Inflation Targeting and Monetary Integration in European Emerging Market Economies

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Diplom-Volkswirt Felix Hammermann
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Dekan: Professor Dr. Helmut Herwartz
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I dedicate this thesis to my parents and siblings.

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Contents

List of Tables

List of Figures

List of Abbreviations

AR	Autoregressive Process
CEEC	Central and Eastern European Countries
CIS	Commonwealth of Independent States
CPI	Consumer Price Index
EBRD	European Bank for Reconstruction and Development
EIU	Economist Intelligence Unit
EMU	Economic and Monetary Union
ERM 2	European Exchange Rate Mechanism
EU	European Union
GDP	Gross Domestic Product
GMM	Generalized Method of Moments
IFS	International Financial Statistics
IMF	International Monetary Fund
IRF	Impulse Response Function
IT	Inflation Targeting
MCI	Monetary Conditions Index
NEER	Nominal Effective Exchange Rate
NOEM	New Open Economy Macroeconomics
OECD	Organisation for Economic Co-operation and Development
OLS	Ordinary Least Squares
PPI	Producer Price Index
RBC	Real Business Cycle
REER	Real Effective Exchange Rate
SSCU	State Statistics Committee of Ukraine
VAR	Vector Autoregression
VECM	Vector Error Correction Model
WDI	World Development Indicators
WEO	World Economic Outlook

1 Introduction

In May 2004, the first cohort of former transition countries in central and eastern Europe joined the European Union. Unlike Denmark and the United Kingdom, these countries do not have the choice to opt-out. New entrants will join the Economic and Monetary Union (EMU) and—at some point in the future—adopt the euro. As part of the preconditions for entering EMU, the Maastricht criteria demand candidate countries to stay for two years in the European Exchange Rate Mechanism (ERM 2). ERM 2 implies that inflation targeting countries like the Czech Republic, Hungary, Poland and, more recently, Slovakia have to conduct monetary policy under an intermediate exchange rate regime, i.e., they have not only to target inflation, but also—at least to some extent—the exchange rate.

Inflation targeting has gained importance as an attractive framework for monetary policy. According to its proponents it is supposed to meet financial markets' criteria of prudent monetary policy (Bernanke, Laubach, Mishkin, and Posen 1999, Loayza and Soto 2002, Woodford 2003a). Besides the focus on price stability, inflation targeting emphasizes the institutional framework. That institutional framework builds on independence, transparency, and accountability. The background for such an institutional setting is the problem of dynamic inconsistency and the problem of the inflation bias (Kydland and Prescott 1977, Barro and Gordon 1983). Both problems originate from incentives to the central bank to create surprise inflation in order to increase employment. As a consequence of rational agents recognizing these incentives, central banks that conduct monetary policy in a discretionary way end up with suboptimally high inflation rates. Based on its rigors institutional framework, inflation targeting helped to anchor inflation expectations and thereby reduce inflation persistently.

In addition, intermediate regimes have been discredited because experience in both Latin American and East Asian emerging market economies in the 1990s revealed shortcomings in pegged exchange rates. Speculative attacks forced the break down of soft pegs and led leading economists to the conclusion that only the "corner solutions" (Fischer 2001) could be

sustainable.[1] It may be argued that the breakdown of the old ERM system with narrow bands for bilateral exchange rate fluctuations already led to wider bands, which—so far—have not been tested. However, the experience with speculative attacks in Latin America and East Asia demonstrate that even more generous bands for the exchange rate may be challenged by the market and are almost impossible to be defended without external support, which the European Central Bank is not obliged to provide to a significant extent.[2] Hence, accession countries may face increasing macroeconomic risks during transition into EMU if the potential trade-off between inflation and exchange rate targeting materializes.

However, critics of the concept of inflation targeting call this new monetary policy strategy just "conservative window-dressing" (Romer 2001: 509). Others claim that independent of the label attached to monetary policy the importance of the exchange rate for macroeconomic institutions in emerging market economies demands for intermediate regimes (Williamson 2000). While the goals of monetary policy in industrialized countries are well described by domestic price and output stability, internal monetary stability in emerging market economies may be argued to depend largely on the prevailing global situation, which goes beyond domestic conditions and can hardly be influenced by them. Thus, central banks in emerging market economies would have to take the exchange rate into account in order to sustain macroeconomic stability. If this would be the case, ERM 2 would even reduce macroeconomic risks for former transition countries on their way into monetary union.

All in all, the existence of a trade-off between inflation and exchange rate targeting is open to debate. The different strategies chosen by the former transition countries in central and eastern Europe provide a case in point. While the most advanced country, Slovenia, already made its way into EMU and the smaller Baltic countries sustained their currency boards and hard pegs in order to join the euro as soon as possible, the larger countries like the Czech

[1] Hard pegs are not discussed in this study. The crisis in Argentina revealed that hard pegs other than monetary unions still contain a high risk and demand a high degree of fiscal consolidation (Mussa 2002). As a consequence of the collapse of the Argentine currency board and the fact that dollarization and monetary union are either not desirable or not feasible, hard peg regimes lost their charm. For European countries, this aversion seems to be less relevant since even the currency board arrangements in some smaller EU countries are supposed to end up as soon as possible in the monetary union (Ghosh, Gulde, and Wolf 2000).

[2] Article 4 of treaty of Maastricht explicitly states that exchange rate policy is subordinated to maintain price stability in the euro area.

Republic, Hungary, Poland, and Slovakia implemented inflation targeting regimes. For this latter group of countries, which may still be classified as emerging market economies, the way to a hard peg in a monetary union leads through the possibly insecure terrain of a soft peg in ERM 2.

In this respect the chapters of this study answer three questions:

- What explains the choice of inflation targets in European emerging market economies (Chapter 2)?
- What is the role of the exchange rate in European emerging market economies' monetary policy reaction functions under inflation targeting (Chapter 3)?
- What are the costs and benefits of an additional exchange rate target under an intermediate exchange rate regime like ERM 2 (Chapter 4)?

Chapter 2 demonstrates that some central and eastern European countries (CEEC) have reached a stage in transition where they can credibly commit to low inflation rates. The analysis in this chapter assumes that inflation outcomes are determined by nonmonetary factors because this is what constrains central banks in their monetary policy as it would be technically possible for central banks to eliminate inflation by monetary restriction. Hence, the assumption made is that the choice of an explicit or implicit intermediate target is penultimate and reflects a central bank's preferences over inflation and output as well as the political and economic environment and expectations that it faces. Thus, when a central bank chooses, e.g., a pegged exchange rate, it implicitly chooses to import the inflation target from the country providing the anchor currency. Against this background, a panel estimation of 19 transition countries reveals the factors underlying the choice of their implicit inflation targets.

For the period between the years 2000 and 2005 inflation rates differ substantially between the central and eastern European countries and the western CIS countries Belarus, Moldova, Russia and Ukraine (CIS-West). While the CEEC and also the eastern CIS countries have been able to reach and maintain inflation rates below 10 percent, the CIS-West countries have all converged at a higher rate of about 10 percent. Panel regressions linking inflation outcomes to a set of independent variables reveal that the central banks in Russia, Ukraine,

Belarus and Moldova may aim at comparatively high inflation rates mainly to make up for, and to perhaps exploit, lagging internal and external liberalization in their economies.

We conclude that moves to liberalize the economy and encourage more openness could directly help to support lower inflation targets in the medium term, and indirectly reduce incentives for central banks to pursue inflationary policies. Moreover, efforts to improve central bank independence could also pay dividends in durably reducing CIS-West inflation. In this context, eliminating the multiple objectives CIS-West central banks now face and providing for a stronger mandate to target inflation would be a good start. For the CEEC, these results imply that the reform process, they have undergone during accession to the EU, created the preconditions for supporting ambitious inflation targets, which, in turn, are a necessary precondition for adopting the euro area inflation target under ERM 2.

Chapter 3 analyzes to which extent the central banks of emerging market economies in central Europe and Latin America that adopted an inflation targeting framework consider the exchange rate in their monetary policy reaction function. As a first step, we demonstrate that a generalized reaction function can be derived, which represents four alternative monetary policy strategies: inflation targeting, Taylor rule, monetary conditions index, and managed floating. These strategies cover the range between pure inflation targeting combined with fully flexible exchange rates and managed exchange rates implying a balanced role of inflation and exchange rate targeting. The generalized reaction function, an extended Taylor rule that takes into account inflation, the output gap, and the exchange rate, is the basis for the estimation of a vector autoregressive model (VAR). The advantage of this approach is that it does not only investigate the reaction function but also the transmission mechanism of inflation, output, and the exchange rate shocks to the interest rate. Additionally, it allows us to distinguish between the announced monetary policy of targeting only inflation (de jure policy) and the actually conducted policy (de facto policy) with respect to additionally targeting the exchange rate.

The results reveal that there is a significant but diminishing role for exchange rate targeting linked to the progress of complementary economic reforms implemented by a country. In Chile, the forerunner of inflation targeting in emerging market economies and a country with

a comparably long tradition in prudent macroeconomic policies, monetary policy is predominately committed to reducing and later keeping inflation down. The exchange rate seems to be managed only during periods of pressure from "contagion" crises. This stage has not yet been reached by the central European countries. The Czech interest rate responds to inflation on the very short horizon, slightly later to output and to the exchange rate on the medium horizon. Poland, like Brazil and Colombia, reveal a clear regime change towards inflation targeting, but with still some importance attached to the exchange rate. Monetary policy in Hungary and Mexico remains highly driven by the development of the exchange rate. Obviously, the value given to an external anchor for domestic monetary policy differs among inflation targeters.

Chapter 4 analyzes optimal monetary policy in a New Keynesian three-country model. The chapter focuses in particular on potential costs of targeting the exchange rate to the euro while being integrated to a third country to various degrees. Recent empirical research points to the relevance of the integration of production structures by outsourcing and offshoring for the transmission of business cycle shocks and the conduct of monetary policy (Burstein, Kurz and Tesar 2008, Imbs 2004, Kalemli-Ozcan, Sørensen, and Yosha 2003, Kose, Otrok and Whiteman 2003, Kose, Prasad, and Terrones 2003). This is especially relevant for the countries that will join the euro area, which will have to stay for two years in ERM 2, and which will, therefore, have to target inflation and the euro exchange rate simultaneously. These countries are significantly integrated with third countries through the production chain. We examine how shocks originating from the third country affect not only the accession country directly but also monetary integration under ERM 2 in a New Keynesian framework.

The underlying research program is rooted in the real business cycle (RBC) theory with explicit microfoundations, i.e., rationally optimizing agents who make forward-looking decisions and thereby take into account dynamic and intertemporal effects. In contrast to the pure RBC theory, these models incorporate Keynesian building blocks with nominal rigidities in prices and wages and imperfect competition in goods and factor markets. This new class of models, known as "New Neoclassical Synthesis" (Goodfriend and King 1997), "New Keynesian" (Clarida, Galí, and Gertler 1999) or "Neo-Wicksellian" (Woodford

2003b), are taken to the open economy in the "New Open Economy Macroeconomics (NOEM)" (Lane 2001, Sarno 2001, Bowman and Doyle 2003).

We assess the cost of a central European inflation targeting central bank to move to an intermediate regime like ERM 2 and thereby target additionally the exchange rate. We contrast two different loss functions. The first—traditional—loss function is based on the variability of output and inflation. Under the second loss function, the central bank has to keep the exchange rate within the bands given by ERM 2 and include additionally the variability of the exchange rate. By minimizing the loss function of the central bank we derive the corresponding optimal monetary policy rule. The results show that a Taylor rule augmented for an exchange rate term is a good simple rule if the loss function assumes an additional benefit of exchange rate smoothing in line with a strategy of monetary integration into the euro area under ERM 2. However, the loss increases with the degree of integration of the accession country with the third country.

All in all, the study reveals important aspects about the potential for implementing inflation targeting frameworks in European emerging market economies and the possibilities to sustain this framework in the phase of accession to the euro area. An extended Taylor rule appears to be the most adequate rule to guide monetary policy during various stages of economic reform and transition. Such a rule supports balancing alternative targets for monetary policy as long as insufficient internal and external liberalization does not permit ambitious inflation targets and fully flexible exchange rates. Such a rule also allows balancing inflation, output, and exchange rate targets in intermediate exchange rate regimes like ERM 2 even given different degrees of integration with third countries. These results are consistent with a rather medium-term strategy observed in the larger new EU member countries among the CEEC. In these countries' monetary strategy targeting the exchange rate to some extent already plays a significant role for entering ERM 2.

2 The Central Banks' Choice of Inflation Targets: What Explains Persistent Inflation Differentials across European Emerging Market Economies?

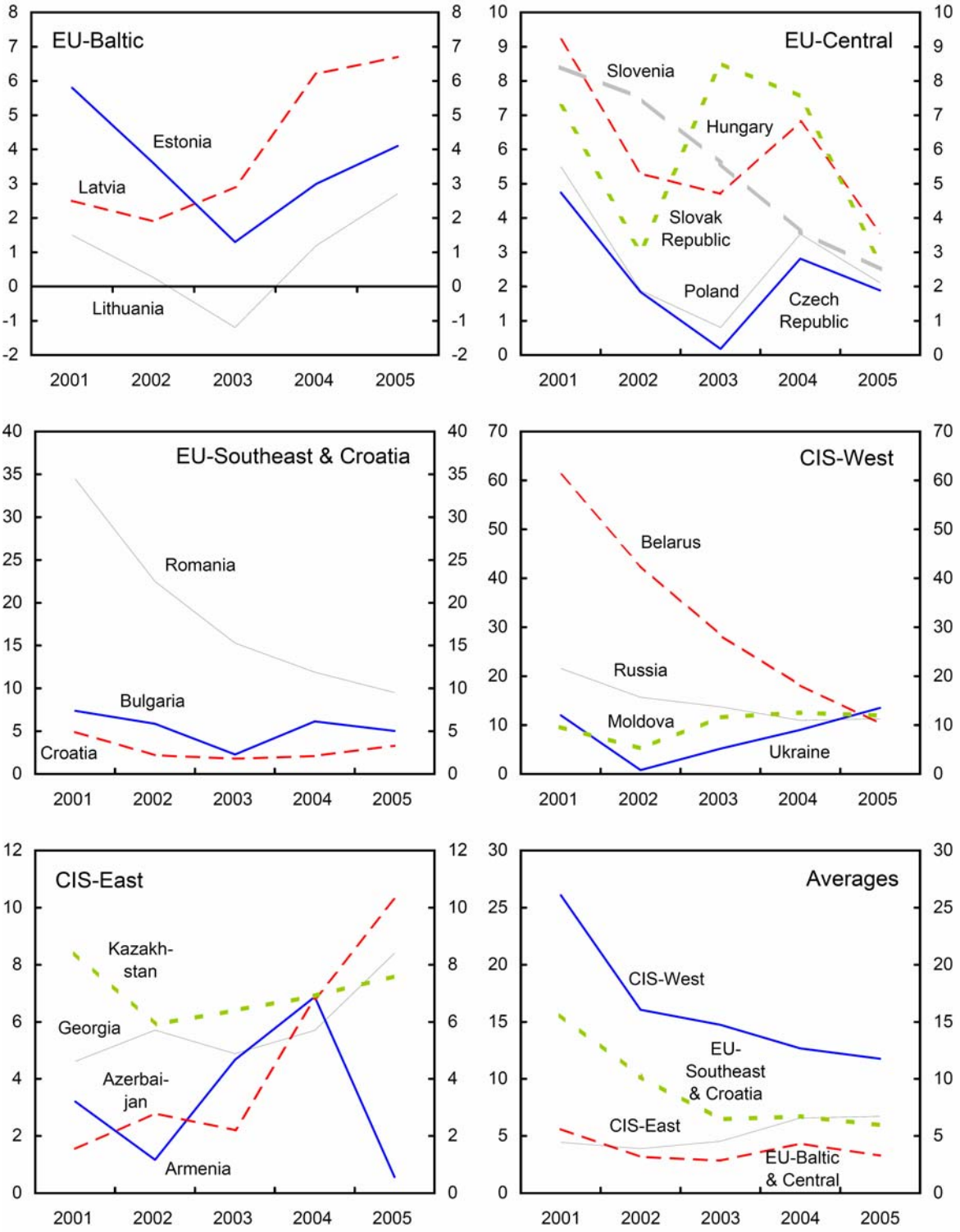
2.1 Introduction

Over 2000-05 a substantial gap opened in the inflation performance of different groups of European emerging market economies (Figure 2.1). Inflation in the central and eastern Europe countries (CEEC)[3] has declined to below 10 percent and remained there. The eastern CIS countries (CIS-East) have also reached inflation rates below 10 percent. However, Belarus, Moldova, Russia, and Ukraine (the CIS-West) have all converged on a higher inflation rate—around 10 percent.

Studies of inflation for individual countries in the CIS-West have emphasized the role of monetary policy and the exchange rate framework (Appendix Table A2.1). For Belarus and Ukraine the link has been made between monetary aggregates and inflation (Pelipas 2006, Lissovolik 2003, Leheyda 2005), whereas for Russia inflation has been seen as determined by exchange rate policy (due to unsterilized foreign exchange intervention) (Granville and Mallick 2006, Ohnsorge and Oomes 2004). These individual country studies have not examined what has motivated the central banks to choose their monetary regimes. Cross-country studies have modeled inflation outcomes as the result of central banks' choice problems (Appendix Table A2.2). Cottarelli, Griffiths, and Moghadam (1998) examine the influence of various incentives on inflation outcomes in transition economies, but in a much earlier time period (1992-95). Mafi-Kreft and Kreft (2006) confirm the well-known role of hard pegs like currency board arrangements in reducing inflation, and find that central bank independence and the prospect of early accession to the EU reduced inflation in the CEEC in contrast to the CIS. Aisen and Veiga (2006) focus on the relation between inflation and political instability in a comprehensive sample of developing and advanced countries.

[3] CEEC comprises the new member states of the European Union only. However, for the purpose of our study we also include Croatia because of its status as a candidate country.

Figure 2.1: Inflation in CEEC and CIS in Percent, 2001-05



Source: EBRD.

In this chapter we apply the cross-country choice/incentive approach to help illuminate possible explanations for the divergent CIS-West and CEEC inflation paths. Continuing high inflation in Russia, Ukraine, Belarus and Moldova suggests that incentive problems may not have been resolved. Our set of explanatory variables covers the key costs and benefits of inflation for transition economies, and key institutional features in them. Our panel also covers eastern CIS countries, which provide a useful control given their lower inflation rates and lagging institutional development relative to the CEEC. The rest of the chapter is organized as follows. The second section examines CPI measurement issues, and rejects the notion that the inflation differential could simply be a statistical artifact. The third and fourth sections discuss the approach to modeling the inflation choice of a central bank, the specific variables to be used in our model, and the panel estimation technique. The fifth section discusses the estimation results and the sixth section uses them to examine the source of higher inflation rates in Russia, Ukraine, Belarus and Moldova, and whether high inflation could persist. The final section discusses policy implications for these countries.

2.2 Could the Inflation Differential Simply Be a Statistical Artifact?

Measuring the true rate of CPI inflation is not straightforward for any country. In the U.S., the Boskin Commission (Boskin et al. 1996, 1998, Gordon 2000, 2006) found an upward bias in the consumer price index driven by four effects:

- quality effect;
- substitution effect;
- new goods effect;
- outlet effect.

The quality effect stems from changes in the quality of a good that may lead to price increases that are misconceived as price inflation. The substitution effect relates to changes in consumption patterns: In response to relative price increases, consumers may switch to similar but cheaper products. The new consumption pattern is not always reflected in updated weights for the consumer price index. The new goods effect arises when new goods are included in the CPI consumption basket only with a delay. Finally, the outlet effect stems

from the difficulties faced by official price collectors in reflecting consumers' moves towards shopping at cheap outlets such as hypermarkets.

Direct studies of mismeasurement bias in transition countries are few, and do not seem to indicate any unusual transition mismeasurement effect. Filer and Hanousek (2003) find that Czech inflation may be overestimated by more than 4 percent due to neglected new goods and the quality bias, but no studies are available on Russia, Ukraine, Belarus or Moldova. Accounting for the level of inflation, the Czech estimate is broadly in line with relative magnitudes measured for advanced economies (Table 2.1). We thus look more directly at the possibility of a larger bias in Russia, Ukraine, Belarus and Moldova.

Table 2.1: Estimates of Inflation Bias in Advanced Economies and the Czech Republic in Percentage Points per Year

		Product Substitution Bias	New Product and Quality Bias	Outlet Substitution Bias	Total Bias	Average Inflation
USA	Boskin et al. (1996)	0.4	0.6	0.1	0.8-1.6	2.8
Canada	Crawford (1998)	0.1	0.5	0.1	0.7	5.1
UK	Cunningham (1996)	0.0-0.1	0.2-0.5	0.1-0.3	0.4-0.8	2.9
Germany	Hoffmann (1999)	0.1	0.3	<0.1	0.5	1.1
Japan	Shiratsuka (1999)	0.1	0.7	0.1	0.9	0.6
Switzerland	Brachinger et al. (1999)	0.4	0.1-0.2	0.0	0.5-0.6	1.0
Typical index	Diewert (1997)	0.2	>0.4	0.3	>0.8	N.A.
Czech Republic 1996-97	Filer and Hanousek (2000)	0.8-1.2	1.0	0.7	2.5-2.9	8.7
Czech Republic 1990-99	Filer and Hanousek (2003)	>0.8	3.9	0.1	>4.8	13.2

Source: Filer and Hanousek (2003).
Note: Average inflation based on three years starting 5 years prior to the publication date of the paper to allow for publication lags except for Filer and Hanousek (2000), which used actual figures for 1996 and 1997, and Filer and Hanousek (2003), which used compound rates from 1999 to 2001.

Differences in CPI-basket related biases do not appear to play a significant role in the inflation differential. The CPI baskets do differ considerably between the CEEC and CIS-West, with the latter showing a much higher share of food (Table 2.2). And a common feature of the household surveys used to generate the CPI basket is that expenditures on food are overrepresented (IMF 2005a, Revenko 2006). However, even if inflation were to be measured with weights closer to the standardized weights used in EU new member states, CIS-West inflation rates would remain high relative to the CEEC (Table 2.3).

Table 2.2: Weights of Sectors in CPI, 2001-05

	Food		Nonfood Goods		Services	
	2001	2005	2001	2005	2001	2005
Czech Republic	0.29	0.30	0.41	0.39	0.31	0.32
Estonia	0.33	0.30	0.39	0.41	0.28	0.29
Hungary	0.29	0.28	0.41	0.43	0.30	0.29
Latvia	0.36	0.33	0.38	0.39	0.25	0.28
Lithuania	0.44	0.36	0.39	0.40	0.17	0.24
Poland	0.36	0.28	0.40	0.44	0.23	0.28
Slovak Republic	0.30	0.25	0.44	0.42	0.26	0.33
Slovenia	0.26	0.24	0.44	0.43	0.30	0.33
EU-Baltic & Central	**0.33**	**0.29**	**0.41**	**0.41**	**0.26**	**0.29**
Bulgaria	0.47	0.42	0.36	0.37	0.17	0.21
Romania	0.46	0.45	0.40	0.40	0.14	0.15
EU-Southeast	**0.47**	**0.43**	**0.38**	**0.38**	**0.15**	**0.18**
Belarus	0.64	0.54	0.19	0.25	0.17	0.21
Moldova	0.57	0.46	0.26	0.30	0.17	0.24
Russia	0.55	0.46	0.31	0.33	0.14	0.21
Ukraine	0.64	0.64	0.14	0.17	0.22	0.20
CIS-West	**0.60**	**0.52**	**0.22**	**0.26**	**0.17**	**0.22**

Source: Eurostat, SSCU, IMF estimates.
Note: No disaggregate data available for Croatia.

Table 2.3: Inflation Based on Different Weights,
Belarus, Moldova, Russia, Ukraine, 2001-05

	Belarus		Moldova		Russia		Ukraine	
	Actual	Benchmark	Actual	Benchmark	Actual	Benchmark	Actual	Benchmark
2001	63.7	78.9	10.0	8.1	21.7	24.2	12.3	9.1
2002	42.9	49.6	5.3	5.9	16.0	19.7	0.8	1.3
2003	28.5	34.1	11.8	11.3	13.7	16.0	5.2	3.9
2004	18.3	16.9	12.6	12.4	10.9	11.6	9.0	6.9
2005	10.4	9.0	12.7	11.8	12.5	13.2	13.5	10.2

Source: SSCU, IMF estimates.
Note: Benchmark inflation is based on 30 percent food, 40 percent nonfood goods, and 30 percent services.

Similarly, different price mismeasurement biases do not seem to be a relevant issue. For example, the Global Retail Development Index developed by A.T. Kearney (2006) indicates low market saturation in Russia, Ukraine, Belarus and Moldova, and no evident change

relative to central and eastern European countries over the past few years. While outlet bias could become more of an issue in the future—international retailers have started to move into these countries with modern hypermarkets and discount stores—it is unlikely that it contributed significantly to a higher relative bias in their past inflation.

Finally, IMF assessments of data quality do not point to any significant differences in CPI data techniques or quality. Eurostat supervised the implementation of western European standards in the CEEC. IMF Reports on the observance of standards and codes (data modules) indicate only minor shortcomings relative to international standards in compiling the CPI in Russia, Ukraine, Belarus and Moldova.[4]

2.3 Modeling the Inflation Differential

To better understand the inflation differential, we construct an empirical model of inflation outcomes in transition countries. We estimate a cross-country panel following the approach of Cottarelli, Griffiths, and Moghadam (1998), Mafi-Kreft and Kreft (2006) and Aisen and Veiga (2006). To the extent that this cross-country model reproduces the inflation differential, we can assess the factors that may be pushing central banks in Russia, Ukraine, Belarus and Moldova to choose higher inflation rates, and even ask whether any change could be expected in these factors, and thus the differential, going forward.

The empirical model takes its cue from the standard central bank choice problem laid out in the time consistency literature (see, for example, Barro and Gordon 1983). Thus the central bank may be understood as having targets for inflation and unemployment, which may not be consistent. These in turn reflect underlying structural features of the economy, and possibly trade-offs against other less formal central bank aims (e.g. financial stability).[5] The central

[4] See http://www.imf.org/external/np/rosc/rosc.asp for details on CPI statistical techniques used in Russia, Ukraine, Belarus, and Moldova. Moldova does not follow international standards for proper techniques in imputation of missing and new observations. To the extent missing observations are associated with scarcity of an item in question, this may understate the CPI, which would imply the inflation differential is understated.

[5] See Rodríguez Palenzuela, Camba-Méndez, and Garcia (2003) for a full discussion of factors affecting the choice of optimal inflation rate. See Cukierman (1992) for a discussion of the various motives that may impinge on a central banks' inflation choice. Besides employment, these can include fiscal revenues, external competitiveness and financial stability.

(continued…)

bank is also understood to have a short-run incentive to use surprise inflation to raise output, and a more pronounced incentive would be associated with higher inflation expectations in equilibrium as agents internalize the central bank's choice problem.[6] Finally, the central bank is understood to have certain weights on output and inflation stabilization in its welfare function. The political and institutional environment in which monetary policy operates helps determine these. In this approach, the actual inflation outcome also reflects a variety of unanticipated shocks to aggregate demand and supply. That is, in the short run a central bank may not be able to offset inflation impulses.

The model's focus on the factors underlying the inflation choice is not to imply that intermediate targets do not matter. The assumption we make is that the choice of intermediate target is penultimate, and reflects a central bank's preferences over inflation and output, as well as the political and economic environment and expectations that it faces. Thus when a central bank chooses a pegged exchange rate, it effectively chooses to import an inflation process, and our model attempts to shed light on this latter choice.[7]

Turning to the inflation choice, to capture *structural features* that may bear on transition central banks' unemployment and inflation targets, we use a number of variables. A key feature of the transition has been the need to shift labor from overstaffed state enterprises and collective farms to more productive uses (see, e.g., Schiff et al. 2006).[8] Central banks may have resisted unemployment due to massive sectoral labor shifts by ensuring significant

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[6] The short-run trade-off will depend, among other things, on the variance of nominal relative to real shocks (but not directly on the level of inflation). Thus a sustained attempt by a transition country central bank to exploit the output-inflation trade-off would essentially eliminate it. For this reason, we confine our sample to the post-hyperinflation period. More generally, when the game between the central bank and agents is in an equilibrium, there is no further incentive for the central bank to shock the economy with unanticipated inflation, and thus to change the relative variances. What matters then are other structural influences on the trade-off, which help determine the level of inflation at which the incentive disappears. We model these.

[7] Technical factors may constrain a central bank's choice of monetary regime. However, those central banks, which lack the capacity to manage a flexible exchange rate regime, are not doomed to import an inflation process; they can always manage inflation via an adjustable peg.

[8] In transition economies, structural changes—obsolescence of capital and disorganization on one side of the transition recession and massive productivity gains on the other—are likely to have been much more important than cyclical issues over the last 10 years. For these reasons it is difficult to estimate the output gap or other capacity measures for these economies, and indeed data in these areas is very incomplete. Our variables for structural unemployment pressures are thus reasonable controls for capacity pressures.

(continued…)

credit growth to the economy via a loose monetary policy. Directed credit from the banking system was in fact a consideration in Belarus and Ukraine at least during the first half of the sample period (see IMF 2005b,c). We use the EBRD's index of enterprise restructuring, and the share of agriculture in GDP to capture this potential influence on inflation.[9] As transition proceeds, and productivity and wages in the traded goods sector rise, so too would wages in the nontradables sector, putting pressure on the overall price level. We capture this (the Balassa-Samuelson effect) via labor productivity growth in the manufacturing sector, interacted with the exchange rate regime.[10] A central bank may be also concerned about distortions to after tax real factor returns created via the interaction of inflation with the tax code. We construct a dummy variable for the presence of a flat income tax—which should be less distortionary—to capture this possible influence.

Other influences on targets are fiscal sustainability, external sustainability, and developments of the financial market. If the government is constrained by a large debt stock—with interest payments crowding out other budget expenditures and rollover considerations creating a flow financing problem—fiscal dominance may hold. A central bank may prefer to maintain loose credit conditions, accentuating fiscal impulses to aggregate demand, rather than risk that monetary tightening produced default, and all the economic turmoil that could bring. We take the stock of government debt as a ratio to GDP as an indication of fiscal sustainability, and use a one-period lag to address potential endogeneity problems.[11] A central bank may attempt to reduce a persistent current account deficit by devaluing the currency. This, via pass-

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[9] Other authors like Cukierman, Edwards, and Tabellini (1992) argue that the agricultural sector is difficult to tax and therefore consider it to be a fiscal motive. This interpretation makes some sense for developing economies with low revenue ratios and large informal agricultural sectors. For the transition economies that we consider, where the agricultural sector is much more organized (e.g., collective farms) and where tax ratios are generally in the 30–40 percent of GDP range, it is not a very compelling interpretation.

[10] Existing empirical evidence suggests that if anything this should work against an inflation differential. Égert, Halpern, and MacDonald (2006) suggest that among the CEEC the highest Balassa-Samuelson effect may be found in Hungary and Poland (up to 2 percent) and the lowest in the Czech Republic and Latvia (close to zero). Égert (2005) finds the effect to be 0.7 percent for Russia and for 0.5 percent for Ukraine.

[11] Many of the transition economies have had large contingent liabilities at one point or another. The so-called lost savings in the CIS from the early 1990s hyperinflation are an example. These liabilities would provide an additional incentive towards inflation, since these have typically not been indexed. Data limitations preclude their use.

through of higher import prices, will raise the inflation rate in the near term.[12] The pay-off is a reduced risk of external crisis in the medium term (which would involve overshooting depreciation, very high inflation, and large output losses). We use the current account balance as a share of GDP, lagged one period, to capture this effect. A greater degree of financial market development could reduce the need for a central bank to keep monetary conditions loose in support of high credit growth. We take the EBRD's bank reform index and securities market development index as possible measures.[13] At the same time, central banks may independently seek to smooth interest rates or exchange rates to keep intermediation smooth. We use the standard deviation of interest and exchange rates to capture this potential influence.

To capture the incentive a transition country central bank may have to inflate in the short run, known as the *output-inflation trade-off,* we use several variables identified as important in the literature, namely the degree of openness, competition, and price liberalization. The real benefits of surprise inflation decline with openness, since competitiveness and net exports suffer (Romer 1993). We measure openness by the EBRD's trade and foreign exchange system liberalization index. Additionally, Rogoff (2003) notes that competition tends to make prices and wages more flexible, reducing the real effects of unanticipated monetary policy. There is thus less incentive for central banks to inflate. We use the EBRD's competition policy index to capture this potential influence. When prices and wages are rigid, the real effects of unanticipated monetary policy become larger (though effects may also show up as shortages of goods). We focus on price rigidities here since we do not have good labor market data for many transition economies (wage indexation, contract duration, and centralization of wage bargaining). There are good economic reasons why price rigidities may exist even in market economies (e.g., menu costs), but these are likely dominated by

[12] If all prices are raised in proportion to the exchange rate depreciation, there would be no real depreciation, and no incentive to use this channel. This could occur in a fully dollarized economy, but none of the transition economies fits this mold for the time period in question.

[13] Financial dollarization is one reflection of financial market development that could impact inflation outcomes, for instance by creating an incentive for a central bank to minimize exchange rate movements (this would prevent impacts on agents' balance sheets, but would also transmit external disturbances to the economy). However, dollarization can also reflect expectations of inflation (see Levy Yeyati 2006), and due to this endogeneity issue, is not modeled here.

pure price liberalization considerations in our set of formerly planned transition economies. We use the EBRD's price liberalization index as a measure.

We aim to capture the weight a central bank places on inflation in its objective function by controlling for key features of the *institutional environment.* Political instability, like frequent elections and unstable governments, may reduce a central bank's horizon, and enhance its focus on output and employment outcomes. In equilibrium, this would lead to higher inflation expectations and inflation. A preliminary look at World Bank data suggests that while this could explain inflation, it will not likely explain the gap: Table 2.4 shows a clear gap between the CEEC and the CIS countries, but in a direction that favors higher inflation in the CEEC. A greater degree of central bank independence may lead a central bank to target a very low level of inflation, and to put more weight on inflation outcomes (ignoring potential short-run trade-offs with output). To control for endogeneity concerns—that low inflation leads central banks towards a stronger institutional framework—we take the initially prevailing degree of central bank independence. Data are drawn from Cukierman, Miller and Neyapti (2002), and measure only the legal degree of independence, which may differ from independence in practice.[14] EU accession, and the requirements this imposes on economic policies, may directly encourage low inflation (for instance, the conditions for adoption of the euro) and indirectly help import monetary policy credibility. We capture this influence via a dummy variable that takes a value of 1 once accession talks begin.

[14] Up-to-date data on central bank independence is in any event unavailable. IMF staff reports on Article IV consultations (Bassett 2003, IMF 2005b, 2005c, 2006) as well as the assessments by the Economist Intelligence Unit (EIU 2005, 2006) suggest that central bank independence is still lacking in the CIS-West.

Table 2.4: Government Stability in Transition Countries, Averages 1999-2003

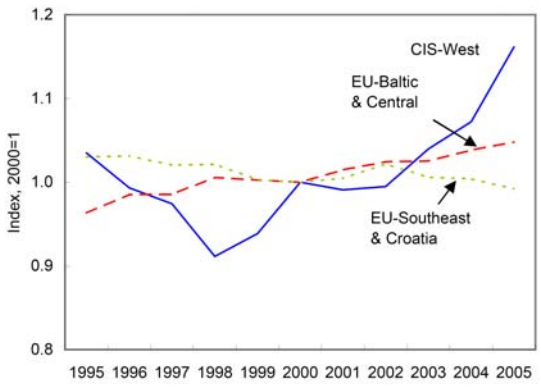
	EU-Baltic & Central	EU-Southeast & Croatia	CIS-West
Years chief executive in office 1/	4.20	2.80	5.40
Changes in effective executive 2/	0.30	0.27	0.15
Herfindahl index for the government 1/	0.56	0.73	0.56
Government fractionalization 1/	0.45	0.27	0.44
Party fractionalization index 2/	0.76	0.75	0.70

1/ Database of Political Institutions, World Bank.
2/ Cross National Time Series Data Archive.

Finally, we link inflation choices to outcomes by controlling for the following key (in the transition context) *exogenous shocks* to aggregate demand and supply.[15] Improvements in the terms of trade will provide a boost to domestic demand, which may prove hard for a central bank to offset. For instance, rising export prices will tend to work their way back to factor prices, potentially fueling cost-push inflation. Looking at the data, there is an asymmetry in terms of trade developments between the CEEC and CIS-West, suggesting this could be an issue (Figure 2.2). Changes in the harvest are potentially important supply shocks for economies where the agricultural sector still looms large. And these can quickly drive up overall prices, given the weight of food in transition countries' CPI. A preliminary look at the data, however, suggests that the pattern of production across country groupings does not vary greatly (Figure 2.3). Changes in administered prices will raise the price of the existing level of output as long as there are downward nominal rigidities in non-administered prices. They are a direct supply shock, which cannot easily be offset by central banks in the short run. In the absence of cross-country data on the impact of individual administered price increases, we capture this potential impact indirectly, via the change in the share of administered prices.

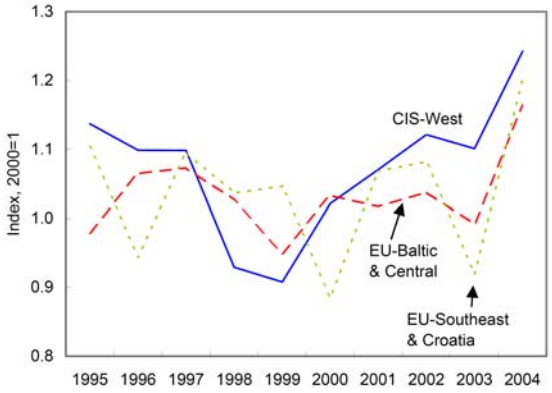
[15] Other unanticipated demand shocks could come via the government (unforeseen and rapid fiscal loosening), or via consumers and investors (unrelated to terms of trade gains, and showing up in large unexpected capital inflows). Given lags in fiscal policy formulation and implementation, we do not see unanticipated fiscal shocks as a key issue. Given our annual data, we would also expect monetary policy to be able to react to slower-to-materialize consumption and investment shocks, leaving in practice a small unanticipated component.

Figure 2.2: Terms of Trade



Source: IMF WEO.

Figure 2.3: Crop Production



Source: World Bank, World Development Indicators.

2.4 Empirical Methodology and Results

For our inflation measure, we follow Cukierman, Miller, and Neyapti (2002) and Mafi-Kreft and Kreft (2006) and use the depreciation rate in the real value of money:

$$d_{i,t} = \frac{\pi^{d}_{i,t}}{1+\pi^{d}_{i,t}}; \qquad i = 1,\ldots, N \text{ and } t = 1,\ldots, T, \tag{2.1}$$

where $\pi^{d}_{i,t}$ is the CPI inflation rate in decimals in country i at time t. Using the depreciation rate of money rather than the logarithm of inflation avoids deflationary periods dropping out of the sample. The data for our variables cover 19 transition countries for the years 1995-2004. One advantage of limiting the sample to transition countries is the element of a natural experiment—the countries in question share an economic and cultural legacy that would otherwise be difficult to control for. The country coverage, in particular the inclusion of the CIS-East, provides for variation along important data dimensions. The sample period does exclude the early 1990s, to avoid the complications presented by the hyperinflations that occurred in many transition countries. Appendix Table A2.3 gives full details about data sources. Panel unit root tests allow us to assume that the rate of depreciation of money is stationary (Table 2.5). Thus, we do not run into spurious correlations from neglected cointegration relationships. Tests for unit roots in other variables cannot reject stationarity.

Table 2.5: Panel Unit Root Tests for the Rate of Depreciation of Money

Method	Statistic	Probability	Observations	Result
Null: Assumption of a common unit root process				
Levin, Lin, and Chu *t*	-6.21	0	171	I(0)
Breitung *t*-statistic	-2.24	0.01	152	I(0)
Null: Assumption of an individual unit root process				
Im, Pesaran, and Shin W stat	-3.69	0	171	I(0)
ADF – Fisher χ^2	83.15	0	171	I(0)
PP – Fisher χ^2	197	0	190	I(0)
Null: Assumption of no common unit root process				
Hadri *Z*-statistic	6.84	0	209	I(1)

We estimate a panel in levels with time fixed effects. Setting X as the vector of explanatory variables, β as the vector of parameters to be estimated, v as the time specific effect, and ε is the error term, we have

$$d_{i,t} = X'_{i,t}\beta + \nu_t + \varepsilon_{i,t}. \tag{2.2}$$

A country fixed-effects model could minimize the risk of omitted variables bias, but would discard information on the levels of the variables, and for our purpose it is important to preserve this information in cross-sectional differences. Time fixed effects allow us to capture the common part of the ongoing transition process, and the cross-sectional correlation stemming from international financial markets and contagion during the financial crises in Russia and earlier in Bulgaria and Romania. We consider models without time fixed effects and with country fixed effects as part of our robustness checks.

We use an estimator with panel corrected standard errors. Ordinary least squares (OLS) is optimal if error processes are homoskedastic and all error processes are independent of each other. However, in our sample we know that panel heteroskedasticity and contemporaneous correlation are likely to arise. Serial correlation (inflation persistence) may also be present due to indexation, and could be modeled in a dynamic panel[16]

[16] We drop the fixed effect, ν_t, in the following exposition.

(2.3) $$d_{i,t} = d_{i,t-1} + X'_{i,t}\beta + \varepsilon_{i,t}$$

by including a lagged dependent variable, $d_{i,t-1}$. The lagged dependent variable is correlated with the error term, $\varepsilon_{i,t}$, in both fixed effect and random effect estimations. Arellano and Bond (1991) suggest to estimate the panel in first differences

(2.4) $$\Delta d_{i,t} = \rho \Delta d_{i,t-1} + \Delta X'_{i,t}\beta + \Delta\varepsilon_{i,t}\,.$$

New correlations show up now between the first differences of the lagged dependent variable, $\Delta d_{i,t-1}$, and the error term, $\Delta\varepsilon_{i,t}$, and autocorrelation of the error term. Arellano and Bond (1991) develop a GMM estimator that solves the problem with the help of instrumental variables. First, one needs to disentangle the explanatory variables, X, into strictly exogenous variables, X^{exo}, and endogenous variables, X^{end},

(2.5) $$\Delta d_{i,t} = \rho\,\Delta d_{i,t-1} + \Delta X_{i,t}^{exo'}\beta_1 + \Delta X_{i,t}^{end'}\beta_2 + \Delta\varepsilon_{i,t}\,.$$

The Arellano-Bond dynamic panel estimator uses the following instruments: levels of the dependent variable, d_i, lagged two and more periods; levels of the endogenous variables, X^{end}, lagged two and more periods; first differences of the strictly exogenous variables, X^{exo}, which are used as their own instruments.

When the dependent variable and/or the independent variables are persistent, a system GMM estimator is preferable following Blundell and Bond (1998). This estimation combines the moment conditions implied by the first differences equation with the additional moment conditions implied by the levels equation. Underlying assumption is that the explanatory variables are uncorrelated with the individual effects (see Arellano and Bover 1995). Then, lagged differences of the explanatory variables and of the dependent variable may be valid instruments for the level estimation.

However, in our set up we would run into the problem of weak instruments, since the instrumental variables are to some extent correlated with the time fixed effects (see Stock, Wright, and Yogo 2002). The small number for our cross section also does not lend itself to a dynamic framework (see Roodman 2006).

Instead, we may assume that the errors follow an autoregressive process of order one, AR(1),

(2.6) $$\varepsilon_{i,t} = \rho\varepsilon_{i,t-1} + \eta_{i,t}\,.$$

We rewrite the model (2.3) with ρ as the AR parameter, η is an independent and identically distributed error process and *L* denotes the lag operator

(2.7) $$d_{i,t} = X'_{i,t}\beta + \frac{\eta_{i,t}}{1-\rho L}$$

(2.8) $$d_{i,t} = X'_{i,t}\beta + \varepsilon_{i,t} + \rho\varepsilon_{i,t-1}$$

(2.9) $$d_{i,t} = X'_{i,t}\beta + \rho d_{i,t-1} - \rho X'_{i,t-1}\beta + \eta_{i,t}\,.$$

From (2.9) we see that two models (2.3) and (2.7) include both a lagged dependent variable. The difference between the two models is in the speed of adjustment to a level change in *X*. If *X* has rather an immediate one-time impact like a regime change, the adjustment is quick (small ρ) and the two formulations do not differ. The AR parameter may be estimated and thereby the problem of serial correlation can be eliminated by applying the procedures suggested by Cochrane and Orcutt (1949) and Prais and Winsten (1954).

The error term is assumed to be heteroskedastic and contemporaneously correlated, with a different but stable variance for each of the *N* countries. Thus, the Gauss-Markov assumption of $\Omega = \sigma^2 I$ is violated and the covariance for an OLS estimate of β is not anymore given by $(X'X)^{-1}$ but

(2.10) $$(X'X)^{-1}(X'\Omega X)(X'X)^{-1}.$$

Similar to the procedure suggested by White (1980), Beck and Katz (1995) derive panel corrected standard errors by using the *T* replicates of the OLS residuals and then taking the square root of

(2.11) $$(X'X)^{-1}\,X'\left(\frac{E'E}{T}\otimes I_T\right)X\,(X'X)^{-1},$$

where *E* is the vector of error terms and $\otimes$ the Kronecker product. If the underlying residuals are contemporaneously correlated and panel-heteroskedastic, the contemporary covariances are the diagonal elements (see Beck 2008, Beck and Katz 2004, and Kittel 1999). We thus follow Beck and Katz (1995) and Edwards (2001) and use panel corrected errors and consider dynamic panel models as part of our robustness checks.

2.5 The Empirical Model

Our baseline specification was chosen for its reasonably high explanatory power, its parsimony and for the robustness of the results (Table 2.6, column 1). The baseline model fits the data and captures the overall inflation gap reasonably well (Figure 2.4). It considers the role of structural features, for factors underlying the output-inflation trade-off, and exogenous shocks in accounting for different inflation outcomes. Figure 2.5 visualizes the magnitudes of the variables by reporting their absolute contribution to fitted inflation in percent. The variables do not vary much over time—at least in the full sample of countries. On average the largest contributors are the constant explaining around 40 percent of inflation and price liberalization with 35 percent. These two variables actually offset each other to a large extent due to opposite signs. Openness contributes about 12 percent, fiscal sustainability 6 percent, and the terms of trade 5 percent. Different from the other variables is the declining contribution of the time fixed effect from 8 percent down to zero. In what follows, we discuss each set of explanatory variables, bringing in results using the additional variables discussed in the third section in turn.

Figure 2.4: Fitted versus Actual Inflation, 1996-2004

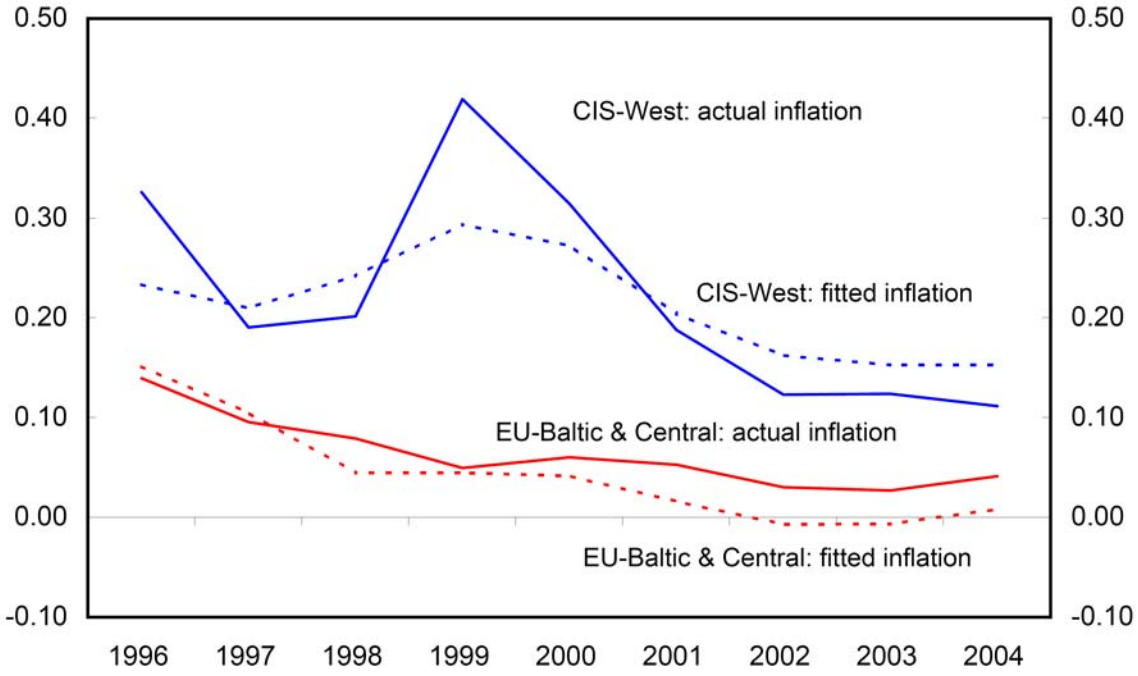
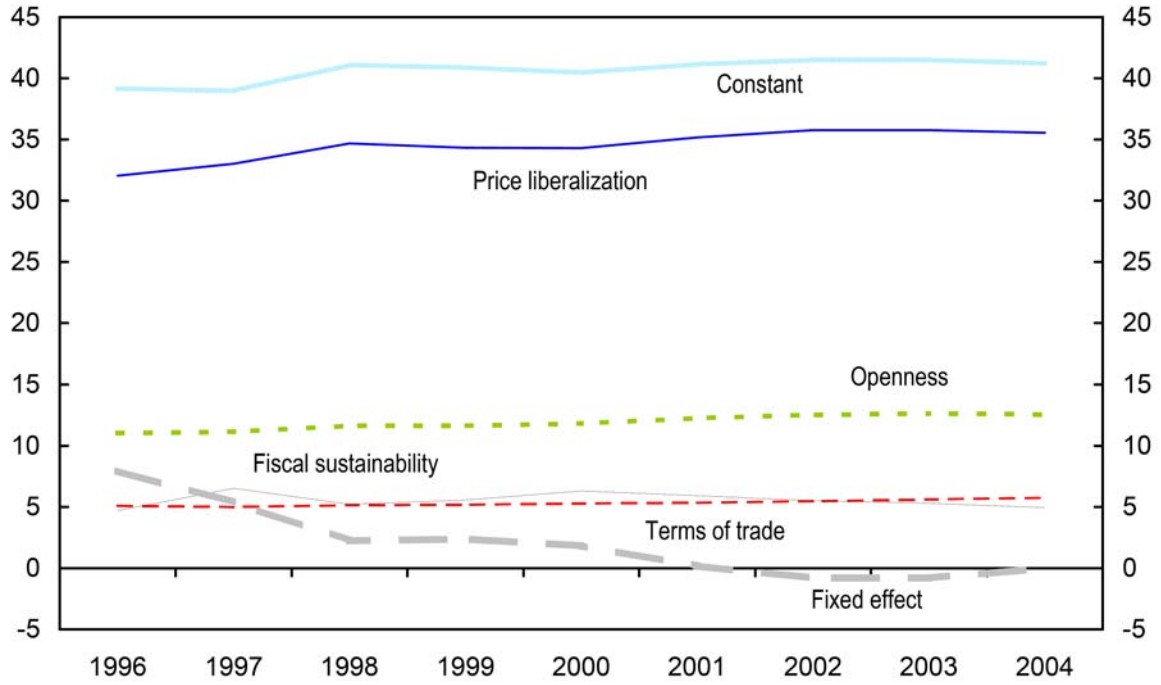


Figure 2.5: Factors Explaining Inflation, 1996-2004



The empirical model suggests that a central bank's incentive towards higher short-run inflation is a key reason for observed outcomes (Table 2.6, columns 1-3). Countries that have not gone as far in liberalizing their internal markets, and countries that have not opened up as much externally tend to have higher inflation. The estimated impacts are robust across specifications and highly significant. They are also considerable in magnitude: all else equal, an improvement of one point on the price liberalization index reduces inflation by about 12-13 percent, and an improvement of 1 point in the openness index reduces inflation by about 4½ percent. To make this concrete, the increasing price regulation in Belarus between 1997 and 2004 (from 4.00, representing comprehensive price liberalization, to 2.67, representing less than significant progress and a large role for non-market state procurement) would have added some 16 percent to inflation, all else equal. Raising openness from 3 to 3.67, and removing remaining trade restrictions, as Ukraine did between 2001 and 2005, would have lowered inflation by about 3 percent, all else equal. Results with the competition variable—which is strongly correlated with the other two—were not robust, but were signed in the expected manner when the other two were excluded from the estimation.

Table 2.6: Results from Panel Regressions: Role of Output-Inflation Trade-Off and Exogenous Factors

	1	2	3	4	5
Structural features					
Fiscal sustainability	0.212 ***	0.213 ***	0.231 ***	0.211 ***	0.206 ***
	[0.024]	[0.024]	[0.027]	[0.024]	[0.040]
Output-inflation trade-off					
Price liberalization	-0.123 ***	-0.133 ***		-0.123 ***	-0.134 ***
	[0.029]	[0.028]		[0.028]	[0.045]
Openness	-0.044 ***	-0.057 ***		-0.044 ***	-0.045 **
	[0.014]	[0.014]		[0.013]	[0.022]
Competition		0.060 ***	-0.013		
		[0.015]	[0.027]		
Exogenous shocks					
Terms of trade	0.076 *	0.111 ***	0.009	0.086 **	0.122 **
	[0.035]	[0.035]	[0.046]	[0.040]	[0.049]
Harvest				-0.038	
				[0.044]	
Change in administered price share					-0.069 **
					[0.030]
No. of Observations	171	171	171	171	138
No. of Countries	19	19	19	19	16
Adjusted R^2	0.724	0.736	0.574	0.726	0.748

Standard errors in brackets: *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Unanticipated shocks to supply and demand are important determinants of cross-country inflation outcomes (Table 2.6, columns 1-2 and 4-5). An improvement in the terms of trade of 10 percent raises inflation by about 0.8 percent, and this effect is significant and robust across various model specifications (interacting the terms of trade with the exchange rate regime did not modify this effect). The harvest has the right sign when added to the model, but was not significant. Finally, a decrease in the share of administered prices increases inflation (by about 0.7 percent for every 10 percent decrease in the change in the share). However, we cannot use this model to understand the inflation gap, since the one-fifth of the sample that drops out is heavily concentrated in the western CIS countries.

The evidence on the institutional environment is mixed (Table 2.7). Consistent with Aisen and Veiga (2006), the addition of a political stability measure to the model raises explanatory power substantially. Moving one step from a stable government such as the one in Russia (scoring 11) toward a less stable government such as the one in Poland (scoring 6) increases

inflation by about 1 percent. However, the larger model including political/institutional variables omitted one country (Georgia), and did not prove as robust, making it less adequate as a baseline.[17] De jure central bank independence, when used in place of political stability in the model, is correctly signed; however, with the same approach, our EU accession dummy is incorrectly signed (Table 2.7, columns 2 and 4). Both are imprecisely estimated. Intuition suggests that these variables are related to the openness index (which has a strong institutional component), and when this control for incentives is removed from the model, central bank independence becomes significant and EU accession takes the expected sign (Table 2.7, columns 3 and 5). In sum, our data and model suggest that pressures on central banks matter, and institutions may matter, but the latter may not be necessary to control inflation, if the incentive to inflate is small.

Table 2.7: Results from Panel Regressions: Role of Institutions

	T2.6-1	1	2	3	4	5
Structural features						
Fiscal sustainability	0.212 ***	0.208 ***	0.214 ***	0.214 ***	0.210 ***	0.210 ***
	[0.024]	[0.027]	[0.024]	[0.024]	[0.025]	[0.025]
Output-inflation trade-off						
Price liberalization	-0.123 ***	-0.154 ***	-0.125 ***	-0.159 ***	-0.120 ***	-0.171 ***
	[0.029]	[0.032]	[0.030]	[0.037]	[0.029]	[0.035]
Openness	-0.044 ***	-0.037 ***	-0.044 ***		-0.057 ***	
	[0.014]	[0.012]	[0.016]		[0.013]	
Institutional environment						
Political stability		-0.008 *				
		[0.004]				
Central bank independence			-0.016	-0.122 **		
			[0.075]	[0.054]		
EU accession					0.034	-0.007
					[0.024]	[0.024]
Exogenous shocks						
Terms of trade	0.076 *	0.147 ***	0.077 **	0.750 *	0.079 **	0.060
	[0.035]	[0.043]	[0.038]	[0.041]	[0.032]	[0.040]
No. of Observations	171	140	171	171	171	171
No. of Countries	19	18	19	19	19	19
Adjusted R^2	0.724	0.782	0.724	0.708	0.707	0.686

Standard errors in brackets: *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

[17] Since average political stability in Russia, Ukraine, Belarus and Moldova does not differ greatly from average political stability in central and eastern European states, the analysis of the gap is not materially effected by using the more parsimonious model as a baseline.

Fiscal considerations appear to create a stronger motivation for higher inflation, but other influences on optimal inflation and output targets do not add to the empirical model (Table 2.8). The level of lagged government debt is correctly signed, varies little in magnitude, and is significant at the 1 percent level across the whole range of specifications. An increase in the government debt ratio of 10 percent would be associated with inflation being 2 percent higher in the subsequent period. Financial market development—a more reformed banking system or a more developed securities market—points strongly to lower inflation across countries, but only when used without price liberalization and openness in the model, with which it is strongly correlated (columns 8-11 and Appendix Table A2.4).[18] Results with variables capturing unemployment pressures—the index of enterprise restructuring and the agricultural share—are similar: only when price liberalization is not in the model are they correctly signed (but even so they are still insignificant)(columns 3-6). Pure optimal inflation considerations—the Balassa-Samuelson effect, and tax-related labor supply distortions—have no significance (columns 1-2). Finally, considerations of external sustainability are correctly signed, but insignificant (column 7). Progress with other structural reforms has been highly correlated in transition economies with progress in internal and external liberalization, and separate effects, if they exist, cannot be distinguished in our sample.

The overall results are reasonably robust across country groupings, time, and alternative econometric specifications. Dropping eastern CIS countries does not affect the main conclusions. The coefficients keep their sign, but the terms of trade turn insignificant (Appendix Table A2.5, column 2). However, this is not unexpected considering that it implies to exclude Azerbaijan and Kazakhstan, oil-producing countries where terms of trade movements have been important. The model still fits the data reasonably well once we constrain the sample to the post-crises years, 2000-04, and for the most part signs and the size of coefficients are reasonably robust (Appendix Table A2.5, column 3). The exception is the government debt variable. However, this again is not unexpected, since the earlier period contained all of the government debt crises. Including country fixed effects does not effect

[18] We do not find interest or exchange rate smoothing to be important influences. These results are available on request.

the signs or significance of any of the variables. However, with the fixed effects largely negative, the size of the price liberalization and government debt impacts become larger. Excluding the time fixed effects (i.e., implementing a random effects model) does not affect the signs of any of the variables. However, the terms of trade, which can be understood as a type of time effect particular to certain countries in the sample, are no longer significant. Again, these results are understandable (see Appendix Table A2.5, columns 4-5). Moving to a dynamic panel[19] does not affect the significance of the fiscal sustainability and internal liberalization variables (see Appendix Table A2.5, columns 6-7). External liberalization and the terms of trade retain their sign and magnitude, but are no longer significant. The size of each coefficient falls, as would be expected. Lagged inflation is significant, but the coefficient of below 0.3 is small compared to other studies where lagged inflation is above 0.6 (see Aisen and Veiga 2006). This latter result is not surprising given the size of our dataset and the large number of instrumental variables required in the dynamic panel set up.

[19] All dependent variables are treated as strictly exogenous, since in other specifications the number of instruments exceeds the cross section dimension. The Arrelano-Bond test (not reported) indicates that all the dynamic panel regressions are free of serial correlation.

Table 2.8: Results from Panel Regressions: Role of Structural Features

	T2.6-1	1	2	3	4	5	6	7	8	9	10	11
Structural features												
Balassa-Samuelson		-0.015										
		[0.067]										
Tax system (labor supply)			0.017									
			[0.023]									
Unemployment pressure--industry				0.021	-0.024							
				[0.020]	[0.021]							
Unemployment pressure--agriculture						-0.234 ***	0.021					
						[0.073]	[0.103]					
Fiscal sustainability	0.212 ***	0.211 ***	0.213 ***	0.217 ***	0.225 ***	0.223 ***	0.209 ***	0.208 ***	0.210 ***	0.233 ***	0.217 ***	0.219 ***
	[0.024]	[0.026]	[0.025]	[0.024]	[0.026]	[0.025]	[0.034]	[0.025]	[0.024]	[0.027]	[0.022]	[0.028]
External sustainability								0.033				
								[0.106]				
Financial market development--banks									0.006	-0.065 ***		
									[0.016]	[0.021]		
Financial market development--securities											0.031 ***	-0.037 **
											[0.008]	[0.016]
Output-inflation trade-off												
Price liberalization	-0.123 ***	-0.122 ***	-0.127 ***	-0.133 ***		-0.124 ***		-0.122 ***	-0.125 ***		-0.123 ***	
	[0.029]	[0.030]	[0.030]	[0.032]		[0.028]		[0.030]	[0.031]		[0.028]	
Openness	-0.044 ***	-0.045 ***	-0.042 ***	-0.052 ***	-0.060 ***	-0.048 ***	-0.079 ***	-0.043 ***	-0.048 ***		-0.056 ***	
	[0.014]	[0.014]	[0.015]	[0.011]	[0.014]	[0.013]	[0.021]	[0.015]	[0.014]		[0.013]	
Exogenous shocks												
Terms of trade	0.076 *	0.079 **	0.073 **	0.077 **	0.048	0.102 ***	0.037	0.069 **	0.080 **	0.009	0.096 ***	0.000
	[0.035]	[0.039]	[0.034]	[0.035]	[0.035]	[0.039]	[0.060]	[0.034]	[0.036]	[0.042]	[0.036]	[0.042]
No. of Observations	171	170	171	171	171	171	171	171	171	171	171	171
No. of Countries	19	19	19	19	19	19	19	19	19	19	19	19
Adjusted R^2	0.724	0.724	0.720	0.728	0.666	0.714	0.506	0.694	0.723	0.663	0.724	0.596

Standard errors in brackets: *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

2.6 What Does the Model Say about the Inflation Differential?

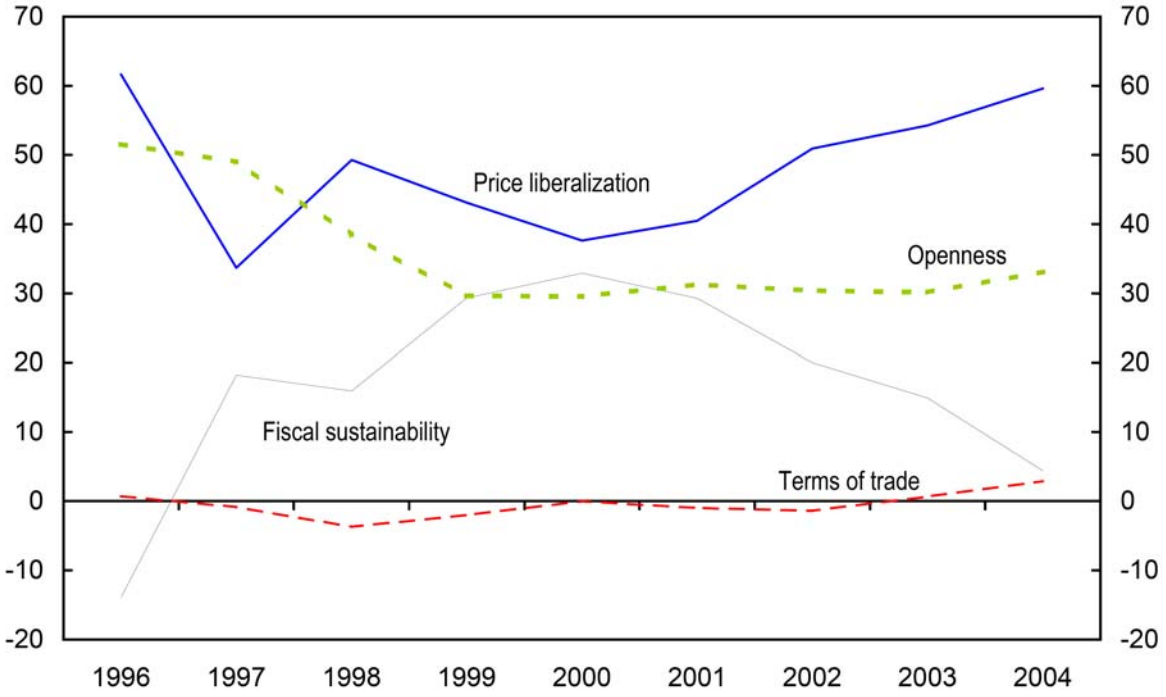
The empirical model provides a vehicle through which to address the possible sources of the inflation differential. The difference between the CIS-West and EU (Baltic and Central) average for each variable, $\bar{x}$, times the coefficient, β_x, over the fitted inflation gap gives the contribution of each variable in percent:[20]

$$\frac{\beta_x \cdot \left(\bar{x}_{CIS-West,t} - \bar{x}_{EU-Baltic\&Central,t}\right)}{\bar{\hat{d}}_{CIS-West,t} - \bar{\hat{d}}_{EU-Baltic\&Central,t}} \cdot 100. \tag{2.11}$$

Using this decomposition, it can be seen that the key motivations for higher inflation in Russia, Ukraine, Belarus and Moldova appear to have been evolving of late (Figure 2.6). Differences in the degree of price liberalization—which can broadly be understood to capture the level of internal liberalization of the economy—appear to have always been an important motivation towards higher inflation, but have grown of late to explain some 60 percent of the gap. Differences in the degree of external liberalization could explain a further 33 percent of the gap at present. Differences in the fiscal environment, once important, appear to have receded as a factor underlying the gap. The impact of terms of trade differences has been very mild to-date for the CIS as a whole (reflecting, in part, their diverse experiences with terms of trade shocks).

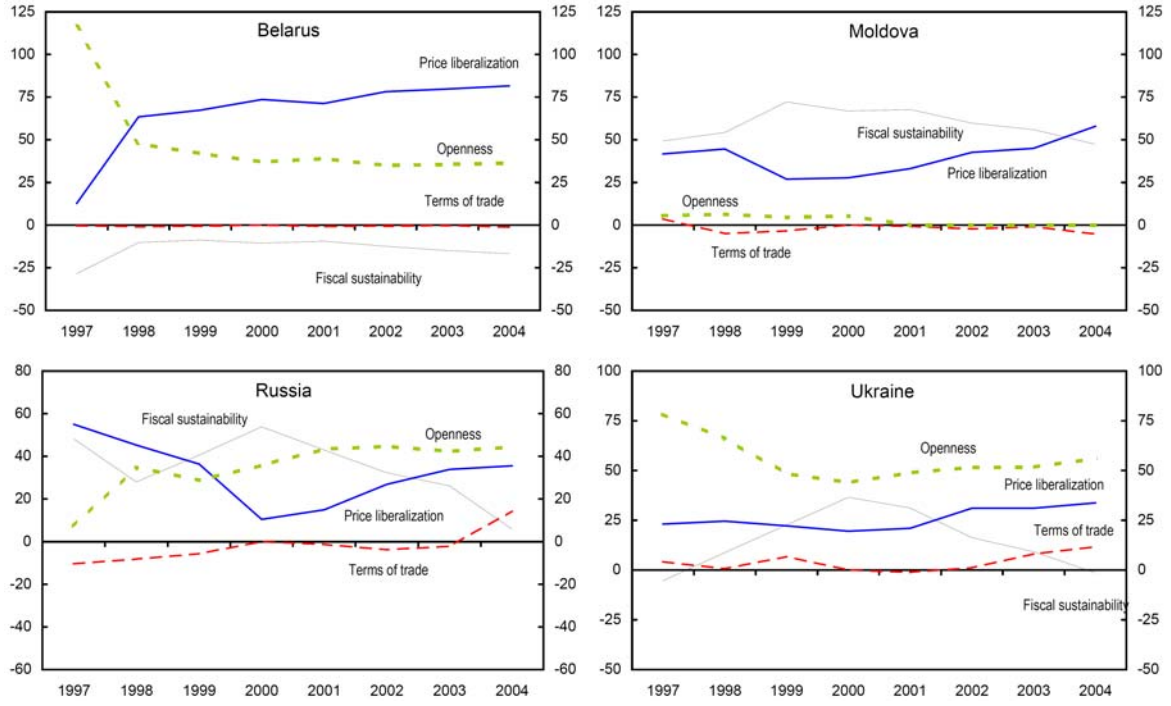
[20] Using the actual inflation gap has no impact on the relative importance of the explanatory variables.

Figure 2.6: Factors Underlying Inflation Differentials, 1996-2004



The details for individual CIS-West countries reflect these findings with some nuances (Figure 2.7). Belarus is very much in line with the CIS-West average. Russia and Ukraine differ to the extent that positive terms of trade movements are more important of late (at 30 and 25 percent of the gap respectively). In turn, a reduction in government debt appears to play a much larger role in offsetting other factors and providing an incentive towards a smaller gap. For Moldova, differences in external liberalization appear to play a limited role, while issues of fiscal sustainability and internal liberalization may be crucial.

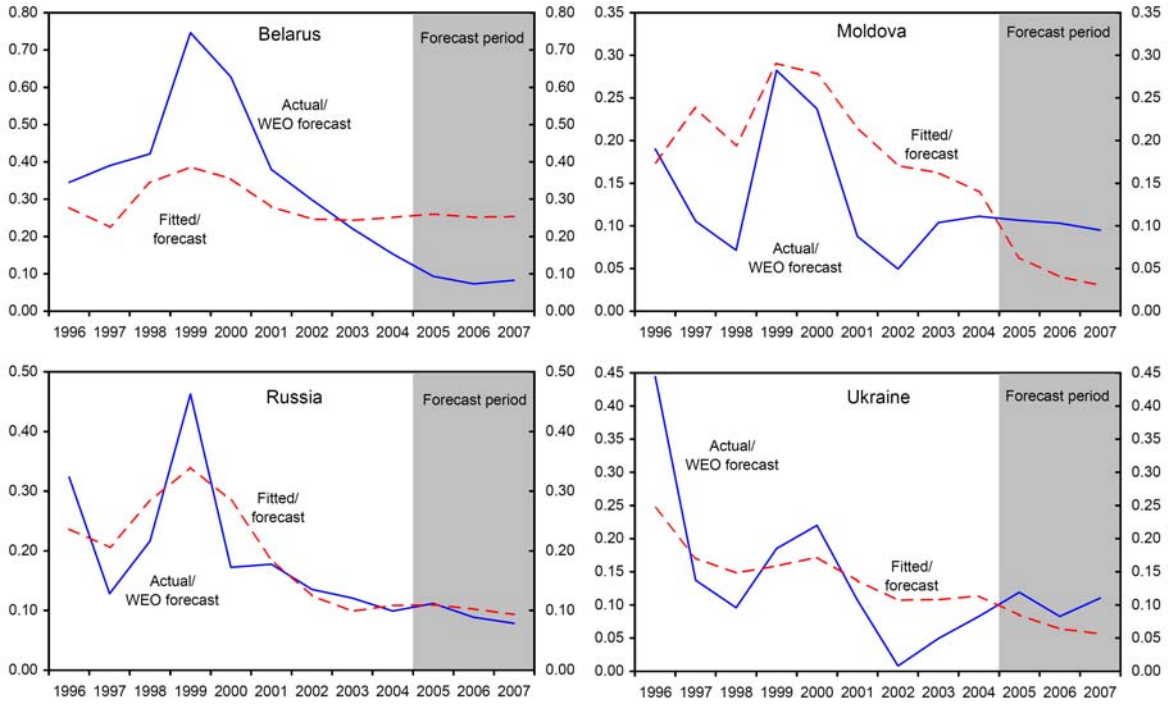
Figure 2.7: Factors Underlying Inflation Differences by Country, 1997-2004



The model can also be used to assess whether CIS-West central banks would have been expected to target lower inflation beyond the estimation period. The out-of-sample values for variables (2005-06) reflect outturns available through the IMF's WEO database, and EBRD indicators updated through 2005. The forecasts for 2007 are based on forecasts in the September 2006 WEO (inflation, government debt, terms of trade); for other indicator variables, the last available value was simply held constant. For the time dummies, they are assumed equal to zero over the forecast horizon (very close to their actual value during the last three years of the sample period). For 2005-06, the model foresaw little to no inflation gap reduction in Russia and Belarus (Figure 2.8, left column). The model fits the Russian data well and suggests that the central bank of Russia would have targeted inflation of about 10 percent (largely in line with the outcome), reflecting especially developments with the terms of trade. The model does not fit the Belarusian data as well. It suggests that the central bank of Belarus would have been content with continued high inflation (around 25 percent and significantly above the outcome), reflecting very slow liberalization of the economy. The model suggests that the central banks in Moldova and Ukraine would have targeted a reduction in inflation to about 5 percent in 2005-06, closing most of the inflation differential (Figure 2.8, right column). In Moldova, this would have reflected the impact of reforms

undertaken in the context of an IMF program; for Ukraine, this would have reflected the impact of reforms introduced in 2005 after the Orange Revolution (especially the removal of import tariffs and of import and export restrictions). In the event, the inflation gap proved persistent for both Moldova and Ukraine. The model captures underlying inflation excluding administered price changes, and cannot pick up the large change in administered energy prices consequent on the move to market pricing for gas imports from Russia. The out-of-sample forecast for 2007 may also provide some insight into the credibility gap that the central banks in these countries may yet face (Figure 2.8). The model suggests that Russia would reduce inflation only very gradually, consistent with the IMF's WEO forecasts. For Belarus, where the model fits less precisely, the inflation choice is seen as well above 2006 levels, and the then prevailing IMF forecast. The model suggests that if these countries announced near-term inflation targets at EU transition country levels, they could face a credibility problem. Moldova and Ukraine are projected by the model to want to reduce their gap considerably, even exceeding WEO forecasts. These results are again, however, subject to a caveat on administered prices, which for Belarus, Moldova and Ukraine would likely reduce the difference between model and WEO forecasts significantly.

Figure 2.8: Out of Sample Model Forecasts, Belarus, Moldova, Russia, Ukraine, 1996-2007



2.7 Summary and Conclusions

The recent inflation differential between Russia, Ukraine, Belarus and Moldova and central European emerging market economies can be modeled reasonably well using a central bank incentive approach. Panel estimation based on 19 transition countries suggests that—in contrast to CEEC and CIS-East—central banks in Russia, Ukraine, Belarus and Moldova appear to have reason to choose higher inflation rates due in some cases to fiscal pressures, but mainly to make up for, and to perhaps exploit, lagging internal and external liberalization in their economies. Out-of-sample forecasts, based on projected developments in the terms of trade, in the underlying structure of these economies, and assuming no changes in institutions, suggest that incentives towards inflation may be diminishing on the back of recent reforms, but not to the point where inflation levels below 5 percent could credibly be announced as targets.

Durably closing the gap will likely require some solution to these countries' incentive problems. First, moves to liberalize the economy and encourage more openness could

directly help reduce inflation in the medium term, consistent with experience in other economies, and indirectly reduce incentives for central banks to pursue inflationary policies. Second, the model provides some evidence that structural reforms more generally—the data cannot distinguish which in particular, but does suggest some importance for reforms to promote faster financial market development and to eliminate labor market overhangs—could also help reduce incentives for central banks to pursue the looser monetary and credit policies, which lead to higher inflation. Third, as emphasized in the literature and suggested by the empirical model, political pressures also matter for inflation outcomes. Thus, efforts to improve central bank independence could also pay dividends in durably reducing CIS-West inflation. In this context, eliminating the multiple objectives CIS-West central banks now face and providing for a stronger mandate to target inflation would be a good start.

More generally, the results also confirm that the preconditions for more ambitious inflation targets are given in the CEEC. It can be assumed that the process of accession to the EU guided reforms that provided the framework for low inflation targets and macroeconomic stability. It is, therefore, rather surprising that the group of CIS-East countries tamed inflation without being the subject of the European integration process but possibly helped by (for some countries) abundant availability of natural resources guaranteeing fiscal sustainability.

Appendix

Table A2.1: Recent Studies of Inflation in Russia, Ukraine, Belarus and Moldova

Authors	Technique	Variables	Sample	Key Findings
Belarus				
Pelipas (2006)	Cointegration	- M1 - real money - CPI - real industrial production - exchange rate - refinancing rate	1992Q1–2003Q4	All monetary variables influence inflation in the short run. Monetary gap has a significant impact on inflation in the long run.
Russia				
Granville and Mallick (2006)	VECM	- CPI - exchange rate - refinancing rate - M2	1993M5–2004M5	Interest rate is not the monetary policy instrument. For the subperiod 2000 2003 inflation was determined by exchange rate policy rather than fiscal policy.
Ohnsorge and Oomes (2004)	Error correction model	- effective broad money (with foreign currency in circulation) - NEER - CPI - various monetary aggregates	1996Q4–2004Q1	De-dollarization explains increase in money aggregates without corresponding increase in inflation since 2003.
Vdovichenko and Voronina (2006)	- GMM (forward-looking monetary policy rule) - OLS - monetary policy as a system of intervention and sterilization	- CPI - industrial production - unemployment - real exchange rate - gold reserves - foreign currency reserves - external debt payments (interventions are reserves minus debt payments) - funds placed by commercial banks on central bank accounts - government's deposits placed with the central bank (both proxies for sterilization)	2000M1–2003M9	The authors find that the central bank beside anti-inflationary communication targeted the exchange rate level and its volatility.
Ukraine				
Leheyda (2006)	Cointegration VAR	- CPI and PPI - industrial production - M2 - unit labor costs - NEER and REER - foreign price level - seasonal dummies - administrative decisions	1997M1–2003M12	The author finds that inflation is determined by exchange rate, inflation inertia, and lagged money supply. Money supply depends on foreign currency inflow.
Lissovolik (2003)	Cointegration VAR	- domestic wages - nominal exchange rate - CPI (headline and services) - base and broad money - interest rates - real GDP - barter transactions	1993M1–2002M12	Cointegration found between CPI and broad money for whole sample but not for 1996-2002 subsample.
Siliverstovs and Bilan (2005)	VAR - no cointegration found - estimated in first differences - Granger causality	- CPI - average wage per capita - M2 - cash market spread as proxy for expected devaluation - seasonal dummies - 1998M9 dummy	1996M1–2003M11	The authors find that changes in devaluation expectations determine price developments, while money supply growth is negligible.

Table A2.2: Recent Cross-Country Studies on Inflation.

Authors	Technique	Variables	Sample	Key Findings
Aisen and Veiga (2006)	System GMM panel, Blundell-Bond (1998)	- CPI - government crisis - cabinet changes - index of economic freedom - polity scale - agriculture - trade openness - growth in real GDP per capita - real overvaluation - growth of oil prices - U.S. treasury bill rate - seigniorage	1960–99 100 countries	The authors find evidence that inflation and seigniorage increase with different measures of political instability.
Cottarelli, Griffiths, and Moghadam (1998)	Dynamic panel, Arellano-Bond (1991)	- CPI - unemployment rate - EBRD transition indicators - IMF questionnaire on wage bargaining, wage indexation, banking sector, central bank independence, government debt - trade openness - current account - fiscal deficit - exchange rate regime - relative price changes - base money	1993–96 Countries: - 22 OECD - 10 CEEC - 15 CIS	Significant variables are fiscal deficit, exchange rate regime, wage indexation, central bank independence and from the EBRD indicators only price liberalization and banking sector reform and in a smaller subsample relative price changes.
Mafi-Kreft and Kreft (2006)	Panel (fixed effects)	- rate of depreciation in real value of money - central bank independence - exchange rate flexibility - hard peg dummy - fast track to EU dummy - fiscal balance - real GDP growth - trade openness - value added of agriculture	1995–2001 25 transition countries	The authors find that hard pegs (currency board arrangements) reduce inflation.

Table A2.3: Data Sources

Description	Source
Consumer prices index (annual average)	EBRD and IMF WEO forecast
Price liberalization index	EBRD
General government gross debt in percent of GDP	EBRD and IMF WEO forecast
Competition policy	EBRD
Change in labor productivity in industry	EBRD
Coarse exchange rate regime classification	IMF
Governance and enterprise restructuring	EBRD
Flat tax dummy	Ivanova, Keen, and Klemm (2005) and http://www.euractiv.com/en/taxation/flat-tax/article-136190
Share of agriculture in percent of GDP	EBRD and WDI
Current account in percent of GDP	EBRD
Bank reform and interest rate liberalization	EBRD
Trade and foreign exchange system	EBRD
Securities markets and non-bank financial institutions	EBRD
Exchange rate vis-à-vis key currency	IFS
Interest rate	IFS
Weighted index of central bank independence	Cukierman, Miller, and Neyapti (2002)
Terms of trade in goods and services	IMF WEO
Change in the share of administered prices	EBRD
Government stability indicator	PRS Group
Crop production index	WDI

Table A2.4: Data Correlation Matrices

Table A2.4a: Correlations for Variables Used in Regressions in Table 2.6

	Inflation	Fiscal sust.	Price liberaliz.	Openness	Competition	Terms of trade	Admin. Prices	Harvest
Inflation	1.00	0.33	-0.49	-0.54	-0.33	-0.10	-0.17	-0.10
Fiscal sust.		1.00	0.02	0.14	-0.02	-0.08	-0.22	-0.07
Price liberaliz.			1.00	0.68	0.37	0.03	0.02	-0.12
Openness				1.00	0.43	0.08	0.07	0.03
Competition					1.00	-0.08	-0.09	0.06
Terms of trade						1.00	0.16	0.31
Admin. prices							1.00	-0.01
Harvest								1.00

Table A2.4b: Correlations for Variables Used in Regressions in Table 2.7

	Inflation	Fiscal sust.	Price liberaliz.	Openness	Gov't stability	CBI	EU Accession	Terms of trade	Admin. prices
Inflation	1.00	0.33	-0.49	-0.54	0.03	-0.10	-0.37	-0.10	-0.17
Fiscal sust.		1.00	0.02	0.14	-0.08	0.10	-0.13	-0.08	-0.22
Price liberaliz.			1.00	0.68	-0.25	0.06	0.46	0.03	0.02
Openness				1.00	-0.32	0.21	0.53	0.08	0.07
Gov't stability					1.00	-0.23	-0.25	0.06	0.06
CBI						1.00	0.20	0.10	-0.10
EU Accession							1.00	-0.05	0.04
Terms of trade								1.00	0.16
Admin. prices									1.00

Table A2.4c: Correlations for Variables Used in Regressions in Table 2.8

	Inflation	Balassa-Sam.	Flat tax dummy	Ent. restructure	Agri. Share	Fiscal sust.	External sust.	Bank reform	Sec. mkt reform	Price liberaliz.	Openness	Terms of trade
Inflation	1.00	-0.13	-0.20	-0.46	0.34	0.33	0.19	-0.50	-0.36	-0.49	-0.54	-0.10
Balassa-Sam.		1.00	0.04	-0.07	0.03	-0.12	0.11	-0.07	0.01	0.08	0.00	0.29
Flat tax dummy			1.00	0.27	-0.28	-0.30	0.08	0.25	0.17	0.28	0.14	0.01
Ent. restructure				1.00	-0.60	0.00	-0.04	0.87	0.77	0.65	0.70	0.03
Agri. Share					1.00	0.32	-0.32	-0.57	-0.73	-0.19	-0.26	0.03
Fiscal sust.						1.00	0.13	-0.03	-0.09	0.02	0.14	-0.08
External sust.							1.00	-0.10	0.13	-0.17	-0.18	0.09
Bank reform								1.00	0.76	0.64	0.73	0.01
Sec. mkt reform									1.00	0.34	0.44	0.06
Price liberaliz.										1.00	0.68	0.03
Openness											1.00	0.08
Terms of trade												1.00

Table A2.5: Robustness of Econometric Results

	1 Baseline T2.6-1	2 without CIS-East	3 only 2000 to 2004	4 with country fixed effects	5 without time fixed effects	6 Dynamic panels	7
Structural features							
Fiscal sustainability	0.212 *** [0.024]	0.221 *** [0.029]	0.016 [0.016]	0.272 *** [0.028]	0.216 *** [0.029]	0.170 * [0.096]	0.190 ** [0.073]
Output-inflation trade-off							
Price liberalization	-0.123 *** [0.029]	-0.117 *** [0.031]	-0.128 *** [0.032]	-0.177 *** [0.037]	-0.157 *** [0.031]	-0.091 * [0.044]	-0.127 *** [0.041]
Openness	-0.044 *** [0.014]	-0.049 ** [0.020]	-0.035 *** [0.012]	-0.047 ** [0.020]	-0.040 *** [0.014]	-0.024 [0.038]	-0.020 [0.045]
Exogenous shocks							
Terms of trade	0.076 ** [0.035]	0.148 [0.128]	0.050 [0.100]	0.084 * [0.047]	0.066 [0.053]	0.017 [0.144]	0.028 [0.146]
Change in administered price share							-0.036 [0.063]
Lagged dependent variable							
Inflation (-1)						0.290 ** [0.104]	0.217 ** [0.101]
No. of Observations	171	135	95	171	171	171	138
No. of Countries	19	15	19	19	19	19	16
Adjusted R^2	0.724	0.678	0.706	0.689	0.620	...	...

Standard errors in brackets: *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

3 Evaluating the Role of the Exchange Rate in Inflation Targeting Regimes of Latin American and European Emerging Market Economies

3.1 Introduction

In this chapter, the hypothesis is tested that exchange rate policy still matters even in inflation targeting regimes. Exchange rate policy in the 1990s was characterized by the reduction of the number of soft pegs, described in the "corner solutions" literature (Fischer 2001). Countries moved either towards hard pegs or, increasingly, towards flexible exchange rates. A second characteristic was the implementation of inflation targeting as a new monetary policy strategy. This strategy provided an attractive framework as it was supposed to meet financial markets' criteria of prudent monetary policy. But critics called this new monetary policy strategy just "conservative window-dressing" (Romer 2001: 509). Others claimed that, independent of the label attached to monetary policy, the importance of the exchange rate for macroeconomic management in emerging markets demands intermediate regimes (Williamson 2000). Thus, there is no consensus in the literature with respect to monetary policy in combination with flexible exchange rate regimes.

Latin American and eastern European Countries provide cases in point. The Latin American countries were disturbed by crises such as the Mexican Crisis in 1994 and the Brazilian Crisis in 1998. These crises lead to a change in monetary policy as the rather fixed exchange rate regimes were not anymore sustainable. In 1999 Brazil, Colombia, and Mexico introduced inflation targeting as a new monetary policy strategy—following the example of Chile. The Chilean central bank had announced its first inflation target in September 1990 and subsequently built up a successful disinflation record (Morandé 2002, Schmidt-Hebbel and Tapia 2002, 2003).

The central European countries had to deal with the structural change from centrally planned to market economies. In the beginning of the 1990s, price liberalization and the abolition of price controls led to hyperinflation. The situation got stabilized by pegging the exchange rate. When the economies were ready for a more independent monetary policy the central banks of

the Czech Republic (1998), Poland (1998), and Hungary (2001) introduced inflation targeting.

In order to analyze the role of exchange rate policy in (de jure) inflation targeting regimes, the second section develops a generalized reaction function that describes four alternative monetary policy strategies: inflation targeting, Taylor rule, monetary conditions index, and managed floating. This generalized reaction function uses the short-term interest rate as the instrument of monetary policy. Targets of monetary policy are inflation, output, and the exchange rate. The generalized reaction function is estimated using a vector autoregressive model (VAR) for the seven Latin America and CEEC countries in the third section. As described above, all the selected countries mastered disinflation, moved to flexible exchange rates, and introduced inflation targeting as a new monetary policy strategy. The VAR analysis investigates the role of the pass-through of the exchange rate on prices and the reaction of the interest rate to shocks in inflation, output gap and the exchange rate. The fourth section concludes on the role of exchange rate policy in de jure inflation targeting regimes.

3.2 A Generalized Reaction Function

Although inflation targeting, Taylor rule, monetary conditions index, and managed floating are often discussed as alternative strategies for monetary policy under flexible exchange rate regimes, it can be shown that the reaction functions resulting for these strategies have a common structure.

Svensson (1999: 626) suggests conducting "flexible inflation targeting," which allows accommodating real disturbances. Hence, the reaction function of flexible inflation targeting does not only rely on the deviation of actual inflation from its target but also takes into account a term for the output gap (Svensson 1999: 628) and is, therefore, identical with the Taylor rule (Taylor 1993):[21]

[21] Svensson's (1999) reaction function for flexible inflation targeting is actually a forward-looking rule based on forecasts for inflation and output gap. Taylor's (1993) reaction function refers to the real interest rate. The

(continued…)

(3.1) $$i_t = \bar{i} + \gamma\left(\pi_t - \bar{\pi}\right) + \lambda\left(y_t - \bar{y}_t\right).$$

On the left hand side of the reaction function is the short-term nominal interest rate, i_t, which depends on $\bar{i}$, the equilibrium interest rate, as well as on deviations of the inflation rate, π_t, and output, y_t, from their target values $\bar{\pi}$ and $\bar{y}_t$, respectively. While $\bar{\pi}$ is determined politically, $\bar{y}_t$ is potential output and, hence, $y_t - \bar{y}_t$ the output gap.

The monetary conditions index (MCI) is an indicator of the stance of monetary policy, which does not only consider an output target but also the influence of the exchange rate on inflation. The nominal interest rate, i_t, is set in order to equalize actual MCI and optimal MCI. Freedman (1996: 75) builds the MCI on the assumption of an aggregate demand relation, where output, y, depends essentially on the real interest rate, r, and the real exchange rate, q:

(3.2) $$y_t = \alpha r_t + \beta q_t .$$

Although various measures of the MCI exist, it essentially depends on a weighted combination of real interest rate and real exchange rate[22] deviations from some neutral levels, $\bar{r}$, and, $\bar{q}$. The MCI can then be written as

(3.3) $$MCI_t = r_t - \bar{r} - \delta\left(q_t - \bar{q}\right).$$

The weight $\delta = \beta/\alpha$ measures the relative demand impact of the interest rate and the exchange rate. The parameters α and β have to be estimated from a macroeconomic model similar to (3.2). Therefore, when setting the interest rate, the impact of the exchange rate on the stance of monetary policy is taken into account.[23] It is also worth noting that, for instance, a rise in the interest rate does not only lead directly to a more restrictive monetary policy but leads, via the uncovered interest rate parity, to an appreciation of the exchange rate, which

---

Fisher equation, $r_t = i_t - \pi_t$, allows expressing the Taylor rule in terms of the nominal interest rate, i.e., the policy instrument of the central bank. With respect to the empirical analysis this simplified version is used.

[22] Here, an increase corresponds to a depreciation of the exchange rate.

[23] See Gerlach and Smets (2000) for an explicit analysis of the conduct of monetary policy using an MCI.

also leads to a more restrictive stance of monetary policy. Consequently, both channels increase the actual MCI.

Bofinger and Wollmershäuser (2001: 26) develop an expression for the optimal MCI, where the reaction function of the optimal MCI is given by[24]

$$MCI_t^{opt} = \gamma\left(\pi_t - \bar{\pi}\right) + \lambda\left(y_t - \bar{y}_t\right). \tag{3.4}$$

Now, equalizing the actual MCI (3.3) and the optimal MCI (3.4) leads to

$$r_t - \bar{r} - \delta\left(q_t - \bar{q}\right) = \gamma\left(\pi_t - \bar{\pi}\right) + \lambda\left(y_t - \bar{y}_t\right) \tag{3.5}$$

and after rearranging

$$r_t = \bar{r} + \gamma\left(\pi_t - \bar{\pi}\right) + \lambda\left(y_t - \bar{y}_t\right) + \delta\left(q_t - \bar{q}\right). \tag{3.6}$$

Finally, the reaction function of a monetary conditions index in terms of the nominal interest rate, i_t, is obtained using the Fisher equation, $r_t = i_t - \pi_t$:

$$i_t = \bar{i}_t + \gamma\left(\pi_t - \bar{\pi}\right) + \lambda\left(y_t - \bar{y}_t\right) + \delta\left(q_t - \bar{q}\right), \tag{3.7}$$

where $\bar{i}_t = \bar{r} + \pi_t$ is the nominal equivalent to an equilibrium real interest rate $\bar{r}$. Actually, equation (3.7) is an extension of the Taylor rule (3.1).[25] In addition to inflation and output, it takes into account deviations of the exchange rate from its equilibrium value. Thus, the reaction function of the MCI corresponds to a Taylor rule with an exchange rate term (Romer 2001: 503).

Unlike an MCI regime, managed floating considers the exchange rate as an additional instrument of monetary policy. Nevertheless, it can be shown that reaction function (3.7) can serve as a generalized form also representing managed floating. Bofinger (2001: 418) sets up the following reaction function for the interest rate

[24] Bofinger and Wollmershäuser (2001: 26) incorporate also a neutral level of the MCI on the right hand side. Here, the neutral level is assumed to be zero.

[25] See Clarida, Galí, and Gertler (1998: 1041) for Taylor rules extended by the exchange rate.

$$i_t = \pi_t + \frac{1}{1-\delta}\left(MCI_t^{opt} - \delta r_t^* - \delta e_t - \bar{r} + \delta(q_{t-1} - \bar{q})\right), \tag{3.8}$$

where π_t is the domestic inflation rate, r_t^* is the foreign interest rate, e_t is the risk premium on the expected depreciation, $\bar{r}$ is a neutral level of the real interest rate, δ is the relative impact of the interest rate and the exchange rate in the monetary conditions index as in (3.3), and $(q_{t-1} - \bar{q})$ is the deviation of the lagged real exchange rate from a neutral level. The optimal MCI (3.4) is substituted into the reaction function for the nominal interest rate (3.8)

$$i_t = \pi_t + \frac{1}{1-\delta}\left[\gamma(\pi_t - \bar{\pi}) + \lambda(y_t - \bar{y}) - \delta r_t^* - \delta e_t + \bar{r} + \delta(q_{t-1} - \bar{q})\right] \tag{3.9}$$

and after rearranging

$$i_t = \pi_t + \frac{1}{1-\delta}\left[\bar{r} + \gamma(\pi_t - \bar{\pi}) + \lambda(y_t - \bar{y})\right] + \frac{\delta}{1-\delta}\left[(q_{t-1} - \bar{q}) - r_t^* - e_t\right]. \tag{3.10}$$

Some substitutions permit to rearrange this reaction function for easier interpretation. First, the uncovered interest rate parity is given by

$$\Delta s_t + e_t = i_t - i_t^*, \tag{3.11}$$

where Δs_t is the targeted depreciation[26] of the nominal exchange rate and e_t is a risk premium on the expected depreciation. The uncovered interest rate parity can also be written in real terms:

$$\Delta q_t = r_t - r_t^* - e_t. \tag{3.12}$$

Second, the real exchange rate of the preceding period can be expressed as today's exchange rate minus the depreciation:

$$q_{t-1} = q_t - \Delta q_t. \tag{3.13}$$

Using these substitutions, the reaction function for the interest rate becomes

[26] As the depreciation is controlled by the central bank, no expectation operator is needed. The remaining risk is covered by the risk premium e_t.

$$i_t = \pi_t + \frac{1}{1-\delta}\left[\bar{r} + \gamma(\pi_t - \bar{\pi}) + \lambda(y_t - \bar{y})\right] - \frac{\delta}{1-\delta}\left[r_t - (q_t - \bar{q})\right]. \tag{3.14}$$

The reaction function (3.14) is simplified for $\delta = 0$. Because $\delta = \beta/\alpha$ the simplification corresponds to $\beta = 0$, i.e., the exchange rate has no impact on aggregate demand (3.2),[27] and then the reaction function (3.14) is equivalent to the Taylor rule (3.1). Generally, the exchange rate does have some impact on aggregate demand in an open economy and, therefore, $\delta \neq 0$. Then, the two fractions before each term in brackets add up to unity

$$\frac{1}{1-\delta} + \left(-\frac{\delta}{1-\delta}\right) = 1. \tag{3.15}$$

Hence, the reaction function (3.14) can be interpreted as a weighted mixture of the Taylor rule and some form of a monetary conditions index (3.3) because the term in the second bracket consists of the interest rate and the exchange rate. These two variables determine the stance of monetary policy.[28]

All in all, the reaction functions of flexible inflation targeting (3.1), the MCI (3.7), and managed floating (3.14) show the same structure as the Taylor rule. Hence, the most generalized version of a reaction function is an extended Taylor rule that allows taking into account inflation, the output gap, and the exchange rate[29]

$$i_t = \bar{i}_t + \gamma(\pi_t - \bar{\pi}) + \lambda(y_t - \bar{y}_t) + \delta(q_t - \bar{q}). \tag{3.7}$$

Managed floating does not fit as easily into this generalization as inflation targeting and the MCI. However, managed floating remains a close relative and equation (3.7) is taken as a starting point for the analysis of monetary policy.

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[27] This assumption could be thought of as the closed economy case.

[28] An increase in the weight β of the exchange rate in aggregate demand (3.2) leads to an increase of the weight of the exchange rate in the reaction function (3.14). Freedman (1996: 75) suggests δ=1/3 for the Canadian MCI. Hence, the first term in the reaction function (3.14) has the weight 1.5 and the second term has the weight –0.5.

[29] The reaction function may also contain lags of the interest rate, if central banks choose to smooth interest rates (Goodfriend 1991).

3.3 Empirical Evidence from Inflation Targeting Regimes

3.3.1 VAR Models for Countries in Latin America and Central Europe

The analysis applied in this paper builds on Gottschalk and Moore (2001) who estimated a vector autoregressive model (VAR) in levels for analyzing the role of instruments and targets in the transmission mechanism of monetary policy in the case of Poland.[30] VARs have proven to be useful for the analysis of monetary policy because they look at the dynamics within the economy and the relations between instruments and targets of monetary policy (Christiano, Eichenbaum, and Evans 1999, von Hagen, Hayo, and Fender 2002: 8). In contrast to traditional empirical analysis, this method treats all variables as endogenous and allows analyzing dynamic responses to shocks.

Impulse response analysis maps the adjustment path in time for a variable to a given shock with the size of one standard deviation. The impulse response functions are reported for twenty steps or for twelve steps, i.e., one year, in the case of particularly short samples. Error bands assess the statistical significance of the impulse responses. These error bands are computed by a Monte Carlo simulation following Sims and Zha (1999). These authors illustrate that the use of conventional error bands with one or two standard errors can be misleading as impulse responses have highly asymmetrical distributions. The analysis here follows their suggestion and uses fractiles of 0.16 and 0.84 instead of a one standard deviation band.

VARs are characterized by very little a priori restrictions. This implies that they show correlations rather than causalities (Cooley and LeRoy 1985, Stock and Watson 2001: 102). However, the derivation of a generalized reaction function in Section 3.2 has shown that the restrictions on parameters are already rather complex if one starts with ad hoc assumptions about targets and instruments of monetary policy. The VAR analysis assumes that these restrictions are largely unknown. A priori information is only required with respect to the question which variables are treated as exogenous and which variables are treated as endogenous.

[30] The estimation strategy builds—among others—on Sims (1992) and Christiano, Eichenbaum, and Evans (1996).

Ranking all variables from the most exogenous to the most endogenous and applying the Choleski decomposition allows identifying the dynamic responses to a shock. It is assumed that the first variable responds only to its own shock, the second variable responds to the first variable plus to a shock to the second variable, and so on. Finally, the last variable reacts without delay to all shocks, but disturbances to this variable have no contemporaneous effect on the other variables.

The empirical analysis builds on the estimation of two different setups. Each VAR is estimated ones in levels and ones in gaps.[31] The estimation in levels reveals the interdependences of basic macro variables. More specific, it allows evaluating the pass-through of exchange rate shocks on prices. The estimation in gaps departs from the generalized reaction function (3.7). All variables are expressed in gaps, i.e., as deviations from their respective target or equilibrium values, if the equilibrium interest rate, $\bar{i}_t$, is transferred to the left hand side of (3.7):

$$i_t - \bar{i}_t = \gamma\left(\pi_t - \bar{\pi}\right) + \lambda\left(y_t - \bar{y}_t\right) + \delta\left(q_t - \bar{q}\right). \tag{3.16}$$

The interest rate gap, $i_t - \bar{i}_t$, is equivalent to a change in the interest rate and, thus, a change in the policy instrument is determined by the gaps of inflation, output, and the exchange rate.

According to the decomposition procedure the general setup of the models consists of a system of four to five equations—depending on the inclusion of a foreign shock. This system represents the structure of the economy. On the left hand side of each equation is successively one of the four to five variables, i.e., foreign shock or foreign shock gap, domestic prices or domestic inflation gap, domestic output or domestic output gap, exchange rate or exchange rate gap, and domestic interest rate or domestic interest rate gap. On the right hand side are always all four to five variables with their lags. In the system in gaps the last equation has the most endogenous variable, the interest rate gap, on the left hand side:

$$\begin{aligned} \textit{Interest Rate Gap}_t = {} & \textit{Foreign Shock Gap}_{t-n} + \textit{Inflation Gap}_{t-n} + \textit{Output Gap}_{t-n} \\ & + \textit{Exchange Rate Gap}_{t-n} + \textit{Interest Rate Gap}_{t-n}, \end{aligned} \tag{3.17}$$

[31] See Marcet (2005) on estimating VARs in first differences.

where *n* goes from zero to the chosen lag length. Thus, equation (3.17) can be interpreted as a variation of the generalized reaction function (3.16) augmented by a foreign shock and—due to the included lags—interest rate smoothing. The foreign shock is used as a proxy for activity in the respective trading partners of Chile, Mexico, and Poland, and, therefore, separates domestic shocks from foreign shocks in the VARs. In each case the target or equilibrium values have been approximated using Hodrick-Prescott filtered values. This procedure implies that the central banks react systematically to a deviation of inflation, output and exchange rate from their trend. Where available, the inflation gap was calculated as the deviation of actual inflation from the announced inflation target.[32] To capture the disinflation process, the monthly values between two annual targets were approximated by a linear trend. All variables, except the interest rate, are based on logs.

Foreign Shock is approximated by German industrial production in the case of Poland and U.S. industrial production in the case of Chile and Mexico. *Prices and Inflation* are constructed using the consumer price index. *Output* builds on the domestic industrial production. All price indexes and industrial production indexes are seasonally adjusted in order to remove autocorrelation in the VAR.[33]

The last two variables of the model are considered to be potential policy instruments. *Exchange Rate* is incorporated as the exchange rate channel might be a key channel for monetary transmission in open economies (Fung 2002). Further, the exchange rate is considered a policy instrument as most countries maintain some form of peg or crawling band and, thus, may have managed their exchange rate during the 1990s. For the Latin American economies the nominal exchange rate vis-à-vis the U.S. dollar and for the central European economies the nominal exchange rate vis-à-vis the euro and ecu is applied. As the Chilean and Polish exchange rate policies exercise a crawling band based on a basket of foreign currencies the nominal effective exchange rate is chosen.[34] Finally, *Interest Rate*

[32] For Brazil and the Czech Republic the inflation targets are taken form Corbo and Schmidt-Hebbel (2003: 47-9) and augmented by EIU Country Profiles and EIU Country Reports.

[33] For Mexico the VAR was estimated with seasonal dummy variables.

[34] In the figures an increase in both, nominal exchange rate as well as nominal effective exchange rate, refers to a depreciation.

represents the policy instrument of the central bank and is approximated by the short-term interest rate.[35]

The VARs have been checked for misspecifications. Table 3.1 and Table 3.2 show the specifications of the VARs for the countries in Latin America and central Europe, respectively. All VARs are estimated with a lag order between three and five to eliminate autocorrelation. The fact that the selected countries moved towards inflation targeting and more exchange rate flexibility indicates a regime change.[36] Hence, for Brazil, Colombia, and Poland two separate VARs are estimated and evaluated with respect to the regime change. The inclusion of dummy variables for outliers does not change the results qualitatively. Rather, they improve the residual properties of the system. The full sets of all impulse response functions are given in the Appendix Figures A3.1 to A3.20.

Table 3.1: VAR Specifications for Countries in Latin America

Country	Levels/Gaps	Lags	Sample	Dummies
Brazil	Levels	5	9409–9812	9503, 9607, 9711, 9809
Brazil	Gaps	5	9409–9812	9607, 9711, 9809
Brazil	Levels	5	9901–0310	0007, 0012, 0204, 0210
Brazil	Gaps	4	9901–0310	–
Chile	Levels	3	9101–0212	9801, 9809, 0107
Chile	Gaps	3	9201–0212	9809, 0107
Colombia	Levels	5	9505–9906	9510, 9512, 9703, 9708, 9806
Colombia	Gaps	4	9505–9906	–
Colombia	Levels	5	9909–0303	0105, 0203, 0211
Colombia	Gaps	3	9909–0303	0203, 0211
Mexico	Levels	3	9601–0309	seasonal, 9704, 9809, 0204
Mexico	Gaps	3	9601–0309	seasonal, 9809

[35] All data is taken from IMF International Financial Statistics, with the following exceptions. The industrial production of Brazil (britot.g), Colombia (cbitot.h), and Poland (poindprna) are taken from datatream. Also retrieved from datastream is the historic exchange rate of the euro, respectively the ecu, vis-à-vis the U.S. dollar (ecunit$). The Chilean industrial production and the Chilean interest rate (interbank average interest rate on non-indexed deposit under 30 days) are taken from the Central Bank of Chile. The Hungarian interest rate is the refinancing rate (1190) provided by the Vienna Institute for International Economic Studies.

[36] The date for the start of inflation targeting is taken from Fracasso, Genberg, and Wyplosz (2003: 2).

Table 3.2: VAR Specifications for Countries in Central Europe

Country	Levels/Gaps	Lags	Sample	Dummies
Czech Republic	Levels	4	9801–0309	9807, 9912
Czech Republic	Gaps	4	9801–0309	–
Hungary	Levels	4	9010–0309	9309, 9406, 9502, 0306
Hungary	Gaps	4	9010–0309	9309, 9406, 9502, 0306
Poland	Levels	3	9201–9802	–
Poland	Gaps	3	9201–9802	–
Poland	Levels	3	9803–0212	–
Poland	Gaps	3	9803–0212	–

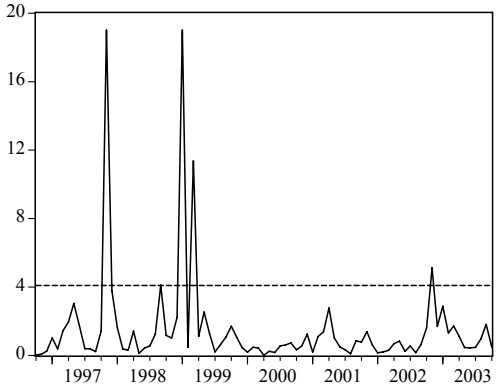
3.3.2 Evidence from Latin America—Case Studies

Case Study Brazil

Monetary policy in Brazil during the 1990s was characterized by several disinflation processes. In the beginning of the decade the Collor Plan, launched in March 1990, helped to stabilize prices for some time. But the disinflation failed and Brazil suffered under hyperinflation once again by 1993. In July 1994 the next disinflation process named Real Plan used the exchange rate as a nominal anchor. The pressure on the exchange rate increased and lead to the Brazilian Crisis in the second half of 1998. In January 1999 Brazil moved to a floating exchange rate and in June 1999 the central bank introduced inflation targeting as new monetary policy strategy. The test on structural breaks supports to estimate two separate VARs. The Chow test on structural breaks in Figure 3.1 shows a clear break point in January 1999. Two separate VARs have been estimated starting after the introduction of the Real Plan in September 1994 to December 1998 and starting with the floating exchange rate regime in January 1999 to October 2003.[37]

[37] The starting date of the sample follows Minella (2003: 610). His VAR estimations last from September 1994 only till December 2000. The longer dataset till October 2003 reveals a policy switch and permits to compare two samples of different monetary policy strategies.

Figure 3.1: 1-Step Chow Test for Brazil



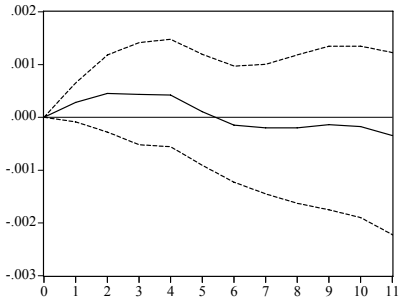
Note: The graph shows the F-statistic of the 1-Step Chow test for the VAR system. The horizontal line provides the 1% significance level for the critical value F(4,28) = 4.084. Break points are 1997:11, 1999:01, 1999:03, and 2002:11.

For both sample periods the VARs are estimated in levels and in gaps. The estimation in levels reveals significant difference in the pass-through of the exchange rate on prices (Figure 3.2). In the first sample period from September 1994 to December 1998 the impulse response of prices is insignificant. In contrast, in the second sample period from January 1999 to October 2003 prices react strongly to shocks in the exchange rate. The response has its peak seven to eight months after the shock. Thus, moving to flexible exchange rates increased the vulnerability of domestic prices by exchange rate shocks.

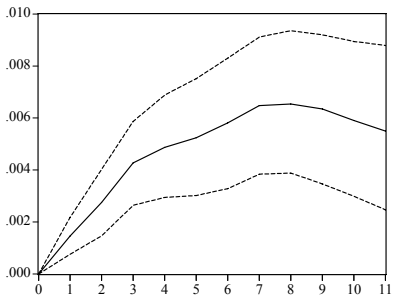
Figure 3.2: Pass-Through in Brazil

Impulse Response Functions of Prices to Exchange Rate Shock

Sample Sept. 1994 to Dec. 1998



Sample Mar. 1999 to Oct. 2003

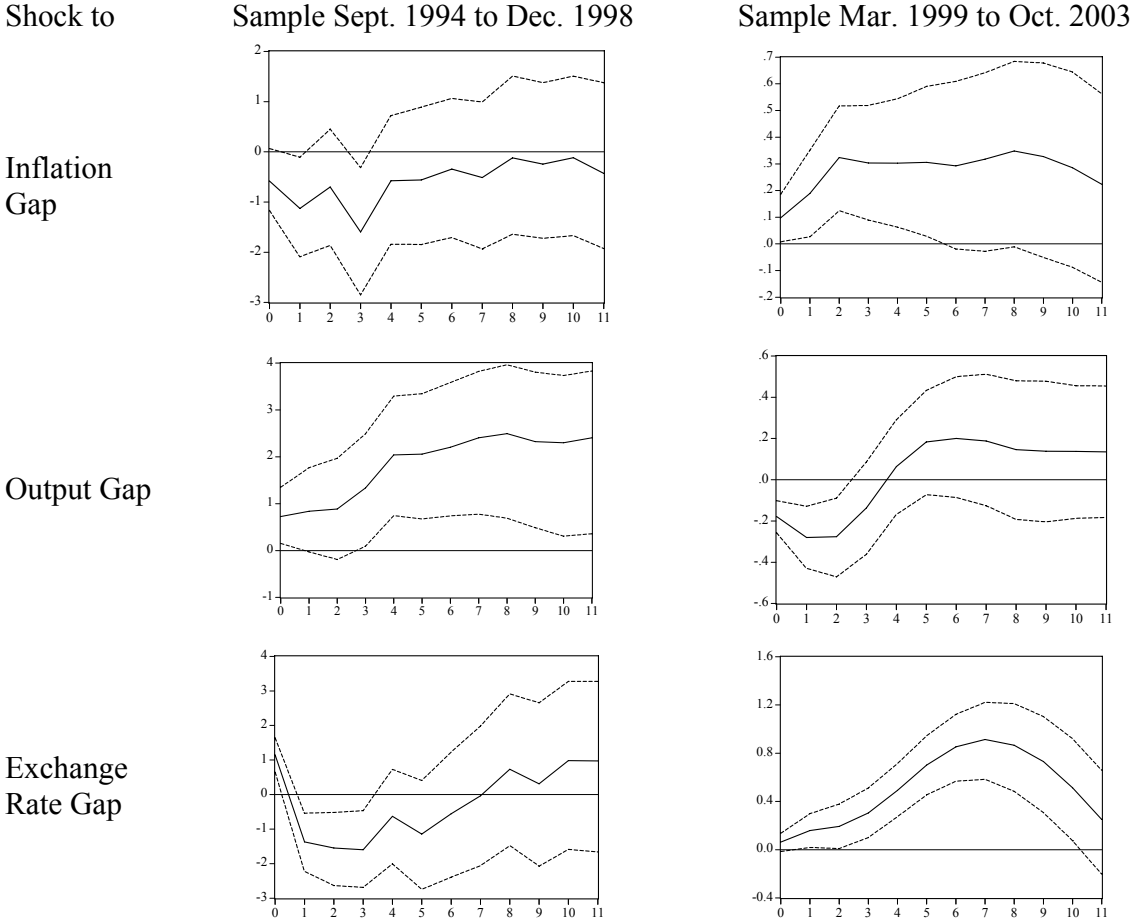


Source: Appendix Figures A3.1 and A3.3.

Figure 3.3 shows the reaction function of the central bank, i.e., the impulse response functions of the interest rate gap after a shock in the inflation gap, output gap, and the exchange rate gap. In the first sample period the interest rate does react to a positive shock in

inflation. However, the impulse response function remains insignificant. The response of the interest rate to a shock in the output gap is significant. A positive shock in the output gap leads to an increase in the interest rate. A depreciation of the currency, i.e., a positive shock in the exchange rate gap, induces an instantaneous increase in the interest rate. After the following drop the response becomes insignificant. The immediate interest rate response is driven by exchange rate policy. In its responses to inflation, output and exchange rate shocks the interest rate as the policy instrument supports the exchange rate peg vis-à-vis the U.S. dollar.

Figure 3.3: Impulse Responses of the Interest Rate Gap in Brazil



Source: Appendix Figures A3.2 and A3.4.

In the second sample period the interest rate shows a clear response to a shock in inflation. A positive deviation of the inflation rate from the inflation target is answered by an increase in the interest rate. The interest rate reacts to a positive shock in the output gap with a decline in

the first two periods. Afterwards, the response is positive but insignificant. The first part of the interest rate response remains counterintuitive. A depreciation of the exchange rate leads to a delayed increase in the interest rate. The increase has its peak in the seventh month following the shock. The weak immediate response reveals that the exchange rate was not tried to be influenced on the short horizon. Rather, the timing of the peak in the seventh month coincides with the peak of the pass-through of the exchange rate shock on prices. Thus, the pattern suggests that the policy instrument is applied in order to fight inflation pressure driven by an exchange rate appreciation. Overall, the three impulse response functions are in line with the prevailing exchange rate policy and monetary policy. The exchange rate floats and the interest rate is set to accomplish the inflation target. However, the response to inflation as well as the exchange rate actually suggests the application of a monetary conditions index.

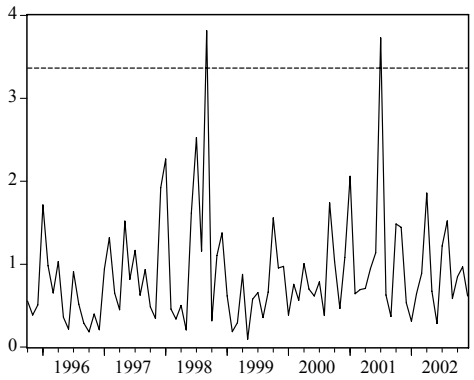
Case Study Chile

In the case of Chile the test for stability for the sample period January 1992 to December 2002 reveals some breaks but no structural change (Figure 3.4). These break points are driven by outliers in the residuals and fixed by dummy variables.

The estimation of the VAR in levels shows that the reactions of the interest rate support the reactions of the interest rate gap in the second VAR. The analysis of the pass-through in Figure 3.5 reveals some impact of the exchange rate on prices, but the impulse response fails to be significant.[38]

[38] See also Noton (2003) on the pass-through in Chile.

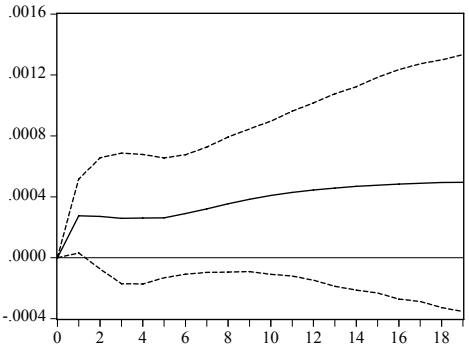
Figure 3.4: 1-Step Chow Test for Chile



Note: The graph shows the F-statistic of the 1-Step Chow test for the VAR system. The horizontal line provides the 1% significance level for the critical value F(5,57) = 3.3655. Break points are 1998:09 and 2001:07.

Figure 3.5: Pass-Through in Chile

Impulse Response Function of Prices to Exchange Rate Shock

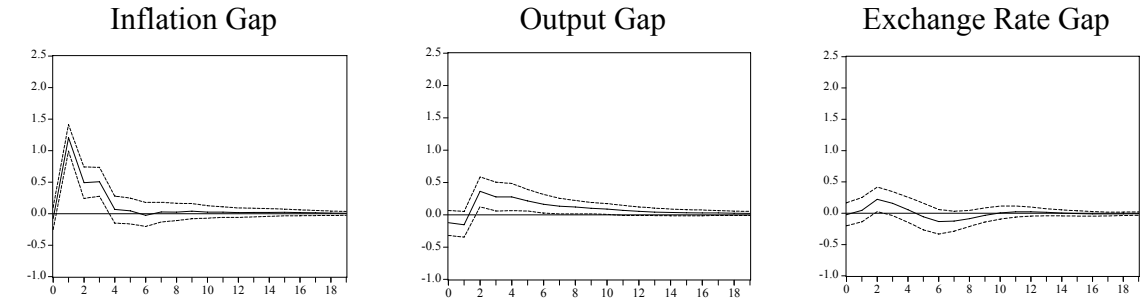


Source: Appendix Figure A3.5, inverted response.

The impulse responses of the interest rate gap to shocks in the inflation gap, the output gap, and the exchange rate gap in Figure 3.6 show a very clear pattern. A positive shock in inflation leads to a clear increase in the interest rate as implied by an inflation targeting regime. This result together with the findings that there was no structural break in the sample suggests that an inflation targeting regime has been in place for the full sample period as claimed by the Chilean central bank, which announced its first inflation target in September 1990. In contrast, the IMF study by Schaechter, Stone, and Zelmer (2000) classifies Chile to

have introduced full-fledged inflation targeting only in September 1999, when Chile implemented the complete framework of an inflation targeting regime.[39]

Figure 3.6: Impulse Responses of the Interest Rate Gap in Chile



Source: Appendix Figure A3.6, inverted response for the exchange rate gap.

A positive output shock leads to a slight increase in the interest rate. Thus, this result suggests some weak form of output smoothing. Schmidt-Hebbel and Tapia (2002: 140-42), who estimate a monetary policy reaction function in the tradition of Clarida (2001) and Clarida, Galí, and Gertler (2000), find the coefficient of the output gap significant, similar to the coefficient of the inflation rate and indicating an anti-cyclical policy.

A depreciation of the exchange rate has hardly any impact on the interest rate. Schmidt-Hebbel and Tapia (2002: 142) find in their estimated reaction function a significant coefficient for a reaction of the interest rate to an exchange rate misalignment. Thus, the central bank reacts to an overvaluation with a lower interest rate.

However, evaluating the role of the exchange rate has some caveats. The data contains some clear outliers. When the exchange rate came under pressure due to the Asian and Russian crises the interest rate was increased abruptly.[40] This exchange rate policy is not captured within the VAR as the outliers are removed by dummy variables. Further, the exchange rate

[39] Of course, this empirical analysis does not investigate whether the legal framework is consistent with formal inflation targeting.

[40] Jonas and Mishkin (2003) call this focus on the exchange rate in the case of the Asian crisis a "serious policy mistake." The Central Bank of Chile was afraid of loosing its credibility in the face of financial turmoil. Actually, the Asian crisis was a terms of trade shock where the central bank should have eased monetary policy. Instead, the increase in the interest rate led to an undershooting of the inflation target and Chile's first recession of the 1990s.

can be influenced by other instruments and policies (Wickham 2002: 22). Schmidt-Hebbel and Tapia (2002: 139) report, for instance, that sterilized foreign exchange interventions took place even in the second half of 2001—a long time after the exchange rate regime moved to a formal pure float. Actually, Levy-Yeyati and Sturzenegger (2005: 1618) state in their study on de facto exchange rate regimes: "Interestingly, in the case of Chile the classification indicates that it has virtually run a pure float since the early 80s, in spite of a complex system of crawling pegs and exchange rate bands that were finally discontinued in 1999." As in the case of Poland, the properties of a pure float were fulfilled by the exchange rate much earlier than the official classification. This delay with respect to the announcement of flexible exchange rates might be due to some political economy arguments (Morandé and Tapia 2002: 7), as each system creates pressure groups that support the status quo.

Williamson (2000: 40-5) discusses the role of the Chilean capital controls. He concludes that these capital controls allowed the central bank some autonomy in setting the interest rate. Thus, the interest rate could be set with the target of keeping domestic inflation down and it was not limited in its effectiveness by capital inflows and corresponding exchange rate movements.

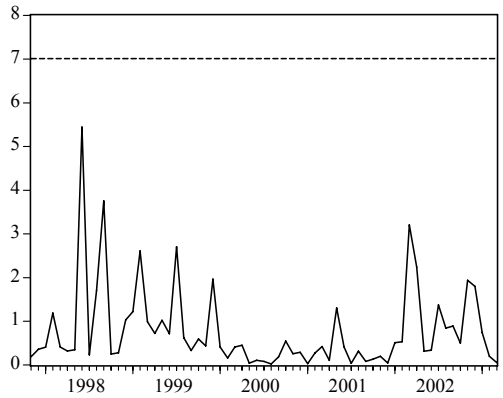
Summing up, Chilean monetary policy is consistent with inflation targeting. There is some evidence for exchange rate smoothing but the lack of a short-run reaction of the interest rate to an exchange rate shock does not support monetary policy strategies like the monetary conditions index or some form of managed floating. The outliers identified in the stability test reveal that the interest rate instrument was used when the currency came under pressure from contagion crisis. Additional policy instruments, such as interventions in the foreign exchange market, might have been used even recently to influence the exchange rate. Altogether, the crawling band was not a nominal anchor; rather, it might have been a "parachute" that remained folded, in case contagion from crises in other countries threatened.

Case Study Colombia

In September 1999 Colombia stopped using the exchange band and introduced inflation targeting as new monetary policy strategy. These events motivate to split the sample at that date although the Chow test in Figure 3.7 does not indicate a structural break. The first

sample period covers May 1995 to June 1999 and the second sample period covers September 1999 to March 2003. Now, the estimations allow evaluating differences in the responses of the two sample periods.

Figure 3.7: 1-Step Chow Test for Colombia



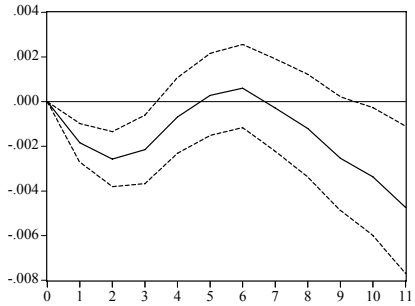
Note: The graph shows the F-statistic of the 1-Step Chow test for the VAR system. The horizontal line provides the 1% significance level for the critical value F(4,8) = 7.01. Break points at the 5% significance level are 1998:06, 1998:09, and 2002:03.

Figure 3.8 shows the pass-through effect derived from the estimations in levels for the two samples. In the first sample period from May 1994 to June 1999, the depreciation of the exchange rate induces a reduction of the price level. Afterwards, prices increase although this part of the impulse response is insignificant. In the second sample period from September 1999 to March 2003 prices increase following an exchange rate shock. Again, this response is hardly significant. Overall, the conclusions on the pass-through from the evaluation of these impulse response functions have to remain weak.

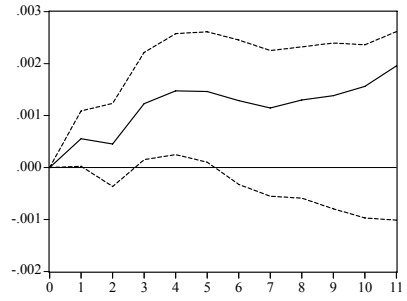
Figure 3.8: Pass-Through in Colombia

Impulse Response Functions of Prices to Exchange Rate Shock

Sample May 1994 to June 1999



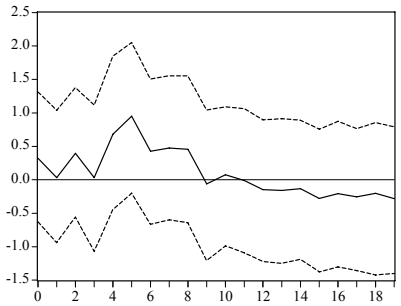
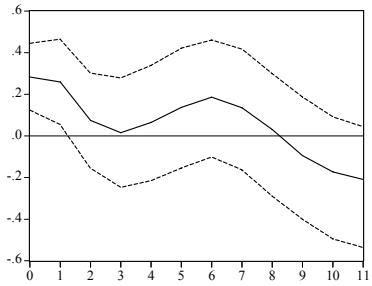
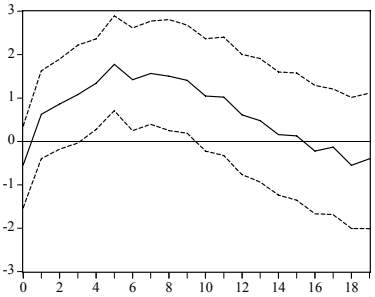
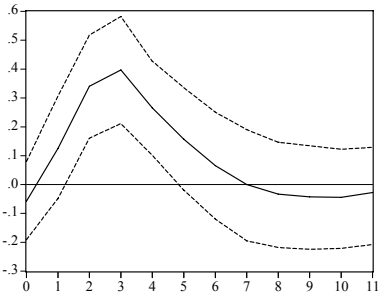
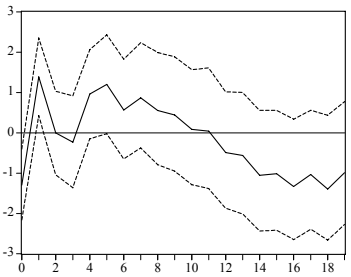
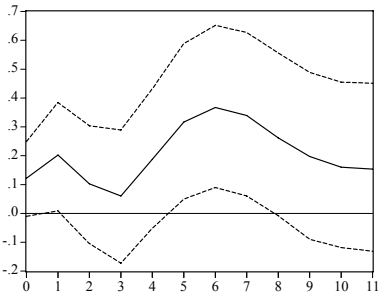
Sample Sept. 1999 to Mar. 2003



Source: Appendix Figures A3.7 and A3.9.

Evaluating the reaction of the interest rate gap in Figure 3.9 reveals stronger results. In the first sample period the response of the interest rate to a shock in the inflation gap is insignificant. The response to the output gap is only significant between the fourth and the ninth period following the shock, indicating that positive output shocks are answered by an increase in the interest rate with a delay of about half a year. Concerning a shock in the deviation of the exchange rate from its trend, the interest rate drops slightly instantaneous but rises significantly in the following period. Afterwards, the impulse response remains insignificant. The increase in the interest rate supports some form of exchange rate management.

Figure 3.9: Impulse Responses of the Interest Rate Gap in Colombia

Shock to	Sample May 1995 to June 1999	Sample Sept. 1999 to Mar. 2003
Inflation Gap		
Output Gap		
Exchange Rate Gap		

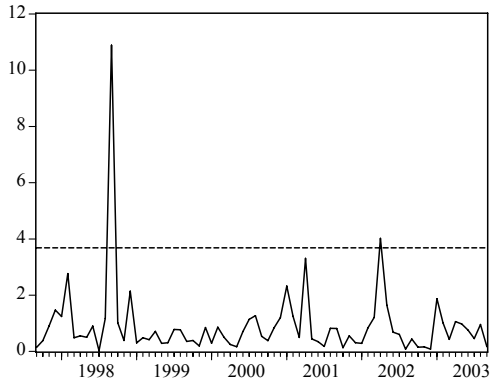
Source: Appendix Figures A3.8 and A3.10.

In the second sample period the interest rate increases immediately in case of a shock to the inflation rate. Subsequently, the response is insignificant. The response to the output gap has a clear peak in the third month, whereas the response to the exchange rate gap has its peak in the sixth month after the shock. The immediate response to inflation and the strong response to the output gap are more in line with a Taylor rule than with strict inflation targeting. Still, the exchange rate matters on a medium horizon. Comparing the two samples reveals an increase in the role of inflation and a move in the denotation of the exchange rate from the short to the medium horizon. Thus, Colombia heads credibly towards inflation targeting as long as the inflation targets are met.

Case Study Mexico

Mexico was heavily disturbed by a crisis in 1994. The following years do not show a structural break in the Chow test in Figure 3.10 except for a strong outlier due to the Brazilian Crisis in September 1998. In January 1999 the central bank introduced inflation targeting as new monetary policy strategy. Due to the constraints on the length of the sample by the Mexican Crisis and the missing structural break in the Chow test only a single VAR is estimated. Otherwise, the sample preceding inflation targeting would have been to short. Therefore, the VAR covers the sample period from January 1996 to September 2003.

Figure 3.10: 1-Step Chow Test for Mexico

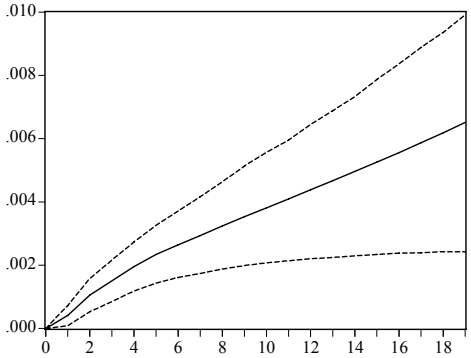


Note: The graph shows the F-statistic of the 1-Step Chow test for the VAR system. The horizontal line provides the 1% significance level for the critical value F(4,56) = 3.686. Break points are 1998:09 and 2002:04.

Figure 3.11 shows the reaction function of prices to a shock in the exchange rate. The pattern indicates a strong pass-through of the exchange rate on prices. The shock does not return to the zero line and die out due to non-stationarity of prices.

Figure 3.11: Pass-Through in Mexico

Impulse Response Function of Prices to Exchange Rate Shock



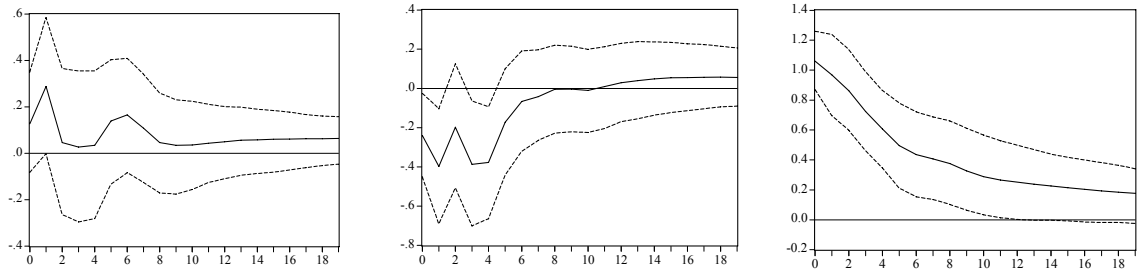
Source: Appendix Figure A3.11.

The impulse response functions of the interest rate gap for the VAR estimated in gaps are displayed in Figure 3.12. Although the interest rate does increase after a shock in the inflation gap this response just fails to be significant. A positive output gap shock leads to a reduction of the interest rate gap. This pattern is not consistent with the expected response of a Taylor rule. The response of the interest rate to the exchange rate shock is positive and very significant. The interest rate changes immediately with the shock.

Figure 3.12: Impulse Responses of the Interest Rate Gap in Mexico

Shock to

Inflation Gap | Output Gap | Exchange Rate Gap



Source: Appendix Figure A3.12.

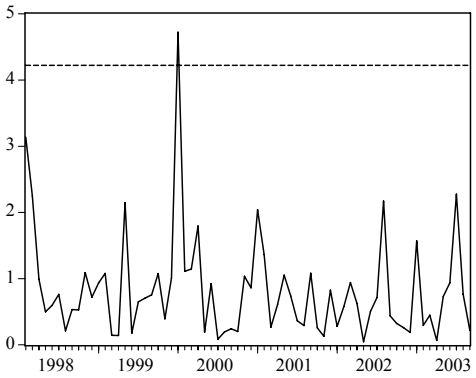
Summing up, with respect to the Mexican Crisis it was not possible to spilt the sample and to analyze the introduction of inflation targeting in contrast to the previous monetary policy. The response to inflation remained weak, whereas the response to the deviation from the trend of the exchange rate was particularly strong. Monetary policy is highly driven by the development of the exchange rate. Inflation and output are not relevant.

3.3.3 Evidence from Central Europe—Case Studies

Case Study Czech Republic

The sample period for the Czech Republic starts with the introduction of inflation targeting in January 1998 and covers till September 2003. The Chow break point test in Figure 3.13 reveals a single outlier in January 2000. This outlier does not coincide with changes in exchange rate or monetary policies.

Figure 3.13: 1-Step Chow Test for the Czech Republic

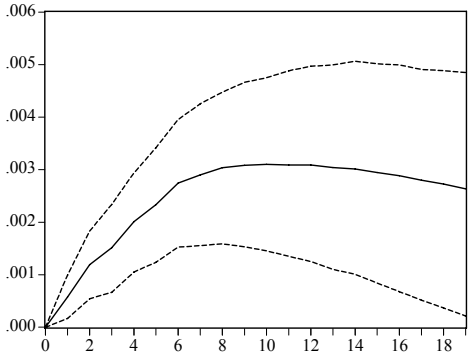


Note: The graph shows the F-statistic of the 1-Step Chow test for the VAR system. The horizontal line provides the 1% significance level for the critical value F(4,24) = 4.22. Break point is 2000:01.

Figure 3.14 shows the response of prices to an exchange rate shocks in the VAR estimation in levels. As in the case of other emerging market economies the exchange rate has a significant pass-through on prices. The impact peaks three quarters after the shock. Then the shock begins to die out.

Figure 3.14: Pass-Through in the Czech Republic

Impulse Response Function of Prices to Exchange Rate Shock



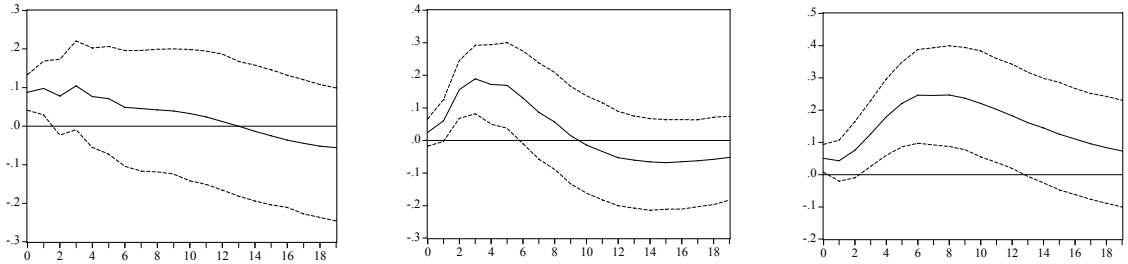
Source: Appendix Figure A3.13.

The impulse responses of the interest rate gap to shocks in the inflation gap, the output gap, and the exchange rate gap in Figure 3.15 also show a very clear pattern. A positive shock in the deviation of inflation from its target leads to an instantaneous increase of the interest rate. This response lasts only on a very short horizon and becomes insignificant in the second month following the shock.

Figure 3.15: Impulse Responses of the Interest Rate Gap in the Czech Republic

Shock to

Inflation Gap | Output Gap | Exchange Rate Gap



Source: Appendix Figure A3.14.

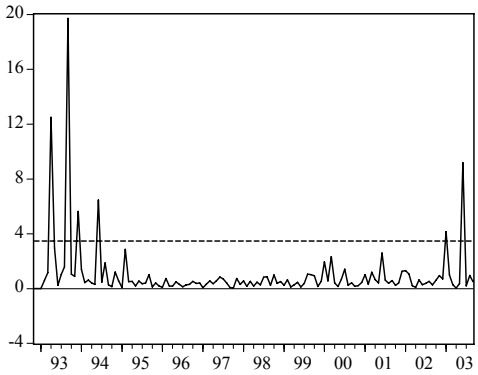
The interest rate also increases due to a shock to the output gap. Here, the peak of the response is a quarter of a year after the shock. A shock in the exchange rate leads to an increase in the interest rate on the medium horizon. The response's peak is half a year after the shock. All three variables have an impact on the monetary policy instrument but not in the same way. The timing goes from the most instantaneous reaction of the interest rate to inflation over the three months delayed reaction to the output gap to the six months delayed

reaction to the exchange rate. Despite a strong commitment towards the inflation target, the central bank pays attention to the developments of output and the exchange rate.

Case Study Hungary

In Figure 3.16 the Chow break point test does not indicate a structural break when the Hungarian central bank introduces inflation targeting in July 2001. The period of inflation targeting itself remains yet to short to contrast different monetary policy strategies in Hungary. The estimations cover October 1990 to September 2003.

Figure 3.16: 1-Step Chow Test for Hungary

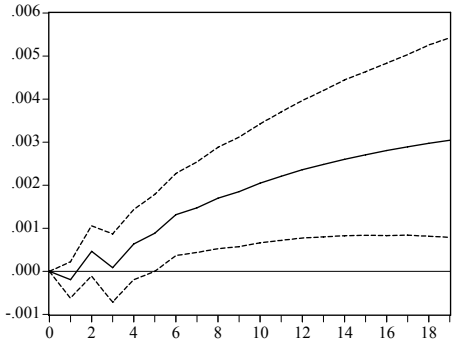


Note: The graph shows the F-statistic of the 1-Step Chow test for the VAR system. The horizontal line provides the 1% significance level for the critical value F(4,128) = 3.479. Break points are 1993:04, 1993:09, 1993:12, 1994:06, 2003:01, and 2003:06.

The pass-through is shown in Figure 3.17. There is no immediate effect of the exchange rate on prices. In fact, there is a delay of half a year until prices increase due to a shock in the exchange rate.

Figure 3.17: Pass-Through in Hungary

Impulse Response Function of Prices to Exchange Rate Shock



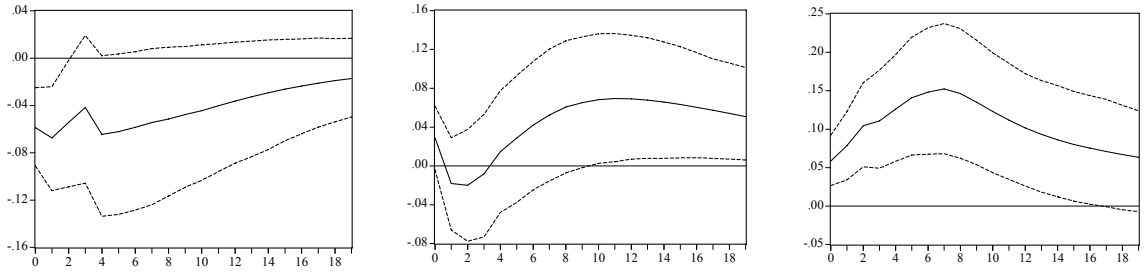
Source: Appendix Figure A3.15.

Figure 3.18 shows the responses of the interest rate gap from the estimation of the VAR in gaps. A shock to the inflation gap is answered by an immediate decline in the interest rate before the response gets insignificant. This response does not suggest a strong commitment of the central bank to keep inflation down. The rather long sample covers only a small period of inflation targeting. The preceding monetary policy strategies seem to dominate the estimation.

Figure 3.18: Impulse Responses of the Interest Rate Gap in Hungary

Shock to

Inflation Gap | Output Gap | Exchange Rate Gap



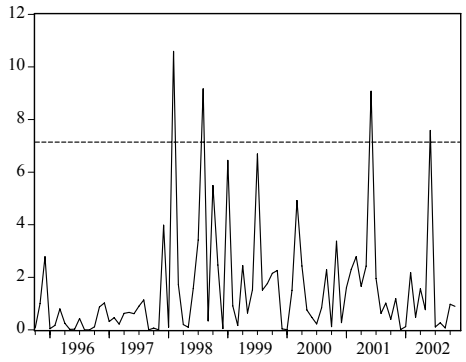
Source: Appendix Figure A3.16.

The instantaneous reaction of the interest rate to the output gap is positive but insignificant. It takes almost a year till the response becomes significant again. The response of the interest rate to a shock in the exchange rate gap is positive and significant. The peak of the response is in the seventh month after the shock. The impulse responses reveal that exchange rate movements determine the monetary policy instrument in Hungary.

Case Study Poland

The empirical analysis for Poland is in part a re-estimation of Gottschalk and Moore (2001) with two deviations.[41] First, the VAR is also estimated using gaps and not only levels of the variables. Second, the VAR is estimated with an extended dataset from January 1992 to December 2002. The longer dataset reveals a policy switch in the sample period when checking the stability of the system. The Chow break point test for the equation of the exchange rate gap detects some breaks in the sample. Figure 3.19 shows that the first break point is in March 1998.

Figure 3.19: 1-Step Chow Test for Poland



Note: The graph shows the F-statistic of the 1-Step Chow test for the equation of the exchange rate gap. The horizontal line provides the 1% significance level for the critical value F(1,55) = 7.1375. Break points are 1998:03, 1998:09, 2001:07, and 2002:07.

This finding is consistent with Polish exchange rate policy. There was a period of inconsistency between the de jure and the de facto exchange rate regime because the Polish central bank introduced the new monetary policy strategy already in 1998, but the official move to pure floating exchange rates took place in April 2000 as it had to be undertaken jointly by the Cabinet and the central bank (Kokoszczyński 2002: 201-202). However, on February 26th 1998 the crawling band around the exchange rate was widened and Wickham (2002: 22) also finds a break point on that date based on the volatility in the variance of returns in daily exchange rate data. Summing up, between February 1998 and March 1998 is a plausible break point. Two separate VARs have been estimated. The first VAR covers the

[41] See Wróbel and Pawłowska (2002) for evidence on the monetary transmission mechanism in Poland between 1995 and 2002.

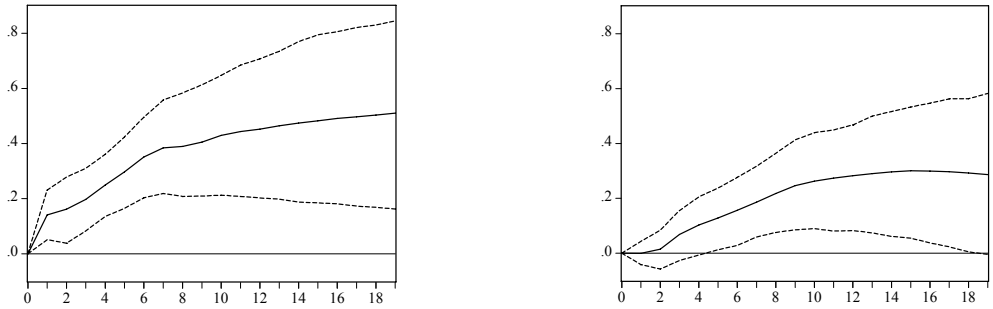
sample period from January 1992 to February 1998 and the second VAR covers the sample period from March 1998 to December 2002. In September 1998 the Polish central bank announced its first inflation target. Thus, the second sample period is not only characterized by a flexible exchange rate regime but also by an inflation targeting regime.

The estimations of the VARs in levels show that the pass-through significantly differs between the two samples (Figure 3.20). In the first sample, the exchange rate shock itself is more persistent than in the second sample. Prices react in the first sample sooner to the exchange rate shock than in the second sample. Both responses are in line with the prevailing exchange rate regimes. In the first sample period the currency is pegged to the nominal exchange rate. Under fixed exchange rates, exchange rate changes were expected to be permanent and have been passed on to prices. Whereas in the second sample period with flexible exchange rates, exchange rate changes were expected to be temporary. Hence, there have been moderate price adjustments, but a short-run impact of the nominal—and because of delayed price adjustment also real—exchange rate shock.

Figure 3.20: Pass-Through in Poland

Impulse Response Functions of Prices to Exchange Rate Shock

Sample Jan. 1992 to Feb. 1998 Sample Mar. 1998 to Dec. 2002



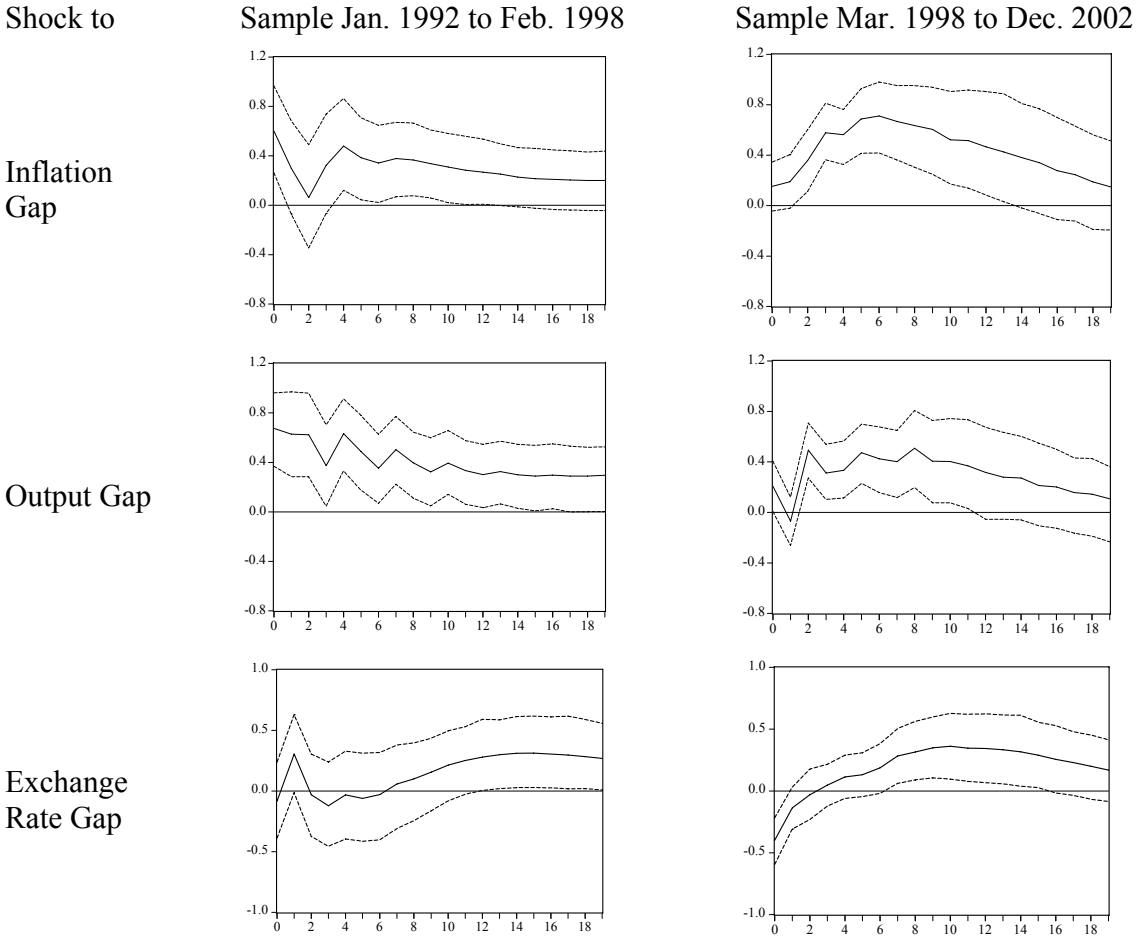
Source: Appendix Figures A3.17 and A3.19, inverted responses.

Figure 3.21 shows the reaction function of the central bank. In the first sample the interest rate reacts to a positive shock in inflation. Immediately after the spike, one observes an insignificant drop of the interest rate before the shock then dies out smoothly. The response of the interest rate to a shock in the output gap is quite clear. A positive shock in output is answered by an increase of the interest rate. Gottschalk and Moore (2001: 32) get a qualitatively similar result. They interpret the output shock as a demand shock and monetary

policy responding with a tightening of monetary conditions. Thus, the impulse response of the interest rate to the shock in output suggests some form of activist monetary policy. Overall, these two impulse responses support the disinflation process along with output smoothing, but both responses are not very pronounced.

A depreciation, i.e., a positive shock in the exchange rate gap, is answered by an immediate increase of the interest rate, but this response fails to be significant. Two periods later the interest rate moves in the opposite direction. The immediate rise in the interest rate after the exchange rate shock is supported by the estimations in levels and the results in Gottschalk and Moore (2001). Gottschalk and Moore (2001: 33) interpret the response of the interest rate as a reaction of the central bank to offset the loosening in the monetary conditions caused by the exchange rate shock. Their interpretation can be augmented to the possibility of managing the exchange rate by interest rate policy. The immediate increase of the interest rate offsets the depreciation, while the later decline of the interest rate smoothes exchange rate adjustment. Hence, the interest rate policy is consistent with the prevailing crawling band. The exchange rate is the nominal anchor during the disinflation process. Therefore, the interest rate is set in order to keep the exchange rate in its band.

Figure 3.21: Impulse Responses of the Interest Rate Gap in Poland



Source: Appendix Figures A3.18 and A3.20, inverted responses for the exchange rate gap.

In the second sample, the interest rate responds with a clear increase to a shock in inflation. The behavior is consistent with what one would expect of the new monetary policy regime, namely, inflation targeting. Also, the response of the interest rate to a shock in output is significant, although the immediate response is not significant. Based on these two impulse response functions the central bank might apply a Taylor rule with a strong weight on inflation deviations.

A depreciation of the exchange rate is answered with an increase of the interest rate on the medium horizon. In contrast to the earlier sample there is no immediate reaction. The pattern suggests that the exchange rate is in the new regime not as important as in the earlier regime in the short run. Monetary policy is committed to keeping inflation down. Still, the central

bank pays some attention to the development of the exchange rate, as suggested by the delayed but persistent response. Thus, the conducted monetary policy would be also consistent with a monetary conditions index or some form of managed floating.

Overall, Poland shows a remarkable regime change. Not only that there is a clear break point between February and March 1998, but also the impulse response functions in the two samples show substantial differences in their adjustment patterns. Inflation gains as a target of monetary policy, whereas the role of the exchange rate moves from the short run to the medium run.

3.4 Conclusions

A generalized reaction function, where changes in the interest rate—approximated by the interest rate gap—are determined by the deviations of inflation rate, output, and exchange rate from their respective targets, has been analyzed with the help of vector autoregressive models (VARs) for economies in Latin America and the CEEC. All these seven emerging market economies have in common that they mastered disinflation and introduced inflation targeting. The analysis revealed the importance of different variables for monetary policy and, in particular, the role of the exchange rate in inflation targeting regimes.

With respect to the empirical analysis the countries are assorted into three groups. First, there are those economies were it was possible to identify a structural break. Comparing the two different sample periods of Brazil, Colombia, and Poland reveals a change in the central bank's conduct of monetary policy with the introduction of inflation targeting. Second, there is Chile with the most experienced inflation targeting central bank in a group on its own. Third, there are those economies with constraints on the length of the sample. In Mexico it is due to the Mexican Crisis in 1994, in the Czech Republic the currency came into existence only in January 1993, and in Hungary inflation targeting was introduced just in July 2001. In these three countries it was not possible to estimate and compare two separate samples.

Concerning the pass-through of the exchange rate on prices, the first group of countries does not show a common pattern in the pass-through. Most noticeable are the differences between Brazil and Poland. As Brazil moved from a fixed to a flexible exchange rate regime the pass-through of the exchange rate on prices increased. The flexible exchange rate revealed the importance of exchange rate movements for the economy. In contrast, the pass-through reduced in Poland. With the shift towards flexible exchange rates the economy become more mature and thus less vulnerable by exchange rate movements. In Colombia the evaluation of the pass-through remains weak. In the second group, i.e., in Chile, the exchange rate has only a weak impact on prices. This pattern indicates—similar to the second sample period of Poland—a stable and mature economy. In the third group, consisting of the Czech Republic, Hungary, and Mexico, the pass-through is strong again. Exchange rates matter for the development of prices.

Concerning the role of the exchange rate in the reaction function of the central bank, the countries in the first group share a similar pattern. Most prominent are the results for Brazil. Compared to the first sample period, the second sample period revealed an increase in the strength of the reaction of the interest rate gap to a shock in the inflation gap. This response is well in line with the introduction of inflation targeting. However, the exchange rate still matters. The peak of the response moved from an immediate reaction towards the medium horizon. Responses in Colombia and Poland are alike. In Chile, monetary policy was characterized by the gradual reduction of inflation with the help of announced inflation targets. This policy is captured well in the empirical results by the very strong response of the interest rate to shocks in inflation. At the same time, Chile was still operating under a crawling band. But this exchange rate policy is not captured by the VAR. Outliers suggest that only special events such as crises in other countries led to more active exchange rate management. In the third group, the responses of the Czech interest rate gap have a conspicuous timing. The impulse response functions reveal the operating horizon of the central bank for each variable. The response to inflation is on the very short horizon, slightly later is the response to output and the response to the exchange rate is on the medium horizon. In Hungary and Mexico, the reactions of the interest rate are dominated by a strong response to exchange rate shocks. In both countries, monetary policy is driven by the development of the exchange rate.

Summing up, Chile's inflation targeting framework has already a strong reputation. Poland and—to a weaker degree—Brazil, Colombia, and the Czech Republic changed their monetary policy and introduced inflation targeting. Yet, these countries track the development of their exchange rates, whereas monetary policy in Hungary and Mexico is still dominated by the exchange rate.

The three CEEC central banks actually implemented inflation targeting regimes and—although to different degrees—did not give up exchange rate targeting completely. As mentioned in Chapter 2, this is consistent with a medium-term strategy of monetary integration into the European Economic and Monetary Union (EMU). However, stabilizing the exchange rate against the euro may have a cost depending on integration with third countries.

Appendix

Figure A3.1: Impulse Response Functions for Brazil, 1994:09 – 1998:12

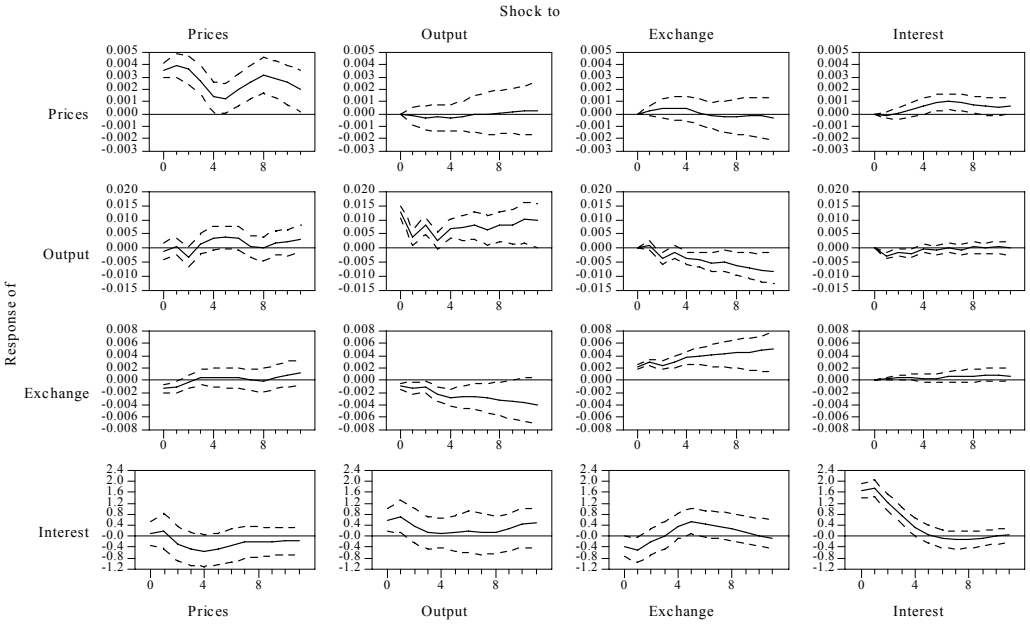


Figure A3.2: IRFs for Brazil Using Gaps, 1994:09 – 1998:12

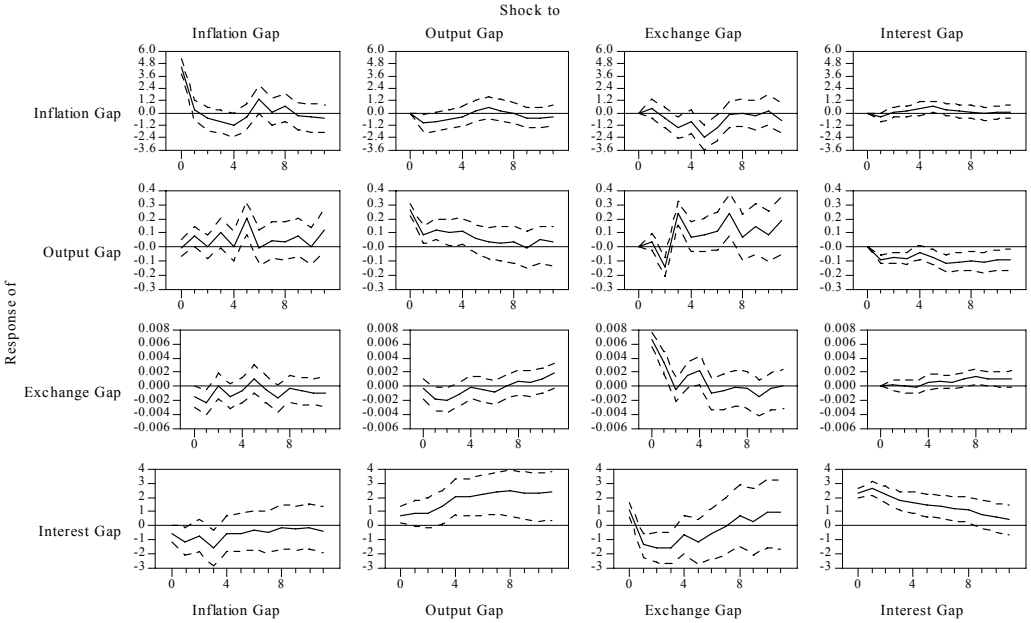


Figure A3.3: Impulse Response Functions for Brazil, 1999:03 – 2003:10

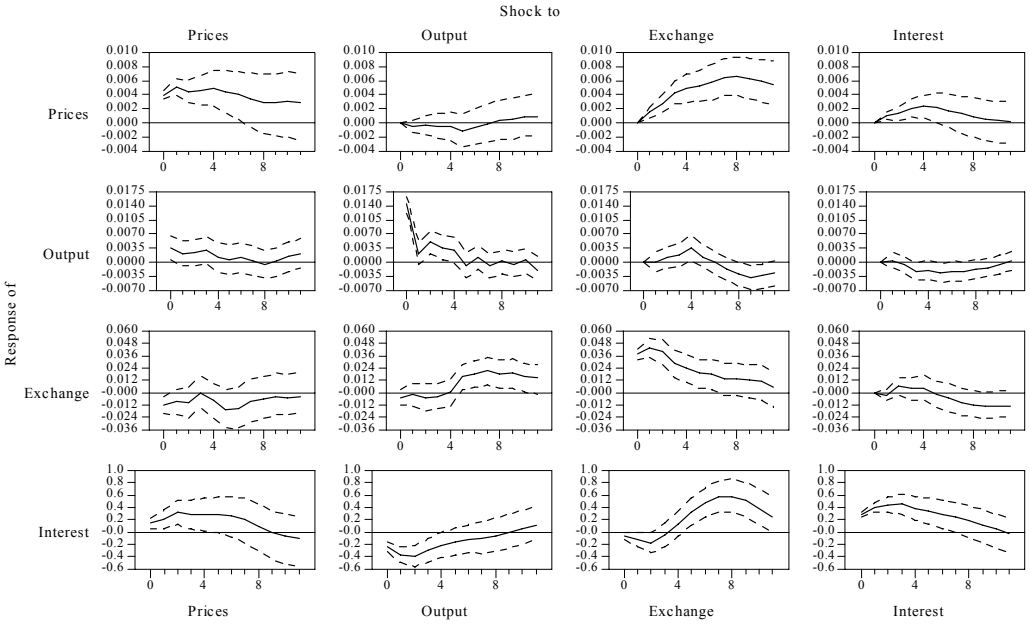


Figure A3.4: IRFs for Brazil Using Gaps, 1999:03 – 2003:10

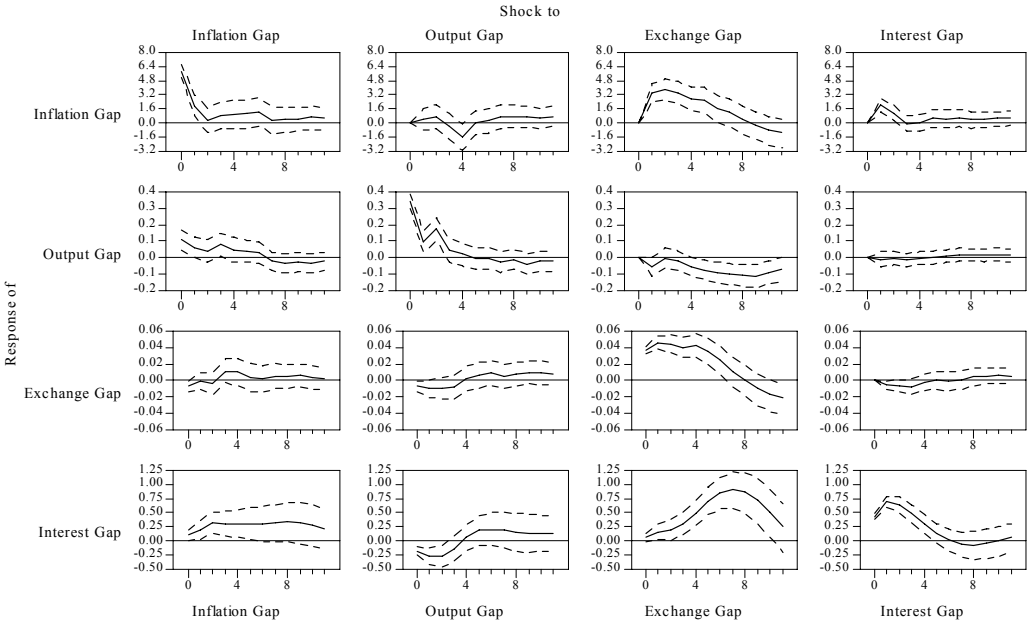
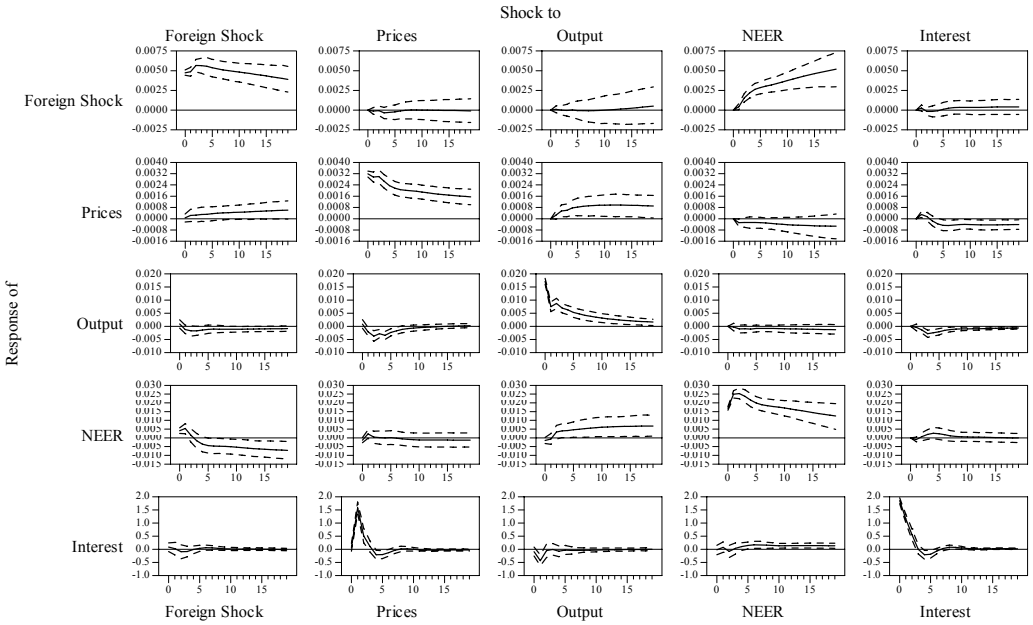


Figure A3.5: Impulse Response Functions for Chile, 1991:01 – 2002:12



Source: Hammermann (2005: Appendix Figure A3a).

Figure A3.6: IRFs for Chile Using Gaps, 1992:01 – 2002:12

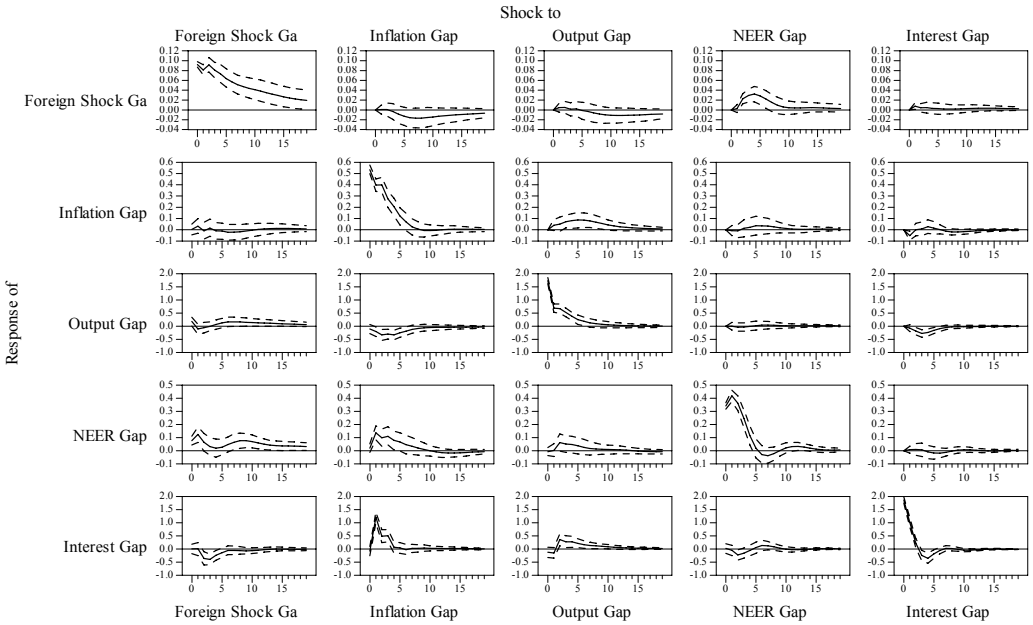


Figure A3.7: Impulse Response Functions for Colombia, 1995:05 – 1999:06

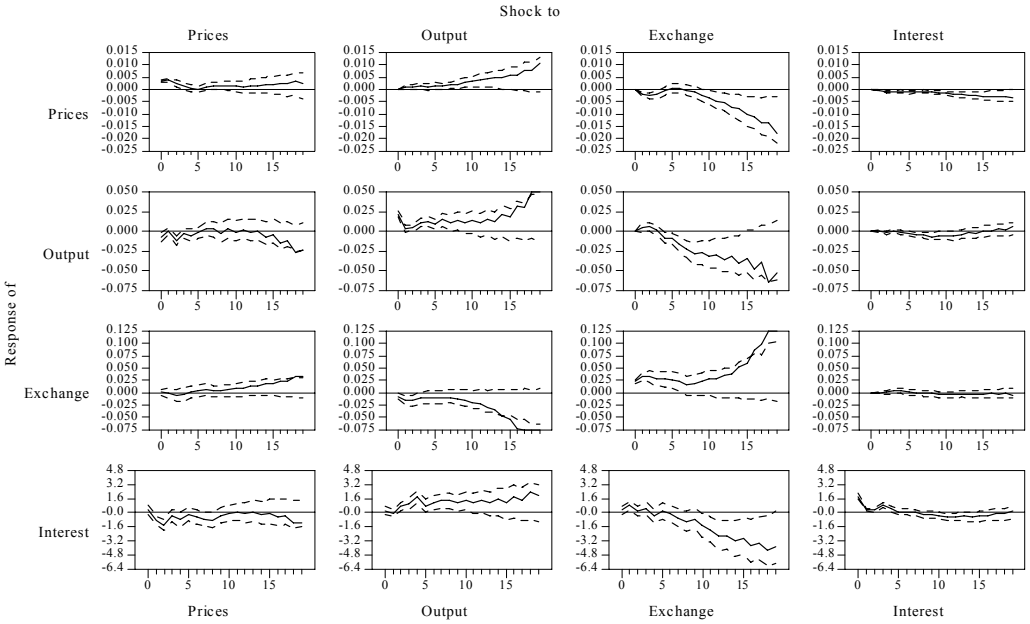


Figure A3.8: IRFs for Colombia Using Gaps, 1995:05 – 1999:06

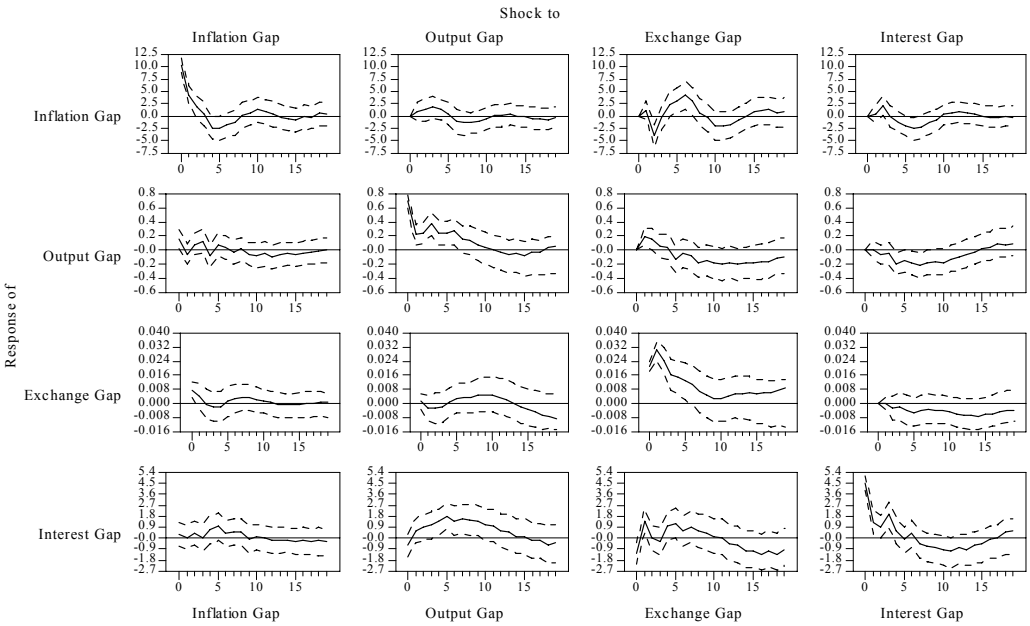


Figure A3.9: Impulse Response Functions for Colombia, 1999:09 – 2003:03

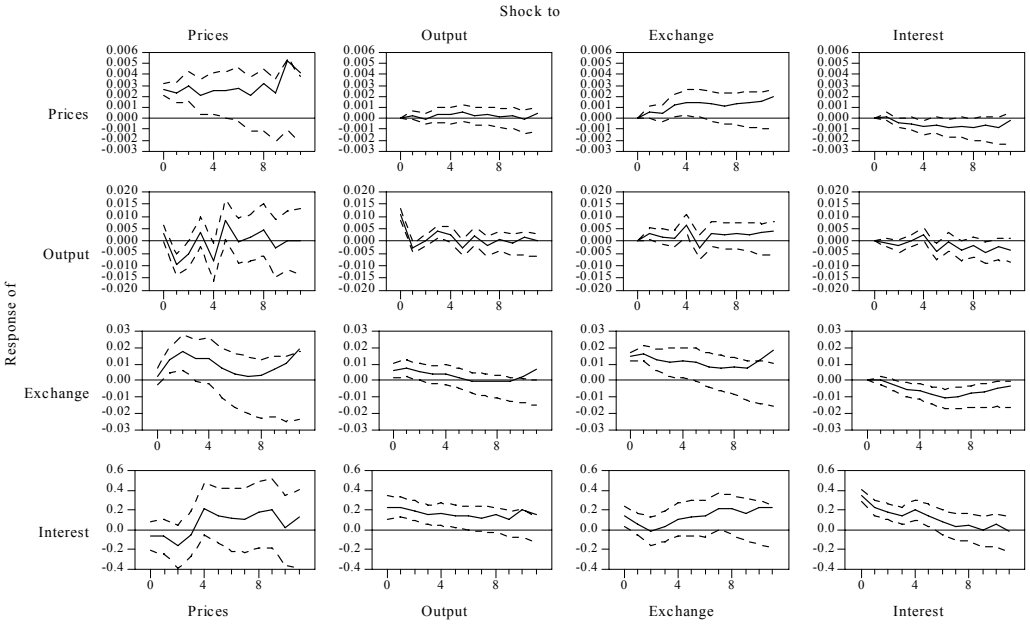


Figure A3.10: IRFs for Colombia Using Gaps, 1999:09 – 2003:03

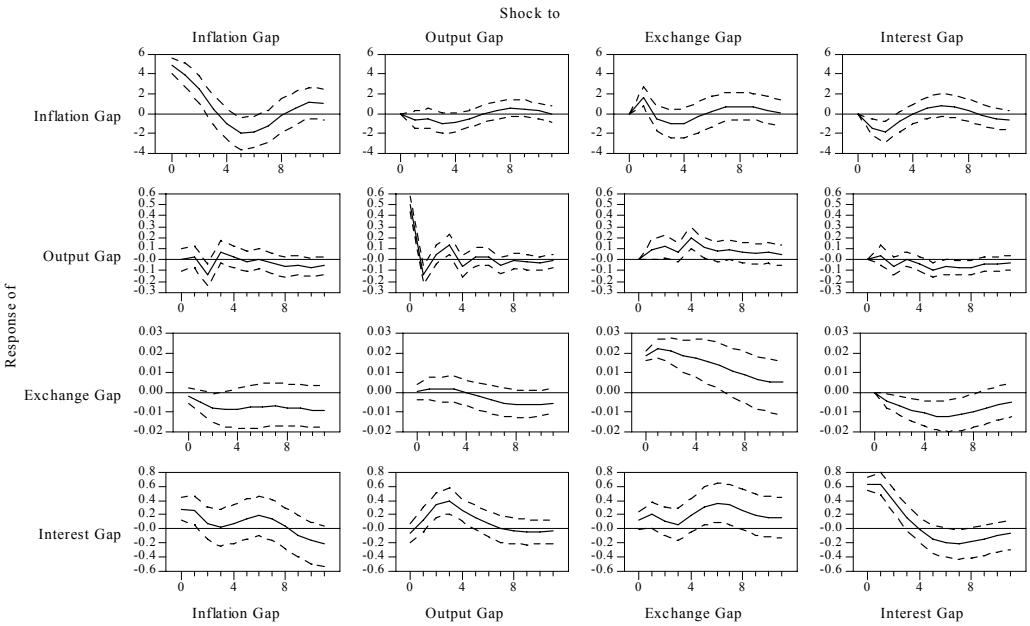


Figure A3.11: Impulse Response Functions for Mexico, 1996:01 – 2003:09

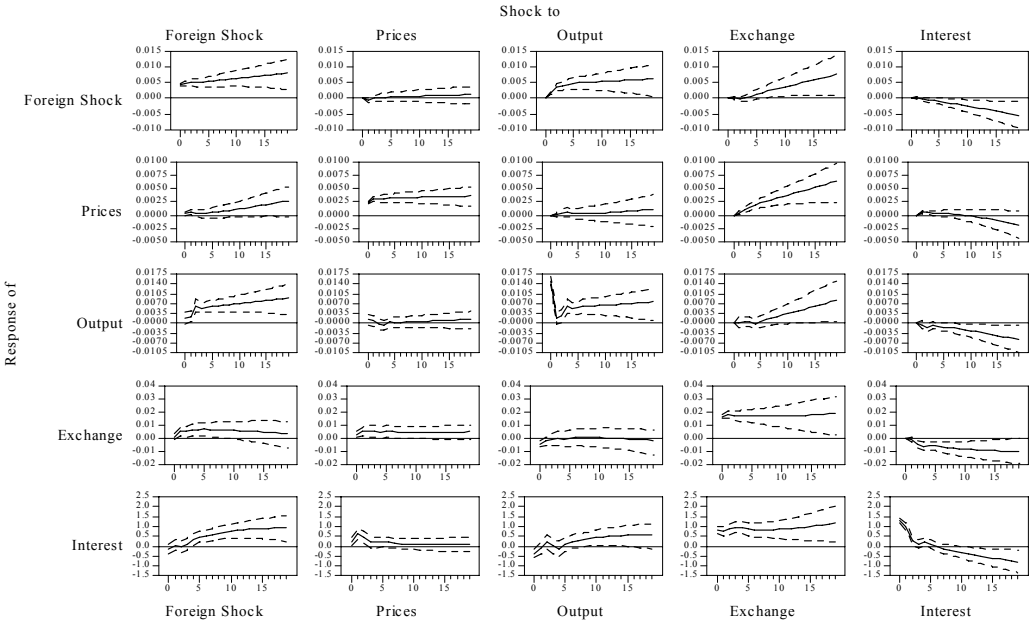


Figure A3.12: IRFs for Mexico Using Gaps, 1996:01 – 2003:09

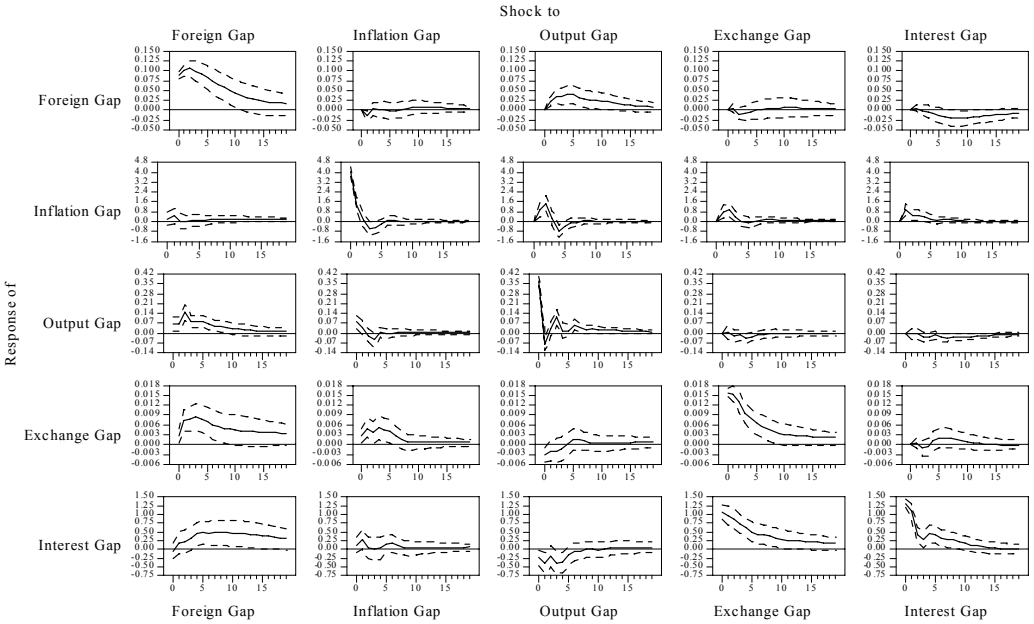


Figure A3.13: Impulse Response Functions for the Czech Republic, 1998:01 – 2003:09

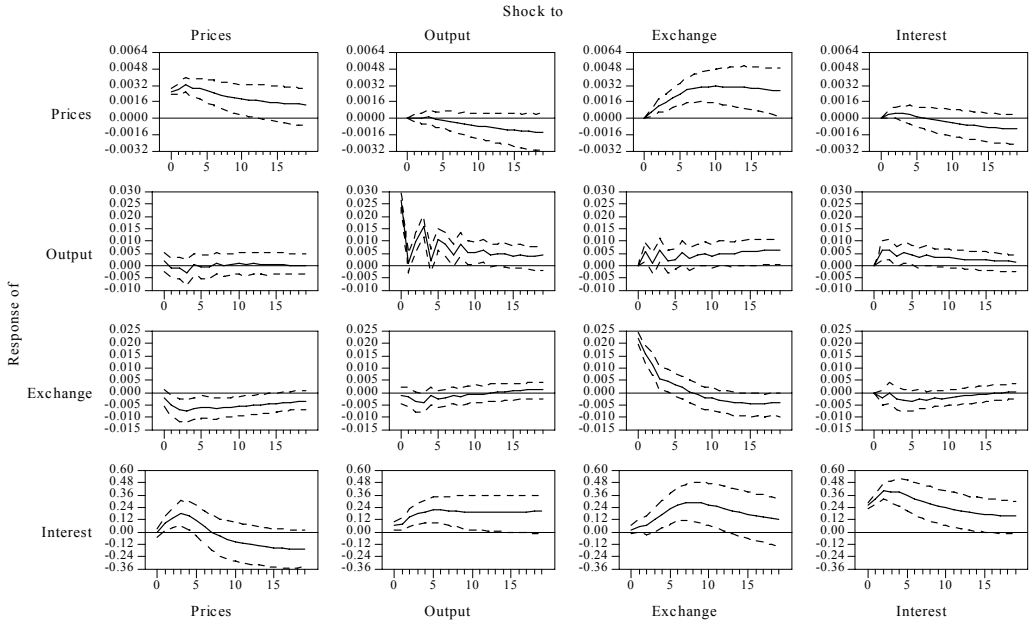


Figure A3.14: IRFs for the Czech Republic Using Gaps, 1998:01 – 2003:09

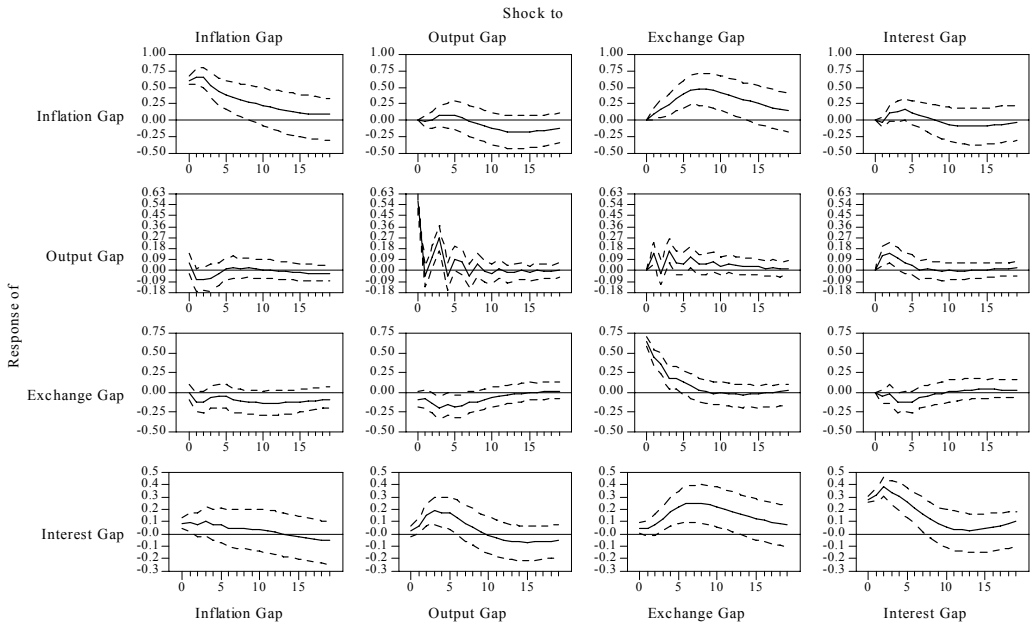


Figure A3.15: Impulse Response Functions for Hungary, 1990:01 – 2003:09

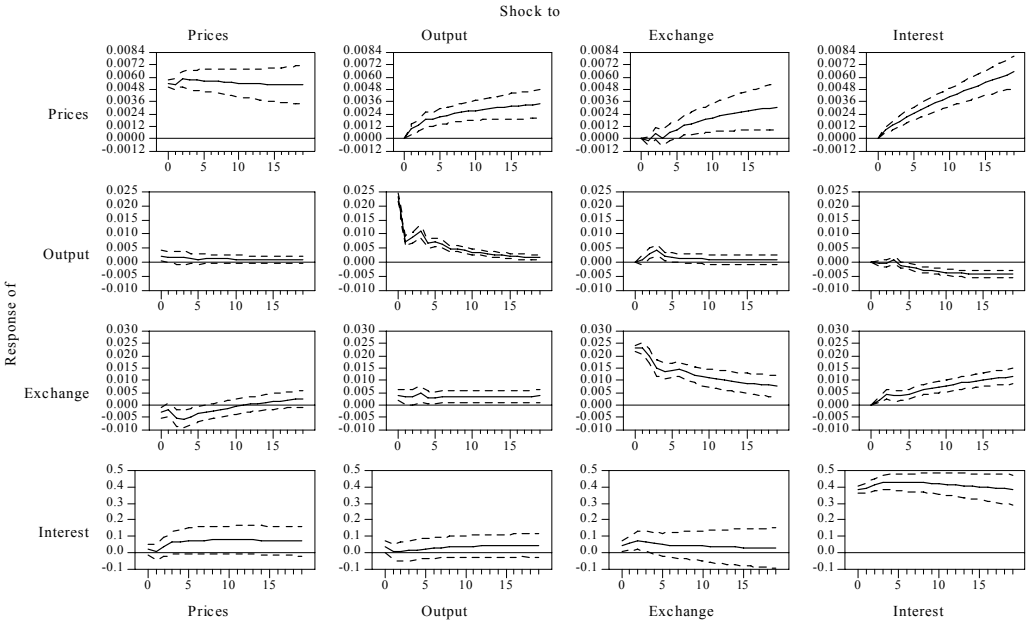


Figure A3.16: IRFs for Hungary Using Gaps, 1990:01 – 2003:09

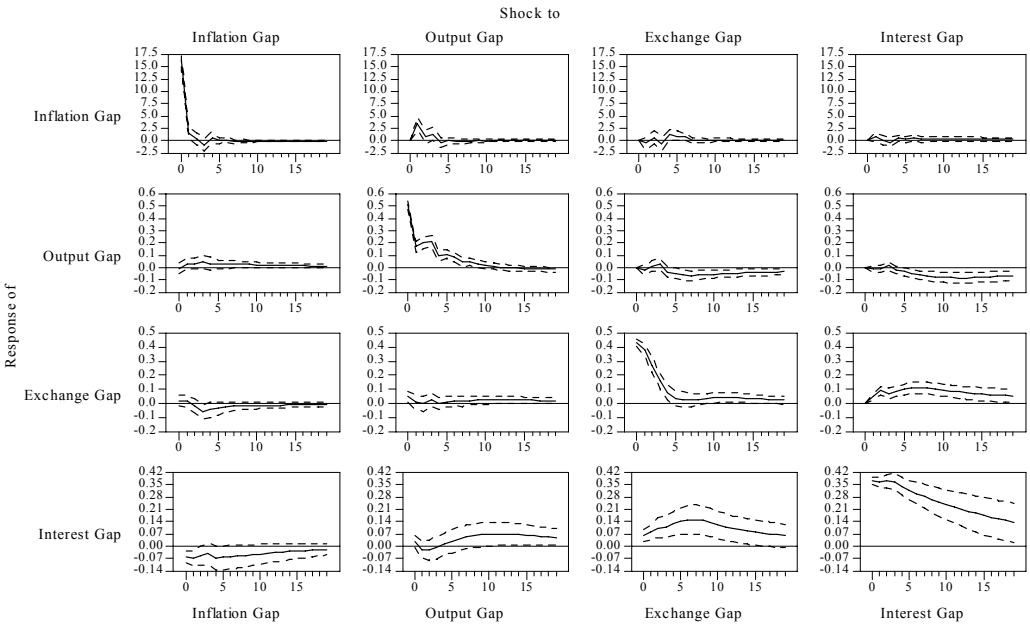
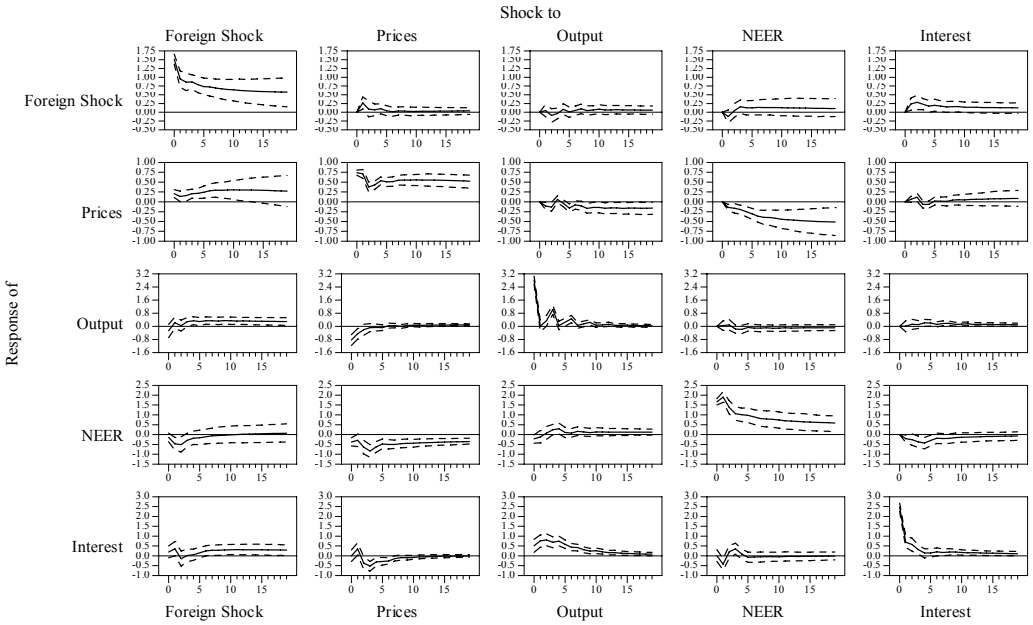


Figure A3.17: Impulse Response Functions for Poland, 1992:01 – 1998:02



Source: Hammermann (2005: Appendix Figure A1a).

Figure A3.18: IRFs for Poland Using Gaps, 1992:01 – 1998:02

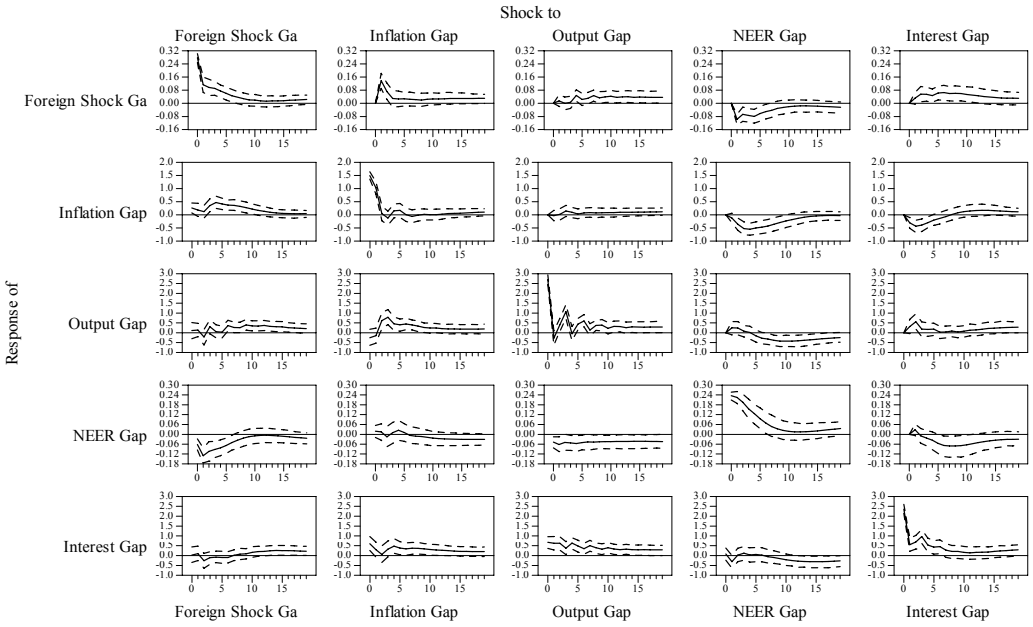
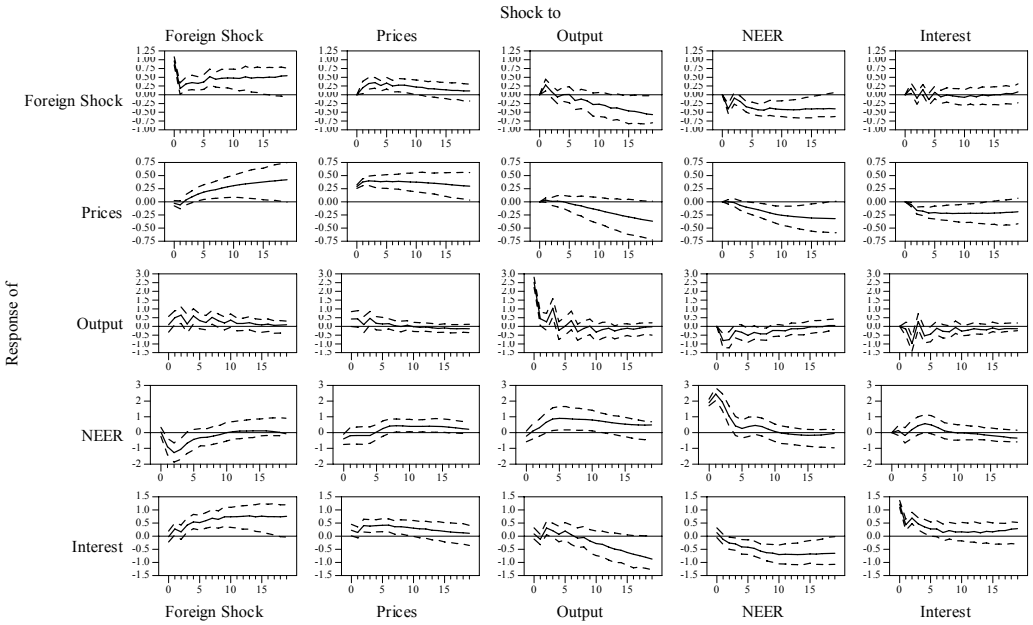
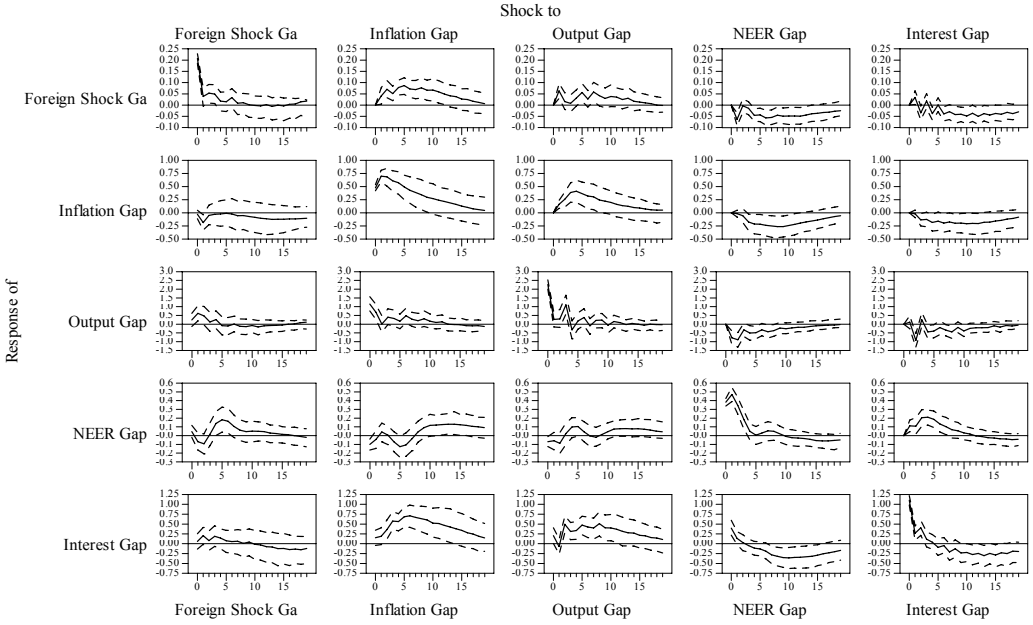


Figure A3.19: Impulse Response Functions for Poland, 1998:03 – 2002:12



Source: Hammermann (2005: Appendix Figure A2a).

Figure A3.20: IRFs for Poland Using Gaps, 1998:03 – 2002:12



4 Inflation Targeting and Monetary Integration under ERM 2: Modeling Third-Country Effects from Production Sharing

4.1 Introduction

In this chapter, we analyze the potential cost of regional monetary integration, i.e., targeting the exchange rate to an anchor currency, while being at the same time integrated with third countries. Recent empirical papers point to the relevance of outsourcing and offshoring for the transmission of business cycle shocks and thereby for the conduct of monetary policy (see, among others, Fontagné and Freudenberg 1999, Imbs 2004, Kalemli-Ozcan, Sørensen, and Yosha 2003, Kose, Otrok and Whiteman 2003, Kose, Prasad, and Terrones 2003, IMF 2007). This is especially true for the countries that are about to join the European Economic and Monetary Union (EMU), which have to stay for two years in the European Exchange Rate Mechanism (ERM 2) and therefore target inflation and the exchange rate simultaneously. These countries are significantly integrated with third countries through the production chain. How do external shocks originating from a third country affect monetary policy under ERM 2?

Up to now, the empirical analysis had to focus on the early years characterized by inflation stabilization. The central and eastern European countries (CEEC) had to deal with the structural change from centrally planned to market economies. In the beginning of the 1990s, price liberalization and the abolition of price controls led to hyperinflation. The situation got stabilized by pegging the exchange rate. When the economies were ready for a more independent monetary policy, the central banks of the Czech Republic (1998), Poland (1998), Hungary (2001), and Slovakia (2005)[42] introduced inflation targeting. Mohanty and Klau (2005) and Frömmel and Schobert (2006) estimate Taylor rules in these inflation targeting countries. Krusec (2005) investigates the transmission mechanism under inflation targeting in a VECM, whereas Chapter 3 showed the reaction of the interest rate to shocks in inflation, output, and the exchange rate in a VAR.

[42] Slovakia had an implicit inflation target since 1999.

Empirical studies could not yet analyze how central banks actually solve the trade-off between inflation and exchange rate stabilization under ERM 2 (see Orlowski and Rybinski 2006).[43] In November 2005, Slovakia was the first inflation targeting country to move into ERM 2. In the next years, other countries of the CEEC that currently target the inflation rate may join ERM 2 and thus target the exchange rate at the same time. ERM 2 allows only for exchange rate movements within a ±15 percent band. After two successful years in ERM 2 and fulfillment of the other convergence criteria specified in the treaty of Maastricht, countries would adopt the euro. In any case, the results of Chapter 3 support the assumption that even the inflation targeters among the CEEC did not completely give up exchange rate targeting, which was, of course, more dominant during the initial phase of disinflation and structural change. Incorporating the exchange rate in the conduct of monetary policy may be interpreted as simulating ERM 2, while still avoiding its strictly binding rules.

The theoretical literature looks either at the strategic role of targeting the exchange rate in competitive devaluations and its role as a shock absorber or rejects the role for targeting the exchange rate as the central bank should target domestic inflation and allow the exchange rate to float (Clarida, Galí, and Gertler 2001). Svensson (2000) shows in his simulations that a Taylor rule without an exchange rate term is a robust monetary policy strategy despite ignoring part of the available information. De Paoli (2006) finds only for implausibly high elasticities of substitution that exchange rate targeting enters the optimal monetary policy (see also Galí 2008).

Nonetheless, the existing models are incomplete as they do not address any potential trade-off between the strategic role of the exchange rate and achieving an announced inflation target given third-country effects. As shown in Table 4.1, central European inflation targeting countries are well integrated into production sharing with the euro area, documented by high trade shares, but a substantial share of their trade is still with the rest of the world (see also Crespo and Fontoura 2007).

[43] The currency board countries Estonia, Latvia, and Lithuania are already in ERM 2; Slovenia adopted the euro in January 2007.

Table 4.1: Trade Shares and Openness of Central European Inflation Targeting Countries and the Euro Area in 2006

	Imports		Exports		Openness
	Euro Area	Rest of the World	Euro Area	Rest of the World	
Czech Republic	0.59	0.41	0.58	0.42	1.31
Hungary	0.52	0.48	0.55	0.45	1.35
Poland	0.56	0.44	0.53	0.47	0.69
Slovakia	0.42	0.58	0.52	0.49	1.56
Average IT Countries	0.52	0.48	0.54	0.46	1.23
Euro Area	0.49	0.51	0.50	0.50	0.65

Source: Eurostat and IFS.
Note: Openness is measured as imports and exports over GDP.

Because a two-country model cannot adequately cope with potential third-country effects, we develop a New Keynesian three-country model. As a starting point, we take a three-country model in which Teo (2005) analyzes the choice of invoice currency in Asia but that does not address the role of monetary policy. We, therefore, integrate Teo's three-country framework into a model that analyzes monetary policy in a New Keynesian two-country model developed by Monacelli (2001). As a result, we are able to analyze regional monetary integration based on a dynamic, microfounded model allowing for third-country effects. Additionally, following Corsetti, Pesenti, Roubini, and Tille (2000), we incorporate a nested consumption basket to distinguish between goods produced in two periphery countries and a core country.

The model develops a set of equations for each of the three countries *A, B,* and *C*. Country *A* is the country under consideration and may represent a CEEC country integrating into EMU by targeting the euro as its anchor currency. Country *B* constitutes the third country, which is not directly affected by the European integration process and, therefore, stays outside ERM 2. Country *B* may represent either another transition country like Russia or the rest of the world. Both countries *A* and *B* together form the periphery, while country *C* represents the core country, which also provides the anchor currency for country *A*. Hence, country *C* features the euro area.

We analyze monetary policy in this setting given alternative degrees of integration between the euro area (core country *C*), an inflation targeting accession country like the Czech

Republic, Hungary, Poland, and Slovakia (country *A*), and a non-EU country or the rest of the world as the third country (country *B*). As a short-cut for modeling production linkages we use alternative elasticities in consumption baskets. This framework allows us to trace the transmission mechanism of a third-country technology shock. Evaluating the central bank loss function for different simple rules and determining the corresponding optimal simple rules allows us to assess the costs of ERM 2 and derive the implications of production sharing for monetary policy. As a robustness check, we introduce home bias in consumption.

Our results show that a Taylor rule extended for an exchange rate term is a good simple rule for different kinds of integration with a third country. Nevertheless, the costs of ERM 2 in terms of a traditional central bank loss function are considerable. The chapter proceeds as follows. In the second section, we derive the New Keynesian three-country model. In the third section, we present the simulation results. The fourth section concludes.

4.2 A New Keynesian Three-Country Model for Analyzing Optimal Monetary Rules under ERM 2

The New Keynesian three-country model incorporates explicit microfoundations with dynamic and intertemporal effects as well as Keynesian building blocks such as nominal rigidities and monopolistic competition. Following the microfoundation of the model, all individuals are at the same time consumers and producers. In their role as consumers, we look at the demand side of the economy. The representative household maximizes its utility by deciding about its optimal consumption. In the individuals' role as producers, we look at the supply side of the economy. The representative firm maximizes its profits by deciding about the product's optimal price. Putting the resulting optimality conditions together, the New Keynesian model is made of three core equations for each country. First, the dynamic IS curve stems form the household's choice on the optimal consumption path. Second, the New Keynesian Phillips curve stems from the firm's choice on the optimal price. Third, the model is closed with a monetary policy rule. These three equations form together with market clearing and international risk sharing the three-country model. The derivation follows along the lines of Galí and Monacelli (2005).

4.2.1 Households

The representative household in country *i, i=A, B, C,* maximizes the expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t^i \left(C_t^i, L_t^i \right), \tag{4.1}$$

where β is the subjective discount factor, C is consumption and L are hours worked, with the respective period utility

$$U_t^i = \frac{1}{1-\mu} {C_t^i}^{1-\mu} - \frac{1}{1+\gamma} {L_t^i}^{1+\gamma}, \tag{4.2}$$

where μ is the risk aversion and γ the inverse of the elasticity of labor supply. Households consume from a nested consumption basket as in Corsetti, Pesenti, Roubini, and Tille (2000). The utility-based consumption basket distinguishes between goods from the periphery P and the center C, where n_P is the size of the periphery relative to the center

$$C_t^i = \left[(n_P)^{1/\theta} \left(C_{P,t}^i \right)^{(\theta-1)/\theta} + (1-n_P)^{1/\theta} \left(C_{C,t}^i \right)^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)}. \tag{4.3}$$

The periphery itself is made up of the two countries A and B, where n_A is the size of country A relative to country B. The consumption basket for the periphery is given by

$$C_{P,t}^i = \left[(n_A)^{1/\psi} \left(C_{A,t}^i \right)^{(\psi-1)/\psi} + (1-n_A)^{1/\psi} \left(C_{B,t}^i \right)^{(\psi-1)/\psi} \right]^{\psi/(\psi-1)}. \tag{4.4}$$

Taken together, the size of country A is $n_P n_A$, of country B is $n_P (1- n_A)$ and of the center C is $(1- n_P)$. The nested consumption basket for country *i, i=A,B,C,* is

$$\begin{aligned} C_t^i = & \left[(n_P)^{1/\theta} \left(\left[(n_A)^{1/\psi} \left(C_{A,t}^i \right)^{(\psi-1)/\psi} + (1-n_A)^{1/\psi} \left(C_{B,t}^i \right)^{(\psi-1)/\psi} \right]^{\psi/(\psi-1)} \right)^{(\theta-1)/\theta} \right. \\ & \left. + (1-n_P)^{1/\theta} \left(C_{C,t}^i \right)^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)}. \end{aligned} \tag{4.5}$$

Introducing home bias in consumption changes the trade shares. If consumers in each country prefer home goods over foreign goods, the degree of openness, η, is smaller than one. The nested consumption baskets for each country depend on the intratemporal elasticity of substitution between goods from the center versus the periphery, θ, as well as on the intratemporal elasticity of substitution between goods from country A versus country B, i.e.,

between the two goods from the periphery, ψ. Thus, the nested consumption baskets with home bias are given by

$$
\begin{aligned}
C_t^A = & \left[\left[1-\eta\left(1-n_P\right)\right]^{1/\theta}\left(\left[1-\eta\left(1-n_A\right)\right]^{1/\psi}\left(C_{A,t}^A\right)^{(\psi-1)/\psi}\right.\right. \\
& \left.\left.+\left[\eta\left(1-n_A\right)\right]^{1/\psi}\left(C_{B,t}^A\right)^{(\psi-1)/\psi}\right]^{\psi/(\psi-1)}\right)^{(\theta-1)/\theta} \\
& +\left[\eta\left(1-n_P\right)\right]^{1/\theta}\left(C_{C,t}^A\right)^{(\theta-1)/\theta}\Bigg]^{\theta/(\theta-1)}
\end{aligned} \tag{4.6}
$$

$$
\begin{aligned}
C_t^B = & \left[\left[1-\eta\left(1-n_P\right)\right]^{1/\theta}\left(\left[\left(\eta n_A\right)^{1/\psi}\left(C_{A,t}^B\right)^{(\psi-1)/\psi}\right.\right.\right. \\
& \left.\left.\left.+\left(1-\eta n_A\right)^{1/\psi}\left(C_{B,t}^B\right)^{(\psi-1)/\psi}\right]^{\psi/(\psi-1)}\right)^{(\theta-1)/\theta} \\
& +\left[\eta\left(1-n_P\right)\right]^{1/\theta}\left(C_{C,t}^B\right)^{(\theta-1)/\theta}\Bigg]^{\theta/(\theta-1)}
\end{aligned} \tag{4.7}
$$

$$
\begin{aligned}
C_t^C = & \left[\left(\eta n_P\right)^{1/\theta}\left(\left[\left(n_A\right)^{1/\psi}\left(C_{A,t}^C\right)^{(\psi-1)/\psi}\right.\right.\right. \\
& \left.\left.\left.+\left(1-n_A\right)^{1/\psi}\left(C_{B,t}^C\right)^{(\psi-1)/\psi}\right]^{\psi/(\psi-1)}\right)^{(\theta-1)/\theta} \\
& +\left[\eta\left(1-n_P\right)\right]^{1/\theta}\left(C_{C,t}^C\right)^{(\theta-1)/\theta}\Bigg]^{\theta/(\theta-1)}.
\end{aligned} \tag{4.8}
$$

The consumption indexes for the three countries *i, i=A,B,C,* are given by

$$
C_{A,t}^i = \left[\left(\frac{1}{n_P n_A}\right)^{1/\phi}\int_0^{n_P n_A}\left(C_{A,t}^i\left(z\right)\right)^{\phi-1/\phi}dz\right]^{\phi/(\phi-1)} \tag{4.9}
$$

$$
C_{B,t}^i = \left[\left(\frac{1}{n_P\left(1-n_A\right)}\right)^{1/\phi}\int_{n_P n_A}^{n_P}\left(C_{B,t}^i\left(z\right)\right)^{\phi-1/\phi}dz\right]^{\phi/(\phi-1)} \tag{4.10}
$$

$$
C_{C,t}^i = \left[\left(\frac{1}{1-n_P}\right)^{1/\phi}\int_{n_P}^{1}\left(C_{C,t}^i\left(z\right)\right)^{\phi-1/\phi}dz\right]^{\phi/(\phi-1)}, \tag{4.11}
$$

where the subscript refers to the country of production and the superscript refers to the country of consumption. The elasticity of substitution for goods of different brands *z,* i.e., "varieties" is given by ϕ.

Prices

The utility-based price indexes are defined analogously to the consumption basket and are therefore broken down into the overall index

$$P_t^i = \left[n_P\left(P_{P,t}^i\right)^{1-\theta} + \left(1-n_P\right)\left(P_{C,t}^i\right)^{1-\theta}\right]^{1/(1-\theta)} \tag{4.12}$$

and the sub-index for prices of goods from the two periphery countries

$$P_{P,t}^i = \left[n_A\left(P_{A,t}^i\right)^{1-\psi} + \left(1-n_A\right)\left(P_{B,t}^i\right)^{1-\psi}\right]^{1/(1-\psi)}. \tag{4.13}$$

As with the consumption basket, introducing home bias renders the consumer price index for each country

$$\begin{aligned} P_t^A = {} & \left[\left[1-\eta\left(1-n_P\right)\right]\left(\left[\left[1-\eta\left(1-n_A\right)\right]\left(P_{A,t}^A\right)^{1-\psi} + \eta\left(1-n_A\right)\left(P_{B,t}^A\right)^{1-\psi}\right]^{1/(1-\psi)}\right)^{1-\theta}\right. \\ & \left. + \eta\left(1-n_P\right)\left(P_{C,t}^A\right)^{1-\theta}\right]^{1/(1-\theta)} \end{aligned} \tag{4.14}$$

$$\begin{aligned} P_t^B = {} & \left[\left[1-\eta\left(1-n_P\right)\right]\left(\left[\eta n_A\left(P_{A,t}^B\right)^{1-\psi} + \left(1-\eta n_A\right)\left(P_{B,t}^B\right)^{1-\psi}\right]^{1/(1-\psi)}\right)^{1-\theta}\right. \\ & \left. + \eta\left(1-n_P\right)\left(P_{C,t}^B\right)^{1-\theta}\right]^{1/(1-\theta)} \end{aligned} \tag{4.15}$$

$$P_t^C = \left[\eta n_P\left(\left[n_A\left(P_{A,t}^C\right)^{1-\psi} + \left(1-n_A\right)\left(P_{B,t}^C\right)^{1-\psi}\right]^{1/(1-\psi)}\right)^{1-\theta} + \left(1-\eta n_P\right)\left(P_{C,t}^C\right)^{1-\theta}\right]^{1/(1-\theta)} \tag{4.16}$$

with

$$P_{A,t}^i = \left[\frac{1}{n_P n_A}\int_0^{n_P n_A}\left(P_{A,t}^i(z)\right)^{1-\phi} dz\right]^{1/(1-\phi)} \tag{4.17}$$

$$P_{B,t}^i = \left[\frac{1}{n_P\left(1-n_A\right)}\int_{n_P n_A}^{n_P}\left(P_{B,t}^i(z)\right)^{1-\phi} dz\right]^{1/(1-\phi)} \tag{4.18}$$

$$P_{C,t}^i = \left[\frac{1}{1-n_P}\int_{n_P}^{1}\left(P_{C,t}^i(z)\right)^{1-\phi} dz\right]^{1/(1-\phi)} \tag{4.19}$$

where $P_{i,t}^i$ is the price index of domestic goods or producer price index (PPI) in country *i*.

Exchange Rates

The terms of trade between country i and country i' are defined as the relative price of imports

$$ToT^{i}_{i',t} = \frac{P^{i}_{i',t}}{P^{i}_{i,t}}, \tag{4.20}$$

i.e., the price of imported goods (produced in country i') in home (country i) currency relative to the price of home goods.

For convenience, we define the following CPI-PPI ratios, $g_{i,t}(ToT)$, for each country i. The expression links CPI prices and domestic prices to the terms of trade (Faia and Monacelli 2004, 2007)

$$\begin{aligned}\frac{P^{A}_{t}}{P^{A}_{A,t}} = &\left[\left[1-\eta\left(1-n_{P}\right)\right]\left[\left(\left[1-\eta\left(1-n_{A}\right)\right]+\eta\left(1-n_{A}\right)\left(ToT^{A}_{B,t}\right)^{1-\psi}\right)^{1/(1-\psi)}\right]^{1-\theta}\right. \\ &\left.+\eta\left(1-n_{P}\right)\left(ToT^{A}_{C,t}\right)^{1-\theta}\right]^{1/(1-\theta)} \equiv g_{A,t}\left(ToT\right)\end{aligned} \tag{4.21}$$

$$\begin{aligned}\frac{P^{B}_{t}}{P^{B}_{B,t}} = &\left[\left[1-\eta\left(1-n_{P}\right)\right]\left[\left(\eta\, n_{A}\left(ToT^{B}_{A,t}\right)^{1-\psi}+\left(1-\eta\, n_{A}\right)\right)^{1/(1-\psi)}\right]^{1-\theta}\right. \\ &\left.+\eta\left(1-n_{P}\right)\left(ToT^{B}_{C,t}\right)^{1-\theta}\right]^{1/(1-\theta)} \equiv g_{B,t}\left(ToT\right)\end{aligned} \tag{4.22}$$

$$\begin{aligned}\frac{P^{C}_{t}}{P^{C}_{C,t}} = &\left[\eta n_{P}\left[\left(n_{A}\left(ToT^{C}_{A,t}\right)^{1-\psi}+\left(1-n_{A}\right)\left(ToT^{C}_{A,t}\right)^{1-\psi}\right)^{1/(1-\psi)}\right]^{1-\theta}\right. \\ &\left.+\left(1-\eta n_{P}\right)\right]^{1/(1-\theta)} \equiv g_{C,t}\left(ToT\right).\end{aligned} \tag{4.23}$$

Households maximize their lifetime utility subject to the period budget constraint,

$$\sum_{h^{t+1}} Q_{t,t+1}\frac{1}{S^{C}_{i,t}}B^{i}_{t+1} + P^{i}_{t}\,C^{i}_{t} = \frac{1}{S^{C}_{i,t}}B^{i}_{t} + W^{i}_{t}\,L^{i}_{t} + \Psi^{i}_{t}, \tag{4.24}$$

where nominal bonds, B^{i}_{t}, are state-contingent claims denominated only in country C's currency, $S^{i'}_{i,t}$ is the nominal exchange rate defined as the price of currency i' in currency i, $Q_{t,t+1} \equiv Q\left(h^{t+1}\middle|h^{t}\right)$ is the period-t price of one unit of country C's currency in state h^{t+1} divided

by the probability of occurrence of that state,[44] W_t^i is the nominal wage; L_t^i are hours worked, and Ψ_t^i are nominal profits of the domestic monopolistic firms, which shares are owned by domestic residents. Complete markets ensure stationarity of bonds and consumption (Schmitt-Grohé and Uribe 2003).

Deriving Demand Functions

First, we derive the individual demand functions for good *z* as well as the aggregate demand functions from the intratemporal cost minimization of the household's consumption across different countries *i* and brands *z*. The detailed optimization is given in the technical appendix.

Individual demand functions for country *i, i = A, B, C:*

$$C_{A,t}^i(z) = \left(\frac{P_{A,t}^i(z)}{P_{A,t}^i}\right)^{-\phi} (n_P n_A)^{-1} C_{A,t}^i \tag{4.25}$$

$$C_{B,t}^i(z) = \left(\frac{P_{B,t}^i(z)}{P_{B,t}^i}\right)^{-\phi} (n_P(1-n_A))^{-1} C_{B,t}^i \tag{4.26}$$

$$C_{C,t}^i(z) = \left(\frac{P_{C,t}^i(z)}{P_{C,t}^i}\right)^{-\phi} (1-n_P)^{-1} C_{C,t}^i . \tag{4.27}$$

Aggregate demand functions for country *i, i = A, B, C:*

$$C_{A,t}^i = v_A^i \left(\frac{P_{A,t}^i}{P_{P,t}^i}\right)^{-\psi} \left(\frac{P_{P,t}^i}{P_t^i}\right)^{-\theta} C_t^i , \tag{4.28}$$

$$C_{B,t}^i = v_B^i \left(\frac{P_{B,t}^i}{P_{P,t}^i}\right)^{-\psi} \left(\frac{P_{P,t}^i}{P_t^i}\right)^{-\theta} C_t^i , \tag{4.29}$$

$$C_{C,t}^i = v_C^i \left(\frac{P_{C,t}^i}{P_t^i}\right)^{-\theta} C_t^i , \tag{4.30}$$

where the trade shares are given by Table 4.2.

---

[44] Each asset in the portfolio B_{t+1}^i pays one unit of country *C*'s currency at time *t*+1 and in state h^{t+1}.

Table 4.2: Trade Shares for Countries *A, B,* and *C*

Country *A*	Country *B*	Country *C*
$v_A^A = [1-\eta(1-n_P)][1-\eta(1-n_A)]$	$v_A^B = [1-\eta(1-n_P)]\eta n_A$	$v_A^C = \eta n_P n_A$
$v_B^A = [1-\eta(1-n_P)]\eta(1-n_A)$	$v_B^B = [1-\eta(1-n_P)](1-\eta n_A)$	$v_B^C = \eta n_P (1-n_A)$
$v_C^A = \eta(1-n_P)$	$v_C^B = \eta(1-n_P)$	$v_C^C = 1-\eta n_P$

Second, we solve the intertemporal optimization by maximizing the lifetime utility function (4.1) subject to the budget constraint (4.24). Combining the first order conditions for consumption and labor determines the consumption labor trade-off and, thereby, the nominal wage

$$W_t^i = \frac{-\partial U_t^i / \partial C_t^i}{\partial U_t^i / \partial L_t^i} = \frac{\left(L_t^i\right)^\gamma}{\left(C_t^i\right)^{-\mu}} P_t^i . \tag{4.31}$$

Combining the first order conditions for consumption and bonds leads to the familiar Euler equation

$$\beta \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\mu} \frac{P_t^i}{P_{t+1}^i} = \frac{S_{i,t+1}^C}{S_{i,t}^C} Q_{t,t+1} . \tag{4.32}$$

Taking expectations conditional on the information available in period t and defining the gross nominal interest rate, R_t^i, on a riskless one-period bond in country i as

$$R_t^i \equiv \frac{1}{E_t\left\{\left(S_{i,t+1}^C / S_{i,t}^C\right) Q_{t,t+1}\right\}}, \tag{4.33}$$

we rewrite the Euler equation as

$$\beta \, R_t^i \, E_t \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\mu} \frac{P_t^i}{P_{t+1}^i} \right\} = 1. \tag{4.34}$$

The Euler equation (4.34) leads to the dynamic IS curve, where today's consumption depends on tomorrow's consumption and the interest rate.

International Risk Sharing

The three countries are linked by international risk sharing. Combining the Euler equation (4.32) for two countries i and i'

$$\beta\left(\frac{C_{t+1}^{i}}{C_{t}^{i}}\right)^{-\mu}\frac{P_{t}^{i}}{P_{t+1}^{i}}=\beta\left(\frac{C_{t+1}^{i'}}{C_{t}^{i'}}\right)^{-\mu}\frac{S_{i',t}^{i}\,P_{t}^{i'}}{S_{i',t+1}^{i}\,P_{t+1}^{i'}} \tag{4.35}$$

and defining the real exchange rate as

$$RER_{i',t}^{i}\equiv\frac{S_{i',t}^{i}\cdot P_{t}^{i'}}{P_{t}^{i}} \tag{4.36}$$

the following risk sharing condition holds in every period

$$C_{t}^{i}=C_{t}^{i'}\cdot\left(RER_{i',t}^{i}\right)^{1/\mu}. \tag{4.37}$$

The real exchange rate may be written in terms of trade

$$RER_{i',t}^{i}=ToT_{i',t}^{i}\cdot g_{i',t}\left(ToT\right)\cdot g_{i,t}\left(ToT\right)^{-1}. \tag{4.38}$$

In a three country model we have two risk sharing conditions expressed in terms of trade

$$C_{t}^{A}=C_{t}^{B}\left(ToT_{B,t}^{A}\right)^{1/\mu}\,g_{B,t}\left(ToT\right)^{1/\mu}\,g_{A,t}\left(ToT\right)^{-1/\mu} \tag{4.39}$$

and

$$C_{t}^{A}=C_{t}^{C}\left(ToT_{C,t}^{A}\right)^{1/\mu}\,g_{C,t}\left(ToT\right)^{1/\mu}\,g_{A,t}\left(ToT\right)^{-1/\mu}. \tag{4.40}$$

If purchasing power parity holds, which is the case if there is no home bias in consumption, the real exchange rate in (4.37) is one and international risk sharing implies perfect consumption smoothing across countries, i.e.,

$$C_{t}^{i}=C_{t}^{i'}. \tag{4.41}$$

4.2.2 Firms

Differences in production and thereby in economic integration along the production chain are modeled via the consumption basket to keep the supply side of the model simple. We also

abstract from capital as labor is the only input factor. Each monopolistically competitive firm *z* in country *i* produces

$$Y_t^i(z) = F_t^i \, L_t^i(z), \tag{4.42}$$

where $Y_t^i(z)$ is the output of firm *z* in period *t* and F_t^i is country specific productivity that follows a first order autoregressive process

$$F_t^i = \rho^i F_{t-1}^i + \varepsilon_t^i. \tag{4.43}$$

Firms maximize profits subject to the production function, demand, and staggered producer currency pricing à la Calvo (1983), where α is the degree of price stickiness. The optimal price of variety *z* is

$$\widetilde{P}_{i,t}^i(z) = \frac{\phi}{\phi - 1} \cdot \frac{E_t \sum_{\tau=0}^{\infty} \alpha^\tau \; \rho_{t,t+\tau} \; Y_{t+\tau}^i(z) \, MC_{t+\tau}^i(z)}{E_t \sum_{\tau=0}^{\infty} \alpha^\tau \; \rho_{t,t+\tau} \; Y_{t+\tau}^i(z)}, \tag{4.44}$$

where $MC_t^i(z) = W_t^i / F_t^i$ are nominal marginal costs given by the Lagrange multiplier associated with the demand constraint, $Y_t^i(z)$ is total ("world") output of country *i*'s firm *z,* and $\phi/(\phi - 1)$ is the mark-up of prices over marginal costs. The technical appendix gives the detailed derivation of this optimality condition and how it leads to the New Keynesian Phillips curve.

Market Clearing

Market clearing requires the demand and supply of goods to be equalized in each of the three countries

$$\begin{aligned} Y_t^A = {} & v_A^A \left(ToT_{P,t}^A\right)^{\psi-\theta} \left(g_A(ToT)\right)^\theta C_t^A \\ & + v_B^A \left(ToT_{P,t}^B\right)^{\psi-\theta} \left(ToT_{B,t}^A\right)^{\psi} \left(g_B(ToT)\right)^\theta C_t^B \\ & + v_C^A \left(ToT_{P,t}^C\right)^{\psi-\theta} \left(ToT_{C,t}^A\right)^{\psi} \left(g_C(ToT)\right)^\theta C_t^C \end{aligned} \tag{4.45}$$

$$\begin{aligned} Y_t^B = {} & v_A^B \left(ToT_{P,t}^A\right)^{\psi-\theta} \left(ToT_{A,t}^B\right)^{\psi} \left(g_A(ToT)\right)^\theta C_t^A \\ & + v_B^B \left(ToT_{P,t}^B\right)^{\psi-\theta} \left(g_B(ToT)\right)^\theta C_t^B \\ & + v_C^B \left(ToT_{P,t}^C\right)^{\psi-\theta} \left(ToT_{C,t}^B\right)^{\psi} \left(g_C(ToT)\right)^\theta C_t^C \end{aligned} \tag{4.46}$$

$$
\begin{aligned}
Y_t^C &= v_A^C \left(ToT_{C,t}^A\right)^{-\theta} \left(g_A\left(ToT\right)\right)^{\theta} C_t^A \\
&+ v_B^C \left(ToT_{C,t}^B\right)^{-\theta} \left(g_B\left(ToT\right)\right)^{\theta} C_t^B \\
&+ v_C^C \left(g_C\left(ToT\right)\right)^{\theta} C_t^C .
\end{aligned} \tag{4.47}
$$

4.2.3 Solving the Model

The model cannot be solved analytically. We follow the strategy of linear approximation of the model's nonlinear equations established in the real business cycle literature (e.g., Campbell 1994, Uhlig 1999). Kydland and Prescott (1982) proposed taking a linear-quadratic approximation to the true model around its steady state. This method is valid in the special case of Kydland and Prescott (1982) but may not be generalized. Kim and Kim (2007) demonstrate in a small model that loglinearizing the model before deriving the optimality conditions (first order conditions) leads to inaccurate results. Such a "naïve" (Benigno and Woodford 2007) linear-quadratic problem is not correct up to first order. King, Plosser, and Rebelo (1988, 2002) use instead a loglinear-quadratic approximation of the model's (nonlinear) optimality conditions.

However, the limitations of this method need to be kept in mind. Ascari and Merkl (2007) show the shortcomings of loglinearizations in a model with a disinflation experiment. The economy moves from one steady state with high inflation to a new steady state with low inflation. Especially with respect to inflation of the starting point (the first steady state) of the disinflation experiment, the economic results may differ dramatically.

Summing up, loglinear approximations are suitable if (i) the nonlinearities are not essential (in contrast to models in finance where risk matters) and (ii) the model analyzes only small deviations from the steady state. In our model, we derived the optimality conditions for the household in Section 4.2.1 and for the firm in Section 4.2.2, i.e., before taking the loglinear approximation. Next, we loglinearize the model around a steady state with no inflation.

Like a closed economy, the model's core equations are an IS curve, a Phillips curve and a monetary policy rule for each country. In an open economy, we need additionally goods market clearing for each country as consumption may deviate from output and, to link the three economies, two international risk sharing conditions. For convenience, we use two

shorthand notations. First, the real marginal costs, $\hat{mc}_t^i$, and, second, the frequently used CPI-PPI ratio, $\hat{g}_{i,t}(ToT)$. To analyze monetary policy under ERM 2, we also back out CPI inflation, $\hat{\pi}_t^i$, and the change in the nominal exchange rate, $\Delta\hat{S}_{i',t}^i$, where an increase corresponds to a change in the rate of depreciation of the currency in country *i*. Country *A* is the accession country. Equations of the loglinearized model for country *A*:

(4.48) $$\hat{C}_t^A = \hat{C}_{t+1}^A - \frac{1}{\mu}\hat{R}_t^A + \frac{1}{\mu}\hat{\pi}_{A,t+1}^A + \frac{v_B^A}{\mu}\hat{ToT}_{B,t+1}^A - \frac{v_B^A}{\mu}\hat{ToT}_{B,t}^A + \frac{v_C^A}{\mu}\hat{ToT}_{C,t+1}^A - \frac{v_C^A}{\mu}\hat{ToT}_{C,t}^A$$

(4.49) $$\hat{\pi}_{A,t}^A = \beta\,\hat{\pi}_{A,t+1}^A + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\,\hat{mc}_t^A$$

(4.50) $$\hat{mc}_t^A = \mu\hat{C}_t^A + \gamma\hat{Y}_t^A - (1+\gamma)\hat{F}_t^A + \hat{g}_{A,t}(ToT)$$

(4.51) $$\hat{g}_{A,t}(ToT) = v_B^A\hat{ToT}_{B,t}^A + v_C^A\hat{ToT}_{C,t}^A$$

(4.52) $$\hat{R}_t^A = \varphi_\pi\,\hat{\pi}_t^A + \varphi_Y\,\hat{Y}_t^A + \varphi_S\,\Delta\hat{S}_{C,t}^A$$

(4.53)
$$\begin{aligned}\hat{Y}_t^A = {} & v_A^A\hat{C}_t^A + \theta v_A^A\,\hat{g}_{A,t}(ToT) - (\psi-\theta)v_B^A\left[1-\eta(1-n_A)\right]\hat{ToT}_{B,t}^A \\ & + v_B^A\hat{C}_t^B + \theta v_B^A\,\hat{g}_{B,t}(ToT) + (\psi-\theta)v_B^A\,\eta\,n_A\,\hat{ToT}_{A,t}^B + \psi v_B^A\,\hat{ToT}_{B,t}^A \\ & + v_C^A\hat{C}_t^C + \theta v_C^A\,\hat{g}_{C,t}(ToT) + (\psi-\theta)v_C^A\,n_A\,\hat{ToT}_{A,t}^C \\ & + (\psi-\theta)v_C^A(1-n_A)\hat{ToT}_{B,t}^C + \psi v_C^A\,\hat{ToT}_{C,t}^A\end{aligned}$$

(4.54) $$\hat{\pi}_t^A = \hat{\pi}_{A,t}^A + v_B^A\hat{ToT}_{B,t}^A - v_B^A\hat{ToT}_{B,t-1}^A + v_C^A\hat{ToT}_{C,t}^A - v_C^A\hat{ToT}_{C,t-1}^A$$

(4.55) $$\Delta\hat{S}_{C,t}^A = \hat{ToT}_{C,t}^A - \hat{ToT}_{C,t-1}^A + \hat{\pi}_{A,t}^A - \hat{\pi}_{C,t}^C$$

Consumption in this economy is determined by expected consumption, the interest rate, expected domestic inflation as well as expected changes in the terms of trade between country *A* and countries *B* and *C*, the dynamic IS curve (4.48). Inflation, in turn, is determined by the forward-looking Phillips curve (4.49), which reflects the impact of marginal costs on inflation. To guarantee determinacy the monetary policy rule (4.52) needs to respond to inflation by more than one to one and thereby fulfill the Taylor principle (Woodford 2003b). A productivity shock for country *A* would reduce marginal costs (4.50) and thereby reduce inflation. The central bank would react to lower inflation with an interest

rate cut. Consumers would respond to a lower interest rate with increases in today's consumption. In a closed economy, the changes in consumption would lead to equal changes in output. In an open economy, the lower interest rate improves also the terms of trade and thereby the increase in domestic output is shared with the rest of the world (goods market clearing, equation 4.53).

The equations for countries *B* and *C* mirror those for country *A* except for the international risk sharing condition (equations 4.56 and 4.57). In a three-country model, we need two risk sharing conditions to link financial markets

$$\hat{C}_t^A = \hat{C}_t^B + \frac{1}{\mu}\hat{T}oT_{B,t}^A + \frac{1}{\mu}g_{B,t}(ToT) - \frac{1}{\mu}g_{A,t}(ToT) \tag{4.56}$$

$$\hat{C}_t^A = \hat{C}_t^C + \frac{1}{\mu}\hat{T}oT_{C,t}^A + \frac{1}{\mu}g_{C,t}(ToT) - \frac{1}{\mu}g_{A,t}(ToT). \tag{4.57}$$

Calibration

The numerical solution requires calibrating the structural parameters of the model (Table 4.3). Most parameters are standard in the real business cycle literature. The discount factor β equals 0.99, so that the steady state annual real interest rate is 4 percent. The period utility function has a risk aversion of 2 to depart from the case of logarithmic utility[45] and an inverse of the labor supply of 3. The Calvo price staggering parameter is set to 0.75, implying an average frequency of price adjustment of four quarters (Galí 2003).

In the three-country model, we explicitly consider different types of production sharing. The elasticity of substitution between goods from two countries follows Backus, Kehoe and Kydland (1995). We take this conventional assumption for the elasticity between goods from the core country *C* and the two periphery countries *A* and *B* and set θ equal to 1.5. Within the two periphery countries, we distinguish two scenarios. Countries *A* and *B* producing complements implies vertical integration along the production chain. We follow Burstein, Kurz and Tesar (2008) and model complements with a relatively low elasticity of substitution as ψ equals 0.05. Their calibration is based on survey data on outsourcing and offshoring in

[45] External shocks would not affect the home country under complete markets and logarithmic utility (Ghironi 2006: 429).

eastern Europe reported in Marin (2006). Horizontal production sharing implies countries producing substitutes. To model strong substitutes, we follow De Paoli (2006) and set ψ equal to 3.

As we focus on the qualitative results, we do not calibrate the model for specific countries. In the benchmark calibration, each country's size is 1/3 and there is no home bias in consumption, i.e., η equals 1. We relax this assumption and introduce home bias as a robustness check (η equals 0.5).

Table 4.3: Calibration of the Model

Structural Parameters		Calibration
α	Calvo staggering	0.75
β	subjective discount factor	0.99
γ	inverse of the elasticity of labor supply	3
μ	risk aversion	2
θ	elasticity of substitution between country *C* vs. *P*	1.5
ψ	elasticity of substitution between country *A* vs. *B*	0.05 (complements) 3 (substitutes)
ρ	autocorrelation of technology shock process	0.9
η	degree of openness	1 (completely open) 0.5 (with home bias)
n_P	size of country *C* vs. *P*	2/3
n_A	size of country *A* vs. *B*	0.5

4.3 Simulation Results

4.3.1 Technology Shock in a Two-Country Model

In order to provide a benchmark for the value added of this model, we briefly revisit a technology shock in a two-country model, where the two countries produce complements in one case and strong substitutes in the other case.[46] We analyze the impulse responses with respect to consumption, output, and domestic inflation. The technology shock process is modeled by a first order autoregressive process to productivity in country *B*

[46] In this two-country example both central banks target domestic inflation.

(4.58) $$\hat{F}_t^B = \rho^B \hat{F}_{t-1}^B + \varepsilon_t^B,$$

where the persistence parameter ρ^B is set to 0.9 (see Faia 2007).

As already pointed out by Mundell (1961), monetary policy faces difficulties of dealing with asymmetric shocks in a monetary union. If the two countries produce complements to each other, output would increase in both countries (Figure 4.1, Table 4.4). The technology shock, however, reduces inflation only in country *B.* In country *A,* higher consumption and a higher production increase marginal costs (4.50) and thereby domestic inflation increases (4.49). Thus, the inflation rates in the two countries would be negatively correlated. If, in contrast, the two countries produce strong substitutes, output would be negatively correlated (Figure 4.2). The technology shock would again reduce the price of products from country *B*. The consumers in country *A* would switch from their own products to the cheaper imports from country *B*. Consequently, output would fall only in country *A*, but inflation would fall in both countries. For both cases, we may distinguish between a central bank of the monetary union that is dominated by one country and a central bank of the union that targets the average of the two countries. Obviously, given the negative correlations in either inflation or output, a monetary policy in favor of one country would be inadequate for the other country. If the central bank bases its decisions on the union-wide aggregates, the negative correlations would partly offset each other and none of the two countries would benefit from an appropriate monetary policy.

Table 4.4: Correlation Coefficients of Inflation and Output in a Two-Country Model

	Inflation	Output
Complements	-0.24	0.69
Substitutes	0.07	-1.10

Figure 4.1: Technology Shock in a Two-Country Model, Complements

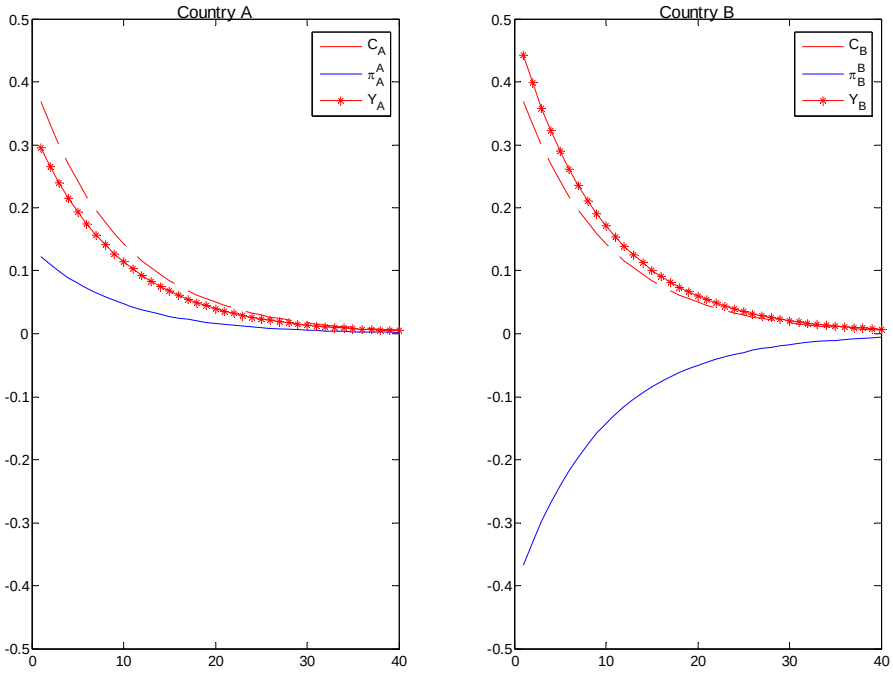
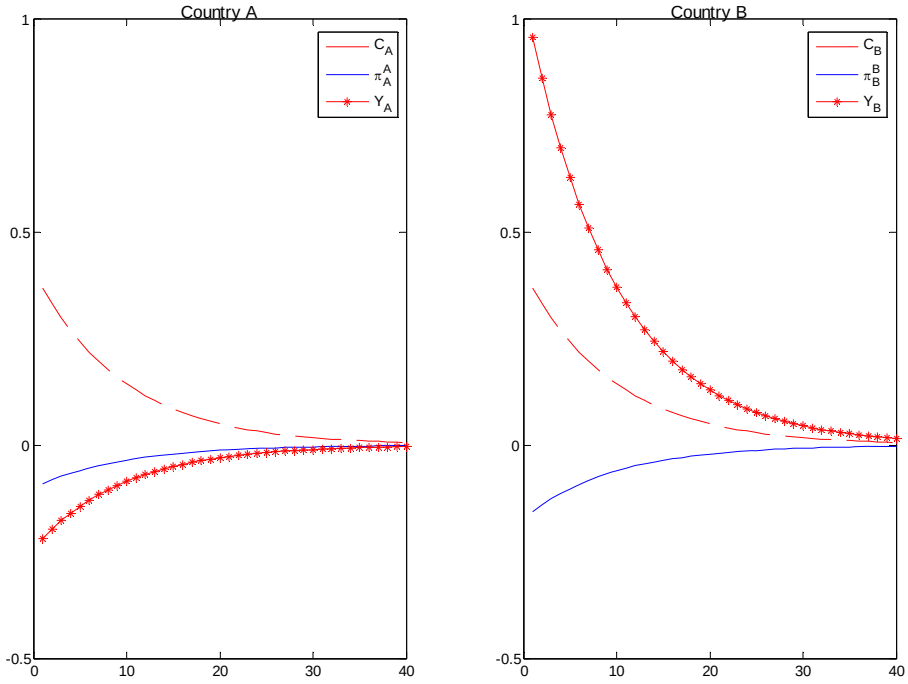


Figure 4.2: Technology Shock in a Two-Country Model, Substitutes



4.3.2 Technology Shock in a Three-Country Model

The three-country model with a nested consumption basket studies the transmission of a technology shock in the third country (country *B*) and its implications for monetary policy in the accession country (country *A*).[47] The model addresses the run up to a monetary union as the central bank in country *A* may have to target the exchange rate vis-à-vis the anchor country *C*. Further, the model emphasizes the implications of asymmetric production sharing among the three countries. Countries *A* and *C* are not affected by the third-country shock in the same way. Thus, under fully flexible exchange rates, the exchange rate between countries *A* and *C* would also bear part of the adjustment. In particular, this model distinguishes between countries *A* and *B* producing complements and strong substitutes.

We evaluate the effects of eight different simple monetary policy rules (4.52) for country *A,* the euro area accession country. Rules 1 and 2 in Table 4.5 stand for strict inflation targeting, where rule 1 targets the domestic (i.e., core) inflation rate and rule 2 targets the inflation of the consumer price index (CPI). Their respective Taylor rule counterparts are rules 3 and 4, including output targeting. Rules 5 to 8 add an exchange rate term to rules 1 to 4 in order to investigate the implications of exchange rate targeting under ERM 2. For the impulse responses presented in the graphs, we focus on the traditional CPI inflation Taylor rule (rule 4) and adding an exchange rate term for ERM 2 (rule 8).

Countries *B* and *C* conduct strict domestic inflation targeting, $\varphi_{\pi_i^i}$ equals 1.5. The central bank in country *B* neither mitigates the effect of the technology shock on output nor second-round effects from abroad showing up in CPI inflation. Otherwise, positive weights for output targeting in the monetary policy rule would reduce the shock in country *B* and, hence, also its impact on the other countries. Countries *B* and *C* are assumed to produce substitutes throughout the simulations. Hence, their output reactions are negatively correlated. Again, a positive weight for output targeting in the monetary policy rule of country *C* would dampen

[47] In principle, the model could be extended to the analysis of monetary shocks. Such a shock could be implemented by a reduction in the third country's interest rate, which could be interpreted as a competitive devaluation or contagion following a financial crisis. The evaluation of simple monetary policy rules could give insights on the strategic costs of ERM 2 and giving up a floating exchange rate for the accession country. We leave this for future research and focus on a third-country technology shock.

the (inverse) shock in the anchor currency country. Any weight on output targeting in the monetary policy rules of countries *B* and *C* would lead to biased results with respect to the analysis of monetary policy in country *A*.

Table 4.5: Simple Monetary Policy Rules for Country *A*

Monetary Policy Rule		$\varphi_{\pi_i^i}$	φ_{π^i}	φ_Y	φ_S
Rule 1	domestic inflation targeting	1.5	–	–	–
Rule 2	CPI inflation targeting	–	1.5	–	–
Rule 3	domestic inflation Taylor rule	1.5	–	0.5	–
Rule 4	CPI inflation Taylor rule	–	1.5	0.5	–
Rule 5	domestic inflation targeting under ERM 2	1.5	–	–	0.5
Rule 6	CPI inflation targeting under ERM 2	–	1.5	–	0.5
Rule 7	domestic inflation Taylor rule under ERM 2	1.5	–	0.5	0.5
Rule 8	CPI inflation Taylor rule under ERM 2	–	1.5	0.5	0.5

As standard in the literature, we evaluate the different monetary policy rules based on the loss function of the central bank. The central bank wishes to minimize volatility in output and inflation (Monacelli 2005).[48] The loss function is defined as

$$L_0^{CB} = E_0 \sum_{t=0}^{\infty} \delta^t \left(\lambda_\pi \hat{\pi}_t^2 + \lambda_Y \hat{Y}_t^2 \right), \tag{4.59}$$

where $0 < \delta < 1$ is a discount factor. Woodford (2003b) shows that such a loss function is consistent with maximizing a social utility function in a closed economy setup. Both, society as well as the Maastricht criteria specify inflation measured by the CPI. As weights in the loss function we take $\lambda_\pi = \lambda_Y = 0.5$.

However, although such a loss function is meaningful as a benchmark for the evaluation of monetary policy, it would be incomplete for the case of regional monetary integration. If the central bank in country *A* would not give any positive weight to exchange rate smoothing, it would be difficult to establish the case for integrating into EMU. Hence, we assume that the central bank minimizes the volatility in the exchange rate as an additional objective. In case

[48] In principle, it would be possible to calculate welfare as a second-order approximation to the representative agent's utility function, see Benigno (2004), Pappa (2004), De Paoli (2006), Lipińska (2006), and Liu and Pappa (2008). We take the perspective of the policy maker and focus on the central bank's loss function.

the central bank operates under ERM 2, the loss function with a positive weight on exchange rate smoothing could be defined as

$$L_0^{ERM\,2} = E_0 \sum_{t=0}^{\infty} \delta^t \left(\lambda_\pi \hat{\pi}_t^2 + \lambda_Y \hat{Y}_t^2 + \lambda_S \Delta \hat{S}_t^2 \right), \tag{4.60}$$

where $\lambda_\pi = 1$ and $\lambda_Y = \lambda_S = 0.5$. The value of the intertemporal loss functions approaches the unconditional mean of the period loss functions as $\delta \to 1$, which equals the sum of the unconditional variances of the policy objectives.

Thus, the two loss functions for country *A* are

$$\textit{Loss CB:} \quad 0.5 \left(\pi^A \right)^2 + 0.5 \left(\hat{Y}^A \right)^2 \tag{4.61}$$

$$\textit{Loss ERM 2:} \quad \left(\pi^A \right)^2 + 0.5 \left(\hat{Y}^A \right)^2 + 0.5 \left(\Delta \hat{S}_C^A \right)^2. \tag{4.62}$$

Technology Shock I: Countries *A* and *B* Producing Complements

We start by analyzing the case where countries *A* and *B*, the two periphery countries, produce complements, while the periphery goods and country *C*'s goods are substitutes. A common feature for all countries stems from international risk sharing. Consequently, in the absence of home bias in consumption, the technology shock is evenly distributed and leads to equal increases in consumption in the three countries (see equation 4.41).

First, we consider the impulse responses according to rule 4, when country *A* conducts a CPI inflation Taylor rule without exchange rate smoothing (Figure 4.3, left column). Following the technology shock in country *B*, output in that country increases. Country *A*, producing complements to country *B*, benefits by a significant increase in its output. Country *C* in contrast looses output as its consumers substitute home goods by the products from the periphery countries *A* and *B,* taking also upward pressure from prices. As a consequence, inflation decreases in all countries. However, the decline in prices is more pronounced in country *A* than in country *C*. The inflation differential leads to an increased appreciation of country *A*'s currency vis-à-vis country *C*'s currency (Figure 4.3, right column, dashed line).

Second, we turn to the case when country *A*'s central bank operates under ERM 2 (Figure 4.3, middle column). The exchange rate between country *A* and *C* is not allowed to adjust to the full extent as the central bank in country *A* conducts a CPI inflation Taylor rule

augmented for exchange rate smoothing (rule 8). As implicitly uncovered interest rate parity holds in the model, the central bank of country *A* has to reduce its interest rate in order to counter the appreciation vis-à-vis the anchor currency, thereby driving up inflation at home. Table 4.6 shows how volatility in domestic as well as CPI inflation increases when comparing the rules without an exchange rate term (rules 1 to 4) with their ERM 2 counterparts (rules 5 to 8). Hence, with relatively stable exchange rates, domestic inflation in country *A* increases sharply, displaying the costs of an asymmetric shock under exchange rate targeting.

Figure 4.3: Technology Shock in a Three-Country Model, Complements

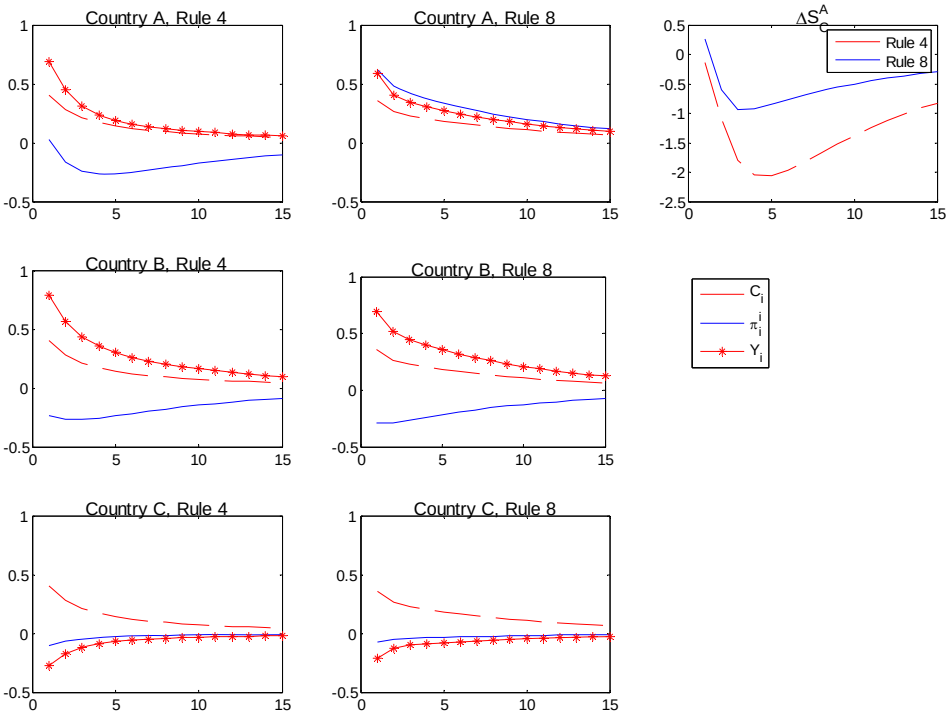


Table 4.6: Performance of Simple Monetary Policy Rules, Complements

Rule		Standard Deviation					Loss	
		π_A^A	π^A	Y^A	ΔS_C^A	C^A	CB	ERM 2
Rule 1	domestic inflation targeting	0.33	1.35	0.8	4.59	0.56	1.23	12.67
Rule 2	CPI inflation targeting	0.46	0.37	1.11	4.71	0.70	0.68	11.85
Rule 3	domestic inflation Taylor rule	0.19	1.57	0.70	5.76	0.52	1.49	19.34
Rule 4	CPI inflation Taylor rule	0.77	1.03	0.99	5.96	0.64	1.02	19.33
Rule 5	domestic inflation ERM 2	1.65	1.86	1.01	1.69	0.67	2.24	5.40
Rule 6	CPI inflation ERM 2	1.51	1.43	1.10	1.91	0.71	1.63	4.48
Rule 7	domestic Taylor ERM 2	1.39	1.71	0.96	2.28	0.65	1.92	5.98
Rule 8	CPI Taylor ERM 2	1.24	1.22	1.05	2.44	0.68	1.30	5.02

Table 4.7: Ratio of Loss for Joining ERM 2, Complements

Ratio	Rule	Loss CB	Loss ERM 2
Rule 5 / Rule 1	domestic inflation targeting	1.82	0.43
Rule 6 / Rule 2	CPI inflation targeting	2.39	0.38
Rule 7 / Rule 3	domestic inflation Taylor rule	1.29	0.31
Rule 8 / Rule 4	CPI inflation Taylor rule	1.28	0.26

To analyze the relative change in the central bank loss of joining ERM 2, we construct the ratio of the loss for each monetary policy rule with an exchange rate term over its counterpart without that term (Table 4.7). Moving to ERM 2 more than doubles the loss for the traditional central bank loss function (Loss CB) as it increases up to 2.39-times. For the two Taylor rules 3 and 4 moving to their ERM 2 counterparts still increases the loss but by less than 30 percent. If the central bank adapts ERM 2 as its policy targets (Loss ERM 2), the loss drops when adding the exchange rate to the policy rule to half or almost a quarter of the loss without exchange rate term (Table 4.7, last column).

Technology Shock II: Countries *A* and *B* Producing Substitutes

As it was in the case with countries *A* and *B* producing complements, the technology shock in country *B* increases consumption in all three countries by the same amount due to perfect international risk sharing when the two periphery countries produce strong substitutes (Figure 4.4). This time, however, the technology shock goes one to one into country *B*'s output. As not only country *A* but also country *C* produces substitutes to country *B,* output declines in both countries. Without a reaction of monetary policy to the shock, CPI inflation would decline in all three countries. However, country *A* conducting monetary policy with a CPI inflation Taylor rule (rule 4), the central bank reacts evenly to inflation and output

(Figure 4.4, left column). Table 4.8 shows how the volatility in domestic and CPI inflation as well as output falls when moving from strict inflation targeting (rules 1 and 2) to a Taylor rule (rules 3 and 4). As a result of the central bank lowering the interest rate to accommodate the decline in output, inflation in country *A* increases slightly. In the case of substitutes, the technology shock in country *B* affects only a little the exchange rate between countries *A* and *C*. The magnitude of the exchange rate reaction to the external shock is small with the peak of the devaluation at only 0.43 standard deviations compared to the peak of the appreciation at 2.07 standard deviations in the case of complements. Thus, moving to a monetary policy rule consistent with ERM 2, where the interest rate reacts also to movements in the exchange rate (rule 8), exhibits less changes in the impulse responses as seen before in the case of complements in Figure 4.3. Table 4.8 shows that volatility in the domestic and CPI inflation as well as output has to increase in order to reduce volatility in the exchange rate (rules 1 to 4 versus rules 5 to 8).

Figure 4.4: Technology Shock in a Three-Country Model, Substitutes

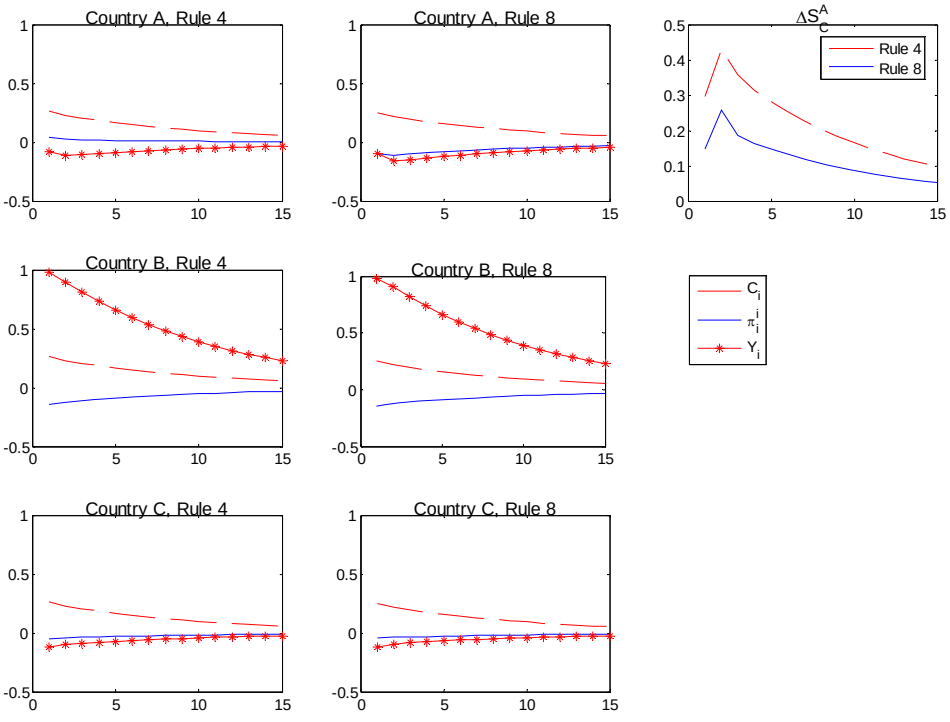


Table 4.8: Performance of Simple Monetary Policy Rules, Substitutes

Rule		Standard Deviation					Loss	
		π_A^A	π^A	Y^A	ΔS_C^A	C^A	CB	ERM 2
Rule 1	domestic inflation targeting	0.16	0.19	0.38	0.63	0.56	0.092	0.3104
Rule 2	CPI inflation targeting	0.18	0.20	0.35	0.65	0.57	0.081	0.3095
Rule 3	domestic inflation Taylor rule	0.08	0.10	0.30	0.96	0.58	0.050	0.5121
Rule 4	CPI inflation Taylor rule	0.07	0.06	0.28	0.95	0.59	0.042	0.4992
Rule 5	domestic inflation ERM 2	0.41	0.44	0.45	0.30	0.55	0.199	0.3403
Rule 6	CPI inflation ERM 2	0.43	0.45	0.43	0.31	0.55	0.194	0.3423
Rule 7	domestic Taylor ERM 2	0.26	0.28	0.40	0.51	0.56	0.121	0.2901
Rule 8	CPI Taylor ERM 2	0.27	0.29	0.39	0.51	0.56	0.117	0.2908

Table 4.9: Ratio of Loss for Joining ERM 2, Substitutes

Ratio	Rule	Loss CB	Loss ERM 2
Rule 5 / Rule 1	domestic inflation targeting	2.17	1.10
Rule 6 / Rule 2	CPI inflation targeting	2.40	1.11
Rule 7 / Rule 3	domestic inflation Taylor rule	2.41	0.57
Rule 8 / Rule 4	CPI inflation Taylor rule	2.79	0.58

Concerning the consequences for the loss function (Loss CB), adding the exchange rate more than doubles the loss for all four rules (Table 4.9). However, the loss for the classical CPI inflation Taylor rule increases 2.79-times when the exchange rate term is incorporated. For the strict inflation targeting rules (rules 1 and 2), the loss for an ERM 2 central bank (Loss ERM 2) is virtually alike with or without the exchange rate term as the ratio is close to 1. For the two Taylor rules (rules 3 and 4), the loss halves.

The evaluation of simple monetary policy rules leads us to the following conclusions (Table 4.10). For countries *A* and *B* producing complements the CPI inflation Taylor rule with exchange rate smoothing (rule 8), shows in the ranking a remarkably good performance for Loss ERM 2. Under Loss CB, this rule even outperforms the domestic inflation Taylor rule (rule 3). For countries *A* and *B* producing substitutes, each of the monetary policy rules without an exchange rate term performs better than the rules that are in line with ERM 2 under the traditional central bank loss function. This result does not hold when adjusting the central bank loss function to take into account exchange rate smoothing (Loss ERM 2). The last two columns of Table 4.10 serve as a robustness check in case the kind of production sharing is not known. Under a traditional central bank loss function (Loss CB) the best performance is given by rule 4, the traditional CPI inflation Taylor rule. In case the central

bank operates under ERM 2 (Loss ERM 2), the best results are achieved by the CPI inflation Taylor rule with exchange rate smoothing (rule 8).

Table 4.10: Ranking of Simple Monetary Policy Rules

Rule		Complements		Substitutes		Sum	
		Loss CB	Loss ERM 2	Loss CB	Loss ERM 2	Loss CB	Loss ERM 2
Rule 1	domestic inflation targeting	3	6	4	4	7	10
Rule 2	CPI inflation targeting	1	5	3	3	4	8
Rule 3	domestic inflation Taylor rule	5	8	2	8	7	16
Rule 4	CPI inflation Taylor rule	2	7	1	7	3	14
Rule 5	domestic inflation ERM 2	8	3	8	5	16	8
Rule 6	CPI inflation ERM 2	6	1	7	6	13	7
Rule 7	domestic Taylor ERM 2	7	4	6	1	13	5
Rule 8	CPI Taylor ERM 2	4	2	5	2	9	4

Robustness Checks

As a robustness check, we introduce home bias in consumption in all three countries by setting the parameter for openness, η, equal to 0.5. The home bias affects also the nested consumption basket as we could think of a bias not only towards home goods but also towards goods from the periphery countries generally. The altered trade shares are shown in Table 4.11. Country *A* trades not anymore as much as before with countries *B* and *C*. The technology shock in country *B* affects country *A* less, leading to strictly lower losses for the central bank. The ranking of the eight monetary policy rules alters only little in the case of complements and not at all for substitutes (Table 4.12 and Appendix A4.1).

Table 4.11: Trade Shares under Home Bias in Consumption

Country *A*	Country *B*	Country *C*
$v_A^A = 0.6250$	$v_A^B = 0.2083$	$v_A^C = 0.1667$
$v_B^A = 0.2083$	$v_B^B = 0.6250$	$v_B^C = 0.1667$
$v_C^A = 0.1667$	$v_C^B = 0.1667$	$v_C^C = 0.6667$

Note: Trade shares are based on relative country size $n_A = 0.5$ and $n_P = 2/3$ and degree of openness $\eta = 0.5$.

Table 4.12: Ranking of Simple Monetary Policy Rules, Home Bias

Rule		Complements		Substitutes		Sum	
		Loss CB	Loss ERM 2	Loss CB	Loss ERM 2	Loss CB	Loss ERM 2
Rule 1	domestic inflation targeting	2	5	4	4	6	9
Rule 2	CPI inflation targeting	1	6	3	3	4	9
Rule 3	domestic inflation Taylor rule	4	7	2	8	6	15
Rule 4	CPI inflation Taylor rule	3	8	1	7	4	15
Rule 5	domestic inflation ERM 2	8	3	8	5	16	8
Rule 6	CPI inflation ERM 2	7	1	7	6	14	7
Rule 7	domestic Taylor ERM 2	6	4	6	1	12	5
Rule 8	CPI Taylor ERM 2	5	2	5	2	10	4

4.3.3 Optimal Simple Rules in a Three-Country Model

In this section, we identify optimal monetary policy rules by minimizing the central bank loss function. The Matlab routine `fminsearch` is an unconstrained nonlinear optimization using the Simplex search method of Lagarias, Reeds, Wright, and Wright (1998). The method does not rely on numerical or analytical gradients and may therefore handle discontinuity. The search starts from initial values and may only give a local solution. We take the parameters of the respective simple monetary policy rules specified in Table 4.5 as starting values. As the method is unconstrained, it searches the entire parameter space and may find optimal coefficients minimizing the loss for implausible policy parameters.

To circumvent this problem, we follow Schmitt-Grohé and Uribe (2007) and conduct additionally a grid search.[49] Rules 3 to 6 are evaluated on a grid in 0.05 steps for $\varphi_{\pi_i^i}$ or φ_{π^i} from 1 to 3 and for φ_Y or φ_S from 0 to 1.51. The grid for rules 7 and 8 is in 0.1 steps for $\varphi_{\pi_i^i}$ or φ_{π^i} from 1 to 3 and for φ_Y or φ_S from 0 to 1.51. Our interpretation focuses on the optimal simple rules based on that grid search. We limit our attention on the traditional Taylor rule (rule 4) and its ERM 2 counterpart with an exchange rate term (rule 8).[50]

[49] Rules 1 and 2 are only one-dimensional and are therefore not evaluated on a grid. The density of the grid needs to be adjusted in the case of three parameters to avoid the well-known curse of dimensionality (Miranda and Fackler 2002).

[50] The results with home bias in consumption are extremely similar (Appendix Tables A4.5 to A4.8).

The traditional central bank loss function (Loss CB) minimizes the volatility in CPI inflation and output. All results in Tables 4.13 and 4.15, including those based on `fminsearch`, in general adhere to the Taylor principle (Woodford 2003b). Most of the time, there is no role for targeting the exchange rate as the respective coefficient is zero. Only in the case of complements, rules 6 and 8 find a positive but small coefficient for the exchange rate term (Table 4.13). In the analysis of simple monetary policy rules, we already saw that the exchange rate between countries *A* and *C* is hardly affected in the case of substitutes. However, in the case of complements, the exchange rate bears part of the macroeconomic adjustment. The exchange rate plays a more important role in the model with complements than in the model with substitutes. The analysis of optimal simple rules confirms the findings of Svensson (2003: 442), who states: "A first obvious problem for a Taylor-type rule, with or without interest-rate smoothing, is that, if there are other important state variables than inflation and the output gap, it will not be optimal." The numerical analysis reveals that the exchange rate approximates other important state variables of the model in the case of complements but not in the case of substitutes. Having an additional policy parameter available pays off as even a small exchange rate coefficient of 0.1 reduces the central bank loss to 0.64 down from 0.67 (Table 4.13, rules 4 and 8).

If the loss function captures the requirements of ERM 2 (Loss ERM 2) and therefore the central bank aims to additionally reduce volatility in the exchange rate, the monetary policy rules with the lowest loss include the exchange rate as a target. If the central bank includes an exchange rate in the monetary policy rule, the Taylor principle is occasionally violated (Tables 4.14 and 4.16). For a technology shock being the only disturbance, the optimal coefficients are in general lower for production sharing via complements than via substitutes. In the latter case, the exchange rate has the same weight as inflation (Table 4.16, rule 8). This result applies also with home bias in consumption (Appendix Table A4.8).

Table 4.13: Optimal Simple Rules, Complements, Loss CB

Rule	Method	$\varphi_{\pi_i^i}$	φ_{π^i}	φ_Y	φ_S	Loss CB	Variance	
							π^A	Y^A
Rule 1	Fminsearch	1.29565	–	–	–	1.219640	1.7629	0.6764
Rule 2	Fminsearch	–	2.01042	–	–	0.669571	0.0508	1.2883
Rule 3	Fminsearch	1.29552	–	8.98453e-005	–	1.219640	1.7629	0.6764
	Grid search	1.3	–	0	–	1.219650	1.7642	0.6752
Rule 4	Fminsearch	–	1.00253	-0.162455	–	0.646189	0.0510	1.2414
	Grid search	**–**	**2**	**0**	**–**	**0.669573**	**0.0517**	**1.2875**
Rule 5	Fminsearch	1.03148	–	–	-0.0280402	1.189650	1.7064	0.6729
	Grid search	1.3	–	–	0	1.219650	1.7642	0.6752
Rule 6	Fminsearch	–	1.15006	–	0.0268152	0.636567	0.1259	1.1472
	Grid search	–	1.6	–	0.05	0.645318	0.0692	1.2215
Rule 7	Fminsearch	1.02709	–	0.0222517	-0.0244431	1.193740	1.7142	0.6733
	Grid search	1.3	–	0	0	1.219650	1.7642	0.6752
Rule 8	Fminsearch	–	0.986577	-0.0645642	0.0128559	0.636252	0.1226	1.1499
	Grid search	**–**	**2**	**0.3**	**0.1**	**0.637657**	**0.1208**	**1.1545**

Table 4.14: Optimal Simple Rules, Complements, Loss ERM 2

Rule	Method	$\varphi_{\pi_i^i}$	φ_{π^i}	φ_Y	φ_S	Loss ERM 2	Variance		
							π^A	Y^A	ΔS_C^A
Rule 1	Fminsearch	1	–	–	–	5.81508	2.2870	0.9289	6.1273
Rule 2	Fminsearch	–	2.06158e+012	–	–	9.04763	0.0000	1.3926	16.7027
Rule 3	Fminsearch	1.00213	–	-0.0851034	–	4.90540	3.3575	1.1037	1.9921
	Grid search	1	–	0.05	–	7.05050	1.9478	0.8415	9.3639
Rule 4	Fminsearch	–	11.3824	-8.45006	–	5.70174	1.0793	1.7714	7.4735
	Grid search	**–**	**3**	**0**	**–**	**9.89654**	**0.0164**	**1.3329**	**18.4274**
Rule 5	Fminsearch	0.97303	–	–	0.0240273	4.93290	3.3747	1.0976	2.0187
	Grid search	1	–	–	0.05	4.96559	3.3362	1.0840	2.1748
Rule 6	Fminsearch	–	0.993576	–	0.130897	4.39416	2.1007	1.3280	3.2589
	Grid search	–	1	–	0.15	4.40392	2.2836	1.3322	2.9084
Rule 7	Fminsearch	0.848377	–	0.385686	0.12972	5.04889	3.3308	1.0636	2.3725
	Grid search	1	–	0.1	0.1	5.04632	3.2389	1.0560	2.5589
Rule 8	Fminsearch	–	0.900737	0.0663188	0.0875105	4.39003	2.0795	1.3330	3.2880
	Grid search	**–**	**1**	**0.1**	**0.2**	**4.40475**	**2.3042**	**1.2828**	**2.9182**

Table 4.15: Optimal Simple Rules, Substitutes, Loss CB

Rule	Method	$\varphi_{\pi_i^i}$	φ_{π^i}	φ_Y	φ_S	Loss CB	Variance	
							π^A	Y^A
Rule 1	Fminsearch	6.32156e+006	–	–	–	0.0570222	0.0055	0.1086
Rule 2	Fminsearch	–	2.47116e+012	–	–	0.0401281	0.0000	0.0803
Rule 3	Fminsearch	30.1037	–	11.853	–	0.0495594	0.0143	0.0848
	Grid search	2.55	–	1	–	0.0495594	0.0143	0.0848
Rule 4	Fminsearch	–	1.18711e+009	736567	–	0.0401213	0.0000	0.0802
	Grid search	**–**	**3**	**0.95**	**–**	**0.0405818**	**0.0051**	**0.0760**
Rule 5	Fminsearch	3.16537e+008	–	–	-1.25152e+006	0.0570221	0.0056	0.1085
	Grid search	3	–	–	0	0.0642976	0.0095	0.1191
Rule 6	Fminsearch	–	6.46762e+007	–	-4.5584e+006	0.0371697	0.0050	0.0693
	Grid search	–	3	–	0	0.0480413	0.0035	0.0926
Rule 7	Fminsearch	1.04055	–	0.23504	-0.046206	0.0508892	0.0146	0.0872
	Grid search	1.1	–	1.5	0.3	0.0467120	0.0119	0.0816
Rule 8	Fminsearch	–	1.3221	-0.244637	-0.212216	0.0310472	0.0117	0.0504
	Grid search	**–**	**3**	**0.9**	**0**	**0.0405892**	**0.0043**	**0.0769**

Table 4.16: Optimal Simple Rules, Substitutes, Loss ERM 2

Rule	Method	$\varphi_{\pi_i^i}$	φ_{π^i}	φ_Y	φ_S	Loss ERM 2	Variance		
							π^A	Y^A	ΔS_C^A
Rule 1	Fminsearch	1.2746	–	–	–	0.294700	0.0774	0.1697	0.2650
Rule 2	Fminsearch	–	1.30269	–	–	0.293541	0.0827	0.1439	0.2779
Rule 3	Fminsearch	1.27441	–	0.00012667	–	0.294700	0.0774	0.1697	0.2650
	Grid search	1.2	–	0.05	–	0.294783	0.0739	0.1679	0.2739
Rule 4	Fminsearch	–	0.99373	0.205343	–	0.290535	0.0795	0.1503	0.2717
	Grid search	**–**	**1**	**0.2**	**–**	**0.290590**	**0.0814**	**0.1509**	**0.2675**
Rule 5	Fminsearch	2.9138	–	–	0.798454	0.289506	0.0781	0.1592	0.2637
	Grid search	2.9	–	–	0.8	0.289510	0.0788	0.1596	0.2618
Rule 6	Fminsearch	–	1.36127	–	0.0319936	0.293514	0.0829	0.1442	0.2771
	Grid search	–	1.4	–	0.05	0.293536	0.0814	0.1437	0.2806
Rule 7	Fminsearch	2.95509	–	-0.0589136	0.773123	0.289506	0.0781	0.1592	0.2637
	Grid search	2.2	–	1.1	1.3	0.289507	0.0781	0.1592	0.2636
Rule 8	Fminsearch	–	0.699423	1.64128	0.974668	0.289414	0.0783	0.1575	0.2647
	Grid search	**–**	**1**	**1.5**	**1**	**0.289519**	**0.0771**	**0.1553**	**0.2694**

4.4 Conclusions

In this chapter, we analyze monetary policy given alternative degrees of integration between the euro area, an inflation targeting accession country like the Czech Republic, Hungary, Poland, and Slovakia, and a non-EU country as the third country.

The model developed in the chapter is based on Teo (2005) and Monacelli (2001) and allows us to simulate the impact of a technology shock originating in a third country on an accession country considering feedback from the euro area as well. The New Keynesian three-country model incorporates explicit microfoundations with dynamic and intertemporal effects as well as Keynesian building blocks such as nominal rigidities and monopolistic competition. The core equations, i.e., the IS curve, the Phillips curve, and the monetary policy rule, are derived following Galí and Monacelli (2005). As a short-cut for modeling production linkages, we use alternative elasticities in consumption baskets. This allows us to trace the transmission mechanism of a third-country technology shock in the alternative cases that a third country produces complements or substitutes, i.e., that a third country is vertically or horizontally integrated with the accession country.

Evaluating the central bank loss function without ERM 2 (Loss CB) and with ERM 2 (Loss ERM 2) under strict inflation targeting and Taylor rules as well as both types of rules extended by exchange rate smoothing allows us to assess the costs of ERM 2 and derive the implications of production sharing for monetary policy. Optimal rules are derived by minimizing the central bank loss function. The Matlab routine `fminsearch` uses the entire

parameter space and therefore tends to find optimal coefficients minimizing the loss for implausible policy parameters. To circumvent this problem, we follow Schmitt-Grohé and Uribe (2007) and conduct additionally a grid search.

The simulations reveal the following results. First, we revisit an asymmetric shock in a two-country model. A common monetary policy cannot address the implications of a technology shock on inflation and output adequately. Under production sharing via complements, inflation in one country is negatively correlated with inflation in the other country. If the two countries produce strong substitutes, output is negatively correlated.

Second, our simulation of a third-country technology shock show that the exchange rate between the accession country and the anchor currency country bears part of the adjustment if production sharing is via complements, but only little if the accession country and the third-country produce strong substitutes.

Third, our evaluation of simple monetary policy rules reveals that targeting the exchange rate, if the central bank has no exchange rate objective (Loss CB), increases the loss. The loss more than doubles in case the accession country and the third country produce substitutes. If the loss of the central bank takes into account the volatility of the exchange rate (Loss ERM 2), the CPI inflation Taylor rule augmented for exchange rate smoothing performs well under any type of production sharing.

Fourth, our results for optimal monetary policy rules depend on the assumed loss function of the central bank. For a traditional central bank loss function (Loss CB), minimizing volatility in CPI inflation and output, the exchange rate generally does not matter. However, having an additional policy parameter still pays off in the case of complements as even a small exchange rate coefficient of 0.1 reduces the central bank loss. If the loss function captures the requirements of ERM 2 (Loss ERM 2), i.e., exchange rate smoothing is an objective of the central bank, allowing for a reaction of the interest rate to the exchange rate lowers the loss. Further, the optimal coefficients are lower for production sharing via complements than via substitutes. In the latter case, the exchange rate has in fact the same coefficient in the optimal monetary policy rule as inflation.

All in all, the results show that a Taylor rule extended for an exchange rate term is a good simple rule for different kinds of integration with a third country and even in case the loss function does not account for a preference for exchange rate smoothing.

Appendix

A4.1 Technology Shock with Home Bias in Consumption

Figure A4.1: Technology Shock in a Three-Country Model, Complements, Home Bias ($\eta = 0.5$)

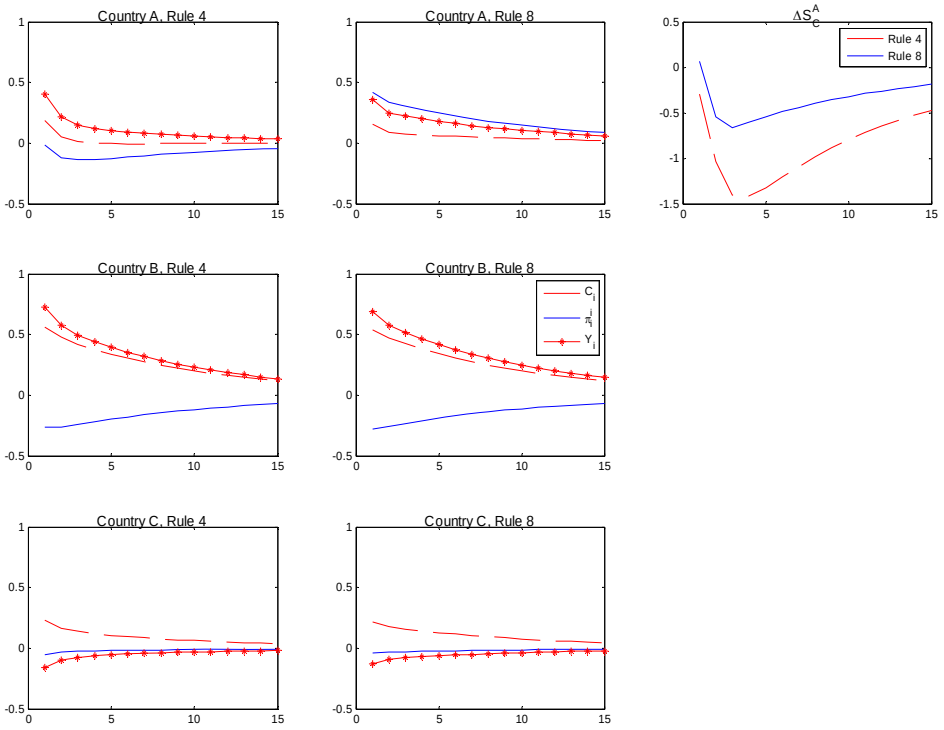


Table A4.1: Performance of Simple Monetary Policy Rules, Complements, Home Bias

Rule		Standard Deviation					Loss	
		π_A^A	π^A	Y^A	ΔS_C^A	C^A	CB	ERM 2
Rule 1	domestic inflation targeting	0.20	0.60	0.48	3.06	0.09	0.2976	5.1600
Rule 2	CPI inflation targeting	0.18	0.26	0.63	3.20	0.25	0.2342	5.3750
Rule 3	domestic inflation Taylor rule	0.11	0.69	0.41	3.70	0.04	0.3210	7.3843
Rule 4	CPI inflation Taylor rule	0.37	0.56	0.54	3.82	0.19	0.3039	7.7696
Rule 5	domestic inflation ERM 2	1.15	1.18	0.67	1.19	0.26	0.9209	2.3208
Rule 6	CPI inflation ERM 2	1.07	1.02	0.71	1.29	0.29	0.7781	2.1325
Rule 7	domestic Taylor ERM 2	0.96	1.03	0.63	1.56	0.22	0.7278	2.4756
Rule 8	CPI Taylor ERM 2	0.88	0.86	0.67	1.63	0.25	0.5964	2.2995

Table A4.2: Ratio of Loss for Joining ERM 2, Complements, Home Bias

Ratio	Rule	Loss CB	Loss ERM 2
Rule 5 / Rule 1	domestic inflation targeting	3.09	0.45
Rule 6 / Rule 2	CPI inflation targeting	3.32	0.40
Rule 7 / Rule 3	domestic inflation Taylor rule	2.27	0.34
Rule 8 / Rule 4	CPI inflation Taylor rule	1.96	0.30

Figure A4.2: Technology Shock in a Three-Country Model, Substitutes, Home Bias ($\eta = 0.5$)

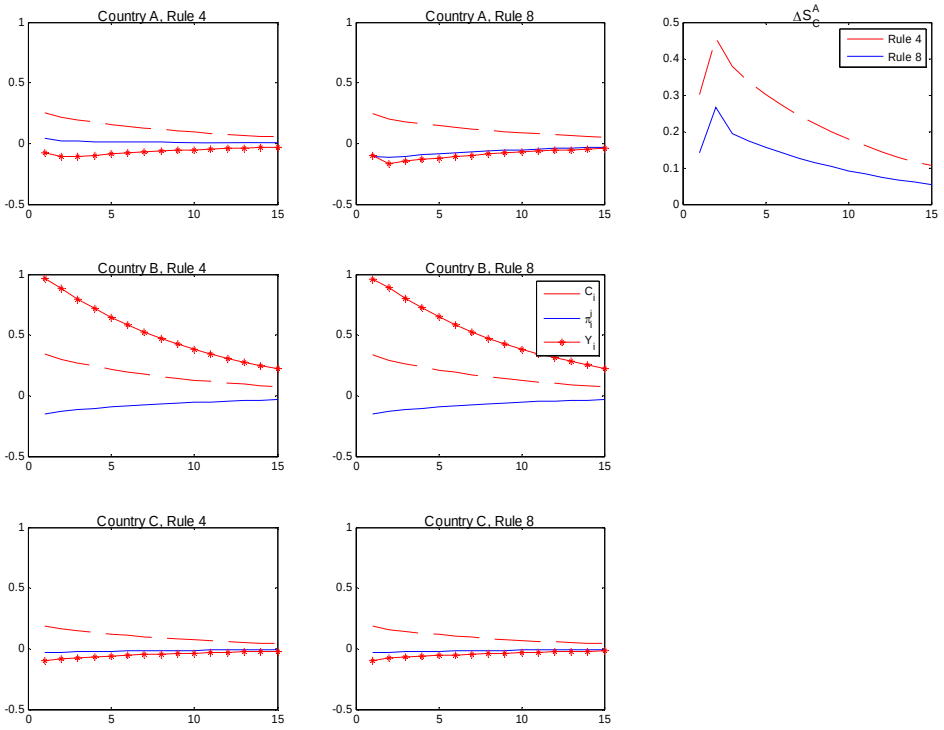


Table A4.3: Performance of Simple Monetary Policy Rules, Substitutes, Home Bias

Rule		Standard Deviation					Loss	
		π_A^A	π^A	Y^A	ΔS_C^A	C^A	CB	ERM 2
Rule 1	domestic inflation targeting	0.15	0.18	0.37	0.70	0.53	0.0842	0.3438
Rule 2	CPI inflation targeting	0.18	0.19	0.35	0.70	0.54	0.0777	0.3423
Rule 3	domestic inflation Taylor rule	0.08	0.09	0.29	1.02	0.56	0.0467	0.5732
Rule 4	CPI inflation Taylor rule	0.06	0.06	0.28	1.02	0.56	0.0405	0.5603
Rule 5	domestic inflation ERM 2	0.43	0.45	0.45	0.33	0.51	0.1993	0.3531
Rule 6	CPI inflation ERM 2	0.44	0.46	0.43	0.33	0.51	0.1976	0.3573
Rule 7	domestic Taylor ERM 2	0.28	0.30	0.40	0.53	0.52	0.1229	0.3097
Rule 8	CPI Taylor ERM 2	0.29	0.30	0.39	0.54	0.53	0.1214	0.3115

Table A4.4: Ratio of Loss for Joining ERM 2, Substitutes, Home Bias

Ratio	Rule	Loss CB	Loss ERM 2
Rule 5 / Rule 1	domestic inflation targeting	2.37	1.03
Rule 6 / Rule 2	CPI inflation targeting	2.54	1.04
Rule 7 / Rule 3	domestic inflation Taylor rule	2.63	0.54
Rule 8 / Rule 4	CPI inflation Taylor rule	3.00	0.56

A4.2 Optimal Simple Rules with Home Bias in Consumption

Table A4.5: Optimal Simple Rules, Complements, Home Bias, Loss CB

Rule	Method	$\varphi_{\pi_i^i}$	φ_{π^i}	φ_Y	φ_S	Loss CB	Variance	
							π^A	Y^A
Rule 1	Fminsearch	2.03489	–	–	–	0.293288	0.3740	0.2126
Rule 2	Fminsearch	–	1.23384	–	–	0.232098	0.1003	0.3639
Rule 3	Fminsearch	2.03205	–	0.000667867	–	0.293288	0.3740	0.2126
	Grid search	1.4	–	0.15	–	0.293288	0.3738	0.2127
Rule 4	Fminsearch	–	1.00097	-0.0493207	–	0.223295	0.0883	0.3583
	Grid search	–	1.25	0	–	0.232109	0.0978	0.3664
Rule 5	Fminsearch	1.03481	–	–	-0.0316297	0.286932	0.3597	0.2142
	Grid search	2.05	–	–	0	0.293289	0.3743	0.2123
Rule 6	Fminsearch	–	0.99373	–	0.00569856	0.222809	0.0971	0.3485
	Grid search	–	1.25	–	0	0.232109	0.0978	0.3664
Rule 7	Fminsearch	7.59237	–	-3.66024	-0.458846	0.225457	0.1097	0.3413
	Grid search	1.2	–	0.2	0	0.293290	0.3743	0.2123
Rule 8	Fminsearch	–	0.989931	0.0432973	0.0106893	0.222696	0.1048	0.3406
	Grid search	–	1.7	0.6	0.1	0.222947	0.1062	0.3397

Table A4.6: Optimal Simple Rules, Complements, Home Bias, Loss ERM 2

Rule	Method	$\varphi_{\pi_i^i}$	φ_{π^i}	φ_Y	φ_S	Loss ERM 2	Variance π^A	Variance Y^A	Variance ΔS_C^A
Rule 1	Fminsearch	1	–	–	–	2.60228	0.7560	0.3753	3.3172
Rule 2	Fminsearch	–	4.28486e+012	–	–	4.29760	0.0000	0.5821	8.0131
Rule 3	Fminsearch	1.00535	–	-0.0910312	–	2.21439	1.3348	0.4734	1.2857
	Grid search	1	–	0.05	–	3.09735	0.5607	0.3298	4.7436
Rule 4	Fminsearch	–	1.00481	-0.244666	–	2.14450	0.8653	0.6207	1.9376
	Grid search	–	3	0	–	4.75461	0.0156	0.4947	8.9833
Rule 5	Fminsearch	0.975484	–	–	0.0222794	2.22138	1.3389	0.4708	1.2942
	Grid search	1	–	–	0.05	2.23029	1.3931	0.4753	1.1992
Rule 6	Fminsearch	–	0.936282	–	0.0580603	2.10596	1.0055	0.5617	1.6393
	Grid search	–	1.05	–	0.15	2.10948	1.0482	0.5441	1.5785
Rule 7	Fminsearch	0.887574	–	0.288784	0.0970124	2.24154	1.3328	0.4619	1.3555
	Grid search	1	–	0.1	0.1	2.24232	1.3547	0.4645	1.3108
Rule 8	Fminsearch	–	0.935602	0.00232331	0.0584814	2.10590	1.0045	0.5610	1.6418
	Grid search	–	1	0	0.1	2.11069	0.9604	0.5437	1.7569

Table A4.7: Optimal Simple Rules, Substitutes, Home Bias, Loss CB

Rule	Method	$\varphi_{\pi_i^i}$	φ_{π^i}	φ_Y	φ_S	Loss CB	Variance π^A	Variance Y^A
Rule 1	Fminsearch	5.03317e+006	–	–	–	0.0528229	0.0040	0.1016
Rule 2	Fminsearch	–	7.91617e+012	–	–	0.0381636	0.0000	0.0763
Rule 3	Fminsearch	31.1817	–	11.8537	–	0.0463182	0.0123	0.0803
	Grid search	1.3	–	0.5	–	0.0463182	0.0123	0.0803
Rule 4	Fminsearch	–	3.48621e+009	2.16309e+006	–	0.0381575	0.0000	0.0763
	Grid search	–	3	0.95	–	0.0389490	0.0050	0.0729
Rule 5	Fminsearch	2.70357e+008	–	–	-1.06894e+006	0.0528167	0.0041	0.1015
	Grid search	3	–	–	0	0.0593840	0.0075	0.1112
Rule 6	Fminsearch	–	2.07149e+008	–	-1.30764e+007	0.0353397	0.0046	0.0661
	Grid search	–	3	–	0	0.0465716	0.0035	0.0896
Rule 7	Fminsearch	1.03753	–	0.22828	-0.0427491	0.0474732	0.0124	0.0825
	Grid search	1	–	1.5	0.3	0.0437715	0.0076	0.0799
Rule 8	Fminsearch	–	2.67328e+007	1.1226e+006	-1.36116e+006	0.0357505	0.0042	0.0673
	Grid search	–	3	1	0	0.0389605	0.0058	0.0721

Table A4.8: Optimal Simple Rules, Substitutes, Home Bias, Loss ERM 2

Rule	Method	$\varphi_{\pi_i^i}$	φ_{π^i}	φ_Y	φ_S	Loss ERM 2	Variance π^A	Variance Y^A	Variance ΔS_C^A
Rule 1	Fminsearch	1.2197	–	–	–	0.312806	0.0894	0.1673	0.2795
Rule 2	Fminsearch	–	1.25255	–	–	0.312953	0.0958	0.1477	0.2866
Rule 3	Fminsearch	1.21923	–	0.000315802	–	0.312806	0.0894	0.1673	0.2795
	Grid search	1.15	–	0.05	–	0.312846	0.0868	0.1661	0.2859
Rule 4	Fminsearch	–	0.99447	0.184219	–	0.310872	0.0930	0.1521	0.2837
	Grid search	–	1.05	0.15	–	0.311291	0.0886	0.1492	0.2962
Rule 5	Fminsearch	2.40439	–	–	0.61751	0.309497	0.0916	0.1575	0.2783
	Grid search	2.45	–	–	0.65	0.309506	0.0927	0.1578	0.2758
Rule 6	Fminsearch	–	1.12256	–	-0.0719897	0.312789	0.0949	0.1477	0.2880
	Grid search	–	1.25	–	0	0.312962	0.0971	0.1482	0.2836
Rule 7	Fminsearch	2.52512	–	-0.149392	0.569528	0.309497	0.0916	0.1575	0.2783
	Grid search	1.9	–	0.6	0.8	0.309497	0.0916	0.1576	0.2782
Rule 8	Fminsearch	–	-5.30875	16.5453	8.96801	0.309433	0.0913	0.1587	0.2776
	Grid search	–	1	1.5	1	0.310027	0.0946	0.1555	0.2754

5 Conclusions

This study addressed a potential trade-off between inflation and exchange rate targeting in former transition countries, which now may be labeled emerging market economies and which prepare for entry into the Economic and Monetary Union (EMU). Among this group of countries, some implemented inflation targeting regimes. These countries may face increasing macroeconomic risks when entering the possibly insecure terrain of a soft peg in the European Exchange Rate Mechanism (ERM 2). Against this background, the study analyzed the choice of inflation targets (Chapter 2), the (de facto) role of exchange rate policy in monetary strategies (Chapter 3), and the potential costs of an additional exchange rate target under an intermediate exchange rate regime like ERM 2 (Chapter 4).

Chapter 2 went beyond the traditional explanations of inflation such as revealing the monetary transmission mechanism from monetary aggregates, interest rate, or exchange rate to inflation or the role of fiscal policy and central bank independence. Emerging market economies with flexible exchange rates that achieved low inflation tend to have indeed central banks where the government changed the central bank law, granted their central bank independence and often these central banks then adopted inflation targeting as monetary policy strategy. The analysis on the differences among the central and eastern European economies, however, allows us to conclude that these changes might in fact be the key stone of a successful transition from a centrally planed to a market economy. Only once these countries reformed their economies, abandoned administered prices and opened themselves to international markets, their economies were ready for persistently low inflation.

This research on the choice of implicit inflation targets in transition countries has implications for other developing countries with high inflation. Once the most common reason for high inflation, namely unsustainable fiscal policy, is under control, other factors in the structure of the economy might still interfere with a market economy. The results might move the current focus on the implementation of certain laws that grant independence to the central bank with a commitment to low inflation toward the structural reforms in the economy. The results for transition countries emphasize for instance price liberalization and openness as key factors. Progress in these fields seems to be a prerequisite to the introduction of low inflation policies. Hence, the approach taken in Chapter 2 should be generalized to all

developing countries. An in-depth analysis would bring forth the individual countries' reform potential and thereby help to focus structural reforms before taking the rather small step of granting independence to the central bank.

Chapter 3 addressed two main issues on inflation targeting in Latin American and European emerging market economies. First, the estimations revealed differences in the speed of transmission of exchange rate shocks among the seven economies. Second, the empirical analysis allowed us to distinguish between what central banks announce (their de jure policy) and what they do (the de facto policy). All central banks adopted inflation targeting as their monetary strategy. The regressions showed that despite the common strategy, the actual conduct differs especially with respect to the reaction of the interest rate to exchange rate shocks. As part of the analysis, we were able to pin down whether announcement of changes in the strategy coincided with changes in the conduct of monetary policy. Understanding what is actually done in contrast to announced policy is a brick stone for central bank watching. The research agenda in this field is already well established. Central bank watchers regularly re-estimate the transmission mechanism in vector autoregressive models and trace changes in monetary policy with break point tests. Additionally, estimating different versions of Taylor rules allows them to identify the central bank's relative weights put on inflation, output, and the exchange rate. All in all, this research has implications for transparency and accountability of central banks. The results verify central bank policies and thereby contribute to reliable monetary policy that stabilizes financial markets and thus the entire economy.

Chapter 4 investigated the challenge of joining the euro area for central banks currently operating under an inflation targeting framework. Those central banks in central Europe will have to enter ERM 2 and target inflation and the exchange rate simultaneously. In a three-country model, we focused on the implications of production sharing with third countries for monetary policy under an intermediate exchange rate regime like ERM 2. The analysis showed that stabilizing the exchange rate vis-à-vis the euro implies a higher loss to the central bank under vertical integration with a third country (producing complements) than under horizontal integration (producing substitutes). Among a set of simple monetary policy rules, the Taylor rule augmented for an exchange rate term turned out to have a reasonable performance under any form of integration.

Further research would shift modeling of production sharing via the consumption basket, i.e., the demand side of the economy, to the supply side by introducing internationally traded intermediate goods. In contrast to the present study, which shed light on qualitative differences, a calibrated model would allow us to quantify the implications of third-country effects from production sharing for monetary integration under ERM 2.

6 References

Aisen, A. and F.J. Veiga (2006). Does Political Instability Lead to Higher Inflation? A Panel Data Analysis. *Journal of Money, Credit and Banking* 38 (5): 1379-90.

Arellano, M. and S. Bond (1991). Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations. *Review of Economic Studies* 58 (2): 277-97.

Arellano, M. and O. Bover (1995). Another Look at the Instrumental Variable Estimation of Error-Component Models. *Journal of Econometrics* 68 (1): 29-51.

Ascari, G. and C. Merkl (2007). Real Wage Rigidities and the Cost of Disinflations. IZA Discussion Paper 3049. Bonn, Germany.

A.T. Kearney (2006). Emerging Market Priorities for Global Retailers.

Backus, D.K., P.J. Kehoe, and F.E. Kydland (1995). International Business Cycles: Theory vs. Evidence. In T.F. Cooley (ed.), *Frontiers of Business Cycle Research.* Princeton University Press. Princeton, NJ.

Barro, R.J. and D.B. Gordon (1983). A Positive Theory of Monetary Policy in a Natural Rate Model. *Journal of Political Economy* 91 (4): 589-610.

Bassett, S. (2003). The Feasibility of Inflation Targeting in Ukraine. IMF Country Report 03/173. International Monetary Fund. Washington, DC, pp. 37-48.

Beck, N. (2008). Time-Series Cross-Section Methods. In J.M. Box-Steffensmeier, H.E. Brady, and D. Collier (eds.), *The Oxford Handbook of Political Methodology.* Oxford University Press. Oxford, UK, forthcoming.

Beck, N. and J.N. Katz (1995). What to Do (and not to Do) with Time-Series Cross-Section Data. *American Political Science Review* 89 (3): 634-47.

Beck, N. and J.N. Katz (2004). Time-Series Cross-Section Issues: Dynamics, 2004. Mimeo, New York University. New York.

Benigno, P. (2004). Optimal Monetary Policy in a Currency Area. *Journal of International Economics* 63 (2): 293-320.

Benigno, P. and M. Woodford (2007). Linear-Quadratic Approximation of Optimal Policy Problems. Mimeo. Columbia University. New York.

Bernanke, B., T. Laubach, F. Mishkin, and A. Posen (1999). *Inflation Targeting: Lessons from the International Experience.* Princeton University Press. Princeton, NJ.

Blundell, R. and S. Bond (1998). Initial Conditions and Moment Restrictions in Dynamic Panel Data Models. *Journal of Econometrics* 87 (1): 115-43.

Bofinger, P. (2001). *Monetary Policy: Goals, Institutions, Strategies, and Instruments.* Oxford University Press. Oxford, UK.

Bofinger, P. and T. Wollmershäuser (2001). Managed Floating: Understanding the New International Monetary Order. CEPR Discussion Paper 3064. London, UK.

Boskin, M.J., E.R. Dulberger, R.J. Gordon, Z. Griliches, and D.W. Jorgenson (1996). Toward a More Accurate Measure of the Cost of Living. Final Report to the U.S. Senate Finance Committee from the Advisory Commission to study the Consumer Price Index.

Boskin, M.J., E.R. Dulberger, R.J. Gordon, Z. Griliches, and D.W. Jorgenson (1998). Consumer Prices, the Consumer Price Index, and the Cost of Living. *Journal of Economic Perspectives* 12 (1): 3-26.

Bowman, D. and B. Doyle (2003). New Keynesian Open-Economy Models and Their Implications for Monetary Policy. Board of Governors of the Federal Reserve System. International Finance Discussion Paper 762. Washington, DC.

Brachinger, H.W., B. Schips, and W. Stier (1999). Expertise zur Relevanz des "Boskin-Reports" für den schweizerischen Landesindex der Konsumentenpreise. Bundesamt für Statistik. Neuchatel, Switzerland.

Burstein, A., C. Kurz, and L. Tesar (2008). Trade, Production Sharing, and the International Transmission of Business Cycles. *Journal of Monetary Economics,* forthcoming.

Calvo, G. (1983). Staggered Prices in a Utility-Maximizing Framework. *Journal of Monetary Economics* 12 (3): 383-98.

Campbell, J.Y. (1994). Inspecting the Mechanism: An Analytical Approach to the Stochastic Growth Model. *Journal of Monetary Economics* 33 (3): 463-506.

Christiano, L.J., M. Eichenbaum, and C.L. Evans (1996). The Effects of Monetary Policy Shocks: Evidence from the Flow of Funds. *Review of Economics and Statistics* 78 (1): 16-34.

Christiano, L.J., M. Eichenbaum, and C.L. Evans (1999). Monetary Policy Shocks: What Have We Learned and to What End? In J.B. Taylor and M. Woodford (eds.),

Handbook of Macroeconomics. Volume 1. Elsevier. Amsterdam, Netherlands, pp. 65-148.

Clarida, R.H. (2001). The Empirics of Monetary Policy Rules in Open Economies. *International Journal of Finance and Economics* 6 (4): 315-23.

Clarida, R., J. Galí, and M. Gertler (1998). Monetary Policy Rules in Practice: Some International Evidence. *European Economic Review* 42 (6): 1033-67.

Clarida, R., J. Galí, and M. Gertler (2000). Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory. *Quarterly Journal of Economics* 115 (1): 147-80.

Clarida, R., J. Galí, and M. Gertler (2001). Optimal Monetary Policy in Open versus Closed Economies: An Integrated Approach. *American Economic Review* 91 (2): 248-52

Cochrane, D. and G.H. Orcutt (1949). Application of Least Squares Regression to Relationships Containing Autocorrelated Error Terms. *Journal of the American Statistical Association* 44 (245): 32-61.

Cooley, T.F. and S.F. LeRoy (1985). Atheoretical Macroeconomics: A Critique. *Journal of Monetary Economics* 16 (3): 283-308.

Corbo, V. and K. Schmidt-Hebbel (2003). Inflation Targeting in Latin America. In J.A. González, V. Corbo, A.O. Krueger, and A. Tornell (eds.), *Latin American Macroeconomic Reforms: The Second Stage.* University of Chicago Press. Chicago, IL, pp. 19-77.

Corsetti, G., P. Pesenti, N. Roubini, and C. Tille (2000). Competitive Devaluations: Toward a Welfare-Based Approach. *Journal of International Economics* 51 (1): 217-41.

Cottarelli, C., M. Griffiths, and R. Moghadam (1998). The Nonmonetary Determinants of Inflation: A Panel Data Study. IMF Working Papers 98/23. International Monetary Fund. Washington, DC.

Crawford, A. (1998). Measurement Biases in the Canadian CPI: An Update. *Bank of Canada Review* Spring, pp. 38-56.

Crespo, N. and M.P. Fontoura (2007). Integration of CEECs into EU Market: Structural Change and Convergence. *Journal of Common Market Studies* 45 (3): 611-32.

Cukierman, A. (1992). *Central Bank Strategy, Credibility, and Independence: Theory and Evidence.* MIT Press. Cambridge, MA.

Cukierman, A., S. Edwards, and G. Tabellini (1992). Seigniorage and Political Instability. *American Economic Review* 82 (3): 537-55.

Cukierman, A., G. P. Miller, and B. Neyapti (2002). Central Bank Reform, Liberalization and Inflation in Transition Economies—an International Perspective. *Journal of Monetary Economics* 49 (2): 237-64.

Cunningham, A. (1996). Measurement Bias in Price Indices: An Application to the UK's RPI. Bank of England Working Paper Series 47. Bank of England. London, UK.

De Paoli, B. (2006). Monetary Policy and Welfare in a Small Open Economy. CEP Discussion Paper 639. Centre for Economic Performance. London, UK.

Diewert, E.W. (1997). Comment. In T. Bresnahan and R.J. Gordon (eds.), *The Economics of New Goods.* University of Chicago Press. Chicago, IL, pp. 423-33.

Edwards, S. (2001). Dollarization: Myths and Realities. *Journal of Policy Modeling* 23 (3): 249-65.

Égert, B. (2005). Equilibrium Exchange Rates in South-Eastern Europe, Russia, Ukraine and Turkey: Healthy or (Dutch) Diseased? *Economic Systems* 29 (2): 205-41.

Égert, B., L. Halpern, and R. MacDonald (2006). Equilibrium Exchange Rates in Transition Economies: Taking Stock of the Issues. *Journal of Economic Surveys* 20 (2): 257-324.

EIU (2005). *Belarus Country Profile 2005.* Economic Intelligence Unit. London, UK.

EIU (2006). *Ukraine Country Profile 2006.* Economic Intelligence Unit. London, UK.

EIU (various issues). *Country Profile* (various countries). Economist Intelligence Unit. London, UK.

EIU (various issues). *Country Report* (various countries). Economist Intelligence Unit. London, UK.

Faia, E. (2007). Financial Differences and Business Cycle Co-Movements in a Currency Area. *Journal of Money, Credit and Banking* 39 (1): 151-85.

Faia, E. and T. Monacelli (2004). Ramsey Monetary Policy and International Relative Prices. European Central Bank Working Paper 344. European Central Bank. Frankfurt, Germany.

Faia, E. and T. Monacelli (2007). Optimal Monetary Policy in a Small Open Economy with Home Bias. *Journal of Money, Credit and Banking,* forthcoming.

Filer, R.K. and J. Hanousek (2000). Output Changes and Inflationary Bias in Transition. *Economic Systems* 24 (3): 285-94.

Filer, R.K. and J. Hanousek (2003). Inflationary Bias in Middle to Late Transition Czech Republic. *Economic Systems* 27 (4): 367-76.

Fischer, S. (2001). Exchange Rate Regimes: Is the Bipolar View Correct? *Journal of Economic Perspectives* 15 (2): 3-24.

Fontagné, L. and M. Freudenberg (1999). Endogenous Symmetry of Shocks in a Monetary Union. *Open Economies Review* 10 (3): 263-87.

Fracasso, A., H. Genberg, and C. Wyplosz (2003). *How Do Central Banks Write? An Evaluation of Inflation Reports by Inflation Targeting Central Banks.* Geneva Reports on the World Economy Special Report 2. International Center for Monetary and Banking Studies. Geneva, Switzerland.

Freedman, C. (1996). The Use of Indicators and of the Monetary Conditions Index in Canada. In Bank of Canada (ed.), *The Transmission of Monetary Policy in Canada.* Ottawa, Canada. pp. 67-79.

Frömmel, M. and F. Schobert (2006). Monetary Policy Rules in Central and Eastern Europe. University of Hanover Discussion Paper 341. Hanover, Germany.

Fung, B.S.C. (2002). A VAR Model Analysis of the Effects of Monetary Policy in East Asia. BIS Working Paper 119. Bank for International Settlements. Basle, Switzerland.

Galí, J. (2003). New Perspectives on Monetary Policy, Inflation, and the Business Cycle. In M. Dewatripont, L.P. Hansen, and S.J. Turnovsky (eds.), *Advances in Economics and Econometrics: Theory and Applications.* Cambridge University Press. Cambridge, UK, pp. 151-97.

Galí, J. (2008). Monetary Policy and the Open Economy. In J. Galí, *Monetary Policy, Inflation, and the Business Cycle.* Princeton University Press. Princeton, NJ, pp. 149-89.

Galí, J. and T. Monacelli (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economic Studies* 72 (3): 707-34.

Gerlach, S. and F. Smets (2000). MCIs and Monetary Policy. *European Economic Review* 44 (9): 1677-700.

Ghironi, F. (2006). Macroeconomic Interdependence under Incomplete Markets. *Journal of International Economics* 70 (2): 428-50.

Ghosh, A.R., A.-M. Gulde, and H.C. Wolf (2000). Currency Boards: More than a Quick Fix? *Economic Policy* 15 (31): 270-335.

Goodfriend, M. (1991). Interest Rates and the Conduct of Monetary Policy. *Carnegie-Rochester Conference Series on Public Policy* 34: 7-30.

Goodfriend, M. and R.G. King (1997). The New Neoclassical Synthesis and the Role of Monetary Policy. In B. Bernanke and J. Rotemberg (eds.), *NBER Macroeconomics Annual* 12. MIT Press. Cambridge, MA, pp. 231-83.

Gordon, R.J. (2000). The Boskin Commission Report and its aftermath. NBER Working Paper 7759. Cambridge, MA.

Gordon, R.J. (2006). The Boskin Commission Report: A Retrospective One Decade Later. NBER Working Paper 12311. Cambridge, MA.

Gottschalk, J. and D. Moore (2001). Implementing Inflation Targeting Regimes: The Case of Poland. *Journal of Comparative Economics* 29 (1): 24-39.

Granville, B. and S. Mallick (2006). Monetary Policy in Russia. In L. Vinhas de Souza and O. Havrylyshyn (eds.), *Growth Resumption in the CIS.* Elsevier. Amsterdam, Netherlands, pp. 73-89.

Hammermann, F. (2005). Do Exchange Rates Matter in Inflation Targeting Regimes? Evidence from a VAR Analysis for Poland and Chile. In R.J. Langhammer and L. Vinhas de Souza (eds.), *Monetary Policy and Macroeconomic Stabilization in Latin America.* Springer. Berlin, Germany, pp. 115-48.

Hoffmann, J. (1999). Problems of Inflation Measurement in Germany: An Update. Mimeo, Eurostat Conference, Cardiff, UK, September 1999.

Imbs, J. (2004). Trade, Finance, Specialization, and Synchronization. *Review of Economics and Statistics* 86 (3): 723-34.

IMF (2005a). Republic of Belarus: Report on the Observance of Standards and Codes—Data Module, Response by the Authorities, and Detailed Assessments Using the Data Quality Assessment Framework. IMF Country Report 05/29. International Monetary Fund. Washington, DC.

IMF (2005b). Republic of Belarus: 2005 Article IV Consultation—Staff Report. IMF Country Report 05/214. International Monetary Fund. Washington, DC.

IMF (2005c). Ukraine: 2005 Article IV Consultation and Ex Post Assessment of Longer-Term Program Engagement—Staff Reports. IMF Country Report 05/415. International Monetary Fund. Washington, DC.

IMF (2006). Republic of Belarus: 2006 Article IV Consultation—Staff Report. IMF Country Report 06/314. International Monetary Fund. Washington, DC.

IMF (2007). Decoupling the Train? Spillovers and Cycles in the Global Economy. Chapter 4 in International Monetary Fund (ed.), *World Economic Outlook: April 2007*. Washington, DC.

International Financial Statistics (various issues). International Monetary Fund. Washington, DC.

Ivanova, A., M. Keen, and A. Klemm (2005). The Russian 'Flat Tax' Reform. *Economic Policy* 20 (43): 397-444.

Jonas, J. and F.S. Mishkin (2003). Inflation Targeting in Transition Economies: Experience and Prospects. In M. Woodford (ed.), *Inflation Targeting*. NBER Conference Series. University of Chicago Press. Chicago, IL.

Kalemli-Ozcan, S., B.E. Sørensen, and O. Yosha (2003). Risk Sharing and Industrial Specialization: Regional and International Evidence. *American Economic Review* 93 (3): 903-18.

Kim, J. and S.H. Kim (2007). Two Pitfalls of Linearization Methods. *Journal of Money, Credit and Banking* 39 (4): 995-1001.

King, R.G., C.I. Plosser, and S.T. Rebelo (1988). Production, Growth and Business Cycles I: The Basic Neoclassical Model. *Journal of Monetary Economics* 21 (2-3): 195-232.

King, R.G., C.I. Plosser, and S.T. Rebelo (2002). Production, Growth and Business Cycles: Technical Appendix. *Computational Economics* 20 (1-2): 87-116.

Kittel, B. (1999). Sense and Sensitivity in Pooled Analysis of Political Data. *European Journal of Political Research* 35 (2): 225-53.

Kokoszczyński, R. (2002). Monetary Policy in Central and Eastern Europe: The Case for Direct Inflation Targeting. In G. Tumpel-Gugerell, L. Wolfe, and P. Mooslechner

(eds.), *Completing Transition: The Main Challenges.* Springer. New York, pp. 191-202.

Kose, M.A., C. Otrok, and C.H. Whiteman (2003). International Business Cycles: World, Region, and Country-Specific Factors. *American Economic Review* 93 (4): 1216-39.

Kose, M.A., E. Prasad, and M. Terrones (2003). How Does Globalization Affect the Synchronization of Business Cycles? *American Economic Review* 93 (2): 56-62.

Krusec, D. (2005). Is Inflation Targeting Effective? Monetary Transmission in Poland, the Czech Republic, Slovakia and Hungary. Mimeo. European University Institute. Firenze, Italy.

Kydland, F.E. and E.C. Prescott (1977). Rules Rather than Discretion: The Inconsistency of Optimal Plans. *Journal of Political Economy* 85 (3): 473–91.

Kydland, F.E. and E.C. Prescott (1982). Time to Build and Aggregate Fluctuations. *Econometrica* 50 (6): 1345-70.

Lagarias, J.C., J.A. Reeds, M.H. Wright, and P.E. Wright (1998). Convergence Properties of the Nelder-Mead Simplex Method in Low Dimensions. *SIAM Journal of Optimization* 9 (1): 112-47.

Lane, P.R. (2001). The New Open Economy Macroeconomics: A Survey. *Journal of International Economics* 54 (2): 235-66.

Leheyda, N. (2006). Determinants of Inflation in Ukraine: A Cointegration Approach. In Vinhas de Souza, L. and P. De Lombaerde (eds.), *The Periphery of the Euro: Monetary and Exchange Rate Policy in CIS Countries.* Ashgate. Aldershot, UK, pp. 313-46.

Levy Yeyati, E. (2006). Financial Dollarization: Evaluating the Consequences. *Economic Policy* 21 (45): 61-118.

Levy-Yeyati, E. and F. Sturzenegger (2005). Classifying Exchange Rate Regimes: Deeds vs. Words. *European Economic Review* 49 (6): 1603-35.

Lipińska, A. (2006). The Maastricht Convergence Criteria and Optimal Monetary Policy for the EMU Accession Countries. Mimeo. London School of Economics. London, UK.

Lissovolik, B. (2003). Determinants of Inflation in a Transition Economy: The Case of Ukraine. IMF Working Papers 03/126. International Monetary Fund. Washington, DC.

Liu, Z. and E. Pappa (2008). Gains from International Monetary Policy Coordination: Does It Pay to Be Different? *Journal of Economic Dynamics and Control* 32 (7): 2085-117.

Loayza, N. and R. Soto (2002), *Inflation Targeting: Design, Performance, Challenges.* Central Bank of Chile. Santiago, Chile.

Mafi-Kreft, E. and S. Kreft (2006). Importing Credible Monetary Policy: A Way for Transition Economies to Fight Inflation? *Economics Letters* 92 (1): 1-6.

Marcet, A. (2005). Overdifferencing VAR's Is Ok. Paper presented at CREI conference "Macroeconomics and Reality, 25 Years later", Barcelona, Spain, April 2005.

Marin, D. (2006). A New International Division of Labor in Europe: Outsourcing and Offshoring to Eastern Europe. *Journal of the European Economic Association* 4 (2-3): 612-22.

Minella, A. (2003). Monetary Policy and Inflation in Brazil (1975-2000): A VAR Estimation. *Revista Brasileira de Economia* 57 (3): 605-35.

Miranda, M.J. and P.L. Fackler (2002) *Applied Computational Economic and Finance.* MIT Press. Cambridge, MA.

Mohanty, M.S. and M. Klau (2005). Monetary Policy Rules in Emerging Market Economies: Issues and Evidence. In R.J. Langhammer and L. Vinhas de Souza (eds.), *Monetary Policy and Macroeconomic Stabilization in Latin America.* Springer. Berlin, Germany, pp. 205-45.

Monacelli, T. (2001). New International Monetary Arrangements and the Exchange Rate. *International Journal of Finance and Economics* 6 (4): 389-400.

Morandé, F.G. (2002). A Decade of Inflation Targeting in Chile: Developments, Lessons, and Challenges. In N. Loayza and R. Soto (eds.), *Inflation Targeting: Design, Performance, Challenges.* Central Bank of Chile. Santiago, Chile, pp. 583-626.

Morandé, F.G. and M. Tapia (2002). Exchange Rate Policy in Chile: From the Band to Floating and Beyond. Working Paper 152. Central Bank of Chile. Santiago, Chile.

Mundell, R.A. (1961). A Theory of Optimal Currency Areas. *American Economic Review* 51 (4): 657-65.

Mussa, M. (2002). *Argentina and the Fund: From Triumph to Tragedy.* Institute for International Economics. Washington, DC.

Noton, C. (2003). The Pass-Through from Depreciation to Inflation: Chile 1986-2001. *Estudios de Economía* 30 (1): 133-55.

Ohnsorge, F. and N. Oomes (2004). Inflation, Money Demand, and De-Dollarization. IMF Country Report 04/316. International Monetary Fund. Washington, DC, pp. 10-42.

Orlowski, L.T. and K. Rybinski (2006). Implications of ERM2 for Poland's Monetary Policy. *Economic Systems* 30 (4): 346-65.

Pappa, E. (2004). Do the ECB and the Fed really Need to Cooperate? Optimal Monetary Policy in a Two-Country World. *Journal of Monetary Economics* 51 (4): 573-779.

Pelipas, I. (2006). Money Demand and Inflation in Belarus: Evidence from Cointegrated VAR. *Research in International Business and Finance* 20 (2): 200-14.

Prais, S. and C. Winsten (1954). Trend Estimation and Serial Correlation. Cowles Commission Discussion Paper 383. Chicago, IL.

Revenko, A. (2006). The Consumer Price Index: Trust It, but Test It. *Zerkalo Nedeli,* 8, No. 587.

Rodríguez Palenzuela, D., G. Camba-Méndez, and J.A. Garcia (2003). Relevant Economic Issues Concerning the Optimal Rate of Inflation. European Central Bank Working Paper 278. European Central Bank. Frankfurt, Germany.

Rogoff, K. (2003). Globalization and Global Disinflation. *Economic Review—Federal Reserve Bank of Kansas City* 88 (4): 45-79.

Romer, D. (1993). Openness and Inflation: Theory and Evidence. *Quarterly Journal of Economics,* 108 (4): 869-903.

Romer, D. (2001). *Advanced Macroeconomics.* 2nd edition. McGraw-Hill. New York.

Roodman, D. (2006). How to Do xtabond2: An Introduction to "Difference" and "System" GMM in Stata. Working Paper 103. Center for Global Development. Washington, DC.

Sarno, L. (2001). Toward a New Paradigm in Open Economy Modeling: Where Do We Stand? *Review Federal Reserve Bank St. Louis* 83 (3): 21-36.

Schaechter, A., M.R. Stone, and M. Zelmer (2000). Adopting Inflation Targeting: Practical Issues for Emerging Market Countries. Occasional Paper 202. International Monetary Fund. Washington, DC.

Schiff, J.A., P. Egoumé-Bossogo, M. Ihara, T. Konuki, and K. Krajnyák (2006). Labor Market Performance in Transition: The Experience of Central and Eastern Europe. IMF Occasional Paper 248. International Monetary Fund. Washington, DC.

Schmidt-Hebbel, K. and M. Tapia (2002). Inflation Targeting in Chile. *North American Journal of Economics and Finance* 13 (2): 125-46.

Schmidt-Hebbel, K. and M. Tapia (2003). Towards Floating and Inflation Targeting in Chile. Paper presented at CEPR/Deutsche Bank Research Workshop "Managed Floating – An Alternative to the Two Corner Solutions?". January 30–31, 2003. Kronberg/Taunus, Germany.

Schmitt-Grohé, S. and M. Uribe (2003). Closing Small Open Economy Models. *Journal of International Economics* 61 (1): 163-85.

Schmitt-Grohé, S. and M. Uribe (2007). Optimal Simple and Implementable Monetary and Fiscal Rules. *Journal of Monetary Economics* 54 (6): 1702-25.

Shiratsuka, S. (1999). Measurement Errors and Quality-Adjustment Methodology: Lessons from the Japanese CPI. *Economic Perspectives* 23 (2): 2-13.

Sims, C.A. (1992). Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy. *European Economic Review* 36 (5): 975-1011.

Sims, C.A. and T. Zha (1999). Error Bands for Impulse Responses. *Econometrica* 67 (5): 1113-55.

Siliverstovs, B. and O. Bilan (2005). Modeling Inflation Dynamics in Transition Economies: The Case of Ukraine. *Eastern European Economics* 43 (6): 66-81.

Stock, J.H. and M.W. Watson (2001). Vector Autoregressions. *Journal of Economic Perspectives* 15 (4): 101-15.

Stock, J.H., J.H. Wright, and M. Yogo (2002). A Survey of Weak Instruments and Weak Identification in Generalized Method of Moments. *Journal of Business and Economic Statistics* 20 (4): 518-29.

Svensson, L.E.O. (1999). Inflation Targeting as a Monetary Policy Rule. *Journal of Monetary Economics* 43 (3): 607-54.

Svensson, L.E.O. (2000). Open-Economy Inflation Targeting. *Journal of International Economics* 50 (1): 155-83.

Svensson, L.E.O. (2003). What Is Wrong with Taylor Rules? Using Judgement in Monetary Policy through Targeting Rules. *Journal of Economic Literature* 41 (2): 426-77.

Taylor, J.B. (1993). Discretion Versus Policy Rules in Practice. *Carnegie-Rochester Conference Series on Public Policy* 39: 195-214.

Teo, W.L. (2005). Should East Asia's Currencies Be Pegged to the Yen? The Role of Invoice Currency. Mimeo. Johns Hopkins University. Baltimore, MD.

Uhlig, H. (1999). A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily. In R. Marimon and A. Scott (eds.), *Computational Methods for the Study of Dynamic Economies.* Oxford University Press. Oxford, UK, pp. 30-61.

Vdovichenko, A.G. and Voronina, V.G. (2006). Monetary Policy Rules and their Application in Russia. *Research in International Business and Finance,* 20 (2): 145-62.

Von Hagen, J., B. Hayo, and I. Fender (2002). Geldtheorie, Geldpolitik und Finanzmärkte. In K.F. Zimmermann (ed.), *Neue Entwicklungen in der Wirtschaftswissenschaft.* Physica-Verlag. Heidelberg, Germany, pp. 1-42.

White, H. (1980). A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity. *Econometrica* 48 (4): 817-38.

Wickham, P. (2002). Do "Flexible" Exchange Rates of Developing Countries Behave Like the Floating Exchange Rates of Industrialized Countries? IMF Working Paper 02/82. International Monetary Fund. Washington, DC.

Williamson, J. (2000). Exchange Rate Regimes for Emerging Markets: Reviving the Intermediate Option. Policy Analyses in International Economics 60. Institute for International Economics. Washington, DC.

Wróbel, E. and M. Pawłowska (2002). Monetary Transmission in Poland: Some Evidence on Interest and Credit Channels. Paper 24. National Bank of Poland. Warsaw, Poland.

Woodford, M. (2003a). *Inflation Targeting.* NBER Conference Series. University of Chicago Press. Chicago, IL.

Woodford, M. (2003b). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press. Princeton, NJ.

7 Technical Appendix to Chapter 4

7.1 Households

7.1.1 Utility Functions and Consumption Indexes

The representative household in country i, $i = A, B, C$ (e.g., EMU accession country, the rest of the world, euro area) maximizes the expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t^i(C_t^i, L_t^i), \tag{7.1}$$

with the respective period utility

$$U_t^i = \frac{1}{1-\mu} {C_t^i}^{1-\mu} - \frac{1}{1+\gamma} {L_t^i}^{1+\gamma}. \tag{7.2}$$

Consumption Basket

The household's utility-based consumption basket distinguishes between goods from the periphery, P, and the center, C,

$$C^i = \left[(n_p)^{\frac{1}{\theta}} \left(C_P^i\right)^{\frac{\theta-1}{\theta}} + (1-n_P)^{\frac{1}{\theta}} \left(C_C^i\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

where n_P is the size of the periphery relative to the center and θ is the intratemporal elasticity of substitution between goods from the center versus the periphery.[1]

The basket of goods produced in the periphery, i.e., in the two countries A and B, is defined as

$$C_P^i = \left[(n_A)^{\frac{1}{\psi}} \left(C_A^i\right)^{\frac{\psi-1}{\psi}} + (1-n_A)^{\frac{1}{\psi}} \left(C_B^i\right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}},$$

where n_A is the size of country A relative to country B and ψ is the intratemporal elasticity of substitution between goods from country A versus country B, i.e., between the two goods from the periphery.

[1] For simplicity, we drop the time subscript.

Taken together the two consumption baskets form the nested consumption basket

$$\begin{aligned} C^i &= \left[(n_p)^{\frac{1}{\theta}} \left(\left[(n_A)^{\frac{1}{\psi}} \left(C_A^i \right)^{\frac{\psi-1}{\psi}} + (1-n_A)^{\frac{1}{\psi}} \left(C_B^i \right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \right)^{\frac{\theta-1}{\theta}} \right. \\ &\quad \left. + (1-n_P)^{\frac{1}{\theta}} \left(C_C^i \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}. \end{aligned}$$

Consumption Basket with Home Bias

Each country prefers home goods over foreign goods, the so-called home bias in consumption. The degree of openness, η, renders the consumption basket of each country

$$\begin{aligned} C^A &= \left[[1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(\left[[1-\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_A^A \right)^{\frac{\psi-1}{\psi}} \right. \right. \right. \\ &\quad \left. \left. \left. + [\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_B^A \right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \right)^{\frac{\theta-1}{\theta}} + [\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_C^A \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \end{aligned} \tag{7.3}$$

$$\begin{aligned} C^B &= \left[[1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(\left[(\eta n_A)^{\frac{1}{\psi}} \left(C_A^B \right)^{\frac{\psi-1}{\psi}} + (1-\eta n_A)^{\frac{1}{\psi}} \left(C_B^B \right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \right)^{\frac{\theta-1}{\theta}} \right. \\ &\quad \left. + [\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_C^B \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \end{aligned} \tag{7.4}$$

$$\begin{aligned} C^C &= \left[(\eta n_P)^{\frac{1}{\theta}} \left(\left[(n_A)^{\frac{1}{\psi}} \left(C_A^C \right)^{\frac{\psi-1}{\psi}} + (1-n_A)^{\frac{1}{\psi}} \left(C_B^C \right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \right)^{\frac{\theta-1}{\theta}} \right. \\ &\quad \left. + (1-\eta n_P)^{\frac{1}{\theta}} \left(C_C^C \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}. \end{aligned} \tag{7.5}$$

The trade shares are given by

Country A	
$v_A^A =$	$1-[1-\eta(1-n_P)][\eta(1-n_A)]-\eta(1-n_P)$
$=$	$[1-\eta(1-n_P)][1-\eta(1-n_A)]$
$v_B^A =$	$[1-\eta(1-n_P)][\eta(1-n_A)]$
$v_C^A =$	$\eta(1-n_P)$

	Country B
$v_A^B =$	$[1-\eta(1-n_P)]\,\eta n_A$
$v_B^B =$	$1-[1-\eta(1-n_P)]\,\eta n_A-\eta(1-n_P)$
$=$	$(1-\eta n_A)\,[1-\eta(1-n_P)]$
$v_C^B =$	$\eta(1-n_P)$

	Country C
$v_A^C =$	$\eta n_P n_A$
$v_B^C =$	$\eta n_p(1-n_A)$
$v_C^C =$	$1-\eta n_P$

Consumption Index

The consumption indexes for the three countries i, $i = A, B, C$, are given by

$$C_A^i = \left[\left(\frac{1}{n_P n_A}\right)^{\frac{1}{\phi}} \int_0^{n_P n_A} \left(C_A^i(z)\right)^{\frac{\phi-1}{\phi}} dz\right]^{\frac{\phi}{\phi-1}} \tag{7.6}$$

$$C_B^i = \left[\left(\frac{1}{n_P(1-n_A)}\right)^{\frac{1}{\phi}} \int_{n_P n_A}^{n_P} \left(C_B^i(z)\right)^{\frac{\phi-1}{\phi}} dz\right]^{\frac{\phi}{\phi-1}} \tag{7.7}$$

$$C_C^i = \left[\left(\frac{1}{1-n_P}\right)^{\frac{1}{\phi}} \int_{n_P}^{1} \left(C_C^i(z)\right)^{\frac{\phi-1}{\phi}} dz\right]^{\frac{\phi}{\phi-1}}, \tag{7.8}$$

where the subscript refers to the country of production and ϕ is the elasticity of substitution for goods of different brands z, i.e., "varieties". The size of each country is given by

	Country A	Country B	Country C
Size	$n_P n_A$	$n_P(1-n_A)$	$1-n_P$

7.1.2 Price Index

Consumer Price Index

The utility-based price indexes are defined analogously to the consumption basket and are therefore broken down into the overall index

$$P^i = \left[n_P \left(P_P^i \right)^{1-\theta} + (1 - n_P) \left(P_C^i \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{7.9}$$

and the sub-index for prices of goods from the two periphery countries

$$P_P^i = \left[n_A \left(P_A^i \right)^{1-\psi} + (1 - n_A) \left(P_B^i \right)^{1-\psi} \right]^{\frac{1}{1-\psi}}. \tag{7.10}$$

As with the consumption basket, introducing home bias renders the consumer price index for each country

$$\begin{aligned} P^A &= \Big[[1 - \eta (1 - n_P)] \Big[\Big([1 - \eta (1 - n_A)] \left(P_A^A \right)^{1-\psi} \\ &\quad + \eta (1 - n_A) \left(P_B^A \right)^{1-\psi} \Big)^{\frac{1}{1-\psi}} \Big]^{1-\theta} + \eta (1 - n_P) \left(P_C^A \right)^{1-\theta} \Big]^{\frac{1}{1-\theta}} \end{aligned} \tag{7.11}$$

$$\begin{aligned} P^B &= \Big[[1 - \eta (1 - n_P)] \Big[\Big(\eta n_A \left(P_A^B \right)^{1-\psi} \\ &\quad + (1 - \eta n_A) \left(P_B^B \right)^{1-\psi} \Big)^{\frac{1}{1-\psi}} \Big]^{1-\theta} + \eta (1 - n_P) \left(P_C^B \right)^{1-\theta} \Big]^{\frac{1}{1-\theta}} \end{aligned} \tag{7.12}$$

$$\begin{aligned} P^C &= \Big[\eta n_P \Big[\Big(n_A \left(P_A^C \right)^{1-\psi} + (1 - n_A) \left(P_B^C \right)^{1-\psi} \Big)^{\frac{1}{1-\psi}} \Big]^{1-\theta} \\ &\quad + (1 - \eta n_P) \left(P_C^C \right)^{1-\theta} \Big]^{\frac{1}{1-\theta}} \end{aligned} \tag{7.13}$$

and

$$P_A^i = \left[\frac{1}{n_P n_A} \int_0^{n_P n_A} \left(P_A^i(z) \right)^{1-\phi} dz \right]^{\frac{1}{1-\phi}} \tag{7.14}$$

$$P_B^i = \left[\frac{1}{n_P (1 - n_A)} \int_{n_P n_A}^{n_P} \left(P_B^i(z) \right)^{1-\phi} dz \right]^{\frac{1}{1-\phi}} \tag{7.15}$$

$$P_C^i = \left[\frac{1}{(1-n_P)} \int_{n_P}^{1} \left(P_C^i(z) \right)^{1-\phi} dz \right]^{\frac{1}{1-\phi}}, \tag{7.16}$$

where P_i^i is the price index for domestic goods in country i or producer price index, PPI.

7.1.3 Exchange Rates

Nominal Exchange Rate

The nominal exchange rate between country i and country i' is defined as $S_{i',t}^i$, expressed as the price of currency i' in currency i.[2] Market clearing for the foreign exchange market requires

$$S_{i'}^i = \frac{1}{S_i^{i'}} \tag{7.17}$$

and that exchange rates can be expressed in their cross rates

$$S_C^A = S_B^A \cdot S_C^B. \tag{7.18}$$

Real Exchange Rate

The real exchange rate between country i and country i' is defined as the relative CPI

$$RER_{i',t}^i = \frac{S_{i',t}^i \cdot P_t^{i'}}{P_t^i}. \tag{7.19}$$

Terms of Trade

The terms of trade between country i and country i' are defined as the relative price of imports

$$ToT_{i',t}^i = \frac{P_{i',t}^i}{P_{i,t}^i}, \tag{7.20}$$

i.e., the price of imported goods (produced in country i') in home (country i) currency relative to the price of home goods (Corsetti et al. 2000: 222).

[2]The nominal exchange rate is not defined in terms of the two countries' CPIs as purchasing power parity does not hold (De Paoli 2006: 7).

Alternatively, using the law of one price,[3] $P^{i}_{i',t} = S^{i}_{i',t} \cdot P^{i'}_{i',t}$, it is the nominal exchange rate times the producer price of imports (i.e., the foreign price of foreign goods) over the home price of home goods

$$ToT^{i}_{i',t} = \frac{S^{i}_{i',t} \cdot P^{i'}_{i',t}}{P^{i}_{i,t}}. \tag{7.21}$$

The terms of trade of one country are the inverse for the respective trade partner country, which follows from the inverse of the nominal exchange rate (7.17)

$$ToT^{i}_{i',t} = \frac{S^{i}_{i',t} \cdot P^{i'}_{i',t}}{P^{i}_{i,t}} = \frac{1}{S^{i'}_{i,t}} \cdot \frac{P^{i'}_{i',t}}{P^{i}_{i,t}} = \frac{1}{\frac{S^{i'}_{i,t} \cdot P^{i}_{i,t}}{P^{i'}_{i',t}}} = \frac{1}{ToT^{i'}_{i,t}}. \tag{7.22}$$

Cross rates for the terms of trade follow from the cross rates for the nominal exchange rate (7.18)

$$\begin{aligned} ToT^{A}_{C,t} &= ToT^{A}_{B,t} \cdot ToT^{B}_{C,t} \\ \frac{S^{A}_{C,t} \cdot P^{C}_{C,t}}{P^{A}_{A,t}} &= \frac{S^{A}_{B,t} \cdot P^{B}_{B,t}}{P^{A}_{A,t}} \cdot \frac{S^{B}_{C,t} \cdot P^{C}_{C,t}}{P^{B}_{B,t}} \\ \frac{S^{A}_{C,t} \cdot P^{C}_{C,t}}{P^{A}_{A,t}} &= \frac{S^{A}_{B,t} \cdot S^{B}_{C,t} \cdot P^{C}_{C,t}}{P^{A}_{A,t}} \\ S^{A}_{C,t} &= S^{A}_{B,t} \cdot S^{B}_{C,t}. \end{aligned} \tag{7.23}$$

Terms of trade relations for the periphery may be expressed in terms of the

[3] However, this does not imply purchasing power parity $P^{i}_{t} = S^{i}_{i',t} \cdot P^{i'}_{t}$.

relations of the countries A, B, and C.

$$
\begin{aligned}
ToT_P^A &= \frac{P_P^A}{P_A^A} = \frac{\left(\left[1 - \eta \left(1 - n_A \right) \right] \left(P_A^A \right)^{1-\psi} + \eta \left(1 - n_A \right) \left(P_B^A \right)^{1-\psi} \right)^{\frac{1}{1-\psi}}}{P_A^A} \\
ToT_P^A &= \frac{\left(\left[1 - \eta \left(1 - n_A \right) \right] \left(P_A^A \right)^{1-\psi} + \eta \left(1 - n_A \right) \left(S_B^A \cdot P_B^B \right)^{1-\psi} \right)^{\frac{1}{1-\psi}}}{P_A^A} \\
\left(ToT_P^A \right)^{1-\psi} &= \left[1 - \eta \left(1 - n_A \right) \right] \underbrace{\frac{\left(P_A^A \right)^{1-\psi}}{\left(P_A^A \right)^{1-\psi}}}_{1} + \eta \left(1 - n_A \right) \frac{\left(S_B^A \cdot P_B^B \right)^{1-\psi}}{\left(P_A^A \right)^{1-\psi}} \\
\left(ToT_P^A \right)^{1-\psi} &= \left[1 - \eta \left(1 - n_A \right) \right] + \eta \left(1 - n_A \right) \underbrace{\left(\frac{S_B^A \cdot P_B^B}{P_A^A} \right)^{1-\psi}}_{ToT_B^A} \\
\left(ToT_P^A \right)^{1-\psi} &= \left[1 - \eta \left(1 - n_A \right) \right] + \eta \left(1 - n_A \right) \left(ToT_B^A \right)^{1-\psi} \\
ToT_P^A &= \left[\left[1 - \eta \left(1 - n_A \right) \right] + \eta \left(1 - n_A \right) \left(ToT_B^A \right)^{1-\psi} \right]^{\frac{1}{1-\psi}} \qquad (7.24)
\end{aligned}
$$

$$
\begin{aligned}
ToT_P^B &= \frac{P_P^B}{P_B^B} = \frac{\left(\eta n_A \left(P_A^B \right)^{1-\psi} + \left(1 - \eta n_A \right) \left(P_B^B \right)^{1-\psi} \right)^{\frac{1}{1-\psi}}}{P_B^B} \\
\left(ToT_P^B \right)^{1-\psi} &= \eta n_A \frac{\left(S_A^B \cdot P_A^A \right)^{1-\psi}}{\left(P_B^B \right)^{1-\psi}} + \left(1 - \eta n_A \right) \underbrace{\frac{\left(P_B^B \right)^{1-\psi}}{\left(P_B^B \right)^{1-\psi}}}_{1} \\
\left(ToT_P^B \right)^{1-\psi} &= \eta n_A \underbrace{\left(\frac{S_A^B \cdot P_A^A}{P_B^B} \right)^{1-\psi}}_{ToT_A^B} + \left(1 - \eta n_A \right) \\
ToT_P^B &= \left[\left(1 - \eta n_A \right) + \eta n_A \left(ToT_A^B \right)^{1-\psi} \right]^{\frac{1}{1-\psi}} \qquad (7.25)
\end{aligned}
$$

$$
\begin{aligned}
ToT_P^C &= \frac{P_P^C}{P_C^C} = \frac{\left(n_A \left(P_A^C\right)^{1-\psi} + (1-n_A)\left(P_B^C\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}}{P_C^C} \\
\left(ToT_P^C\right)^{1-\psi} &= n_A \frac{\left(P_A^C\right)^{1-\psi}}{\left(P_C^C\right)^{1-\psi}} + (1-n_A)\frac{\left(P_B^C\right)^{1-\psi}}{\left(P_C^C\right)^{1-\psi}} \\
\left(ToT_P^C\right)^{1-\psi} &= n_A \left(\frac{P_A^C}{P_C^C}\right)^{1-\psi} + (1-n_A)\left(\frac{P_B^C}{P_C^C}\right)^{1-\psi} \\
\left(ToT_P^C\right)^{1-\psi} &= n_A \underbrace{\left(\frac{S_A^C \cdot P_A^A}{P_C^C}\right)^{1-\psi}}_{ToT_A^C} + (1-n_A)\underbrace{\left(\frac{S_B^C \cdot P_B^B}{P_C^C}\right)^{1-\psi}}_{ToT_B^C} \\
ToT_P^C &= \left[n_A \left(ToT_A^C\right)^{1-\psi} + (1-n_A)\left(ToT_B^C\right)^{1-\psi}\right]^{\frac{1}{1-\psi}} \qquad (7.26)
\end{aligned}
$$

Taken together, the equations for the inverse of the terms of trade (7.22), the cross rate (7.23), and the three expressions for the terms of trade related to the periphery, (7.24), (7.25), and (7.26), reduce the number of necessary terms of trade expressions to only two.

Purchasing Power Parity

If home bias is present, the law of one price does not imply purchasing power parity. For simplicity, we only show this result for the center-periphery CPI (7.9).

$$P^P \neq S_C^P \cdot P^C$$

$$
\begin{aligned}
\Leftrightarrow \quad & \left[(1-\eta(1-n_P))\left(P_P^P\right)^{1-\theta} + \eta(1-n_P)\left(P_C^P\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \\
& \neq S_C^P \cdot \left[\eta n_P \left(P_P^C\right)^{1-\theta} + (1-\eta n_P)\left(P_C^C\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \\
\Leftrightarrow \quad & (1-\eta(1-n_P))\left(P_P^P\right)^{1-\theta} + \eta(1-n_P)\left(P_C^P\right)^{1-\theta} \\
& \neq (S_C^P)^{1-\theta} \cdot \left[\eta n_P \left(P_P^C\right)^{1-\theta} + (1-\eta n_P)\left(P_C^C\right)^{1-\theta}\right]
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & (1-\eta(1-n_P))\left(P_P^P\right)^{1-\theta}+\eta(1-n_P)\left(P_C^P\right)^{1-\theta} \\
& \neq (S_C^P)^{1-\theta}\eta n_P\left(P_P^C\right)^{1-\theta}+(S_C^P)^{1-\theta}(1-\eta n_P)\left(P_C^C\right)^{1-\theta} \\
\Leftrightarrow \quad & (1-\eta(1-n_P))\left(P_P^P\right)^{1-\theta}+\eta(1-n_P)\left(P_C^P\right)^{1-\theta} \\
& \neq \eta n_P \underbrace{(S_C^P)^{1-\theta}(P_P^C)^{1-\theta}}_{(P_P^P)^{1-\theta}}+(1-\eta n_P)\underbrace{(S_C^P)^{1-\theta}(P_C^C)^{1-\theta}}_{(P_C^P)^{1-\theta}}
\end{aligned}
$$

Apply law of one price $P_C^P = S_C^P \cdot P_C^C$

$$
\begin{aligned}
\Leftrightarrow \quad & (1-\eta(1-n_P))\left(P_P^P\right)^{1-\theta}+\eta(1-n_P)\left(P_C^P\right)^{1-\theta} \\
& \neq \eta n_P(P_P^P)^{1-\theta}+(1-\eta n_P)(P_C^P)^{1-\theta}
\end{aligned}
$$

Only if there is no home bias in consumption, $\eta = 1$,

$$
\begin{aligned}
\Leftrightarrow \quad & (1-(1-n_P))\left(P_P^P\right)^{1-\theta}+(1-n_P)\left(P_C^P\right)^{1-\theta} \\
& = n_P(P_P^P)^{1-\theta}+(1-n_P)(P_C^P)^{1-\theta} \\
\Leftrightarrow \quad & n_P\left(P_P^P\right)^{1-\theta}+(1-n_P)\left(P_C^P\right)^{1-\theta} = n_P(P_P^P)^{1-\theta}+(1-n_P)(P_C^P)^{1-\theta}
\end{aligned}
$$

are the weights equal across countries and purchasing power parity holds

$$P^i = S_{i'}^i \cdot P^{i'}.$$

CPI-PPI Ratio

For convenience, we derive the CPI-PPI ratios for each country. The expression links CPI prices and domestic prices (PPI) to the terms of trade.

CPI-PPI Ratio for Country A Rewriting the consumer price index (7.11) for country A

$$
\begin{aligned}
P^A \quad = \quad & \Big[[1-\eta(1-n_P)]\Big[\Big([1-\eta(1-n_A)]\left(P_A^A\right)^{1-\psi} \\
& +\eta(1-n_A)\left(P_B^A\right)^{1-\psi}\Big)^{\frac{1}{1-\psi}}\Big]^{1-\theta}+\eta(1-n_P)\left(P_C^A\right)^{1-\theta}\Big]^{\frac{1}{1-\theta}}
\end{aligned}
$$

$$\begin{aligned}\left(P^A\right)^{1-\theta} &= [1-\eta(1-n_P)]\left[\left([1-\eta(1-n_A)]\left(P_A^A\right)^{1-\psi}\right.\right.\\ &\quad\left.\left.+\eta(1-n_A)\left(P_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta}+\eta(1-n_P)\left(P_C^A\right)^{1-\theta}.\end{aligned}$$

Divide both sides by $\left(P_A^A\right)^{1-\theta}$

$$\begin{aligned}\frac{\left(P^A\right)^{1-\theta}}{\left(P_A^A\right)^{1-\theta}} &= \frac{1}{\left(P_{A,t}^A\right)^{1-\theta}}[1-\eta(1-n_P)]\left[\left([1-\eta(1-n_A)]\left(P_A^A\right)^{1-\psi}\right.\right.\\ &\quad\left.\left.+\eta(1-n_A)\left(P_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta}+\eta(1-n_P)\frac{\left(P_C^A\right)^{1-\theta}}{\left(P_A^A\right)^{1-\theta}}\end{aligned}$$

$$\begin{aligned}\left(\frac{P^A}{P_A^A}\right)^{1-\theta} &= [1-\eta(1-n_P)]\left[\left(\frac{1}{\left(P_A^A\right)^{1-\theta}}\right)^{\frac{1}{1-\theta}}\left([1-\eta(1-n_A)]\left(P_A^A\right)^{1-\psi}\right.\right.\\ &\quad\left.\left.+\eta(1-n_A)\left(P_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta}+\eta(1-n_P)\left(\frac{P_C^A}{P_A^A}\right)^{1-\theta}\end{aligned}$$

$$\begin{aligned}\left(\frac{P^A}{P_A^A}\right)^{1-\theta} &= [1-\eta(1-n_P)]\left[\frac{1}{P_A^A}\left([1-\eta(1-n_A)]\left(P_A^A\right)^{1-\psi}\right.\right.\\ &\quad\left.\left.+\eta(1-n_A)\left(P_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta}+\eta(1-n_P)\left(\frac{P_C^A}{P_A^A}\right)^{1-\theta}\end{aligned}$$

$$\begin{aligned}\left(\frac{P^A}{P_A^A}\right)^{1-\theta} &= [1-\eta(1-n_P)]\left[\left(\left(\frac{1}{P_A^A}\right)^{1-\psi}[1-\eta(1-n_A)]\left(P_A^A\right)^{1-\psi}\right.\right.\\ &\quad\left.\left.+\left(\frac{1}{P_A^A}\right)^{1-\psi}\eta(1-n_A)\left(P_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta}+\eta(1-n_P)\left(\frac{P_C^A}{P_A^A}\right)^{1-\theta}\end{aligned}$$

$$\begin{aligned}\left(\frac{P^A}{P_A^A}\right)^{1-\theta} &= [1-\eta(1-n_P)]\left[\left([1-\eta(1-n_A)]\left(\frac{P_A^A}{P_A^A}\right)^{1-\psi}\right.\right.\\ &\quad\left.\left.+\eta(1-n_A)\left(\frac{P_B^A}{P_A^A}\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta}+\eta(1-n_P)\left(\frac{P_C^A}{P_A^A}\right)^{1-\theta}.\end{aligned}$$

Applying the definition of the terms of trade (7.20)

$$\left(\frac{P^A}{P_A^A}\right)^{1-\theta} = [1-\eta(1-n_P)]\left[\left([1-\eta(1-n_A)]\underbrace{\left(\frac{P_A^A}{P_A^A}\right)^{1-\psi}}_{1}\right.\right.$$
$$\left.\left.+\eta(1-n_A)\underbrace{\left(\frac{P_B^A}{P_A^A}\right)^{1-\psi}}_{ToT_B^A}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + \eta(1-n_P)\underbrace{\left(\frac{P_C^A}{P_A^A}\right)^{1-\theta}}_{ToT_C^A}$$

$$\left(\frac{P^A}{P_A^A}\right)^{1-\theta} = [1-\eta(1-n_P)]\left[\left([1-\eta(1-n_A)]+\eta(1-n_A)\left(ToT_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta}$$
$$+\eta(1-n_P)\left(ToT_C^A\right)^{1-\theta}.$$

Define new function $g_A(ToT)$

$$\frac{P^A}{P_A^A} = \left[[1-\eta(1-n_P)]\left[\left([1-\eta(1-n_A)]+\eta(1-n_A)\left(ToT_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta}\right.$$
$$\left.+\eta(1-n_P)\left(ToT_C^A\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \equiv g_A(ToT). \tag{7.27}$$

CPI-PPI Ratio for Country B Rewriting the consumer price index (7.12) for country B

$$P^B = \left[[1-\eta(1-n_P)]\left[\left(\eta n_A\left(P_A^B\right)^{1-\psi}\right.\right.\right.$$
$$\left.\left.\left.+(1-\eta n_A)\left(P_B^B\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta}+\eta(1-n_P)\left(P_C^B\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

$$\frac{\left(P^B\right)^{1-\theta}}{\left(P_B^B\right)^{1-\theta}} = [1-\eta(1-n_P)]\left[\left(\frac{1}{\left(P_B^B\right)^{1-\theta}}\right)^{\frac{1}{1-\theta}}\left(\eta n_A\left(P_A^B\right)^{1-\psi}\right.\right.$$
$$\left.\left.+(1-\eta n_A)\left(P_B^B\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta}+\eta(1-n_P)\frac{\left(P_C^B\right)^{1-\theta}}{\left(P_B^B\right)^{1-\theta}}$$

$$\left(\frac{P^B}{P_B^B}\right)^{1-\theta} = [1-\eta(1-n_P)]\left[\left(\left(\frac{1}{P_B^B}\right)^{1-\psi}\eta n_A\left(P_A^B\right)^{1-\psi} + \left(\frac{1}{P_B^B}\right)^{1-\psi}(1-\eta n_A)\left(P_B^B\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + \eta(1-n_P)\left(\frac{P_C^B}{P_B^B}\right)^{1-\theta}$$

$$\left(\frac{P^B}{P_B^B}\right)^{1-\theta} = [1-\eta(1-n_P)]\left[\left(\eta n_A\underbrace{\left(\frac{P_A^B}{P_B^B}\right)}_{ToT_A^B}^{1-\psi} + (1-\eta n_A)\underbrace{\left(\frac{P_B^B}{P_B^B}\right)}_{1}^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + \eta(1-n_P)\underbrace{\left(\frac{P_C^B}{P_B^B}\right)}_{ToT_C^B}^{1-\theta}$$

$$\left(\frac{P^B}{P_B^B}\right)^{1-\theta} = [1-\eta(1-n_P)]\left[\left(\eta n_A\left(ToT_A^B\right)^{1-\psi} + (1-\eta n_A)\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + \eta(1-n_P)\left(ToT_C^B\right)^{1-\theta}.$$

Define new function $g_B(ToT)$

$$\frac{P^B}{P_B^B} = \left[[1-\eta(1-n_P)]\left[\left(\eta n_A\left(ToT_A^B\right)^{1-\psi} + (1-\eta n_A)\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + \eta(1-n_P)\left(ToT_C^B\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \equiv g_B(ToT). \tag{7.28}$$

CPI-PPI Ratio for Country C Rewriting the consumer price index (7.13) for country C

$$P^C = \left[\eta n_P\left[\left(n_A\left(P_A^C\right)^{1-\psi} + (1-n_A)\left(P_B^C\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + (1-\eta n_P)\left(P_C^C\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

$$\frac{\left(P^C\right)^{1-\theta}}{\left(P_C^C\right)^{1-\theta}} = \eta n_P \left[\left(\frac{1}{\left(P_C^C\right)^{1-\theta}}\right)^{\frac{1}{1-\theta}} \left(n_A \left(P_A^C\right)^{1-\psi} + (1-n_A)\left(P_B^C\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + (1-\eta n_P) \underbrace{\frac{\left(P_C^C\right)^{1-\theta}}{\left(P_C^C\right)^{1-\theta}}}_{1}$$

$$\left(\frac{P^C}{P_C^C}\right)^{1-\theta} = \eta n_P \left[\left(\left(\frac{1}{P_C^C}\right)^{1-\psi} n_A \left(P_A^C\right)^{1-\psi} + \left(\frac{1}{P_C^C}\right)^{1-\psi} (1-n_A)\left(P_B^C\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + (1-\eta n_P)$$

$$\left(\frac{P^C}{P_C^C}\right)^{1-\theta} = \eta n_P \left[\left(n_A \underbrace{\left(\frac{P_A^C}{P_C^C}\right)}_{ToT_A^C}{}^{1-\psi} + (1-n_A) \underbrace{\left(\frac{P_B^C}{P_C^C}\right)}_{ToT_B^C}{}^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + (1-\eta n_P).$$

Define new function $g_C(ToT)$

$$\frac{P^C}{P_C^C} = \left[\eta n_P \left[\left(n_A \left(ToT_A^C\right)^{1-\psi} + (1-n_A)\left(ToT_B^C\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + (1-\eta n_P)\right]^{\frac{1}{1-\theta}} \equiv g_C(ToT). \tag{7.29}$$

CPI-PPI Ratio for Periphery P Rewriting the consumer price index for periphery P

$$\begin{aligned}
P^P &= \left[\left[1-\eta\left(1-n_P\right)\right]\left(P_P^P\right)^{1-\theta}+\eta\left(1-n_P\right)\left(P_C^P\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \\
\left(P^P\right)^{1-\theta} &= \left[1-\eta\left(1-n_P\right)\right]\left(P_P^P\right)^{1-\theta}+\eta\left(1-n_P\right)\left(P_C^P\right)^{1-\theta} \\
\frac{\left(P^P\right)^{1-\theta}}{\left(P_P^P\right)^{1-\theta}} &= \left[1-\eta\left(1-n_P\right)\right]\underbrace{\frac{\left(P_P^P\right)^{1-\theta}}{\left(P_P^P\right)^{1-\theta}}}_{1}+\eta\left(1-n_P\right)\frac{\left(P_C^P\right)^{1-\theta}}{\left(P_P^P\right)^{1-\theta}} \\
\left(\frac{P^P}{P_P^P}\right)^{1-\theta} &= \left[1-\eta\left(1-n_P\right)\right]+\eta\left(1-n_P\right)\underbrace{\left(\frac{P_C^P}{P_P^P}\right)^{1-\theta}}_{ToT_C^P} \\
\frac{P^P}{P_P^P} &= \left[\left[1-\eta\left(1-n_P\right)\right]+\eta\left(1-n_P\right)\left(ToT_C^P\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \equiv g_P\left(ToT\right).
\end{aligned}$$

RER-ToT Relation

Rearranging the terms of trade definition (7.21) to isolate the nominal exchange rate

$$ToT_{i',t}^{i}\frac{P_{i,t}^{i}}{P_{i',t}^{i'}}=S_{i',t}^{i}.$$

Substituting this nominal exchange rate expression in the definition of the real exchange rate (7.19)

$$\begin{aligned}
RER_{i',t}^{i} &= \frac{ToT_{i',t}^{i}\frac{P_{i,t}^{i}}{P_{i',t}^{i'}}\cdot P_t^{i'}}{P_t^i} \\
&= ToT_{i',t}^{i}\frac{P_t^{i'}}{P_{i',t}^{i'}}\frac{P_{i,t}^{i}}{P_t^i} \\
&= ToT_{i',t}^{i}\underbrace{\frac{P_t^{i'}}{P_{i',t}^{i'}}}_{g_{i'}(ToT)}\underbrace{\left(\frac{P_t^i}{P_{i,t}^i}\right)^{-1}}_{g_i(ToT)} \\
RER_{i',t}^{i} &= ToT_{i',t}^{i}\cdot g_{i'}(ToT)\cdot g_i(ToT)^{-1} \qquad (7.30) \\
RER_{i',t}^{i} &= ToT_{i',t}^{i}\frac{g_{i'}(ToT)}{g_i(ToT)}.
\end{aligned}$$

See Faia and Monacelli (2004: 7, eq. 16).

7.1.4 Budget Constraint

The household is constrained by the period budget constraint, where bonds B_t^i are only denominated in country A's currency[4]

$$\sum_{h^{t+1}} Q_{t,t+1} S_{A,t}^i B_{t+1}^i + P_t^i C_t^i = S_{A,t}^i B_t^i + W_t^i L_t^i + \Psi_t^i,$$

which corresponds to

$$\sum_{h^{t+1}} Q_{t,t+1} \frac{1}{S_{i,t}^A} B_{t+1}^i + P_t^i C_t^i = \frac{1}{S_{i,t}^A} B_t^i + W_t^i L_t^i + \Psi_t^i. \tag{7.31}$$

Budget constraint for country A, i.e., the home country[5]

$$\sum_{h^{t+1}} Q_{t,t+1} B_{t+1}^A + P_t^A C_t^A = B_t^A + W_t^A L_t^A + \Psi_t^A. \tag{7.32}$$

Budget constraint for country B, i.e., the foreign country

$$\sum_{h^{t+1}} Q_{t,t+1} \frac{1}{S_{B,t}^A} B_{t+1}^B + P_t^B C_t^B = \frac{1}{S_{B,t}^A} B_t^B + W_t^B L_t^B + \Psi_t^B. \tag{7.33}$$

Budget constraint for country C, i.e., the other foreign country

$$\sum_{h^{t+1}} Q_{t,t+1} \frac{1}{S_{C,t}^A} B_{t+1}^C + P_t^C C_t^C = \frac{1}{S_{C,t}^A} B_t^C + W_t^C L_t^C + \Psi_t^C. \tag{7.34}$$

Interest Rate

The gross nominal interest rate, R_t^i, on a riskless one-period bond in country i is defined as (see Faia and Monacelli 2004: 6)

$$R_t^i \equiv \frac{1}{E_t \left\{ \frac{S_{i,t+1}^A}{S_{i,t}^A} Q_{t,t+1} \right\}} \tag{7.35}$$

[4]See Faia and Monacelli (2004). The choice of country in which the bonds are denominated does not matter due to complete markets. The budget constraint in Galí and Monacelli (2005) is in real terms. Following Galí and Monacelli (2005: 710) there is no money in the budget constraint, nor in the utility function, as monetary policy is specified in terms of an interest rate rule. Teo (2005) uses bond adjustment costs instead of complete markets to ensure stationarity.

[5]Notice that $S_{A,t}^A = 1$.

Variables

C_t^i is aggregate consumption

L_t^i are labor hours

B_{t+1}^i are nominal state contigent security denominated in country A's currency $Q_{t,t+1} \equiv Q\left(h^{t+1} \mid h^t\right)$ is the period-t price of a claim to one unit of country A's currency in state h^{t+1} divided by the probability of occurence of that state; each asset in the portfolio B_{t+1}^i pays one unit of country A's currency at time $t+1$ and in state h^{t+1} (see Faia and Monacelli 2004).

$S_{i',t}^i$ is the nominal exchange rate between country i and country i', expressed as the price of country i' in currency i, i.e., an increase in $S_{i',t}^i$ corresponds to a depreciation of currency i against currency i'.

R_t^i is the gross nominal interest rate on a riskless one-period bond in country i.

W_t^i is the nominal wage rate

P_t^i is the consumer price index

Ψ_t^i is the dividend of domestic households from domestic firms

Elasticities

μ risk aversion: 2

γ inverse of the elasticity of labor supply: 3

θ elasticity of substitution for goods from the center versus the periphery, i. e., intratemporal elasticity: 1.5

ψ elasticity of substitution for goods from country A versus country B: 0.05 for complements and 3 for substitutes

ϕ elasticity of substitution for goods of different brands z, i. e., "varieties"

Parameters

β subjective discount factor: 0.99

η degree of openness: 1 (completely open) or 0.5 (with home bias)

7.1.5 Household's Utility Maximization

Maximize (7.1) subject to (7.31)

$$\begin{aligned} \max_{\{C_t^i(z),\, L_t^i(z)\}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t U_t^i(C_t^i, L_t^i) \\ \text{s.t.} \quad & \sum_{h^{t+1}} Q_{t,t+1} \frac{1}{S_{i,t}^A} B_{t+1}^i + P_t^i C_t^i = \frac{1}{S_{i,t}^A} B_t^i + W_t^i L_t^i + \Psi_t^i \end{aligned}$$

First Order Conditions (FOC)

FOCs are the same for all three countries i, $i = A, B, C$.

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial C_t^i} &= \frac{1-\mu}{1-\mu} \left(C_t^i\right)^{-\mu} - \lambda_t^i P_t^i = 0 \\ &\Leftrightarrow \underbrace{\left(C_t^i\right)^{-\mu}}_{\frac{\partial U_t^i}{\partial C_t^i}} = \lambda_t^i P_t^i \qquad (7.36) \\ \frac{\partial \mathcal{L}}{\partial L_t^i} &= -\frac{1+\gamma}{1+\gamma} \left(L_t^i\right)^{\gamma} + \lambda_t^i W_t^i = 0 \\ &\Leftrightarrow \underbrace{\left(L_t^i\right)^{\gamma}}_{-\frac{\partial U_t^i}{\partial L_t^i}} = \lambda_t^i W_t^i \qquad (7.37) \\ \frac{\partial \mathcal{L}}{\partial B_t^i} &= -\lambda_t^i \left[-\frac{1}{S_{i,t}^A}\right] - \frac{1}{\beta} \lambda_{t-1}^i Q_{t-1,t} \frac{1}{S_{i,t-1}^A} = 0 \\ &\Leftrightarrow \lambda_t^i \frac{1}{S_{i,t}^A} = \frac{1}{\beta} \lambda_{t-1}^i \frac{1}{S_{i,t-1}^A} Q_{t-1,t} \qquad \mid \cdot\beta \text{ and iterate forward} \\ &\Leftrightarrow \beta \lambda_{t+1}^i \frac{1}{S_{i,t+1}^A} = \lambda_t^i \frac{1}{S_{i,t}^A} Q_{t,t+1} \qquad (7.38) \\ &\Leftrightarrow \beta \frac{\lambda_{t+1}^i}{\lambda_t^i} \frac{S_{i,t}^A}{S_{i,t+1}^A} = Q_{t,t+1} \end{aligned}$$

Equilibrium Equations

The equilibrium conditions stem from the first order conditions.

Consumption Labor Trade-Off Using (7.36)

$$\begin{aligned} \frac{\partial U_t^i}{\partial C_t^i} &= \lambda_t^i P_t^i \\ \Leftrightarrow \quad \lambda_t^i &= \frac{\partial U_t^i}{\partial C_t^i}\frac{1}{P_t^i} = \left(C_t^i\right)^{-\mu}\frac{1}{P_t^i} \end{aligned}$$

and (7.37)

$$\begin{aligned} -\frac{\partial U_t^i}{\partial L_t^i} &= \lambda_t^i W_t^i \\ \Leftrightarrow \quad \lambda_t^i &= -\frac{\partial U_t^i}{\partial L_t^i}\frac{1}{W_t^i} = \left(L_t^i\right)^{\gamma}\ \frac{1}{W_t^i}. \end{aligned}$$

Substitute for λ_t^i

$$\begin{aligned} \left(L_t^i\right)^{\gamma}\ \frac{1}{W_t^i} &= \left(C_t^i\right)^{-\mu}\frac{1}{P_t^i} \\ \left(L_t^i\right)^{\gamma}\ P_t^i &= \left(C_t^i\right)^{-\mu} W_t^i. \end{aligned} \tag{7.39}$$

Nominal Wage

$$W_t^i = \frac{-\partial U_t^i/\partial L_t^i}{\partial U_t^i/\partial C_t^i}\, P_t^i = \frac{\left(L_t^i\right)^{\gamma}}{\left(C_t^i\right)^{-\mu}}\, P_t^i. \tag{7.40}$$

Real Wage

$$\frac{W_t^i}{P_t^i} = \frac{-\partial U_t^i/\partial L_t^i}{\partial U_t^i/\partial C_t^i} = \left(C_t^i\right)^{\mu}\left(L_t^i\right)^{\gamma}. \tag{7.41}$$

Euler Equation for Bonds Deriving the Euler equation for bonds. Taking λ_t^i from consumption FOC

$$\lambda_t^i = \left(C_t^i\right)^{-\mu}\frac{1}{P_t^i}.$$

Iterating forward one period

$$\lambda_{t+1}^i = \left(C_{t+1}^i\right)^{-\mu}\frac{1}{P_{t+1}^i}.$$

Plugging these two λ-expressions into the FOC for bonds (7.38)

$$\begin{aligned}
\overbrace{\beta(C^i_{t+1})^{-\mu}\frac{1}{P^i_{t+1}}}^{\lambda^i_{t+1}}\frac{1}{S^A_{i,t+1}} &= \overbrace{(C^i_t)^{-\mu}\frac{1}{P^i_t}}^{\lambda^i_t}\frac{1}{S^A_{i,t}}Q_{t,t+1} \\
\beta\frac{(C^i_{t+1})^{-\mu}\frac{1}{S^A_{i,t+1}P^i_{t+1}}}{(C^i_t)^{-\mu}\frac{1}{S^A_{i,t}P^i_t}} &= Q_{t,t+1} \\
\beta\left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\mu}\frac{P^i_t}{P^i_{t+1}}\frac{S^A_{i,t}}{S^A_{i,t+1}} &= Q_{t,t+1} \qquad (7.42) \\
\beta\left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\mu}\frac{P^i_t}{P^i_{t+1}} &= \frac{S^A_{i,t+1}}{S^A_{i,t}}Q_{t,t+1}.
\end{aligned}$$

Applying the definition of the gross nominal interest rate in equation (7.35)

$$\beta\left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\mu}\frac{P^i_t}{P^i_{t+1}} = \frac{1}{R^i_t}. \qquad (7.43)$$

Deriving International Risk Sharing Euler equation (7.42) for home country A

$$\beta\left(\frac{C^A_{t+1}}{C^A_t}\right)^{-\mu}\frac{P^A_t}{P^A_{t+1}} = Q_{t,t+1}.$$

Euler equation for generic foreign country i

$$\beta\left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\mu}\frac{P^i_t}{P^i_{t+1}}\frac{S^A_{i,t}}{S^A_{i,t+1}} = Q_{t,t+1}.$$

Equalize the two Euler equations due to the existence of complete markets for nominal state contingent securities

$$\beta\left(\frac{C^A_{t+1}}{C^A_t}\right)^{-\mu}\frac{P^A_t}{P^A_{t+1}} = Q_{t,t+1} = \beta\left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\mu}\frac{S^A_{i,t}P^i_t}{S^A_{i,t+1}P^i_{t+1}}. \qquad (7.44)$$

Divide by β and P^A_t, multiply with P^A_{t+1} and apply definition of the real exchange rate (7.19),

$$\begin{aligned}
\left(\frac{C^A_{t+1}}{C^A_t}\right)^{-\mu} &= \left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\mu}\underbrace{\frac{S^A_{i,t}P^i_t}{P^A_t}}_{RER^A_{i,t}}\underbrace{\frac{P^A_{t+1}}{S^A_{i,t+1}P^i_{t+1}}}_{\left(RER^A_{i,t+1}\right)^{-1}} \\
\left(\frac{C^A_{t+1}}{C^A_t}\right)^{-\mu} &= \left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\mu}\left(\frac{RER^A_{i,t+1}}{RER^A_{i,t}}\right)^{-1}. \qquad (7.45)
\end{aligned}$$

Imposing that purchasing power parity (PPP)[6] holds at all times the real exchange rate is redundant

$$\begin{aligned} P_t^A &= S_{i,t}^A P_t^i \\ 1 &= \frac{S_{i,t}^A P_t^i}{P_t^A} = RER_{i,t}^A. \end{aligned} \tag{7.46}$$

Thus, there is complete risk sharing. The level of consumption is the same in all three countries

$$\begin{aligned} \left(\frac{C_{t+1}^A}{C_t^A}\right)^{-\mu} &= \left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\mu} \\ \left(\frac{C_t^i}{C_t^A}\right)^{-\mu} &= \left(\frac{C_{t+1}^i}{C_{t+1}^A}\right)^{-\mu}. \end{aligned}$$

By iterating this equation one period forward

$$\left(\frac{C_{t+1}^i}{C_{t+1}^A}\right)^{-\mu} = \left(\frac{C_{t+2}^i}{C_{t+2}^A}\right)^{-\mu}$$

and substituting we obtain

$$\left(\frac{C_t^i}{C_t^A}\right)^{-\mu} = \left(\frac{C_{t+2}^i}{C_{t+2}^A}\right)^{-\mu}$$

and in general for time $t+k$

$$\left(\frac{C_t^i}{C_t^A}\right)^{-\mu} = \left(\frac{C_{t+k}^i}{C_{t+k}^A}\right)^{-\mu}.$$

The relation holds in every period[7]

$$\underbrace{\left(C_t^A\right)^{-\mu}}_{U_c\left(C_t^A\right)} = \underbrace{\left(C_t^i\right)^{-\mu}}_{U_c\left(C_t^i\right)} \tag{7.47}$$

$$C_t^A = C_t^i = C_t \tag{7.48}$$

[6] See also Galí and Monacelli (2005: 713, eq. 16).

[7] Notice, one actually has to take into account the initial cross-country distribution of wealth. Here, we assume symmetric initial conditions, i.e., zero net foreign asset holdings and an ex ante identical environment. See Galí and Monacelli (2005: 713) and Monacelli (2001: 391, eq. 14).

and the level of consumption is the same in all three economies.

If PPP does not hold, due to home bias in consumption, we continue from equation (7.45)

$$\begin{aligned}\left(\frac{C^A_{t+1}}{C^A_t}\right)^{-\mu} &= \left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\mu}\left(\frac{RER^A_{i,t+1}}{RER^A_{i,t}}\right)^{-1}\\ \left(\frac{C^i_t}{C^A_t}\right)^{-\mu} &= \left(\frac{C^i_{t+1}}{C^A_{t+1}}\right)^{-\mu}\left(\frac{RER^A_{i,t+1}}{RER^A_{i,t}}\right)^{-1}.\end{aligned}$$

By iterating this equation one period forward

$$\left(\frac{C^i_{t+1}}{C^A_{t+1}}\right)^{-\mu} = \left(\frac{C^i_{t+2}}{C^A_{t+2}}\right)^{-\mu}\left(\frac{RER^A_{i,t+2}}{RER^A_{i,t+1}}\right)^{-1}$$

and substituting we obtain

$$\begin{aligned}\left(\frac{C^i_t}{C^A_t}\right)^{-\mu} &= \left(\frac{C^i_{t+2}}{C^A_{t+2}}\right)^{-\mu}\left(\frac{RER^A_{i,t+2}}{RER^A_{i,t+1}}\right)^{-1}\left(\frac{RER^A_{i,t+1}}{RER^A_{i,t}}\right)^{-1}\\ \left(\frac{C^i_t}{C^A_t}\right)^{-\mu} &= \left(\frac{C^i_{t+2}}{C^A_{t+2}}\right)^{-\mu}\left(\frac{RER^A_{i,t+2}}{RER^A_{i,t}}\right)^{-1}\end{aligned}$$

and in general for time $t+k$

$$\begin{aligned}\left(\frac{C^i_t}{C^A_t}\right)^{-\mu} &= \left(\frac{C^i_{t+k}}{C^A_{t+k}}\right)^{-\mu}\left(\frac{RER^A_{i,t+k}}{RER^A_{i,t}}\right)^{-1}\\ \left(\frac{C^i_t}{C^i_{t+k}}\right)^{-\mu} &= \left(\frac{C^A_t}{C^A_{t+k}}\right)^{-\mu}\frac{RER^A_{i,t}}{RER^A_{i,t+k}}.\end{aligned}$$

The relation holds in every period

$$\begin{aligned}\underbrace{\left(C^i_t\right)^{-\mu}}_{U_c\left(C^i_t\right)} &= \underbrace{\left(C^A_t\right)^{-\mu}}_{U_c\left(C^A_t\right)}RER^A_{i,t}\\ \left(C^A_t\right)^{\mu} &= \left(C^i_t\right)^{\mu}\cdot RER^A_{i,t}\\ C^A_t &= C^i_t\cdot\left(RER^A_{i,t}\right)^{\frac{1}{\mu}}.\end{aligned} \tag{7.49}$$

See Galí and Monacelli (2005: 713, eq. 17). Substitute expression of the real exchange rate by equation (7.30) in terms of trade.

For country A and B

$$\begin{aligned} C_t^A &= C_t^B \cdot \left(RER_{B,t}^A\right)^{\frac{1}{\mu}} \\ C_t^A &= C_t^B \cdot \left(ToT_{B,t}^A \frac{g_B(ToT)}{g_A(ToT)}\right)^{\frac{1}{\mu}} \\ C_t^A &= C_t^B \left(ToT_{B,t}^A\right)^{\frac{1}{\mu}} g_B(ToT)^{\frac{1}{\mu}} g_A(ToT)^{-\frac{1}{\mu}}. \end{aligned} \tag{7.50}$$

For country A and C

$$\begin{aligned} C_t^A &= C_t^C \cdot \left(RER_{C,t}^A\right)^{\frac{1}{\mu}} \\ C_t^A &= C_t^C \cdot \left(ToT_{C,t}^A \frac{g_C(ToT)}{g_A(ToT)}\right)^{\frac{1}{\mu}} \\ C_t^A &= C_t^C \left(ToT_{C,t}^A\right)^{\frac{1}{\mu}} g_C(ToT)^{\frac{1}{\mu}} g_A(ToT)^{-\frac{1}{\mu}}. \end{aligned} \tag{7.51}$$

7.1.6 Deriving Demand Functions

The demand functions stem from a cost minimization of the household when consuming goods from different countries, i, and different brands, z.

Individual Demand Function A

Notice, the demand function for individual brands z depends only on the size of the country and not on the trade share. Hence, home bias does not enter.

$$\begin{aligned} \max_{C_A^i(z)} \quad & C_A^i = \left[(n_P n_A)^{-\frac{1}{\phi}} \int_0^{n_P n_A} \left(C_A^i(z)\right)^{\frac{\phi-1}{\phi}} dz\right]^{\frac{\phi}{\phi-1}} \\ \text{s.t.} \quad & X_t \geq \int_0^{n_P n_A} P_A^i(z)\, C_A^i(z)\, dz \end{aligned}$$

Transform into Lagrangian $\mathcal{L} = f(x) - \lambda \cdot h(x)$

$$h(x) \equiv X_t - \int_0^{n_P n_A} P_A^i(z)\, C_A^i(z)\, dz \geq 0$$

$$\mathcal{L} = \left[(n_P n_A)^{-\frac{1}{\phi}} \int_0^{n_P n_A} \left(C_A^i(z)\right)^{\frac{\phi-1}{\phi}} dz\right]^{\frac{\phi}{\phi-1}} - \lambda \left[X_t - \int_0^{n_P n_A} P_A^i(z)\, C_A^i(z)\, dz\right].$$

First order condition

$$\frac{\partial \mathcal{L}}{\partial C_A^i(z)} = \frac{\phi}{\phi-1}\left[(n_P n_A)^{-\frac{1}{\phi}} \int_0^{n_P n_A} \left(C_A^i(z)\right)^{\frac{\phi-1}{\phi}} dz\right]^{\overbrace{\frac{\phi-(\phi-1)}{\phi-1}}^{\frac{\phi}{\phi-1}-1}}$$

$$\cdot (n_P n_A)^{-\frac{1}{\phi}} \frac{\phi-1}{\phi} \left(C_A^i(z)\right)^{\overbrace{\frac{\phi-1}{\phi}-1}^{\frac{\phi-1-\phi}{\phi}}} \frac{1}{\frac{dC_A^i(z)}{dz}} - \lambda(-1)\cdot P_A^i(z)\frac{1}{\frac{dC_A^i(z)}{dz}} = 0 \quad \Big| \cdot \frac{dC_A^i(z)}{dz}$$

$$\Leftrightarrow \underbrace{\left[(n_P n_A)^{-\frac{1}{\phi}} \int_0^{n_P n_A} \left(C_A^i(z)\right)^{\frac{\phi-1}{\phi}} dz\right]^{\frac{1}{\phi-1}}}_{(C_A^i)^{\frac{1}{\phi}}} (n_P n_A)^{-\frac{1}{\phi}} \left(C_A^i(z)\right)^{-\frac{1}{\phi}} + \lambda P_A^i(z) = 0$$

$$\Leftrightarrow -\lambda P_A^i(z) = \left(C_A^i\right)^{\frac{1}{\phi}} (n_P n_A)^{-\frac{1}{\phi}} \left(C_A^i(z)\right)^{-\frac{1}{\phi}}$$

$$\Leftrightarrow -\lambda P_A^i(z) C_A^i(z) = \left(C_A^i\right)^{\frac{1}{\phi}} (n_P n_A)^{-\frac{1}{\phi}} \left(C_A^i(z)\right)^{\frac{\phi-1}{\phi}}.$$

Taking integrals

$$\Leftrightarrow -\lambda \underbrace{\int_0^{n_P n_A} P_A^i(z) C_A^i(z)\, dz}_{X_t = P_A^i C_A^i} = \underbrace{\int_0^{n_P n_A} (n_P n_A)^{-\frac{1}{\phi}} \left(C_A^i(z)\right)^{\frac{\phi-1}{\phi}} dz}_{\left(C_A^i\right)^{\frac{\phi-1}{\phi}}} \left(C_A^i\right)^{\frac{1}{\phi}}$$

$$\Leftrightarrow -\lambda P_A^i C_A^i = \left(C_A^i\right)^{\frac{\phi-1}{\phi}} \left(C_A^i\right)^{\frac{1}{\phi}}$$

$$\Leftrightarrow -\lambda P_A^i = \left(C_A^i\right)^{\frac{\phi-1+1-\phi}{\phi}} = \left(C_A^i\right)^0 = 1$$

$$\Leftrightarrow \lambda = -\frac{1}{P_A^i}.$$

Substitute λ in FOC

$$\Leftrightarrow (-1)\cdot \underbrace{\left(-\frac{1}{P_A^i}\right)}_{\lambda} P_A^i(z) = \left(C_A^i\right)^{\frac{1}{\phi}} (n_P n_A)^{-\frac{1}{\phi}} \left(C_A^i(z)\right)^{-\frac{1}{\phi}}$$

$$\Leftrightarrow \left(C_A^i(z)\right)^{\frac{1}{\phi}} = \left(\frac{P_A^i(z)}{P_A^i}\right)^{-1} (n_P n_A)^{-\frac{1}{\phi}} \left(C_A^i\right)^{\frac{1}{\phi}}$$

$$\Leftrightarrow C_A^i(z) = \left(\frac{P_A^i(z)}{P_A^i}\right)^{-\phi} (n_P n_A)^{-1} C_A^i. \tag{7.52}$$

Individual Demand Function B

$$\begin{aligned} \max_{C_B^i(z)} \quad & C_B^i = \left[\left((1-n_A)\, n_P \right)^{-\frac{1}{\phi}} \int_{n_P n_A}^{n_P} \left(C_B^i(z) \right)^{\frac{\phi-1}{\phi}} dz \right]^{\frac{\phi}{\phi-1}} \\ \text{s.t.} \quad & X_t \geq \int_{n_P n_A}^{n_P} P_B^i(z)\, C_B^i(z)\, dz \end{aligned}$$

Transform into Lagrangian $\mathcal{L} = f(x) - \lambda \cdot h(x)$

$$h(x) \equiv X_t - \int_{n_P n_A}^{n_P} P_B^i(z)\, C_B^i(z)\, dz \geq 0$$

$$\mathcal{L} = \left[\left((1-n_A)\, n_P \right)^{-\frac{1}{\phi}} \int_{n_P n_A}^{n_P} \left(C_B^i(z) \right)^{\frac{\phi-1}{\phi}} dz \right]^{\frac{\phi}{\phi-1}} - \lambda \left[X_t - \int_{n_P n_A}^{n_P} P_B^i(z)\, C_B^i(z)\, dz \right].$$

First order condition

$$\frac{\partial \mathcal{L}}{\partial C_B^i(z)} = \frac{\phi}{\phi-1} \left[\left((1-n_A)\, n_P \right)^{-\frac{1}{\phi}} \int_{n_P n_A}^{n_P} \left(C_B^i(z) \right)^{\frac{\phi-1}{\phi}} dz \right]^{\overbrace{\frac{\phi-(\phi-1)}{\phi-1}}^{\frac{\phi}{\phi-1}-1}}$$

$$\cdot \left((1-n_A)\, n_P \right)^{-\frac{1}{\phi}} \frac{\phi-1}{\phi} \left(C_B^i(z) \right)^{\overbrace{\frac{\phi-1}{\phi}-1}^{\frac{\phi-1-\phi}{\phi}}} \frac{1}{\frac{dC_B^i(z)}{dz}} - \lambda\,(-1) \cdot P_B^i(z) \frac{1}{\frac{dC_B^i(z)}{dz}} = 0 \quad \Big| \cdot \frac{dC_B^i(z)}{dz}$$

$$\begin{aligned} \Leftrightarrow \quad & \underbrace{\left[\left((1-n_A)\, n_P \right)^{-\frac{1}{\phi}} \int_{n_P n_A}^{n_P} \left(C_B^i(z) \right)^{\frac{\phi-1}{\phi}} dz \right]^{\frac{1}{\phi-1}}}_{\left(C_B^i\right)^{\frac{1}{\phi}}} \left((1-n_A)\, n_P \right)^{-\frac{1}{\phi}} \left(C_B^i(z) \right)^{-\frac{1}{\phi}} \\ & + \lambda\, P_B^i(z) = 0 \\ \Leftrightarrow \quad & -\lambda\, P_B^i(z) = \left(C_B^i \right)^{\frac{1}{\phi}} \left((1-n_A)\, n_P \right)^{-\frac{1}{\phi}} \left(C_B^i(z) \right)^{-\frac{1}{\phi}} \\ \Leftrightarrow \quad & -\lambda\, P_B^i(z)\, C_B^i(z) = \left(C_B^i \right)^{\frac{1}{\phi}} \left((1-n_A)\, n_P \right)^{-\frac{1}{\phi}} \left(C_B^i(z) \right)^{\frac{\phi-1}{\phi}}. \end{aligned}$$

Taking integrals

$$\Leftrightarrow \quad -\lambda \underbrace{\int_{n_P n_A}^{n_P} P_B^i(z)\, C_B^i(z)\, dz}_{X_t = P_B^i\, C_B^i} = \underbrace{\int_{n_P n_A}^{n_P} \left((1-n_A)\, n_P\right)^{-\frac{1}{\phi}} \left(C_B^i(z)\right)^{\frac{\phi-1}{\phi}} dz}_{\left(C_B^i\right)^{\frac{\phi-1}{\phi}}} \left(C_B^i\right)^{\frac{1}{\phi}}$$

$$\Leftrightarrow \quad -\lambda\, P_B^i\, C_B^i = \left(C_B^i\right)^{\frac{\phi-1}{\phi}} \left(C_B^i\right)^{\frac{1}{\phi}}$$

$$\Leftrightarrow \quad -\lambda\, P_B^i = \left(C_B^i\right)^{\frac{\phi-1+1-\phi}{\phi}} = \left(C_B^i\right)^0 = 1$$

$$\Leftrightarrow \quad \lambda = -\frac{1}{P_B^i}.$$

Substitute λ in FOC

$$\Leftrightarrow \quad (-1) \cdot \underbrace{\left(-\frac{1}{P_B^i}\right)}_{\lambda} P_B^i(z) = \left(C_B^i\right)^{\frac{1}{\phi}} \left(\left(1-n_A\right) n_P\right)^{-\frac{1}{\phi}} \left(C_B^i(z)\right)^{-\frac{1}{\phi}}$$

$$\Leftrightarrow \quad \left(C_B^i(z)\right)^{\frac{1}{\phi}} = \left(\frac{P_B^i(z)}{P_B^i}\right)^{-1} \left(\left(1-n_A\right) n_P\right)^{-\frac{1}{\phi}} \left(C_B^i\right)^{\frac{1}{\phi}}$$

$$\Leftrightarrow \quad C_B^i(z) = \left(\frac{P_B^i(z)}{P_B^i}\right)^{-\phi} \left(\left(1-n_A\right) n_P\right)^{-1} C_B^i. \tag{7.53}$$

Individual Demand Function C

$$\begin{aligned} \max_{C_C^i(z)} \quad & C_C^i = \left[(1-n_P)^{-\frac{1}{\phi}} \int_{n_P}^1 \left(C_C^i(z)\right)^{\frac{\phi-1}{\phi}} dz\right]^{\frac{\phi}{\phi-1}} \\ \text{s.t.} \quad & X_t \geq \int_{n_P}^1 P_C^i(z)\, C_C^i(z)\, dz \end{aligned}$$

Transform into Lagrangian $\mathcal{L} = f(x) - \lambda \cdot h(x)$

$$h(x) \equiv X_t - \int_{n_P}^1 P_C^i(z)\, C_C^i(z)\, dz \geq 0$$

$$\mathcal{L} = \left[(1-n_P)^{-\frac{1}{\phi}} \int_{n_P}^1 \left(C_C^i(z)\right)^{\frac{\phi-1}{\phi}} dz\right]^{\frac{\phi}{\phi-1}} - \lambda \left[X_t - \int_{n_P}^1 P_C^i(z)\, C_C^i(z)\, dz\right].$$

First order condition

$$\frac{\partial \mathcal{L}}{\partial C_C^i(z)} = \frac{\phi}{\phi-1} \left[(1-n_P)^{-\frac{1}{\phi}} \int_{n_P}^1 \left(C_C^i(z)\right)^{\frac{\phi-1}{\phi}} dz\right]^{\overbrace{\frac{\phi-(\phi-1)}{\phi-1}}^{\frac{\phi}{\phi-1}-1}}$$

$$\cdot \left(1-n_P\right)^{-\frac{1}{\phi}} \frac{\phi-1}{\phi} \left(C_C^i(z)\right)^{\overbrace{\frac{\phi-1-\phi}{\phi}}^{\frac{\phi-1}{\phi}-1}} \frac{1}{\frac{dC_C^i(z)}{dz}} - \lambda\,(-1) \cdot P_C^i(z) \frac{1}{\frac{dC_C^i(z)}{dz}} = 0 \quad \Big| \cdot \frac{dC_C^i(z)}{dz}$$

$$\Leftrightarrow \quad \underbrace{\left[(1-n_P)^{-\frac{1}{\phi}} \int_{n_P}^{1} \left(C_C^i(z)\right)^{\frac{\phi-1}{\phi}} dz\right]^{\frac{1}{\phi-1}}}_{(C_C^i)^{\frac{1}{\phi}}} (1-n_P)^{-\frac{1}{\phi}} \left(C_C^i(z)\right)^{-\frac{1}{\phi}} + \lambda P_C^i(z) = 0$$

$$\Leftrightarrow \quad -\lambda P_C^i(z) = \left(C_C^i\right)^{\frac{1}{\phi}} (1-n_P)^{-\frac{1}{\phi}} \left(C_C^i(z)\right)^{-\frac{1}{\phi}}$$

$$\Leftrightarrow \quad -\lambda P_C^i(z)\, C_C^i(z) = \left(C_C^i\right)^{\frac{1}{\phi}} (1-n_P)^{-\frac{1}{\phi}} \left(C_C^i(z)\right)^{\frac{\phi-1}{\phi}}.$$

Taking integrals

$$\Leftrightarrow \quad -\lambda \underbrace{\int_{n_P}^{1} P_C^i(z)\, C_C^i(z)\, dz}_{X_t = P_C^i\, C_C^i} = \underbrace{\int_{n_P}^{1} (1-n_P)^{-\frac{1}{\phi}} \left(C_C^i(z)\right)^{\frac{\phi-1}{\phi}} dz}_{\left(C_C^i\right)^{\frac{\phi-1}{\phi}}} \left(C_C^i\right)^{\frac{1}{\phi}}$$

$$\Leftrightarrow \quad -\lambda P_C^i\, C_C^i = \left(C_C^i\right)^{\frac{\phi-1}{\phi}} \left(C_C^i\right)^{\frac{1}{\phi}}$$

$$\Leftrightarrow \quad -\lambda P_C^i = \left(C_C^i\right)^{\frac{\phi-1+1-\phi}{\phi}} = \left(C_C^i\right)^0 = 1$$

$$\Leftrightarrow \quad \lambda = -\frac{1}{P_C^i}.$$

Substitute λ in FOC

$$\Leftrightarrow \quad (-1) \cdot \underbrace{\left(-\frac{1}{P_C^i}\right)}_{\lambda} P_C^i(z) = \left(C_C^i\right)^{\frac{1}{\phi}} (1-n_P)^{-\frac{1}{\phi}} \left(C_C^i(z)\right)^{-\frac{1}{\phi}}$$

$$\Leftrightarrow \quad \left(C_C^i(z)\right)^{\frac{1}{\phi}} = \left(\frac{P_C^i(z)}{P_C^i}\right)^{-1} (1-n_P)^{-\frac{1}{\phi}} \left(C_C^i\right)^{\frac{1}{\phi}}$$

$$\Leftrightarrow \quad C_C^i(z) = \left(\frac{P_C^i(z)}{P_C^i}\right)^{-\phi} (1-n_P)^{-1}\, C_C^i. \tag{7.54}$$

7.1.7 Aggregate Demand Function

Due to the nested consumption basket and the introduction of home bias in consumption the aggregate demand functions have to be derived individually.

Aggregate Demand Function for Country A

Maximize total expenditure, Z, consisting of country A's CPI price index, P^A, times total consumption, C^A.

$$\begin{aligned}
\max_{C^A} \quad & Z = C^A\, P^A \\
\text{s.t.} \quad & C^A = \Bigg[[1-\eta\,(1-n_P)]^{\frac{1}{\theta}} \Bigg(\Bigg[[1-\eta\,(1-n_A)]^{\frac{1}{\psi}} \left(C_A^A\right)^{\frac{\psi-1}{\psi}} \\
& \quad + [\eta\,(1-n_A)]^{\frac{1}{\psi}} \left(C_B^A\right)^{\frac{\psi-1}{\psi}} \Bigg]^{\frac{\psi}{\psi-1}} \Bigg)^{\frac{\theta-1}{\theta}} + [\eta\,(1-n_P)]^{\frac{1}{\theta}} \left(C_C^A\right)^{\frac{\theta-1}{\theta}} \Bigg]^{\frac{\theta}{\theta-1}}
\end{aligned}$$

Transform into Lagrangian $\mathcal{L} = f(x) - \lambda \cdot h(x)$

$$f(x) \equiv C^A\, P^A = C_A^A\, P_A^A + C_B^A\, P_B^A + C_C^A\, P_C^A$$

$$\begin{aligned}
h(x) \equiv & \Bigg[[1-\eta\,(1-n_P)]^{\frac{1}{\theta}} \Bigg(\Bigg[[1-\eta\,(1-n_A)]^{\frac{1}{\psi}} \left(C_A^A\right)^{\frac{\psi-1}{\psi}} \\
& + [\eta\,(1-n_A)]^{\frac{1}{\psi}} \left(C_B^A\right)^{\frac{\psi-1}{\psi}} \Bigg]^{\frac{\psi}{\psi-1}} \Bigg)^{\frac{\theta-1}{\theta}} + [\eta\,(1-n_P)]^{\frac{1}{\theta}} \left(C_C^A\right)^{\frac{\theta-1}{\theta}} \Bigg]^{\frac{\theta}{\theta-1}} - C^A \geq 0
\end{aligned}$$

$$\begin{aligned}
\mathcal{L} = & \; C_A^A\, P_A^A + C_B^A\, P_B^A + C_C^A\, P_C^A \\
& -\lambda \Bigg\{ \Bigg[[1-\eta\,(1-n_P)]^{\frac{1}{\theta}} \Bigg(\Bigg[[1-\eta\,(1-n_A)]^{\frac{1}{\psi}} \left(C_A^A\right)^{\frac{\psi-1}{\psi}} \\
& \quad + [\eta\,(1-n_A)]^{\frac{1}{\psi}} \left(C_B^A\right)^{\frac{\psi-1}{\psi}} \Bigg]^{\frac{\psi}{\psi-1}} \Bigg)^{\frac{\theta-1}{\theta}} + [\eta\,(1-n_P)]^{\frac{1}{\theta}} \left(C_C^A\right)^{\frac{\theta-1}{\theta}} \Bigg]^{\frac{\theta}{\theta-1}} - C^A \Bigg\}.
\end{aligned}$$

First order conditions for country A

Country A's demand for periphery P's goods

$$\begin{aligned}
\mathcal{L} = & \; C_P^A\, P_P^A + C_C^A\, P_C^A \\
& -\lambda \Bigg\{ \Bigg[[1-\eta\,(1-n_P)]^{\frac{1}{\theta}} \left(C_P^A\right)^{\frac{\theta-1}{\theta}} \\
& \quad + [\eta\,(1-n_P)]^{\frac{1}{\theta}} \left(C_C^A\right)^{\frac{\theta-1}{\theta}} \Bigg]^{\frac{\theta}{\theta-1}} - C^A \Bigg\}
\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial C_P^A} &= P_P^A - \lambda \frac{\theta}{\theta-1} \underbrace{\Big[[1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^A\right)^{\frac{\theta-1}{\theta}} }_{\overbrace{\quad}^{\frac{\theta-(\theta-1)}{\theta-1}}} \\ &\quad \underbrace{+ [\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_C^A\right)^{\frac{\theta-1}{\theta}} \Big]^{\overbrace{\quad}^{\frac{1}{\theta-1}}}}_{(C^A)^{\frac{1}{\theta}}} \\ &\quad \cdot \frac{\theta-1}{\theta} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^A\right)^{\overbrace{-\frac{1}{\theta}}^{\frac{\theta-1-\theta}{\theta}}} = 0\end{aligned}$$

$$\begin{aligned}&\Leftrightarrow \quad P_P^A = \lambda \frac{\theta}{\theta-1} (C^A)^{\frac{1}{\theta}} \frac{\theta-1}{\theta} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^A\right)^{-\frac{1}{\theta}} \\ &\Leftrightarrow \quad P_P^A = \lambda (C^A)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^A\right)^{-\frac{1}{\theta}} \\ &\Leftrightarrow \quad P_P^A C_P^A = \lambda (C^A)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^A\right)^{\frac{\theta-1}{\theta}} \qquad (7.55)\end{aligned}$$

$$\begin{aligned}P_P^A &= \lambda (C^A)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^A\right)^{-\frac{1}{\theta}} \\ P_P^A &= \lambda \left(\frac{C^A}{C_P^A}\right)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \\ \lambda \left(\frac{C^A}{C_P^A}\right)^{\frac{1}{\theta}} &= [1-\eta(1-n_P)]^{-\frac{1}{\theta}} P_P^A \qquad (7.56)\end{aligned}$$

Country A's demand for country A's goods

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_A^A} \;=\; & P_A^A - \lambda \frac{\theta}{\theta-1} \underbrace{\Bigg[[1-\eta(1-n_P)]^{\frac{1}{\theta}} \Bigg(\Bigg[[1-\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_A^A\right)^{\frac{\psi-1}{\psi}} \right.}} \\
& \underbrace{\left. + [\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_B^A\right)^{\frac{\psi-1}{\psi}} \Bigg]^{\frac{\psi}{\psi-1}} \Bigg)^{\frac{\theta-1}{\theta}} + [\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_C^A\right)^{\frac{\theta-1}{\theta}} \Bigg]^{\overbrace{\frac{\theta-(\theta-1)}{\theta-1}}^{\frac{1}{\theta-1}}}}_{(C^A)^{\frac{1}{\theta}}} \\
& \cdot \frac{\theta-1}{\theta} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \\
& \cdot \underbrace{\left(\left[[1-\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_A^A\right)^{\frac{\psi-1}{\psi}} + [\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_B^A\right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \right)^{\overbrace{\frac{\theta-1-\theta}{\theta}}^{-\frac{1}{\theta}}}}_{\left(C_P^A\right)^{-\frac{1}{\theta}}} \\
& \cdot \frac{\psi}{\psi-1} \underbrace{\Bigg[[1-\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_A^A\right)^{\frac{\psi-1}{\psi}} + [\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_B^A\right)^{\frac{\psi-1}{\psi}} \Bigg]^{\overbrace{\frac{\psi-(\psi-1)}{\psi-1}}^{\frac{1}{\psi-1}}}}_{\left(C_P^A\right)^{\frac{1}{\psi}}} \\
& \cdot \frac{\psi-1}{\psi} [1-\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_A^A\right)^{\overbrace{-\frac{1}{\psi}}^{\frac{\psi-1-\psi}{\psi}}} = 0
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & P_A^A = \lambda \frac{\theta}{\theta-1} (C^A)^{\frac{1}{\theta}} \frac{\theta-1}{\theta} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^A\right)^{-\frac{1}{\theta}} \\
& \qquad \cdot \frac{\psi}{\psi-1} \left(C_P^A\right)^{\frac{1}{\psi}} \frac{\psi-1}{\psi} [1-\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_A^A\right)^{-\frac{1}{\psi}} \\
\Leftrightarrow \quad & P_A^A = \lambda (C^A)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^A\right)^{-\frac{1}{\theta}} \left(C_P^A\right)^{\frac{1}{\psi}} [1-\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_A^A\right)^{-\frac{1}{\psi}} \\
\Leftrightarrow \quad & P_A^A = \lambda (C^A)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} [1-\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_P^A\right)^{\frac{1}{\psi}-\frac{1}{\theta}} \left(C_A^A\right)^{-\frac{1}{\psi}} \\
\Leftrightarrow \quad & P_A^A C_A^A = \lambda (C^A)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} [1-\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_P^A\right)^{\frac{1}{\psi}-\frac{1}{\theta}} \left(C_A^A\right)^{\frac{\psi-1}{\psi}}
\end{aligned} \tag{7.57}
$$

$$
\begin{aligned}
P_A^A &= \lambda (C^A)^{\frac{1}{\theta}} \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(C_P^A\right)^{-\frac{1}{\theta}} \left(C_P^A\right)^{\frac{1}{\psi}} \left[1-\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}} \left(C_A^A\right)^{-\frac{1}{\psi}} \\
P_A^A &= \underbrace{\lambda \left(\frac{C^A}{C_P^A}\right)^{\frac{1}{\theta}}}_{(7.56)} \left(\frac{C_P^A}{C_A^A}\right)^{\frac{1}{\psi}} \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left[1-\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}
\end{aligned}
$$

$$
\begin{aligned}
P_A^A &= \left[1-\eta\left(1-n_P\right)\right]^{-\frac{1}{\theta}} P_P^A \left(\frac{C_P^A}{C_A^A}\right)^{\frac{1}{\psi}} \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left[1-\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}} \\
\left(C_A^A\right)^{\frac{1}{\psi}} &= \left[1-\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}} \left(\frac{P_A^A}{P_P^A}\right)^{-1} \left(C_P^A\right)^{\frac{1}{\psi}} \\
C_A^A &= \left[1-\eta\left(1-n_A\right)\right] \left(\frac{P_A^A}{P_P^A}\right)^{-\psi} C_P^A \qquad (7.58)
\end{aligned}
$$

Country A's demand for country B's goods

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_B^A} &= P_B^A - \lambda \frac{\theta}{\theta-1} \underbrace{\Bigg[\left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \Bigg(\Bigg[\left[1-\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}} \left(C_A^A\right)^{\frac{\psi-1}{\psi}}}_{} \right. \\
&\quad \underbrace{\left. + \left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}} \left(C_B^A\right)^{\frac{\psi-1}{\psi}} \Bigg]^{\frac{\psi}{\psi-1}} \Bigg)^{\frac{\theta-1}{\theta}} + \left[\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(C_C^A\right)^{\frac{\theta-1}{\theta}} \Bigg]^{\overbrace{\frac{\theta-(\theta-1)}{\theta-1}}^{\frac{1}{\theta-1}}}}_{(C^A)^{\frac{1}{\theta}}} \\
&\quad \cdot \frac{\theta-1}{\theta} \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}
\end{aligned}
$$

$$\cdot\underbrace{\left(\left[\left[1-\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}\left(C_A^A\right)^{\frac{\psi-1}{\psi}}+\left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}\left(C_B^A\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}\right)^{\overbrace{-\frac{1}{\theta}}^{\frac{\theta-1-\theta}{\theta}}}}_{\left(C_P^A\right)^{-\frac{1}{\theta}}}$$

$$\cdot\frac{\psi}{\psi-1}\underbrace{\left[\left[1-\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}\left(C_A^A\right)^{\frac{\psi-1}{\psi}}+\left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}\left(C_B^A\right)^{\frac{\psi-1}{\psi}}\right]^{\overbrace{\frac{1}{\psi-1}}^{\frac{\psi-(\psi-1)}{\psi-1}}}}_{\left(C_P^A\right)^{\frac{1}{\psi}}}$$

$$\cdot\frac{\psi-1}{\psi}\left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}\left(C_B^A\right)^{\overbrace{-\frac{1}{\psi}}^{\frac{\psi-1-\psi}{\psi}}}=0$$

$$\begin{aligned}
\Leftrightarrow\quad & P_B^A=\lambda\frac{\theta}{\theta-1}\left(C^A\right)^{\frac{1}{\theta}}\frac{\theta-1}{\theta}\left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left(C_P^A\right)^{-\frac{1}{\theta}}\\
& \quad\cdot\frac{\psi}{\psi-1}\left(C_P^A\right)^{\frac{1}{\psi}}\frac{\psi-1}{\psi}\left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}\left(C_B^A\right)^{-\frac{1}{\psi}}\\
\Leftrightarrow\quad & P_B^A=\lambda(C^A)^{\frac{1}{\theta}}\left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left(C_P^A\right)^{-\frac{1}{\theta}}\left(C_P^A\right)^{\frac{1}{\psi}}\left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}\left(C_B^A\right)^{-\frac{1}{\psi}}\\
\Leftrightarrow\quad & P_B^A=\lambda(C^A)^{\frac{1}{\theta}}\left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}\left(C_P^A\right)^{\frac{1}{\psi}-\frac{1}{\theta}}\left(C_B^A\right)^{-\frac{1}{\psi}}\\
\Leftrightarrow\quad & P_B^AC_B^A=\lambda(C^A)^{\frac{1}{\theta}}\left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}\left(C_P^A\right)^{\frac{1}{\psi}-\frac{1}{\theta}}\left(C_B^A\right)^{\frac{\psi-1}{\psi}}
\end{aligned}\tag{7.59}$$

$$\begin{aligned}
P_B^A &= \lambda(C^A)^{\frac{1}{\theta}}\left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left(C_P^A\right)^{-\frac{1}{\theta}}\left(C_P^A\right)^{\frac{1}{\psi}}\left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}\left(C_B^A\right)^{-\frac{1}{\psi}}\\
P_B^A &= \underbrace{\lambda\left(\frac{C^A}{C_P^A}\right)^{\frac{1}{\theta}}}_{(7.56)}\left(\frac{C_P^A}{C_B^A}\right)^{\frac{1}{\psi}}\left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}
\end{aligned}$$

$$\begin{aligned}
P_B^A &= P_P^A\left[1-\eta\left(1-n_P\right)\right]^{-\frac{1}{\theta}}\left(\frac{C_P^A}{C_B^A}\right)^{\frac{1}{\psi}}\left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}\\
\left(C_B^A\right)^{\frac{1}{\psi}} &= \left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}\left(\frac{P_B^A}{P_P^A}\right)^{-1}\left(C_P^A\right)^{\frac{1}{\psi}}\\
C_B^A &= \eta\left(1-n_A\right)\left(\frac{P_B^A}{P_P^A}\right)^{-\psi}C_P^A
\end{aligned}\tag{7.60}$$

Country A's demand for country C's goods

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial C_C^A} &= P_C^A - \lambda \frac{\theta}{\theta-1} \underbrace{\Bigg[[1-\eta(1-n_P)]^{\frac{1}{\theta}} \Bigg(\Bigg[[1-\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_A^A\right)^{\frac{\psi-1}{\psi}} \\ &\quad + \underbrace{[\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_B^A\right)^{\frac{\psi-1}{\psi}} \Bigg]^{\frac{\psi}{\psi-1}} \Bigg)^{\frac{\theta-1}{\theta}} + [\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_C^A\right)^{\frac{\theta-1}{\theta}} \Bigg]^{\overbrace{\frac{\theta-(\theta-1)}{\theta-1}}^{\frac{1}{\theta-1}}}}_{(C^A)^{\frac{1}{\theta}}} \\ &\quad \cdot \frac{\theta-1}{\theta} [\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_C^A\right)^{\overbrace{-\frac{1}{\theta}}^{\frac{\theta-1-\theta}{\theta}}} = 0\end{aligned}$$

$$\begin{aligned}&\Leftrightarrow \quad P_C^A = \lambda \frac{\theta}{\theta-1} (C^A)^{\frac{1}{\theta}} \frac{\theta-1}{\theta} [\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_C^A\right)^{-\frac{1}{\theta}} \\ &\Leftrightarrow \quad P_C^A = \lambda (C^A)^{\frac{1}{\theta}} [\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_C^A\right)^{-\frac{1}{\theta}} \\ &\Leftrightarrow \quad P_C^A C_C^A = \lambda (C^A)^{\frac{1}{\theta}} [\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_C^A\right)^{\frac{\theta-1}{\theta}} \qquad (7.61)\end{aligned}$$

$$\begin{aligned}P_C^A &= \lambda (C^A)^{\frac{1}{\theta}} [\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_C^A\right)^{-\frac{1}{\theta}} \\ P_C^A &= \lambda \left(\frac{C^A}{C_C^A}\right)^{\frac{1}{\theta}} [\eta(1-n_P)]^{\frac{1}{\theta}} \qquad (7.62)\end{aligned}$$

Intermediate Step: Deriving the Lagrange Multiplier Using definition of total expenditure

$$Z = C^A P^A = C_A^A P_A^A + C_B^A P_B^A + C_C^A P_C^A$$

and FOC expressions for $C_A^A P_A^A$, $C_B^A P_B^A$, and $C_C^A P_C^A$.

$$\begin{aligned}C^A P^A &= \underbrace{\lambda (C^A)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} [1-\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_P^A\right)^{\frac{1}{\psi}-\frac{1}{\theta}} \left(C_A^A\right)^{\frac{\psi-1}{\psi}}}_{(7.57)} \\ &\quad + \underbrace{\lambda (C^A)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} [\eta(1-n_A)]^{\frac{1}{\psi}} \left(C_P^A\right)^{\frac{1}{\psi}-\frac{1}{\theta}} \left(C_B^A\right)^{\frac{\psi-1}{\psi}}}_{(7.59)} \\ &\quad + \underbrace{\lambda (C^A)^{\frac{1}{\theta}} [\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_C^A\right)^{\frac{\theta-1}{\theta}}}_{(7.61)}\end{aligned}$$

$$
\begin{aligned}
&= \lambda(C^A)^{\frac{1}{\theta}}\left[\left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left(C_P^A\right)^{\frac{1}{\psi}-\frac{1}{\theta}}\left(\left[1-\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}\left(C_A^A\right)^{\frac{\psi-1}{\psi}}\right.\right.\\
&\quad\left.\left.+\left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}}\left(C_B^A\right)^{\frac{\psi-1}{\psi}}\right)+\left[\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left(C_C^A\right)^{\frac{\theta-1}{\theta}}\right]\\
&= \lambda(C^A)^{\frac{1}{\theta}}\left[\left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\underbrace{\left(C_P^A\right)^{\frac{1}{\psi}-\frac{1}{\theta}}\left(C_P^A\right)^{\frac{\psi-1}{\psi}}}_{\left(C_P^A\right)^{\frac{\theta-1}{\theta}}}+\left[\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left(C_C^A\right)^{\frac{\theta-1}{\theta}}\right]\\
&= \lambda(C^A)^{\frac{1}{\theta}}\left[\left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left(C_P^A\right)^{\frac{\theta-1}{\theta}}+\left[\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left(C_C^A\right)^{\frac{\theta-1}{\theta}}\right]\\
&\quad \left(C_P^A\right)^{\frac{1}{\psi}}\left(C_P^A\right)^{\frac{\psi-1}{\psi}}\left(C_P^A\right)^{-\frac{1}{\theta}}=\left(C_P^A\right)^{\frac{1+\psi-1}{\psi}}\left(C_P^A\right)^{-\frac{1}{\theta}}=\left(C_P^A\right)^{1}\left(C_P^A\right)^{-\frac{1}{\theta}}=\left(C_P^A\right)^{\frac{\theta-1}{\theta}}\\
&= \lambda\ (C^A)^{\frac{1}{\theta}}\underbrace{\left[\left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left(C_P^A\right)^{\frac{\theta-1}{\theta}}+\left[\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left(C_C^A\right)^{\frac{\theta-1}{\theta}}\right]}_{(C^A)^{\frac{\theta-1}{\theta}}}\\
&= \lambda\left(C^A\right)^{\frac{1}{\theta}}(C^A)^{\frac{\theta-1}{\theta}}
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow C^A P^A &= \lambda\, C^A\\
\Leftrightarrow P^A &= \lambda
\end{aligned}
\tag{7.63}
$$

Holds for all consumption baskets $P^i = \lambda$ for $i = A, B, C$.

Summary: Aggregate Demand Functions for Country A Using (7.63) to derive country A's demand for goods from the periphery P

$$
\begin{aligned}
\underbrace{P^A}_{\lambda}\left(\frac{C^A}{C_P^A}\right)^{\frac{1}{\theta}} &= \left[1-\eta\left(1-n_P\right)\right]^{-\frac{1}{\theta}}P_P^A\\
\left(C_P^A\right)^{\frac{1}{\theta}} &= \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left(\frac{P_P^A}{P^A}\right)^{-1}\left(C^A\right)^{\frac{1}{\theta}}\\
C_P^A &= \left[1-\eta\left(1-n_P\right)\right]\left(\frac{P_P^A}{P^A}\right)^{-\theta}C^A.
\end{aligned}
\tag{7.64}
$$

Country A's demand for country A's goods

$$\begin{aligned} C_A^A &= [1-\eta(1-n_A)]\left(\frac{P_A^A}{P_P^A}\right)^{-\psi} \underbrace{[1-\eta(1-n_P)]\left(\frac{P_P^A}{P^A}\right)^{-\theta} C^A}_{C_P^A} \\ C_A^A &= \underbrace{[1-\eta(1-n_A)][1-\eta(1-n_P)]}_{v_A^A}\left(\frac{P_A^A}{P_P^A}\right)^{-\psi}\left(\frac{P_P^A}{P^A}\right)^{-\theta} C^A \end{aligned} \tag{7.65}$$

$$\begin{aligned} v_A^A &\equiv 1-[1-\eta(1-n_P)][\eta(1-n_A)]-\eta(1-n_P) \\ &= 1-[1-\eta+\eta n_P][\eta-\eta n_A]-\eta+\eta n_P \\ &= 1-\left[\eta-\eta n_A-\eta^2+\eta^2 n_A+\eta^2 n_P-\eta^2 n_P n_A\right]-\eta+\eta n_P \\ &= 1-\eta+\eta n_A+\eta^2-\eta^2 n_A-\eta^2 n_P+\eta^2 n_P n_A-\eta+\eta n_P \\ &= 1-\eta+\eta n_P-\eta+\eta^2-\eta^2 n_P+\eta n_A-\eta^2 n_A+\eta^2 n_P n_A \\ &= [1-\eta+\eta n_A][1-\eta+\eta n_P] \\ &= [1-\eta(1-n_P)][1-\eta(1-n_A)] \end{aligned}$$

Country A's demand for country B's goods

$$\begin{aligned} C_B^A &= \eta(1-n_A)\left(\frac{P_B^A}{P_P^A}\right)^{-\psi} \underbrace{[1-\eta(1-n_P)]\left(\frac{P_P^A}{P^A}\right)^{-\theta} C^A}_{C_P^A} \\ C_B^A &= \underbrace{\eta(1-n_A)[1-\eta(1-n_P)]}_{v_B^A}\left(\frac{P_B^A}{P_P^A}\right)^{-\psi}\left(\frac{P_P^A}{P^A}\right)^{-\theta} C^A \end{aligned} \tag{7.66}$$

Country A's demand for country C's goods

$$\begin{aligned} P_C^A &= \underbrace{P^A}_{\lambda}\left(\frac{C^A}{C_C^A}\right)^{\frac{1}{\theta}}[\eta(1-n_P)]^{\frac{1}{\theta}} \\ \left(C_C^A\right)^{\frac{1}{\theta}} &= [\eta(1-n_P)]^{\frac{1}{\theta}}\left(\frac{P_C^A}{P^A}\right)^{-1}\left(C^A\right)^{\frac{1}{\theta}} \\ C_C^A &= \underbrace{\eta(1-n_P)}_{v_C^A}\left(\frac{P_C^A}{P^A}\right)^{-\theta} C^A \end{aligned} \tag{7.67}$$

Aggregate Demand Function for Country B

Maximize total expenditure, Z, consisting of country B's CPI price index, P^B, times total consumption, C^B.

$$\begin{aligned}
\max_{C^B} \quad & Z = C^B\, P^B \\
\text{s.t.} \quad & C^B = \Bigg[[1-\eta\,(1-n_P)]^{\frac{1}{\theta}} \left(\left[(\eta n_A)^{\frac{1}{\psi}} \left(C_A^B\right)^{\frac{\psi-1}{\psi}} + (1-\eta n_A)^{\frac{1}{\psi}} \left(C_B^B\right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \right)^{\frac{\theta-1}{\theta}} \\
& + [\eta\,(1-n_P)]^{\frac{1}{\theta}} \left(C_C^B\right)^{\frac{\theta-1}{\theta}} \Bigg]^{\frac{\theta}{\theta-1}}
\end{aligned}$$

Transform into Lagrangian $\mathcal{L} = f(x) - \lambda \cdot h(x)$

$$f(x) \equiv C^B\, P^B = C_A^B\, P_A^B + C_B^B\, P_B^B + C_C^B\, P_C^B$$

$$\begin{aligned}
h(x) \quad \equiv \quad & \Bigg[[1-\eta\,(1-n_P)]^{\frac{1}{\theta}} \left(\left[(\eta n_A)^{\frac{1}{\psi}} \left(C_A^B\right)^{\frac{\psi-1}{\psi}} + (1-\eta n_A)^{\frac{1}{\psi}} \left(C_B^B\right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \right)^{\frac{\theta-1}{\theta}} \\
& + [\eta\,(1-n_P)]^{\frac{1}{\theta}} \left(C_C^B\right)^{\frac{\theta-1}{\theta}} \Bigg]^{\frac{\theta}{\theta-1}} - C^B \geq 0
\end{aligned}$$

$$\begin{aligned}
\mathcal{L} \quad = \quad & C_A^B\, P_A^B + C_B^B\, P_B^B + C_C^B\, P_C^B \\
& -\lambda \Bigg\{ \Bigg[[1-\eta\,(1-n_P)]^{\frac{1}{\theta}} \left(\left[(\eta n_A)^{\frac{1}{\psi}} \left(C_A^B\right)^{\frac{\psi-1}{\psi}} + (1-\eta n_A)^{\frac{1}{\psi}} \left(C_B^B\right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \right)^{\frac{\theta-1}{\theta}} \\
& + [\eta\,(1-n_P)]^{\frac{1}{\theta}} \left(C_C^B\right)^{\frac{\theta-1}{\theta}} \Bigg]^{\frac{\theta}{\theta-1}} - C^B \Bigg\}.
\end{aligned}$$

First order conditions for country B

Country B's demand for periphery P's goods

$$\begin{aligned}
\mathcal{L} \quad = \quad & C_P^B\, P_P^B + C_C^B\, P_C^B \\
& -\lambda \Bigg\{ \Big[[1-\eta\,(1-n_P)]^{\frac{1}{\theta}} \left(C_P^B\right)^{\frac{\theta-1}{\theta}} \\
& + [\eta\,(1-n_P)]^{\frac{1}{\theta}} \left(C_C^B\right)^{\frac{\theta-1}{\theta}} \Big]^{\frac{\theta}{\theta-1}} - C^B \Bigg\}
\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial C_P^B} &= P_P^B - \lambda \frac{\theta}{\theta-1} \underbrace{\Big[[1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^B\right)^{\frac{\theta-1}{\theta}}}_{\overbrace{\frac{\theta-(\theta-1)}{\theta-1}}^{\frac{1}{\theta-1}}} \\ &\quad \underbrace{+ [\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_C^B\right)^{\frac{\theta-1}{\theta}} \Big]^{\frac{1}{\theta-1}}}_{(C^B)^{\frac{1}{\theta}}} \\ &\quad \cdot \frac{\theta-1}{\theta} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^B\right)^{\overbrace{\frac{\theta-1-\theta}{\theta}}^{-\frac{1}{\theta}}} = 0\end{aligned}$$

$$\begin{aligned}\Leftrightarrow \quad & P_P^B = \lambda \frac{\theta}{\theta-1} (C^B)^{\frac{1}{\theta}} \frac{\theta-1}{\theta} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^B\right)^{-\frac{1}{\theta}} \\ \Leftrightarrow \quad & P_P^B = \lambda (C^B)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^B\right)^{-\frac{1}{\theta}} \\ \Leftrightarrow \quad & P_P^B C_P^B = \lambda (C^B)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^B\right)^{\frac{\theta-1}{\theta}} \end{aligned} \tag{7.68}$$

$$\begin{aligned} P_P^B &= \lambda (C^B)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^B\right)^{-\frac{1}{\theta}} \\ P_P^B &= \lambda \left(\frac{C^B}{C_P^B}\right)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \\ \lambda \left(\frac{C^B}{C_P^B}\right)^{\frac{1}{\theta}} &= [1-\eta(1-n_P)]^{-\frac{1}{\theta}} P_P^B \end{aligned} \tag{7.69}$$

Country B's demand for country A's goods

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_A^B} &= P_A^B - \lambda \frac{\theta}{\theta-1} \underbrace{\Bigg[[1-\eta(1-n_P)]^{\frac{1}{\theta}} \Bigg(\Bigg[(\eta n_A)^{\frac{1}{\psi}} \left(C_A^B\right)^{\frac{\psi-1}{\psi}}}_{} \\
&\quad + \underbrace{(1-\eta n_A)^{\frac{1}{\psi}} \left(C_B^B\right)^{\frac{\psi-1}{\psi}} \Bigg]^{\frac{\psi}{\psi-1}} \Bigg)^{\frac{\theta-1}{\theta}} + [\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_C^B\right)^{\frac{\theta-1}{\theta}} \Bigg]^{\overbrace{\frac{\theta-(\theta-1)}{\theta-1}}^{\frac{1}{\theta-1}}}}_{(C^B)^{\frac{1}{\theta}}} \\
&\quad \cdot \frac{\theta-1}{\theta} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \\
&\quad \cdot \underbrace{\Bigg(\Bigg[(\eta n_A)^{\frac{1}{\psi}} \left(C_A^B\right)^{\frac{\psi-1}{\psi}} + (1-\eta n_A)^{\frac{1}{\psi}} \left(C_B^B\right)^{\frac{\psi-1}{\psi}} \Bigg]^{\frac{\psi}{\psi-1}} \Bigg)^{\overbrace{\frac{\theta-1-\theta}{\theta}}^{-\frac{1}{\theta}}}}_{\left(C_P^B\right)^{-\frac{1}{\theta}}} \\
&\quad \cdot \frac{\psi}{\psi-1} \underbrace{\Bigg[(\eta n_A)^{\frac{1}{\psi}} \left(C_A^B\right)^{\frac{\psi-1}{\psi}} + (1-\eta n_A)^{\frac{1}{\psi}} \left(C_B^B\right)^{\frac{\psi-1}{\psi}} \Bigg]^{\overbrace{\frac{\psi-(\psi-1)}{\psi-1}}^{\frac{1}{\psi-1}}}}_{\left(C_P^B\right)^{\frac{1}{\psi}}} \\
&\quad \cdot \frac{\psi-1}{\psi} (\eta n_A)^{\frac{1}{\psi}} \left(C_A^B\right)^{\overbrace{\frac{\psi-1-\psi}{\psi}}^{-\frac{1}{\psi}}} = 0
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow \quad P_A^B &= \lambda \frac{\theta}{\theta-1} (C^B)^{\frac{1}{\theta}} \frac{\theta-1}{\theta} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^B\right)^{-\frac{1}{\theta}} \\
&\quad \cdot \frac{\psi}{\psi-1} \left(C_P^B\right)^{\frac{1}{\psi}} \frac{\psi-1}{\psi} (\eta n_A)^{\frac{1}{\psi}} \left(C_A^B\right)^{-\frac{1}{\psi}} \\
\Leftrightarrow \quad P_A^B &= \lambda (C^B)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} \left(C_P^B\right)^{-\frac{1}{\theta}} \left(C_P^B\right)^{\frac{1}{\psi}} (\eta n_A)^{\frac{1}{\psi}} \left(C_A^B\right)^{-\frac{1}{\psi}} \\
\Leftrightarrow \quad P_A^B &= \lambda (C^B)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} (\eta n_A)^{\frac{1}{\psi}} \left(C_P^B\right)^{\frac{1}{\psi}-\frac{1}{\theta}} \left(C_A^B\right)^{-\frac{1}{\psi}} \\
\Leftrightarrow \quad P_A^B C_A^B &= \lambda (C^B)^{\frac{1}{\theta}} [1-\eta(1-n_P)]^{\frac{1}{\theta}} (\eta n_A)^{\frac{1}{\psi}} \left(C_P^B\right)^{\frac{1}{\psi}-\frac{1}{\theta}} \left(C_A^B\right)^{\frac{\psi-1}{\psi}}
\end{aligned}$$

$$
\begin{aligned}
P_A^B &= \lambda (C^B)^{\frac{1}{\theta}} \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(C_P^B\right)^{-\frac{1}{\theta}} \left(C_P^B\right)^{\frac{1}{\psi}} \left(\eta n_A\right)^{\frac{1}{\psi}} \left(C_A^B\right)^{-\frac{1}{\psi}} \\
P_A^B &= \underbrace{\lambda \left(\frac{C^B}{C_P^B}\right)^{\frac{1}{\theta}}}_{(7.69)} \left(\frac{C_P^B}{C_A^B}\right)^{\frac{1}{\psi}} \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(\eta n_A\right)^{\frac{1}{\psi}} \\
P_A^B &= \left[1-\eta\left(1-n_P\right)\right]^{-\frac{1}{\theta}} \; P_P^B \left(\frac{C_P^B}{C_A^B}\right)^{\frac{1}{\psi}} \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(\eta n_A\right)^{\frac{1}{\psi}} \\
\left(C_A^B\right)^{\frac{1}{\psi}} &= \left(\eta n_A\right)^{\frac{1}{\psi}} \left(\frac{P_A^B}{P_P^B}\right)^{-1} \left(C_P^B\right)^{\frac{1}{\psi}} \\
C_A^B &= \eta n_A \left(\frac{P_A^B}{P_P^B}\right)^{-\psi} C_P^B \qquad (7.70)
\end{aligned}
$$

Country B's demand for country B's goods

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_B^B} &= P_B^B - \lambda \frac{\theta}{\theta-1} \underbrace{\left[\left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(\left[\left(\eta n_A\right)^{\frac{1}{\psi}} \left(C_A^B\right)^{\frac{\psi-1}{\psi}}\right.\right.\right.}_{} \\
&\quad \underbrace{\left.\left.\left. + \left(1-\eta n_A\right)^{\frac{1}{\psi}} \left(C_B^B\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}\right)^{\frac{\theta-1}{\theta}} + \left[\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(C_C^B\right)^{\frac{\theta-1}{\theta}}\right]^{\overbrace{\frac{\theta-(\theta-1)}{\theta-1}}^{\frac{1}{\theta-1}}}}_{(C^B)^{\frac{1}{\theta}}} \\
&\quad \cdot \frac{\theta-1}{\theta} \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \\
&\quad \underbrace{\cdot \left(\left[\left(\eta n_A\right)^{\frac{1}{\psi}} \left(C_A^B\right)^{\frac{\psi-1}{\psi}} + \left(1-\eta n_A\right)^{\frac{1}{\psi}} \left(C_B^B\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}\right)^{\overbrace{\frac{\theta-1-\theta}{\theta}}^{-\frac{1}{\theta}}}}_{\left(C_P^B\right)^{-\frac{1}{\theta}}} \\
&\quad \underbrace{\cdot \frac{\psi}{\psi-1} \left[\left(\eta n_A\right)^{\frac{1}{\psi}} \left(C_A^B\right)^{\frac{\psi-1}{\psi}} + \left(1-\eta n_A\right)^{\frac{1}{\psi}} \left(C_B^B\right)^{\frac{\psi-1}{\psi}}\right]^{\overbrace{\frac{\psi-(\psi-1)}{\psi-1}}^{\frac{1}{\psi-1}}}}_{\left(C_P^B\right)^{\frac{1}{\psi}}} \\
&\quad \cdot \frac{\psi-1}{\psi} \left(1-\eta n_A\right)^{\frac{1}{\psi}} \left(C_B^B\right)^{\overbrace{-\frac{1}{\psi}}^{\frac{\psi-1-\psi}{\psi}}} = 0
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad P_B^B &= \lambda \frac{\theta}{\theta-1} \left(C^B\right)^{\frac{1}{\theta}} \frac{\theta-1}{\theta} \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(C_P^B\right)^{-\frac{1}{\theta}} \\
&\quad \cdot \frac{\psi}{\psi-1} \left(C_P^B\right)^{\frac{1}{\psi}} \frac{\psi-1}{\psi} \left(1-\eta n_A\right)^{\frac{1}{\psi}} \left(C_B^B\right)^{-\frac{1}{\psi}} \\
\Leftrightarrow \quad P_B^B &= \lambda (C^B)^{\frac{1}{\theta}} \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(C_P^B\right)^{-\frac{1}{\theta}} \left(C_P^B\right)^{\frac{1}{\psi}} \left(1-\eta n_A\right)^{\frac{1}{\psi}} \left(C_B^B\right)^{-\frac{1}{\psi}} \\
\Leftrightarrow \quad P_B^B &= \lambda (C^B)^{\frac{1}{\theta}} \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(1-\eta n_A\right)^{\frac{1}{\psi}} \left(C_P^B\right)^{\frac{1}{\psi}-\frac{1}{\theta}} \left(C_B^B\right)^{-\frac{1}{\psi}} \\
\Leftrightarrow \quad P_B^B C_B^B &= \lambda (C^B)^{\frac{1}{\theta}} \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(1-\eta n_A\right)^{\frac{1}{\psi}} \left(C_P^B\right)^{\frac{1}{\psi}-\frac{1}{\theta}} \left(C_B^B\right)^{\frac{\psi-1}{\psi}} \quad (7.71)
\end{aligned}
$$

$$
\begin{aligned}
P_B^B &= \lambda (C^B)^{\frac{1}{\theta}} \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(C_P^B\right)^{-\frac{1}{\theta}} \left(C_P^B\right)^{\frac{1}{\psi}} \left(1-\eta n_A\right)^{\frac{1}{\psi}} \left(C_B^B\right)^{-\frac{1}{\psi}} \\
P_B^B &= \underbrace{\lambda \left(\frac{C^B}{C_P^B}\right)^{\frac{1}{\theta}}}_{(7.69)} \left(\frac{C_P^B}{C_B^B}\right)^{\frac{1}{\psi}} \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(1-\eta n_A\right)^{\frac{1}{\psi}}
\end{aligned}
$$

$$
\begin{aligned}
P_B^B &= \left[1-\eta\left(1-n_P\right)\right]^{-\frac{1}{\theta}} P_P^B \left(\frac{C_P^B}{C_B^B}\right)^{\frac{1}{\psi}} \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(1-\eta n_A\right)^{\frac{1}{\psi}} \\
\left(C_B^B\right)^{\frac{1}{\psi}} &= \left(1-\eta n_A\right)^{\frac{1}{\psi}} \left(\frac{P_B^B}{P_P^B}\right)^{-1} \left(C_P^B\right)^{\frac{1}{\psi}} \\
C_B^B &= \left(1-\eta n_A\right) \left(\frac{P_B^B}{P_P^B}\right)^{-\psi} C_P^B \qquad (7.72)
\end{aligned}
$$

Country B's demand for country C's goods

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_C^B} &= P_C^B - \lambda \frac{\theta}{\theta-1} \underbrace{\left[\left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(\left[\left(\eta n_A\right)^{\frac{1}{\psi}} \left(C_A^B\right)^{\frac{\psi-1}{\psi}}\right.\right.\right.}_{} \\
&\quad \underbrace{\left.\left.\left. + \left(1-\eta n_A\right)^{\frac{1}{\psi}} \left(C_B^A\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}\right)^{\frac{\theta-1}{\theta}} + \left[\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(C_C^A\right)^{\frac{\theta-1}{\theta}}\right]^{\overbrace{\frac{\theta-(\theta-1)}{\theta-1}}^{\frac{1}{\theta-1}}}}_{(C^B)^{\frac{1}{\theta}}} \\
&\quad \cdot \frac{\theta-1}{\theta} \left[\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(C_C^B\right)^{\overbrace{-\frac{1}{\theta}}^{\frac{\theta-1-\theta}{\theta}}} = 0
\end{aligned}
$$

$$
\begin{aligned}
&\Leftrightarrow \quad P_C^B = \lambda \frac{\theta}{\theta-1} \left(C^B\right)^{\frac{1}{\theta}} \frac{\theta-1}{\theta} \left[\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(C_C^B\right)^{-\frac{1}{\theta}} \\
&\Leftrightarrow \quad P_C^B = \lambda \left(C^B\right)^{\frac{1}{\theta}} \left[\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(C_C^B\right)^{-\frac{1}{\theta}} \\
&\Leftrightarrow \quad P_C^B C_C^B = \lambda \left(C^B\right)^{\frac{1}{\theta}} \left[\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(C_C^B\right)^{\frac{\theta-1}{\theta}}
\end{aligned}
$$

$$
\begin{aligned}
P_C^B &= \lambda \left(C^B\right)^{\frac{1}{\theta}} \left[\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(C_C^B\right)^{-\frac{1}{\theta}} \\
P_C^B &= \lambda \left(\frac{C^B}{C_C^B}\right)^{\frac{1}{\theta}} \left[\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}
\end{aligned}
$$

Summary: Aggregate Demand Functions for Country B Using (7.63) to derive country B's demand for goods from the periphery P

$$
\begin{aligned}
\underbrace{P^B}_{\lambda} \left(\frac{C^B}{C_P^B}\right)^{\frac{1}{\theta}} &= \left[1-\eta\left(1-n_P\right)\right]^{-\frac{1}{\theta}} P_P^B \\
\left(C_P^B\right)^{\frac{1}{\theta}} &= \left[1-\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \left(\frac{P_P^B}{P^B}\right)^{-1} \left(C^B\right)^{\frac{1}{\theta}} \\
C_P^B &= \left[1-\eta\left(1-n_P\right)\right] \left(\frac{P_P^B}{P^B}\right)^{-\theta} C^B
\end{aligned} \tag{7.73}
$$

Country B's demand for country A's goods

$$
\begin{aligned}
C_A^B &= \eta n_A \left(\frac{P_A^B}{P_P^B}\right)^{-\psi} \underbrace{\left[1-\eta\left(1-n_P\right)\right] \left(\frac{P_P^B}{P^B}\right)^{-\theta} C^B}_{C_P^B} \\
C_A^B &= \underbrace{\eta n_A \left[1-\eta\left(1-n_P\right)\right]}_{v_A^B} \left(\frac{P_A^B}{P_P^B}\right)^{-\psi} \left(\frac{P_P^B}{P^B}\right)^{-\theta} C^B
\end{aligned} \tag{7.74}
$$

Country B's demand for country B's goods

$$
\begin{aligned}
C_B^B &= \left(1-\eta n_A\right) \left(\frac{P_B^B}{P_P^B}\right)^{-\psi} \underbrace{\left[1-\eta\left(1-n_P\right)\right] \left(\frac{P_P^B}{P^B}\right)^{-\theta} C^B}_{C_P^B} \\
C_B^B &= \underbrace{\left(1-\eta n_A\right) \left[1-\eta\left(1-n_P\right)\right]}_{v_B^B} \left(\frac{P_B^B}{P_P^B}\right)^{-\psi} \left(\frac{P_P^B}{P^B}\right)^{-\theta} C^B
\end{aligned} \tag{7.75}
$$

$$
\begin{aligned}
v_B^B &\equiv (1-\eta n_A)\left[1-\eta\left(1-n_P\right)\right] \\
&= (1-\eta n_A)\left[1-\eta+\eta n_P\right] \\
&= 1-\eta+\eta n_P-\eta n_A+\eta^2 n_A-\eta^2 n_P n_A \\
&= 1-\eta n_A+\eta^2 n_A-\eta^2 n_A n_P-\eta+\eta n_P \\
&= 1-\left[1-\eta+\eta n_P\right]\eta n_A-\eta+\eta n_P \\
&= 1-\left[1-\eta\left(1-n_P\right)\right]\eta n_A-\eta\left(1-n_P\right)
\end{aligned}
$$

Country B's demand for country C's goods

$$
\begin{aligned}
P_C^B &= \underbrace{P^B}_{\lambda}\left(\frac{C^B}{C_C^B}\right)^{\frac{1}{\theta}}\left[\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}} \\
\left(C_C^B\right)^{\frac{1}{\theta}} &= \left[\eta\left(1-n_P\right)\right]^{\frac{1}{\theta}}\left(\frac{P_C^B}{P^B}\right)^{-1}\left(C^B\right)^{\frac{1}{\theta}} \\
C_C^B &= \underbrace{\eta\left(1-n_P\right)}_{v_C^B}\left(\frac{P_C^B}{P^B}\right)^{-\theta} C^B \qquad (7.76)
\end{aligned}
$$

Aggregate Demand Function for Country C

Maximize total expenditure, Z, consisting of country C's CPI price index, P^C, times total consumption, C^C.

$$
\begin{aligned}
\max_{C^C} \quad & Z = C^C P^C \\
\text{s.t.} \quad & C^C = \left[(\eta n_P)^{\frac{1}{\theta}}\left(\left[(n_A)^{\frac{1}{\psi}}\left(C_A^C\right)^{\frac{\psi-1}{\psi}}+(1-n_A)^{\frac{1}{\psi}}\left(C_B^C\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}\right)^{\frac{\theta-1}{\theta}}\right. \\
& \left.+(1-\eta n_P)^{\frac{1}{\theta}}\left(C_C^C\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}
\end{aligned}
$$

Transform into Lagrangian $\mathcal{L} = f(x) - \lambda \cdot h(x)$

$$
f(x) \equiv C^C P^C = C_A^C P_A^C + C_B^C P_B^C + C_C^C P_C^C
$$

$$
\begin{aligned}
h(x) \equiv & \left[(\eta n_P)^{\frac{1}{\theta}}\left(\left[(n_A)^{\frac{1}{\psi}}\left(C_A^C\right)^{\frac{\psi-1}{\psi}}+(1-n_A)^{\frac{1}{\psi}}\left(C_B^C\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}\right)^{\frac{\theta-1}{\theta}}\right. \\
& \left.+(1-\eta n_P)^{\frac{1}{\theta}}\left(C_C^C\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} - C^C \geq 0
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L} &= C_A^C\, P_A^C + C_B^C\, P_B^C + C_C^C\, P_C^C \\
&\quad -\lambda \left\{ \left[(\eta n_P)^{\frac{1}{\theta}} \left(\left[(n_A)^{\frac{1}{\psi}} \left(C_A^C\right)^{\frac{\psi-1}{\psi}} + (1-n_A)^{\frac{1}{\psi}} \left(C_B^C\right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \right)^{\frac{\theta-1}{\theta}} \right.\right. \\
&\quad \left.\left. + (1-\eta n_P)^{\frac{1}{\theta}} \left(C_C^C\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} - C^C \right\}.
\end{aligned}
$$

First order conditions for country C

Country C's demand for periphery P's goods

$$
\begin{aligned}
\mathcal{L} &= C_P^C\, P_P^C + C_C^B\, P_C^B \\
&\quad -\lambda \left\{ \left[(\eta n_P)^{\frac{1}{\theta}} \left(C_P^C\right)^{\frac{\theta-1}{\theta}} \right.\right. \\
&\quad \left.\left. + (1-\eta n_P)^{\frac{1}{\theta}} \left(C_C^C\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} - C^C \right\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_P^C} &= P_P^C - \lambda \frac{\theta}{\theta-1} \underbrace{\left[(\eta n_P)^{\frac{1}{\theta}} \left(C_P^C\right)^{\frac{\theta-1}{\theta}} \right.}_{\overbrace{\quad}^{\frac{\theta-(\theta-1)}{\theta-1}}} \\
&\quad \underbrace{\left. + (1-\eta n_P)^{\frac{1}{\theta}} \left(C_C^C\right)^{\frac{\theta-1}{\theta}} \right]^{\overbrace{\frac{1}{\theta-1}}}}_{(C^C)^{\frac{1}{\theta}}} \\
&\quad \cdot \frac{\theta-1}{\theta} (\eta n_P)^{\frac{1}{\theta}} \left(C_P^C\right)^{\overbrace{-\frac{1}{\theta}}^{\frac{\theta-1-\theta}{\theta}}} = 0
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & P_P^C = \lambda \frac{\theta}{\theta-1} (C^C)^{\frac{1}{\theta}} \frac{\theta-1}{\theta} (\eta n_P)^{\frac{1}{\theta}} \left(C_P^C\right)^{-\frac{1}{\theta}} \\
\Leftrightarrow \quad & P_P^C = \lambda (C^C)^{\frac{1}{\theta}} (\eta n_P)^{\frac{1}{\theta}} \left(C_P^C\right)^{-\frac{1}{\theta}} \\
\Leftrightarrow \quad & P_P^C C_P^C = \lambda (C^C)^{\frac{1}{\theta}} (\eta n_P)^{\frac{1}{\theta}} \left(C_P^C\right)^{\frac{\theta-1}{\theta}}
\end{aligned}
$$

$$
\begin{aligned}
P_P^C &= \lambda (C^C)^{\frac{1}{\theta}} (\eta n_P)^{\frac{1}{\theta}} \left(C_P^C\right)^{-\frac{1}{\theta}} \\
P_P^C &= \lambda \left(\frac{C^C}{C_P^C} \right)^{\frac{1}{\theta}} (\eta n_P)^{\frac{1}{\theta}} \\
\lambda \left(\frac{C^C}{C_P^C} \right)^{\frac{1}{\theta}} &= (\eta n_P)^{-\frac{1}{\theta}} P_P^C \qquad (7.77)
\end{aligned}
$$

Country C's demand for country A's goods

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_A^C} &= P_A^C - \lambda \frac{\theta}{\theta-1} \underbrace{\Bigg[(\eta n_P)^{\frac{1}{\theta}} \Bigg(\Bigg[(n_A)^{\frac{1}{\psi}} \left(C_A^C\right)^{\frac{\psi-1}{\psi}}}_{} \\
&\quad \underbrace{+ \left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}} \left(C_B^C\right)^{\frac{\psi-1}{\psi}} \Bigg]^{\frac{\psi}{\psi-1}} \Bigg)^{\frac{\theta-1}{\theta}} + \left(1-\eta n_P\right)^{\frac{1}{\theta}} \left(C_C^C\right)^{\frac{\theta-1}{\theta}} \Bigg]^{\overbrace{\frac{\theta-(\theta-1)}{\theta-1}}^{\frac{1}{\theta-1}}}}_{\left(C^C\right)^{\frac{1}{\theta}}} \\
&\quad \cdot \frac{\theta-1}{\theta} \left(\eta n_P\right)^{\frac{1}{\theta}} \\
&\quad \cdot \underbrace{\Bigg(\Bigg[(n_A)^{\frac{1}{\psi}} \left(C_A^C\right)^{\frac{\psi-1}{\psi}} + \left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}} \left(C_B^C\right)^{\frac{\psi-1}{\psi}} \Bigg]^{\frac{\psi}{\psi-1}} \Bigg)^{\overbrace{\frac{\theta-1-\theta}{\theta}}^{-\frac{1}{\theta}}}}_{\left(C_P^C\right)^{-\frac{1}{\theta}}} \\
&\quad \cdot \frac{\psi}{\psi-1} \underbrace{\Bigg[(n_A)^{\frac{1}{\psi}} \left(C_A^C\right)^{\frac{\psi-1}{\psi}} + \left[\eta\left(1-n_A\right)\right]^{\frac{1}{\psi}} \left(C_B^C\right)^{\frac{\psi-1}{\psi}} \Bigg]^{\overbrace{\frac{\psi-(\psi-1)}{\psi-1}}^{\frac{1}{\psi-1}}}}_{\left(C_P^C\right)^{\frac{1}{\psi}}} \\
&\quad \cdot \frac{\psi-1}{\psi} \left(n_A\right)^{\frac{1}{\psi}} \left(C_A^C\right)^{\overbrace{-\frac{1}{\psi}}^{\frac{\psi-1-\psi}{\psi}}} = 0
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow \quad P_A^C &= \lambda \frac{\theta}{\theta-1} \left(C^C\right)^{\frac{1}{\theta}} \frac{\theta-1}{\theta} \left(\eta n_P\right)^{\frac{1}{\theta}} \left(C_P^C\right)^{-\frac{1}{\theta}} \\
&\quad \cdot \frac{\psi}{\psi-1} \left(C_P^C\right)^{\frac{1}{\psi}} \frac{\psi-1}{\psi} \left(n_A\right)^{\frac{1}{\psi}} \left(C_A^C\right)^{-\frac{1}{\psi}} \\
\Leftrightarrow \quad P_A^C &= \lambda (C^C)^{\frac{1}{\theta}} \left(\eta n_P\right)^{\frac{1}{\theta}} \left(C_P^C\right)^{-\frac{1}{\theta}} \left(C_P^C\right)^{\frac{1}{\psi}} \left(n_A\right)^{\frac{1}{\psi}} \left(C_A^C\right)^{-\frac{1}{\psi}} \\
\Leftrightarrow \quad P_A^C &= \lambda (C^C)^{\frac{1}{\theta}} \left(\eta n_P\right)^{\frac{1}{\theta}} \left(n_A\right)^{\frac{1}{\psi}} \left(C_P^C\right)^{\frac{1}{\psi}-\frac{1}{\theta}} \left(C_A^C\right)^{-\frac{1}{\psi}} \\
\Leftrightarrow \quad P_A^C C_A^C &= \lambda (C^C)^{\frac{1}{\theta}} \left(\eta n_P\right)^{\frac{1}{\theta}} \left(n_A\right)^{\frac{1}{\psi}} \left(C_P^C\right)^{\frac{1}{\psi}-\frac{1}{\theta}} \left(C_A^C\right)^{\frac{\psi-1}{\psi}}
\end{aligned}$$

$$
\begin{aligned}
P_A^C &= \lambda (C^A)^{\frac{1}{\theta}} \left(\eta n_P\right)^{\frac{1}{\theta}} \left(C_P^A\right)^{-\frac{1}{\theta}} \left(C_P^A\right)^{\frac{1}{\psi}} \left(n_A\right)^{\frac{1}{\psi}} \left(C_A^C\right)^{-\frac{1}{\psi}} \\
P_A^C &= \underbrace{\lambda \left(\frac{C^C}{C_P^C}\right)^{\frac{1}{\theta}}}_{(7.77)} \left(\frac{C_P^C}{C_A^C}\right)^{\frac{1}{\psi}} \left(\eta n_P\right)^{\frac{1}{\theta}} \left(n_A\right)^{\frac{1}{\psi}}
\end{aligned}
$$

$$
\begin{aligned}
P_A^C &= \left(\eta n_P\right)^{-\frac{1}{\theta}} P_P^C \left(\frac{C_P^C}{C_A^C}\right)^{\frac{1}{\psi}} \left(\eta n_P\right)^{\frac{1}{\theta}} \left(n_A\right)^{\frac{1}{\psi}} \\
\left(C_A^C\right)^{\frac{1}{\psi}} &= \left(n_A\right)^{\frac{1}{\psi}} \left(\frac{P_A^C}{P_P^C}\right)^{-1} \left(C_P^C\right)^{\frac{1}{\psi}} \\
C_A^C &= n_A \left(\frac{P_A^C}{P_P^C}\right)^{-\psi} C_P^C \qquad (7.78)
\end{aligned}
$$

Country C's demand for country B's goods

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_B^C} &= P_B^C - \lambda \frac{\theta}{\theta - 1} \underbrace{\Bigg[\left(\eta n_P\right)^{\frac{1}{\theta}} \Bigg(\Bigg[\underbrace{\left(n_A\right)^{\frac{1}{\psi}} \left(C_A^C\right)^{\frac{\psi-1}{\psi}}}_{} \\
&\quad + \left(1 - n_A\right)^{\frac{1}{\psi}} \left(C_B^C\right)^{\frac{\psi-1}{\psi}} \Bigg]^{\frac{\psi}{\psi-1}} \Bigg)^{\frac{\theta-1}{\theta}} + \left(1 - \eta n_P\right)^{\frac{1}{\theta}} \left(C_C^C\right)^{\frac{\theta-1}{\theta}} \Bigg]^{\overbrace{\frac{\theta-(\theta-1)}{\theta-1}}^{\frac{1}{\theta-1}}}}_{(C^C)^{\frac{1}{\theta}}} \\
&\quad \cdot \frac{\theta - 1}{\theta} \left(\eta n_P\right)^{\frac{1}{\theta}} \\
&\quad \cdot \underbrace{\left(\left[\left(n_A\right)^{\frac{1}{\psi}} \left(C_A^C\right)^{\frac{\psi-1}{\psi}} + \left(1 - n_A\right)^{\frac{1}{\psi}} \left(C_B^C\right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \right)^{\overbrace{\frac{\theta-1-\theta}{\theta}}^{-\frac{1}{\theta}}}}_{\left(C_P^C\right)^{-\frac{1}{\theta}}} \\
&\quad \cdot \frac{\psi}{\psi - 1} \underbrace{\left[\left(n_A\right)^{\frac{1}{\psi}} \left(C_A^C\right)^{\frac{\psi-1}{\psi}} + \left(1 - n_A\right)^{\frac{1}{\psi}} \left(C_B^C\right)^{\frac{\psi-1}{\psi}} \right]^{\overbrace{\frac{\psi-(\psi-1)}{\psi-1}}^{\frac{1}{\psi-1}}}}_{\left(C_P^C\right)^{\frac{1}{\psi}}} \\
&\quad \cdot \frac{\psi - 1}{\psi} \left(1 - n_A\right)^{\frac{1}{\psi}} \left(C_B^C\right)^{\overbrace{-\frac{1}{\psi}}^{\frac{\psi-1-\psi}{\psi}}} = 0
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad P_B^C &= \lambda \frac{\theta}{\theta-1} \left(C^C\right)^{\frac{1}{\theta}} \frac{\theta-1}{\theta} \left(\eta n_P\right)^{\frac{1}{\theta}} \left(C_P^C\right)^{-\frac{1}{\theta}} \\
&\quad \cdot \frac{\psi}{\psi-1} \left(C_P^C\right)^{\frac{1}{\psi}} \frac{\psi-1}{\psi} \left(1-n_A\right)^{\frac{1}{\psi}} \left(C_B^C\right)^{-\frac{1}{\psi}} \\
\Leftrightarrow \quad P_B^C &= \lambda (C^C)^{\frac{1}{\theta}} \left(\eta n_P\right)^{\frac{1}{\theta}} \left(C_P^C\right)^{-\frac{1}{\theta}} \left(C_P^C\right)^{\frac{1}{\psi}} \left(1-n_A\right)^{\frac{1}{\psi}} \left(C_B^C\right)^{-\frac{1}{\psi}} \\
\Leftrightarrow \quad P_B^C &= \lambda (C^C)^{\frac{1}{\theta}} \left(\eta n_P\right)^{\frac{1}{\theta}} \left(1-n_A\right)^{\frac{1}{\psi}} \left(C_P^C\right)^{\frac{1}{\psi}-\frac{1}{\theta}} \left(C_B^C\right)^{-\frac{1}{\psi}} \\
\Leftrightarrow \quad P_B^C C_B^C &= \lambda (C^C)^{\frac{1}{\theta}} \left(\eta n_P\right)^{\frac{1}{\theta}} \left(1-n_A\right)^{\frac{1}{\psi}} \left(C_P^C\right)^{\frac{1}{\psi}-\frac{1}{\theta}} \left(C_B^C\right)^{\frac{\psi-1}{\psi}}
\end{aligned}
$$

$$
\begin{aligned}
P_B^C &= \lambda (C^A)^{\frac{1}{\theta}} \left(\eta n_P\right)^{\frac{1}{\theta}} \left(C_P^A\right)^{-\frac{1}{\theta}} \left(C_P^A\right)^{\frac{1}{\psi}} \left(1-n_A\right)^{\frac{1}{\psi}} \left(C_B^C\right)^{-\frac{1}{\psi}} \\
P_B^C &= \underbrace{\lambda \left(\frac{C^C}{C_P^C}\right)^{\frac{1}{\theta}}}_{(7.77)} \left(\frac{C_P^C}{C_B^C}\right)^{\frac{1}{\psi}} \left(\eta n_P\right)^{\frac{1}{\theta}} \left(1-n_A\right)^{\frac{1}{\psi}}
\end{aligned}
$$

$$
\begin{aligned}
P_B^C &= \left(\eta n_P\right)^{-\frac{1}{\theta}} P_P^C \left(\frac{C_P^C}{C_B^C}\right)^{\frac{1}{\psi}} \left(\eta n_P\right)^{\frac{1}{\theta}} \left(1-n_A\right)^{\frac{1}{\psi}} \\
\left(C_B^C\right)^{\frac{1}{\psi}} &= \left(1-n_A\right)^{\frac{1}{\psi}} \left(\frac{P_B^C}{P_P^C}\right)^{-1} \left(C_P^C\right)^{\frac{1}{\psi}} \\
C_B^C &= \left(1-n_A\right) \left(\frac{P_B^C}{P_P^C}\right)^{-\psi} C_P^C \qquad (7.79)
\end{aligned}
$$

Country C's demand for country C's goods

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_C^C} &= P_C^C - \lambda \frac{\theta}{\theta-1} \underbrace{\Bigg[\left(\eta n_P\right)^{\frac{1}{\theta}} \Bigg(\Bigg[\left(n_A\right)^{\frac{1}{\psi}} \left(C_A^C\right)^{\frac{\psi-1}{\psi}}}_{} \\
&\quad \underbrace{+ \left[\eta \left(1-n_A\right)\right]^{\frac{1}{\psi}} \left(C_B^C\right)^{\frac{\psi-1}{\psi}} \Bigg]^{\frac{\psi}{\psi-1}} \Bigg)^{\frac{\theta-1}{\theta}} + \left(1-\eta n_P\right)^{\frac{1}{\theta}} \left(C_C^C\right)^{\frac{\theta-1}{\theta}} \Bigg]^{\overbrace{\frac{\theta-(\theta-1)}{\theta-1}}^{\frac{1}{\theta-1}}}}_{(C^C)^{\frac{1}{\theta}}} \\
&\quad \cdot \frac{\theta-1}{\theta} \left(1-\eta n_P\right)^{\frac{1}{\theta}} \overbrace{\left(C_C^C\right)^{-\frac{1}{\theta}}}^{\frac{\theta-1-\theta}{\theta}} = 0
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & P_C^C = \lambda \frac{\theta}{\theta-1} \left(C^C\right)^{\frac{1}{\theta}} \frac{\theta-1}{\theta} \left(1-\eta n_P\right)^{\frac{1}{\theta}} \left(C_C^C\right)^{-\frac{1}{\theta}} \\
\Leftrightarrow \quad & P_C^C = \lambda (C^C)^{\frac{1}{\theta}} \left(1-\eta n_P\right)^{\frac{1}{\theta}} \left(C_C^C\right)^{-\frac{1}{\theta}} \\
\Leftrightarrow \quad & P_C^C C_C^C = \lambda (C^C)^{\frac{1}{\theta}} \left(1-\eta n_P\right)^{\frac{1}{\theta}} \left(C_C^C\right)^{\frac{\theta-1}{\theta}}
\end{aligned}
$$

$$
\begin{aligned}
P_C^C &= \lambda (C^C)^{\frac{1}{\theta}} \left(1-\eta n_P\right)^{\frac{1}{\theta}} \left(C_C^C\right)^{-\frac{1}{\theta}} \\
P_C^C &= \lambda \left(\frac{C^C}{C_C^C}\right)^{\frac{1}{\theta}} \left(1-\eta n_P\right)^{\frac{1}{\theta}}
\end{aligned}
$$

Summary: Aggregate Demand Functions for Country C Using (7.63) to derive country C's demand for goods from the periphery P

$$
\begin{aligned}
\underbrace{P^C}_{\lambda} \left(\frac{C^C}{C_P^C}\right)^{\frac{1}{\theta}} &= \left(\eta n_P\right)^{-\frac{1}{\theta}} P_P^C \\
\left(C_P^C\right)^{\frac{1}{\theta}} &= \left(\eta n_P\right)^{\frac{1}{\theta}} \left(\frac{P_P^C}{P^C}\right)^{-1} \left(C^C\right)^{\frac{1}{\theta}} \\
C_P^C &= \eta n_P \left(\frac{P_P^C}{P^C}\right)^{-\theta} C^C
\end{aligned} \tag{7.80}
$$

Country C's demand for country A's goods

$$
\begin{aligned}
C_A^C &= n_A \left(\frac{P_A^C}{P_P^C}\right)^{-\psi} \underbrace{\eta n_P \left(\frac{P_P^C}{P^C}\right)^{-\theta} C^C}_{C_P^C} \\
C_A^C &= \underbrace{n_A \eta n_P}_{v_A^C} \left(\frac{P_A^C}{P_P^C}\right)^{-\psi} \left(\frac{P_P^C}{P^C}\right)^{-\theta} C^C
\end{aligned} \tag{7.81}
$$

Country C's demand for country B's goods

$$
\begin{aligned}
C_B^C &= \left(1-n_A\right) \left(\frac{P_B^C}{P_P^C}\right)^{-\psi} \underbrace{\eta n_P \left(\frac{P_P^C}{P^C}\right)^{-\theta} C^C}_{C_P^C} \\
C_B^C &= \underbrace{\left(1-n_A\right) \eta n_P}_{v_B^C} \left(\frac{P_B^C}{P_P^C}\right)^{-\psi} \left(\frac{P_P^C}{P^C}\right)^{-\theta} C^C
\end{aligned} \tag{7.82}
$$

Country C's demand for country C's goods

$$\begin{aligned} P_C^C &= \underbrace{P^C}_{\lambda}\left(\frac{C^C}{C_C^C}\right)^{\frac{1}{\theta}}\left(1-\eta n_P\right)^{\frac{1}{\theta}} \\ \left(C_C^C\right)^{\frac{1}{\theta}} &= \left(1-\eta n_P\right)^{\frac{1}{\theta}}\left(\frac{P_C^C}{P^C}\right)^{-1}\left(C^C\right)^{\frac{1}{\theta}} \\ C_C^C &= \underbrace{\left(1-\eta n_P\right)}_{v_C^C}\left(\frac{P_C^C}{P^C}\right)^{-\theta} C^C \end{aligned} \tag{7.83}$$

7.1.8 Individual Demand in Terms of CPI Basket

$$\begin{aligned} C_A^A(z) &= \left(\frac{P_A^A(z)}{P_A^A}\right)^{-\phi}\left(n_P n_A\right)^{-1}\underbrace{v_A^A\left(\frac{P_A^A}{P_P^A}\right)^{-\psi}\left(\frac{P_P^A}{P^A}\right)^{-\theta} C^A}_{C_A^A} \\ C_A^A(z) &= \frac{1}{n_P n_A} v_A^A\left(\frac{P_A^A(z)}{P_A^A}\right)^{-\phi}\left(\frac{P_A^A}{P_P^A}\right)^{-\psi}\left(\frac{P_P^A}{P^A}\right)^{-\theta} C^A \end{aligned} \tag{7.84}$$

$$\begin{aligned} C_A^B(z) &= \left(\frac{P_A^B(z)}{P_A^B}\right)^{-\phi}\left(n_P n_A\right)^{-1}\underbrace{v_A^B\left(\frac{P_A^B}{P_P^B}\right)^{-\psi}\left(\frac{P_P^B}{P^B}\right)^{-\theta} C^B}_{C_A^B} \\ C_A^B(z) &= \frac{1}{n_P n_A} v_A^B\left(\frac{P_A^B(z)}{P_A^B}\right)^{-\phi}\left(\frac{P_A^B}{P_P^B}\right)^{-\psi}\left(\frac{P_P^B}{P^B}\right)^{-\theta} C^B \end{aligned} \tag{7.85}$$

$$\begin{aligned} C_A^C(z) &= \left(\frac{P_A^C(z)}{P_A^C}\right)^{-\phi}\left(n_P n_A\right)^{-1}\underbrace{v_A^C\left(\frac{P_A^C}{P_P^C}\right)^{-\psi}\left(\frac{P_P^C}{P^C}\right)^{-\theta} C^C}_{C_A^C} \\ C_A^C(z) &= \frac{1}{n_P n_A} v_A^C\left(\frac{P_A^C(z)}{P_A^C}\right)^{-\phi}\left(\frac{P_A^C}{P_P^C}\right)^{-\psi}\left(\frac{P_P^C}{P^C}\right)^{-\theta} C^C \end{aligned} \tag{7.86}$$

$$\begin{aligned} C_B^A(z) &= \left(\frac{P_B^A(z)}{P_B^A}\right)^{-\phi}\left(\left(1-n_A\right)n_P\right)^{-1}\underbrace{v_B^A\left(\frac{P_B^A}{P_P^A}\right)^{-\psi}\left(\frac{P_P^A}{P^A}\right)^{-\theta} C^A}_{C_B^A} \\ C_B^A(z) &= \frac{1}{\left(1-n_A\right)n_P} v_B^A\left(\frac{P_B^A(z)}{P_B^A}\right)^{-\phi}\left(\frac{P_B^A}{P_P^A}\right)^{-\psi}\left(\frac{P_P^A}{P^A}\right)^{-\theta} C^A \end{aligned} \tag{7.87}$$

$$C_B^B(z) = \left(\frac{P_B^B(z)}{P_B^B}\right)^{-\phi} ((1-n_A)\,n_P)^{-1} \underbrace{v_B^B \left(\frac{P_B^B}{P_P^B}\right)^{-\psi} \left(\frac{P_P^B}{P^B}\right)^{-\theta} C^B}_{C_B^B}$$

$$C_B^B(z) = \frac{1}{(1-n_A)\,n_P} v_B^B \left(\frac{P_B^B(z)}{P_B^B}\right)^{-\phi} \left(\frac{P_B^B}{P_P^B}\right)^{-\psi} \left(\frac{P_P^B}{P^B}\right)^{-\theta} C^B \tag{7.88}$$

$$C_B^C(z) = \left(\frac{P_B^C(z)}{P_B^C}\right)^{-\phi} ((1-n_A)\,n_P)^{-1} \underbrace{v_B^C \left(\frac{P_B^C}{P_P^C}\right)^{-\psi} \left(\frac{P_P^C}{P^C}\right)^{-\theta} C^C}_{C_B^C}$$

$$C_B^C(z) = \frac{1}{(1-n_A)\,n_P} v_B^C \left(\frac{P_B^C(z)}{P_B^C}\right)^{-\phi} \left(\frac{P_B^C}{P_P^C}\right)^{-\psi} \left(\frac{P_P^C}{P^C}\right)^{-\theta} C^C \tag{7.89}$$

$$C_C^A(z) = \left(\frac{P_C^A(z)}{P_C^A}\right)^{-\phi} (1-n_P)^{-1} \underbrace{v_C^A \left(\frac{P_C^A}{P^A}\right)^{-\theta} C^A}_{C_C^A}$$

$$C_C^A(z) = \frac{1}{1-n_P} v_C^A \left(\frac{P_C^A(z)}{P_C^A}\right)^{-\phi} \left(\frac{P_C^A}{P^A}\right)^{-\theta} C^A \tag{7.90}$$

$$C_C^B(z) = \left(\frac{P_C^B(z)}{P_C^B}\right)^{-\phi} (1-n_P)^{-1} \underbrace{v_C^B \left(\frac{P_C^B}{P^B}\right)^{-\theta} C^B}_{C_C^B}$$

$$C_C^B(z) = \frac{1}{1-n_P} v_C^B \left(\frac{P_C^B(z)}{P_C^B}\right)^{-\phi} \left(\frac{P_C^B}{P^B}\right)^{-\theta} C^B \tag{7.91}$$

$$C_C^C(z) = \left(\frac{P_C^C(z)}{P_C^C}\right)^{-\phi} (1-n_P)^{-1} \underbrace{v_C^C \left(\frac{P_C^C}{P^C}\right)^{-\theta} C^C}_{C_C^C}$$

$$C_C^C(z) = \frac{1}{1-n_P} v_C^C \left(\frac{P_C^C(z)}{P_C^C}\right)^{-\phi} \left(\frac{P_C^C}{P^C}\right)^{-\theta} C^C \tag{7.92}$$

7.1.9 Goods Market Clearing

Goods Market Clearing in Country A

Goods market clearing in levels in country A for variety z

$$Y_t^A(z) = n_P n_A C_{A,t}^A(z) + n_P(1-n_A)\,C_{A,t}^B(z) + (1-n_P)\,C_{A,t}^C(z)$$

$$
\begin{aligned}
Y^A(z) &= n_P n_A \left(n_P n_A\right)^{-1} v_A^A \left(\frac{P_A^A(z)}{P_A^A}\right)^{-\phi} \left(\frac{P_A^A}{P_P^A}\right)^{-\psi} \left(\frac{P_P^A}{P^A}\right)^{-\theta} C^A \\
&\quad + n_P\left(1-n_A\right)\left(n_P n_A\right)^{-1} v_A^B \left(\frac{P_A^B(z)}{P_A^B}\right)^{-\phi} \left(\frac{P_A^B}{P_P^B}\right)^{-\psi} \left(\frac{P_P^B}{P^B}\right)^{-\theta} C^B \\
&\quad + \left(1-n_P\right)\left(n_P n_A\right)^{-1} v_A^C \left(\frac{P_A^C(z)}{P_A^C}\right)^{-\phi} \left(\frac{P_A^C}{P_P^C}\right)^{-\psi} \left(\frac{P_P^C}{P^C}\right)^{-\theta} C^C
\end{aligned}
$$

Substitute trade shares

$$
n_P n_A \left(n_P n_A\right)^{-1} v_A^A = v_A^A
$$

$$
\begin{aligned}
& n_P\left(1-n_A\right)\left(n_P n_A\right)^{-1} v_A^B \\
= \;& n_P\left(1-n_A\right)\left(n_P n_A\right)^{-1}\left[\left[1-\eta\left(1-n_P\right)\right]\eta n_A\right] \\
= \;& \left[1-\eta\left(1-n_P\right)\right]\frac{\eta n_A n_P\left(1-n_A\right)}{n_P n_A} \\
= \;& \left[1-\eta\left(1-n_P\right)\right]\eta\left(1-n_A\right) = v_B^A
\end{aligned}
$$

$$
\begin{aligned}
& \left(1-n_P\right)\left(n_P n_A\right)^{-1} v_A^C \\
= \;& \left(1-n_P\right)\left(n_P n_A\right)^{-1}\left[\eta n_A n_P\right] \\
= \;& \left(1-n_P\right)\frac{\eta n_A n_P}{n_P n_A} \\
= \;& \eta\left(1-n_P\right) = v_C^A
\end{aligned}
$$

$$
\begin{aligned}
Y^A(z) &= v_A^A \left(\frac{P_A^A(z)}{P_A^A}\right)^{-\phi} \left(\frac{P_A^A}{P_P^A}\right)^{-\psi} \left(\frac{P_P^A}{P^A}\right)^{-\theta} C^A \\
&\quad + v_B^A \left(\frac{P_A^B(z)}{P_A^B}\right)^{-\phi} \left(\frac{P_A^B}{P_P^B}\right)^{-\psi} \left(\frac{P_P^B}{P^B}\right)^{-\theta} C^B \\
&\quad + v_C^A \left(\frac{P_A^C(z)}{P_A^C}\right)^{-\phi} \left(\frac{P_A^C}{P_P^C}\right)^{-\psi} \left(\frac{P_P^C}{P^C}\right)^{-\theta} C^C.
\end{aligned}
$$

Using the definition of the law of one price

$$
\begin{aligned}
P_{A,t}^B(z) &= \frac{P_{A,t}^A(z)}{S_{B,t}^A} \\
P_{A,t}^C(z) &= \frac{P_{A,t}^A(z)}{S_{C,t}^A}
\end{aligned}
$$

$$
\begin{aligned}
P_{A,t}^{B} &= \frac{P_{A,t}^{A}}{S_{B,t}^{A}} \\
P_{A,t}^{C} &= \frac{P_{A,t}^{A}}{S_{C,t}^{A}}
\end{aligned}
$$

and expressing all prices in terms of country A's currency

$$
\begin{aligned}
Y^{A}(z) &= v_{A}^{A}\left(\frac{P_{A}^{A}(z)}{P_{A}^{A}}\right)^{-\phi}\left(\frac{P_{A}^{A}}{P_{P}^{A}}\right)^{-\psi}\left(\frac{P_{P}^{A}}{P^{A}}\right)^{-\theta}C^{A} \\
&\quad + v_{B}^{A}\left(\frac{\frac{P_{A}^{A}(z)}{S_{B}^{A}}}{\frac{P_{A}^{A}}{S_{B}^{A}}}\right)^{-\phi}\left(\frac{P_{A}^{B}}{P_{P}^{B}}\right)^{-\psi}\left(\frac{P_{P}^{B}}{P^{B}}\right)^{-\theta}C^{B} \\
&\quad + v_{C}^{A}\left(\frac{\frac{P_{A}^{A}(z)}{S_{C}^{A}}}{\frac{P_{A}^{A}}{S_{C}^{A}}}\right)^{-\phi}\left(\frac{P_{A}^{C}}{P_{P}^{C}}\right)^{-\psi}\left(\frac{P_{P}^{C}}{P^{C}}\right)^{-\theta}C^{C}
\end{aligned}
$$

$$
\begin{aligned}
Y^{A}(z) &= v_{A}^{A}\left(\frac{P_{A}^{A}(z)}{P_{A}^{A}}\right)^{-\phi}\left(\frac{P_{A}^{A}}{P_{P}^{A}}\right)^{-\psi}\left(\frac{P_{P}^{A}}{P^{A}}\right)^{-\theta}C^{A} \\
&\quad + v_{B}^{A}\left(\frac{P_{A}^{A}(z)}{P_{A}^{A}}\right)^{-\phi}\left(\frac{P_{A}^{B}}{P_{P}^{B}}\right)^{-\psi}\left(\frac{P_{P}^{B}}{P^{B}}\right)^{-\theta}C^{B} \\
&\quad + v_{C}^{A}\left(\frac{P_{A}^{A}(z)}{P_{A}^{A}}\right)^{-\phi}\left(\frac{P_{A}^{C}}{P_{P}^{C}}\right)^{-\psi}\left(\frac{P_{P}^{C}}{P^{C}}\right)^{-\theta}C^{C}
\end{aligned}
$$

$$
\begin{aligned}
Y^{A}(z) &= \left(\frac{P_{A}^{A}(z)}{P_{A}^{A}}\right)^{-\phi}\left[v_{A}^{A}\left(\frac{P_{A}^{A}}{P_{P}^{A}}\right)^{-\psi}\left(\frac{P_{P}^{A}}{P^{A}}\right)^{-\theta}C^{A}\right. \\
&\quad + v_{B}^{A}\left(\frac{P_{A}^{B}}{P_{P}^{B}}\right)^{-\psi}\left(\frac{P_{P}^{B}}{P^{B}}\right)^{-\theta}C^{B} \\
&\quad \left. + v_{C}^{A}\left(\frac{P_{A}^{C}}{P_{P}^{C}}\right)^{-\psi}\left(\frac{P_{P}^{C}}{P^{C}}\right)^{-\theta}C^{C}\right].
\end{aligned}
$$

Aggregate over all varieties z

$$
\begin{aligned}
Y^{A} &= \left[\int_{0}^{1}\left(Y^{A}(z)\right)^{\frac{\phi-1}{\phi}}dz\right]^{\frac{\phi}{\phi-1}} \\
&= \left[\int_{0}^{1}\left(\left(\frac{P_{A}^{A}(z)}{P_{A}^{A}}\right)^{-\phi}\left[v_{A}^{A}\left(\frac{P_{A}^{A}}{P_{P}^{A}}\right)^{-\psi}\left(\frac{P_{P}^{A}}{P^{A}}\right)^{-\theta}C^{A}\right.\right.\right. \\
&\quad \left.\left.\left. + v_{B}^{A}\left(\frac{P_{A}^{B}}{P_{P}^{B}}\right)^{-\psi}\left(\frac{P_{P}^{B}}{P^{B}}\right)^{-\theta}C^{B} + v_{C}^{A}\left(\frac{P_{A}^{C}}{P_{P}^{C}}\right)^{-\psi}\left(\frac{P_{P}^{C}}{P^{C}}\right)^{-\theta}C^{C}\right]\right)^{\frac{\phi-1}{\phi}}dz\right]^{\frac{\phi}{\phi-1}}.
\end{aligned}
$$

Use $\int_0^1 \left(\left(\frac{P_A^A(z)}{P_A^A} \right)^{-\phi} dz \right)^{\frac{\phi-1}{\phi}} = 1$

$$\begin{aligned} Y^A &= v_A^A \left(\frac{P_A^A}{P_P^A} \right)^{-\psi} \left(\frac{P_P^A}{P^A} \right)^{-\theta} C^A \\ &\quad + v_B^A \left(\frac{P_A^B}{P_P^B} \right)^{-\psi} \left(\frac{P_P^B}{P^B} \right)^{-\theta} C^B \\ &\quad + v_C^A \left(\frac{P_A^C}{P_P^C} \right)^{-\psi} \left(\frac{P_P^C}{P^C} \right)^{-\theta} C^C. \end{aligned}$$

Use law of one price $P_P^i = S_P^i \cdot P_P^P$, expand by CPI P^i, and use definition of CPI-PPI relation $\frac{P^i}{P_i^i} = g_i(ToT)$ to rewrite the following six expressions

$$\frac{P_A^A}{P_P^A} = \frac{\frac{P^A \cdot P_A^A}{P^A}}{\frac{P^P \cdot \overbrace{S_P^A \cdot P_P^P}^{P_P^A}}{P^P}} = \frac{P^A \cdot g_A(ToT)^{-1}}{P^P \cdot S_P^A \cdot g_P(ToT)^{-1}} = \underbrace{\left(\frac{S_P^A \cdot P^P}{P^A} \right)^{-1}}_{RER_P^A} \frac{g_A(ToT)^{-1}}{g_P(ToT)^{-1}}$$

$$\begin{aligned} \frac{P_A^B}{P_P^B} &= \frac{\frac{P_A^A}{S_B^A}}{S_P^B \cdot P_P^P} = \frac{\frac{P_A^A}{S_B^A} \cdot \frac{P^A}{P^A}}{S_P^B P_P^P \cdot \frac{P^P}{P^P}} = \frac{\frac{P^A}{S_B^A}}{S_P^B P^P} \cdot \frac{\frac{P_A^A}{P^A}}{\frac{P_P^P}{P^P}} \\ &= \frac{P^B}{P^B} \cdot \frac{\frac{P^A}{S_B^A}}{S_P^B P^P} \cdot \frac{g_A(ToT)^{-1}}{g_P(ToT)^{-1}} \\ &= \frac{P^B}{S_P^B P^P} \cdot \frac{P^A}{S_B^A P^B} \cdot \frac{g_A(ToT)^{-1}}{g_P(ToT)^{-1}} \\ &= \underbrace{\left(\frac{S_P^B P^P}{P^B} \right)^{-1}}_{RER_P^B} \cdot \underbrace{\left(\frac{S_B^A P^B}{P^A} \right)^{-1}}_{RER_B^A} \cdot \frac{g_A(ToT)^{-1}}{g_P(ToT)^{-1}} \\ &= \left(RER_P^B \right)^{-1} \left(RER_B^A \right)^{-1} \frac{g_A(ToT)^{-1}}{g_P(ToT)^{-1}} \end{aligned}$$

$$\begin{aligned} \frac{P_A^C}{P_P^C} &= \frac{\frac{P_A^A}{S_C^A}}{S_P^C P_P^P} = \frac{\frac{P^A}{S_C^A}}{S_P^C P^P} \frac{\frac{P_A^A}{P^A}}{\frac{P_P^P}{P^P}} = \frac{\frac{P^A}{S_C^A P^C}}{\frac{S_P^C P^P}{P^C}} \frac{g_A(ToT)^{-1}}{g_P(ToT)^{-1}} \\ &= \left(RER_P^C \right)^{-1} \left(RER_C^A \right)^{-1} \frac{g_A(ToT)^{-1}}{g_P(ToT)^{-1}} \end{aligned}$$

$$\frac{P_P^A}{P^A} = \frac{\overbrace{S_P^A \cdot P_P^P}^{P_P^A}}{P^A} \cdot \frac{P^P}{P^P} = \frac{S_P^A \cdot P^P}{P^A} \cdot \frac{P_p^P}{P^P} = RER_P^A \cdot g_P \left(ToT\right)^{-1}$$

$$\frac{P_P^B}{P^B} = \frac{\overbrace{S_P^B \cdot P_P^P}^{P_P^B}}{P^B} \cdot \frac{P^P}{P^P} = \frac{S_P^B \cdot P^P}{P^B} \cdot \frac{P_p^P}{P^P} = RER_P^B \cdot g_P \left(ToT\right)^{-1}$$

$$\frac{P_P^C}{P^C} = \frac{\overbrace{S_P^C \cdot P_P^P}^{P_P^C}}{P^C} \cdot \frac{P^P}{P^P} = \frac{S_P^C \cdot P^P}{P^C} \cdot \frac{P_p^P}{P^P} = RER_P^C \cdot g_P \left(ToT\right)^{-1}.$$

Plug in

$$\begin{aligned} Y^A &= v_A^A \left(\left(RER_P^A\right)^{-1} \frac{g_A \left(ToT\right)^{-1}}{g_P \left(ToT\right)^{-1}} \right)^{-\psi} \left(RER_P^A \, g_P \left(ToT\right)^{-1}\right)^{-\theta} C^A \\ &\quad + v_B^A \left(\left(RER_P^B\right)^{-1} \left(RER_B^A\right)^{-1} \frac{g_A \left(ToT\right)^{-1}}{g_P \left(ToT\right)^{-1}} \right)^{-\psi} \left(RER_P^B \, g_P \left(ToT\right)^{-1}\right)^{-\theta} C^B \\ &\quad + v_C^A \left(\left(RER_P^C\right)^{-1} \left(RER_C^A\right)^{-1} \frac{g_A \left(ToT\right)^{-1}}{g_P \left(ToT\right)^{-1}} \right)^{-\psi} \left(RER_P^C \, g_P \left(ToT\right)^{-1}\right)^{-\theta} C^C \end{aligned}$$

$$\begin{aligned} Y^A &= v_A^A \left(RER_P^A \frac{g_A \left(ToT\right)}{g_P \left(ToT\right)} \right)^{\psi} \left(RER_P^A \, g_P \left(ToT\right)^{-1}\right)^{-\theta} C^A \\ &\quad + v_B^A \left(RER_P^B \, RER_B^A \frac{g_A \left(ToT\right)}{g_P \left(ToT\right)} \right)^{\psi} \left(RER_P^B \, g_P \left(ToT\right)^{-1}\right)^{-\theta} C^B \\ &\quad + v_C^A \left(RER_P^C \, RER_C^A \frac{g_A \left(ToT\right)}{g_P \left(ToT\right)} \right)^{\psi} \left(RER_P^C \, g_P \left(ToT\right)^{-1}\right)^{-\theta} C^C. \end{aligned}$$

Rewrite the real exchange rate in terms of trade as in equation (7.30)

$$RER_P^A = ToT_P^A \frac{g_P(ToT)}{g_A(ToT)}, \quad RER_P^B = ToT_P^B \frac{g_P(ToT)}{g_B(ToT)}, \quad RER_P^C = ToT_P^C \frac{g_P(ToT)}{g_C(ToT)},$$

$$RER_B^A = ToT_B^A \frac{g_B(ToT)}{g_A(ToT)}, \quad RER_C^A = ToT_C^A \frac{g_C(ToT)}{g_A(ToT)}$$

$$
\begin{aligned}
Y^A &= v_A^A \left(ToT_P^A \frac{g_P(ToT)}{g_A(ToT)} \frac{g_A\left(ToT\right)}{g_P\left(ToT\right)} \right)^{\psi} \left(ToT_P^A \frac{g_P(ToT)}{g_A(ToT)} \frac{1}{g_P\left(ToT\right)} \right)^{-\theta} C^A \\
&+ v_B^A \left(ToT_P^B \frac{g_P(ToT)}{g_B(ToT)} ToT_B^A \frac{g_B(ToT)}{g_A(ToT)} \frac{g_A\left(ToT\right)}{g_P\left(ToT\right)} \right)^{\psi} \\
&\cdot \left(ToT_P^B \frac{g_P(ToT)}{g_B(ToT)} \frac{1}{g_P\left(ToT\right)} \right)^{-\theta} C^B \\
&+ v_C^A \left(ToT_P^C \frac{g_P(ToT)}{g_C(ToT)} ToT_C^A \frac{g_C(ToT)}{g_A(ToT)} \frac{g_A\left(ToT\right)}{g_P\left(ToT\right)} \right)^{\psi} \\
&\cdot \left(ToT_P^C \frac{g_P(ToT)}{g_C(ToT)} \frac{1}{g_P\left(ToT\right)} \right)^{-\theta} C^C
\end{aligned}
$$

$$
\begin{aligned}
Y^A &= v_A^A \left(ToT_P^A \right)^{\psi} \left(ToT_P^A \frac{1}{g_A\left(ToT\right)} \right)^{-\theta} C^A \\
&+ v_B^A \left(ToT_P^B \, ToT_B^A \right)^{\psi} \left(ToT_P^B \frac{1}{g_B\left(ToT\right)} \right)^{-\theta} C^B \\
&+ v_C^A \left(ToT_P^C \, ToT_C^A \right)^{\psi} \left(ToT_P^C \frac{1}{g_C\left(ToT\right)} \right)^{-\theta} C^C
\end{aligned}
$$

$$
\begin{aligned}
Y^A &= v_A^A \left(ToT_P^A \right)^{\psi-\theta} \left(g_A(ToT) \right)^{\theta} C^A \\
&+ v_B^A \left(ToT_P^B \right)^{\psi-\theta} \left(ToT_B^A \right)^{\psi} \left(g_B(ToT) \right)^{\theta} C^B \\
&+ v_C^A \left(ToT_P^C \right)^{\psi-\theta} \left(ToT_C^A \right)^{\psi} \left(g_C(ToT) \right)^{\theta} C^C. \qquad (7.93)
\end{aligned}
$$

Substituting periphery terms of trade by equations (7.24), (7.25), and (7.26)

$$
\begin{aligned}
Y^A &= v_A^A \left[\left[1 - \eta \left(1 - n_A \right) \right] + \eta \left(1 - n_A \right) \left(ToT_B^A \right)^{1-\psi} \right]^{\frac{\psi-\theta}{1-\psi}} \left(g_A(ToT) \right)^{\theta} C^A \\
&+ v_B^A \left[\left(1 - \eta n_A \right) + \eta n_A \left(ToT_A^B \right)^{1-\psi} \right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_B^A \right)^{\psi} \left(g_B(ToT) \right)^{\theta} C^B \\
&+ v_C^A \left[n_A \left(ToT_A^C \right)^{1-\psi} + \left(1 - n_A \right) \left(ToT_B^C \right)^{1-\psi} \right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_C^A \right)^{\psi} \left(g_C(ToT) \right)^{\theta} C^C.
\end{aligned}
$$

See Faia and Monacelli (2004: 12, eq. 42). Substituting the CPI-PPI relations (7.27), (7.28) and (7.29) would be possible. It seems to be easier to define those

relations as part of the system later.

$$
\begin{aligned}
Y^A &= v_A^A \left[[1-\eta(1-n_A)] + \eta(1-n_A) \left(ToT_B^A\right)^{1-\psi} \right]^{\frac{\psi-\theta}{1-\psi}} \\
&\cdot \left[[1-\eta(1-n_P)] \left[\left([1-\eta(1-n_A)] + \eta(1-n_A) \left(ToT_B^A\right)^{1-\psi} \right)^{\frac{1}{1-\psi}} \right]^{1-\theta} \right. \\
&\quad \left. + \eta(1-n_P) \left(ToT_C^A\right)^{1-\theta} \right]^{\frac{\theta}{1-\theta}} C^A \\
&+ v_B^A \left[(1-\eta n_A) + \eta n_A \left(ToT_A^B\right)^{1-\psi} \right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_B^A\right)^{\psi} \\
&\cdot \left[[1-\eta(1-n_P)] \left[\left(\eta n_A \left(ToT_A^B\right)^{1-\psi} + (1-\eta n_A) \right)^{\frac{1}{1-\psi}} \right]^{1-\theta} \right. \\
&\quad \left. + \eta(1-n_P) \left(ToT_C^B\right)^{1-\theta} \right]^{\frac{\theta}{1-\theta}} C^B \\
&+ v_C^A \left[n_A \left(ToT_A^C\right)^{1-\psi} + (1-n_A) \left(ToT_B^C\right)^{1-\psi} \right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_C^A\right)^{\psi} \\
&\cdot \left[\eta n_P \left[\left(n_A \left(ToT_A^C\right)^{1-\psi} + (1-n_A) \left(ToT_B^C\right)^{1-\psi} \right)^{\frac{1}{1-\psi}} \right]^{1-\theta} \right. \\
&\quad \left. + (1-\eta n_P) \right]^{\frac{\theta}{1-\theta}} C^C
\end{aligned}
$$

Goods Market Clearing in Country B

Goods market clearing in levels in country B for variety z

$$
Y_t^B(z) = n_P n_A C_{B,t}^A(z) + n_P(1-n_A) C_{B,t}^B(z) + (1-n_P) C_{B,t}^C(z)
$$

$$
\begin{aligned}
Y^B(z) &= \underbrace{n_P n_A \left((1-n_A) n_P\right)^{-1} v_B^A}_{v_A^B} \left(\frac{P_B^A(z)}{P_B^A}\right)^{-\phi} \left(\frac{P_B^A}{P_P^A}\right)^{-\psi} \left(\frac{P_P^A}{P^A}\right)^{-\theta} C^A \\
&+ \underbrace{n_P(1-n_A) \left((1-n_A) n_P\right)^{-1} v_B^B}_{v_B^B} \left(\frac{P_B^B(z)}{P_B^B}\right)^{-\phi} \left(\frac{P_B^B}{P_P^B}\right)^{-\psi} \left(\frac{P_P^B}{P^B}\right)^{-\theta} C^B \\
&+ \underbrace{(1-n_P) \left((1-n_A) n_P\right)^{-1} v_B^C}_{v_C^B} \left(\frac{P_B^C(z)}{P_B^C}\right)^{-\phi} \left(\frac{P_B^C}{P_P^C}\right)^{-\psi} \left(\frac{P_P^C}{P^C}\right)^{-\theta} C^C
\end{aligned}
$$

Substitute trade shares

$$\begin{aligned} Y^B(z) &= v_A^B \left(\frac{P_B^A(z)}{P_B^A}\right)^{-\phi} \left(\frac{P_B^A}{P_P^A}\right)^{-\psi} \left(\frac{P_P^A}{P^A}\right)^{-\theta} C^A \\ &+ v_B^B \left(\frac{P_B^B(z)}{P_B^B}\right)^{-\phi} \left(\frac{P_B^B}{P_P^B}\right)^{-\psi} \left(\frac{P_P^B}{P^B}\right)^{-\theta} C^B \\ &+ v_C^B \left(\frac{P_B^C(z)}{P_B^C}\right)^{-\phi} \left(\frac{P_B^C}{P_P^C}\right)^{-\psi} \left(\frac{P_P^C}{P^C}\right)^{-\theta} C^C. \end{aligned}$$

Using definitions of law of one price

$$\begin{aligned} P_{B,t}^A(z) &= \frac{P_{B,t}^B(z)}{S_{A,t}^B} = S_{B,t}^A \cdot P_{B,t}^B(z) \\ P_{B,t}^C(z) &= \frac{P_{B,t}^B(z)}{S_{C,t}^B} = \frac{P_{B,t}^B(z)}{S_{A,t}^B \cdot S_{C,t}^A} = \frac{P_{B,t}^B(z)}{\frac{1}{S_{B,t}^A} \cdot S_{C,t}^A} = \frac{S_{B,t}^A \cdot P_{B,t}^B(z)}{S_{C,t}^A} \end{aligned}$$

$$\begin{aligned} P_{B,t}^A &= S_{B,t}^A \cdot P_{B,t}^B \\ P_{B,t}^C &= \frac{S_{B,t}^A \cdot P_{B,t}^B}{S_{C,t}^A} \end{aligned}$$

$$\begin{aligned} Y^B(z) &= v_A^B \left(\frac{S_B^A \cdot P_B^B(z)}{S_B^A \cdot P_B^B}\right)^{-\phi} \left(\frac{P_B^A}{P_P^A}\right)^{-\psi} \left(\frac{P_P^A}{P^A}\right)^{-\theta} C^A \\ &+ v_B^B \left(\frac{P_B^B(z)}{P_B^B}\right)^{-\phi} \left(\frac{P_B^B}{P_P^B}\right)^{-\psi} \left(\frac{P_P^B}{P^B}\right)^{-\theta} C^B \\ &+ v_C^B \left(\frac{\frac{S_B^A \cdot P_B^B(z)}{S_C^A}}{\frac{S_B^A \cdot P_B^B}{S_C^A}}\right)^{-\phi} \left(\frac{P_B^C}{P_P^C}\right)^{-\psi} \left(\frac{P_P^C}{P^C}\right)^{-\theta} C^C \end{aligned}$$

$$\begin{aligned} Y_t^B(z) &= v_A^B \left(\frac{P_B^B(z)}{P_B^B}\right)^{-\phi} \left(\frac{P_B^A}{P_P^A}\right)^{-\psi} \left(\frac{P_P^A}{P^A}\right)^{-\theta} C^A \\ &+ v_B^B \left(\frac{P_B^B(z)}{P_B^B}\right)^{-\phi} \left(\frac{P_B^B}{P_P^B}\right)^{-\psi} \left(\frac{P_P^B}{P^B}\right)^{-\theta} C^B \\ &+ v_C^B \left(\frac{P_B^B(z)}{P_B^B}\right)^{-\phi} \left(\frac{P_B^C}{P_P^C}\right)^{-\psi} \left(\frac{P_P^C}{P^C}\right)^{-\theta} C^C \end{aligned}$$

$$
\begin{aligned}
Y^B(z) &= \left(\frac{P_B^B(z)}{P_B^B}\right)^{-\phi}\left[v_A^B\left(\frac{P_B^A}{P_P^A}\right)^{-\psi}\left(\frac{P_P^A}{P^A}\right)^{-\theta}C^A\right. \\
&\quad \left. + v_B^B\left(\frac{P_B^B}{P_P^B}\right)^{-\psi}\left(\frac{P_P^B}{P^B}\right)^{-\theta}C^B + v_C^B\left(\frac{P_B^C}{P_P^C}\right)^{-\psi}\left(\frac{P_P^C}{P^C}\right)^{-\theta}C^C\right].
\end{aligned}
$$

Aggregate over all varieties z

$$
\begin{aligned}
Y^B &= \left[\int_0^1 \left(Y^B(z)\right)^{\frac{\phi-1}{\phi}}dz\right]^{\frac{\phi}{\phi-1}} \\
&= \left[\int_0^1 \left(\left(\frac{P_B^B(z)}{P_B^B}\right)^{-\phi}\left[v_A^B\left(\frac{P_B^A}{P_P^A}\right)^{-\psi}\left(\frac{P_P^A}{P^A}\right)^{-\theta}C^A\right.\right.\right. \\
&\quad \left.\left.\left. + v_B^B\left(\frac{P_B^B}{P_P^B}\right)^{-\psi}\left(\frac{P_P^B}{P^B}\right)^{-\theta}C^B + v_C^B\left(\frac{P_B^C}{P_P^C}\right)^{-\psi}\left(\frac{P_P^C}{P^C}\right)^{-\theta}C^C\right]\right)^{\frac{\phi-1}{\phi}}dz\right]^{\frac{\phi}{\phi-1}}.
\end{aligned}
$$

Use $\int_0^1 \left(\left(\frac{P_B^B(z)}{P_B^B}\right)^{-\phi}dz\right)^{\frac{\phi-1}{\phi}} = 1$

$$
\begin{aligned}
Y^B &= v_A^B\left(\frac{P_B^A}{P_P^A}\right)^{-\psi}\left(\frac{P_P^A}{P^A}\right)^{-\theta}C^A + v_B^B\left(\frac{P_B^B}{P_P^B}\right)^{-\psi}\left(\frac{P_P^B}{P^B}\right)^{-\theta}C^B \\
&\quad + v_C^B\left(\frac{P_B^C}{P_P^C}\right)^{-\psi}\left(\frac{P_P^C}{P^C}\right)^{-\theta}C^C.
\end{aligned}
$$

Use law of one price $P_P^i = S_P^i \cdot P_P^P$, expand by CPI P^i, and use definition of CPI-PPI relation $\frac{P^i}{P_i^i} = g_i(ToT)$ to rewrite the following three expressions

$$
\begin{aligned}
\frac{P_B^A}{P_P^A} &= \frac{\frac{P_B^B}{S_A^B}}{S_P^A P_P^P} = \frac{\frac{P^B}{S_A^B}}{S_P^A P^P}\frac{\frac{P_B^B}{P^B}}{\frac{P_P^P}{P^P}} = \frac{\frac{P^B}{S_A^B P^A}}{\frac{S_P^A P^P}{P^A}}\frac{g_B(ToT)^{-1}}{g_P(ToT)^{-1}} \\
&= \left(RER_A^B\right)^{-1}\left(RER_P^A\right)^{-1}\frac{g_B(ToT)^{-1}}{g_P(ToT)^{-1}}
\end{aligned}
$$

$$
\frac{P_B^B}{P_P^B} = \frac{\frac{P^B \cdot P_B^B}{P^B}}{\frac{P^P \cdot \overbrace{S_P^B \cdot P_P^P}^{P_P^B}}{P^P}} = \frac{P^B \cdot g_B(ToT)^{-1}}{P^P \cdot S_P^B \cdot g_P(ToT)^{-1}} = \underbrace{\left(\frac{S_P^B \cdot P^P}{P^B}\right)^{-1}}_{RER_P^B}\frac{g_B(ToT)^{-1}}{g_P(ToT)^{-1}}
$$

$$\begin{aligned}\frac{P_B^C}{P_P^C} &= \frac{\frac{P_B^B}{S_C^B}}{S_P^C P_P^P} = \frac{\frac{P^B}{S_C^B}}{S_P^C P^P}\frac{\frac{P_B^B}{P^B}}{\frac{P_P^P}{P^P}} = \frac{\frac{P^B}{S_C^B P^C}}{\frac{S_P^C P^P}{P^C}}\frac{g_B\left(ToT\right)^{-1}}{g_P\left(ToT\right)^{-1}} \\ &= \left(RER_C^B\right)^{-1}\left(RER_P^C\right)^{-1}\frac{g_B\left(ToT\right)^{-1}}{g_P\left(ToT\right)^{-1}}.\end{aligned}$$

Plug in

$$\begin{aligned}Y^B &= v_A^B\left(\left(RER_A^B\right)^{-1}\left(RER_P^A\right)^{-1}\frac{g_B\left(ToT\right)^{-1}}{g_P\left(ToT\right)^{-1}}\right)^{-\psi} \\ &\quad\cdot\left(RER_P^A\cdot g_P\left(ToT\right)^{-1}\right)^{-\theta}C^A \\ &\quad+v_B^B\left(\left(RER_P^B\right)^{-1}\frac{g_B\left(ToT\right)^{-1}}{g_P\left(ToT\right)^{-1}}\right)^{-\psi}\left(RER_P^B\cdot g_P\left(ToT\right)^{-1}\right)^{-\theta}C^B \\ &\quad+v_C^B\left(\left(RER_C^B\right)^{-1}\left(RER_P^C\right)^{-1}\frac{g_B\left(ToT\right)^{-1}}{g_P\left(ToT\right)^{-1}}\right)^{-\psi} \\ &\quad\cdot\left(RER_P^C\cdot g_P\left(ToT\right)^{-1}\right)^{-\theta}C^C\end{aligned}$$

$$\begin{aligned}Y^B &= v_A^B\left(\left(RER_A^B\right)\left(RER_P^A\right)\frac{g_B\left(ToT\right)}{g_P\left(ToT\right)}\right)^{\psi}\left(RER_P^A\ g_P\left(ToT\right)^{-1}\right)^{-\theta}C^A \\ &\quad+v_B^B\left(\left(RER_P^B\right)\frac{g_B\left(ToT\right)}{g_P\left(ToT\right)}\right)^{\psi}\left(RER_P^B\ g_P\left(ToT\right)^{-1}\right)^{-\theta}C^B \\ &\quad+v_C^B\left(\left(RER_C^B\right)\left(RER_P^C\right)\frac{g_B\left(ToT\right)}{g_P\left(ToT\right)}\right)^{\psi}\left(RER_P^C\ g_P\left(ToT\right)^{-1}\right)^{-\theta}C^C.\end{aligned}$$

Substitute real exchange rates

$$RER_P^A = ToT_P^A\frac{g_P(ToT)}{g_A(ToT)},\quad RER_P^B = ToT_P^B\frac{g_P(ToT)}{g_B(ToT)},\quad RER_P^C = ToT_P^C\frac{g_P(ToT)}{g_C(ToT)},$$

$$RER_A^B = ToT_A^B\frac{g_A(ToT)}{g_B(ToT)},\quad RER_C^B = ToT_C^B\frac{g_C(ToT)}{g_B(ToT)}$$

$$
\begin{aligned}
Y^B &= v_A^B \left(ToT_A^B \frac{g_A(ToT)}{g_B(ToT)} ToT_P^A \frac{g_P(ToT)}{g_A(ToT)} \frac{g_B\left(ToT\right)}{g_P\left(ToT\right)} \right)^{\psi} \\
&\quad \cdot \left(ToT_P^A \frac{g_P(ToT)}{g_A(ToT)} g_P\left(ToT\right)^{-1} \right)^{-\theta} C^A \\
&\quad + v_B^B \left(ToT_P^B \frac{g_P(ToT)}{g_B(ToT)} \frac{g_B\left(ToT\right)}{g_P\left(ToT\right)} \right)^{\psi} \left(ToT_P^B \frac{g_P(ToT)}{g_B(ToT)} g_P\left(ToT\right)^{-1} \right)^{-\theta} C^B \\
&\quad + v_C^B \left(ToT_C^B \frac{g_C(ToT)}{g_B(ToT)} ToT_P^C \frac{g_P(ToT)}{g_C(ToT)} \frac{g_B\left(ToT\right)}{g_P\left(ToT\right)} \right)^{\psi} \\
&\quad \cdot \left(ToT_P^C \frac{g_P(ToT)}{g_C(ToT)} g_P\left(ToT\right)^{-1} \right)^{-\theta} C^C
\end{aligned}
$$

$$
\begin{aligned}
Y^B &= v_A^B \left(ToT_A^B \; ToT_P^A \right)^{\psi} \left(ToT_P^A \; g_A\left(ToT\right)^{-1} \right)^{-\theta} C^A \\
&\quad + v_B^B \left(ToT_P^B \right)^{\psi} \left(ToT_P^B \; g_B\left(ToT\right)^{-1} \right)^{-\theta} C^B \\
&\quad + v_C^B \left(ToT_C^B \; ToT_P^C \right)^{\psi} \left(ToT_P^C \; g_C\left(ToT\right)^{-1} \right)^{-\theta} C^C
\end{aligned}
$$

$$
\begin{aligned}
Y^B &= v_A^B \left(ToT_A^B \right)^{\psi} \left(ToT_P^A \right)^{\psi-\theta} \left(g_A\left(ToT\right) \right)^{\theta} C^A \\
&\quad + v_B^B \left(ToT_P^B \right)^{\psi-\theta} \left(g_B\left(ToT\right) \right)^{\theta} C^B \\
&\quad + v_C^B \left(ToT_C^B \right)^{\psi} \left(ToT_P^C \right)^{\psi-\theta} \left(g_C\left(ToT\right) \right)^{\theta} C^C. \qquad (7.94)
\end{aligned}
$$

Goods Market Clearing in Country C

Goods market clearing in levels in country C for variety z

$$
Y_t^C\left(z\right) = n_P n_A C_{C,t}^A\left(z\right) + n_P\left(1-n_A\right) C_{C,t}^B\left(z\right) + \left(1-n_P\right) C_{C,t}^C\left(z\right)
$$

$$
\begin{aligned}
Y^C\left(z\right) &= \underbrace{n_P n_A \left(1-n_P\right)^{-1} v_C^A}_{v_A^C} \left(\frac{P_C^A(z)}{P_C^A} \right)^{-\phi} \left(\frac{P_C^A}{P^A} \right)^{-\theta} C^A \\
&\quad + \underbrace{n_P\left(1-n_A\right)\left(1-n_P\right)^{-1} v_C^B}_{v_B^C} \left(\frac{P_C^B(z)}{P_C^B} \right)^{-\phi} \left(\frac{P_C^B}{P^B} \right)^{-\theta} C^B \\
&\quad + \underbrace{\left(1-n_P\right)\left(1-n_P\right)^{-1} v_C^C}_{v_C^C} \left(\frac{P_C^C(z)}{P_C^C} \right)^{-\phi} \left(\frac{P_C^C}{P^C} \right)^{-\theta} C^C
\end{aligned}
$$

$$
\begin{aligned}
Y^C(z) &= v_A^C \left(\frac{P_C^A(z)}{P_C^A}\right)^{-\phi} \left(\frac{P_C^A}{P^A}\right)^{-\theta} C^A \\
&\quad + v_B^C \left(\frac{P_C^B(z)}{P_C^B}\right)^{-\phi} \left(\frac{P_C^B}{P^B}\right)^{-\theta} C^B \\
&\quad + v_C^C \left(\frac{P_C^C(z)}{P_C^C}\right)^{-\phi} \left(\frac{P_C^C}{P^C}\right)^{-\theta} C^C
\end{aligned}
$$

Using definitions of law of one price

$$
\begin{aligned}
P_{C,t}^A(z) &= S_{C,t}^A \cdot P_{C,t}^C(z) \\
P_{C,t}^B(z) &= S_{C,t}^B \cdot P_{C,t}^C(z)
\end{aligned}
$$

$$
\begin{aligned}
P_{C,t}^A &= S_{C,t}^A \cdot P_{C,t}^C \\
P_{C,t}^B &= S_{C,t}^B \cdot P_{C,t}^C
\end{aligned}
$$

$$
\begin{aligned}
Y^C(z) &= v_A^C \left(\frac{S_C^A \cdot P_C^C(z)}{S_C^A \cdot P_C^C}\right)^{-\phi} \left(\frac{P_C^A}{P^A}\right)^{-\theta} C^A \\
&\quad + v_B^C \left(\frac{S_C^B \cdot P_C^C(z)}{S_C^B \cdot P_C^C}\right)^{-\phi} \left(\frac{P_C^B}{P^B}\right)^{-\theta} C^B \\
&\quad + v_C^C \left(\frac{P_C^C(z)}{P_C^C}\right)^{-\phi} \left(\frac{P_C^C}{P^C}\right)^{-\theta} C^C
\end{aligned}
$$

$$
Y^C(z) = \left(\frac{P_C^C(z)}{P_C^C}\right)^{-\phi} \left[v_A^C \left(\frac{P_C^A}{P^A}\right)^{-\theta} C^A + v_B^C \left(\frac{P_C^B}{P^B}\right)^{-\theta} C^B + v_C^C \left(\frac{P_C^C}{P^C}\right)^{-\theta} C^C \right].
$$

Aggregate over all varieties z

$$
\begin{aligned}
Y^C &= \left[\int_0^1 \left(Y^C(z)\right)^{\frac{\phi-1}{\phi}} dz\right]^{\frac{\phi}{\phi-1}} \\
&= \left[\int_0^1 \left(\left(\frac{P_C^C(z)}{P_C^C}\right)^{-\phi} \left[v_A^C \left(\frac{P_C^A}{P^A}\right)^{-\theta} C^A \right.\right.\right. \\
&\quad \left.\left.\left. + v_B^C \left(\frac{P_C^B}{P^B}\right)^{-\theta} C^B + \; v_C^C \left(\frac{P_C^C}{P^C}\right)^{-\theta} C^C \right] \right)^{\frac{\phi-1}{\phi}} dz \right]^{\frac{\phi}{\phi-1}}.
\end{aligned}
$$

Use $\int_0^1 \left(\left(\frac{P_C^C(z)}{P_C^C} \right)^{-\phi} dz \right)^{\frac{\phi-1}{\phi}} = 1$

$$Y^C = v_A^C \left(\frac{P_C^A}{P^A} \right)^{-\theta} C^A + v_B^C \left(\frac{P_C^B}{P^B} \right)^{-\theta} C^B + v_C^C \left(\frac{P_C^C}{P^C} \right)^{-\theta} C^C$$

$$Y^C = v_A^C \underbrace{\left(\frac{S_C^A P^C}{P^A} \frac{P_C^C}{P^C} \right)^{-\theta}}_{RER_C^A (g_C(ToT))^{-1}} C^A + v_B^C \underbrace{\left(\frac{S_C^B P^C}{P^B} \frac{P_C^C}{P^C} \right)^{-\theta}}_{RER_C^B (g_C(ToT))^{-1}} C^B + v_C^C \underbrace{\left(\frac{P_C^C}{P^C} \right)^{-\theta}}_{(g_C(ToT))^{-1}} C^C$$

$$\begin{aligned} Y^C &= v_A^C \left(RER_C^A \right)^{-\theta} \left(g_C \left(ToT \right) \right)^{\theta} C^A + v_B^C \left(RER_C^B \right)^{-\theta} \left(g_C \left(ToT \right) \right)^{\theta} C^B \\ &\quad + v_C^C \left(g_C \left(ToT \right) \right)^{\theta} C^C. \end{aligned}$$

Substitute real exchange rates

$$RER_C^A = ToT_C^A \frac{g_C(ToT)}{g_A(ToT)}, \quad RER_C^B = ToT_C^B \frac{g_C(ToT)}{g_B(ToT)}$$

$$\begin{aligned} Y^C &= v_A^C \left(ToT_C^A \frac{g_C(ToT)}{g_A(ToT)} \right)^{-\theta} \left(g_C \left(ToT \right) \right)^{\theta} C^A \\ &\quad + v_B^C \left(ToT_C^B \frac{g_C(ToT)}{g_B(ToT)} \right)^{-\theta} \left(g_C \left(ToT \right) \right)^{\theta} C^B + v_C^C \left(g_C \left(ToT \right) \right)^{\theta} C^C \end{aligned}$$

$$Y^C = v_A^C \left(\frac{ToT_C^A}{g_A(ToT)} \right)^{-\theta} C^A + v_B^C \left(\frac{ToT_C^B}{g_B(ToT)} \right)^{-\theta} C^B + v_C^C \left(g_C \left(ToT \right) \right)^{\theta} C^C$$

$$Y^C = v_A^C \left(\frac{g_A(ToT)}{ToT_C^A} \right)^{\theta} C^A + v_B^C \left(\frac{g_B(ToT)}{ToT_C^B} \right)^{\theta} C^B + v_C^C \left(g_C \left(ToT \right) \right)^{\theta} C^C. \quad (7.95)$$

7.1.10 Loglinerarizations

Variables without a time subscript are variables in their steady state. Variables with a hat are deviations from that steady state.

Loglinearized CPI

Starting from equation (7.11), the CPI for country A

$$P_t^A = \left[\underbrace{[1-\eta(1-n_P)]}_{v_P}\left[\left(\underbrace{[1-\eta(1-n_A)]}_{v_A}\left(P_{A,t}^A\right)^{1-\psi} + \underbrace{\eta(1-n_A)}_{v_B}\left(P_{B,t}^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + \underbrace{\eta(1-n_P)}_{v_C}\left(P_{C,t}^A\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

$$P_t^P = \left[\left(v_A\left(P_{A,t}^A\right)^{1-\psi} + v_B\left(P_{B,t}^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta}$$

Use new weights v_A, v_B, v_C, and v_P to simplify algebra

$$P_t^A = \left[v_P\left[\left(v_A\left(P_{A,t}^A\right)^{1-\psi} + v_B\left(P_{B,t}^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + v_C\left(P_{C,t}^A\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

$$\left(P_t^A\right)^{1-\theta} = v_P\left[\left(v_A\left(P_{A,t}^A\right)^{1-\psi} + v_B\left(P_{B,t}^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + v_C\left(P_{C,t}^A\right)^{1-\theta}$$

$$\begin{aligned}
&(1-\theta)\left(P^A\right)^{1-\theta-1}dP^A = (1-\theta)\,v_P\left[\left(v_A\left(P_A^A\right)^{1-\psi} + v_B\left(P_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta-1} \\
&\cdot\frac{1}{1-\psi}\left(v_A\left(P_A^A\right)^{1-\psi} + v_B\left(P_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}-1}\cdot(1-\psi)\,v_A\left(P_A^A\right)^{1-\psi-1}dP_A^A \\
&+(1-\theta)\,v_P\left[\left(v_A\left(P_A^A\right)^{1-\psi} + v_B\left(P_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta-1} \\
&\cdot\frac{1}{1-\psi}\left(v_A\left(P_A^A\right)^{1-\psi} + v_B\left(P_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}-1}\cdot(1-\psi)\,v_B\left(P_B^A\right)^{1-\psi-1}dP_B^A \\
&+(1-\theta)\,v_C\left(P_C^A\right)^{1-\theta-1}dP_C^A
\end{aligned}$$

Cancel $(1-\psi)$

$$
\begin{aligned}
(1-\theta)\left(P^A\right)^{1-\theta}\hat{P}_t^A = (1-\theta)\,v_P \underbrace{\left[\left(v_A\left(P_A^A\right)^{1-\psi} + v_B\left(P_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta-1}}_{(P^P)^{-\theta}} \\
\cdot \underbrace{\left(v_A\left(P_A^A\right)^{1-\psi} + v_B\left(P_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}-1}}_{(P^P)^{\psi}} \cdot\, v_A\left(P_A^A\right)^{1-\psi}\hat{P}_{A,t}^A \\
+ (1-\theta)\,v_P \underbrace{\left[\left(v_A\left(P_A^A\right)^{1-\psi} + v_B\left(P_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta-1}}_{(P^P)^{-\theta}} \\
\cdot \underbrace{\left(v_A\left(P_A^A\right)^{1-\psi} + v_B\left(P_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}-1}}_{(P^P)^{\psi}} \cdot\, v_B\left(P_B^A\right)^{1-\psi}\hat{P}_{B,t}^A \\
+ (1-\theta)\,v_C\left(P_C^A\right)^{1-\theta}\hat{P}_{C,t}^A
\end{aligned}
$$

$$
\begin{aligned}
(1-\theta)\left(P^A\right)^{1-\theta}\hat{P}_t^A &= (1-\theta)\,v_P\left(P^P\right)^{-\theta}\cdot\left(P^P\right)^{\psi}\cdot v_A\left(P_A^A\right)^{1-\psi}\hat{P}_{A,t}^A \\
&+ (1-\theta)\,v_P\left(P^P\right)^{-\theta}\cdot\left(P^P\right)^{\psi}\cdot v_B\left(P_B^A\right)^{1-\psi}\hat{P}_{B,t}^A \\
&+ (1-\theta)\,v_C\left(P_C^A\right)^{1-\theta}\hat{P}_{C,t}^A
\end{aligned}
$$

Divide by $(1-\theta)$

$$
\begin{aligned}
\left(P^A\right)^{1-\theta}\hat{P}_t^A &= v_P\left(P^P\right)^{\psi-\theta}\cdot v_A\left(P_A^A\right)^{1-\psi}\hat{P}_{A,t}^A \\
&\quad + v_P\left(P^P\right)^{\psi-\theta}\cdot v_B\left(P_B^A\right)^{1-\psi}\hat{P}_{B,t}^A \\
&\quad + v_C\left(P_C^A\right)^{1-\theta}\hat{P}_{C,t}^A
\end{aligned}
$$

Normalize price levels to one, $P^i = 1$

$$
\hat{P}_t^A = v_P v_A \hat{P}_{A,t}^A + v_P v_B \hat{P}_{B,t}^A + v_C \hat{P}_{C,t}^A
$$

Substitute usual weights

$$
\begin{aligned}
\hat{P}_t^A &= \left[1-\eta\left(1-n_P\right)\right]\left[1-\eta\left(1-n_A\right)\right]\hat{P}_{A,t}^A \\
&\quad + \left[1-\eta\left(1-n_P\right)\right]\eta\left(1-n_A\right)\hat{P}_{B,t}^A + \eta\left(1-n_P\right)\hat{P}_{C,t}^A \qquad (7.96)
\end{aligned}
$$

$$\begin{aligned}\hat{P}_t^B &= [1-\eta(1-n_P)]\,\eta n_A \hat{P}_{A,t}^B \\ &\quad + [1-\eta(1-n_P)]\,(1-\eta n_A)\,\hat{P}_{B,t}^B + \eta(1-n_P)\,\hat{P}_{C,t}^B \end{aligned} \tag{7.97}$$

$$\hat{P}_t^C = \eta n_P n_A \hat{P}_{A,t}^C + \eta n_P(1-n_A)\,\hat{P}_{B,t}^C + (1-\eta n_P)\,\hat{P}_{C,t}^C \tag{7.98}$$

Equations (7.96) to (7.98) are the equivalent to Monacelli (2001: 390, eq. 3).

Loglinearized CPI Inflation Using simplified weights

$$\begin{aligned}\hat{\pi}_t^i &= \hat{P}_t^i - \hat{P}_{t-1}^i \\ &= v_P v_A \hat{P}_{A,t}^i + v_P v_B \hat{P}_{B,t}^i + v_C \hat{P}_{C,t}^i - v_P v_A \hat{P}_{A,t-1}^i - v_P v_B \hat{P}_{B,t-1}^i - v_C \hat{P}_{C,t-1}^i \\ &= v_P v_A \left(\hat{P}_{A,t}^i - \hat{P}_{A,t-1}^i\right) + v_P v_B \left(\hat{P}_{B,t}^i - \hat{P}_{B,t-1}^i\right) + v_C \left(\hat{P}_{C,t}^i - \hat{P}_{C,t-1}^i\right) \\ \hat{\pi}_t^i &= v_P v_A \hat{\pi}_{A,t}^i + v_P v_B \hat{\pi}_{B,t}^i + v_C \hat{\pi}_{C,t}^i \end{aligned} \tag{7.99}$$

Substitute usual weights

$$\begin{aligned}\hat{\pi}_t^A &= [1-\eta(1-n_P)]\,[1-\eta(1-n_A)]\,\hat{\pi}_{A,t}^A \\ &\quad + [1-\eta(1-n_P)]\,\eta(1-n_A)\,\hat{\pi}_{B,t}^A + \eta(1-n_P)\,\hat{\pi}_{C,t}^A \end{aligned} \tag{7.100}$$

$$\begin{aligned}\hat{\pi}_t^B &= [1-\eta(1-n_P)]\,\eta n_A \hat{\pi}_{A,t}^B \\ &\quad + [1-\eta(1-n_P)]\,(1-\eta n_A)\,\hat{\pi}_{B,t}^B + \eta(1-n_P)\,\hat{\pi}_{C,t}^B \end{aligned} \tag{7.101}$$

$$\hat{\pi}_t^C = \eta n_P n_A \hat{\pi}_{A,t}^C + \eta n_P(1-n_A)\,\hat{\pi}_{B,t}^C + (1-\eta n_P)\,\hat{\pi}_{C,t}^C \tag{7.102}$$

Equation (7.99) is the equivalent to Monacelli (2001: 391, eq. 5).

Loglinearized Exchange Rates

Nominal exchange rate between country i and country i' is $\hat{S}_{i',t}^i$.

Real exchange rate between country i and country i' is

$$\widehat{RER}_{i',t}^{\,i} = \hat{S}_{i',t}^i + \hat{P}_t^{i'} - \hat{P}_t^i \tag{7.103}$$

Terms of trade between country i and country i' are

$$\widehat{ToT}^{i}_{i',t} = \hat{P}^{i}_{i',t} - \hat{P}^{i}_{i,t} = \hat{S}^{i}_{i',t} + \hat{P}^{i'}_{i',t} - \hat{P}^{i}_{i,t} \tag{7.104}$$

Summary of terms of trade definitions:

Terms of Trade		Definition 1		Definition 2
$\widehat{ToT}^{A}_{B,t}$	$=$	$\hat{P}^{A}_{B,t} - \hat{P}^{A}_{A,t}$	$=$	$\hat{S}^{A}_{B,t} + \hat{P}^{B}_{B,t} - \hat{P}^{A}_{A,t}$
$\widehat{ToT}^{A}_{C,t}$	$=$	$\hat{P}^{A}_{C,t} - \hat{P}^{A}_{A,t}$	$=$	$\hat{S}^{A}_{C,t} + \hat{P}^{C}_{C,t} - \hat{P}^{A}_{A,t}$
$\widehat{ToT}^{B}_{A,t}$	$=$	$\hat{P}^{B}_{A,t} - \hat{P}^{B}_{B,t}$	$=$	$\hat{S}^{B}_{A,t} + \hat{P}^{A}_{A,t} - \hat{P}^{B}_{B,t}$
$\widehat{ToT}^{B}_{C,t}$	$=$	$\hat{P}^{B}_{C,t} - \hat{P}^{B}_{B,t}$	$=$	$\hat{S}^{B}_{C,t} + \hat{P}^{C}_{C,t} - \hat{P}^{B}_{B,t}$
$\widehat{ToT}^{C}_{A,t}$	$=$	$\hat{P}^{C}_{A,t} - \hat{P}^{C}_{C,t}$	$=$	$\hat{S}^{C}_{A,t} + \hat{P}^{A}_{A,t} - \hat{P}^{C}_{C,t}$
$\widehat{ToT}^{C}_{B,t}$	$=$	$\hat{P}^{C}_{B,t} - \hat{P}^{C}_{C,t}$	$=$	$\hat{S}^{C}_{B,t} + \hat{P}^{B}_{B,t} - \hat{P}^{C}_{C,t}$

Loglinearized CPI-PPI Ratios

Country A Substitute weights in $g_A(ToT)$ to simplify the algebra

$$\begin{aligned} g_A(ToT) \;=\; & \left[\underbrace{[1-\eta(1-n_P)]}_{v_P} \left[\left(\underbrace{[1-\eta(1-n_A)]}_{v_A} + \underbrace{\eta(1-n_A)}_{v_B} \left(ToT^{A}_{B}\right)^{1-\psi} \right)^{\frac{1}{1-\psi}} \right]^{1-\theta} \right. \\ & \left. + \underbrace{\eta(1-n_P)}_{v_C} \left(ToT^{A}_{C}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \end{aligned} \tag{7.105}$$

Derive

$$
\begin{aligned}
dg_{A,t}(ToT) = & \frac{1}{1-\theta}\left[v_P\left[\left(v_A + v_B\left(ToT_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + v_C\left(ToT_C^A\right)^{1-\theta}\right]^{\frac{1}{1-\theta}-1} \\
& \cdot (1-\theta)\, v_P\left[\left(v_A + v_B\left(ToT_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta-1} \\
& \cdot \frac{1}{1-\psi}\left(v_A + v_B\left(ToT_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}-1} \\
& \cdot (1-\psi)\, v_B\left(ToT_B^A\right)^{1-\psi-1} \cdot dToT_{B,t}^A \\
& + \frac{1}{1-\theta}\left[v_P\left[\left(v_A + v_B\left(ToT_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + v_C\left(ToT_C^A\right)^{1-\theta}\right]^{\frac{1}{1-\theta}-1} \\
& \cdot v_C\,(1-\theta)\left(ToT_C^A\right)^{1-\theta-1} \cdot dToT_{C,t}^A
\end{aligned}
$$

$$
\begin{aligned}
g_A(ToT)\,\hat{g}_{A,t}(ToT) = & \left[v_P\left[\left(v_A + v_B\left(ToT_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + v_C\left(ToT_C^A\right)^{1-\theta}\right]^{\frac{1}{1-\theta}-1} \\
& \cdot v_P\left[\left(v_A + v_B\left(ToT_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta-1} \\
& \cdot \left(v_A + v_B\left(ToT_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}-1} \\
& \cdot v_B\left(ToT_B^A\right)^{1-\psi} \cdot \widehat{ToT}_{B,t}^A \\
& + \left[v_P\left[\left(v_A + v_B\left(ToT_B^A\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}\right]^{1-\theta} + v_C\left(ToT_C^A\right)^{1-\theta}\right]^{\frac{1}{1-\theta}-1} \\
& \cdot v_C\left(ToT_C^A\right)^{1-\theta} \cdot \widehat{ToT}_{C,t}^A
\end{aligned}
$$

All steady state values are normalized to one

$$
\begin{aligned}
\hat{g}_{A,t}(ToT) = & \underbrace{\left[v_P\left[\underbrace{(v_A + v_B)}_{1}^{\frac{1}{1-\psi}}\right]^{1-\theta} + v_C\right]^{\frac{1}{1-\theta}-1}}_{1} \\
& \cdot v_P\underbrace{\left[(v_A + v_B)^{\frac{1}{1-\psi}}\right]}_{1}^{1-\theta-1} \cdot \underbrace{(v_A + v_B)}_{1}^{\frac{1}{1-\psi}-1} \cdot v_B\widehat{ToT}_{B,t}^A \\
& + \underbrace{\left[v_P\left[(v_A + v_B)^{\frac{1}{1-\psi}}\right]^{1-\theta} + v_C\right]}_{1}^{\frac{1}{1-\theta}-1} \cdot v_C\widehat{ToT}_{C,t}^A
\end{aligned}
$$

$$\hat{g}_{A,t}(ToT) = v_P v_B \widehat{ToT}^{A}_{B,t} + v_C \widehat{ToT}^{A}_{C,t}$$

Substitute weights

$$\hat{g}_{A,t}(ToT) = [1 - \eta(1 - n_P)]\,\eta(1 - n_A)\,\widehat{ToT}^{A}_{B,t} + \eta(1 - n_P)\,\widehat{ToT}^{A}_{C,t} \tag{7.106}$$

Country B

$$\hat{g}_B(ToT) = [1 - \eta(1 - n_P)]\;\eta n_A \widehat{ToT}^{B}_{A,t} + \eta(1 - n_P)\;\widehat{ToT}^{B}_{C,t} \tag{7.107}$$

Country C

$$\hat{g}_C(ToT) = \eta n_P\, n_A \widehat{ToT}^{C}_{A,t} + \eta n_P(1 - n_A)\;\widehat{ToT}^{C}_{B,t} \tag{7.108}$$

Loglinearized Exhange Rate Relations

We can show that

$$\hat{S}^{i}_{i',t} = -\hat{S}^{i'}_{i,t}. \tag{7.109}$$

The same relation holds for the terms of trade

$$\begin{aligned} \widehat{ToT}^{i}_{i',t} &= -\widehat{ToT}^{i'}_{i,t} \qquad (7.110)\\ \hat{S}^{i}_{i',t} + \hat{P}^{i'}_{i',t} - \hat{P}^{i}_{i,t} &= -\left(\hat{S}^{i'}_{i,t} + \hat{P}^{i}_{i,t} - \hat{P}^{i'}_{C,t}\right)\\ \hat{S}^{i}_{i',t} + \hat{P}^{i'}_{i',t} - \hat{P}^{i}_{i,t} &= -\hat{S}^{i'}_{i,t} - \hat{P}^{i}_{i,t} + \hat{P}^{i'}_{i',t} \end{aligned}$$

and following from (7.109)

$$\begin{aligned} \hat{S}^{i}_{i',t} + \hat{P}^{i'}_{i',t} - \hat{P}^{i}_{i,t} &= \hat{S}^{i}_{i',t} - \hat{P}^{i}_{i,t} + \hat{P}^{i'}_{i',t}\\ 0 &= 0. \end{aligned}$$

Thus, the number of terms of trade definitions reduces to three.

Further, the third nominal exchange rate is given by the other two (7.18)

$$\hat{S}^{A}_{B,t} = \hat{S}^{A}_{C,t} + \hat{S}^{C}_{B,t}. \tag{7.111}$$

The same is true for the terms of trade using (7.111) and the definitions for the terms of trade (7.104)

$$\widehat{ToT}^{A}_{B,t} = \widehat{ToT}^{A}_{C,t} + \widehat{ToT}^{C}_{B,t}. \tag{7.112}$$

Equation (7.112) reduces the number of terms of trade to two.

The real exchange rate may be expressed in terms of trade (7.30)

$$\begin{aligned}
RER^{i}_{i',t} &= ToT^{i}_{i',t} \cdot g_{i',t}(ToT) \cdot g_{i,t}(ToT)^{-1} \\
dRER^{i}_{i',t} &= dToT^{i}_{i',t} \cdot g_{i'}(ToT) \cdot g_{i}(ToT)^{-1} \\
&\quad + ToT^{i}_{i'} \cdot dg_{i',t}(ToT) \cdot g_{i}(ToT)^{-1} \\
&\quad - ToT^{i}_{i'} \cdot g_{i'}(ToT) \cdot g_{i}(ToT)^{-2} dg_{i,t}(ToT)^{-1} \\
\widehat{RER}^{i}_{i',t} &= \widehat{ToT}^{i}_{i',t} + \hat{g}_{i',t}(ToT) - \hat{g}_{i,t}(ToT).
\end{aligned} \tag{7.113}$$

Backing out the nominal exchange rate from equation (7.21)

$$\begin{aligned}
ToT^{i}_{i',t} &= \frac{S^{i}_{i',t} \cdot P^{i'}_{i',t}}{P^{i}_{i,t}} \\
S^{i}_{i',t} &= ToT^{i}_{i',t} \frac{P^{i}_{i,t}}{P^{i'}_{i',t}}.
\end{aligned}$$

Taking first differences

$$S^{i}_{i',t} - S^{i}_{i',t-1} = ToT^{i}_{i',t} \frac{P^{i}_{i,t}}{P^{i'}_{i',t}} - ToT^{i}_{i',t-1} \frac{P^{i}_{i,t-1}}{P^{i'}_{i',t-1}}.$$

Loglinearizing

$$\begin{aligned}
dS^{i}_{i',t} - dS^{i}_{i',t-1} &= dToT^{i}_{i',t} \frac{P^{i}_{i}}{P^{i'}_{i'}} + ToT^{i}_{i'} dP^{i}_{i,t} \frac{1}{P^{i'}_{i'}} - ToT^{i}_{i'} P^{i}_{i} \frac{1}{\left(P^{i'}_{i'}\right)^{2}} dP^{i'}_{i',t} \\
&\quad - \left(dToT^{i}_{i',t-1} \frac{P^{i}_{i}}{P^{i'}_{i'}} + ToT^{i}_{i'} dP^{i}_{i,t-1} \frac{1}{P^{i'}_{i'}} - ToT^{i}_{i'} P^{i}_{i} \frac{1}{\left(P^{i'}_{i'}\right)^{2}} P^{i'}_{i',t-1} \right)
\end{aligned}$$

$$\begin{aligned}
S^{i}_{i'} \hat{S}^{i}_{i',t} - S^{i}_{i'} \hat{S}^{i}_{i',t-1} &= ToT^{i}_{i'} \frac{P^{i}_{i}}{P^{i'}_{i'}} \widehat{ToT}^{i}_{i',t} + ToT^{i}_{i'} \frac{P^{i}_{i}}{P^{i'}_{i'}} \hat{P}^{i}_{i,t} - ToT^{i}_{i'} \frac{P^{i}_{i}}{P^{i'}_{i'}} \hat{P}^{i'}_{i',t} \\
&\quad - ToT^{i}_{i'} \frac{P^{i}_{i}}{P^{i'}_{i'}} \widehat{ToT}^{i}_{i',t-1} - ToT^{i}_{i'} \frac{P^{i}_{i}}{P^{i'}_{i'}} \hat{P}^{i}_{i,t-1} + ToT^{i}_{i'} \frac{P^{i}_{i}}{P^{i'}_{i'}} \hat{P}^{i'}_{i',t-1}
\end{aligned}$$

$$\begin{aligned}
\hat{S}^{i}_{i',t} - \hat{S}^{i}_{i',t-1} &= \widehat{ToT}^{i}_{i',t} + \hat{P}^{i}_{i,t} - \hat{P}^{i'}_{i',t} - \widehat{ToT}^{i}_{i',t-1} - \hat{P}^{i}_{i,t-1} + \hat{P}^{i'}_{i',t-1} \\
\Delta \hat{S}^{i}_{i',t} &= \widehat{ToT}^{i}_{i',t} - \widehat{ToT}^{i}_{i',t-1} + \pi^{i}_{i,t} - \pi^{i'}_{i',t}
\end{aligned} \tag{7.114}$$

Loglinearized Terms of Trade for the Periphery From equation (7.24)

$$\begin{aligned} ToT_{P,t}^{A} &= \left[\left[1-\eta\left(1-n_A\right)\right]+\eta\left(1-n_A\right)\left(ToT_{B,t}^{A}\right)^{1-\psi}\right]^{\frac{1}{1-\psi}} \\ \left(ToT_{P,t}^{A}\right)^{1-\psi} &= \left[1-\eta\left(1-n_A\right)\right]+\eta\left(1-n_A\right)\left(ToT_{B,t}^{A}\right)^{1-\psi} \end{aligned}$$

$$\begin{aligned} \left(1-\psi\right)\left(ToT_{P}^{A}\right)^{1-\psi-1}\cdot dToT_{P,t}^{A} &= \left(1-\psi\right)\eta\left(1-n_A\right)\left(ToT_{B}^{A}\right)^{1-\psi-1}\cdot dToT_{B,t}^{A} \\ \left(ToT_{P}^{A}\right)^{1-\psi}\widehat{ToT}_{P,t}^{A} &= \eta\left(1-n_A\right)\left(ToT_{B}^{A}\right)^{1-\psi}\widehat{ToT}_{B,t}^{A} \\ \widehat{ToT}_{P,t}^{A} &= \eta\left(1-n_A\right)\widehat{ToT}_{B,t}^{A} \end{aligned} \tag{7.115}$$

From equation (7.25)

$$\begin{aligned} ToT_{P}^{B} &= \left[\left(1-\eta n_A\right)+\eta n_A\left(ToT_{A}^{B}\right)^{1-\psi}\right]^{\frac{1}{1-\psi}} \\ \left(ToT_{P}^{B}\right)^{1-\psi} &= \left(1-\eta n_A\right)+\eta n_A\left(ToT_{A}^{B}\right)^{1-\psi} \end{aligned}$$

$$\begin{aligned} \left(1-\psi\right)\left(ToT_{P}^{B}\right)^{1-\psi-1}\cdot dToT_{P,t}^{B} &= \left(1-\psi\right)\eta n_A\left(ToT_{A}^{B}\right)^{1-\psi-1}\cdot dToT_{A,t}^{B} \\ \left(ToT_{P}^{B}\right)^{1-\psi}\widehat{ToT}_{P,t}^{B} &= \eta n_A\left(ToT_{A}^{B}\right)^{1-\psi}\widehat{ToT}_{A,t}^{B} \\ \widehat{ToT}_{P,t}^{B} &= \eta n_A\widehat{ToT}_{A,t}^{B} \end{aligned} \tag{7.116}$$

From equation (7.26)

$$\begin{aligned} ToT_{P}^{C} &= \left[n_A\left(ToT_{A}^{C}\right)^{1-\psi}+\left(1-n_A\right)\left(ToT_{B}^{C}\right)^{1-\psi}\right]^{\frac{1}{1-\psi}} \\ \left(ToT_{P}^{C}\right)^{1-\psi} &= n_A\left(ToT_{A}^{C}\right)^{1-\psi}+\left(1-n_A\right)\left(ToT_{B}^{C}\right)^{1-\psi} \end{aligned}$$

$$\begin{aligned} &\left(1-\psi\right)\left(ToT_{P}^{C}\right)^{1-\psi-1}\cdot dToT_{P,t}^{C} = \\ &n_A\left(ToT_{A}^{C}\right)^{1-\psi-1}\cdot dToT_{A,t}^{C}+\left(1-n_A\right)\left(ToT_{B}^{C}\right)^{1-\psi-1}\cdot dToT_{B,t}^{C} \end{aligned}$$

$$\begin{aligned} \left(ToT_{P}^{C}\right)^{1-\psi}\widehat{ToT}_{P,t}^{C} &= n_A\left(ToT_{A}^{C}\right)^{1-\psi}\widehat{ToT}_{A,t}^{C}+\left(1-n_A\right)\left(ToT_{B}^{C}\right)^{1-\psi}\widehat{ToT}_{B,t}^{C} \\ \widehat{ToT}_{P,t}^{C} &= n_A\widehat{ToT}_{A,t}^{C}+\left(1-n_A\right)\widehat{ToT}_{B,t}^{C} \end{aligned} \tag{7.117}$$

Rewriting the expression $\hat{P}_t^A - \hat{P}_{A,t}^A$ by using the definition for the loglinearized

CPI (7.96)

$$\begin{aligned}
\hat{P}_t^A - \hat{P}_{A,t}^A &= v_P v_A \hat{P}_{A,t}^A + v_P v_B \hat{P}_{B,t}^A + v_C \hat{P}_{C,t}^A - \hat{P}_{A,t}^A \\
&= v_P v_B \, \hat{P}_{B,t}^A + v_C \hat{P}_{C,t}^A + (v_P v_A - 1) \, \hat{P}_{A,t}^A \\
\text{use } (v_P v_A - 1) &= -\left(v_P v_B + v_C^A\right) \quad \text{as } v_P v_A + v_P v_B + v_C = 1 \\
&= v_P v_B \, \hat{P}_{B,t}^A + v_C \hat{P}_{C,t}^A - (v_P v_B + v_C) \, \hat{P}_{A,t}^A \\
&= v_P v_B \underbrace{\left(\hat{P}_{B,t}^A - \hat{P}_{A,t}^A\right)}_{\widehat{ToT}_{B,t}^A} + v_C \underbrace{\left(\hat{P}_{C,t}^A - \hat{P}_{A,t}^A\right)}_{\widehat{ToT}_{C,t}^A} \\
\hat{P}_t^A - \hat{P}_{A,t}^A &= v_P v_B \, \widehat{ToT}_{B,t}^A + v_C \, \widehat{ToT}_{C,t}^A \qquad (7.118) \\
\hat{P}_t^A &= \hat{P}_{A,t}^A + v_P v_B \, \widehat{ToT}_{B,t}^A + v_C \, \widehat{ToT}_{C,t}^A \\
\hat{P}_t^A &= \hat{P}_{A,t}^A + \text{effective ToT } A. \qquad (7.119)
\end{aligned}$$

Equation (7.119) corresponds to equation (13) in Galí and Monacelli (2005: 712), where α is the degree of openness, i.e., the equivalent of $v_P v_B$ and v_C.

Loglinearized Household Equilibrium Conditions

Consumption Labor Trade-Off

$$\hat{W}_t^i = \mu \, \hat{C}_t^i - \gamma \, \hat{L}_t^i + \hat{P}_t^i \qquad (7.120)$$

Euler Equation Loglinearizing the Euler equation (7.43)

$$\hat{C}_t^i = \hat{C}_{t+1}^i - \frac{1}{\mu}\left(\hat{R}_t^i - \hat{\pi}_{t+1}^i\right) \qquad (7.121)$$

Risk Sharing Loglinearizing the risk sharing equation (7.48)

$$\hat{C}_t^i = \hat{C}_t^{i'} \qquad (7.122)$$

Risk sharing without PPP (from equation 7.49)

$$\hat{C}_t^i = \hat{C}_t^{i'} + \frac{1}{\mu} \widehat{RER}_{i',t}^i \qquad (7.123)$$

For country A and B using equation (7.50)

$$C_t^A = C_t^B \left(ToT_{B,t}^A\right)^{\frac{1}{\mu}} g_B(ToT)^{\frac{1}{\mu}} g_A(ToT)^{-\frac{1}{\mu}}$$

$$\hat{C}_t^A = \hat{C}_t^B + \frac{1}{\mu}\widehat{ToT}_{B,t}^A + \frac{1}{\mu}\hat{g}_B(ToT) - \frac{1}{\mu}\hat{g}_A(ToT) \tag{7.124}$$

For country A and C using equation (7.51)

$$C_t^A = C_t^C \left(ToT_{C,t}^A\right)^{\frac{1}{\mu}} g_C(ToT)^{\frac{1}{\mu}} g_A(ToT)^{-\frac{1}{\mu}}$$

$$\hat{C}_t^A = \hat{C}_t^C + \frac{1}{\mu}\widehat{ToT}_{C,t}^A + \frac{1}{\mu}\hat{g}_C(ToT) - \frac{1}{\mu}\hat{g}_A(ToT) \tag{7.125}$$

Loglinearized Goods Market Clearing

Country A

$$\begin{aligned}
Y^A &= v_A^A \left[\left[1-\eta\left(1-n_A\right)\right] + \eta\left(1-n_A\right)\left(ToT_B^A\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(g_A(ToT)\right)^{\theta} C^A \\
&+ v_B^A \left[\left(1-\eta n_A\right) + \eta n_A \left(ToT_A^B\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_B^A\right)^{\psi} \left(g_B(ToT)\right)^{\theta} C^B \\
&+ v_C^A \left[n_A \left(ToT_A^C\right)^{1-\psi} + \left(1-n_A\right)\left(ToT_B^C\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_C^A\right)^{\psi} \left(g_C(ToT)\right)^{\theta} C^C
\end{aligned}$$

$$\begin{aligned}
Y^A &= \left[1-\eta\left(1-n_P\right)\right]\left[1-\eta\left(1-n_A\right)\right] \\
&\cdot \left[\left[1-\eta\left(1-n_A\right)\right] + \eta\left(1-n_A\right)\left(ToT_B^A\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(g_A(ToT)\right)^{\theta} C^A \\
&+ \left[1-\eta\left(1-n_P\right)\right]\left[\eta\left(1-n_A\right)\right] \\
&\cdot \left[\left(1-\eta n_A\right) + \eta n_A \left(ToT_A^B\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_B^A\right)^{\psi} \left(g_B(ToT)\right)^{\theta} C^B \\
&+ \eta\left(1-n_P\right)\left[n_A \left(ToT_A^C\right)^{1-\psi} + \left(1-n_A\right)\left(ToT_B^C\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_C^A\right)^{\psi} \left(g_C(ToT)\right)^{\theta} C^C
\end{aligned}$$

Use shorthands, $\widetilde{v}^i_{i'}$, for weights to simlify algebra.

$$
\begin{aligned}
Y^A = & \underbrace{[1-\eta\,(1-n_P)]}_{\widetilde{v}_P}\underbrace{[1-\eta\,(1-n_A)]}_{\widetilde{v}^A_A} \\
& \cdot \left[\underbrace{[1-\eta\,(1-n_A)]}_{\widetilde{v}^A_A} + \underbrace{\eta\,(1-n_A)}_{\widetilde{v}^A_B}\left(ToT^A_B\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} (g_A(ToT))^{\theta}\; C^A \\
& + \underbrace{[1-\eta\,(1-n_P)]}_{\widetilde{v}_P}\underbrace{[\eta\,(1-n_A)]}_{\widetilde{v}^A_B} \\
& \cdot \left[\underbrace{(1-\eta n_A)}_{\widetilde{v}^B_B} + \underbrace{\eta n_A}_{\widetilde{v}^B_A}\left(ToT^B_A\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT^A_B\right)^{\psi} (g_B(ToT))^{\theta}\; C^B \\
& + \underbrace{\eta\,(1-n_P)}_{\widetilde{v}_C}\left[\underbrace{n_A}_{\widetilde{v}^C_A}\left(ToT^C_A\right)^{1-\psi} + \underbrace{(1-n_A)}_{\widetilde{v}^C_B}\left(ToT^C_B\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT^A_C\right)^{\psi} (g_C(ToT))^{\theta}\; C^C
\end{aligned}
$$

$$
\begin{aligned}
Y^A \;=\; & \widetilde{v}_P\widetilde{v}^A_A\left[\widetilde{v}^A_A + \widetilde{v}^A_B\left(ToT^A_B\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} (g_A(ToT))^{\theta}\; C^A \\
& + \widetilde{v}_P\widetilde{v}^A_B\left[\widetilde{v}^B_B + \widetilde{v}^B_A\left(ToT^B_A\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT^A_B\right)^{\psi} (g_B(ToT))^{\theta}\; C^B \\
& + \widetilde{v}_C\left[\widetilde{v}^C_A\left(ToT^C_A\right)^{1-\psi} + \widetilde{v}^C_B\left(ToT^C_B\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT^A_C\right)^{\psi} (g_C(ToT))^{\theta}\; C^C
\end{aligned}
$$

Variables are $Y^A, C^A, C^B, C^C, ToT^A_B, ToT^B_A, ToT^C_A, ToT^C_B,\; ToT^A_C, g_A(ToT), g_B(ToT),$

and $g_C(ToT)$.

$$
\begin{aligned}
dY_t^A &= \widetilde{v}_P \widetilde{v}_A^A \left[\widetilde{v}_A^A + \widetilde{v}_B^A \left(ToT_B^A\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(g_A(ToT)\right)^{\theta} \cdot dC_t^A \\
&+ \widetilde{v}_P \widetilde{v}_A^A \left[\widetilde{v}_A^A + \widetilde{v}_B^A \left(ToT_B^A\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} C^A \cdot \theta \left(g_A(ToT)\right)^{\theta-1} \cdot dg_{A,t}(ToT) \\
&+ \frac{\psi-\theta}{1-\psi} \widetilde{v}_P \widetilde{v}_A^A \left[\widetilde{v}_A^A + \widetilde{v}_B^A \left(ToT_B^A\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}-1} \left(g_A(ToT)\right)^{\theta} \; C^A \\
&\quad \cdot (1-\psi)\, \widetilde{v}_B^A \left(ToT_B^A\right)^{1-\psi-1} \cdot dToT_{B,t}^A \\
&+ \widetilde{v}_P \widetilde{v}_B^A \left[\widetilde{v}_B^B + \widetilde{v}_A^B \left(ToT_A^B\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_B^A\right)^{\psi} \left(g_B(ToT)\right)^{\theta} \cdot dC_t^B \\
&+ \widetilde{v}_P \widetilde{v}_B^A \left[\widetilde{v}_B^B + \widetilde{v}_A^B \left(ToT_A^B\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_B^A\right)^{\psi} C^B \cdot \theta \left(g_B(ToT)\right)^{\theta-1} \cdot dg_{B,t}(ToT) \\
&+ \widetilde{v}_P \widetilde{v}_B^A \left[\widetilde{v}_B^B + \widetilde{v}_A^B \left(ToT_A^B\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(g_B(ToT)\right)^{\theta} C^B \cdot \psi \left(ToT_B^A\right)^{\psi-1} \cdot dToT_{B,t}^A \\
&+ \frac{\psi-\theta}{1-\psi} \widetilde{v}_P \widetilde{v}_B^A \left[\widetilde{v}_B^B + \widetilde{v}_A^B \left(ToT_A^B\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}-1} \left(ToT_B^A\right)^{\psi} \left(g_B(ToT)\right)^{\theta} \; C^B \\
&\quad \cdot (1-\psi)\, \widetilde{v}_A^B \left(ToT_A^B\right)^{1-\psi-1} \cdot dToT_{A,t}^B \\
&+ \widetilde{v}_C \left[\widetilde{v}_A^C \left(ToT_A^C\right)^{1-\psi} + \widetilde{v}_B^C \left(ToT_B^C\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_C^A\right)^{\psi} \left(g_C(ToT)\right)^{\theta} \cdot dC_t^C \\
&+ \widetilde{v}_C \left[\widetilde{v}_A^C \left(ToT_A^C\right)^{1-\psi} + \widetilde{v}_B^C \left(ToT_B^C\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_C^A\right)^{\psi} C^C \cdot \theta \left(g_C(ToT)\right)^{\theta-1} \cdot dg_{C,t}(ToT) \\
&+ \widetilde{v}_C \left[\widetilde{v}_A^C \left(ToT_A^C\right)^{1-\psi} + \widetilde{v}_B^C \left(ToT_B^C\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(g_C(ToT)\right)^{\theta} \; C^C \cdot \psi \left(ToT_C^A\right)^{\psi-1} \cdot dToT_{C,t}^A \\
&+ \frac{\psi-\theta}{1-\psi} \widetilde{v}_C \left[\widetilde{v}_A^C \left(ToT_A^C\right)^{1-\psi} + \widetilde{v}_B^C \left(ToT_B^C\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}-1} \left(ToT_C^A\right)^{\psi} \left(g_C(ToT)\right)^{\theta} \; C^C \\
&\quad \cdot (1-\psi)\, \widetilde{v}_A^C \left(ToT_A^C\right)^{1-\psi-1} \cdot dToT_{A,t}^C \\
&+ \frac{\psi-\theta}{1-\psi} \widetilde{v}_C \left[\widetilde{v}_A^C \left(ToT_A^C\right)^{1-\psi} + \widetilde{v}_B^C \left(ToT_B^C\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}-1} \left(ToT_C^A\right)^{\psi} \left(g_C(ToT)\right)^{\theta} \; C^C \\
&\quad \cdot (1-\psi)\, \widetilde{v}_B^C \left(ToT_B^C\right)^{1-\psi-1} \cdot dToT_{B,t}^C
\end{aligned}
$$

$$
\begin{aligned}
Y\,\hat{Y}_t^A &= \widetilde{v}_P \widetilde{v}_A^A \left[\widetilde{v}_A^A + \widetilde{v}_B^A \left(ToT_B^A\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(g_A(ToT)\right)^{\theta} \cdot C^A \hat{C}_t^A \\
&+ \widetilde{v}_P \widetilde{v}_A^A \left[\widetilde{v}_A^A + \widetilde{v}_B^A \left(ToT_B^A\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} C^A \cdot \theta \left(g_A(ToT)\right)^{\theta} \cdot \hat{g}_{A,t}(ToT) \\
&+ (\psi-\theta)\, \widetilde{v}_P \widetilde{v}_A^A \left[\widetilde{v}_A^A + \widetilde{v}_B^A \left(ToT_B^A\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}-1} \\
&\quad \cdot \left(g_A(ToT)\right)^{\theta}\, C^A \cdot \widetilde{v}_B^A \left(ToT_B^A\right)^{1-\psi} \cdot \widehat{ToT}_{B,t}^A \\
&+ \widetilde{v}_P \widetilde{v}_B^A \left[\widetilde{v}_B^B + \widetilde{v}_A^B \left(ToT_A^B\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_B^A\right)^{\psi} \left(g_B(ToT)\right)^{\theta} \cdot C^B \hat{C}_t^B \\
&+ \widetilde{v}_P \widetilde{v}_B^A \left[\widetilde{v}_B^B + \widetilde{v}_A^B \left(ToT_A^B\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_B^A\right)^{\psi} C^B \cdot \theta \left(g_B(ToT)\right)^{\theta} \cdot \hat{g}_{B,t}(ToT) \\
&+ \widetilde{v}_P \widetilde{v}_B^A \left[\widetilde{v}_B^B + \widetilde{v}_A^B \left(ToT_A^B\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(g_B(ToT)\right)^{\theta} C^B \cdot \psi \left(ToT_B^A\right)^{\psi} \cdot \widehat{ToT}_{B,t}^A \\
&+ (\psi-\theta)\, \widetilde{v}_P \widetilde{v}_B^A \left[\widetilde{v}_B^B + \widetilde{v}_A^B \left(ToT_A^B\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}-1} \\
&\quad \cdot \left(ToT_B^A\right)^{\psi} \left(g_B(ToT)\right)^{\theta}\, C^B \cdot \widetilde{v}_A^B \left(ToT_A^B\right)^{1-\psi} \cdot \widehat{ToT}_{A,t}^B \\
&+ \widetilde{v}_C \left[\widetilde{v}_A^C \left(ToT_A^C\right)^{1-\psi} + \widetilde{v}_B^C \left(ToT_B^C\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_C^A\right)^{\psi} \left(g_C(ToT)\right)^{\theta} \cdot C^C \hat{C}_t^C \\
&+ \widetilde{v}_C \left[\widetilde{v}_A^C \left(ToT_A^C\right)^{1-\psi} + \widetilde{v}_B^C \left(ToT_B^C\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(ToT_C^A\right)^{\psi} C^C \cdot \theta \left(g_C(ToT)\right)^{\theta} \cdot \hat{g}_{C,t}(ToT) \\
&+ \widetilde{v}_C \left[\widetilde{v}_A^C \left(ToT_A^C\right)^{1-\psi} + \widetilde{v}_B^C \left(ToT_B^C\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}} \left(g_C(ToT)\right)^{\theta}\, C^C \cdot \psi \left(ToT_C^A\right)^{\psi} \cdot \widehat{ToT}_{C,t}^A \\
&+ (\psi-\theta)\, \widetilde{v}_C \left[\widetilde{v}_A^C \left(ToT_A^C\right)^{1-\psi} + \widetilde{v}_B^C \left(ToT_B^C\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}-1} \\
\cdot \quad &\left(ToT_C^A\right)^{\psi} \left(g_C(ToT)\right)^{\theta}\, C^C \cdot \widetilde{v}_A^C \left(ToT_A^C\right)^{1-\psi} \cdot \widehat{ToT}_{A,t}^C \\
&+ (\psi-\theta)\, \widetilde{v}_C \left[\widetilde{v}_A^C \left(ToT_A^C\right)^{1-\psi} + \widetilde{v}_B^C \left(ToT_B^C\right)^{1-\psi}\right]^{\frac{\psi-\theta}{1-\psi}-1} \\
&\quad \cdot \left(ToT_C^A\right)^{\psi} \left(g_C(ToT)\right)^{\theta}\, C^C \cdot \widetilde{v}_B^C \left(ToT_B^C\right)^{1-\psi} \cdot \widehat{ToT}_{B,t}^C
\end{aligned}
$$

All steady state variables are normalized to one.

$$
\begin{aligned}
\hat{Y}_t^A &= \widetilde{v}_P \widetilde{v}_A^A \left[\widetilde{v}_A^A + \widetilde{v}_B^A\right]^{\frac{\psi-\theta}{1-\psi}} \cdot \hat{C}_t^A \\
&\quad + \widetilde{v}_P \widetilde{v}_A^A \left[\widetilde{v}_A^A + \widetilde{v}_B^A\right]^{\frac{\psi-\theta}{1-\psi}} \cdot \theta \cdot \hat{g}_{A,t}(ToT) \\
&\quad + (\psi-\theta)\, \widetilde{v}_P \widetilde{v}_A^A \left[\widetilde{v}_A^A + \widetilde{v}_B^A\right]^{\frac{\psi-\theta}{1-\psi}-1} \cdot \widetilde{v}_B^A \cdot \widehat{ToT}_{B,t}^A \\
&\quad + \widetilde{v}_P \widetilde{v}_B^A \left[\widetilde{v}_B^B + \widetilde{v}_A^B\right]^{\frac{\psi-\theta}{1-\psi}} \cdot \hat{C}_t^B \\
&\quad + \widetilde{v}_P \widetilde{v}_B^A \left[\widetilde{v}_B^B + \widetilde{v}_A^B\right]^{\frac{\psi-\theta}{1-\psi}} \cdot \theta \cdot \hat{g}_{B,t}(ToT) \\
&\quad + \widetilde{v}_P \widetilde{v}_B^A \left[\widetilde{v}_B^B + \widetilde{v}_A^B\right]^{\frac{\psi-\theta}{1-\psi}} \psi \cdot \widehat{ToT}_{B,t}^A \\
&\quad + (\psi-\theta)\, \widetilde{v}_P \widetilde{v}_B^A \left[\widetilde{v}_B^B + \widetilde{v}_A^B\right]^{\frac{\psi-\theta}{1-\psi}-1} \cdot \widetilde{v}_A^B \cdot \widehat{ToT}_{A,t}^B \\
&\quad + \widetilde{v}_C \left[\widetilde{v}_A^C + \widetilde{v}_B^C\right]^{\frac{\psi-\theta}{1-\psi}} \cdot \hat{C}_t^C \\
&\quad + \widetilde{v}_C \left[\widetilde{v}_A^C + \widetilde{v}_B^C\right]^{\frac{\psi-\theta}{1-\psi}} \cdot \theta \cdot \hat{g}_{C,t}(ToT) \\
&\quad + \widetilde{v}_C \left[\widetilde{v}_A^C + \widetilde{v}_B^C\right]^{\frac{\psi-\theta}{1-\psi}} \cdot \psi \cdot \widehat{ToT}_{C,t}^A \\
&\quad + (\psi-\theta)\, \widetilde{v}_C \left[\widetilde{v}_A^C + \widetilde{v}_B^C\right]^{\frac{\psi-\theta}{1-\psi}-1} \cdot \widetilde{v}_A^C \cdot \widehat{ToT}_{A,t}^C \\
&\quad + (\psi-\theta)\, \widetilde{v}_C \left[\widetilde{v}_A^C + \widetilde{v}_B^C\right]^{\frac{\psi-\theta}{1-\psi}-1} \cdot \widetilde{v}_B^C \cdot \widehat{ToT}_{B,t}^C
\end{aligned}
$$

Note

$$
\begin{aligned}
\widetilde{v}_A^A + \widetilde{v}_B^A &= 1 - \eta(1-n_A) + \eta(1-n_A) = 1 \\
\widetilde{v}_A^B + \widetilde{v}_B^B &= 1 - \eta n_A + \eta n_A = 1 \\
\widetilde{v}_A^C + \widetilde{v}_B^C &= n_A + 1 - n_A = 1
\end{aligned}
$$

Substitute

$$
\begin{aligned}
\hat{Y}_t^A &= \widetilde{v}_P \widetilde{v}_A^A \cdot \hat{C}_t^A \\
&\quad + \theta \widetilde{v}_P \widetilde{v}_A^A \cdot \hat{g}_{A,t}(ToT) \\
&\quad + (\psi - \theta)\, \widetilde{v}_P \widetilde{v}_A^A \widetilde{v}_B^A \cdot \widehat{ToT}_{B,t}^A \\
&\quad + \widetilde{v}_P \widetilde{v}_B^A \cdot \hat{C}_t^B \\
&\quad + \theta \widetilde{v}_P \widetilde{v}_B^A \cdot \hat{g}_{B,t}(ToT) \\
&\quad + \psi \widetilde{v}_P \widetilde{v}_B^A \cdot \widehat{ToT}_{B,t}^A \\
&\quad + (\psi - \theta)\, \widetilde{v}_P \widetilde{v}_B^A \widetilde{v}_A^B \cdot \widehat{ToT}_{A,t}^B \\
&\quad + \widetilde{v}_C \cdot \hat{C}_t^C \\
&\quad + \theta \widetilde{v}_C \cdot \hat{g}_{C,t}(ToT) \\
&\quad + \psi \widetilde{v}_C \cdot \widehat{ToT}_{C,t}^A \\
&\quad + (\psi - \theta)\, \widetilde{v}_C \widetilde{v}_A^C \cdot \widehat{ToT}_{A,t}^C \\
&\quad + (\psi - \theta)\, \widetilde{v}_C \widetilde{v}_B^C \cdot \widehat{ToT}_{B,t}^C
\end{aligned}
$$

$$
\begin{aligned}
\hat{Y}_t^A &= \widetilde{v}_P \widetilde{v}_A^A \cdot \hat{C}_t^A + \theta \widetilde{v}_P \widetilde{v}_A^A \cdot \hat{g}_{A,t}(ToT) + (\psi - \theta)\, \widetilde{v}_P \widetilde{v}_A^A \widetilde{v}_B^A \cdot \widehat{ToT}_{B,t}^A \\
&\quad + \widetilde{v}_P \widetilde{v}_B^A \cdot \hat{C}_t^B + \theta \widetilde{v}_P \widetilde{v}_B^A \cdot \hat{g}_{B,t}(ToT) \\
&\quad + \psi \widetilde{v}_P \widetilde{v}_B^A \cdot \widehat{ToT}_{B,t}^A + (\psi - \theta)\, \widetilde{v}_P \widetilde{v}_B^A \widetilde{v}_A^B \cdot \widehat{ToT}_{A,t}^B \\
&\quad + \widetilde{v}_C \cdot \hat{C}_t^C + \theta \widetilde{v}_C \cdot \hat{g}_{C,t}(ToT) + \psi \widetilde{v}_C \cdot \widehat{ToT}_{C,t}^A \\
&\quad + (\psi - \theta)\, \widetilde{v}_C \widetilde{v}_A^C \cdot \widehat{ToT}_{A,t}^C + (\psi - \theta)\, \widetilde{v}_C \widetilde{v}_B^C \cdot \widehat{ToT}_{B,t}^C
\end{aligned}
$$

$$
\begin{aligned}
\hat{Y}_t^A &= [1-\eta(1-n_P)][1-\eta(1-n_A)]\hat{C}_t^A \\
&+\theta[1-\eta(1-n_P)][1-\eta(1-n_A)]\hat{g}_{A,t}(ToT) \\
&+(\psi-\theta)[1-\eta(1-n_P)][1-\eta(1-n_A)]\eta(1-n_A)\widehat{ToT}_{B,t}^A \\
&+[1-\eta(1-n_P)]\eta(1-n_A)\hat{C}_t^B+\theta[1-\eta(1-n_P)]\eta(1-n_A)\hat{g}_{B,t}(ToT) \\
&+\psi[1-\eta(1-n_P)]\eta(1-n_A)\widehat{ToT}_{B,t}^A+(\psi-\theta)[1-\eta(1-n_P)]\eta(1-n_A)\eta n_A\widehat{ToT}_{A,t}^B \\
&+\eta(1-n_P)\hat{C}_t^C+\theta\eta(1-n_P)\hat{g}_{C,t}(ToT)+\psi\eta(1-n_P)\widehat{ToT}_{C,t}^A \\
&+(\psi-\theta)\eta(1-n_P)n_A\widehat{ToT}_{A,t}^C+(\psi-\theta)\eta(1-n_P)(1-n_A)\widehat{ToT}_{B,t}^C \qquad (7.126)
\end{aligned}
$$

Country B

$$
\begin{aligned}
Y^B &= v_A^B\left(ToT_A^B\right)^{\psi}\left(ToT_P^A\right)^{\psi-\theta}\left(g_A(ToT)\right)^{\theta}C^A \\
&+v_B^B\left(ToT_P^B\right)^{\psi-\theta}\left(g_B(ToT)\right)^{\theta}C^B \\
&+v_C^B\left(ToT_C^B\right)^{\psi}\left(ToT_P^C\right)^{\psi-\theta}\left(g_C(ToT)\right)^{\theta}C^C
\end{aligned}
$$

$$
\begin{aligned}
dY_t^B &= v_A^B\left(ToT_A^B\right)^{\psi}\left(ToT_P^A\right)^{\psi-\theta}\left(g_A(ToT)\right)^{\theta}\cdot dC_t^A \\
&+v_A^B\left(ToT_A^B\right)^{\psi}\left(ToT_P^A\right)^{\psi-\theta}\theta\left(g_A(ToT)\right)^{\theta-1}C^A\cdot dg_{A,t}(ToT) \\
&+v_A^B\left(ToT_A^B\right)^{\psi}(\psi-\theta)\left(ToT_P^A\right)^{\psi-\theta-1}\left(g_A(ToT)\right)^{\theta}C^A\cdot dToT_{P,t}^A \\
&+v_A^B\psi\left(ToT_A^B\right)^{\psi-1}\left(ToT_P^A\right)^{\psi-\theta}\left(g_A(ToT)\right)^{\theta}C^A\cdot dToT_{A,t}^B \\
&+v_B^B\left(ToT_P^B\right)^{\psi-\theta}\left(g_B(ToT)\right)^{\theta}\cdot dC_t^B \\
&+v_B^B\left(ToT_P^B\right)^{\psi-\theta}\theta\left(g_B(ToT)\right)^{\theta-1}C^B\cdot dg_{B,t}(ToT) \\
&+v_B^B(\psi-\theta)\left(ToT_P^B\right)^{\psi-\theta-1}\left(g_B(ToT)\right)^{\theta}C^B\cdot dToT_{P,t}^B \\
&+v_C^B\left(ToT_C^B\right)^{\psi}\left(ToT_P^C\right)^{\psi-\theta}\left(g_C(ToT)\right)^{\theta}\cdot dC_t^C \\
&+v_C^B\left(ToT_C^B\right)^{\psi}\left(ToT_P^C\right)^{\psi-\theta}\theta\left(g_C(ToT)\right)^{\theta-1}C^C\cdot dg_{C,t}(ToT) \\
&+v_C^B\left(ToT_C^B\right)^{\psi}(\psi-\theta)\left(ToT_P^C\right)^{\psi-\theta-1}\left(g_C(ToT)\right)^{\theta}C^C\cdot dToT_{P,t}^C \\
&+v_C^B\psi\left(ToT_C^B\right)^{\psi-1}\left(ToT_P^C\right)^{\psi-\theta}\left(g_C(ToT)\right)^{\theta}C^C\cdot dToT_{C,t}^B
\end{aligned}
$$

$$
\begin{aligned}
Y^B\hat{Y}_t^B &= v_A^B\left(ToT_A^B\right)^{\psi}\left(ToT_P^A\right)^{\psi-\theta}\left(g_A\left(ToT\right)\right)^{\theta}C^A\cdot\hat{C}_t^A \\
&+v_A^B\left(ToT_A^B\right)^{\psi}\left(ToT_P^A\right)^{\psi-\theta}\theta\left(g_A\left(ToT\right)\right)^{\theta}C^A\cdot\hat{g}_{A,t}\left(ToT\right) \\
&+v_A^B\left(ToT_A^B\right)^{\psi}\left(\psi-\theta\right)\left(ToT_P^A\right)^{\psi-\theta}\left(g_A\left(ToT\right)\right)^{\theta}C^A\cdot\widehat{ToT}_{P,t}^A \\
&+v_A^B\psi\left(ToT_A^B\right)^{\psi}\left(ToT_P^A\right)^{\psi-\theta}\left(g_A\left(ToT\right)\right)^{\theta}C^A\cdot\widehat{ToT}_{A,t}^B \\
&+v_B^B\left(ToT_P^B\right)^{\psi-\theta}\left(g_B\left(ToT\right)\right)^{\theta}\cdot C^B\hat{C}_t^B \\
&+v_B^B\left(ToT_P^B\right)^{\psi-\theta}\theta\left(g_B\left(ToT\right)\right)^{\theta}C^B\cdot\hat{g}_{B,t}\left(ToT\right) \\
&+v_B^B\left(\psi-\theta\right)\left(ToT_P^B\right)^{\psi-\theta}\left(g_B\left(ToT\right)\right)^{\theta}C^B\cdot\widehat{ToT}_{P,t}^B \\
&+v_C^B\left(ToT_C^B\right)^{\psi}\left(ToT_P^C\right)^{\psi-\theta}\left(g_C\left(ToT\right)\right)^{\theta}\ C^C\cdot\hat{C}_t^C \\
&+v_C^B\left(ToT_C^B\right)^{\psi}\left(ToT_P^C\right)^{\psi-\theta}\theta\left(g_C\left(ToT\right)\right)^{\theta}C^C\cdot\hat{g}_{C,t}\left(ToT\right) \\
&+v_C^B\left(ToT_C^B\right)^{\psi}\left(\psi-\theta\right)\left(ToT_P^C\right)^{\psi-\theta}\left(g_C\left(ToT\right)\right)^{\theta}C^C\cdot\widehat{ToT}_{P,t}^C \\
&+v_C^B\psi\left(ToT_C^B\right)^{\psi}\left(ToT_P^C\right)^{\psi-\theta}\left(g_C\left(ToT\right)\right)^{\theta}C^C\cdot\widehat{ToT}_{C,t}^B
\end{aligned}
$$

$$
\begin{aligned}
\hat{Y}_t^B &= v_A^B\hat{C}_t^A+v_A^B\theta\hat{g}_{A,t}\left(ToT\right)+v_A^B\left(\psi-\theta\right)\widehat{ToT}_{P,t}^A+v_A^B\psi\widehat{ToT}_{A,t}^B \\
&+v_B^B\hat{C}_t^B+v_B^B\theta\hat{g}_{B,t}\left(ToT\right)+v_B^B\left(\psi-\theta\right)\widehat{ToT}_{P,t}^B \\
&+v_C^B\hat{C}_t^C+v_C^B\theta\hat{g}_{C,t}\left(ToT\right)+v_C^B\left(\psi-\theta\right)\widehat{ToT}_{P,t}^C+v_C^B\psi\widehat{ToT}_{C,t}^B
\end{aligned}
$$

$$
\begin{aligned}
\hat{Y}_t^B &= v_A^B\hat{C}_t^A+\theta v_A^B\hat{g}_{A,t}\left(ToT\right)+\left(\psi-\theta\right)v_A^B\eta\left(1-n_A\right)\widehat{ToT}_{B,t}^A+\psi v_A^B\widehat{ToT}_{A,t}^B \\
&+v_B^B\hat{C}_t^B+\theta v_B^B\hat{g}_{B,t}\left(ToT\right)+\left(\psi-\theta\right)v_B^B\eta n_A\widehat{ToT}_{A,t}^B \\
&+v_C^B\hat{C}_t^C+\theta v_C^B\hat{g}_{C,t}\left(ToT\right) \\
&+\left(\psi-\theta\right)v_C^B\left[n_A\widehat{ToT}_{A,t}^C+\left(1-n_A\right)\widehat{ToT}_{B,t}^C\right]+\psi v_C^B\widehat{ToT}_{C,t}^B
\end{aligned}
$$

$$
\begin{aligned}
\hat{Y}_t^B &= v_A^B\hat{C}_t^A+\theta v_A^B\hat{g}_{A,t}\left(ToT\right)+\left(\psi-\theta\right)v_A^B\eta\left(1-n_A\right)\widehat{ToT}_{B,t}^A+\psi v_A^B\widehat{ToT}_{A,t}^B \\
&+v_B^B\hat{C}_t^B+\theta v_B^B\hat{g}_{B,t}\left(ToT\right)+\left(\psi-\theta\right)v_B^B\eta n_A\widehat{ToT}_{A,t}^B \\
&+v_C^B\hat{C}_t^C+\theta v_C^B\hat{g}_{C,t}\left(ToT\right)+\left(\psi-\theta\right)v_C^Bn_A\widehat{ToT}_{A,t}^C \\
&+\left(\psi-\theta\right)v_C^B\left(1-n_A\right)\widehat{ToT}_{B,t}^C+\psi v_C^B\widehat{ToT}_{C,t}^B
\end{aligned}
$$

$$
\begin{aligned}
\hat{Y}_t^B &= [1-\eta(1-n_P)]\,\eta n_A \hat{C}_t^A + \theta\,[1-\eta(1-n_P)]\,\eta n_A \hat{g}_{A,t}(ToT) \\
&+ (\psi-\theta)\,[1-\eta(1-n_P)]\,\eta n_A \eta(1-n_A)\,\widehat{ToT}_{B,t}^A \\
&+ \psi\,[1-\eta(1-n_P)]\,\eta n_A \widehat{ToT}_{A,t}^B \\
&+ (1-\eta n_A)\,[1-\eta(1-n_P)]\,\hat{C}_t^B + \theta(1-\eta n_A)\,[1-\eta(1-n_P)]\,\hat{g}_{B,t}(ToT) \\
&+ (\psi-\theta)(1-\eta n_A)\,[1-\eta(1-n_P)]\,\eta n_A \widehat{ToT}_{A,t}^B \\
&+ \eta(1-n_P)\,\hat{C}_t^C + \theta\eta(1-n_P)\,\hat{g}_{C,t}(ToT) + (\psi-\theta)\,\eta(1-n_P)\,n_A \widehat{ToT}_{A,t}^C \\
&+ (\psi-\theta)\,\eta(1-n_P)(1-n_A)\,\widehat{ToT}_{B,t}^C + \psi\eta(1-n_P)\,\widehat{ToT}_{C,t}^B \qquad (7.127)
\end{aligned}
$$

Country C

$$
\begin{aligned}
Y^C &= v_A^C \left(\frac{g_A(ToT)}{ToT_C^A}\right)^\theta C^A + v_B^C \left(\frac{g_B(ToT)}{ToT_C^B}\right)^\theta C^B + v_C^C\,(g_C(ToT))^\theta\, C^C \\
Y^C &= v_A^C\,(g_A(ToT))^\theta \left(ToT_C^A\right)^{-\theta} C^A + v_B^C\,(g_B(ToT))^\theta \left(ToT_C^B\right)^{-\theta} C^B \\
&\quad + v_C^C\,(g_C(ToT))^\theta\, C^C
\end{aligned}
$$

$$
\begin{aligned}
dY_t^C &= v_A^C\,(g_A(ToT))^\theta \left(ToT_C^A\right)^{-\theta} \cdot dC_t^A \\
&\quad + v_A^C\,(g_A(ToT))^\theta\, C^A \cdot (-\theta)\left(ToT_C^A\right)^{-\theta-1} \cdot dToT_{C,t}^A \\
&\quad + v_A^C \left(ToT_C^A\right)^{-\theta} C^A \cdot \theta\,(g_A(ToT))^{\theta-1} \cdot dg_{A,t}(ToT) \\
&\quad + v_B^C\,(g_B(ToT))^\theta \left(ToT_C^B\right)^{-\theta} \cdot dC_t^B \\
&\quad + v_B^C\,(g_B(ToT))^\theta \cdot (-\theta)\left(ToT_C^B\right)^{-\theta-1} \cdot dToT_{C,t}^B \\
&\quad + v_B^C \left(ToT_C^B\right)^{-\theta} \cdot \theta\,(g_B(ToT))^{\theta-1} \cdot dg_{B,t}(ToT) \\
&\quad + v_C^C\,(g_C(ToT))^\theta \cdot dC_t^C \\
&\quad + v_C^C \cdot \theta\,(g_C(ToT))^{\theta-1} \cdot dg_{C,t}(ToT)
\end{aligned}
$$

$$
\begin{aligned}
Y^C\hat{Y}_t^C &= v_A^C\left(g_A(ToT)\right)^\theta\left(ToT_C^A\right)^{-\theta}\cdot C^A\hat{C}_t^A \\
&+v_A^C\left(g_A(ToT)\right)^\theta C^A\cdot(-\theta)\left(ToT_C^A\right)^{-\theta}\cdot\widehat{ToT}_{C,t}^A \\
&+v_A^C\left(ToT_C^A\right)^{-\theta}C^A\cdot\theta\left(g_A(ToT)\right)^\theta\cdot\hat{g}_{A,t}(ToT) \\
&+v_B^C\left(g_B(ToT)\right)^\theta\left(ToT_C^B\right)^{-\theta}\cdot C^B\hat{C}_t^B \\
&+v_B^C\left(g_B(ToT)\right)^\theta\cdot(-\theta)\left(ToT_C^B\right)^{-\theta}\cdot\widehat{ToT}_{C,t}^B \\
&+v_B^C\left(ToT_C^B\right)^{-\theta}\cdot\theta\left(g_B(ToT)\right)^\theta\cdot\hat{g}_{B,t}(ToT) \\
&+v_C^C\left(g_C\left(ToT\right)\right)^\theta\cdot C^C\hat{C}_t^C \\
&+v_C^C\cdot\theta\left(g_C(ToT)\right)^\theta\cdot\hat{g}_{C,t}(ToT)
\end{aligned}
$$

$$
\begin{aligned}
\hat{Y}_t^C &= v_A^C\hat{C}_t^A+(-\theta)\,v_A^C\widehat{ToT}_{C,t}^A+\theta v_A^C\hat{g}_{A,t}(ToT) \\
&+v_B^C\hat{C}_t^B+(-\theta)\,v_B^C\widehat{ToT}_{C,t}^B+\theta v_B^C\hat{g}_{B,t}(ToT) \\
&+v_C^C\,\hat{C}_t^C+\theta v_C^C\hat{g}_{C,t}(ToT)
\end{aligned}
$$

$$
\begin{aligned}
\hat{Y}_t^C &= \eta n_P n_A\hat{C}_t^A+\eta n_P n_A\,(-\theta)\,\widehat{ToT}_{C,t}^A+\theta\eta n_P n_A\hat{g}_{A,t}(ToT) \\
&+\eta n_p\,(1-n_A)\,\hat{C}_t^B+(-\theta)\,\eta n_p\,(1-n_A)\,\widehat{ToT}_{C,t}^B+\theta\eta n_p\,(1-n_A)\,\hat{g}_{B,t}(ToT) \\
&+(1-\eta n_P)\;\hat{C}_t^C+\theta\,(1-\eta n_P)\,\hat{g}_{C,t}(ToT) \qquad (7.128)
\end{aligned}
$$

7.1.11 Rewriting Some Equations

CPI Inflation in Terms of Trade

Starting from loglinearized CPI Inflation (7.99)

$$
\begin{aligned}
\hat{\pi}_t^A &= v_P v_A\hat{\pi}_{A,t}^A+v_P v_B\hat{\pi}_{B,t}^A+v_C\hat{\pi}_{C,t}^A \\
\hat{\pi}_t^A &= v_P v_A\hat{\pi}_{A,t}^A+v_P v_B\hat{\pi}_{B,t}^A+v_C\hat{\pi}_{C,t}^A \\
\left(\hat{P}_t^A-\hat{P}_{t-1}^A\right) &= v_P v_A\left(\hat{P}_{A,t}^A-\hat{P}_{A,t-1}^A\right) \\
&+v_P v_B\left(\hat{P}_{B,t}^A-\hat{P}_{B,t-1}^A\right)+v_C\left(\hat{P}_{C,t}^A-\hat{P}_{C,t-1}^A\right)
\end{aligned}
$$

Add and subtract $\pm\left(v_P v_B + v_C\right)\left(\hat{P}^A_{A,t} - \hat{P}^A_{A,t-1}\right)$

$$
\begin{aligned}
\hat{\pi}^A_t &= v_P v_B\left(\hat{P}^A_{B,t} - \hat{P}^A_{B,t-1} - \left(\hat{P}^A_{A,t} - \hat{P}^A_{A,t-1}\right)\right) \\
&\quad + v_C\left(\hat{P}^A_{C,t} - \hat{P}^A_{C,t-1} - \left(\hat{P}^A_{A,t} - \hat{P}^A_{A,t-1}\right)\right) \\
&\quad + \underbrace{\left(v_P v_A + v_P v_B + v_C\right)}_{1}\underbrace{\left(\hat{P}^A_{A,t} - \hat{P}^A_{A,t-1}\right)}_{\hat{\pi}^A_{A,t}} \\
&= v_P v_B(\underbrace{\hat{P}^A_{B,t} - \hat{P}^A_{A,t}}_{\widehat{ToT}^A_{B,t}} - \underbrace{\hat{P}^A_{B,t-1} + \hat{P}^A_{A,t-1}}_{\widehat{ToT}^A_{B,t-1}}) \\
&\quad + v_C\underbrace{\left(\hat{P}^A_{C,t} - \hat{P}^A_{A,t} - \hat{P}^A_{C,t-1} + \hat{P}^A_{A,t-1}\right)}_{\Delta\widehat{ToT}^A_{C,t}} + \hat{\pi}^A_{A,t} \\
\hat{\pi}^A_t &= \hat{\pi}^A_{A,t} + v_P v_B \Delta\widehat{ToT}^A_{B,t} + v_C \Delta\widehat{ToT}^A_{C,t} \qquad (7.129)
\end{aligned}
$$

$$
\begin{aligned}
\hat{\pi}^B_t &= \hat{\pi}^B_{B,t} + v_P v_A \Delta\widehat{ToT}^B_{A,t} + v_C \Delta\widehat{ToT}^B_{C,t} \qquad (7.130) \\
\hat{\pi}^C_t &= \hat{\pi}^C_{C,t} + v_P v_A \Delta\widehat{ToT}^C_{A,t} + v_P v_B \Delta\widehat{ToT}^C_{B,t} \qquad (7.131)
\end{aligned}
$$

long version without differences and using all six terms of trade definitions

$$
\begin{aligned}
\hat{\pi}_t^A &= \hat{\pi}_{A,t}^A + \underbrace{\left[1-\eta\left(1-n_P\right)\right]\eta\left(1-n_A\right)}_{v_B^A}\widehat{ToT}_{B,t}^A \\
&\quad -\underbrace{\left[1-\eta\left(1-n_P\right)\right]\eta\left(1-n_A\right)}_{v_B^A}\widehat{ToT}_{B,t-1}^A \\
&\quad +\underbrace{\eta\left(1-n_P\right)}_{v_C^A}\widehat{ToT}_{C,t}^A - \underbrace{\eta\left(1-n_P\right)}_{v_C^A}\widehat{ToT}_{C,t-1}^A \qquad (7.132) \\
\hat{\pi}_t^B &= \hat{\pi}_{B,t}^B + \underbrace{\left[1-\eta\left(1-n_P\right)\right]\,\eta n_A}_{v_A^B}\widehat{ToT}_{A,t}^B \\
&\quad -\underbrace{\left[1-\eta\left(1-n_P\right)\right]\,\eta n_A}_{v_A^B}\widehat{ToT}_{A,t-1}^B \\
&\quad +\underbrace{\eta\left(1-n_P\right)}_{v_C^B}\widehat{ToT}_{C,t}^B - \underbrace{\eta\left(1-n_P\right)}_{v_C^B}\widehat{ToT}_{C,t-1}^B \qquad (7.133) \\
\hat{\pi}_t^C &= \hat{\pi}_{C,t}^C + \underbrace{\left[1-\eta\left(1-n_P\right)\right]\,\eta n_A}_{v_A^B}\widehat{ToT}_{A,t}^C \\
&\quad -\underbrace{\left[1-\eta\left(1-n_P\right)\right]\,\eta n_A}_{v_A^B}\widehat{ToT}_{A,t-1}^C \\
&\quad +\underbrace{\left[1-\eta\left(1-n_P\right)\right]\eta\left(1-n_A\right)}_{v_B^A}\widehat{ToT}_{B,t}^C \\
&\quad -\underbrace{\left[1-\eta\left(1-n_P\right)\right]\eta\left(1-n_A\right)}_{v_B^A}\widehat{ToT}_{B,t-1}^C \qquad (7.134)
\end{aligned}
$$

Euler Equation in Terms of Trade

Country A Starting from loglinearized Euler equation (7.121) and substituting with (7.132)

$$
\begin{aligned}
\hat{C}_t^A &= \hat{C}_{t+1}^A - \frac{1}{\mu}\left(\widehat{R}_t^A - \hat{\pi}_{t+1}^A\right) \\
\hat{C}_t^A &= \hat{C}_{t+1}^A - \frac{1}{\mu}\widehat{R}_t^A \\
&\quad + \frac{1}{\mu}\left(\hat{\pi}_{A,t+1}^A + \left[1-\eta\left(1-n_P\right)\right]\eta\left(1-n_A\right)\widehat{ToT}_{B,t+1}^A\right. \\
&\quad - \left[1-\eta\left(1-n_P\right)\right]\eta\left(1-n_A\right)\widehat{ToT}_{B,t}^A \\
&\quad \left. + \eta\left(1-n_P\right)\widehat{ToT}_{C,t+1}^A - \eta\left(1-n_P\right)\widehat{ToT}_{C,t}^A\right) \\
\hat{C}_t^A &= \hat{C}_{t+1}^A - \frac{1}{\mu}\widehat{R}_t^A + \frac{1}{\mu}\hat{\pi}_{A,t+1}^A \\
&\quad + \frac{\left[1-\eta\left(1-n_P\right)\right]\eta\left(1-n_A\right)}{\mu}\widehat{ToT}_{B,t+1}^A - \frac{\left[1-\eta\left(1-n_P\right)\right]\eta\left(1-n_A\right)}{\mu}\widehat{ToT}_{B,t}^A \\
&\quad + \frac{\eta\left(1-n_P\right)}{\mu}\widehat{ToT}_{C,t+1}^A - \frac{\eta\left(1-n_P\right)}{\mu}\widehat{ToT}_{C,t}^A
\end{aligned} \tag{7.135}
$$

Country B

$$
\begin{aligned}
\hat{C}_t^B &= \hat{C}_{t+1}^B - \frac{1}{\mu}\left(\widehat{R}_t^B - \hat{\pi}_{t+1}^B\right) \text{ substituting (7.133)} \\
\hat{C}_t^B &= \hat{C}_{t+1}^B - \frac{1}{\mu}\widehat{R}_t^B + \frac{1}{\mu}\hat{\pi}_{B,t+1}^B \\
&\quad + \frac{\left[1-\eta\left(1-n_P\right)\right]\eta n_A}{\mu}\widehat{ToT}_{A,t+1}^B - \frac{\left[1-\eta\left(1-n_P\right)\right]\eta n_A}{\mu}\widehat{ToT}_{A,t}^B \\
&\quad + \frac{\eta\left(1-n_P\right)}{\mu}\widehat{ToT}_{C,t+1}^B - \frac{\eta\left(1-n_P\right)}{\mu}\widehat{ToT}_{C,t}^B
\end{aligned} \tag{7.136}
$$

Country C

$$
\begin{aligned}
\hat{C}_t^C &= \hat{C}_{t+1}^C - \frac{1}{\mu}\left(\widehat{R}_t^C - \hat{\pi}_{t+1}^C\right) \text{ substituting (7.134)} \\
\hat{C}_t^C &= \hat{C}_{t+1}^C - \frac{1}{\mu}\widehat{R}_t^C + \frac{1}{\mu}\hat{\pi}_{C,t+1}^C \\
&\quad + \frac{\left[1-\eta\left(1-n_P\right)\right]\eta n_A}{\mu}\widehat{ToT}_{A,t+1}^C - \frac{\left[1-\eta\left(1-n_P\right)\right]\eta n_A}{\mu}\widehat{ToT}_{A,t}^C \\
&\quad + \frac{\left[1-\eta\left(1-n_P\right)\right]\eta\left(1-n_A\right)}{\mu}\widehat{ToT}_{B,t+1}^C \\
&\quad - \frac{\left[1-\eta\left(1-n_P\right)\right]\eta\left(1-n_A\right)}{\mu}\widehat{ToT}_{B,t}^C
\end{aligned} \tag{7.137}
$$

Equations (7.135), (7.136) and (7.137) are equivalent to Monacelli (2001: 393, eq. 22 and eq. 25).

7.2 Firms

The representative firm, z, in country $i, i = A, B, C$ produces

$$Y_t^i(z) = F_t^i \, L_t^i(z). \tag{7.138}$$

7.2.1 Variables

$Y_t^i(z)$ is output of firm z in period t

$L_t^i(z)$ are labor hours employed by firm z

F_t^i is the country specific productivity and follows a first order autoregressive process

$$F_t^i = \rho^F \ln F_{t-1}^i + \epsilon_t^i. \tag{7.139}$$

W_t^i is the wage rate

$MC_t^i(z)$ are marginal cost given by the Lagrange multiplier

$$MC_t^i(z) \equiv \lambda_t^i(z). \tag{7.140}$$

7.2.2 Firm's Cost Minimization

General approach for minimization of $f(x)$ subject to the constraint $h(x) \geq 0$, i. e., a market clearing constraint where supply $\geq$ demand

$$\mathcal{L} = f(x) - \lambda \cdot h(x). \tag{7.141}$$

$$\begin{aligned} \min_{L_h^i(z)} \quad & W_t^i \, L_t^i(z) \\ \text{s.t.} \quad & F_t^i \, L_t^i(z) \geq Y_t^{i,d}(z) \end{aligned}$$

First Order Condition

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial L_t^i(z)} &= W_t^i - \lambda_t^i(z)\,[F_t^i] = 0 \\ &\Leftrightarrow \quad \frac{W_t^i}{F_t^i} = \lambda_t^i(z) \equiv MC_t^i(z) \end{aligned} \tag{7.142}$$

Equation (7.142) defines the nominal marginal cost $MC_t^i(z)$.

Loglinearized Production Function

$$\hat{Y}_t^i(z) = \hat{F}_t^i \; + \; \hat{L}_t^i(z) \tag{7.143}$$

7.2.3 Real Marginal Cost

Take definition of (nominal) marginal cost (7.142)

$$W_t^i = MC_t^i(z) \cdot F_t^i.$$

Substituting W_t^i from (7.40) and solving for $MC_t^i(z)$

$$MC_t^i(z)\, F_t^i = \frac{-\partial U_t^i/\partial L_t^i}{\partial U_t^i/\partial C_t^i} \cdot P_t^i \tag{7.144}$$

$$MC_t^i(z) = \frac{(L_t^i)^{\gamma}}{(C_t^i)^{-\mu}} \frac{1}{F_t^i} P_t^i. \tag{7.145}$$

Dividing by the domestic price index $P_{i,t}^i$ to get real marginal cost[8]

$$mc_t^i(z) \equiv \frac{MC_t^i(z)}{P_{i,t}^i} \tag{7.146}$$

$$mc_t = \frac{-\partial U_t^i/\partial L_t^i}{\partial U_t^i/\partial C_t^i} \frac{P_t^i}{P_{i,t}^i} \frac{1}{F_t^i} = \frac{(L_t^i)^{\gamma}}{(C_t^i)^{-\mu}} \underbrace{\frac{P_t^i}{P_{i,t}^i}}_{g_{i,t}(ToT)} \frac{1}{F_t} = \left(C_t^i\right)^{\mu} \left(L_t^i\right)^{\gamma} F_t^{i\,-1} g_{i,t}\,(ToT)\,. \tag{7.147}$$

Equation (17a) in Monacelli (2001: 392).

$$mc_t = \left(C_t^i\right)^{\mu} \frac{(L_t^i)^{\gamma}}{F_t^i}\, g_i\,(ToT)\, \frac{(F_t^i)^{\gamma}}{(F_t^i)^{\gamma}}$$

[8]Discounting with domestic prices in country i, $P_{i,t}^i$, in contrast to country i's CPI, P_t^i, as the firm's production costs depend only on domestic prices (Galí and Monacelli 2005: 715).

$$mc_t = \left(C_t^i\right)^{\mu} \underbrace{\left(L_t^i\, F_t^i\right)^{\gamma}}_{\left(Y_t^i\right)^{\gamma}} \frac{1}{F_t^i\ \left(F_t^i\right)^{\gamma}}\ g_i\,(ToT)$$

$$mc_t = \left(C_t^i\right)^{\mu}\ \left(Y_t^i\right)^{\gamma}\ \left(F_t^i\right)^{-(1+\gamma)}\ g_i\,(ToT)$$

Loglinearized Real Marginal Cost

$$\widehat{mc}_t = \mu\,\hat{C}_t^i\ +\ \gamma\,\hat{L}_t^i\ -\ \hat{F}_t^i\ +\ \hat{g}_i\,(ToT) \tag{7.148}$$

and adding and subtracting $\pm\gamma\,\hat{F}_t^i$

$$\widehat{mc}_t = \mu\,\hat{C}_t^i\ +\ \gamma\,\hat{L}_t^i\ +\ \gamma\,\hat{F}_t^i\ -\ \hat{F}_t^i - \gamma\,\hat{F}_t + \hat{g}_i\,(ToT) \tag{7.149}$$

substituting the loglinearized production function (7.143) $\hat{Y}_t^i = \hat{F}_t^i\ +\ \hat{L}_t^i$

$$\widehat{mc}_t = \mu\,\hat{C}_t^i\ +\ \gamma\,\hat{Y}_t^i\ -\ (1+\gamma)\,\hat{F}_t^i + \hat{g}_i\,(ToT) \tag{7.150}$$

$$\widehat{mc}_t^A = \mu\,\hat{C}_t^A\ +\ \gamma\,\hat{Y}_t^A\ -\ (1+\gamma)\,\hat{F}_t^A + \hat{g}_A\,(ToT) \tag{7.151}$$

$$\widehat{mc}_t^B = \mu\,\hat{C}_t^B\ +\ \gamma\,\hat{Y}_t^B\ -\ (1+\gamma)\,\hat{F}_t^B + \hat{g}_B\,(ToT) \tag{7.152}$$

$$\widehat{mc}_t^C = \mu\,\hat{C}_t^C\ +\ \gamma\,\hat{Y}_t^C\ -\ (1+\gamma)\,\hat{F}_t^C + \hat{g}_C\,(ToT)\,. \tag{7.153}$$

7.2.4 Optimal Price Setting

The optimal price setting stems from maximizing profits. General formula for (nominal) profits, NP, is revenue, RV, minus total cost, TC,

$$NP(z) = RV(z) - TC(z). \tag{7.154}$$

Here, profits depend on good z. We are actually interested in the optimal price, P, that maximizes profits from selling good z

$$NP_t(P) = P \cdot Y_t(z) - MC_t(z) \cdot L_t(z). \tag{7.155}$$

In order to get the optimal price, we maximize the value of firm z, i. e., we maximize the expected present discounted stream of profits, $V(z)$. As profits are the difference between revenue and total cost, we first derive these two expressions and then maximize the firm's value by choosing the optimal price, P. We look at country A. Price setting in countries B and C follows the same optimization.

Revenue The following equations give the revenue for firm z in country A, indicated by the subscript. The superscript of the prices refers to the currency in which the prices are denominated. Thus, the firm in country A has to transfer its revenue from exporting to countries B and C back to its home currency by multiplying with the respective exchange rate, $S^A_{B,t}$ and $S^A_{C,t}$.

$$\begin{aligned} RV^A_t(z) &= n_P n_A RV^A_{A,t} + n_p\,(1-n_A)\,RV^B_{A,t} + (1-n_P)\,RV^C_{A,t} \\ &= n_P n_A P^A_{A,t}(z)\; Y^A_{A,t}(z) + n_p\,(1-n_A)\,S^A_{B,t}\,P^B_{A,t}(z)\,Y^B_{A,t}(z) \\ &\quad + (1-n_P)\,S^A_{C,t}\,P^C_{A,t}(z)\;Y^C_{A,t}(z) \end{aligned} \tag{7.156}$$

The general formula for the revenue in country i's firm z is

$$RV^i(z) = P^i_{i,t}(z)\,n_i\,Y^i_{i,t}(z) + \sum_{i'} S^i_{i',t}\,P^{i'}_{i,t}(z)\,n_{i'}\,Y^{i'}_{i,t}(z). \tag{7.157}$$

Total Cost Total cost of production for country A's firm z

$$TC^A_t(z) = W^A_t\,L^A_t(z) \tag{7.158}$$

is the nominal wage times labor hours. Rewriting the production function

$$\begin{aligned} & Y^A_t(z) = F^A_t\,L^A_t(z) \\ \Leftrightarrow\quad & L^A_t(z) = \frac{Y^A_t(z)}{F^A_t}, \end{aligned}$$

where $Y^A_t(z)$ is output of country A's firm z in period t and it allows us to substitute for labor hours employed by firm z, $L^A_t(z)$

$$TC^A_t(z) = W^A_t \cdot \frac{Y^A_t(z)}{F^A_t} = \frac{W^A_t}{F^A_t}\;Y^A_t(z). \tag{7.159}$$

Use FOC from cost minimization[9] (7.142)

$$MC^A_t(z) = \frac{W^A_t}{F^A_t}$$

[9]Marginal cost, $MC^i_t(z)$, depend on z as the Lagrange multiplier, λ^i_t, in the firm's cost minimization depends on z.

to get total cost

$$\begin{aligned} TC_t^A(z) &= MC_t^A(z)\, Y_t^A(z) \\ &= MC_t^A(z)\, \left(n_P n_A\, Y_{A,t}^A(z) + n_p\,(1-n_A)\, Y_{A,t}^B(z)\right. \\ &\quad \left. + (1-n_P)\, Y_{A,t}^C(z)\right), \end{aligned} \tag{7.160}$$

where total demand of country A's firm z, $Y_t^A(z)$, is given by the sum of total demand for firm z's products in the three countries

$$Y_t^A(z) = n_P n_A\, Y_{A,t}^B(z) + n_p\,(1-n_A)\, Y_{A,t}^C(z) + (1-n_P)\, Y_{A,t}^A(z). \tag{7.161}$$

We can substitute the total demand from the individual demand function (7.52) as for individual goods z the market clearing condition

$$C_t(z) = Y_t(z) \tag{7.162}$$

holds.

$$Y_{A,t}^A(z) = \frac{1}{n_P n_A}\left(\frac{P_{A,t}^A(z)}{P_{A,t}^A}\right)^{-\phi} Y_{A,t}^A \tag{7.163}$$

$$Y_{A,t}^B(z) = \frac{1}{n_P n_A}\left(\frac{P_{A,t}^B(z)}{P_{A,t}^B}\right)^{-\phi} Y_{A,t}^B \tag{7.164}$$

$$Y_{A,t}^C(z) = \frac{1}{n_P n_A}\left(\frac{P_{A,t}^C(z)}{P_{A,t}^C}\right)^{-\phi} Y_{A,t}^C \tag{7.165}$$

Now, good z's demand does not depend anymore on the quantities of good z but only on aggregate variables such as price level and aggregate output, and the

price of good z, i. e., the variable we are looking for. Total costs are

$$\begin{aligned} TC_t^A(z) &= MC_t^A(z) n_P n_A \overbrace{\frac{1}{n_P n_A}\left(\frac{P_{A,t}^A(z)}{P_{A,t}^A}\right)^{-\phi} Y_{A,t}^A}^{Y_{A,t}^A(z)} \\ &\quad + MC_t^A(z)\, n_p\,(1-n_A) \overbrace{\frac{1}{n_P n_A}\left(\frac{P_{A,t}^B(z)}{P_{A,t}^B}\right)^{-\phi} Y_{A,t}^B}^{Y_{A,t}^B(z)} \\ &\quad + MC_t^A(z)\ (1-n_P) \overbrace{\frac{1}{n_P n_A}\left(\frac{P_{A,t}^C(z)}{P_{A,t}^C}\right)^{-\phi} Y_{A,t}^C}^{Y_{A,t}^C(z)} . \end{aligned} \tag{7.166}$$

The general formula for the total cost of country i's firm z is

$$TC_t^i(z) = MC_t^i(z) \sum_{i'=A,B,C} n_{i'} \frac{1}{n_i}\left(\frac{P_{i,t}^{i'}(z)}{P_{i,t}^{i'}}\right)^{-\phi} Y_{i,t}^{i'} . \tag{7.167}$$

Flexible Prices

Analog to the lecture notes by Monacelli (2007), we derive the optimal price in a framework when all prices are flexible. Here, the firm's optimization problem is static.

Firm's Profit Maximization under Flexible Prices Following Monacelli's lecture notes[10] the firm (in country A) maximizes real profits, RP, i. e., revenue and total cost are discounted with the CPI price index, P_t^A,

$$RP_t(z) = \frac{P_t^A(z)}{P_t^A} Y_t^A(z) - \frac{W_t^A}{P_t^A} L_t^A(z). \tag{7.168}$$

Profits are maximized under two constraints: (i) the production function (7.138), and (ii) the demand functions for individual goods under individual good's market

[10] There is no difference whether nominal or real profits are maximized as the CPI price index drops out in the first order condition.

clearing (7.162). Both constraints are simply substituted into the profits equation. First, rearranging the production function to $L_t^A(z) = \frac{Y_t^A(z)}{F_t^A}$ and substituting

$$W_t^A L_t^A(z) = \frac{W_t^A}{F_t^A} Y_t^A(z) = MC_t^A(z)\, Y_t^A(z) \tag{7.169}$$

leads to

$$\begin{aligned} RP_t(z) \;=\; & \frac{1}{P_t^A}\left[P_{A,t}^A(z)\, n_P n_A\, Y_{A,t}^A(z) + S_{B,t}^A\, P_{A,t}^B(z)\, n_p\,(1-n_A)\; Y_{A,t}^B(z)\right. \\ & + S_{C,t}^A\, P_{A,t}^C(z)\;(1-n_P)\; Y_{A,t}^C(z) \\ & -MC_t^A(z)\;\left(n_P n_A\, Y_{A,t}^A(z)\right. \\ & \left.\left. +n_p\,(1-n_A)\; Y_{A,t}^B(z) + \;(1-n_P)\; Y_{A,t}^C(z)\right)\right]. \end{aligned} \tag{7.170}$$

Second, plugging in the demand functions (7.163) to (7.165) leads to

$$\begin{aligned} RP_t^A \;=\; & \frac{1}{P_t^A}\left\{ P_{A,t}^A(z) n_P n_A \overbrace{\frac{1}{n_P n_A}\left(\frac{P_{A,t}^A(z)}{P_{A,t}^A}\right)^{-\phi}}^{Y_{A,t}^A(z)} Y_{A,t}^A \right. \\ & + S_{B,t}^A\, P_{A,t}^B(z) n_p\,(1-n_A) \overbrace{\frac{1}{n_P n_A}\left(\frac{P_{A,t}^B(z)}{P_{A,t}^B}\right)^{-\phi}}^{Y_{A,t}^B(z)} Y_{A,t}^B \\ & + \;S_{C,t}^A\, P_{A,t}^C(z)\,(1-n_P) \overbrace{\frac{1}{n_P n_A}\left(\frac{P_{A,t}^C(z)}{P_{A,t}^C}\right)^{-\phi}}^{Y_{A,t}^C(z)} Y_{A,t}^C \\ & - \;MC_t^A(z)\left[n_P n_A \overbrace{\frac{1}{n_P n_A}\left(\frac{P_{A,t}^A(z)}{P_{A,t}^A}\right)^{-\phi}}^{Y_{A,t}^A(z)} Y_{A,t}^A \right. \\ & + \;n_p\,(1-n_A) \overbrace{\frac{1}{n_P n_A}\left(\frac{P_{A,t}^B(z)}{P_{A,t}^B}\right)^{-\phi}}^{Y_{A,t}^B(z)} Y_{A,t}^B \\ & \left.\left. + \;(1-n_P) \overbrace{\frac{1}{n_P n_A}\left(\frac{P_{A,t}^C(z)}{P_{A,t}^C}\right)^{-\phi}}^{Y_{A,t}^C(z)} Y_{A,t}^C \right] \right\}. \end{aligned} \tag{7.171}$$

Derived from the first order conditions, the three prices $P^A_{A,t}$, $P^B_{A,t}$ and $P^C_{A,t}$ are

$$\begin{aligned}
\frac{\partial\, RP_t(z)}{\partial P^A_{A,t}(z)} &= \frac{1}{P^A_t}\left[(1-\phi)(P^A_{A,t}(z))^{-\phi}(P^A_{A,t})^{\phi}Y^A_{A,t}\right. \\
&\quad \left. - MC^A_t(z)\, Y^A_{A,t}(-\phi)(P^A_{A,t}(z))^{-\phi-1}(P^A_{A,t})^{\phi}\right] = 0 \\
&\Leftrightarrow \frac{1}{P^A_t}Y^A_{A,t}\,(P^A_{A,t})^{\phi}\,\left[(1-\phi)(P^A_{A,t}(z))^{-\phi}\right. \\
&\quad \left. -(-\phi)\, MC^A_t(z)\,(P^A_{A,t}(z))^{-\phi-1}\right] = 0 \\
&\qquad\qquad \Big| \div \left[\frac{1}{P^A_t}Y^A_{A,t}\,(P^A_{A,t})^{\phi}\right] \\
&\Leftrightarrow (1-\phi)(P^A_{A,t}(z))^{-\phi} + \phi\, MC^A_t(z)\,(P^A_{A,t}(z))^{-\phi-1} = 0 \\
&\mid \div (P^A_{A,t}(z))^{-\phi} \\
&\Leftrightarrow (1-\phi) + \phi\, MC^A_t(z)\,(P^A_{A,t}(z))^{-1} = 0 \\
&\mid \cdot P^A_{A,t}(z) \quad \mid -\phi\, MC^A_t(z) \\
&\Leftrightarrow (1-\phi)P^A_{A,t}(z) = -\phi\, MC^A_t(z) \\
&\Leftrightarrow P^A_{A,t}(z) = -\frac{\phi}{1-\phi}\, MC^A_t(z) = \underbrace{\frac{\phi}{\phi-1}}_{\text{mark-up}}\, MC^A_t(z) \qquad (7.172)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial\, RP_t(z)}{\partial P^B_{A,t}(z)} &= \frac{1}{P^A_t}\left[(1-\phi)S^A_{B,t}(P^B_{A,t}(z))^{-\phi}(P^B_{A,t})^{\phi}\frac{n_p\,(1-n_A)}{n_P n_A}Y^B_{A,t}\right. \\
&\quad \left. - MC^A_t(z)\,\frac{n_p\,(1-n_A)}{n_P n_A}Y^B_{A,t}(-\phi)(P^B_{A,t}(z))^{-\phi-1}(P^B_{A,t})^{\phi}\right] = 0 \\
&\Leftrightarrow \frac{1}{P^A_t}\frac{n_p\,(1-n_A)}{n_P n_A}Y^B_{A,t}\,(P^B_{A,t})^{\phi}\,\left[(1-\phi)S^A_{B,t}(P^B_{A,t}(z))^{-\phi}\right. \\
&\quad \left. -(-\phi)\, MC^A_t(z)(P^B_{A,t}(z))^{-\phi-1}\right] = 0 \\
&\qquad\qquad \Big| \div \left[\frac{1}{P^A_t}\frac{n_p\,(1-n_A)}{n_P n_A}Y^B_{A,t}\,(P^B_{A,t})^{\phi}\right] \\
&\Leftrightarrow (1-\phi)S^A_{B,t}(P^B_{A,t}(z))^{-\phi} + \phi\, MC^A_t(z)\,(P^B_{A,t}(z))^{-\phi-1} = 0 \\
&\mid \div (P^B_{A,t}(z))^{-\phi} \\
&\Leftrightarrow (1-\phi)S^A_{B,t} + \phi\, MC^A_t(z)\,(P^B_{A,t}(z))^{-1} = 0 \\
&\mid \cdot P^B_{A,t}(z) \quad \mid -\phi\, MC^A_t(z)
\end{aligned}$$

$$\begin{aligned} \Leftrightarrow \quad & (1-\phi) S^A_{B,t} P^B_{A,t}(z) = -\phi\, MC^A_t(z) \qquad | \div S^A_{B,t} \\ \Leftrightarrow \quad & P^B_{A,t}(z) = -\frac{\phi}{1-\phi}\frac{1}{S^A_{B,t}} MC^A_t(z) = \underbrace{\frac{\phi}{\phi-1}}_{\text{mark-up}} \frac{1}{S^A_{B,t}}\, MC^A_t(z) \end{aligned} \tag{7.173}$$

$$P^C_{A,t}(z) = \underbrace{\frac{\phi}{\phi-1}}_{\text{mark-up}} \frac{1}{S^A_{C,t}}\, MC^A_t(z) \tag{7.174}$$

Summing up, under flexible prices we find that prices are set in the producer's currency and then transferred into the buyer's currency. The producer sets the price above marginal cost as the mark-up is larger than one, when $\phi \geq 1$. The mark-up stems from monopolistic competition in goods markets. Rewriting the mark-up $\frac{\phi}{\phi-1}$ as $\frac{1}{1-\frac{1}{\phi}}$ allows us to see easily that in a perfectly competitive goods market when $\phi \to \infty$ the mark-up goes to one and prices equal marginal cost.

Sticky Prices under Producer Currency Pricing

Calvo pricing with probability of changing the price $(1-\alpha)$.
Analog to Monacelli (2001) and also his lecture notes we derive the optimal price in a New Keynesian framework when exporters' prices are set in the producer currency, i. e., there is full exchange rate pass-through to the buyer. Thus, firm z sets only the price, $\widetilde{P}^A_{A,t}$. Prices in the buyer's currency in period $t+\tau$ are

$$\widetilde{P}^A_{A,t}(z) = \widetilde{P}^A_{A,t}(z) \tag{7.175}$$

$$\widetilde{P}^B_{A,t+\tau}(z) = S^B_{A,t+\tau} \cdot \widetilde{P}^A_{A,t}(z) \tag{7.176}$$

$$\widetilde{P}^C_{A,t+\tau}(z) = S^C_{A,t+\tau} \cdot \widetilde{P}^A_{A,t}(z). \tag{7.177}$$

The optimal price maximizes the expected present discounted stream of profits, i. e., the value of firm z

$$V_t(z) = \sum_{\tau=0}^{\infty} \alpha^\tau\, E_t\, \rho_{t,t+\tau} \left[P_t(z) \cdot Y_{t+\tau}(z) - MC_{t+\tau}(z) \cdot Y_{t+\tau}(z) \right]$$

$$
\begin{aligned}
V_t(z) &= \sum_{\tau=0}^{\infty} \alpha^{\tau} E_t \, \rho^A_{t,t+\tau} \Big[\widetilde{P}^A_{A,t}(z) \, n_P n_A Y^A_{A,t+\tau}(z) \\
&\quad + S^A_{B,t+\tau} \, \widetilde{P}^B_{A,t+\tau}(z) n_p \left(1 - n_A\right) Y^B_{A,t+\tau}(z) \\
&\quad + S^A_{C,t+\tau} \, \widetilde{P}^C_{A,t+\tau}(z) \, \left(1 - n_P\right) Y^C_{A,t+\tau}(z) \\
&\quad - MC^A_{t+\tau}(z) \, \big(n_P n_A Y^A_{A,t+\tau}(z) \\
&\quad + n_p \left(1 - n_A\right) Y^B_{A,t+\tau}(z) + \left(1 - n_P\right) Y^C_{A,t+\tau}(z) \big) \Big] . \qquad (7.178)
\end{aligned}
$$

Using the pricing rules (7.175) to (7.177) to maximize over the optimal price $\widetilde{P}^A_{A,t}$

$$
\begin{aligned}
V_t(z) &= \sum_{\tau=0}^{\infty} \alpha^{\tau} E_t \, \rho^A_{t,t+\tau} \Big[\Big(\widetilde{P}^A_{A,t}(z) - MC^A_{t+\tau}(z) \Big) \, n_P n_A Y^A_{A,t+\tau}(z) \\
&\quad + [\underbrace{S^A_{B,t+\tau} \, S^B_{A,t+\tau}}_{1} \, \widetilde{P}^A_{A,t}(z) \, - MC^A_{t+\tau}(z)] n_p \left(1 - n_A\right) \, Y^B_{A,t+\tau}(z) \\
&\quad + [\underbrace{S^A_{C,t+\tau} \, S^C_{A,t+\tau}}_{1} \, \widetilde{P}^A_{A,t}(z) \, - MC^A_{t+\tau}(z)] \, \left(1 - n_P\right) Y^C_{A,t+\tau}(z) \Big] \qquad (7.179)
\end{aligned}
$$

Plug demand functions (7.163) to (7.165) under the given pricing rules (7.175) to (7.177)

$$
Y^A_{A,t+\tau}(z) = \frac{1}{n_P n_A} \left(\frac{\widetilde{P}^A_{A,t}(z)}{P^A_{A,t+\tau}} \right)^{-\phi} Y^A_{A,t+\tau} \qquad (7.180)
$$

$$
Y^B_{A,t+\tau}(z) = \frac{1}{n_P n_A} \left(\frac{S^B_{A,t+\tau} \, \widetilde{P}^A_{A,t}(z)}{P^B_{A,t+\tau}} \right)^{-\phi} Y^B_{A,t+\tau} \qquad (7.181)
$$

$$
Y^C_{A,t+\tau}(z) = \frac{1}{n_p n_A} \left(\frac{S^C_{A,t+\tau} \widetilde{P}^A_{A,t}\left(z\right)}{P^C_{A,t+\tau}} \right)^{-\phi} Y^C_{A,t+\tau} \qquad (7.182)
$$

into the firm's value

$$
\begin{aligned}
V_t(z) &= \sum_{\tau=0}^{\infty} \alpha^{\tau} E_t \, \rho^A_{t,t+\tau} \Bigg[\Big(\widetilde{P}^A_{A,t}(z) - MC^A_{t+\tau}(z) \Big) \, \frac{n_P n_A}{n_P n_A} \left(\frac{\widetilde{P}^A_{A,t}(z)}{P^A_{A,t+\tau}} \right)^{-\phi} Y^A_{A,t+\tau} \\
&\quad + \Big(\widetilde{P}^A_{A,t}(z) \, - MC^A_{t+\tau}(z) \Big) \, \frac{n_p \left(1 - n_A\right)}{n_P n_A} \left(\frac{S^B_{A,t+\tau} \, \widetilde{P}^A_{A,t}(z)}{P^B_{A,t+\tau}} \right)^{-\phi} Y^B_{A,t+\tau} \\
&\quad + \Big(\widetilde{P}^A_{A,t}(z) \, - MC^A_{t+\tau}(z) \Big) \, \frac{\left(1 - n_P\right)}{n_P n_A} \left(\frac{S^C_{A,t+\tau} \, \widetilde{P}^A_{A,t}(z)}{P^C_{A,t+\tau}} \right)^{-\phi} Y^C_{A,t+\tau} \Bigg] \qquad (7.183)
\end{aligned}
$$

and multiply out to get an expression, which might be derived more easily.

$$
\begin{aligned}
V_t(z) \;=\; \sum_{\tau=0}^{\infty} \alpha^{\tau}\, E_t\, \rho^{A}_{t,t+\tau} \Bigg[& \widetilde{P}^{A}_{A,t}(z)\, \frac{n_P n_A}{n_P n_A} \left(\frac{\widetilde{P}^{A}_{A,t}(z)}{P^{A}_{A,t+\tau}} \right)^{-\phi} Y^{A}_{A,t+\tau} \\
& - MC^{A}_{t+\tau}(z)\, \frac{n_P n_A}{n_P n_A} \left(\frac{\widetilde{P}^{A}_{A,t}(z)}{P^{A}_{A,t+\tau}} \right)^{-\phi} Y^{A}_{A,t+\tau} \\
& + \widetilde{P}^{A}_{A,t}(z) \frac{n_p\,(1-n_A)}{n_P n_A} \left(\frac{S^{B}_{A,t+\tau}\, \widetilde{P}^{A}_{A,t}(z)}{P^{B}_{A,t+\tau}} \right)^{-\phi} Y^{B}_{A,t+\tau} \\
& - MC^{A}_{t+\tau}(z) \frac{n_p\,(1-n_A)}{n_P n_A} \left(\frac{S^{B}_{A,t+\tau}\, \widetilde{P}^{A}_{A,t}(z)}{P^{B}_{A,t+\tau}} \right)^{-\phi} Y^{B}_{A,t+\tau} \\
& + \widetilde{P}^{A}_{A,t}(z)\, \frac{(1-n_P)}{n_P n_A} \left(\frac{S^{C}_{A,t+\tau}\, \widetilde{P}^{A}_{A,t}(z)}{P^{C}_{A,t+\tau}} \right)^{-\phi} Y^{C}_{A,t+\tau} \\
& - MC^{A}_{t+\tau}(z) \frac{(1-n_P)}{n_P n_A} \left(\frac{S^{C}_{A,t+\tau}\, \widetilde{P}^{A}_{A,t}(z)}{P^{C}_{A,t+\tau}} \right)^{-\phi} Y^{C}_{A,t+\tau} \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
V_t(z) \;=\; \sum_{\tau=0}^{\infty} \alpha^{\tau}\, E_t\, \rho^{A}_{t,t+\tau} \Bigg[& \widetilde{P}^{A}_{A,t}(z)\, \widetilde{P}^{A}_{A,t}(z)^{-\phi} \left(P^{A}_{A,t+\tau} \right)^{\phi} \frac{n_P n_A}{n_P n_A} Y^{A}_{A,t+\tau} \\
& - MC^{A}_{t+\tau}(z) \widetilde{P}^{A}_{A,t}(z)^{-\phi} \left(P^{A}_{A,t+\tau} \right)^{\phi} \frac{n_P n_A}{n_P n_A} Y^{A}_{A,t+\tau} \\
& + \widetilde{P}^{A}_{A,t}(z) \frac{n_p\,(1-n_A)}{n_P n_A} \left(S^{B}_{A,t+\tau} \right)^{-\phi} \widetilde{P}^{A}_{A,t}(z)^{-\phi} \left[P^{B}_{A,t+\tau} \right]^{\phi} Y^{B}_{A,t+\tau} \\
& - MC^{A}_{t+\tau}(z) \frac{n_p\,(1-n_A)}{n_P n_A} \left(S^{B}_{A,t+\tau} \right)^{-\phi} \widetilde{P}^{A}_{A,t}(z)^{-\phi} \left[P^{B}_{A,t+\tau} \right]^{\phi} Y^{B}_{A,t+\tau} \\
& + \widetilde{P}^{A}_{A,t}(z)\, \frac{(1-n_P)}{n_P n_A} \left(S^{C}_{A,t+\tau} \right)^{-\phi} \widetilde{P}^{A}_{A,t}(z)^{-\phi} \left(P^{C}_{A,t+\tau} \right)^{\phi} Y^{C}_{A,t+\tau} \\
& - MC^{A}_{t+\tau}(z) \frac{(1-n_P)}{n_P n_A} \left(S^{C}_{A,t+\tau} \right)^{-\phi} \widetilde{P}^{A}_{A,t}(z)^{-\phi} \left(P^{C}_{A,t+\tau} \right)^{\phi} Y^{C}_{A,t+\tau} \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
V_t(z) \;=\; & \sum_{\tau=0}^{\infty} \alpha^{\tau}\, E_t\, \rho^A_{t,t+\tau} \Bigg[\widetilde{P}^A_{A,t}(z)^{1-\phi} \left(P^A_{A,t+\tau}\right)^{\phi} \; \frac{n_P n_A}{n_P n_A} Y^A_{A,t+\tau} \\
& - \widetilde{P}^A_{A,t}(z)^{-\phi} \left(P^A_{A,t+\tau}\right)^{\phi} \; \frac{n_P n_A}{n_P n_A} Y^A_{A,t+\tau} MC^A_{t+\tau}(z) \\
& + \widetilde{P}^A_{A,t}(z)^{1-\phi} \frac{n_p\,(1-n_A)}{n_P n_A} \left(S^B_{A,t+\tau}\right)^{-\phi} \left[P^B_{A,t+\tau}\right]^{\phi} Y^B_{A,t+\tau} \\
& - \widetilde{P}^A_{A,t}(z)^{-\phi} \frac{n_p\,(1-n_A)}{n_P n_A} \left(S^B_{A,t+\tau}\right)^{-\phi} \left[P^B_{A,t+\tau}\right]^{\phi} Y^B_{A,t+\tau} MC^A_{t+\tau}(z) \\
& + \widetilde{P}^A_{A,t}(z)^{1-\phi} \frac{(1-n_P)}{n_P n_A} \left(S^C_{A,t+\tau}\right)^{-\phi} \left(P^C_{A,t+\tau}\right)^{\phi} Y^C_{A,t+\tau} \\
& - \widetilde{P}^A_{A,t}(z)^{-\phi} \frac{(1-n_P)}{n_P n_A} \left(S^C_{A,t+\tau}\right)^{-\phi} \left(P^C_{A,t+\tau}\right)^{\phi} Y^C_{A,t+\tau} MC^A_{t+\tau}(z) \Bigg]
\end{aligned}
$$

First Order Condition

$$
\begin{aligned}
\frac{\partial V_t(z)}{\partial \widetilde{P}^A_{A,t}(z)} \;=\; & E_t \sum_{\tau=0}^{\infty} \alpha^{\tau} \rho^A_{t,t+\tau} \Bigg[(1-\phi)\, \widetilde{P}^A_{A,t}(z)^{-\phi}\, {P^A_{A,t+\tau}}^{\phi}\, \frac{n_P n_A}{n_P n_A} Y^A_{A,t+\tau} \\
& - (-\phi)\, \widetilde{P}^A_{A,t}(z)^{-\phi-1} \left(P^A_{A,t+\tau}\right)^{\phi} \frac{n_P n_A}{n_P n_A} Y^A_{A,t+\tau}\, MC^A_{t+\tau}(z) \\
& + (1-\phi)\, \widetilde{P}^A_{A,t}(z)^{-\phi} S^B_{A,t+\tau}(z)^{-\phi} {P^B_{A,t+\tau}}^{\phi} \frac{n_p\,(1-n_A)}{n_P n_A} Y^B_{A,t+\tau} \\
& - (-\phi)\, \widetilde{P}^A_{A,t}(z)^{-\phi-1} S^B_{A,t+\tau}(z)^{-\phi} \left(P^B_{A,t+\tau}\right)^{\phi} \frac{n_p\,(1-n_A)}{n_P n_A} Y^B_{A,t+\tau}\, MC^A_{t+\tau}(z) \\
& + (1-\phi)\, \widetilde{P}^A_{A,t}(z)^{-\phi} S^C_{A,t+\tau}(z)^{-\phi} {P^C_{A,t+\tau}}^{\phi} \frac{(1-n_P)}{n_P n_A} Y^C_{A,t+\tau} \\
& - (-\phi)\, \widetilde{P}^A_{A,t}(z)^{-\phi-1}\; S^C_{A,t+\tau}(z)^{-\phi} \left(P^C_{A,t+\tau}\right)^{\phi} \\
& \cdot \frac{(1-n_P)}{n_P n_A} Y^C_{A,t+\tau}\, MC^A_{t+\tau}(z) \Bigg] = 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial V_t(z)}{\partial \widetilde{P}^A_{A,t}(z)} \;=\; & E_t \sum_{\tau=0}^{\infty} \alpha^\tau \rho^A_{t,t+\tau} \Big[(1-\phi)\, \widetilde{P}^A_{A,t}(z)^{-\phi}\, {P^A_{A,t+\tau}}^{\phi}\, \frac{n_P n_A}{n_P n_A} Y^A_{A,t+\tau} \\
& + \phi\, \widetilde{P}^A_{A,t}(z)^{-\phi-1} \left(P^A_{A,t+\tau}\right)^{\phi} \frac{n_P n_A}{n_P n_A} Y^A_{A,t+\tau}\, MC^A_{t+\tau}(z) \\
& + (1-\phi)\, \widetilde{P}^A_{A,t}(z)^{-\phi} S^B_{A,t+\tau}(z)^{-\phi} {P^B_{A,t+\tau}}^{\phi} \frac{n_p\,(1-n_A)}{n_P n_A} Y^B_{A,t+\tau} \\
& + \phi\, \widetilde{P}^A_{A,t}(z)^{-\phi-1} S^B_{A,t+\tau}(z)^{-\phi} \left(P^B_{A,t+\tau}\right)^{\phi} \frac{n_p\,(1-n_A)}{n_P n_A} Y^B_{A,t+\tau}\, MC^A_{t+\tau}(z) \\
& + (1-\phi)\, \widetilde{P}^A_{A,t}(z)^{-\phi} S^C_{A,t+\tau}(z)^{-\phi} {P^C_{A,t+\tau}}^{\phi} \frac{(1-n_P)}{n_P n_A} Y^C_{A,t+\tau} \\
& + \phi\, \widetilde{P}^A_{A,t}(z)^{-\phi-1} S^C_{A,t+\tau}(z)^{-\phi} \left(P^C_{A,t+\tau}\right)^{\phi} \\
& \cdot \frac{(1-n_P)}{n_P n_A} Y^C_{A,t+\tau}\, MC^A_{t+\tau}(z) \Big] = 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial V_t(z)}{\partial \widetilde{P}^A_{A,t}(z)} \;=\; & E_t \sum_{\tau=0}^{\infty} \alpha^\tau \rho^A_{t,t+\tau} \Bigg[(1-\phi)\, n_P n_A \underbrace{\left(\frac{\widetilde{P}^A_{A,t}(z)}{P^A_{A,t+\tau}}\right)^{-\phi} \frac{1}{n_P n_A} Y^A_{A,t+\tau}}_{Y^A_{A,t+\tau}(z)} \\
& + \phi\, \widetilde{P}^A_{A,t}(z)^{-1} n_P n_A \underbrace{\left(\frac{\widetilde{P}^A_{A,t}(z)}{P^A_{A,t+\tau}}\right)^{-\phi} \frac{1}{n_P n_A} Y^A_{A,t+\tau}}_{Y^A_{A,t+\tau}(z)} MC^A_{t+\tau}(z) \\
& + (1-\phi)\, n_p\,(1-n_A)\, \underbrace{\widetilde{P}^A_{A,t}(z)^{-\phi} S^B_{A,t+\tau}(z)^{-\phi} {P^B_{A,t+\tau}}^{\phi} \frac{1}{n_P n_A} Y^B_{A,t+\tau}}_{Y^B_{A,t+\tau}(z)} \\
& + \phi\, \widetilde{P}^A_{A,t}(z)^{-1} n_p\,(1-n_A)\, \underbrace{\widetilde{P}^A_{A,t}(z)^{-\phi} S^B_{A,t+\tau}(z)^{-\phi} \left(P^B_{A,t+\tau}\right)^{\phi} \frac{1}{n_P n_A} Y^B_{A,t+\tau}}_{Y^B_{A,t+\tau}(z)} MC^A_{t+\tau}(z) \\
& + (1-\phi)\, (1-n_P)\, \underbrace{\widetilde{P}^A_{A,t}(z)^{-\phi} S^C_{A,t+\tau}(z)^{-\phi} {P^C_{A,t+\tau}}^{\phi} \frac{1}{n_P n_A} Y^C_{A,t+\tau}}_{Y^C_{A,t+\tau}(z)} \\
& + \phi\, \widetilde{P}^A_{A,t}(z)^{-1} (1-n_P) \\
& \cdot \underbrace{\widetilde{P}^A_{A,t}(z)^{-\phi} S^C_{A,t+\tau}(z)^{-\phi} \left(P^C_{A,t+\tau}\right)^{\phi} \frac{1}{n_P n_A} Y^C_{A,t+\tau}}_{Y^C_{A,t+\tau}(z)}\, MC^A_{t+\tau}(z) \Big] = 0
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & E_t \sum_{\tau=0}^{\infty} \alpha^\tau \rho^A_{t,t+\tau} \Big[(1-\phi)\, n_P n_A\, Y^A_{A,t+\tau}(z) + \phi\, n_P n_A\, \widetilde{P}^A_{A,t}(z)^{-1}\, Y^A_{A,t+\tau}(z)\, MC^A_{t+\tau}(z) \\
& + (1-\phi)\, n_p\, (1-n_A)\; Y^B_{A,t+\tau}(z) + \phi\, n_p\, (1-n_A)\; \widetilde{P}^A_{A,t}(z)^{-1}\, Y^B_{A,t+\tau}(z)\, MC^A_{t+\tau}(z) \\
& + (1-\phi)\, (1-n_P)\; Y^C_{A,t+\tau}(z) + \phi\; (1-n_P)\; \widetilde{P}^A_{A,t}(z)^{-1}\, Y^C_{A,t+\tau}(z)\, MC^A_{t+\tau}(z) \Big] = 0 \\
\Leftrightarrow \quad & E_t \sum_{\tau=0}^{\infty} \alpha^\tau \rho^A_{t,t+\tau} \Bigg[(1-\phi) \underbrace{\left(n_P n_A Y^A_{A,t+\tau}(z) + n_p\, (1-n_A)\, Y^B_{A,t+\tau}(z) + (1-n_P)\, Y^C_{A,t+\tau}(z) \right)}_{Y^A_{t+\tau}(z)} \\
& + \phi\; \widetilde{P}^A_{A,t}(z)^{-1} \left(n_P n_A Y^A_{A,t+\tau}(z) + n_p\, (1-n_A)\, Y^B_{A,t+\tau}(z) + (1-n_P)\, Y^C_{A,t+\tau}(z) \right) \\
& \cdot MC^A_{t+\tau}(z) \Big] = 0 \\
\Leftrightarrow \quad & \widetilde{P}^A_{A,t}(z) = -\frac{\phi}{1-\phi} \cdot \frac{E_t \sum_{\tau=0}^{\infty} \alpha^\tau \rho^A_{t,t+\tau}\, Y^A_{t+\tau}(z)\, MC^A_{t+\tau}(z)}{E_t \sum_{\tau=0}^{\infty} \alpha^\tau \rho^A_{t,t+\tau}\, Y^A_{t+\tau}(z)} \\
\Leftrightarrow \quad & \widetilde{P}^A_{A,t}(z) = \frac{\phi}{\phi-1} \cdot \frac{E_t \sum_{\tau=0}^{\infty} \alpha^\tau \rho^A_{t,t+\tau}\, Y^A_{t+\tau}(z)\, MC^A_{t+\tau}(z)}{E_t \sum_{\tau=0}^{\infty} \alpha^\tau \rho^A_{t,t+\tau}\, Y^A_{t+\tau}(z)}
\end{aligned} \tag{7.184}
$$

where $Y^A_{t+\tau}(z)$ is total ("world") output of country A's firm z as in (7.161).

7.3 New Keynesian Model

7.3.1 Domestic Price Indexes and Inflation

Evolution of Aggregate Price Index

Following the lecture notes by Monacelli (2007)

$$
P^i_{i',t} = \left[\alpha \left(P^i_{i',t-1} \right)^{1-\phi} + (1-\alpha) \left(\widetilde{P}^i_{i',t} \right)^{1-\phi} \right]^{\frac{1}{1-\phi}}. \tag{7.185}
$$

Aggregate price level in period t loglinearized

$$
\hat{P}^i_{i',t} = \alpha \hat{P}^i_{i',t-1} + (1-\alpha)\, \widehat{\widetilde{P}}^i_{i',t}. \tag{7.186}
$$

Inflation loglinearized

$$
\pi^i_{i',t} = (1-\alpha) \left(\widehat{\widetilde{P}}^i_{i',t} - \hat{P}^i_{i',t-1} \right). \tag{7.187}
$$

Notice, that inflation is not expressed as its deviation from steady state but in levels.

7.3.2 Phillips Curve

Loglinearized Price-Setting Equation

Starting from the equation for the optimal price (7.184)[11]

$$\widetilde{P}^A_{A,t}(z) = \underbrace{\frac{\phi}{\phi-1}}_{\text{mark-up}} \cdot \frac{E_t \sum_{\tau=0}^{\infty} \alpha^\tau \rho^A_{t,t+\tau} Y^A_{t+\tau}(z)\, MC^A_{t+\tau}(z)}{E_t \sum_{\tau=0}^{\infty} \alpha^\tau \rho^A_{t,t+\tau} Y^A_{t+\tau}(z)}$$

$$\text{define } \tilde{\rho}^i_{t,t+\tau} \equiv \frac{\lambda^i_{t+\tau}}{\lambda^i_t} = \left(\frac{C^i_{t+\tau}}{C^i_t}\right)^{-\mu} \frac{P^i_t}{P^i_{t+\tau}} \qquad \text{see (7.38 and 7.42)}$$

$$\text{and substitute } \rho^A_{t,t+\tau} \text{ by } \beta^\tau\, \tilde{\rho}^A_{t,t+\tau}$$

$$\Leftrightarrow \quad \widetilde{P}^A_{A,t}(z)\, E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \tilde{\rho}^A_{t,t+\tau} Y^A_{t+\tau}(z) =$$

$$\frac{\phi}{\phi-1} E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \tilde{\rho}^A_{t,t+\tau} Y^A_{t+\tau}(z)\, MC^A_{t+\tau}(z). \tag{7.188}$$

Since each firm z sets the same price in equilibrium we can drop the index z and multiply both sides by $\left(P^A_{A,t+\tau}\right)^\phi$

$$\Leftrightarrow \quad \underbrace{\widetilde{P}^A_{A,t}\, E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \tilde{\rho}^A_{t,t+\tau} Y^A_{t+\tau} \left(P^A_{A,t+\tau}\right)^\phi}_{\text{LHS}} =$$

$$\underbrace{\frac{\phi}{\phi-1} E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \tilde{\rho}^A_{t,t+\tau} Y^A_{t+\tau} \left(P^A_{A,t+\tau}\right)^\phi MC^A_{t+\tau}}_{\text{RHS}} \tag{7.189}$$

$$\text{LHS:} \quad \widetilde{P}^A_{A,t}\, E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \tilde{\rho}^A_{t,t+\tau} Y^A_{t+\tau} \left(P^A_{A,t+\tau}\right)^\phi \tag{7.190}$$

$$\text{RHS:} \quad \frac{\phi}{\phi-1} E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \tilde{\rho}^A_{t,t+\tau} Y^A_{t+\tau} \left(P^A_{A,t+\tau}\right)^\phi MC^A_{t+\tau} \tag{7.191}$$

[11] See Teo (2005, Footnote 11).

Loglinearizing LHS (7.190)

$$
\begin{aligned}
t \qquad & \widetilde{P}^A_{A,t} \overbrace{(\alpha\beta)^0}^{1} \overbrace{\tilde{\rho}^A_{t,t}}^{1} Y^A_t \left(P^A_{A,t}\right)^{\phi} \\
t+1 \qquad & + \widetilde{P}^A_{A,t}\, \alpha\beta\, \tilde{\rho}^A_{t,t+1}\, Y^A_{t+1} \left(P^A_{A,t+1}\right)^{\phi} \\
t+2 \qquad & + \widetilde{P}^A_{A,t}\, (\alpha\beta)^2\, \tilde{\rho}^A_{t,t+2}\, Y^A_{t+2} \left(P^A_{A,t+2}\right)^{\phi} \\
t+\tau \qquad & + \ldots \\
& \text{Loglinearize with steady state of } \widetilde{P}^A_A(z) = P^A_A,\ \tilde{\rho}^A = 1 \\
t,\ \widetilde{P}^A_{A,t} \qquad & \widehat{\widetilde{P}}^A_{A,t}\, P^A\, Y^A \left(P^A_A\right)^{\phi} \\
t,\ Y^A_t \qquad & + \hat{Y}^A_{A,t}\, Y^A \left(P^A_A\right)^{1+\phi} \\
t,\ P^A_{A,t} \qquad & + \phi\, \hat{P}^A_{A,t}\, Y^A \left(P^A_A\right)^{1+\phi} \\
t+1,\ \widetilde{P}^A_{A,t} \qquad & + \widehat{\widetilde{P}}^A_{A,t}\, P^A\, \alpha\beta\, Y^A \left(P^A_A\right)^{\phi} \\
t+1,\ Y^A_{t+1} \qquad & + \hat{Y}^A_{A,t+1}\, \alpha\beta\, Y^A \left(P^A_A\right)^{1+\phi} \\
t+1,\ P^A_{A,t+1} \qquad & + \phi\, \hat{P}^A_{A,t+1}\, \alpha\beta\, Y^A \left(P^A_A\right)^{1+\phi} \\
t+1,\ \tilde{\rho}^A_{t,t+1} \qquad & + \hat{\tilde{\rho}}^A_{t,t+1}\, \alpha\beta\, Y^A \left(P^A_A\right)^{1+\phi} \\
t+2,\ \widetilde{P}^A_{A,t} \qquad & + \widehat{\widetilde{P}}^A_{A,t}\, P^A\, \alpha\beta\, Y^A \left(P^A_A\right)^{\phi} \\
t+2,\ Y^A_{t+2} \qquad & + \hat{Y}^A_{A,t+2}\, (\alpha\beta)^2\, Y^A \left(P^A_A\right)^{1+\phi} \\
t+2,\ P^A_{A,t+2} \qquad & + \phi\, \hat{P}^A_{A,t+2}\, (\alpha\beta)^2\, Y^A \left(P^A_A\right)^{1+\phi} \\
t+2,\ \tilde{\rho}^A_{t,t+2} \qquad & + \hat{\tilde{\rho}}^A_{t,t+2}\, (\alpha\beta)^2\, Y^A \left(P^A_A\right)^{1+\phi} \\
t+\tau \qquad & + \ldots
\end{aligned}
$$

Collecting terms

$$
\begin{aligned}
t \qquad & \left(P_A^A\right)^{1+\phi} Y^A \left(\widehat{\widetilde{P}}_{A,t}^A + \hat{Y}_{A,t}^A + \phi\, \hat{P}_{A,t}^A\right) \\
t+1 \qquad & + \left(P_A^A\right)^{1+\phi} Y^A \alpha\beta \left(\widehat{\widetilde{P}}_{A,t}^A + \hat{Y}_{A,t+1}^A + \phi\, \hat{P}_{A,t+1}^A + \hat{\tilde{\rho}}_{t,t+1}^A\right) \\
t+2 \qquad & + \left(P_A^A\right)^{1+\phi} Y^A (\alpha\beta)^2 \left(\widehat{\widetilde{P}}_{A,t}^A + \hat{Y}_{A,t+2}^A + \phi\, \hat{P}_{A,t+2}^A + \hat{\tilde{\rho}}_{t,t+2}^A\right) \\
t+\tau \qquad & + \ldots
\end{aligned}
$$

Simplify

$$
\begin{aligned}
\text{LHS:} \qquad & \left(P_A^A\right)^{1+\phi} Y^A \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau\, \widehat{\widetilde{P}}_{A,t}^A \\
& + \left(P_A^A\right)^{1+\phi} Y^A E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \left(\hat{Y}_{A,t+\tau}^A + \phi\, \hat{P}_{A,t+\tau}^A + \hat{\tilde{\rho}}_{t,t+\tau}^A\right)
\end{aligned}
$$

$$
\begin{aligned}
\text{LHS:} \qquad & \left(P_A^A\right)^{1+\phi} Y^A \frac{1}{1-\alpha\beta}\, \widehat{\widetilde{P}}_{A,t}^A \\
& + \left(P_A^A\right)^{1+\phi} Y^A E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \left(\hat{Y}_{A,t+\tau}^A + \phi\, \hat{P}_{A,t+\tau}^A + \hat{\tilde{\rho}}_{t,t+\tau}^A\right)
\end{aligned}
\tag{7.192}
$$

Loglinearizing RHS (7.191)

$$
\begin{array}{ll}
 & \text{mark-up} \cdot [\\
t & \overbrace{(\alpha\beta)^0}^{1}\,\overbrace{\tilde{\rho}^A_{t,t}}^{1}\, Y^A_t \left(P^A_{A,t}\right)^{\phi} MC^A_t \\
t+1 & + \alpha\beta\, \tilde{\rho}^A_{t,t+1}\, Y^A_{t+1} \left(P^A_{A,t+1}\right)^{\phi} MC^A_{t+1} \\
t+2 & + (\alpha\beta)^2\, \tilde{\rho}^A_{t,t+2}\, Y^A_{t+2} \left(P^A_{A,t+2}\right)^{\phi} MC^A_{t+2} \\
t+\tau & + \ldots]
\end{array}
$$

Following (7.146) substitute $mc^A_t(z) = MC^A_t(z)/P^A_{A,t}$

$$
\begin{array}{ll}
 & \text{mark-up} \cdot [\\
t & Y^A_t \left(P^A_{A,t}\right)^{1+\phi} mc^A_t \\
t+1 & + \alpha\beta\, \tilde{\rho}^A_{t,t+1}\, Y^A_{t+1} \left(P^A_{A,t+1}\right)^{1+\phi} mc^A_{t+1} \\
t+2 & + (\alpha\beta)^2\, \tilde{\rho}^A_{t,t+2}\, Y^A_{t+2} \left(P^A_{A,t+2}\right)^{1+\phi} mc^A_{t+2} \\
t+\tau & + \ldots]
\end{array}
$$

Loglinearize with steady state of $\widetilde{P}^A_A(z) = P^A_A$, $\tilde{\rho}^A = 1$

$$
\begin{array}{ll}
t,\, Y^A_t & \text{mark-up} \cdot [\, \hat{Y}^A_t\, Y^A \left(P^A_A\right)^{1+\phi} mc^A \\
t,\, P^A_{A,t} & + (1+\phi)\, \hat{P}^A_{A,t}\, Y^A \left(P^A_A\right)^{1+\phi} mc^A \\
t,\, mc^A_t & + \widehat{mc}^A_t\, Y^A \left(P^A_A\right)^{1+\phi} mc^A \\
t+1,\, Y^A_{t+1} & + \hat{Y}^A_{t+1}\, \alpha\beta\, Y^A \left(P^A_A\right)^{1+\phi} mc^A \\
t+1,\, P^A_{A,t+1} & + (1+\phi)\, \hat{P}^A_{A,t+1}\, \alpha\beta\, Y^A \left(P^A_A\right)^{1+\phi} mc^A \\
t+1,\, \tilde{\rho}^A_{t,t+1} & + \hat{\tilde{\rho}}^A_{t,t+1}\, \alpha\beta\, Y^A \left(P^A_A\right)^{1+\phi} mc^A \\
t+1,\, mc^A_{t+1} & + \widehat{mc}^A_{t+1}\, \alpha\beta\, Y^A \left(P^A_A\right)^{1+\phi} mc^A \\
t+2,\, Y^A_{t+2} & + \hat{Y}^A_{t+2}\, (\alpha\beta)^2\, Y^A \left(P^A_A\right)^{1+\phi} mc^A \\
t+2,\, P^A_{A,t+2} & + (1+\phi)\, \hat{P}^A_{A,t+2}\, (\alpha\beta)^2\, Y^A \left(P^A_A\right)^{1+\phi} mc^A \\
t+2,\, \tilde{\rho}^A_{t,t+2} & + \hat{\tilde{\rho}}^A_{t,t+2}\, (\alpha\beta)^2\, Y^A \left(P^A_A\right)^{1+\phi} mc^A \\
t+2,\, mc^A_{t+2} & + \widehat{mc}^A_{t+2}\, (\alpha\beta)^2\, Y^A \left(P^A_A\right)^{1+\phi} mc^A \\
t+\tau & + \ldots]
\end{array}
$$

Collecting terms

$$
\begin{aligned}
t \qquad & \text{mark-up} \cdot \left[\left(P_A^A\right)^{1+\phi} Y^A \, mc^A \left(\hat{Y}_t^A \; + \; (1+\phi)\, \hat{P}_{A,t}^A \; + \; \widehat{mc}_t^A\right)\right. \\
t+1 \qquad & + \left(P_A^A\right)^{1+\phi} Y^A \, mc^A \, \alpha\beta \left(\hat{Y}_{t+1}^A + (1+\phi)\hat{P}_{A,t+1}^A + \hat{\tilde{\rho}}_{t,t+1}^A + \widehat{mc}_{t+1}^A\right) \\
t+2 \qquad & + \left(P_A^A\right)^{1+\phi} Y^A \, mc^A \, (\alpha\beta)^2 \left(\hat{Y}_{t+2}^A + (1+\phi)\hat{P}_{A,t+2}^A + \hat{\tilde{\rho}}_{t,t+2}^A + \widehat{mc}_{t+2}^A\right) \\
t+\tau \qquad & \left. + \ldots \right]
\end{aligned}
$$

Simplify

$$
\begin{aligned}
\text{RHS:} \qquad & \left(P_A^A\right)^{1+\phi} Y^A \, mc^A \cdot \text{mark-up} \cdot E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \\
& \cdot \left(\hat{Y}_{t+\tau}^A \; + \; (1+\phi)\, \hat{P}_{A,t+\tau}^A \; + \; \hat{\tilde{\rho}}_{t,t+\tau}^A \; + \; \widehat{mc}_{t+\tau}^A\right) \\
& \text{Cancel mark-up} \; = \frac{1}{mc^A}
\end{aligned}
$$

$$
\text{RHS:} \qquad \left(P_A^A\right)^{1+\phi} Y^A E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \left(\hat{Y}_{t+\tau}^A + (1+\phi)\hat{P}_{A,t+\tau}^A + \hat{\tilde{\rho}}_{t,t+\tau}^A + \widehat{mc}_{t+\tau}^A\right) \tag{7.193}
$$

Putting the Two Sides together: LHS = RHS Combining (7.192) and (7.193)

$$
\begin{aligned}
& \left(P_A^A\right)^{1+\phi} Y^A \frac{1}{1-\alpha\beta}\, \widehat{\widetilde{P}}_{A,t}^A \\
& + \left(P_A^A\right)^{1+\phi} Y^A E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \left(\hat{Y}_{t+\tau}^A + \phi\, \hat{P}_{A,t+\tau}^A + \hat{\tilde{\rho}}_{t,t+\tau}^A\right) \\
= & \\
& \left(P_A^A\right)^{1+\phi} Y^A E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \left(\hat{Y}_{t+\tau}^A + (1+\phi)\hat{P}_{A,t+\tau}^A + \hat{\tilde{\rho}}_{t,t+\tau}^A + \widehat{mc}_{t+\tau}^A\right) \\
& \text{Divide both sides by } \left(P_A^A\right)^{1+\phi} Y^A \\
& \text{and subtract } E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \left(\hat{Y}_{t+\tau}^A + \phi\, \hat{P}_{A,t+\tau}^A + \hat{\tilde{\rho}}_{t,t+\tau}^A\right) \\
\Leftrightarrow \quad & \frac{1}{1-\alpha\beta}\, \widehat{\widetilde{P}}_{A,t}^A \\
= \quad & E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \left(\hat{Y}_{t+\tau}^A - \hat{Y}_{t+\tau}^A + (1+\phi)\hat{P}_{A,t+\tau}^A - \phi\, \hat{P}_{A,t+\tau}^A + \hat{\tilde{\rho}}_{t,t+\tau}^A - \hat{\tilde{\rho}}_{t,t+\tau}^A + \widehat{mc}_{t+\tau}^A\right) \\
\Leftrightarrow \quad & \frac{1}{1-\alpha\beta}\, \widehat{\widetilde{P}}_{A,t}^A = E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \left(\hat{P}_{A,t+\tau}^A + \widehat{mc}_{t+\tau}^A\right) \\
\Leftrightarrow \quad & \widehat{\widetilde{P}}_{A,t}^A = (1-\alpha\beta)\, E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \left(\hat{P}_{A,t+\tau}^A + \widehat{mc}_{t+\tau}^A\right) \qquad (7.194)
\end{aligned}
$$

Equation for Inflation

Rewrite (7.194) in first differences

$$
\widehat{\widetilde{P}}_{A,t}^A = (1-\alpha\beta)\, \left(\widehat{mc}_t^A + \hat{P}_{A,t}^A\right) + \alpha\beta\, (1-\alpha\beta)\, \left(\widehat{mc}_{t+1}^A + \hat{P}_{A,t+1}^A\right). \qquad (7.195)
$$

Recall (7.186)

$$
\hat{P}_{A,t}^A = \alpha \hat{P}_{A,t-1}^A + (1-\alpha)\, \widehat{\widetilde{P}}_{A,t}^A. \qquad (7.196)
$$

Rearrange for later use to

$$
\widehat{\widetilde{P}}_{A,t}^A = \frac{\hat{P}_{A,t}^A - \alpha\, \hat{P}_{A,t-1}^A}{1-\alpha}. \qquad (7.197)
$$

Combine (7.195) with (7.196)

$$
\hat{P}_{A,t}^A = \alpha \hat{P}_{A,t-1}^A + (1-\alpha)\, (1-\alpha\beta)\, \left(\widehat{mc}_t^A + \hat{P}_{A,t}^A\right) + (1-\alpha)\, \alpha\beta\, \widehat{\widetilde{P}}_{A,t+1}^A. \qquad (7.198)
$$

Substitute $\hat{P}^A_{t+1}$ by (7.197)

$$\hat{P}^A_{A,t} = \alpha \hat{P}^A_{A,t-1} + (1-\alpha)(1-\alpha\beta)\left(\widehat{mc}^A_t + \hat{P}^A_{A,t}\right) + (1-\alpha)\,\alpha\beta\,\frac{\hat{P}^A_{A,t+1} - \alpha\,\hat{P}^A_{A,t}}{1-\alpha}$$

$$= \alpha \hat{P}^A_{A,t-1} + (1-\alpha)(1-\alpha\beta)\left(\widehat{mc}^A_t + \hat{P}^A_{A,t}\right) + \alpha\beta\left(\hat{P}^A_{A,t+1} - \alpha\,\hat{P}^A_{A,t}\right) \tag{7.199}$$

$$\hat{P}^A_{A,t} - \alpha \hat{P}^A_{A,t-1} = (1-\alpha)(1-\alpha\beta)\,\widehat{mc}^A_t + (1-\alpha)(1-\alpha\beta)\,\hat{P}^A_{A,t} + \alpha\beta\,\hat{P}^A_{A,t+1} - \alpha^2\beta\,\hat{P}^A_{A,t}.$$

Add $\alpha\,\hat{P}^A_{A,t}$ and subtract $\hat{P}^A_{A,t}$

$$\alpha\,\hat{P}^A_{A,t} - \alpha \hat{P}^A_{A,t-1} = (1-\alpha)(1-\alpha\beta)\,\widehat{mc}^A_t + \alpha\beta\,\hat{P}^A_{A,t+1} + (1-\alpha-\alpha\beta+\alpha^2\beta)\,\hat{P}^A_{A,t} - \alpha^2\beta\,\hat{P}^A_{A,t} - \alpha\,\hat{P}^A_{A,t} + \hat{P}^A_{A,t}$$

$$\alpha\left(\hat{P}^A_{A,t} - \hat{P}^A_{A,t-1}\right) = (1-\alpha)(1-\alpha\beta)\,\widehat{mc}^A_t + \alpha\beta\,\hat{P}^A_{A,t+1} - \alpha\beta\,\hat{P}^A_{A,t}$$

$$\alpha\underbrace{\left(\hat{P}^A_{A,t} - \hat{P}^A_{A,t-1}\right)}_{\widehat{\pi}^A_{A,t}} = (1-\alpha)(1-\alpha\beta)\,\widehat{mc}^A_t + \alpha\beta\underbrace{\left(\hat{P}^A_{A,t+1} - \hat{P}^A_{A,t}\right)}_{\widehat{\pi}^A_{A,t+1}}.$$

Equation for inflation

$$\alpha\,\widehat{\pi}^A_{A,t} = (1-\alpha)(1-\alpha\beta)\,\widehat{mc}^A_t + \alpha\beta\,\widehat{\pi}^A_{A,t+1}$$

$$\widehat{\pi}^A_{A,t} = \beta\,E_t\left\{\widehat{\pi}^A_{A,t+1}\right\} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\,\widehat{mc}^A_t. \tag{7.200}$$

Equation (7.200) is the equivalent to Monacelli (2001: 393, eq. 23).

$$\widehat{\pi}^A_{A,t} = \beta\,E_t\left\{\widehat{\pi}^A_{A,t+1}\right\} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\,\widehat{mc}^A_t \tag{7.201}$$

$$\widehat{\pi}^B_{B,t} = \beta\,E_t\left\{\widehat{\pi}^B_{B,t+1}\right\} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\,\widehat{mc}^B_t \tag{7.202}$$

$$\widehat{\pi}^C_{C,t} = \beta\,E_t\left\{\widehat{\pi}^C_{C,t+1}\right\} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\,\widehat{mc}^C_t \tag{7.203}$$

7.3.3 Monetary Policy

Following Woodford (2003: 24) the central bank conducts monetary policy by setting the interest rate. The interest rate policy rule is exogenous. The central

bank follows a so-called Taylor rule by reacting to deviations of inflation, output and terms of trade

$$\hat{R}_t = \varphi_\pi \widehat{\pi}_t + \varphi_Y \hat{Y}_t + \varphi_{ToT} \widehat{ToT}_t \tag{7.204}$$

where φ_π, φ_Y and φ_{ToT} are parameters with $\varphi_\pi > 1$ to fulfill the Taylor principle and guarantee determinancy (Woodford 2003: 91).

7.3.4 Summary

Summary of Equations

Equation	**Country A**	**Country B**	**Country C**	
Euler	(7.135)	(7.136)	(7.137)	
Phillips	(7.201)	(7.202)	(7.203)	
Marginal Cost	(7.151)	(7.152)	(7.153)	
CPI PPI Ratio	(7.106)	(7.107)	(7.108)	
Monetary Policy Rule	(7.204)	(7.204)	(7.204)	
Goods Market Clearing	(7.126)	(7.127)	(7.128)	
Risk Sharing		(7.124)	(7.125)	
Exog. Shock	(7.139)	(7.139)	(7.139)	
Σ	7	8	8	23

Country	A	Equation
Euler	(7.135)	$\hat{C}_t^A = \hat{C}_{t+1}^A - \frac{1}{\mu}\hat{R}_t^A + \frac{1}{\mu}\hat{\pi}_{A,t+1}^A$
		$+\frac{[1-\eta(1-n_P)]\eta(1-n_A)}{\mu}\widehat{ToT}_{B,t+1}^A - \frac{[1-\eta(1-n_P)]\eta(1-n_A)}{\mu}\widehat{ToT}_{B,t}^A$
		$+\frac{\eta(1-n_P)}{\mu}\widehat{ToT}_{C,t+1}^A - \frac{\eta(1-n_P)}{\mu}\widehat{ToT}_{C,t}^A$
Phillips	(7.201)	$\widehat{\pi}_{A,t}^A = \beta\, E_t\left\{\widehat{\pi}_{A,t+1}^A\right\} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\widehat{mc}_t^A$
Marg.Cost	(7.151)	$\widehat{mc}_t^A = \mu\,\hat{C}_t^A + \gamma\,\hat{Y}_t^A - (1+\gamma)\,\hat{F}_t^A + \hat{g}_A\,(ToT)$
$g_A\,(ToT)$	(7.106)	$\hat{g}_{A,t}(ToT) = [1-\eta\,(1-n_P)]\,\eta\,(1-n_A)\,\widehat{ToT}_{B,t}^A$
		$+\eta\,(1-n_P)\,\widehat{ToT}_{C,t}^A$
Mon. Pol.	(7.204)	$\hat{R}_t^A = \varphi_\pi\widehat{\pi}_{A,t}^A + \varphi_Y\hat{Y}_t^A + \varphi_{ToT}\widehat{ToT}_t$
Output	(7.126)	$\hat{Y}_t^A = [1-\eta\,(1-n_P)]\,[1-\eta\,(1-n_A)]\,\hat{C}_t^A$
		$+\theta\,[1-\eta\,(1-n_P)]\,[1-\eta\,(1-n_A)]\,\hat{g}_{A,t}(ToT)$
		$+(\psi-\theta)\,[1-\eta\,(1-n_P)]\,[1-\eta\,(1-n_A)]\,\eta\,(1-n_A)\,\widehat{ToT}_{B,t}^A$
		$+[1-\eta\,(1-n_P)]\,\eta\,(1-n_A)\,\hat{C}_t^B$
		$+\theta\,[1-\eta\,(1-n_P)]\,\eta\,(1-n_A)\,\hat{g}_{B,t}(ToT)$
		$+\psi\,[1-\eta\,(1-n_P)]\,\eta\,(1-n_A)\,\widehat{ToT}_{B,t}^A$
		$+(\psi-\theta)\,[1-\eta\,(1-n_P)]\,\eta\,(1-n_A)\,\eta n_A\widehat{ToT}_{A,t}^B$
		$+\eta\,(1-n_P)\,\hat{C}_t^C + \theta\eta\,(1-n_P)\,\hat{g}_{C,t}(ToT)$
		$+\psi\eta\,(1-n_P)\,\widehat{ToT}_{C,t}^A$
		$+(\psi-\theta)\,\eta\,(1-n_P)\,n_A\widehat{ToT}_{A,t}^C$
		$+(\psi-\theta)\,\eta\,(1-n_P)\,(1-n_A)\,\widehat{ToT}_{B,t}^C$
Shock	(7.139)	$F_t^A = \rho^F \ln F_{t-1}^A + \epsilon_t^A$

Country	B	Equation
Euler	(7.136)	$\hat{C}_t^B = \hat{C}_{t+1}^B - \frac{1}{\mu}\hat{R}_t^B + \frac{1}{\mu}\hat{\pi}_{B,t+1}^B$
		$+\frac{[1-\eta(1-n_P)]\,\eta n_A}{\mu}\widehat{ToT}_{A,t+1}^B - \frac{[1-\eta(1-n_P)]\,\eta n_A}{\mu}\widehat{ToT}_{A,t}^B$
		$+\frac{\eta(1-n_P)}{\mu}\widehat{ToT}_{C,t+1}^B - \frac{\eta(1-n_P)}{\mu}\widehat{ToT}_{C,t}^B$
Phillips	(7.202)	$\hat{\pi}_{B,t}^B = \beta\, E_t\left\{\hat{\pi}_{B,t+1}^B\right\} + \frac{(1-\alpha)\,(1-\alpha\beta)}{\alpha}\,\widehat{mc}_t^B$
Marg.Cost	(7.152)	$\widehat{mc}_t^B = \mu\,\hat{C}_t^B + \gamma\,\hat{Y}_t^B - (1+\gamma)\,\hat{F}_t^B + \hat{g}_B\,(ToT)$
$g_B\,(ToT)$	(7.107)	$\hat{g}_B\,(ToT) = [1-\eta\,(1-n_P)]\;\eta n_A\widehat{ToT}_{A,t}^B$
		$+\eta\,(1-n_P)\;\widehat{ToT}_{C,t}^B$
Mon. Pol.	(7.204)	$\hat{R}_t^B = \varphi_\pi\hat{\pi}_{B,t}^B + \varphi_Y\hat{Y}_t^B + \varphi_{ToT}\widehat{ToT}_t$
Output	(7.127)	$\hat{Y}_t^B = [1-\eta\,(1-n_P)]\,\eta n_A\hat{C}_t^A$
		$+\theta\,[1-\eta\,(1-n_P)]\,\eta n_A\hat{g}_{A,t}\,(ToT)$
		$+\,(\psi-\theta)\,[1-\eta\,(1-n_P)]\,\eta n_A\eta\,(1-n_A)\,\widehat{ToT}_{B,t}^A$
		$+\psi\,[1-\eta\,(1-n_P)]\,\eta n_A\widehat{ToT}_{A,t}^B$
		$+\,(1-\eta n_A)\,[1-\eta\,(1-n_P)]\,\hat{C}_t^B$
		$+\theta\,(1-\eta n_A)\,[1-\eta\,(1-n_P)]\,\hat{g}_{B,t}\,(ToT)$
		$+\,(\psi-\theta)\,(1-\eta n_A)\,[1-\eta\,(1-n_P)]\,\eta n_A\widehat{ToT}_{A,t}^B$
		$+\eta\,(1-n_P)\,\hat{C}_t^C + \theta\eta\,(1-n_P)\,\hat{g}_{C,t}\,(ToT)$
		$+\,(\psi-\theta)\,\eta\,(1-n_P)\,n_A\widehat{ToT}_{A,t}^C$
		$+\,(\psi-\theta)\,\eta\,(1-n_P)\,(1-n_A)\,\widehat{ToT}_{B,t}^C$
		$+\psi\eta\,(1-n_P)\,\widehat{ToT}_{C,t}^B$
Risk Sharing	(7.124)	$\hat{C}_t^A = \hat{C}_t^B + \frac{1}{\mu}\widehat{ToT}_{B,t}^A + \frac{1}{\mu}\hat{g}_B(ToT) - \frac{1}{\mu}\hat{g}_A(ToT)$
Shock	(7.139)	$F_t^B = \rho^F \ln F_{t-1}^B + \epsilon_t^B$

Country	C	Equation
Euler	(7.137)	$\hat{C}_t^C = \hat{C}_{t+1}^C - \frac{1}{\mu}\hat{R}_t^C + \frac{1}{\mu}\hat{\pi}_{C,t+1}^C$
		$+\frac{[1-\eta(1-n_P)]\,\eta n_A}{\mu}\widehat{ToT}_{A,t+1}^C - \frac{[1-\eta(1-n_P)]\,\eta n_A}{\mu}\widehat{ToT}_{A,t}^C$
		$+\frac{[1-\eta(1-n_P)]\eta(1-n_A)}{\mu}\widehat{ToT}_{B,t+1}^C - \frac{[1-\eta(1-n_P)]\eta(1-n_A)}{\mu}\widehat{ToT}_{B,t}^C$
Phillips	(7.203)	$\hat{\pi}_{C,t}^C = \beta\, E_t\left\{\hat{\pi}_{C,t+1}^C\right\} + \frac{(1-\alpha)\,(1-\alpha\beta)}{\alpha}\,\widehat{mc}_t^C$
Marg.Cost	(7.153)	$\widehat{mc}_t^C = \mu\,\hat{C}_t^C + \gamma\,\hat{Y}_t^C - (1+\gamma)\,\hat{F}_t^C + \hat{g}_C\,(ToT)$
$g_C\,(ToT)$	(7.108)	$\hat{g}_C\,(ToT) = \eta n_P\, n_A \widehat{ToT}_{A,t}^C + \eta n_P\,(1-n_A)\,\widehat{ToT}_{B,t}^C$
Mon. Pol.	(7.204)	$\hat{R}_t^C = \varphi_\pi \hat{\pi}_{C,t}^C + \varphi_Y \hat{Y}_t^C + \varphi_{ToT}\widehat{ToT}_t$
Output	(7.128)	$\hat{Y}_t^C = \eta n_P n_A \hat{C}_t^A$
		$+\eta n_P n_A\,(-\theta)\,\widehat{ToT}_{C,t}^A + \theta\eta n_P n_A \hat{g}_{A,t}(ToT)$
		$+\eta n_p\,(1-n_A)\,\hat{C}_t^B + (-\theta)\,\eta n_p\,(1-n_A)\,\widehat{ToT}_{C,t}^B$
		$+\theta\eta n_p\,(1-n_A)\,\hat{g}_{B,t}(ToT)$
		$+\,(1-\eta n_P)\,\hat{C}_t^C + \theta\,(1-\eta n_P)\,\hat{g}_{C,t}(ToT)$
Risk Sharing	(7.125)	$\hat{C}_t^A = \hat{C}_t^C + \frac{1}{\mu}\widehat{ToT}_{C,t}^A + \frac{1}{\mu}\hat{g}_C(ToT) - \frac{1}{\mu}\hat{g}_A(ToT)$
Shock	(7.139)	$F_t^C = \rho^F \ln F_{t-1}^C + \epsilon_t^C$

Using Shorthands for Trade Shares

Country	A	Equation
Euler	(7.135)	$\hat{C}_t^A = \hat{C}_{t+1}^A - \frac{1}{\mu}\hat{R}_t^A + \frac{1}{\mu}\hat{\pi}_{A,t+1}^A$
		$+\frac{v_B^A}{\mu}\widehat{ToT}_{B,t+1}^A - \frac{v_B^A}{\mu}\widehat{ToT}_{B,t}^A$
		$+\frac{v_C^A}{\mu}\widehat{ToT}_{C,t+1}^A - \frac{v_C^A}{\mu}\widehat{ToT}_{C,t}^A$
Phillips	(7.201)	$\widehat{\pi}_{A,t}^A = \beta\, E_t\left\{\widehat{\pi}_{A,t+1}^A\right\} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\widehat{mc}_t^A$
Marg.Cost	(7.151)	$\widehat{mc}_t^A = \mu\,\hat{C}_t^A + \gamma\,\hat{Y}_t^A - (1+\gamma)\,\hat{F}_t^A + \hat{g}_A(ToT)$
$g_A(ToT)$	(7.106)	$\hat{g}_{A,t}(ToT) = v_B^A\widehat{ToT}_{B,t}^A$
		$+v_C^A\widehat{ToT}_{C,t}^A$
Mon. Pol.	(7.204)	$\hat{R}_t^A = \varphi_\pi\widehat{\pi}_{A,t}^A + \varphi_Y\hat{Y}_t^A + \varphi_{ToT}\widehat{ToT}_t$
Output	(7.126)	$\hat{Y}_t^A = v_A^A\hat{C}_t^A$
		$+\theta v_A^A\hat{g}_{A,t}(ToT)$
		$+(\psi-\theta)\,v_B^A\cdot[1-\eta(1-n_A)]\cdot\widehat{ToT}_{B,t}^A$
		$+v_B^A\hat{C}_t^B$
		$+\theta v_B^A\hat{g}_{B,t}(ToT)$
		$+\psi v_B^A\widehat{ToT}_{B,t}^A$
		$+(\psi-\theta)\,v_B^A\cdot\eta n_A\cdot\widehat{ToT}_{A,t}^B$
		$+v_C^A\hat{C}_t^C + \theta v_C^A\hat{g}_{C,t}(ToT)$
		$+\psi v_C^A\widehat{ToT}_{C,t}^A$
		$+(\psi-\theta)\,v_C^A\cdot n_A\cdot\widehat{ToT}_{A,t}^C$
		$+(\psi-\theta)\,v_C^A\cdot(1-n_A)\cdot\widehat{ToT}_{B,t}^C$
Shock	(7.139)	$F_t^A = \rho^F\ln F_{t-1}^A + \epsilon_t^A$

Country	B	Equation
Euler	(7.136)	$\hat{C}_t^B = \hat{C}_{t+1}^B - \frac{1}{\mu}\hat{R}_t^B + \frac{1}{\mu}\hat{\pi}_{B,t+1}^B$ $+\frac{v_A^B}{\mu}\widehat{ToT}_{A,t+1}^B - \frac{v_A^B}{\mu}\widehat{ToT}_{A,t}^B$ $+\frac{v_C^B}{\mu}\widehat{ToT}_{C,t+1}^B - \frac{v_C^B}{\mu}\widehat{ToT}_{C,t}^B$
Phillips	(7.202)	$\hat{\pi}_{B,t}^B = \beta\, E_t\left\{\hat{\pi}_{B,t+1}^B\right\} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\widehat{mc}_t^B$
Marg.Cost	(7.152)	$\widehat{mc}_t^B = \mu\,\hat{C}_t^B + \gamma\,\hat{Y}_t^B - (1+\gamma)\,\hat{F}_t^B + \hat{g}_B\,(ToT)$
$g_B\,(ToT)$	(7.107)	$\hat{g}_B\,(ToT) = v_A^B\widehat{ToT}_{A,t}^B$ $+v_C^B\,\widehat{ToT}_{C,t}^B$
Mon. Pol.	(7.204)	$\hat{R}_t^B = \varphi_\pi\hat{\pi}_{B,t}^B + \varphi_Y\hat{Y}_t^B + \varphi_{ToT}\widehat{ToT}_t$
Output	(7.127)	$\hat{Y}_t^B = v_A^B\hat{C}_t^A$ $+\theta v_A^B\hat{g}_{A,t}\,(ToT)$ $+(\psi-\theta)\,v_A^B\cdot\eta\,(1-n_A)\cdot\widehat{ToT}_{B,t}^A$ $+\psi v_A^B\widehat{ToT}_{A,t}^B$ $+v_B^B\hat{C}_t^B$ $+\theta v_B^B\hat{g}_{B,t}\,(ToT)$ $+(\psi-\theta)\,v_A^B\cdot(1-\eta n_A)\cdot\widehat{ToT}_{A,t}^B$ $+v_C^B\hat{C}_t^C + \theta v_C^B\hat{g}_{C,t}\,(ToT)$ $+(\psi-\theta)\,v_C^B\cdot n_A\cdot\widehat{ToT}_{A,t}^C$ $+(\psi-\theta)\,v_C^B\cdot(1-n_A)\cdot\widehat{ToT}_{B,t}^C$ $+\psi v_C^B\widehat{ToT}_{C,t}^B$
Risk Sharing	(7.124)	$\hat{C}_t^A = \hat{C}_t^B + \frac{1}{\mu}\widehat{ToT}_{B,t}^A + \frac{1}{\mu}\hat{g}_B(ToT) - \frac{1}{\mu}\hat{g}_A(ToT)$
Shock	(7.139)	$F_t^B = \rho^F \ln F_{t-1}^B + \epsilon_t^B$

Country	**C**	**Equation**
Euler	(7.137)	$\hat{C}_t^C = \hat{C}_{t+1}^C - \frac{1}{\mu}\hat{R}_t^C + \frac{1}{\mu}\hat{\pi}_{C,t+1}^C$ $+\frac{v_A^B}{\mu}\widehat{ToT}_{A,t+1}^C - \frac{v_A^B}{\mu}\widehat{ToT}_{A,t}^C$ $+\frac{v_B^A}{\mu}\widehat{ToT}_{B,t+1}^C - \frac{v_B^A}{\mu}\widehat{ToT}_{B,t}^C$
Phillips	(7.203)	$\hat{\pi}_{C,t}^C = \beta\, E_t\left\{\hat{\pi}_{C,t+1}^C\right\} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\,\widehat{mc}_t^C$
Marg.Cost	(7.153)	$\widehat{mc}_t^C = \mu\,\hat{C}_t^C + \gamma\,\hat{Y}_t^C - (1+\gamma)\,\hat{F}_t^C + \hat{g}_C\,(ToT)$
$g_C\,(ToT)$	(7.108)	$\hat{g}_C\,(ToT) = v_A^C\widehat{ToT}_{A,t}^C + v_B^C\,\widehat{ToT}_{B,t}^C$
Mon. Pol.	(7.204)	$\hat{R}_t^C = \varphi_\pi\hat{\pi}_{C,t}^C + \varphi_Y\hat{Y}_t^C + \varphi_{ToT}\widehat{ToT}_t$
Output	(7.128)	$\hat{Y}_t^C = v_A^C\hat{C}_t^A$ $-\theta v_A^C\widehat{ToT}_{C,t}^A + \theta v_A^C\hat{g}_{A,t}(ToT)$ $+v_B^C\hat{C}_t^B - \theta v_B^C\widehat{ToT}_{C,t}^B$ $+\theta v_B^C\hat{g}_{B,t}(ToT)$ $+v_C^C\,\hat{C}_t^C + \theta v_C^C\hat{g}_{C,t}(ToT)$
Risk Sharing	(7.125)	$\hat{C}_t^A = \hat{C}_t^C + \frac{1}{\mu}\widehat{ToT}_{C,t}^A + \frac{1}{\mu}\hat{g}_C(ToT) - \frac{1}{\mu}\hat{g}_A(ToT)$
Shock	(7.139)	$F_t^C = \rho^F \ln F_{t-1}^C + \epsilon_t^C$

Additional equations for terms of trade using inverse (7.110) and cross rates (7.112).

$$\begin{aligned}
\widehat{ToT}_{A,t}^C &= -\widehat{ToT}_{C,t}^A \\
\widehat{ToT}_{A,t}^B &= -\widehat{ToT}_{B,t}^A \\
\widehat{ToT}_{C,t}^B &= \widehat{ToT}_{A,t}^B + \widehat{ToT}_{C,t}^A \\
\widehat{ToT}_{B,t}^C &= -\widehat{ToT}_{C,t}^B
\end{aligned}$$

Back out CPI inflation based on equations (7.132) to (7.134)

$$\begin{aligned}
\hat{\pi}_t^A &= \hat{\pi}_{A,t}^A + v_B^A\widehat{ToT}_{B,t}^A - v_B^A\widehat{ToT}_{B,t-1}^A + v_C^A\widehat{ToT}_{C,t}^A - v_C^A\widehat{ToT}_{C,t-1}^A \\
\hat{\pi}_t^B &= \hat{\pi}_{B,t}^B + v_A^B\widehat{ToT}_{A,t}^B - v_A^B\widehat{ToT}_{A,t-1}^B + v_C^B\widehat{ToT}_{C,t}^B - v_C^B\widehat{ToT}_{C,t-1}^B \\
\hat{\pi}_t^C &= \hat{\pi}_{C,t}^C + v_A^B\widehat{ToT}_{A,t}^C - v_A^B\widehat{ToT}_{A,t-1}^C + v_B^A\widehat{ToT}_{B,t}^C - v_B^A\widehat{ToT}_{B,t-1}^C
\end{aligned}$$

Back out real exchange rate using (7.113)

$$\widehat{RER}^{i}_{i',t} = \widehat{ToT}^{i}_{i',t} + \hat{g}_{i',t}(ToT) - \hat{g}_{i,t}(ToT)$$

Trade weighted real effective exchange rates

$$\widehat{REER}^{A}_{t} = \frac{v^A_B}{v^A_B + v^A_C}\widehat{RER}^{A}_{B,t} + \frac{v^A_C}{v^A_B + v^A_C}\widehat{RER}^{A}_{C,t}$$

$$\widehat{REER}^{B}_{t} = \frac{v^B_A}{v^B_A + v^B_C}\widehat{RER}^{B}_{A,t} + \frac{v^B_C}{v^B_B + v^B_C}\widehat{RER}^{B}_{C,t}$$

$$\widehat{REER}^{C}_{t} = \frac{v^C_A}{v^C_A + v^C_B}\widehat{RER}^{C}_{A,t} + \frac{v^C_B}{v^C_A + v^C_B}\widehat{RER}^{C}_{B,t}$$

Backing out change in nominal exchange rate (7.114)

$$\Delta\hat{S}^{i}_{i',t} = \widehat{ToT}^{i}_{i',t} - \widehat{ToT}^{i}_{i',t-1} + \pi^{i}_{i,t} - \pi^{i'}_{i',t}$$

$$\Delta\hat{S}^{A}_{B,t} = \widehat{ToT}^{A}_{B,t} - \widehat{ToT}^{A}_{B,t-1} + \pi^{A}_{A,t} - \pi^{B}_{B,t}$$

$$\Delta\hat{S}^{A}_{C,t} = \widehat{ToT}^{A}_{C,t} - \widehat{ToT}^{A}_{C,t-1} + \pi^{A}_{A,t} - \pi^{C}_{C,t}$$

$$\Delta\hat{S}^{B}_{C,t} = \widehat{ToT}^{B}_{C,t} - \widehat{ToT}^{B}_{C,t-1} + \pi^{B}_{B,t} - \pi^{C}_{C,t}$$

Summary of Variables

Variable	**Country A**	**Country B**	**Country C**	
Consumption	$\hat{C}^A$	$\hat{C}^B$	$\hat{C}^C$	
Domestic Inflation	$\hat{\pi}^A_A$	$\hat{\pi}^B_B$	$\hat{\pi}^C_C$	
Marginal Cost	$\widehat{mc}^A$	$\widehat{mc}^B$	$\widehat{mc}^C$	
CPI PPI Ratio	$\hat{g}_A\,(ToT)$	$\hat{g}_B\,(ToT)$	$\hat{g}_C\,(ToT)$	
Interest Rate	$\hat{R}^A$	$\hat{R}^B$	$\hat{R}^C$	
Output	$\hat{Y}^A$	$\hat{Y}^B$	$\hat{Y}^C$	
Terms of Trade		$\widehat{ToT}^A_B$	$\widehat{ToT}^A_C$	
Exog. Shock	$\hat{F}^A$	$\hat{F}^B$	$\hat{F}^C$	
Σ	6+1	7+1	7+1	20+3 = 23

7.4 References

Corsetti, G., P. Pesenti, N. Roubini, and C. Tille (2000). Competitive Devaluations: Toward a Welfare-Based Approach. *Journal of International Economics* 51 (1): 217-41.

De Paoli, B. (2006). Monetary Policy and Welfare in a Small Open Economy. CEP Discussion Paper 639. Centre for Economic Performance. London, UK.

Faia, E. and T. Monacelli (2004). Ramsey Monetary Policy and International Relative Prices. European Central Bank Working Paper 344. European Central Bank. Frankfurt, Germany.

Galí, J. and T. Monacelli (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economic Studies* 72 (3): 707-34.

Monacelli, T. (2001). New International Monetary Arrangements and the Exchange Rate. *International Journal of Finance and Economics* 6 (4): 389-400.

Monacelli, T. (2007). A New Keynesian Optimizing Framework with Nominal Rigidities. Mimeo. IGER, Università Bocconi. Milan, Italy.

Teo, W.L. (2005). Should East Asia's Currencies Be Pegged to the Yen? The Role of Invoice Currency. Mimeo. Johns Hopkins University. Baltimore, MD.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press. Princeton, NJ.

Ich erkläre hiermit an Eides Statt, dass ich meine Doktorarbeit „Inflation Targeting and Monetary Integration in European Emerging Market Economies“ selbstständig angefertigt habe und dass ich alle von anderen Autoren wörtlich übernommenen Stellen, wie auch die sich an die Gedanken anderer Autoren eng anlehnenden Ausführungen meiner Arbeit, besonders gekennzeichnet und die Quellen zitiert habe.

Frankfurt, 4. Februar 2008

Curriculum Vitae

SURNAME	Hammermann
NAME	Jan Felix
DATE OF BIRTH	25 May 1976
PLACE OF BIRTH	Wuppertal, Germany
CITIZENSHIP	German

since 11/2007	Economist, Deutsche Bundesbank, Frankfurt
2002 – 2007	Economist, Kiel Institute for the World Economy, Kiel
since 2002	Doctoral candidate, Christian-Albrechts-Universität zu Kiel
2006	Intern, International Monetary Fund, Washington D.C., USA
2004 – 2005	Visiting student, Universitat Pompeu Fabra, Barcelona, Spain
2001 – 2002	Advanced Studies Certificate in International Economic Policy Research, Kiel Institute for the World Economy, Kiel
1996 – 2001	Diplom-Volkswirt, Rheinische Friedrich-Wilhelms-Universität Bonn
1998 – 1999	Visiting student, University of York, United Kingdom
1995 – 1996	Alternative service, Travelers' Mission, Nuremberg
1995	Abitur, Gertrud-Bäumer-Gymnasium, Remscheid
1992	Visiting student, Christ's College, Christchurch, New Zealand

Frankfurt, February 2008