Canada) where it is found statistically significant in-sample predictability of the equity risk premium, using both the original time series of the term spread and of its frequencies and for different horizons, while for other of countries (Japan, United Kingdom, South Africa and Australia) there is no evidence of in-sample predictability.

3.3 Out-Of-Sample Forecasting

In recent decades, out-of-Sample predictability has become a relevant method of forecasting financial variables. This method is particularly relevant in financial and economic terms because is closed to real-world prediction once it uses information available until the moment of predictability to predict.

We start with an in-sample period from March 1973 until December 1989 in order to make the first OOS estimate. Afterwards, the sample increases by one observation, as the OOS forecast is generated using a sequence of expanding windows and the process runs until the end of the sample.

The three OOS periods considered are: the full OOS period that starts in January 1990 and ends in August 2018. The next two periods are a division of this full period and run from January 1990 to December 2006 and January 1997 till August 2018.

Following Faria and Verona (2018), the equation that represents the h-step-ahead forecast of equity risk premium is given by:

$$\hat{r}_{t:t+h} = \hat{\alpha}_t + \hat{\beta}_t x_t \tag{15}$$

where $\hat{r}_{t:t+h}$ denotes the h-step-ahead forecast equity risk premium of the month t until month t+h, and $\hat{\alpha}_t$ and $\hat{\beta}_t$ represent the OLS estimators of α and β respectively of month t, using observations from the beginning of the dataset until month t.

As in Faria and Verona (2018), the evaluation of OOS forecasting performance is done throughout Campbell and Thomson (2007) R^2_{OS} statistic. As it is standard in the literature, the historical mean (HM) forecast $\bar{r}_t$, which represents the average returns up to time t, is the benchmark model.

The R^2_{OS} statistic is calculated in the following way:

$$R^2_{OS} = 100\left(1 - \frac{MSFE_{PRED}}{MSFE_{HM}}\right) \tag{16}$$

where $MSFE_{PRED}$ is the mean squared forecast error of the predictive model, while $MSFE_{HM}$ represents the mean squared forecast error of the historical mean given by:

$$MSFE_{PRED} = \sum_{t=t_0}^{T-h} (r_{t:t+h} - \hat{r}_{t:t+h})^2 \tag{17}$$

$$MSFE_{HM} = \sum_{t=t_0}^{T-h} (r_{t:t+h} - \bar{r}_{t:})^2 \tag{18}$$

where $\hat{r}_{t:t+h}$ represents the excess return forecast from the model with each of the alternative predictors.

The R^2_{OS} therefore measure the reduction in the mean squared forecast error from the usage of the predictive model relative to the historical mean.

The predictive model outperforms (underperforms) historical average in terms of MSFE if the R^2_{OS} is positive (negative).

Similarly to Rapach, Ringgenberg, and Zhou (2016) and Faria and Verona (2018) the statistical significance of the results is analyzed using the Clark and West (2007) statistic. Here, the null hypothesis is verified when the MSFE of the historical mean is smaller or equal to the MSFE of the predictive model and the alternative hypothesis is validated when the MSFE of the historical mean is greater than the MSFE of the predictive model (H_0: $R^2_{OS} \leq 0$ against H_A: $R^2_{OS} > 0$).

3.3.1 Analysis of Results

The following tables present R_{OS}^2 (in percentages) for the equity risk premium forecast of the full OOS period (1990:01 – 2018:08) for the same forecasting horizons considered in the in-sample analysis, of each predictor. Moreover, asterisks denote the significance of OOS MSFE-adjusted statistic of Clark and West (2007). *** represent significance at 1%, ** denote significance at 5% and * symbolizes significance at 10%.

3.3.1.1 Germany

Predictor	R_{OS}^2				
	h = 1	h = 3	h = 6	h = 12	h =24
TMS_{TS}	0,76**	1,74**	3,09**	5,54**	4,87*
TMS_{HF}	-0,18	-0,63	-0,70	-0,35	-0,07
TMS_{BCF}	0,02	0,06	0,48	2,25*	3,00*
TMS_{LF}	0,96**	2,24**	3,52*	4,87**	3,00

Table 10: Out-of-Sample R-squares for Germany

By analyzing the results, we can state that the original time series (TMS_{TS}) and the low frequency component (TMS_{LF}) of the term spread outperform the historical mean benchmark (positive and statistically significant R_{OS}^2) for almost all forecast horizons. On the other hand, the high frequency (TMS_{HF}) and the business cycle (TMS_{BCF}) components are poor OOS predictors, as they have a negative or non-significant R_{OS}^2 (excepting for forecasting horizons of 12 and 24 months in the business cycle frequency).

3.3.1.2 France

Predictor	R^2_{OS}				
	h = 1	h = 3	h = 6	h = 12	h =24
TMS_{TS}	0,16	1,25**	2,59**	3,48**	6,85**
TMS_{HF}	0,19	0,12	0,04	-0,56	-2,17
TMS_{BCF}	-0,82	-0,88	-0,64	-0,11	-1,63
TMS_{LF}	0,77*	1,41**	1,37**	-2,78	-8,16

Table 11: Out-of-Sample R-squares for France

France has a very similar behavior to Germany but with smaller R^2_{OS}. Again, the original time series appears to be a good OOS predictor with positive R^2_{OS} and significant for forecasting horizons starting at 3 months. However, the low frequency of the term spread loses value for historical mean benchmark for forecasting horizons superior to twelve months. Nevertheless, TMS_{LF} beats the HM benchmark in forecasting horizons up to 6 months. The high and business cycle frequency of the term spread reveal to be a poor OOS predictor of the term spread, which is surprising given the statistical significance of the business cycle in in-sample analysis.

3.3.1.3 Japan

Predictor	R^2_{OS}				
	h = 1	h = 3	h = 6	h = 12	h =24
TMS_{TS}	-2,33	-3,08	-4,03	-10,04	-13,32
TMS_{HF}	-2,14	-1,38	-0,93	-2,31	-7,62
TMS_{BCF}	-1,04	-3,51	-5,63	-9,82	-8,71
TMS_{LF}	-0,83	-3,28	-6,93	-12,87	-23,23

Table 12: Out-of-Sample R-squares for results for Japan

When it comes to the OOS analysis for Japan, all the predictors appear to have no predictive power in all forecasting horizons. The R^2_{OS} are all negative, which means that the predictive model does not beat the historical mean benchmark.

3.3.1.4 United States of America

Predictor	R^2_{OS}				
	h = 1	h = 3	h = 6	h = 12	h =24
TMS_{TS}	-0,89	-1,64	-1,60	0,22	4,32**
TMS_{HF}	-0,74	-0,72	-0,26	-0,45	0,61
TMS_{BCF}	-2,49	-7,24	-10,21	-7,34	1,07
TMS_{LF}	0,31*	1,01**	2,26**	5,74***	11,21***

Table 13: Out-of-Sample R-squares for for U.S

The United States of America is the first country where we can observe clear evidence of the predictability power of the low frequency component of the term spread on all forecasting horizons.

Here, the TMS_{LF} has a positive and statically significant R^2_{OS} for all forecasting horizons, meaning that the smooth component outperforms the historical mean benchmark. This result matches the results obtained by Faria and Verona (2018) who found that "the low frequency component of the term spread, when extracted using wavelet filtering methods, has remarkably robust empirical equity premium OOS forecasting power."

3.3.1.5 United Kingdom

Predictor	R^2_{OS}				
	h = 1	h = 3	h = 6	h = 12	h =24
TMS_{TS}	-0,33	-0,36	-0,07	0,42	-9,89
TMS_{HF}	0,40	-0,65	-1,02	-4,59	-13,51
TMS_{BCF}	-0,16	-0,56	-1,46	-5,11	-15,51
TMS_{LF}	-0,24	-0,48	-0,40	-0,43	-7,20

Table 14: Out-of-Sample R-squares for U.K

The United Kingdom is similar to Japan, in that there is no evidence of forecasting power of any predictor on none forecasting horizon, as the R^2_{OS} are almost all negative and not significant.

3.3.1.6 Canada

Predictor	R^2_{OS}				
	h = 1	h = 3	h = 6	h = 12	h =24
TMS_{TS}	0,58*	1,12*	1,89*	4,11*	9,79**
TMS_{HF}	-0,01	-0,47	-0,21	0,31	2,19
TMS_{BCF}	-0,70	-2,22	-4,42	-4,79	1,33*
TMS_{LF}	0,55*	1,04*	1,83**	3,46**	11,92**

Table 15: Out-of-Sample R-squares for Canada

Canada joins the group of Germany, France and United States. Once more, Canada reflects an improvement of historical mean benchmark model in the original time series and low frequency component of the term spread. We have the highest R^2_{OS} observed in sample, 11.92, significant at 5%.

3.3.1.7 South Africa

Predictor	R^2_{OS}				
	h = 1	h = 3	h = 6	h = 12	h =24
TMS_{TS}	-1,18	-2,98	-4,61	-14,47	-25,70
TMS_{HF}	-2,93	-1,93	-0,34	-2,19	-2,89
TMS_{BCF}	-1,01	-3,96	-9,66	-30,19	-25,03
TMS_{LF}	-0,59	-2,00	-3,09	-7,81	-20,95

Table 16: Out-of-Sample R-squares for South Africa

The R^2_{OS} for South Africa are almost all negative, meaning that the OOS forecast from the term spread or any of its frequencies do not beat historical mean.

3.3.1.8 Australia

Predictor	R^2_{OS}				
	h = 1	h = 3	h = 6	h = 12	h =24
TMS_{TS}	-1,21	-1,80	-1,97	-2,40	-8,99
TMS_{HF}	-1,69	-2,26	-1,23	-1,21	-7,98
TMS_{BCF}	-0,29	-0,35	-0,08	-2,77	-10,12
TMS_{LF}	-0,84	-2,69	-6,38	-12,32	-57,81

Table 17: Out-of-Sample R-squares for Australia

For Australia, all the R^2_{OS} are negative, alluding to the fact that OOS forecasting power of the term spread and its frequencies is weaker than that of the historical mean benchmark.

3.3.2 – Cumulative sum of squared forecast errors

In order to analyze the consistency over time of the out-of-sample performance of the predictors, it is important to look at the dynamics of the difference between the cumulative square forecasting error for the historical mean forecasting and the cumulative square forecasting error when TMS_{TS}, TMS_{HF}, TMS_{BCF} and TMS_{LF} are used as equity risk premium predictors.

The following figures denote the difference between the cumulative square forecasting error for the historical mean forecasting model and the cumulative square forecasting error based on the predictive regression (15) for TMS_{TS} (original time series of the term spread which is denoted by the black line), TMS_{HF} (high frequency of the term spread denoted by the green line), TMS_{BCF} (business cycle frequency of the term spread denoted by the red line) and TMS_{LF} (low frequency of the term spread denoted by the blue line).

The results should be read as follows: when one of the lines rises (falls), the predictive regression correspondent to that line outperforms (underperforms) the historical mean benchmark. If a given line has a positive slope, then it means that this predictive regression consistently outperforms HM. Moreover, when the end point is above the zero-axis line, the R^2_{OS} is positive.

Germany

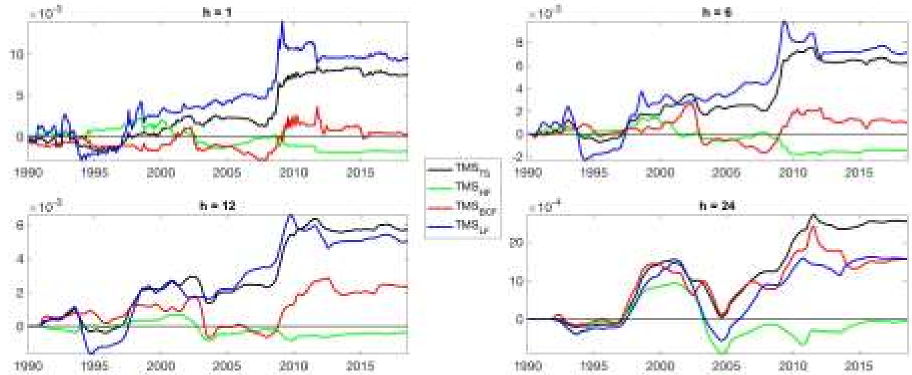

Figure 10: Cumulative sum of squared forecast errors for Germany

France

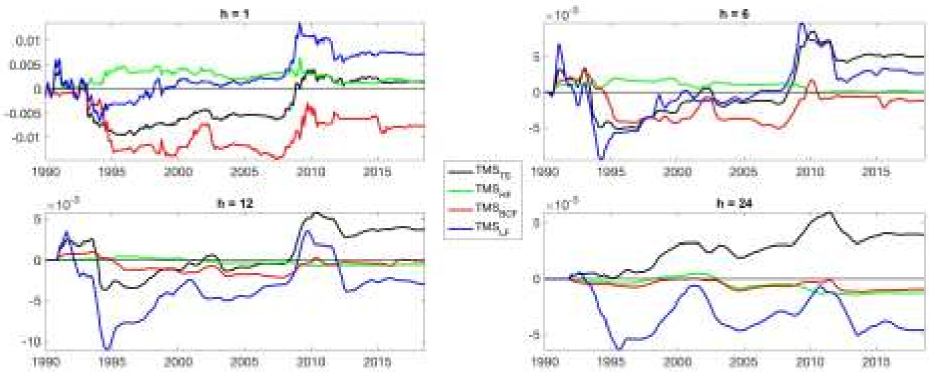

Figure 11: Cumulative sum of squared forecast errors for France

Japan

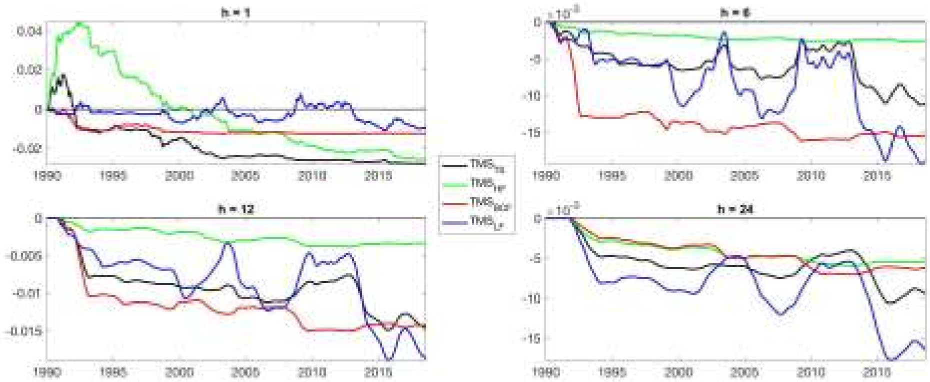

Figure 12: Cumulative sum of squared forecast errors for Japan

United States of America

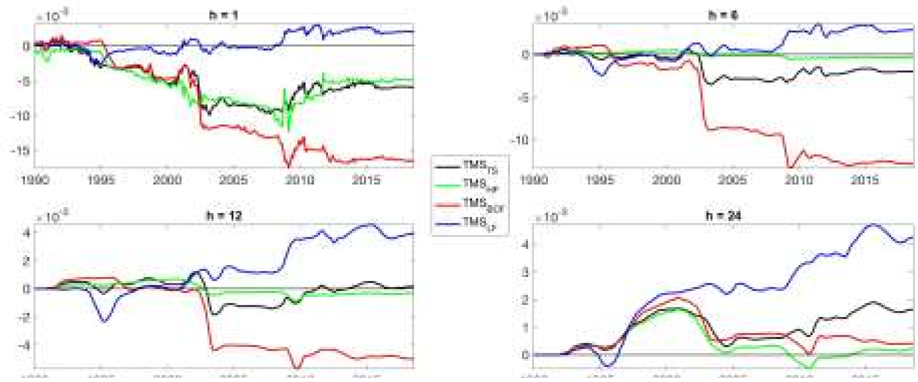

Figure 13: Cumulative sum of squared forecast errors for U.S.

United Kingdom

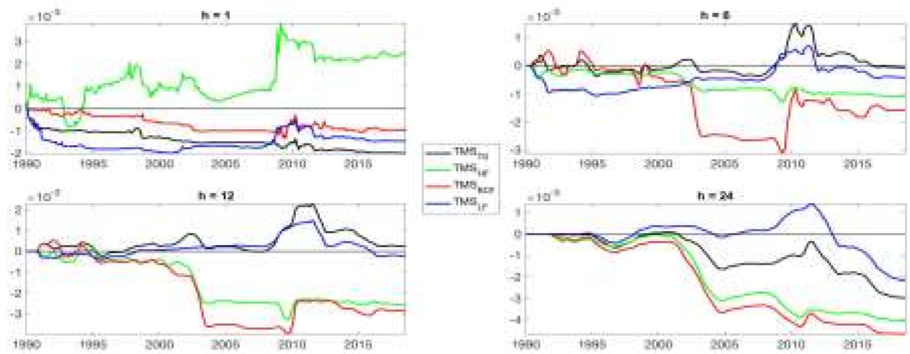

Figure 14: Cumulative sum of squared forecast errors for U.K.

Canada

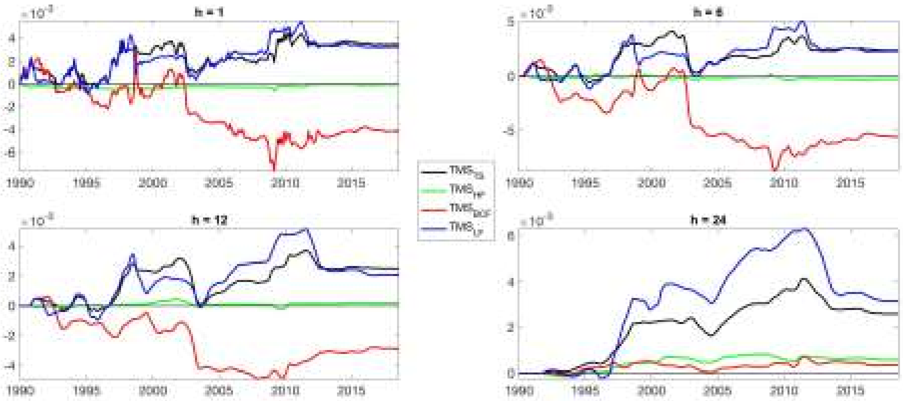

Figure 15: Cumulative sum of squared forecast errors for Canada

South Africa

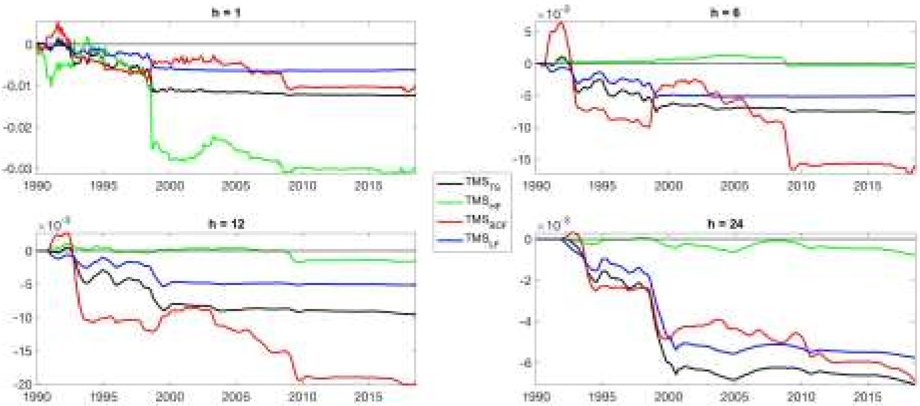

Figure 16: Cumulative sum of squared forecast errors for South Africa

Australia

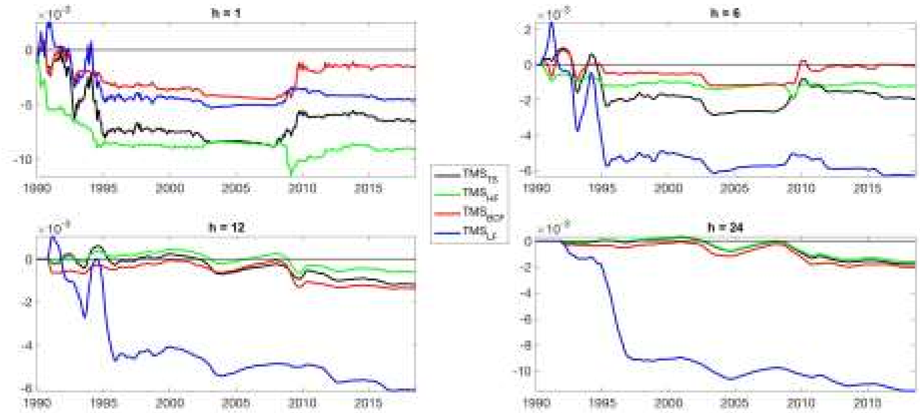

Figure 17: Cumulative sum of squared forecast errors for Australia

From the above graphics, we can see that, in a general way, the predictive regressions outperform the historical mean benchmark in Germany, in France from 2010, in the United States for the low frequency component and in Canada especially for forecasting horizons of 24 months on the original time series and the low frequency component. In what concerns to positive R-squares, they are present in Germany excepting for the high frequency component, in France mainly for forecasting horizons of 1 and 6 months, in the United States principally for the low frequency component and for Canada for forecasting horizons starting at 6 months.

3.4 Robustness Analysis

3.4.1 Different Sample Periods

As a robustness exercise, we studied for the different countries under analysis the predictability power of their equity risk premium of the predictors in different OOS periods. The method is exactly the same as the full OOS period method but now is tested for different subsamples. The full OOS period used in the section 3.3 (1990:01-2018:08) is now divided into two subsamples: the first one runs from January 1990 until December 2006, just before the emergence of the Great Financial Crisis, and the second one runs from January 2007 until August 2018.

The following tables report OOS R-squares (in percentage) of the equity risk premium forecasts at h = 1,3,6,12 and 24 forecasting horizons given by equation (5), for each predictor. Asterisks denote the significance of OOS MSFE-adjusted statistic of Clark and West (2007). *** represent significance at 1%, ** denote significance at 5% and * symbolize significance at 10%.

Germany

Predictor		R^2_{OS}				
		h = 1	h = 3	h = 6	h = 12	h =24
1990-2006	TMS_{TS}	0,32	0,67	1,84	3,51	3,42
	TMS_{HF}	-0,08	0,43	0,00	-0,29	-0,42
	TMS_{BCF}	-0,36	-0,93	-1,07	-0,84	2,24
	TMS_{LF}	0,83*	1,98*	3,60*	5,22*	2,35
2007-2018	TMS_{TS}	1,45**	3,52**	5,09*	9,24*	8,56*
	TMS_{HF}	-0,35	-2,37	-1,83	-0,46	0,81
	TMS_{BCF}	0,63	1,68*	2,97**	7,90**	4,94
	TMS_{LF}	1,16	2,68	3,40	4,23	4,65*

Table 18: Out-of-Sample R-squares of sub-samples for Germany

France

Predictor		R^2_{OS}				
		h = 1	h = 3	h = 6	h = 12	h =24
1990-2006	TMS_{TS}	-0,95	-1,82	-1,30	-0,84	8,58
	TMS_{HF}	0,52	0,63	1,03	-0,58	-1,56
	TMS_{BCF}	-2,27	-3,83	-4,34	-3,51	-1,59
	TMS_{LF}	0,38	0,38*	0,21**	-5,00**	-8,81**
2007-2018	TMS_{TS}	2,07**	6,29**	8,02*	9,45	3,59
	TMS_{HF}	-0,39	-0,73	-1,33	-0,53	-3,31
	TMS_{BCF}	1,66*	3,98**	4,51*	4,58**	-1,71
	TMS_{LF}	1,45	3,10	2,99	0,27	-6,94

Table 19: Out-of-Sample R-squares of sub-samples for France

Japan

Predictor		R^2_{OS}				
		h = 1	h = 3	h = 6	h = 12	h =24
1990-2006	TMS_{TS}	-3,17	-4,08	-4,72	-13,17	-16,23
	TMS_{HF}	-1,59	-1,43	-1,36	-3,14	-11,32
	TMS_{BCF}	-1,57	-5,38	-8,59	-14,09	-11,25
	TMS_{LF}	-0,71	-3,35	-7,69	-14,21	-24,56
1007-2018	TMS_{TS}	-0,90	-1,59	-3,08	-5,76	-8,93
	TMS_{HF}	-3,08	-1,31	-0,33	-1,16	-2,02
	TMS_{BCF}	-0,14	-0,69	-1,53	-3,98	-4,87
	TMS_{LF}	-1,01	-3,17	-5,87	-11,03	-21,20

Table 20: Out-of-Sample R-squares of sub-samples for Japan

United States of America

Predictor		R^2_{OS}				
		h = 1	h = 3	h = 6	h = 12	h =24
1990-2006	TMS_{TS}	-2,87	-5,19	-5,52	-4,07	2,69
	TMS_{HF}	-2,73	-2,87	-0,23	-0,64	0,96
	TMS_{BCF}	-4,05	-11,02	-16,79	-12,62	3,06
	TMS_{LF}	-0,11	-0,03	0,65*	3,24**	10,04***
2007-2018	TMS_{TS}	0,90*	1,71*	1,52	4,73**	7,05***
	TMS_{HF}	1,07	1,31	-0,28	-0,25	0,03
	TMS_{BCF}	-1,07	-3,66	-4,97	-1,81	-2,27
	TMS_{LF}	0,68	1,98*	3,54*	8,36**	13,16***

Table 21: Out-of-Sample R-squares of sub-samples for U.S.A

United Kingdom

Predictor		R^2_{OS}				
		h = 1	h = 3	h = 6	h = 12	h =24
1990-2006	TMS_{TS}	-0,43	-0,65	-0,62	-0,04	-6,79
	TMS_{HF}	0,21	-0,45	-1,44	-8,19	-14,92
	TMS_{BCF}	-0,28	-1,34	-4,62	-12,49	-17,96
	TMS_{LF}	-0,49	-1,05	-0,94	0,77	1,73
2007-2018	TMS_{TS}	-0,18	0,05	0,56	0,94	-15,77
	TMS_{HF}	0,68	-0,95	-0,54	-0,50	-10,83
	TMS_{BCF}	0,02	0,56	2,13	3,31	-10,85
	TMS_{LF}	0,11	0,34	0,21	-1,80	-24,18

Table 22: Out-of-Sample R-squares of sub-samples for U.K

Canada

Predictor		R^2_{OS}				
		h = 1	h = 3	h = 6	h = 12	h =24
1990-2006	TMS_{TS}	0,62*	1,32	2,79	4,45*	18,59***
	TMS_{HF}	-0,08	0,03	-0,06	0,29	4,84**
	TMS_{BCF}	-1,35	-4,27	-10,20	-13,87	1,84
	TMS_{LF}	0,78*	1,88*	3,88**	8,26***	32,29***
2007-2018	TMS_{TS}	0,54	0,88	0,99	3,66	-4,99
	TMS_{HF}	0,08	-1,06	-0,36	0,33	-2,26
	TMS_{BCF}	0,19	0,21	1,36	7,54***	0,49
	TMS_{LF}	0,23	0,05	-0,22	-3,06	-22,30

Table 23: Out-of-Sample R-squares of sub-samples for Canada

South Africa

Predictor		R^2_{OS}				
		h = 1	h = 3	h = 6	h = 12	h =24
1990-2006	TMS_{TS}	-1,48	-3,59	-5,90	-20,79	-39,07
	TMS_{HF}	-3,46	-1,74	0,86*	0,01	-0,60
	TMS_{BCF}	-0,85	-2,85	-7,70	-36,19	-31,01
	TMS_{LF}	-0,81	-2,58	-4,37	-11,50	-32,06
2007-2018	TMS_{TS}	-0,21	-0,89	-1,35	-3,25	-7,15
	TMS_{HF}	-1,21	-2,57	-3,36	-6,10	-6,06
	TMS_{BCF}	-1,54	-7,73	-14,64	-19,54	-16,72
	TMS_{LF}	0,09*	-0,01	0,14	-1,25	-5,53

Table 24: Out-of-Sample R-squares of sub-samples for South Africa

Australia

Predictor		R^2_{OS}				
		h = 1	h = 3	h = 6	h = 12	h =24
1990-2006	TMS_{TS}	-3,16	-6,00	-6,76	-1,32	-2,46
	TMS_{HF}	-3,20	-5,11	-2,94	1,28	-1,73
	TMS_{BCF}	-1,64	-3,56	-2,81	-0,37	-5,45
	TMS_{LF}	-1,86	-5,71	-14,66	-25,76	-139,10
2007-2018	TMS_{TS}	0,78*	1,78	1,16	-3,06	-12,59
	TMS_{HF}	-0,16	0,17	-0,11	-2,75	-11,43
	TMS_{BCF}	1,09*	2,39*	1,70	-4,24	-12,69
	TMS_{LF}	0,19	-0,11	-0,95	-4,04	-13,05

Table 25: Out-of-Sample R-squares of sub-samples for Australia

Germany stills to present most of R^2_{OS} positive in both subsample periods beating the historical mean benchmark on the original time series and low frequency of the term spread. In what concerns to France, the results are slightly different in the sense that the original time series of 1990-2006 period have negative R^2_{OS} for almost all forecasting horizons. Although the results of 2007-2018 match with the full OOS sample results. Moreover, for the low frequency component of the term spread stills presenting positive R^2_{OS}. U.S. presents a similar OOS results to the full sample except for forecasting horizons of 1 and 3 months on the first subsample period. Canada still match full sample OOS results for original time series and low frequency of the term spread. Regarding the countries that did not present forecasting power until this point, the results are not surprising, there is not a pattern that justifies predictability power on none of the predictors used.

3.4.1 J=6 level MODWT MRA

In this subsection, the OOS results are computed using a J=6[11] level MODWT MRA. The sample and the method are exactly the same, except for the fact that the business cycle and the smooth component are computed in a different way.

The following tables report the R_{OS}^2 (in percentage) for the ERP forecasts for the h-month horizon of the full OOS period (1990:01 – 2018:08). The predictors, original time-series and frequency components of the term spread were computed throughout wavelets decomposition using a J=6 MODWT MRA level. Moreover, asterisks denote the significance of OOS MSFE-adjusted statistic of Clark and West (2007). *** represents significance at 1%, ** denotes significance at 5% and * symbolizes significance at 10%.

Germany

Predictor		R_{OS}^2				
		h = 1	h = 3	h = 6	h = 12	h =24
J = 6	TMS_{TS}	0,76**	1,74**	3,09**	5,54**	4,87*
	TMS_{HF}	-0,18	-0,63	-0,70	-0,35	-0,07
	TMS_{BCF}	0,48*	1,38*	2,72**	5,65**	7,14**
	TMS_{LF}	0,38	0,34	0,04	-1,75	-5,14

Table 26: Out-of-Sample R-squares computed with J=6 MODWT MRA for Germany

France

Predictor		R_{OS}^2				
		h = 1	h = 3	h = 6	h = 12	h =24
J = 6	TMS_{TS}	0,16	1,25**	2,59**	3,48**	6,85**
	TMS_{HF}	0,19	0,12	0,04	-0,56	-2,17
	TMS_{BCF}	-0,49	-0,43*	-0,20*	1,36	6,19*
	TMS_{LF}	0,28	0,23	-0,49	-5,56	-18,29

Table 27: Out-of-Sample R-squares computed with J=6 MODWT MRA for France

[11] J has to be chosen such that: $J \leq \log_2 202 \simeq 7{,}7$. So it is possible to run J=6 level MODWT MRA

Japan

Predictor		R^2_{OS}				
		h = 1	h = 3	h = 6	h = 12	h =24
J = 6	TMS_{TS}	-2,33	-3,08	-4,03	-10,04	-13,32
	TMS_{HF}	-2,14	-1,38	-0,93	-2,31	-7,62
	TMS_{BCF}	-0,51	-2,58	-5,28	-10,48	-12,93
	TMS_{LF}	-1,03	-3,57	-6,75	-12,53	-26,40

Table 28: Out-of-Sample R-squares computed with J=6 MODWT MRA for Japan

United States of America

Predictor		R^2_{OS}				
		h = 1	h = 3	h = 6	h = 12	h =24
J = 6	TMS_{TS}	-0,89	-1,64	-1,60	0,22	4,32**
	TMS_{HF}	-0,74	-0,72	-0,26	-0,45	0,61
	TMS_{BCF}	-1,18	-3,43	-4,93	-3,91	1,56
	TMS_{LF}	1,01***	3,10***	5,88***	11,49***	16,07***

Table 29: Out-of-Sample R-squares computed with J=6 MODWT MRA for U.S.

United Kingdom

Predictor		R^2_{OS}				
		h = 1	h = 3	h = 6	h = 12	h =24
J = 6	TMS_{TS}	-0,33	-0,36	-0,07	0,42	-9,89
	TMS_{HF}	0,40	-0,65	-1,02	-4,59	-13,51
	TMS_{BCF}	-0,46	-1,33	-2,93	-7,26	-16,38
	TMS_{LF}	-0,66	-1,82	-2,84	-3,95	-7,59

Table 30: Out-of-Sample R-squares computed with J=6 MODWT MRA for U.K

Canada

Predictor		R^2_{OS}				
		h = 1	h = 3	h = 6	h = 12	h =24
J = 6	TMS_{TS}	0,58**	1,12*	1,89*	4,11*	9,79**
	TMS_{HF}	-0,01	-0,47	-0,21	0,31	2,19
	TMS_{BCF}	-0,32	-1,04	-2,34	-3,09	4,72**
	TMS_{LF}	0,26	0,34	0,90	1,34*	5,58*

Table 31: Out-of-Sample R-squares computed with J=6 MODWT MRA for Canada

South Africa

Predictor		R^2_{OS}				
		h = 1	h = 3	h = 6	h = 12	h =24
J = 6	TMS_{TS}	-1,18	-2,98	-4,61	-14,47	-25,70
	TMS_{HF}	-2,93	-1,93	-0,34	-2,19	-2,89
	TMS_{BCF}	-0,45	-1,63	-3,72	-14,80	-22,77
	TMS_{LF}	-0,68	-2,24	-3,30	-5,99	-11,32

Table 32: Out-of-Sample R-squares computed with J=6 MODWT MRA for South Africa

Australia

Predictor		R^2_{OS}				
		h = 1	h = 3	h = 6	h = 12	h =24
J = 6	TMS_{TS}	-1,21	-1,80	-1,97	-2,40	-8,99
	TMS_{HF}	-1,69	-2,26	-1,23	-1,21	-7,98
	TMS_{BCF}	-0,54	-1,16	-1,70	-2,33	-9,43
	TMS_{LF}	-0,77	-2,21	-4,04	-4,98	-18,04

Table 33: Out-of-Sample R-squares computed with J=6 MODWT MRA for Australia

The conclusions that can be taken from this set of results are very similar to the OOS results computed with a J=5 MODWT MRA. Germany has positive and significant R^2_{OS}, although this significance appears to be stronger on the business cycle-frequency. This can be explained by the fact of the business cycle frequency computed with a J=6 level MODWT MRA capture oscillations between 16 to 128 months whereas the TMS_{BCF} computed with a J=5 level MODWT MRA capture oscillations between 16 to 64 months. France continues to present positive and statically significant R^2_{OS}, although they are more evident on the original time series of the term spread. U.S. present interesting results where is possible to clearly observe the significance of the low frequency of the term spread. This result leads to exactly the same conclusion as Faria and Verona (2018). Canada stills evidencing that the original time series and the low frequency component of the term spread are good predictors with most of R^2_{OS} positive and some of them statically significant. In what concerns to Japan, United Kingdom, South Africa and Australia the results continue to lead to the same conclusion, the R^2_{OS} are almost all negative meaning that the predictors are not good forecasters of the ERP in these countries.

Conclusion

In this thesis was studied the forecasting power of the equity risk premium from the original time-series of the term spread and its different frequency components for eight countries: Germany, France, Japan, U.S., United Kingdom, Canada, South Africa and Australia. To decompose the frequency components of the term spread was used the Maximum Overlap Discrete Wavelet Transform Multiresolution Analysis method and the predictors were tested both in-sample and out-of-sample exercises. In this paper we focus our analysis in the out-of-sample predictability of the equity risk premium and in order to effectively predict ERP in real-time the out-of-sample exercise is the most suitable one.

The major novelty of this thesis is that we extend the equity risk premium analysis beyond the U.S market to seven additional markets: Germany, France, Japan, United Kingdom, Canada, South Africa and Australia. This has not been addressed in the literature so far and we foresee it as highly relevant for both local and internationally diversified equity investors.

We found that the original time-series and the low frequency component of the term spread are a strong and robust out-of-sample equity risk premium predictors for a set of international countries namely: Germany, France, United States of America and Canada. Its out-of-sample forecasting performance is strong for forecasting horizons from one month to two years. Unfortunately, it is not possible to consider the original time series of the term spread or its frequency components good predictors of equity risk premium across all markets studied, although with this research we found out that the equity risk premium and the term spread (and its frequency components) are not independent variables and the analysis of their relation can be very useful for equity risk premium forecasting exercises both in U.S and international markets.

The attractiveness of the term spread is that it is easy to compute from public available data which is crucial for investors with little databases access to use in their asset allocation decisions.

Moreover, in this thesis we show that the term spread can be a good variable to study the prediction of the equity risk premium, not just because of the evidence of its original time-series, but also because of the fact that the frequency domain approach allowed the extraction of the frequency segments that have the highest predictability power. Furthermore, one of the goals of this paper was to compare the U.S results with Faria and Verona (2018) which worked as a robustness analysis of their paper, given that were used different databases and our results support their conclusions.

The principal limitation of this paper was on the searching for data to compute the variables, as it were used different databases to collect the data for the different countries, even though the databases present high levels of correlation (about 98%).

For researchers interested on this topic it is proposed an analysis of the predictability power of the U.S. term spread and its frequency components above the equity risk premium of international countries.

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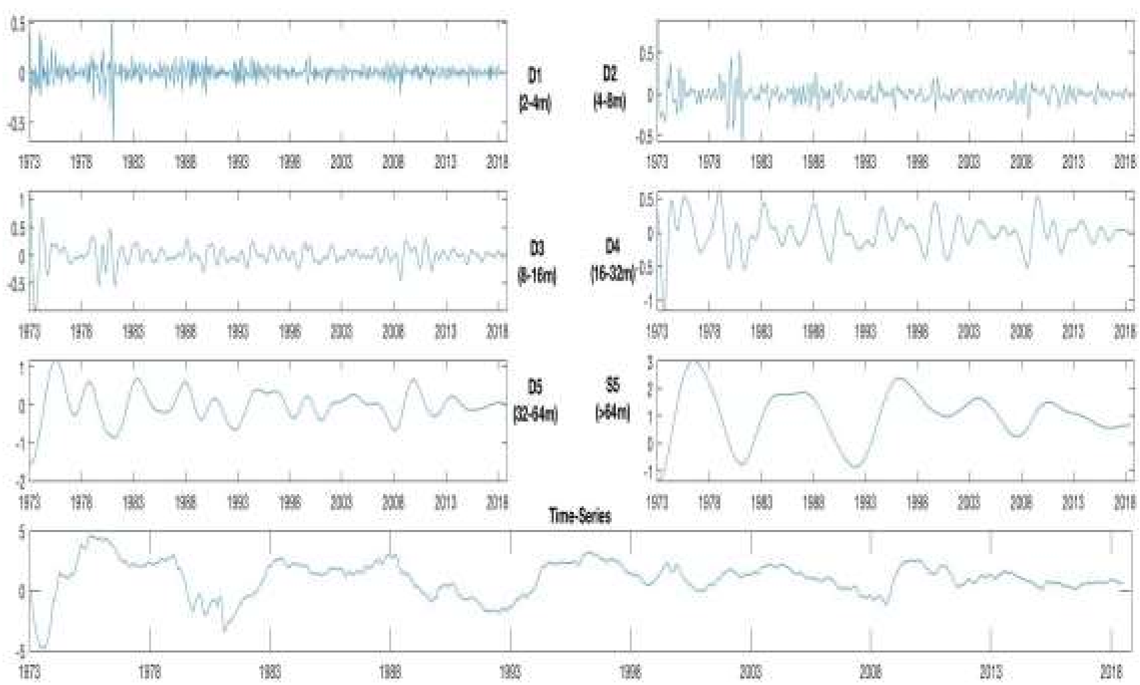
Appendix

Summary Statistics, correlations and detail and smooth component of the term spread for Germany

Variable	Mean	Median	1st percentile	99th percentile	Std. Deviation	Minimum	Maximum
ERP	0,0040	0,0028	-0,1485	0,1283	0,0520	-0,1828	0,7767
TMS_{TS}	1,0092	1,2017	-3,7400	4,3724	1,4990	-4,7900	4,6200
TMS_{HF}	0,0000	0,0062	-1,1119	0,8605	0,3169	-1,4458	2,4628
TMS_{BCF}	0,0000	0,0894	-3,7951	2,7937	1,1236	-4,1735	2,8700
TMS_{LF}	1,0092	1,0478	0,0770	1,6665	0,3998	0,0716	1,6706

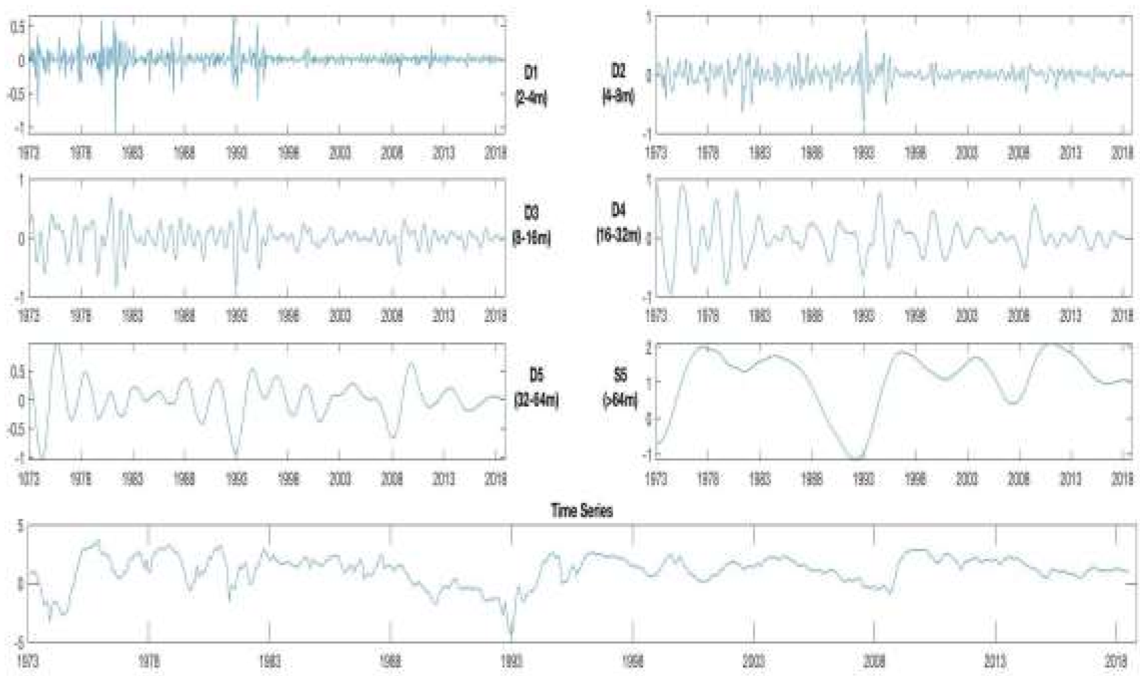
Variable	TMS_TS	TMS_HF	TMS_BCF	TMS_LF
TMS_{TS}	1			
TMS_{HF}	0,4328	1		
TMS_{BCF}	0,9601	0,2740	1	
TMS_{LF}	0,7082	0,0599	0,5722	1



Summary Statistics, correlations and detail and smooth component of the term spread for France

Variable	Mean	Median	1st percentile	99th percentile	Std. Deviation	Minimum	Maximum
ERP	0,0048	0,0041	-0,1452	0,1366	0,0534	-0,1792	0,7677
TMS_{TS}	1,0851	1,3184	-2,7463	3,1500	1,2878	-4,2873	3,7500
TMS_{HF}	0,0000	0,0236	-1,2556	0,8849	0,3950	-2,4533	1,4609
TMS_{BCF}	0,0000	0,1032	-2,3575	1,9400	0,8044	-2,4925	2,1081
TMS_{LF}	1,0851	1,2752	-0,3828	1,6821	0,5628	-0,3878	1,6848

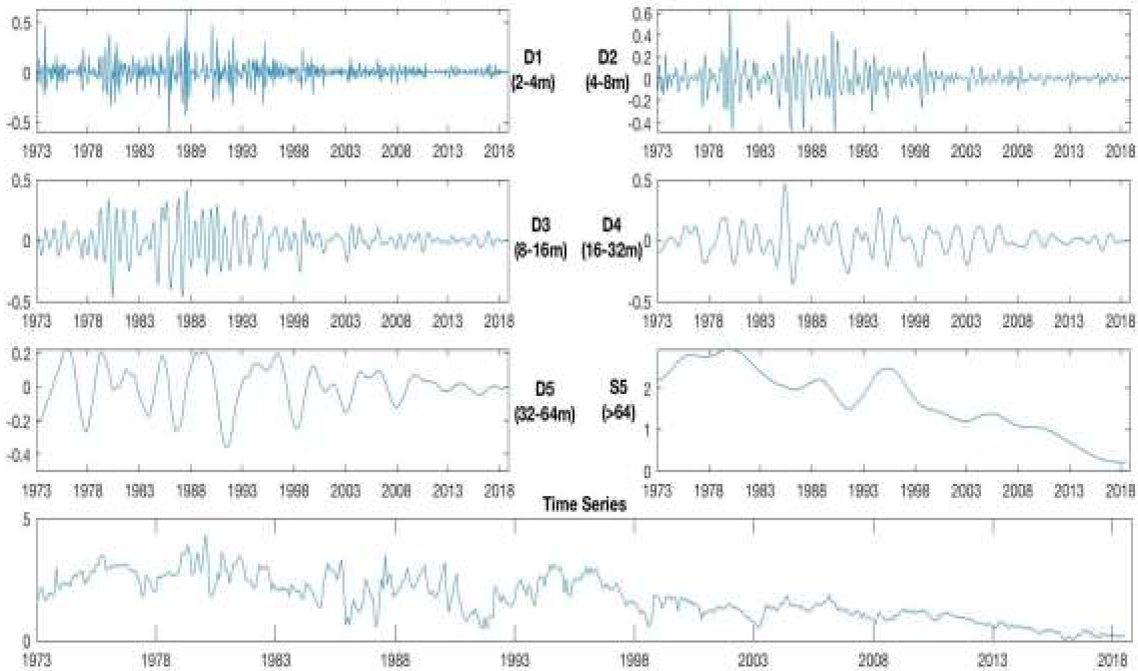
Variable	TMS_TS	TMS_HF	TMS_BCF	TMS_LF
TMS_{TS}	1			
TMS_{HF}	0,5202	1		
TMS_{BCF}	0,8804	0,3281	1	
TMS_{LF}	0,6647	0,0195	0,3548	1



Summary Statistics, correlations and detail and smooth component of the term spread for Japan

Variable	Mean	Median	1st percentile	99th percentile	Std. Deviation	Minimum	Maximum
ERP	0,0052	0,0020	-0,7503	0,6493	0,1941	-1,3254	0,9800
TMS_{TS}	1,7126	1,6340	0,1275	3,5922	0,8984	0,0190	4,3230
TMS_{HF}	0,0000	0,0056	-0,8658	0,8164	0,2819	-1,1872	1,3235
TMS_{BCF}	0,0000	0,0054	-0,9216	0,6697	0,3097	-1,0044	0,7332
TMS_{LF}	1,7126	1,9026	0,2619	2,8159	0,7417	0,2586	2,8176

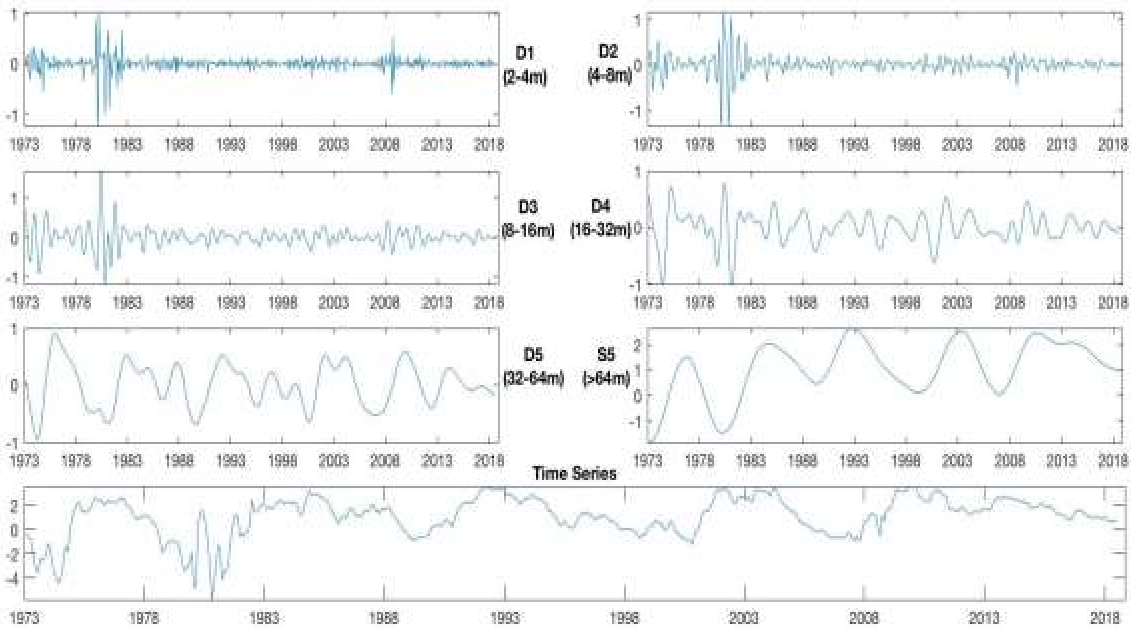
Variable	TMS_TS	TMS_HF	TMS_BCF	TMS_LF
TMS_{TS}	1			
TMS_{HF}	0,3816	1		
TMS_{BCF}	0,4889	0,1890	1	
TMS_{LF}	0,8620	0,0032	0,1028	1



Summary Statistics, correlations and detail and smooth component of the term spread for United States of America

Variable	Mean	Median	1st percentile	99th percentile	Std. Deviation	Minimum	Maximum
ERP	0,0031	0,0028	-0,0889	0,1227	0,0361	-0,2203	0,2131
TMS_{TS}	1,0790	1,3400	-4,2000	3,5000	1,6372	-5,8100	3,5700
TMS_{HF}	0,0000	0,0184	-1,4940	1,5988	0,5057	-3,5283	2,6980
TMS_{BCF}	0,0000	-0,0162	-2,5009	2,1921	1,1124	-2,6775	2,2102
TMS_{LF}	1,0790	1,2840	-0,5040	2,0737	0,6973	-0,5128	2,0755

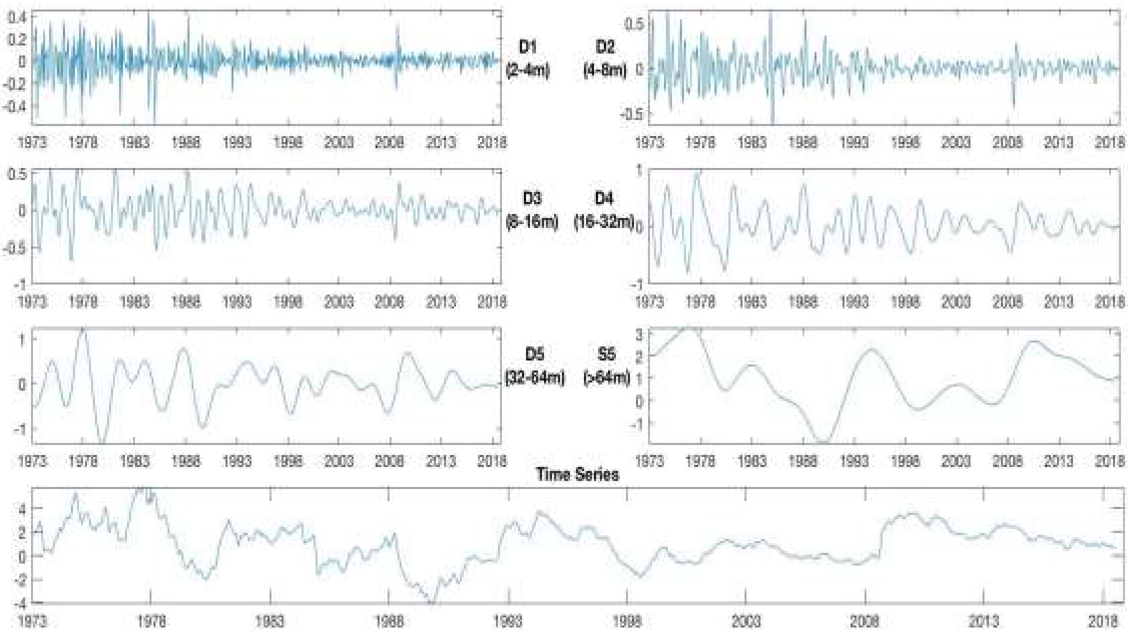
Variable	TMS_TS	TMS_HF	TMS_BCF	TMS_LF
TMS_{TS}	1			
TMS_{HF}	0,4504	1		
TMS_{BCF}	0,8682	0,1964	1	
TMS_{LF}	0,6362	0,0190	0,3008	1



Summary Statistics, correlations and detail and smooth component of the term spread for United Kingdom

Variable	Mean	Median	1st percentile	99th percentile	Std. Deviation	Minimum	Maximum
ERP	0,0037	0,0048	-0,1073	0,1473	0,0638	-0,6297	0,7399
TMS_{TS}	0,9991	1,0956	-3,2461	5,3306	1,6850	-4,0060	5,7837
TMS_{HF}	0,0000	-0,0129	-0,9664	1,1453	0,3692	-1,5369	1,5437
TMS_{BCF}	0,0000	-0,0214	-2,6887	2,5486	1,0495	-2,8233	2,7984
TMS_{LF}	0,9991	1,0324	-0,8457	2,6288	0,9065	-0,8571	2,6366

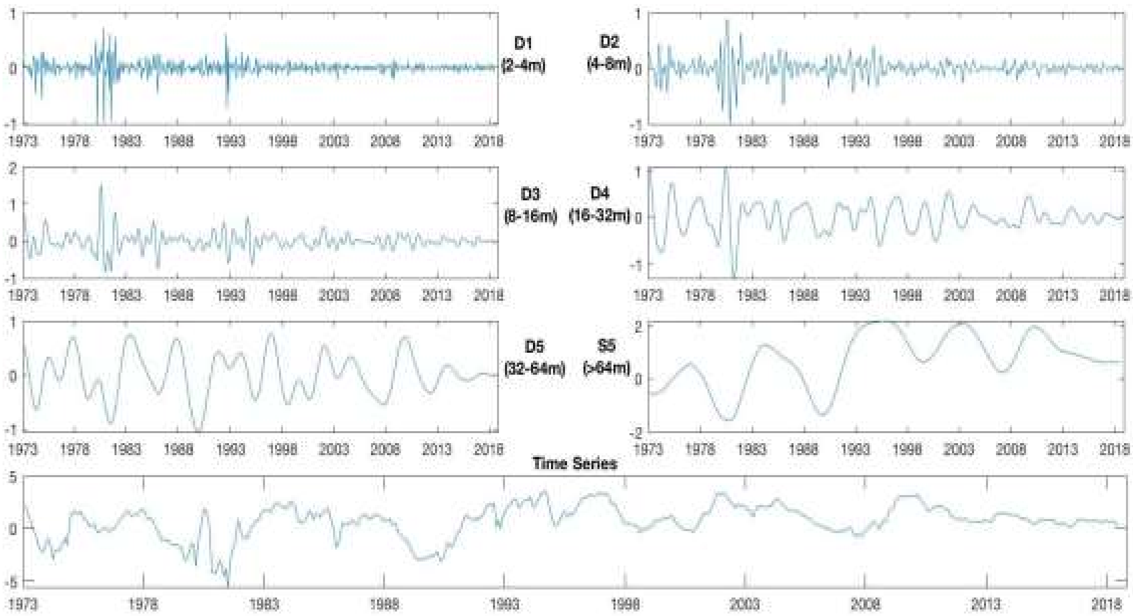
Variable	TMS_TS	TMS_HF	TMS_BCF	TMS_LF
TMS_{TS}	1			
TMS_{HF}	0,4158	1		
TMS_{BCF}	0,8391	0,2966	1	
TMS_{LF}	0,7180	0,0222	0,2813	1



Summary Statistics, correlations and detail and smooth component of the term spread for Canada

Variable	Mean	Median	1st percentile	99th percentile	Std. Deviation	Minimum	Maximum
ERP	0,0028	0,0037	-0,0868	0,0921	0,0279	-0,1067	1,1427
TMS_{TS}	0,6948	0,8207	-3,7824	3,3243	1,5357	-5,5625	3,5750
TMS_{HF}	0,0000	0,0006	-1,5677	1,2385	0,4538	-2,4641	2,5312
TMS_{BCF}	0,0000	0,0480	-2,6583	1,7194	1,0140	-3,0965	1,7690
TMS_{LF}	0,6948	0,8270	-0,5323	1,7755	0,7031	-0,5355	1,7803

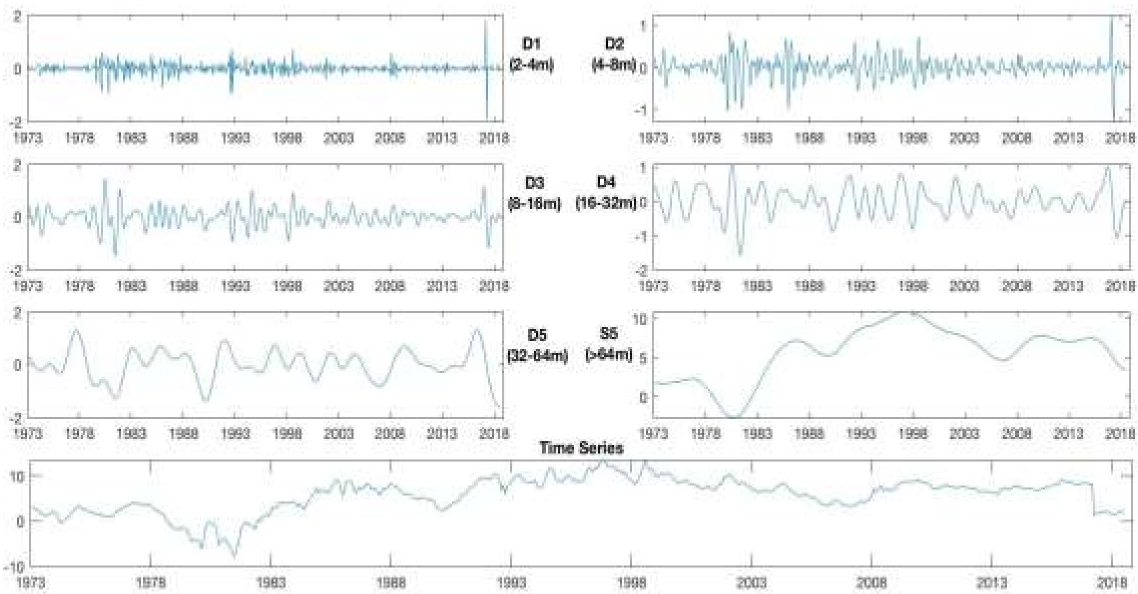
Variable	TMS_TS	TMS_HF	TMS_BCF	TMS_LF
TMS_{TS}	1			
TMS_{HF}	0,4846	1		
TMS_{BCF}	0,8590	0,2745	1	
TMS_{LF}	0,6325	0,0172	0,2568	1



Summary Statistics, correlations and detail and smooth component of the term spread for South Africa

Variable	Mean	Median	1st percentile	99th percentile	Std. Deviation	Minimum	Maximum
ERP	0,0044	0,0069	-0,1032	0,1237	0,0501	-0,7060	0,2468
TMS_{TS}	5,6934	6,8805	-5,7424	12,1876	3,8302	-7,7200	12,4900
TMS_{HF}	0,0000	0,0204	-2,2588	1,7673	0,6717	-3,4152	2,8856
TMS_{BCF}	0,0000	0,2607	-4,3922	2,7975	1,5666	-4,9808	2,9189
TMS_{LF}	5,6934	6,3668	-0,4722	10,0917	3,0252	-0,4907	10,0983

Variable	TMS_TS	TMS_HF	TMS_BCF	TMS_LF
TMS_{TS}	1			
TMS_{HF}	0,2786	1		
TMS_{BCF}	0,6230	0,2310	1	
TMS_{LF}	0,8816	0,0111	0,2196	1



Summary Statistics, correlations and detail and smooth component of the term spread for Australia

Variable	Mean	Median	1st percentile	99th percentile	Std. Deviation	Minimum	Maximum
ERP	0,0028	0,0043	-0,0676	0,0687	0,0273	-0,2010	0,0932
TMS_{TS}	0,1231	0,3300	-5,1640	3,9448	1,7879	-13,3000	4,6100
TMS_{HF}	0,0000	0,0121	-1,8589	1,9089	0,7067	-8,2186	2,4077
TMS_{BCF}	0,0000	0,0014	-3,4178	2,3792	1,1030	-4,1849	2,7105
TMS_{LF}	0,1231	0,0903	-1,2544	1,7498	0,7750	-1,2606	1,7578

Variable	TMS_TS	TMS_HF	TMS_BCF	TMS_LF
TMS_{TS}	1			
TMS_{HF}	0,5714	1		
TMS_{BCF}	0,8358	0,2747	1	
TMS_{LF}	0,5965	0,0153	0,2544	1