International Asset Prices: Empirical Evidence

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A thesis submitted for the degree of

Dr. Sc. Pol.

International Asset Prices: Empirical Evidence

Inaugural-Dissertation zur Erlangung des akademischen Grades
eines Doktors der Wirtschafts- und Sozialwissenschaften
der Wirtschafts- und Sozialwissenschaftlichen Fakultät
der Christian-Albrechts-Universität zu Kiel

vorgelegt von
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aus Bogotá, Kolumbien

Kiel, Juli 2009

Christian-Albrechts-Universität zu Kiel
Wilhelm-Seelig-Platz 1
24118 Kiel
Germany

Gedruckt mit Genehmigung der
Wirtschafts- und Sozialwissenschaftlichen Fakultät
der Christian-Albrechts-Universität zu Kiel

Dekan:
Prof. Dr. Thomas Lux

Erstberichterstattender:
Prof. Dr. Helmut Herwartz

Zweitberichterstattender:
Prof. Dr. Thomas Lux

Tag der Abgabe der Arbeit:
20. Februar 2009

Tag der mündlichen Prüfung:
18. Juni 2009

I would like to dedicate this thesis to my family and friends.

Acknowledgements

I would like to acknowledge the exceptional supervision and guidance of Professor Dr. Helmut Herwartz and Professor Dr. Thomas Lux in the preparation of this thesis. I would also like to acknowledge helpful comments and suggestions from participants of the Global Finance Conference in Hangzhou, China, May 2008 (Chapter 1), 2nd International Real Estate Conference of the International Real Estate Business School in Regensburg, Germany, June 2008 (Chapter 2), 17th Annual International Conference of the American Real Estate and Urban Economics Association in Istanbul, Turkey, July 2008 (Chapter 2), 7th INFINITI Conference on International Finance in Dublin, Ireland, June 2009 (Chapter 3), 2nd International Workshop on Computational and Financial Econometrics in Neuchatel, Switzerland, June 2008 (Chapter 4) and the Econophysics Colloquium of the Kiel Institute for the World Economy in Kiel, Germany, August 2008 (Chapter 4). Financial support of the European Commission under Strep contract 516446 for Chapter 4 is also greatfully acknowledged.

Abstract

The recently witnessed financial turmoil and the current international stability context have demonstrated the need for a deeper understanding about *ex-ante* international asset price fluctuations. Moreover, it is now more evident that very little is known about real estate securities and their long-run and short-run empirical determinants. There is also a lack of knowledge about the impact of US (or other foreign) shocks on international asset markets. Last but not least, there is a need for devising better models to forecast volatility and to manage risk in asset markets. The present thesis is a collection of essays that contribute to the latter issues. To preview some of the main results of the thesis with policy implications, we find that: (i) asset pricing models relevant for developed and emerging markets as well as different asset classes should be evaluated out-of-sample to avoid spurious results that may arise when performing the analysis exclusively in-sample, (ii) equilibrium adjustment, time-varying risk premia, asset return dynamics and contemporaneous equity returns play an important role in describing fluctuations of international real estate security prices, (iii) the aggregate impact (in absolute terms) and the 'goodness of fit' of US monetary policy on international equity and real estate prices is increasing over time, hinting at time-varying world market integration and (iv) analysts would get most efficient approximations of future asset returns and asset volatility when subjecting a broad set of alternative predictors to forecast combinations.

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Preface

The present thesis is a collection of essays written at the University of Kiel during my period as a doctoral candidate in the program 'Quantitative Economics'. The essays have been organized in book format and interconnected in order to make the thesis more readable. As in the case of Chapter 1 which has been already accepted for publication, all other Chapters were written with the end goal to be published in refereed journals. The theme of the dissertation is 'International Asset Prices: Empirical Evidence'. The articles in this thesis are:

1. Chapter 1: Herwartz Helmut and Morales-Arias Leonardo (2009). In-sample and out-of-sample properties of international stock return dynamics conditional on equilibrium pricing factors. *European Journal of Finance*, 15, 1-29.

2. Chapter 2: Morales-Arias Leonardo (2009). An empirical analysis of international real estate securities: Long-run equilibrium, short-run dynamics and contemporaneous dependence. *Working Paper*, University of Kiel, 1-40.

3. Chapter 3: Herwartz Helmut and Morales-Arias Leonardo (2009). On US monetary policy and international asset prices. *Working Paper*, Kiel Institute for the World Economy, 1-38.

4. Chapter 4: Lux Thomas and Morales-Arias Leonardo (2009). Forecasting volatility under fractality, regime-switching, long memory and Student-t innovations. *Working Paper*, Kiel Institute for the World Economy, 1-35.

Citations from the co-authored work in this thesis found in Chapters 1, 3 and 4 should be done from the above articles.

Introduction

The recently witnessed financial turmoil has uncovered the need for a deeper understanding of international asset price fluctuations. In particular, it is now more evident that the accelerated globalization process has made developed financial markets more fragile to potential crises (most notably) in the United States. However, we have also witnessed that emerging markets remained quite resilient to the global financial turmoil at first but that they came under increasing pressure due to the unavailability of financing from abroad, capital outflows and in some cases tighter domestic liquidity conditions (IMF (2008)). The latter outcomes raise some questions about the differences and similarities of asset prices in developed and emerging markets and the need of analyzing and understanding them better. Moreover, it seems also relevant to study international asset price fluctuations from an *ex-ante* rather than an *ex-post* perspective in order to shed light on possible empirical models that may be useful for forward-looking as opposed to retrospective policies.

In the first Chapter of the present thesis we contribute to the latter issues, by conducting a comprehensive analysis of the in-sample and out-of-sample properties of stock return dynamics in 14 developed and 12 emerging markets. We start by formulating an international asset pricing model that decomposes log stock returns into equilibrium pricing factors (accounting and discount factors) and short-run (vector) autoregressive dynamics. Based on this model, we design both in-sample and out-of-sample *panel* modeling techniques to investigate international stock market returns at short and long horizons. To preview some of the results in this Chapter with policy implications we find that: (i) asset pricing models relevant for developed and emerging markets should be evaluated out-of-sample to avoid spurious results that may arise when performing the analysis exclusively in-sample and (ii) analysts would get most efficient approximations of future asset returns when subjecting a broad set of alternative predictors (pricing factor models, the random walk and vector autoregressions) to forecast combinations in both developed and emerging markets.

The current crisis has also shown that very little is known about real estate assets, an asset class that accounts for almost 50% of households' asset portfolio. Most of the empirical finance studies available in the literature have focused on equity and bond markets and very little attention has been given to understanding the characteristics of real estate securities and their interrelationship with other asset classes. Uncovering some of the empirical determinants of international real estate securities can be useful for preventing or managing future crises in these markets.

In the second Chapter of the present thesis we examine a real estate pricing model for a cross-section of international financial markets. We model real estate security returns conditional on the long-run equilibrium error between (log) real estate security prices, equity prices and bond prices as well as conditional on the returns and the time-varying volatility of the latter three asset types. The model also incorporates the time-varying volatility of a constructed 'world' asset factor that mimics changes of regimes in international asset markets and the time-varying covariances of this factor with real estate, equity and bond returns. In addition, we also study the contemporaneous dependence of international real estate security returns to international equity returns. We estimate the model using monthly data for the sample period 01/1996-12/2006 and we design a battery of in-sample and out-of-sample techniques to analyze it. To preview some of the results in this Chapter with policy implications we find that: (i) international real estate, equity and bond markets are financially integrated both within and across countries and (ii) time-varying risk premia, asset return dynamics and contemporaneous equity returns play an important role in describing fluctuations of international real estate security prices.

Another gap in the empirical finance literature relevant to the current international stability context is the lack of understanding about the impact of US monetary policy (or other foreign shocks) on international asset markets. The latter relationship is very important. Domestic monetary authorities could presumably take better decisions in light of a potential international financial crisis if they have a more accurate understanding of the effects of US monetary policy on their asset markets. For instance, coordinated and coherent policy actions across countries could more rapidly and effectively stabilize the global financial system in times of crises (IMF (2008)).

In the third Chapter of the present thesis, we contribute to the understanding of the empirical relationship between US monetary policy and international equity, bond and real estate markets for the sample period 01/1994 to 12/2007. Our empirical approach is based on the recent methodology of Identification through Heteroskedasticity (IH) proposed by Rigobon & Sack (2004). We employ the IH approach and extend it to the international context by means of a heterogeneous panel framework. We consider issues such as controlling for dynamic components in the conditional mean of international asset returns, selection of non-policy dates in the international context, recursive estimation and bootstrap inference. To preview some of the results with policy implications in this Chapter we find that: (i) the impact of US monetary shocks on international equity and bond prices is statistically significant on bordering countries and countries with inflation targeting as well as at the aggregate level and (ii) the aggregate impact (in absolute terms) and the 'goodness of fit' of US monetary policy on international equity and real estate prices is increasing over time.

Last but not least, the present financial crisis has taught practitioners, academics and policy makers that there is a need for devising models, polices and institutions whose purpose is of managing risk as well as surveilling asset markets. For instance, regulators use Value-at-Risk (VaR) calculations to work out how much capital financial institutions need to put aside for troubled times. However, according to The Economist (2009), VaR calculations have been flawed because risk managers have turned a blind eye to 'tail risk', i.e. they have ignored extreme events or the 'fat tails' of the empirical distributions of asset returns as proposed by Mandelbrot (1999). The latter issues could be addressed better with forward-looking volatility models that can predict extreme events more accurately but also take account of other stylized facts of asset returns such as long-range dependence and multifractality.

To this end, in the fourth and last Chapter of this thesis we turn to volatility forecasting in international asset markets. We study the performance of volatility models that incorporate features such as long (short) memory, regime-switching and multifractality along with two competing distributional assumptions of the error component, i.e. Normal vs Student-t. Our precise contribution is twofold. First, we introduce a new model to the family of Markov-Switching Multifractal models of asset returns (MSM), namely, the Markov-Switching Multifractal model of asset returns with Student-t innovations (MSM-t). Second, we perform a comprehensive forecasting analysis of the MSM models as well as other competing volatility models of the GARCH legacy (GARCH, GARCH-t, FIGARCH and FIGARCH-t) in international asset markets. Our cross-sections consist of all-share equity portfolios, bond indices and portfolios of real estate securities at the country level. Furthermore, we investigate whether there is an improvement upon forecasts from single models when optimally combining forecasts obtained from the different models at hand. To preview some of the results with policy implications we find that: (i) empirical forecasts of volatility models with Student-t innovations, multifractal components and long memory are adequate for volatility forecasting in international asset markets and (ii) forecast combinations obtained from the different volatility models considered in this Chapter provide a clear improvement upon forecasts from single models for forecasting international asset market volatility.

There is one major point of intersection that we have found in the aforementioned analyses and that prevails amongst the different streams of the modern empirical finance literature: asset price behavior is difficult to fully understand. The finance community has, nevertheless, arrived to a consensus about the stylized facts of asset prices. Some of the stylized facts applicable in the context of this thesis can be summarized as follows: (i) asset prices are non-stationary, (ii) asset returns are leptokurtic and (iii) asset returns exhibit heteroskedasticity and volatility clustering. However, asset price data at different frequencies may exhibit diverse 'degrees' of items (i), (ii) and (iii). For instance, asset prices at the monthly frequency have mostly been found to be integrated of order one while asset prices at the daily frequency have been found to be fractionally integrated (cf. Baillie & Bollerslev (1994), Gil-Alana (2006)).

Thus, asset prices at the monthly (or lower) frequency may be relevant for establishing a relationship with macroeconomic variables since the latter indicators are usually also found to be integrated of order one. On the other hand, while asset returns at low and high frequencies exhibit items (ii) and (iii), asset returns at higher frequencies such as daily or tic-by-tic may posses more complex characteristics such as long memory and multifractality. Thus, asset prices at high frequencies may be more relevant to the analysis of volatility measurement/forecasting, risk management models or market micro-structure models.

The two sets of characteristics from asset price data at low and high frequency also give rise to two streams in the empirical finance literature. One stream focuses on understanding the relationship between asset prices and the macroeconomy while the other stream focuses on understanding the 'behavioral' or 'phenomenological' aspect of asset prices which may translate at the aggregate level. Following Cochrane (1996, 2006), a major proponent of the macro-finance stream, the macroeconomic risk that drives asset prices and expected returns is a key question of finance. The link between macroeconomic variables and asset returns can be induced from the empirical evidence that many of the variables that have predictive power for asset returns can also predict variables that describe economic activity over the business cycle. Following Lux (2007), a major proponent of the behavioral finance stream, asset prices at high frequencies (e.g. daily or tic-by-tic) are characterized by a set of statistical properties that prevail with surprising uniformity. The design of models that can describe the stylized facts of asset prices at higher frequency draw their inspiration from models in the behavioral finance literature and is closer to models of multi-particle interaction in physics. The basic idea is that the regularities observed in asset prices at high frequencies (e.g. volatility clustering, fat tails, scaling behavior, long memory, etc) can be explained via the microscopic interactions of the constituent parts of a complex system.

In this thesis we obtain results that intersect between the two streams, and which may be employed in tandem to have a better understanding of the behavior of asset prices in general. For instance, subjecting forecasts of either 'theoretical' (equilibrium pricing models) and 'atheoretical' (random walk model, autoregressive models) models to forecast combinations can improve upon their single forecasts. The latter result is in line with the behavioral finance literature which suggests that ex-ante asset returns could be better approximated by combining forecasts of 'fundamentalists' who subject asset return forecasts to fundamental factors (e.g. dividend yields, interest rate variables) and 'chartists' who subject their forecast to atheoretical factors (e.g. autoregressive models). Similar results also apply to volatility modeling: combining forecasts of 'traditional' volatility models (e.g. (FI)GARCH) and the new 'behavioral' volatility models (e.g. MSM) can also improve upon their single forecasts.

Throughout our studies we diagnose a significant degree of heterogeneity in international asset price fluctuations which suggests that it is difficult to determine the 'right' global model. While certain components of international asset prices may be deemed homogeneous (and modeled as such) we also find that they can be composed of various heterogeneous components in the short run which may be determined, e.g. by the asset class and/or the particular cross-section under inspection. For example, we obtain that ex-ante returns of developed stock markets can be explained from theoretical variables while the opposite is true for emerging markets. Additionaly, we find that the risk-return relationship in international real estate securities is country specific, while we diagnose a homogeneous long-run equilibrium relationship across countries for the latter asset class. Similarly, we obtain that the impact of US monetary shocks on international equity markets depends on the asset class considered and on how close is the country to the US or whether it has similar monetary policy frameworks (e.g. inflation targeting). Lastly, our results indicate that the applicability of various volatility models depends on the asset class considered (i.e. equity, bonds or real estate).

We find that a good way to analyze asset pricing models (either 'traditional' or 'behavioral') is by using panel data. The latter is promising in two main directions. From an economic perspective, using panel data allows analysts to avoid spurious conclusions that may arise from using a model that is only informational to a particular country or sector. From an econometric perspective, panel data methods come along with an augmentation of sample information and thus are likely to improve the power of statistical tests.

Another important result that arises from the present thesis is the importance of studying asset pricing models via recursive methods and out-of-sample diagnostics. Most asset pricing studies provide results which are many times sample dependent and that focus exclusively on the in-sample evidence of predictability. While those models are still important to provide a general idea of which factors may contribute to explain asset price fluctuations, from a practical perspective they may lead analysts to reach false conclusions on the applicability of the model to (say) risk management strategies.

In general, the empirical results of this thesis suggest that international asset pricing models should be adaptive and more data-driven. Moreover, they should be analyzed with various data analytical tools and specification tests in order to uncover frictions inherent in asset markets. The latter recommendation is in line with the recent critique to contemporaneous economics by Colander *et al.* (2009) who argue that data-driven models have a better chance of nesting a multivariate, path-dependent data-generating process and relevant macroeconomic theories.

Each Chapter of the present thesis has a corresponding Appendix with a more detailed description of the econometric methods used that are not provided in the main text. Moreover, for Chapter 1 we also provide a simple theoretical model that motivates the (baseline) empirical specifications studied. Given the essay nature of this thesis, there is no dependence in notation throughout the Chapters.

Chapter 1

In-Sample and Out-of-Sample Properties of International Stock Return Dynamics Conditional on Equilibrium Pricing Factors

1.1 Introduction

In the earlier literature on stock return predictability the random walk model was generally used to provide an approximation of log price behavior and to offer a base of support for the Efficient Market Hypothesis (EMH) in its weak form (Alexander (1961), Fama (1965), Godfrey *et al.* (1963)). However, during the 1970's several studies motivated that static models fail to capture time-varying conditional features of stock returns (Fama (1970)). In the 1980's several contributions to the return predictability literature underscore that the random walk poorly describes stock price behavior (Shiller (1981), LeRoy & Porter (1981), Fama & French (1988a), Lo & McKinlay (1988), Poterba & Summers (1988)). Since the 1990's, the general consensus is that asset pricing models which incorporate time-varying factors can explain return dynamics more accurately (Fama (1991), Campbell (2000)). Macroeconomic aggregates, equilibrium relationships and time-varying betas have become, since then, popular candidates employed in return predictability studies to describe expected returns.

Most asset pricing studies that successfully explain expected returns have focused on the *in-sample* evidence of predictability. Variables such as dividend yields, earning yields, and earnings to dividend ratios have shown to have explanatory power for returns especially at longer horizons (Keim & Stambaugh (1986), Campbell & Shiller (1987, 1988), Fama & French (1988a, 1989), Hodrick (1992), Lee (1998)). Other macro variables employed in these type of studies include the term spread, the default spread, expected inflation, consumption to wealth ratios, the log stock price-market index ratio and aggregate output amongst others (Chen *et al.* (1986), Campbell & Mankiw (1989), Marathe & Shawky (1994), Mills (1996)). The selection of the latter variables for models of aggregate stock returns is usually based on the hypothesis that they may affect the two major components of asset prices, i.e. the cash flow component and/or the discount rate component. Recent studies have provided evidence from an intertemporal Capital Asset Pricing Model (ICAPM) that accounting measures have a stonger effect on value stocks while discount rates have a pronounced effect on growth stocks (Campbell & Vuolteenaho (2004), Campbell *et al.* (2005)). In an international context, accounting components such as book-to-market ratios, cash flow-to-price ratios and GDP, coupled with discount factor components such as exchange rates, money supply and interest rates can explain international asset returns (Wongbangpo & Sharma (2002), Hou *et al.* (2006)).

Only a few recent empirical contributions have scrutinized the *out-of-sample* performance of dynamic return models that use some of the latter variables as conditioning factors. In particular, accounting variables such as the log dividend-earnings ratio and book-to-market ratios as well as discount factor variables such as detrended consumption-wealth ratios and term spreads have shown to forecast excess returns in the US (Lamont (1998), Lettau & Ludvigson (2001a), Campbell & Thompson (2007)). In the international market arena, various types of interest rate variables have also provided evidence of out-of-sample forecasting power (Harrasty & Roulet (2000), Rapach *et al.* (2005)).

In this Chapter we conduct a comprehensive analysis of the in-sample and out-of-sample properties of stock return dynamics in 14 developed and 12 emerging markets. We start by formulating an asset pricing model that decomposes log stock returns into equilibrium pricing factors (both accounting and discount factors) and short-run (vector) autoregressive dynamics. Based on this model, we design in-sample and out-of-sample *panel* modeling techniques to investigate international stock market returns conditional on equilibrium relationships at short and long horizons.

We argue that *in-sample* return predictability does not necessarily imply *out-of-sample* return forecastability. When modeling longitudinal processes, the econometrician runs the risk to falsely apply structurally invariant models to data that undergo some shift of the underlying dynamic structure. In such scenarios diagnosed parameter significance might be more a statistical artifact rather than an empirical underpin of the postulated asset pricing model (Neely & Weller (2000), Andrew & Bekaert (2001), Hjalmarsson (2006), Rapach & Wohar (2006)). Moreover, numerous outstanding factors such as lag order selection, the dimensionality of the parameter space, integration and cointegration features influence the performance of predictive regression models in a way that might be hard to control (Lewellen (2004), Campbell & Yogo (2006)). For the latter reason one may alternatively or at least complementary evaluate the ability of postulated models to provide accurate ex-ante forecasts.

Ex-ante forecasting studies of this kind have mainly focused on individual markets without addressing the aggregation issue from a cross sectional perspective. As introduced previously, exploiting the panel dimension is promising in two directions. First, it may uncover if predictive or forecasting content is market specific and thereby guards against potentially spurious conclusions from single market analyses. Second, panel data analysis goes along with an augmentation of sample information and, thus, is likely to improve the power of statistical tests. The latter merit is particularly relevant in our context since financial data exhibit first order dynamics which are typically hidden by relatively large noise components.

The Chapter further addresses the lack of attention that has been given to modeling stock return dynamics in *emerging markets*. Some main characteristics of *emerging markets* can be found in Bekaert & Harvey (2003) (with further references) and may be summarized as follows: (i) average returns and dividend yields usually decline after liberalization, (ii) stock markets may become more volatile due to faster price adjustment in reaction to relevant information, (iii) correlations with the world market increase with more financial integration. These results imply in our context that data for *emerging markets* may suffer from sharp fluctuations and structural breaks. Consequently, it should be more difficult to find a transparent relationship between equity returns and long-run variables.

To preview the results, we find that (i) there is evidence of in-sample signaling from the equilibrium relations but this feature does not appear to translate into out-of-sample forecasting, (ii) rolling window forecasting can better approximate the distributional features of returns in comparison with a recursive scheme, (iii) forecasting with single lagged equilibrium relationships does not play a uniformly significant role in forecasting stock returns, (iv) forecasting with a full model containing all lagged equilibrium relations can outperform a random walk model and a VAR(1) model in developed markets, and (v) linear combinations of forecasts of alternative models reduce forecast uncertainty and improve other performance measures of singular models.

The remainder of the Chapter is organized as follows. The subsequent section presents the theoretical and empirical specification of our model of log returns. Section 1.3 addresses estimation issues. Section 1.4 provides the in-sample diagnostics and the evidence on return predictability. Section 1.5 presents the out-of-sample analyses. Section 1.6 concludes. The formal derivation of the asset pricing model and a more detailed representation of the econometric toolkit are given in Appendix A.

1.2 The model

This section presents a present value model that describes stock price movements from expectations of discounted log earnings-dividend ratio, earnings growth and the log stochastic discount factor. Furthermore, we present an empirical model which is derived from some assumptions on the stochastic behavior of the latter variables.

1.2.1 Theoretical framework

Modern research on asset pricing has been based on the theoretical finding that in the presence of complete markets, there exists a unique stochastic discount factor (SDF) that establishes the relationship between the payoff from assets and the price for all assets in the economy. The major advantage of the SDF framework is its universality. The SDF model is general enough to derive traditional models (CAPM, APT, etc.) as special cases. The SDF framework basically summarizes the idea that rational investors must value uncertain future (state contingent) payoffs to determine the price they are currently willing to pay for a share of stock. The higher the *rate* at which investors discount future payoffs the lower the price they should pay for a share of stock today. The latter considerations can be formalized in more detail following our variations of the models in Campbell & Shiller (1988) and Cochrane (2001).[1] From the first order conditions of a representative agent who maximizes expected discounted utility of consumption one obtains the present value relation,

$$P_t = E_t\left[M_{t+1}X_{t+1}\right], \tag{1.1}$$

where P_t is the *ex-dividend* price of the stock at period t, E_t is the expectations operator conditioning on information at period t and $X_t = P_t + D_t$ is the payoff the investor receives at period t where D_t denotes the dividend payed during period t. In addition, $M_{t+1} = \delta(U'(C_{t+1})/U'(C_t))$ is the stochastic discount factor (SDF henceforth) which generalizes standard discount factor ideas where $U'(C_t)$ is the marginal utility of consumption at period t and δ is a constant discount factor.[2]

Following Campbell (2000) equation (1.1) states that asset prices can be written as a state-price-weighted average of the payoffs in each state of nature. In the absence of arbitrage opportunities, a set of positive state prices and a positive SDF exist. Thus, if markets are complete then state prices and the SDF are unique. Log-linearizing the above expression yields,

$$p_t = E_t\left[\mu_{t+1} + m_{t+1} + x_{t+1}\right], \tag{1.2}$$

[1]Specific details on the derivations are provided in Appendix A.

[2]see Cochrane (2001) or Campbell *et al.* (1997) for further details.

where $p_t = \ln P_t$, $m_{t+1} = \ln M_{t+1}$ and $x_{t+1} = \ln(P_{t+1} + D_{t+1})$ denote the log stock price, the log SDF and the log payoff, respectively, and $\mu_{t+1} = (1/2)\sigma^2_{\mu,t+1}$ is a variable depending on (conditional) second order moments of m_t and $\tilde{x}_t = \ln X_t P^{-1}_{t-1}$. Following Campbell & Shiller (1988) it is possible to show that $x_{t+1} = \ln(P_{t+1} + D_{t+1}) \approx c + \theta p_{t+1} + (1-\theta) d_{t+1}$, where $c = -\ln(\theta) - (1-\theta)\ln(1/\theta - 1)$ is a linearization constant, $d_t = \ln D_t$ and $\theta = 1/(1 + D/P)$ is the average price/payoff ratio. Now define $a_t = \ln A_t$ where A_t is used to denote earnings per share at period t. Using the latter approximation for x_t in (1.2), adding $(1-\theta)\Delta a_{t+1}$ from both sides of (1.2), solving forward for $(p_t - a_t)$ and imposing the (no bubble) transversality condition $\lim_{j\to\infty} \theta^j E_t(p_{t+j} - a_{t+j}) = 0$ yields,

$$p_t - a_t = \frac{c}{1-\theta} + E_t \sum_{j=1}^{\infty} \theta^{j-1} \left[\mu_{t+j} + (\theta - 1)(a_{t+j} - d_{t+j}) + \Delta a_{t+j} + m_{t+j}\right]. \quad (1.3)$$

Equation (1.3) states that the log price-earnings ratio at period t is a linear function of risk-premia μ_t, the log earnings-dividend ratio $(a_t - d_t)$, earnings growth Δa_t and the log SDF m_t. More precisely, (1.3) expresses that if the log price-earnings ratio is high today, investors expect some combination of high risk, low future log earnings-dividend ratio, high earnings growth and high future stochastic discounting.

Note the similarity of this equation with the seminal log-linear present value model in Campbell & Shiller (1988). In this case we have accounted for three new issues. First, we specify an equation where both the deviations between prices and earnings and earnings and dividends are taken into account. According to Campbell (2000) there is an increasing interest in using earnings rather than dividends as driving variables in present value models of stock prices since dividends are more difficult to observe. Lee (1998) and Lamont (1998) show the importance of including the relationship between earnings and dividends in present value models. Secondly, we formalize a relation between stock prices and a log SDF rather than between stock prices and log returns. The former is more general since the log SDF can be specified depending on the interest at hand (i.e. consumption or a factor model). Finally, the model comprises a variable μ_t that depends on second order moments and could be modeled to account for risk premia.

1.2.2 Empirical specification

The model in (1.3) implies that if p_t, a_t and d_t are integrated of order one, $I(1)$, i.e. stationary after taking first differences, then the linear combinations $p_t - a_t$ and $a_t - d_t$ must be $I(0)$, for the right and left hand side of (1.3) to be balanced. In that case, log price-earnings and log earnings-dividend ratios are cointegrated with a unit cointegrating parameter, briefly denoted $CI(1, -1)$. In addition, the model describes log price-earnings movements from expectations of the variables in the right hand side of (1.3). As shown in Appendix A, it is possible to derive from (1.3) a linear model of the form,

$$\Delta p_t = v_t + u_t, \tag{1.4}$$

where $\Delta p_t = \ln P_t - \ln P_{t-1}$ is the (log) asset return, v_t is the time-varying mean and u_t is a zero-mean disturbance term (or 'shock') which could be modelled depending on the interest at hand. In the present Chapter, the time-varying mean is specified as,

$$v_t = \mu + \alpha_1 q_{t-1} + \alpha_2 g_{t-1} + \alpha_3 h_{t-1} + \alpha_4 y_{t-1} + \alpha_5 s_{t-1}, \tag{1.5}$$

where $q_t = (p_t - a_t)$, $g_t = (a_t - d_t)$ and h_t, y_t and s_t are three observable $I(0)$ equilibrium relationships that proxy the evolution of m_t (the log SDF). For the definitions of the discount factors h_t, y_t and s_t we employ three popular equilibrium relations used in the literature although the specification for the log SDF can be augmented to account for more equilibrium pricing factors. The first factor we consider is the difference of the log stock price and the log market portfolio index, that is we let $h_t = p_t - p_t^m$, $p_t^m = \ln P_t^m$. If expected stock returns and expected market portfolio returns are $I(0)$ and both p_t and p_t^m are $I(1)$ then their linear combination should be $I(0)$, or in other words the log stock price and the log market index are $CI(1, -1)$.[3] As a second factor we regard the term spread, $y_t = (r_t^l - r_t^s)$, the difference from a l-period (r_t^l) and a s-period rate of interest (r_t^s), $l > s$. According to the expectations hypothesis, y_t should equal a weighted average of future expected changes in the s-period interest rate. If these

[3]See Mills (1999), chapter 7 for further details.

changes are $I(0)$ and r_t^l and r_t^s are $I(1)$ then the latter are $CI(1,-1)$.[4] Lastly, we consider an interest differential from the l-period domestic (r_t^l) and foreign (r_t^{fn}) rate, $s_t = (r_t^l - r_t^{fn})$. According to the uncovered interest rate parity s_t should equal expectations of future changes in the spot exchange rate.[5] If changes in the spot exchange rate are $I(0)$ and r_t^l and r_t^{fn} are $I(1)$ then the domestic and foreign long term interest rate are $CI(1,-1)$. Using the definitions for h_t, y_t and s_t in (1.5) we get the empirical model,

$$\Delta p_t = \mu + \alpha' f_{t-1} + u_t, \tag{1.6}$$

where $f_t = (p_t - a_t, a_t - d_t, p_t - p_t^m, r_t^l - r_t^s, r_t^l - r_t^{fn})'$ and $\alpha = (\alpha_1, \ldots, \alpha_5)'$ is the vector of parameters attached to the equilibrium pricing factors. According to (1.6), returns are explained by the lagged log price-earnings ratio (PE), the log earnings-dividend ratio (ED), the log stock price-market index ratio (PPM), the term spread (TES) and the interest differential (IND). Moreover, we note that this empirical model for log price changes can be subdivided into two main groups of equilibrium pricing factors, namely, an 'accounting' component (i.e. financial indicators) $q_t = (p_t - a_t)$ and $g_t = (a_t - d_t)$, and a discount factor component, $h_t = (p_t - p_t^m)$, $y_t = (r_t^l - r_t^s)$ and $s_t = (r_t^l - r_t^{fn})$.

1.3 Econometric methodology

In this section we provide a brief description of the data employed in our study. We also present the econometric implementation of the empirical model in (1.6) for short to long horizon returns.

[4] see Campbell *et al.* (1997), chapter 10 and 11 for further details.
[5] see Daniels & VanHoose (1999) chapter 7 for further details.

Developed markets	T_i	Emerging markets	T_i
Australia	269	Argentina	138
Austria	228	Chile	133
Belgium	301	Colombia	143
Canada	301	Czech Republic	132
France	298	Mexico	175
Germany	301	Pakistan	150
Ireland	228	Poland	131
Italy	228	Philippines	193
Japan	290	South Africa	288
Netherlands	228	Sri Lanka	192
Norway	276	Thailand	216
Spain	215	Turkey	178
Switzerland	301		
United Kingdom	300		
Total	3764		2069

Table 1.1: Number of available observations for developed and emerging markets. At the aggregate level 3022 and 1433 ($h = 1$), 2994 and 1409 ($h = 3$), 2952 and 1373 ($h = 6$) ex-ante forecast errors are available for developed and emerging markets, respectively.

1.3.1 The data

We analyze time series data for $N = 14$ developed markets and $N = 12$ emerging markets. Let $i = 1, ..., N$ indicate the cross sectional entities. In principle, we attempt to collect data at the monthly frequency over the sample period 01/01/1980 to 10/01/2005. Particularly in the case of emerging markets, however, the number of observations differs across economies. Thus, we analyze unbalanced panels and let T_i denote the number of observations available for each cross sectional entity. The specific markets and available observations are listed in Table 1.1.

Datastream calculated total market stock price indices and price/earnings and dividend yields accruing to these indices for each individual country were collected. We use the Datastream world stock price index as a proxy for the market portfolio. In the case of the interest variables we gather three month treasury yields for the short-term and ten year government bond yields for the long-term interest rates from the IMF International Statistics and Datastream. Due to unavailability of this type of data for some particular *emerging markets* we gather certificates of deposits or money market rates for the short term interest rates and fixed term rates for long-term interest rates. We employ the US government 10-year bond yield as a proxy for the long-term foreign interest rate. We do not include the US in the analysis for three reasons: First, including the US would obtain an extremely 'unbalanced' panel in terms of market capitalization. Secondly, as a pricing factor our international interest differential does not apply for the US. Finally we recall that most available studies of this type concentrate on the US. All (log) stock price indices, dividends, earnings and interest rates are treated a priori as $I(1)$.

1.3.2 A model of international stock returns for short and long horizons

As shown in Appendix A, assuming that p_{it}, a_{it} and d_{it} and the interest rates in (1.6) are $I(1)$, their joint dynamics can be given in form of a vector error correction model of order $p-1$, VECM($p-1$), for the ($K = 7$)-dimensional vector $z_{it} = (p_{it}, a_{it}, d_{it}, p_t^m, r_{it}^s, r_{it}^l, r_t^{fn})'$. According to (1.6), we can formalize five ($r = 1, ..., 5$) cointegrating relationships in the matrix of over-identifying restrictions of the VECM($p-1$) system, namely, the stationarity of the log PE, ED and PPM ratio, the term spread and the interest rate differential. Since our interest is in the analysis of conditional returns we pick out the first equation of the VECM, i.e.

$$\Delta p_{it} = \mu_{1,i} + \alpha'_{1,i} f_{it-1} + \sum_{j=1}^{p-1} b'_{1j,i} \Delta z_{it-j} + u_{1,it}, \tag{1.7}$$

where $\alpha_{1,i} = (\alpha_{11,i}, \ldots, \alpha_{15,i})'$ is a vector of Error Correction (EC) coefficients of cointegrating relations attached to $f_{it} = (p_{it} - a_{it}, a_{it} - d_{it}, p_{it} - p_t^m, r_{it}^l - r_{it}^s, r_{it}^l - r_t^{fn})'$ and $b_{1j,i}$ are $K \times 1$ vectors of autoregressive parameters attached to Δz_{it-j}, $j = 1, \ldots, p-1$. Note that (1.6) is obtained as a special case of (1.7) if the VAR order is one (i.e. $p = 1$).

Many studies confirm that some of the equilibrium relations in (1.7) have stronger signaling strength for log returns at longer horizons (Fama & French (1988b, 1989), Hodrick (1992), Lettau & Ludvigson (2001b)). Adopting a similar approach to the long horizon framework of the finance literature we reparameterize (1.7) by replacing on the left hand side one period returns by higher horizon counterparts, i.e.

$$\Delta p_{it}^{(h)} = \mu_{1,i} + \alpha'_{1,i} f_{it-1} + \sum_{j=1}^{p-1} b'_{1j,i} \Delta z_{it-j} + u_{1,it}^{(h)}, \tag{1.8}$$

where $\Delta p_{it}^{(h)} = \sum_{l=1}^{h} \Delta p_{it+l-1}$, and, by construction $\Delta p_{it}^{(1)} = \Delta p_{it}$. Note that the explanatory variables in (1.8) are exactly the same as in (1.7). With regard to the model parameters in (1.8) we note that they differ from the corresponding quantities in (1.7) but do not indicate model dependence in our notation. Several approaches have been proposed in the literature to deal with long-horizon regressions and models of overlapping data (Hansen & Hodrick (1980), Newey & West (1987), Hodrick (1992), Goetzmann & Jorion (1993), Nelson & Kim (1993)). In this study we design different in-sample and out-of-sample *panel* techniques to analyze such types of (conditional) short and long horizon regression models.

1.4 In-sample analysis

Apart from providing some empirical time series features, in-sample analyses aim to figure out if particular pricing factors explain stock returns significantly and uniformly over the considered cross sections. To address the latter issue we employ Error Correction Models (ECMs) as given in (1.8) for $h = 1, 3, 6$. The $p = 2$ order is chosen to account for possible serial error correlation when $h = 1$. The following paragraphs briefly mention the applied econometric techniques and then deliver the corresponding empirical results. Technical details about the performed tests are found in Appendix A.

1.4.1 Stationarity analysis

Model (1.7) requires that the imposed cointegrating relations deliver stationary residual series. Thus, we first consider if the actual PE, ED, and PPM ratios, TES, and IND are in fact stationary. We test for the prevalence of stochastic trends governing the latter variables both at the individual and the aggregate levels of *developed* or *emerging markets*. At the individual level we test by means of augmented Dickey Fuller (ADF) statistics (Dickey & Fuller (1979)). The test regressions are specified throughout with a constant and one lag of the dependent variable. At the aggregate level we employ the Levin *et al.* (2002) (LLC) and the Im *et al.* (2003) (IPS) panel unit root tests.

1.4.2 Predictability analysis

In addition to stationarity of the cointegrating vectors, stability of the model in (1.7), requires that the error correction parameter $\alpha_{11,i}$ corresponding to the PE ratio is less than zero. In other words, log stock price changes must decrease (increase) in time t to correct positive (negative) PE ratios (or 'valuation gaps') in $t - 1$. Note that for the remaining equilibrium relations (the ED ratio, the PPM ratio, TES and IND) their specific signs are not a-priori restricted by the presumption of model stability. To study the predictive strength of the equilibrium relationships at short and longer horizons we consider different statistics for the model in (1.7) at the single market and aggregate levels. In-sample analyses comprise:

1. *Single equation models*: At the single market level we infer if single coefficient estimates $\hat{\alpha}_{1r,i}$, $i = 1, \ldots, N$, $r = 1, \ldots, 5$, differ significantly from zero.

2. *Cross sectional Wald test*: At the aggregate level, we test the null hypothesis that EC parameters governing the impact of particular pricing factors ($\alpha_{1r,i}$) are jointly zero over the cross section dimension, i.e. $H_{0,r} : \alpha_{1r,i} = 0$, $i = 1, ..., N$. The Wald statistic is denoted λ_c.

3. *Mean group inference*: To further study the overall signaling strength of EC terms we perform mean group (MG) estimation (Pesaran & Smith (1995)). The MG estimator is denoted $\hat{\alpha}_{MG}$.

	ADF		IPS		LLC	
	$< c_{0.05}$	$< c_{0.10}$	Stat	p-value	Stat	p-value
	Developed markets					
PE	3	4	-3.571	0.000	-2.314	0.010
ED	3	3	-3.870	0.000	-0.474	0.317
TES	8	9	-6.653	0.000	-3.195	0.000
IND	4	7	-3.839	0.000	-1.142	0.126
PPM	2	3	-2.440	0.007	0.003	0.501
	Emerging markets					
PE	4	6	-5.176	0.000	-2.753	0.003
ED	2	4	-3.843	0.000	-1.967	0.024
TES	11	11	-8.570	0.000	-8.159	0.000
IND	4	5	-3.228	0.000	0.282	0.611
PPM	2	3	-4.064	0.000	-5.160	0.000

Table 1.2: Unit root test results by equilibrium relation. ADF critical values $c_{0.05}$, $c_{0.10}$ are -2.86, -2.57, respectively. As stated PE is the log price-earnings ratio $(p_{it} - a_{it})$, ED is the log earnings-dividend ratio $(a_{it} - d_{it})$, TES is the term spread $(r^l_{it} - r^s_{it})$, IND is the interest differential $(r^l_{it} - r^{fn}_t)$ and PPM is the log stock price-market index ratio $(p_{it} - p^m_t)$.

	$\lvert t_{\hat{\alpha}}\rvert > 1.96$	> 1.64,ov.	$\hat{\alpha} < 0$	λ_c	$\hat{\alpha}_{MG}$	t_{MG}	$\lvert t_{\hat{\alpha}}\rvert > 1.96$	> 1.64,ov.	$\hat{\alpha} < 0$	λ_c	$\hat{\alpha}_{MG}$	t_{MG}
EC	Developed markets						Emerging markets					
	$h = 1$											
PE	1	1, 0, 0	9	11.21	-0.008	**-2.185**	4	5, 1, 0	8	**64.35**	-0.022	-1.215
ED	1	2, 0, 0	8	17.67	-0.007	-0.818	4	4, 0, 0	8	**31.71**	-0.005	-0.366
TES	3	5, 0, 0	9	**30.74**	-0.001	-1.012	3	4, 0, 0	4	**27.31**	0.0006	0.306
IND	3	7, 0, 0	11	**29.23**	-0.004	**-3.372**	1	3, 0, 0	6	**25.16**	-0.0007	-0.494
PPM	6	9, 0, 0	13	**60.69**	-0.047	**-5.025**	2	2, 0, 0	7	**30.07**	-0.036	-1.812
	$h = 3$											
PE	2	2, 0, 0	9	13.616	-0.027	**-2.178**	5	5, 0, 0	7	**49.19**	-0.063	-1.227
ED	1	3, 0, 0	10	16.126	-0.035	-1.649	4	4, 1, 0	9	**22.76**	-0.024	-0.594
TES	6	6, 0, 0	7	**35.00**	-0.0008	-0.196	3	4, 0, 0	6	14.71	-0.002	-0.731
IND	3	6, 0, 0	9	**26.03**	-0.009	**-3.080**	2	5, 0, 0	6	**21.47**	0.0001	0.042
PPM	9	10, 0, 0	12	**65.62**	-0.140	**-4.512**	2	2, 0, 0	8	**26.11**	-0.082	-1.665
	$h = 6$											
PE	4	4, 2, 1	10	20.81	-0.068	**-2.703**	6	6, 3, 2	7	**48.94**	-0.129	-1.464
ED	2	5, 1, 0	11	15.49	-0.100	**-2.859**	5	6, 1, 2	11	20.43	-0.094	-1.301
TES	5	6, 0, 1	9	**31.20**	-0.002	-0.302	3	3, 0, 1	6	9.52	-0.008	-1.556
IND	5	6, 0, 1	9	**26.83**	-0.016	**-2.727**	2	7, 1, 1	6	15.85	0.001	0.280
PPM	9	9, 3, 5	11	**56.25**	-0.220	**-3.727**	3	3, 0, 1	7	**24.49**	-0.111	-1.421

Table 1.3: In-sample analysis results for developed and emerging markets. The single equation ECM is $\Delta p_{it}^{(h)} = \mu_{1,i} + \alpha' f_{it-1} + b_{11,i}' \Delta z_{it-1} + u_{1t}^{(h)}$ with $f_{it} = (p_{it} - a_{it}, a_{it} - d_{it}, p_{it} - p_t^m, r_{it}^l - r_{it}^s, r_{it}^l - r_t^{fn})'$ for $h = 1, 3, 6$. Critical values for λ_c are taken from the $\chi^2(N)$ distribution. 'ov.' reports the 'overlap' of cross sections where for a particular pricing factor the absolute in-sample t-ratio exceeds 1.64 and ex-ante ratio $\overline{dr}_i$ is 'significantly' below unity. The latter counts are collected for the recursive, rolling forecasting schemes.

1.4.3 In-sample results

Table 1.2 presents the results on the stationarity features of the equilibrium relations in (1.7). We refrain from reporting detailed ADF statistics over all markets but just show the number of test statistics per EC relation that are smaller than the 5% and 10% critical value, respectively. Moreover, we document the IPS and LLC panel unit root test statistics. Selected results for estimated ECMs are given in Table 1.3. We provide the number of $t-$ratios of the EC coefficients exceeding 1.96 and 1.64 in absolute value, respectively, and the number of negative coefficient estimates. The results for the cross-sectional Wald test and MG inference are also reported. The following discussion throughout refers to the 5% significance level.

1.4.3.1 Stationarity tests

For 2 out of 14 *developed markets* the null hypothesis of a nonstationary PPM ratio can be rejected whereas evidence in favor of nonstationary terms spreads is much weaker. For the latter variable the null hypothesis of a stochastic trend is rejected e.g. for 8 *developed markets*. With respect to *emerging markets* and depending on the considered cointegrating relation the number of significant test statistics varies between 2 (ED, PPM) and 11 (TES). Testing against a heterogeneous alternative, the IPS test obtains that a panel unit root is rejected in both cross sections for each cointegrating relation. The homogeneous LLC test, however, yields evidence of nonstationarity for ED, PPM and IND in *developed markets* and IND in *emerging markets*.

1.4.3.2 Predictability tests

From the results for in-sample fitting (Table 1.3) it is seen that most estimated EC coefficients in the case of *developed markets* show a negative sign over all postulated cointegrating relationships. With respect to short horizon returns ($h = 1$) we find that between 9 (PE) or 13 (PPM) *developed markets* show parameter estimates which are less than zero. With regard to single market results we obtain that at short horizons, between 1 (PE) and 6 (PPM) *developed markets* yield an error correction coefficient which is significant according to the respective heteroskedasticity consistent t-ratios. When modeling stock returns at higher horizons, the number of significant coefficients is between 2 (PE) and 9 (PPM) for $h = 3$ and 4 (PE) and 9 (PPM) for $h = 6$ according to autocorrelation and heteroskedasticity consistent t-ratios. In *emerging markets* 4 to 8 (out of 12) single markets show negative EC coefficients for $h = 1$. Thus, significance of error correction dynamics at short and medium horizons is less often diagnosed in *emerging* markets in comparison with *developed markets*.

At the aggregate level MG ($\hat{\alpha}_{MG}$) inference reveals that the PE, PPM and IND variables have a significantly negative effect on log returns at short horizons in *developed markets*. At longer horizons also the ED ratio exerts a significantly negative impact. The signaling strength of the above mentioned equilibrium relationships confirms previous studies by Mills (1996), Lee (1998), Lamont (1998) and Lettau & Ludvigson (2001b) amongst others. With respect to *emerging markets*, no lagged equilibrium relationship has a significant effect on market returns neither at short nor at longer horizons. The insignificance of the MG estimator hints at marked heterogeneity characterizing the cross section of emerging markets. For *developed markets* the joint null hypothesis $H_{0,r} : \alpha_{1r,i} = 0$, $i = 1, \ldots, N$, is rejected with the Wald test λ_c for discount factors PPM, TES, and IND at horizons $h = 1, 3, 6$. As for *emerging markets* $H_{0,r} : \alpha_{1r,i} = 0$, can be rejected for all the cointegrating relations when modeling short horizon returns ($h = 1$).

Summarizing in-sample estimation results we find heterogeneity with respect to return predictability from particular equilibrium pricing factors amongst *developed* and *emerging* equity markets. Accounting factors (PE and ED ratios) appear to have more indicative power in *emerging* than in *developed* markets while discount factors appear to signal stock returns in both cross sections. It is worthwhile mentioning that all in-sample results are obtained presuming structural model invariance. Significance of EC coefficients might be spurious owing to neglected model instability. Given the former results on the LLC test one should a-priori be careful in interpreting in-sample inference. From this perspective it appears preferable to conduct ex-ante forecasting to assess the potential of pricing factors for modeling stock returns.

1.5 Out-of-sample analysis

Model assessment based on the forecasting performance may be preferred over in-sample fitting since issues such as the number of model parameters, integration and cointegration features, regime switching and lag selection affect the estimation of predictive regression models non trivially (Lewellen (2004), Campbell & Yogo (2006)). This section describes the ex-ante forecasting design and evaluation methods. Forecasting results are then discussed in Section 1.5.3. Technical details on forecast diagnosis and combination are provided in Appendix A.

Model	$\Delta p_{it}^{(h)} = \mu_{1,i} + \alpha'_{1,i} f_{it-1} + \sum_{j=1}^{p-1} b'_{1j,i} \Delta z_{it-j} + u_{1,it}^{(h)}$
RW	$\alpha'_{1,i} = 0$, $p = 1$
VA1	$\alpha'_{1,i} = 0$, $p = 2$
VA2	$\alpha'_{1,i} = 0$, $p = 3$
EC1	$f_{it} = (p_{it} - a_{it})$ (PE), $p = 2$
EC2	$f_{it} = (a_{it} - d_{it})$ (DE), $p = 2$
EC3	$f_{it} = (r_{it}^l - r_{it}^s)$ (TES), $p = 2$
EC4	$f_{it} = (r_{it}^l - r_t^{fn})$ (IND), $p = 2$
EC5	$f_{it} = (p_{it} - p_t^m)$ (PPM), $p = 2$
EC6	$f_{it} = (p_{it} - a_{it}, a_{it} - d_{it})'$ (accounting factors), $p = 2$
EC7	$f_{it} = (p_{it} - p_t^m, r_{it}^l - r_{it}^s, r_{it}^l - r_t^{fn})'$ (discount factors), $p = 2$
EC8	$f_{it} = (p_{it} - a_{it}, a_{it} - d_{it}, p_{it} - p_t^m, r_{it}^l - r_{it}^s, r_{it}^l - r_t^{fn})'$, $p = 2$
	Singular forecasts entering the combination
CO1	EC1 to EC5 (single factors)
CO2	EC6, EC7 (joint factors)
CO3	EC1, EC2 (accounting factors)
CO4	EC3, EC4, EC5 (discount factors)
CO5	RW, VA1, VA2, EC1 to EC5
CO6	RW, VA1, VA2, EC6 to EC8

Table 1.4: Control models (upper panel) and combined forecasting models (lower panel)

1.5.1 The forecasting design

1.5.1.1 Forecasting schemes

We apply a recursive and a rolling windows forecasting procedure. Let $\tau - 1$ denote the last time instance available to implement a single forecast iteration. The first forecasting scheme is applied recursively by increasing the in-sample period from $\tau - 1 = T_{min} = 48$ to $\tau - 1 = T_i - h$ (see Table 1 for T_i). In the second scheme of rolling window forecasting we fix a window of size $T_{min} = 48$ which is moved over the subsample $\tau - 1 = T_{min}, \ldots, \tau - 1 = T_i - h$. Let $\Delta\hat{p}_{i,\tau}^{(h)}$ denote an ex-ante forecast for $\Delta p_{i,\tau}^{(h)}$. Forecast errors are denoted

$$\hat{u}_{i,\tau}^{(h)} = \Delta p_{i,\tau}^{(h)} - \Delta\hat{p}_{i,\tau}^{(h)},\ \tau = 49, \ldots, T_i - h + 1. \tag{1.9}$$

A priori, rolling forecasting schemes are likely better immunized against the adverse effects of falsely imposing structural invariance as they build upon parameter homogeneity within a 'small' time window of observations. As a consequence, the former results on unit root testing might not be representative for each subsample used below for forecasting exercises. Note that after iteration $\mathcal{T}_i = T_i - h - 47$ forecast errors enter the cross model comparisons.

1.5.1.2 Control strategies

In order to study which particular (set of) cointegrating relation(s) contributes best to forecasting stock returns we follow two strategies. We formalize a benchmark model where only a constant and first order autoregressive variables contained in Δz_{it-1} are used for stock return forecasting. Since such a model specification leaves the EC framework, we refer to it as VAR(1) in first differences (VA1). The latter model is suitable as a benchmark specification since augmenting it sequentially with EC terms might signal cointegration and indicate the most relevant equilibrium relationships entering price formation.

Firstly, we augment VA1 with each cointegrating relation separately and obtain 5 forecasting models, denoted EC1 to EC5. Secondly VA1 is augmented with the accounting factors (i.e., PE, ED), the discount factors (i.e., PPM, TES, IND) and both factor sets jointly. Joint factor strategies are denoted EC6, EC7 and EC8, respectively. The particular advantage of controlling for EC terms in this way is that we can investigate which component of returns (single, accounting, discount or joint components) is the strongest contributor to forecasting accuracy. The upper panel of Table 1.4 summarizes the control models.

1.5.1.3 Combining forecasts

A particular insight from the methodological literature on forecasting is that it is often preferable to combine alternative forecasts in a linear fashion and thereby obtain a new predictor (Granger (1989), Aiolfi & Timmermann (2006)). A detailed (cross sectional) encompassing analysis is beyond the scope of this Chapter. Instead of formally testing forecast complementarity we directly address the performance of combined forecasts. We consider six sets of forecasting models to enter forecast combinations. The lower panel of Table 1.4 provides the model references for combined forecasts and the underlying singular forecasts.

Singular forecasts entering CO1 to CO4 focus on the forecasting strength of equilibrium pricing factors. The last two combination schemes (CO5 and CO6) might be more interesting for practical purposes of return forecasting since they also exploit the informational content of atheoretical model formalizations.

1.5.2 Forecast evaluation

A central purpose of this study is to examine conditional return models out-of-sample from a cross sectional perspective. Diagnosing return forecasts is complicated owing to forecast error heteroskedasticity. At horizons $h = 3, 6$, forecast errors are also subjected to serial correlation. For the latter reasons we also consider nonparametric inferential tools. The out-of-sample evaluation techniques comprise:

1. *Serial correlation*: We consider a serial correlation statistic robust under heteroskedasticity denoted $\omega_{i,l}$ for lag l. The overall hypothesis of no serial correlation ($h = 1$) is tested by aggregating log p-values via a Fisher (1932) test denoted Ω_l.

2. *Distributional features*: We investigate if forecasts $\Delta\hat{p}_{i\tau}$ match key properties of the distribution of realized returns $\Delta p_{i\tau}$ by means of a variant of the

Henrikkson–Merton statistic denoted hm_i (Henriksson & Merton (1981), Cumby & Modest (1987)). Sensible forecasting models should deliver hm_i statistics in excess of unity. An aggregated Henrikkson Merton statistic ($\mathcal{HM}$) is computed from return forecasts over both data dimensions.

3. *Absolute and relative accuracy*: We rank competing models according to the mean absolute forecast error (MAFE) relative to the benchmark specification. A relative MAFE $\overline{dr}_i(\bullet) < 1$ hints at an inferior forecasting performance of the benchmark. At the aggregate level we determine overall MAFEs over both data dimensions, denoted $\bar{d}(0)$ and $\mathcal{DR}$.

4. *Outperformance of the benchmark model*: To contrast the benchmark against a particular model we consider the frequency of obtaining a smaller absolute forecast error from the former which is denoted $\mathcal{P}$. Over the panel dimension, N market specific relative MAFEs are obtained. From these, we count the relative MAFEs $\overline{dr}_i(\bullet)$ less than unity denoted $\mathcal{S}$.

5. *Testing for a unit ratio*: Borrowing from the derivation of the IPS test (Im *et al.* (2003)) a panel statistic to test $H_0 : \text{dr} = 1$ against $H_1 : \text{dr} < 1$ is performed. The test statistic is denoted $\mathcal{TR}$ and is approximately Gaussian under H_0.

Model	hm_i	$\omega_{i,1}$	Ω_1	$\Omega_{1,p}$	hm_i	hm_i	hm_i	$\omega_{i,1}$	Ω_1	$\Omega_{1,p}$	hm_i	hm_i
h	1				3	6	1				3	6
	Developed markets						Emerging markets					
	Recursive forecasting											
RW	0	2	47.98	.011	0	0	0	0	25.28	.390	1	1
VA1	4	0	17.28	.943	1	1	1	1	27.05	.302	2	1
VA2	5	1	23.66	.699	0	3	2	1	30.65	.164	0	2
EC1	6	1	18.53	.912	4	8	5	1	26.72	.318	3	4
EC2	7	0	16.58	.956	1	2	2	1	24.52	.432	3	0
EC3	4	0	17.42	.940	2	3	2	0	25.37	.386	1	3
EC4	4	0	17.02	.948	1	3	2	1	27.14	.298	1	1
EC5	7	0	16.11	.964	6	6	0	1	25.97	.355	2	1
EC6	4	1	17.94	.928	5	8	1	0	22.71	.537	4	4
EC7	6	1	20.45	.848	5	8	1	0	18.42	.782	4	2
EC8	5	0	18.91	.901	7	10	3	0	17.39	.831	4	7
	Rolling windows											
RW	0	2	48.69	.009	1	2	1	0	24.55	.431	4	5
VA1	11	0	21.41	.808	3	8	7	0	14.29	.940	4	4
VA2	7	0	20.65	.840	1	6	4	0	16.52	.869	4	4
EC1	8	1	34.05	.199	8	11	4	0	21.27	.623	5	6
EC2	10	0	24.03	.680	6	11	4	0	16.01	.888	4	2
EC3	7	0	26.26	.559	4	10	5	0	14.11	.944	4	9
EC4	9	0	26.26	.559	4	11	3	0	14.11	.944	4	6
EC5	11	1	22.90	.738	11	14	2	0	21.13	.631	3	4
EC6	6	1	41.12	.052	8	12	3	2	33.33	.261	5	8
EC7	9	2	42.79	.037	11	13	2	1	27.06	.302	3	9
EC8	7	5	73.88	.003	14	14	4	2	47.40	.003	6	11

Table 1.5: Diagnostic statistic for return forecasts. $\Omega_{1,p}$ lists p-values for Ω_1. The entries below hm_i and $\omega_{i,1}$ report the number of markets where the null hypothesis is rejected at the 5% significance level.

re h	VA1	VA2	RW	EC1	EC2	EC3	EC4	EC5	EC6	EC7	EC8	CO1	CO2	$\widetilde{\text{CO}}1$	$\widetilde{\text{CO}}2$	CO3	CO4	CO5	CO6
	$\mathcal{HM}$: Testing the informational content of contingency tables																		
1 1	1.113	1.102	*1.017*	1.124	1.117	1.098	1.100	1.127	1.097	1.114	1.092	1.119	1.109	1.116	1.121	1.114	1.112	1.098	1.090
1 3	1.055	1.049	*1.009*	1.105	1.049	1.059	1.067	1.116	1.103	1.114	1.136	1.084	1.112	1.072	1.123	1.083	1.059	1.060	1.082
1 6	1.064	1.079	*1.020*	1.154	1.077	1.078	1.089	1.138	1.156	1.212	1.240	1.117	1.186	1.143	1.197	1.133	1.117	1.088	1.145
0 1	1.167	1.135	*1.031*	1.164	1.169	1.145	1.150	1.191	1.141	1.171	1.119	1.177	1.178	1.181	1.185	1.170	1.171	1.154	1.151
0 3	1.096	1.076	1.051	1.149	1.121	1.098	1.121	1.154	1.158	1.189	1.246	1.156	1.202	1.157	1.205	1.133	1.145	1.137	1.180
0 6	1.136	1.149	1.090	1.216	1.161	1.170	1.208	1.232	1.225	1.250	1.329	1.244	1.299	1.228	1.285	1.196	1.235	1.195	1.262
	$\mathcal{P}$: Empirical probabilities of getting a smaller absolute forecast error as from the benchmark model VA1																		
1 1	–	0.468	0.476	0.478	0.475	0.483	0.477	0.479	0.463	0.468	0.455	0.474	0.468	0.481	0.475	0.480	0.488	0.510	0.502
0 1	–	0.431	0.503	0.482	0.476	0.440	0.486	0.485	0.461	0.441	0.421	0.494	0.475	0.497	0.468	0.482	0.485	**0.544**	**0.525**
1 3	–	0.475	0.498	0.499	0.467	0.468	0.493	0.501	0.485	0.496	0.470	**0.512**	0.495	0.502	0.505	0.497	**0.524**	**0.526**	**0.523**
0 3	–	0.420	**0.530**	0.493	0.470	0.469	0.481	0.492	0.475	0.485	0.478	**0.542**	**0.521**	**0.531**	**0.519**	0.517	**0.529**	**0.562**	**0.560**
1 6	–	0.476	0.481	0.512	0.475	0.486	0.485	**0.521**	0.501	**0.525**	0.500	**0.543**	**0.540**	**0.520**	**0.536**	0.519	**0.545**	**0.535**	**0.543**
0 6	–	0.450	0.503	0.503	0.492	0.489	0.486	0.507	0.502	**0.527**	**0.557**	**0.585**	**0.579**	**0.566**	**0.570**	**0.550**	**0.556**	**0.593**	**0.610**
	$\bar{d}(0)$	$\mathcal{DR}$: Overall MAFE estimates relative to the benchmark																	
1 1	0.041	1.040	1.014	1.008	1.012	1.007	1.010	1.011	1.022	1.027	1.048	1.006	1.013	1.006	1.012	1.006	1.006	0.995	**0.966**
	(2.719)	(0.593)	(0.830)	(0.312)	(0.279)	(0.197)	(0.245)	(0.389)	(0.406)	(0.564)	(0.724)	(0.218)	(0.381)	(0.225)	(0.381)	(0.234)	(0.214)	(0.312)	(0.456)
0 1	0.044	1.126	0.964	1.029	1.025	1.030	1.019	1.015	1.057	1.068	1.134	1.005	1.029	1.006	1.030	1.015	1.005	0.997	0.973
	(2.817)	(1.100)	(1.103)	(0.586)	(0.512)	(0.482)	(0.480)	(0.658)	(0.793)	(1.063)	(1.309)	(0.367)	(0.749)	(0.399)	(0.750)	(0.436)	(0.393)	(0.367)	(0.600)
1 3	0.078	1.040	0.988	1.004	1.022	1.008	1.011	1.008	1.025	1.016	1.041	0.996	1.002	0.999	1.001	1.002	0.995	0.984	0.950
	(3.757)	(0.572)	(0.781)	(0.526)	(0.449)	(0.389)	(0.431)	(0.636)	(0.633)	(0.831)	(1.091)	(0.347)	(0.602)	(0.363)	(0.591)	(0.388)	(0.355)	(0.334)	(0.435)
0 3	0.084	1.132	0.934	1.021	1.028	1.019	1.021	1.003	1.043	1.020	1.032	0.971	0.968	0.972	0.970	0.991	0.974	0.980	**0.921**
	(3.872)	(1.027)	(1.012)	(0.835)	(0.704)	(0.723)	(0.668)	(0.947)	(1.079)	(1.245)	(1.502)	(0.492)	(0.926)	(0.553)	(0.944)	(0.601)	(0.545)	(0.487)	(0.693)
1 6	0.117	1.027	1.004	0.992	1.023	1.002	1.013	0.994	1.011	0.982	0.993	0.978	0.971	0.983	0.966	0.988	0.981	0.977	0.927
	(4.370)	(0.633)	(0.795)	(0.615)	(0.536)	(0.417)	(0.582)	(0.830)	(0.764)	(1.029)	(1.223)	(0.474)	(0.748)	(0.503)	(0.760)	(0.482)	(0.480)	(0.404)	(0.491)
0 6	0.124	1.098	0.957	0.990	1.010	0.998	1.011	0.965	0.978	0.941	0.872	0.926	0.891	0.930	0.897	0.954	0.933	0.958	**0.869**
	(4.509)	(1.071)	(1.009)	(0.909)	(0.846)	(0.864)	(0.870)	(1.111)	(1.204)	(1.340)	(1.412)	(0.575)	(1.015)	(0.634)	(1.049)	(0.690)	(0.674)	(0.590)	(0.748)

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re	h	VA1	VA2	RW	EC1	EC2	EC3	EC4	EC5	EC6	EC7	EC8	CO1	CO2	$\widetilde{\text{CO}}1$	$\widetilde{\text{CO}}2$	CO3	CO4	CO5	CO6
		$\mathcal{TR}$: Testing the null hypothesis H_0 : dr $= 1$ against H_1 : dr < 1																		
1	1	–	6.890	1.924	4.122	4.749	4.086	4.597	3.421	6.040	5.767	7.374	3.423	4.371	3.130	4.093	3.962	3.527	-1.174	-0.333
0	1	–	11.46	**-3.047**	5.242	5.374	6.589	4.255	2.049	7.192	6.917	10.49	1.929	3.954	2.105	4.053	3.932	1.677	**-7.185**	**-4.488**
1	3	–	7.221	-1.425	1.445	4.634	2.203	3.029	1.082	4.151	2.109	4.027	-0.299	0.880	0.189	0.571	1.682	-1.184	-4.267	-4.077
0	3	–	12.84	**-6.683**	3.089	4.765	3.469	3.682	0.449	3.891	1.911	2.405	**-5.215**	**-3.367**	**-4.381**	**-2.960**	-0.767	**-4.399**	**-11.37**	**-11.49**
1	6	–	4.826	0.782	0.301	6.258	0.618	5.375	-1.787	4.678	**-3.820**	-1.009	**-9.515**	**-7.814**	**-7.494**	**-9.154**	-2.584	**-11.05**	**-6.424**	**-11.71**
0	6	–	8.626	**-3.646**	-0.273	3.321	0.670	3.564	**-4.474**	**-2.110**	**-4.793**	**-9.615**	**-18.36**	**-14.45**	**-15.59**	**-13.62**	-8.836	**-15.56**	**-15.61**	**-22.31**
		$\mathcal{S}$: Counts of relative MAFE measures being less than unity																		
1	1	–	0	4	2	1	2	1	2	1	1	0	3	2	3	2	3	1	9	8
0	1	–	0	10	1	1	0	2	3	0	0	0	6	0	4	0	2	4	**13**	**13**
1	3	–	0	9	6	0	4	3	6	3	3	3	6	5	6	6	5	10	**12**	**12**
0	3	–	0	**14**	4	2	4	2	8	2	5	3	**13**	**11**	**12**	10	7	**11**	**14**	**14**
1	6	–	1	7	9	4	5	4	7	8	8	8	10	**11**	**11**	**12**	9	9	10	**12**
0	6	–	0	**13**	6	5	6	4	9	7	9	**13**	**14**	**14**	**14**	**13**	**13**	**13**	**14**	**14**

Table 1.6: Performance statistics for alternative forecasting models for developed market returns against the VA1 benchmark. re indicates if the forecasting scheme is recursive (1) or rolling (0). CO and $\widetilde{\text{CO}}$ indicate alternative information sets to underling the weighting schemes in combined forecasts. In the panel for $\mathcal{HM}$ statistics *italic* entries indicate *insignificance* of the respective quantity. Bold entries for $\mathcal{TR}$, $\mathcal{P}$ and $\mathcal{S}$ indicate significance at the 5% level. Bold entries given for $\mathcal{DR}$ indicate the best performing forecast model by horizon. Numbers in parentheses are standard errors ($\times$ 100).

re	h	RW	VA1	VA2	EC1	EC2	EC3	EC4	EC5	EC6	EC7	EC8	CO1	CO2	$\widetilde{\text{CO1}}$	$\widetilde{\text{CO2}}$	CO3	CO4	CO5	CO6
		$\mathcal{P}$: Empirical probabilities of getting a smaller absolute forecast error as from the benchmark RW																		
1	6	–	**0.519**	0.501	0.517	0.498	0.508	0.496	**0.530**	0.502	**0.534**	0.513	**0.535**	**0.535**	**0.519**	**0.546**	0.517	**0.539**	**0.529**	**0.541**
0	6	–	0.497	0.454	0.493	0.481	0.489	0.479	0.500	0.490	**0.520**	**0.543**	**0.537**	**0.561**	**0.532**	**0.553**	**0.525**	**0.542**	**0.531**	**0.573**
		$\bar{d}(0)$	$\mathcal{DR}(\bullet)$: Overall MAFE estimates relative to the benchmark																	
1	6	0.118	0.996	1.024	0.988	1.019	0.999	1.009	0.990	1.008	0.978	0.990	0.975	0.968	0.979	0.963	0.985	0.977	0.970	0.953
		(4.311)	(0.789)	(0.975)	(0.914)	(0.901)	(0.860)	(0.933)	(1.066)	(1.001)	(1.165)	(1.327)	(0.838)	(0.967)	(0.848)	(0.972)	(0.840)	(0.847)	(0.960)	(1.261)
0	6	0.119	1.045	1.147	1.035	1.055	1.043	1.056	1.008	1.022	0.983	0.911	0.968	0.931	0.972	0.937	0.997	0.975	0.966	0.906
		(4.311)	(1.101)	(1.588)	(1.265)	(1.310)	(1.337)	(1.342)	(1.416)	(1.415)	(1.493)	(1.507)	(1.073)	(1.208)	(1.095)	(1.234)	(1.138)	(1.144)	(0.575)	(0.821)
		$\mathcal{TR}$: Testing the null hypothesis $H_0 : \mathrm{dr} = 1$ against $H_1 : \mathrm{dr} < 1$																		
1	1	–	**-2.085**	2.549	-0.821	-0.496	-1.243	-0.724	-0.551	0.618	1.346	3.422	-1.253	-0.231	-1.222	-0.330	-1.079	-1.259	**-3.056**	**-2.519**
0	1	–	2.809	10.36	5.154	4.720	4.994	4.293	3.700	6.909	7.093	10.57	3.317	4.887	3.411	4.933	4.122	3.246	0.078	0.800
1	3	–	1.339	5.482	1.901	4.032	2.169	2.772	2.007	4.186	2.701	4.520	1.197	1.802	1.494	1.584	2.018	0.686	-0.919	-1.549
0	3	–	6.139	13.17	7.185	7.903	6.799	7.264	5.185	7.884	5.829	6.214	3.491	2.720	3.546	2.981	5.333	3.527	1.633	-1.710
1	6	–	-0.684	2.500	-1.109	1.894	-0.791	1.194	-1.415	1.195	**-2.703**	-1.055	**-3.128**	**-3.717**	**-2.486**	**-4.330**	-1.485	**-3.325**	**-4.833**	**-7.714**
0	6	–	3.614	8.850	3.015	4.206	3.011	4.727	0.836	1.576	-0.710	**-5.435**	**-2.555**	**-5.257**	**-2.041**	**-4.699**	-0.172	-1.925	**-3.805**	**-10.20**

Table 1.7: Selected performance measures for forecasting *developed markets'* returns. The benchmark is RW. For further notes see Table 1.6.

re h	VA1	VA2	RW	EC1	EC2	EC3	EC4	EC5	EC6	EC7	EC8	CO1	CO2	$\widetilde{\text{CO}}1$	$\widetilde{\text{CO}}2$	CO3	CO4	CO5	CO6
	$\mathcal{HM}$: Testing the informational content of contingency tables																		
1 1	1.060	*1.046*	*0.988*	1.078	1.070	1.068	1.066	*0.998*	1.058	*1.029*	*1.042*	*1.050*	*1.051*	1.067	*1.048*	1.087	1.058	*1.044*	1.060
1 3	*1.043*	*1.011*	*1.022*	1.066	1.064	1.053	1.070	*0.986*	1.056	*1.032*	1.069	1.063	1.068	1.069	1.059	1.065	*1.051*	1.057	1.056
1 6	*1.015*	*1.023*	*1.021*	1.077	*1.047*	1.049	1.078	*0.980*	1.095	*1.036*	1.120	*1.038*	1.091	1.040	1.088	*1.053*	*1.038*	*1.010*	1.076
0 1	1.069	1.075	*1.039*	1.076	1.061	1.093	*1.053*	*1.028*	1.068	1.088	*1.049*	1.070	1.104	1.082	1.101	1.081	1.091	1.065	1.079
0 3	1.110	1.107	1.135	1.106	1.106	1.109	1.102	1.051	1.098	1.070	1.131	1.111	1.119	1.097	1.122	1.094	1.111	1.114	1.130
0 6	1.119	1.107	1.167	1.149	1.114	1.215	1.175	1.074	1.143	1.193	1.255	1.182	1.191	1.199	1.192	1.147	1.186	1.151	1.220
	$\mathcal{P}$: Empirical probabilities of getting a smaller absolute forecast error as from the benchmark model VA1																		
1 1	–	0.468	0.506	0.494	0.494	0.468	0.482	0.447	0.488	0.449	0.418	0.496	0.478	0.507	0.476	0.506	0.482	**0.531**	0.506
0 1	–	0.421	**0.546**	0.482	0.469	0.500	0.498	0.461	0.455	0.448	0.416	0.507	0.479	0.487	0.475	0.477	0.504	**0.560**	**0.534**
1 3	–	0.461	**0.549**	0.499	0.524	0.497	0.488	0.444	0.501	0.449	0.454	0.519	0.495	0.515	0.493	**0.528**	0.496	**0.544**	**0.527**
0 3	–	0.433	**0.551**	0.490	0.467	0.493	0.495	0.450	0.470	0.456	0.458	0.517	0.514	0.518	0.505	0.505	0.515	**0.553**	**0.539**
1 6	–	0.468	0.524	0.515	0.510	0.489	0.495	0.445	0.505	0.441	0.472	**0.532**	0.502	0.521	0.500	0.523	0.495	0.519	**0.527**
0 6	–	0.439	**0.548**	0.500	0.486	0.538	0.503	0.457	0.494	0.492	**0.542**	**0.597**	**0.537**	**0.588**	**0.531**	**0.530**	**0.583**	**0.570**	**0.568**
	$\overline{d}(0)$	$\mathcal{DR}(\bullet)$: Overall MAFE estimates relative to the benchmark																	
1 1	0.072	1.091	0.935	1.003	1.014	1.006	1.012	1.029	1.031	1.060	1.091	0.998	1.028	0.999	1.028	1.002	1.002	0.976	0.997
	(4.525)	(1.661)	(1.484)	(0.585)	(0.530)	(0.841)	(0.775)	(0.677)	(0.901)	(1.327)	(1.614)	(0.445)	(0.902)	(0.513)	(0.911)	(0.446)	(0.556)	(0.522)	(0.768)
0 1	0.077	1.156	**0.872**	1.017	1.026	1.005	1.025	1.037	1.051	1.088	1.165	0.995	1.032	0.999	1.034	1.007	0.999	0.946	0.963
	(4.746)	(3.019)	(1.996)	(0.884)	(0.696)	(0.977)	(1.118)	(1.031)	(1.234)	(1.988)	(2.784)	(0.591)	(1.260)	(0.673)	(1.397)	(0.633)	(0.755)	(0.802)	(1.211)
1 3	0.134	1.065	0.927	0.987	1.003	1.009	1.005	1.043	1.015	1.059	1.068	0.981	1.005	0.982	1.005	0.984	0.994	0.966	0.967
	(5.839)	(1.353)	(1.317)	(0.949)	(0.827)	(1.511)	(1.206)	(1.092)	(1.358)	(1.820)	(2.019)	(0.739)	(1.247)	(0.856)	(1.247)	(0.709)	(0.898)	(0.683)	(0.905)
0 3	0.139	1.146	**0.893**	0.997	1.027	1.010	1.022	1.046	1.026	1.051	1.087	0.967	0.981	0.968	0.985	0.982	0.985	0.953	0.945
	(6.011)	(2.268)	(1.470)	(1.306)	(0.994)	(1.432)	(1.341)	(1.449)	(1.670)	(2.069)	(2.481)	(0.799)	(1.503)	(0.810)	(1.515)	(0.912)	(0.962)	(0.774)	(1.153)
1 6	0.188	1.055	0.939	0.965	0.986	1.013	0.991	1.051	0.988	1.061	1.033	0.960	0.969	0.963	0.971	0.959	0.987	0.957	0.936
	(6.630)	(1.303)	(1.087)	(1.142)	(1.118)	(1.701)	(1.463)	(1.400)	(1.746)	(2.168)	(2.356)	(0.932)	(1.662)	(1.007)	(1.643)	(0.959)	(1.106)	(0.739)	(1.205)
0 6	0.191	1.143	0.906	0.960	1.023	0.994	1.000	1.034	0.983	1.008	0.953	0.919	0.908	0.916	0.913	0.948	0.950	0.922	**0.885**
	(6.853)	(2.277)	(1.367)	(1.662)	(1.391)	(1.938)	(1.612)	(1.728)	(2.069)	(2.150)	(2.453)	(0.984)	(1.711)	(1.061)	(1.723)	(1.277)	(1.199)	(0.828)	(1.201)

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re	h	VA1	VA2	RW	EC1	EC2	EC3	EC4	EC5	EC6	EC7	EC8	CO1	CO2	$\widetilde{\text{CO}}1$	$\widetilde{\text{CO}}2$	CO3	CO4	CO5	CO6
		$\mathfrak{TR}$: Testing the null hypothesis H_0 : dr = 1 against H_1 : dr < 1																		
1	1	–	6.246	**-5.140**	2.363	1.961	2.958	2.508	4.644	3.848	5.163	6.390	0.503	3.581	0.549	3.605	1.278	1.683	-4.441	-0.554
0	1	–	7.282	**-8.266**	2.509	4.399	1.234	2.674	4.496	4.740	5.395	7.897	0.013	3.004	0.392	2.993	2.250	0.248	**-7.254**	**-3.580**
1	3	–	4.957	**-6.126**	-0.230	0.172	0.665	0.732	3.643	1.165	3.357	3.417	-1.758	-0.082	-1.824	0.108	-1.553	0.471	**-4.978**	**-3.783**
0	3	–	7.051	**-7.429**	0.417	3.253	0.820	2.413	3.754	1.993	2.791	3.594	**-3.647**	-0.794	**-3.355**	-0.621	-0.851	-0.990	**-6.041**	**-4.485**
1	6	–	4.481	**-4.893**	**-3.951**	-2.364	3.890	0.892	8.076	-0.213	6.350	3.285	**-6.859**	**-2.189**	**-4.958**	**-1.830**	**-7.114**	-0.817	**-6.974**	**-6.834**
0	6	–	6.966	**-6.051**	**-4.406**	3.543	**-4.463**	1.123	3.526	-0.341	0.014	**-2.277**	**-15.10**	**-8.619**	**-13.84**	**-7.873**	**-7.958**	**-9.470**	**-10.01**	**-11.36**
		$\mathcal{S}$: Counts of relative MAFE measures being less than unity																		
1	1	–	1	12	3	3	3	3	1	1	0	0	5	0	6	0	2	2	10	8
0	1	–	0	**12**	4	0	5	2	1	1	1	0	5	2	4	2	2	6	**11**	**10**
1	3	–	0	**12**	5	6	6	4	1	5	2	2	7	3	7	3	8	5	**10**	9
0	3	–	0	**12**	6	1	3	4	2	1	3	0	9	6	8	5	6	7	**12**	**10**
1	6	–	1	**12**	9	8	4	5	2	7	2	5	8	7	**10**	8	9	5	**10**	8
0	6	–	0	**12**	7	3	7	7	3	7	6	**10**	**12**	**10**	**12**	**10**	**10**	**12**	**12**	**12**

Table 1.8: Performance statistics for alternative forecasting models for emerging market returns against the VA1 benchmark. For further notes see Table 1.6.

re h	RW	VA1	VA2	EC1	EC2	EC3	EC4	EC5	EC6	EC7	EC8	CO1	CO2	$\widetilde{\text{CO1}}$	$\widetilde{\text{CO2}}$	CO3	CO4	CO5	CO6
	$\mathcal{P}$: Empirical probabilities of getting a smaller absolute forecast error as from the benchmark model																		
1 6	–	0.476	0.450	0.496	0.488	0.452	0.492	0.418	0.487	0.428	0.451	0.490	0.482	0.483	0.474	0.497	0.457	0.468	0.481
0 6	–	0.452	0.402	0.490	0.465	0.473	0.456	0.436	0.468	0.460	0.511	0.519	0.511	0.509	0.509	0.495	0.490	0.484	0.522
	$\bar{d}(0)$	$\mathcal{DR}(\bullet)$: Overall MAFE estimates relative to the benchmark																	
1 6	0.176	1.065	1.123	1.028	1.050	1.078	1.056	1.119	1.052	1.129	1.100	1.023	1.032	1.025	1.034	1.022	1.051	1.028	1.008
	(6.360)	(1.232)	(1.835)	(1.574)	(1.634)	(2.146)	(1.862)	(1.755)	(2.056)	(2.355)	(2.569)	(1.446)	(1.927)	(1.484)	(1.910)	(1.487)	(1.581)	(1.505)	(2.498)
0 6	0.173	1.104	1.262	1.059	1.129	1.097	1.104	1.141	1.084	1.113	1.051	1.015	1.002	1.011	1.008	1.046	1.049	1.024	0.979
	(6.293)	(1.665)	(2.867)	(2.210)	(2.137)	(2.586)	(2.201)	(2.224)	(2.498)	(2.547)	(2.832)	(1.768)	(2.165)	(1.811)	(2.173)	(1.967)	(1.906)	(1.345)	(1.918)
	$\mathcal{TR}$: Testing the null hypothesis $H_0 : \mathrm{dr} = 1$ against $H_1 : \mathrm{dr} < 1$																		
1 1	–	4.773	7.239	5.116	5.124	5.124	5.166	6.497	6.255	6.835	8.020	4.816	6.013	4.791	5.991	4.922	5.065	3.658	4.661
0 1	–	7.189	10.593	7.829	8.162	7.135	7.765	8.622	8.971	9.134	11.157	7.173	8.041	7.238	8.033	7.648	7.174	5.598	6.302
1 3	–	5.582	7.400	4.665	4.881	5.023	5.146	7.054	5.470	6.373	6.413	4.284	4.798	4.227	4.938	4.358	5.004	3.394	3.143
0 3	–	6.514	9.710	6.141	7.233	5.855	6.898	7.927	6.920	7.127	7.535	4.558	5.012	4.668	5.138	5.534	5.406	4.169	3.621
1 6	–	4.648	6.081	3.184	3.682	5.162	4.035	8.536	4.732	7.709	5.837	2.549	3.212	2.766	3.261	2.499	4.260	1.981	0.561
0 6	–	5.191	8.691	3.711	6.151	4.086	5.186	6.412	4.694	4.248	1.903	1.269	0.545	1.215	0.834	2.943	2.483	1.483	-1.176

Table 1.9: Selected performance measures for forecasting *emerging markets'* returns. The benchmark is RW. For further notes see Table 1.6.

1.5.3 Out-of-sample results

Apart from uncovering the predictive content of pricing factors the forecasting exercises allow numerous directions to compare model accuracy. Conditioning on the benchmark VA1 we first contrast rolling window and recursive forecasting. In the second place we compare VA1 against specifications having an extended (VA2) or reduced (RW) autoregressive order. Then, we address the forecasting content of models including single pricing factors (EC1 to EC5). Forecasting accuracy achieved when augmenting the benchmark jointly with accounting or discount factors or both is also discussed. Finally, we turn to combined forecasts. At a disaggregated level we only provide selective results on forecast error correlation and distributional properties. Detailed results are provided at the aggregate level of *developed* and *emerging markets*, respectively. The following discussions refer to the 5% significance level.

1.5.3.1 Rolling windows vs. recursive forecasting

Considering VA1 employed for recursive and rolling window forecasting at *developed markets* it turns out that for all $h = 1, 3, 6$, $\mathcal{HM}$ statistics (Table 1.6) are in favor of the rolling window approach while recursive forecasting obtains slightly smaller MAFEs. The hypothesis of no serial correlation of $h = 1$ forecast errors cannot be rejected for *developed markets*. From the aggregate $\mathcal{HM}$ statistics it is evident that the VA1 benchmark is suitable to approximate distributional features of stock returns. The only insignificant $\mathcal{HM}$ statistics of the VA1 (1.05, 1.02) are reported for recursive forecasting of *emerging market* returns at higher horizons ($h = 3, 6$).

Contrasting some of the statistics for *developed* and *emerging markets* in Tables 1.6 and 1.8, it turns out that the relative merits of the rolling window scheme are more apparent for *emerging markets*. For instance, with regard to benchmark forecasting at the high horizon ($h = 6$) $\mathcal{HM}$ statistics are in favor of rolling window forecasting (1.119 against 1.015) while at the same time MAFEs reported for recursive modeling are only slightly below the rolling window measure (0.1891 vs. 0.1894). The latter results hint at a higher likelihood of structural instabilities of *emerging markets*' return dynamics in comparison with *developed markets*.

Overall, we find that rolling windows forecasting is slightly superior to recursive prediction. Therefore, the following discussions mainly refer to former scheme.

1.5.3.2 Modifying the (vector) autoregressive order

Contrasting the VA1 against VA2 it turns out that higher order dynamics do not improve forecasting accuracy. We obtain a few slightly improved $\mathcal{HM}$ statistics but notice some cases where VA2 performs worse in comparison with VA1. For instance, forecasting *developed markets*' returns ($h = 1, 3$) obtains $\mathcal{HM}$ statistics for the VA2 which are about 0.02 less than the corresponding measures of the VA1. The inferior performance of VA2 is also visible from relative MAFEs that exceed unity significantly.

While the forecasting performance of the benchmark model is superior to the VA2 it is of immediate interest if a reduction of the autoregressive order improves forecasting accuracy. The RW model formalizes conditional returns merely in terms of a constant implying a drift term to govern the (log) price process. Coupled with rolling windows, RW might be seen as a local drift specification. Conditional on the set of rolling window forecasts, a reduction of the autoregressive order obtains smaller $\mathcal{HM}$ statistics over all forecast horizons in the case of *developed markets* (Table 1.6). Moreover, at the cross sectional level we diagnose serial correlation of rolling RW based forecasts ($h = 1$) in the case of *developed markets* (Table 1.5).

With regard to *emerging markets*, the local drift model yields higher $\mathcal{HM}$ statistics as VA1 for $h = 1, 3, 6$, and serial correlation ($h = 1$) cannot be diagnosed at the aggregate or single market levels (Table 1.5). Relative MAFEs signal that the local drift model outperforms VA1 significantly at all horizons, $h = 1, 3, 6$, and for both cross sections (Tables 1.6, 1.8). The strength of the rolling RW can also be diagnosed when counting the cross section members having a relative MAFE ($\overline{dr_i}$) below unity. Forecasting medium to high range returns yields $\mathcal{S} = 13$ ($h = 6$) or $\mathcal{S} = 14$ ($h = 3$) for *developed markets*. With respect to *emerging markets* $\mathcal{S}$ attains its maximum of $\mathcal{S} = N = 12$. The latter counts are throughout significant against the VA1. Assessing the overall performance of RW and VA2 relative to VA1 by means of the panel statistic $\mathcal{TR}$, it is confirmed that VA2 performs significantly worse in comparison to VA1 and local drift based forecasts outperform the benchmark at all horizons and for both cross sections.

1.5.3.3 Single factor forecasting

Augmenting VA1 with single pricing factors mostly improves $\mathcal{HM}$ statistics documented for return forecasts of *developed markets*. Conditional on $h = 6$, PE and PPM ratios improve the distributional features of ex-ante forecasts since the $\mathcal{HM}$ statistics increase in comparison with VA1. Two further statistics hint at potential forecasting strength of the PPM ratio. The relative MAFE estimate from this particular single factor model is significantly below unity (0.965). Moreover, according to $\mathcal{TR} = -4.474$ a unit MAFE ratio is rejected with conventional significance.

For rolling window forecasting in *emerging markets* the general impression is similar. In forecasting higher order returns ($h = 6$) EC1 (PE) and EC3 (TES) outperform VA1. Including the latter factor raises the $\mathcal{HM}$ statistic from 1.11 to 1.21. Finally, owing to $\mathcal{TR} = -4.463$ the null hypothesis of a unit MAFE ratio computed against VA1 is rejected. The PE ratio obtains a similar $\mathcal{TR}$-statistic (-4.406) which also hints at forecasting strength of this factor for the cross section of *emerging markets*. Both augmentations of VA1 do not achieve the average forecasting performance of RW. MAFE ratios computed against VA1 are .919, .969 and .998 for RW, EC1 and EC3, respectively.

The latter comparisons of single factor models against VA1 deserve a word of caution. It turns out that $\mathcal{TR}$-statistics become throughout insignificant when the rolling RW is the benchmark in *emerging markets* (Table 1.9). It is noteworthy that the local trend model is not atheoretical as it is likely to exploit time-varying risk premia. From the literature on conditionally heteroskedastic processes originating with Engle (1982), periods of lower and higher stock market volatility are known to alternate. In turn, volatility clustering implies time varying risk premia which, in our setting, might be captured by local drifts.

1.5.3.4 Joint factor forecasting

Rolling window forecasting for *developed markets* by means of model specifications comprising accounting (EC6) and discount factors (EC7) and both (EC8) jointly, yields some improved $\mathcal{HM}$ measures. For instance, in case of $h = 6$ conditioning on all pricing factors (EC8) attains an $\mathcal{HM}$ statistic of almost 1.33 which clearly exceeds the benchmark 1.136 (VA1). Interestingly, it appears as if it were more the discount factors that carry forecasting power in *developed markets*. Concentrating merely on accounting factors (EC6) the $\mathcal{HM}$ statistic drops back to 1.22 while modeling with joint discount factors (EC7) obtains 1.25. Similarly, augmenting VA1 with the discount (accounting) factors jointly we obtain $\mathcal{P} = 0.525$ ($\mathcal{P} = 0.502$) of all $h = 3$ predictions drawn from model EC7 (EC6). The model including all lagged equilibrium relationships (EC8) attains the smallest MAFE in case $h = 6$ over all competing specifications discussed so far. Measured against VA1 the $\mathcal{DR}$-ratio for EC8 is 0.872 indicating forecasting power of pricing factor models at the cross sectional level. Moreover, we find that the full factor model not only outperforms the VA1 but also the RW benchmark (Tables 1.6 and 1.7). Contrasting model EC8 against the RW yields significant performance statistics, $\mathcal{TR} = -5.435$, $\mathcal{P} = 0.543$ and $\mathcal{DR} = 0.911$ ($h = 6$).

The latter results have to be mitigated with respect to *emerging markets*' returns. It still holds that the rolling EC8 outperforms VA1 in case $h = 6$. Contrasting EC8 against RW, however, the latter specification remains superior. This result hints at the difficulty to extract forecasting information from long run variables in these markets.

1.5.3.5 Combined forecasts I

Forecast combinations are performed with weights conditional on the entire sequence of forecast errors available in the forecast origin (CO$\bullet$) or conditional on the 20 most recent realized errors ($\widetilde{\text{CO}\bullet}$). Since both weighting schemes turn out to perform similar our discussion focuses on CO$\bullet$. For details about forecast combinations we refer to Table 1.4.

Combined forecasts show higher frequencies $\mathcal{P}$ as documented for singular model forecasts (Tables 1.6, 1.8). The latter estimates vary between 55.0% and

57.9% for *developed* and 53.0% and 59.7% for *emerging markets*. Similar frequencies are obtained for *developed markets* when contrasting combined and RW based rolling forecasts. With regard to *emerging markets*, forecast combination fails to outperform RW (Table 1.9), thus, in what follows we focus on *developed markets*.

Overall, combined forecasts deliver $\mathcal{TR}$-statistics that are significant when constructed against the VA1 (varying between -8.836 and -18.36). Measured against RW, $\mathcal{TR}$ varies between -0.172 and -5.257. Surprisingly, however, the relative MAFEs do not attain the performance measures of EC8 being $\mathcal{DR}$ =.872 (.911) when computed against the VA1 (RW) model. At the first sight the latter results appear conflicting. The standard errors reported for the $\mathcal{DR}$-statistics reveal that combined forecasts show less forecast uncertainty as EC8. The latter models' $\mathcal{DR}$ measure of 0.87, for instance, shows a standard error of 1.412 implying an underlying variance which is about twice the variance of the $\mathcal{DR}$ statistic reported for CO2 combining EC6 (joint accounting factors) and EC7 (joint discount factors). Forecast combinations generated from singular forecasts EC1 to EC5 reduce forecast uncertainty even further (CO1).

Comparing forecast combinations of EC1 and EC2 (CO3) and of EC3 to EC5 (CO4), underpins that discount factors carry more information than accounting variables. Measured against RW, CO4 yields a $\mathcal{DR}$-statistic of 0.975 which is significantly less than unity. In comparison, the corresponding statistic for CO3 is only slightly smaller than unity (0.997).

The results on the performance of combined forecasts are encouraging in the sense that linear combinations of alternative predictors appear to reduce forecast uncertainty and might also improve other performance measures attached to the combined singular ingredients. From these exercises we find strong and outperforming competitors of RW in case of *developed markets* but not for the cross-section of *emerging markets*.

1.5.3.6 Combined forecasts II

As a final approach we consider forecast combinations obtained from a set of singular forecasts building upon both, ECMs and 'atheoretical' models such as

RW, VA1 and VA2. The first set of forecasts entering the combination strategy (CO5) consists of RW, VA1, VA2 and single factor models EC1 to EC5. The second combination strategy (CO6) comprises RW, VA1, VA2 and joint factor models EC6 to EC8.

Combined forecasts obtained along the latter lines yield the best forecasting accuracy over all competitors considered so far in the case of *developed markets*. The rolling CO6 strategy obtains in case $h = 6$ a MAFE ratio of $\mathcal{DR}$ =.869 which is below the measure provided for EC8 (0.872). In addition, the standard errors are almost twice as large for the singular forecast (EC8) as for the combined forecasts. As a consequence, CO6 is more efficient since it reduces both the MAFE and forecast uncertainty. The empirical probabilities of obtaining a smaller absolute forecast error from combined forecasts in comparison with VA1 are significant throughout ($\mathcal{P} = 0.61$ ($h = 6$), $\mathcal{P} = 0.56$ ($h = 3$), $\mathcal{P} = 0.525$ ($h = 1$), Table 1.6). Based upon the $\mathcal{TR}$-statistic, the combined forecast outperforms VA1 significantly at all horizons with the 'least significant' statistic being $\mathcal{TR} = -4.488$. Testing against the RW benchmark (Table 1.7) also hints at a markedly better performance of combined forecasts in case $h = 6$ and, to some lesser extent also at the medium horizon ($h = 3$, $\mathcal{TR} = -1.710$). Taking combined forecasts (CO5) from the set comprising RW, VA1, VA2 and EC1 to EC5 obtains slightly inferior ex-ante measures than CO6.

With respect to *emerging markets*, combined forecasts uniformly outperform the VA1 benchmark (Table 1.8). Contrasting combined forecasts against the RW benchmark is still supportive for the latter modeling venue (Table 1.9). A few statistics hint at (insignificantly) superior forecasting performance of combined rolling forecasts compared with the RW. Focusing on high horizon returns ($h = 6$) the relative MAFE ($\mathcal{DR}$) is below unity, $\mathcal{TR} = -1.176$ and $\mathcal{P} = 0.522$.

1.5.3.7 In-sample fitting vs. out-of-sample forecasting

The discussion of empirical results has been addressing both in-sample fitting and out-of-sample forecasting. It is of immediate interest to uncover if in-sample results provide some guidance on the forecasting strength of pricing factors. Addressing the latter issue is complicated since estimation results are obtained for the entire sample while forecast performance measures reflect a sequence of empirical models each conditioning on smaller information sets.

In-sample estimates could offer misleading advice for forecasting since some pricing factors show significant EC coefficient estimates but single pricing factors rarely improve VA1 forecasts at the aggregate level. To give a further indication of the latter issue we refer to Table 1.3, which displays the number of cross section members with EC coefficients that have absolute $t-$ratios in excess of 1.64. Taking the latter as an indication of forecastability one would expect that for particular cross section members those pricing factors help to improve the forecasting accuracy of VA1. To measure the impact of a particular pricing factor on forecastability at a single market level we check if the model and country specific MAFE ratio, $\overline{dr}_i$, is 'significantly' below unity. Along the latter lines we count how often a single factor forecasting model (EC1 to EC5) delivers MAFE ratios that fulfill the one sided criterion $\overline{dr}_i + 1.64\sqrt{\widehat{\text{Var}}[\overline{dr}_i]} < 1$. Ideally, one should observe a high overlap of cross sections where a particular pricing factor yields significant coefficient estimates in-sample and forecasting power according to a 'small' $\overline{dr}_i$ measure.

Table 1.3 provides counts of cross sections where in-sample and out-of sample results point into the same direction. After determining 'significant' $t-$ratios we count for which cross section members, forecasting schemes obtain 'significantly' small MAFE ratios. It turns out that not merely for $h = 1$ or $h = 3$ the latter overlap is small but also for the case $h = 6$ where numerous pricing factors are found to improve the VA1 forecasts conditional on single markets. For instance, with respect to the PPM ratio 9 (out of 14) *developed markets* show a $t-$ratio in excess of 1.64 but for only 5 (rolling windows) of these 9 markets the MAFE ratio is 'significantly' below unity as well. Similar failures of in-sample modeling to signal forecastability are reported for other pricing factors or for the cross section of *emerging markets*.

1.6 Concluding remarks

In this Chapter we examined the in-sample and out-of-sample dynamic features of stock market returns conditional on equilibrium pricing factors for *developed* and *emerging markets*. We put forward an asset pricing model that decomposes log stock returns into accounting and discount equilibrium pricing factors and short run (vector) autoregressive dynamics. Based on this model, we study if different equilibrium relationships can (i) signal (or predict) stock return behavior within sample and whether they can (ii) significantly improve (either jointly, individually or via combination) out-of-sample return forecasts.

Overall, we find that forecasting with single lagged equilibrium relationships does not play a uniformly significant role in explaining future stock return behavior but that forecasting with a full model containing all lagged equilibrium relations can outperform both a random walk model with local drift and pure VARs. The forecast content of (joint) pricing factors is more obvious for *developed* in comparison to *emerging markets*. Regarding the former, we find that accounting variables such as price to earnings and earnings to dividend ratios contribute less to ex-ante return models than discount factors such as term spreads, international interest or stock index differentials. Concerning *emerging markets*, a random walk model with local drift appears to describe best ex-ante returns. This result hints at two main possibilities: First, time varying risk premia might be a crucial factor in forecasting return behavior in *emerging markets*. Second, sharp structural variations prevent asset return models that account for long run variables to forecast *emerging markets'* returns. Moreover, we find that linear combination of forecasts of alternative models reduces forecast uncertainty and improves average forecasting precision. In particular, we find that an analyst would get most efficient approximations of future returns when subjecting a broad set of alternative predictors (including pricing factor models, the random walk and vector autoregressions) to forecast combinations.

Our approach to modeling stock returns conditional on equilibrium relationships has various policy dimensions. First, we contribute to the rising debate if predictive regression models can in fact be useful to describe expected asset returns. Our results put forward that analysts should treat these type of models with caution. That is, the postulated relationships should be analyzed out-of-sample to avoid spurious results that may arise when performing the analysis exclusively in-sample. Second, we exploit the augmentation of information that comes along with panel data and design modeling methods to examine conditional returns. Lastly, we believe that further research should be done for emerging financial markets. Our findings show that financial data of emerging markets may incorporate large noisy components and possibly regime-switching coupled with time-varying risk premia. An interesting extension to this study would be the analysis of time-varying risk premia in asset returns at a cross-country level with particular attention to other type of assets (fixed income, real estate, etc.) and cross-sections (developed vs. emerging markets).

Chapter 2

An Empirical Analysis of International Real Estate Securities: Long-run Equilibrium, Short-run Dynamics and Contemporaneous Dependence

2.1 Introduction

As introduced in the previous Chapter, financial economists have long been interested in analyzing asset price behavior based on different equilibrium asset pricing models. The Sharpe (1964) and Lintner (1965)'s Capital Asset Pricing Model (CAPM), the Arbitrage Pricing Theory (APT) of Ross (1976), the present value models of Campbell & Shiller (1987, 1988) and the stochastic discount factor model of Hansen & Singleton (1982) have been some of the most predominant models in the asset pricing literature since the late 1960's. However, asset pricing studies have been mostly applied to equity and bond markets while leaving aside real estate security markets. The importance of understanding the behavior of real estate assets can be induced from the fact that total real estate accounts for about half of the world's wealth (Corgel *et al.* (2000)). Moreover, from the perspective of a long term investor, real estate assets could be a more prudent investment vehicle since they may better balance risk/reward and market variability (cf. Viceira & Campbell (2002)). Thus, not studying real estate price behavior means neglecting the understanding of a very important asset class and a major source of wealth. Nevertheless, very little is known about the factors that may explain real estate security prices in equilibrium and their short-run run deviations from equilibrium (e.g. time-varying risk-premia, asset return dynamics and contemporaneous dependence).

The latter gap is tackled in the present Chapter where a real estate pricing model is examined for a cross-section of international financial markets. We model real estate security returns conditional on the long-run equilibrium error between (log) real estate security prices, equity prices and bond prices as well as conditional on the returns and the time-varying volatility of the latter three asset types. The model also incorporates the time-varying volatility of a constructed 'world' asset factor and the time-varying covariances of this factor with real estate, equity and bond returns. In addition, we also consider whether contemporaneous equity returns can explain international real estate returns. The model is estimated using monthly data for the sample period 01/1996-12/2006 and a battery of in-sample and out-of-sample techniques are used to analyze it.

We hypothesize and examine an empirical equilibrium relationship between real estate security prices, equity prices and bond prices for the following reasons. First, real estate securities are shares of listed real estate firms whose major bulk of business is in the operations of commercial and residential real estate. Since real estate securities are traded in the stock market, they should have an empirical relationship with the domestic all-share equity index (or another proxy for the domestic asset market portfolio) under capital asset pricing considerations. Second, the activities of listed real estate firms usually includes leasing, real estate development and tenant services. Since the main activities of listed real estate firms will be strongly related to interest rates, real estate securities should also have a strong empirical relationship with the bond market, and thus, a bond price index.

The empirical equilibrium relationship between real estate security prices, equity prices and bond prices viewed from an international *panel* cointegration perspective allows us to uncover the degree of financial integration between these three asset markets within and across international markets. Financial markets are integrated when assets have the same theoretical price (after adjusting for risk) irrespective of the market, otherwise, markets are said to be segmented (Bekaert & Harvey (1995)). The CAPM or the APT have been the most popular models used in the finance literature to test whether asset markets are integrated. A modern way to analyze the degree of financial integration between asset markets is by studying whether asset prices are cointegrated, i.e. by studying whether they follow a long-run equilibrium relationship. The intuition is that if asset markets are integrated then asset prices should, up to a scalar, be driven by a common stochastic trend (cf. Mills (1996), Alexander (1999)).

There are a few available studies that analyze the long-run relationship between *equity* prices and variables such as dividends, earnings, industrial production, interest rates and foreign stock prices from a *panel* cointegration perspective (Nasseh & Strauss (2000, 2004), Bohl & Siklos (2004), Pan (2007)). There are also some available studies that analyze whether there is a long run relationship between real estate prices or bond prices and some of the aforementioned as well as other economic indicators (Wilson *et al.* (1996), Chaudry *et al.* (1999), Gallin (2006), Taipalus (2006), Ciner (2007)). However, there is, in general, a lack of studies that model the long run relationship of real estate security prices, equity prices and bond prices from an international *panel* cointegration perspective.

If real estate prices cointegrate with equity and bond prices and correctly adjust to short-run deviations from this equilibrium, what other factors can predict these short-run deviations? We hypothesize that time-varying second order moments of asset returns such as volatility and covariances with a common factor can predict real estate returns (i.e. log real estate price changes). There are both empirical and theoretical arguments that can be put forward to support the latter hypothesis. A well known *empirical* argument is the consensus amongst the finance community that asset returns are conditionally heteroskedastic which implies time variability in their second order moments. Thus, if second order moments are time-varying it is intuitively plausible that they may affect the conditional mean of asset returns. One *theoretical* argument is that time-varying second order moments in-mean result from the time-varying risk premia required by risk-averse agents. Investors form expectations of future return behavior based on the 'riskiness' of the asset in question, since they must be compensated for the additional risk they undertake. The latter argument gives rise to the usual risk-return relationship which has recently recaptured momentum in empirical asset pricing studies of equity markets (cf. Ludvigson & Ng (2007), Bollerslev *et al.* (2008)). Moreover, if asset markets are financially integrated, then one should expect an inter-relationship as well as an intra-relationship between asset prices, returns and second order moments. Thus, time-varying second order moments have both strong theoretical and empirical arguments why they should be employed when studying short-run fluctuations of real estate security prices. Nevertheless, there seems to be a lack of attention to examining the tradeoff between asset risk and real estate returns especially at the cross-country level.

A prominent candidate model that could be used to analyze the explanatory power of time-varying second order moments in-mean of real estate returns amongst a cross-section of countries is the intertemporal CAPM (ICAPM). The latter model has already been tested in the world context and it has been found that a significant (conditional) variation in covariance risk exists between country stock returns and a world stock return (Harvey (1991)). Other studies have employed the ICAPM framework with other data sets or econometric methodologies and have found evidence of the significant effect of time-varying second order moments on the time-varying mean of asset returns in single markets (Attanasio (1991), Turtle *et al.* (1994), Hafner & Herwartz (1998)).

However, we see two main problems with the ICAPM and related models which we tackle in this Chapter. First, there are several empirical anomalies of the CAPM which suggest that also the ICAPM might not be a particularly accurate model to explain a cross-section of international real estate returns. Several factors such as market value, payout ratios, earnings or dividend yields have been found to explain various cross-sections of asset returns (Sattman (1980), Banz (1981), Rosenberg *et al.* (1985), Bandhari (1988), Fama & French (1992)). Return differentials of portfolios constructed to mimic factors related to size, book-to-market measures and term structure components (a term premium and a default premium) can capture strong common variation in returns in equity and bond markets (Fama & French (1993, 1995)). These results have been recently tested within conditional asset pricing frameworks and it has been found that the conditional moments of some of the latter variables can explain the cross section of equity, bond and real estate returns in the US (Dunne (1999), Downs (2000), Lettau & Ludvigson (2001b), Campbell & Vuolteenaho (2004), Downs & Patterson (2005)). The latter findings are still an open question for the cross-section of real estate returns at the country level. In this study we contribute to this open question by constructing a 'world' asset factor that accounts for return differentials in portfolios that mimic switches in fundamentals inherent in international asset prices (high versus low price/dividend ratios for real estate security prices, high versus low price/earnings ratios for equity prices and high versus low term spreads for bond prices). Furthermore, we test whether its second order moments (its time-varying variance and its time-varying covariance with the cross-section of international equity, bond and real estate market returns) can explain the cross section of international real estate returns.

Second, as criticized in the recent empirical finance literature, asset pricing models such as the ICAPM or related multivariate volatility-in-mean models can many times impose restrictive parametric assumptions and they often suffer from a curse of dimensionality problem. To circumvent the latter constraints, empirical finance models of conditional (multivariate) volatility or models containing time-varying second order moments in-mean have recently been addressed via realized volatility (RV) methods (cf. Andersen *et al.* (2003), Ludvigson & Ng (2007), Bollerslev *et al.* (2008)). The RV approach allows conditional asset pricing models to be tested flexibly with, say, panel data or recursive methods. In this study we employ the RV approach to analyze the risk-return relationship in international real estate securities.

Apart from uncovering the conditional explanatory power of particular variables for international real estate returns, we also contribute to the understanding of the *contemporaneous* interactions between international real estate security returns and international equity returns. The latter relationship is important from an economic perspective since various studies have shown that there is a causal relationship between investor sentiment and all-share equity returns hinting that the latter may be used as a natural proxy for the former (cf. Neal & Wheatley (1998), Lee *et al.* (2002), Brown & Cliff (2004), Lux (2008b)). The intuition is that people take rising asset prices as a sign of confidence and a reason to invest in (say) mutual funds, retirement accounts or simply to take new bets. From a behavioral finance perspective, proxies for investor sentiment can account for 'noise trader risk', 'animal spirits' and 'fads and fashions' which are important components needed to characterize asset return variation (cf. DeLong *et al.* (1990)).

To preview the results, we find for the panel of countries considered that: (i) real estate, equity and bond prices are cointegrated, (ii) a *homogeneous* long-run equilibrium relationship between real estate, equity and bond prices exists and real estate prices adjust (on average) to deviations from this equilibrium, (iii) time-varying second order moments of real estate, equity, bond returns and the constructed world factor can explain real estate returns, (iv) there is a dynamic relationship between real estate and bond returns, and that (v) there are contemporaneous interactions between real estate returns and equity returns. These results put forward that there is evidence of financial integration both within and across the international asset markets considered and that risk pricing variables play an important role in describing short-run real estate price fluctuations. In general, our findings suggest that a way to understand international real estate securities is by analyzing their interactions with equity markets and bond markets.

This Chapter is organized as follows. The next section introduces the empirical model analyzed. Section 2.3 addresses the econometric methodology employed and designed for the empirical analyses. Section 2.4 reports the results and the last section concludes. Details on issues not addressed in the main text can be found in Appendix B.

2.2 The model

In this section we present a baseline empirical specification to study international real estate securities. In what follows, real estate price $P_{h,t}$ is defined as the price of a portfolio of real estate securities. Equity price $P_{s,t}$ is defined as the price of a share of an all-share equity portfolio. Bond price $P_{b,t}$ is defined as the price of a long-term government (n-period) bond. In what follows let $p_{\bullet,t} = \ln P_{\bullet,t}$ be the log price of asset $\bullet = h, s, b$. Moreover, log returns are defined as $\Delta p_{\bullet,t} = \ln P_{\bullet,t} - \ln P_{\bullet,t-1}$ and the general model of real estate returns as:

$$\Delta p_{h,t} = \upsilon_t + u_{h,t}, \tag{2.1}$$

where υ_t is a time-varying mean and $u_{h,t}$ is a zero-mean disturbance term (or 'shock'). As shown in the previous Chapter, it is possible to derive the time-varying mean υ_t from an equilibrium asset pricing framework. Our main interest here is not to present a new theory but rather to perform the empirical analysis of the following specification of the time-varying mean of real estate returns:

$$\upsilon_t = \alpha + \phi h_{t-1} + \pi' \lambda_{t-1} + \nu_t. \tag{2.2}$$

In this model α is a constant, ϕ is the coefficient of equilibrium adjustment which should satisfy $\phi < 0$ and the variable $h_t = p_{h,t} - \beta_1 p_{s,t} - \beta_2 p_{b,t}$ defines the 'equilibrium' error between (log) real estate prices ($p_{h,t}$), equity prices ($p_{s,t}$) and bond prices ($p_{b,t}$). In that case, we say that the log real estate price, the log equity price and the log bond price are cointegrated with long-run parameters β_1 and β_2, briefly denoted $CI(1, -\beta_1, -\beta_2)$. Intuitively, from an asset pricing perspective, the linear combination h_t should capture (up to a constant) expectations between differentials in risk premia and payoffs from each asset $\bullet = h, s, b$ as well as stochastic discounting (cf. Campbell & Shiller (1988), Lettau & Ludvigson (2001a), Cochrane (2001)).

The vector $\lambda_t = (\sigma^2_{h,t}, \sigma^2_{s,t}, \sigma^2_{b,t}, \sigma^2_{f,t}, \sigma_{hf,t}, \sigma_{sf,t}, \sigma_{bf,t})'$ collects time-varying second order moments where $\sigma^2_{\bullet,t}$ denotes the time-varying volatility of the returns of asset $\bullet = h, s, b$ and $\sigma^2_{f,t}$ is the time-varying volatility of a (common) asset return factor f_t. Moreover, the variables $\sigma_{\bullet f,t}$ are the time-varying covariances of the factor f_t with the returns of asset $\bullet = h, s, b$. In this context, the parameter vector π attached to λ_t is the price of variance and covariance risk to real estate returns. As introduced in Chapter 1, the inclusion of the common factor f_t can be motivated by modern asset pricing theory: under complete markets there exists a (common) discount factor that is a linear combination of the return of different asset types and whose conditional moments price time series as well as cross-sectional variation of all assets (cf. Cochrane (2001), Campbell *et al.* (1997)). Note also that many theoretical models of the finance literature predict a positive risk-return tradeoff. Although some empirical evidence supports the latter relationship, several studies have found a strong negative relationship (cf. Black (1976), Campbell & Hentschel (1992)).[1] In our context, this implies that it is difficult to have a-priori knowledge of the exact signs of the parameters in the vector π.

Furthermore, we analyze two different specifications for ν_t. The first one assumes that ν_t can be explained only by dynamics of asset returns $\bullet = h, s, b$, i.e.

$$\nu_t = \sum_{j=1}^{p} \varsigma_j' \Delta z_{t-j}, \tag{2.3}$$

where $\Delta z_t = (\Delta p_{h,t}, \Delta p_{s,t}, \Delta p_{b,t})'$ and ς_j is a $k \times 1$ vector of parameters for each lag j. Model (2.1) coupled with (2.2) and (2.3) allows to analyze whether short-run (log) real estate price changes (i.e. log real estate returns) can be conditioned upon the equilibrium error of real estate prices h_t, the second order moments in λ_t and lags of asset returns contained in Δz_t. In this scenario we deal with a predictive model since the evolution of real estate returns at t is strictly conditional on information available at period $t-1, ..., t-p$. The model can be seen as a single equation error correction model 'augmented' by an additional vector of explanatory variables λ_t. The model is also interesting in the sense that it conditions real estate security returns upon different 'layers' of asset prices, namely, log asset prices (h_t), returns (Δz_t) and volatility (λ_t).

The second model for ν_t augments the specification in (2.3) with contemporaneous equity returns, i.e.[2]

$$\nu_t = \eta \Delta p_{s,t} + \sum_{j=1}^{p} \varsigma_j' \Delta z_{t-j}. \tag{2.4}$$

Model (2.1) coupled with (2.2) and (2.4) introduces a possible simultaneity issue in the contemporaneous relationship between equity returns ($\Delta p_{s,t}$) and real

[1] see Campbell *et al.* (1997) Chap. 12 for a comprehensive discussion on this issue. This has lead to propositions such as the *leverage hypothesis* or the *volatility feed-back hypothesis* due to Black (1976) and Campbell & Hentschel (1992) respectively.

[2] A previous version of this Chapter included contemporaneous and dynamic interactions of real estate returns ($\Delta p_{h,t}$) with changes in a sentiment index. However, the results turned out insignificant so we report only model (2.4).

estate returns ($\Delta p_{h,t}$) which would complicate the identification of η. Thus, a careful identification strategy should be employed. The empirical model can also be written as,

$$\Delta p_{h,t} = \alpha + \phi h_{t-1} + \pi' \lambda_{t-1} + \nu_t + u_{h,t}, \tag{2.5}$$

coupled with either (2.3) or (2.4). Thus, we propose the analysis of log real estate returns at period t via three major components: (i) a 'theoretical' component made up of the lagged equilibrium error (h_{t-1}) as well as lagged time-varying risk pricing variables (λ_{t-1}), (ii) an 'atheoretical' component made up of dynamics of asset returns ($\Delta z_{t-1}, ..., \Delta z_{t-p}$) and (iii) contemporaneous equity returns ($\Delta p_{s,t}$).

2.3 Econometric methodology

2.3.1 The data

The empirical specifications for log real estate returns introduced in the previous section are examined from an international panel perspective for the following reasons. First, as introduced previously, studies of real estate security prices have been mostly applied to individual markets, particularly the US. Second, a useful and interesting asset pricing model should hold for single markets as well as for a cross-section of markets, otherwise one may fall in the trap of generalizing a model that might be only informational to a particular market. Third, the main interest here is in uncovering the long-run, dynamic and contemporaneous components that can explain the cross-section of international real estate securities. Let $i = 1, ..., N$ indicate the respective cross sectional entities (countries) and $t = 1, ..., T$ the *monthly* time periods. The sample covered for the analysis runs from 01/1996 to 12/2006 which is interesting as we evaluate the model before the September 2008 financial crisis. For instance, if the coefficient of equilibrium adjustment ϕ_i is positive, then there is an 'explosive' behavior of real estate securities in relation to equity and bond prices in country i.

We have two main datasets. The first dataset is a panel of $N = 14$ developed countries and is used to evaluate the real estate pricing model proposed in

(2.5). The countries are: Australia, Belgium, Canada, Denmark, France, Germany, Italy, Japan, Netherlands, Norway, Spain, Sweden, United Kingdom and United States. Countries were chosen upon data availability for the sample period covered. The data is used to compute the returns $\Delta p_{h,it}$ and the equilibrium error h_{it} and consists of Datastream calculated indices of real estate securities, all-share equity (total market) and 10-year (benchmark) government bonds at the *monthly* frequency.

The second dataset consists of financial data (price indices, price-earnings ratios and dividend yields) for real estate, equity and bond markets at both the *daily* and *yearly* frequency which is also collected from Datastream. It also consists of *yearly* data on GDP as well as *yearly* long-term and short-term government bond yields collected from Datastream or the IMF international statistics. The *daily* data is employed to compute realized volatilities of asset returns $\hat{\sigma}^2_{\bullet,it}$ for $\bullet = h, s, b$. The daily data is also used to compute a common 'world' asset factor $\hat{f}_{dt}$ where d denotes a *daily* observation in a particular month t. The daily frequency of the factor allows us to compute its realized volatility $\hat{\sigma}^2_{f,t}$ and the realized covariances of the factor with asset returns $\hat{\sigma}_{\bullet f,it}$ for $\bullet = h, s, b$. The *yearly* data is also used in the computation of the common factor as explained in Appendix B.

2.3.2 Stationarity and cointegration analysis

The real estate pricing model in (2.5) presumes that the composite equilibrium error $h_{it} = p_{h,it} - \beta_i' \mathbf{x}_{it}$ with $\mathbf{x}_{it} = (p_{s,it}, p_{b,it})'$ is $I(0)$. As a first step, we should analyze whether the variables $p_{h,it}$, $p_{s,it}$ and $p_{b,it}$ are integrated of order one, i.e. $I(1)$. However, in the present Chapter, the order of integration of the latter variables is not explicitly tested since it is quite well known in the finance literature that asset prices at the *monthly* frequency are mostly $I(1)$.

If $p_{h,it}$, $p_{s,it}$ and $p_{b,it}$ are in fact $I(1)$ then, under the null of *no cointegration*, the residual h_{it} will also be $I(1)$. To test the latter null hypothesis, we employ the panel cointegration procedure proposed by Pedroni (2004) who recently developed a residual-based panel cointegration test for models with more than one independent variable. Both the Phillips-Peron (PP) and the Augmented Dickey-Fuller (ADF) versions of the Group and Panel t-statistics of Pedroni's test are employed. The null hypothesis of *no cointegration* is the same for both group and panel statistics but the alternative hypothesis differs. The group statistic assumes a heterogeneous alternative while the panel statistic assumes a homogeneous alternative. As a means of comparison we also employ the Levin *et al.* (2002) (LLC) panel unit root test to test the null hypothesis of *unit root*.

2.3.3 Common factor, time-varying second order moments and equilbrium error

The strategy for estimation consists of: (i) estimating a common 'world' asset factor $\hat{f}_{dt}$ from international daily asset returns, (ii) estimating the vector of second order moments $\hat{\lambda}_{it}$ by means of realized volatility methods, (iii) estimating the error correction term $\hat{h}_{it}$. The flow chart of the strategy for estimation and testing can be represented as follows,

Common factor $(\hat{f}_{dt})$ $\rightarrow$ Realized volatility $(\hat{\lambda}_{it})$ $\rightarrow$

Long-run coefficients (β_i)=$\left\{\begin{matrix}\text{Pooled}\,_{(\hat{\beta})} \\ \text{Mean group}\,_{(\bar{\beta})}\end{matrix}\right\}$ $\rightarrow$ Hausmman test $(H_0 : \beta_i = \beta)$

$\rightarrow$ Equilibrium error $(\hat{h}_{it})$= $\left\{\begin{matrix} p_{h,it} - \hat{\beta}_1 p_{s,it} - \hat{\beta}_2 p_{b,it} \text{ vs.} \\ p_{h,it} - \hat{\beta}_{1,i} p_{s,it} - \hat{\beta}_{2,i} p_{b,it}. \end{matrix}\right\}$

In what follows we describe each step in greater detail although the explanation is kept condensed to save on space. A more precise treatment of the methods employed can be looked up in the references herein.

Equity Markets		Bond Markets	Real Estate Markets
Australia	Argentina	Australia	Austria
Austria	Brasil	Belgium	Belgium
Belgium	Chile	Canada	Canada
Canada	China	Denmark	Denmark
Denmark	Colombia	France	France
Finland	Cyprus	Germany	Germany
France	Czech Republic	Italy	Italy
Germany	Greece	Japan	Japan
Hong Kong	Hungary	Netherlands	Netherlands
Ireland	India	Norway	Norway
Italy	Mexico	Spain	Spain
Japan	Pakistan	Sweden	Sweden
Luxemburg	Peru	United Kingdom	United Kingdom
New Zealand	Poland	United States	United States
Norway	Russia		
Spain	South Africa		
Sweden			
Switzerland			
United Kingdom			
United States			

Table 2.1: International equity, bond and real estate markets for factor construction

2.3.3.1 The common factor

We construct a factor that mimics changes in regimes in international real estate, equity and bond price fundamentals. The factor is common to all markets (i.e. countries) and is of the form,

$$\hat{f}_{dt} = \hat{f}_{h,dt} + \hat{f}_{s,dt} + \hat{f}_{b,dt}, \tag{2.6}$$

where d is a daily observation in month t. Each factor $\hat{f}_{\bullet,dt}$ for asset $\bullet = h, s, b$ represents (log) return differentials between country portfolios constructed based on certain criteria. We select criteria that aim to proxy the 'fair' price of international assets, namely, price-dividend ratios (PD) for real estate, price-earnings ratios (PE) for equity and term spreads (TS) for bond markets. The latter approach is in line with previous findings that returns of portfolios constructed based on fundamental measures can significantly explain variation of expected returns (cf. Fama & French (1993, 1995)).[3] Moreover, we also employ GDP (in Dollar terms) to control for the 'value' of the countries included in the portfolios. This is in line with recent findings that also value measures can significantly explain time variation of expected returns (cf. Campbell & Vuolteenaho (2004), Campbell *et al.* (2005)).

The constructed 'world' asset factor $\hat{f}_{dt}$ accounts for total asset return differentials of portfolios composed of countries which have high versus low fundamentals (PD/PE/TS) corresponding to the three $\bullet = h, s, b$ markets after controlling for the 'value' of their economies (GDP). Moreover, the factor should account dynamically for the changes in (yearly) fundamentals in international asset markets since it is computed by rebalancing every year the groups of countries in the portfolios used to calculate the return differentials. To save on space, technical details on the construction of the factor $\hat{f}_{dt}$ are provided in Appendix B.

[3] We employ PD ratios for real estate markets as opposed to PE ratios for two main reasons. First we have a lack of observations of PE ratios for some of the markets considered. Second, and most importantly, real estate securities should have a strong dividend-ratio component if the portfolios are mostly composed of Real Estate Investment Trusts (REITs). To qualify as REITs a real estate company must agree to pay out in dividends at least 90% of its taxable profit. With REIT status, a company can avoid paying corporate income tax. See *www.reit.com* for more detailed information.

It is noteworthy that, due to data availability, we include as many countries as possible for *equity* markets in the construction of the common factor to have a better representation of international asset markets. For instance, an 'unbalanced' panel in the case of equity markets allows us to include emerging equity markets which may account for large fluctuations in world asset markets (cf. Bekaert & Harvey (2003)). The countries considered for the factor construction are presented in Table 2.1.

2.3.3.2 Time-varying second order moments

The panel version of the model of real estate returns as specified in (2.5) accounts for time-varying second order moments contained in the vector λ_{it} for each i. Note that a possible estimation approach for the specification in (2.1) would be a 4-dimensional volatility-in-mean model. However, as mentioned previously, such models would be difficult to implement with panel data or recursive estimation. For instance, with the BEKK(1,1,1) representation of Engle & Kroner (1995) for the time-varying variance this would imply the estimation of at least 50 parameters for each i. The estimation would even be more cumbersome, for instance, in the case of a multivariate stochastic volatility-in-mean model with panel data.[4] Thus, for the purpose of this study, we follow the non-parametric realized volatility (RV) approach proposed by Andersen *et al.* (2003) who provide a general framework for the measurement and modeling of volatility and covariance distributions. Realized volatility estimates allows one to use traditional time series methods for analysing empirical finance models.

In our context, the realized variance-covariance matrix for each $i = 1, ..., N$ and $t = 1, 2, ..., T$ time periods is given by:

$$\hat{\Sigma}_{it} = \sum_{d=1}^{D} \mathbf{v}_{i,dt}\mathbf{v}'_{i,dt}, \tag{2.7}$$

where $\mathbf{v}_{i,dt} = (\Delta p_{h,idt}, \Delta p_{s,idt}, \Delta p_{b,idt}, \hat{f}_{dt})'$ and d is a daily observation in a particular month t. Given the estimates for each month t we obtain $\hat{\lambda}_{it} = (\hat{\sigma}_{11,it}, \hat{\sigma}_{22,it}, \hat{\sigma}_{33,it}$

[4]As a preliminary analysis we estimated 2-dimensional and 3-dimensional volatility-in-mean models for a panel of international asset markets. However, we ran into problems of non-convergence for many countries and the results were mostly insignificant.

$, \hat{\sigma}_{44,t}, \hat{\sigma}_{41,it}, \hat{\sigma}_{42,it}, \hat{\sigma}_{43,it})' = (\hat{\sigma}^2_{h,it}, \hat{\sigma}^2_{s,it}, \hat{\sigma}^2_{b,it}, \hat{\sigma}^2_{f,t}, \hat{\sigma}_{hf,it}, \hat{\sigma}_{sf,it}, \hat{\sigma}_{bf,it})'$. Thus, the procedure boils down to simply computing volatility and covariance estimates at month t from a higher frequency of the data, in this case daily data. It is noteworthy, however, that the RV approach assumes that the variables in $\mathbf{v}_{i,dt}$ follow (semi)-martingale processes for the second moment estimates to be consistent. This requirement applies to our context empirically since it is well known in the finance literature that daily asset returns follow approximately martingales (cf. Ding *et al.* (1993), Bollerslev & Mikkelsen (1996), Baillie (1996)). It also applies for $\hat{f}_{dt}$ since it is constructed from linear combinations of portfolios' asset returns.

2.3.3.3 Equilibrium error

In order to estimate the long-run coefficients in the error term h_{it} with panel data, one may consider a variety of recently available methodologies such as the panel Autoregressive Distributed Lag (ADL) model proposed in Pesaran *et al.* (1999) and Pesaran & Smith (1995), the Fully Modified OLS (FMOLS) estimators proposed by Pedroni (2001, 2004) and the dynamic OLS (DOLS) estimators proposed by Kao & Chiang (2000) and Mark & Sul (2003). In general, Monte Carlo studies have shown that the DOLS model produces less volatile estimates with a relatively smaller sample bias and should be preferred in single equation models (cf. Maddala & Kim (1998), Kao & Chiang (2000)).[5] Thus, for the purpose of this study we consider a DOLS(a) model for $i = 1, 2, ..., N$ groups and $t = 1, 2, ..., T$ *monthly* time periods which takes the form:

$$p_{h,it} = \beta_{0,i} + \beta_i' \mathbf{x}_{it} + \delta_i' \Delta \mathbf{y}_{it} + \varepsilon_{it}. \tag{2.8}$$

In our case β_i is a $k \times 1$ vector of long-run coefficients attached to $\mathbf{x}_{it} = (p_{s,it}, p_{b,it})'$, $\delta_i = (\delta'_{i,-a_i}, ..., \delta'_{i,0}, ..., \delta'_{i,+a_i})'$ is a $(2a_i + 1)k$-dimensional vector of projection parameters attached to $\Delta \mathbf{y}_{it} = (\Delta \mathbf{x}'_{it-a_i}, ..., \Delta \mathbf{x}'_{it}, ..., \Delta \mathbf{x}'_{it+a_i})'$ which is a $(2a_i + 1)$ k-dimensional vector of leads and lags of the first differences of the variables in $\mathbf{x}_{it}$. The leads and lags are included to control for the endogenous feedback effects of the regressors. The Mean Group and Panel DOLS approaches are employed for

[5] We performed a Hausmann test to discriminate between the ADL, FMOLS and DOLS estimators and the result favors the DOLS estimator.

the estimation of the long-run coefficients with panel data (cf. Kao & Chiang (2000), Pedroni (2001), Mark & Sul (2003)). While the Mean Group DOLS approach assumes a heterogeneous vector of long-run coefficients for each member i of the cross-section, the Panel DOLS approach assumes a homogeneous vector of long-run coefficients. The Mean Group and Panel DOLS estimators are denoted $\bar{\beta}$ and $\hat{\beta}$, respectively. More details on the estimators are found in Appendix B.

We discriminate between the Mean Group and the Panel DOLS estimators by testing the null hypothesis of long-run parameter homogeneity $H_0 : \beta_i = \beta$ via a Hausmann test denoted $\mathcal{H}$. The equilibrium error is then computed as $\hat{h}_{it} = p_{h,it} - \hat{\beta}_1 p_{s,it} - \hat{\beta}_2 p_{b,it}$ if H_0 holds and $\hat{h}_{it} = p_{h,it} - \hat{\beta}_{1,i} p_{s,it} - \hat{\beta}_{2,i} p_{b,it}$ if H_0 is rejected. Note also that if H_0 holds, then the equilibrium relationship between (log) real estate, equity and bond prices holds *homogeneously* across the panel of countries considered.

2.3.4 International real estate returns

In this section we present the estimation and testing strategy of the panel version of model (2.5) given the estimated variables $\hat{\lambda}_{it}$ and $\hat{h}_{it}$. The model analyzed for $i = 1, ..., N$ and $t = 1, ..., T$ is given by[6]

$$\Delta p_{h,it} = \alpha_i + \phi_i \hat{h}_{it-1} + \pi_i' \hat{\lambda}_{it-1} + \nu_{it} + u_{h,it}, \tag{2.9}$$

where α_i represents the fixed effects, ϕ_i is the error correction coefficient measuring the speed of adjustment toward the long-run equilibrium, π_i is the vector attached to the volatility components capturing the 'price of risk'. In what follows, we consider in more detail the estimation and testing techniques for the two models describing ν_{it}.

[6] We also experimented with the threshold error correction proposed in Enders & Siklos (2001). However, we found no evidence in favor of threshold error correction.

2.3.4.1 Dynamic analysis

In this section we consider the first model of ν_{it} for $i = 1, ..., N$ and $t = 1, ..., T$ which is given by:

$$\nu_{it} = \sum_{j=1}^{p} \varsigma'_{j,i} \Delta z_{it-j}, \tag{2.10}$$

where $\varsigma_{j,i}$ is a 3×1 parameter vector attached to $\Delta z_{it} = (\Delta p_{h,it}, \Delta p_{s,it}, \Delta p_{b,it})'$ for each lag j. Model (2.9) coupled with (2.10) can be estimated via OLS. In what follows, let $\hat{\psi}_{m,i} = (\hat{\alpha}_i, \hat{\phi}_i(\hat{\beta}), \hat{\pi}'_i, \hat{\varsigma}'_{1i}, ..., \hat{\varsigma}'_{pi})'$ be the vector of OLS estimates obtained from (2.9) if $H_0 : \beta_i = \beta$ holds and let $\hat{\psi}_{n,i} = (\hat{\alpha}_i, \hat{\phi}_i(\hat{\beta}_i), \hat{\pi}'_i, \hat{\varsigma}'_{1i}, ..., \hat{\varsigma}'_{pi})'$ be the vector of OLS estimates if $H_0 : \beta_i = \beta$ is rejected. For simplicity let $\hat{\psi}_{r,li}$ for $r = m, n$ denote a particular coefficient l contained in the latter vectors. Standard errors of the estimates $\hat{\psi}_{r,li}$ are computed with White (1980)'s heteroskedasticity robust covariance estimator to account for the heteroskedasticity in the innovations $u_{h,it}$ found empirically in most models of asset returns.

At the panel dimension, we perform Mean Group inference with respect to the above model along the lines of Pesaran *et al.* (1999) and Pesaran & Smith (1995). The Pooled Mean Group (PMG) estimator is given by $\bar{\psi}_{PMG} = (\bar{\alpha}, \bar{\phi}(\hat{\beta}), \bar{\pi}', \bar{\varsigma}'_1, ..., \bar{\varsigma}'_p)'$ and is computed from $\hat{\psi}_{m,i}$ while the Mean Group (MG) estimator is given by $\bar{\psi}_{MG} = (\bar{\alpha}, \bar{\phi}(\hat{\beta}_i), \bar{\pi}', \bar{\varsigma}'_1, ..., \bar{\varsigma}'_p)'$ and is computed from $\hat{\psi}_{n,i}$. Both estimators can be obtained by averaging the individual market estimates $\hat{\psi}_{r,i}$ for $r = m, n$ and they are computed from our estimation strategy depending on the result of the Hausmann test ($\bar{\psi}_{PMG}$ if $H_0 : \beta_i = \beta$ holds otherwise $\bar{\psi}_{MG}$).[7]

In order to evaluate the robustness of the estimates and some of the statistics used to evaluate the model we perform bootstrap inference. The bootstrap distribution is obtained under the null hypothesis $H_0 : \alpha_i = \phi_i = \pi'_i = \varsigma'_{j,i} = 0$ in (2.9) and (2.10) so that $\Delta p_{h,it} = u_{h,it}$. The procedure consists of:

1. Generating bootstrap returns $\Delta p^b_{h,it}$ by randomly drawing with replacement from the set of returns $\{\Delta p_{h,i1}, ..., \Delta p_{h,iT}\}$ for each $i = 1, ..., N$.

[7] Additionally, we employed efficient GMM to estimate a pooled version of (2.9) across the panel to obtain a homogeneous set of parameters and we employed a Hausmann test to discriminate between the homogeneous and heterogeneous estimators. The test favors the heterogeneous panel estimators.

2. Re-estimating the parameters $\hat{\psi}^b_{r,i}$, $\bar{\psi}^b_{\bullet}$ as well as the (adjusted) degree of explanation (R^2_i) of the regression denoted R^2_{ib}.

3. Steps 1 and 2 are repeated, say 500 times, and confidence intervals are obtained by using $\gamma/2$ and $(1-\gamma/2)$ quantiles of the bootstrap distribution.

Note that since the bootstrap distribution is obtained under the null $H_0 : \alpha_i = \phi_i = \pi'_i = \varsigma'_{j,i} = 0$, the estimates $\hat{\psi}_{r,i}$, $\bar{\psi}_{\bullet}$ and R^2_i that lie *outside* the bootstrap confidence bands can be seen as statistically significant.

Most empirical studies of asset return predictability focus on the in-sample performance of asset pricing models. As mentioned in the previous Chapter and rectified in our findings in the international market context, recent studies have found that in-sample predictability does not necessarily imply out-of-sample forecastability (cf. Andrew & Bekaert (2001), Hjalmarsson (2006), Campbell & Thompson (2007)). Treating asset pricing models only in-sample may lead to spurious conclusions about the empirical underpinning of the asset pricing model at hand. Although the main interest in this Chapter is not out-of-sample forecasting but rather to uncover some of the long-run, dynamic and contemporaneous determinants of international real estate returns, the diagnostic approach treats selected out-of-sample tests at the individual market level and at the aggregate level.

For the out-of-sample analysis we employ rolling window forecasting. A window of size $T_{min} = 48$ is fixed and moved over the subsample $\tau - 1 = T_{min}, \ldots, \tau - 1 = T - 1$.[8] Similar to Lettau & Ludvigson (2001a), the long-run parameter vector $\hat{\beta}_i$ is estimated via DOLS(2) at each window.[9] Let $\Delta\hat{p}_{h,i\tau}$ denote an ex-ante forecast for $\Delta p_{h,i\tau}$ conditional on information available in $\tau - 1$. Forecast errors are computed as,

$$\hat{u}_{h,i\tau} = \Delta p_{h,i\tau} - \Delta\hat{p}_{h,i\tau}, \ \tau = 49, \ldots, T. \tag{2.11}$$

[8]We experimented with other window sizes but the results remain qualitatively the same.

[9]The results do not change qualitatively when using the one-step ADL estimator proposed in Pesaran *et al.* (1999) and Pesaran & Smith (1995) or the FMOLS estimator proposed by Pedroni (2004).

The out-of-sample analysis consists on testing the behavior of the above forecast errors or of the single forecasts $\Delta\hat{p}_{h,i\tau}$. Technical details of the in-sample and out-of-sample tests employed are found in Appendix B.

2.3.4.2 Contemporaneous analysis

In this section we consider the alternative model of ν_{it} for $i = 1, ..., N$ and $t = 1, ..., T$ which is given by:

$$\nu_{it} = \eta\Delta p_{s,it} + \sum_{j=1}^{p} \varsigma'_{j,i}\Delta z_{it-j}. \tag{2.12}$$

Model (2.12) introduces a contemporaneous feature which complicates the analysis of model (2.9). First, we assume a homogeneous contemporaneous parameter η attached to $\Delta p_{s,it}$. Second, the relationship between contemporaneous real estate returns $\Delta p_{h,it}$ and equity returns $\Delta p_{s,it}$ can be characterized by a simultaneity problem since we do not know the exact direction of the causality between the two variables. Consequently, OLS estimation of η in model (2.9) would yield biased estimates. The problem is approached via 'Identification through Heteroskedasticity' (IH) developed by Rigobon (2003). We start by considering the following reduced form model,

$$y_{it} = \Gamma_i x_{it} + e_{it}, \tag{2.13}$$

where $y_{it} = (\Delta p_{h,it}, \Delta p_{s,it})'$, Γ_i is a $2 \times k$ matrix of reduced form parameters attached to $x_{it} = (1, \hat{h}_{it-1}, \hat{\lambda}'_{it-1}, \Delta z'_{it-1}, ..., \Delta z'_{it-p})'$ and $e_{it} = A^{-1}u_{it}$ are the reduced form residuals where,

$$A = \begin{bmatrix} 1 & -\eta \\ -\rho & 1 \end{bmatrix}, \tag{2.14}$$

and $u_{it} = (u_{h,it}, u_{s,it})'$ are the structural shocks which are assumed heteroskedastic but uncorrelated. Thus, in this set up, the reduced form residuals e_{it} contain the contemporaneous parameters of interest. Defining $\Omega_i = E[[e_{h,it}\, e_{s,it}]' \cdot [e_{h,it}\, e_{s,it}]]$, it can be shown that the later *unconditional* variance-covariance matrix contains the contemporaneous parameters η and ρ as well as the structural variances of $u_{h,it}$ and $u_{s,it}$:

$$\Omega_i = \zeta \begin{bmatrix} \sigma_{h,i}^2 + \eta^2\sigma_{s,i}^2 & \rho\sigma_{h,i}^2 + \eta\sigma_{s,i}^2 \\ & \rho^2\sigma_{h,i}^2 + \sigma_{s,i}^2 \end{bmatrix}, \tag{2.15}$$

where $\zeta = (1-\eta\rho)^{-2}$. It is worth noting that the covariance matrix Ω_i can also be assumed to be time-varying (Rigobon (2003)). The above analytical expression for the variance-covariance matrix of the reduced form residuals $e_{h,it}$ and $e_{s,it}$ exemplifies the identification problem of introducing the contemporaneous effects of equity returns on real estate returns in model (2.9) since we end up with four unknowns (the contemporaneous parameters and the two variances) but only three equations.

Following Rigobon (2003), assume that the data can be split into two (or more) distinct sets assuming that there are two regimes in the variances of the structural shocks, i.e. high and low volatility (heteroskedasticity). In addition, it is assumed that the structural parameters are stable across regimes. These assumptions imply that one may estimate $\hat{\Omega}_{i,R}$ for at least $R = 1, 2$ where $\sigma_{h,i1}^2/\sigma_{s,i1}^2 \neq \sigma_{h,i2}^2/\sigma_{s,i2}^2$ and obtain 6 moments from the data (2 variances and 1 covariance from $\hat{\Omega}_{i,1}$ and 2 variances and 1 covariance from $\hat{\Omega}_{i,2}$) which should be explained by 6 parameters of interest (the 2 contemporaneous parameters η or ρ and the 4 structural moments). We end up with 6 equations and 6 unknowns that enable us to identify η and ρ. To exploit regime changes in the panel, the 'algorithm' to estimate A is along the lines of Lee *et al.* (2004) and consists of:

1. Estimating (2.13) for each unit i separately and recovering the reduced form residuals $\hat{e}_{it}$ for each i. Estimating the unconditional covariance matrix $\hat{\Omega}_i$ of the reduced form residuals and splitting the units $i = 1, ..., N$ into four groups $g = 1, ..., 4$: high/low variance of 'conditionally centered' real estate returns $\hat{e}_{h,it}$ and high/low variance of 'conditionally centered' equity returns $\hat{e}_{s,it}$. High variance is defined as values above the median while low variance are values below the median.

2. For each of the four groups $g = 1, ..., 4$ we estimate 'Mean Group' covariance matrices, denoted $\hat{\Omega}_g$. This procedure would provide 12 sample moments

(4 covariances and 8 variances from $\hat{\Omega}_g$, $g = 1, .., 4$) which need to be explained by 10 parameters (8 structural moments and the 2 contemporaneous coefficients η and ρ).

3. One can employ GMM or non-linear least squares to estimate the 10 parameters of interest by minimizing the distance of the sample moments from their theoretical counterparts.

Model (2.13) allows to perform impulse response analysis to study the contemporaneous effect of a shock to the structural innovations on the variables in y_{it}. One may estimate the structural impulse responses for each of units i, for the sub-samples $g = 1, .., 4$ or for the entire sample since the matrix of coefficients $\hat{A}$ is assumed to be invariant to the different regimes. The estimated structural impulse responses for all $i = 1, ..., N$ and $t = 1, ..., T$ would be nonlinear functions of the estimate $\hat{A}$ (the matrix of structural parameters) and $\hat{B}_{ji}$ (the submatrix of coefficients attached to lags j of $\Delta p_{h,it}$ and $\Delta p_{s,it}$):

$$\hat{\vartheta}_{lm,i}(h) = f(\hat{A}, \hat{B}_{i1}, ..., \hat{B}_{ip}), \tag{2.16}$$

where $\hat{\vartheta}_{lm,i}(h)$ represents the response of variable l to an impulse on innovation m, h periods ago in country i. 'Mean group' impulse responses are estimated as $\bar{\vartheta}_{lm}(h) = N_g^{-1} \sum_i \hat{\vartheta}_{lm,i}(h)$ for each of the groups $g = 1, ..., 4$. Bootstrap confidence bands for the impulse responses are computed along the lines of Benkwitz *et al.* (2001). The latter procedured is described in Appendix B.

2.4 Results

This section discusses the results of the real estate pricing model discussed in the previous section. The next two subsections discuss the cointegration and long-run estimation results as well as a summary of the realized volatility and common factor results. The last two subsections address the individual results and the aggregate results of the international real estate return model, respectively. The following discussion refers to the 5% level of statistical significance unless otherwise stated.

Country	LAG	NWB	ADF$_i$	PP$_i$	$\hat{\beta}_{1i}$	$\hat{\beta}_{2i}$
AUL	0	0	**-4.410**	**-4.390**	**0.806**	**1.295**
			[0.000]	[0.000]	(4.968)	(21.56)
BEL	0	5	**-3.376**	**-3.486**	**0.209**	**1.353**
			[0.009]	[0.006]	(8.745)	(21.02)
CAN	3	4	**-2.262**	**-2.899**	**0.387**	**3.554**
			[0.009]	[0.014]	(3.856)	(12.66)
DEN	0	2	**-2.009**	**-2.020**	**0.847**	**4.014**
			[0.043]	[0.042]	(3.526)	(6.330)
FRA	7	2	*-1.716*	-1.578	**0.319**	**3.553**
			[0.082]	[0.108]	(3.839)	(19.21)
GER	2	3	**-2.027**	**-2.047**	**1.906**	**2.477**
			[0.041]	[0.039]	(7.380)	(4.378)
ITA	0	6	**-1.972**	*-1.870*	**0.612**	**2.081**
			[0.047]	[0.006]	(5.337)	(9.906)
JAP	2	4	-0.799	-0.732	**0.888**	-0.100
			[0.367]	[0.397]	(3.943)	(−0.224)
NET	1	3	*-1.744*	-1.604	**0.166**	**1.662**
			[0.077]	[0.102]	(3.090)	(11.72)
NOR	0	0	**-2.709**	**-2.709**	**0.644**	**3.518**
			[0.008]	[0.007]	(7.106)	(16.02)
SPA	1	4	-0.619	-0.507	**0.508**	**2.743**
			[0.447]	[0.495]	(2.516)	(11.06)
SWE	0	2	-1.315	-1.439	0.209	**3.549**
			[0.174]	[0.139]	(1.528)	(9.047)
GRB	3	5	-0.641	-0.243	**0.438**	**1.596**
			[0.438]	[0.597]	(2.619)	(4.235)
USA	3	6	-0.795	-1.130	**0.360**	**2.338**
			[0.371]	[0.234]	(3.000)	(5.988)

Table 2.2: Unit root and long run estimation results per country (t-ratios in parentheses, p-values in brackets). Note: ADF$_i$ is the Augmented Dickey-Fuller unit root test statistic with Aikaike Information Criteria (AIC) selected lag of the dependent variable given by LAG. PP$_i$ is the Philipps-Peron unit root test with Newey-West Barlett kernel with the bandwidth given by NWB. The ADF$_i$ and PP$_i$ unit root tests are for the estimated error $\hat{h}_{it} = p_{h,it} - \hat{\beta}_i' \mathbf{x}_{it}$. Estimation of the long-run parameters $\hat{\beta}_{1i}$ and $\hat{\beta}_{2i}$ is done via a fixed effect DOLS(2) model for each i.

Statistic	P-ADF	P-PP	G-ADF	G-PP	LLC
	7.323	**3.393**	**9.852**	**4.367**	**-4.448**
	[0.000]	[0.001]	[0.000]	[0.000]	[0.000]
LR est./Hauss.	$\bar{\beta}_1$	$\bar{\beta}_2$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\mathcal{H}$
	0.593	**2.402**	**0.696**	**2.253**	0.137
	(4.954)	(7.678)	[0.000]	[0.015]	[0.711]

Table 2.3: Cointegration and long-run estimation results at the aggregate level (t-ratios in parentheses, p-values in brackets). Note: P-ADF (P-PP) is the Augmented Dickey-Fuller (Phillips Perron) version of Pedroni's *panel* cointegration statistic, G-ADF (G-PP) is the Augmented Dickey-Fuller (Phillips Perron) version of Pedroni's *group* cointegration statistic. LLC is the Levin *et al.* (2002) panel unit root test. $\bar{\beta}$ is the vector of long-run coefficients computed via (fixed effects) Mean Group Panel DOLS(2), $\hat{\beta}$ is the vector of long-run coefficients computed via (fixed effects) Pooled DOLS(2) and $\mathcal{H}$ is the Hausmann statistic.

2.4.1 Cointegration and long-run estimation results

As stated previously, we hypothesized a priory that (log) real estate ($p_{h,it}$), equity ($p_{s,it}$) and bond ($p_{b,it}$) indices are all $I(1)$. Thus, finding a stationary linear combination of these variables would indicate that a long run equilibrium error exists between the latter asset prices. As a preliminary stationarity analysis, Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests were performed for the equilibrium error $\hat{h}_{it}$ at the single market level (Table 2.2). The results of the ADF and PP unit root tests suggest that the latter equilibrium error is stationary for particular markets thus hinting at a stationary relationship between the three asset types considered. More precisely, the null of unit root can be rejected at a 5% level of significance in 7 (6) out of the 14 markets under inspection with the ADF (PP) test, namely, Australia, Belgium, Canada, Denmark, Germany, Italy and Norway. However, since $\hat{h}_{it}$ is estimated, the ADF and PP tests can only give a hint about the stationarity of the latter equilibrium error since critical values of the tests might change.

Thus, at the aggregate level, it is preferable to test the null of *no cointegration* between (log) real estate ($p_{h,it}$), equity ($p_{s,it}$) and bond ($p_{b,it}$) indices via the panel cointegration tests proposed by Pedroni (2004) which take account of appropriate asymptotic and finite sample properties. The null of *no cointegration* can be rejected at the 5% significance level with the Panel and the Group statistics for both ADF and PP versions of Pedroni's test (Table 2.3: P-ADF=7.323, P-PP=3.393, G-ADF=9.852, G-PP=4.367). Moreover, the null hypothesis of non-stationarity for the equilibrium error can also be rejected with the LLC panel unit root test at a 5% level of significance (Table 2.3: LLC=-4.448). Thus, we find strong evidence of a long-run relationship between (log) real estate security prices, equity prices and bond prices for the cross-section of international asset markets. This finding hints at the possibility that the latter three asset markets are financially integrated (at least on average) within the countries at hand.

With respect to the estimation of long run-parameters, we fit a DOLS(2) model based on the result of the Akaike Information Criterion (AIC). We obtain a positive and statistically significant relationship at the 5% level between log real estate security prices and log equity prices (log bond prices) in 13 (13) out of the 14 international asset markets (Table 2.2). The positive and statistically significant relationship between the three assets at the single market level is confirmed at the aggregate level via the Mean Group and panel DOLS estimators (Table 2.3). The result on the Hausmann test $\mathcal{H} = 0.14$ is in favor of the pooled DOLS estimator of long-run coefficients which hints at homogeneity in the postulated equilibrium relationship for real estate security prices amongst international markets. This finding also favors the argument that real estate, equity and bond markets are financially integrated across the countries under inspection. Moreover, the homogeneity in the long-run relationship between (log) real estate, equity and bond prices across countries also suggests that increases in (log) equity or bond prices have a positive and homogeneous marginal effect on international real estate prices. The elasticity of real estate prices with respect to equity prices is estimated to be $\hat{\beta}_1 = 0.7$ (p-value=0.000) while the elasticity with respect to bond prices is estimated to be $\hat{\beta}_2 = 2.25$ (p-value=0.015).

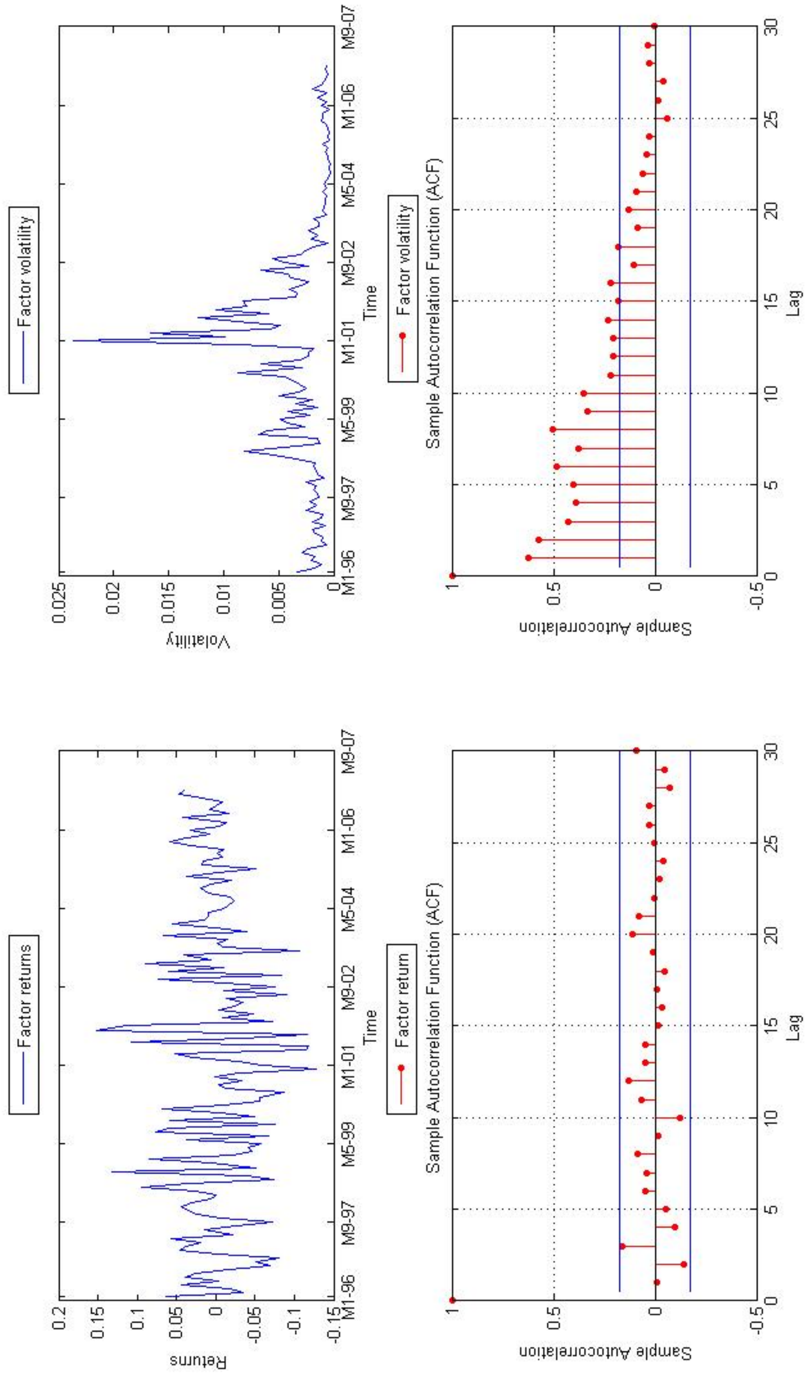

Figure 2.1: Factor returns and factor volatility

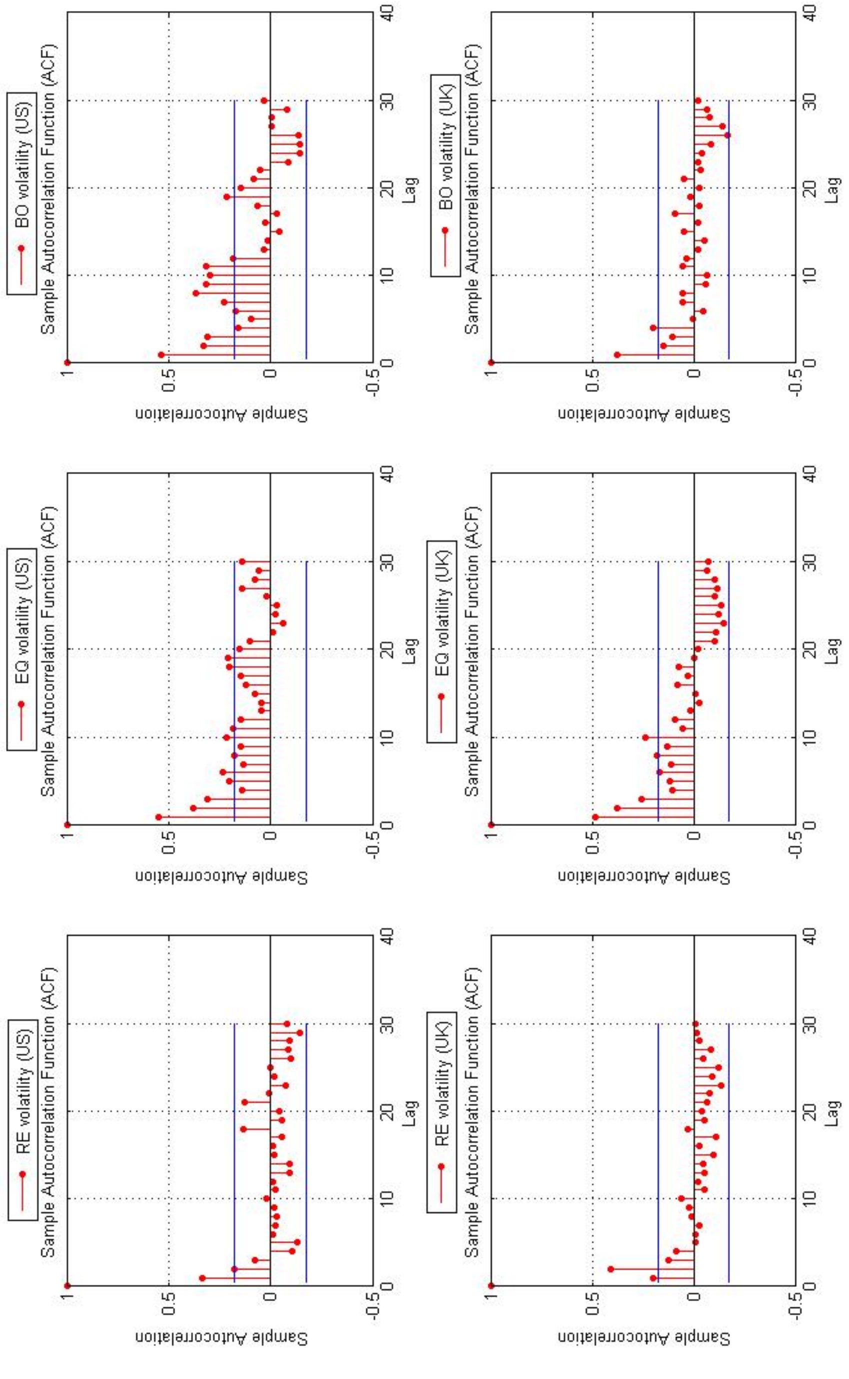

Figure 2.2: Autocorrelation functions of realized volatilities for United States and United Kingdom

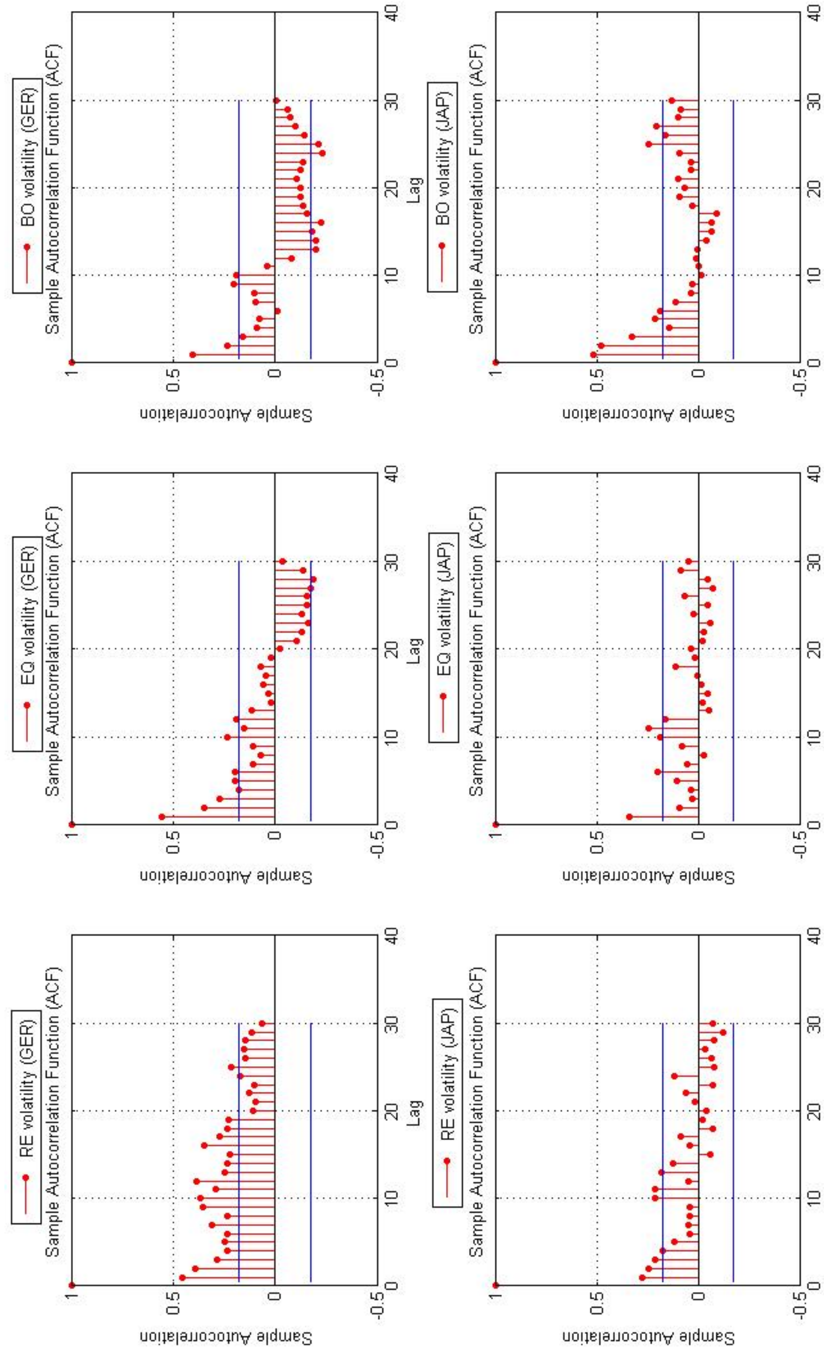

Figure 2.3: Autocorrelation functions of realized volatilities for Germany and Japan

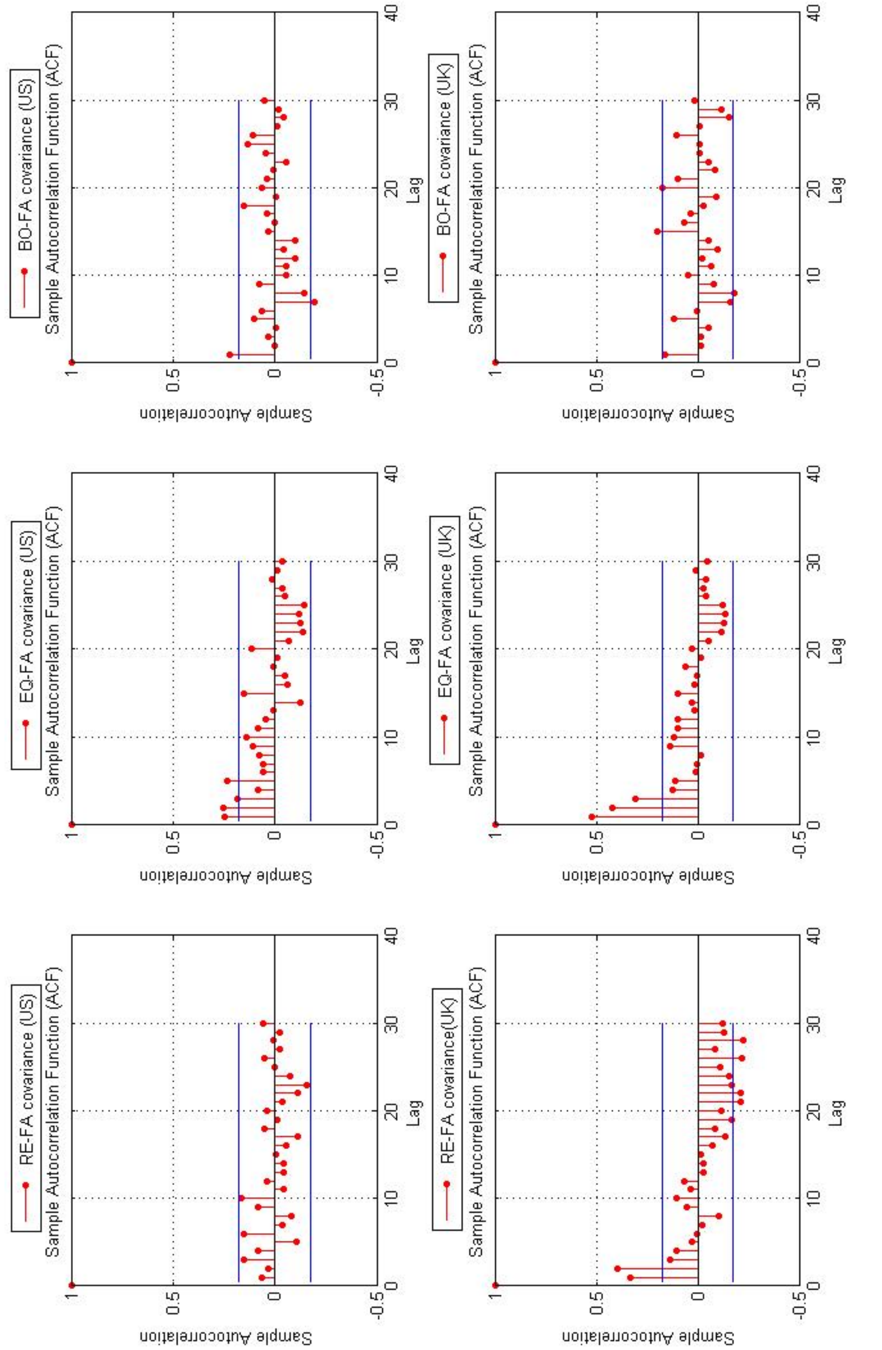

Figure 2.4: Autocorrelation functions of realized covariances with the factor for United States and United Kingdom

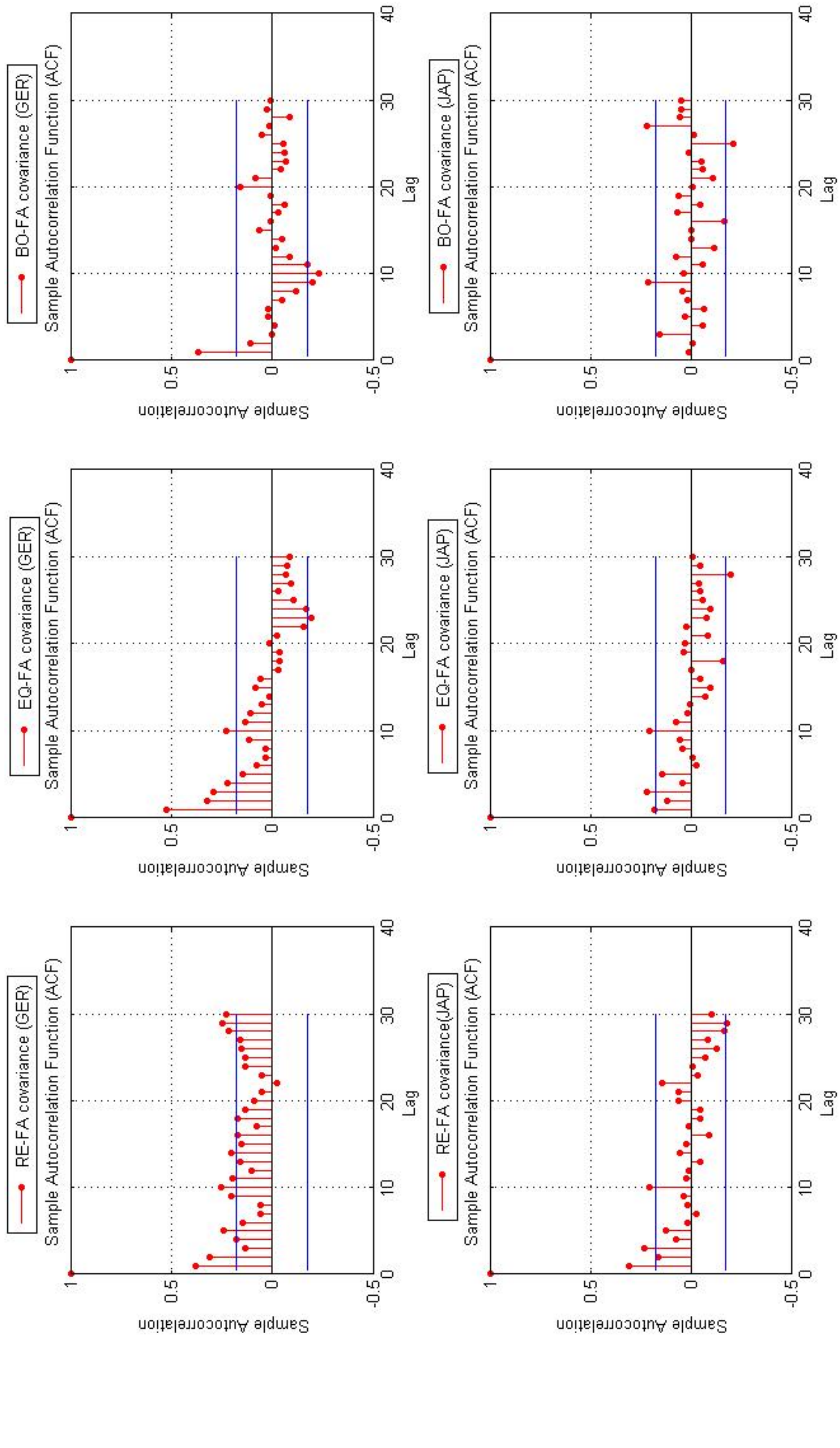

Figure 2.5: Autocorrelation functions of realized covariances with the factor for Germany and Japan

2.4.2 Common factor and volatility results

Figure 2.1 shows the returns and volatility of the computed world asset factor as well as the respective autocorrelation functions. The figures show that the factor follows a martingale-type process: it cannot be predicted from its own past although its volatility appears to be highly predictable down to 16 lags, i.e. $\hat{f}_t \sim (0, \sigma^2_{f,t})$. The factor also captures the increase in volatility in international financial markets during the financial turmoil of the Nasdaq crash and of the period on or after September 11, 2001. Figures 2.2 to 2.5 show the corresponding autocorrelation functions of the realized volatility ($\hat{\sigma}^2_{\bullet,it}$) and covariance estimates ($\hat{\sigma}_{\bullet f,it}$) for some of the main players of world financial markets, namely, the US, UK, Japan and Germany although the same type of figures result for all other markets inspected. The figures show that volatility estimates of real estate, equity and bond returns are predictable for the most part. The autocorrelation lags of the latter volatility estimates that are statistically significant at the 5% level range between 1 to 20 depending on the market and the asset type considered. With respect to the covariance estimates, the predictability finding is less obvious although there is still evidence of autocorrelation a the 5% significance level for 1 to 5 lags for most country estimates.

Country	$\hat{\phi}_i$	$\hat{\phi}_i^b$	$\hat{\pi}_i^{no}$	$\hat{\varsigma}_i^{no}$	arch$_i$	JB$_i$	$\tau_{IN,1i}$	$\tau_{IN,4i}$	ϖ_i	R_i^2	$\tau_{OS,i}^{be}$	$\tau_{OS,i}$	fe$_i$	hm$_i$
AUL	-0.025	-0.068	2, 2	0, 3	**26.92**	0.772	0.524	2.512	**36.74**	**0.078**	1.874	0.455	1.266	1.052
	(−0.472)				[0.000]	[0.680]	[0.469]	[0.643]	[0.000]	0.070	[0.171]	[0.500]	[0.103]	
BEL	-0.004	-0.018	1, 1	1, 3	**12.22**	0.539	0.096	1.165	**36.36**	**0.101**	0.978	**5.249**	0.427	1.010
	(−0.328)				[0.032]	[0.764]	[0.756]	[0.884]	[0.000]	0.074	[0.322]	[0.022]	[0.335]	
CAN	**-0.078**	**-0.049**	1, 3	1, 3	**11.48**	0.840	0.614	1.958	**38.94**	**0.198**	0.042	1.297	-0.163	1.110
	(−2.183)				[0.042]	[0.657]	[0.433]	[0.743]	[0.000]	0.083	[0.838]	[0.255]	[0.565]	
DEN	-0.021	-0.022	1, 3	0, 3	**12.27**	*4.901*	0.005	7.324	**70.95**	**0.117**	0.637	0.150	**4.674**	0.928
	(−1.329)				[0.031]	[0.086]	[0.946]	[0.120]	[0.000]	0.085	[0.424]	[0.699]	[0.000]	
FRA	-0.010	-0.025	3, 3	1, 3	6.674	0.611	0.817	*8.074*	**46.87**	**0.105**	0.035	0.558	**1.730**	**1.328**
	(−0.543)				[0.246]	[0.736]	[0.366]	[0.089]	[0.000]	0.070	[0.851]	[0.455]	[0.042]	
GER	**-0.040**	**-0.026**	1, 4	1, 2	**12.77**	0.398	0.669	2.020	**42.97**	**0.187**	0.004	**4.070**	0.747	0.995
	(−1.765)				[0.026]	[0.819]	[0.413]	[0.732]	[0.000]	0.076	[0.952]	[0.044]	[0.228]	
ITA	**-0.096**	**-0.052**	0, 4	0, 1	**16.09**	*4.911*	0.271	**9.637**	**26.13**	**0.138**	0.077	**12.29**	0.539	1.052
	(−2.559)				[0.007]	[0.086]	[0.603]	[0.047]	[0.010]	0.091	[0.782]	[0.000]	[0.295]	

Table 2.4: Single market results - Part I (t-ratios in parentheses, p-values in brackets). Note: the real estate return model for each i is $\Delta p_{h,it} = \alpha_i + \phi_i \hat{h}_{it-1} + \pi_{1i}\hat{\sigma}^2_{h,it-1} + \pi_{2i}\hat{\sigma}^2_{s,it-1} + \pi_{3i}\hat{\sigma}^2_{b,it-1} + \pi_{4i}\hat{\sigma}^2_{f,t-1} + \pi_{5i}\hat{\sigma}_{hf,it-1} + \pi_{6i}\hat{\sigma}_{sf,it-1} + \pi_{7i}\hat{\sigma}_{bf,it-1} + \upsilon_{11,i}\Delta p_{h,it-1} + \upsilon_{21,i}\Delta p_{s,it-1} + \upsilon_{31,i}\Delta p_{b,it-1} + u_{h,it}$ with $\hat{h}_{it} = p_{h,it} - \hat{\beta}'\mathbf{x}_{it}$. $\hat{\phi}_i$ is the estimate of the coefficient of equilibrium adjustment and $\hat{\phi}_i^b$ its 90% quantile of the bootstrap distribution. π_i^{no} (ς_i^{no}) denotes the number of risk (dynamic) pricing coefficients attached to λ_{it} (Δz_{it}) that have robust t-ratios greater than 1.64 in absolute value followed by the number of risk (dynamic) pricing coefficients that lie *outside* the 90% confidence interval of the bootstrap distribution. In-sample statistics: arch$_i$ is the ARCH-LM conditional heteroskedasticity statistic with five lags, JB$_i$ is the Jarque-Bera normality test for the standardized residuals $\hat{u}_{h,it}(\sum_{d=1}^{D}\Delta p^2_{h,idt})^{-1/2} = \hat{u}_{h,it}/\hat{\sigma}_{h,it}$, $\tau_{IN,li}$ is the in-sample Wald statistic for serial correlation of the residuals $\hat{u}_{h,it}$ which is robust under heteroskedasticity with lags $l = 1$ and $l = 4$, ϖ_i is the Wald test of joint coefficient restriction $\hat{\psi}_{m,1i} = \hat{\psi}_{m,2i} = ... = 0$ which is robust under heteroskedasticity, R_i^2 is the (adjusted) degree of explanation of the regression with its corresponding bootstrap critical value under the null hypothesis $\hat{\psi}_{m,1i} = \hat{\psi}_{m,2i} = ... = 0$. (*continued in Table 2.5*).

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Country	$\hat{\phi}_i$	$\hat{\phi}_i^b$	$\hat{\pi}_i^{no}$	$\hat{\varsigma}_i^{no}$	arch$_i$	JB$_i$	$\tau_{IN,1i}$	$\tau_{IN,4i}$	ϖ_i	R_i^2	$\tau_{OS,i}^{be}$	$\tau_{OS,i}$	fe$_i$	hm$_i$
JAP	-0.032	-0.039	3, 3	0, 0	6.816	2.579	0.094	2.328	**39.09**	**0.164**	0.222	0.541	**2.153**	**1.277**
	(-1.142)				[0.235]	[0.275]	[0.759]	[0.676]	[0.000]	0.072	[0.638]	[0.462]	[0.016]	
NET	-0.010	-0.029	3, 2	1, 3	3.605	1.540	0.396	*7.949*	**97.21**	**0.183**	0.039	*3.841*	*1.332*	1.114
	(-0.631)				[0.608]	[0.463]	[0.529]	[0.093]	[0.000]	0.072	[0.844]	[0.050]	[0.091]	
NOR	-0.057	**-0.056**	2, 4	0, 3	5.115	2.442	0.442	*8.332*	**32.32**	0.068	0.164	1.608	-0.737	0.956
	(-1.480)				[0.402]	[0.294]	[0.506]	[0.080]	[0.001]	0.077	[0.685]	[0.205]	[0.769]	
SPA	-0.002	-0.037	4, 4	2, 3	6.785	1.760	0.035	2.790	**45.82**	**0.112**	0.668	0.109	**2.167**	**1.255**
	(-0.074)				[0.237]	[0.415]	[0.851]	[0.593]	[0.000]	0.066	[0.414]	[0.742]	[0.015]	
SWE	**-0.038**	**-0.028**	3, 3	1, 3	3.680	0.489	0.472	5.769	**58.96**	**0.168**	0.152	0.312	-0.217	**1.216**
	(-1.781)				[0.596]	[0.783]	[0.492]	[0.217]	[0.000]	0.077	[0.696]	[0.577]	[0.586]	
GRB	-0.006	-0.030	2, 2	0, 3	1.425	2.466	0.466	2.255	**111.2**	**0.201**	1.656	0.343	**4.686**	**1.151**
	(-0.323)				[0.921]	[0.291]	[0.495]	[0.689]	[0.000]	0.084	[0.198]	[0.558]	[0.000]	
USA	0.021	-0.033	1, 3	1, 2	3.372	3.121	0.438	2.173	**36.34**	**0.135**	0.006	0.015	*1.489*	**1.211**
	(0.913)				[0.642]	[0.210]	[0.508]	[0.704]	[0.000]	0.081	[0.936]	[0.901]	[0.068]	

Table 2.5: Single Market Results - Part II (t-ratios in parentheses, p-values in brackets). (*continued from Table 2.4*) Out-of-sample statistics: $\tau_{OS,i}$ ($\tau_{OS,i}^{be}$) is the out-of-sample Wald statistic of first order serial correlation of forecasting errors of the full model (benchmark) which is robust under heteroskedasticity, fe$_i$ is the forecasting encompassing statistic robust under heteroskedasticity and hm$_i$ is the variant of the Hendrickson-Merton statistic.

Test	F-arch	F-JB	F-$\tau_{1,IN}$	F-$\tau_{4,IN}$	F-ϖ	F-τ_{OS}	F-τ_{OS}^{be}	F-fe	HM	$\hat{\eta}$	$\hat{\rho}$	
Stat.	**70.46**	27.38	16.33	33.66	**232.0**	**49.47**	16.62	**101.9**	**1.127**	**0.329**	**0.347**	
	[0.000]	[0.498]	[0.961]	[0.212]	[0.000]	[0.007]	[0.956]	[0.000]		(2.272)	(2.468)	
$\Delta p_{h,it}$	$\hat{\alpha}_i$	$\hat{\phi}_i$	$\hat{\pi}_{1i}$	$\hat{\pi}_{2i}$	$\hat{\pi}_{3i}$	$\hat{\pi}_{4i}$	$\hat{\pi}_{5i}$	$\hat{\pi}_{6i}$	$\hat{\pi}_{7i}$	$\hat{\varsigma}_{11i}$	$\hat{\varsigma}_{21i}$	$\hat{\varsigma}_{31i}$
$\bar{\psi}_{PMG}$	**-0.268**	**-0.028**	23.54	**-269.5**	432.2	**-1.826**	**-43.99**	**33.35**	**135.1**	-0.006	-0.026	**0.236**
	(−3.015)	(−3.353)	(0.515)	(−4.309)	(0.767)	(−3.643)	(−2.288)	(2.226)	(2.191)	(−0.195)	(−0.812)	(3.596)
CI_1	0.142	0.013	53.50	60.90	972.7	1.020	20.98	21.75	100.5	0.051	0.055	0.151
CI_2	-0.128	-0.014	-50.47	-57.31	-947.2	-0.945	-20.64	-24.04	-89.19	-0.051	-0.059	-0.152
ω	*22.82*	**24.83**	**57.46**	**95.37**	15.57	**35.11**	**36.42**	**29.26**	**24.29**	16.57	9.707	**32.55**
	[0.063]	[0.036]	[0.000]	[0.000]	[0.340]	[0.001]	[0.001]	[0.010]	[0.042]	[0.280]	[0.783]	[0.003]
$\hat{\kappa}$	**-0.018**	**-0.196**	0.002	**-0.003**	0.001	**-0.307**	-0.004	**0.009**	**0.003**	0.004	-0.031	**0.019**
	(−4.879)	(−5.448)	(1.084)	(−3.867)	(1.155)	(−4.474)	(−1.637)	(2.290)	(3.899)	(0.117)	(−1.401)	(2.721)
$\hat{\psi}_{m,i} < 0$	12	13	5	14	7	11	11	4	4	9	7	2
$\|t\| > 1.96$	2	2	3	6	0	3	3	3	1	1	1	3
$\|t\| > 1.64$	3	4	3	9	1	4	4	3	3	3	1	4
$\hat{\psi}^b_{m,i}(.95)$	3	4	6	9	1	4	6	3	2	1	1	3
$\hat{\psi}^b_{m,i}(.90)$	4	5	7	9	4	4	9	5	6	5	2	4
$Q_{.05}$	-	5	6	10	3	2	9	9	8	4	6	3

Table 2.6: Aggregate results (t-ratios in parentheses, p-values in brackets). Note: the real estate return model for each i is $\Delta p_{h,it} = \alpha_i + \phi_i \hat{h}_{it-1} + \pi_{1i}\hat{\sigma}^2_{h,it-1} + \pi_{2i}\hat{\sigma}^2_{s,it-1} + \pi_{3i}\hat{\sigma}^2_{b,it-1} + \pi_{4i}\hat{\sigma}^2_{f,t-1} + \pi_{5i}\hat{\sigma}_{hf,it-1} + \pi_{6i}\hat{\sigma}_{sf,it-1} + \pi_{7i}\hat{\sigma}_{bf,it-1} + \upsilon_{11,i}\Delta p_{h,it-1} + \upsilon_{21,i}\Delta p_{s,it-1} + \upsilon_{31,i}\Delta p_{b,it-1} + u_{h,it}$ with $\hat{h}_{it} = p_{h,it} - \hat{\beta}'\mathbf{x}_{it}$. F-$\bullet$ is the Fisher test for $\bullet$ = arch, JB, $\tau_{l,IN}$, ϖ, τ_{OS}, τ_{OS}^{be}, fe and HM is the aggregate Hendrickson-Merton statistic. $\bar{\psi}_{PMG} = (\bar{\alpha}, \bar{\phi}(\hat{\beta}), \bar{\pi}', \bar{\varsigma}_1')'$ is the Pooled Mean Group estimator obtained by averaging $\hat{\psi}_{m,i} = (\hat{\alpha}_i, \hat{\phi}_i(\hat{\beta}), \hat{\pi}_i', \hat{\varsigma}_{1i}')'$, CI_j are the 95% bootstrap confidence intervals of the Pooled Mean Group estimator $\bar{\psi}_{PMG}$, ω is the Cross-sectional Wald statistic robust under heteroskedasticity and $\hat{\kappa}$ is the cross-sectional regression estimate. $|t| > 1.96$ ($|t| > 1.64$) denotes the number (across countries) of robust t-ratios from the coefficients $\hat{\psi}_{m,i}$ that are greater than 1.96 (1.64) in absolute value. $\hat{\psi}^b_{m,i}(.95)$ ($\hat{\psi}^b_{m,i}(.90)$) denotes the number of coefficients $\hat{\psi}_{m,i}$ which lie *outside* the 95% (90%) bootstrap confidence interval. $Q_{.05}$ denotes the number of countries in which the explanatory variables from the set $k_{it} = \left\{\hat{h}_{it}, \hat{\sigma}^2_{h,it}, \hat{\sigma}^2_{s,it}, \hat{\sigma}^2_{b,it}, \hat{\sigma}^2_{f,it}, \hat{\sigma}_{hf,it}, \hat{\sigma}_{sf,it}, \hat{\sigma}_{bf,it}, \Delta p_{h,it}, \Delta p_{s,it}, \Delta p_{b,it}\right\}$ have predictability power for real estate returns $\Delta p_{h,it}$ according to the Q-test (see Appendix B) at the 5% significance level. The parameters η and ρ are the contemporaneous coefficients in (2.14) estimated via Identification through Heteroskedasticity (for country groups see Figure 2.6).

2.4.3 Real estate returns

2.4.3.1 Single market results

The *individual* market analysis for real estate returns consists of estimating the model in (2.9) for each i and performing different in-sample and out-of-sample tests as well as residual diagnostics. The model is implemented with $p = 1$ based on the AIC. The vector of parameters is given by the vector containing the pooled long-run estimates $\hat{\psi}_{m,i} = (\hat{\alpha}_i, \hat{\phi}_i(\hat{\beta}), \hat{\pi}'_i, \hat{\varsigma}'_{1i})'$ given the result on the Hausmann statistic $\mathcal{H}$ outlined in the previous section that the null of long-run parameter homogeneity cannot be rejected.

In the first instance we test the null hypothesis that a particular coefficient estimate $\hat{\psi}_{m,li}$ attached to the set of explanatory variables in (2.9) and (2.10) is equal to zero. Note that model (2.9) is stable as long as the coefficient ϕ_i is less than zero. Tables 2.4 and 2.5 show that 13 out of the 14 countries under inspection show a negative error correction coefficient estimate. According to robust t-ratios, 4 out of the 13 negative coefficients are statistically significant at the 5% level if a one-sided test is performed. More precisely, the equilibrium error $\hat{h}_{it}$ can signal real estate returns in Canada, Germany, Italy and Sweden. According to the bootstrap distribution under the null hypothesis of no error correction, we obtain that the error correction coefficient in Norway is also statistically significant. Interestingly, the coefficient of equilibrium adjustment $\hat{\phi}_i$ for the United States is positive which points out to the overvaluation of real estate markets in comparison to equity and bond markets in the latter country for the sample period covered.

As previously mentioned, the sign of the coefficients in the vectors π_i and $\varsigma_{j,i}$ are not a-priori restricted by the presumption of model stability or unambiguous economic arguments. In these lines, we diagnose at the 5% significance level that the number of statistically significant coefficients in π_i attached to the vector of risk-premia λ_{it} is largest when conditioning real estate returns upon the volatility of equity returns (6) and lowest when conditioning real estate returns on the volatility of bond returns (0) according to the robust t-ratios (Table 2.6). Interestingly, lagged volatility of equity returns has a negative effect on real estate returns in 14 out of the 14 countries under inspection. Moreover, 9 of the 14 countries show that lagged equity volatility has a significantly negative effect on real estate returns at conventional significance levels. This puts forward that lagged equity volatility seems to play a very important role in pricing risk of international real estate securities. Indeed, the effect of equity volatility is more frequent statistically significant across countries than the effect of real estate volatility (Table 2.6). Overall, lagged time-varying risk-premia have explanatory power for real estate returns at the 10% level in all international markets considered according to the t-ratios and bootstrap confidence bands (π_i^{no}, Tables 2.4 and 2.5).

In the case of asset return dynamics included in the vector Δz_{it}, we obtain that lagged bond returns seem to best explain real estate return variation out of the three asset return variables considered. In particular, according to robust t-ratios, past bond returns show explanatory power for real estate returns at the 5% level in 3 out of the 14 countries considered while lagged equity and real estate returns show explanatory power in 1 out of the 14 countries (Table 2.6). Overall, past returns of real estate, equity and bond markets can explain real estate return variation at the 10% level in most countries except for Japan according to the t-ratios and bootstrap confidence bands (ς_i^{no}, Tables 2.4 and 2.5). The results on the explanatory power of the independent variables considered to characterize the dynamics of international real estate returns is qualitatively the same (or better) when considering the number of coefficients that lie *outside* the bootstrap confidence intervals or the Q-statistic found in Campbell & Yogo (2006) (Table 2.6).

In addition to significant tests on single coefficients, we also test the null hypothesis that the coefficients of the model are *jointly* equal to zero. The typical F-test would not be appropriate in this case since we are dealing with an asset return model with conditionally heteroskedastic residuals. Instead, the null hypothesis $\hat{\psi}_{m,1i} = \hat{\psi}_{m,2i} = ... = 0$ is tested via a Wald test with White (1980)'s heteroskedasticity robust covariance estimator denoted ϖ_i. The results on this test show that the null hypothesis of the coefficients being jointly equal to zero can be rejected at the 5% level in all the 14 countries at hand (Tables 2.4 and 2.5). Thus, we find that the equilibrium error (h_{it}), the time-varying second order moments (λ_{it}) and the (vector) autoregressive dynamics (Δz_{it}) can jointly explain one-period ahead real estate returns.

A study by Foster *et al.* (1997) proposes to assess the goodness of fit of an asset pricing model by determining the maximal adjusted-R^2 of a predictive return regression. The maximal R^2 in our context, is the 95% cutoff value of the degree of explanation from the predictive regression under the null hypothesis that all the coefficients are equal to zero. As a final in-sample predictability test at the single market level we compute the in-sample adjusted-R_i^2 of the model in each country and compute bootstrap critical values for the latter statistic under the null hypothesis $\hat{\psi}_{m,1i} = \hat{\psi}_{m,2i} = ... = 0$. The results of this exercise show that the R_i^2 statistic is significantly larger than the bootstrap critical values in most countries except for Norway. This result is in line with the previous result of the Wald test (Tables 2.4 and 2.5). Thus, there is strong evidence of real estate return predictability from the regressors considered and thus evidence against the Efficient Market Hypothesis (EMH) in international real estate markets. Interestingly, there is no evidence of first order serial correlation ($\tau_{IN,1i}$) for any of the markets but there is some evidence of 4th order serial correlation ($\tau_{IN,4i}$) at the 10% level in countries like France, Italy, Netherlands and Norway.

The out-of-sample analysis consists of three main tests, namely, (i) a test on the distributional features of the real estate return forecasts, (ii) a test on serial correlation of forecast errors and (iii) a forecast encompassing test of forecast errors. The test on distributional features denoted hm_i is the variant of the Henrikkson–Merton statistic (Henriksson & Merton (1981), Cumby & Modest (1987)) introduced in the previous Chapter. The results of the test on distributional features are presented in Tables 2.4 and 2.5 and show that the proposed model is convenient to approximate distributional features of ex-ante real estate returns in countries like France, Japan, Spain, Sweden, the UK and the USA where the hm_i is greater than the 5% critical value 1.15.

The test of forecast encompassing denoted fe_i is taken from Harvey *et al.* (1998). Forecasts $\Delta\hat{p}_{h,i\tau}(M1)$ obtained from a particular benchmark scheme, are said to encompass forecasts $\Delta\hat{p}_{h,i\tau}(M0)$ corresponding to our model if the latter contains no useful information improving the ex-ante approximation of $\Delta p_{h,i\tau}$. Not useful in this context means that a linear combination between the forecasts $\Delta\hat{p}_{h,i\tau}(M1)$ and $\Delta\hat{p}_{h,i\tau}(M0)$ has a mean squared error (MSE) not significantly smaller than that of $\Delta\hat{p}_{h,i\tau}(M1)$ alone. The benchmark specification is the model in (2.9) with $\phi_i = 0$ and $\theta_i' = 0$. The vector Δz_{it} is included in order to (i) account for possible serial correlation that would result from a simple drift model and (ii) to study the marginal forecasting contribution of the equilibrium error ($\hat{h}_{it}$) and the risk-pricing variables ($\hat{\lambda}_{it}$) (i.e. the 'theoretical' pricing variables).

It is important to note that forecasting models should deliver serially uncorrelated sequences of forecasting errors. Thus, we also perform a (heteroskedaticity robust) Wald test for the forecasting errors $\hat{u}_{h,i\tau}(M0)$ and $\hat{u}_{h,i\tau}(M1)$. The out-of-sample serial correlation statistic of the model (benchmmark) is given by $\tau_{OS,i}$ ($\tau_{OS,i}^{be}$) in Tables 2.4 and 2.5. The results on the forecast encompassing exercise show that the benchmark specification *does not* forecast encompass the full model in Denmark, France, Japan, Spain and the UK where the null hypothesis of forecast encompassing is rejected at the 5% level of significance (Tables 2.4 and 2.5). Moreover, there is no evidence of first order serial correlation in the forcasting errors of the latter countries. This result suggests that forecast combinations obtained from real estate returns conditioned upon an equilibrium error and time-varying risk and those obtained from atheoretical dynamics could improve ex-ante forecasts in some countries.

The results at the single market level indicate that there is a high degree of heterogeneity amongst international markets with respect to the explanatory power of the different independent variables considered in the model of real estate returns. However, the results uncover that real estate markets are inefficient for the sample period analyzed since we diagnose both in-sample and out-of-sample evidence of predictability (from the equilibrium error ($\hat{h}_{it}$), risk-premia ($\hat{\lambda}_{it}$) and asset return dynamics (Δz_{it})) within the cross-section of countries considered.

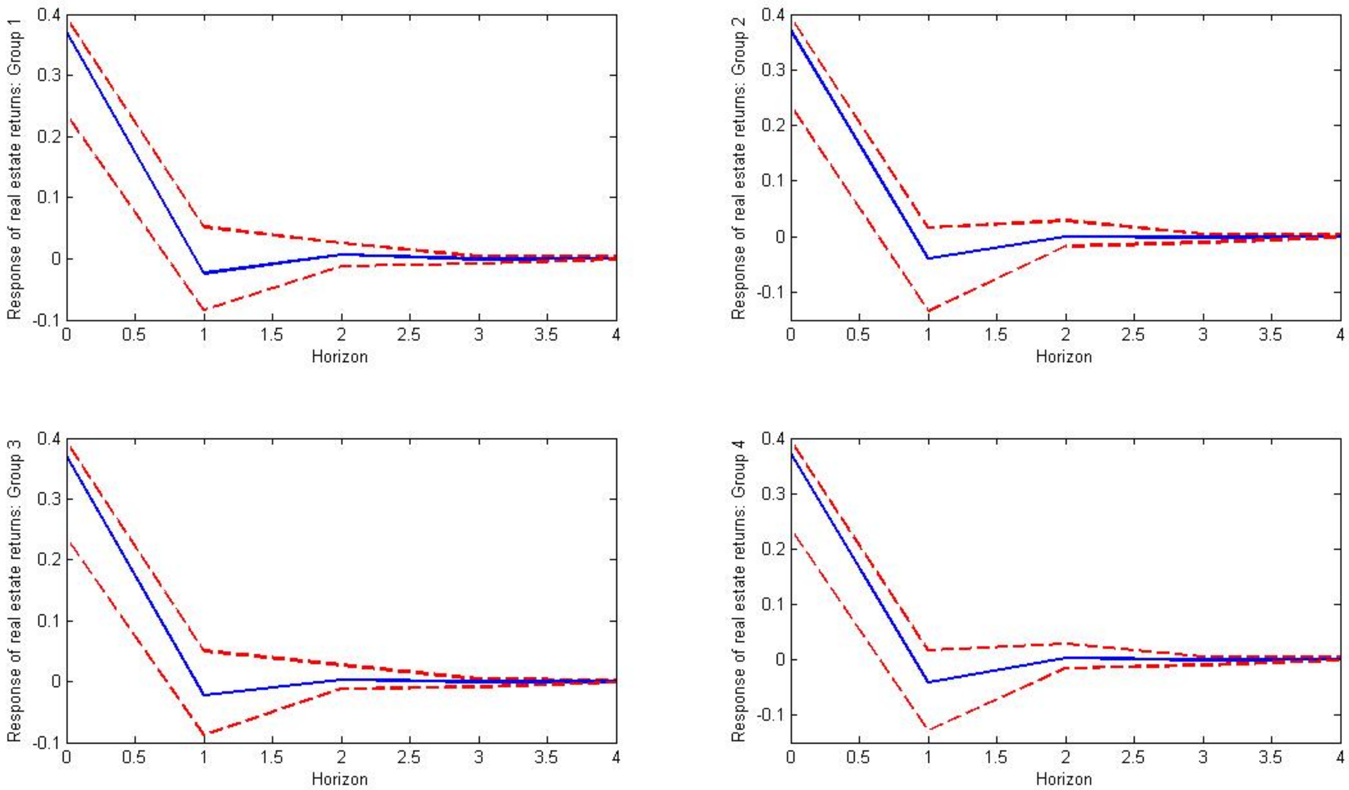

Figure 2.6: Response of real estate returns to a (one standard error) shock in equity returns. Note: Group 1 ('high volatility' of real estate markets): AUL, BEL, CAN, FRA, GRB, NET, USA, Group 2 ('low volatility' of real estate markets): DEN, ITA, GER, JAP, NOR, SPA, SWE, Group 3 ('high volatility' of equity markets): AUL, CAN, BEL, DEN, GRB, JAP, USA, Group 4 ('low volatility' of equity markets): FRA, GER, ITA, NET, NOR, SPA, SWE.

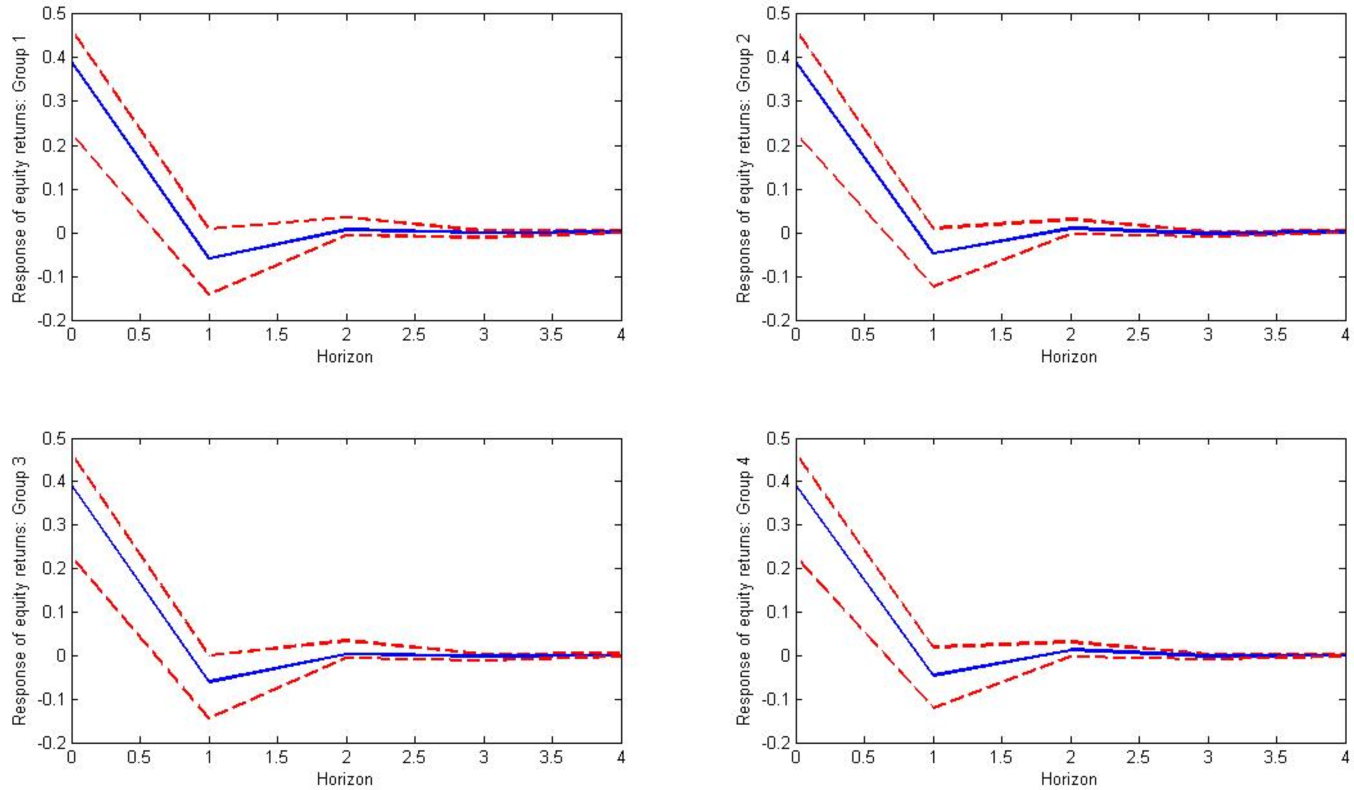

Figure 2.7: Response of equity returns to a (one standard error) shock in real estate returns. Note: For further notes on country groups see Figure 2.6.

2.4.3.2 Aggregate results

The *aggregate* market analysis consists, as a first instance, in estimating model (2.9) via mean group inference. As mentioned earlier the result of the Hausmann test ($\mathcal{H}$) leads to the acceptance of the null hypothesis $H_0 : \beta_i = \beta$. Therefore, the mean group inference is done from the vector of estimated coefficients $\hat{\psi}_{m,i} = (\hat{\alpha}_i, \hat{\phi}_i(\hat{\beta}), \hat{\pi}_i', \hat{\varsigma}_{1i}')'$ and the panel estimator is given by the *Pooled* Mean Group estimator $\bar{\psi}_{PMG} = (\bar{\alpha}, \bar{\phi}(\hat{\beta}), \bar{\pi}', \bar{\varsigma}_1')'$. The results are displayed in Table 2.6. The magnitude of the PMG error correction coefficient $\bar{\phi} = 2.8$ is statistically significant at the 5% level and shows that a one percent increase in the $\hat{h}_{it}$ ratio today would lead on average to a decrease of 2.8 basis points in (log) real estate prices tomorrow. Thus, according to the model, real estate prices had a relatively slow adjustment to deviations from equity and bond prices in the sample period analyzed.

The aggregate effect of the risk pricing variables on real estate returns diagnosed by the PMG estimator differs. Lagged equity volatility and lagged world factor volatility have on average a statistically significant negative effect on real estate returns at the 5% level. However, lagged time-varying volatilities of real estate and bond returns have on average no statistically significant effect on real estate returns. With respect to lagged time-varying covariances of equity returns and bond returns (real estate returns) with the 'world' asset factor, we obtain that they have on average a statistically significant positive (negative) effect on international real estate returns at the 5% level. In the case of asset return dynamics we find that at the aggregate level, only lagged bond returns can significantly explain real estate returns on average at the 5% level. Thus, it appears as if (lagged) time-varying volatility of bond returns does not explain future real estate returns on average while their first order dynamics do. The latter result also suggests that monetary policy may significantly affect (one-period ahead) real estate returns. Moreover, we obtain that the statistically significant PMG coefficients according to t-ratios, also lie outside the bootstrap confidence bands (CI_1, CI_2) which supports the PMG inference.

We also perform the usual cross-sectional regression approach in the finance literature to check the robustness of the aggregate measures. The latter strategy is used to study whether the $\hat{\psi}_{m,li}$'s estimates attached to a particular pricing variable l for each unit i can explain average real estate returns. An asset pricing model should explain not only time series variation of expected returns but also the cross-section of average returns. The estimated cross-section coefficient is denoted $\hat{\kappa}$ and the results are presented in Table 2.6. The cross-sectional regression estimates are mostly in agreement with the PMG inference with respect to the aggregate effect of the regressors on the cross-section of real estate returns (both in terms of signs and statistical significance at the 5% level). That is, we find that the coefficients $\hat{\psi}_{m,li}$ attached to the (lagged) equilibrium error, equity volatility, factor volatility, covariances with the factor and bond returns can significantly explain the cross-section of average real estate returns.

As a final in-sample test at the aggregate level, we test the null hypothesis that short-run parameters governing the impact of a particular pricing variable l in $\hat{\psi}_{m,li}$ are jointly zero over the cross section dimension (i.e. $H_0 : \psi_{m,li} = 0$ for $i = 1, ..., N$) via a robust cross-sectional Wald test introduced in the previous Chapter. The test statistic is given by ω and the results are also presented in Table 2.6. As in the case of the cross-sectional regressions, the results on the cross-sectional Wald test also confirm the statistically significant results of the PMG estimation. That is, lagged values of the equilibrium error, equity volatility, factor volatility, covariances with the factor and bond returns have a statistically significant effect on the cross-section of real estate returns at the 5% level. Moreover, the latter test also shows that the coefficient attached to lagged volatility of real estate returns is different from zero over the cross-section at the 5% level providing evidence that past real estate volatility can explain real estate returns at the aggregate level.

The out-of-sample analysis shows that the model approximates distributional features of real estate returns at the aggregate level since the statistic HM = 1.127 is above its 5% critical value 1.05. As for the forecast encompassing test, the Fisher (1932) test F-fe = 101.9 (p-value=0.000) indicates that the ex-ante approximation of $\Delta p_{h,it}$ may be improved at the aggregate level by combining forecasts of a 'theoretical' specification (containing $\hat{h}_{it}$ and $\hat{\lambda}_{it}$) and an 'atheoretical' one (containing only Δz_{it}).

Lastly, we consider the results of the contemporaneous analysis which are presented in Table 2.6. The findings indicate that there is a positive and statistically significant dual contemporaneous relationship at the 5% level between equity returns and real estate returns in the international markets considered (Table 2.6: η=0.329 (2.272), ρ=0.347 (2.468)). The latter finding can be interpreted as evidence of contagion effects from shocks of equity markets to real estate markets and vice-versa. Figures 2.6 and 2.7 show the impulse responses. Note that the contemporaneous response (the y-axis value when the horizon is zero) is homogeneous for the regime groups $g = 1, 2, 3, 4$. Overall, the dynamic effect of real estate returns from the shock on equity returns dissipates rapidly after about 2 to 3 months.

The in-sample and out-of-sample aggregate results of the formulated real estate pricing model point out to the same direction as the individual market results. That is, (log) real estate price changes can be conditionally explained by the equilibrium error $\hat{h}_{it}$, (selected) time-varying second order moments in $\hat{\lambda}_{it}$ and bond return dynamics in Δz_{it}. Interestingly, we find that there is a strong statistically significant relationship between equity markets and real estate security markets in the international context via volatility, log prices and contemporaneous returns. The empirical relationship of real estate securities with bond markets appears to be via bond prices or bond return dynamics rather than via bond volatility.

2.5 Concluding remarks

This Chapter analyzed empirically a real estate pricing model for a cross-section of international asset markets. The main findings can be summarized as follows. First, real estate security prices, equity prices and bond prices are cointegrated in the panel of countries according to the results of different panel cointegration tests as well as panel unit root tests.

Second, a homogeneous long-run equilibrium relationship between international real estate, equity and bond prices exists and real estate prices adjust (on average) significantly to deviations from this equilibrium. Both the cointegration test results and the significant long-run homogeneous relationship between the three assets under inspection suggest financial integration amongst these asset markets both within particular countries and across the panel. These results also suggest that increases in equity or bond prices have a positive and *homogeneous* marginal effect on international real estate security prices. The elasticities of real estate security prices with respect to stock (bond) prices are estimated to be less (greater) than one.

Third, we find strong evidence that second order moments of real estate, equity, bonds and a constructed world factor are time-varying and that they can predict real estate security returns. The latter results imply that (i) risk pricing variables play an important role when formulating models for real estate securities and that (ii) analysts may exploit the relatively simple approach of realized volatility methods to build appropriate models for international asset pricing and risk management.

Fourth, additional asset return dynamics such as past bond returns can significantly signal real estate returns which suggests that monetary policy may influence real estate security prices. Lastly, there is evidence of a (dual) contemporaneous relationship between international equity returns and real estate returns but the impact of shocks dissipates quickly. Overall, our findings suggest that a way to understand international real estate securities is by analyzing their comovements with equity markets and bond markets.

Chapter 3

International Transmission of Shocks: US Monetary Policy and International Asset Prices

3.1 Introduction

In this Chapter we turn to the analysis of the relationship between monetary policy in the United States (US) and asset prices which has recently attracted the attention of economists. The relationship has already captured momentum at a national level since understanding the responsiveness of US asset prices to US monetary policy allows policymakers and market participants to build appropriate models for the distribution of risk bearing. In a world of financially interconnected markets, a relationship between US monetary policy and international asset prices may also exist. In this context, international asset prices may presumably respond not only to their domestic monetary policy shocks but also to US monetary policy shocks. This suggests that monetary policy authorities around the world would be able to take better decisions if they have an accurate understanding of the response of their asset markets to a monetary shock in the US. For instance, it would allow policy makers to devise coherent and coordinated strategies in light of adverse contagion effects that may arise from international financial crises.

In this Chapter we contribute to the understanding of the empirical relationship between US monetary policy and international equity, bond and real estate markets for the sample period 01/1994 to 12/2007. Our empirical approach is based on the recent methodology of Identification through Heteroskedasticity (IH) proposed by Rigobon & Sack (2004). We employ the IH approach and extend it to the international context by means of a heterogeneous panel framework. We consider issues such as controlling for dynamic components in the conditional mean of international asset returns, selection of non-policy dates in the international context, recursive estimation and bootstrap inference.

The empirical nature of our approach originates from the absence of a broad consensus on the stylized facts of the relationship between asset prices and monetary policy. The difficulty in arriving to such a consensus stems from the fact that, already at national level, the relationship may well depend on the specification of the model used, and in some cases, it is not transparent whether targeting asset prices may have desirable effects (cf. Bernanke & Woodford (1997), Bullard & Schaling (2002), Geromichalos *et al.* (2007)). Moreover, if the central bank's goal is price stability, and this is interpreted as stability of the price of current consumption, as opposed to stability of the price of current vs. future consumption, there might be no reason for a central bank to influence asset prices. Focusing on asset prices when setting monetary policy might be only relevant to the extent that they may signal inflationary or deflationary pressures (cf. Bernanke & Gertler (2000, 2004)). On the other hand, an active reaction of the monetary authority to asset price developments may help prevent bubbles and could even be welfare improving (cf. Cecchetti *et al.* (2000), Carlstrom & Fuerst (2007)).

Asset prices should theoretically play, nevertheless, a major role as providers of valuable information on expectations about future discount factors, which suggests that they may have an empirical relationship with the monetary policy rate (cf. Campbell & Shiller (1987, 1988), Vickers (1999)). Indeed, recent studies have identified a significant response of US asset markets to US monetary policy shocks (Cochrane & Piazessi (2002), Rigobon & Sack (2004)). The significant impact of monetary policy on equity prices could be explained by the effect of revisions on forecasted equity risk premia (cf. Bernanke & Kuttner (2005)). It can also be argued that changes in asset prices and changes in the short-term interest rate can affect each other contemporaneously via channels such as through expectations of the future output path and inflation (Rigobon & Sack (2002), Chadha *et al.* (2004)).

The empirical evidence of a (contemporaneous) relationship between asset prices and monetary policy found for the US suggests that in a world of highly interconnected financial markets, monetary shocks that affect US asset prices could also affect international asset prices. For instance, foreign investors who hold US assets will be affected by US monetary policy shocks which could in turn affect their investment decisions with respect to foreign and domestic assets. However, while central banks still hold control over inflation, long term bond prices and equity prices are determined by global supply and demand forces (Rogoff (2006), Bernanke (2007)). Thus, it is not obvious whether (growing) financial integration can strengthen or weaken the effects of monetary policy on asset prices at home or abroad. This issue is of particular importance since capital market rates are one of the most important channels through which monetary policy makers may influence the real economy and inflationary pressures. Therefore, a clearer understanding of the effect of US monetary policy on international asset prices can improve the view on how monetary authorities worldwide could coordinate actions to influence asset markets when necessary.

The relationship between US or domestic monetary policy and international asset prices has already been analyzed but following different empirical approaches. It has been found that international equity prices react negatively in response to interest rate surprises by the US monetary authority, and that the response's variation is mainly related to financial integration with the US (Wongswan (2005)). Moreover, it seems that asset prices react strongest to domestic monetary policy shocks but that there are also substantial international spillovers between money, bond, equity markets and exchange rates within and between the US and the Euro area (Ehrmann *et al.* (2005)). The evidence on the relationship between monetary policy and asset prices in countries like the United Kingdom (UK), Japan and the European Union (EU) is tenuous particularly in periods after the 1990s (Furlanetto (2008)). Nevertheless, there is recent evidence that certain equity markets in the EU react significantly to monetary surprises of the European Central Bank (ECB) (Bohl *et al.* (2008)). The latter findings suggest that the impact of US (domestic) monetary shocks across (within) border might be country dependent as well as time dependent.

So far, little attention has been given to studying the effect of US monetary shocks on international asset prices from a parsimonious model that allows the analyst to estimate the impact for single markets as well as at the aggregate level. We argue that a US monetary policy shock might have different influences on international asset markets (e.g. due to financial integration, distance, distinct monetary policy frameworks, prudential rules, etc), thus, it is preferable to assume heterogeneity in the relationship and estimate the aggregate impact accordingly. Previous studies have not focused on analyzing the impact of US monetary policy on international asset prices using recursive estimation at the panel dimension. In this study we propose recursive analysis to understand whether the aggregate impact of US monetary policy is time-varying across international asset markets. The latter approach is in line with recent empirical and theoretical evidence that international market integration may be time-varying (cf. Bekaert & Harvey (1995), DeRoon & DeJong (2005), Pavlova & Rigobon (2009), Pachenko & Wu (2009)). From an econometric perspective, recursive analysis via heterogeneous panels may also safeguard the analyst against structural changes present in the data and, thus, prevent her from arriving at spurious conclusions that may arise from single markets or fixed time periods. As in the previous Chapter, we partially try to overcome the general lack of attention to real estate markets by accounting for indices of real estate securities at the country level.

To preview some of our main results we find that: (i) the IH approach is in general more appropriate than the popular event study approach for identification of the impact of US monetary policy on international asset prices, (ii) the equity markets of Mexico and Canada have a statistically significant response to US monetary shocks hinting at a proximity effect, (iii) the estimated impact of US monetary policy is heterogeneous across countries but statistically significant at the aggregate level in equity and bond markets and (iv) the aggregate impact (in absolute terms) and the 'goodness of fit' of US monetary policy on international equity and real estate markets seems to be increasing over time. Our results suggest that the empirical relationship between US monetary policy and international asset prices is sample and country specific and that a way to safeguard the analysis against spurious conclusions is to aggregate the impact recursively.

The Chapter is organized as follows. The next section presents the empirical specification studied. Section 3.3 describes the dataset and the econometric methodology used. Section 3.4 presents and analyzes the results of the study. The last section concludes.

3.2 The model

This section presents a baseline model to analyze the impact of US monetary policy on international asset prices. US monetary policy is taken as the US Federal Reserve's decisions to change the US short term interest rate (US STIR henceforth). In what follows let equity price $P_{s,t}$ be the price of a domestic all-share equity portfolio at time t. Bond price $P_{b,t}$ is defined as the price of a domestic long-term government bond. Real estate price $P_{h,t}$ is defined as the price of a domestic portfolio of real estate securities. Moreover, let r_t^* denote the US STIR. To save on notation, we employ $P_t = P_{\bullet,t}$ henceforth to refer to asset $\bullet = s, h, b$ and $P_t^* = P_{\bullet,t}^*$ to refer to the US counterpart of asset $\bullet = s, h, b$. The empirical relationship between US monetary policy and international asset prices is expressed as,

$$\Delta p_t = \alpha \Delta r_t^* + \phi' x_t + z_t + \eta_t, \tag{3.1}$$

where $\Delta p_t = \ln P_t - \ln P_{t-1}$ is the (log) return of the domestic asset, $\Delta r_t^* = r_t^* - r_{t-1}^*$ is the change in the US STIR, x_t is a $k \times 1$ vector of predetermined variables, z_t is an exogenous shock and η_t is the shock to the asset price. The vector x_t could contain, for instance, dynamics of Δp_t and Δr_t^*, equilibrium relationships, exchange return dynamics, inflation dynamics, etc. The variable z_t could capture conditions such as changes in risk preferences, changes in sentiment, liquidity shocks and macroeconomic shocks not captured by x_t (cf. Rigobon & Sack (2002)). In addition, we could expect a-priori a negative value for α since asset prices should have a negative relationship with discount rates (cf. Campbell & Shiller (1987, 1988)).

The estimation of (3.1) faces two major challenges. On the one hand, the variables Δp_t and Δr_t^* could be characterized as endogenous in this set up since (log) international asset price changes may be affected by changes in the US STIR and vice-versa. The intuition from the causality $\Delta p_t \rightarrow \Delta r_t^*$ might not be clear at first. However, in a world of integrated financial markets, we could expect, for instance, the causality $\Delta p_t \rightarrow \Delta p_t^*$ (cf. Ehrmann *et al.* (2005)) and thus $\Delta p_t^* \rightarrow \Delta r_t^*$ (cf. Rigobon & Sack (2002, 2004), Chadha *et al.* (2004)) where $\Delta p_t^* = \ln P_t^* - \ln P_{t-1}^*$. In this set up, if Δr_t^* is endogenous then the OLS estimate of α will be biased. On the other hand, the omission of the variables in x_t or z_t constitutes a severe problem, given that these variables also carry important information to characterize the relationship between international asset prices and US monetary policy. Therefore, a careful identification strategy has to be employed.

Borrowing from Rigobon & Sack (2002, 2004), the following bivariate system of equations is considered to characterize the endogenous relationship between Δp_t and Δr_t^* as well as the macroeconomic variable(s) in x_t:

$$\Delta r_t^* = \beta \Delta p_t + \phi_1' x_t + \varphi z_t + \epsilon_t, \tag{3.2}$$

$$\Delta p_t = \alpha \Delta r_t^* + \phi_2' x_t + z_t + \eta_t, \tag{3.3}$$

where ϵ_t is the monetary policy shock, and z_t is supposed as a common shock. The shocks ϵ_t and η_t are assumed to have no serial correlation and to be independent with each other and with the common shock z_t (Rigobon & Sack (2002, 2004)). Equations (3.2) and (3.3) cannot be estimated consistently using OLS due to the presence of simultaneous equations and omitted variables. For instance, in a simpler scenario of $\phi_1 = \phi_2 = 0$, if OLS is used to estimate (3.3), then it can be shown that one would obtain simultaneity bias if $\beta \neq 0$ and $\sigma_\eta^2 > 0$ and omitted variables bias if $\varphi \neq 0$ and $\sigma_z^2 > 0$ where $\sigma_\bullet^2$ is used to denote the variance of $\bullet = \eta, z$. The system of equations in (3.2) and (3.3) can also be expressed in reduced form,

$$y_t = \upsilon_t + u_t, \tag{3.4}$$

where $y_t = (\Delta r_t^*, \Delta p_t)'$, $u_t = (u_{1t}, u_{2t})'$ and $\upsilon_t = \Gamma x_t$ defines the time-varying mean where Γ is a $2 \times k$ matrix of reduced form parameters attached to x_t. The reduced form residuals in u_t contain the contemporaneous parameters of interest, i.e. $u_{1t} = (1-\alpha\beta)^{-1}[(\beta+\varphi)z_t+\beta\eta_t+\epsilon_t]$ and $u_{2t} = (1-\alpha\beta)^{-1}[(1+\alpha\varphi)z_t+\eta_t+\alpha\epsilon_t]$.

The main parameter under inspection is α which can be estimated consistently by the IH approach introduced in Rigobon (2003) and extended in other empirical studies of monetary policy (cf. Normandin & Phaneuf (2004), Lane & Lutkepohl (2009)). In our context, IH consists in looking at changes in the co-movements of the US STIR and international asset prices when the variance of one of the shocks in the system is known to shift while the parameters of equations (3.2) and (3.3) are assumed to remain stable. Following Rigobon & Sack (2004), we use two sub-samples, F ('policy dates') and $\tilde{F}$ ('non-policy dates'). It can be shown that the assumptions,

$$\sigma^2_{\epsilon,F} > \sigma^2_{\epsilon,\tilde{F}},\ \sigma^2_{\eta,F} = \sigma^2_{\eta,\tilde{F}},\ \sigma^2_{z,F} = \sigma^2_{z,\tilde{F}}, \tag{3.5}$$

on the variances of the shocks must hold for the IH estimator to be consistent. These conditions imply that the importance of policy shocks is larger in the sub-sample F. An important point is that the variance of the policy shock must not become infinitely large, but only increase relative to the variances of other periods preceding or following the shock. One may use, for instance, days of the US Federal Open Market Committee (FOMC) meeting and of the Chairman's semi-annual monetary policy testimony to the US Congress (so-called Humphrey Hawkins Report) to identify circumstances in which the conditions (3.5) are plausible (cf. Rigobon & Sack (2004)).

Let $\Omega_F = E[[u_{1t}\, u_{2t}]' \cdot [u_{1t}\, u_{2t}]|t \in F]$ and $\Omega_{\tilde{F}} = E[[u_{1t}\, u_{2t}]' \cdot [u_{1t}\, u_{2t}]|t \in \tilde{F}]$. The analytical expression for the difference in the latter covariance matrices under the assumptions in (3.5) can be shown to satisfy,

$$\Omega_D = \Omega_F - \Omega_{\tilde{F}} = \lambda \begin{bmatrix} 1 & \alpha \\ \alpha & \alpha^2 \end{bmatrix}, \tag{3.6}$$

with $\lambda = (\sigma^2_{\epsilon,F} - \sigma^2_{\epsilon,\tilde{F}})(1-\alpha\beta)^{-2}$. Thus, α can be identified from the change in the covariance matrix since $\alpha = \Omega_{D,12}\Omega^{-1}_{D,11}$, $\alpha = \Omega_{D,22}\Omega^{-1}_{D,12}$ or $\alpha = (\Omega_{D,22}\Omega^{-1}_{D,11})^{-1/2}$. Within the above framework of heteroskedasticity, endogeneity and omitted variables it is possible to devise a simple parameter stability test. More precisely, we can check whether the parameter α is stable within the sample of policy and non-policy dates by testing whether the determinant of the difference in *conditional* covariance of the two subsamples F and $\tilde{F}$ is zero (DCC test) or simply by comparing the two estimates $\hat{\alpha} = \hat{\Omega}_{D,12}\hat{\Omega}_{D,11}^{-1}$ and $\hat{\alpha} = \hat{\Omega}_{D,22}\hat{\Omega}_{D,12}^{-1}$ (Rigobon (2000)).

3.3 Econometric methodology

This section consists of four subsections. In the first subsection the dataset used to carry out the analysis is described. In the second subsection the estimation strategy is discussed. The third and fourth subsections are devoted, respectively, to issues such as the selection of non-policy dates, comparison of estimated impacts, bootstrap inference and the recursive analysis.

3.3.1 Data

The empirical specification introduced in the previous section to study the response of international asset prices to US monetary policy (US Federal Reserve's decisions to change the US STIR) is examined from a panel perspective. Let $i = 1, ..., N$ indicate in the following the respective cross sectional entities (countries). The sample covered for our analysis runs from 01/1994 to 12/2007 at the *daily* frequency. The asset price data consists of *daily* Datastream calculated all-share (total market) equity indices ($N = 29$), 10-year (benchmark) government bond indices ($N = 19$), real estate security indices ($N = 14$) and exchange rate data (currency in country i/US dollar) for all the different economies considered. Countries were selected upon data availability for the sample period covered.

Previous studies that analyze the relationship between US monetary and asset prices have used monetary policy rate measures such as the Federal Funds Futures rate (cf. Rigobon & Sack (2004)), the Federal Funds rate (cf. Bernanke & Kuttner (2005), Cochrane & Piazessi (2002)) and the 3-month Treasury rate (cf. Rigobon & Sack (2002)). Similar to the Federal Funds Futures rate, the 3-month Treasury rate adjusts daily to capture changes in market expectations about monetary policy over the near term (Rigobon & Sack (2002)). In this study, the 3-month Treasury rate allows to capture changes in market expectations over the short term when inferring the response of international asset prices to changes in the US STIR. Moreover, as an interesting by-product, the interaction of international asset prices and the US Treasury rate allows to analyze empirically the 'flight to quality' effect, i.e. the switch between risky (e.g. international equity) and riskless (e.g. US STIR) assets, which has recently attracted the attention of economists (cf. Pachenko & Wu (2009), Pavlova & Rigobon (2009)). Therefore, for the purpose of this study, we employ the US 3-month Treasury rate at the *daily* frequency which is also collected from Datastream.

The policy dates are taken from the Federal Reserve's web-site, where the meeting calendar, minutes and statements of the Federal Open Market Committee are published (*http://www.federalreserve.gov/monetarypolicy/fomc.htm*). In order to reduce the adverse effects of 'outliers' we concentrate on scheduled policy dates. Overall, a total number of 116 policy dates are employed.

3.3.2 Estimation methodology

As previously introduced, the impact of US monetary policy on international asset prices is studied via a heterogeneous panel framework. The panel version of the model presented in (3.4) is given by,

$$y_{it} = \upsilon_{it} + u_{it}, \tag{3.7}$$

where $y_{it} = (\Delta r_t^*, \Delta p_{it})'$, contains the change in the US STIR Δr_t^* and the return $\Delta p_{it} = \ln P_{\bullet,it} - \ln P_{\bullet,it-1}$ of asset $\bullet = s, h, b$ for $i = 1, ..., N$ countries and $t = 1, ..., T$ daily time periods. The vector $u_{it} = (u_{1t}, u_{2it})'$ contains the reduced form residuals, more precisely, $u_{1t} = (1 - \alpha_i\beta_i)^{-1}[(\beta_i + \varphi)z_t + \beta_i\eta_{it} + \epsilon_t]$, $u_{2it} = (1 - \alpha_i\beta_i)^{-1}[(1 + \alpha_i\varphi)z_t + \eta_{it} + \alpha_i\epsilon_t]$ and α_i is the parameter of interest. The model for the time-varying mean $\upsilon_{it} = \Gamma_i x_{it}$ is specified as,

$$\upsilon_{it} = \upsilon_i + A_{i,1}y_{it-1} + ... + A_{i,p}y_{it-p} + B_{i,1}f_{it-1} + ... + B_{i,p}f_{it-p}, \tag{3.8}$$

where $f_{it} = (\Delta s_{it}, \Delta p_t^*)'$ is a vector of exogenous components containing the country i/US dollar exchange rate return $\Delta s_{it} = \ln S_{it} - \ln S_{it-1}$ and the return on the US equity/bond/real estate index, i.e. $\Delta p_t^* = \ln P_{\bullet,t}^* - \ln P_{\bullet,t-1}^*$ for $\bullet = s, b, h$. Therefore, the lagged variables in y_{it} and f_{it} allow to control for 'conditional dependencies' of international asset markets to (i) the country i equity/bond/real estate return, (ii) the change in the US STIR, (iii) the country i/US exchange rate return and (iv) the US equity/bond/real estate return. Model (3.7) coupled with (3.8) reduces to a vector autoregression with exogenous components of order p for each cross-section member (henceforth VARX(p)). It is noteworthy that the conditional mean υ_{it} accounts for expectations (conditional on information available in time $t-1,...,t-p$) of the variables contained in y_{it} and f_{it}.

The number of lags in (3.8) may be chosen as usual from a sequential test of the null $H_0 : p_{max}$ vs. $H_1 : p_{max} - 1$ or via Aikaike and Schwarz information criteria. Given the optimal lag $\hat{p}$, model (3.7) is estimated and the reduced form residuals denoted $\hat{u}_{it}$ are obtained. The response of the 'conditionally centered' (log) returns ($\hat{u}_{2it}$) of international equity, bond and real estate assets to US STIR changes ($\hat{u}_{1t}$) is estimated by employing the Instrumental Variable (IV) and Generalized Method of Moments (GMM) estimators proposed in Rigobon & Sack (2004) as well as the popular Event Study (ES) estimator. Specific issues not detailed here may be looked up in the article by Rigobon & Sack (2004). Our approach extends the latter estimation strategy to analyze the empirical relationship between US monetary policy and international asset prices within a heterogeneous panel framework.

In what follows let $\Delta r^* = [\hat{u}'_{1F} \ \hat{u}'_{1\tilde{F}_i}]'$ and $\Delta p_i = [\hat{u}'_{2iF} \ \hat{u}'_{2i\tilde{F}_i}]'$ be $(T_F + T_{\tilde{F}_i}) \times 1 = \mathcal{T}_i \times 1$ partitioned vectors containing (conditionally centered) US STIR changes and (log) asset price changes for country i, respectively, for the sample of policy dates $t \in F$ and the sample of non-policy dates $t \in \tilde{F}_i$. The latter $\tilde{F}_i = \left\{t_{\tilde{1}}, t_{\tilde{2}}, ..., t_{\tilde{F}_i}\right\}$ may consist of the days preceding, following or surrounding those included in the former $F = \{t_1, t_2, ..., t_F\}$. Once the sample of non-policy dates $\tilde{F}_i$ is chosen, the coefficient α_i for $i = 1, ..., N$ can be estimated via three-stage least squares (3SLS),

$$\hat{\alpha}_{IV,ij} = (\Delta r^{*'} q_j \Delta r^*)^{-1} \Delta r^{*'} q_j \Delta p_i, \tag{3.9}$$

where $q_j = w_j (w'_j \hat{\Sigma}_i w_j)^{-1} w'_j$ for $j = 1, 2$ with $w_1 = [\hat{u}'_{1F} \ -\hat{u}'_{1\tilde{F}_i}]'$ or $w_2 = [\hat{u}'_{2iF} \ -\hat{u}'_{2i\tilde{F}_i}]'$ the two possible vectors of instruments and $\hat{\Sigma}_i = \text{diag}\,[\hat{\varepsilon}_i^2]$ with $\hat{\varepsilon}_i$ the residuals of the two-stage least squares regression (2SLS).[1] It is noteworthy that in the case that the number of observations in the sets F and $\tilde{F}_i$ differ, each subsample in Δr^*, Δp_i and w_j has to be divided by the square root of the total number of dates in each particular set. The degree of explanation $R^2_{IV,i}$ of the IV regression is obtained as proposed in Pesaran & Smith (1994). The $R^2_{IV,i}$ provides a measure for the 'goodness of fit' in the response of international asset prices to US monetary policy.

In addition to IV estimation outlined above, GMM estimation may also be naturally employed given the analytical expression for the moment conditions in (3.6). The two parameters to be estimated via the GMM approach are α_i, which is the main parameter of interest, and $\lambda_i \equiv (\sigma^2_{\epsilon,F} - \sigma^2_{\epsilon,\tilde{F}_i})(1-\alpha_i\beta_i)^{-2}$, which gives a measure of the heteroskedasticity in the data. Thus, in GMM estimation the parameter vector $\zeta_i = (\alpha_{GMM,i}, \lambda_i)'$ may be obtained. In our context, the GMM estimator for $i = 1, ..., N$ is given by,

$$\hat{\zeta}_{\mathcal{T}_i} = \arg\min_{\zeta_i \in \Phi} b_{\mathcal{T}_i}(\zeta_i)' A_{\mathcal{T}_i} b_{\mathcal{T}_i}(\zeta_i), \tag{3.10}$$

with Φ the parameter space, $b_{\mathcal{T}_i}(\zeta_i)$ the vector of differences between sample moments and analytical moments and $A_{\mathcal{T}_i}$ a positive definite and possibly random weighting matrix. Moreover, $\hat{\zeta}_{\mathcal{T}_i}$ is consistent and asymptotically Normal under suitable 'regularity conditions' (cf. Harris & Matyas (1999)). The J_i-statistic is available in GMM estimation as a measure of the goodness of fit of the model.

In contrast to the IV and GMM estimators, the popular ES estimator addresses the identification problem by focusing on periods immediately surrounding changes in the US STIR. In our context, the ES approach obtains the following estimator for $i = 1, ..., N$:

$$\hat{\alpha}_{ES,i} = (\hat{u}'_{1F}\hat{u}_{1F})^{-1}\hat{u}'_{1F}\hat{u}_{2iF}. \tag{3.11}$$

The degree of explanation of the ES regression denoted $R^2_{ES,i}$ may also be computed in the usual way from the OLS residuals of regression (3.11). A Hausmann

[1] Note that the two stage least squares estimator (2SLS) estimator is numerically identical to $\hat{\alpha}_{IV,ij} = (w'_j\Delta r^*)^{-1}w'_j\Delta p_i$ which is the estimator provided in Rigobon & Sack (2004) since there is one independent variable and one instrument.

test denoted $\mathcal{H}_i$ can be used in order to test the null hypothesis $H_0 : \alpha_{IV,i} = \alpha_{ES,i}$. Under the null hypothesis $\mathcal{H}_i$ is $F_{1,\mathcal{T}_i-1}$-distributed. Rejection of H_0 would indicate that the ES estimator is biased. Alternatively, it could indicate that the variance of the policy shock in the sample of policy dates F is not sufficiently large for near identification to hold (Rigobon & Sack (2004)).

In our panel approach we may regard heterogeneity in the estimators since we could expect different reactions to US monetary policy amongst the countries at hand and the distinct asset classes, i.e. we may assume that $\alpha_{\bullet,i} = \alpha_\bullet + \tau_{\bullet,i}$ for $\bullet = IV, ES, GMM$ where $\tau_{\bullet,i}$ is a cross-sectionally *iid* disturbance. Therefore, it is suitable under the latter assumption to aggregate the impact of US monetary policy via the Mean Group (MG) estimator proposed by Pesaran & Smith (1995). In order to account for heteroskedasticity across country estimates we perform a *standardized* MG estimation, i.e.

$$\bar{\alpha}_\bullet = N^{-1} \sum_{i=1}^{N} \hat{\alpha}_{\bullet,i} \hat{\sigma}_{\bullet,i}^{-1}, \tag{3.12}$$

for $\bullet = IV, ES, GMM$ where the standard errors $\hat{\sigma}_{\bullet,i} = \widehat{\mathrm{Var}}[\hat{\alpha}_{\bullet,i}]^{1/2}$ are robust under heteroskedasticity. It is worthwhile noting that the above estimators rely on consistency of single market estimates and provide a guidance on the 'average impact' of US monetary policy on international asset prices under parameter heterogeneity. In addition, we estimate a 'Mean Group Difference' (MGD),

$$\bar{\alpha}_{MGD} = N^{-1} \sum_{i=1}^{N} (\hat{\alpha}_{IV,i} \hat{\sigma}_{IV,i}^{-1} - \hat{\alpha}_{ES,i} \hat{\sigma}_{ES,i}^{-1}), \tag{3.13}$$

in order to compare the heteroskedasticity based estimator and the ES estimator.

3.3.3 Selection of non-policy dates and comparison of estimated impacts

In the international context the selection of the non-policy dates $\tilde{F}_i$ is complicated in the sense that for certain countries (particularly those of other continents than America) it is difficult to know which set $\tilde{F}_i$ leads to the most informative change in covariance $\Omega_{D,i}$ for IV and GMM estimation. Our practical solution for this problem is to 'search' for the best window of non-policy dates $\tilde{F}_i$ for each i around the policy dates F. To motivate the selection of non-policy dates, note that the regression model in (3.1) could be transformed and estimated by OLS once an analyst has access to the true impact parameter α. This transformed regression is supposed to offer, conditional on x_t and z_t the highest accuracy in whitening the data. Transforming the regression model (3.1) with inefficient or even biased estimates of α is expected to worsen the model's fitting accuracy. Therefore, competing estimates of α extracted from alternative sets of non-policy dates may be ranked according to an R^2 criterion derived from a transformed model representation. More precisely, the 'algorithm' to select the most appropriate set of instruments for each of the countries in the panel and for each of the asset markets consists of:

1. Obtaining a vector of non-policy dates $\tilde{F}_i = (F-1, F-2, ..., F-h_{\bullet,i})'$ for $\bullet = IV, GMM$ constructed by stacking different sets of non-policy dates for a maximum of $h_{\bullet,i}$ days preceding the policy dates. At each horizon $h_{\bullet,i}$ the estimates $\hat{\alpha}^h_{\bullet,i}$ for $\bullet = IV, GMM$ are obtained and the model,
$$\mu_{it} = \phi_i' x_{it} + \gamma_i t + \eta_{it}, \tag{3.14}$$
is estimated for $i = 1, ..., N$ and $t = 1, ..., T$ *daily* time periods via OLS where $\mu_{it} = \Delta p_{it} - \hat{\alpha}^h_{\bullet,i} \Delta r^*_t$, $x_{it} = (y'_{it-1}, ..., y'_{it-p}, f'_{it-1}, ..., f'_{it-p})'$ and t is a time trend which 'proxies' the common shock.[2]

2. Computing the R^2 of regression (3.14) and choosing the $\hat{h}_{\bullet,i}$ that corresponds to the maximum quantity from the set $\left\{R^2_{\bullet,i1}, ..., R^2_{\bullet,ih}\right\}$ to obtain an optimal set of non-policy dates $\tilde{F}^*_i = (F-1, F-2, ..., F-\hat{h}_{\bullet,i})'$.[3]

The above procedure is intuitive as a set of non-policy dates is chosen which, conditional on the candidate parameter $\hat{\alpha}^h_{\bullet,i}$, the vector x_{it} and the time trend t, minimizes the distance between the international (log) asset price change (Δp_{it})

[2] We also constructed a common factor from international asset return data by means of Principal Component Analysis and used it to proxy the common shock. However, experimentations along these lines resulted in qualitatively similar results to the simple model with a time trend.

[3] Note that the set of non-policy dates $\tilde{F}^*_i$ does not intersect with the set of policy dates F.

and the change in the US STIR (Δr_t^*). We also experimented with alternative maximum values for $h_{\bullet,i}$ but found that $h_{\bullet,i} > 3$ produces results that are much more heterogeneous and volatile than $h_{\bullet,i} = 3$. Therefore, $h_{\bullet,i} = 3$ was chosen.

In order to test whether the set of selected non-policy dates $\tilde{F}_i^*$ satisfies the heteroskedasticity requirement (3.6), the null hypothesis $H_0 : \Omega_{D,i} = 0$ versus the alternative hypothesis $H_1 : \Omega_{D,i} \neq 0$ is tested by means of the likelihood ratio statistic:

$$\mathcal{G}_i = \mathcal{T}_i \ln \frac{|\hat{\Omega}_{i\mathcal{T}_i}|}{|\hat{\Omega}_{iF}|^g |\hat{\Omega}_{i\tilde{F}_i^*}|^{1-g}}, \tag{3.15}$$

where $\hat{\Omega}_{i\mathcal{T}_i}$ is the estimated covariance matrix for the sample containing both the policy dates $t \in F$ and the sample of non-policy dates $t \in \tilde{F}_i^*$ and $g = T_F/\mathcal{T}_i$. The statistic (3.15) has an asymptotic χ^2 distribution with $2^{-1}m(m+1)$ degrees of freedom where $m \times m$ is the dimension of the covariance matrix (Galeano & Pena (2007)). Rejection of the null hypothesis $H_0 : \Omega_{D,i} = 0$ would provide evidence of heteroskedasticity in the subsamples F and $\tilde{F}_i^*$ and, thus, supports the applied identification scheme.

The 'auxiliary regression' in (3.14) may also be applied with the ES estimator $\hat{\alpha}_{ES,i}$ to obtain a goodness of fit measure denoted $R^2_{ES,ih}$ for consistency of notation. In this way one may discriminate between estimators across the panel by comparing the quantities $R^2_{IV,ih}$, $R^2_{GMM,ih}$ (which correspond to the estimated impacts $\hat{\alpha}_{\bullet,i}$ and horizons $\hat{h}_{\bullet,i}$ for $\bullet = IV, GMM$) and $R^2_{ES,ih}$.[4] It is also important to note that many exchange markets were closed in countries that are not in the American continent at the time of the monetary policy announcements (around 14:00 US Eastern Time). For this reason, one could argue that the (conditionally) centered returns $\hat{u}_{2iF}$ and $\hat{u}_{2i\tilde{F}_i}$ of country i which is not in the American continent, should be modified to $\hat{u}_{2iF+1}$ (IV, GMM, ES) and $\hat{u}_{2i\tilde{F}_i+1}$ (IV, GMM), respectively, in order to account for the timing effect of the shock. To control for the timing issue in non-American asset markets, we also used the 'auxiliary regression' in (3.14) above with the estimates $\hat{\alpha}^s_{\bullet,i}$ for $\bullet = IV, ES, GMM$, which

[4] Note that the quantities $R^2_{\bullet,ih}$ for $\bullet = IV, GMM, ES$ obtained from (3.14) are different from the quantities $R^2_{\bullet,i}$ for $\bullet = IV, ES$ which are obtained from the regressions in (3.9) and (3.11), respectively.

were obtained by employing $\hat{u}_{2iF+s}$ (IV, GMM, ES) and $\hat{u}_{2i\tilde{F}_i^*+s}$ (IV, GMM) for $s = 0, 1$. However, only in very few cases (5 out of all countries and all asset markets) we found that $s = 1$, and in those cases, parameter estimates were more volatile. Moreover, it is also plausible that the impact of FOMC announcements on asset markets occurred during overnight trading (cf. Wongswan (2005)). Thus we decided to keep $s = 0$ uniformly.

3.3.4 Bootstrap inference, parameter stability and recursive estimation

Studying the impact of US monetary policy shocks on international asset prices is complicated due to the large amount of factors that may affect the relationship. Nevertheless, in our set up, we control for (i) conditional effects of different explanatory variables in the time-varying mean υ_{it}, (ii) a common shock z_t, (iii) regime changes and heteroskedasticity ($\Omega_{D,i}$) and (iv) selection of non-policy dates ($\tilde{F}_i$). Generally, our analysis is conditional on the set of policy dates and economic states and dynamics surrounding these time points governing the stochastic properties of model estimates and diagnostics. To disentangle random and structural features of the statistical outcomes we adopt resampling techniques to generate pseudo samples of policy dates. With them at hand, we build empirical distributions of the statistical tools employed. The resampling algorithm consists of:

1. Drawing with replacement from the set of policy dates $F = \{t_1, t_2, ..., t_F\}$ to obtain a new set of bootstrap policy dates $F_b = \{t_{1b}, t_{2b}, ..., t_{Fb}\}$.

2. Obtaining the 'optimal' set of non-policy dates $\tilde{F}_{ib}^*$ for IV and GMM estimation and using $\tilde{F}_{ib}^*$ and F_b to compute bootstrap quantities $\hat{\alpha}_{\bullet,ib}$, $\bar{\alpha}_{\bullet,b}$ for $\bullet = IV, ES, GMM, MGD$ and $R^2_{\bullet,ib}$ for $\bullet = IV, ES$.

3. Repeating steps 1 to 2 for 500 times and obtaining bootstrap confidence intervals from the $\gamma/2$ and $(1-\gamma/2)$ quantile of the bootstrap distribution.

The upper quantiles of R^2_{ib} corresponding to the regressions (3.9) and (3.11), also referred to as R^2_i upper bound, are usually proposed in the literature to evaluate the adequacy of asset pricing relationships (cf. Foster *et al.* (1997)).

As introduced previously, it is possible within the IH framework to inspect whether the impact of US monetary policy on international asset prices is stable over the sample period analyzed. One simply has to compare the $\hat{\alpha}_{IV,ij}$ estimators obtained with the two set of instruments $w_1 = [\hat{u}'_{1F} \; - \hat{u}'_{1\tilde{F}_i}]'$ or $w_2 = [\hat{u}'_{2iF} \; - \hat{u}'_{2i\tilde{F}_i}]'$. A significant difference between these estimates would hint at parameter instability. To study this issue in more detail across the panel, a 'mean group difference' in analogy to (3.13) is computed with the two different estimates $\hat{\alpha}_{IV,ij}$ obtained with the instruments w_1 or w_2.

In order to safeguard the overall analysis from possible parameter instability, the impact of US monetary policy on international asset prices is also studied via a recursive *standardized* MG estimation procedure. The latter procedure consists on fixing a window of size $T_{F,min} = 72$ of policy dates which is moved over the subsample $\tau = T_{F,min}, \ldots, \tau = T_F$. At each window, the parameter $\hat{\alpha}_{\bullet,i}$ for each i, and subsequently, the MG estimate $\bar{\alpha}_{\bullet}$ are computed for $\bullet = IV, ES, GMM$. The optimal set of non-policy dates $\tilde{F}_i^*$ is also computed at each window as explained in the previous section to obtain the estimates $\hat{\alpha}_{\bullet,i}$ for $\bullet = IV, GMM$. Thus, a vector of MG impacts $\bar{\alpha}_{\bullet,rw} = (\bar{\alpha}_{\bullet,1}, ..., \bar{\alpha}_{\bullet,RW})'$ is obtained which can be analyzed to understand the magnitude of the impact over different sub-samples. Since the new MG estimate at each window is based on past and new shocks, it also allows us to study the response of international asset markets to US monetary policy over time. Notably, the rolling window approach is consistent with the IH approach under the null hypothesis of a stable impact α_i in every sub-sample.

A rolling window 'Mean Group R^2' is also computed from $R^2_{\bullet,i}$ for $\bullet = IV, ES$ obtained from the regressions (3.9) and (3.11) along the latter lines i.e, $\bar{R}^2_{\bullet,rw} = (\bar{R}^2_{\bullet,1}, ..., \bar{R}^2_{\bullet,RW})'$. Both rolling window estimates $\bar{\alpha}_{\bullet,rw}$ and $\bar{R}^2_{\bullet,rw}$ allow to uncover whether the actual degree of market integration is time-varying as suggested in recent studies (cf. DeRoon & DeJong (2005), Pachenko & Wu (2009)).

3.4 Results

This section discusses the results of the impact of US monetary policy on international asset prices. Empirical results are reported in Tables 3.1 to 3.7. Figures 3.1, 3.2 and 3.3 display graphically the empirical densities of the estimates

$\hat{\alpha}_{\bullet,i}\hat{\sigma}_{\bullet,i}^{-1}$ for $\bullet = IV, ES, GMM$ along with the recursive results. In what follows, we mostly refer to statistical significance at the 10% level.

	VARX(3)		IV				ES				GMM			
Country	v_{it}^{no}	$\mathcal{P}_i$	$\hat{\alpha}_{i1}$	$\hat{\sigma}_i$	$CI_{i,1}$	$CI_{i,2}$	$\hat{\alpha}_i$	$\hat{\sigma}_i$	$CI_{i,1}$	$CI_{i,2}$	$\hat{\alpha}_i$	$\hat{\sigma}_i$	$CI_{i,1}$	$CI_{i,2}$
Australia	9	44.175	**-2.103**	1.046	-5.301	-0.891	-0.754	0.597	-1.655	0.112	**-1.948**	0.842	-4.213	-0.787
Austria	5	28.491	-0.455	1.140	-2.863	0.859	0.245	0.629	-0.715	1.085	-0.386	0.810	-2.576	0.880
Belgium	5	44.530	0.320	1.286	-1.185	3.274	0.626	0.819	-0.310	2.098	0.562	1.002	-11.948	3.423
Canada	4	29.769	-3.371	2.509	-12.858	0.291	**-1.557**	1.267	-3.546	-0.027	**-4.762**	2.086	-9.844	-1.762
Chile	5	36.238	0.755	1.967	-2.751	2.818	1.008	1.192	-0.819	2.218	0.387	1.360	-2.775	2.706
China	7	51.914	3.280	4.470	-3.653	16.377	-0.964	1.777	-3.674	0.757	0.645	2.212	-4.274	10.461
Colombia	5	20.803	-1.170	1.398	-5.353	1.342	-0.770	0.643	-1.672	0.046	-0.740	0.792	-3.731	1.087
Denmark	8	30.364	**-2.672**	1.283	-4.753	-0.933	-1.037	0.898	-2.019	0.703	**-2.562**	0.855	-4.284	-0.911
Finland	6	29.203	**-6.646**	3.079	-12.987	-2.779	-2.227	2.171	-4.381	1.361	**-4.220**	1.841	-8.279	-0.784
France	7	37.356	-0.422	1.335	-2.425	1.178	0.595	0.694	-0.360	1.610	-0.069	0.887	-1.845	1.383
Germany	9	42.715	-0.080	1.497	-2.405	2.015	1.042	0.824	-0.001	2.185	-0.100	2.445	-1.367	1.796
Greece	6	19.250	-2.542	1.892	-5.474	0.133	-0.815	1.401	-2.350	1.376	**-2.722**	1.179	-6.230	0.206
H. Kong	9	27.627	-0.598	2.651	-3.530	3.595	0.058	1.314	-1.352	2.230	-0.290	1.710	-1.245	5.724
Hungary	10	28.574	-2.471	4.731	-6.997	8.667	-2.138	3.126	-5.355	3.794	-1.635	3.015	-5.910	8.693
India	7	17.879	-0.978	3.600	-11.405	5.100	0.239	1.535	-2.169	2.183	0.360	1.563	-4.085	3.284
Ireland	6	27.398	**-2.686**	1.583	-5.611	-0.301	-0.704	1.092	-1.879	1.158	**-2.408**	1.110	-4.616	-0.075
Italy	6	43.203	**-3.073**	1.860	-5.939	-0.675	-1.639	1.144	-3.137	0.092	**-2.767**	1.630	-5.660	-0.730
Japan	5	45.003	-2.085	1.755	-4.929	0.460	-0.117	1.307	-1.480	2.257	**-2.610**	1.197	-6.460	0.796
Luxembourg	8	28.759	-1.176	2.326	-3.150	4.164	-1.804	1.229	-3.036	0.286	-0.908	1.559	-3.101	4.665
Mexico	6	24.207	**-6.010**	4.234	-26.780	-0.126	**-2.876**	2.142	-6.373	-0.523	**-5.486**	3.118	-29.415	-0.128
N. Zealand	3	22.443	**-2.246**	1.246	-4.695	-0.631	-0.650	0.819	-1.539	0.601	**-2.304**	0.872	-5.084	-0.863
Norway	5	36.112	0.165	1.265	-1.860	1.780	**0.993**	0.686	0.221	2.105	0.392	0.844	-1.486	2.007
Peru	7	26.409	-0.435	2.506	-5.191	5.124	-0.783	1.230	-2.365	0.993	-2.235	1.469	-5.138	1.658
Spain	9	26.333	**-2.625**	1.771	-6.347	-0.326	-0.510	1.029	-1.742	1.032	-1.750	1.596	-4.722	0.370
S. Africa	5	14.842	-0.397	2.361	-2.504	5.837	-0.483	1.532	-1.911	2.344	-0.384	1.483	-2.484	4.240
Sweden	5	32.383	**-3.284**	1.657	-6.848	-1.329	-1.046	1.092	-2.421	0.553	**-2.983**	1.374	-6.725	-1.090
Switzerland	8	46.450	-0.734	1.438	-2.157	2.304	0.011	0.991	-1.062	1.724	-0.389	0.979	-1.201	2.274
U. Kingdom	8	38.752	**-2.064**	1.251	-4.095	-0.106	-0.680	0.785	-1.507	0.746	-1.009	1.969	-9.385	0.374
Venezuela	8	70.815	**9.776**	5.812	1.736	40.185	**3.570**	2.595	0.963	8.080	1.788	1.478	-0.235	39.352

Table 3.1: Country estimates for equity markets. v_{it}^{no} is the number of coefficients in the time-varying mean v_{it} of the VARX(3) which have robust t-ratios greater than 1.64 in absolute value, $\mathcal{P}_i$ is the Portmanteau test of the null hypothesis of no 6th order serial correlation in the residuals of the VARX(3) which is $\chi^2(48)$ distributed, $\hat{\alpha}_{\bullet,i}$ ($\hat{\sigma}_{\bullet,i}$) is the point estimate (heteroskedasticity robust standard error) for country i with $\bullet = IV, ES, GMM$, $CI_{i,j}$ are the 90% bootstrap confidence bands. Note that the estimate $\hat{\alpha}_{IV,i1}$ is obtained with the set of instruments w_1. Entries in **bold** are statistically significant at the 10% level according to the asymptotic and/or bootstrap distribution.

	$\Omega_{D,i}$			IV			ES		GMM			
Country	$\mathcal{G}_i$	$\mathcal{H}_i$	p-val	$\hat{h}_i$	R^2_{ih}	R^2_{ib}	R^2_{ih}	R^2_{ib}	$\hat{h}_i$	R^2_{ih}	J_i-stat	p-val
Australia	**64.359**	2.463	0.118	2	0.276	0.037	0.283	0.040	2	0.269	0.545	0.460
Austria	**64.275**	0.543	0.462	2	0.092	0.011	0.094	0.018	2	0.089	0.795	0.373
Belgium	**71.616**	0.095	0.758	2	0.097	0.008	0.100	0.030	2	0.095	**3.926**	0.048
Canada	**44.910**	0.702	0.403	1	0.020	0.036	0.021	0.069	3	0.021	1.607	0.205
Chile	**60.793**	0.026	0.872	2	0.068	0.019	0.068	0.067	2	0.067	1.648	0.199
China	**45.108**	1.070	0.302	1	0.005	0.008	0.004	0.011	3	0.005	0.398	0.528
Colombia	**35.935**	0.104	0.747	1	0.077	0.011	0.076	0.024	2	0.077	0.467	0.494
Denmark	**63.206**	**3.181**	0.076	2	0.197	0.032	0.202	0.041	2	0.193	0.856	0.355
Finland	**82.584**	**4.098**	0.044	2	0.144	0.057	0.146	0.079	2	0.143	**3.694**	0.055
France	**95.030**	0.795	0.373	2	0.115	0.005	0.117	0.023	2	0.112	**4.889**	0.027
Germany	**94.409**	0.806	0.370	2	0.109	0.006	0.111	0.037	2	0.107	**5.976**	0.015
Greece	**69.322**	1.845	0.176	2	0.080	0.025	0.080	0.026	2	0.080	**3.898**	0.048
H. Kong	**79.136**	0.081	0.776	2	0.184	0.007	0.185	0.018	2	0.183	**2.833**	0.092
Hungary	**63.107**	0.009	0.925	2	0.094	0.033	0.094	0.077	2	0.094	0.357	0.550
India	**30.816**	0.140	0.709	1	0.041	0.012	0.040	0.027	3	0.041	1.828	0.176
Ireland	**65.102**	**2.992**	0.085	2	0.163	0.030	0.166	0.030	2	0.160	0.987	0.320
Italy	**81.862**	0.957	0.329	2	0.054	0.022	0.055	0.061	2	0.053	**4.745**	0.029
Japan	**68.281**	**2.826**	0.094	2	0.134	0.023	0.136	0.027	2	0.133	1.483	0.223
Luxembourg	**62.471**	0.101	0.751	2	0.087	0.029	0.088	0.086	2	0.086	0.471	0.492
Mexico	**30.856**	0.736	0.392	1	0.026	0.033	0.025	0.095	3	0.026	0.845	0.358
N. Zealand	**65.429**	**2.884**	0.091	2	0.160	0.039	0.163	0.033	2	0.156	0.019	0.889
Norway	**65.779**	0.608	0.436	2	0.116	0.004	0.118	0.021	2	0.114	2.604	0.107
Peru	**31.615**	0.025	0.873	1	0.038	0.013	0.037	0.041	3	0.038	**3.431**	0.064
Spain	**83.652**	2.153	0.144	2	0.074	0.023	0.074	0.025	2	0.073	**7.539**	0.006
S. Africa	**64.435**	0.002	0.962	2	0.125	0.023	0.126	0.052	2	0.123	**4.263**	0.039
Sweden	**80.682**	**3.225**	0.074	2	0.124	0.020	0.126	0.033	2	0.122	2.471	0.116
Switzerland	**81.809**	0.511	0.476	2	0.099	0.008	0.100	0.024	2	0.097	**4.248**	0.039
U. Kingdom	**94.371**	2.017	0.157	2	0.131	0.017	0.134	0.034	2	0.128	**10.145**	0.001
Venezuela	**46.867**	1.424	0.234	1	0.052	0.045	0.052	0.057	3	0.052	2.039	0.153

Table 3.2: Selected statistics for equity markets. $\mathcal{G}_i$ are the statistics from the likelihood ratio test of the null $H_0 : \Omega_{D,i} = 0$ which is $\chi^2(3)$ distributed, $\mathcal{H}_i$ is the Hausmann test of the null $H_0 : \alpha_{IV,i} = \alpha_{ES,i}$ and corresponding p-value, $\hat{h}_{\bullet,i}$ is the estimated horizon of non-policy dates via the $\bullet = IV, GMM$ methodology, $R^2_{\bullet,ih}$ is the degree of explanation from the 'auxiliary regression' in (3.14) with $\hat{\alpha}_{\bullet,i}$ for $\bullet = IV, GMM, ES$, $R^2_{\bullet,ib}$ are the bootstrap upper 10% quantile from the $R^2_{\bullet,i}$ of the regression via $\bullet = IV, ES$ in (3.9) and (3.11), J_i-stat is the statistic of the J-test in GMM estimation and corresponding p-value. Entries in **bold** are statistically significant at the 10% level.

3.4.1 Single market results

Point estimates of equity markets for the sample of policy and non-policy dates between 01/1994 and 12/2007 are displayed in Table 3.1. The distinct lag selection tests performed (sequential likelihood ratio, AIC and SIC) for the VARX(p) pointed to three lags in most of the countries and the three asset markets so we have set $p = 3$ throughout. The VARX(3) estimation shows that the number of coefficients in the time-varying mean υ_{it} that are significantly different from zero at a 10% level ranges from 3 to 10. The latter result puts forward that there are some significant conditional effects from the dynamic variables considered in the time-varying mean υ_{it} and thus evidence of significant expectation effects for which one needs to control. Moreover, no serial correlation was diagnosed at the 5% level of significance in the residuals of the VARX(3) in any of the countries according to the Portmanteau statistic $\mathcal{P}_i$ which supports the use of the estimated reduced form residuals. As expected, the GMM and IV approaches usually produce similar point estimates. In contrast, the ES approach yields estimates that are many times different from the heteroskedasticity based estimates in terms of magnitude and precision. Thus, in what follows, comparisons between estimators are mostly done between IV and ES.

The sample of international equity markets consists of 29 units. Amongst the 29 equity markets, 24 (5) show a negative (positive) impact from US monetary policy shocks with IV estimation. The number of positive point estimates increases to 10 when considering ES estimation. A very apparent feature of the results is the heterogeneity in the value of the estimates. Amongst the countries with a positive coefficient, the response to monetary policy estimated via IV (ES) ranges from 0.17 in Norway to 9.78 in Venezuela (0.01 in Switzerland to 3.57 in Venezuela). Amongst those countries with a negative coefficient obtained via IV (ES), the heterogeneity ranges from -0.08 in Germany to -6.65 in Finland (-0.1 in Japan to -2.88 in Mexico). Overall, we find that 13 (2) countries show a statistically significant negative impact at the 10% level according to the bootstrap confidence bands and/or robust standard errors in IV and GMM (ES) estimation, namely: Australia, Canada, Denmark, Finland, Greece, Ireland, Italy, Japan, Mexico, New Zealand, Spain, Sweden and the United Kingdom (Canada and Mexico).

Interestingly, US monetary policy has a statistically significant negative effect on the equity market of Mexico and Canada according to the bootstrap confidence bands of the heteroskedasticity and event study estimators. In fact, Mexico and Canada are the countries with the largest negative response to the Federal Reserve's policy announcements in the American continent. This result speaks in favor of a proximity effect in the impact of US monetary policy on equity markets across border. The equity markets in Australia, Japan and the United Kingdom also show a significant response to US monetary policy shocks hinting at a size (market capitalization) effect. Another interesting finding is the significant response to US monetary policy in Scandinavian equity markets which is evident in both IV and GMM estimation. The only Scandinavian country with an insignificant impact to US monetary policy estimated via IV or GMM is that of Norway. However, the impact of the Norwegian equity market to US monetary policy is significantly positive at the 10% level with the ES estimator according to the bootstrap confidence bands. Another interesting case is the statistically significant response of the New Zealand equity market to US monetary policy since the latter was the first country in the world to introduce inflation targeting. In fact, most of the equity markets with a statistically significant response to US monetary policy follow inflation targeting. The latter findings show that the response of international equity markets to US monetary policy may be related to distance, financial integration and similar monetary policy frameworks as suggested by previous studies (cf. Wongswan (2005), Ehrmann *et al.* (2005)).

Selected diagnostic statistics for equity markets are shown in Table 3.2. The null hypothesis $\Omega_{D,i} = 0$ is rejected at the 5% level in all markets which suggests heteroskedasticity between the sample of policy and non-policy dates and thus supports the IH scheme. The result on the Hausmann test $\mathcal{H}_i$, shows that we are able to formally reject the hypothesis $\alpha_{IV,i} = \alpha_{ES,i}$ at the 10% level in Denmark, Finland, Ireland, Japan, New Zealand and Sweden. The estimated horizon $\hat{h}_{IV,i}$ ($\hat{h}_{GMM,i}$) in equity markets is usually equal to 1 (3) in countries inside the American continent and equal to 2 (2) in countries located outside the American continent. Interestingly, the R^2_{ih} of the 'auxiliary regression' in (3.14) is generally higher for IV and GMM estimates in comparison with ES estimates in countries of the American continent while the opposite is true when considering non-American countries. The bootstrap upper 10% quantiles R^2_{ib} corresponding to the R^2_i measures obtained from the IV and ES regressions in (3.9) and (3.11), respectively, show a higher accuracy of fit for the countries with a statistically significant impact to US monetary policy. Moreover, the J_i-statistic is statistically insignificant at the 5% level in 20 out of 29 international equity markets.

	VARX(3)		IV				ES				GMM			
Country	υ_{it}^{no}	$\mathcal{P}_i$	$\hat{\alpha}_{i1}$	$\hat{\sigma}_i$	$CI_{i,1}$	$CI_{i,2}$	$\hat{\alpha}_i$	$\hat{\sigma}_i$	$CI_{i,1}$	$CI_{i,2}$	$\hat{\alpha}_i$	$\hat{\sigma}_i$	$CI_{i,1}$	$CI_{i,2}$
Australia	9	40.963	-0.495	0.642	-1.616	0.411	-0.242	0.375	-0.737	0.274	-0.454	0.450	-1.631	0.411
Austria	6	18.765	**1.157**	0.425	0.623	2.500	0.229	0.256	-0.240	0.475	**0.860**	0.294	0.445	1.580
Belgium	9	36.888	**0.707**	0.453	0.088	1.883	0.017	0.316	-0.478	0.343	**0.709**	0.377	-0.034	1.811
Canada	2	23.688	**-1.411**	1.148	-4.469	-0.241	**-1.654**	1.063	-3.425	-0.737	-0.694	0.837	-14.820	0.552
Denmark	9	35.595	**0.934**	0.523	0.287	2.785	0.078	0.243	-0.259	0.473	0.552	0.345	-0.126	1.443
Finland	7	44.514	0.612	0.542	-0.116	1.802	-0.038	0.395	-0.623	0.388	0.474	0.355	-0.211	1.349
France	9	18.712	**0.794**	0.547	0.061	2.125	0.103	0.364	-0.416	0.561	**0.710**	0.372	0.078	1.754
Germany	8	21.086	**0.949**	0.438	0.433	2.354	-0.008	0.277	-0.453	0.264	**0.483**	0.273	0.044	0.975
Ireland	8	28.199	0.612	0.584	-0.115	1.936	0.005	0.387	-0.548	0.469	0.368	0.371	-0.337	1.258
Italy	7	30.095	**0.907**	0.555	0.170	2.770	0.095	0.290	-0.363	0.507	0.445	0.360	-0.265	1.339
Japan	6	19.969	**1.316**	0.583	0.608	3.354	**0.825**	0.419	0.370	1.599	**0.931**	0.412	0.422	2.900
Netherlands	8	28.119	0.315	0.557	-0.438	1.199	-0.038	0.397	-0.600	0.382	0.071	0.341	-0.575	0.742
N. Zealand	6	23.836	1.370	1.039	-0.395	2.319	0.614	0.768	-0.769	1.311	**1.291**	0.565	-0.591	2.049
Norway	5	49.113	0.507	0.769	-0.384	2.727	-0.065	0.488	-0.663	0.709	0.562	0.594	-0.204	2.742
Portugal	6	18.741	**0.981**	0.520	0.391	2.540	0.300	0.301	-0.125	0.673	**1.000**	0.434	0.335	2.385
Spain	10	20.383	**0.709**	0.565	0.039	2.108	-0.148	0.352	-0.762	0.247	0.715	0.445	-0.103	1.940
Sweden	5	59.939	**1.104**	0.630	0.303	2.519	0.351	0.350	-0.208	0.804	**0.822**	0.440	0.176	1.632
Switzerland	7	27.049	**0.848**	0.450	0.354	2.371	0.225	0.251	-0.094	0.617	**0.857**	0.446	0.290	2.595
U. Kingdom	4	33.536	0.441	0.753	-0.447	1.758	-0.250	0.564	-1.011	0.338	0.826	0.543	-0.999	0.886

Table 3.3: Country estimates for bond markets. υ_{it}^{no} is the number of coefficients in the time-varying mean υ_{it} of the VARX(3) which have robust t-ratios greater than 1.64 in absolute value, $\mathcal{P}_i$ is the Portmanteau test of the null hypothesis of no 6th order serial correlation in the residuals of the VARX(3) which is $\chi^2(48)$ distributed, $\hat{\alpha}_{\bullet,i}$ ($\hat{\sigma}_{\bullet,i}$) is the point estimate (heteroskedasticity robust standard error) for country i with $\bullet = IV, ES, GMM$, $CI_{i,j}$ are the 90% bootstrap confidence bands. Note that the estimate $\hat{\alpha}_{IV,i1}$ is obtained with the set of instruments w_1. Entries in **bold** are statistically significant at the 10% level according to the asymptotic and/or bootstrap distribution.

	$\Omega_{D,i}$			IV			ES		GMM			
Country	$\mathcal{G}_i$	$\mathcal{H}_i$	p-val	$\hat{h}_i$	R^2_{ih}	R^2_{ib}	R^2_{ih}	R^2_{ib}	$\hat{h}_i$	R^2_{ih}	J_i-stat	p-val
Australia	**48.331**	0.237	0.627	3	0.387	0.014	0.387	0.024	3	0.387	0.249	0.617
Austria	**62.732**	**7.473**	0.007	3	0.150	0.045	0.149	0.030	3	0.150	2.311	0.128
Belgium	**53.820**	**4.528**	0.034	3	0.103	0.023	0.102	0.018	3	0.103	0.037	0.848
Canada	**66.606**	0.316	0.575	3	0.003	0.053	0.003	0.220	3	0.003	**12.012**	0.001
Denmark	**69.246**	**3.413**	0.066	3	0.244	0.039	0.243	0.023	3	0.244	**4.197**	0.041
Finland	**54.771**	**3.068**	0.081	3	0.077	0.027	0.077	0.034	3	0.077	0.682	0.409
France	**53.655**	**2.848**	0.093	3	0.068	0.029	0.068	0.030	3	0.068	0.155	0.694
Germany	**65.849**	**7.984**	0.005	3	0.164	0.037	0.163	0.021	3	0.164	**6.767**	0.009
Ireland	**53.906**	1.935	0.166	3	0.055	0.025	0.054	0.030	3	0.055	1.970	0.160
Italy	**73.710**	**2.946**	0.087	3	0.101	0.031	0.100	0.026	3	0.101	**5.625**	0.018
Japan	**84.403**	1.460	0.228	2	0.032	0.050	0.032	0.067	2	0.032	**4.188**	0.041
Netherlands	**51.541**	0.816	0.367	3	0.065	0.018	0.064	0.035	3	0.065	1.579	0.209
N. Zealand	**53.533**	1.171	0.280	3	0.337	0.068	0.337	0.104	3	0.337	0.153	0.696
Norway	**53.324**	0.929	0.336	3	0.121	0.033	0.120	0.035	3	0.121	0.105	0.746
Portugal	**53.122**	2.591	0.109	3	0.052	0.027	0.052	0.030	3	0.052	0.013	0.911
Spain	**53.086**	**3.772**	0.053	3	0.110	0.021	0.110	0.023	3	0.110	0.002	0.965
Sweden	**71.576**	2.069	0.152	3	0.066	0.027	0.066	0.040	3	0.066	**5.510**	0.019
Switzerland	**60.358**	**2.784**	0.097	3	0.077	0.028	0.003	0.017	3	0.077	1.696	0.193
U. Kingdom	**58.781**	1.918	0.167	3	0.033	0.021	0.033	0.043	3	0.033	**9.958**	0.002

Table 3.4: Selected statistics for bond markets. $\mathcal{G}_i$ are the statistics from the likelihood ratio test of the null $H_0 : \Omega_{D,i} = 0$ which is $\chi^2(3)$ distributed, $\mathcal{H}_i$ is the Hausmann test of the null $H_0 : \alpha_{IV,i} = \alpha_{ES,i}$ and corresponding p-value, $\hat{h}_{\bullet,i}$ is the estimated horizon of non-policy dates via the $\bullet = IV, GMM$ methodology, $R^2_{\bullet,ih}$ is the degree of explanation from the 'auxiliary regression' in (3.14) with $\hat{\alpha}_{\bullet,i}$ for $\bullet = IV, GMM, ES$, $R^2_{\bullet,ib}$ are the bootstrap upper 10% quantile from the $R^2_{\bullet,i}$ of the regression via $\bullet = IV, ES$ in (3.9) and (3.11), J_i-stat is the statistic of the J-test in GMM estimation and corresponding p-value. Entries in **bold** are statistically significant at the 10% level.

As for equity markets, the results for the set of 19 international bond markets show that there are also conditional effects from the dynamic variables included in the time-varying mean υ_{it} as shown by the number of coefficients significant at the 10% level (Table 3.3). Furthermore, there is no evidence of serial correlation in the residuals of the VARX(3). The IV and GMM estimates show that most bond markets, except for Canada and Autralia, have a positive impact to US monetary policy. The ES estimator also obtains mostly positive impacts except for a few more negative cases than in IV or GMM. Out of the 17 countries with a positive coefficient 11 (9) show a statistically significant positive impact at the 10% level according to the bootstrap confidence bands and/or robust standard errors in IV (GMM) estimation. The only statistically significant positive coefficient at the 10% level in the case of ES estimation is that of Japan. However, both IV and ES estimators produce similar point estimates for Canada which are found to be negative and statistically significant at the 10% level according to the bootstrap confidence bands. Note that, while Rigobon & Sack (2004) use bond yields and changes in Eurodollar futures as opposed to bond returns, they find a positive response of US bond markets to US monetary policy shocks. Nevertheless, the direction of the relationship between bond returns and interest rates may vary because revisions in expected risk-adjusted bond returns and expected inflation may have offseting effects (cf. Campbell & Ammer (1993)).

Similar to equity markets, the null $\Omega_{D,i} = 0$ is rejected in all countries for bond markets at the 5% significance level (Table 3.4). The null hypothesis $\alpha_{IV,i} = \alpha_{ES,i}$ is rejected at the 10% level according to the Hausmann statistic $\mathcal{H}_i$ in 9 out of the 19 bond markets. The estimated horizons $\hat{h}_{IV,i}$ and $\hat{h}_{GMM,i}$ are equal to 3 days prior to the shock in all countries except for Japan. The $R^2_{\bullet,ih}$ measures for $\bullet = IV, GMM$ from the 'auxiliary regression' are either greater than or equal to $R^2_{ES,ih}$ obtained from ES estimates. Moreover, the J_i-statistic is statistically insignificant at the 5% level in 12 bond markets.

	VARX(3)		IV				ES				GMM			
Country	υ_{it}^{no}	$\mathcal{P}_i$	$\hat{\alpha}_{i1}$	$\hat{\sigma}_i$	$CI_{i,1}$	$CI_{i,2}$	$\hat{\alpha}_i$	$\hat{\sigma}_i$	$CI_{i,1}$	$CI_{i,2}$	$\hat{\alpha}_i$	$\hat{\sigma}_i$	$CI_{i,1}$	$CI_{i,2}$
Australia	4	61.975	-1.398	1.179	-3.657	0.194	-0.830	0.780	-1.938	0.177	**-1.873**	0.866	-10.944	-0.418
Austria	5	38.457	-1.141	1.115	-3.695	0.077	0.052	0.445	-0.565	0.724	-0.247	0.474	-3.488	0.161
Belgium	5	38.356	**2.000**	1.283	0.335	6.228	0.769	0.829	-0.334	2.121	**2.466**	1.177	0.293	26.160
Canada	7	23.982	-1.500	2.806	-4.184	4.874	-0.748	1.893	-2.570	2.827	-1.776	1.737	-4.993	4.475
Denmark	6	20.081	**-2.792**	2.206	-9.216	-0.574	-1.579	1.567	-4.478	0.012	-0.527	0.599	-6.231	0.511
France	5	32.600	0.141	1.323	-1.674	2.723	0.412	0.841	-0.617	1.824	0.140	0.945	-1.922	2.551
Germany	4	50.743	**-5.035**	3.642	-15.849	-1.477	-1.942	2.107	-5.715	0.021	**-4.783**	3.185	-14.830	-0.855
Italy	7	22.119	-1.090	2.423	-4.958	2.681	-1.005	1.502	-3.295	0.929	-0.742	1.422	-4.945	4.291
Japan	7	30.001	-3.641	3.515	-8.615	2.405	0.875	2.819	-1.911	6.148	-2.369	2.432	-7.259	3.173
N. Zealand	10	33.459	2.252	1.497	-0.232	4.967	1.141	0.795	-0.223	1.890	3.205	1.802	-1.894	4.715
Norway	8	29.861	-0.844	1.933	-4.234	1.888	-0.233	1.127	-1.598	1.392	-0.777	1.393	-4.118	1.948
Spain	7	25.836	0.059	1.814	-2.580	3.417	0.197	1.149	-1.175	1.829	-0.110	1.238	-2.608	2.647
Sweden	5	27.595	-1.663	2.472	-4.003	2.791	-0.189	1.823	-2.081	3.038	-1.451	1.598	-5.502	2.629
U. Kingdom	3	49.139	-1.592	2.017	-4.445	2.271	-0.206	1.443	-1.743	2.651	-1.649	1.255	-4.167	2.076

Table 3.5: Country estimates for real estate markets. υ_{it}^{no} is the number of coefficients in the time-varying mean υ_{it} of the VARX(3) which have robust t-ratios greater than 1.64 in absolute value, $\mathcal{P}_i$ is the Portmanteau test of the null hypothesis of no 6th order serial correlation in the residuals of the VARX(3) which is $\chi^2(48)$ distributed, $\hat{\alpha}_{\bullet,i}$ $(\hat{\sigma}_{\bullet,i})$ is the point estimate (heteroskedasticity robust standard error) for country i with $\bullet = IV, ES, GMM$, $CI_{i,j}$ are the 90% bootstrap confidence bands. Note that the estimate $\hat{\alpha}_{IV,i1}$ is obtained with the set of instruments w_1. Entries in **bold** are statistically significant at the 10% level according to the asymptotic and/or bootstrap distribution.

	$\Omega_{D,i}$			IV			ES		GMM			
Country	$\mathcal{G}_i$	$\mathcal{H}_i$	p-val	$\hat{h}_i$	R^2_{ih}	R^2_{ib}	R^2_{ih}	R^2_{ib}	$\hat{h}_i$	R^2_{ih}	J_i-stat	p-val
Australia	**64.779**	0.412	0.522	2	0.078	0.023	0.078	0.036	2	0.078	**4.461**	0.035
Austria	**323.682**	1.362	0.244	2	0.011	0.004	0.010	0.021	2	0.011	1.943	0.163
Belgium	**67.136**	1.582	0.210	3	0.021	0.036	0.021	0.034	3	0.021	**9.185**	0.002
Canada	**60.774**	0.132	0.717	2	0.032	0.026	0.031	0.053	2	0.032	0.212	0.645
Denmark	**100.857**	0.611	0.435	2	0.019	0.030	0.019	0.041	2	0.019	**3.514**	0.061
France	**63.674**	0.070	0.791	2	0.032	0.012	0.032	0.025	2	0.032	0.019	0.892
Germany	**66.315**	1.084	0.299	2	0.011	0.026	0.011	0.028	2	0.011	0.299	0.584
Italy	**51.425**	0.002	0.964	3	0.018	0.011	0.018	0.025	3	0.018	2.309	0.129
Japan	**65.149**	**4.622**	0.033	2	0.053	0.024	0.053	0.046	2	0.053	1.829	0.176
N. Zealand	**98.111**	0.766	0.382	3	0.066	0.025	0.066	0.055	3	0.066	**4.102**	0.043
Norway	**48.690**	0.151	0.698	3	0.033	0.007	0.033	0.010	3	0.033	0.431	0.512
Spain	**61.786**	0.010	0.922	2	0.017	0.010	0.017	0.024	2	0.017	1.732	0.188
Sweden	**62.815**	0.780	0.378	2	0.027	0.015	0.027	0.034	2	0.028	0.154	0.695
U. Kingdom	**61.797**	0.968	0.326	2	0.035	0.029	0.035	0.049	2	0.035	0.008	0.930

Table 3.6: Selected statistics for real estate markets. $\mathcal{G}_i$ are the statistics from the likelihood ratio test of the null $H_0 : \Omega_{D,i} = 0$ which is $\chi^2(3)$ distributed, $\mathcal{H}_i$ is the Hausmann test of the null $H_0 : \alpha_{IV,i} = \alpha_{ES,i}$ and corresponding p-value, $\hat{h}_{\bullet,i}$ is the estimated horizon of non-policy dates via the $\bullet = IV, GMM$ methodology, $R^2_{\bullet,ih}$ is the degree of explanation from the 'auxiliary regression' in (3.14) with $\hat{\alpha}_{\bullet,i}$ for $\bullet = IV, GMM, ES$, $R^2_{\bullet,ib}$ are the bootstrap upper 10% quantile from the $R^2_{\bullet,i}$ of the regression via $\bullet = IV, ES$ in (3.9) and (3.11), J_i-stat is the statistic of the J-test in GMM estimation and corresponding p-value. Entries in **bold** are statistically significant at the 10% level.

As for equity and bond markets, the panel of 14 international real estate markets shows significant conditional effects in the time-varying mean υ_{it} and no evidence of serial correlation (Table 3.5). Out of the 14 international real estate markets considered, we obtain that 10 (11) of them show a negative effect to US monetary policy shocks via IV (GMM). The latter count is somewhat lower in ES estimation where we find 8 negative coefficients. There is a statistically significant negative impact in Australia, Denmark and Germany and a statistically significant positive impact in Belgium at the 10% level when considering the bootstrap confidence bands for the estimates obtained from the heteroskedasticity based estimators (IV and GMM). ES estimates are insignificant throughout. The insignificant response of real estate markets to monetary policy found here confirms findings of previous studies (cf. Furlanetto (2008)). Results on selected diagnostics for real estate markets are qualitatively similar to those of equity and bond markets (Table 3.6).

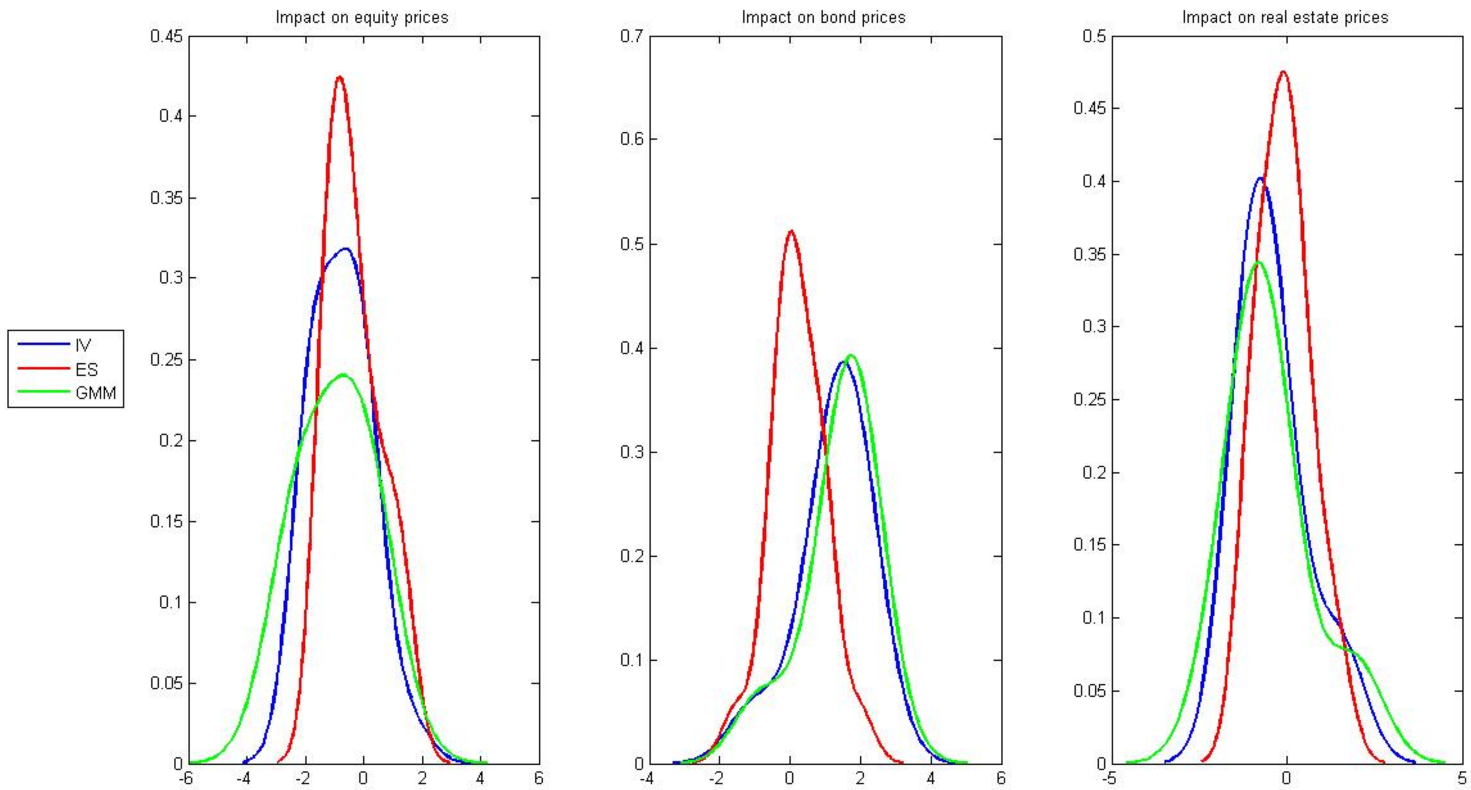

Figure 3.1: Densities of the impact of US monetary policy on international asset prices. Note: densities are estimated from the standardized estimates $\hat{\alpha}_{\bullet,i}\hat{\sigma}_{\bullet,i}^{-1}$ for $i = 1, ..., N$ and $\bullet = IV, ES, GMM$ via a Kernel function.

3.4.2 Aggregate results

As can be noted from the preceding discussion, point estimates are generally similar between IV and GMM methods but in many instances dissimilar to those obtained from the ES method. This may be observed in the estimated densities of the standardized estimates $\hat{\alpha}_{\bullet,i}\hat{\sigma}^{-1}_{\bullet,i}$ for each of the international asset markets considered under the three different methodologies used (Figure 3.1). In general, the heteroskedasticity based estimators (IV and GMM) work 'best' in relation to the ES estimator in bond markets and similar in equity and real estate markets according to the R^2_{ih} measures and the Hausmann statistics $\mathcal{H}_i$. Overall, IV and GMM estimation produce estimates that are more frequently found statistically significant than ES estimation according to the bootstrap distribution and/or asymptotic (robust) standard errors. Nevertheless, there is one particularly interesting result for heteroskedasticity or event study estimators: Mexico and Canada have a statistically significant negative response to US monetary policy.

	Equity					Bonds					Real Estate				
MG	IV	ES	GMM	MGD1	MGD2	IV	ES	GMM	MGD1	MGD2	IV	ES	GMM	MGD1	MGD2
$\bar{\alpha}_\bullet$	**-0.780**	**-0.330**	**-0.979**	**-0.450**	**-1.252**	**1.246**	0.218	**1.408**	**1.028**	**1.896**	-0.398	-0.076	-0.459	**-0.323**	-1.065
$\hat{\sigma}_\bullet$	0.178	0.165	0.215	0.126	0.344	0.224	0.177	0.233	0.131	0.338	0.249	0.195	0.312	0.136	1.319
CI_1	-1.419	-1.120	-1.607	-0.759	-2.147	0.479	-0.655	0.285	0.636	0.775	-0.927	-0.535	-1.296	-0.583	-5.289
CI_2	-0.101	0.356	-0.151	-0.108	-0.085	2.136	1.208	2.312	1.576	3.269	0.063	0.433	0.129	-0.120	0.307

Table 3.7: Mean group estimates per asset market. $\bar{\alpha}_\bullet$ ($\hat{\sigma}_\bullet$) is the *standardized* Mean Group estimator (asymptotic standard error) via $\bullet = IV, ES, GMM$ methodology and the 'Mean Group Difference' estimators $\bullet = MGD1, MGD2$, CI_j are the 90% *standardized* Mean Group bootstrap confidence bands. Note that $MGD1$ refers to the mean group difference between the IV estimates and the ES estimates and $MGD2$ refers to the mean group difference between the IV estimates with the instruments w_1 and w_2. Entries in **bold** are statistically significant at the 10% level according to the asymptotic and/or bootstrap distribution.

Another clear feature of the preceding findings is that the sensitiveness of the response of asset prices to US monetary policy is country specific in terms of magnitude and precision. The estimates for each country also vary with the considered asset. This confirms our a priori hypothesis of heterogeneity in the postulated panel relationship. The results of the standardized MG estimates in Table 3.7 put forward a negative average impact in equity markets, a positive average impact in bond markets and a negative average impact in real estate markets with all three estimators considered. The average (standardized) impact is statistically significant at the 10% level in equity and bond markets with the IV and GMM estimators according to the bootstrap confidence bands and asymptotic standard errors. In the case of the ES estimator, the average impact is statistically significant at the 10% level in equity markets according to the asymptotic standard error. The negative and statistically significant response of international equity markets to changes in the US STIR provides evidence of the 'flight to quality' effect as proposed in recent theoretical and empirical models of the international propagation of shocks (cf. Pavlova & Rigobon (2009), Pachenko & Wu (2009)).

Interestingly, there is evidence of a statistically significant mean group difference at the 10% level between IV and ES estimates in all three asset markets considered ($MGD1$). Moreover, we diagnose a statistically significant mean group difference at the 10% level between the IV estimates $\hat{\alpha}_{IV,ij}$ obtained from the two different sets of instruments w_1 and w_2 in equity and bond markets ($MGD2$) according to the bootstrap confidence bands and asymptotic standard errors. The latter result hints at instability in the empirical relationship between US monetary policy and international equity and bond prices for the sample 01/1994 to 12/2007.

Thus, it seems preferable from the latter findings (high heterogeneity and possible parameter instability) to study the response of international asset prices to US monetary policy by taking different and smaller windows of policy dates to estimate the impact. In order to minimize parameter instability, one could in principle analyze different sub-samples based on information about events in the economies that might have caused structural breaks. However, this would be cumbersome to implement (and possibly misleading) given the high degree of heterogeneity found amongst cross-sectional members and across asset types. Alternatively, as previously described, we aggregated the impact with a *standardized* rolling window MG procedure which allows us to have an aggregate view of the unstable vs. stable periods and the overall direction of the impact over time.

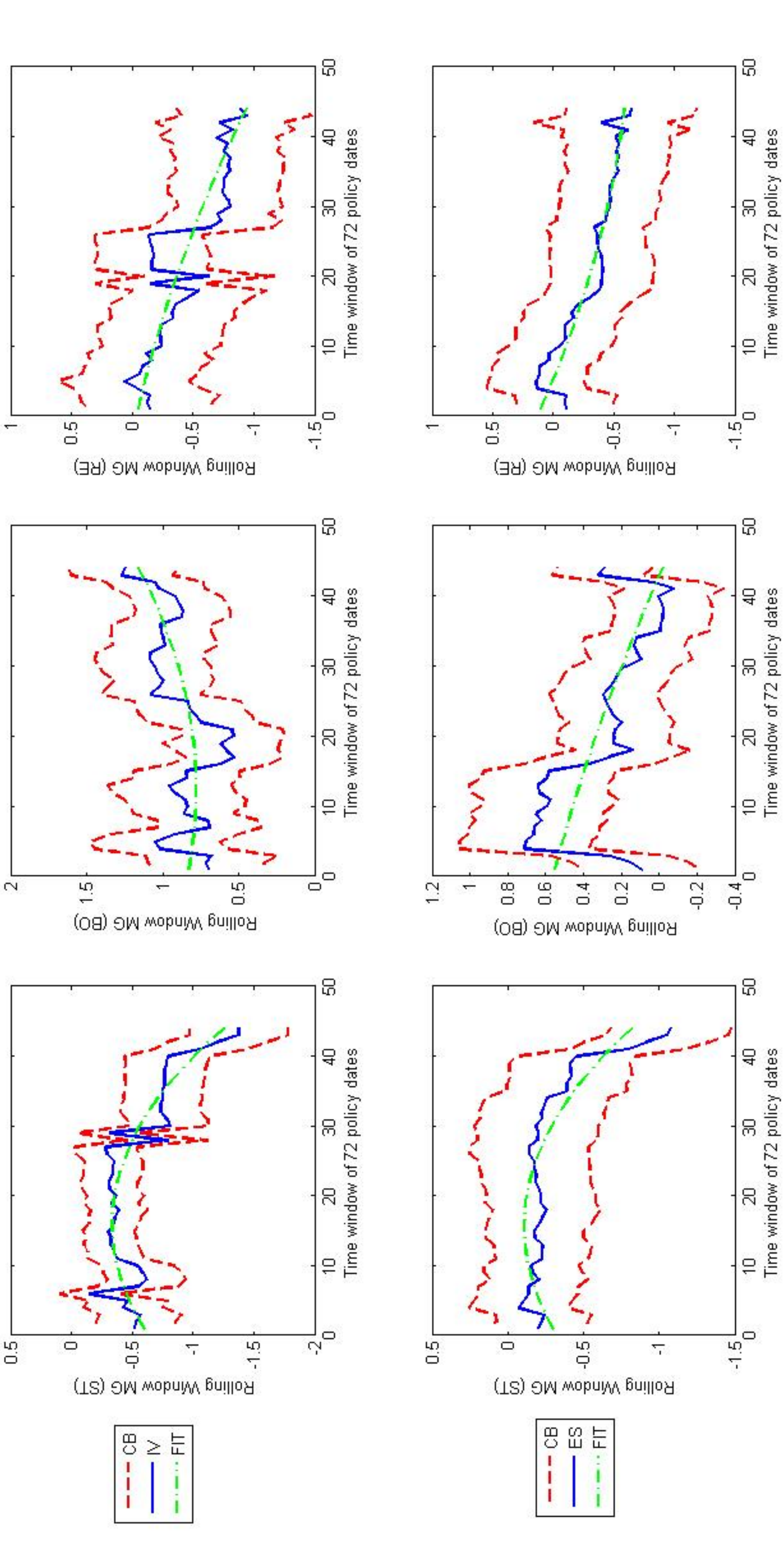

Figure 3.2: Rolling window Mean Group impact of US monetary policy on international asset prices. Note: CB (dashed red lines) denote the asymptotic 90% confidence bands, FIT (dashed and dotted green line) is a polynomial fit of the IV and ES rolling window Mean Group impacts.

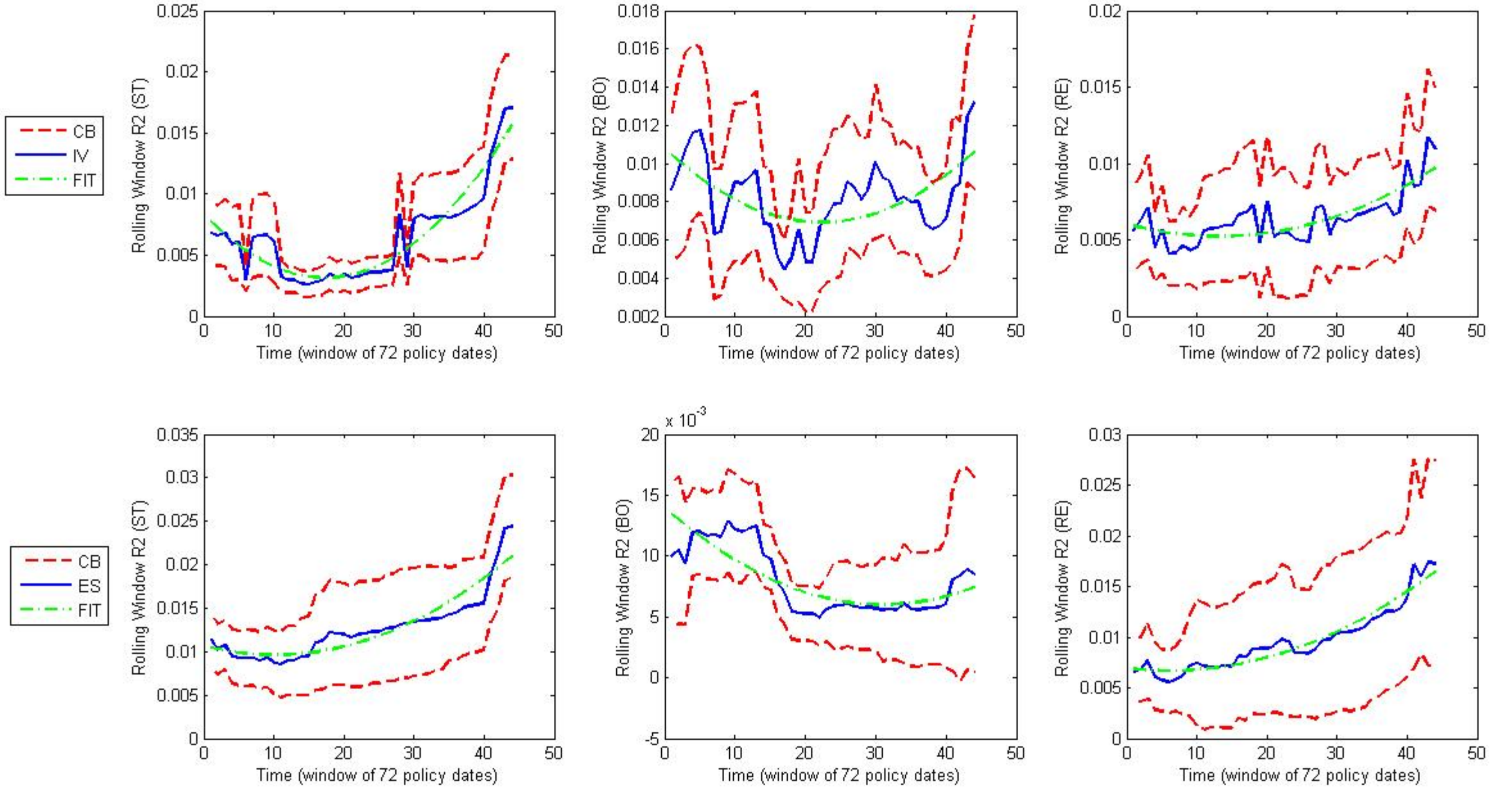

Figure 3.3: Rolling window Mean Group R^2 of the impact of US monetary policy on international asset prices. Note: CB (dashed red lines) denote the asymptotic 90% confidence bands, FIT (dashed and dotted green line) is a polynomial fit of the IV and ES rolling window Mean Group R^2s.

Figure 3.2 shows graphically the results of the rolling window MG impact of US monetary policy on international asset prices. We present only the plots of the IV and ES estimators for space considerations, although similar pictures to those of IV can be obtained when employing the GMM estimator. Interestingly, the response of international asset prices to US monetary policy is significantly time-varying at the 10% level for most of the windows in IV estimation in the three asset markets. Morever, we find that the impact of US monetary policy on international equity and real estate markets is negative for most of the time windows and it shows a downward trend in IV and ES estimation. With respect to bond markets the relationship is positive for all time windows in IV estimation and shows a positive trend. For ES estimation there is also a positive relationship with bond markets for most time windows but the trend is slightly negative.

The result on the stability test $MGD2$ and the time variability of the (standardized) MG impact suggests that the relationship between US monetary policy and international asset prices is sample specific. Thus, not considering recursive estimation will be misleading in the sense that one would not obtain a 'true' distribution of US monetary policy impacts in international asset markets when focusing only on particular samples. In addition, Figure 3.3 also shows the rolling window R^2s computed from the $R^2_{\bullet,i}$ measures for $\bullet = IV, ES$. The picture clearly shows that the 'goodness of fit' in the response of international equity and real estate markets to US monetary policy has been increasing over time while it shows no clear trend for bond markets. From an econometric perspective, our findings on the time variability of the aggregate impacts and goodness of fit suggest that one should treat distributions over the entire sample as in Figure 3.1 with caution, as they might change over time. From an economic perspective, our results also confirm recent theoretical propositions and empirical findings of the time-varying dependencies of asset markets (cf. DeRoon & DeJong (2005), Pavlova & Rigobon (2009), Pachenko & Wu (2009)).

3.5 Concluding remarks

In this Chapter we estimated the response of international asset prices to US monetary policy. The problems of endogeneity and omitted variables were addressed by using a careful identification strategy proposed by Rigobon & Sack (2004). We extend the latter study to investigate the empirical relationship between US monetary policy and international asset prices via a heterogeneous panel approach. We consider new issues such as controlling for conditional dynamic effects in the postulated relationship, instrument selection in the heterogeneous panel context, recursive estimation and bootstrap inference.

In general, IV and GMM estimation are more appropriate for identification of the impact of US monetary policy on international asset prices than the popular ES estimator at the aggregate level. The results also indicate that, for the entire sample period covered (01/1994 to 12/2007), the Federal Reserve's policy announcements had heterogeneous impacts in international equity, bond and real estate markets. In international equity markets there is mostly a negative response to US monetary policy. The largest negative and statistically significant coefficients in the American continent are those of Mexico and Canada which suggests a proximity effect to shocks from the US. At the aggregate level, the MG impact of equity markets is negative and statistically significant. In the case of bond markets the relationship appears to be positive and statistically significant at the aggregate level and for most country estimates. Real estate markets obtain at the aggregate level a negative impact to US monetary policy shocks although not statistically significant.

We diagnose evidence of instability in the postulated empirical relationship between US monetary policy and international asset prices. A careful inspection of the MG estimates over time by means of a rolling window procedure shows that the (average) impact is negative for equity and real estate markets and positive for bond markets. While the time-varying MG impact appears to be increasing in absolute value over time for the latter two markets, it shows no clear trend for the former. Interestingly, we find that the 'goodness of fit' in the response of international equity and real estate prices to US monetary policy seems to be increasing over time. These results also confirm recent findings of time-varying dependencies in international asset markets.

The results of this study are important because they present a general pattern for the response of international asset prices to US monetary policy. The empirical approach allows to quantify and thus compare the effects across countries, asset markets, estimators and sample periods. In this sense, this Chapter contributes to the understanding of the empirical relationship between US monetary policy and international asset prices. Several opportunities are also raised for further research. For instance, a similar exercise could be done to study the response of international asset prices to ECB policy shocks and comparing the impact with that of the US Federal Reserve. Moreover, one could also analyze the impact of US monetary policy on exchange rates. We leave these issues for future exploration.

Chapter 4

Forecasting Volatility in International Asset Markets: Fractality, Regime-Switching, Long Memory and Student-t Innovations

4.1 Introduction

In the last three Chapters we have analyzed various issues with respect to international asset returns. In this Chapter we turn to the analysis of volatility in international equity, bond and real estate markets. Volatility forecasting is of crucial importance for financial practitioners and academics. Accurate forecasts of volatility allow analysts to build appropriate models for risk management such as portfolio allocation, Value-at-Risk, option and futures pricing, etc. For these reasons, scholars have devoted a great deal of attention to developing parametric as well as non-parametric models to forecast future volatility (cf. Andersen *et al.* (2005a) for a recent review on volatility modeling and Poon & Granger (2003) for a review on volatility forecasting).

In this Chapter, we are interested in the performance of a new type of volatility model, the so-called Markov-Switching Multifractal Model (MSM) vis-*à*-vis its more time honored competitors from the (Generalized) Autoregressive Conditional Heteroskedasticity (GARCH) family. The former model is a causal analog of the earlier non-causal Multifractal Model of Asset Returns (MMAR) due originally to Calvet *et al.* (1997).

In contrast to mainstream volatility models, the MSM model can accommodate, by its very construction, the feature of *multifractality* via its hierarchical, multiplicative structure with heterogeneous components. *Multifractality* refers to the variations in the *scaling behavior* of various moments or to different degrees of long-term dependence of various moments. Long-term dependence in various moments of (mainly financial) data have been reported in several studies by economists and physicists so that this feature now counts as a well established stylized fact (cf. Ding *et al.* (1993), Lux (1996), Mills (1997), Lobato & Savin (1998), Schmitt *et al.* (1999), Vassilicos *et al.* (2004)). Empirical research in finance also provides us with more direct evidence in favor of the hierarchical structure of multifractal cascade models (cf. Muller (1997)).

It seems plausible that the higher degree of flexibility of MSM models in capturing different degrees of temporal dependence of various moments could also facilitate volatility forecasting. Indeed, recent studies have shown that the MSM models can forecast future volatility more accurately than traditional long memory and regime-switching models of the (G)ARCH family such as Fractionally Integrated GARCH (FIGARCH) and Markov-Switching GARCH (MSGARCH) (cf. Calvet & Fisher (2004), Lux & Kaizoji (2007), Lux (2008a)).

It is also worthwhile to emphasize the intermediate nature of MSM models between 'true' long-memory and regime-switching. It has been pointed out that it is hard to distinguish empirically between both types of structures and that even single regime-switching models could easily give rise to apparent long memory (cf. Granger & Terasvirta (1999)). MSM models generate what has been called 'long-memory over a finite interval' and in certain limits converges to a process with 'true' long-term dependence.[1] The MSM model combines features

[1] In contrast to the combinatorial MMAR of Calvet *et al.* (1997), the MSM model has no asymptotic power-law behavior of its autocorrelation function. However, depending on the number of volatility components, a pre-asymptotic hyperbolic decay of the autocorrelation might be so pronounced as to be practically indistinguishable from 'true' long memory (cf. Liu

of both types of generating mechanisms in a very parsimonious way. The flexible regime-switching nature of the MSM model might also allow to integrate seemingly unusual time periods such as the Japanese bubble of the 1980s in a very convenient manner without resorting to dummies or specifically designed regimes (cf. Lux & Kaizoji (2007)). Nevertheless, the finance literature has only scarcely exploited MSM models so far. Most efforts with respect to volatility modeling have been directed towards refinements of GARCH-type models, stochastic volatility models and more recently realized volatility models (cf. Andersen & Bollerslev (1998), Andersen *et al.* (2003), Andersen *et al.* (2005b), Abraham *et al.* (2007)).

Up until now, the scarce literature on MSM models of volatility has only considered the Gaussian distribution for return innovations. However, recent studies have shown that out-of-sample forecasts of volatility models with Student-t innovations might improve upon those resulting from volatility models with Gaussian innovations (cf. Rossi & Gallo (2006), Chuang *et al.* (2007), Wu & Shieh (2007)). In addition, there could also be an interaction between the modeling of *fat tails* and dependency in volatility: if more extreme realisations are covered by a fat-tailed distribution, the estimates of the parameters measuring serial dependence of higher moments might change which also alters the forecasting capabilities of an estimated model.

In this Chapter we examine the performance of various volatility models from the MSM and GARCH families along with two competing distributional assumptions of the error component, i.e. Normal vs Student-t. Our precise contribution is twofold. First, we introduce a new model to the family of MSM models, the Markov-Switching Multifractal model of asset returns with Student-t innovations (MSM-t). This model is an extension of the MSM model with Normal innovations which can be estimated via Maximum Likelihood (ML) or Generalized Method of Moments (GMM) (cf. Calvet & Fisher (2004), Lux (2008a)). Forecasting can be performed via Bayesian updating (ML) or best linear forecasts together with the generalized Levinson-Durbin algorithm (GMM). We investigate the in-sample and out-of-sample performance of the MSM-t model via Monte Carlo simulations.

et al. (2007)).

Second, we perform a forecasting analysis of MSM vs (FI)GARCH models with Normal and Student-t innovations. By contrasting both sets of models we are able to empirically evaluate the performance of models which incorporate different characterizations of the latent volatility process: the MSM models which take account of multifractality, Markov-switching and (apparent) long memory against more traditional models of the GARCH legacy (GARCH, GARCH-t, FIGARCH and FIGARCH-t) which take account of short/long memory and autoregressive components. Furthermore, the wide variety of models considered here provides an interesting platform to study empirical out-of-sample complementarities between models via forecast combinations.

The cross-sections chosen for our empirical analysis consist of all-share equity indices, bond indices and real estate security indices at the country level. Similar to previous Chapters, we believe that the use of panel data is promising in two main aspects when evaluating volatility models. First, in order not to generalize its usefulness, an interesting volatility model should perform adequately for a cross-section of markets and different asset classes. Second, testing volatility models for a cross-section of markets comes along with an augmentation of sample information and thus provides more power to statistical tests.

To preview some of our results, we confirm that ML and GMM estimation are both suitable for MSM-t models. We also find that using GMM plus linear forecasts leads to minor losses in efficiency compared to optimal Bayesian forecasts based on ML estimates. This justifies using the former approach in our empirical exercise which reduces computational costs significantly. Moreover, empirical *panel* forecasts of MSM-t models show an improvement over the alternative MSM models with Normal innovations in terms of mean absolute forecast errors while they seem to deteriorate for (FI)GARCH models with Student-t innovations in relation to their Gaussian counterparts. In terms of mean absolute errors, the MSM-t dominates all other models at long forecasting horizons for all asset classes. Lastly, forecast combinations obtained from the different MSM and (FI)GARCH models considered provide a clear improvement upon forecasts from single models.

The Chapter is organized as follows. The next section introduces the general framework of volatility modeling. Section three and four provide a short review of the MSM and (FI)GARCH volatility models. Section five presents the Monte

Carlo experiments performed with respect to the MSM-t models. Section six addresses the results of our comprehensive *panel* empirical analysis of the different volatility models under inspection. The last section concludes with some final remarks.

4.2 Theoretical framework of volatility

In a seminal study of conditional heteroskedasticity in economic variables, Engle (1982) introduced the autoregressive conditional heteroskedasticity (ARCH) model. Since then, a wide range of volatility models have been proposed to forecast future volatility. The following specification of financial returns is usually considered in asset pricing models of volatility,

$$\Delta p_t = \upsilon_t + u_t, \tag{4.1}$$

where $\Delta p_t = \ln P_t - \ln P_{t-1}$, $\ln P_t$ is the log asset price and $\upsilon_t = E_{t-1}\Delta p_t$ is the conditional mean of the return series and u_t is a disturbance term. In this Chapter we consider a simple first order autoregressive model to describe the conditional mean, i.e.

$$\upsilon_t = \mu + \rho\Delta p_{t-1}, \tag{4.2}$$

and the disturbance term is modelled as,

$$u_t = \sigma_t \varepsilon_t. \tag{4.3}$$

Different assumptions can be used for the distribution of ε_t. For example, we may assume a Normal distribution, Student-t distribution, Logistic distribution, mixed diffusion, etc (cf. Chuang *et al.* (2007)). For the purpose of this Chapter we consider two competing types of distributions for the innovations ε_t, namely, a Normal distribution and a Student-t distribution. Defining $x_t = \Delta p_t - \upsilon_t$, the 'centered' returns are modelled as,

$$x_t = \sigma_t \varepsilon_t. \tag{4.4}$$

From the above general framework of volatility different parametric and non-parametric representations can be assumed for the latent volatility process σ_t.

In what follows, we describe the new family of MSM volatility models as well as the more time-honored GARCH-type volatility models for the characterization of σ_t. Since the former models are a very recent addition to the family of volatility models, we devote most of the next sections to describing them and keep the explanation on the alternative volatility models short to save on space.

4.3 Markov-Switching Multifractal models

4.3.1 Volatility specifications

The basic principle for construction of multifractal models in statistical physics is a cascading process of iterative splitting of initially uniform probability mass into more and more heterogeneous subsets. Starting with a uniform distribution over a certain interval, one splits this interval into two subintervals that receive fractions, say π_1 and $1 - \pi_1$, of the overall mass. In the next step, the same procedure is repeated for the newly created subsets so that one ends up with four intervals with probability mass π_1^2, $\pi_1(1-\pi_1)$ and $(1-\pi_1)^2$, respectively.[2] In principle, this process can be repeated at *infinitum*. One thus obtains a hierarchical structure of components, where smaller ones (smaller whirls emanating from larger ones in its application to turbulent flows in physics) emanate from the higher levels of the hierarchy via this probabilistic split of energy. By its very construction, a combinatorial multifractal along the above lines exhibits different degrees of scaling or long-term dependence for different powers of the resulting measure.

Calvet *et al.* (1997) proceeded one step further by proposing the MMAR: a compound stochastic process as a data generating mechanism for financial prices in which a multifractal cascade plays the role of a time transformation or time-varying scale function of the variance of the incremental process.[3] In their model, an incremental Brownian motion is subordinate to the cumulative distribution function of a multifractal measure. However, this multifractal component is of

[2] The two intervals in the center both have measure $\pi_1(1-\pi_1)$

[3] Cf. also Mandelbrot (1999) and Mandelbrot & Hudson (2004) for a non-technical introduction to the subject and the relationship between the MMAR and multifractal models in turbulence.

combinatorial rather than causal nature (it is actually identical to the model proposed for turbulent flows by Mandelbrot (1974)). Unfortunately, the MMAR suffers from non-stationarity since the combinatorial construction of the multifractal measure is restricted to a (predefined) bounded interval (in space or time). This severe limitation has been overcome by the development of the MSM model.

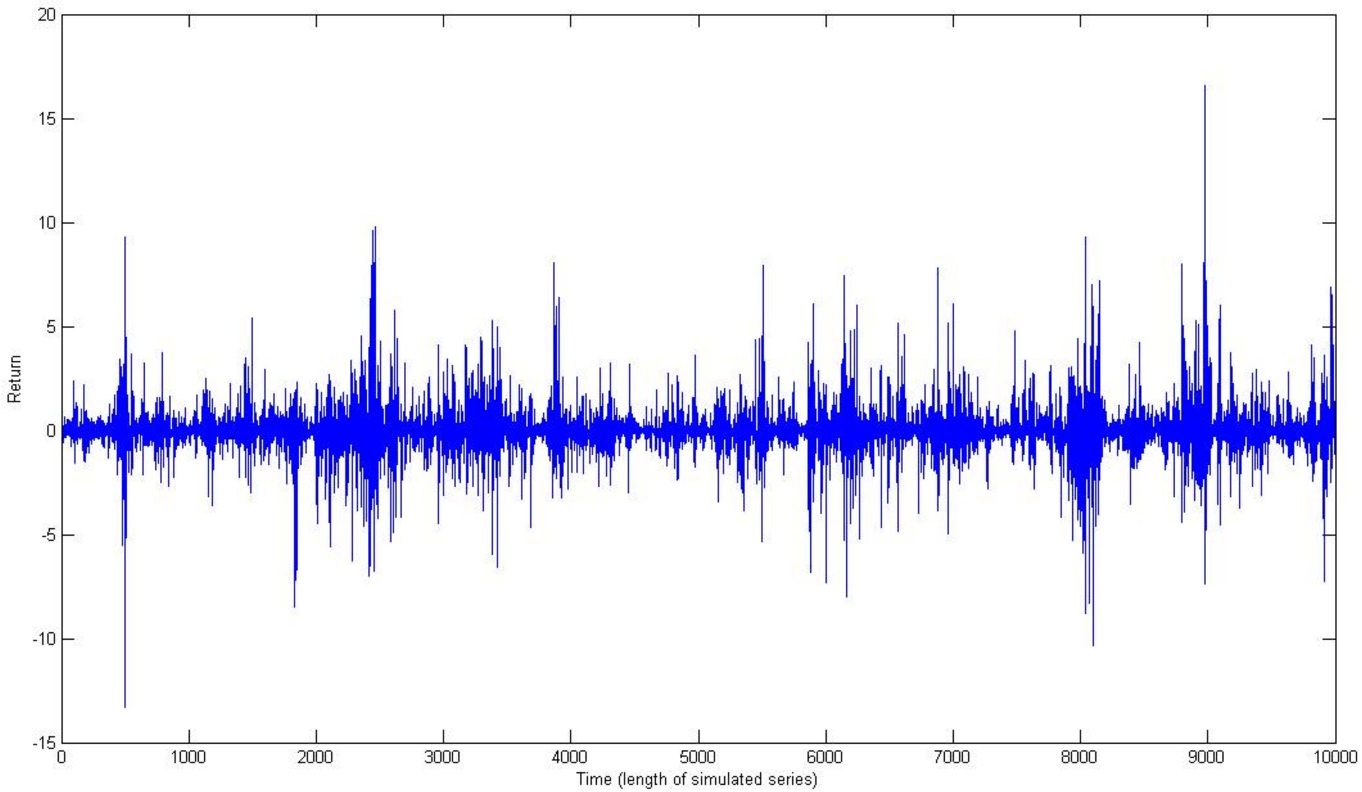

Figure 4.1: Simulation of a Binomial Markov-Switching Multifractal Model (Normal innovations) with $k = 10$, $m_0 = 1.5$, $\sigma = 1.3$, $\gamma_k = 0.5$, $b = 2$. Note: the model was calibrated according to the empirical estimates.

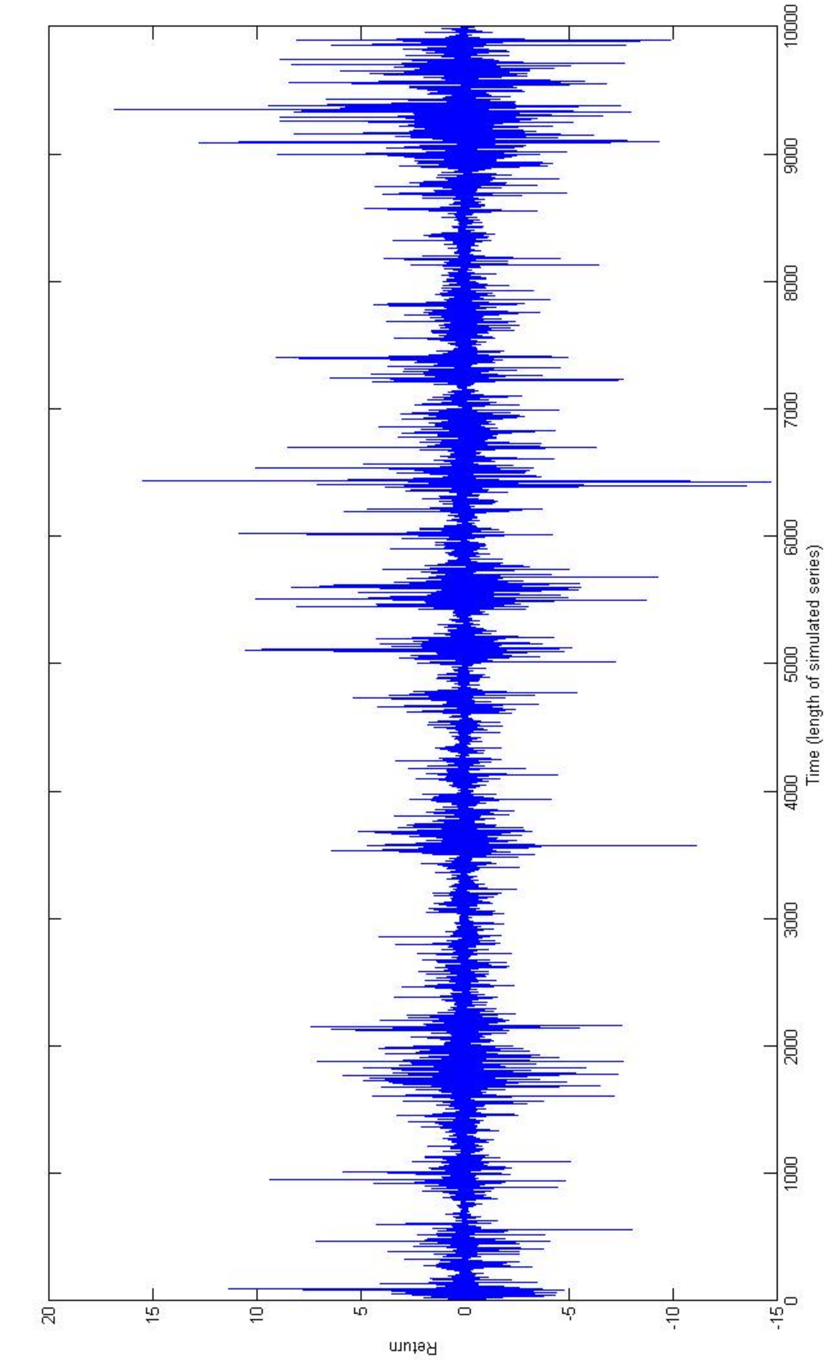

Figure 4.2: Simulation of a Binomial Markov-Switching Multifractal Model (Student-t innovations) with $k = 10$, $m_0 = 1.5$, $\sigma = 1.3$, $\nu = 6$, $\gamma_k = 0.5$, $b = 2$. Note: the model was calibrated according to the empirical estimates.

Instantaneous volatility σ_t in the MSM framework is determined by the product of k volatility components or multipliers $M_t^{(1)}, M_t^{(2)}, \ldots, M_t^{(k)}$ and a scale factor σ:

$$\sigma_t^2 = \sigma^2 \prod_{i=1}^{k} M_t^{(i)}. \tag{4.5}$$

Following the basic hierarchical principle of the multifractal approach, each volatility component will be renewed at time t with a probability γ_i depending on its rank within the hierarchy of multipliers and remains unchanged with probability $1 - \gamma_i$. Calvet & Fisher (2001) propose to formalize transition probabilities according to:

$$\gamma_i = 1 - (1 - \gamma_k)^{(b^{i-k})}, \tag{4.6}$$

which guarantees convergence of the discrete-time version of the MSM to a Poissonian continuous-time limit. In principle, γ_k and b are parameters to be estimated. Note that (4.6) or its restricted versions imply that different multipliers $M_t^{(i)}$ of the product (4.5) have different mean life times. However, previous applications have often used pre-specified parameters γ_k and b in equation (4.6) in order to restrict the number of parameters (cf. Lux (2008a)). The MSM model is fully specified once we have determined the number k of volatility components and their distribution.

In the small body of available literature, the multipliers $M_t^{(i)}$ have been assumed to follow either a Binomial or a Lognormal distribution. Since one could normalize the distribution so that $E[M_t^{(i)}] = 1$, only one parameter has to be estimated for the distribution of volatility components. In this Chapter we explore the Binomial and Lognormal specifications for the distribution of multipliers. Following Calvet & Fisher (2004), the Binomial MSM (BMSM) is characterized by Binomial random draws taking the values m_0 and $2 - m_0$ ($1 \leq m_0 < 2$) with equal probability (thus, guaranteeing an expectation of unity for all $M_t^{(i)}$). The model, then, is a Markov switching process with 2^k states. In the Lognormal MSM (LMSM) model, multipliers are determined by random draws from a Lognormal distribution with parameters λ and s, i.e.

$$M_t^{(i)} \sim LN(-\lambda, s^2). \tag{4.7}$$

Normalisation via $E[M_t^{(i)}] = 1$ leads to

$$\exp(-\lambda + 0.5s^2) = 1, \tag{4.8}$$

from which a restriction on the shape parameter can be inferred: $s = \sqrt{2\lambda}$. Hence, the distribution of volatility components is parameterized by a one-parameter family of Lognormals with the normalization restricting the choice of the shape parameter. It is noteworthy that the dynamic structure imposed by (4.5) and (4.6) provides for a rich set of different regimes with an extremely parsimonious parameterization. For increasing k there is, indeed, no limit to the number of regimes considered without any increase in the number of parameters to be estimated. Figures 4.1 and 4.2 show simulated x_t series with the BMSM as underlying volatility model with Normal and Student-t innovations, respectively.

4.3.2 Estimation and forecasting

The pioneering approach for the estimation of the early combinatorial MMAR was the 'scaling' estimator adopted from statistical physics (Calvet *et al.* (1997)). While Calvet & Fisher (2002) develop refined estimators and diagnostic tests for the earlier MMAR, its non-causal nature makes it intrinsically difficult to apply to financial data. As a consequence, although the potential of this new approach for generating multi-scaling in returns is not shared by traditional models in finance, rigorous comparison of its performance to, for example, GARCH processes as a candidate alternative, is hampered by the lack of statistical theory for the parameter estimates and statistical tools for comparison of alternative models. A study by Lux (2004) demonstrates the unreliability of the so-called 'scaling' estimator adopted from the physics literature. At least for the subordinate multifractal processes of Calvet *et al.* (1997) and Calvet & Fisher (2002), this popular estimator is shown to give extremely volatile parameter estimates and tests based on this estimator are frequently found unable to reject the null hypothesis of multifractal behavior for unifractal processes.

In a seminal study by Calvet & Fisher (2004), an ML estimation approach was proposed for the BMSM model which overcomes the problems of the 'scaling' estimator of the MMAR. The log likelihood function in its most general form may be expressed as,

$$L(x_1, ..., x_T; \varphi) = \sum_{t=1}^{T} \ln g(x_t | x_1, ..., x_{t-1}), \tag{4.9}$$

where $g(x_t | x_1, ..., x_{t-1})$ is the likelihood function of the MSM model with various distributional assumptions. The parameter vector of the BMSM with Gaussian innovations is given by $\varphi = (m_0, \sigma)'$. On the other hand, the parameter vector of the BMSM with Student-t innovations is given by $\varphi = (m_0, \sigma, \nu)'$ where ν $(2 < \nu < \infty)$ is the distributional parameter accounting for the degrees of freedom in the density function of the Student-t distribution. When ν approaches infinity, we obtain a Normal distribution. Thus, the lower ν, the 'fatter' the tail.

The greatest advantage of the ML procedure is that, as a by-product, it allows one to obtain optimal forecasts via Bayesian updating of the conditional probabilities $\Omega_t = \mathcal{P}(M_t = m^i | x_1, ..., x_t)$ for the unobserved volatility states $m^i, i = 1, ..., 2^k$. Although the ML algorithm was a huge step forward for the analysis of MSM models, it is restrictive in the sense that it works only for discrete distributions of the multipliers and is not applicable for, e.g. the alternative proposal of a Lognormal distribution. Due to the potentially large state space (we have to take into account transitions between 2^k distinct states), ML estimation also encounters bounds of computational feasibility for specifications with more than about $k = 10$ volatility components in the Binomial case.

To overcome the lack of practicability of ML estimation, Lux (2008a) introduced a GMM estimator that is universally applicable to all possible specifications of MSM processes. In particular, it can be used in all those cases where ML is not applicable or computationally unfeasible. In the GMM framework for MSM models, the vector of BMSM parameters φ is obtained by minimizing the distance of empirical moments from their theoretical counterparts, i.e.

$$\widehat{\varphi}_T = \arg\min_{\varphi \in \Phi} f_T(\varphi)' A_T f_T(\varphi), \tag{4.10}$$

with Φ the parameter space, $f_T(\varphi)$ the vector of differences between sample moments and analytical moments, and A_T a positive definite and possibly random weighting matrix. Moreover, $\hat{\varphi}_T$ is consistent and asymptotically Normal if suitable 'regularity conditions' are fulfilled (cf. Harris & Matyas (1999)). Within this GMM framework it becomes also possible to estimate the LMSM model. In the case of the LMSM model, the parameter vector $\vartheta = (\lambda, \sigma)'$ ($\vartheta = (\lambda, \sigma, \nu)'$) replaces φ in (4.10) when Normal (Student-t) innovations are assumed.

In order to account for the proximity to long memory characterizing MSM models, Lux (2008a) proposed to use log differences of absolute returns together with the pertinent analytical moment conditions, i.e.

$$\xi_{t,T} = \ln |x_t| - \ln |x_{t-T}|. \tag{4.11}$$

The above variable only has nonzero autocovariances over a limited number of lags. To exploit the temporal scaling properties of the MSM model, covariances of various moments over different time horizons are chosen as moment conditions, i.e.

$$\mathrm{Mom}\,(T, q) = E\left[\xi^q_{t+T,T} \cdot \xi^q_{t,T}\right], \tag{4.12}$$

for $q = 1, 2$ and $T = 1, 5, 10, 20$ together with $E\left[x_t^2\right] = \sigma^2$ for identification of σ in the MSM model with Normal innovations. In the case of the MSM-t model, two sets of moment conditions are utilized in addition to (4.12), namely, one that considers $E\left[|x_t|\right]$ (GMM1) and the other one that considers $E\left[|x_t|\right]$, $E\left[x_t^2\right]$ and $E\left[|x_t^3|\right]$ (GMM2). Details on moment conditions are provided in Appendix D.

We follow most of the literature by using the inverse of the Newey-West estimator of the variance-covariance matrix as the weighting matrix for GMM1. We also adopt an iterative GMM scheme updating the weighting matrix until convergence of both the parameter estimates and the variance-covariance matrix of moment conditions is obtained. However, we note that including the third moment ($E\left[|x_t^3|\right]$) for data generated from a Student-t distribution would not guarantee convergence of the sequence of weighting matrices under our choice of the inverse of the Newey-West (or any other) estimate of the variance-covariance matrix. Therefore, estimates based on the usual choice of the weighting matrix would not be consistent. Thus we simply resort to using the identity matrix for GMM2 which guarantees consistency as all the regularity conditions required for GMM are met.

Since GMM does not provide us with information on conditional state probabilities, we cannot use Bayesian updating and have to supplement it with a different forecasting algorithm. To this end, we use best linear forecasts (cf. Brockwell & Davis (1991), c.5) together with the generalized Levinson-Durbin algorithm developed by Brockwell & Dahlhaus (2004). We first have to consider the zero-mean time series,

$$X_t = x_t^2 - E[x_t^2] = x_t^2 - \widehat{\sigma}^2, \tag{4.13}$$

where $\widehat{\sigma}$ is the estimate of the scale factor σ. Assuming that the data of interest follow a stationary process $\{X_t\}$ with mean zero, the best linear h-step forecasts are obtained as

$$\widehat{X}_{n+h} = \sum_{i=1}^{n} \phi_{ni}^{(h)} X_{n+1-i} = \boldsymbol{\phi}_n^{(h)} \mathbf{X}_n, \tag{4.14}$$

where the vectors of weights $\boldsymbol{\phi}_n^{(h)} = (\phi_{n1}^{(h)}, \phi_{n2}^{(h)}, ..., \phi_{nn}^{(h)})'$ can be obtained from the analytical auto-covariances of X_t at lags h and beyond. More precisely, $\boldsymbol{\phi}_n^{(h)}$ are any solution of $\boldsymbol{\Psi}_n \boldsymbol{\phi}_n^{(h)} = \boldsymbol{\kappa}_n^{(h)}$ where $\boldsymbol{\kappa}_n^{(h)} = (\kappa_{n1}^{(h)}, \kappa_{n2}^{(h)}, ..., \kappa_{nn}^{(h)})'$ denote the autocovariance of X_t and $\boldsymbol{\Psi}_n = [\kappa(i-j)]_{i,j=1,...,n}$ is the variance-covariance matrix.

4.4 Generalized Autoregressive Conditional Heteroskedasticity models

4.4.1 Volatility specifications

We shortly turn to the 'competing' GARCH type volatility models to describe σ_t. The most common GARCH(1,1) model assumes that the volatility dynamics is governed by,

$$\sigma_t^2 = \omega + \alpha x_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{4.15}$$

where the unconditional variance is given by $\sigma^2 = \omega(1-\alpha-\beta)^{-1}$ and the restrictions on the parameters are $\omega > 0, \alpha, \beta \geq 0$ and $\alpha + \beta < 1$. Various extensions to (4.15) have been considered in the financial econometrics literature. One of the major additions to the GARCH family are models that allow for long-memory in the specification of volatility dynamics. The FIGARCH model introduced by Baillie *et al.* (1996) expands the variance equation of the GARCH model by considering fractional differences. As in the case of (4.15) we restrict our attention to one lag in both the autoregressive term and in the moving average term. The FIGARCH(1,d,1) is given by,

$$\sigma_t^2 = \omega + \left[1 - \beta L - (1 - \delta L)(1 - L)^d\right] x_t^2 + \beta \sigma_{t-1}^2, \tag{4.16}$$

where L is a lag operator, d is the parameter of fractional differentiation and the restrictions on the parameters are $\beta - d \leq \delta \leq (2 - d)3^{-1}$ and $d(\delta - 2^{-1}(1 - d)) \leq \beta(d - \beta + \delta)$. The major advantage of model (4.16) is that the Binomial expansion of the fractional difference operator introduces an infinite number of past lags with hyperbolically decaying coefficients for $0 < d < 1$. For $d = 0$, the FIGARCH model reduces to the standard GARCH(1,1) model while for $d = 1$ the model reduces to an IGARCH(1,1) model. Note that in contrast to the MSM model, both GARCH and FIGARCH are *unifractal* models. While GARCH exhibits only short-term dependence (i.e. exponential decay of autocorrelations of moments) FIGARCH has homogeneous hyperbolic decay of the autocorrelation of its moments characterized uniquely by the parameter d.

4.4.2 Estimation and forecasting

The GARCH and FIGARCH models can be estimated via standard (Quasi) ML procedures as in (4.9). In the case of the GARCH(1,1) the parameter vector, say θ, replaces φ in (4.9), where $\theta = (\omega, \alpha, \beta)'$ ($\theta = (\omega, \alpha, \beta, \nu)'$) is the vector of parameters if Normal (Student-t) innovations are assumed. The h-step ahead forecast representation of the GARCH(1,1) is given by,

$$\hat{\sigma}_{t+h}^2 = \hat{\sigma}^2 + (\hat{\alpha} + \hat{\beta})^{h-1} \left[\hat{\sigma}_{t+1}^2 - \hat{\sigma}^2\right], \tag{4.17}$$

where $\hat{\sigma}^2 = \hat{\omega}(1 - \hat{\alpha} - \hat{\beta})^{-1}$. In the case of the FIGARCH(1,d,1) the parameter vector, say ψ, replaces φ (4.9), where $\psi = (\omega, \alpha, \delta, d)'$ ($\psi = (\omega, \alpha, \delta, d, \nu)'$) is the vector of parameters if Normal (Student-t) innovations are assumed. Note that in practice, the infinite number of lags with hyperbolically decaying coefficients introduced by the Binomial expansion of the fractional difference operator $(1-L)^d$

must be truncated. We employ a lag truncation at 1000 steps as in Lux & Kaizoji (2007). The h-period ahead forecasts of the FIGARCH(1,d,1) model can be obtained most easily by recursive substitution, i.e.

$$\hat{\sigma}^2_{t+h} = \hat{\omega}(1-\hat{\beta})^{-1} + \eta(L)\hat{\sigma}^2_{t+h-1}, \tag{4.18}$$

where $\eta(L) = 1-(1-\hat{\beta}L)^{-1}(1-\hat{\delta}L)(1-L)^{\hat{d}}$ can be calculated from the recursions $\eta_1 = \hat{\delta}-\hat{\beta}+\hat{d}$, $\eta_j = \hat{\beta}\eta_j + [(j-1-\hat{d})j^{-1}-\hat{\delta}]\pi_{j-1}$ where $\pi_j \equiv \pi_{j-1}(j-1-\hat{d})j^{-1}$ are the coefficients in the MacLaurin series expansion of the fractional differencing operator $(1-L)^d$.

4.5 Monte Carlo analysis

Monte Carlo studies with the MSM-t were performed along the lines of Calvet & Fisher (2004) and Lux (2008a) in order to shed light on parameter estimation and out-of-sample forecasting via the MSM-t vis-à-vis the MSM model with Normal innovations. Monte Carlo experiments are reported in Tables 4.1 through 4.3.

		$\nu = 5$									$\nu = 6$								
		ML	GMM1	GMM2	ML	GMM1	GMM2	ML	GMM1	GMM2	ML	GMM1	GMM2	ML	GMM1	GMM2	ML	GMM1	GMM2
		$m_0 = 1.3$			$m_0 = 1.4$			$m_0 = 1.5$			$m_0 = 1.3$			$m_0 = 1.4$			$m_0 = 1.5$		
	$\bar{m}_0$	1.294	1.215	1.344	1.395	1.348	1.434	1.496	1.469	1.527	1.295	1.166	1.338	1.395	1.314	1.428	1.496	1.449	1.520
	FSSE	0.024	0.123	0.141	0.024	0.092	0.109	0.023	0.056	0.071	0.024	0.127	0.135	0.024	0.103	0.102	0.023	0.060	0.072
	RMSE	0.025	0.149	0.148	0.024	0.105	0.114	0.024	0.064	0.076	0.024	0.184	0.140	0.024	0.134	0.105	0.024	0.079	0.075
	$\bar{\nu}$	4.997	4.903	4.826	5.030	4.914	4.824	5.089	4.939	4.844	6.033	4.904	4.970	6.104	4.903	4.966	6.337	4.927	4.984
T=2,500	FSSE	0.675	0.771	0.922	0.806	0.784	0.929	1.028	0.802	0.961	1.022	0.730	0.997	1.285	0.746	1.009	3.133	0.764	1.024
	RMSE	0.674	0.776	0.937	0.805	0.788	0.945	1.031	0.803	0.972	1.021	1.317	1.433	1.288	1.326	1.444	3.147	1.317	1.441
	$\bar{\sigma}$	0.997	0.967	0.902	0.999	0.965	0.898	1.003	0.968	0.888	0.997	0.918	0.852	0.999	0.913	0.849	1.004	0.916	0.842
	FSSE	0.093	0.104	0.173	0.125	0.133	0.195	0.162	0.165	0.222	0.092	0.095	0.153	0.125	0.125	0.172	0.163	0.156	0.199
	RMSE	0.093	0.109	0.198	0.125	0.137	0.219	0.162	0.168	0.248	0.092	0.125	0.213	0.124	0.152	0.229	0.163	0.177	0.254
	$\bar{m}_0$	1.298	1.222	1.332	1.399	1.357	1.427	1.499	1.471	1.518	1.299	1.171	1.327	1.399	1.327	1.419	1.499	1.453	1.512
	FSSE	0.016	0.102	0.115	0.016	0.058	0.076	0.016	0.040	0.056	0.016	0.109	0.105	0.016	0.066	0.075	0.016	0.042	0.055
	RMSE	0.016	0.129	0.119	0.016	0.072	0.080	0.016	0.049	0.059	0.016	0.169	0.108	0.016	0.098	0.077	0.016	0.063	0.057
	$\bar{\nu}$	5.090	4.691	4.684	5.108	4.692	4.689	5.136	4.701	4.696	6.146	4.714	4.842	6.184	4.708	4.839	6.244	4.723	4.847
T=5,000	FSSE	0.520	0.578	0.777	0.591	0.590	0.790	0.691	0.603	0.801	0.772	0.533	0.837	0.902	0.546	0.844	1.104	0.563	0.854
	RMSE	0.527	0.655	0.838	0.600	0.665	0.848	0.703	0.672	0.856	0.784	1.392	1.429	0.919	1.402	1.435	1.129	1.395	1.434
	$\bar{\sigma}$	1.002	0.954	0.902	1.003	0.954	0.900	1.005	0.957	0.896	1.002	0.907	0.854	1.003	0.905	0.854	1.005	0.908	0.852
	FSSE	0.066	0.076	0.144	0.088	0.095	0.156	0.113	0.117	0.172	0.065	0.069	0.124	0.088	0.089	0.135	0.113	0.110	0.153
	RMSE	0.065	0.088	0.174	0.088	0.105	0.185	0.113	0.125	0.201	0.065	0.116	0.191	0.088	0.130	0.199	0.113	0.143	0.212
	$\bar{m}_0$	1.298	1.239	1.330	1.399	1.364	1.422	1.499	1.476	1.515	1.299	1.188	1.321	1.399	1.338	1.414	1.499	1.459	1.509
	FSSE	0.012	0.067	0.078	0.012	0.039	0.053	0.012	0.028	0.039	0.012	0.083	0.075	0.012	0.042	0.053	0.012	0.029	0.038
	RMSE	0.012	0.091	0.084	0.012	0.053	0.058	0.012	0.036	0.042	0.012	0.140	0.078	0.012	0.075	0.055	0.012	0.050	0.039
	$\bar{\nu}$	4.993	4.500	4.491	4.996	4.504	4.496	5.005	4.510	4.505	5.997	4.556	4.655	6.007	4.556	4.655	6.027	4.563	4.664
T=10,000	FSSE	0.345	0.427	0.572	0.402	0.433	0.578	0.475	0.442	0.587	0.512	0.385	0.619	0.609	0.392	0.630	0.733	0.399	0.642
	RMSE	0.345	0.657	0.765	0.402	0.658	0.766	0.475	0.659	0.768	0.511	1.494	1.480	0.608	1.497	1.485	0.732	1.492	1.482
	$\bar{\sigma}$	0.996	0.946	0.894	0.995	0.947	0.892	0.994	0.948	0.888	0.996	0.900	0.849	0.995	0.900	0.848	0.994	0.902	0.845
	FSSE	0.048	0.059	0.112	0.065	0.071	0.122	0.082	0.086	0.137	0.048	0.053	0.097	0.064	0.066	0.106	0.082	0.081	0.119
	RMSE	0.048	0.079	0.154	0.065	0.089	0.163	0.083	0.101	0.177	0.048	0.113	0.179	0.065	0.119	0.185	0.082	0.127	0.195

Table 4.1: Monte Carlo ML and GMM estimation of the Binomial MSM-t model with $k = 8$, $\sigma = 1$, $\nu = 5, 6$ and $m_0 = 1.3, 1.4, 1.5$. FSSE: finite sample standard error (e.g. FSSE= $[\sum_{s=1}^{S}(m_{0,s} - \bar{m}_0)^2/S]^{1/2}$), RMSE: root mean squared error (e.g. RMSE= $[\sum_{s=1}^{S}(m_{0,s} - m_0)^2/S]^{1/2}$). The entries $\bar{m}_0$, $\bar{\sigma}$ and $\bar{\nu}$ denote the mean of the estimated parameters over S=400 Monte Carlo runs.

		$\nu = 5$						$\nu = 6$					
		GMM1	GMM2	GMM1	GMM2	GMM1	GMM2	GMM1	GMM2	GMM1	GMM2	GMM1	GMM2
		$m_0 = 1.3$		$m_0 = 1.4$		$m_0 = 1.5$		$m_0 = 1.3$		$m_0 = 1.4$		$m_0 = 1.5$	
	$\bar{m}_0$	1.226	1.333	1.360	1.427	1.475	1.519	1.175	1.325	1.330	1.419	1.456	1.513
	FSSE	0.103	0.117	0.058	0.080	0.040	0.057	0.110	0.111	0.067	0.079	0.043	0.056
	RMSE	0.127	0.122	0.071	0.084	0.048	0.060	0.166	0.113	0.097	0.081	0.062	0.057
	$\bar{\nu}$	4.705	4.627	4.711	4.628	4.726	4.642	4.732	4.776	4.727	4.777	4.743	4.794
BMSM	FSSE	0.591	0.705	0.602	0.710	0.610	0.721	0.546	0.761	0.560	0.770	0.572	0.783
	RMSE	0.660	0.797	0.667	0.801	0.668	0.804	1.380	1.440	1.390	1.445	1.381	1.438
	$\bar{\sigma}$	0.951	0.884	0.951	0.874	0.954	0.857	0.899	0.839	0.897	0.831	0.902	0.816
	FSSE	0.116	0.172	0.150	0.198	0.189	0.231	0.107	0.157	0.141	0.180	0.178	0.212
	RMSE	0.126	0.208	0.158	0.234	0.194	0.272	0.147	0.224	0.174	0.247	0.203	0.280
		$\lambda = 0.05$		$\lambda = 0.10$		$\lambda = 0.15$		$\lambda = 0.05$		$\lambda = 0.10$		$\lambda = 0.15$	
	$\bar{\lambda}$	0.030	0.064	0.079	0.113	0.129	0.163	0.023	0.067	0.069	0.116	0.119	0.165
	FSSE	0.022	0.047	0.027	0.048	0.028	0.049	0.019	0.045	0.025	0.047	0.027	0.050
	RMSE	0.030	0.049	0.034	0.050	0.035	0.051	0.033	0.048	0.040	0.049	0.041	0.052
	$\bar{\nu}$	4.540	4.539	4.601	4.563	4.629	4.574	4.730	4.840	4.684	4.868	4.693	4.891
LMSM	FSSE	0.799	0.911	0.775	0.905	0.773	0.900	0.700	0.894	0.680	0.910	0.672	0.904
	RMSE	0.921	1.020	0.871	1.004	0.856	0.995	1.449	1.464	1.481	1.452	1.469	1.430
	$\bar{\sigma}$	0.922	0.844	0.921	0.821	0.920	0.780	0.886	0.822	0.875	0.799	0.873	0.765
	FSSE	0.119	0.224	0.159	0.258	0.192	0.281	0.105	0.184	0.148	0.222	0.179	0.253
	RMSE	0.142	0.273	0.178	0.314	0.207	0.356	0.155	0.256	0.194	0.299	0.219	0.344

Table 4.2: Monte Carlo GMM estimation of the Binomial and Lognormal MSM-t models with $T = 5,000$, $k = 10$, $\sigma = 1$, $\nu = 5, 6$, $m_0 = 1.3, 1.4, 1.5$ and $\lambda = 0.05, 0.10, 0.15$. FSSE: finite sample standard error (e.g. FSSE= $[\sum_{s=1}^{S}(m_{0,s} - \bar{m}_0)^2/S]^{1/2}$), RMSE: root mean squared error (e.g. RMSE= $[\sum_{s=1}^{S}(m_{0,s} - m_0)^2/S]^{1/2}$). The entries $\bar{m}_0$, $\bar{\sigma}$ and $\bar{\nu}$ denote the mean of the estimated parameters over S=400 Monte Carlo runs.

4.5.1 In-sample analysis

Table 4.1 shows the result of the Monte Carlo simulations of the BMSM-t via ML estimation and the two sets of moment conditions for GMM estimation (GMM1, GMM2) with a relatively small number of multipliers $k = 8$ for which ML is still feasible. The Binomial parameters are set to $m_0 = 1.3, 1.4, 1.5$ and the sample sizes are given by $T_1 = 2,500$, $T_2 = 5,000$ and $T_3 = 10,000$. As mentioned previously the admissible parameter range for m_0 is $m_0 \in [1, 2]$ and the volatility process collapses to a constant if the latter parameter hits its lower boundary 1. The parameter corresponding to the Student-t distribution is set to $\nu = 5$ and $\nu = 6$. As in the case of Lux (2008a), the main difference in our simulation set up to the one proposed in Calvet & Fisher (2004) is that we fix the parameters of the transition probabilities in (4.6) to $b = 2$ and $\gamma_k = 0.5$ which reduces the number of parameters for estimation to only three.

The simulation results show (as expected) that GMM estimates of m_0 are in general less efficient in comparison to ML estimates. The finite sample standard error (FSSE) and root mean squared error (RMSE) of the GMM estimates with $\nu = 5$ show that the estimated parameters for m_0 are more variable with lower T and smaller 'true' values of m_0. As in the case of the MSM model with Normal innovations, biases and MSEs of the ML estimates for m_0 are found to be essentially independent of the true parameter values $m_0 = 1.3, 1.4, 1.5$. With respect to GMM estimates with the two different sets of moment conditions (GMM1, GMM2), both the bias and the MSEs decrease as we increase m_0 from 1.3 to 1.5. Interestingly, when the degrees of freedom are increased from $\nu = 5$ to $\nu = 6$ we find an overall increase in the bias and MSEs of m_0 via GMM1 while the bias and MSEs of m_0 via GMM2 decrease.

ML estimates of the distributional parameter ν show a relatively small bias although it seems to slightly increase for larger m_0 at $T = 2,500$. The variability of ν via ML is also found to be more pronounced than the variability of the Binomial parameter m_0. GMM estimates of ν have a larger bias and MSEs in comparison to ML estimates. As we move from $\nu = 5$ to $\nu = 6$, we find that the bias and MSEs of the parameter ν estimated via GMM1 and GMM2 become larger.

The quality of the estimates of the scale parameter σ at $\nu = 5$ is very similar under ML and GMM1 particularly when the sample size is increased. With respect to estimates of σ via GMM2, we find that the bias is somewhat larger in comparison to ML and GMM1. As in the case of the MSM model with Normal innovations, we find that the MSEs of σ increase for higher m_0 while they are more or less unchanged as we move from $\nu = 5$ to $\nu = 6$.

Table 4.2 displays the results of the Monte Carlo analysis of the MSM-t model with a setting that makes ML estimation computationally infeasible, that is, the BMSM-t and LMSM-t models with $k = 10$.[4] As in the previous experiments, the simulations are performed with $m_0 = 1.3, 1.4, 1.5$, $\nu = 5, 6$ and the same logic is applied to the LMSM model for which the location parameter of the continuous distribution is set to $\lambda = 0.05, 0.1, 0.15$. Note that the admissible space for λ is $\lambda \in [0, \infty)$. As in the case of the Binomial parameter m_0, when the Lognormal parameter hits its lower boundary at 0, the volatility process collapses to a constant. To save on space, the simulations are only presented with $T = 5,000$.

The results of the simulations indicate that the Binomial parameter m_0 estimated via GMM1 or GMM2 are practically invariant to higher number of components k, both in terms of bias and MSEs for the parameter values $m_0 = 1.3, 1.4, 1.5$ and $\nu = 5$. The bias and MSEs of m_0 usually increase in GMM1 as we increase the degrees of freedom from $\nu = 5$ to $\nu = 6$. As in the BMSM model with Normal innovations, the bias and the variability of σ increase with k as it becomes hard to discriminate between very long-lived volatility components and the constant scale factor (cf. Lux (2008a)). The distributional parameter ν is found to be relatively invariant for GMM1 and GMM2 when $k = 10$ in relation to $k = 8$. Bias and MSEs of the distributional parameter ν increase for GMM1 and GMM2 as we move from $\nu = 5$ to $\nu = 6$. In the LMSM-t model, we find that biases and MSEs for λ at $\nu = 5$ are somewhat larger for GMM1 than GMM2. As for the BMSM-t, bias and MSEs of λ are relatively invariant for larger k buy they usually increase as we move from $\nu = 5$ to $\nu = 6$.

Summing up, we find that the Monte Carlo simulations for the in-sample performance of the Binomial and Lognormal MSM models with Student-t in-

[4]Simulation results for $k = 15, 20$ are qualitatively similar and can be provided upon request.

novations 'point' into the same direction as those of Lux (2008a) for the MSM model with Gaussian innovations: while GMM is less efficient than ML, it comes with moderate biases and moderate standard errors. The efficiency of both GMM algorithms also appear quite insensitive with respect to the number of multipliers.

		$\nu = 5$									$\nu = 6$								
		ML	GMM1	GMM2	ML	GMM1	GMM2	ML	GMM1	GMM2	ML	GMM1	GMM2	ML	GMM1	GMM2	ML	GMM1	GMM2
	h	$m_0 = 1.3$			$m_0 = 1.4$			$m_0 = 1.5$			$m_0 = 1.3$			$m_0 = 1.4$			$m_0 = 1.5$		
MSE	1	0.971	0.988	0.990	0.957	0.978	0.983	0.947	0.974	0.979	0.964	0.987	0.985	0.947	0.972	0.975	0.937	0.966	0.971
		(0.012)	(0.011)	(0.013)	(0.019)	(0.016)	(0.018)	(0.025)	(0.021)	(0.022)	(0.013)	(0.013)	(0.016)	(0.020)	(0.017)	(0.021)	(0.025)	(0.022)	(0.025)
	5	0.981	0.992	0.994	0.974	0.986	0.989	0.971	0.985	0.988	0.976	0.991	0.990	0.969	0.982	0.985	0.966	0.981	0.983
		(0.009)	(0.008)	(0.011)	(0.013)	(0.012)	(0.013)	(0.016)	(0.014)	(0.015)	(0.010)	(0.010)	(0.013)	(0.014)	(0.013)	(0.015)	(0.017)	(0.015)	(0.017)
	10	0.986	0.994	0.996	0.981	0.990	0.992	0.980	0.989	0.992	0.983	0.993	0.993	0.978	0.987	0.989	0.977	0.987	0.989
		(0.008)	(0.007)	(0.009)	(0.011)	(0.010)	(0.011)	(0.013)	(0.012)	(0.013)	(0.009)	(0.008)	(0.011)	(0.012)	(0.011)	(0.013)	(0.014)	(0.013)	(0.014)
	20	0.991	0.996	0.998	0.988	0.993	0.995	0.987	0.993	0.995	0.988	0.995	0.996	0.985	0.991	0.993	0.985	0.991	0.993
		(0.006)	(0.006)	(0.009)	(0.009)	(0.008)	(0.010)	(0.011)	(0.010)	(0.011)	(0.007)	(0.007)	(0.010)	(0.010)	(0.009)	(0.011)	(0.012)	(0.011)	(0.012)
MAE	1	0.955	0.972	0.938	0.923	0.960	0.923	0.892	0.952	0.909	0.952	0.971	0.931	0.918	0.954	0.913	0.885	0.944	0.898
		(0.061)	(0.047)	(0.064)	(0.084)	(0.065)	(0.072)	(0.108)	(0.082)	(0.082)	(0.059)	(0.042)	(0.059)	(0.083)	(0.062)	(0.068)	(0.106)	(0.079)	(0.078)
	5	0.969	0.977	0.942	0.951	0.971	0.931	0.935	0.968	0.922	0.967	0.975	0.936	0.947	0.966	0.923	0.930	0.963	0.914
		(0.056)	(0.046)	(0.064)	(0.079)	(0.061)	(0.072)	(0.102)	(0.076)	(0.081)	(0.055)	(0.041)	(0.059)	(0.077)	(0.059)	(0.068)	(0.101)	(0.074)	(0.077)
	10	0.977	0.980	0.944	0.964	0.976	0.935	0.954	0.976	0.928	0.975	0.978	0.938	0.961	0.972	0.928	0.951	0.972	0.921
		(0.053)	(0.045)	(0.065)	(0.074)	(0.059)	(0.072)	(0.096)	(0.072)	(0.081)	(0.051)	(0.040)	(0.058)	(0.073)	(0.057)	(0.067)	(0.095)	(0.070)	(0.076)
	20	0.984	0.982	0.946	0.976	0.982	0.939	0.972	0.983	0.933	0.983	0.981	0.940	0.975	0.978	0.932	0.969	0.979	0.926
		(0.048)	(0.044)	(0.065)	(0.067)	(0.055)	(0.072)	(0.087)	(0.068)	(0.081)	(0.046)	(0.040)	(0.058)	(0.066)	(0.053)	(0.066)	(0.086)	(0.066)	(0.075)

Table 4.3: Monte Carlo Assessment of Bayesian vs. Best Linear Forecasts, Binomial MSM Specification for $k = 8$, $\nu = 5, 6$ and $m_0 = 1.3, 1.4, 1.5$. MSE: mean square errors and MAE: mean absolute errors. MSEs and MAEs are given in percentage of the MSEs and MAEs of a naive forecast using the in-sample variance. All entries are averages over 400 Monte Carlo runs (with standard errors given in parentheses). In each run, an overall sample of 10,000 entries has been split into an in-sample period of 5,000 entries for parameter estimation and an out-of-sample period of 5,000 entries for evaluation of forecasting performance. ML stands for parameter estimation based on the maximum likelihood procedure and pertinent inference on the probability of the 2^k states of the model. GMM1 uses parameters estimated by GMM with moment set 1, while GMM2 implements parameters estimated via GMM with moment set 2.

4.5.2 Out-of-sample analysis

Table 4.3 shows the forecasting results from optimal forecasts (ML) and best linear forecasts (GMM) of the BMSM-t model. The out-of-sample MC analysis is performed within the same framework as the in-sample analysis when comparing ML and GMM procedures. That is, we set $k = 8$ and evaluate the forecasts for the BMSM-t model with parameters $m_0 = 1.3, 1.4, 1.5$, $\sigma = 1$ and $\nu = 5, 6$. In our Monte Carlo experiments, we also imposed a lower boundary $\underline{\nu} = 4.05$ as a constraint in the GMM estimates as otherwise forecasting with the Levinson-Durbin algorithm would have been impossible.

In the forecasting simulations we set $T = 10,000$ and use $T = 5,000$ for in-sample estimation and $T = 5,000$ for out-of-sample forecasting in order compare them with the results of the Gaussian MSM models in Lux (2008a). The forecasting performance of the models is evaluated with respect to their mean squared errors (MSE) and mean absolute errors (MAE) standardized relative to the in-sample variance which implies that values below 1 indicate improvement against a constant volatility model. Relative MSE and MAE are averages over 400 simulation runs.

The results basically show that, similarly as for the Gaussian MSM models, the loss in forecasting accuracy when employing GMM as opposed to ML is small particularly when compared against GMM2. Thus, the lower efficiency of GMM does not impede its forecasting capability in connection with the Levinson-Durbin algorithm. Both MSE and MAE measures show improvement when the parameter m_0 increases from 1.3 to 1.5 while they seem to deteriorate for longer horizons although only marginally. Interestingly, we find that GMM2 based forecasts even improve in terms of MAEs relative to ML based forecasts for $h \geq 5$ so that it appears entirely justified to resort to the computationally parsimonious GMM2 estimation and linear forecasts in our subsequent empirical part.[5]

[5]Out-of-sample forecasts of the lognormal MSM-t models behave very similar, but they are not displayed here because of the lack of a ML benchmark.

Equity Markets		Bond Markets	Real Estate Markets
Austria	Italy	Australia	Austria
Argentina	Norway	Belgium	Canada
Belgium	Mexico	Canada	France
Canada	New Zealand	Denmark	Germany
Chile	Japan	France	Italy
Denmark	South Africa	Germany	Japan
Finland	Spain	Ireland	New Zealand
France	Sweden	Netherlands	Spain
Germany	Thailand	Sweden	Sweden
Greece	Turkey	United Kingdom	United Kingdom
Hong Kong	United Kingdom	United States	United States
India	United States		South Africa
Ireland			

Table 4.4: Equity, Bond and Real Estate markets for the empirical analysis. Countries were chosen upon data availability for the sample period 01/1990 to 01/2008. We employ Datastream calculated (total market) stock indices, 10-year benchmark government bond indices and real estate security indices.

4.6 Empirical analysis

In this section we turn to the results of our empirical application to compare the in-sample and out-of-sample performance of the different volatility models discussed previously. We follow a similar approach to the *panel* empirical analysis of volatility forecasting performed for the Tokyo Stock Exchange in Lux & Kaizoji (2007). However, here we concentrate on three new different cross-sections of asset markets, namely, all-share stock indices ($N = 25$), 10-year government bond market indices ($N = 11$), and real estate security indices ($N = 12$) at the cross-country level. The sample runs from 01/1990 to 01/2008 at the daily frequency which leads to 4697 observation from which 2,500 are used for in-sample estimation and the remaining observations for out-of-sample forecasting. The data is obtained from Datastream and the countries were chosen upon data availability for the sample period covered. Specific countries for each of the three asset markets are presented in Table 4.4. In the following discussions we refer to statistical significance at the 5% level throughout.

		GARCH							FIGARCH								
		Normal innovations			Student-t innovations				Normal innovations				Student-t innovations				
Mkt.		$\bar{\omega}$	$\bar{\beta}$	$\bar{\alpha}$	$\bar{\omega}$	$\bar{\beta}$	$\bar{\alpha}$	$\bar{\nu}$	$\bar{\omega}$	$\bar{\beta}$	$\bar{\delta}$	$\bar{d}$	$\bar{\omega}$	$\bar{\beta}$	$\bar{\delta}$	$\bar{d}$	$\bar{\nu}$
ST	MG	0.081	0.844	0.112	0.061	0.855	0.122	5.800	0.164	0.271	0.065	0.334	0.101	0.497	0.222	0.428	5.783
		(0.022)	(0.014)	(0.007)	(0.017)	(0.013)	(0.013)	(0.297)	(0.033)	(0.079)	(0.068)	(0.029)	(0.026)	(0.037)	(0.038)	(0.028)	(0.296)
	Min	0.006	0.587	0.054	0.003	0.626	0.040	3.183	0.027	-0.938	-0.915	0.001	0.009	0.039	-0.111	0.286	3.282
	Max	0.491	0.939	0.175	0.425	0.957	0.373	9.387	0.683	0.851	0.762	0.832	0.591	0.873	0.905	0.933	9.608
BO	MG	0.005	0.903	0.075	0.004	0.907	0.078	4.760	0.014	0.558	0.216	0.422	0.009	0.614	0.253	0.443	4.824
		(0.002)	(0.014)	(0.010)	(0.001)	(0.011)	(0.011)	(0.244)	(0.008)	(0.084)	(0.064)	(0.044)	(0.002)	(0.029)	(0.021)	(0.029)	(0.227)
	Min	0.012	0.777	0.031	0.001	0.814	0.028	3.013	0.002	-0.208	-0.378	0.200	0.002	0.437	0.168	0.282	3.533
	Max	0.021	0.946	0.154	0.012	0.951	0.159	6.364	0.095	0.786	0.388	0.685	0.028	0.739	0.344	0.577	6.383
RE	MG	0.064	0.877	0.084	0.064	0.782	0.111	4.512	0.132	0.434	0.332	0.229	0.132	0.308	0.104	0.367	4.831
		(0.013)	(0.019)	(0.011)	(0.018)	(0.074)	(0.011)	(0.324)	(0.031)	(0.154)	(0.159)	(0.037)	(0.035)	(0.149)	(0.115)	(0.069)	(0.255)
	Min	0.004	0.716	0.023	0.002	0.014	0.043	2.000	0.027	-0.598	-0.720	0.029	0.017	-0.506	-0.577	0.044	3.295
	Max	0.137	0.969	0.143	0.192	0.956	0.176	6.256	0.364	0.923	0.968	0.425	0.364	0.926	0.599	0.999	6.529

Table 4.5: GARCH and FIGARCH in-sample estimates. MG: mean group parameter estimates (with standard errors in parentheses) of GARCH, GARCH-t, FIGARCH and FIGARCH-t models for $N = 25$ international stock market indices (ST), $N = 11$ international 10-year government bond indices (BO), $N = 12$ international real estate security indices (RE). Min: minimum estimated parameter value in the cross-section. Max: maximum estimated parameter value in the cross-section.

		BMSM					LMSM				
		Normal innovations		Student-t innovations			Normal innovations		Student-t innovations		
Mkt.		$\bar{m}_0$	$\bar{\sigma}$	$\bar{m}_0$	$\bar{\sigma}$	$\bar{\nu}$	$\bar{\lambda}$	$\bar{\sigma}$	$\bar{\lambda}$	$\bar{\sigma}$	$\bar{\nu}$
ST	MG	1.435	1.273	1.388	0.808	4.821	0.118	1.273	0.107	0.794	4.921
		(0.019)	(0.113)	(0.033)	(0.080)	(0.192)	(0.012)	(0.113)	(0.017)	(0.087)	(0.190)
	Min	1.264	0.681	1.033	0.449	4.050	0.036	0.681	0.001	0.331	4.050
	Max	1.619	2.932	1.681	2.257	6.779	0.261	2.932	0.341	2.217	6.752
BO	MG	1.487	0.418	1.435	0.252	4.580	0.159	0.418	0.140	0.245	4.588
		(0.037)	(0.029)	(0.053)	(0.027)	(0.291)	(0.025)	(0.029)	(0.034)	(0.028)	(0.289)
	Min	1.234	0.293	1.123	0.160	4.050	0.032	0.293	0.008	0.135	4.050
	Max	1.642	0.601	1.693	0.485	7.089	0.289	0.601	0.362	0.482	7.084
RE	MG	1.594	1.376	1.646	0.765	4.900	0.307	1.374	0.268	0.692	5.192
		(0.041)	(0.143)	(0.063)	(0.094)	(0.128)	(0.085)	(0.143)	(0.056)	(0.109)	(0.127)
	Min	1.393	0.682	1.430	0.487	4.365	0.086	0.682	0.137	0.158	4.574
	Max	1.912	2.287	1.945	1.503	5.555	1.137	2.282	0.741	1.416	6.067

Table 4.6: Binomial and Lognormal MSM in-sample estimates. MG: mean group parameter estimates (with standard errors in parentheses) of the BMSM, BMSM-t, LMSM and LMSM-t models for $N = 25$ international stock market indices (ST), $N = 11$ international 10-year government bond indices (BO), $N = 12$ international real estate security indices (RE). Min: minimum estimated parameter value in the cross-section. Max: maximum estimated parameter value in the cross-section.

4.6.1 In-sample analysis

For our in-sample analysis we account for a constant and an AR(1) term in the conditional mean of the return data as in (4.4). Results of the Mean Group (MG) estimates of the parameters of the (FI)GARCH and MSM models explained in previous sections are reported in Table 4.5 and Table 4.6, respectively. MG estimates are obtained by averaging individual market estimates. We also report minimum and maximum values of the estimates obtained to have an idea about the distribution of the parameters across the countries under inspection.

We find in the case of the GARCH model that there is on average a statistically significant effect of past volatility on current volatility ($\bar{\beta}$) and of past squared innovations on current volatility ($\bar{\alpha}$) in all three markets at the 5% significance level (Table 4.5). The results are qualitatively the same with respect to the estimates $\bar{\beta}$ and $\bar{\alpha}$ in the case of the GARCH-t. The distributional parameter ($\bar{\nu}$) is on average greater than 4 in all three markets and statistically significant.

Taking into account long memory and Student-t innovations via the FIGARCH specification we find that there is a statistically significant average effect of past volatility ($\bar{\beta}$) and past squared innovations ($\bar{\delta}$) on current volatility in all three markets. FIGARCH also provides evidence for the presence of long memory as given by the MG estimate of the differencing parameter $\bar{d}$ in the three cross-sections (Table 4.5). When we consider the FIGARCH-t we find the same qualitative results for the average impact of the parameters $\bar{\beta}$, $\bar{\delta}$ and $\bar{d}$ as in the FIGARCH and the same qualitative results of the distributional parameter $\bar{\nu}$ as with the GARCH-t model.

In-sample estimation of the BMSM and LMSM models with Normal and Student-t innovations is restricted to GMM since ML estimation with panel data requires a tremendous amount of time for $k > 8$. The estimation procedure for the MSM models consists in estimating the models for each country in each of the stock, bond and real estate markets for a cascade level of $k = 10$. The choice of the number of cascade levels is motivated by previous findings of very similar parameter estimates for all k above this benchmark (Liu *et al.* (2007), Lux (2008a)). Note, however, that forecasting performance might nevertheless improve for $k > 10$ and proximity to temporal scaling of empirical data might be closer. Our choice of the specification $k = 10$ is, therefore, a relatively conservative one.[6]

For space considerations we only present the results of the MSM-t models estimated with the second set of moment conditions (GMM2) given that we found that this set of moment conditions produced more accurate forecasts in the Monte Carlo Simulations. We have also restricted the parameter ν by using 4.05 as a lower bound in order to employ best linear forecasts. Nevertheless, we found very few cases where $\nu < 4$ in both (FI)GARCH and MSM models.

With respect to the BMSM model, the mean Binomial parameter $\bar{m}_0$ is statistically different from the benchmark case $\bar{m}_0 = 1$ in all three markets (Table 4.6). In the LMSM model, we find that the mean Lognormal parameter $\bar{\lambda}$ has a value which is statistically different from zero in all three asset markets. In the case of the BMSM-t, the mean distributional parameter $\bar{\nu}$ obtains a value that is statistically significant in all the three markets. Considering the LMSM-t we obtain similar qualitative results as for the BMSM-t in terms of the average parameters $\bar{\sigma}$ and $\bar{\nu}$.

Summarizing the in-sample results at the aggregate level, we find that there is (on average) a statistically significant effect of past volatility and long-memory on current volatility as well as evidence of multifractal volatility and *fat tails* in return innovations. It is also noteworthy, that in many cases, the mean multifractal parameters $\bar{m}_0$ and $\bar{\lambda}$ turn out to be different for the models with Student-t innovations from those with Normal innovations. Since higher m_0 and λ lead to more heterogeneity and, therefore, more extreme observations, we see a trade-off between parameters for the fat-tailed innovations and those governing temporal dependence of volatility. What differences these variations in multifractal parameters make for forecasting, is investigated below.

4.6.2 Out-of-sample analysis

In this section we turn to the discussion of the out-of-sample results. Forecasting horizons are set to 1, 5, 20, 50 and 100 days ahead. We have used only one set of

[6] In our case, results for $k = 15$ and $k = 20$ are practically the same as with $k = 10$. Details are available upon request.

in-sample parameter estimates and have not re-estimated the models via rolling window schemes because of the computational burden that one encounters with respect to ML estimation of the FIGARCH models. We have also experimented with different subsamples but we have found no qualitative difference with respect to the current in-sample and out-of-sample window split which is roughly about half for in-sample estimation and half for out-of-sample forecasting.

In order to compare forecasts across models we use the principle of relative MSE and MAE as previously mentioned. That is, the MSE and MAE corresponding to a particular model are given in percentage of a naive predictor using historical volatility (i.e. the sample mean of squared returns of the in-sample period). We also report the number of statistically significant improvements of a particular model against a benchmark specification via the Diebold & Mariano (1995) test. The latter test allows to test the null hypothesis that two competing models have statistically equal forecasting performance. Details about the computation of MSEs and MAEs with panel data and corresponding standard errors and the count test based on the Diebold & Mariano (1995) statistics are provided in Appendix D.

		MSE								MAE							
		Normal innovations				Student-t innovations				Normal innovations				Student-t innovations			
Mkt.	h	GARCH	FIGA	BMSM	LMSM	GARCH	FIGA	BMSM	LMSM	GARCH	FIGA	BMSM	LMSM	GARCH	FIGA	BMSM	LMSM
ST	1	0.873	**0.863**	0.948	0.944	0.883	0.870	0.947	0.948	**0.796**	0.797	0.940	0.935	0.803	0.797	0.825	0.819
		(0.018)	(0.019)	(0.007)	(0.008)	(0.022)	(0.024)	(0.008)	(0.008)	(0.004)	(0.004)	(0.001)	(0.001)	(0.005)	(0.005)	(0.003)	(0.003)
	5	0.919	**0.903**	0.972	0.971	0.962	0.923	0.949	0.953	0.825	0.824	0.962	0.962	0.850	0.837	0.826	**0.822**
		(0.012)	(0.012)	(0.004)	(0.004)	(0.014)	(0.012)	(0.008)	(0.008)	(0.004)	(0.004)	(0.001)	(0.001)	(0.005)	(0.004)	(0.003)	(0.003)
	20	0.928	**0.906**	0.972	0.972	0.981	0.912	0.949	0.954	0.867	0.859	0.964	0.964	0.944	0.890	0.826	**0.822**
		(0.010)	(0.012)	(0.004)	(0.004)	(0.013)	(0.011)	(0.008)	(0.008)	(0.003)	(0.003)	(0.001)	(0.001)	(0.004)	(0.004)	(0.003)	(0.003)
	50	0.945	**0.917**	0.973	0.973	1.039	0.918	0.950	0.954	0.911	0.892	0.964	0.964	1.089	0.944	0.827	**0.823**
		(0.008)	(0.011)	(0.004)	(0.004)	(0.015)	(0.010)	(0.008)	(0.008)	(0.003)	(0.003)	(0.001)	(0.001)	(0.004)	(0.003)	(0.003)	(0.003)
	100	0.958	**0.934**	0.973	0.973	1.181	0.940	0.950	0.954	0.943	0.926	0.964	0.964	1.283	1.000	0.827	**0.822**
		(0.007)	(0.009)	(0.004)	(0.004)	(0.032)	(0.008)	(0.008)	(0.008)	(0.002)	(0.002)	(0.001)	(0.001)	(0.006)	(0.003)	(0.003)	(0.003)
BO	1	0.838	**0.830**	1.023	1.018	0.840	0.832	0.981	0.975	0.791	**0.764**	1.011	1.006	0.793	0.768	0.843	0.825
		(0.008)	(0.009)	(0.002)	(0.002)	(0.008)	(0.009)	(0.005)	(0.006)	(0.003)	(0.003)	(0.001)	(0.001)	(0.003)	(0.003)	(0.003)	(0.003)
	5	0.856	**0.833**	1.047	1.047	0.856	0.837	0.984	0.979	0.818	**0.776**	1.032	1.032	0.822	0.781	0.845	0.827
		(0.007)	(0.008)	(0.003)	(0.003)	(0.007)	(0.008)	(0.005)	(0.006)	(0.003)	(0.003)	(0.001)	(0.001)	(0.003)	(0.003)	(0.003)	(0.003)
	20	0.910	**0.846**	1.049	1.049	0.922	0.852	0.983	0.977	0.894	**0.809**	1.034	1.034	0.923	0.825	0.844	0.826
		(0.005)	(0.008)	(0.003)	(0.003)	(0.004)	(0.007)	(0.005)	(0.006)	(0.002)	(0.003)	(0.001)	(0.001)	(0.002)	(0.003)	(0.003)	(0.003)
	50	0.975	**0.870**	1.050	1.050	1.072	0.884	0.980	0.975	0.974	0.853	1.034	1.034	1.079	0.882	0.842	**0.824**
		(0.002)	(0.007)	(0.003)	(0.003)	(0.005)	(0.006)	(0.006)	(0.006)	(0.001)	(0.002)	(0.001)	(0.001)	(0.002)	(0.002)	(0.003)	(0.003)
	100	1.031	**0.906**	1.051	1.051	1.267	0.932	0.977	0.971	1.042	0.907	1.034	1.035	1.245	0.950	0.839	**0.820**
		(0.002)	(0.005)	(0.003)	(0.003)	(0.013)	(0.004)	(0.006)	(0.006)	(0.001)	(0.002)	(0.001)	(0.001)	(0.003)	(0.002)	(0.003)	(0.003)
RE	1	0.827	**0.821**	0.929	0.921	0.839	0.832	0.911	0.910	0.706	0.685	0.897	0.886	0.675	**0.654**	0.691	0.680
		(0.026)	(0.027)	(0.011)	(0.013)	(0.025)	(0.027)	(0.016)	(0.017)	(0.005)	(0.005)	(0.002)	(0.002)	(0.005)	(0.005)	(0.004)	(0.005)
	5	0.858	**0.850**	0.964	0.963	0.880	0.860	0.920	0.925	0.736	0.708	0.932	0.930	0.713	**0.681**	0.697	0.687
		(0.021)	(0.022)	(0.006)	(0.006)	(0.019)	(0.021)	(0.015)	(0.015)	(0.004)	(0.005)	(0.001)	(0.001)	(0.005)	(0.005)	(0.004)	(0.005)
	20	0.879	**0.863**	0.966	0.965	0.911	0.876	0.921	0.925	0.800	0.746	0.934	0.932	0.801	0.727	0.697	**0.687**
		(0.018)	(0.020)	(0.006)	(0.006)	(0.014)	(0.018)	(0.015)	(0.015)	(0.004)	(0.004)	(0.001)	(0.001)	(0.004)	(0.005)	(0.004)	(0.005)
	50	0.897	**0.869**	0.966	0.966	0.944	0.890	0.921	0.926	0.865	0.775	0.934	0.932	0.896	0.780	0.696	**0.687**
		(0.014)	(0.018)	(0.006)	(0.006)	(0.008)	(0.015)	(0.015)	(0.015)	(0.003)	(0.004)	(0.001)	(0.001)	(0.004)	(0.004)	(0.004)	(0.005)
	100	0.926	**0.875**	0.965	0.964	0.999	0.925	0.914	0.919	0.932	0.799	0.933	0.932	0.992	0.856	0.694	**0.684**
		(0.008)	(0.018)	(0.006)	(0.006)	(0.007)	(0.010)	(0.016)	(0.016)	(0.002)	(0.004)	(0.001)	(0.001)	(0.004)	(0.004)	(0.004)	(0.005)

Table 4.7: Forecasting results of MSM and (FI)GARCH models. The table shows panel MSE and MAE with standard errors in parentheses relative to naive forecasts of historical volatility by asset market considered at horizons $h = 1, 5, 20, 50, 100$. Entries in **bold** denote the model with the lowest MSE or MAE at each horizon h.

		MSE								MAE							
		Normal innovations				Student-t innovations				Normal innovations				Student-t innovations			
Mkt.	h	GARCH	FIGA	BMSM	LMSM	GARCH	FIGA	BMSM	LMSM	GARCH	FIGA	BMSM	LMSM	GARCH	FIGA	BMSM	LMSM
ST	1	20	23	22	22	21	21	11	10	17	15	22	22	16	14	25	25
	5	18	20	14	15	18	19	9	8	15	13	14	14	13	11	25	25
	20	17	19	10	10	15	17	9	8	14	11	14	14	9	10	25	25
	50	11	18	10	10	13	13	9	8	10	10	14	14	6	9	25	25
	100	7	17	10	10	9	11	9	7	8	9	14	14	4	5	25	25
BO	1	10	9	9	9	8	8	6	6	9	9	9	9	8	9	10	10
	5	9	9	5	5	8	8	6	6	9	9	7	7	8	9	10	10
	20	9	9	4	4	7	8	6	6	9	9	6	6	7	8	10	10
	50	6	8	4	4	3	7	6	6	5	7	6	6	2	7	10	10
	100	1	5	4	4	0	4	6	6	3	5	6	6	0	5	10	10
RE	1	10	9	10	10	9	9	7	7	9	10	11	11	9	10	12	11
	5	9	9	10	10	7	9	6	6	7	9	9	9	7	9	12	11
	20	7	9	9	9	4	8	6	5	8	9	9	9	5	9	12	11
	50	5	10	9	9	5	9	6	5	5	9	9	9	2	9	12	11
	100	4	9	8	8	2	7	6	5	3	9	9	9	2	8	12	11

Table 4.8: Average forecasting accuracy of alternative volatility models. The table shows the number of improvements for single models against historical volatility via the Diebold & Mariano (1995) test for $N = 25$ international stock market indices (ST), $N = 11$ international 10-year government bond indices (BO), $N = 12$ international real estate security indices (RE) at horizons $h = 1, 5, 20, 50, 100$.

4.6.2.1 Single models

Results of the average relative MSE and MAE and corresponding standard errors of the out-of-sample forecasts from the different models are reported in Table 4.7. We find that at short horizons, $h = 1$, GARCH and FIGARCH models obtain lower average MSE and MAE than the BMSM and LMSM models in all three markets. However, the variability of the MSE and MAE in the MSM models are in general lower than those of the (FI)GARCH models. At horizons over 20 days, GARCH models (with Normal or Student-t innovations) show a deterioration in MSE and MAE measures hinting at their inability to accurately forecast volatility at higher horizons. In contrast, MSE and MAE resulting from the FIGARCH models (with Normal or Student-t innovations) are in general lower and more stable across horizons.

With respect to the MSM models (with Normal or Student-t innovations) we find that they produce MSEs and MAEs which are lower than one in stock and real estate markets. We also find that the forecasts from the MSM models are much more homogeneous and less variable across horizons than those of the (FI)GARCH models whose forecasts usually deteriorate as the horizon is increased beyond $h = 20$. Comparing forecasts of the BMSM versus LMSM we find that the models produce qualitatively similar forecasts in terms of MSEs and MAEs.

Diagnosing forecasts from the models with Normal vs. Student-t innovations, we find that neither GARCH nor FIGARCH models produce lower MSEs and MAEs on average over the three markets when Student-t innovations are employed. Results are different in MSM models for which we find Student-t innovations to improve forecasting precision over all three markets in particular at horizons $h \geq 20$. In fact, Table 4.8 shows that, in terms of MAEs, there is a larger number of statistically significant improvements against historical volatility with the BMSM-t and LMSM-t models in comparison to their Gaussian and (FI)GARCH counterparts. Interestingly, the LMSM-t outperforms all other models in all markets in terms of MAEs when $h \geq 50$ and seems to provide for a sizable gain in forecasting accuracy at long horizons.

Note that the MSM models also showed some sensitivity of parameter estimates on the distributional assumptions (Normal vs. Student-t). As it seems, the volatility models react quite differently to different distributions of innovations ε_t: on the one hand, the transition to Student-t was not reflected in remarkable changes of estimated parameters for (FI)GARCH models and their forecasting performance, if anything, slightly deteriorates under fat-tailed innovations. On the other hand, the effect of distributional assumptions on MSM parameters was more pronounced and their forecasting performance appears to be superior under Student-t innovations throughout our samples. Taken together, we see different patterns of interaction of conditional and unconditional distributional properties. This indicates that alternative models may capture different facets of the dependency in second moments so that there would be a potential gain from combining forecasts (a topic explored below).

		MSE									MAE								
		(FI)GARCH			MSM			(FI)GARCH-MSM			(FI)GARCH			MSM			(FI)GARCH-MSM		
	h	CO1	CO2	CO3	CO4	CO5	CO6	CO7	CO8	CO9	CO1	CO2	CO3	CO4	CO5	CO6	CO7	CO8	CO9
ST	1	**0.864**	0.868	0.869	0.936	0.932	0.931	0.877	0.878	0.872	0.792	0.785	0.786	0.830	0.818	0.817	0.750	0.751	**0.746**
		(0.021)	(0.020)	(0.020)	(0.009)	(0.010)	(0.010)	(0.017)	(0.017)	(0.018)	(0.005)	(0.005)	(0.005)	(0.002)	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)
	5	0.909	0.913	0.919	0.946	0.942	0.941	0.906	0.905	**0.905**	0.822	0.810	0.814	0.835	0.821	0.821	0.764	**0.764**	0.764
		(0.012)	(0.012)	(0.012)	(0.008)	(0.009)	(0.009)	(0.013)	(0.013)	(0.013)	(0.004)	(0.004)	(0.004)	(0.002)	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)
	20	**0.905**	0.910	0.921	0.947	0.942	0.942	0.913	0.913	0.910	0.859	0.836	0.845	0.836	0.822	0.822	0.781	**0.780**	0.783
		(0.012)	(0.012)	(0.011)	(0.008)	(0.009)	(0.009)	(0.012)	(0.012)	(0.012)	(0.003)	(0.003)	(0.004)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
	50	**0.912**	0.914	0.915	0.947	0.943	0.943	0.921	0.920	0.915	0.892	0.857	0.866	0.836	0.822	0.822	0.800	**0.798**	0.801
		(0.011)	(0.011)	(0.011)	(0.008)	(0.009)	(0.009)	(0.011)	(0.011)	(0.012)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
	100	0.926	0.924	0.924	0.948	0.943	0.943	0.931	0.928	**0.922**	0.922	0.878	0.887	0.835	0.821	0.821	0.820	**0.811**	0.815
		(0.010)	(0.011)	(0.011)	(0.008)	(0.009)	(0.009)	(0.011)	(0.011)	(0.012)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
BO	1	**0.829**	0.829	0.830	0.947	0.949	0.949	0.842	0.845	0.844	0.763	0.766	0.768	0.851	0.850	0.850	0.721	**0.720**	0.721
		(0.009)	(0.009)	(0.009)	(0.005)	(0.005)	(0.005)	(0.010)	(0.010)	(0.010)	(0.003)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)
	5	**0.833**	0.834	0.835	0.951	0.953	0.953	0.845	0.847	0.847	0.774	0.778	0.780	0.854	0.853	0.853	0.727	**0.726**	0.727
		(0.008)	(0.008)	(0.008)	(0.005)	(0.005)	(0.005)	(0.010)	(0.010)	(0.010)	(0.003)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
	20	**0.844**	0.846	0.848	0.951	0.953	0.953	0.852	0.854	0.852	0.808	0.812	0.816	0.853	0.852	0.852	0.741	**0.740**	0.742
		(0.008)	(0.008)	(0.007)	(0.005)	(0.005)	(0.005)	(0.009)	(0.009)	(0.009)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
	50	0.867	0.870	0.874	0.948	0.951	0.951	0.858	0.861	**0.858**	0.851	0.855	0.863	0.851	0.850	0.850	0.758	**0.757**	0.759
		(0.007)	(0.006)	(0.006)	(0.005)	(0.005)	(0.005)	(0.009)	(0.009)	(0.009)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
	100	0.899	0.901	0.907	0.945	0.948	0.948	0.866	0.869	**0.866**	0.901	0.902	0.915	0.848	0.847	0.847	0.773	**0.772**	0.776
		(0.005)	(0.005)	(0.005)	(0.006)	(0.006)	(0.006)	(0.009)	(0.009)	(0.009)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
RE	1	**0.823**	0.823	0.823	0.881	0.880	0.879	0.828	0.829	0.828	0.653	0.655	0.654	0.696	0.686	0.687	0.596	0.594	**0.590**
		(0.027)	(0.027)	(0.027)	(0.020)	(0.020)	(0.020)	(0.027)	(0.027)	(0.027)	(0.005)	(0.005)	(0.005)	(0.004)	(0.004)	(0.004)	(0.005)	(0.005)	(0.005)
	5	0.850	0.851	0.854	0.896	0.895	0.895	0.850	0.849	**0.849**	0.674	0.676	0.675	0.702	0.691	0.692	0.606	0.602	**0.599**
		(0.023)	(0.023)	(0.022)	(0.018)	(0.018)	(0.018)	(0.024)	(0.024)	(0.024)	(0.005)	(0.005)	(0.005)	(0.004)	(0.004)	(0.004)	(0.005)	(0.005)	(0.005)
	20	0.860	0.860	0.863	0.897	0.895	0.895	0.858	0.857	**0.857**	0.703	0.705	0.706	0.702	0.692	0.692	0.613	0.606	**0.605**
		(0.021)	(0.021)	(0.021)	(0.018)	(0.018)	(0.018)	(0.023)	(0.023)	(0.023)	(0.005)	(0.005)	(0.005)	(0.004)	(0.004)	(0.004)	(0.005)	(0.005)	(0.005)
	50	0.864	0.863	0.865	0.897	0.896	0.896	0.859	0.857	**0.856**	0.727	0.728	0.731	0.701	0.691	0.691	0.614	0.607	**0.607**
		(0.019)	(0.020)	(0.019)	(0.018)	(0.018)	(0.018)	(0.023)	(0.023)	(0.023)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.005)	(0.005)	(0.005)
	100	0.868	0.867	0.868	0.890	0.888	0.888	0.851	0.850	**0.849**	0.750	0.752	0.756	0.699	0.689	0.689	0.614	0.606	**0.606**
		(0.019)	(0.019)	(0.019)	(0.020)	(0.020)	(0.020)	(0.025)	(0.025)	(0.025)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.005)	(0.005)	(0.005)

Table 4.9: Results of forecast combinations from MSM and (FI)GARCH models. The table shows panel MSE and MAE with standard errors in parentheses relative to naive forecasts of historical volatility by asset market considered at horizons $h = 1, 5, 20, 50, 100$. Entries in **bold** denote the model with the lowest MSE or MAE at horizon h.

		MSE									MAE								
		(FI)GARCH			MSM			(FI)GARCH-MSM			(FI)GARCH			MSM			(FI)GARCH-MSM		
	h	CO1	CO2	CO3	CO4	CO5	CO6	CO7	CO8	CO9	CO1	CO2	CO3	CO4	CO5	CO6	CO7	CO8	CO9
	1	23	23	23	20	20	20	23	23	23	15	16	16	25	25	25	24	24	24
	5	19	18	18	11	11	11	22	22	22	14	16	15	25	25	25	24	24	24
ST	20	19	20	19	10	10	10	20	20	20	12	14	14	25	25	25	23	23	24
	50	19	21	21	10	10	10	19	19	20	9	15	15	25	25	25	24	24	24
	100	19	22	21	10	10	10	18	18	20	9	14	14	25	25	25	24	24	24
	1	9	10	10	7	7	7	8	8	8	9	9	9	10	10	10	11	11	11
	5	9	9	9	6	6	6	8	8	9	9	9	9	10	10	10	11	11	11
BO	20	9	9	9	6	6	6	7	7	9	9	9	8	10	10	10	11	11	11
	50	8	9	8	6	6	6	7	7	8	7	7	7	10	10	10	11	11	11
	100	5	6	6	6	6	6	7	7	8	6	5	5	10	10	10	11	11	10
	1	9	9	9	9	9	9	9	9	9	10	10	9	12	12	12	12	12	12
	5	9	9	8	8	6	6	10	10	10	9	9	9	12	12	12	12	12	12
RE	20	9	9	9	8	6	6	8	8	8	9	9	9	12	12	12	12	12	12
	50	9	10	10	8	6	6	8	8	9	9	10	9	12	12	12	12	12	12
	100	9	10	9	8	6	6	7	6	7	9	10	8	12	12	12	12	12	12

Table 4.10: Average forecasting accuracy of combination models. The table shows the number of improvements for combination models against historical volatility via the Diebold & Mariano (1995) test for $N = 25$ international stock market indices (ST), $N = 11$ international 10-year government bond indices (BO), $N = 12$ international real estate security indices (RE) at horizons $h = 1, 5, 20, 50, 100$. The combination models are CO1:FIGARCH+FIGARCH-t, CO2:GARCH+FIGARCH+FIGARCH-t, CO3:GARCH+GARCH-t+FIGARCH+FIGARCH-t, CO4:BMSM+LMSM-t, CO5:BMSM-t+BMSM+LMSM-t, CO6:BMSM-t+BMSM+LMSM-t+LMSM, CO7:FIGARCH+LMSM-t, CO8:BMSM-t+LMSM-t+FIGARCH, CO9:BMSM-t+LMSM-t+FIGARCH+FIGARCH-t.

4.6.2.2 Combined forecasts

A particular insight from the methodological literature on forecasting is that it is often preferable to combine alternative forecasts in a linear fashion and thereby obtain a new predictor (cf. Granger (1989), Aiolfi & Timmermann (2006)). We analyze forecast complementarities of (FI)GARCH and MSM models by addressing the performance of combined forecasts. The forecast combinations are computed by assigning each single forecast a weight equal to a model's empirical frequency of minimizing the absolute or squared forecast error over realized past forecasts. To take account of structural variation we update the weighting scheme over the 20 most recent forecast errors so that despite linear combinations of forecasts, the influence of various components is allowed to change over time via flexible weights (for details see Appendix D).

Tables 4.9 and 4.10 report the results of the forecasting combination exercise. Our forecast combination strategy consists in considering whether forecast combinations of (FI)GARCH models, MSM models or both families of models lead to an improvement upon forecasts from single models. Our results put forward that they generally do. This is in line with the empirical result of Lux & Kaizoji (2007) that the rank correlations of forecasts obtained from certain volatility models are quite low, hinting at room for improvement upon forecasts from single models with forecast combinations.

We start by considering the results of the forecast combinations of (FI)GARCH models (Tables 4.9 and 4.10). Three different combination strategies are presented denoted CO1, CO2 and CO3. The first combination strategy (CO1) is given by the (weighted) linear combination between FIGARCH and FIGARCH-t forecasts. The latter combination gives an idea how FIGARCH forecasts can be complemented by considering a fat tailed distribution. We find an improvement in terms of MSEs from CO1 over single forecasts of the FIGARCH and the FIGARCH-t models at horizons $h = 20$, $h = 50$ and $h = 100$ in the different asset markets under inspection. The same result applies when we consider the forecast combinations GARCH+FIGARCH+FIGARCH-t (CO2) and GARCH+GARCH-t+FIGARCH+FIGARCH-t (CO3) although only for stock and real estate markets at higher horizons. CO2 and CO3 hint at how forecast could be improved when considering short memory along with long memory and fat tails. In terms of MAEs, CO1 can improve upon forecasts of the single (FI)GARCH specifications at all horizons in all markets. The forecast combinations CO2 and CO3 can also improve upon forecasts of the single (FI)GARCH models in terms of MAEs at all horizons in stock markets and real estate markets.

The second set of forecasts combinations considered result from the MSM models which are given by BMSM+LMSM-t (CO4), BMSM+BMSM-t+LMSM-t (CO5) and BMSM+BMSM-t+LMSM+LMSM-t (CO6). The latter forecast combinations allow to analyze the complementarities that arise when one combines models with different assumptions regarding the distribution of the multifractal parameter as well as the tails of the innovations. The results indicate that there is an improvement upon forecasts of single models in all three markets particularly against those obtained from the MSM models with Normal innovations both in terms of MSEs and MAEs. The improvement obtained from forecasts combinations is immediately evident in the case of bond markets where the MSEs and MAEs become less than one. We also find that the combination of forecasts in the MSM models does not translate into more variable MSEs or MAEs, a feature that speaks in favor of optimally combining single models' ingredients.

The last set of forecasts combinations examined are those resulting from MSM models and FIGARCH models. The combinatorial strategies are given by FIGARCH+LMSM-t (CO7), BMSM-t+LMSM-t+FIGARCH (CO8), BMSM-t+LMSM-t+FIGARCH+FIGARCH-t (CO9). The latter forecast combinations allow to analyze the complementarities of two families of volatility models which assume two distinct distributions of the innovations along with different characteristics for the latent volatility process: (FI)GARCH models which account for short/long memory and autoregressive components and MSM models which account for multifractality, regime-switching and apparent long memory. Interestingly, the improvement upon forecasts of single models from the MSM-FIGARCH strategy is somewhat more evident than in the previous strategies. In terms of MSEs, for instance, we generally find a statistically significant improvement over historical volatility more frequently in stock, bond and real estate markets when comparing CO7, CO8, CO9 against single models (Tables 4.8 and 4.10). We also find that the variability of the combinations of MSM and FIGARCH models does not change too much with respect to single models.

Summing up, we find that the forecast combinations between FIGARCH, MSM or both types of models lead to improvements in forecasting accuracy upon forecasts of single models. In particular, we find that the forecasting strategy FIGARCH-MSM seems to be the most successful one in relation to single models or the other combination strategies - a feature that could be exploited in real time for risk management strategies. The particular usefulness of this combination strategy appears plausible given the flexibility of the MSM model in capturing varying degrees of long-term dependence and the added flexibility of FIGARCH for short horizon dependencies via its AR and MA parameters which are not accounted for in MSM models.

4.7 Concluding remarks

In this Chapter we examined the in-sample and out-of-sample performance of volatility models that incorporate different features characterizing the latent volatility process (long vs. short memory, regime-switching and multifractality) as well as distributional regularities of returns (*fat tails*). More precisely, we consider two major sets of 'competitors', namely, volatility specifications from the new MSM models and the popular (FI)GARCH models along with Normal or Student-t innovations. We introduce a new member to the family of MSM models that accounts for Student-t innovations. This new model allows to study whether there is an improvement in forecasting accuracy vis-à-vis the existing MSM model with Normal innovations and the (FI)GARCH models with Normal or Student-t innovations. The MSM-t model can be estimated either via ML or GMM. The suitability of ML and GMM estimation for MSM models with Student-t innovations is analyzed via Monte Carlo simulations. We conduct a comprehensive empirical study using country data on all-share equity indices, 10-year government bond indices and real estate security indices. In addition, we explore whether we may improve forecasting accuracy by constructing forecast combinations of the various models under inspection.

In-sample Monte Carlo experiments of the MSM-t model behave similarly like the MSM model with Normal innovations indicating that ML and GMM estimation are both suitable for estimating the new Binomial (ML and GMM) and Lognormal (GMM) MSM-t models. The out-of-sample Monte Carlo analysis shows that best linear forecasts are qualitatively similar to optimal forecasts so that the computationally advantageous strategy of GMM estimation of parameters plus linear forecasts can be adapted without much loss of efficiency. The in-sample empirical analysis shows that there is strong evidence of long memory and multifractality in international equity markets, bond markets and real estate markets as well as evidence of *fat tails*. The out-of-sample empirical analysis puts forward that GARCH models are less precise in accurately forecasting volatility for horizons greater than 20 days. This problem is not encountered once long-memory is incorporated via the FIGARCH model which produces MSEs and MAEs that are generally less than one.

The recently introduced MSM models with Normal innovations produce forecasts that improve upon historical volatility, but are in some cases inferior to FIGARCH with Normal innovations. However, two additional observations shed more positive light on the capabilities of MSM models for forecasting volatility. First, adding *fat tails* typically improves forecasts from MSM models while the same change of specification has, if anything, a negative effect for the (FI)GARCH models. While MSM-t is somewhat inferior to FIGARCH under the MSE criterion, it is superior under the MAE criterion at long horizons across all markets. Second, our forecasting combination exercise showed particularly sizable gains from combining FIGARCH and MSM in various ways. Therefore, both models appear to capture somewhat different facets of the latent volatility and can be sensibly used in tandem to improve upon forecasts of single models.

Conclusions

This thesis has analyzed different facets of international asset prices. Each Chapter was devoted to studying specific issues not scrutinized in the finance literature so far while providing useful results for the general understanding of international asset price fluctuations. Our concluding remarks will focus on the main policy implications that can be drawn out of our comprehensive empirical analysis of international asset prices in light of the current world financial context. In addition, we will highlight some of the main results that fall in the intersection of the 'mainstream' asset pricing literature and the behavioral asset pricing literature as well as topics that could be analyzed deeper in future research.

First, we find that there are clear differences between the in-sample and out-of-sample properties of international stock market returns between developed and emerging financial markets. While the former show a clearer relationship with macroeconomic (equilibrium) relations, the latter suggest a strong relationship with (time-varying) risk-premia. These results are intuitive as emerging markets have much more impediments to accurate price discovery (e.g. delays), less transparent data construction, lack of prudential rules, political and exchange rate instability, etc. which may affect the relationship with macroeconomic factors non-trivially. A common result with respect to both developed and emerging stock market returns is that a strong evidence of predictability from particular variables does not imply that the models will also perform well out-of-sample. The sample of data in question might be characterized by too many underlying factors such as regime changes, parameter uncertainty, model selection, heterogeneity, etc. Therefore, analysts should be careful in drawing conclusions about the applicability of asset pricing models for forecasting given some 'promising' in-sample results. We argue that recursive estimation and forecast combinations of both ‘theoretical’ and ‘atheoretical’ models can lead to a clearer understanding of international equity price fluctuations. In general, our results suggest that forward looking and combinatorial rather than retrospective and single market analyses should be put into practice more often to circumvent spurious conclusions and to possibly prevent future financial crises.

Second, we find that real estate security prices have a strong empirical relationship with equity and bond prices in the long-run and that the relationship is homogeneous amongst international markets. This finding hints at financial integration between these markets both within and across the inspected cross-section members. Interestingly, equity volatility and bond returns have predictive power for international real estate returns hinting again at the strong relationship with equity and bond markets. World asset volatility and contemporaneous equity returns also explain returns of real estate securities. These finding suggest that analysts can better understand and predict real estate security price fluctuations by studying their strong empirical relationship with equity and bond markets as well as global risk variables. Given the general lack of attention that has been given to (international) real estate security markets, our results highlight some important empirical determinants that may help prevent future crises in these markets.

Third, we find that monetary shocks in the US can be transmitted abroad. In particular, we find that there is evidence of a statistically significant impact of US monetary policy on the equity markets of Mexico and Canada hinting at a spillover effect to nearby countries. The impact is also found to be statistically significant in equity markets that are financially integrated to the US and countries with inflation targeting. Moreover, we find that US monetary policy shocks can have a significant impact on world equity and bond markets on average. However, we find some evidence of instability in the empirical relationship between US monetary policy and international asset prices. A closer look at the response of international asset prices to US monetary policy reveals that the aggregate impact is increasing on absolute terms over time. There is also evidence that there is an upward trend in the ‘goodness’ of fit of the effect of US monetary policy on international equity and real estate markets. The latter results speak in favor of more coordinated actions of monetary authorities around the world (specially those with inflation targeting or close to the US) when conducting monetary policy. This would allow market participants and governments to deal better with the negative effects of financial crises.

Fourth, we obtain that 'behavioral' volatility models that incorporate different features characterizing the stylized facts of asset prices such as fractality, regime-switching and fat tails combined with more 'mainstream' volatility models of the GARCH legacy can lead to better volatility forecasts in international asset markets. The latter results are important to strengthen risk management systems of international financial markets. For instance, analysts could use forecast combinations of various volatility models to formulate better Value-at-Risk frameworks and thus undertake investments that lead to more stable risk-adjusted returns. This would potentially minimize the propensity to future crises.

The findings in this thesis also show some points of intersection between the macro-finance and the behavioral finance stream. In particular, the findings of regime changes in international asset markets, heterogeneity in short-run fluctuations of international asset prices but homogeneous long-run relationships, contagion effects (monetary policy shocks and shocks to equity markets), superior forecasting performance of asset returns via combinations of 'theoretical' (equilibrium pricing) and 'atheoretical' (random walk, autoregressive) models, superior forecasting performance of volatility via combinations of 'mainstream' (GARCH and FIGARCH) and 'behavioral' (MSM) models can be easily seen as common ground in the two streams. We see the perspective of building a bridge between the two streams by designing methods that could be useful to both lines of thought such as the use of recursive estimation, aggregation under parameter heterogeneity, analyzing contagion effects, combination strategies, modeling multifractality and fat tails, increasing robustness via panel data, amongst others.

There are also some recommendations that can be drawn out of the empirical results of this work. More precisely we recommend: (i) forward looking and aggregate rather than retrospective and single market analyses, (ii) a closer attention to real estate assets and their empirical determinants, (iii) more coordinated actions of monetary authorities around the world when conducting policy, and (iv) combinatorial strategies to formulate better Value-at-Risk frameworks and thus undertake more stable and prudent investments.

This thesis also raises some questions for future research. For instance, a more detailed analysis with respect to the relationship between time-varying risk premia and stock returns of emerging markets would uncover whether the risk-return relationship is stronger than (say) the relationship with domestic and/or global variables. Another interesting research question is whether profitable trading opportunities exist within the real estate security market given its relatively recent existence and inefficiencies found at the international level. This could be done via statistical arbitrage strategies. Given our results on the increasing impact of US monetary policy shocks on international asset markets, it would also be interesting to analyze whether there is an empirical relationship when augmenting different countries' Taylor rule with the US monetary policy rate. Last but not least, it seems relevant in the current financial stability context to analyze which volatility models lead to better risk management practices. For instance, one could study the relative performance of different volatility models for Value-at-Risk via Monte Carlo simulations and empirical applications.

Appendix A

Appendix to Chapter 1

A.1 Representative investor problem

A representative investor faces the maximization problem,

$$\text{Max;} \quad E_0 \left[\sum_{t=0}^{\infty} \delta^t U(C_t) \right],$$

subject to

$$C_t + \nu_{t+1} P_t = W_t + \nu_t X_t,$$

where C_t and W_t are the agent's consumption and wealth at period t, respectively, P_t is the *ex-dividend* price of the stock at period t and X_t is the payoff the investor receives at period t. Formally, let $X_t = P_t + D_t$, where D_t denotes the dividend payed during period t. Lastly, ν_t denotes the number of shares held at the *beginning* of period t. The Lagrangean is given by,

$$L_t = E_0 \sum_{t=0}^{\infty} \delta^t \left[U(C_t) - \lambda_t \left\{ C_t + \nu_{t+1} P_t - W_t - \nu_t X_t \right\} \right],$$

and the first order conditions,

$$\delta^{-t} \frac{\partial L_t}{\partial C_t} = U'(C(t)) - \lambda_t = 0, \; \delta^{-t} \frac{\partial L_t}{\partial \nu_{t+1}} = -\lambda_t P_t + \delta E_t \lambda_{t+1} X_{t+1} = 0. \quad \text{(A.1)}$$

Using (A.1) yields (1.1), where $M_{t+1} = \delta(U'(C_{t+1})/U'(C_t))$. Assuming that M_t and $\tilde{X}_t = P_{t-1}^{-1}X_t$ are jointly log normal, log-linearizing (1.1) and rearranging yields,

$$p_t = E_t\left[m_{t+1} + x_{t+1}\right] + (1/2)\,\mathrm{Var}_t\left[m_{t+1} + \tilde{x}_{t+1}\right], \tag{A.2}$$

where $p_t = \ln P_t$, $m_{t+1} = \ln M_{t+1}$, $x_{t+1} = \ln\left(P_{t+1} + D_{t+1}\right)$ and $\tilde{x}_{t+1} = x_{t+1} - p_t$. Defining $\mu_{t+1} = (1/2)\,\mathrm{Var}_t\left[m_{t+1} + \tilde{x}_{t+1}\right] = 1/2\sigma^2_{\mu,t+1}$ we obtain (1.2). The term x_{t+1} can also be written as $x_{t+1} = \ln\left(1 + e^{p_{t+1}-d_{t+1}}\right) + d_{t+1}$ where $d_t = \ln D_t$. A Taylor Series expansion to the term $\ln\left(1 + e^{p_{t+1}-d_{t+1}}\right)$ around an equilibrium point $P/D = e^{p-d}$ yields,

$$\begin{aligned}
\ln\left(1 + e^{p_{t+1}-d_{t+1}}\right) &\approx \ln\left(1 + \left(\frac{P}{D}\right)\right) + \frac{1}{1+P/D}\cdot\frac{1}{D}\cdot P\cdot\left(p_{t+1} - p\right) \\
&\quad + \frac{1}{1+P/D}\cdot -\frac{P}{D^2}\cdot D\cdot\left(d_{t+1} - d\right) \\
&= \ln\left(1 + \left(\frac{P}{D}\right)\right) - \left(\frac{P/D}{1+P/D}\right)\ln\left(\frac{P}{D}\right) \\
&\quad + \left(\frac{P/D}{1+P/D}\right)\left(p_{t+1} - d_{t+1}\right) = c + \theta(p_{t+1} - d_{t+1}),
\end{aligned} \tag{A.3}$$

with $c = -\ln(\theta) - (1-\theta)\ln(1/\theta - 1)$ and $\theta = 1/(1 + D/P)$. Using the latter approximation in x_{t+1} and rearranging yields $x_{t+1} = c + \theta p_{t+1} + (1-\theta)d_{t+1}$. Substituting in (1.2) yields,

$$p_t = c + E_t\left[\mu_{t+1} + \theta p_{t+1} + (1-\theta)d_{t+1} + m_{t+1}\right]. \tag{A.4}$$

Define $a_t = \ln A_t$ where A_t is used to denote earnings per share at period t. Adding $(1-\theta)(a_{t+1} - a_t)$ from both sides of (A.4) and rearranging we obtain,

$$p_t - a_t = c + E_t\left[\mu_{t+1} + (\theta - 1)(a_{t+1} - d_{t+1}) + \Delta a_{t+1} + \theta(p_{t+1} - a_{t+1}) + m_{t+1}\right].$$

Solving forward for $p_t - a_t$ and by the law of iterated expectations $E_t\left[E_{t+1}\left[Z\right]\right] = E_t\left[Z\right]$,

$$p_t - a_t = \frac{c}{1-\theta} + E_t\sum_{j=1}^{\infty}\theta^{j-1}\left[\mu_{t+j} + (\theta-1)(a_{t+j} - d_{t+j}) + \Delta a_{t+j} + m_{t+j}\right] + b_t \tag{A.5}$$

Imposing the (no bubble) transversality condition $b_t = \lim_{j\to\infty} \theta^j E_t(p_{t+j} - a_{t+j}) = 0$ in (A.5) we obtain equation (1.3). We start by defining $g_t = a_t - d_t$ and presume that risk premia μ_t, earnings growth Δa_t and g_t behave according to the following models:

$$\mu_{t+1} = \bar{\mu}, \tag{A.6}$$

$$\Delta a_{t+1} = \epsilon_{a,t+1}, \tag{A.7}$$

$$g_{t+1} = \eta g_t + \epsilon_{g,t+1},\ \eta < 1. \tag{A.8}$$

Furthermore, we assume that the log SDF can be specified by,

$$-m_{t+1} = \bar{m} + h_t + y_t + s_t + v_t + \epsilon_{m,t+1} \tag{A.9}$$

$$h_{t+1} = \phi_1 h_t + \epsilon_{h,t+1}, \tag{A.10}$$

$$y_{t+1} = \phi_2 y_t + \epsilon_{y,t+1}, \tag{A.11}$$

$$s_{t+1} = \phi_3 s_t + \epsilon_{s,t+1}, \tag{A.12}$$

$$v_{t+1} = \phi_4 v_t + \epsilon_{v,t+1}, \tag{A.13}$$

$$\epsilon_{m,t+1} = \varphi \epsilon_{v,t+1},\ \phi_i < 1,\ i = 1,2,3,4,$$

where we let $\epsilon_{\bullet,t} \sim \text{iid}(0, \sigma^2_{\bullet})$, $\bullet = a, g, h, g, s, v$ for simplicity. Moreover, h_t, y_t and s_t are observable $I(0)$ equilibrium pricing factors and v_t is some 'unobservable' $I(0)$ factor. We work with the negative of the log stochastic discount factor for convenience given that the log SDF is the negative of the log return (see Campbell *et al.* (1997)). A motivation for non-fundamental or non-observable component of stock prices can be found in Lee (1998). Noting that for an AR(1) process, $z_{t+1} = \phi z_t + \epsilon_{zt+1}$ we have that $z_{t+i} = \phi^i z_t + \sum_{j=1}^{i} \phi^{i-j} \epsilon_{zt+j}$ replacing (A.6)-(A.13) in (1.3) we obtain,

$$p_t - a_t = \frac{\hat{\mu} - \bar{m}}{1-\theta} + \frac{\eta(\theta-1)g_t}{1-\eta\theta} - \frac{h_t}{1-\phi_1\theta} - \frac{y_t}{1-\phi_2\theta} - \frac{s_t}{1-\phi_3\theta} - \frac{v_t}{1-\phi_4\theta},$$

where $\hat{\mu} = c + \bar{\mu}$. Defining $\dot{\mu} = (\hat{\mu} - \bar{m})/(1-\theta)$, $\pi_1 = \eta(\theta-1)/(1-\eta\theta)$, $\pi_2 = 1/(1-\phi_1\theta)$, $\pi_3 = 1/(1-\phi_2\theta)$, $\pi_4 = 1/(1-\phi_3\theta)$ and $\pi_5 = 1/(1-\phi_4\theta)$. Solving for v_t and plugging in (A.13) we obtain,

$$\begin{aligned} &-(p_{t+1} - a_{t+1}) + \dot{\mu} + \pi_1 g_{t+1} - \pi_2 h_{t+1} - \pi_3 y_{t+1} - \pi_4 s_{t+1} \\ &= -\phi_4(p_t - a_t) + \phi_4\dot{\mu} + \phi_4\pi_1 g_t - \phi_4\pi_2 h_t - \phi_4\pi_3 y_t - \phi_4\pi_4 s_t + \pi_5\epsilon_{v,t+1}. \end{aligned}$$

Using (A.8) and (A.10)-(A.12) to replace for g_{t+1}, h_{t+1}, y_{t+1}, s_{t+1} and solving for $(p_{t+1} - a_{t+1})$ above we obtain,

$$\begin{aligned}(p_{t+1} - a_{t+1}) &= (1-\phi_4)\dot{\mu} + \phi_4(p_t - a_t) + (\eta - \phi_4)\pi_1 g_t \\ &+(\phi_4-\phi_1)\pi_2 h_t + (\phi_4-\phi_2)\pi_3 y_t + (\phi_4-\phi_3)\pi_4 s_t \\ &+ \underbrace{(\pi_1\epsilon_{g,t+1} + \pi_2\epsilon_{h,t+1} + \pi_3\epsilon_{y,t+1} + \pi_4\epsilon_{s,t+1} + \pi_5\epsilon_{v,t+1})}_{\epsilon_{pa,t+1}}.\end{aligned} \tag{A.14}$$

Now write stock prices changes as,

$$(p_{t+1} - p_t) = (p_{t+1} - a_{t+1}) - (p_t - a_t) + (a_{t+1} - a_t). \tag{A.15}$$

Replacing (A.14) in (A.15) using $\Delta a_{t+1} = \epsilon_{a,t+1}$ we obtain,

$$\begin{aligned}\Delta p_{t+1} &= (1-\phi_4)\dot{\mu} + (\phi_4 - 1)(p_t - a_t) + (\eta - \phi_4)\pi_1 g_t \\ &+(\phi_4-\phi_1)\pi_2 h_t + (\phi_4-\phi_2)\pi_3 y_t + (\phi_4-\phi_3)\pi_4 s_t + u_{t+1},\end{aligned}$$

where $u_{t+1} = \epsilon_{pa,t+1} + \epsilon_{a,t+1}$ and since all errors are *iid* by construction we let $u_t \sim \mathrm{iid}(0, \sigma_u^2)$. Similarly,

$$\Delta p_{t+1} = \mu + \alpha_1 q_t + \alpha_2 g_t + \alpha_3 h_t + \alpha_4 y_t + \alpha_5 s_t + u_{t+1}, \tag{A.16}$$

where we define $q_t = (p_t - a_t)$ and the parameters are given by $\mu = (1-\phi_4)\dot{\mu}$, $\alpha_1 = (\phi_4 - 1)$, $\alpha_2 = (\eta - \phi_4)\pi_1$, $\alpha_3 = (\phi_4 - \phi_1)\pi_2$, $\alpha_4 = (\phi_4 - \phi_2)\pi_3$ and $\alpha_5 = (\phi_4 - \phi_3)\pi_4$.

A.2 A VECM representation of the model

Consider a VAR(p) representation for the (7×1) vector $z_{it} = (p_{it}, a_{it}, d_{it}, p_t^m, r_{it}^s, r_{it}^l, r_t^{fn})'$,

$$z_{it} = \delta_{it} + A_{1,i} z_{it-1} + A_{2,i} z_{it-2} + \ldots + A_{p,i} z_{it-p} + u_{it}, \tag{A.17}$$

with $A_{j,i}$, $j = 1, \ldots, p$, denoting $K \times K$ matrices of coefficients, δ_{it} is a linear trend term and u_{it} is a $K \times 1$ vector white noise process, $u_{it} \sim (0, \Omega_i)$. The corresponding VECM($p-1$) is

$$\Delta z_{it} = \mu_i + \Pi_i z_{it-1} + \Gamma_{1,i}\Delta z_{it-1} + \ldots + \Gamma_{p-1,i}\Delta z_{it-p+1} + u_{it}, \tag{A.18}$$

where $\mu_i = \Delta\delta_{it}$, $\Pi_i = (A_{1,i} + \ldots + A_{p,i} - I_K) = \alpha_i\beta$. The assumptions on potential equilibrium relationships linking the variables in z_{it} lead to over-identifying restrictions in the vector of error correction (EC) terms $\Pi_i z_{it-1}$,

$$\Pi_i = \alpha_i \cdot \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}. \tag{A.19}$$

Hence, five cointegrating relationships ($r = 1, ..., 5$) are formalized in (A.19) namely, the stationarity of the log PE, ED and PPM ratio, the term spread and the interest rate differential. Picking out the first equation of the VECM leads to (1.7).

A.3 Descriptions of in-sample and out-of-sample tests

A.3.1 In-sample tests

1. *Single equation models*: At the single market level our analysis consists of testing $\hat{\alpha}_{1r,i} = 0$, $i = 1, \ldots, N$, $r = 1, \ldots, 5$, by means of robust covariance estimators for $h = 1$ (White (1980)) and for $h = 3, 6$ (Newey & West (1987)).

2. *Cross sectional Wald test*: To infer on joint significance of $\hat{\alpha}_{1r,i} = 0$, $i = 1, ..., N$, we employ a stacked version of single market equations obtaining a 'block diagonal' regression design and determine a Wald-statistic

$$\lambda_c = (G\hat{\boldsymbol{\alpha}}_p)' \left[G\widehat{\mathrm{Cov}}_c \left[\hat{\boldsymbol{\alpha}}_p\right] G' \right]^{-1} (G\hat{\boldsymbol{\alpha}}_p), \; c = 1, 2, \tag{A.20}$$

where G is a $(N \times N(K+R+1))$ matrix formalizing the null hypothesis $H_{0,r}$: $\hat{\alpha}_{1r,i} = 0$, $i = 1, ..., N$. Moreover, $\hat{\boldsymbol{\alpha}}_p$ is a vector of dimension $N(K + R + 1) \times 1$, with $K + R + 1 = 13$, collecting the parameters of the cross sectional model in stacked form. In (A.20), $\widehat{\text{Cov}}_1[\hat{\boldsymbol{\alpha}}_p]$ ($\text{Cov}_2[\hat{\boldsymbol{\alpha}}_p] = h \cdot \text{Cov}_1[\hat{\boldsymbol{\alpha}}_p]$) denotes White (1980) (Newey & West (1987)) robust covariance estimator for $h = 1$ ($h = 3, 6$). Critical values for the Wald statistic are taken from the $\chi^2(N)$ distribution.

3. *Mean group inference:* The mean group (MG) estimator proposed by Pesaran & Smith (1995) and corresponding t-statistics are,

$$\bar{\alpha}_{r,MG} = \frac{1}{N}\sum_{i=1}^{N}\hat{\alpha}_{1r,i},\ t_{r,MG} = \bar{\alpha}_{r,MG}/\sqrt{\widehat{\text{Var}}\left[\bar{\alpha}_{r,MG}\right]},$$
$$\widehat{\text{Var}}\left(\bar{\alpha}_{r,MG}\right) = \sum_{i=1}^{N}\left(\hat{\alpha}_{1r,i} - \bar{\alpha}_{r,MG}\right)^2/(N(N-1)),\ r = 1, ..., 5. \quad \text{(A.21)}$$

A.3.2 Out-of-sample tests

To reduce complexity of notation we skip the horizon dimension (h) below when referring to forecasts or forecast errors.

1. *Serial correlation:* The auxiliary regression of the sequences $\hat{u}_{i\tau}$ for ($h = 1$) is given by

$$\hat{u}_{i\tau} = c + \kappa_1\hat{u}_{i\tau-1} + \ldots + \kappa_l\hat{u}_{i\tau-l} + v_{i\tau}, \tau = T_{min} + 1, \ldots, T_i, \quad \text{(A.22)}$$

We test the hypothesis $H_0 : \kappa_1 = \kappa_2 = \ldots = \kappa_l = 0$ by means of a Wald-test

$$\omega_{i,l} = \hat{\kappa}'(\text{Cov}[\hat{\kappa}])^{-1}\hat{\kappa} \xrightarrow{d} \chi^2(l), \quad \text{(A.23)}$$

where $\hat{\kappa} = (\hat{\kappa}_1, \hat{\kappa}_2, \ldots, \hat{\kappa}_l)'$. $\text{Cov}[\hat{\kappa}]$ is consistent under heteroskedasticity and lag orders $l = 1, 4$ are considered. The overall hypothesis of no serial correlation at the aggregate level for ($h = 1$) is tested by aggregating log p-values, i.e.

$$\Omega_l = 2\sum_{i=1}^{N}\ln(\psi(\omega_{i,l})) \sim \chi^2(2N), \quad \text{(A.24)}$$

where $\psi(\bullet)$ is the complement of the $\chi^2(l)$ distribution function (Fisher (1932)). Owing to contemporaneous correlation over stock markets the $\chi^2(2N)$ distribution in (A.24) might not hold. However, since all the models are applied to the same cross section the Fisher test is still useful for model comparison.

2. *Distributional features:* The Henrikkson–Merton statistic is

$$\begin{aligned}\mathrm{hm}_i &= \left(\sum_\tau \mathcal{I}(\Delta p_{i\tau} \geq 0)\right)^{-1} \sum_\tau \mathcal{I}(\Delta \hat{p}_{i\tau} \geq 0 \wedge \Delta p_{i\tau} \geq 0) \\ &+ \left(\sum_\tau \mathcal{I}(\Delta p_{i\tau} < 0)\right)^{-1} \sum_\tau \mathcal{I}(\Delta \hat{p}_{i\tau} < 0 \wedge \Delta p_{i\tau} < 0). \quad \text{(A.25)}\end{aligned}$$

where $\mathcal{I}(\bullet)$ is an indicator function. The hm_i statistic is the sum of the conditional probabilities of correctly forecasting a positive or negative price change. We use critical values simulated from 10000 sequences of bivariate Gaussian variables taking the number of available predictions per stock market into account. An aggregated Henrikkson Merton statistic ($\mathcal{HM}$) is computed from return forecasts over both data dimensions.

3. *Absolute and relative average accuracy:* Let '0' and '$\bullet$' indicate a benchmark and a particular competing model, respectively. The MAFE of the benchmark is

$$\bar{d}_i(0) = \mathcal{T}_i^{-1} \sum_\tau d_{i\tau}(0),\ d_{i\tau}(0) = |\hat{u}_{i\tau}(0)|. \quad \text{(A.26)}$$

The relative MAFE is

$$\overline{dr}_i(\bullet) = \frac{\bar{d}_i(\bullet)}{\bar{d}_i(0)},\ \bar{d}_i(\bullet) = \mathcal{T}_i^{-1} \sum_\tau d_{i\tau}(\bullet). \quad \text{(A.27)}$$

The variance of this ratio is

$$\begin{aligned}\widehat{\mathrm{Var}}[\overline{dr}_i(\bullet)] &= \frac{\widehat{\mathrm{Var}}[\bar{d}_i(\bullet)]}{\bar{d}_i(0)^2} + \widehat{\mathrm{Var}}[\bar{d}_i(0)]\left(\frac{\bar{d}_i(\bullet)}{\bar{d}_i(0)^2}\right)^2 \\ &\quad -2\widehat{\mathrm{Cov}}[\bar{d}_i(\bullet), \bar{d}_i(0)]\frac{\bar{d}_i(\bullet)}{\bar{d}_i(0)^3}, \quad \text{(A.28)}\end{aligned}$$

where empirical (co)variances of $\bar{d}_i(0)$ and $\bar{d}_i(\bullet)$, are determined in the usual way. In addition to single market MAFEs, we determine overall MAFE measures as in (A.26) and (A.27) over both data dimensions. The respective statistics are denoted $\bar{d}(0)$ and $\mathcal{DR}$.

4. *Outperformance of the benchmark model:* To contrast a particular model against a benchmark we consider the frequency

$$\mathcal{P} = \mathcal{T}^{-1} \sum_{i,\tau} \mathcal{I}(d_{i\tau}(\bullet) < d_{i\tau}(0)), \ \mathcal{T} = \sum_{i} \mathcal{T}_i. \tag{A.29}$$

   Under equal model accuracy $\mathcal{P} \in [0.5 \pm 2\sqrt{0.25/\mathcal{T}}]$ with 5% significance. Moreover, we count the relative MAFEs less than unity, $\mathcal{S} = \sum_i \mathcal{I}(\overline{dr}_i < 1)$. Under equal model performance $\mathcal{S}$ can be seen as the number of successes over N draws from a binomial distribution with $p = 0.5$. Some selected binomial success probabilities for $N = 12$ are: $p(\mathcal{S} \geq 11) = 3.174\text{E-}03$, $p(\mathcal{S} \geq 10) = 0.019$, $p(\mathcal{S} \geq 9) = 0.073$ while for $N = 14$ are: $p(\mathcal{S} \geq 12) = 6.47\text{E-}03$, $p(\mathcal{S} \geq 11) = 0.029$, $p(\mathcal{S} \geq 10) = 0.090$.

5. *Testing for a unit ratio:* The panel statistic to test $H_0 : \text{dr} = 1$ against $H_1 : \text{dr} < 1$ is

$$\mathcal{TR} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} (\overline{dr}_i - 1)/\sqrt{\widehat{\text{Var}}[\overline{dr}_i]} \approx N(0,1). \tag{A.30}$$

A.4 Forecast combinations

Let $\Delta\hat{p}_{i\tau}(1), \Delta\hat{p}_{i\tau}(2), \ldots, \Delta\hat{p}_{i\tau}(M)$ denote a set of predictors for $\Delta p_{i\tau}$ from model m. We assign each single forecast a weight equal to a models' empirical frequency of minimizing the absolute forecast error over realized errors available in the forecast origin $\tau - 1$. Formally,

$$\Delta\hat{p}_{i\tau}(\text{CO}\bullet) = \sum_{m=1}^{M} g_{i\tau}(m)\Delta\hat{p}_{i\tau}(m), \tag{A.31}$$

where

$$g_{i\tau}(m) = (\tau - T_{min} - h - 1)^{-1} \sum_{v=T_{min}+1}^{\tau-1-h} \mathcal{I}(d_{iv}(m) = \min_j d_{iv}(j), \ j = 1, \ldots, M), \tag{A.32}$$

with $d_{i\tau} = |\hat{u}_{i\tau}|$ and $\mathcal{I}(.)$ denoting an indicator function. To update the weighting scheme according to more recent performance measures, we adopt a second weighting with $\tilde{g}_{i,\tau}(m)$ determined over the 20 most recent forecast errors, i.e.

$$\Delta\hat{p}_{i\tau}(\widetilde{\text{CO}\bullet}) = \sum_{m=1}^{M} \tilde{g}_{i\tau}(m)\Delta\hat{p}_{i\tau}(m), \tag{A.33}$$

where

$$\tilde{g}_{i,\tau}(m) = \begin{cases} g_{i,\tau}(m) & \text{if} \quad \tau - 1 - h \leq T_{min} + 20 \\ 20^{-1} \sum_{\upsilon=\tau-20-h}^{\tau-1-h} \mathcal{I}(d_{i\upsilon}(m) = \min_j d_{i\upsilon}(j)) & \text{otherwise.} \end{cases} \tag{A.34}$$

Appendix B

Appendix to Chapter 2

B.1 Construction of the common factor

The construction of the common 'world' asset factor is as follows. We rank 14/36/14 countries for each year y from 1996 to 2006 according to their GDP value (in US dollar terms) for the $\bullet = h, s, b$ markets. We then split the countries into two groups, with the breakpoint being the yearly GDP median. For each GDP group (big and small), we then rank the $\bullet = h, s, b$ markets by their annual price-dividend (PD), price-earnings (PE) and term spreads (TS) of year $y - 1$. The markets are again split, with the breakpoint being the yearly median of PD/PE/TS in each GDP group. Four asset market groups are obtained: PO= B/H, B/L, S/H and S/L, indicating big (B) or small (S) countries determined by their GDP value with high (H) or low (L) fundamental ratios determined by PD/PE/TS. Thus, the markets are split according to their PD/PE/TS, given that they are in the group of big or small countries.

We compute daily returns for each of the $\bullet = h, s, b$ country indices in the portfolios B/H, B/L, S/H and S/L that were constructed in each year y. In what follows, let $dy = 1, ..., DY$ denote a *daily* observation d within *year* y. Let $\boldsymbol{\Delta p}_{\bullet,dy}(PO) = (\Delta p_{\bullet,1dy}, ..., \Delta p_{\bullet,Ndy})'$ be the N-dimensional vector of daily log returns for each of the portfolios PO =B/H, B/L, S/H and S/L constructed in year

y for the $\bullet = s, h, b$ markets. We compute the unconditional covariance matrix of $\boldsymbol{\Delta p}_{\bullet,dy}(PO)$ denoted $\hat{\Pi}$ and we employ Factor Analysis (FA) in order to calculate the weights attached to a particular market return contained in $\boldsymbol{\Delta p}_{\bullet,dy}(PO)$ at each year y. FA allows us to decompose the covariance matrix as $\Pi = bb' + \Psi$ where b is a $N \times 1$ vector of factor loadings and Ψ is a $N \times N$ diagonal matrix containing specific variances. Given the estimated covariance matrix $\hat{\Pi}$, Maximum Likelihood is used to estimate b and Ψ under the constraint $b'\Psi b = \Theta$ where Θ is a diagonal matrix. Using the Maximum Likelihood estimators $\hat{b}$ and $\hat{\Psi}$, we compute the 'optimal return' of the portfolios PO =B/H, B/L, S/H and S/L at each dy for each market $\bullet = h, s, b$, i.e.

$$\Delta\hat{p}_{\bullet,dy}(PO) = \tilde{w}'\boldsymbol{\Delta p}_{\bullet,dy}(PO),\ \tilde{w} = a^{-1}\hat{w},\ a = \iota'\hat{w},\ \hat{w} = \left(\hat{b}'\hat{\Psi}^{-1}\hat{b}\right)^{-1}\hat{b}'\hat{\Psi}^{-1}, \tag{B.1}$$

where ι is a $N \times 1$ vector of ones. This computation is repeated for each year y and for each of the portfolios PO in the $\bullet = h, s, b$ markets. For each of the markets we compute the simple average of the returns of the two portfolios B/H and S/H at each day dy and subtract it to the simple average of the two returns of the two portfolios S/L and B/L at each day dy,

$$\hat{f}_{\bullet,dy} = (\Delta\hat{p}_{\bullet,dy}(BH) + \Delta\hat{p}_{\bullet,dy}(SH))/2 - (\Delta\hat{p}_{\bullet,dy}(BL) + \Delta\hat{p}_{\bullet,dy}(SL))/2. \tag{B.2}$$

The latter procedure gives a factor for each of the $\bullet = h, s, b$ markets respectively, that accounts for return differentials of portfolios sorted by high and low PD/PE/TS (after controlling for GDP) at each year y. The factor for all y in each $\bullet = h, s, b$ market is computed by stacking all $\hat{f}_{\bullet,dy}$ for each year y to obtain $\hat{f}_{\bullet,d}$. Finally, the common factor for all three $\bullet = h, s, b$ markets is computed by taking a sum of each of the individual factors, formally

$$\hat{f}_d = \hat{f}_{h,d} + \hat{f}_{s,d} + \hat{f}_{b,d}. \tag{B.3}$$

Let $\hat{f}_{dt}$ be the estimated factor for a daily observation d in month t. The factor at month t can be computed as $\hat{f}_t = \sum_{d=1}^{D} \hat{f}_{dt}$.

B.2 Mean group and panel DOLS

1. *Mean Group DOLS estimation*: The MG DOLS estimator $\bar{\beta}$ is computed by averaging the conventional DOLS estimates $\hat{\beta}_i$ obtained for $i = 1, ..., N$, i.e.

$$\bar{\beta} = \frac{1}{N} \sum_{i=1}^{N} \hat{\beta}_i, \; \widehat{\text{Cov}} \left[\bar{\beta} \right] = \sum_{i=1}^{N} (\hat{\beta}_i - \bar{\beta})(\hat{\beta}_i - \bar{\beta})'/(N(N-1)). \tag{B.4}$$

   Individual t-ratios for each i are computed with Newey & West (1987)'s heteroskedasticity and autocorrelation consistent covariance estimator (HAC).

2. *Panel DOLS estimation*: The panel estimator is obtained via two main steps. First, the variables in (2.8) are centered. Let the centered variables be given by $\tilde{p}_{it}$, $\tilde{\mathbf{x}}_{it}$ and $\Delta\tilde{\mathbf{y}}_{it}$. Second, the following pooled regression is performed,

$$\hat{\xi} = \left(\sum_{i=1}^{N} \sum_{t=1}^{T} \tilde{q}_{it} \tilde{q}_{it}' \right)^{-1} \left(\sum_{i=1}^{N} \sum_{t=1}^{T} \tilde{q}_{it} \tilde{p}_{h,it} \right), \tag{B.5}$$

   where $\hat{\xi} = (\hat{\beta}', \hat{\delta}_1', ..., \hat{\delta}_N')'$ is the estimated vector of parameters, $\tilde{q}_{it}$ is a $2k(1 + \sum_{i=1}^{N} a_i)$ dimensional vector whose first k elements are $\tilde{\mathbf{x}}_{it}$ and the elements $k(1 + \sum_{j=1}^{i-1}(2a_j + 1)) + 1$ to $k(1 + \sum_{j=1}^{i}(2a_j + 1))$ are $\Delta\tilde{\mathbf{y}}_{it}$ with zeros elsewhere. The latter procedure allows to estimate a homogeneous long run vector of coefficients while allowing for heterogeneous estimates for the leads and lags. Mark & Sul (2003) propose a suitable Wald test for testing the restriction $R\beta = \beta_{re}$. Interested readers are referred to the article by Mark & Sul (2003) for specifics about the Wald test.

B.3 Descriptions of in-sample and out-of-sample tests

In this section we describe the in-sample and out-of-sample tests employed in Chapter 2. To reduce the complexity of notation, in what follows we refrain from using $r = m, n$ in $\psi_{r,i}$ although every test described would be applied with either the pooled DOLS ($\hat{\beta}$) or single market DOLS ($\hat{\beta}_i$) long-run coefficient estimates in the real estate equation (2.9) depending on the result of the Hausmann test $\mathcal{H}$ of the null hypothesis $H_0 : \beta = \beta_i$.

B.3.1 In-sample tests

1. *Q-test*: Several studies have shown that the t-test is inappropriate when the predictor variable is persistent and/or its innovations are highly correlated with returns (Cavanagh *et al.* (1995), Stambaugh (1999) and Lewellen (2004)). Campbell & Yogo (2006) recently propose a robust Q-test that corrects for the latter issues. In our context, the Q-test for variable $l = 1, ..., L$ and cross-section $i = 1, ..., N$ in its more general version considers the model,

$$\Delta p_{h,it} = \alpha_i + \psi_{li} k_{it-1} + u_{it}, \tag{B.6}$$

$$k_{it} = \rho_i k_{it-1} + v_{it}, \tag{B.7}$$

$$b(L) v_{it} = e_{it}, \tag{B.8}$$

where $b(L) = \sum_{j=1}^{p-1} b_j L^j$ with $b_0 = 1$ and $b(1) \neq 1$ and the variable k_{it} is taken from the set of variables $k_{it} = (\hat{h}_{it}, \hat{\sigma}^2_{h,it}, \hat{\sigma}^2_{s,it}, \hat{\sigma}^2_{b,it}, \hat{\sigma}^2_{f,it}, \hat{\sigma}_{hf,it}, \hat{\sigma}_{sf,it}, \hat{\sigma}_{bf,it}, \Delta p_{h,it}, \Delta p_{s,it}, \Delta p_{b,it})$. It is assumed that all the roots of $b(L)$ are less than one in absolute value and that ρ_i is less than but very close to one (i.e. local-to-unity). The equations (B.7) and (B.8) together imply that Δk_{it} follows an ADF type of regression with autoregressive coefficients $\tau_{i,j}$ for $j = 1, ..., p-1$. The Q-statistic is then given by,

$$Q(\psi_{i,0}, \rho_i) = \frac{\sum_{t=1}^{T} \tilde{k}_{it-1}[\Delta p_{h,it} - \psi_{0,li} k_{it-1} - \frac{\sigma_{iue}}{\sigma_{ie}\omega_i} v_{it}] + Z}{\sigma_{iu}(1-\gamma_i^2)^{1/2}(\sum_{t=1}^{T} \tilde{k}^2_{it-1})^{1/2}}, \tag{B.9}$$

where $Z = \frac{T}{2}\frac{\sigma_{iue}}{\sigma_{ie}\omega_i}(\omega_i^2 - \sigma_{iv}^2)$. The tilde is used to denote a centered process, the moments σ^2_{iu}, σ^2_{ie}, σ^2_{iv}, $\sigma_{i,ue}$ correspond to variances and covariances of u_{it}, v_{it} and e_{it} and $\gamma_i = \sigma_{i,ue}/(\sigma_{iu}\sigma_{ie})$ and $\omega_i^2 = \sigma_{ie}/(1 - \sum_{j=1}^{p-1} \tau_{i,j})^2$. When there are no short run dynamics in (B.8) (i.e. $b(1)$ so that $\omega_i^2 = \sigma^2_{iv} = \sigma^2_{ie}$) the Q-statistic applies for the case where ρ_i is not close to unity. Under the null $\psi_{0,li} = 0$ the above test is Gaussian. Although the Q-test is asymptotically pivotal, the test is infeasible as it requires knowledge of ρ_i and of the nuissance parameters ω_i^2 and σ_{iv}^2 to compute the test statistic. A feasible version that replaces the latter parameters with consistent estimators can be used and has the same asymptotic distribution. Details on the empirical implementation of the feasible Q-test can be looked up in the paper by Campbell & Yogo (2006).

2. *Wald-test*: The Wald-test is used for the test of joint significance of the parameters in model (2.9) and for the tests on serial correlation (in- and out-of-sample). To test for serial correlation under heteroskedasticity we employ the auxiliary regression in (A.22) with $\hat{u}_{h,it}$ or $\hat{u}_{h,i\tau}$ replacing $\hat{u}_{i\tau}$ and we use the test in (A.23) which gives the statistic denoted $\tau_{\bullet,li}$ for $\bullet = IN, OS$ (see Appendix A). In the case of the joint significance test, we test the null $\hat{\psi}_{1i} = \hat{\psi}_{3i} = ... = 0$. The Wald test in (A.23) is employed with $\hat{\psi}_i$ replacing $\hat{\kappa}$ which gives the statistic denoted ϖ_i.

3. *Fisher test*: The Fisher statistics F-$\bullet$ are computed via the Fisher test presented in (A.24). In our context, the $\chi^2(2N)$ distribution might not hold due to contemporaneous correlation over real estate markets. However, this test is useful to provide an aggregate measure of the individual statistics.

4. *Cross-sectional Wald-test*: To test the null hypothesis $H_0 : \psi_{li} = 0$, $i = 1, ..., N$, we determine a Wald-statistic as in (A.20) with $\hat{\boldsymbol{\psi}}$ replacing $\hat{\boldsymbol{\alpha}}_p$. $\widehat{\text{Cov}}[\hat{\boldsymbol{\psi}}]$ is computed with White (1980)'s robust covariance estimator.

5. *Cross-sectional regressions*: The cross-sectional regression model for variable $l = 1, ..., L$ and cross-section $i = 1, ..., N$ in our case is given by,

$$\Delta\bar{p}_{h,i} = \kappa\hat{\psi}_{li} + u_i. \tag{B.10}$$

This regression model is also known as a two-pass regression estimate since one must first obtain the time series estimate $\hat{\psi}_{li}$ and then perform the cross-sectional regression. The usual approach to estimate κ is via OLS or GLS. Following Cochrane (2001), OLS is sometimes much more robust than GLS in these type of regression of asset returns but the GLS regression should improve efficiency. However, in order to estimate the model in (B.10) via GLS one must estimate and invert the variance covariance matrix of u_i which is quite difficult in practice. Therefore, we employ OLS with White (1980) robust covariance estimator.

B.3.2 Out-of-sample tests

1. *Hendrickson-Merton statistic*: The variant of the Henrikkson–Merton statistic (Henriksson & Merton (1981), Cumby & Modest (1987)) presented in (A.25) is employed with $\Delta p_{h,i\tau}$ replacing $\Delta p_{i\tau}$.

2. *Forecast encompassing test*: Testing the null hypothesis of forecast encompassing is addressed in Harvey *et al.* (1998). It can be shown that forecast encompassing of model M1 by model M0 implies that in the regression,

$$\hat{u}_{h,i\tau}(M0) = \chi_i(\hat{u}_{h,i\tau}(M0) - \hat{u}_{h,i\tau}(M1)) + v_{it}, \qquad \text{(B.11)}$$

   the parameter χ_i is equal to zero against the alternative that is greater than zero. Accordingly, testing the null hypothesis that M0 forecast encompasses M1 appears straightforward since (B.11) could be seen as a usual regression model offering significance tests for the parameter χ_i. As argued by Harvey *et al.* (1998), however, the practical implementation of such significance tests will likely have to account for possible serial correlation (in case of medium to long term forecasts) or heteroskedasticity. Robust standard errors for OLS based estimates $\hat{\chi}_i$ are provided by Harvey *et al.* (1998). We concentrate in this paper on heteroskedasticity consistent test statistics. Harvey *et al.* (1998) derive the following test statistic:

$$\mathrm{fe}_i \quad = \quad \sqrt{\mathcal{T}}\bar{d}_i/\hat{q}, \ \hat{q} = \mathcal{T}^{-1}\sum_{\tau}(\hat{u}_{h,i\tau}(M0) - \hat{u}_{h,i\tau}(M1))^2\hat{v}_{i\tau}^2. \qquad \text{(B.12)}$$

   where $\bar{d}_i = \mathcal{T}^{-1}\sum_{\tau}(d_{i\tau}(M0) - d_{i\tau}(M1))$ with $d_{h,i\tau}(\bullet) = \hat{u}_{i\tau}(\bullet)^2$. Under the null hypothesis of forecast encompassing fe_i is asymptotically Gaussian. The aggregation is done via the Fisher test as in (A.24).

B.4 Bootstrap impulse responses

The procedure for computing bootstrap impulse responses consists of:

(a) Estimating the model in (2.13) and obtaining the parameters in (2.15) following steps 1 to 3 outlined in the estimation algorithm of section 2.3.4.2.

(b) Generating bootstrap residuals $\hat{e}_{it}^{b}$ by randomly drawing with replacement from the set of residuals, $\{\hat{e}_{i1}, ..., \hat{e}_{iT}\}$ where $\hat{e}_{it} = y_{it} - \hat{\Gamma}_i x_{it}$.

(c) Constructing bootstrap time series y_{it}^{b} recursively using the representation in (2.13) .

(d) Obtaining bootstrap parameters $\hat{A}^{b}, \hat{B}_{ij}^{b}$ from the generated data and computing the bootstrap impulse responses $\hat{\vartheta}_{lm,i}^{b}(h)$ and $\bar{\vartheta}_{lm}^{b}(h)$.

(e) Steps 1 to 4 are repeated say 500 times, and the confidence intervals are obtained by using $\gamma/2$ and $(1 - \gamma/2)$ quantiles of the bootstrap distribution.

Appendix C

Appendix to Chapter 3

C.1 Identification through heteroskedasticity

In this subsection we explain briefly the IH approach with unobservable shocks introduced by Rigobon & Sack (2004). Performing an OLS regression of equation (3.3) (assuming for simplicity $\phi_1 = \phi_2 = 0$) would yield the mean parameter,

$$E\hat{\alpha} = \alpha + (1 - \alpha\beta)\frac{\beta\sigma_\eta^2 + (\beta + \varphi)\sigma_z^2}{\sigma_\varepsilon^2 + \beta^2\sigma_\eta^2 + (\beta + \varphi)^2\sigma_z^2}. \tag{C.1}$$

Thus, the estimate $\hat{\alpha}$ would be biased due to simultaneity bias if $\beta \neq 0$ and $\sigma_\eta^2 > 0$ and omitted variable bias if $\varphi \neq 0$ and $\sigma_z^2 > 0$. We recall that the reduced form residuals of the reduced form model in (3.4) are given by:

$$u_{1t} = (1 - \alpha\beta)^{-1}[(\beta + \varphi)z_t + \beta\eta_t + \varepsilon_t], \tag{C.2}$$

$$u_{2t} = (1 - \alpha\beta)^{-1}[(1 + \alpha\varphi)z_t + \eta_t + \alpha\varepsilon_t]. \tag{C.3}$$

The variance-covariance matrix of the above reduced form residuals is then given by:

$$\dot{\Omega} = \dot{\lambda} \begin{bmatrix} (\beta+\varphi)^2\sigma_z^2 + \beta^2\sigma_\eta^2 + \sigma_\varepsilon^2 & \xi(\beta+\varphi)\sigma_z^2 + \beta\sigma_\eta^2 + \alpha\sigma_\varepsilon^2 \\ \xi(\beta+\varphi)\sigma_z^2 + \beta\sigma_\eta^2 + \alpha\sigma_\varepsilon^2 & \xi^2\sigma_z^2 + \sigma_\eta^2 + \alpha^2\sigma_\varepsilon^2 \end{bmatrix}, \quad \text{(C.4)}$$

where $\dot{\lambda} = (1-\beta\alpha)^{-2}$ and $\xi = (1+\alpha\varphi)$. Furthermore, assume that there is heteroskedasticity in the data and that there are two regimes F and $\tilde{F}$ with corresponding variances Ω_F and $\Omega_{\tilde{F}}$. It is also assumed that the parameters β and α are stable across regimes and that the assumptions (3.5) on the shocks hold. Then, the difference in covariances $\Omega_D = \Omega_F - \Omega_{\tilde{F}}$ gives (3.6).

Appendix D

Appendix to Chapter 4

D.1 Generalized Method of Moments estimation of MSM models

In this section we summarize the closed-form solutions of selected moments for estimation of the Binomial and the Lognormal models via GMM which are provided in Lux (2008a). Details about the derivation of the moment conditions not shown here may be looked up in the latter article. In what follows let:

$$\mu_t = \prod_{i=1}^{k} M_t^{(i)}, \tag{D.1}$$

and denote the multifractal volatility process and $\eta_{t,T}$ its log increments:

$$\eta_{t,T} = \ln(\mu_t) - \ln(\mu_{t-T}) = \sum_{i=1}^{k} \ln(M_t^{(i)}) - \sum_{i=1}^{k} \ln(M_{t-T}^{(i)}). \tag{D.2}$$

D.1.1 Moments of the Binomial model

After some algebra it can be shown that:

$$E\left[\eta_{t+T,T}\eta_{t,T}\right] = -(\ln(m_0)-\ln(2-m_0))^2 \cdot \sum_{i=1}^{k}\frac{1}{4}\left(1-(1-\gamma_i)^T\right)^2, \tag{D.3}$$

$$\begin{aligned} E\left[\eta^2_{t+T,T}\eta^2_{t,T}\right] = &\left\{\sum_{i=1}^{k}\left(\frac{1}{2}\left(1-(1-\gamma_i)^T\right)\sum_{j=1}^{k}\frac{1}{2}\left(1-(1-\gamma_j)^T\right)\right)\right. \\ &\left. + 2\sum_{i=1}^{k}\left(\frac{1}{4}\left(1-(1-\gamma_i)^T\right)^2\sum_{j=1,j\neq i}^{k}\frac{1}{4}\left(1-(1-\gamma_j)^T\right)^2\right)\right\}\chi, \end{aligned} \tag{D.4}$$

and

$$E\left[\eta^2_{t+T,T}\right] = \sum_{i=1}^{k}\frac{1}{2}(1-(1-\gamma_i)^T)\cdot(\ln(m_0)-\ln(2-m_0))^2, \tag{D.5}$$

where $\chi = (\ln(m_0)-\ln(2-m_0))^4$. For the Binomial MSM with Normal innovations we identify σ^2 via $E[x_t^2] = \sigma^2$. For the Binomial MSM with Student-t innovations we have:

$$E\left[|x_t|\right] = \frac{\sigma\cdot\Xi_1\cdot\nu^{1/2}\cdot\Gamma(0.5(\nu-1))}{\pi^{1/2}\cdot\Gamma(0.5\nu)}, \tag{D.6}$$

$$E\left[x_t^2\right] = \frac{\sigma^2\cdot\nu}{(\nu-2)}, \tag{D.7}$$

$$E\left[|x_t^3|\right] = \frac{\sigma^3\cdot\Xi_2\cdot\nu^{3/2}\cdot\Gamma(2)\cdot\Gamma(0.5(\nu-3))}{\Gamma(0.5)\cdot\Gamma(0.5\nu)}, \tag{D.8}$$

where $\Xi_1 = \left(0.5m_0^{1/2}+0.5(2-m_0)^{1/2}\right)^k$, $\Xi_2 = \left(0.5m_0^{3/2}+0.5(2-m_0)^{3/2}\right)^k$ and $\Gamma(\bullet)$ is the Gamma function. In order to compute linear forecasts we also need the second moment and the auto-covariances of the volatility process:

$$E\left[\mu_t^2\right] = \left(0.5(m_0^2+(2-m_0)^2)\right)^k, \tag{D.9}$$

and

$$\begin{aligned} E\left[\mu_{t+T}\mu_t\right] = &\prod_{i=1}^{k}\left\{0.5\left(1-(1-\gamma_i)^T\right)m_0\left(2-m_0\right)\right. \\ &\left.+\left((1-\gamma_i)^T+0.5\left(1-(1-\gamma_i)^T\right)\right)\left(0.5m_0^2+0.5\left(2-m_0\right)^2\right)\right\}. \end{aligned} \tag{D.10}$$

D.1.2 Moments of the Lognormal model

Similar to the Binomial case, after some algebra we may arrive to:

$$E\left[\eta_{t+T,T}\eta_{t,T}\right] = -\sum_{i=1}^{k}\left(1-(1-\gamma_i)^T\right)^2 s^2, \tag{D.11}$$

$$\begin{aligned} E\left[\eta_{t+T,T}^2\eta_{t,T}^2\right] &= \sum_{i=1}^{k}\left(1-(1-\gamma_i)^T\right)^2 \cdot 6s^4 \\ &+\left\{\sum_{i=1}^{k}\left(\left(1-(1-\gamma_i)^T\right)\sum_{j=1,j\neq i}^{k}\left(1-(1-\gamma_j)^T\right)\right)\right\}\cdot 4s^4 \\ &+\left\{\sum_{i=1}^{k}\left(\left(1-(1-\gamma_i)^T\right)^2\sum_{j=1,j\neq i}^{k}\left(1-(1-\gamma_j)^T\right)^2\right)\right\}2s^4, \end{aligned} \tag{D.12}$$

and

$$E\left[\eta_{t+T,T}^2\right] = \sum_{j=1}^{k}\left(1-(1-\gamma_i)^T\right)\cdot 2s^2. \tag{D.13}$$

For the Lognormal MSM with Normal innovations we identify σ^2 via $E[x_t^2] = \sigma^2$. For the Lognormal MSM with Student-t innovations we employ (D.6)-(D.8) with $\Xi_1 = \exp(-0.25\cdot k\cdot\lambda)$ and $\Xi_2 = \exp(0.75\cdot k\cdot\lambda)$. Analogously to the Binomial model, we need the second moment and the auto-covariances of the volatility process for the linear forecasts:

$$E\left[\mu_t^2\right] = \exp(2\lambda\cdot k), \tag{D.14}$$

and

$$E\left[\mu_{t+T}\mu_t\right] = \prod_{i=1}^{k}\left\{\left(1-(1-\gamma_i)^T\right)+(1-\gamma_i)^T\exp\left(2\cdot\lambda\right)\right\}. \tag{D.15}$$

D.1.3 Moments of the compound process

Recalling the innovations of the compound process:

$$\xi_{t,T} = \ln|x_t| - \ln|x_{t-T}|.$$

We have:

$$E\left[\xi_{t+T,T}\xi_{t,T}\right] = 0.25 \cdot E\left[\eta_{t+T,T}\eta_{t,T}\right] + \left(E\left[\ln|\varepsilon_t|\right]\right)^2 - E\left[\left(\ln|\varepsilon_t|\right)^2\right], \tag{D.16}$$

and

$$\begin{aligned} E\left[\xi^2_{t+T,T}\xi^2_{t,T}\right] &= 0.25^2 \cdot E\left[\eta^2_{t+T,T}\eta^2_{t,T}\right] - \left\{E\left[\eta^2_{t,T}\right] - E\left[\eta_{t+T,T}\eta_{t,T}\right]\right\} \\ &\quad \cdot \left\{\left(E\left[\ln|\varepsilon_t|\right]\right)^2 - E\left[\left(\ln|\varepsilon_t|\right)^2\right]\right\} + 3 \cdot E\left[\left(\ln|\varepsilon_t|\right)^2\right]^2 \\ &\quad -4 \cdot E\left[\ln|\varepsilon_t|\right] E\left[\left(\ln|\varepsilon_t|\right)^3\right] + E\left[\left(\ln|\varepsilon_t|\right)^4\right]. \end{aligned} \tag{D.17}$$

The log moments of the standard Normal (Student-t) variates ε_t can be computed using the Gamma function and its derivatives (see below). Inserting (D.3)-(D.5) for the Binomial model and (D.11)-(D.13) for the Lognormal model in (D.16) and (D.17), we obtain the analytical expressions for the moment conditions in (4.12).

D.1.4 Log moments of the Normal distribution

The first log moment $E[\ln|\varepsilon_t|]$ is given by:

$$\begin{aligned} E[\ln|\varepsilon|] &= \int_{-\infty}^{\infty} \ln|\varepsilon| \frac{1}{\sqrt{2\pi}} \exp(\frac{-\varepsilon^2}{2}) d\varepsilon \\ &= \frac{1}{\sqrt{2\pi}} \left\{ \frac{1}{\sqrt{2}} \Gamma'(0.5) - \frac{1}{\sqrt{2}} \ln(2)\Gamma(0.5) \right\}, \end{aligned} \tag{D.18}$$

where $\Gamma(\bullet)$ is the Gamma function. The second log moment $E[\ln|\varepsilon_t|^2]$ is given by:

$$\begin{aligned} E[\ln|\varepsilon|^2] &= \int_{-\infty}^{\infty} (\ln|\varepsilon|)^2 \frac{1}{\sqrt{2\pi}} \exp(\frac{-\varepsilon^2}{2}) d\varepsilon \\ &= \frac{1}{4\sqrt{\pi}} \Gamma''(0.5) - \frac{1}{2\sqrt{\pi}} \ln(2)\Gamma'(0.5) \qquad \text{(D.19)} \\ &\quad + \frac{1}{4\sqrt{\pi}} \ln(2)^2 \Gamma(0.5), \qquad \text{(D.20)} \end{aligned}$$

where $\Gamma'(\bullet)$ denotes the first derivative of the Gamma function. The third log moment $E[(\ln|\varepsilon_t|)^3]$ is given by:

$$\begin{aligned}
E[\ln|\varepsilon|^3] &= \int_{-\infty}^{\infty} (\ln|\varepsilon|)^3 \frac{1}{\sqrt{2\pi}} \exp(\frac{-\varepsilon^2}{2}) d\varepsilon \\
&= \frac{1}{8\sqrt{\pi}}\Gamma'''(0.5) - \frac{3}{8\sqrt{\pi}}\ln(2)\Gamma''(0.5) + \frac{3}{8\sqrt{\pi}}\ln(2)^2\Gamma'(0.5) \\
&\quad - \frac{1}{8\sqrt{\pi}}\ln(2)^3\Gamma(0.5), \qquad \text{(D.21)}
\end{aligned}$$

where $\Gamma''(\bullet)$ and $\Gamma'''(\bullet)$ denote the second and third derivative of the Gamma function, respectively. The fourth log moment $E[(\ln|\varepsilon_t|)^4]$ is given by:

$$\begin{aligned}
E[\ln|\varepsilon|^4] &= \int_{-\infty}^{\infty} (\ln|\varepsilon|)^4 \frac{1}{\sqrt{2\pi}} \exp(\frac{-\varepsilon^2}{2}) d\varepsilon \\
&= \frac{1}{16\sqrt{\pi}}\Gamma^{(4)}(0.5) - \frac{4}{16\sqrt{\pi}}\ln(2)\Gamma'''(0.5) + \frac{6}{16\sqrt{\pi}}\ln(2)^2\Gamma''(0.5) \\
&\quad - \frac{4}{16\sqrt{\pi}}\ln(2)^3\Gamma'(0.5) + \frac{1}{16\sqrt{\pi}}\ln(2)^4\Gamma(0.5). \qquad \text{(D.22)}
\end{aligned}$$

where $\Gamma^{(4)}(\bullet)$ denotes the fourth derivative of the Gamma function.

D.1.5 Log moments of the Student-t distribution

This subsection draws on Lee (2007) who computed log moments of the Student-t distribution with Mathematica 4.2. The first log moment $E[\ln|\varepsilon_t|]$ is given by:

$$\begin{aligned}
E[\ln|\varepsilon|] &= \int_{-\infty}^{\infty} \ln|\varepsilon| \frac{\Gamma(\frac{1}{2}(\nu+1))}{\sqrt{\nu\pi}\Gamma(\frac{1}{2}\nu)\left(1+\frac{x^2}{\nu}\right)^{(\nu+1)/2}} d\varepsilon \\
&= -\frac{\left(\Gamma[1+\frac{\nu}{2}](-\Gamma'[1] + \ln(4) - \ln(\nu) + \psi_0(\frac{\nu}{2}))\right)}{\nu\Gamma[\frac{\nu}{2}]}, \qquad \text{(D.23)}
\end{aligned}$$

where the Euler constant $-\Gamma'(-1)$, has the numerical value $0.57721566490\cdots$ and $\psi_n(x) = \frac{d^n}{dx^n}\frac{\Gamma'(x)}{\Gamma(x)} = \frac{d^n}{dx^n}\psi_0(x)$ is the polygamma function. The second log moment $E[\ln|\varepsilon_t|^2]$ is given by:

$$\begin{aligned}
E[\ln|\varepsilon|^2] &= \int_{-\infty}^{\infty} \ln|\varepsilon|^2 \frac{\Gamma(\frac{1}{2}(\nu+1)}{\sqrt{\nu\pi}\Gamma(\frac{1}{2}\nu)\big(1+\frac{x^2}{\nu}\big)^{(\nu+1)/2}} d\varepsilon \\
&= \frac{1}{8\nu\Gamma(\frac{\nu}{2})}\Big(-8\Gamma(1+\frac{\nu}{2})\big((-\Gamma'(1)+\ln(4))\ln(\nu) \\
&\quad -(-\Gamma'(1)+\ln(4)-\ln(\nu))\psi_0(\frac{\nu}{2})\big) + \nu\Gamma(\frac{\nu}{2})\big(2(-\Gamma'(1))^2 \\
&\quad +\pi^2+4(-\Gamma'(1))\ln(4)+2\ln(4)^2+2\ln(\nu)^2+2\psi_0(\frac{\nu}{2})^2 \\
&\quad +2\psi_1(\frac{\nu}{2})\big)\Big). \qquad \text{(D.24)}
\end{aligned}$$

The third log moment $E[(\ln|\varepsilon_t|)^3]$ is given by:

$$\begin{aligned}
E[\ln|\varepsilon|^3] &= \int_{-\infty}^{\infty} \ln|\varepsilon|^3 \frac{\Gamma(\frac{1}{2}(\nu+1)}{\sqrt{\nu\pi}\Gamma(\frac{1}{2}\nu)\big(1+\frac{x^2}{\nu}\big)^{(\nu+1)/2}} d\varepsilon \\
&= \frac{1}{16}\Big\{ -(-\Gamma'(1)+\ln(4)-\ln(\nu))\big(3\pi^2+2(-\Gamma'(1)+\ln(4))^2 \\
&\quad +2\ln(\nu)\big(-2(-\Gamma'(1)+\ln(4))+\ln(\nu)\big)\big) \\
&\quad +\psi_0(\frac{\nu}{2})\Big(-3(\pi^2+2(-\Gamma'(1)+\ln(4))^2)+12(-\Gamma'(1)+\ln(4))\ln(\nu) \\
&\quad -6\ln(\nu)^2-2\psi_0(\frac{\nu}{2})\big(3(-\Gamma'(1)+\ln(4) \\
&\quad -\ln(\nu))+\psi_0(\frac{\nu}{2})\big)\Big)-6\big(-\Gamma'(1)+\ln(4)-\ln(\nu) \\
&\quad +\psi_0(\frac{\nu}{2})\big)\psi_1(\frac{\nu}{2})-2\psi_2(\frac{\nu}{2})-28\zeta(3)\Big\}, \qquad \text{(D.25)}
\end{aligned}$$

where $\zeta(3)$ is the Riemann zeta function with a value of 1.2020569032. The fourth log moment $E[(\ln|\varepsilon_t|)^4]$ is given by:

$$\begin{aligned}
E[\ln|\varepsilon|^4] &= \int_{-\infty}^{\infty} \ln|\varepsilon|^4 \frac{\Gamma(\frac{1}{2}(\nu+1)}{\sqrt{\nu\pi}\Gamma(\frac{1}{2}\nu)\big(1+\frac{x^2}{\nu}\big)^{(\nu+1)/2}} d\varepsilon \\
&= \frac{1}{64}\Bigg\{ -(-4\Gamma'(1)^4 + 12(-\Gamma'(1))^2\pi^2 + 7\pi^4 \\
&\quad +8(-\Gamma'(1))(2(-\Gamma'(1))^2+3\pi^2)\ln(4) \\
&\quad +12(2(-\Gamma'(1))^2+\pi^2)\ln(4)^2 + 16(-\Gamma'(1))\ln(4)^3 + 4\ln(4)^4 \\
&\quad -4(2(-\Gamma'(1)) + \ln(16) - \ln(\nu))\ln(\nu)(3\pi^2 + 2(-\Gamma'(1)+\ln(4))^2 \\
&\quad -2(-\Gamma'(1)+\ln(4))\ln(\nu) + \ln(\nu)^2) + 16(-\Gamma'(1)+\ln(4) \\
&\quad -\ln(\nu))\psi_0(\frac{\nu}{2})^3 + 4\psi_0(\frac{\nu}{2})^4 + 12\big(\pi^2 + 2(-\Gamma'(1)+\ln(4)-\ln(\nu))^2 \\
&\quad +2\ln(\nu)(-2(-\Gamma'(1)+\ln(4)) + \ln(\nu))\big)\psi_1(\frac{\nu}{2}) + 12\psi_1(\frac{\nu}{2})^2 \\
&\quad +12\psi_0(\frac{\nu}{2})^2\big(\pi^2 + 2(-\Gamma'(1)+\ln(4))^2 + \ln(\nu)(-2(-\Gamma'(1)+\ln(4)) \\
&\quad +\ln(\nu)) + 2\psi_1(\frac{\nu}{2})\big) + 4\psi_3(\frac{\nu}{2}) + 16(-\Gamma'(1)+\ln(4)-\ln(\nu))(\psi_2(\frac{\nu}{2}) \\
&\quad +14\zeta(3)) + 8\psi_0(\frac{\nu}{2})\big((-\Gamma'(1)+\ln(4)-\ln(\nu))(3\pi^2 + 2(-\Gamma'(1) \\
&\quad +\ln(4))^2 + 2\ln(\nu)(-2(-\Gamma'(1)+\ln(4)) + \ln(\nu))) + 6(-\Gamma'(1) \\
&\quad +\ln(4)-\ln(\nu))\psi_1(\frac{\nu}{2}) + 2\psi_2(\frac{\nu}{2}) + 28\zeta(3)\big)\Bigg\}. \qquad \text{(D.26)}
\end{aligned}$$

D.2 Maximum Likelihood estimation of MSM models

Let $M_t = (M_{1t}, M_{2t}, ..., M_{kt})'$ be a $1 \times k$ vector with 2^k different possible values $m^1, ..., m^{2^k}$. The (log) Likelihood function is given by:

$$\begin{aligned} L(x_1, ..., x_T; \varphi) &= \sum_{t=1}^{T} \ln g(x_t | x_1, ..., x_{t-1}) & \text{(D.27)} \\ &= \sum_{t=1}^{T} \ln \left[\sum_{i=1}^{2^k} \mathcal{P}(M_t = m^i | x_1, ..., x_{t-1}) \cdot g(x_t | M_t = m^i) \right] \\ &= \sum_{t=1}^{T} \ln \left[(\Omega_{t-1} \mathcal{A}) \cdot g(x_t | M_t = m^i) \right]. & \text{(D.28)} \end{aligned}$$

The term $\Omega_t = \mathcal{P}(M_t = m^i | x_1, ..., x_t)$ defines the conditional probabilities over the unobserved states $m^1, ..., m^{2^k}$. The transition matrix $\mathcal{A} = (a_{i,j})_{1 \leq i,j \leq 2^k}$ has components $a_{i,j}$ given by:

$$\begin{aligned} \mathcal{P}(M_{t+1} = m^j | M_t = m^i) &= \Pi_{k=1}^{K} [(1 - \gamma_k) \cdot 1_{\{m_k^i = m_k^j\}} \\ &+ \gamma_k \cdot \mathcal{P}(M = m_k^j)], & \text{(D.29)} \end{aligned}$$

where m_k^i denotes the m-th component of vector m^i and $1_{\{m_k^i = m_k^j\}}$ is the dummy variable equal to 1 if $m_k^i = m_k^j$ and 0 otherwise. Let $\mathcal{F}_n[\bullet]$ and $\mathcal{F}_s[\bullet]$ denote the densities of a standard Normal and Student-t, respectively. Thus, we have that the density function of x_t conditional on μ_t for the Binomial MSM with Normal innovations is given by $g(x_t | M_t = m^i) = [\sigma \mu_t]^{-1} \mathcal{F}_n \left[x_t [\sigma \mu_t]^{-1} \right]$ and that of the Binomial model with Student-t innovations is given by $g(x_t | M_t = m^i) = [\sigma \mu_t]^{-1} \mathcal{F}_s \left[x_t [\sigma \mu_t]^{-1} \right]$. Finally, Ω_t may be updated as

$$\Omega_{t+1} = \frac{g(x_t | M_t = m^i) \odot (\Omega_t \cdot \mathcal{A})}{[g(x_t | M_t = m^i) \odot (\Omega_t \cdot \mathcal{A})] \, \iota'}, \tag{D.30}$$

where ι is a 1×2^k vector of ones and $\odot$ is the Hadamard product.

D.3 Forecast evaluation

D.3.1 Forecast combinations

Let $\hat{\sigma}^2_{i\tau}(1), \hat{\sigma}^2_{i\tau}(2), \ldots, \hat{\sigma}^2_{i\tau}(M)$ denote a set of predictors for $\sigma^2_{i\tau}$ from model m and $i = 1, ..., N$. We assign each single forecast a weight equal to a model's empirical frequency of minimizing the absolute forecast error over realized errors available in the forecast origin $\tau - 1$. To take account of structural variation the weights are determined over the 20 most recent forecast errors. Formally, the combined forecast are computed as in (A.31) and (A.33) in Appendix A with $\hat{\sigma}^2_{i\tau}$ replacing $\Delta\hat{p}_{i\tau}$.

D.3.2 Relative MSE and MAE

To define absolute and relative measures let '0' and '•' indicate a benchmark (historical volatility) and a particular competing model ((FI)GARCH, MSM, CO1,...,CO9), respectively. Forecast errors are given by:

$$\hat{e}_{i\tau}(0) = \sigma^2_{i\tau} - \hat{\sigma}^2_i, \ \hat{e}_{i\tau}(\bullet) = \sigma^2_{i\tau} - \hat{\sigma}^2_{i\tau}, \tag{D.31}$$

where $\sigma^2_{i\tau}$ are the squared residuals obtained after linear filtering of returns (using the in-sample means and first-order autocorrelations), $\hat{\sigma}^2_i$ is the historical volatility estimate (in-sample variance of filtered returns) and $\hat{\sigma}^2_{i\tau}$ is the volatility forecast of the competing model ((FI)GARCH, MSM, CO1,...,CO9). The MSE and MAE of the benchmark (historical volatility) specification are:

$$\bar{d}_i(0) = \mathcal{T}^{-1} \sum_{\tau} d_{i\tau}(0), \ d_{i\tau}(0) = \hat{e}_{i\tau}(0)^2 \text{ or } d_{i\tau}(0) = |\hat{e}_{i\tau}(0)|. \tag{D.32}$$

The average performance of a competing model specification could be given in relation to $\bar{d}_i(0)$, obtaining relative MSEs or MAEs as:

$$\overline{dr}_i(\bullet) = \frac{\bar{d}_i(\bullet)}{\bar{d}_i(0)}, \ \bar{d}_i(\bullet) = \mathcal{T}^{-1} \sum_{\tau} d_{i\tau}(\bullet). \tag{D.33}$$

We determine overall panel MSEs and MAEs measures as in (D.33) over both data dimensions. The panel MSEs or MAEs are denoted $\mathcal{DR}$. The variance of (D.33) denoted Var[$\mathcal{DR}$] is estimated as in (A.28) from the full set of forecasting errors.

D.3.3 Average forecasting accuracy

To contrast the performance of a particular forecasting model '0' and some alternative specification, '$\bullet$', in terms of the achieved MSEs and MAEs we employ the Diebold & Mariano (1995) statistic for $i = 1, ..., N$, i.e.

$$dm_i = \bar{d}_i / \sqrt{\widehat{\mathrm{Var}}[\bar{d}_i]},\ \bar{d}_i = \mathcal{T}^{-1} \sum_{\tau} d_{i\tau},\ d_{i\tau} = d_{i\tau}(0) - d_{i\tau}(\bullet), \qquad \text{(D.34)}$$

where $\widehat{\mathrm{Var}}[\bar{d}_i]$ is the Newey & West (1987)'s heteroskedasticity and autocorrelation consistent variance estimator with Andrews (1991) procedure for bandwidth selection. Under the null hypothesis of statistically equal forecasting performance of '0' and '$\bullet$' the statistic dm_i should have an asymptotic standard Normal distribution. Significantly negative test statistics hint at a superior forecasting performance achieved by '0'. Note that over a set of alternative forecasting schemes used as '$\bullet$' cross comparison of models is implicitly obtained from contrasting several models against one specific forecasting scheme '0'. At the panel dimension we count the number of improvements of model '0' against the benchmark '$\bullet$' at the 5% significance level denoted $\mathcal{S}$. Under equal forecasting performance, $\mathcal{S}$ can be seen as the number of successes over N draws from a binomial distribution with a 5% probability of success. Some selected binomial success probabilities are: $N = 25$, $p(\mathcal{S} \geq 3) = 0.0341$; $N = 11$, $p(\mathcal{S} \geq 3) = 0.0016$; $N = 12$, $p(\mathcal{S} \geq 3) = 0.0022$.

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Eidesstattliche Erklärung

Ich erkläre hiermit, an Eides Statt, dass ich meine Doktorarbeit 'International Asset Prices: Empirical Evidence' selbständig und ohne fremde Hilfe angefertigt habe und dass ich alle von anderen Autoren wörtlich übernommenen Stellen, wie auch die sich an die Gedanken anderer Autoren eng anlehnenden Ausführungen meiner Arbeit, besonders gekennzeichnet und die Quellen nach den mir angegebenen Richtlinien zitiert habe.

Kiel, Juli 2009

Leonardo Morales-Arias