Access Regulation under Asymmetric Information about Demand*

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Abstract

We study the impact of access regulation in a telecommunications market on an entrant's decision whether to invest in a network or ask for access when the regulator cannot observe its potential demand. Since the entrant has incentives to not compete vigorously right after entry in order to convince the regulator that it needs cheap access in the future, the regulator must set access prices which tend to be distorted (lower or higher) as compared to first best. Still, this is better than committing to ignore *ex post* demand information. Consulting the entrant earlier about its expectations improves welfare and may help to achieve the first best.

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1 Introduction

Telecommunications markets are often characterized by the presence of an operator, the previously state-owned firm, that enjoys significant market power due to high entry barriers. These result mainly from the large investments in terms of money and time that the building of a new network requires, which makes it difficult for a new operator to enter the market as a full facility-based competitor, at least in the short run.

To solve this problem, regulatory authorities may require the incumbent to make parts of its network available to firms that want to enter the market but have no capacity to build a complete network. Operators can then enter as service-based competitors by using the incumbent's network to supply their services, paying an access price in return.

However, in the long run, the regulators' main objective is to have facility-based competition since it increases the incentives for product innovation and cost reduction. Moreover, only when firms have their own network can competition be sustainable without the intervention of a regulator.

The objectives of creating competition in the short run and having facility-based competition in the long run were traditionally seen as contradictory. Only recently, the European Commission started claiming that there is no trade-off between these two types of competition, based on the "Investment Ladder Theory", see Cave and Vogelsang (2003). According to this theory, service-based competition can lead to facility-based competition if it is managed correctly. Initially, regulators should encourage entry into retail markets where significant market power was found by setting low access prices. Then, over time, once entrants have consolidated their position in the market and investment costs are lower, regulators should start to increase access prices. Entrants are then expected to respond to these increases by investing in their own network.

Yet, the "Investment Ladder Theory" is difficult to implement due to the asymmetry of information which characterizes telecommunications markets. As Oldale and Padilla (2004) argue, operators generally have superior information about the quality of their products and the reaction of consumers, while regulators can only infer entrants' demand by the observation of their performance after entry. Hence, when regulators observe that entrants are still dependent on cheap access to compete in the market, because demand is low, they continue to set low access prices. However, if entrants are already doing well, with a large client base, regulators can ban access in order to induce them to invest. This gives the wrong incentives to entrants since it penalizes entrants that do well, while protecting the ones that do badly. As a result, given that regulators are not able to observe entrants' potential demand but only their realized demand, entrants may have incentives to shirk, i.e. to target a lower demand by investing less in advertising or by adopting a less aggressive behavior in the market, in order to induce regulators to set lower access prices in the future.

In the following, we argue that this introduces distortions even when regulators take their decision before competition and can commit to it, since the resulting optimal outcome are access prices which depend on demand observations.

There are other motives that may lead entrants to shirk. For instance, Pénard (2000) shows that an entrant invests less in capacity in order not to provoke an aggressive response by the incumbent, practicing what is known in the literature as "judo economics".

This problem of incentives can be explored in the context of signaling games. There are few applications of this type of games to a telecommunications regulation context. Sarmento (2003), for instance, considers a model where the telecommunications incumbent uses its price as a signal about the demand size of potential entrants in order to induce the regulator to allow only a small number of entrants. Contrary to her paper, in our model, the receiver of the signal, the regulator, is the first to move by setting the access prices. Thus, he can use the Revelation Principle, due to Myerson (1979), to construct a direct-revelation mechanism with a truth-telling equilibrium. This is similar to the mechanisms used by regulators in contexts of asymmetric information about costs described in Laffont and Tirole (1993, 1994).

We construct a dynamic framework with two periods of telecommunications competition, two operators (an incumbent and an entrant), and asymmetric information about demand, to analyze the impact of access regulation on the entry strategies of the entrant. Similarly to Bourreau and Dogan (2005), the entrant can compete in the market as a service-based competitor by asking for access to the incumbent's network, or as a facility-based competitor by building an own network. We introduce the asymmetry of information by assuming that the entrant observes the state of demand after entry, which can be either low or high, while the regulator has just prior beliefs about the probability of each state occurring. Yet, the regulator can observe the demand captured by the entrant at the end of the first period.

In case of a low demand state, we assume that it is too expensive for the entrant to invest in an own network, and thus it will always need access to the incumbent's network to be able to compete. On the other hand, when demand is high, the entrant may opt to invest in a new infrastructure after asking for access in the first period. First-best access prices will then involve inducing the entrant to invest in the second period in the case of a high demand state, and promoting service-based competition in both periods in the case of a low demand state.

We first show that welfare is higher if the regulator sets second-period access prices contingent on first-period demand observation which induce the entrant to reveal the demand state. However, this introduces some distortions: When the entrant's profit decreases faster (slower) in the access price for a higher demand, the first-period access price set by the regulator is lower (higher) than the one he would set if the entrant did not have incentives to shirk. Additionally, in order to make the gains from shirking lower, the regulator sets a higher second-period access price contingent on low demand. In an extreme case, if the probability of low demand is sufficiently low, the access price may be such that a low-type entrant leaves the market. The second-period access price contingent on high demand may or may not force a high-type entrant to invest. The latter is optimal when the probability of a high demand state is low enough. Still this is better than committing not to look.

Next, we extend our model to the case where investment is never viable, and show that most of our previous conclusions continue to hold since a high-type entrant still wants lower rather than higher access prices. When investment is viable in both states the regulator simply needs to ban access in the second period, and therefore there are no incentives to shirk.

Finally, we introduce a consultation stage which takes place before the entrant starts competing. In this stage the regulator asks the entrant about its expected demand level. This allows him to set *ex ante* first-period access prices contingent on the answer given by the entrant, and second-period access prices contingent on the concordance of the observed demand with the entrant's answer, so that if the entrant is caught lying it can be penalized. Due to the increase in the regulator's knowledge, this consultation stage improves welfare, and the complete information first best may be possible to achieve, although under restrictive circumstances.

The remainder of the paper is organized as follows. We describe the model in Section 2. In Section 3 we determine the first-best access prices, while in Section 4 we solve the regulator's problem under asymmetric information. In Section 5 we consider the cases where investment is (not) possible in both states. In Section 6 we introduce a consultation stage before competition starts, and in Section 7 we conclude.

2 The Model

We introduce a model with two periods of telecommunications competition and asymmetric information about demand. There are two firms that compete in this market: the incumbent, which is a vertically integrated firm and has a network and a retail business, and the entrant, which only has a retail business. Future profits are discounted by a factor $\delta \leq 1$.

In the first period, the entrant can compete in the market as a service-based competitor by buying access to the incumbent's infrastructure and paying in return an access price which consists on a usage charge r. In the second period, the entrant can opt between competing as a service-based competitor or as a facility-based competitor by investing in an own network. We assume that the entrant can only invest in the second period because it does not have the capacity to enter the market as a facility-based competitor in the short run. This is equivalent to assume that investment cost in the first period is too high for an investment to be viable.

Finally, we introduce a regulator who sets the access prices for both periods in order to maximize social welfare. We assume that the access prices are the only instrument available to the regulator. This corresponds closely to current European practice.

The demand faced by the entrant can be high (H) or low (L) with a probability of α and $1 - \alpha$, respectively. The entrant learns the demand state after entry, and we assume that the sunk cost of entry is sufficiently low as compared to expected profits, so that the it always enters, although it may decide not to compete in one of the periods. On the other hand, the regulator does not observe the demand state. He only has prior beliefs equal to $(\alpha, 1 - \alpha)$ about the probability of each state occurring. This is a realistic assumption because the regulator does not have perfect information about the expected reaction of consumers after the entry of a new firm. Yet, we assume that at the end of the first period the regulator can observe the entrant's captured demand.

The regulator sets the access prices for both periods at the beginning of the game and commits to them until the end. Hence, while he cannot revise the access prices after observing demand in the first period, he can set second-period access prices contingent on the demand captured by the entrant in the first period. He thus sets r_1, r_{2L} and r_{2H}, which are, respectively, the access price for the first period, the access price for the second period if the entrant's demand in the first period is low, and if it is high. If the entrant does not compete in the first period the regulator sets r_2 for the second period.

Given that the regulator does not observe the state of demand, but only the realized demand, the entrant in the high demand state can choose to target low demand in the first period in order to signal to the regulator that it is a low-type entrant. It just needs to make less effort in advertising, or compete less intensively on prices. It can even decide not to compete in the first period if, for the access price set, the low-type entrant best response is to stay out of the market. In both these cases we say that the high-type entrant is shirking (X). This may be profit-maximizing, despite the lower profits in the first period, if it allows the entrant to benefit from a low access price in the second period, as it will be confused with a low-type entrant. The low-type entrant cannot do anything except to capture low demand.

We now introduce the notation we will use for the rest of the paper.

In both periods, if the high-type entrant decides to compete as a service-based competitor and captures high demand its profit is given by $\pi_S^H(r)$ and welfare by $w_S^H(r)$. If, on the other hand, the high-type entrant shirks, these are given, respectively, by $\pi_S^X(r)$ and $w_S^X(r)$. If the high-type entrant decides to compete as a facility-based competitor in the second period, profit and welfare are, respectively, π_F^H and w_F^H.

If the low-type entrant competes as a service-based competitor its profit is $\pi_S^L(r)$ and welfare is $w_S^L(r)$, both in period 1 and 2, while if it competes as a facility-based competitor in the second period they are given, respectively, by π_F^L and w_F^L.

Finally, in monopoly welfare is given by w_M.

Before proceeding, we need to make some assumptions.

Service-based competition

First of all, the access prices set by the regulator can never be such that the incumbent does not want to operate its network since in that case there would not be any firm in the market. Hence, we assume that the access price for a given $i = H, X, L$ cannot be lower than $r_{\min}^i$, which is the limit below which the incumbent's profit is negative.

The entrant's profit during service-based competition is decreasing and differentiable in r, and it is negative for a sufficiently high access price:

$$\frac{d\pi_S^i(r)}{dr} < 0, \ \ \pi_S^i\left(r_{\max}^i\right) = 0 \text{ with } r_{\max}^i > r_{\min}^i, \ \forall i = H, X, L. \tag{1}$$

This implies that an entrant which is not shirking does not ask for access if the access price is above $r_{\max}^i$. Its profit in the first period is then:

$$\Pi_1^i(r) = \begin{cases} \pi_S^i(r) & if \quad r_{\min}^i \leq r \leq r_{\max}^i \\ 0 & if \quad r > r_{\max}^i \end{cases}, \ \forall i = H, L. \tag{2}$$

If the high-type entrant shirks, it does not ask for access for $r > r^L_{\max}$, otherwise it would be detected by the regulator. Thus, its first-period profit is:

$$\Pi^X_1(r) = \begin{cases} \pi^X_S(r) & if \quad r^X_{\min} \leq r \leq r^L_{\max} \\ 0 & if \quad r > r^L_{\max} \end{cases}. \tag{3}$$

Note that $\Pi^X_1(r)$ can be negative if $r^L_{\max} > r^X_{\max}$. Indeed, it can be profit-maximizing for the high-type entrant to incur negative profits in the first period by shirking because it expects to obtain benefits from a lower access price in the second period.

We also assume that, for a given access price, the entrant's profit during service-based competition is higher when it captures high demand as compared with profits when it captures low demand:

$$\pi^H_S(r) > \max\left\{\pi^X_S(r), \pi^L_S(r)\right\}, \ \forall r \in \left[r^H_{\min}, r^H_{\max}\right]. \tag{4}$$

This assumption together with (1) implies that:

$$r^H_{\max} > \max\left\{r^L_{\max}, r^X_{\max}\right\}. \tag{5}$$

Regarding welfare, we assume that during service-based competition, and for a given access price, it is higher when the entrant captures high demand:

$$w^H_S(r) > \max\left\{w^X_S(r), w^L_S(r)\right\}, \ \forall r \in \left[r^H_{\min}, r^H_{\max}\right]. \tag{6}$$

Additionally, we assume that in each state the welfare function is differentiable in r, strictly concave and has a unique maximizer r^*_i for $i = H, L$. We leave open which of r^*_L or r^*_H is larger, but we assume that they are both higher than $r_{\min} \equiv \max\left\{r^H_{\min}, r^L_{\min}\right\}$.

Finally, we assume that, whatever the demand state, welfare is higher with duopoly than with monopoly:

$$w^i_S(r) > w_M \quad \text{for } r \in \left[r^i_{\min}, r^i_{\max}\right], \ \forall i = H, L. \tag{7}$$

Example

We can use as an example which fulfills these assumptions about service-based competition the partial-consumer participation model used in Vareda (2007). In this paper, demand functions are similar to Bowley (1924), and given by:

$$q_I = \frac{a_I - p_I + \theta p_E - \theta a_E}{1 - \theta^2} \tag{8}$$

$$q_E = \frac{a_E - p_E + \theta p_I - \theta a_I}{1 - \theta^2}, \tag{9}$$

where (a_k, q_k, p_k) are the reservation price, the number of subscribers and the subscription price of operator k, with $k = I$ for the incumbent and $k = E$ for the entrant. $\theta \in (0, 1)$ indicates the degree of substitutability between the services offered by the two operators. The incumbent incurs a constant marginal cost c while the entrant pays r per unit to the incumbent. Firms compete in prices (see Appendix A).

The Hotelling model, as in Bourreau and Dogan (2005), does not satisfy our assumptions since the entrant's profit is insensitive to the access price for a large interval of prices. Thus, in their model, one cannot consider many welfare effects.

Facility-based competition

In the second period the entrant can build an own network. This investment costs the entrant I, and for now, until Section 5, we assume that:

$$\pi_F^H \geq I > \pi_F^L. \tag{10}$$

It follows that the low-type entrant will never want to invest in an own network, while the high-type entrant may find it optimal. Indeed, if the regulator sets a sufficiently high access price for the second period, the high-type entrant prefers to invest rather than ask for access.

Lemma 1 *There is a unique* $\overline{r}_H \in \left[r_{\min}^H, r_{\max}^H\right]$ *such that:*

$$\pi_F^H - I \geq \pi_S^H(r), \tag{11}$$

if and only if $r \geq \overline{r}_H$.

Proof. The RHS of (11) is strictly decreasing in r by assumption, while the LHS is constant. Given assumption (10), we know that $\pi_F^H - I \geq \pi_S^H\left(r_{\max}^H\right) = 0$. Thus, there will be at most one $\overline{r}_H \in \left[r_{\min}^H, r_{\max}^H\right]$ such that $\pi_F^H - I = \pi_S^H(\overline{r}_H)$. If $\pi_F^H - I > \pi_S^H\left(r_{\min}^H\right)$, then $\overline{r}_H = r_{\min}^H$. ■

Thus $\overline{r}_H$ is the lowest access price where the entrant prefers to invest in an own network. Below this, service-based competition is the best option for the high-type entrant.

Assuming that in case of indifference the high-type entrant chooses to invest, its profit in the second period is:

$$\Pi_2^H(r) = \begin{cases} \pi_S^H(r) & if \quad r_{\min}^H \leq r < \overline{r}_H \\ \pi_F^H - I & if \quad r \geq \overline{r}_H \end{cases}, \tag{12}$$

while the profit of a low-type entrant is similar to the first-period's profit, i.e. $\Pi_2^L(r) = \Pi_1^L(r)$.

Timing of the game:

i) At moment zero, the entrant enters the market and Nature selects demand size, H or L, with probability α and $1-\alpha$, respectively, with $\alpha \in [0,1]$. The entrant observes Nature's choice.

ii) At the beginning of the first period, the regulator sets the access prices for both periods (r_1, $r_{2,}$ r_{2L} and r_{2H}) given prior beliefs $(\alpha, 1-\alpha)$ about the probability of each state of demand.

iii) The entrant decides if it stays out of the market or if it competes in the first period as a service-based competitor by asking for access at r_1. The high-type entrant can additionally decide between targeting low or high demand.

iv) First-period payoffs are realized. If the entrant competes in the first period, the regulator observes the entrant's captured demand and decides whether r_{2L} or r_{2H} will hold in period 2. If the entrant does not compete, the regulator sets r_2.

v) The entrant chooses between staying out of the market, asking for access at the given access price or building an own network at a cost of I.

vi) Second period payoffs are realized.

3 First-best access prices

3.1 Complete information

In this section we determine the first-best access prices in case of complete information, i.e. when the regulator observes Nature's choice immediately after it takes place.

According to assumption (1), first-period welfare for each demand state if the entrant does not shirk is:

$$W_1^i(r) = \begin{cases} w_S^i(r) & if \quad r_{\min}^i \leq r \leq r_{\max}^i \\ w_M & if \quad r > r_{\max}^i \end{cases}, \ \forall i = H, L. \tag{13}$$

Given our assumption $w_S^i(r_{\max}^i) > w_M$, these welfare functions are not continuous at $r_{\max}^i$, and can be plotted as in Figure 1.

Lemma 2 *When the regulator has complete information about demand, the first-period first-best access price for the high demand state is $r_{1H}^{fb} = r_H^*$, and for the low demand state is $r_{1L}^{fb} = r_L^*$.*

Proof. Given (7) it is always better to have two firms competing rather than one. The result then follows from our assumption about the existence of a unique maximum on each service-based competition welfare function. ■

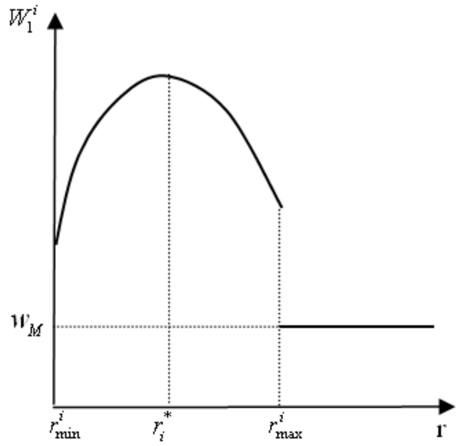

Figure 1

In the second period, welfare in the low demand state is similar to the first period since we assume that the low-type entrant never invests in an own network. Hence:

$$W_2^L(r) = W_1^L(r). \tag{14}$$

As for the high demand state, the entrant invests if the access price is sufficiently high. Thus, the second-period welfare in the high demand state is:

$$W_2^H(r) = \begin{cases} w_S^H(r) & if \quad r_{\min}^H \leq r < \overline{r}_H \\ w_F^H - I & if \quad r \geq \overline{r}_H \end{cases}. \tag{15}$$

According to this, a perfectly informed regulator prefers to have facility-based competition in the high demand state if and only if:

$$w_F^H - I > w_S^H(r), \ \forall r \in \left[r_{\min}^H, \overline{r}_H\right). \tag{16}$$

We assume that (16) holds, since this corresponds to regulator's preferences for facility-based competition in the long run. Thus, second-period welfare in the high demand state can be plotted as in Figure 2.[1]

Just looking at Figures 1 and 2 it is easy to determine the first-best access prices for the second period.

[1] This is represented for $\overline{r}_H > r_H^*$. If $\overline{r}_H < r_H^*$, the second branch starts before the first reaches its maximum.

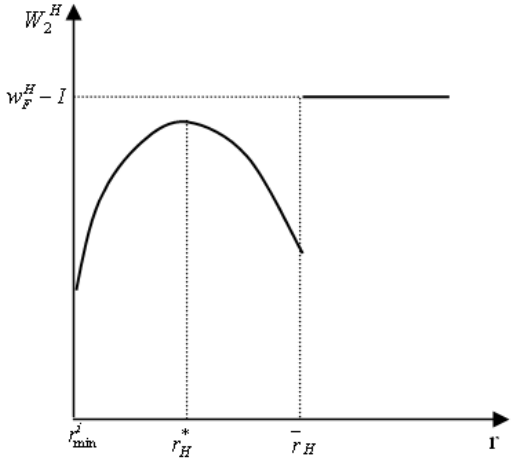

Figure 2

Lemma 3 *When the regulator has complete information about demand, the first-best access price for the second period is* $r_{2H}^{fb} \geq \overline{r}_H$ *in case of high demand, and* $r_{2L}^{fb} = r_L^*$ *in case of low demand.*

Proof. According to assumption (16) the regulator prefers to induce the high-type entrant to invest. Thus, given Lemma 1, he sets $r_{2H} \geq \overline{r}_H$. In case of low demand, it is impossible to force investment, thus the regulator sets the access price which maximizes welfare under service-based competition. ■

This result explains the incentives for strategic behavior by the high-type entrant. Indeed, given that the regulator prefers to set a higher access price for the second period in case of high demand, the high-type entrant may have incentives to shirk in the first period in order not to be obliged to invest, and benefit from a low access price in the second period.

3.2 Incomplete information in first period only

We now determine the socially optimal access prices when the regulator does not observe Nature's choice at the beginning of the game, but observes the true state of demand after the first period. This is equivalent to a context where the high-type entrant cannot shirk, and thus the demand it captures in the first period is a perfect signal of the demand state.

In the second period, the regulator has perfect information, therefore the socially optimal access price for each demand state is equal to the complete information case. However, for the first period the regulator can only maximize expected welfare given prior beliefs.

Lemma 4 *If $r_H^* \leq r_{\max}^L$, when the regulator can only observe the state of demand after the first period, the first-period socially optimal access price is:*

$$r_E^* = \arg\max \left\{ \alpha W_1^H(r) + (1-\alpha) W_1^L(r) \right\}, \tag{17}$$

and both types of entrant ask for access.

Proof. Given that $w_S^H(r)$ and $w_S^L(r)$ are strictly concave for $r < r_{\max}^L$, expected welfare is also strictly concave in this interval. Moreover, at $r_{\max}^L$ and $r_{\max}^H$ there are downward jumps by assumption (7). Thus, if $r_H^* < r_{\max}^L$, $\alpha W_1^H(r) + (1-\alpha) W_1^L(r)$ has a unique maximum $r_E^* \in \left(r_{\min}, r_{\max}^L\right)$ since $\left.\frac{d\left(\alpha W_1^H(r)+(1-\alpha)W_1^L(r)\right)}{dr}\right|_{r>r_{\max}^L} < 0$, and $r_i^* > r_{\min}$, $\forall i = H, L$. Therefore both types of entrant ask for access (see Figure 3). ■

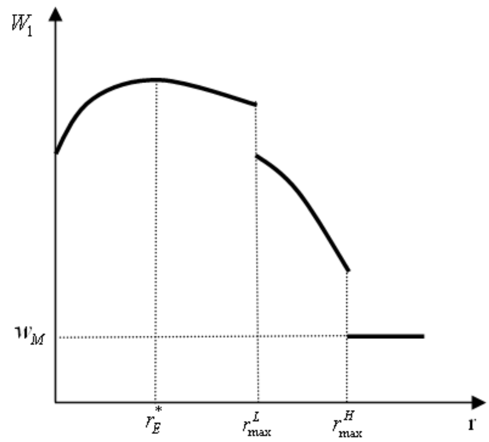

Figure 3

Given that it maximizes expected welfare, r_E^* will be somewhere between r_H^* and r_L^*. If $r_H^* > r_{\max}^L$, we could have $r_E^* > r_{\max}^L$, and the low-type entrant would not ask for access in the first period. However, for the rest of the paper we assume that $r_H^* \leq r_{\max}^L$.

4 Optimal regulation under asymmetric information

In this section we solve our model under the assumption that the regulator can never observe the true state of demand, and takes his decisions about the access prices at the beginning of the game. Hence, he sets the access price for the first period only based on prior beliefs about the probability of each demand state. However, as he can observe the demand captured by the entrant at the end of the first period, the second-period access prices can be set contingent on this observation. We further assume that the regulator has the ability to commit to these access prices until the end of the game.

Note that the complete information first-best access prices are impossible to set in this scenario since the regulator is not able to set first period access prices contingent on any signal about demand state. The best he can do is to achieve the incomplete information socially optimal outcome.

As we said before, in both periods, the low-type entrant will only choose between asking for access or staying out of the market. On the other hand, the high-type entrant can behave strategically in the first period, i.e. choose its target demand by comparing discounted profits if it shirks or if it captures high demand. It will prefer to shirk in the first period, sacrificing some profits, if the discounted benefit from paying an access price contingent on low demand in the second period is sufficiently high, i.e.

$$\delta\left(\Pi_2^H\left(r_{2L}\right)-\Pi_2^H\left(r_{2H}\right)\right)>\Pi_1^H\left(r_1\right)-\Pi_1^X\left(r_1\right). \quad (18)$$

If the regulator ignored this incentive for shirking by the high-type entrant, he would set the incomplete information socially optimal access prices, since he would believe that the demand observed in the first period was a perfect signal of the real state of demand. In this case, the high-type entrant would not shirk if and only if:

$$\delta\left(\Pi_2^H\left(r_L^*\right)-\pi_F^H+I\right)\leq\Pi_1^H\left(r_E^*\right)-\Pi_1^X\left(r_E^*\right). \quad (19)$$

This certainly happens if r_L^* is higher than $\overline{r}_H$, as in this case the high-type entrant would also invest in the second period for the access price contingent on low demand. Even if $r_L^*<\overline{r}_H$, it is possible to achieve the incomplete information socially optimal outcome when for r_E^* the losses from shirking are high enough. Otherwise, the high-type entrant would prefer to target low demand.

However, the regulator understands this behavior by the entrant. Therefore, if he wants the high-type entrant to reveal the true state of demand he must set access prices such that revealing the state of demand is the entrant's best response. Indeed, the regulator can apply the Revelation Principle by constructing a direct-revelation mechanism with a truth-telling equilibrium as in Laffont and Tirole (1993, 1994). The resulting welfare is the best that the regulator can achieve by taking into account the entrant's demand in the first period. This may come at some cost since he will have to leave additional rents to the high-type entrant as compared to the first best, while other outcomes are even worse. The only alternative is to commit to ignore first-period demand, on which we will comment below.

Proposition 5 *The separating equilibrium is socially preferred to a pooling equilibrium where the high-type entrant shirks.*

Proof. Suppose that $(r_1^{pe}, r_{2L}^{pe}, r_{2H}^{pe})$ are the access prices leading to a pooling equilibrium. This is dominated by a separating equilibrium as the regulator can always set $r_{2L} = r_{2H} = r_{2L}^{pe}$ and the high-type entrant does not shirk. In this case, welfare is the same in the second period, but it is higher in the first period. ∎

The regulator then solves the following direct-revelation mechanism problem:

$$\max_{r_1, r_{2L}, r_{2H}} \left\{\alpha W_1^H(r_1) + (1-\alpha) W_1^L(r_1) + \delta\left(\alpha W_2^H(r_{2H}) + (1-\alpha) W_2^L(r_{2L})\right)\right\}, \tag{20}$$

subject to the conditions that:

$$\begin{aligned} \Pi_1^H(r_1) - \Pi_1^X(r_1) &\geq \delta\left(\Pi_2^H(r_{2L}) - \Pi_2^H(r_{2H})\right) && (21) \\ r_1 &\geq r_{\min} && (22) \\ r_{2H} &\geq r_{\min}^H && (23) \\ r_{2L} &\geq r_{\min}^L. && (24) \end{aligned}$$

The first restriction in the regulator's problem is the incentive compatibility constraint and guarantees that the best strategy for the high-type entrant is to target high demand. Whenever (19) does not hold, (21) becomes active and the regulator has to introduce distortions as compared to the socially optimal access prices. The other three restrictions are only to assure that the incumbent operates its network in both periods.

We do not introduce participation constraints for the entrant since it may be optimal for the regulator to leave one type of entrant out of the market. For instance, in order to force the high-type entrant to reveal demand and invest, the regulator may be forced to set r_{2L} higher than $r_{\max}^L$, and thus a low-type entrant will not ask for access in the second period. The impact of having the entrant not participating in the market is included in the welfare and profit functions, which are also defined for the range of access prices where the entrant does not compete.

The Lagrangian of this problem is:

$$\begin{aligned}\pounds &= \alpha W_1^H(r_1) + (1-\alpha) W_1^L(r_1) + \delta\left(\alpha W_2^H(r_{2H}) + (1-\alpha) W_2^L(r_{2L})\right) \\ &\quad + \lambda\left[\Pi_1^H(r_1) - \Pi_1^X(r_1) - \delta\left(\Pi_2^H(r_{2L}) - \Pi_2^H(r_{2H})\right)\right], \qquad (25)\end{aligned}$$

with the following first-order conditions:

$$\frac{d\left(\alpha W_1^H(r_1) + (1-\alpha) W_1^L(r_1)\right)}{dr_1} + \lambda \frac{d\left(\Pi_1^H(r_1) - \Pi_1^X(r_1)\right)}{dr_1} \leq 0, r_1 \geq r_{\min}, \frac{\partial \pounds}{\partial r_1} r_1 = 0 \qquad (26)$$

$$\alpha \frac{dW_2^H(r_{2H})}{dr_{2H}} + \lambda \frac{d\Pi_2^H(r_{2H})}{dr_{2H}} \leq 0, r_{2H} \geq r_{\min}^H, \frac{\partial \pounds}{\partial r_{2H}} r_{2H} = 0 \qquad (27)$$

$$(1-\alpha) \frac{dW_2^L(r_{2L})}{dr_{2L}} - \lambda \frac{d\Pi_2^H(r_{2L})}{dr_{2L}} \leq 0, r_{2L} \geq r_{\min}^L, \frac{\partial \pounds}{\partial r_{2L}} r_{2L} = 0 \qquad (28)$$

$$\delta\left(\Pi_2^H(r_{2L}) - \Pi_2^H(r_{2H})\right) \leq \Pi_1^H(r_1) - \Pi_1^X(r_1), \lambda \geq 0, \frac{\partial \pounds}{\partial \lambda} \lambda = 0. \qquad (29)$$

Given that we only have generic functional forms for profit and welfare, we can just indicate the direction of the distortions as compared to the first best.

Proposition 6 *If* $\frac{d\left(\pi_S^H(r) - \pi_S^X(r)\right)}{dr} < (>) 0$ *for all* $r \in \left(r_{\min}^H, r_{\max}^H\right)$, *the optimal separating equilibrium access price for the first period is* $r_1^{se} \leq (\geq) r_E^*$. *If* $\frac{d\left(\pi_S^H(r) - \pi_S^X(r)\right)}{dr} = 0$ *for all* $r \in \left(r_{\min}^H, r_{\max}^H\right)$ *then* $r_1^{se} = r_E^*$.

Proof. First-order condition (26) is defined on three branches, the first ending at $r_{\max}^L$ and the second at $r_{\max}^H$ (see Figure 3).

For $\frac{d\left(\pi_S^H(r) - \pi_S^X(r)\right)}{dr} < 0$, if $\lambda > 0$ then the first-branch local maximum $\widetilde{r}_1$ is lower than r_E^* because expected welfare is strictly concave. This dominates the local maxima on the second and third branch since at these the LHS of (26) is negative. Hence, $r_1^{se} < r_E^*$. For $\frac{d\left(\pi_S^H(r) - \pi_S^X(r)\right)}{dr} = 0$, and by similar arguments, we find that $r_1^{se} = r_E^*$. Finally, for $\frac{d\left(\pi_S^H(r) - \pi_S^X(r)\right)}{dr} > 0$, if $\lambda > 0$ then the first-branch local maxima is $\widetilde{r}_1 > r_E^*$. Hence, $r_1^{se} > r_E^*$. If, for any case, $\lambda = 0$ then $r_1^{se} = r_E^*$. ■

When the losses from shirking are lower (higher) the higher is the access price, the regulator should set a first-period access price lower (higher) than the socially optimal one under incomplete information. This lower (higher) access price helps to discourage the high-type entrant from shirking since it increases its losses from targeting low demand in the first period. If the effect of r on profit is the same whatever the demand captured by the entrant, then the regulator should not introduce any distortion on the access price since this would not deter the entrant from shirking.

The most reasonable of these three conditions to be verified is to have the high demand profit decreasing faster in r than the low demand profit, since the higher is the access price the lower is the margin the entrant gains per unit sold, and thus the lower are the gains from capturing high demand. This is also the condition which holds in the example given in Section 2. Therefore, for the rest of the paper we will assume that:

$$\frac{d\left(\pi_S^H(r) - \pi_S^X(r)\right)}{dr} < 0 \text{ for all } r \in \left(r_{\min}^H r_{\max}^H\right), \tag{30}$$

and thus, according to Proposition 6, $r_1^{se} \leq r_E^*$.

Proposition 7 *The optimal separating equilibrium access price for the second period contingent on high demand is either $r_{2H}^{se} \leq r_H^*$ or $r_{2H}^{se} = \overline{r}_H$, while the one contingent on low demand is $r_{2L}^{se} \geq r_L^*$. It is possible that $r_{2L}^{se} > r_{\max}^L$.*

Proof. First-order condition (27) is defined on two branches, with the first ending at $\overline{r}_H$ (see Figure 2). The local maximum on the first branch is $\widetilde{r}_{2H} \leq r_H^*$ by (1), and because the welfare function is strictly concave. $\overline{r}_H$ is the local maximum on the second branch since there (27) does not depend on r. Therefore, r_{2H}^{se} is either $\widetilde{r}_{2H}$ or $\overline{r}_H$.

First-order condition (28) is defined on three branches since welfare is not continuous at $r_{\max}^L$ and profit is not differentiable at $\overline{r}_H$.

First assume that $r_L^* < \overline{r}_H \leq r_{\max}^L$. In this case, the first branch ends at $\overline{r}_H$ and its local maximum is $\widetilde{r}_{2L} \geq r_L^*$ by (1) and by the strictly concavity of the welfare function. Any $r_{2L} \geq \overline{r}_H$ cannot be optimal since (21) is no longer active and welfare is lower. If $r_L^* < r_{\max}^L < \overline{r}_H$, the first branch ends at $r_{\max}^L$ and its local maximum is again $\widetilde{r}_{2L} \geq r_L^*$. The local maximum on the second branch is its superior limit since $\left.\frac{\partial \pounds}{\partial r_{2L}}\right|_{r \geq r_{\max}^L} > 0$, but this is dominated by $\overline{r}_H$ the local maximum on the third branch. If $r_L^* \geq \overline{r}_H$, then (21) is not binding in equilibrium, and the regulator sets r_L^*, the first-branch local maximum. Therefore, r_{2L}^{se} is either $\widetilde{r}_{2L} \geq r_L^*$ or $\overline{r}_H > r_{\max}^L$. ■

If it is optimal to force investment by the high-type entrant the regulator should set $\overline{r}_H$, or simply ban access. However, inducing the high-type entrant to reveal demand may be too expensive because of the distortions introduced in the low demand state. Thus, the it may be better not to induce the high-type entrant to invest but to continue to ask for access. This can be the case, for instance, when the probability of a low demand state occurring is very high, since in this case the distortions introduced in the low demand state have a high weight in the regulator's objective function and, as a consequence, the regulator should set an access price the closest possible to r_L^*. Hence, given restriction (21), the regulator may be forced to allow the high-type entrant to continue to ask for access so that $\Pi_2^H(r_{2H})$ is high enough.

Regarding the access price contingent on low demand, to discourage the high-type entrant from shirking, it should be higher than the first-best one, so that the second-period gains from shirking are lower. If $r_{\max}^L$ is very low, the access price the regulator should set may even be such that the low-type entrant does not ask for access in the second period. This may be optimal when the probability of a high demand state occurring is high, since in this case the regulator should definitely force the high-type entrant to invest, and be less concerned with the distortions introduced in the low demand state.

We have shown in Proposition 6 that no competition in the first period is not optimal, since $r_1^{se} \leq r_E^* < r_{\max}^L$. Hence r_2^{se} is off the equilibrium path. However, the regulator should define it such that any type of entrant does not prefer to stay out in the first period, i.e.

$$\Pi_1^i\left(r_1^{se}\right) \geq \delta\left(\Pi_2^i\left(r_2^{se}\right) - \Pi_2^i\left(r_{2i}^{se}\right)\right), \ \forall i = H, L. \tag{31}$$

When the regulator commits not to look at first-period demand, there are no incentives to shirk by the high-type entrant since it cannot influence the second-period access price. However, the regulator sets the latter only based on prior beliefs, just as he does for the first-period access price. Therefore, there is a possible trade-off.

Proposition 8 *Looking at first-period demand leads to higher welfare than committing not to look.*

Proof. The access prices set by the regulator when he does not look at first-period demand give rise to a separating equilibrium since the high-type entrant does not shirk. Therefore, they are part of the opportunity set of problem (20) to (24). ■

We can then conclude that looking at demand in the first period is always preferred to not looking despite the eventual distortions introduced in the first period and the corresponding rents left to the high-type entrant to obtain information about demand. This is true since looking at first period demand allows the regulator to use this information and set second-period access prices better suited to the respective demand state.

As a final remark, note that the equilibrium access prices that result from the direct-revelation mechanism are not time-consistent. Indeed, in this equilibrium, the regulator learns the real state of demand at the end of the first period, and thus the *ex-post* socially optimal access prices for the second period are different. Only when the regulator can set the incomplete information first-best access prices without inducing to shirk, will second-period access prices be *ex-post* optimal.

Social welfare is, however, lower when the regulator cannot commit to his decisions since the high-type entrant has more incentives to shirk in order to influence the regulator's *ex-post* beliefs, and therefore it ends up investing less often.[2] This is nothing more than the ratchet effect known from the literature.

5 Variations on investment

Until now we have assumed that an investment is viable only for the high-type entrant, see assumption (10). In this section we change this assumption.

Case 1: Non-viability of investment in both states
This situation corresponds to the following assumption:

$$I > \max\left\{\pi_F^L, \pi_F^H\right\}. \tag{32}$$

In this case, the regulator is not able to induce an entrant to invest in an own network using the access price. Thus, he can only promote service-based competition, which implies that the first-best access price for the second period is r_L^* or r_H^* depending on whether demand is low or high, respectively.

With asymmetric information about demand, when the regulator sets the incomplete information socially optimal access prices, the high-type entrant shirks if:

$$\delta\left(\Pi_2^H\left(r_L^*\right) - \Pi_2^H\left(r_H^*\right)\right) > \Pi_1^H\left(r_E^*\right) - \Pi_1^X\left(r_E^*\right). \tag{33}$$

If this holds, the regulator should introduce distortions similar to the ones introduced in the previous section. We just need to assume that $\overline{r}_H = +\infty$, and the results come out immediately from Propositions 6 and 7: The regulator should set $r_1^{se} \leq r_E^*$ and $r_{2L}^{se} \geq r_L^*$ in order to discourage shirking by the high-type entrant, while the second-period access price contingent on high demand should now be $r_{2H}^{se} \leq r_H^*$ since he cannot force the high-type entrant to invest.

[2]Whenever the commitment equilibrium is not time-consistent, one can show that in the no-commitment equilibrium the high-type entrant shirks at least with some positive probability.

Case 2: Viability of investment in both states

Both types of entrants may find it optimal to invest in their own infrastructure in the second period if:

$$\min\left\{\pi_F^L, \pi_F^H\right\} \geq I. \tag{34}$$

Similarly to Proposition 1, there is an access price above which the low-type entrant prefers to invest, i.e. there is a unique $\overline{r}_L \in \left[r_{\min}^L, r_{\max}^L\right]$ defined as the lowest r such that:

$$\pi_F^L - I \geq \pi_S^L(r). \tag{35}$$

Given the regulator's preferences for facility-based competition in the long run, we just consider the case where both investments are socially desirable, i.e.

$$w_F^i - I \geq w_S^i(r) \quad \text{for } r \leq \overline{r}^i,\ \forall i = L, H. \tag{36}$$

In this case, the regulator should ban access in the second period so that any type of entrant invests. He can then set r_E^* in the first period, since there are no incentives to shirk.

6 Consulting the entrant before competition starts

In this section we introduce a consultation stage where the regulator asks the entrant about the demand it believes it will capture. This consultation stage takes place before competition starts, but after the entrant learns demand through the conduction of a market study. This may bring some gains as compared to the solution in Section 4 since it already allows the regulator to set first-period access prices contingent on the answer given by the entrant, and therefore the complete information first best may be possible to achieve. The regulator can also set second-period access prices contingent on the fact whether demand level observed later is consistent with what the entrant said at the beginning. Note that the regulator does not ask about the state of demand since he would not be able to observe a lie if the high-type entrant said that demand state is low and shirked in the first period.

We assume that the regulator announces the price schedule first, with automatic selection of the prices given the answer. This is, like this the regulator can give better incentives as compared to the case where he first asks the entrant and then announces the prices *ex post*. In this latter case, since the regulator would only define the lying punishment after it had occurred, he would not have incentives to use it, and thus the entrant would more often lie.

The regulator then sets r_{1L} and r_{1H} which are, respectively, the access price for the first period if the entrant says it will capture high or low demand, r_{2Lt} and r_{2Ht} which are second-period access prices if he observes low or high demand, respectively, and this is in accordance with the entrant's answer, and r_{2Lf} and r_{2Hf} if it is not. It follows that the consultation stage is not "cheap talk" since, before the second period, the regulator can check if the entrant has said the truth about demand.

First note that the regulator prefers the entrant not to lie, since only then he will obtain any information. Therefore, he will penalize the entrant for lying. He can do this by banning access in the second period whenever he observes a lie. In the low demand case this punishment implies that the entrant will leave the market, but in the high demand case this will only induce the entrant to invest in an own network.

Thus, in order for the high-type entrant not to lie, the access prices set by the regulator must be such that the gains from lying are lower than the losses of being caught, i.e.

$$\delta\left(\Pi_2^H\left(r_{2Ht}\right) - \pi_F^H + I\right) \geq \Pi_1^H\left(r_{1L}\right) - \Pi_1^H\left(r_{1H}\right). \tag{37}$$

If $r_{2Ht} \geq \overline{r}_H$, r_{2Hf} will not constitute any penalty for lying since the high-type entrant invests in both cases, the LHS of (37) is zero. In this case, for $r_{1L} < r_{1H}$ the high-type entrant lies for sure. Therefore, the regulator must set $r_{1L} \geq r_{1H}$.

If $r_{2Ht} < \overline{r}_H$ the regulator effectively penalizes the high-type entrant when he catches it lying, the LHS of (37) is positive. Therefore he has some freedom to set first-period access prices contingent on the answer given by the entrant with $r_{1L} < r_{1H}$.

The regulator has also to take into account the high-type entrant incentives for shirking. Hence, similarly to Section 4, he needs to set access prices such that:

$$\Pi_1^H\left(r_{1H}\right) - \Pi_1^X\left(r_{1L}\right) \geq \delta\left(\Pi_2^H\left(r_{2Lt}\right) - \Pi_2^H\left(r_{2Ht}\right)\right). \tag{38}$$

The only difference between this and (21) is that in this case the first period access price differs according to the entrant's answer.

From these two restrictions we obtain:

$$\Pi_1^H\left(r_{1H}\right) + \delta\Pi_2^H\left(r_{2Ht}\right) \geq \max\left\{\Pi_1^H\left(r_{1L}\right) + \delta\left(\pi_F^H - I\right), \Pi_1^X\left(r_{1L}\right) + \delta\Pi_2^H\left(r_{2Lt}\right)\right\}. \tag{39}$$

If the regulator ignored the incentives to lie and shirk by the high-type entrant, and set the first-best access prices, he would only achieve the first best if $r_L^* \geq r_H^*$, i.e. if in the first period the high-type entrant prefers to tell the truth. Moreover, the high-type entrant would not shirk if and only if

$$\Pi_1^H(r_H^*) - \Pi_1^X(r_L^*) \geq \delta\left(\Pi_2^H(r_L^*) - \pi_F^H + I\right), \tag{40}$$

i.e. when $\overline{r}_H \leq r_L^*$ (RHS of (40) is zero), or when the first period losses are higher than the gains from shirking (the LHS of (40) is high enough).

Whenever $r_L^* < r_H^*$ or (40) do not hold, the regulator has to introduce distortions similar to the ones introduced in Section 4 to discourage the high-type entrant from lying and shirking.

Proposition 9 *If* $\delta \geq \frac{\Pi_1^L(r_{1H}^{cs})-\Pi_1^L(r_{1L}^{cs})}{\Pi_2^L(r_{2Lt}^{cs})}$, *when the regulator can consult the entrant before competition in the first-period, optimal access prices are* $r_{1L}^{cs} \geq r_L^*$, $r_{1H}^{cs} \leq r_H^*$, $r_{2Lt}^{cs} \geq r_L^*$, *and* $r_{2Ht}^{cs} = \overline{r}_H$ *or* $r_{2Ht}^{cs} \leq r_H^*$.

Proof. See Appendix B. ■

The regulator should set a higher r_{1L} and a lower r_{1H} in order to increase the losses from shirking in the first period. These also help prevent a lie by the high-type entrant. Then, he should set a higher r_{2Lt} to decrease the gains from shirking, while r_{2Ht}^{cs} can be such that the high-type entrant invests or not.

Note that, we need to assure that the low-type entrant also does not have incentives to lie. Indeed, if $r_{1H}^{cs} < r_{1L}^{cs}$ the low-type entrant may have incentives to say demand is high despite the punishment it will suffer in the second period, i.e. we can have:

$$\Pi_1^L(r_{1H}^{cs}) - \Pi_1^L(r_{1L}^{cs}) > \delta\Pi_2^L(r_{2Lt}^{cs}). \tag{41}$$

In this case, this restriction would become active, and we could even have $r_{1L}^{cs} < r_L^*$, $r_{1H}^{cs} > r_H^*$, and/or $r_{2Lt}^{cs} < r_L^*$ (see discussion in Appendix B).

We will now compare this with the context where there is no consultation stage, which was the case of previous sections.

Proposition 10 *Consulting the entrant before competition leads to (weakly) higher total welfare than basing access prices on realized demand in the first period only.*

Proof. We can prove this by arguments of revealed preferences since the separating equilibrium obtained in Section 4 is part of the opportunity set of this game. Just note that the regulator can set $r_{1L} = r_{1H} = r_1^{se}$, $r_{2Lt} = r_{2Lf} = r_{2L}^{se}$ and $r_{2Ht} = r_{2Hf} = r_{2H}^{se}$, and obtain the same welfare level.

■

The existence of a consultation stage may then help to improve welfare. Only when $r_{1L}^{cs} = r_{1H}^{cs}$, there will be no gains in asking about demand, and welfare will be equal to the case where access prices are only based on realized demand. This happens when $r_{2Ht}^{cs} = \overline{r}_H$ and the restriction $r_{1L} \geq r_{1H}$ is biding, i.e. when the regulator wants to force the high-type entrant to invest, but its gains from lying at first best are very high, since r_L^* is much lower than r_H^*. In all other cases, the consultation stage helps the regulator to define better access prices right from the beginning of the game, and thus strictly improves welfare.

Note that this section's results are equal to the ones we would obtain if instead of asking the entrant about its expected demand, the regulator asked it to select one of two possible contracts. These two contracts would be composed by the access price to be paid in the first period, and the access price to be paid in the second period if demand captured in the first period was low or high, i.e., $R_j = \left(r_1^j, r_{2L}^j, r_{2H}^j\right)$, with $j = A, B$. Contract A could then be designed to be selected by a low-type entrant, and contract B by a high-type entrant, inducing the entrant to reveal the demand state through its contract selection. The restrictions that would assure that each entrant made the selection correspondent to its type would be the same as with the consultation stage interpretation. Thus, the contract designed to be selected by the low-type entrant would be $R_A = \left(r_{1L}^{cs}, r_{2Lt}^{cs}, r_{2Hf}^{cs}\right)$, and the contract designed to be selected by the high-type entrant would be $R_B = \left(r_{1H}^{cs}, r_{2Lf}^{cs}, r_{2Ht}^{cs}\right)$.

7 Conclusion

The existence of asymmetric information about demand when the regulator sets prices for the entrant's access to the incumbent's infrastructure creates incentives for the entrant to target a lower demand. Indeed, given that the regulator wants to induce an entrant to invest in an own infrastructure, he can ban access after observing that the entrant has the capability to invest. Thus an entrant in a high demand state may prefer to target low demand (shirk) in order to signal to the regulator that it still needs cheap access. This problem persists even if the regulator commits to access prices *ex ante* since optimal regulation involves setting future access prices contingent on observed demand.

The regulator solves this problem of incentives by committing to access prices which are distorted in relation to the first best. When the decrease in the entrant's profit resulting from a high access price is higher in the high-demand state, the regulator should set a lower access price for the first period in order to increase the initial losses from shirking. Additionally, he should set a higher second-period access price contingent on low demand so that the gains from shirking are lower. In the high demand state the entrant may or may not be induced to invest. Still, social welfare will be higher than if the regulator were to ignore the entrant's demand.

Finally, we also show that a consultation stage before competition starts, where the regulator asks the entrant about the demand it will capture, may improve welfare since it allows the regulator to already set first-period access prices contingent on demand. Moreover, given that the regulator sets *ex-ante* access prices which induce the entrant to tell the truth about the state of demand, he may even be able to induce to the complete information first best.

In future research we intend to relax the assumption that the entrant has a perfect signal about the state of demand. It will also be interesting to consider the role of the incumbent in this process. Indeed, if it was possible for the regulator to ask the incumbent about demand, he could design a mechanism to obtain additional information. This is interesting because the incumbent and the entrant may have opposite interests.

Appendix A - Example

Given demand functions (8) and (9), we assume that $a_E = \lambda_i a_I$, with $i = H, X, L$ and $\lambda_H > \max\{\lambda_L, \lambda_X\}$, i.e. to a lower demand state corresponds a lower reservation price. We further assume, for simplicity, that $c = 0$. Solving the price competition game as in Vareda (2007), welfare and entrant's profit are:

$$W_S(\lambda, r) = \frac{1}{2}\frac{\left(12 + 2\theta^4 - 9\theta^2\right)\left(\lambda^2 + 1\right) - 2\theta\left(8 - \theta^3\right)\lambda}{\left(1 - \theta^2\right)\left(4 - \theta^2\right)^2} a_I^2 + \frac{\theta^3 + 3\theta^2\lambda - 4\lambda}{\left(4 - \theta^2\right)^2} r a_I - \frac{1}{2}\frac{5\theta^2 + 4}{\left(4 - \theta^2\right)^2} r^2$$

$$\pi_S(\lambda, r) = \frac{\left(\left(2\lambda - \theta^2\lambda - \theta\right)a_I - 2\left(1-\theta^2\right)r\right)^2}{\left(1-\theta^2\right)\left(4-\theta^2\right)^2},$$

and all our assumptions hold:

$$\frac{\partial \pi_S(\lambda, r)}{\partial r} = -4\frac{\left(2\lambda - \theta^2\lambda - \theta\right)a_I - 2\left(1-\theta^2\right)r}{\left(4-\theta^2\right)^2} < 0$$

$$\frac{\partial \pi_S(\lambda, r)}{\partial \lambda} = \frac{\left(2\lambda - \theta^2\lambda - \theta\right)a_I - 2\left(1-\theta^2\right)r}{\left(1-\theta^2\right)\left(4-\theta^2\right)^2}2\left(2-\theta^2\right)a_I > 0$$

$$\frac{\partial^2 \pi_S(\lambda, r)}{\partial r \partial \lambda} = -4\frac{2-\theta^2}{\left(4-\theta^2\right)^2}a_I < 0$$

$$\frac{\partial W_S(\lambda, r)}{\partial \lambda} = \frac{1}{2}\frac{\left(\left(2\theta^4 - 9\theta^2 + 12\right)\lambda + \left(3\theta^3 - 8\theta\right)\right)a_I - \left(1-\theta^2\right)\left(4-3\theta^2\right)r}{\left(1-\theta^2\right)\left(4-\theta^2\right)^2}a_I > 0$$

$$W_S(\lambda, r) > W^M = \frac{1}{2}a_I^2.$$

Appendix B - Proof of Proposition 9

We have already argued it cannot be optimal if the entrant lies since in this case the regulator would be better off not listening to the answer. Thus, in order to discourage a lie, the regulator sets the highest possible access prices when he catches the entrant lying, i.e. $r_{2Lf}^{cs} = r_{2Hf}^{cs} = +\infty$.

To find the remaining optimal access prices $\left(r_{1L}^{cs}, r_{1H}^{cs}, r_{2Lt}^{cs}, r_{2Ht}^{cs}\right)$ we need to consider the following Lagrangian:

$$\begin{aligned}
\pounds &= \alpha W_1^H(r_{1H}) + (1-\alpha)W_1^L(r_{1L}) + \delta\left(\alpha W_2^H(r_{2Ht}) + (1-\alpha)W_2^L(r_{2Lt})\right) \\
&\quad +\lambda_1\left[\Pi_1^H(r_{1H}) + \delta\Pi_2^H(r_{2Ht}) - \Pi_1^X(r_{1L}) - \delta\Pi_2^H(r_{2Lt})\right] \\
&\quad +\lambda_2\left[\Pi_1^H(r_{1H}) + \delta\Pi_2^H(r_{2Ht}) - \Pi_1^H(r_{1L}) - \delta\left(\pi_F^H - I\right)\right].
\end{aligned}$$

First-order conditions are:

$$\alpha\frac{dW_1^H(r_{1H})}{dr_{1H}} + (\lambda_1 + \lambda_2)\frac{d\Pi_1^H(r_{1H})}{dr_{1H}} \le 0, r_{1H} \ge r_{\min}^H, \frac{\partial \pounds}{\partial r_{1H}}r_{1H} = 0 \tag{42}$$

$$(1-\alpha)\frac{dW_1^L(r_{1L})}{dr_{1L}} - \lambda_1\frac{d\Pi_1^X(r_{1L})}{dr_{1L}} - \lambda_2\frac{d\Pi_1^H(r_{1L})}{dr_{1L}} \le 0, r_{1L} \ge r_{\min}^L, \frac{\partial \pounds}{\partial r_{1L}}r_{1L} = 0 \tag{43}$$

$$\alpha \frac{dW_2^H(r_{2Ht})}{dr_{2Ht}} + (\lambda_1 + \lambda_2) \frac{d\Pi_2^H(r_{2Ht})}{dr_{2Ht}} \leq 0, r_{2Ht} \geq r_{\min}^H, \frac{\partial \pounds}{\partial r_{2Ht}} r_{2Ht} = 0 \quad (44)$$

$$(1-\alpha) \frac{dW_2^L(r_{2Lt})}{dr_{2Lt}} - \lambda_1 \frac{d\Pi_2^H(r_{2Lt})}{dr_{2Lt}} \leq 0, r_{2Lt} \geq r_{\min}^L, \frac{\partial \pounds}{\partial r_{2Lt}} r_{2Lt} = 0 \quad (45)$$

$$\Pi_1^H(r_{1H}) + \delta \Pi_2^H(r_{2Ht}) \geq \Pi_1^X(r_{1L}) + \delta \Pi_2^H(r_{2Lt}), \lambda_1 \geq 0, \frac{\partial \pounds}{\partial \lambda_1} \lambda_1 = 0 \quad (46)$$

$$\Pi_1^H(r_{1H}) + \delta \Pi_2^H(r_{2Ht}) \geq \Pi_1^H(r_{1L}) + \delta \left(\pi_F^H - I\right), \lambda_2 \geq 0, \frac{\partial \pounds}{\partial \lambda_2} \lambda_2 = 0. \quad (47)$$

From (42) and (43), the optimal access prices for the first period are, respectively, $r_{1H}^{cs} \leq r_H^*$ and $r_{1L}^{cs} \geq r_L^*$ by (1) and using arguments similar to the ones used in Proposition 7. From (44) we obtain $r_{2Ht}^{cs} = \overline{r}_H$ or $r_{2Ht}^{cs} \leq r_H^*$ as in Proposition 7, and given (45) we have $r_{2Lt}^{cs} \geq r_L^*$.

Note that these are only valid if the low-type entrant does not lie, i.e., if $\Pi_1^L(r_{1L}^{cs}) + \delta \Pi_2^L(r_{2Lt}^{cs}) \geq \Pi_1^L(r_{1H}^{cs})$, which we assume to hold. On the contrary, we would have to include this as a restriction on the regulator's problem, which would add another term in conditions (42), (43) and (45). This term would have the opposite signal of the terms associated to the other multipliers, and therefore, we could have a solution with $r_{1L}^{cs} < r_L^*$, $r_{1H}^{cs} > r_H^*$, and/or $r_{2Lt}^{cs} < r_L^*$.

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