DOCTORAL PROGRAMME

Information Management

Specialization in Statistics and Econometrics

Advanced Survival Modelling for Consumer Credit Risk Assessment: Addressing recurrent events, multiple outcomes and frailty

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Abstract

This thesis worked on the application of advanced survival models in consumer credit risk assessment, particularly to address issues of recurrent delinquency (or default) and recovery (cure) events as well as multiple risk events and frailty. Each chapter (2 to 5) addressed a separate problem and several key conclusions were reached. Chapter 2 addressed the neglected area of modelling recovery from delinquency to normal performance on retail consumer loans taking into account the recurrent nature of delinquency and also including time-dependent macroeconomic variables. Using data from a lending company in Zimbabwe, we provided a comprehensive analysis of the recovery patterns using the extended Cox model. The findings vividly showed that behavioural variables were the most important in understanding recovery patterns of obligors. This confirms and underscores the importance of using behavioural models to understand the recovery patterns of obligors in order to prevent credit loss. The findings also strongly revealed that the falling real gross domestic product, representing a deteriorating economic situation significantly explained the diminishing rate of recovery from delinquency to normal performance among consumers. The study pointed to the urgent need for policy measures aimed at promoting economic growth for the stabilisation of consumer welfare and the financial system at large.

Chapter 3 extends the work in chapter 2 and notes that, even though multiple failure-time data are ubiquitous in finance and economics especially in the credit risk domain, it is unfortunate that naive statistical techniques which ignore the subsequent events are commonly used to analyse such data. Applying standard statistical methods without addressing the recurrence of the events produces biased and inefficient estimates, thus offering erroneous predictions. We explore various ways of modelling and forecasting recurrent delinquency and recovery events on consumer loans. Using consumer loans data from a severely distressed economic environment, we illustrate and empirically compare extended Cox models for ordered recurrent recovery events. We highlight that accounting for multiple events proffers detailed information, thus providing a nuanced understanding of the recovery prognosis of delinquents. For ordered indistinguishable recurrent recovery events, we recommend using the Andersen and Gill (1982) model since it fits these assumptions and performs well on predicting recovery.

Chapter 4 extends chapters 2 and 3 and highlight that rigorous credit risk analysis is not only of significance to lenders and banks but is also of paramount importance for sound regulatory and economic policy making. Increasing loan impairment or delinquency, defaults and mortgage foreclosures signals a sick economy and generates considerable financial stability concerns. For lenders and banks, the accurate estimation of credit risk parameters remains essential for pricing, profit testing, capital provisioning as well as for managing delinquents. Traditional credit scoring models such as the logit regression only provide estimates of the lifetime probability of default for a loan but cannot identify the existence of cures and or other movements. These methods lack the ability to characterise the progression of borrowers over time and cannot utilise all the available data to understand the recurrence of risk events and possible occurrence of multiple loan outcomes.

In this paper, we propose a system-wide multi-state framework to jointly model state occupations and the transitions between normal performance (current), delinquency, prepayment, repurchase, short sale and foreclosure on mortgage loans. The probability of loans transitioning to and from the various states is estimated in a discrete-time multi-state Markov model with seven allowable states and sixteen possible transitions. Additionally, we investigate the relationship between the probability of loans transitioning to and from various loan outcomes and loan-level covariates. We empirically test the performance of the model using the US single-family mortgage loans originated during the first quarter of 2009 and were followed on their monthly repayment performance until the third quarter of 2016. Our results show that the main factors affecting the transition into various loan outcomes are affordability as measured by debt-to-income ratio, equity as marked by loan-to-value ratio, interest rates and the property type.

In chapter 5, we note that there has been increasing availability of consumer credit in Zimbabwe, yet the credit information sharing systems are not as advanced. Using frailty survival models on credit bureau data from Zimbabwe, the study investigates the possible underestimation of credit losses under the assumption of independence of default event times. The study found that adding a frailty term significantly improved the models, thus indicating the presence of unobserved heterogeneity. The major policy recommendation is for the regulator to institute appropriate policy frameworks to allow robust and complete credit information sharing and reporting as doing so will significantly improve the functioning of the credit market.

List of Publications

1. Chamboko, R., and Bravo, J. M. (2016). On the modelling of prognosis from delinquency to normal performance on retail consumer loans. Risk Management, 18(4), 264-287.
2. Chamboko, R., and Bravo, J.M (forthcoming). Modelling and forecasting recurrent recovery events on consumer loans. Accepted for publication in the International Journal of Applied Decision Sciences.
3. Chamboko, R., and Bravo, J.V.M (forthcoming). Frailty correlated default on retail consumer loans in Zimbabwe. Accepted for publication in the International Journal of Applied Decision Sciences.
4. Chamboko, R., and Bravo, J.V.M (forthcoming). A multi-state approach to modelling intermediate events and multiple mortgage loan outcomes. Submitted to an internal journal.

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Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Motivation

Credit risk is defined as the risk of an economic loss resulting from a counterparty failing to meet its contractual obligations (Jorion, 2003). It is a function of the credit exposure (exposure at default), the probability of a risk event (probability of default) and the fractional possible loss given a default event (and its converse is the value that can be recovered in case of default). Credit risk is the most important risk banks consider as credit granting remain the major income generating activity for them. As such, credit risk modelling continues to be an important task for such institutions.

The need to improve and strengthen credit risk modelling has intensified as regulators tightened regulatory and compliance requirements responding to the recent global financial crises. The introduction of the Basel II/III accord also intensified the appetite for better models as banks, especially those using the internal ratings based (IRB) approach to estimate capital requirements are required to compute the probability of default (PD), exposure at default (EAD) and loss given default (LGD). The financial crises also meant massive credit loss for lenders and resulted in consumer's ability to honour their loan commitments seriously affected with some economies still struggling to recuperate.

In emerging markets and developing countries, there has been a rapid growth of microfinance institutions and digital platforms such as mobile money extending unsecured loans to the poor or those near so, as well as a proliferation of retail and payroll lending. To compound the problem, developing countries and emerging markets also suffer from limited credit information sharing and less advanced credit information systems. The combination of these has resulted in massive over indebtedness among impoverished populations (FSDA, 2016; Fanta, Mutsonziwa, Berkowitz, & Goosen, 2017; SAHRC, 2017; Totolo, 2018; Izaguirre, Kaffenberger & Mazer, 2018).

It is therefore imperative that sound credit risk analysis is of major importance to lenders, banks regulators and economic policy makers as it helps gauge the state of a financial system and the performance of the economy. Growing loan impairments, delinquency, defaults and mortgage foreclosures are indications of a failing economy. The inability of consumers to service their loans and the growing indebtedness among the vulnerable and poor populations in developing countries and emerging markets increases the need to understand the repayment behaviour, including the recovery prognosis of these distressed borrowers.

Rigorous studies on the recovery of distressed borrowers are scarce, yet they are essential as they may contribute to a better understanding of consumers' over indebtedness, depths of financial distress, and the level of impairment in a financial system. Beside, such insights shed light into the state of consumer welfare since the performance of the economy is correlated to that of the financial sector (Beck, Levine and Loayza, 2000; Demirgüç-Kunt and Maksimovic, 1998; King and Levine, 1993). A neglect on this subject may result in failure to detect the deterioration of assets especially in the shadow banking system and may result in financial instability.

1.2 Statistical anomalies and contribution of the thesis

This thesis worked on the application of advanced survival modelling for consumer credit risk assessment, particularly addressing issues of recurrent events, multiple outcomes and frailty. Due to the above described financial hardships or distress and over indebtedness among consumers, the recurrence of delinquency or default events and hence recovery becomes an important subject. Even though recurrent events or multiple failure-time data are common in the credit risk domain, practitioners and researchers usually consider one event and thus do not fully utilise such data to enhance the understanding of the problem. Thus, unsuitable statistical techniques which ignore the recurrence of events are commonly used to analyse such data. Ignoring the recurrent nature of the events or the correlation structure of the data and applying standard statistical methods produces biased and inefficient estimates, thus resulting in wrong decisions. Also, these recurrent events are rarely modelled taking into account time-dependent macroeconomic variables.

Chapter 2 therefore presents a pioneering paper "on the modelling of prognosis from delinquency to normal performance on retail consumer loans" taking into account the recurrent nature of delinquency and including time-dependent covariates. The paper was published in the journal Risk Management. Chapter 3 extends the work covered in chapter 2 and to a greater depth looks at methods for "modelling and forecasting recurrent recovery events on consumer loans" and the paper was accepted for publication in the International Journal of Applied Decision Sciences.

When it comes to mortgage loans, numerous possible loan outcomes are also possible. Unfortunately, traditional approaches to credit risk modelling consider one risk event at a time and use standard classification methods such as logistic regression or standard survival models. These methods lack the ability to characterise the progression of borrowers over time and cannot utilise all the available data to understand the recurrence of risk events, cure or recovery among borrowers and the possible occurrence of multiple loan outcomes. Chapter 4 therefore introduces a novel system-wide framework and implemented "a multi-state approach to modelling intermediate events and multiple mortgage loan outcomes". The paper is under review in an international journal.

On another note, in developing countries and emerging markets, where there is increasing availability of consumer credit, it is unfortunate that credit information sharing systems are not as advanced and there is incompleteness and limitations in credit information sharing. Thus, fitting standard credit risk models without all important variables included may fail to detect default clustering especially due to unobserved heterogeneity. This may lead to the underestimation of possible credit losses, thus, banks and other lending institutions making inadequate capital provisions to cover for the actual loses. Chapter 5 therefore looks into issues of incomplete and limited credit information sharing which may result in "frailty correlated default on retail consumer loans", especially due to unobserved heterogeneity. The paper was also accepted for publication in the International Journal of Applied Decision Sciences.

1.3 Conclusion

This chapter summarised the motivation and statistical issues underpinning the four studies carried out under this thesis. The next four chapters (chapter 2 to chapter 5) individually addresses a specific problem and chapter 6 provides a summary of the thesis and concludes.

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Chapter 2

On the modelling of prognosis from delinquency to normal performance on retail consumer loans

This chapter is made of a paper which was published in the journal titled Risk Management and recommended citation is as follows: Chamboko, R., and Bravo, J. M. (2016). On the modelling of prognosis from delinquency to normal performance on retail consumer loans. Risk Management, 18(4), 264-287.

Abstract

This paper addressed the neglected area of modelling recovery from delinquency to normal performance on retail consumer loans taking into account the recurrent nature of delinquency and also including time dependent macroeconomic variables. Using data from a lending company in Zimbabwe, we provided a comprehensive analysis of the recovery patterns using the extended Cox model. The findings vividly showed that behavioural variables were the most important in understanding recovery patterns of obligors. This confirms and underscores the importance of using behavioural models to understand the recovery patterns of obligors in order to prevent credit loss. The study also points to the urgent need for policy measures aimed at promoting economic growth for the stabilisation of consumer welfare and the financial system at large.

Keywords: consumer credit risk; non-performing loans; recurrent events, delinquency; recovery; counting process; survival analysis

2.1 Introduction

Credit granting remains the main income generating activity for banks and lending institutions. Thus, credit risk modelling continues to be an important exercise for such entities. The appetite for good credit risk models also increased due to intensified regulation after the recent economic failures and the introduction of Basell II which allowed banks to use the internal rating based (IRB) approach to determine the appropriate levels of capital commensurate with their lending risk. Credit scoring is useful for banks and other lending institutions for two main purposes, namely initial scoring and behaviour scoring (Sarlija et al. 2009). Initial scoring provides a basis for lending institutions to score new credit applicants to decide on whether to grant a loan to an applicant and as well to estimate the down payment and interest rates for different clients. A good scoring model will therefore allow a lending institution to extend credit to less riskier clients, thus maximising profitability (Einav et al. 2013). These models rely on data provided by the client at application (Bellotti and Crook, 2013) and or possibly from other sources such as credit bureaus.

On the other hand, behavioural scoring models are those which allow institutions to predict the repayment behaviour or performance of their clients for purposes of capital management, customer relationship management, profitability forecasting, setting risk based collections and recovery strategies among other reasons (Sarlija et al. 2009; Malik and Thomas, 2012; Baesens et al. 2003). This approach uses data provided by the applicant at loan application as well as information generated by the contact of the client with the bank over time. When time varying behavioural variables and or macroeconomic variables are included, the model becomes dynamic (Bellotti and Crook, 2013).

In the recent years, there has been bourgeoning literature on research focused on improving the predictive performance of different credit risk assessment methods and in some instances with some illusion of progress as illuminated by Hand (2006). Also important is the fact that research focused on behavioural assessment of existing loan obligors also increased as seen above. Even though there is increasing research on behavioural scoring, little research is available on modelling recovery from delinquency to normal performance on loan obligors (Ha, 2010; Ho Ha and Krishnan, 2012). Due to moral hazard and changes in the macroeconomic environment or other circumstances, it may happen that approved loan obligors may show a totally different behaviour than predicted. Such obligors need be to be managed appropriately to avoid credit loss. This neglected subject is very important as it provides lending institutions especially in distressed economic environments with practical information on how best to deal with delinquents to avoid credit loss and to inform risk based collection and recovery strategies (Sarlija et al. 2009).

Using data from a lending company in Zimbabwe, we model delinquents to: 1) determine factors associated with recovery from delinquency to normal performance, 2) estimate the time to recovery and to 3) build a recovery prediction model for retail consumers using survival analysis. This study is specifically important because it uses data from a country experiencing severe economic distress as evidenced by the high proportion of non-performing loans (NPLs). Findings of this study provides the much needed relevant and practical information to guide the development of sound credit risk management policies, risk-based delinquents management, recovery and collection strategies. Specifically, for Zimbabwe, the study is very timely as it comes at a time when the Reserve Bank of Zimbabwe (RBZ) is putting in place measures to resolve NPLs in the banking sector including the acceleration of the establishment of a credit registry and the institution of an entity- Zimbabwe Asset Management Corporation (ZAMCO) to deal with NPLs. Equally important, banking institutions are also starting to put in place credit recovery units and strategies (RBZ, 2015). Overall, the findings of this study may be useful to institutions in the lending business, banks, credit bureaus and policy makers. The findings of the study may also provide relevant insights to researchers and practitioners in any other country with a deteriorating macroeconomic environment and troubling non-performing loans.

Following the above discussion, we contribute to the body of literature by providing a comprehensive analysis of how to model recovery from delinquency to normal performance using survival analysis in the context of retail consumer loans taking into account the recurrent nature of delinquency and also including time dependent macroeconomic variables. Though highly neglected (Gupta et al. 2015), the recurrence of delinquency is a practical and common situation in consumer loan portfolios especially in distressed economic environments. To our knowledge, this is the first paper to apply survival analysis to model the recovery of retail consumer loans delinquents taking into account the recurrent nature of delinquency and also including time dependent macroeconomic variables. Bellotti and Crook (2013) mentioned the possibility of clustering of event times as a result of the same individual having several accounts, but they did not take it into account in their study. Even though Ha (2010) as well as Ho Ha and Krishnan (2012) used a Cox proportional hazard (PH) model to model recoverable credit of retail consumers, their studies were silent on both the recurrent nature of delinquency and the use of time dependent macroeconomic variables. In our approach, we achieved this by fitting extended Cox models to take care of time dependent covariates and specifically using the counting process approach of Andersen and Gill (1982) to deal with the recurrent events.

In this study, an obligor was deemed to be in a delinquent state if one had missed one or more payments. On the other hand, recovery to normal performance was marked by the inception of payment after being delinquent. Non-performing loans refers to loans whose principal amount, interest or both has not be paid for more than 90 days (substandard, doubtful and loss categories). The rest of the paper is structured as follows: Section 2 gives an overview of credit risk in Zimbabwe, section 3 provides a taxonomy of statistical methods for credit risk assessment, section 4 describes in detail the methods used to implement the credit recovery prediction model. Section 5 details the results of the study whilst section 6 discusses the implications of the results on delinquency and credit risk management. Lastly, section 7 concludes and summarises the study limitations and provides hints on further research.

2.2 Overview of credit risk in Zimbabwe

It is now more than a decade since Zimbabwe has plunged into economic distress and has not recuperated since then. After a world record high annual inflation of around 231 million percent in 2008 and loss of confidence in own currency by the locals, in January 2009, the country adopted a multi-currency system which marked an improved state of the economy (RBZ, 2010). This system allowed the use of multiple currencies including the Pound Sterling, South Africa Rand, Botswana Pula, with official transactions being carried out though the United States Dollar (USD) (RBZ, 2010). At the same time, a number of distressed and failing banks were closed whilst a number of microfinance institutions were born. As shown in Table 2.1, four commercial banks, three merchant banks and one building society closed down whilst the number microfinance institutions rose from 95 to 147 between 2009 and 2015. The increase in the number of the microfinance institutions was viewed as a positive development as they were envisaged to close the funding gap especially for small business and individuals who could not access funds from commercial banks. However, this development had its repercussions as loans disbursed by these institutions were very expensive with interest rates reaching 40% in the midst of numerous consumer complaints and with challenges on regulation compliance (RBZ, 2015).

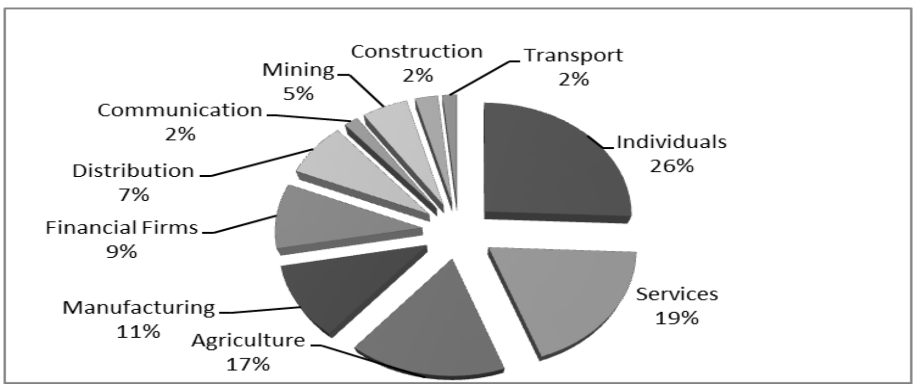
Table 2. 1: Structure of the banking sector (2009 vs 2015)

	Number as at December 2009	**Number as at June 2015**
Commercial Banks	17	13
Merchant Banks	4	1
Building Societies	4	3
Savings Bank	1	1
Microfinance institutions	95	147

Source: RBZ (2015). Mid-Term Monetary Policy Statement. July 2015
RBZ (2010). Monetary Policy Statement. January 2010

Since then, there was an increase in lending from about $639.28 million in December 2009 to four billion in June 2015 (RBZ, 2015; RBZ, 2010). As depicted in Figure 2.1, lending was largely for individuals constituting 26% of total credit (RBZ, 2015) and this also partly explains why our study focuses on consumer loans.

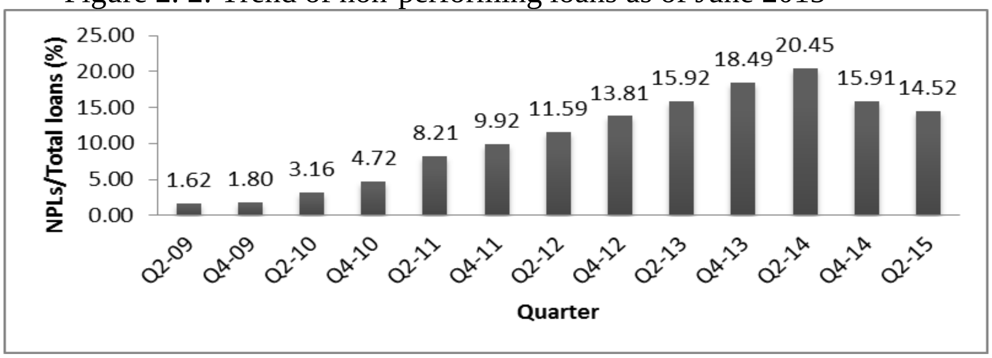
Figure 2. 1: Sectoral distribution of loans as of June 2015



Source: RBZ (2015). Mid-Term Monetary Policy Statement. July 2015.

Due to severe macroeconomic challenges adversely affecting the ability of obligors to pay back as well as high interest rates and poor risk management practices within banks and other lending institutions, credit risk remains a significant challenge facing the sector as reflected by rising non-performing loans (RBZ, 2015). As shown in Figure 2.2, NPLs rose from 1.6 % in June 2009 surpassing the watch list (10%) in June 2012 and continued to spike beyond the close monitoring threshold (15%) in June 2013. The quality of the assets continued to deteriorate until NPLs reached 20.45 % in June 2014 (RBZ, 2015).

Figure 2. 2: Trend of non-performing loans as of June 2015



Source: RBZ (2015). Mid-Term Monetary Policy Statement. July 2015

This situation was also worsened by the absence of a credit registry which can provide essential information about borrowers to compliment underwriting efforts in the country. It is against this background that banks, lending companies and retailers offering credit lines in this environment should draw the best they can from their clients' data to understand their behaviour and manage them accordingly in order to minimise credit loss as well as setting risk-based collection and recovery strategies.

2.3 Credit risk models

2.3.1 Actuarial versus market-based credit risk assessment methods

The two common approaches used for credit risk assessment are the actuarial/statistical based methods and market-based approaches. Actuarial methods provide objective rates of default and use historical data to generate such. Such methods can be applied for both consumers and corporate credit risk assessment. Market-price approaches are used for corporate credit risk assessment and these refer to risk-neutral default probabilities implied from traded market prices of assets such as debt, equity or credit derivatives (Jorison, 2003).

2.3.2 A taxonomy of statistical methods for consumer credit risk assessment

The transition from interview based underwriting to data driven credit scoring dates back to Altman (1968) when the Z score discriminant analysis model was published. Despite being very handy and simple, this model has its limitations including assuming independent variables to be continuous and normally distributed, an assumption which is rarely met (Kalbfleisch and Prentice, 1980). Secondly, it is more sensible to use it when one has an idea of the priori probabilities, a situation which is not always guaranteed (Noh et al. 2005). Besides, it only classifies obligors but cannot provide estimates of the probabilities related to the classification. All these short comings can be solved when one uses logistic regression as the model of choice and thus has become the industry standard, (Professional Risk Managers International Association [PRMIA], 2011). The logistic model assumes that the predicted outcome is a linear function of the predictors through a logit transformation. Several comparison studies have been done to assess the competitiveness of logistic regression in the context of credit risk estimation and mostly the model proved to be very competitive (Stepanova and Thomas, 2002; Fantazzini and Figini, 2009; Noh et al. 2005).

Artificial Neural Networks (ANNs), decision trees, support vector machines and genetic programing have also been successfully applied in credit risk assessment (Hand and Henley, 1997; Huang et al. 2006). ANNs, which are an artificial intelligence method (Russell and Norvig, 2003), were a great success due to their ability to overcome the proportionality and linearity constraints as is the case with parametric models (Bentz and Merunka, 2000) as well as the ability to deal with data whose structure is poorly understood (Hand and Henley, 1997). The decision tree method is a non- parametric approach which uses a decision tree to map observations of an individual to conclusions about the individual's class. Even though it works with both continuous and categorical variables, it is not well suited for continuous variables. Support vector machines are regarded as the best when dealing with small sample classification and regression (Chen et al. 2012). Genetic programming is regarded as an evolutionary algorithm since it finds its roots in biological evolution. Even though this technique is very powerful and provides good performance in terms of accuracy and error rate (Rampone et al. 2013), it is a very complex method.

Even though all the above discussed methods have been successfully applied to credit risk assessment, they have been mostly applied to default probability estimation and were rarely applied to recovery prognostic modelling. It should also be born in mind that in this context, the phenomena of interest (delinquency and hence recovery) is recurrent and most of the above discussed methods are not suited to handle such events. In addition, these models are static, making them inferior to dynamic credit risk assessment models such as survival models since they lack the ability to handle time dependent covariates (Noh et al. 2005).

2.3.3 Survival models for credit risk assessment

The last two decades experienced a shift from static models towards the use of dynamic methods such as survival analysis for credit risk modelling. Among other reasons, survival models have shown to be more informative compared to the other static statistical and machine learning approaches since they do not only predict whether an event will occur but also when it is likely to occur (Tong et al. 2012). Survival analysis also allows the prediction of default probabilities for many different time periods (Einav et al. 2013), thus providing lending institutions important information which facilitates decision making and action taking to prevent default (Lim and Sohn, 2007). Besides, it allows the analysis of the seasoning effects where the probability of default or recovery varies with time as the loan matures (Tong et al. 2012).

Early published research on credit risk assessment using survival analysis methods dates back to Narain et al., (1992). Banasik et al., (1999) further developed the idea using the accelerated life exponential model. A few years later Stepanova and Thomas (2002) also explored survival analysis methods for personal loan data analysis. In a closely related subject, Whalen (1991) evaluated the application of the Cox proportional hazard model in predicting bank failure. Lando (1994) proposed the use of proportional hazard model to model time until bond default. Henebry (1997) also used the proportional hazard model to assess the role of cash flow variables in predicting bank failure. As noted by Tong et al., (2012), the semi-parametric Cox PH model (Cox, 1972) has been widely used survival model is the field. However, there is limited literature on the application of such models on recovery prognosis.

Ha (2010) as well as Ho Ha and Krishnan (2012) successfully used the Cox proportional hazard (PH) model in a hybrid approach to predict recovery on credit cards debts. Overall, their approach showed satisfactory results with good model calibration. However, these studies are different from our approach in the sense that they did not address the typical problem of recurrence of delinquency. Furthermore, attention was only paid at discriminating clients based on the potential to recover and no focus was paid to the effects of macroeconomic conditions to the overall recovery of obligors. In this study, we addressed these essential issues.

2.4 Prognostic recovery model

2.4.1 Data and stylised facts

Data was obtained from a microfinance institution in Zimbabwe under a non-disclosure agreement. This included loans initiated from October 2013 to June 2015. Variables in the dataset made available to us included demographic variables such as gender, marital status, number of dependents, city of residence and some loan variables including loan amount, instalment size, loan term, loan originating branch. The data set also had details on the payment status on each month for every obligor up to September 2015. Based on this, we also computed new behavioural variables including number of missed payments, number of delinquency spells, average duration of delinquency spell, average time interval between delinquency spells, and ratio of delinquent amount to the loan amount. Even though these behavioural variables changed over time, they were fixed at estimation. We also sourced data on some macroeconomic variables including lending rates and real gross domestic product (RGDP) growth from the RBZ reports for the same period. Presented in Table 2.2 are the variables used for modelling recovery. These are categorised into demographic or applicant variables (AV), loan variables (LV), behavioural variables (BV), macroeconomic variables (MV), vintage (Vint) and the event status.

Table 2. 2: Variables used for recovery modelling

Classification	Variable	Description
Applicant Variables	Gender	Gender: Male = 1 Female = 2
	Marital	Marital status: Single = 1, Married = 2, Divorced =3, Widowed =4, Undisclosed =5
	Dep	Number of dependents
	Prov	Province of residence (1-10)
Loan Variables	Loan	Loan amount
	Ins	Instalment Size
	Term	Loan term
	Org	Loan Originator (1-5)
Vintage	Vint	Vintage (Q4-2013 – Q2-2015)
Behavioural Variables	Total_Del	Number of missed payments
	Aver_Del	Average duration of delinquency spell
	Interval	Number/interval of delinquency spell
	Aver_Len_Spel	Average time interval between delinquency spells
	Bal_to_Loan	Ratio of delinquent amount to the contract amount
Macroeconomic Variables	Rates	Bank interest rate/Lending rates
	RGDP	RGDP growth
Status Event	Rec	Outcome: Recover =1, delinquent = 0

The data set comprised of 4575 loan obligors who were followed for a period of up to 23 months. About 2% had not missed any payment and were excluded from the data set. The remaining 4485 had missed at least a single payment and were considered for the recovery modelling process. The extraction of delinquents was followed by data pre-processing which involved creating additional behavioural variables as described above. This was followed by converting the data into the recurrent event data layout which allows implementing the counting process survival analysis technique. The recovery modelling approach and how this feeds into risk-based delinquents and credit risk management are described in detail in the next sections.

2.4.2 Recovery modelling using survival analysis

There are two important concepts in survival analysis and these are the survivor function and the hazard function which we here denote as $S(t)$ and $h(t)$ respectively. Let T denote a random variable representing the survival time of an obligor in a delinquency spell. Suppose that T has a probability distribution with underlying density function $f(t)$, T can be characterised by its cumulative distribution function defined by;

$$F(t) = P(T < t) = \int_0^t f(u)du \text{ , } t > 0. \tag{2.1}$$

That is, for any value of $t > 0$, $F(t)$ is the probability that the survival time will be less than some value t. The survival function $S(t)$ is defined as the probability that the survival time will be greater than or equal to t and is expressed as follows:

$$S(t) = P(T \geq t) = 1 - F(t). \tag{2.2}$$

This is interpreted as the probability of surviving from the time of origin to time beyond t. The hazard function, $h(t)$ is a measure of the instantaneous potential for the event to occur per unit time given that one has survived up to time t. That is the probability that one will recover at time t given that one has been in delinquency until that time. This function $h(t)$ is also known as the hazard rate or in this particular case, recovery intensity. It is expressed as follows:

$$h(t) = \lim_{\delta t \to 0} \left\{ \frac{P(t \leq T < t + \delta t \mid T \geq t)}{\delta t} \right\}. \tag{2.3}$$

In the standard Cox PH model, the effect of covariates is multiplicative with respective to the hazard as follows:

$$h(t, X) = h_0(t) \exp\left[\sum_{i=1}^{p} \beta_i X_i \right] \tag{2.4}$$

The model states that for an individual i at time t, the hazard is a product of the baseline hazard function $h_0(t)$ and the exponential, e of the sum of the linear function $\beta_i X_i$, where X_1 to X_p are explanatory variables. The parameters, β_i of this model can be estimated using the maximum likelihood approximation. Importantly, this model is time independent and as such cannot handle time varying covariates such as macroeconomic variables which are known to have a significant influence on credit risk (Castro, 2013). To solve this problem, an extended Cox model was used to accommodate time dependent variables (Kleinbaum and Klein 2011). The extended Cox model is mathematical expressed follows:

$$h(t, X(t)) = h_0(t)\exp\left[\sum_{i=1}^{p_1} \beta_i X_i + \sum_{j=1}^{p_2} \delta_j X_j(t)\right] \qquad (2.5)$$

where the value of $X_j(t)$ determines the hazard at any given time t and δ_j is the coefficient of $X_j(t)$.

Since the problem at hand is of recurrent identical events (no order of importance), we used the counting process approach (Andersen et al. 1993) to analyse the phenomena. Importantly, the difference in the application of the Cox PH model for analysing recurrent event data and the usual non-recurrent event data (one observation per subject) is on handling the multiple time intervals belonging to the same individual in the formation of the likelihood function maximised for the Cox model. Additionally, in the counting process approach, individuals continue to be in the risk set until all time intervals are completed as opposed to the non-recurrent data were individuals are taken out of the risk set when the event or censorship occurs.

However, the counting process approach, as is the case with the treatment of non-recurrent event data in the standard Cox model, treats the multiple records from each individual as independent even though this is not the case since they belong to the same individual (they are correlated observations). To remedy this problem, the robust estimation is used to account for the correlations of events contributed by the same individual. This process involves the adjustment of the regression coefficients variance estimates (Kleinbaum and Klein 2011) as pioneered by Zeger and Liang (1986) and adapted for recurrent event data by Lin and Wei (1989). Mathematically, the robust variance estimator is expressed as follows:

$$\hat{R}\left(\hat{\beta}\right) = \hat{Var}\left(\hat{\beta}\right)\left[\hat{R}_s^{'}\,\hat{R}_s\right]\hat{Var}\left(\hat{\beta}\right) \tag{2.6}$$

where $\hat{Var}\left(\hat{\beta}\right)$ represents the information matrix and $\hat{R}$ represents the matrix of score residuals.

2.4.3 Model building and model fit

The model building process was done in two steps. The first step involved univariate analysis in which the significance of each covariate was evaluated at the 0.25 significance level as seen in Hosmer et al., (2013). In the second step, all variables significant at 0.25 level were candidates for the multivariate analysis. In the model building exercise, care was given on the purpose of the model. First, we were interested in the effect of each covariate on the hazard function, thus to determine their effects on the recovery of obligors. Second, we were also interested in a model with a minimum combination of variables with a good level of accuracy to predict recovery. Knowing that adding a variable in the model is associated with an increase in its standard error and heavy reliance on observed data, changes in the -2 log likelihood and their significance at the

10% level with appropriate degrees of freedom were used to compare and select the best among the nested models.

2.4.4 Model Evaluation

To evaluate the performance of the model in identifying obligors with the ability to recover from delinquency, we computed the area under the receiver operating characteristic (AUROC) curve for both in-sample (70%) and out of sample (30%) data as seen in Kelly and O'Malley (2016) and Kruppa et al., (2013) . Ideally, if the model exhibits a good calibration, that is the predicted values match the observed values, the area under the curve approaches one. Even though there are no conventional cut-offs, an area under the receiver operating characteristic (ROC) curve of 0.5 shows no discrimination capability whilst greater or equal to 0.7 is deemed acceptable (Hosmer et al. 2013).

2.5 Results and Discussion

Presented in the next three subsections are the results of the descriptive analysis, univariate analysis and multivariate analysis. The first section shows the descriptive statistics which aids the understanding of the data and what to expect in the model. The univariate analysis shows the effect of the individual covariates on the hazard function as well as the strength of the association between covariates and recovery. Lastly, the multivariate analysis section provides the effects of the covariates on the hazard function after controlling for other factors and the strength of the association between covariates and recovery. This section also provides results from competing models, assessment of model fit and overall model performance. Key results are also discussed.

2.5.1. Descriptive Statistics

Table 2.3 shows the distribution of the obligors on various categorical applicant and loan variables. Largely, obligors were married (62%) with the smallest proportion being widowed (2%). Also, 62% were male. Loans were initiated by five originators with the greatest provider having initiated 43% of the loans and the least having initiated 11%. An equal proportion of the obligors (19%) started their contracts in Q4-2013 and Q1-2014 and this gradually decreased to 4% in Q2-2015. For confidentiality and commercial reasons we did not show the distribution by provinces.

Table 2. 3: Distribution of obligors on applicant and loan variables

Variable	Category	%	Variable	Category	%
Marital Status	Single	27	Vintage	Q4-2013	19
				Q1-2014	19
	Married	62		Q2-2014	18
	Widowed	2		Q3-2014	16
	Divorced	3		Q4-2014	15
	Undisclosed	4		Q1-2015	8
Loan Originator	A	11		Q2-2015	4
	B	43	**Gender**	Male	62
	C	13		Female	38
	D	13			
	E	20			

Source: Author's calculations

Table 2.4 provides a summary of descriptive statistics on selected continuous behavioural, applicant, loan and macroeconomic variables. On average obligors had two dependents, with an average loan term of eleven months. Total delinquency ranged from one month to twenty three months with an average of eight months. The minimum number of delinquency spells was one and a maximum of nine spells with an average of 2.4. The average delinquency period was around two months and also taking on average two months between delinquent spells. The mean of the ratio of delinquent amount to loan amount stood at 0.3. Individual lending rates ranged from

12.46% to 14.35% averaging at 14.16%. RGDP growth ranged from -0.082 to 0.746 with an average of 0.311. Also, for confidentiality and commercial reasons we did not provide summaries for some loan variables such as loan amount, delinquent amounts and instalment size.

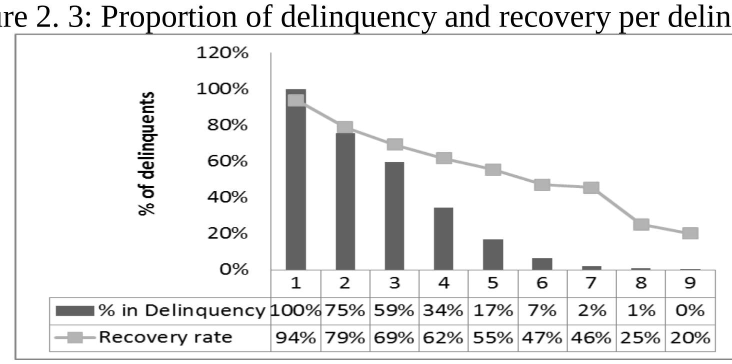
Table 2. 4: Descriptive statistics on selected continuous covariates

Variable	Minimum	Maximum	Mean	SD
Number of dependents	0	11	1.8	1.49
Loan Term	2	24	11.3	2.6
Total delinquency period	1	23	7.7	4.8
Number/Interval of delinquency spell	1	9	2.4	1.4
Average delinquency period	1	23	2.4	2.4
Average length of period between delinquency spells	0	15	2.1	1.6
Ratio of delinquent amount to loan amount	0	1	0.3	0.28
Bank rate/Lending rates	12.46	14.35	14.16	0.24
RGDP growth	-0.082	0.746	0.311	0.285

Source: Author's calculations

As shown in Figure 2.3, recurrence of delinquency was common in this portfolio as evidenced by 75% of the delinquents falling into a second delinquency spell and about 69% entering a third one. It is also shown that about 94% of the delinquents recovered the first delinquency spell whilst of those who entered a second spell, 79% of them recovered. Only about 25% of those who entered an eighth spell recovered whilst only 20% of those who fell into a ninth spell recovered.

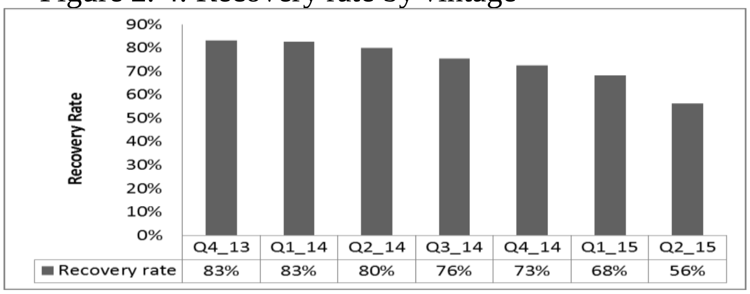
Figure 2. 3: Proportion of delinquency and recovery per delinquency spell



Source: Author's preperation

Shown in Figure 2.4 is that recovery reduced with vintage. About 83% of the loans initiated in the fourth quarter of 2013 and the first quarter of 2014 recovered whilst about 68% and 56% of those initiated in the first and second quarter of 2015 had recovered by the time of the study.

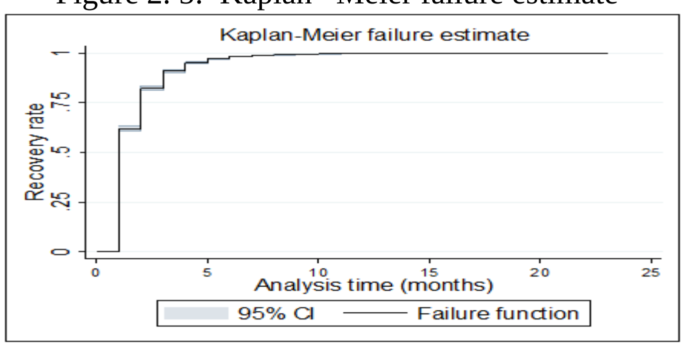
Figure 2. 4: Recovery rate by vintage



Source; Authors' preparation

Figure 2.5 shows the Kaplan-Meier failure estimate. It is evident that the majority (62%) of the obligors escaped delinquency in the first month of the delinquency spell and by the 5th month, 97% of the obligors had escaped delinquency.

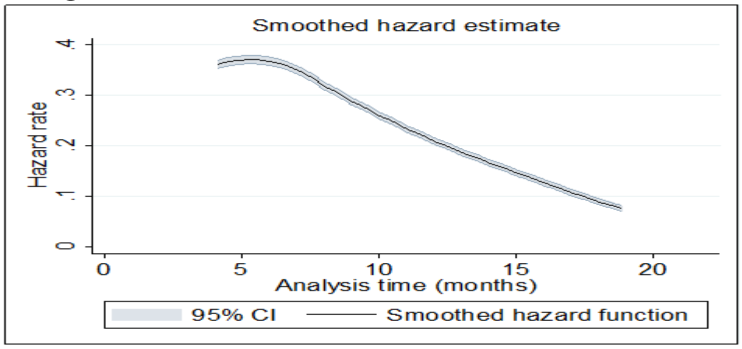
Figure 2. 5: Kaplan –Meier failure estimate



Source: Author's estimation

Also shown in Figure 2.6 is that the instantaneous potential to recover from delinquency to normal performance (recovery intensity) peaked on the fifth month and rapidly fell afterwards.

Figure 2. 6: Hazard rate estimate



Source: Author's estimation

Overall, the time to recovery was favourable since about half of the obligors survived in a delinquent state for at most one month (median survival time) and also 75% of the obligors survived in that state for not more than two months (75^{th} survival time). Equivalently, only 25% of the obligors continued to be in a delinquent state beyond two months. Also, only 10% survived a delinquent state beyond three months, whilst about 5% survived beyond four months and lastly only 1% surviving beyond eight months.

2.5.2 Univariate analysis

As depicted in Table 2.5, the results of the univariate association between covariates and recovery show that all variables except for instalment were significant at 0.25 significance level and were therefore considered for multivariate analysis. By inspecting the signs of the estimates of all variables we had a theoretical understanding on, we found that they behaved (affected the hazard function) in the expected way except for vintage and number of delinquency spells which however behaved as expected after controlling for other factors.

Table 2. 5: Univariate association between covariates and recovery

Variable	Category	Estimate	Robust Std error	P-value
Sex	Male	ref	-	-
	Female	0.1043603	0.0247944	< 0.001***
Marital status	Married	ref	-	-
	Single	-0.114100	0.0287267	< 0.001***
	Widowed	-0.076864	0.0882609	0.384
	Divorced	-0.006087	0.0645252	0.925
	Undisclosed	-0.050239	0.0677819	0.459
Loan Originator	A	ref	-	-
	B	0 .113714	0.0418703	0.007***
	C	-0.378555	0.0511747	< 0.001***
	D	-0.020664	0.0455885	0.650
	E	0 .055708	0.0498452	0.264
Province	1	ref	-	-
	2	-0.139692	0.0377985	< 0.001 ***
	3	0.2202252	0.0696408	0.002***
	4	-0.102662	0.0562061	0.068 *
	5	-0.034297	0.0451185	0.447
	6	-0.165476	0.0630858	0.009 **
	7	0.0027181	0.0579109	0.963
	8	-0.610550	0.2560448	0.017 **
	9	-0.498261	0.1848247	0.007 ***
	10	-0.108298	0.0930368	0.244
Number of Dependents	0	ref	-	-
	1	0.0779142	0.0372447	0.036
	2	0.1441986	0.0349575	< 0.001***
	3	0.1596687	0.0386408	< 0.001***
	4+	0.1490700	0.0409979	< 0.001***
Vintage	Q4-13	ref	-	-
	Q1-14	-0.065565	0.041251	0.112
	Q2-14	-0.041741	0.0413952	0.313
	Q3-14	-0.120120	0.0411678	0.004
	Q4-14	0.1033444	0.0389957	0.008 ***
	Q1-15	0.1815568	0.0415008	< 0.001***
	Q2-15	0 .122259	0.0456256	0.007
Loan Term		0.0382129	0.0042843	< 0.001***
Loan Amount		0.0001151	.0000234	< 0.001***
Instalment		-0.000187	.0002736	0.493
RGDP growth		0.1669354	0.0368064	< 0.001***
Bank/Lending rates		-0.055845	0.0411125	0.174
Number/Interval of delinquency spell	1	ref	-	-
	2	1.138007	.045397	< 0.001***
	3	1.578226	.0494477	< 0.001***
	4	1.932355	.0562663	< 0.001***
	5	2.349516	.069642	< 0.001***
	6-9	2.813066	.0915531	< 0.001***
Total missed payments		-0.115634	0.0021844	< 0.001***
Average delinquency period		-0.388813	0.0044875	< 0.001***
Average length of period between spells		0.1220318	0.0062645	< 0.001***
Delinquent amount to loan ratio		-0.651852	0.0332287	< 0.001***

*sig @ 0.1, **sig @ 0.05, ***sig @ 0.01

2.5.3 Multivariate recovery model

Results from the extended Cox model fitted using the counting process approach are presented in Table 2.6. After 6 iterations the log likelihood converged at -78723.879 ($p < 0.001$). Covariates were considered significant at 10%, 5% and 1% level. Key results are also here discussed.

i. Marital status was the only applicant variable which significantly impacted the hazard function. Compared to the married, the divorced ($p = 0.036$) had a significantly comprised ability to recover. However, by inspecting the hazard ratio, we found that the strength of the relationship was not strong since the married were only 1.03 times more likely to recover than the divorced. The recovery behaviour of the rest was not significantly different from that of the married. Even though females had a better chance to recover than men, this was not statistically significant ($p = 0.188$). Equally important, the provinces of obligors were not an important factor on their ability to recover.

ii. Significant BVs included, number of delinquency spells, ratio of delinquent amount to loan amount ($p < 0.001$), average delinquency period ($p < 0.001$) and average length of period between delinquency spells ($p < 0.001$). As one would expect, an increase on the average delinquency period negatively impacted the ability to recover. Also, a higher delinquent amount to loan amount ratio had a negative effect on recovery. This makes perfect sense since obligors with a comprised ability to payback would have a higher value of this ratio. The number of delinquency spell in the sequence of the spells was also statistically significant. As expected, repeatedly falling into a delinquent state reduced the chances of recovering. For instance it was easier to recover from the first delinquency spell than second and third spell and so on. On the other hand, a higher average period between delinquency spells was the desired situation and it positively affected the ability to recovery. This could be because clients who take long to fall into a delinquent state are good clients and as such have a better chance to recover. The total number of missed payments was omitted from the model since it was highly correlated with the average delinquency period ($r = 0.7$).

iii. Loan variables which were significant included the loan originator and the loan term. We found that, in some way, the recovery behaviour of the obligors was linked to the loan originator. Evidently, obligors linked to originators B ($p < 0.001$), D ($p = 0.010$), and E ($p = 0.010$) were more likely to recover when compared to those from A. On the other hand those linked to C ($p = 0.089$) were less likely to recover. This important pattern in the recovery behaviour tells a story on either substandard underwriting practices or inadequate risk management practices by some originators. Longer terms were also associated with better recovery chances ($p < 0.01$). Instalment size was not included in the model as it was insignificant in the univariate analysis ($p = 0.493$). Besides, it was correlated with the loan amount ($r = 0.77$).

iv. Only two macroeconomic variables were available for inclusion in the model and these were RGDP growth and lending rates. As expected, the falling RGDP significantly impacted on the ability of the delinquents to recover ($p < 0.001$). The variable lending rates was omitted from the multivariate model since it was correlated with RGDP growth ($r = 0.74$). Also, when included, it behaved the opposite of the expected and for sure we can`t expect recovery to improve with an increase on lending rates. It therefore makes sense that RGDP growth could be the probable important macroeconomic variable since a falling GDP signals poor economic performance which leads to poor recovery and not falling lending rates.

v. Vintage was also significant in the multivariate model. After controlling for other factors, loans initiated in Q4-13 had the best chance to recover compared to all the subsequent quarters. Also, it was shown that by the passage of every quarter the chances of recovery reduced. This result disagrees with the findings of Kelly and O'Malley, (2016) who found a lower chance to default and a higher chance to cure on younger mortgage loans in Ireland. This situation could be explained by two reasons; one being the worsening macroeconomic environment as seen above and secondly, a change in the risk appetite by the lender and relaxed underwriting standards to accept more borrowers.

Table 2. 6: Multivariate association between covariates and recovery

Variable	Category	Estimate	Hazard ratio	Robust Std error	P-value
Sex	Male	ref	-	-	-
	Female	.0089606	1.009	0.00687	0.188
Marital status	Married	ref			
	Single	0.0035795	1.00543	0.0086275	0.678
	Widowed	0.016131	1.018886	0.0236435	0.495
	Divorced	-0.0324652	0.966454	0.0169777	0.036**
	Undisclosed	0.0127854	1.018586	0.0167586	0.446
Province	1	ref	-	-	-
	2	0.0108564	1.021698	0.011891	0.361
	3	0.0240229	0.9921453	0.021504	0.264
	4	-0.0087757	0.9968856	0.016221	0.588
	5	-0.003176	1.005298	0.01173	0.787
	6	0.003442	1.005298	0.015106	0.820
	7	-0.0120173	0.9877884	0.015511	0.438
	8	-0.0369097	0.9642466	0.056337	0.512
	9	-0.0200868	0.9861154	0.042442	0.636
	10	-0.0205409	0.9834748	0.025050	0.412
Number of Dependents	0	ref	-	-	-
	1	0.0099680	1.010018	0.0112883	0.372
	2	0.0113057	1.01137	0.0113772	0.315
	3	0.0193915	1.019581	0.0127444	0.121
	4+	0.0181307	1.018296	0.0137751	0.180
Loan Originator	A	ref	-	-	-
	B	0.049187	1.0504170	0.0117373	<0.001***
	C	-0.021235	0.9789889	0.0138405	0.089*
	D	0.036517	1.0371920	0.013879	0.010**
	E	0.037569	1.0382840	0.0145506	0.015**
Loan Term		0.007602	1.0076310	0.0016389	<0.001***
Loan Amount		-1.77e-06	1.00000	0.0000185	0.814
Vintage	Q4-13	ref	-	-	-
	Q1-14	-.0127287	0.9873520	.0174284	0.471
	Q2-14	-.0724697	0.9300939	.0237645	0.005***
	Q3-14	-.1672292	0.8460056	.0266985	<0.001***
	Q4-14	-.2075075	0.8126071	.0282029	<0.001***
	Q1-15	-.2652605	0.7670061	.0277194	<0.001***
	Q2-15	-.3633173	0.6953658	.0284221	<0.001***
RGDP growth		0.1810574	1.201788	0.0377812	<0.001***
Number/Interval of delinquency spell	1	ref	-	-	-
	2	-.2944409	0.7449480	0.0285458	<0.001***
	3	-.4777065	0.6202042	0.0262908	<0.001***
	4	-.5948737	0.5516322	0.0280753	<0.001***
	5	-.5176923	0.5958941	0.0398652	<0.001***
	6-9	-.3464235	0.7072129	0.0619938	<0.001***
Average delinquency period		-0.4096095	0.675245	0.0062541	<0.001***
Average length of period between delinquency spells		0.0138495	1.008388	0.0027599	<0.001***
Delinquent amount to loan ratio		-0.2701777	0.8063792	0.0215665	<0.001***

*sig @ 0.1, **sig @ 0.05, ***sig @ 0.01

2.5.3.1 Model fit

To identify the model which best fits the data, covariates which significantly improved the model fit were identified and retained by inspecting the significance of the change in in the -2 log likelihood after adding new covariates. For AVs, only adding marital status resulted in a significant change in -2 log likelihood. Adding the average delinquency period, number of delinquency spells and ratio of delinquent amount to loan amount also resulted in significant changes in the -2 log likelihood, thus were retained as BVs. In the category of MVs, only RGDP growth was important whilst term and loan originator were the important LVs. Nested models built with the combination of these groups of covariates (AV, BV, LV, MV and Vint) were compared. Again, the significance of the change in -2 log likelihood after adding a set of new covariates was used to evaluate improvement in model fit. The results presented in Table 2.7 show that the model with BVs, MVs and vintage had the best fit as it presented the greatest change in -2 log likelihood with a minimum set of covariates.

Table 2. 7: Model fit assessment

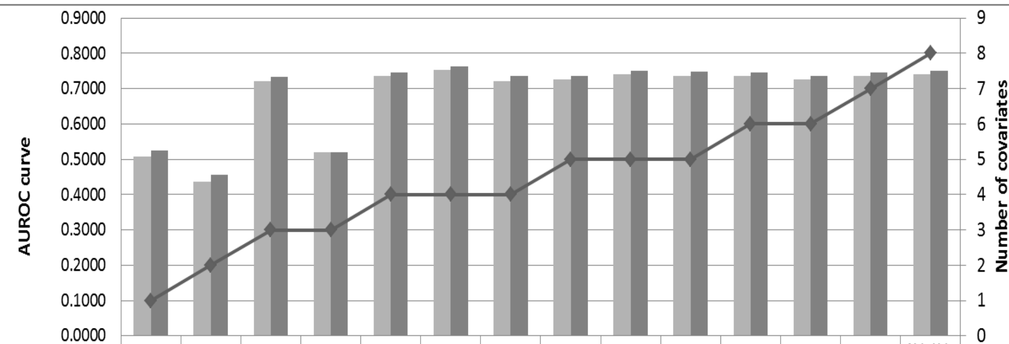
Model	Number of covariates	Change in -2 log likelihood	P value
AV	1	11.576	0.01480
AV+MV	2	20.562	< 0.001
BV	3	2204.858	< 0.001
AV+LV	3	146.776	< 0.001
BV+MV	**4**	**2269.269**	**< 0.001**
BV+Vint	**4**	**2269.887**	**< 0.001**
AV+BV	4	2206.282	< 0.001
BV+LV	5	2226.003	< 0.001
BV+MV+Vint	**5**	**2277.061**	**< 0.001**
AV+BV+MV	5	2269.524	< 0.001
BV+LV+MV	6	2273.29	< 0.001
AV+LV+BV	6	2227.04	< 0.001
AV+LV+BV+MV	7	2273.501	< 0.001
AV+LV+BV+MV+Vint	8	2279.554	< 0.001

*sig @ 0.1, **sig @ 0.05, ***sig @ 0.01

2.5.3.2 Model Evaluation

We computed the in-sample and out of sample AUROC curve for the various competing models discussed above and the results are presented in Figure 2.7. The in-sample data set had 3140 delinquents whilst the out of sample data comprised of 1345 delinquents.

Figure 2. 7: Model performance evaluation

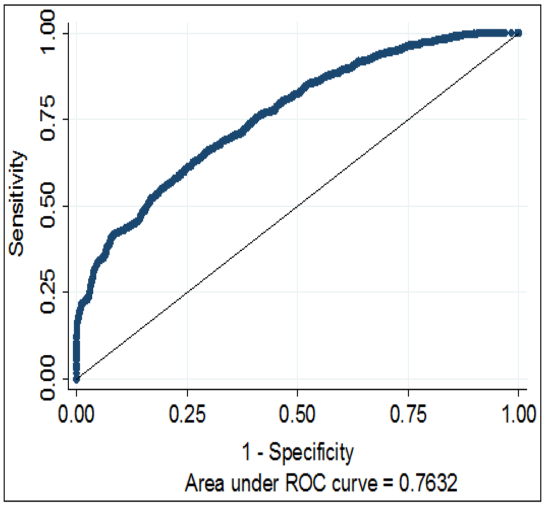


	AV	AV+MV	BV	AV+LV	BV+MV	BV+Vint	AV+BV	BV+LV	BV+MV +Vint	AV+BV+ MV	BV+LV+ MV	AV+LV+ BV	AV+LV+ BV+MV	AV+LV+ BV+MV +Vint
AUROC in-sample	0.5074	0.4373	0.7210	0.5209	0.7359	0.7540	0.7220	0.7253	0.7421	0.7361	0.7351	0.7261	0.7353	0.7415
AUROC out of sample	0.5248	0.4562	0.7345	0.5200	0.7470	0.7632	0.7352	0.7360	0.7498	0.7476	0.7464	0.7371	0.7471	0.7505
Number of covariates	1	2	3	3	4	4	4	5	5	5	6	6	7	8

Source: Authors' estimations

The results showed that the model with behavioural variables and vintage was the best model with minimum covariates in terms of discrimination capability with an AUROC curve of 0.7479 and 0.7632 in-sample and out of sample respectively. Figure 2.8 graphically shows the ROC curve for the BV+Vint model on the out of sample data. In this case the three behavioural variables were; number of delinquency spells, average delinquency period and ratio of delinquent amount to loan amount.

Figure 2. 8: Out of sample ROC curve (BV+Vint)



Source: Authors' estimation

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Even though we had picked the model with BVs, MVs and vintage as best in terms of model fit, we see that in terms of prediction both in sample and out of sample, this model did not exceed that with BVs and vintage. It is also evident that BVs carried the greatest discrimination information and were highly predictive. Alone, the model with BVs was very competitive producing an AUROC curve (0.7345) in an acceptable threshold (more than 0.7). This finding and result ii above corroborate that of Bellotti and Crook, (2013) who found BVs, not only significantly improving model fit but also translating into better forecasting of credit card defaults in United Kingdom. This underscores the fact that understanding the repayment behaviour of the obligors is key to the identification of those likely to recover from delinquency and when it is likely to happen.

Vintage carried important discrimination information. This suggests a great shift in the risk appetite and or changes in the underwriting standards to accept more clients. On another hand, the worsening macroeconomic environment could be responsible to the deteriorating quality of assets.

Even though the falling RGDP is systemic risk affecting all obligors equally and thus, does not help the model to discriminate, its inclusion in the model makes it dynamic, thus providing important information to understand recovery patterns with changes in economic performance as also articulated by Kelly and O'Malley (2016) with respect to the Irish mortgages. The same was also stressed by Castro (2013) who concluded that credit risk was significantly influenced by the macroeconomic outlook in Greece, Portugal Italy, Spain and Ireland. This finding is very essential in setting the appropriate risk appetite and for pricing of loans.

2.6 Implications on delinquency and credit risk management

The fact that only 2% of the 4575 obligors were non-delinquent confirms the riskiness of the environment banks and lending institutions are operating in. In this case, it was evident that delinquency was the 'new normal' with such a limited number of loan obligors finishing their loan commitments without missing a payment. This situation confirms that the country continues to be in the throes of an ailing economic environment. In such an environment, banks and lending institutions need to institute appropriate data driven measures to understand the repayment behaviour of obligors in order to avoid losses.

Even though recovery time was generally favourable, about 25% of the obligors continued to be in a delinquent state beyond two months whilst 10% survived beyond three months and 5% beyond four months. Greatest attention should be paid to those obligors whose predicted longevity in the delinquency state is beyond two and three months. Early interaction with distressed obligors may help to improve the recovery of such.

The findings vividly showed that recovery was mainly explained by average time one stayed in the delinquency state, the number of delinquency spells and the ratio of the delinquent amount to loan amount. This brings to the fore the importance of studying the behavioural aspects of obligors in order to understand the timing of their recovery or lack of it so that appropriate action can be taken at the right time. It was easier for delinquents to escape the first delinquency spell than the subsequent once. This emphasise the importance of early engagement of delinquents to avoid falling into numerous subsequent delinquency spells which are difficult to recover from.

Lower recovery was associated with a high delinquent amount to loan ratio. As such, there is need to emphasise the importance of paying greater care and management of such portions of the loan portfolio. The fact that loan origination improved model fit and carried discrimination information shows that either underwriting standards and or risk management practices were partly responsible for the bad quality of assets held. This finding emphasises the point that the quality of assets could be improved by improving underwriting standards, loan conditions and credit risk management practices.

The longer the terms were, the higher were the chances of recovery. This finding underpins the importance of determining the optimal term to reduce distress. A deteriorating macro-economic environment as captured by persistent negative RGDP growth affected the ability of consumers to recover. As such, the need for urgent policy measures aimed at promoting economic growth and the stabilisation of consumer welfare as well as the financial system as a whole cannot be overemphasised. Given that recovery diminished every quarter, lenders need to strike an informed balance between risk appetite and the developments on the macroeconomic landscape to avoid losses.

2.7 Conclusion

In this paper, we addressed the neglected area of modelling recovery from delinquency to normal performance on retail consumer loans taking into account the recurrent nature of delinquency and also including time dependent macroeconomic variables. We provided a comprehensive analysis of the recovery patterns using the extended Cox model through the counting process approach. We demonstrated that the recurrence of delinquency is a common but neglected situation which increasingly becomes important during times of financial distress as the ability of consumers to honour their loan commitments is seriously compromised. This is particularly timely and relevant for Zimbabwe as the country continues to stagger in an economically distressed environment. We concluded that the falling RGDP representing a debilitating economic situation significantly explains why recovery from delinquency was diminishing. As such, policy measures aimed at unleashing the economic fortunes of the country and bolstering consumer welfare are very urgent. Equally important, lenders need to revisit their underwriting standards and credit risk management practices to mitigate losses. The study underscores the importance of behavioural assessment of delinquents to understand their recovery patterns for optimal timing and decision making in their management.

The main limitation of the study was on the dataset. First, it was small, and only from a microfinance institution, making it difficult to generalise the findings to consumer loans portfolios in Zimbabwe. Conducting a similar study using a larger dataset from different lenders may be beneficial. Secondly, other known important variables such as the employment category and income were not made available to us. We opine that these variables could have helped to improve the discrimination capability of the model. Also, this dataset only comprised of retail borrowers, as such, including other types of consumer loans categories may help to understand the problem better. Furthermore, behavioural variables used in the study were fixed at estimation, further research may consider including dynamic behavioural variables to improve prediction precision. Further research may also consider exploring the changes in marginal probabilities due to changes on the macroeconomic variables.

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Chapter 3

Modelling and forecasting recurrent recovery events on consumer loans

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Abstract

Even though multiple failure-time data are ubiquitous in finance and economics especially in the credit risk domain, it is unfortunate that naive statistical techniques which ignore the subsequent events are commonly used to analyse such data. Applying standard statistical methods without addressing the recurrence of the events produces biased and inefficient estimates, thus offering erroneous predictions. We explore various ways of modelling and forecasting recurrent delinquency and recovery events on consumer loans. Using consumer loans data from a severely distressed economic environment, we illustrate and empirically compare extended Cox models for ordered recurrent recovery events. We highlight that accounting for multiple events proffers detailed information, thus providing a nuanced understanding of the recovery prognosis of delinquents. For ordered indistinguishable recurrent recovery events, we recommend using the Andersen and Gill (1982) model since it fits these assumptions and performs well on predicting recovery.

Keywords: Variance-corrected models; frailty models; multi-state models; Cox model; recurrent events; delinquency; recovery; consumer loans; credit risk.

JEL Classifications: C01: C41; C53; E51; G21; G28

3.1 Introduction

The Basel II and III Accords allowed banks to calculate their capital requirements by the standardized approach or by the internal ratings based (IRB) approach. The need to compute bank capital requirements and the possibility to use internal models demanded the development of more sophisticated modelling approaches and better models of credit risk. Specifically, institutions using the IRB approach are required to compute the probability of default (PD), exposure at default (EAD) and loss given default (LGD). Nevertheless, literature shows that most of the studies, even those using survival models which gained momentum in the recent past (Tong et al., 2012) were focused on modelling PD (Banasik et al., 1999; Bellotti and Crook, 2013; Malik and Thomas, 2010; Noh et al, 2005; Sarlija et al., 2009; Stepanova and Thomas, 2002; Tong et al., 2012).

The least attention given to the modelling of LGD compared to PD is explained by Zhang and Thomas (2012) and Frye and Jacobs (2012) as due to (i) the lack of data and the shortness of time series, (ii) the censored character of the data, which are not easily handled by classical regression techniques, and (iii) the different reasons why the debtors defaulted, which in turn leads to a different repayment patterns. Modelling LGD has been relatively well developed for corporate credit than for consumer credit. In corporate lending LGD can be obtained from analysing historical bond losses (Altman et al., 1977) or simply implied from the market value of the traded defaulted bonds (Altman and Eberhart, 1994). For secured consumer loans, two approaches are common. One is to use the implied historical LGD through the quantification of the realised losses (RL) of a particular portfolio segment. By estimating the PD of that segment, it becomes possible to estimate the LGD since $RL = LGD * PD$ (Zhang and Thomas, 2012). The second approach involves modelling the collection process (Dermine and de Carvalho, 2006) where the probability of a repossession and the value of the repossessed asset is estimated.

For unsecured consumer loan portfolios, modelling LGD is much difficult since there is no recoverable collateral. As a result, modelling LGD sorely depends on the ability or willingness of the obligors to repay, recover or cure. Obtaining the estimates of the cure or recovery rate (RR), thus makes it possible to quantify the LGD since $LGD = 1 - RR$.

There is little research available on the recovery patterns of obligors, especially consumers whose ability to repay their loans has been seriously affected by the major changes on the macroeconomic landscape around the world due to the recent global financial crises failures (Ozkan and Unsal, 2012; Whelan, 2013). Also, with the proliferation of microfinance institutions and mobile money platforms extending credit (usually unsecured) to low income consumers who oftentimes are already in poverty or near so, as well as the increasing payroll lending in emerging markets, and massive over indebtedness (Fanta et al., 2017; Totolo, 2018; Izaguirre et al., 2018; South African Human Rights Commission, 2017; Financial Sector Deepening Africa, 2016), there is increasing need to understand the repayment behaviour, including the recovery prognosis of these distressed obligors.

Rigorous studies on the recovery of distressed borrowers are not only essential for banks and lenders, but also for regulators and policy makers. Such studies may contribute to a better understanding of consumers' over indebtedness, financial distress, and the level of impairment in a financial system. Furthermore, for policy makers and regulators, it is of paramount importance to have enhanced theoretical and empirical methods to detect potential misalignments in the financial system (Aboura and Roye, 2017). Such insights also tell a story on the state of consumer welfare since the performance of the financial sector is correlated to the performance of the economy (Beck et al., 2000; Demirgüç-Kunt and Maksimovic, 1998; King and Levine, 1993). Besides, such information undoubtedly provides useful indications on the degree of impairment and depths of financial distress which can be good signs of alarm should the financial sector be in shock (Castro, 2013). A blind eye on this may fail to detect the deterioration of assets especially in the shadow banking system and may lead to financial instability.

Other than the scarcity of studies on the recovery patterns on consumer loans, there is hardly any research which studied the recovery patterns of consumer loan obligors taking into cognisance the recurrence of delinquency and hence recovery events. Researchers find it easy and convenient to model the time until the first event or current event status and apply standard statistical, data mining or machine learning methods. These methods ignore the occurrence of subsequent events. In contrast, survival models have the ability to handle recurrent events. Besides, they also offer other merits of being dynamic, thus allowing the handling of time dependent covariates (Bellotti and Crook, 2013) the ability to forecast multiple periods (Tong et al., 2012), the ability to handle censored information (Noh et al., 2005), allowing the investigation of seasoning effects (Tong et al., 2012) and the capability of not only predicting whether or not an event (recovery) will occur but when it will occur (Banasik et al., 1999).

In fact, modelling the time until the first event or the event status at estimation (i.e. whether someone has recovered or not) using standard statistical methods such as the logit model or even the standard Cox model (Cox, 1972) means that information about the repayment behaviour of the obligors after the first or before the final observation is wasted. These approaches thus do not effectively make use of the available information. For instance, someone may be in a recovery state at the time of estimation but having experienced delinquency and recovery events several times before the final observed event. This becomes very relevant in over-borrowed or distressed economic environments where delinquency is a ubiquitous phenomenon (Chamboko and Bravo, 2016). Applying standard statistical methods without addressing the recurrence of the events produces biased and inefficient estimates, thus producing erroneous predictions. Employing appropriate methods which account for multiple events proffers detailed information thus providing a nuanced understanding of the recovery prognosis of delinquents.

This paper builds on the study by Chamboko and Bravo (2016) which provided a comprehensive analysis of recovery prognosis considering the recurrence of delinquency and recovery events. In this study, we further explore methods for modelling time to multiple recovery events per individual. We demonstrate the importance of distinguishing between multiple events which have a distinct ordering and those without and how this affects the choice of the model and the implications on the results. Assuming there is order on the occurrence of events, we fit the Andersen and Gill (1982) counting process [AG-CP] model, the Prentice, Williams and Patterson (1981) counting process [PWP-CP] model and the Wei, Lin and Weissfeld (1989) [WLW] model.

These models are applied to a data set from a severely distressed economic environment and the results are compared and discussed.

Importantly, by applying these extended Cox models, we circumvent the pitfall of using the standard Cox model which is only suitable for modelling the time to a single event since these models can model multiple outcome events occurring on a single subject by adjusting for within individual correlation. For the purpose of this study, being delinquent means having missed one or more payments and this includes those technically in a default state, whilst recovery is when a delinquent resume making payments.

The paper is structured as follows. In section 2, we provide a review of statistical methods for modelling recurrent events. Section 3 presents the results of the recovery prediction models. Section 4 discusses the study findings and concludes.

3.2 Methods for handling recurrent events data

Even though multiple failure-time data are ubiquitous in finance and specifically the credit risk domain, it is unfortunate that naive statistical techniques are commonly used to analyse such data. This is despite the fact that there is considerable literature on the modelling of recurrent events in other fields such as medicine (Andersen and Gill, 1982; Lin and Wei, 1989; Moulton and Dibley, 1997; Kelly and Lim, 2000; Shen et al., 2016; Ding and Sun, 2017; Li-An et al., 2018: Bailey et al., 2016 and Duan and Fu, 2015). As far as the authors know, no research is available on the modelling of recurrent events in the context of unsecured consumer loans. This section therefore provides a comprehensive review of methods for handling recurrent events to guide further research in recovery modelling. However, not all methods herein discussed are applied since some of them do not suit the data used in this study.

The commonly used method for analysing survival data is the Cox proportional hazards model (Kleinbaum and Klein, 2011). This has also been the case on the application of survival models in consumer credit risk assessment (Tong et al., 2012). Be that as it may, this model is suited for modelling the time to only one event of interest due to the independence of failure times assumption which states that the timing of failure events is independent. In the case that more than one event is of interest (recurrent events), applying such a model will violate this assumption since observations contributed by a subject would be correlated on that subject. Applying the standard Cox model on recurrent events data leads to incorrect decision as the null hypothesis will be rejected more than it should be due to artificial narrowing of confidence intervals of the estimates (Amorim and Cai, 2014).

3.2.1 Variance corrected models

As noted above, the contribution of multiple events by an individual violate the traditional independence events assumption for the Cox model. To account for the unobserved heterogeneity, variance corrected models were introduced and these solve this problem by use of robust standard errors or stratification (Box-Steffensmeier and De Boef, 2006).

The Andersen–Gill model

The AG-CP model extends the Cox model. It is used when events are ordered but of the same type or indistinguishable. This model assumes that past events explain the correlation of event times for an individual. It is most suited when correlation among events is as a result of measured covariates (Moulton and Dibley, 1997). The approach usually uses the robust estimation to adjust the variances of the estimated regression coefficients to account for lack of independence on events contributed by the same individual. It is usually implemented when the overall objective is to assess the effect of relevant predictors on the intensity of the reoccurrence of events. Even though the model assumes that recurrent events follow a non-homogenous Poisson process, such Markovian assumptions can be relaxed thus allowing the model to handle time varying covariates (Wei and Glidden, 1997). This therefore allows the model to include the number of prior recurrences, thus capturing the dependence structure for the recurrent times (Wei and Glidden, 1997). The AG-CP model thus has numerous merits including the ability to handle time-dependent covariates, recurrent events and discontinuous risk events. Mathematically, the AG-CP model is formulated as follows:

$$\lambda_{ik}\left(t / X, \beta\right) = \mathrm{I}_{ik}\left(t\right)\lambda_0\left(t\right)\exp\left(x_{ik}\beta\right) \tag{3.1}$$

where $\lambda_{ik}(t)$ is the hazard function for the k^{th} event of the i^{th} subject at time t; $\lambda_0(t)$ represents the common baseline hazard for all events over time; X_{ik} represents the vector of p covariates for the i^{th} individual; β is a fixed vector of p coefficients; I_{ik} is a predictable process taking values in {1,0} indicating when the i^{th} individual is under observation.

The Prentice, Williams and Patterson (conditional risk set) model

The PWP-CP model is similar to the AG-CP model but stratified by events. This approach is applicable when the events of interest are ordered. The approach analyses the ordered multiple events by stratification depending on the number of events observed during the follow up (Prentice et al., 1981). In this approach, the time until the first event influences the composition of the risk set for subsequent events. All individuals form the risk set for the first stratum. However, only those who experienced the first event form the risk set for the subsequent stratum. An advantage of using this model is its ability to provide both event specific and overall effects of each explanatory variable (Amorim and Cai, 2014). The formula is expressed as follows:

$$\lambda_{ik}\left(t/X,\beta\right)=\mathrm{I}_{ik}\left(t\right)\lambda_{0k}\left(t\right)\exp\left(x_{ik}\beta\right), \tag{3.2}$$

where $\lambda_{0k}(t)$ represents the event-specific baseline hazard for the k^{th} event over time. In this model, a subject is assumed not to be at risk for a subsequent event until a current event has terminated.

The Wei, Lin, and Weissfeld (marginal risk set) model

The WLW is founded on the concept of marginal risk sets. This means that for the event k, at time t, the marginal risk set is composed of those individuals under observation that had not had event k at time t. This model ignores the ordering of events and treats them as were unordered competing risk events (Castañeda and Gerritse, 2010). The maximum number of events a delinquent experience becomes the number of strata for that individual. Using covariate interactions, the model allows for a separate underlying hazard function per event and strata. For an individual i, the hazard function for the j^{th} event is given as follows:

$$\lambda_{ij}\left(t\,|\,X,\beta\right)=\lambda_{0j}\left(t\right)\exp\left(\chi_{ij}\left(t\right)\beta_{j}\right) \tag{3.3}$$

Generalised estimating equations (GEE) using the Poisson distribution

The Poisson regression model is applied to analysing count data when the dependent variable represents the number of independent events that occur during a fixed period of time (Prentice, Williams and Patterson, 1981). The GEE Poisson model estimates the same model, however, allowing for dependence within clusters (Sagara et al., 2014). Therefore, it is appropriate to model recurrent events within a subject, such as in longitudinal data. The regression coefficients are considered as refits and iteratively correct for the within-subject correlation. The conditional mean of the number of events Y can be represented as follows:

$$Ln(Y / X, \beta) = X_i \beta \quad , \tag{3.4}$$

where Ln denotes the natural logarithm, the canonical link between the conditional mean of Y and the linear predictors $X_i \beta = \beta_0 + \beta_1 X_1 + ... + \beta_n X_n$.

3.2.2 Random effects or frailty models

The frailty model (Vaupel et al., 1979), also known as the random effects approach introduces a frailty or random covariate into the model to accounts for unmeasured or unobserved factors which multiplicatively modifies the hazard function of a subject or cluster (Kleinbaum and Klein, 2011; Amorim and Cai, 2014). The idea of frailty can be traced back to the work of Greenwood and Yule (1920) and more developed by Clayton (1978). The term itself was later introduced by Vaupel et al. (1979) who applied it in univariate survival models and its multivariate applications were promoted by Clayton (1978) and was extensively applied by Hougard (2000). Frailty models are an extension of the Cox PH model. They come in two flavours; the univariate frailty and the multivariate frailty. In the univariate case, the frailty component measures heterogeneity as a result of unobserved subject-specific factors that cannot be explained by observed covariates

(Kleinbaum and Klein 2011; Amorim and Cai, 2014). That is, individuals may have different risk levels, even after controlling for known risk factors, because of some relevant but unobserved covariates (Chamboko and Bravo, forthcoming). The frailty parameter therefore incorporates the effect of these unknown covariates.

In the multivariate setting, the frailty model treats repeated events as a special case of the unit level heterogeneity (Box-Steffensmeier and De Boef, 2006). The frailty component is included to account for heterogeneity as a result of unobserved subject-specific factors that may cause within subject correlation. For recurrent events, it induces dependence on the repeated event times (Kelly and Lim, 2000). For the multivariate case, the shared frailty model is the commonly used model (Amorim and Cai 2014) with the shared frailty component assumed to follow a gamma distribution with a mean of one and unknown variance (Therneau and Grambsch, 2000). The gamma distribution is widely used because of its mathematical tractability.

In this situation with recurrent events, clusters are formed, of which each cluster will contain observations from the same individual, to deal with in-subject correlation. In this case, it is not multiple individual sharing the same cluster. Rather, it is multiple observations representing the same subject sharing the same frailty. The model assumes that event times are independent given the frailty values. Unlike variance corrected models which account for the within-subject correlation by using the robust variance estimators to adjust the standard errors of the coefficient estimates, in frailty models, the shared frailty is included into the model and affects the estimated coefficients and the standard errors. The model can be written as follows:

$$\lambda_{ik}\left(t \mid X, \beta, u\right) = u_i \lambda_{ik}\left(t\right) = \lambda_0\left(t\right) \exp\left(x_{ik}\beta + u_i\right) \tag{3.5}$$

where λ_{ik} represents the conditional hazard function for the k^{th} observation from the i^{th} cluster conditional on u_i; $\lambda_0(t)$ is the baseline hazard function; β is the fixed effects vector of dimension p; X_{ik} is the vector of p covariates; u_i is the i^{th} cluster random effect. Observations in the same cluster u share the same frailty factor. Similar to the Cox model and extensions, the baseline hazard function for the frailty model does not adjust by event. However, unlike the Cox model the coefficient estimates of covariates effect from the frailty model may change if a significant random effect is detected.

3.2.3 Multi-state models

Multi-state models (MSM) are an extension of the competing risk models which deals with situations when there is movement from one initial state to multiple and competing absorbing states (Putter et al., 2007). A simplified MSM may have only two states; a transient state and an absorbing state. Through time, an individual can move from one state to another, and such transition may happen through intermediate states. As illustrated in Figure 3.1, if the transition states are of the same type, such becomes an MSM for recurrent events. As in Figure 3.1, a MSM is graphically, represented using boxes and arrows where boxes represent the states whilst the arrows between the boxes represent the transitions.

Figure 3. 1: Illustration of a multistate model for recurrent events

The occurrence of an event, which is also referred to as a transition is central to a MSM which is characterised by the estimation of between states transition probabilities and transition intensities which show the instantaneous potential to transition to another state conditional to occupying another state (Meira-Machado et al., 2009). The transition intensities can be estimated using the AG-CP approach, the PWP model or the Cox model (Andersen and Keiding, 2002).

3.3 Statistical Analyses and Results

3.3.1 Data

The data set used in this study was drawn from a microfinance consumer loan portfolio from Zimbabwe, a country experiencing severe economic distress. It comprises of 4575 delinquents who had missed one or more payments during the loan period. The delinquents' repayment behaviour was followed for a period of up to 23 months, from October 2013 to September 2015. For analysis, the data were converted to the wide format where each row represents one individual at risk of a certain event as illustrated in Figure 3.2. Taking for instance person 57 who had four delinquency spells (intervals) during the repayment period, the same individual therefore occupies four rows. As shown by the status of 1, this individual recovered all the delinquency spells and it took two and four months for this individual to escape the second and forth delinquency spells respectively.

Figure 3. 2: Recurrent events data structure

Person	Interval	Status	Time
57	1	1	1
57	2	1	2
57	3	1	1
57	4	1	4
58	1	1	2
58	2	1	1
58	3	1	1
58	4	1	5
58	5	1	2
58	6	0	1
59	1	1	9
59	2	1	2
59	3	0	5

Notes: Without showing all the variables in the dataset, the extract in Figure 3.2 shows how data is structured in the long format to allow for recurrent event data analysis.

3.3.2 Results

In this section, we present the descriptive statistics on the study variables. We also show the ubiquitousness of delinquency and recovery in the portfolio. We further illustrate the results from three models used to analyse multiple failure-time data namely: (i) the AG-CP model, (ii) PWP-CP model and (iii) the WLW model. Since this study builds from the study by Chamboko and Bravo (2016), only the variables which resulted in the best model fit and discrimination in that study are used. These variables were average delinquency period (months), balance-to-loan ratio, vintage and recovery interval and their summary statistics are given in Table 3.1 and Table 3.2.

Table 3. 1: Summary statistics

Variable	**Mean**	**SD**	**Minimum**	**Maximum**
Average delinquency period (months)	2.4	2.4	1	23
Balance-to-loan ratio	0.3	0.28	0	1

Notes: Average duration of delinquency and balance-to-loan ratio computed for the period October 2013 to September 2015.

As shown in Table 3.1, the average duration of delinquency spells ranged from 1 month to 23 months with an average of 2.4 months and a standard deviation of 2.4 months as well. The balance-to-loan ratio ranged from 0 to 1 with a mean of 0.3 and a standard deviation of 0.28. Table 3.2 shows the distribution of obligors on loan origination period (vintage) which was almost balanced across the quarters.

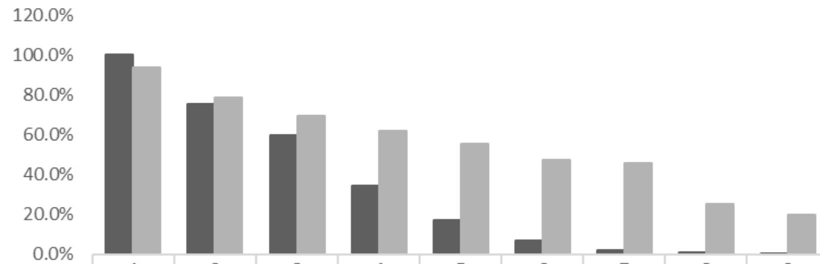
Table 3. 2: Descriptive statistics

Variable	**Category**	**Per cent**
Vintage	Q4-13	19
	Q1-14	19
	Q2-14	18
	Q3-14	16
	Q4-14	15
	Q1 & 2-15	10

Notes: Vintage refers to the specific quarter and year the loan was acquired and this range from quarter 4, 2013 to quarter 2, 2015.

In this portfolio delinquency seemed to be the "new normal", with only 2% of the obligors having not been delinquent (missed a payment) and thus, were excluded from the analysis to remain with 4485 delinquent obligors. As depicted in Figure 3.3 below, some obligors experienced up-to nine delinquency spells (delinquency intervals). On the first delinquent spell, which all experienced, about 94% of them recovered. About 75% experienced the second spell whilst 59% and 34% experienced the third and fourth delinquency spells. Among these 79%, 69% and 61% respectively, recovered. Relatively, a smaller fraction experienced the sixth to the ninth delinquency spells and a range of 47% down to 20% recovered from that.

Figure 3. 3: Delinquency and recovery (cure) rate per delinquency spell



	1	2	3	4	5	6	7	8	9
% in Delinquency	100.0%	75.4%	59.4%	34.3%	16.8%	6.6%	2.0%	0.6%	0.1%
Cure fraction or Recovery rate per delinquency spell	93.8%	78.8%	69.3%	61.7%	55.4%	47.1%	45.6%	25.0%	20.0%

Source: Author's preparation

3.3.2.1 Unordered multiple events

There are unordered events of the same type and unordered events of different types (competing risks). For events of the same type, correlated failure times may arise in studies of individuals or corporates/enterprises working or operating in the same industry, sector or region. Secondly, such correlated failure times may be observed when the same individuals experience the same event repeatedly. However, in the repeated events scenario, such events are likely to have an order even though there may not be a clear theoretical underpinning to explain the relationship between events. For unordered events of the same type, a shared frailty model can be applied to correct for the correlation due to multiple events contributed by the same individual, (Wei and Glidden, 1997). In this study, the event of interest (recovery) is occurring repeatedly on an individual and it makes sense to say that events are ordered since the second event can only occur after the first event.

3.3.2.2 Ordered multiple events

As discussed above, we assert that events observed in this study were ordered events of the same type occurring repeatedly on the same individual, that is, the second event could only occur after the first event and so on. Candidate models for analysing such ordered failure-time data include three extended Cox models namely, the AG-CP, the PWP-CP and the WLW models. We present the results from these different models as below.

The AG-CP model

Table 3.3 shows the parameter estimates for the AG-CP model. The results show that the average delinquency period was a significant predictor of recovery ($p < 0.001$) whilst the balance-to-loan ratio was not ($p = 0.128$). On vintage, the recovery of obligors whose loans were initiated during quarter 1, 2014 ($p = 0.756$) and quarter 2, 2014 ($p = 0.536$) were not significantly different from those initiated during quarter 4, 2013. However, the recovery of obligors whose loans were initiated from quarter 3, 2014 until quarter 2, 2015 was significantly different from those initiated in quarter 4, 2013. The estimates show that by the passage of each quarter, the chances of recovering diminished. Considering the number of delinquency spells and the recovery thereof, the results show that it was more difficult for obligors to recover from subsequent delinquency spells that the first one (ie, the estimate for the recovery interval, 2, 3 and 4 are more negative compared to interval 1).

Table 3. 3: Parameter estimates for the AG-CP model

Variable	Category	Estimate	Hazard Ratio	Robust Standard Error	P- value
Average delinquency period	-	-0.2782	0.7572	0.0098	<0.001
Balance to loan ratio	-	-0.0859	0.9177	0.0569	0.128
Vintage	Q4-13	-	-	-	-
	Q1-14	0.0123	1.0124	0.0396	0.756
	Q2-14	-0.0240	0.9763	0.0388	0.536
	Q3-14	-0.1970	0.8212	0.0389	<0.001
	Q4-14	-0.1701	0.8430	0.0385	<0.001
	Q1 & 2-15	-0.2381	0.7882	0.0454	<0.001
Recovery Interval	1	-	-	-	-
	2	-0.8218	0.4396	0.0828	<0.001
	3	-1.5765	0.2066	0.1980	<0.001
	4+	-1.6135	0.1992	0.4306	<0.001

Notes: Parameter estimates for the AG-CP model considering the observation period October 2013-September 2015.
Source: authors estimations

The PWP-CP model

Table 3.4 shows the results obtained from fitting the PWP-CP model using the same data and variables as in the AG-CP, except that the number of the event (recovery interval) was used for stratification. The results are not so different from those produced by the AG-CP model showing that the average delinquency period ($p < 0.001$) was a significant predictor of recovery whilst the balance-to-loan ratio was not ($p = 0.180$). In the same way on vintage, the results show that the recovery of obligors whose took loans during quarter 1, 2014 ($p = 0.960$) and quarter 2, 2014 ($p = 0.273$) were not significantly different from those initiated during quarter 4, 2013, whilst the recovery of the rest of the obligors differed significantly from those of Q1 2014.

Table 3. 4: Parameter estimates for the PWP-CP model

Variable	Category	Estimate	Hazard Ratio	Robust Standard Error	P- value
Average delinquency period	-	-0.2846	0.7523	0.0096	<0.001
Balance to loan ratio	-	-0.0757	0.9271	0.0565	0.180
Vintage	Q4-13	-	-	-	-
	Q1-14	0.0020	1.002	0.0445	0.960
	Q2-14	-0.0427	0.9582	0.0380	0.273
	Q3-14	-0.2171	0.8045	0.03872	<0.001
	Q4-14	-0.1903	0.8267	0.0383	<0.001
	Q1&2-15	-0.2600	0.7710	0.0455	<0.001

Stratified by recovery interval

Notes: Parameter estimates for the PWP-CP model considering the observation period October 2013 to September 2015, stratified by recovery interval. Source: Author's estimation

The WLW model

Consistent with the AG-CP and PWP-CP models, the average delinquency period was also statistically significant ($p < 0.001$) in the WLW model (Table 3.5). Different from the other two models, the balance-to-loan ratio was statistically significant in the WLW model ($p = 0.003$). Furthermore, results from using the WLW model show that the recovery of obligors whose loans were initiated in Q1 ($p = 0.022$) and Q2 2014 ($p < 0.001$) were significantly different from those initiated in Q1 2014 contrary to the AG-CP and PWP-CP models.

Table 3. 5: Parameter estimates for the WLW Model

Variable	Category	Estimate	Hazard Ratio	Robust Standard Error	P- value
Average delinquency period	-	-0.3947	0.6739	0.0089	<0.001
Balance to loan ratio	-	-0.0982	0.9064	0.0329	0.003
Vintage	Q4-13	-	-	-	-
	Q1-14	-0.0412	0.9596	0.01725	0.0220
	Q2-14	-0.1428	0.8669	0.0160	<0.001
	Q3-14	-0.3047	0.7373	0.01512	<0.001
	Q4-14	-0.3310	0.7182	0.01591	<0.001
	Q1&2-15	-0.436	0.6464	0.1876	<0.001

Stratified by recovery interval

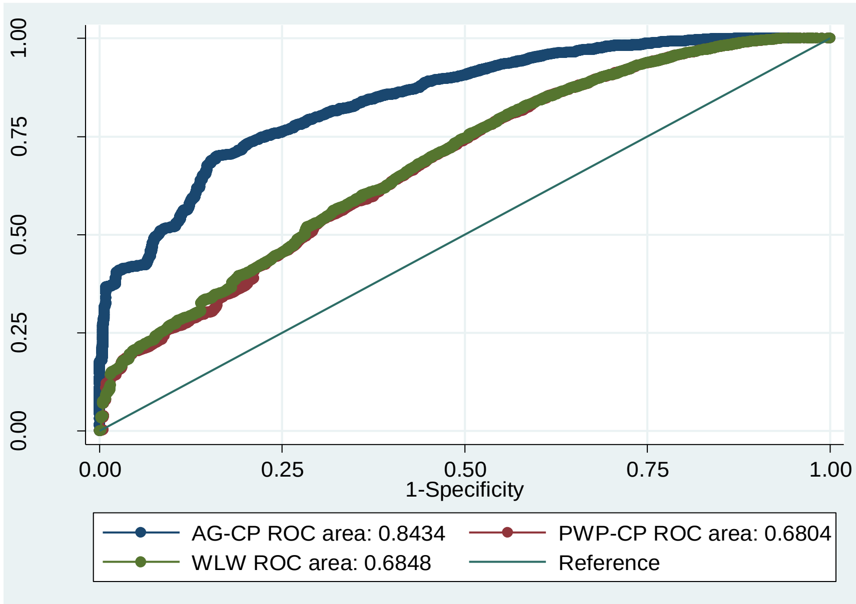
Notes: Parameter estimates for the WLW model considering the observation period October 2013 to September 2015, stratified by recovery interval. Author's estimation.

3.3.2.3 Model evaluation and comparison

The above described models were fitted on a training data set and produced the recorded results. The same models were evaluated in terms of their prediction performance on a test data set which constituted 30 percent of the entire data set. Since the problem at hand is a classification problem using appropriate recurrent events models, the area under the curves (AUC) for each receiver operating characteristics (ROC) curve were used to evaluate the predictive performance of the models. Presented in Figure 3.4 is a comparison of the three ROCs.

The ROC curve is a common measure of a model's discriminant ability in the cross section. An AUC of 1 would mean that the model perfectly predicts the observed classes whilst an AUC of 0.5 would suggest no difference from randomness. The commonly accepted minimum AUC for a model to be deemed to have good discriminant ability is 0.7 (Hosmer et al., 2013). Graphically, the further to the left the curve is, the better the performance of the classifier whilst the closer it becomes to the 45-degree line, the less accurate it is. As shown in Figure 3.4, the ROC curve for the AG-CP model was furthest to the left with an AUC of 0.8434 compared to the PWP-CP and WLW which scored AUC of 0.6804 and 0.6848 respectively.

Figure 3. 4: Area under the ROC curves.



Source: Author's estimation

As shown in Table 3.6, a statistical comparison of the AUCs suggest that they are not equal (chi-square = 1009.82, p < 0.001). With an AUC of 0.8434, the AG-CP model performed the best in predicting recovery of obligors compared to the PWP-CP and WLW models with AUC below the minimum acceptable discrimination threshold of 0.7.

Table 3. 6: Test for equality of AUC

Model	**ROC Area**	**Standard Error**	**95% Confidence Interval**
Andersen and Gill model	0.8434	0.0045	0.83448 : 0.85224
Prentice, Williams and Patterson model	0.6804	0.0066	0.66738 : 0.69334
Wei, Lin, and Weissfeld model	0.6848	0.0065	0.67196 : 0.69762
Ho: area(AG-CP) = area(PWP-CP) = area(WLW), chi2(2) = 1009.82 Prob>chi2 = 0.0000			

Source: Author's estimation

3.4 Discussion and Conclusion

Even though several methods have been proposed for the analysis of recurrent events data, practitioners and researchers continue to use inappropriate methods such as logistic regression or the standard Cox model to analyse such data (Gill et al., 2009; Guo et al., 2008). We emphasise that methods meant for analysing time to first event or merely categorical outcomes cannot be used to investigate the effects of covariates on the occurrence or re-occurrence of multiple events (Dancourt et al., 2004; Gill et al., 2009). This is particularly relevant for modelling and foresting recovery or cure events due to the recurrence nature.

The Poisson model and other count data models, such as negative binomial offer a simple way to analyse recurrent events. However, their pitfall is that they ignore the time between the events and only focus on the number of events per given time. Besides, they do not allow the investigation of changes in event occurrence due to changes in exposure over time (Amorim and Cai, 2014). In multi-state models, the fact that an individual can move from one state to another through time allows multi-state models to capture all possible transitions including through intermediate events, thus providing an ideal way to model recurrent recovery events. This approach provides very rich information on delinquency and recovery, thus allowing for better precision on the prediction of the prognosis of delinquents. However, in the current study, these models were not included in the comparison. The shared frailty model offers a convenient way to model recurrent events by incorporating the shared frailty into the model thus affecting the estimated coefficients and the standard errors. However, this model is not suitable for handling ordered recurrent events.

This study explored various ways of modelling and forecasting recurrent events with applications on recurrent delinquency and recovery events on consumer loans. The choice of the correct model to use differs with the objective the investigation seeks to address. Looking at the data and the underlying assumptions we illustrated the appropriateness of several methods and compared the results. Considering that recurrent events were ordered, we illustrated the application of some variance corrected models namely the AG-CP model, the PWP-CP model and the WLW model. Since this was for illustrative reasons other applicable models such as multi-state models were not used.

The AG-CP and PWP-CP models can be applied to analysing ordered recurrent events of the same type whilst the WLW model can be applied to both analysing recurrent events of the same type and recurrent events of different types (Castaneda and Garritse, 2010). The PWP model makes use of a time-varying strata, allowing the underlying hazard to vary by event. Thus, the PWP model is sometimes referred to as a stratified AG model. The WLW model differs from the PWP model on the definition of the risk set. The WLW model allows individuals to be part of the risk set up to the maximum number of events observed on individuals, an approach which may lead to overestimation of covariates effects (Kelly and Lim, 2000).

With correct model specification, the AG and PWP models produces unbiased estimates and with the same sample size produces almost the same estimates, whilst the WLW model produces biased estimates and may require a larger sample (Castañeda and Gerritse, 2010). The AG-CP and the PWP-CP models are considered to be more efficient compared to the WLW model (Therneau and Grambsch, 2000). This is consistent with our results above with the AG-CP and PWP-CP models producing almost similar estimates and different from the WLW model. However, on application of the model on out of sample (test) data, the AG-CP model performed the best compared to the PWP and the WLW models. These results are corroborated by literature which shows that the AG-CP model requires few assumptions and is as robust as the original Cox model (Sagara et al., 2014; Therneau and Grambsch, 2000; Andersen and Gill, 1982).

Unlike standard statistical methods, these approaches do not only make provision for making inferences about the effects of covariates on recovery or cure times, but also provide insights on the relationship between recovery times and or the effects of prior recovery events on the likelihood of future recovery. In conclusion, we recommend that researchers and practitioners carry out a proper inspection of the type and order of the recurrent events for choosing the model to use since these models work with different assumptions and produce different results. For modelling and forecasting recurrent recovery events which in this paper we identified as ordered and indistinguishable, we recommend using the Andersen and Gill (1982) model since it fits these assumptions well and as well performed best on prediction.

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Chapter 4

A multi-state approach to modelling intermediate events and multiple mortgage loan outcomes

This chapter is made of a paper which was submitted and currently under review in an international journal.

Abstract

Rigorous credit risk analysis is not only of significance to lenders and banks but is also of paramount importance for sound regulatory and economic policy making. Increasing loan impairment or delinquency, defaults and mortgage foreclosures signals a sick economy and generates considerable financial stability concerns. For lenders and banks, the accurate estimation of credit risk parameters remains essential for pricing, profit testing, capital provisioning as well as for managing delinquents. Traditional credit scoring models such as the logit regression only provide estimates of the lifetime probability of default for a loan but cannot identify the existence of cures and or other movements. These methods lack the ability to characterise the progression of borrowers over time and cannot utilise all the available data to understand the recurrence of risk events and possible occurrence of multiple loan outcomes. In this paper, we propose a system-wide multi-state framework to jointly model state occupations and the transitions between normal performance (current), delinquency, prepayment, repurchase, short sale and foreclosure on mortgage loans. The probability of loans transitioning to and from the default state is estimated in a discrete-time multi-state Markov model with seven allowable states and sixteen possible transitions. Additionally, we investigate the relationship between the probability of loans transitioning to and from various loan outcomes and loan-level covariates. We empirically test the

performance of the model using the US single-family mortgage loans originated during the first quarter of 2009 and were followed on their monthly repayment performance until the third quarter of 2016. Our results show that the main factors affecting the transition into various loan outcomes are affordability as measured by debt-to-income ratio, equity as marked by loan-to-value ratio, interest rates and the property type.

Keywords. Credit risk; survival analysis; multi-state models; delinquency; default; foreclosure; recovery. *JEL Classifications:* C01: C41; C53; C58; E51; G21

4.1 Introduction

The critical role of the mortgage market in triggering the recent global financial crisis has led to a surge in policy interest, bank regulation and academic research in this area. Encouraged by regulators, banks now devote significant resources in developing internal credit risk models to better quantify expected credit losses and to assign the mandatory economic capital. Rigorous credit risk analysis is not only of significance to lenders and banks, but is also of paramount importance for sound economic policy making (Kelly and O'Malley, 2016) and regulation as it provides a good check on the "health" of a financial system and at large, the course of the economy (Castro, 2013). Increasing loan impairment or delinquency, defaults and mortgage foreclosures signals a sick economy and generates considerable financial stability concerns.

Given the recent global financial turmoil and economic failures many economies are still struggling to escape the aftermath (Mesnard et al., 2016; Ozkan and Unsal, 2012; Whelan, 2013). Consumer welfare has been jeopardised, and the ability of consumers to honour their loan commitments has been severely affected. Consequently, many economies experienced rising impaired loans which are chocking the financial sector and the overall performance of their economies (Whelan, 2013). For instance, by end of 2015, in Europe alone, Greece and Cyprus reported non-performing loans ratios of more than 40% whilst Romania, Italy, Portugal, Bulgaria, Croatia, Hungary, and Ireland had gross NPL ratio ranging between 10% and 20% (Mesnard et al., 2016).

The regulatory changes brought by the revised Basel Accords (subsequently adopted by national legislation in many countries and regions, e.g., the US Regulatory Capital Rules and the European Capital Requirement Directives) introduced stronger risk management requirements for banks. The main instruments of these regulations are the minimum capital requirements, the supervisory control mechanisms and the market discipline. Under this new regulation, the capital requirements are tightly coupled to estimated credit portfolio losses. According to the Basel II/III "internal ratings-based" (IRB) approach, financial institutions are allowed to use their own internal risk measures for key drivers of credit risk as key inputs in providing loss estimates for the mortgage book and in computing capital requirements (Basel, 2006). To assess the bank's credit risk exposure and provide appropriate loss estimates for the mortgage book, three risk measures are required: (i) the size of exposure at default (EAD), (ii) the probability of default (PD) and (iii) the loss given default (LGD). The expected credit loss or impairment calculation rules imposed by the recently adopted IFRS9 standard require financial institutions to calculate expected loss for the banking book over the entire life of the exposures, conditional on macroeconomic factors, on a point-in-time basis, i.e., recalibrating PDs where necessary to reflect the effects of the current economic conditions.

Traditional credit scoring models applying single-period classification techniques (e.g., logit, probit) to classify credit customers into different risk groups and to estimate the probability of default are still the most popular data mining techniques used in the industry. Despite their popularity, scoring models such as the logit regression can only provide an estimate of the lifetime probability of default for a loan but cannot identify the existence of cures and or other competing transitions and their relationship to loan-level and macro covariates, and do not provide insight on the timing of default, the cure from default, the time since default and time to collateral repossession (Gaffney et al., 2014; Lessmann et al., 2015). Forecasting the dynamic behaviour of the delinquency status of the borrower in critical for the development of customer profit scores.

To better assess the bank's credit risk exposure, we argue that in distressed economic environments the focus of credit risk modelling should go beyond modelling defaults and foreclosures as the only main outcomes and should explicitly allow for other loan-level transitions both into and out of default together with other state occupation experienced by obligors. For instance, it will be of paramount importance and informative for policy to understand the progression of mortgagors from normal performance to delinquency, and subsequently to default and foreclosure as well as the cure from delinquency or default to normal performance. For the US market as an example, understanding of cures is particularly important to gauge if strategies implemented by lenders and other policy interventions such as the US Treasury Department's Home Affordable Refinance Program (HARP) and the Federal Housing Administration's Home Affordable Modification Program (FHA-HAMP) were effective in cutting down losses and preventing ruthless default by underwater mortgagors (Liu and Sing, 2018; Tracy and Wright, 2016).

Also, the transition into other competing and absorbing states such as early or prepayment, repurchase or short sale is of great importance as they are significant competing risks that can affect the profitability and solvency of lending institutions. For lenders and banks, the accurate estimation of these remain essential for pricing, profit forecasting, capital provision as well as for managing delinquents to avoid negative loan outcomes. The introduction of explicit links between PD and mortgages' transitions allows us to avoid the use of exogenous assumptions to model the relationship between static discrete-time PDs and dynamic cash flows.

Several methods are used for consumer credit risk assessment. Logistic regression is the industry standard (Crook et al., 2007; Noh et al., 2005; Lessmann et al., 2015). Bajari et al. (2008) developed a US sub-prime market scoring model using a bivariate probit model allowing borrowers to default either because the mortgage to equity ratio exceeds a certain threshold (due to, e.g., falling home prices) such that default increases their lifetime wealth or because of short-term budget constraints (due to insufficient income and/or lack of access to other forms of credit). Discriminant analysis, decision trees, support vector machines, artificial neural networks, genetic programming and standard models using external ratings provided by external credit assessment institutions have also been successfully applied (Arminger et al., 1997; Hand & Henley, 1997; Baesens et al., 2003; Kruppa et al., 2013; Lessmann et al., 2015; Butaru et al., 2016; Abellán and Castellano, 2017).

Above these, other methods such as survival models have been identified as superior than the former due to their ability to incorporate time varying covariates such as macroeconomic conditions which affect performance on loan payment over time and the ability to forecast event occurrence (default, recovery, prepayment, foreclosure) in the next instant of time, given that the event has not occurred until that time (Bellotti and Crook, 2013; Chamboko and Bravo, 2016).

Commonly, survival models have been used to model the risk of defaulting (Bellotti and Crook, 2013; Noh et al., 2005; Sarlija et al., 2009; Tong et al., 2012; Chamboko and Bravo, forthcoming). A handful of studies have also used the same to model foreclosure on mortgages (Gerardi et al., 2008) and also recovery from delinquency to current or normal performance (Ha, 2010; Ho Ha and Krishnan 2012; Chamboko and Bravo, 2016; Chamboko and Bravo, forthcoming). The competing risks survival framework has also been used to model the competing risks of early payment and default on loan contracts (Deng et al., 1996; Stepanova and Thomas, 2002). The American (put) option-based model of default has also been widely used in the USA (Kau et al., 1992; Deng et al., 2000) and UK markets (Ncube and Satchell, 1994). These models define default as an American option on the house price with a strike price set equal to mortgage value and assume that the borrower will default immediately when the value of the property drops to the level of the mortgage value (Kelly and O’Malley, 2016).

Even though these static statistical methods and the standard survival models have been a success story in credit risk modelling, they do not have the capacity to handle the transition to and from various states, an approach useful when the requirement is to jointly estimate the probability of occupying various states and the corresponding transitions. Motivated by the fixed income market transition based credit risk assessment methods such as the McKinsey's CreditPortfolioView (McKinsey and Company, 1998) and JP Morgan´s Creditmetrics (Gupton et al., 1997), in this paper we address this gap and contribute to the body of literature by proposing a system-wide framework to jointly model state occupations and the transitions between normal performance, delinquency, prepayment, repurchase, short sale and foreclosure on mortgage loans.

The probability of loans transitioning to and from the default state is estimated in a multi-state Markov model with seven allowable states and sixteen possible transitions. In comparison to a classical Logit/Probit framework, the multi-state covariate-driven transition-based framework provides a richer framework for computing expected loss estimates in mortgage books and offers a number of advantages compared to alternative credit risk models. Our work is novel in the sense that it goes beyond modelling default or foreclosure as the only risk on mortgages or the competing risks of prepayment and foreclosure to include transient states of delinquency to allow the modelling of recovery or cure from distress. We also introduce additional absorbing states of short sale and repurchase. Our approach goes beyond portfolio-level transition models which use a discrete-time cohort approach employing simple summing techniques and historical data to estimate the probability that a loan rating will downgrade or upgrade within a specified time, typically one year. In fact, the multi-state approach used in this paper permits an individual estimation of the contribution of loan-specific and macro-level covariates on the movement between the different states.

We circumvent the use of simplified transition approaches, which lack the ability to detect the deterioration of the quality of a mortgage portfolio. In fact, within a year, a mortgage holder may experience severe delinquency spells and recover by the end of the year amounting to a zero-transition probability if simplified transition approaches are used, thus failing to measure one important aspect of risk which credit deterioration. We employ the continuous approach which captures the occurrence of non-absorbing intermediate (recurrent) events, thus allowing for the understanding of the deterioration of the portfolio before the final and absorbing events such as foreclosure take place. Additionally, we investigate the relationship between the probability of loans transitioning to and from various loan outcomes and loan-level covariates. We empirically test the performance of the model using the US single-family mortgage loans originated during the first quarter of 2009 and were followed on their monthly repayment performance until the third quarter of 2016.

The rest of the paper is structured as follows; section 2 review literature on the factors affecting repayment of mortgage loans and commonly applied methods whilst section 3 presents the methodological approaches used. In section 4, the statistical analysis and results are presented and section 5 provides a summary of the study and concludes.

4.2 Literature review

A plethora of studies have looked at factors affecting mortgage repayment with most of them focusing on default or foreclosure especially in the USA real estate market and a sizable number on the UK market. Taking mortgage default as a (put) option, early literature used the Black and Scholes (1973) pioneered contingent claims framework. Using this approach, the key drivers of default were home values and interest rates (Gerardi et al., 2013). Riddiough (1991) provided early insights on the modelling of "trigger events" such as job loss, health shocks, divorce and other accidents. Similarly, Kau et al. (1993), Deng et al. (1996) and many other researchers assessed effects of these trigger events on default and foreclosure and produced mixed findings on the factors which matters the most. Schwartz and Toroush (1993) reported loan vintage and housing index returns volatility as the keys drivers of observed default behaviour. Deng et al. (2000) argued that negative events such as job losses and divorces were significant predictors of mortgage default. Using data on mortgages originated between 2003 and 2007, Mayer et al. (2009) found unemployment and house prices as the key predictors of delinquency in the USA market.

In response to the mortgage default and foreclosure crises which began in 2007, an increased number of researchers analysed and documented numerous factors as the determinants of the observed default and foreclosure behaviour. One of the key hypothesis regarding the causes of mortgage delinquency is that home owners will not continue servicing a mortgage if they enter into negative equity, i.e., if the value of the property falls below the mortgage value (Kau et al., 1992; Kelly and O'Malley, 2016). In this approach, a mortgage is seen as an American option with strike price equal to mortgage value and the property being the underlying asset. It is assumed that the borrower will default immediately when the option enters into the in-the-money zone, i.e., when value of the property drops below the mortgage value. This was also referred to as ruthless or strategic default, a term used to describe a borrower in a position of negative equity who chooses to default despite having enough financial resources to continue servicing the mortgage (Gerardi et al., 2013).

Chan et al. (2014) found loan and individual characteristics such as borrowers credit history, current loan-to-value, race, ethnicity and income are key drivers of foreclosure. Guiso et al. (2009) found severe negative equity, gender, future employment expectations, race and morality as key determinants of ruthless or strategic default. Declining house prices which led to negative home equity and long-term unemployment rates were also found to be key drivers of observed mortgage defaults, foreclosures and housing vacancies (Jones et al., 2016; Tian et al., 2016). Using hazard models, Gabriel and Agarwal (2015) investigated the dynamics of mortgage defaults by introducing time-varying covariates and found that borrowers were mostly sensitive to negative equity during harsh economic times.

Foote et al. (2008) also assessed this concept of negative equity on mortgage default decisions and found that some mortgagors who were in negative equity did not default and argued that this could partly be explained by price expectations. Consistent with that, Foote et al. (2008) also indicated that mortgagors who were in negative equity and defaulted could have done so not only because of negative equity but because of a “double trigger” effect (negative equity combined with some adverse event such as loss of employment, health problems, death of spouse, divorce etc). In agreement with the double trigger hypothesis, numerous other studies also documented that mortgagors could be in negative equity and still not default (Elul et al. 2010; Bhutta et al. 2010).

Using UK data, Aron and Muellbauer (2010) concluded that experiencing negative equity is just one of the fundamental economic drivers of payment delinquency, along with the debt service ratio and the unemployment rates. Danis and Pennington-Cross (2005) conclude that delinquency predominantly leads to termination of a loan through prepayment, while negative equity leads to termination through default. Again, using UK data, Aron and Muellbauer (2016) found that the aggregate debt-service ratio, the proportion of mortgages in negative equity and the unemployment rate have significant effects on aggregate rates of repossessions and arrears. Tian et al. (2016) documented that household and local unemployment rates were key drivers of mortgage defaults.

Carranza and Estrada (2013) found home prices and debt balances as the key determinants of mortgage default on Colombia.

Another key factor explaining mortgage delinquency is the inability to reimburse. This is likely to happen when a reduction in disposable income, often triggered by unemployment spells or family events (e.g., divorce, death of a spouse) reduces the capacity to continue servicing the mortgage. Gerardi et al. (2007), Bhutta et al. (2010), Elul et al. (2010), Bajari et al. (2008), Fuster and Willen (2012) also emphasized the role of cash flow problems or illiquidity as an important factor explaining the inability to continue paying a mortgage loan as someone who is highly illiquid may not be able to find the cash to make the loan payment and may find it costly to wait for house prices to recover. Fuster and Willen (2012) documented the effect of payment size as an important determinant of default and cure (recovery from delinquency) stating as high as 40 % reduction in default rates due to a 2-percentage point reduction in interest rates. Similarly, they found a 75% increase in the cure hazard caused by a 2-2.5 percentage point reduction in interest rates. Tracy and Wright (2016) found that a reduction in a typical monthly payment under the HARP would reduce credit losses by 56 basis points.

Malik and Thomas (2012) developed a Markov chain model based on behavioural scores to establish the credit risk of portfolios of consumer loans. The authors used behavioural scores calculated on a regular basis by consumer lenders as analogues of ratings in corporate credit risk and no application or behavioural variables were included in the study. Grimshaw and Alexander (2011) applied a Markov chain model to subprime loans to forecast the probability of moving between two adjacent time periods into selected states (‘current’, ‘delinquent’, ‘paid-off’) using

Bayes and empirical Bayes estimation methods for the transition matrix, but did not analyse the marginal effects of particular variables on specific transition probabilities.

Jarrow et al. (1997) used a continuous-time Markov chain model to assess the risk of structured finance securities where the states of the Markov chain are the bond rating and the transition matrix (called in this context migration matrix) expresses the probability of migrating to another rating level. Bajari et al. (2013) estimated a dynamic structural model of subprime borrowers' default behavior to study the relative importance of different drivers of default and concluded that principal write downs have a significant effect on borrowers' default behaviour and welfare. Danis and Pennington-Cross (2005) used a two-step procedure and a seemingly unrelated bivariate probit model of mortgage outcomes to estimate probabilities of prepaid and default. The authors concluded that very delinquent loans are more likely to prepay than to default and that prepayment rates increase substantially as delinquency intensity increases.

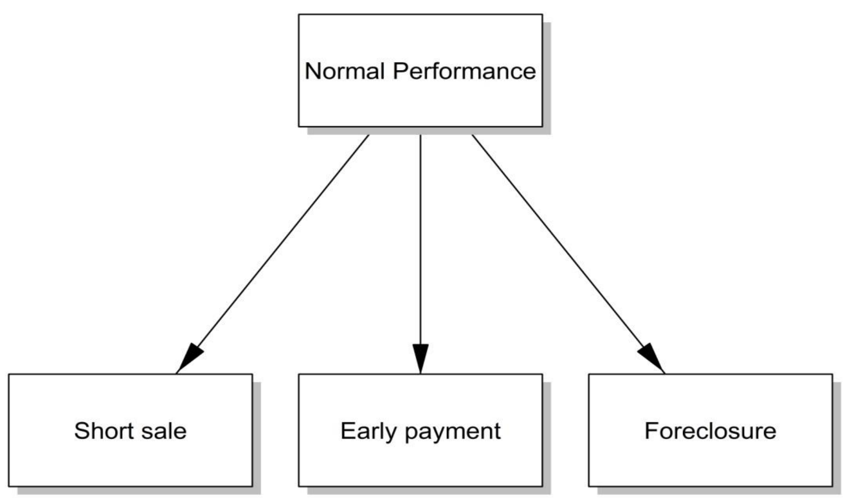
As observed in the literature above, most of the studies investigated mortgage delinquency, default or foreclosure as the main outcomes of interest. Limited literature is available on the modelling of the cure from delinquency to normal performance as well as the transition to other states such as early payment, and none on repurchase and short sale as mortgage loan outcomes. Usually these studies modelled one outcome of interest at any time, thus missing on the dynamics of the portfolio with the passage of time. By modelling the occurrence of multiple loan events at the same time and their recurrences we offer a much richer perspective than these traditional approaches.

4.3 Modelling intermediate events and multiple loan outcomes

4.3.1 Competing risks

Competing risks generalise the standard survival analysis (Beyersmann et al., 2012) and refers to a situation where there is more than one cause of failure or outcomes (i.e., foreclosure, short sale or early payment). As such, competing risks models are meant to deal with situations where there is one initial state and multiple and mutually exclusive absorbing states (Putter et al., 2007). Among the competing risks, only the first one to occur is observed. Diagrammatically, a simple competing risks model is presented as follows (Figure 4.1).

Figure 4. 1: A competing risks scenario with three causes of failure



Source: Author’s preparation

Since this approach only observes the first event as result of the competing risks, subsequent events, if available, are not considered. For instance, a typical mortgage loan gets into delinquency and default before foreclosure, thus having intermediate states which are neither initial nor absorbing. Should the intermediate and later events be of interest, the competing risks approach will fail to fully utilise the available data to understand loan repayment behaviour. In such instances, extensions of competing risk models, particularly multistate models are applicable.

4.3.2 Multi-state models

Multi-state models are an extension of the competing risk models (Putter, et al., 2007). They allow the modelling of events of different types as well as both intermediate and subsequent events. It is often assumed that a multi-state model is a Markov model with the Markov property stating that the transition rate is independent of both the states visited prior the current state and the sojourn time (length of stay in current state). In other words, the future depends on history only through the present. In this paper we focus on discrete-time Markov chains with a finite set of states since mortgage data is observed at discrete-time intervals. It is assumed that the state space of the model describes all the different situations in which the loan can be in. The assumption that there is a simple stochastic model of the dynamics of the credit status allows us then to compute the expected future credit status of each loan.

Andersen and Keiding (2002) describe a multi-state process as a stochastic process $(X(t),\ t \in T)$ with a finite state space $S = \{1, \ldots p\}$ and with right continuous sample path: $X(t+) = X(t)$, with $T = [0, \tau]$ or $[0, \tau)$ with $\tau \leq +\infty$. The process has initial distribution $\pi_h(0) = Prob(X(0) = i)$, $i \in S$. The multi-state process $X(.)$ generate a history X_t consisting of the observation of the process in the interval $[0, t]$. With this history, we can define transition probabilities by:

$$P_{ij}(s,t) = Prob(X(t) = j | X(t) = j, X_{S^-}) \qquad (4.1)$$

where $P_{ij}(s,t)$ is the probability of being in state j at time t given that the subject was in state i at time s. If T is the time needed to reach state j from i, the transition intensity (hazard rate) of $i \rightarrow j$ transition is given by (Andersen and Keiding 2002).

$$\lambda_{ij}(t) = lim_{\Delta t \to 0} \left(\frac{P_{ij}(t \leq T < t + \Delta t | T \geq t)}{\Delta t} \right) \tag{4.2}$$

A state $i \in S$ is absorbing if all $t \in T$, $i \in S$, $j \neq i$, $\lambda_{ij}(t) = 0$, otherwise i is transient. Where there is no transition, transition intensities will be zero for all t. The cumulative transition from $i \rightarrow j$ is given by Nelson-Aalen estimators as;

$$\Lambda_{ij}(t) = \int_0^t \lambda_{ij}(u) du = \widehat{\Lambda}_{ij}(t) = \sum_{sxt} \frac{\mathrm{N}_{ij}(s)}{\mathrm{Y}_i(s)} \tag{4.3}$$

Where $\mathrm{N}_{ij}(s)$ represents the number of transitions observed from state i to state j at time s, and $\mathrm{Y}_i(s)$ represents the number of uncensored individuals in state i at time s. The transition probabilities can be expressed in form of a matrix as

$$P(s,t) = \prod_{u \epsilon (s,t)} (I + \Delta\ \Lambda\,(u)) \tag{4.4}$$

with $P_{ij}(s,t)$ entries, where I is the identity matrix and $\Delta\Lambda(u)$ is a matrix containing elements $\Delta\Lambda_{ij}(u)$, representing the change in the cumulative transition rate between states i and j at time u. The Aalen-Johansen estimator (Aalen and Johansen 1978) is used to estimate the transition probabilities. The Aalen-Johansen estimator $\hat{P}(s,t)$ is derived by replacing $\Lambda(u)$ with the matrix $\widehat{\Lambda}(u)$ of Nelson-Aalen estimators $\left(\widehat{\Lambda}_{ij}(t)\right)$ as below

$$\hat{P}(s,t) = \prod_{u\epsilon(s,t)} \left(I + \Delta\widehat{\Lambda}(u)\right) \tag{4.5}$$

where $\Delta\widehat{\Lambda}(u)$ is the observed change in matrix $\widehat{\Lambda}(u)$, which are basically increments in the at time u, representing the estimates of the cumulative hazard of transitioning from state i to state j j at time u.

For mortgage repayments, all individuals start in the same state (performing/current), thus we can have M, the initial state (at time 0) for the multi-state model. We can also define the cumulative incidence function (CIF) for states Z as follows:

$$CIF_{\mathrm{Z}}(t) = P(T_z \leq t) = P_{\mathrm{MZ}}(0,t) = \left[\prod_{u\epsilon(0,t)} \left(I + \Delta\widehat{\Lambda}(u)\right)\right]_{M,Z} \tag{4.6}$$

where T_z is the time to transition from any other state to state Z.

To analyse the relationship between the time-varying characteristics of borrowers and loans and their transition rates in a multi-state model we model particular transition intensities as functions of both acquisition and performance explanatory variables. To be more specific, we use a proportional hazards Cox model and a multiplicative structure with a common baseline $i \to j$ transition intensity $P_{ij,0}(s,t)$. For an individual mortgage contract, m, with time-fixed covariates $X_m = X_{mk}$, the transition intensity is modelled as

$$P_{ij}^{(m)}(s,t) = P_{ij}^{(0)}(s,t)exp\left(\beta_{ij}' X_m\right) \tag{4.7}$$

where the proportionality factor $exp(\beta_{ijm})$ quantifies the effect of a particular covariate X_{mk} on the transition intensity. We use R package mstate to fit the multi-state model and to estimate the state or transition probabilities $P(s,t)$ and the cumulative transition rates $\Lambda(u)$ whilst the survival package was used for fitting the transition specific prognostic survival models.

4.4 Statistical Analysis and Results

4.4.1 Data

The study analysed 383 770 mortgage contracts from the Fannie Mae single-family mortgage loans originated during the first quarter of 2009 and were followed on their monthly repayment performance until the third quarter of 2016. This is an interesting period to analyse the credit performance of US mortgage loans, following the collapse of Lehman Brothers and the entrance of Fannie Mae & Freddie Mac into Conservatorship in September 2008, the creation of the Troubled Asset Relief Program (TARP) in October 2008, the signing of the Dodd-Frank Wall Street Reform and Consumer Protection Act legislation in July 2010, regulating the financial industry, preventing predatory mortgage lending and make it easier for consumers to understand the terms of a mortgage before finalizing the contract. During this period, cash sales peak, home prices hit bottom and foreclosure rates reached record high levels with close to 1,178 million by mid 2010. The delinquency rate only dropped below 4 percent for first the time in March 2015 since the start of the crisis.

Fannie Mae provides loan performance data on part of its single-family mortgage loans to promote better understanding of the credit performance of Fannie Mae mortgage loans. The population includes two datasets: original loans and loans subsequently refinanced through HARP. The primary dataset contains a subset of Fannie Mae's 30-year and less, fully amortizing, full documentation, single-family, conventional fixed-rate mortgages. Credit performance data can be used to model credit risk from loan acquisition through property disposition (Fannie Mae, 2018). Static acquisition and a selected number of dynamic performance variables were considered for modelling the various loan outcomes. Presented in Table 4.1 and Table 4.2 are the loan acquisition and loan performance variables respectively.

Table 4. 1: Acquisition Variables

Acquisition Variables	Description/values	Allowable values
Loan Identifier	A unique identifier for the mortgage loan.	
Origination Date		MM/YYYY
Loan Purpose	An indicator that denotes if a mortgage loan in a pool is either a purchase money mortgage or refinance mortgage.	P = Purchase, C = Cash-out Refinance R = No Cash-out Refinance, U = Refinance-Not Specified
Product Type	A code that denotes if a mortgage loan is a fixed-rate or adjustable-rate mortgage.	FRM – Fixed-rate mortgage loan
Property State	A two-letter abbreviation indicating the state	
Property Type		SF = Single-Family, CO = Condo, CP = Co-Op, MH = Manufactured Housing, PU = PUD
Relocation Mortgage Indicator		Y = Yes, N = No
Channel/ Origination Type	Retail (R), broker (B) or correspondent (C)	R, B, C
Borrower Credit Score		300 – 850, Blank (if Credit Score is < 300 or > 850 or unknown)
Co-Borrower Credit Score		300 – 850, Blank (if Credit Score is < 300 or > 850 or unknown)
First Payment Date	The date of the first scheduled mortgage loan payment to be made by the borrower	MM/YYYY
Debt-to-Income (DTI) Ratio	A ratio calculated at origination derived by dividing the borrower's total monthly obligations (including housing expense) by his or her stable monthly income.	1% - 64% Blank (if DTI is = 0, or ≥ 65 or unknown)
First-Time Home Buyer Indicator	Denotes whether a borrower or co-borrower qualifies as a first-time homebuyer.	Y = Yes N = No, U = Unknown
Number of Borrowers	The number of individuals obligated to repay the mortgage loan.	1 - 10
Number of Units	The number of units comprising the related mortgaged property.	1-4
Occupancy Status	An indicator that denotes how the borrower used the mortgaged property at the origination date of the mortgage (principal residence, second home or investment property).	P = Principal, S = Second, I = Investor, U = Unknown
Original Combined Loan-to-Value (CLTV)	The CLTV reflects the loan-to-value ratio inclusive of all loans secured by a mortgaged property on the origination date of the underlying mortgage loan.	• 0% - 200% • Blank (if CLTV is > 200 or unknown)
Original Interest Rate	The original interest rate on a mortgage loan	Blank = Unknown
Original Loan Term	The number of months in which regularly scheduled borrower payments are due	301 - 419
Original Loan-to-Value (LTV)	The Original LTV reflects the loan-to-value ratio of the loan amount secured by a mortgaged property on the origination date of the underlying mortgage loan.	0% - 97% Blank = Unknown
Original Unpaid Principal Balance (UPB)	The original amount of the mortgage loan as indicated by the mortgage documents.	

Source: Fannie Mae

Table 4. 2: Performance Variables

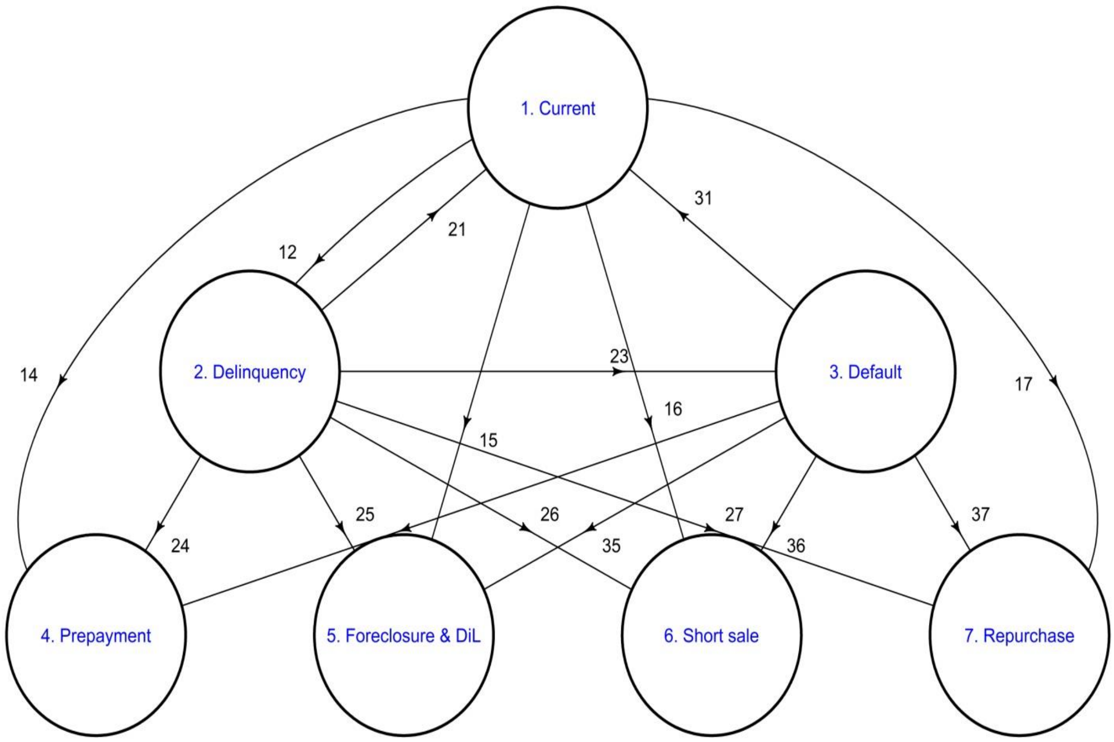
Performance Variables	**Description/values**	**Allowable values**
Current Loan Delinquency Status	The number of days, represented in months, the obligor is delinquent	0 = Current, or less than 30 days past due, 1 = 30 – 59 days. 2 = 60 – 89, 3 = 90 – 119. X = Unknown. Sequence continues thereafter for every 30 day period.
Modification Flag		Y = Yes, N = No
Monthly Reporting Period		MM/DD/YYYY
Loan Age	Months after loan origination	
Zero balance code	A code indicating the reason the mortgage loan's balance was reduced to zero.	01 = Prepaid or Matured, 02 = Third Party Sale 03 = Short Sale, 06 = Repurchased, 09 = Deed-in-Lieu, REO Disposition, 15 = Note Sale, 16 = Reperforming Loan Sale

Source: Fannie Mae

4.4.2 Conceptual Framework

As depicted in Figure 4.2, we propose a multi-state model framework with seven allowable states numbered 1 to 7 with 16 possible transitions. The state numbered 1 is the initial state and states 4 to 7 are absorbing (final) whilst states numbered 1 to 3 are transient (intermediate). The transition from one state to the other is presented by arrows from state $i \rightarrow j$.

Figure 4. 2: A multi-state model framework for analysing mortgage loans data.



Source: Author's preparation.

Table 4.3 defines and describes the different stages of delinquency - performing included in the Fannie Mae dataset. Some states (e.g., default) refer to a "terminal" status, i.e., not subject to change, while others (e.g., delinquency) indicate that loan status may change as it continues to move through its lifecycle.

Table 4. 3: States descriptions

State	Description
Current/normal performance	Less than 30 days overdue on payments
Delinquency	Is the state when a loan obligor had missed payments for 30 to 59 days.
Default	Should the obligor miss payments for 60 days or more consecutively, such is defined as default.
Prepayment	Occurs when a loan obligor pays a loan on a shorter period than contractually agreed.
Mortgage foreclosure	This occurs when the lender repossesses a property due to the failure of borrower to pay in time and or in full instalments.
Deed-in-Lieu, REO Disposition	Is a transaction where the homeowner voluntarily transfers title to the property to the lender in exchange for a release from the mortgage obligation.
Short sale	Also known as a pre-foreclosure sale, is when one sells a home for less than the balance remaining on a mortgage. If the mortgage company agrees to a short sale, one can sell a home and pay off all (or a portion of) mortgage balance with the proceeds.
Recovery or cure	This is a transition. It entails resumption of payments after being delinquent or in default.

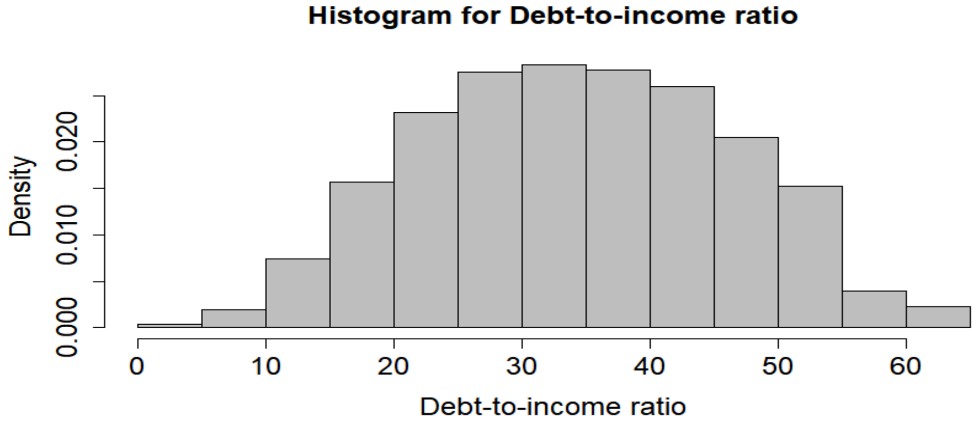
Source: Fannie Mae

4.4.3 Results and discussion

4.4.3.1 Descriptive Statistics

Table 4.4 and Table 4.5 present the characteristics of the borrowers, loans, properties purchased and some borrower behavioural characteristics. On the borrowers, only about 5% were first time home buyers, with most of the contracts owned by two borrowers (median number of borrowers =2). The credit score of the borrowers ranged from 508 to 850, averaging at 763 whilst that of the co-borrowers were almost the same ranging from 505 to 850 averaging at 769.3. The debt-to-income ranged from 1% to 64%, averaging at 33%. As depicted in Figure 4.3, the majority of the borrower’s DTI was between 20% and 50%.

Figure 4. 3: Distribution of debt-to-income ratio.



Source: Author's preparation.

Looking at the loan and property characteristics as well as the purpose, we observed that, even though most of the borrowers were not first home buyers, the majority (93%) used the mortgaged property as their own residence as indicated at the origination date. Only 15% were purchase money mortgages with the other 85% as refinance mortgages. Almost three quarters of the properties were single family properties, and about 20% being planned urban developments and close to 7% as condominium houses. After acquiring the mortgage, almost all borrowers did not relocate from that residency and have also not sought modification of the mortgage.

The monetary value of the properties had a median value of $215 000. Using the loan-to-value figures, the loan amounts averaged close to 70% of the value of the property, ranging from as low as 3% to a maximum of 97% of the property value and the mean loan term was about 30 years (360 months). The origination interest rate ranged from 1.88% to 8.625% averaging at 5%.

Table 4. 4: Descriptive statistics for categorical covariates

Class of variables	Variable	Category	Percent
Loan and borrower characteristics	Channel	Broker	13.6
		Correspondent	31.8
		Retail	54.6
	First Time Home Buyer Indicator	Yes	5.3
		No	94.7
Property characteristics and purpose	Loan Purpose	Purchase	15.3
		No Cash-out Refinance	53.4
		Cash-out Refinance	31.3
	Occupancy Status	Principal	93.2
		Second	4.0
		Investor	2.9
	Property Type	Condo	6.8
		Co-op	0.4
		Manufactured Housing	0.2
		Planned Urban Development	19.5
		Single Family	73.1
	Number of Units	1	98.7
		2	1.1
		3	0.1
		4	0.1
Behavioural variables	Relocation	Yes	0.2
		No	99.8
	Modification Flag	Yes	0.3
		No	99.7

Source: Author's preparation.

Table 4. 5: Descriptive statistics for continuous covariates

Variable	Min	1st quantile	Median	Mean	3rd quartile	Max	Std dev
Borrower characteristics							
Borrowers ‘Credit Score	508.0	741.0	773.0	763.0	793.0	850.0	40.06
Co-borrower’s Credit Score	505.0	751.0	779.0	769.3	797.0	850.0	36.99
Debt-to-Income Ratio	1	24	32	33.08	42.0	64.0	11.81
Number of Borrowers	1.0	1.0	2.0	1.61	2.0	7.0	0.498
Loan characteristics							
Combined Loan-to-Value	3.0	61.0	75.0	79.99	80.0	103.0	15.62
Loan-to-Value	3.0	60.0	73.0	68.77	80.0	97.0	15.65
Original Loan Term (months)	301	360	360	359.9	360	360	1.88
Unpaid Principal Balance	10000	147000	215000	235223	308000	950000	113370.6
Original Interest Rate	1.88	4.75	4.875	4.98	5.125	8.625	0.361
Loan Age (months)	0	30	41.0	45.77	58.0	92.0	24.63
Behavioural variables							
Number of transitions	0	1.0	1.0	1.27	1	63.0	2.156
Sojourned time (months)	1.0	26	40.0	43.61	54	92.0	25.96

Source: Author's preparation.

4.4.3.2 Transition matrix

The transition matrix (Table 4.6) summarises the state occupation probabilities or transitions experienced by borrowers during the study period. The results show that only 11.6% of the borrowers stayed current on their payments during the study follow-up period whilst 16.9% progressed into a delinquent state. Most of the borrowers (71.4%) transitioned from current to prepayment. This high prepayment finding is echoed by Danis and Pennington-Cross (2005) who concluded that prepayment rates increase substantially as delinquency intensity increases.

For those who entered a delinquency state, 70.6% recovered (cured) whilst 27.1% defaulted and 2.3% progressed to prepayment. Of those who reached a default state, 74.9% cured, thus transitioning back to the current state. The high recovery or cure for those both in delinquency and default states is a desirable situation which in this case signalled the ability of borrowers to escape financial distress. This could be attributed to various lenders strategies and policy interventions such as HARP which were implemented in the aftermath of the financial crises to cut down losses and preventing ruthless default by underwater mortgagors (Liu and Sing, 2018). Another 12.7% of those who defaulted progressed to foreclosure, whilst another 6% and 5.4% transitioned to short sale and prepaid respectively.

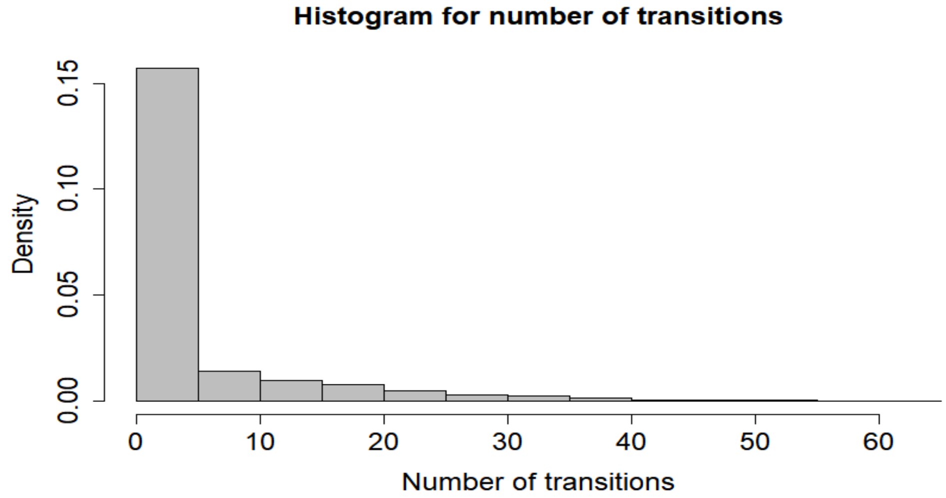
Table 4. 6: Transition matrix

		To						
		Current	Delinquent	Default	Repurchase	Prepaid	Foreclosure	Short sale
From	Current	0.116	0.169	0.0	0.001	0.714	0.0	0.0
	Delinquent	0.706	0.0	0.271	0.0	0.023	0.0	0.0
	Default	0.749	0.0	0.0	0.011	0.054	0.127	0.06

Source: Author's estimation.

Figure 4.4 shows the distribution of the number of transitions experienced by borrowers. Only 11.6% of the borrowers had no transition at all, whilst about 54% transitioned (experienced event) only once, thus Figure 4.4 is skewed towards zero transitions. The direction or the states where the borrowers transitioned to are described in the transition matrix above (Table 4.6).

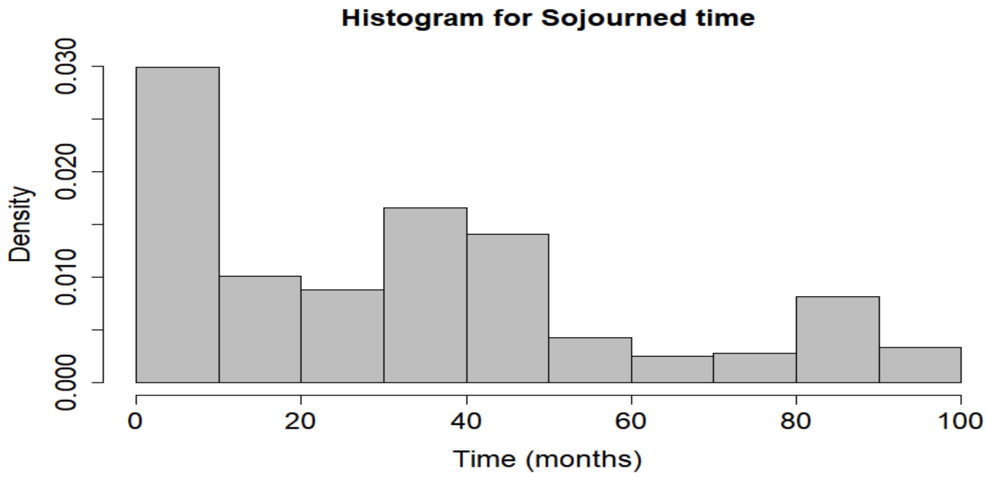
Figure 4. 4: Number of transitions



Source: Author's preparation

Figure 4.5 shows that the time spent in a state before experiencing an event or transitioning to another state. This ranged from one month to 92 months (Table 4.5). About 21% stayed in a state for three months or less before experiencing an event. On average, one stayed in a state for about 40 months (median sojourned time) before transiting into another state (Table 4.5). Further analysis shows that on average, borrowers stayed in the "current" state for about 40 months before transitioning whilst they stayed about 7 months in a default state before transitioning into an absorbing state (foreclosure, repurchase, short sale or prepaid).

Figure 4. 5: Sojourned time

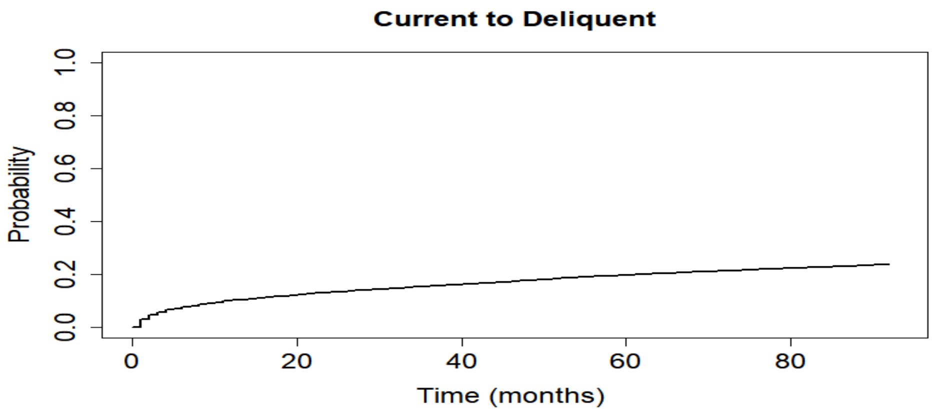


Source: Author's preparation

4.4.3.3 Cumulative incidence functions

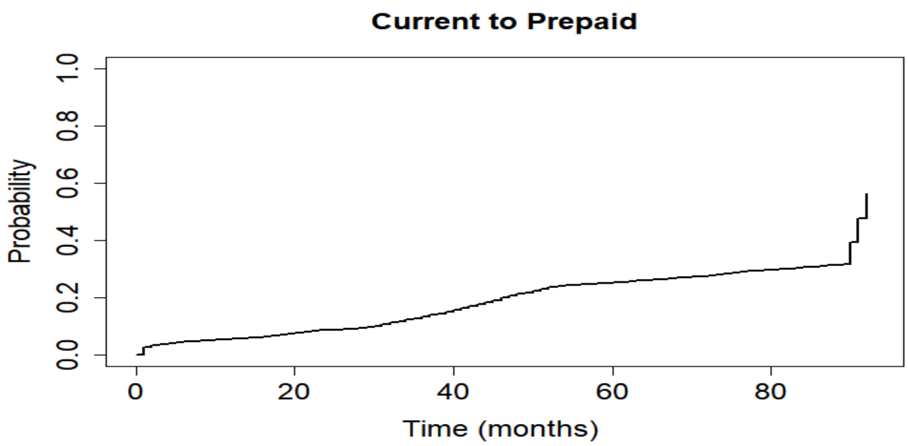
Since the current framework is not for a single risk event occurring per individual, instead it is a multi-state scenario involving recurrent and multiple events, and at any transient state, a borrower had competing risks likely to cause the occurrence of an event, the cause specific failure probabilities are therefore best described by cumulative incidence curves. The cumulative incidence functions in Figure 4.6 to Figure 4.9 below quantify the cause-specific failure probabilities for selected transitions. In this case, the dependent censoring arising from the competing causes renders the Kaplan-Meier estimator (Kaplan and Meier, 1958) inappropriate. As shown in Figure 4.6, the probability of borrowers in a current state to transition to a delinquent state remained low (below 20%) across the duration of the study. However, the probability of transitioning from current to a prepayment state started low during the first months of a mortgage and gradually rose to around 20% after 40 months and at last a sudden spike towards the end of the study period (Figure 4.6). This pattern could be explained by the existence of step-down prepayment penalties which make it expensive to prepay during the early years of the mortgage contract.

Figure 4. 6: Cumulative incidence function (Current to delinquent)



Source: Author's estimation.

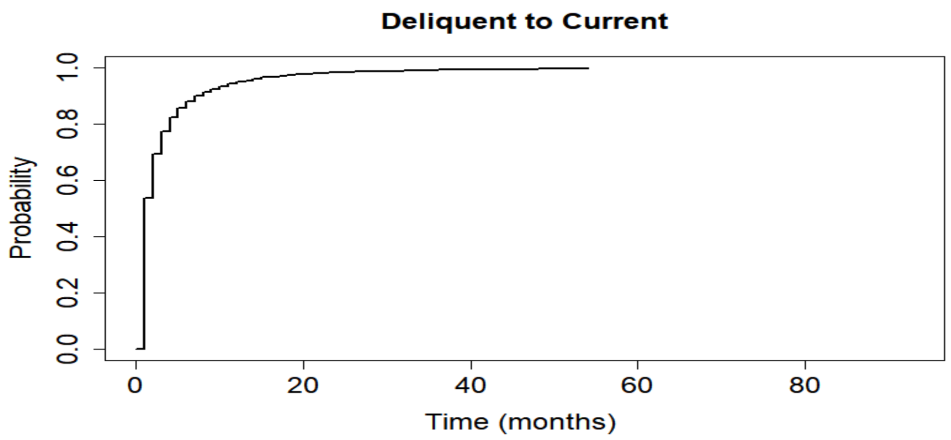
Figure 4. 7: Cumulative incidence function (Current to prepaid)



Source: Author's estimation.

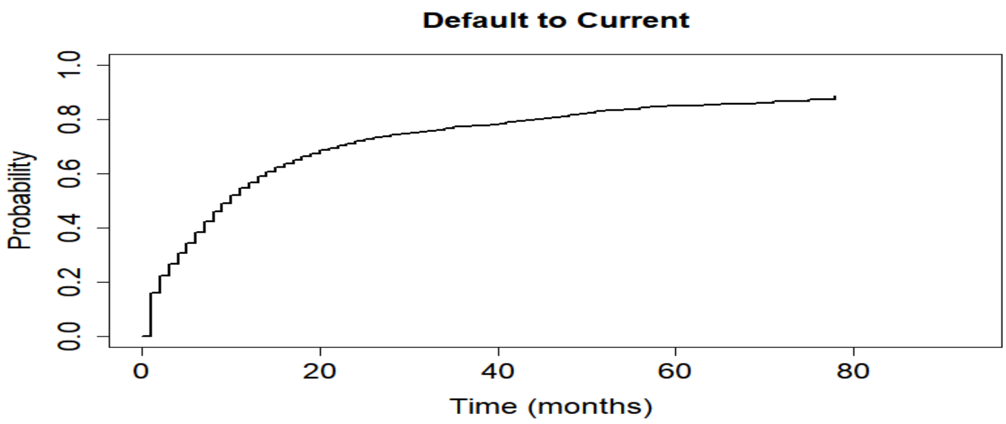
As illustrated on Figure 4.8, borrowers who entered a delinquent state had a very high and encouraging probability to revert to a current state within a month or two. We also note that, for the borrowers who entered a default state, they also had encouragingly high probabilities of reverting to current (Figure 4.9). The chances gradually rose during the first 20 months and then stagnated.

Figure 4. 8: Cumulative incidence function (Delinquent to Current)



Source: Author's estimation.

Figure 4. 9: Cumulative incidence function (Default to current)



Source: Author's estimation.

4.4.3.4 Prognostic factors for event specific transitions

In this section we present the results obtained on the estimation of prognostic factors (covariates) for event specific transitions using the proportional hazards model. We selected the most important transitions as shown in the transition matrix (Table 4.6) and these are current to delinquent and or default, current to prepaid, delinquent to current and default to current.

Current to prepayment transitions

Table 4.7 present the results on the prognostic factors for the transition from normal performance (current) to prepayment. The results suggest that higher borrower's credit score ($p < 0.001$), higher co-borrower credit score ($p < 0.001$), increased number of borrowers ($p < 0.001$), a higher debt-to-income ratio ($p = 0.0122$) and being a first-time home buyer ($p < 0.001$) significantly reduced the propensity to transition from a current state to prepayment. Equally important, longer loan terms ($p = 0.0168$), having more units ($p < 0.001$), and a higher loan-to-value ratio ($p < 0.001$) also reduced the chances of transitioning from current to prepayment whilst higher original principal balances ($p < 0.001$) and higher original interest rates ($p < 0.001$) increased the chances to transition into prepayment.

With respect to the property type securing the mortgage loan, the results show that, compared to condominiums, single-family home ($OR = 1.1,\ p < 0.001$) and planned urban development ($OR = 1.1,\ p < 0.001$) were more likely to transition from current to prepayment and the opposite was true for cooperative share ($OR = 0.78,\ p < 0.001$) and manufactured home ($OR = 0.65,\ p < 0.001$). The results also show that purchase money mortgages were less likely to transition from current to prepayment compared to refinance mortgages ($OR = 0.979,\ p = 0.0076$). Similarity, borrowers who had their mortgage loans modified were twice more likely to transition from a current state to prepayment ($OR = 2.37,\ p < 0.001$). In any case, modifying the loan could mean having issues with the loan or inability to repay, thus incentivising the need to get rid of the mortgage. Mortgage loans made to borrowers whose employers relocate their employees were more likely to transition to prepayment ($OR = 1.29,\ p < 0.001$).

Table 4. 7: Predictors of current to prepaid transitions

Covariate	**Parameter estimate**	**Standard Error(SE)**	**Hazard ratio (HR)**
Modification Y	0.863***	0.0145	2.37
Purpose P	-0.0208***	0.00778	0.979
Purpose R	0.08***	0.00435	1.08
Property Type CP	-0.249***	0.0517	0.78
Property Type MH	-0.424***	0.0408	0.654
Property Type PU	0.0988***	0.00111	1.10
Property Type SF	0.112***	0.0104	1.12
Relocation Y	0.253***	0.0442	1.29
Occupancy P	0.399***	0.0128	1.49
Occupancy S	0.274***	0.0155	1.32
First time home buyer U	-0.553*	0.2.36	0.575
First time home buyer Y	-0.104***	0.0124	0.901
Units	-0.283***	0.0143	0.753
Channel C	0.0267***	0.00635	1.03
Channel R	-0.0933***	0.00587	0.911
Co-borrower credit score	-0.00163***	0.0000712	0.998
Borrower credit score	-0.00174***	0.0000706	0.998
DTI	-0.000439*	0.000175	1.00
Number of borrowers	-0.0999***	0.0181	0.905
LTV	-0.00358***	0.000128	0.996
Loan term	-0.00234*	0.000977	0.998
Original principal balance	0.00000188***	0.0000000177	1.00
Original interest rate	0.644***	0.00551	1.90

*** significant at 1%, ** significant at 5%, *significant at 10%
Source: Author's estimation.

On loan origination channel used, loans originated by retailers were less likely to transition to prepayment compared to those originated by brokers $(OR = 0.911,\ p < 0.001)$ and the opposite was true for those originated by correspondents $(OR = 1.03,\ p < 0.001)$. Considering how the borrower used the mortgaged property at the origination date, the results suggest that those who used property as the principal residence $(OR = 1.49,\ p < 0.001)$ or second home $(OR = 1.32,\ p < 0.001)$ were more likely to transition to prepayment compared to those who regarded these as investment property.

Current to delinquency and default transitions

Table 4.8 presents the results on the factors affecting the transition from current to a delinquency state suggesting difficulties in repaying mortgage loans. Considering the definition applied in this study, delinquency is the state when a loan obligor had missed payments for 30 to 59 days whilst consecutively missing payments for 60 days or more was defined as having defaulted. It is therefore reasonable to say that the factors which affect the transition from current to delinquency are the same for the delinquency to default transition since it is the same path with at most one-month difference.

As expected, higher credit scores for borrowers $(p < 0.001)$ or co-borrowers $(p < 0.001)$ reduced the propensity to transition to a delinquency state. Similarly, being a first-time home buyer $(p < 0.001)$ significantly reduced the propensity to get into a delinquency state. As also expected, a higher debt-to-income ratio $(p < 0.001)$ longer loan term $(p < 0.001)$, higher loan-to-value ratio $(p < 0.001)$, higher original loan amount $(p < 0.001)$ and higher original interest rate $(p < 0.001)$ increased the chances of borrowers to transition into a delinquency state.

Mortgage loans originated by retailers $(OR = 0.785,\ p < 0.001)$ or by correspondents $(OR = 0.897,\ p < 0.001)$ were less likely to get into a delinquency state compared to those originated by brokers. The purpose of the mortgaged property at the origination date was also an important factor explaining transiting into delinquency. Borrowers with properties which were their principal residence $(OR = 1.10,\ p = 0.003)$ had higher chances to fall into delinquency compared to those who regarded these as investment property. On the property type securing the mortgage loan, borrowers who had planned urban developments $(OR = 0.993,\ p = 0.855)$, cooperative share $(OR = 1.13,\ p = 0.402)$ and manufactured homes $(OR = 0.938,\ p = 0.565)$ were not significantly different from those with condominiums whilst single-family home buyers $(OR = 1.1,\ p < 0.001)$ were more likely to transition from current to a delinquency state.

Compared to cash-out refinance, purchase money $(OR = 0.819,\ p < 0.001)$ and no cash-out refinance $(OR = 0.852,\ p < 0.001)$ mortgages were less likely to transition into a delinquency state. Borrowers who had their mortgage loans modified were five times more likely to transition from current to a delinquency state $(OR = 5.57,\ p < 0.001)$. Being a borrower, whose employers relocate their employees $(p = 0.713)$ and the number of borrowers per contract $(p = 0.713)$ were not important factors explaining the transition to delinquency.

Table 4. 8: Predictors **of current to delinquent/default transitions**

Covariate	**Parameter estimate**	**Standard Error (SE)**	**Hazard Ratio (HR)**
Modification Y	1.72 ***	0.0299	5.57
Purpose P	-0.199 ***	0.0136	0.819
Purpose R	-0.16***	0.0136	0.852
Property Type CP	0.123	0.147	1.13
Property Type MH	-0.0636	0.110	0.938
Property Type PU	-0.00682	0.0372	0.993
Property Type SF	0.190***	0.0343	1.21
Relocation Y	-0.0495	0.135	0.952
Occupancy P	0.094***	0.0335	1.10
Occupancy S	0.0218	0.04.41	1.02
First time home buyer U	-0.11	0.0113	0.92
First time home buyer Y	-0.234***	0.0355	0.791
Units	-0.1.58***	0.0361	0.854
Channel C	-0.109 ***	0.0192	0.897
Channel R	-0.242***	0.0179	0.785
Co-borrower credit score	-0.00714***	0.000197	0.993
Borrower credit score	-0.0112***	0.000199	0.989
DTI	0.0214***	0.000558	1.02
Number of borrowers	-0.0186	0.0518	0.982
Loan-to-Value	0.012***	0.000462	1.01
Loan term	0.0294***	0.00525	1.03
Original principal balance	6.62e-07***	5.89e-08	1.00
Original interest rate	0.39***	0.048	1.48

*** significant at 1%, ** significant at 5%, *significant at 10%
Source: Authors' estimation

Delinquent to current transition (cure or recovery)

Table 4.9 presents the factors affecting the transition from a delinquency state to a current state. This transition represents the cure or recovery of distressed borrowers back normal performance (current). It was evident that higher borrowers' credit scores significantly increased the chances of borrowers to cure from a delinquency state as expected ($p < 0.001$). Similarly, increasing the number of borrowers per loan also increased the chances of recovering ($p < 0.001$). As also expected, higher debt-to-income ratio ($p < 0.001$), higher loan-to-value ratio ($p < 0.001$), and higher original interest rate ($p < 0.001$) reduced the chances of recovering for the distressed borrowers.

Other factors which were significant include the type of the property, the purpose of the loan, the type of occupancy and the origination channel. Mortgage loans originated by retailers $(OR = 0.941,\ p < 0.001)$ or by correspondents $(OR = 0.978,\ p < 0.001)$ were less likely to recover compared to those originated by brokers. Borrowers with properties which were their principal residence $(OR = 0.818,\ p < 0.001)$ or second home $(OR = 0.91,\ p = 0.0193)$ had lower chances of recovering from delinquency compared to those who regarded these as investment property.

On the property type securing the mortgage loan, borrowers who had planned urban developments $(OR = 0.95,\ p < 0.001)$ and single-family home buyers $(OR = 0.927,\ p = 0.015)$ were less likely to recover compared to those with condominiums. Those with cooperative share $(OR = 1.23,\ p = 0.024)$ were more likely to recover whilst those with manufactured homes $(OR = 1.04,\ p = 0.6876)$ were not significantly different from those with condominiums. Compared to cash-out refinance, purchase money $(OR = 1.16,\ p < 0.001)$ and no cash-out refinance $(OR = 1.06,\ p < 0.001)$ mortgages were less likely to recover from a delinquency state. In this transition, it was shown that the loan term $(p = 0.8768)$, the original principal balance $(p = 0.9732)$, being a borrower whose employer relocate their employees $(p = 0.4639)$, being a first-time home buyer $(p = 0.445)$ and the number of units $(p = 0.592)$ were not significant.

Table 4. 9: Predictors of the delinquent to current transitions

Covariate	Parameter estimate	Standard Error (SE)	Hazard Ratio (HR)
Purpose P	0.152***	0.0147	1.16
Purpose R	0.0564***	0.00983	1.06
Property Type CP	0.207***	0.0862	1.23
Property Type MH	0.0346	0.0861	1.04
Property Type PU	0.0515**	0.0212	0.95
Property Type SF	-0.0762***	0.0188	0.927
Relocation Y	0.0773	0.106	1.08
Occupancy P	-0.201	0.0216	0.818
Occupancy S	-0.0783**	0.0491	0.925
First time home buyer Y	-0.0172	0.0202	0.985
Units	-0.0116	0.0216	0.988
Channel C	-0.0219*	0.0129	0.978
Channel R	-0.08612***	0.0123	0.941
Borrower credit score	0.00202***	0.0000895	1.0
DTI	-0.00811***	0.000383	0.992
Number of borrowers	0.0378***	0.00865	1.04
LTV	-0.00596***	0.000317	0.994
Loan term	0.000631	0.00407	1.0
Principal balance	-0.00000000143	0.0000000672	1.0
Original interest rate	-0.141***	0.00998	0.868

*** significant at 1%, ** significant at 5%, *significant at 10%
Source: Author's estimation

Default to current transition (cure or recovery)

Table 4.10 presents the factors explaining recovery of distressed borrowers from a default state back to a current state. This transition also represents the cure or recovery of distressed borrowers back normal performance (current). As expected, higher borrowers' credit scores ($p < 0.001$) and higher number of borrowers per loan significantly increased the chances of borrowers to recover. As also expected, higher debt-to-income ratio ($p < 0.001$) higher loan-to-value ratio ($p < 0.001$), higher principal balance ($p < 0.001$), higher number of units ($p < 0.001$) and higher original interest rate ($p < 0.001$) reduced the chances of recovering for the defaulted borrowers.

Compared to cash-out refinance, purchase money ($OR = 1.36,\ p < 0.001$) were more likely to recover whilst no cash-out refinance ($OR = 1.03,\ p < 0.001$) mortgages were not significantly different. It was also shown that the loan term ($p = 0.3534$), the origination channel, being a first-time home buyer ($p = 0.1125$) and the type of occupancy were not significant factors.

Table 4. 10: Predictors of the default to current transitions

Covariate	**Parameter estimate**	**Standard Error (SE)**	**Hazard Ratio (HR)**
Purpose P	0.305***	0.0367	1.36
Purpose R	0.0281	0.0245	1.03
Property Type CP	-0.0112	0.228	0.989
Property Type MH	0.0199	0.223	1.02
Property Type PU	0.236***	0.0535	1.27
Property Type SF	0.242***	0.0417	1.27
Relocation Y	0.604***	0.216	1.83
Occupancy P	0.0404	0.0573	1.04
Occupancy S	0.00872	0.0832	1.01
First time home buyer Y	-0.0753	0.0475	0.927
Units	-0.189***	0.0542	0.828
Channel C	0.0266	0.0315	1.03
Channel R	-0.0284	0.0303	0.972
Borrower credit score	0.00174***	0.000219	1.03
DTI	-0.00415***	0.000983	0.996
Number of borrowers	0.0846***	0.0219	01.09
LTV	-0.00936***	0.00147	0.991
Loan term	-0.00842	0.0091	0.992
Principal balance	-0.000000981***	0.000000018	1.0
Original interest rate	-0.294***	0.0401	0.804

*** significant at 1%, ** significant at 5%, *significant at 10%
Source: Author's estimation

4.5 Summary and conclusion

Sound credit risk analysis remains essential for lenders, banks, policy makers and regulators. This becomes more so especially in the aftermath of the world credit crises when it is important to understand the state of consumer welfare and their ability to honour their loan commitments and recover from distress. The traditional approaches to credit risk modelling consider one risk event at a time and use standard classification methods such as logistic regression or standard survival model or other machine learning classification approaches. These methods lack the ability to characterise the progression of a borrower over time and cannot utilise all the available data to understand the recurrence of risk events and possible occurrence of multiple loan outcomes.

In this paper, we adopt a multi-state approach to modelling the progression of borrowers from one state to another to fully understand the dynamics of a cohort of borrowers over time. We introduce a multi-state framework with seven allowable states namely, current (normal performance), delinquent, default, repurchase, foreclosure, short sale and prepaid. We test this framework using the Fannie Mae data with a cohort of borrowers whose loans where initiated during the first quarter of 2009 and were followed for 92 months until the third quarter of 2016. The transition matrix shows that about 11.6% of the borrowers did not transition to any other state but remained current on their payments during the study follow-up period. Conversely this means, the other 88.4% of the borrowers transitioned into some risky state according to the contractual agreement. As high as 71.4% of the borrowers transitioned from the current state into prepaid, thus having paid the mortgage loan faster than the contractually agreed time. This shows a massive risk to lenders as potential income from interest is lost because of prepayment. However, the probability of prepayment rose towards the end of the study follow up period suggesting that prepayment penalties could be into effect helping to preserve the contract provisions for the first few years.

The results also show that 16.9% of the borrowers missed at least one payment, thus transitioned from current into a delinquent state. Of those who entered a delinquent state, most of them (70.6%) recovered (cured), however noting that another 27.1% defaulted. Importantly, about three quarters of those who entered a default state also recovered to a current state. Overall, there were reasonable recovery rates for those who entered delinquency and default state signalling the ability of borrowers to recover from financial distress. Even though recovery of the defaulters was reasonably high, another quarter transitioned to an absorbing state with most of them (12.7%) progressing into foreclosure, whilst 6% and 5.4% respectively transitioned to short sale and prepaid.

In terms of the factors affecting the transition into various loan outcomes, we see the cross-cutting importance of affordability as measured by debt-to-income ratio, equity as marked by loan-to-value ratio, interest rates, and the property type. Based on the transition matrix, the most import transitions were current to prepaid, current to delinquent and or default, delinquent to current and default to current.

On prepayment, the study shows that higher borrowers' credit score, higher number of borrowers, higher debt-to-income ratio, and being a first-time home buyer, longer loan terms, having more units and a higher loan-to-value ratio significantly reduced the chances of transitioning to prepayment. On the other hand, higher original principal balances, higher original interest rates, having a single-family home or planned urban development, loan modification and relocation increased the chances to transition into prepayment.

For the transitions to delinquency and default, higher borrowers' credit scores, being a first-time home buyer, having the mortgage loans originated by a retailer or correspondent reduced the probability to transition to a delinquency or default state. On the other side, higher debt-to-income ratios, longer loan terms, higher loan-to-value ratios, higher original loan amount and higher original interest rates, being a single-family home buyer, being a borrower with property as the principal residence increased the chances of borrowers to transition into a delinquency or default state.

Recovery or cure was an important aspect of this study. We see that for both recovery from delinquency or default, higher borrower's credit scores and more borrowers per contract increased the chances of recovering. On the other hand, higher debt-to-income ratio, higher loan-to-value and higher original interest rates reduced the borrower's chances of recovering.

We conclude that jointly modelling the described state occupations and transitions allows a system-wide helicopter view which provides a holistic understanding of the dynamics of a mortgage loan portfolio more than just modelling prepayment, foreclosure, delinquency, recovery, repurchase, and short sale separately. We there recommend that during times of economic distress, the focus of credit risk modelling should go beyond modelling defaults and foreclosures as the only main outcomes, especially on mortgage loans as other transitions and state occupation experienced by borrowers becomes critically essential. Further research may consider incorporating other macro-economic variables such as employment rates, housing prices among others since past research shows their importance in explaining repayment behaviour.

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Chapter 5

Frailty correlated default on retail consumer loans in Zimbabwe

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Abstract

There has been increasing availability of consumer credit in Zimbabwe, yet the credit information sharing systems are not as advanced. Using frailty survival models on credit bureau data from Zimbabwe, this study investigates the possible underestimation of credit losses under the assumption of independence of default event times. The study found that adding a frailty term significantly improved the models, thus indicating the presence of unobserved heterogeneity. The major policy recommendations are for the regulator to institute appropriate policy frameworks to allow robust and complete credit information sharing and reporting as doing so will significantly improve the functioning of the credit market.

Key Words: Default clustering; Frailty; Credit risk; Expected losses; Unobserved heterogeneity; Survival models

JEL classifications: C41, C58, D81, E51, G23, G32

5.1 Background

Credit risk models have vastly improved in the past five decades, allowing lenders to assess potential credit loss on their portfolios, thus making capital provisions for expected losses. With the introduction of Basel II and III, most banks have sophisticated their credit risk models to meet the regulatory capital requirements. However, there has not been similar incentives for other retail lenders, yet they are not immune to the risk of borrowers defaulting on their credit.

There has been increasing availability of consumer credit in many developing markets including Zimbabwe (RBZ, 2016), yet the credit information sharing systems are not as advanced (Soledad and Peria, 2014). This mismatch poses the risk of credit loss to lenders as well as the risk of financial distress and over indebtedness to the borrowers, thus obstructing the functioning of the credit market. There is evidence that credit information sharing reduces moral hazard and adverse selection as well as increasing the obligors' incentives to repay their loans (Büyükkarabacak and Valev, 2012; Brown and Zehnder, 2007; Doblas-Madrid and Minetti, 2013). In environments, characterised by limited credit information sharing and incomplete data, there is potential risk of default clustering, specifically frailty correlated defaults due to incomplete data rendering some important variables being unavailable for modelling as well as due to bad borrowers being able to get multiple loans yet having limited capacity to repay.

Default clustering has always been a major source of risk for lending institutions (Qi et al., 2014; Jarrow and Yu, 2001). However, despite its acknowledged importance, discussions around default correlation have hardly progressed from being qualitative (Nickerson and Griffin, 2017; Anagnostou et al., 2018). An understanding of default clustering is very important for risk management as financial institutions such as banks must make capital provision to survive default losses (Chen and Hardle, 2015). In advanced financial markets, the understanding of default clustering of underlying assets is essential for portfolio management and pricing of credit derivatives such as credit default swaps (Azizpour et al., 2018; Kao et al., 2015; Hull and White, 2001; Zhou, 2001; Das et al., 2006). Default clustering can be attributed to three components; macro-economic, frailty and contagion (Koopman et al., 2011, 2012). It is well documented that individuals or firms default rates depend on the prevailing macroeconomic conditions (Castro, 2013; Duffie et al., 2007; Figlewski et al., 2012). Equally important, the state of the macro economy also influences the credit quality of defaultable firms (Guo and Wang, 2018).

The contagion effect of defaults is when defaulting firms weaken related firms which they do business with. In other words, the failure of some may have a cascading or contagion effect (Azizpour and Giesecke, 2008; Jorion and Zhang, 2009). Following the recent global financial crises, it was established that the role of contagion was underestimated and not well captured (Kwon and Lee, 2018; Pourkhanali et al, 2016). The fall of the Lehman Brothers in September 2008 was illustrative as it resulted in the fall of many other firms in the financial industry (Azizpour et al, 2018). As such, industry effects are a major source of credit risk (Schwaab et al, 2016). Contagion models therefore attempt to understand the interaction between firms which the default intensity of one firm tend to jumpily increase as a result of another firm defaulting (Guo and Wang, 2018; Dong et al., (2016).

Frailty correlated default is usually attributed to unobserved heterogeneity, that is unobserved or important factors being omitted from the model. Such omitted or unmodeled variables may turn to cause defaults (Nickerson and Griffin, 2017). Researchers therefore attempt to quantify the additional variation in default intensities due to unobserved or important variables being omitted from the model or other effects that are not easily quantifiable (Duffie et al., 2007; Koopman et al., 2011, 2012). In this case the frailty factor captures default clustering beyond what can be accounted for by observed individual or firm specific factors and macro-economic variables (Duffie et al, 2009).

Despite the great progress made in credit risk modelling, standard credit risk models often ignore the presence of default clustering (Koopman et al., 2011) and are therefore unable to fully explain the observed default clustering and consistently tend to underestimate extreme default losses (Chen and Wu, 2014; Das et al., 2007; Koopman et al., 2012). In fact, it is often assumed that default event times are independent and hence the use of traditional classification statistical methods or the standard Cox model. Introducing a frailty factor captures a meaningful component of the common variation in default rates and its presence increases the default rate volatility compared to a model without it thus shifting the probability mass. As such, ignoring default clustering leads to the underestimation of losses, thus lenders having inadequate capital provisions to buffer the actual losses (Giesecke and Weber, 2004; Foglia et al., 2009).

Given the limitations in credit information sharing in Zimbabwe as well as the incompleteness of credit information (limited variables), there is a possibility of the violation of the assumption that default event times are independent. Using credit bureau data, we investigate this assumption by using a frailty survival model to ascertain if adding a frailty term significantly improved the model. We also investigate the possible underestimation of default hazard under the assumption of independence of default event times. Lastly, we also investigate the effects of various known covariates on default events.

The contribution of this paper to literature is twofold. First, it provides evidence of frailty correlated default on retail consumer credit in Zimbabwe, which is characterised by limited or incomplete credit information sharing. This suggests possible underestimation of credit loses which has implications of the pricing of loans and loss provisioning. Second, and particularly for Zimbabwe, this is the first paper to provide evidence on prognostic factors for default particularly on store credit. The rest of the paper is structured as follows; section 2 provides an overview of frailty modelling through survival analysis. Section 3 presents and discuss the results, section 4 discuss the implications of the findings and section 5 concludes.

5.2 Modelling frailty through survival models

A frailty model generalises the survival regression model, thus an extension of the Cox proportional hazard model (Cox, 1972). Beyond the observed covariates, a frailty model accounts for unmeasured or unobserved factors which multiplicatively modifies the hazard function of a subject or cluster (Amorim and Cai, 2015; Kleinbaum and Klein, 2011). Frailty models can be classified into two broad categorises, (1) frailty models for univariate data (with univariate survival time as end point) and (2) shared frailty models for multivariate data (multivariate survival end points). Though the two have similarities, they are conceptually different. In shared frailty multivariate situation, the frailty variance measures the level of dependence between event times within a cluster whilst for the univariate frailty case, the frailty variance is a measure of unobserved heterogeneity. In this study, the focus is on univariate survival data, and therefore no attention is paid to multivariate frailty models.

Univariate frailty

Reality is that the population is not homogenous, individuals or firms are different and poses varied risk to lenders. Univariate frailty models take this into account and acknowledge that heterogeneity can be explained by observed covariates, but, in the event that important factors are unobservable or unknown, it results in unobserved heterogeneity. The idea is to suppose that different obligors possess different frailties or risk profiles, and those who are more frail or riskier will experience the default event earlier than those who are less risky. The frailty is not directly estimated from the data, instead, it is assumed to have a unit mean and finite variance, which is estimated (Gutierrez, 2002). When the estimated frailty (α) is more than one, subjects are said to have experienced an increased hazard of failure (risk of event) and are considered more frail or susceptible for reasons unexplained by observed covariates and are more likely to fail (Gutierrez, 2002). Conversely, an estimated frailty less than one suggest a lower risk of failure or less frail. The notion of frailty can be traced back to the work on accident proneness by Greenwood and Yule (1920). However, the concept of frailty itself was introduced by Vaupel et al. (1979) in mortality studies. Lancaster, (1979) also used the frailty concept in the unemployment duration studies.

Traditionally, if all factors are known, these can be included for analysis in the Cox proportional hazard model which take the form:

$$h(t, X) = h_0(t)\exp\left[\sum_{i=1}^{p} \beta_i X_i\right] \tag{5.1}$$

where $h_0(t)$ denotes the baseline hazard function, X_i is the vector of measured covariates and β_i is the vector of regression parameters which can be estimated using the maximum likelihood approximation. The univariate frailty model therefore extends the Cox model so that the hazard of a subject, in addition, now depends on an unobservable variable α , acting multiplicatively of the hazard function $h_0(t)$ as follows:

$$h(t, \alpha, X) = \alpha h_0(t)\exp\left[\sum_{i=1}^{p} \beta_i X_i\right] \tag{5.2}$$

If the frailty α has a probability density function $g(\alpha)$, then the conditional survivor function of the model which represents the fraction of subjects surviving until time t is expressed as follows:

$$S_\theta(t) = \int_0^\infty \{S(t)\}^\alpha g(\alpha) d\alpha \tag{5.3}$$

When α follows a gamma distribution with mean one and variance θ

$$g(\alpha) = \frac{\alpha^{1/\theta - 1} \exp(-\alpha/\theta)}{\Gamma(1/\theta)\theta^{1/\theta}} \tag{5.4}$$

Such that (3) changes to

$$S_\theta(t) = [1 - \theta In\{S(t)\}]^{-1/\theta} \tag{5.5}$$

If α is distributed as an inverse Gaussian,

$$g(\alpha) = \left(\frac{1}{2\pi\theta\alpha^3}\right)^{1/2} \exp\left\{\frac{-1}{2\theta}\left(\alpha - 2 + \frac{1}{\alpha}\right)\right\} \tag{5.6}$$

Such that (3) changes to

$$S_\theta(t) = \exp\left\{\frac{1}{\theta}\left(1 - [1 - 2\theta In\{S(t)\}]^{1/2}\right)\right\} \tag{5.7}$$

5.3 Empirical results and discussion

5.3.1 Data

We use data from a credit bureau in Zimbabwe for loans originated from 2010 to 2016. The data is particularly from borrowers on store credit which includes credit on household furniture and other electrical appliances. The data set comprises of 16363 unique borrowers (see Table 5.1). Variables with data included (1) application variables; age, gender, income, debt-to-income ratio (DTI), (2) loan variables; loan amount, loan term, instalment size (3) behavioural variables; number of missed payments, and average length of delinquency spells.

5.3.2 Descriptive statistics

Table 5.1 below shows the distribution of the sample studied across categorical variables as well as the default rate across the categories. The overall default rate was alarmingly high at 24 percent with men (25.9%) defaulting more than women (21.5%). Default was also higher for higher income groups yet it decreased with increasing age. Contrary to literature, default was highest for individuals with lowest DTI (25.4%) and the reverse was true.

Table 5. 1: Distribution of borrowers across categorical variables

Variable	Category	Sample representation	Default rate
Gender	Male	58%	25.9%
	Female	42%	21.5%
Age group	18-34	28%	25.8%
	35-44	41%	24.1%
	45+	31%	22.2%
Income group ($)	<300	24%	22.2%
	301-500	39%	21.9%
	501-1000	23%	27.9%
	1001+	15%	26.3%
Debt-to-Income ratio	<0.35	93%	25.4%
	0.36-0.49	2%	11.1%
	>0.5	5%	3.3%
All	16363	100%	24%

Author's estimation

Presented in Table 2 are the descriptive statistics for the continuous covariates. Borrowers ranged from as young as 19 years to as old as 72 years with an average age of 41years. The loan amounts ranged from $100 to $9462 with a mean of $903.74 whilst loan terms varied from a month to three years with a mean of 11 months. Incomes of borrowers ranged from $100 to $6000 with an average income of $652.53.

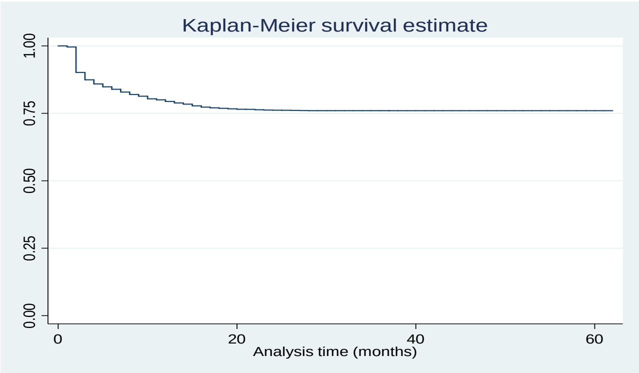
Table 5. 2: Descriptive statistics for continuous covariates

Variable	Min	Max	Mean	Std dev
Loan Amount	100	9462.08	903.7427	880.88
Loan Term	1	36	11.12467	6.1930
Instalment size	7.21	1000	101.72	113.5985
Number of missed payments	0	9	1.146428	1.7823
Average duration of delinquency spell	0	32	2.0186	5.0919
Income	100	6000	652.5278	659.4738
Debt-to-Income ratio	0.0060	1.5	0.1935	0.1777
Age	19	72	41.0516	9.0258

Source: Author's estimation

Shown in Figure 5.1 is the Kaplan-Meier survival curve which illustrates that most of the default events happened in the first few months of the loan contracts.

Figure 5. 1: Probability of surviving a default event



Source: Author's estimation

5.3.3 Variable selection and model fit

To determine candidate variables for the model which best explain the observed default behaviour, we inspected the effect of individual variables on the hazard function using the Weibull with gamma frailty model. Variables which were significant at the 0.25 significance level in the univariate model were retained (Hosmer et al., 2013). Loan amount was insignificant ($p = 0.321$), and in any case, it was positively correlated with loan term ($r = 0.4313$) and instalment size ($r = 0.4354$). We then inspected the changed in the model -2 loglikelihood after adding a set of variables as defined in table Table 5.3 and Table 5.4. By adding application variables in the model, there was a significant change in the -2log likelihood signifying improvement in model fit. Though adding loan variables in the model, significantly improved model fit, the change in -2log likelihood was the minimum among all set of variables. Lastly, adding behavioural variables offered great improvement in model fit as supported by a large change in -2log likelihood and was statistically significant.

Table 5. 3: Model fit

Set of variables (model)	**-2log likelihood**
null	-16823.89
AV	-14155.267 ***
AV+LV	-14110.648 ***
AV+LV+BV	-12469.285***

*** significant at 0.01 level

5.3.4 The role of the frailty component

Table 5.4 shows the results from the three models; Weibull with gamma frailty, Weibull with inverse Gaussian frailty and the Weibull without frailty. The results from the two frailty models show little differences with respect to the choice of frailty distribution. However, the gamma frailty model had slightly higher estimates for most of the covariates. Importantly, both models show that the likelihood ratio test of theta = 0 were statistically significant ($p < 0.001$), suggesting the presence of unobserved heterogeneity. When a significant random effect is detected, unlike the Weibull without frailty, the coefficient estimates of covariates from the frailty models change. This is evident in Table 5.4 with Weibull without frailty model coefficients different from those of the frailty models, in many cases with lower estimates. The results are consistent with Bijwaard, (2014) who stated that ignoring frailty tend to bias the effect of the covariates towards zero. The finding also echo that of Fernandes and Artes (2016) who found a significant role of a latent unobservable factor in explaining defaults among small firms in Brazil. This also agrees with Giesecke and Weber (2004) and Foglia et al. (2009) who also observed that ignoring default clustering leads to the underestimation of losses which have massive implications on loan pricing and capital adequacy.

Potentially important variables such as interest rates, the complete debt load on all loans of borrowers, employment status, house ownership, fixed telephone line indicator and number of dependents, among others were not available. As such the absence of such important variables may result in biased default estimates of potential or existing borrowers, thus rendering the use of frailty models essential.

Table 5. 4: Default prognostic factors

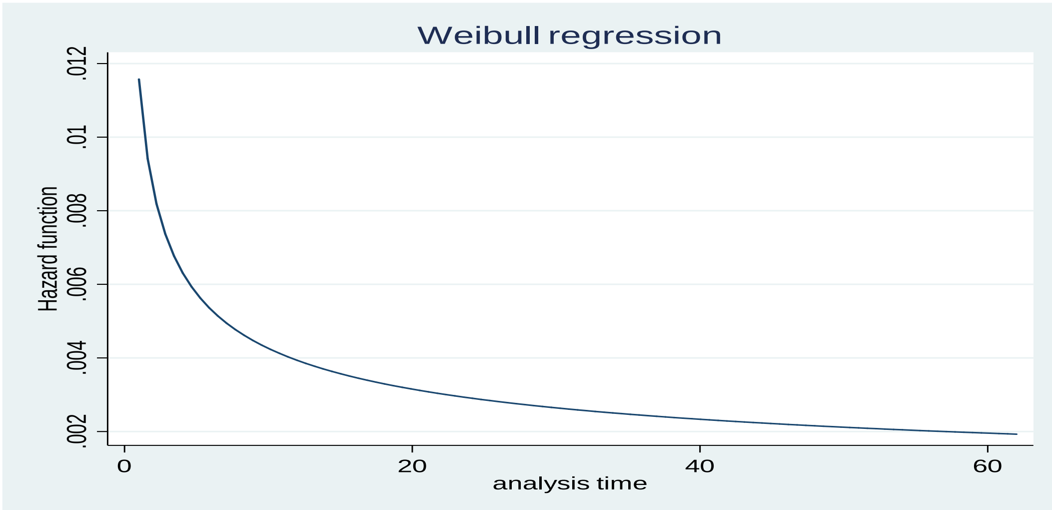
	Variable	*Weibull with gamma frailty* Estimate	Std error	Hazard ratio	*Weibull with inverse-Gaussian frailty* Estimate	Std error	Hazard ratio	*Weibull without frailty* Estimate	Std error	Hazard ratio
Application variables	Male	-	-	-	-	-		-	-	-
	Female	-0.337***	0.049	0.714	-0.272***	0.046	0.762	-0.160***	0.033	0.852
	Age 18-34	-	-	-	-	-	-	-	-	-
	Age 35-44	-0.172***	0.057	0.842	-0.195***	0.054	0.823	0.169***	0.039	0.844
	Age 45+	-0.292***	0.061	0.747	-0.265***	0.058	0.767	-0.191***	0.042	0.826
	DTI < 0.35	-	-	-	-	-	-	-	-	-
	DTI 0.35-0.49	-0.502**	0.204	0.605	-0.495**	0.194	0.609	-0.359**	0.150	0.698
	DTI > 0.5	-1.212***	0.243	0.297	-1.449***	0.239	0.235	-1.384***	0.211	0.250
	Income <300	-	-	-	-	-	-	-	-	-
	Income 300-500	-0.297***	0.063	0.742	-2.553***	0.059	0.775	-0.154***	0.063	0.857
	Income 501-1000	0.040	0.701	1.041	0.052	0.066	1.053	0.051	0.070	1.052
	Income >1000	0.007	0.089	1.007	-0.003	0.084	0.996	-0.007	0.089	0.992
Loan Variables	Loan Term	0.032***	0.004	1.033	0.0001***	0.004	1.024	0.0085***	0.003	1.008
	Instalment size	0.0002	0.0003	1.000	0.0233	0.0003	1.000	0.00001	0.0002	1.000
Behavioural variables	Number of missed payments	0.941***	0.024	2.563	0.7429***	0.0162	2.102	0.486***	0.007	1.626
	Average delinquency period	0.0001	0.004	1.000	0.0239***	0.0036	1.024	0.029***	0.002	1.029
Sharpe	p	0.823			0.7661					
Variance	theta	1.719			3.222					
Hypothesis test	LR test of theta = 0. Chi-square	614.75			436.12					
	LR test of theta = 0. p-value	<0.001			<0.001					

***significant at the 0.01 level, ** significant at the 0.05 level, * significant at 0.1 level
Source: Author's estimation

By comparing the predicted hazards for the three models (Figure 5.2, Figure 5.3 and Figure 5.4), it is evident that the Weibull with gamma frailty had higher predicted hazard estimates for the first few months with a maximum close to 0.012 followed by the Weibull with inverse Gaussian frailty which also had higher predicted hazard estimates during the first few months compared to the Weibull without frailty with maximum values slight above 0.09 and 0.06 respectively. These results further illustrate that a significant frailty term in the frailty models affected the model coefficients such that the predicted hazards are higher during the early months of the loan contracts due to the presents of frail individual who default early and the hazard rates stabilises afterwards since the at-risk population will be composed of less frail individuals.

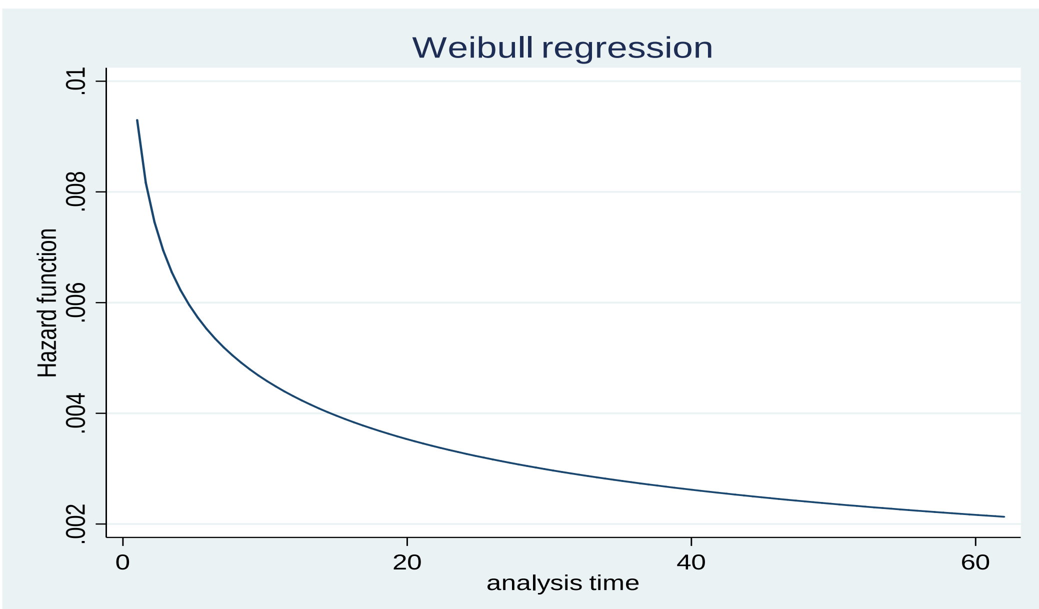
This means that ignoring the frailty component fail to characterise the early defaults which imply greater exposure at default (EAD) for the lender, thus underestimating the expected losses (Elliott and Shen, 2015). Azizpour et al. (2014) also made similar conclusions noting that models with contagion and frailty clustering sources produced better default forecasts than models with only observable macroeconomic factors. They showed that models without the frailty effect understated the default forecast. In both cases of the frailty model the sharpe parameter p was less than one, suggesting that with time, the hazard was decreasing (see Table 5.4).

Figure 5. 2: Weibull with gamma frailty



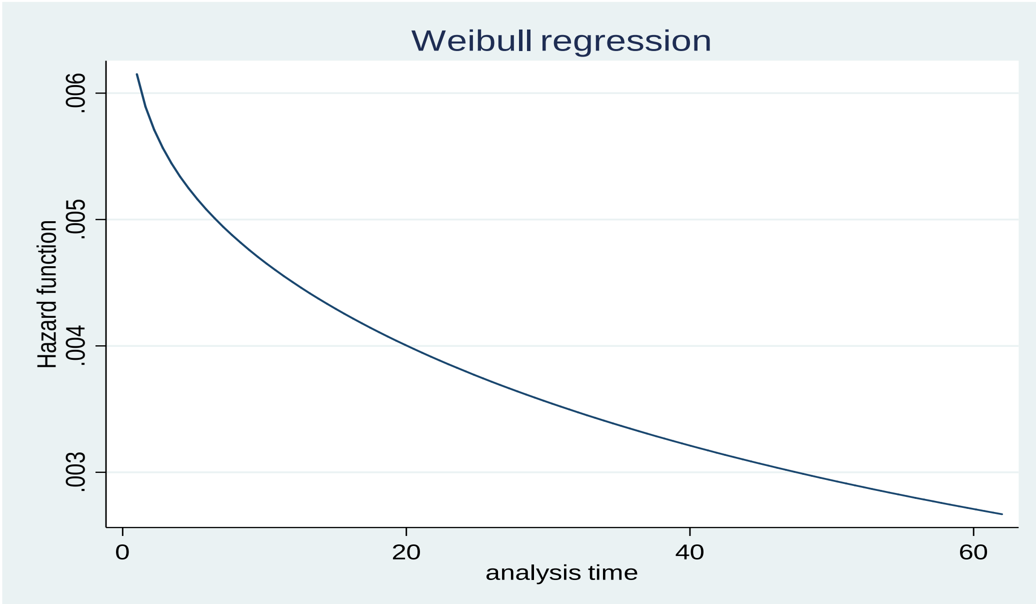
Source: Author's estimation

Figure 5. 3: Weibull with inverse Gaussian frailty



Source: Author's estimation

Figure 5. 4: Weibull without frailty



Source: Author's estimation

5.3.5 Default prognostic factors

Using the results from the Weibull with Gamma frailty model, we discuss the default prognostic factors. The model coefficient for gender of -0.337 and a hazard ratio (HR) of 0.714 which are statistically significant ($p < 0.01$) shows that women were about 29 percent less likely to default on store credit than their male counterparts. The results collaborate that of Chamboko and Bravo, (2016) who found women more likely to recover from delinquency than their male counterparts. This finding is an important realisation given the drive to improve the participation of women in the use of financial services in developing countries. The results on age suggest that the youth (18-34 years) are riskier than the middle aged (35-44 years) and the aged (45 years and above) borrowers (HR = 0.842, $p < 0.01$ and HR = 0.747, $p < 0.01$ respectively). The findings above are also supported by that of (Chamboko, et al, 2017) who also observed that Zimbabwean men and younger individuals who were oftentimes single were more likely to be financially distressed.

The results show that the lowest income earners (< \$300) were more likely to default compared to the \$300 to \$500 earners ($p < 0.01$). However, increasing income after this bracket was associated with higher default rates, thus, those who earned above \$500 were not statistically different from those earning below \$300. This finding point to the fact that credit assessment based on income earned without a complete picture of the entire individual debt load can be misleading. Unfortunately, this study used the debt load for the current loan only and as such did not account for the other loans an individual had. The debt-to-income ratio covariate suggested that increasing debt exposure was associated with statistically significant lower default rates. This is contrary conventional wisdom since increasing debt relative to income results in financial strain. A look at the relationship between income and loan amount showed a weak negative correlation between loan amount and income ($r = -0.2096$). This shows that mostly low-income earners had a relatively higher debt load yet they were less likely to default. This finding suggests a moral issue on loan repayment in Zimbabwe with poorer individuals exhibiting a higher moral obligation to repay their debts.

On loan variables and behavioural variables, longer loan terms were associated with higher default rates as evidenced by a positive coefficient of 0.032 which was statistically significant ($p < 0.01$). The number of past missed payments was a significant predictor of future default behaviour (coefficient of 0.941, $p < 0.01$). Instalment sizes and the duration of delinquency spells were also statistically insignificant.

5.4 Implications for policy and practice

The study results show the presence of unobserved heterogeneity, which suggest that the information being collected and shared does not contain all necessary and complete variables to allow accurate assessment of individuals credit risk. It is therefore of paramount policy importance to ensure that that there is a policy framework in place to improve and strengthen credit information reporting and sharing. Literature shows strong evidence that the collection, sharing and reporting of credit information play a vital component for the proper functioning of a financial market (Büyükkarabacak and Valev, 2012; Giannetti and Jentzsch, 2013). Literature also shows that most credit market failures are mostly attributed to the steep information gradient between borrowers and lenders (Doblas-Madrid and Minetti, 2013).

Equally important, lenders in such environments characterised by inadequate credit information sharing need to be cognisant that using incomplete information, lacking some essential variables to assess potential and existing borrowers' risk can lead to the underestimation of credit losses resulting in mispricing of loans and inadequate loss provisioning. As such, there is equal impetus for regulators, lenders and borrowers to strengthen the sharing of credit information. Proper credit information allow lenders to assess the risk of borrowers accurately, thus extending credit to less risky clients, reducing the cost of borrowing to the consumers and improving repayment rates (Doblas-Madrid and Minetti, 2013; Soledad and Peria, 2014).This also leads to reduced financial distress and over indebtedness, thus creating healthy credit markets (Soledad and Peria, 2014).

5.5 Conclusion

Given the incompleteness and limitations in the credit information sharing in Zimbabwe, this study investigated the possible underestimation of losses under the assumption of independence of default event times. This stems from that fact that standard credit risk models often ignore the presence of default clustering, yet there is the possibility of the violation of the assumption when important factors are not observed or available for modelling. The study also sought to provide evidence on the predictors of default on store credit in Zimbabwe. Using frailty survival models on credit bureau data from Zimbabwe, the study found that adding a frailty term significantly improved the model, thus indicating the presence of unobserved heterogeneity. As such, using standard statistical models without a frailty component may fail to account for the presence of frail individuals who are likely to fail early thus underestimating the expected losses.

As predictors of default on store credit in Zimbabwe, the study shows that longer loan terms were associated with higher default rates and women were less likely to default than men. Repayment behaviour in the past remained the major predictor of default behaviour with those having missed many payments in the past more likely to default on current contracts, reinforcing the importance of lenders conducting credit checks before extending loans. The lowest income earners and the relatively higher earners had equal chances of defaulting reinforcing the fact that credit assessment based on income alone without complete information about the borrower's debt load and past repayment information is misleading. In fact, borrowers with a higher debt load who were often poorer exhibited a higher moral obligation to repay their debts.

The major policy recommendations are for the regulator in Zimbabwe to institute appropriate policy frameworks to allow robust and complete credit information sharing and reporting as doing so will significantly improve the functioning of credit markets.

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Chapter 6

Summary and Conclusions

This thesis worked on the application of advanced survival models in consumer credit risk assessment, particularly to address issues of recurrent delinquency (or default) and recovery (cure) events as well as multiple risk events and frailty. Each chapter (2 to 5) addressed a separate problem and several key conclusions were reached.

Chapter 2 dealt with the modelling of recurrent delinquency (or default) and recovery events on consumers loans using data from a distressed economic environment of Zimbabwe, incorporating time dependent macroeconomic variables. The study provided a comprehensive analysis of the recovery patterns using an extended Cox model – the Andersen and Gill counting process approach. Even though recovery (cure) modelling remains a neglected subject with little attention paid to the problem, the paper emphasised that the recurrence of delinquency and default events is a ubiquitous phenomenon in troubled economies, hence the modelling of the corresponding recovery or cure events becomes essential. With the ailing economy of Zimbabwe, the study demonstrated the important role of recovery modelling in understanding the state of consumer welfare and the depth of financial distress in the economy.

The results strongly revealed that the falling real gross domestic product, representing a deteriorating economic situation significantly explained the diminishing rate of recovery from delinquency to normal performance among consumers. The findings had great industry and policy relevancy especially about the need to balance risk appetite and the developments on the macroeconomic environment as well as the urgent need for sound economic policies to relieve consumers from the economic hardships. The study also documented other factors affecting recovery and emphasised the important role of recovery and behavioural modelling especially for delinquents to inform appropriate and timely action to manage delinquents and mitigate credit losses.

Chapter 3 extended the work in chapter 2 and reviewed various methods for handling recurrent recovery events. The study emphasised that the modelling and forecasting of recovery events is so important especially now given the recent global financial crises and the rise of unsecured lending to the poor in developing countries and emerging markets which have left consumers in a compromised position to owner their loan commitments. With increased financial distress, repeatedly falling into delinquency and defaults events becomes common, thus making multiple failure-time data common in the credit risk domain. The study notes that inappropriate statistical techniques such as such as logistic regression or the standard Cox model which ignore the subsequent events are commonly used to analyse such data. This is despite the existence and wide application of several methods to analyse recurrent events data, especially in the medical field. The study reviewed such methods as the GEE Poisson regression models, random effects or frailty models, multi-state models and some variance corrected models, namely the AG-CP model, the PWP-CP model and the WLW model.

To empirically illustrate the application of these models, the study employed consumer loans data from a severely distressed economic environment of Zimbabwe. The choice of the correct model to use differs with the objective the investigation seeks to address. Looking at the data and the underlying assumptions the study illustrated the appropriateness of several methods and compared the results. Considering that events of interest (recovery) were ordered and taking into account the structure of the data, the study illustrated and empirically compared the AG-CP model, the PWP-CP model and the WLW model. The study concluded that for ordered indistinguishable recurrent recovery events, the use of the Andersen and Gill (1982) model is recommended since it fits the assumptions and performs well on predicting recovery.

The study emphasised that methods meant for analysing time to first event or merely categorical outcomes cannot be used to investigate the effects of covariates on the occurrence or re-occurrence of multiple events and applying standard statistical methods without addressing the recurrence of the events produces biased and inefficient estimates. The study highlights that accounting for multiple events proffers detailed information, thus providing a nuanced understanding of the recovery prognosis of delinquents.

Chapter 4 built on the work in chapter 2 and chapter 3 and reemphasise the importance of sound credit risk analysis for lenders, banks, policy makers and regulators especially in the aftermath of the world credit crises when it is important to understand the state of consumer welfare and their ability to honour their loan commitments and recover from distress. The study highlighted that traditional approaches to credit risk modelling consider one risk event at a time and use standard classification methods such as logistic regression or standard survival model or other machine learning classification approaches. These methods lack the ability to characterise the progression of a borrower over time and cannot utilise all the available data to understand the recurrence of risk events and possible occurrence of multiple loan outcomes.

The chapter goes beyond the modelling of recurrent events to develop a system wide framework to model recurrent risk events, cure and transition into multiple states (loan outcomes) to fully understand the dynamics of a cohort of borrowers over time. The study used seven allowable states namely, current (normal performance), delinquent, default, repurchase, foreclosure, short sale and prepaid. The framework was tested using the Fannie Mae data with a cohort of borrowers whose loans where initiated during the first quarter of 2009 and were followed for 92 months until the third quarter of 2016. The probability of loans transitioning to and from the various states is estimated in a discrete-time multi-state Markov model with seven allowable states and sixteen possible transitions. Results showed massive risk to lenders due to potential income from interest being lost because of prepayment. In addition, the results also showed reasonable recovery rates for those who entered delinquency and default state signalling the ability of borrowers to recover from financial distress. In terms of the factors affecting the transition into various loan outcomes, the cross-cutting importance of affordability as measured by debt-to-income ratio, equity as marked by loan-to-value ratio, interest rates, and the property type was observed.

It was concluded that jointly modelling the described state occupations and transitions allows a system-wide helicopter view which provides a holistic understanding of the dynamics of a mortgage loan portfolio more than just modelling prepayment, foreclosure, delinquency, recovery, repurchase, and short sale separately. Furthermore, it was recommended that during times of economic distress, the focus of credit risk modelling should go beyond modelling defaults and foreclosures as the only main outcomes, especially on mortgage loans as other transitions and state occupation experienced by borrowers becomes critically essential.

Chapter 5 dwelt on the important aspect of incomplete data and limited credit information sharing and risk reporting affecting developing countries and emerging economies. Particularly for Zimbabwe, the study sought to investigate if information collected for credit risk assessment contained all the important variables needed to perform adequate credit risk assessment to inform consumer lending. In other words, to test the possible underestimation of credit losses under the assumption of independence of default event times. The study also sought to provide evidence on the predictors of default on store credit in Zimbabwe.

Using frailty survival models on credit bureau data from Zimbabwe, the study found that adding a frailty term significantly improved the model, thus indicating the presence of unobserved heterogeneity. As such, using standard statistical models without a frailty component may fail to account for the presence of frail individuals who are likely to fail early thus underestimating the expected losses. The study also documented that loan terms, gender, income, past repayment behaviour in the past were the major predictors of the observed default behaviour. The key policy recommendation was for the regulator in Zimbabwe to institute appropriate policy frameworks to allow robust and complete credit information sharing and reporting as doing so will significantly improve the functioning of credit markets.