Reducing Mixed-Integer to Zero-One Programs

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Dipl.-Kfm. Marcel Büther
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Dekan	Professor Dr. Helmut Herwartz
Erstberichterstattender	Professor Dr. Andreas Drexl
Zweitberichterstattender	PD Dr. Uwe Jensen
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Contents

Abbreviations

eGAP	elastic generalized assignment problem
eSSCFLP	elastic single source capacitated facility location problem
GAP	generalized assignment problem
IP	integer program
KP	knapsack problem
KPC	knapsack problem with a single continuous variable
LP	linear program
MIP	mixed-integer program
SSCFLP	single source capacitated facility location problem

1 Introduction into the Topic

During the last decades, there has been a lot of success in the development of efficient algorithms for several optimization problems in the operational research community. This is supported on the one hand by the rapid development of the hard- and software, that is available at relatively small cost and on the other hand by the arising importance of the optimization of all processes of a company to ensure its competitiveness on local and a fortiori on global markets. Therefore, a company can not only consider the internal processes, it has to manage the whole supply chain. That is the management of material and information flows both in and between facilities, such as vendors, manufacturing and assembly plants and distribution centers, see Thomas and Griffin [13]. Due to the very complex interconnections and dependencies of the elements of such a supply chain, its optimization in reasonable time requires an underlying mathematical program which can then be solved by efficient algorithms using modern hard- and software.

Such a mathematical program can be built e.g. only by integer variables, which is then called integer program, IP for short. Naturally, the binary variables, a special case of the integer ones, are included as well. A brief excerpt from these programs is given by the following basic IPs: the knapsack problem, KP for short, see e.g. Martello et al. [8], Martello et al. [9] and Pisinger [11], the generalized assignment problem, GAP for short, see e.g. Savelsbergh [12], Díaz and Fernández [4] and Yagiura et al. [15], et cetera. These are called basic IPs, because they can be used for

- optimizing rudimental real–life problems, applied one-to-one or adjusted just a little to the specific case
- solving relaxations of more complex problems, where the complexity corresponds to the number of constraints.

Although most of the research of the last decades has focused on the IPs, these programs are often not sufficient: for most real–life applications, unfortunately, the environment can not be represented appropriate, neither by such basic IPs nor by more complex ones with more constraints and more integer variables. This challenge can only be solved by using additionally continuous variables in the LP. The corresponding optimization problem is then called mixed-integer program, MIP for short.

To show the need of using MIPs instead of IPs, we present a simple example, the knapsack problem with a single continuous variable, KPC for short, which is based on the classic knapsack problem, KP. Therefore, the objective is to choose items in order to maximize profit without exceeding a given capacity of an arbitrary ressource. While this capacity can not be influenced according to the KP, the KPC considers the opportunity to extend or reduce, respectively, the available capacity. Extending capacity reduces profit while reducing capacity increases the profit. A real–life application for the KPC can be found in determining the optimal choice of investment projects for a one-period time horizon with the opportunity to lend or place money on the capital market. The given budget at the beginning of the period is the limited resource, which is reduced if a project is selected. For every project, the investor gains a pay-off at the end of the period. So far, this problem is similar to the classic KP. But in real–life, normally one can use the capital market in order to get money for realizing profitable projects or to place money for returning a dividend. Thus, for the KPC the objective is to maximize the overall pay-off, which results directly from the projects and from the capital market. Certainly, one has to observe additionally the cost that occur if the given budget was not sufficient and money had to be let on the capital market. Note that the KPC can be extended easily to represent even better the reality by either introducing further periods or different interest rates for lending and placing money.

For optimizing MIPs it is of course possible to develop always a new algorithm for each of them. But obviously, this is very expensive: On the one hand, based on the endless number of potentially occurring MIPs and on the other hand, based on the complexity of a MIP itself. It is well–known, that several IPs are NP-hard, NP-hardness follows straightforwardly for these MIPs, that are based on the corresponding IP, e.g. the KPC based on the KP, see among others Nauss [10]. This fact implies, that already finding a feasible solution might require a lot of computational efforts which is obviously a burden if the MIP should be optimized.

Additionally, there are several approved approaches available for optimizing IPs, e.g. efficient algorithms and very powerful commercial solvers. Thus, instead of developing a new approach for each MIP, the basic idea of this thesis was to check, if it is admirable to reduce a MIP to the corresponding integer one, e.g. the KPC to the KP, in order to apply the well-known algorithms for it.

For the KPC, some related work can be found in Kellerer et al. [6], who reduced a IP to another well–known one: the bounded knapsack problem to the classic KP. In comparison to the classic KP, this problem offers the possibility to select one item more often up to an upper bound. In the same work as well as in Martello and Toth [7] one can get also some first insights in the replacement of continuous coefficients by integer ones for the KP. But to the best of our knowledge, there is no work available so far which combines both approaches and moreover, which transfers the results to more complex MIPs and analyzes the resulting problems.

Since the KPC is one of the basic MIPs, our research started on reducing it to the KP. The theoretical results are then carried over to the elastic generalized assignment problem, eGAP for short, which is the classic GAP extended by continuous variables to adjust the capacities. In addition to the reduction of the eGAP to GAP, we present an heuristic, which combines several well–known elements for the optimization of a standard GAP in a new way. Of course, all developed approaches have been implemented and computer based tested to proof the efficiency. The corresponding articles for all research topics will be described in the next chapter.

2 Survey of the underlying Articles

In the following we will give an overview of the three articles, on which this thesis is based. For each, we present briefly:

- the complete title and author names
- the abstract of the article
- the interconnection between the articles
- the content and the main results
- the current status of publication

All three articles are currently available in "Manuskripte aus den Instituten für Betriebswirtschaftslehre" at the Christian-Albrechts-University at Kiel.

2.1 Reducing the 0-1 Knapsack Problem with a Single Continuous Variable to the Standard 0-1 Knapsack Problem

Marcel Büther, Dirk Briskorn (2007): Reducing the 0-1 Knapsack Problem with a Single Continuous Variable to the Standard 0-1 Knapsack Problem, Manuskripte aus den Instituten für Betriebswirtschaftslehre der Universität Kiel, No. 629

Abstract 1. *The 0-1 knapsack problem with a single continuous variable (KPC) is a natural extension of the binary knapsack problem (KP), where the capacity is not any longer fixed but can be extended which is expressed by a continuous variable. This variable might be unbounded or restricted by a lower or upper bound, respectively. This paper concerns techniques in order to reduce several variants of KPC to KP which enables us to employ approaches for KP. We propose both, an equivalent reformulation and a heuristic one bringing along less computational effort. We show that the heuristic reformulation can be customized in order to provide solutions having an objective value arbitrarily close to the one of the original problem.*

Obviously, this article has been developed together with Dirk Briskorn, where the workload has been split equally. First, we highlight special cases of the KPC, which result from different restrictive bounds on the continuous variable. Additionally, we show, that the KPC can always be optimized by solving at most three standard KPs. If therefore all coefficients are integers, each KP can be handled very efficient by the algorithm `combo`. This algorithm was developed by Martello et al. [8] and it is currently the state–of–the–art approach for standard KPs with integer coefficients, see Martello et al. [9].

But, the main importance of this paper for the whole thesis results from its fundamental behavior regarding the treatment of a MIP. Here, we present for the first time the replacement of a continuous variable by several binary ones. Since we limit the solution space by using binary instead of continuous variables, we only get an upper bound on the objective function value for a maximization problem. However, as already mentioned above, we are able to show, that this bound can be arbitrarily close to the optimal objective function value by adapting the IP-reformulation appropriate. Since the KPC is one of the basic MIPs, it is only built by the objective and one constraint, we used it as a starting point for transferring the developed idea of replacing a continuous variable to handle more complex MIPs. This is the content of our next articles.

2.2 Reducing the Elastic Generalized Assignment Problem to the Standard Generalized Assignment Problem

Marcel Büther (2007): Reducing the Elastic Generalized Assignment Problem to the Standard Generalized Assignment Problem, Manuskripte aus den Instituten für Betriebswirtschaftslehre der Universität Kiel, No. 632

Abstract 2. *The elastic generalized assignment problem (eGAP) is a natural extension of the generalized assignment problem (GAP) where the capacities are not fixed but* *can be adjusted which is expressed by continuous variables. These variables might be unbounded or restricted by a lower or upper bound, respectively. This paper concerns techniques in order to reduce several variants of eGAP to GAP which enables us to employ standard approaches for the GAP. This results into an heuristic, which can be customized in order to provide solutions having an objective value arbitrarily close to the optimal one.*

While there exist several efficient algorithms for the GAP, see e.g. Yagiura et al. [15], the eGAP has not been studied much so far, although it is of great practical relevance. To the best of our knowledge, there is only one recent paper by Nauss [10] for a special case of the eGAP. It concerns a branch&bound algorithm to solve small but hard instances. To handle larger instances, we transferred the same replacement techniques for the continuous variables by binary ones, like developed for the KP.

This article has been developed on my own and the main result of the replacement of the continuous variables can be summarized as follows: the modified eGAP can be reduced to a standard GAP, which enables us to use well-known approaches one–to–one. Thus, in this work, we passed the IP reformulation of the eGAP as well as the eGAP itself to the state–of–the–art commercial branch&bound solver CPLEX by ILOG. The occurring computation times and the obtained bounds on the objective are not worse for the reformulation than that, given by the original MIP, using the same time limit. In contrast, since the commercial solver is optimized for standard IPs, in some cases it can present better results based on the IP than on the MIP.

2.3 Beam Search for the Elastic Generalized Assignment Problem

Marcel Büther (2008): Beam Search for the Elastic Generalized Assignment Problem, Manuskripte aus den Instituten für Betriebswirtschaftslehre der Universität Kiel, No. 634

Abstract 3. *The elastic generalized assignment problem (eGAP) is a natural extension of the generalized assignment problem (GAP) where the capacities are not any longer fixed but can be adjusted which is expressed by continuous variables. These variables might be unbounded or restricted by a lower or upper bound, respectively. This paper describes an algorithm based on beam search, combined with Lagrangian relaxation and local search to provide strong lower as well as upper bounds for the eGAP.*

The article is based on the results of the previous work. When solving the IP reformulation of the eGAP, the used commercial solver was not able to solve all instances to optimality due to memory lacks. Of course, the same occurred for the MIP in even more cases. Thus, we developed an heuristic, based on beam search, see Valente and Alves [14], a truncated version of the classical branch&bound algorithm. It is working on the IP reformulation, because then we are able to apply solution methodologies one–to–one that have been already successfully developed and tested for the GAP, like Lagrangian relaxation, see e.g. Baker and Sheasby [1] and local search, see e.g. Díaz and Fernández [4].

The results of the computational study can be summarized as follows: The obtained lower bounds by Lagrangian relaxation on the IP-reformulation are already better than that, delivered by the commercial solver based on the corresponding LP–relaxation of the MIP. Since we set the parameters for beam search very restrictive in this work, the obtained upper bounds of our heuristic are not yet as good as those obtained by CPLEX. But for future work we can adapt our parameter settings furthermore and thus enlarge the search to obtain better bounds. This is not straightforward possible for CPLEX, too, because CPLEX runs out of memory even for small instances where the beam search algorithm has an almost constant memory demand.

Hence, this paper just points out the potential of our beam search algorithm, which can be improved furthermore by redefining the used parameters and by additionally including other approved building blocks for the standard GAP, like e.g. scatter search, see Glover et al. [5].

3 Summary and Outlook

Summarizing the results of all three articles of this thesis, we can determine for the MIPs regarded, that the reduction to a standard IP provides several advantages and possibilities: It enables us to use well–known and approved approaches instead of developing always a new algorithm for each MIP. This treatment of a MIP reduces the analytic and computational effort for optimizing it.

Based on the results so far we will focus amongst others on the following three topics in the future:

i So far we tested our reformulation only on a few well–known classes of hard instances of the GAP, see Yagiura et al. [15], modified to eGAP according to

Nauss [10]. These classes will be extended furthermore regarding all parameters of the problem to confirm the present results. Additionally we will try to built instance generators for even harder MIP instances for the eGAP as well as for the KPC, which can not be handled at all by the commercial solver and strictly require applying approaches like introduced in this work.

ii Our beam search heuristic contains several well–known approaches for the GAP. Regarding the computational results therefore, we notice, that we can redefine the parameter settings furthermore to obtain even better bounds on the objective function value. Moreover, we will try to transfer other, not yet regarded approaches for solving classic IPs, see above.

iii Finally, we will examine further MIPs, whether the results obtained so far, can be transferred therefor, too. One example for an additional MIP is the elastic single source capacitated facility location problem, eSSCFLP for short. This problem is a natural extension of the single source capacitated facility location problem, SSCFLP, see e.g. Díaz and Fernández [3] or Barceló et al. [2] for a detailed description. Briefly, the SSCFLP aims at determining the optimal selection of facility locations, each with a given capacity limit in order to serve all customers at minimum cost: fixed cost for operating a facility as well as assignment cost. Like for the eGAP, the eSSCFLP considers the opportunity to adjust the capacities.

References

[1] B. M. Baker and J. Sheasby. Accelerating the convergence of subgradient optimisation. *European Journal of Operational Research*, 117:136–144, 1999.

[2] J. Barceló, E. Fernandez, A. Hallefjord, and K. O. Jörnsten. Lagrangean relaxation and constraint generation procedures for capacitated plant location problems with single sourcing. *OR Spektrum*, 12:79–88, 1990.

[3] J. A. Díaz and E. Fernández. A branch-and-price algorithm for the single source capacitated facility location problem. *Journal of the Operational Research Society*, 53:728–740, 2002.

[4] J. A. Díaz and E. Fernández. A tabu search algorithm for the generalized assignment problem. *European Journal of Operational Research*, 132:22–38, 2001.

[5] F. Glover, M. Laguna, and R. Martí. Fundamentals of scatter search and path relinking. 2000.

[6] H. Kellerer, U. Pferschy, and D. Pisinger. *Knapsack Problems*. Springer Berlin, 2004.

[7] S. Martello and P. Toth. *Knapsack Problems: Algorithms and Computer Implementations*. John Wiley and Sons, 1990.

[8] S. Martello, D. Pisinger, and P. Toth. Dynamic programming and strong bounds for the 0-1 knapsack problem. *Management Science*, 45(3):414–424, 1999.

[9] S. Martello, D. Pisinger, and P. Toth. New trends in exact algorithms for the 0-1 knapsack problem. *European Journal of Operational Research*, 123:325–332, 2000.

[10] R. M. Nauss. The elastic generalized assignment problem. *Journal of the Operational Research Society*, 55:1333–1341, 2004.

[11] D. Pisinger. Where are the hard knapsack problems? *Computers & Operations Research*, 32(9):2271–2284, 2005.

[12] P. Savelsbergh. A branch-and-price algorithm for the generalized assignment problem. *Operations Research*, 45(6):831–841, 1997.

[13] D. J. Thomas and P. M. Griffin. Coordinated supply chain management. *European Journal of Operational Research*, 94:1–15, 1996.

[14] J. M. S. Valente and R. A. F. S. Alves. Beam search algorithms for the single machine total weighted tardiness scheduling problem with sequence-dependent setups. *Computers & Operations Research*, 35:2388–2405, 2008.

[15] M. Yagiura, T. Ibaraki, and F. Glover. An ejection chain approach for the generalized assignment problem. *INFORMS Journal on Computing*, 16(2):133–151, 2004.

CURRICULUM VITAE

Persönliche Daten

Name	**Marcel Büther**
Titel	Diplom Kaufmann
Adresse	Kiebitzweg 28a 22869 Schenefeld, Deutschland
Geburtstag	13.02.1981 in Eckernförde
Nationalität	Deutsch
Familienstand	ledig, keine Kinder

Ausbildungsdaten

Christian-Albrechts-Universität zu Kiel

07/06 – 07/07	Promotion im Fach Operations Research / Produktion und Logistik **magna cum laude** (1,0) Promotionsthema: Reducing Mixed-Integer to Zero-One Programs Graduiertenförderung durch Stipendien nach der Landes-verordnung
04/03 – 11/05	Hauptstudium der Betriebswirtschaftslehre **Diplom Kaufmann** **„sehr gut“** (1,4) als Jahrgangsbester Schwerpunkte: Controlling, Produktion & Logistik und Quantitative Wirt-schaftsforschung Diplomarbeitsthema: Lagrangean relaxation and path relinking for the single source capacitated facility location problem „sehr gut“ (1,0)
10/01 – 03/03	Grundstudium der Betriebswirtschaftslehre Vordiplom „gut“ (1,6) in BWL und VWL

Gymnasium Altenholz

09/91 – 06/00	Abitur „gut“ (2,0)

Auslandsaufenthalt

Freiwilliges Praktikum

02/06 – 07/06	Airbus S.A.S., Toulouse, Frankreich Praktikum im Procurement für Composite Materials Identifikation von potentiellen Einsparungen in Höhe von rund 8 Mio € pro Jahr

Praktische Erfahrung

Berufserfahrung

ab 01/08	Büther Consulting Freiberuflicher Unternehmensberater

Freiwillige Praktika

03/05 – 04/05	Airbus Deutschland GmbH, Hamburg Werkstudent in Vollzeit im Programm- und Versions-management des A380 Monitorring der Flugzeuge sowie Ressourcenplanung
08/04 – 09/04	Airbus Deutschland GmbH, Hamburg Werkstudent in Vollzeit im Lean Management bei der Werks-koordination des Airbus Manufacturing Systems Evaluierung über das gesamte Werk Hamburg
09/02 – 10/02	Ingenieurbüro Meise-Schulze-Göbel, Eckernförde Allgemeines betriebwirtschaftliches Praktikum

Wissenschaftliche Tätigkeiten

07/06 – 07/08	Lehrstuhl für Produktion und Logistik: Betreuung und Bewertung von Seminaren und Diplomarbeiten
10/03 – 03/06	Lehrstuhl für Produktion und Logistik: - Tutor für Operations Research (100 Teilnehmer) - ab 01/06 Wissenschaftliche Hilfskraft
10/03 – 03/06	Lehrstuhl für Statistik und Empirische Wirtschaftsforschung: - Wissenschaftliche Hilfskraft - ab 04/04 Betreuung des Kurses Statistik mit EXCEL 2000 (70 Teilnehmer)

Wehrdienst

09/00 – 06/01	1. Panzerflugabwehrkanonenlehrbataillon 6, Lütjenburg

Kenntnisse und Fähigkeiten

Sprachen	Deutsch (Muttersprache)
	Englisch (sehr gut)
	Französisch (gut)
EDV Kenntnisse	Microsoft Office: Access (SQL), Excel, Outlook, PowerPoint und Word
	C, C++, Pascal und Java
	CPLEX und Matlab
	LaTeX, SAP (Business Data Warehouse)

Aktivitäten

Ehrenamtliches Engagement

seit 02/08	Aktives Mitglied der Freiwilligen Feuerwehr Schenefeld
seit 09/01	Aktives Mitglied der Freiwilligen Feuerwehr Kiel-Schilksee
10/04 – 01/08	3. Vorsitzender und Kassenwart der gemeinnützigen Landjugendgruppe Dorfjugend Alt-Schilksee
Sport	Joggen, Krafttraining, Tauchen
Hobbies	Hundesport

Schenefeld, den 12. Juli 2008