Entscheidungsverhalten unter Risiko: Experimentelle Studien und neue theoretische Ansätze

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Kapitel 1

Deutsche

Zusammenfassung

1 Einleitung und Überblick

Bis zum Ende der siebziger Jahre wurde die Entscheidungstheorie unter Risiko sehr stark durch die Erwartungsnutzentheorie geprägt. Gemäß der grundlegenden Arbeit von von Neumann und Morgenstern (1947) repräsentiert der Erwartungsnutzen das Entscheidungsverhalten aller Personen, deren Präferenzen vollständig, transitiv und stetig sind sowie das Unabhängigkeitsaxiom erfüllen. Während Vollständigkeit und Stetigkeit im Wesentlichen als technische Bedingungen angesehen werden können, restringieren die Transititvität und das Unabhängigkeitsaxiom die Präferenzen in erheblichem Maße: Transitivität verlangt, dass aus einer Präferenz der Alternative A gegenüber B und der Präferenz von B gegenüber C auch eine Präferenz von A gegenüber C folgen muss. Diese Annahme ist notwendig, wenn – wie in der Erwartungsnutzentheorie – jede Alternative durch eine reelle Zahl bewertet werden soll, da die Ordnungsrelation auf den reellen Zahlen auch transitiv ist. Das Unabhängigkeitsaxiom verlangt, dass eine Präferenz zwischen A und B bestehen bleibt, wenn bei beiden Alternativen eine Wahrscheinlichkeitsmischung mit einer dritten Alternative C durchgeführt wird. So folgt aus einer Präferenz von A gegenüber B beispielsweise, dass ein Münzwurf, bei dem man A oder C erhält, einem Münzwurf, bei dem man B oder C erhält, vorgezogen werden muss. Das Unabhängigkeitsaxiom bewirkt, dass der Erwartungsnutzen linear in den Wahrscheinlichkeiten ist, während die Konsequenzen durch eine in der Regel nicht-lineare Nutzenfunktion bewertet werden.

Aufgrund der zentralen Bedeutung beider Axiome für die Erwartungsnutzentheorie ist es kaum verwunderlich, dass sie relativ bald nach dem Erscheinen der Arbeit von von Neumann und Morgenstern experimentell überprüft wurden, siehe beispielsweise Preston & Baratta (1948), Allais (1953), Edwards (1955) und Tversky (1969). Bereits diese Studien zeigen, dass beide Axiome durchaus kritisch zu sehen sind, da sie durch das Entscheidungsverhalten vieler Personen systematisch verletzt werden. Zahlreiche Folgestudien haben diese Evidenz der frühen Studien im Wesentlichen bestätigt, weshalb die Erwartungsnutzentheorie aus deskriptiver Sicht nicht sehr erfolgreich ist.

Ein typisches experimentelles Design (der sogenannte common ratio Effekt) besteht aus den folgenden zwei Paaren, wobei p die jeweilige Gewinnwahrscheinlichkeit angibt:

A: 3000 EUR, p =1 versus B: 4000 EUR, p = 0.8,
0 EUR, p = 0.2

A*: 3000 EUR, p = 0.25, versus B*: 4000 EUR, p = 0.2,
0 EUR, p = 0.75 0 EUR, p = 0.8

Der Erwartungsnutzen impliziert, dass entweder im ersten Problem A und im zweiten A* gewählt wird oder B und B* gewählt werden. Ein Großteil der Probanden wählt jedoch gewöhnlich A und B*, was das Unabhängigkeitsaxiom verletzt.

Derartige Verstöße gegen die Erwartungsnutzentheorie bilden die Motivation für die Entwicklung zahlreicher alternativer Theorien, die durch eine Abschwächung der Axiome des Erwartungsnutzens versuchen, eine realistischere Modellierung des Entscheidungsverhaltens zu generieren. Die bekannteste Alternative ist zur Zeit wohl die Prospect Theorie (Kahneman & Tversky, 1979, Tversky & Kahneman, 1992), bei der das Unabhängigkeitsaxiom u.a. aufgrund einer nicht-linearen Wahrscheinlichkeitsgewichtung verletzt wird. Die Wahrscheinlichkeitsgewichtung berücksichtigt empirische Beobachtungen, dass kleine Wahrscheinlichkeiten bzw. die Wahrscheinlichkeiten extremer Konsequenzen häufig überbewertet werden. Sowohl die experimentelle Überprüfung der Axiome der Erwartungsnutzentheorie als auch die Modellierung der Verletzungen dieser Axiome im Rahmen der Prospect Theorie sind Gegenstand der vorliegenden Arbeit. Während in Teil A neue Experimente vorgestellt werden, die insbesondere die Möglichkeit von Entscheidungsfehlern in die Analyse einbeziehen, werden in Teil B im Wesentlichen neue Varianten der Prospect Theorie abgeleitet und charakterisiert.

Die Aufsätze in Teil A stellen neue Experimente zum Test des Unabhängigkeitsaxioms vor. Das neuartige an diesen Experimenten ist, dass explizit die Möglichkeit von Entscheidungsfehlern berücksichtigt wird. In wiederholten Experimenten wurde beobachtet, dass Probanden bei binären Entscheidungsproblemen in der ersten Wiederholung eine andere Alternative als in der zweiten Wiederholung wählen. Geht man von stabilen Präferenzen aus und berücksichtigt, dass nur eine der Wiederholungen für die Auszahlung relevant ist (in der Regel durch Zufallsauswahl festgelegt), muss der Proband in einer der Wiederholungen einen Fehler begangen haben, indem er die weniger präferierte Alternative gewählt hat. Die Modellierung solcher Fehler erfordert, dass eine stochastische

Komponente in das zugrundegelegte Modell integriert wird. Die einfachste Möglichkeit ist hier ein Fechner-Modell, bei dem eine normalverteilte Zufallsvariable zu der Nutzenfunktion addiert wird und die Summe aus Nutzenfunktion und Zufallsvariable das Entscheidungsverhalten determiniert. Ein solches Fechner-Modell bildet die Grundlage für die Hypothesen, die in Kapitel 2 untersucht werden. Der Nachteil des Fechner-Modells ist, dass man eine bestimmte Nutzenfunktion unterstellen muss, zu der die Fehlerterme addiert werden. Verwendet man hier beispielsweise den Erwartungsnutzen, unterstellt man implizit, dass die wahren Präferenzen Transitivität und das Unabhängigkeitsaxiom erfüllen. Dies bedeutet, dass man auf Grundlage eines derartigen Modells zwar Verletzungen beider Axiome untersuchen kann, es jedoch nicht möglich ist Aussagen dahingehend zu treffen, ob eine Abschwächung der Axiome zu einer signifikant besseren Erklärung der Daten führt. Aus diesem Grund basiert Kapitel 3 auf einem allgemeineren Fehlermodell, dem sogenannten true-and-error Modell. Dieses Modell erlaubt beliebige Präferenzen zwischen den einzelnen Alternativen, die mit jeweils konstanten Fehlerwahrscheinlichkeiten das Entscheidungsverhalten bestimmen. Damit ist das Modell im Gegensatz zu bekannten anderen Fehlermodellen (siehe u.a. Thurstone (1927), Luce (1959) oder Busemeyer & Townsend (1993)) neutral in Bezug auf Transitivität, d.h. es wird keine Annahme darüber getroffen, ob die wahren Präferenzen transitiv sind oder nicht. Das gleiche gilt für die Erfüllung des Unabhängigkeitsaxioms.

Die Aufsätze in Teil B widmen sich im Wesentlichen der Analyse bzw. Weiterentwicklung der Prospect Theorie. Eine Ausnahme bildet Kapitel 4, in dem das für Teil A wichtige Thema der Entscheidungsfehler noch einmal aus theoretischer Sicht betrachtet wird. Während die bisherigen Fechner-Modelle für Entscheidungen unter Risiko von Fehlern bei der Bewertung von Konsequenzen ausgehen, werden in Kapitel 4 Fehler bei der Wahrnehmung von Wahrscheinlichkeiten analysiert. Bei den Weiterentwicklungen der Prospect Theorie wird in Kapitel 5 zunächst die Modellierung der Risikoeinstellung in der Prospect Theorie genauer analysiert, da die Literatur in dieser Hinsicht bisher unvollständig war. Es zeigt sich, dass aufgrund der Wahrscheinlichkeitsgewichtung Risikoaversion in der Prospect Theorie sogar mit einer konvexen Nutzenfunktion vereinbar ist. Dies ist jedoch nur der Fall, wenn die Gewichtungsfunktionen Unstetigkeiten aufweisen, was jedoch bei Wahrscheinlichkeiten, die gegen null oder eins konvergieren, nicht unrealistisch ist.

In Kapitel 6 wird eine Variante der Prospect Theorie mit (stückweise) linearer Nutzenfunktion entwickelt. In der Erwartungsnutzentheorie wird die Risikoeinstellung einer

Person alleine durch die Krümmung der Nutzenfunktion bestimmt, d.h. Risikoaversion folgt aus abnehmendem Grenznutzen und damit einer konkaven Nutzenfunktion. Es ist jedoch unrealistisch, dass die Nutzenfunktion gerade für kleine Geldbeträge, wie sie typischerweise in Experimenten verwendet werden, eine bedeutende Krümmung aufweist. So legen zahlreiche Studien nahe, dass die Nutzenfunktion für kleinere Beträge annähernd linear ist (siehe u.a. Edwards (1955) und Lopes (1995)), was im Rahmen der Erwartungsnutzentheorie risikoneutrales Verhalten impliziert, d.h. es wird einfach die Lotterie mit dem höchsten Erwartungswert gewählt. In der Prospect Theorie ist dies anders, da das Risikoverhalten auch durch die Wahrscheinlichkeitsgewichtung beeinflusst wird. Das in Kapitel 6 abgeleitete Modell ist aufgrund der linearen Nutzenfunktion sehr einfach und eignet sich daher gut für empirische Studien. Insbesondere die zu Grunde liegende Axiomatik ist weit weniger komplex als in der allgemeinen Prospect Theorie und kann daher leichter getestet werden.

In Kapitel 7 wird schließlich ein neues Modell der Prospect Theorie mit dem Ziel entwickelt, empirische Beobachtungen zu erklären, die sich durch die bisherigen Varianten der Prospect Theorie nicht modellieren lassen. Eine solche Beobachtung ist das Preference Reversal Phänomen, das erstmals von Lichtenstein & Slovic (1971) beschrieben wurde und noch heute ein intensiv diskutiertes Thema darstellt. Beim Preference Reversal Phänomen werde die Präferenzen auf zwei Arten ermittelt, durch direkte Auswahl zwischen den Alternativen und durch die Zuweisung minimaler Verkaufspreise. Lichtenstein und Slovic haben nun beobachtet, dass beim Vergleich einer sicheren und einer riskanten Alternative viele Personen die sichere Alternative in einer direkten Auswahl bevorzugen würden, für die riskante Alternative aber einen höheren Verkaufspreis verlangen. Ein solcher Preference Reversal ist weder mit dem Erwartungsnutzen noch im Rahmen der bisherigen Modelle der Prospect Theorie erklärbar. Das neue Modell in Kapitel 7 ermöglicht die Erklärung des Preference Reversal Phänomens durch den Einbezug zustandsabhängiger Referenzpunkte. Bereits in der ursprünglichen Version der Prospect Theorie aus dem Jahr 1979 spielen Referenzpunkte eine wichtige Rolle und sind im Vergleich zum Erwartungsnutzen die wichtigste Verallgemeinerung neben der Wahrscheinlichkeitsgewichtung. Dabei wird angenommen, dass der Entscheidungsträger in Abhängigkeit vom gegebenen Entscheidungsproblem einen Referenzpunkt bildet und negative Abweichungen von diesem Referenzpunkt (Verluste) stärker gewichtet werden als positive Abweichungen (Gewinne). In der Regel wird davon ausgegangen, dass das Anfangsvermögen des Entscheidungsträgers seinen Referenzpunkt bildet. Da in den bisherigen Varianten der Prospect Theorie der Referenzpunkt jedoch deterministisch ist, führt ein stochastisches Anfangsvermögen zu

Problemen. Gerade aber bei der Festlegung des minimalen Verkaufspreises einer riskanten Alternative ist das Anfangsvermögen stochastisch. Kapitel 7 zeigt, dass die Erweiterung der Prospect Theorie um stochastische Referenzpunkte hinreichend ist, um das Preference Reversal zu erklären. Zudem ist das Modell auch mit den übrigen typischen Verstößen gegen den Erwartungsnutzen vereinbar, weshalb es für zukünftige empirische Studien sehr geeignet erscheint.

2 Beschreibung der einzelnen Kapitel

Diese kumulative Dissertation besteht neben der deutschen Zusammenfassung aus sechs weiteren Kapiteln, von denen die ersten beiden (Teil A) experimentelle Studien darstellen, während Kapitel 4-7 (Teil B) neue theoretische Ansätze des Entscheidungsverhaltens unter Risiko entwickeln.

Kapitel 2 (Testing Expected Utility Plus Noise) stellt ein neues Experiment zur Überprüfung der Implikationen des Unabhängigkeitsaxiom dar, in dem explizit die Möglichkeit von Entscheidungsfehlern berücksichtigt wird. Nehmen wir an, dass die Präferenzen eines Entscheidungsträgers durch eine Nutzenfunktion repräsentiert werden können. Dies impliziert, dass eine Funktion V existiert, so dass

(1) $V(S, R) > 0$ (< 0)

gilt, falls der Entscheidungsträger bei der Wahl zwischen den Alternativen S und R die Alternative S der Alternative R vorzieht (R gegenüber S vorzieht). Bei der Möglichkeit von Entscheidungsfehlern kann es jedoch passieren, dass der Entscheidungsträger R wählt, obwohl er gemäß seinen wahren Präferenzen (d.h. gemäß der Nutzenfunktion V) eigentlich S gegenüber R vorzieht. Eine einfache Möglichkeit, derartige Fehler zu modellieren ist die Einführung eines Störterms ε, so dass die Auswahl zwischen S und R von ε beeinflusst werden kann. Präziser formuliert, wird der Entscheidungsträger S (R) wählen, falls

(2) $V(S, R) + \varepsilon > 0$ (< 0)

gilt. Es wird gewöhnlich angenommen, dass die Zufallsvariable ε symmetrisch verteilt um einen Erwartungswert von null ist. In Kapitel 2 muss jedoch lediglich angenommen werden, dass der Median von ε null beträgt, d.h. Entscheidungsfehler treten immer mit einer Wahrscheinlichkeit auf, die geringer als 50% ist. Zudem kann in Kapitel 2 auf die Annahme verzichtet werden, dass die Verteilung von ε für alle Auswahlprobleme identisch ist.

Bei Entscheidungen unter Risiko sind die Alternativen durch Lotterien gegeben. Im folgenden wird eine Lotterie S durch einen Vektor $S = (x_1, s_1; x_2, s_2; \ldots; x_n, s_n)$ repräsentiert. Diese Notation bedeutet, dass der Entscheidungsträger die Konsequenz x_i mit der Wahrscheinlichkeit s_i erhält. Der Einfachheit halber gehen wir nur von endlichen Lotterien, d.h. endlichen Werten von n aus. Zudem wird angenommen, dass die Präferenzen die Axiome der Erwartungsnutzentheorie erfüllen. Dies impliziert die Existenz einer von Neumann-Morgenstern Nutzenfunktion u, so dass V(S, R) durch die Differenz der Erwartungsnutzen von S und R gegeben ist:

(3) $V(S, R) = \Sigma_i\, u(x_i)(s_i - r_i)$.

In anderen Worten wird angenommen, dass das Auswahlverhalten durch den Erwartungsnutzen und einen stochastischen Fehlerterm modelliert werden kann. Ziel der experimentellen Studie ist es, dieses Modell empirisch zu überprüfen.

Grundlage des experimentellen Designs sind 28 common consequence und common ratio Effekte. Beide Effekte bestehen aus zwei Lotteriepaaren (S und R sowie S* und R*), so dass der Erwartungsnutzen entweder die Präferenz von S gegenüber R und S* gegenüber R* oder die Präferenz von R und R* impliziert. Die Auswahlmuster S und R* sowie R und S* verletzen dagegen den Erwartungsnutze, falls keine Entscheidungsfehler vorliegen. Die Innovation des experimentellen Designs in Kapitel 2 ist die Tatsache, dass den Probanden alle Entscheidungsprobleme dreimal präsentiert werden. Die Antworten eines Probanden zu einem gegebenen common consequence oder common ratio Effekt können nun durch einen Vektor dargestellt werden, bei dem die ersten drei Einträge die jeweilige Wahl zwischen S und R bei den drei Wiederholungen und die letzen drei Einträge die jeweilige Wahl zwischen S* und R* darstellen. (S, R, S, R*, R*, S*) bedeutet beispielsweise, dass ein Proband in der ersten Runde S und R*, in der zweiten Runde R und R* sowie in der dritten Runde S und S* gewählt hat. Das Entscheidungsmuster eines Probanden erfüllt Wiederholungskonsistenz, wenn die ersten drei Einträge und die letzten drei Einträge identisch sind, d.h. es gilt (S, S, S, S*, S*, S*), (R, R, R, R*, R*, R*), (S, S, S, R*, R*, R*), oder (R, R, R, S*, S*, S*). Ist die Wiederholungskonsistenz verletzt, muss der Proband zwangsläufig einen Entscheidungsfehler begangen haben, wenn sich die Präferenzen (wie angenommen) im Laufe des Experimentes nicht ändern.

Betrachten wir den Fall, dass ein Entscheidungsträger gemäß seiner wahren Präferenzen S gegenüber R und wegen der Konsistenz mit dem Erwartungsnutzen auch S* gegenüber R* vorzieht. Ferner betrage die Wahrscheinlichkeit eines Ausfahlfehlers α bei der Wahl zwischen S und R und β bei der Wahl zwischen S* und R*. Dies impliziert, dass das Auswahlmuster (S, S, S, S*, S*, S*) mit der Wahrscheinlichkeit $(1 - \alpha)^3(1 - \beta)^3$, (R, R, R, R*, R*, R*) mit der Wahrscheinlichkeit $\alpha^3\beta^3$, (S, S, S, R*, R*, R*) mit der Wahrscheinlichkeit $(1 - \alpha)^3\beta^3$ und (R, R, R, S*, S*, S*) mit der Wahrscheinlichkeit $\alpha^3(1 - \beta)^3$beobachtet wird Da dies die einzigen vier Auswahlmuster sind, die Wiederholungskonsistenz erfüllen, lässt sich die Wahrscheinlichkeit p_{rc} berechnen, mit der eine Verletzung des Unabhängigkeitsaxioms beim Vorliegen von Wiederholungskonsistenz auftreten. Analog lässt sich auch die Wahrscheinlichkeit p_{ri} berechnen, mit der Verletzungen des Unabhängigkeitsaxioms auftreten, wenn Wiederholungskonsistenz nicht erfüllt ist. Es stellt sich heraus, dass p_{ri} für alle zulässigen Werte von α und β höher ist als p_{rc}, d.h. das Modell impliziert, dass Verletzungen des Unabhängigkeitsaxioms beim Vorliegen von Wiederholungskonsistenz seltener beobachtet werden. Genau dieses Resultat wird in Kapitel 2 experimentell überprüft. Die Ergebnisse für alle 28 common consequence und common ratio Effekte finden sich in Tabelle 1. In der zweiten (dritten) Spalte wird die Verletzungsrate des Unabhängigkeitsaxioms in den Fällen angegeben, in denen Wiederholungskonsistenz verletzt (erfüllt) ist. Nur in zwei von 28 Fällen ist die Verletzungsrate beim Vorliegen von Wiederholungskonsistenz (leicht) höher, in 26 Fällen ist sie niedriger. Von diesen 26 Fällen sind 20 signifikant auf dem 5%-Niveau. Dies wird aus der letzen Spalte deutlich, in der die Ergebnisse eines zweiseitigen Wilcoxon-Tests angegeben werden (*** steht dabei für ein Signifikanzniveau von 1%, ** für 5% und * für 10%). Somit unterstützt das vorgestellte Experiment das Modell des Erwartungsnutzens mit Entscheidungsfehlern.

Auch Kapitel 3 (Testing Violations of Independence Conditions in the Presence of Errors and Splitting Effects) beruht auf einem experimentellen Design, in dem Probanden wiederholt identische Auswahlprobleme bearbeiten müssen. Während das zu Grunde liegende Modell in Kapitel 2 auf dem Erwartungsnutzen beruht, wird in Kapitel 3 das sogenannte true and error Modell verwendet. Betrachtet man die Auswahl zwischen zwei Alternativen , A und B, ist die Wahrscheinlichkeit, dass ein Individuum B wählt (P(B)) in diesem Modell durch $P(B) = p(1 - e) + (1 - p)e$ gegeben. Dabei bezeichnet p die Wahrscheinlichkeit, dass das Individuum B tatsächlich präferiert während e die Wahrscheinlichkeit eines Fehlers darstellt. Das Modell besagt somit, dass das Individuum B entweder wählt, wenn es B tatsächlich präferiert und keinen Fehler macht oder aber eigentlich A präferiert, bei der Auswahl jedoch einen Fehler begeht.

Tabelle1: Ergebnisse von Kapitel 2

Problem	Wiederholungs-Inkonsistenz [#]	Wiederholungs-konsistenz [#]	Differenz	Signifikanz-niveau
1	69.05% [7]	18.18% [11]	51%	**
2	53.33% [5]	56.25% [16]	-3%	
3	47.92% [8]	26.67% [15]	21%	*
4	26.67% [5]	25.00% [12]	2%	
5	43.75% [8]	33.33% [9]	10%	
6	63.89% [6]	26.67% [15]	37%	*
7	33.33% [6]	5.56% [18]	28%	***
8	30.56% [6]	0.00% [16]	31%	***
9	39.58% [8]	7.14% [14]	32%	***
10	50.00% [10]	55.56% [9]	-6%	
11	48.61% [12]	60.00% [10]	-11%	
12	50.00% [10]	11.11% [9]	39%	***
13	53.33% [10]	0.00% [11]	53%	***
14	47.22% [6]	0.00% [14]	47%	***
15	45.83% [4]	0.00% [15]	46%	***
16	22.22% [6]	5.56% [18]	17%	**
17	27.78% [6]	0.00% [18]	28%	***
18	29.17% [8]	0.00% [15]	29%	***
19	46.67% [5]	5.26% [19]	41%	**
20	47.62% [7]	6.25% [16]	41%	***
21	33.33% [4]	0.00% [19]	33%	**
22	37.50% [8]	7.69% [13]	30%	***
23	41.67% [8]	15.38% [13]	26%	**
24	28.57% [7]	15.38% [13]	13%	
25	38.89% [6]	6.25% [16]	33%	***
26	48.81% [14]	0.00% [8]	49%	***
27	54.17% [8]	14.29% [14]	40%	***
28	58.97% [13]	0.00% [9]	59%	***

Um zu zeigen, wie dieses Modell zum Test des Unabhängigkeitsaxioms verwendet werden kann, betrachten wir die hypothetischen Ergebnisse eines Experiments zum common ratio Effekt (siehe Abschnitt 1.1) in Tabelle 2.

Tabelle 2: Ein Experiment zum common ratio Effekt

	A*	B*
A	51	23
B	11	15

In diesem Experiment haben 34 Probanden das Unabhängigkeitsaxiom verletzt, indem sie A und B* gewählt haben (23 Probanden) bzw. B und A* (11 Probanden). Aufgrund dieser asymmetrischen Verletzungsraten könnte man davon ausgehen, dass die Verletzungen nicht alleine auf Fehlern beruhen. Auch der entsprechende statistische Test würde eine signifikante Verletzung des Unabhängigkeitsaxioms anzeigen. Dennoch können die Ergebnisse in Tabelle 2 auch alleine auf Fehlern beruhen. Um dies zu zeigen, erweitern wir das true and error Modell wie folgt: Da es beim common ratio Effekt vier mögliche Auswahlmuster gibt, betrachten wir vier mögliche wahre Präferenzmuster, p_{AA*}, p_{AB*}, p_{BA*} und p_{BB*}. Dabei gibt bspw. p_{AA*} die Wahrscheinlichkeit an, dass man gemäß seiner wahren Präferenzen A gegenüber B und A* gegenüber B* vorzieht. Somit ist die Wahrscheinlichkeit, dass die wahren Präferenzen gegen das Unabhängigkeitsaxiom verstoßen, durch $p_{AB*} + p_{BA*}$ gegeben. Bezeichnen wir mit e die Wahrscheinlichkeit eines Fehlers bei der Wahl zwischen A und B und e* die Wahrscheinlichkeit eines Fehlers bei der Wahl zwischen A* und B*, ergibt sich für die Wahrscheinlichkeit, dass man bspw. die Auswahl von A und B* beobachtet:

(4) $$P(AB^*) = p_{AA^*}(1 - e)e' + p_{AB^*}(1 - e)(1 - e') + p_{BA^*}ee' + p_{BB^*}e(1 - e').$$

Analog lässt sich die Wahrscheinlichkeit der drei anderen Auswahlmuster berechnen. Somit ist das Modell durch vier Gleichungen charakterisiert, mit denen man fünf Parameter schätzen möchte, nämlich e und e* sowie die Wahrscheinlichkeiten, dass die wahren Präferenzen einem bestimmten Muster entsprechen (p_{AA*}, p_{AB*}, p_{BA*} und p_{BB*}). Da sich diese vier Wahrscheinlichkeiten aber zu eins addieren müssen, gibt es nur drei freie Parameter. Da sich mit vier Gleichungen nicht fünf Parameter schätzen lassen, ist das Modell unterdeterminiert. Für die Werte von Tabelle 2 gibt es somit viele Lösungen, die die Daten perfekt erklären, zwei davon sind in Tabelle 3 dargestellt.

Tabelle 3: Mögliche Parameterkonstellationen

Parameter	Modell 1: EN	Modell 2: EN verletzt
P_{AA^*}	0.80	0.67
P_{AB^*}	0.00	0.17
P_{BA^*}	0.00	0.00
P_{BB^*}	0.20	0.16
e	0.10	0.15
e′	0.30	0.15

Bei Modell 1 können die Voraussagen des Erwartungsnutzens (EN) durch die Annahme ungleicher Fehlerwahrscheinlichkeiten erfüllt werden (es gilt $p_{AB^*} + p_{BA^*} = 0$), während bei Annahme gleicher Fehlerwahrscheinlichkeiten EN im Modell 2 verletzt wird. Da beide Modelle die Daten jedoch perfekt abbilden, lässt sich nicht sagen, welches Modell zu bevorzugen ist. Um weiter gehende Aussagen zu treffen, integrieren wir Wiederholungen in das experimentelle Design. Nehmen wir an, dass ein Proband die Auswahl zwischen A und B zweimal durchführen muss. Dann ist die Wahrscheinlichkeit, dass dieser beim ersten Mal A und beim zweiten Mal B wählt, durch

(5) $P(AB) = pe(1 - e) + (1 - p)(1 - e)e = e(1 - e)$

gegeben, wobei p wiederum die Wahrscheinlichkeit angibt, dass der Proband gemäß der wahren Präferenzen B vorzieht. Werden nun beide Auswahlprobleme, das zwischen A und B und das zwischen A* und B* wiederholt durchgeführt, gibt es insgesamt 16 möglich Auswahlmuster aber immer noch fünf zu schätzende Parameter. Somit lässt sich eindeutig testen, ob die Hypothese $p_{AB^*} + p_{BA^*} = 0$ abgelehnt werden kann.

Tabelle 4 enthält die Ergebnisse unserer Tests für verschiedene Unabhängigkeitsbedingungen. Das Unabhängigkeitsaxiom der Erwartungsnutzentheorie wird durch vier sogenannte common consequence Effekte (CCE1-4) und zwei bereits angesprochene common ratio Effekte (CRE 1 und 2) getestet. Die daran anschließenden unteren Zeilen in Tabelle 4 beschreiben Tests von schwächeren Unabhängigkeitsbedingungen, die in Alternativen zur Erwartungsnutzentheorie verwendet werden. Der Index s in der ersten Spalte von Tabelle 4 bedeutet, dass es sich um eine sogenannte „split version“ handelt. Dies bedeutet, dass beide Lotteiren eines Auswahlproblems so dargestellt sind, dass es die gleiche Anzahl möglicher Ereignisse mit identischen

Wahrscheinlichkeiten gibt. Die split version des in Abschnitt 1.1 dargestellten common ratio Effekt ist daher die folgende:

A_S: 3000 EUR, p = 0.8.	versus	B_S: 4000 EUR, p = 0.8,
3000 EUR, p = 0.2		0 EUR, p = 0.2
A_S*: 3000 EUR, p = 0.2,	versus	B_S*: 4000 EUR, p = 0.2,
3000 EUR, p = 0.05		0 EUR, p = 0.05
0 EUR, p = 0.75		0 EUR, P = 0.75

Tabelle 4: Tests verschiedener Unabhängigkeitsbedingungen

Eigenschaft	p_{AA*}	p_{AB*}	p_{BA*}	p_{BB*}	e	e´	Test
CCE1	0.44	0.02	**0.30**	0.24	0.15	0.11	**20.36****
$CCE1_S$	0.52	**0.20**	0.00	0.28	0.13	0.16	**12.77****
CCE2	0.02	0.00	**0.10**	0.88	0.02	0.08	**15.33****
$CCE2_S$	0.09	0.03	**0.05**	0.84	0.07	0.07	**7.61***
CCE3	0.25	**0.21**	0.16	0.39	0.16	0.12	**18.69****
$CCE3_S$	0.52	**0.24**	0.00	0.25	0.13	0.16	**12.63****
CCE4	0.67	0.01	**0.29**	0.02	0.14	0.09	**21.96****
$CCE4_S$	0.80	0.01	0.02	0.17	0.14	0.12	0.82
CRE1	0.25	0.00	**0.64**	0.11	0.11	0.07	**44.64****
$CRE1_S$	0.44	0.00	**0.46**	0.10	0.15	0.05	**27.21****
CRE2	0.57	0.00	**0.20**	0.23	0.14	0.11	**18.00****
$CRE2_S$	0.84	0.02	0.01	0.12	0.17	0.12	0.45
UTI	0.06	0.01	**0.52**	0.40	0.13	0.18	**18.76****
UTI_S	0.17	0.01	0.00	0.82	0.14	0.18	0.05
LTI	0.04	0.00	0.14	0.82	0.05	0.15	3.96
LTI_S	0.04	0.00	0.01	0.95	0.06	0.08	0.24
UCI	0.14	0.08	0.13	0.66	0.13	0.22	3.88
UCI_S	0.13	0.09	0.02	0.76	0.13	0.09	5.07
LDI	0.94	0.00	0.00	0.06	0.02	0.05	0.00
UDI	0.16	0.02	0.03	0.79	0.09	0.10	1.80

Anmerkung: * entspricht einem Signifikanzniveau von 5%, ** einem Signifikanzniveau von 1%.

Es ist leicht zu erkennen, dass die split version nur eine andere Darstellungsweise ist, die Lotterien aber mathematisch identisch sind, d.h., die Wahrscheinlichkeiten der einzelnen Auszahlungen sind unverändert. Dennoch kann diese unterschiedliche Darstellungsweise das Entscheidungsverhalten systematisch beeinflussen (wir sprechen dann von einem Splitting Effekt). So wirkt die Lotterie A* im Verglich zu B* in der split version evt. attraktiver, da die 3000 nun zweimal genannt wird, währen bei B* die 0 nun zweimal genannt wird.

Tabelle 4 zeigt, dass das Unabhängigkeitsaxiom der Erwartungsnutzentheorie in nahezu allen Tests signifikant verletzt wird. Bei den schwächeren Unabhängigkeitsbedingungen ist dagegen nur eine signifikante Verletzung beobachtbar. Interessant ist auch, dass substantielle Splitting Effekt vorliegen. So sinkt bspw. bei CCE1 die Wahrscheinlichkeit des Musters BA* von 0.3 auf null, während die Wahrscheinlichkeit der umgekehrten Verletzung des Unabhängigkeitsaxioms (AB*) von 0.02 auf 0.2 ansteigt.

Kapitel 4 widmet sich auch dem Thema der Entscheidungsfehler, diesmal allerdings aus einer theoretischen Perspektive. Da die meisten Personen wohl eher gewohnt sind, Geldbeträge statt Wahrscheinlichkeiten zu evaluieren, erscheint es realistisch, dass ihnen bei der Bewertung von Wahrscheinlichkeiten eher Fehler unterlaufen als bei der Bewertung von Geldbeträgen. Insofern erscheint es sinnvoll im Gegensatz zu dem in Kapitel 2 verwendeten Fechner-Modell, bei dem die Fehlerterme zur Nutzenskala addiert werden, Fehlerterme zu den Wahrscheinlichkeiten zu addieren. Dabei wird angenommen, dass auch die fehlerhaft bewerteten Wahrscheinlichkeiten die klassischen Bedingungen der Wahrscheinlichkeitsrechnung erfüllen müssen, insbesondere müssen alle Wahrscheinlichkeiten zwischen null und eins liegen und sich zu eins aufaddieren. Unter diesen Annahmen impliziert das Modell, dass kleine Wahrscheinlichkeiten im Durchschnitt übergewichtet und große Wahrscheinlichkeiten im Durchschnitt untergewichtet werden. Damit bildet das Modell eine Erklärung für die Wahrscheinlichkeitsgewichtungsfunktion in der Prospect Theory. Zudem wird gezeigt, dass das Modell einige typische Verletzungen des Erwartungsnutzens erklären kann.

Die Kapitel 5-7 sind theoretische Analysen, die sich mit Weiterentwicklungen der (kumulativen) Prospect Theorie beschäftigen. Um die Prospect Theory zu charakterisieren, betrachten wir ein Modell der Unsicherheit, bei dem einer von s = 1, 2, …, S Umweltzuständen eintreten kann. Die subjektive oder objektive Wahrscheinlichkeit des Umweltzustandes s wird mit p_s bezeichnet. Eine Lotterie L ist nun durch die Auszahlungen in den einzelnen Umweltzuständen charakterisiert. Bezeichnet man die Auszahlung im Umweltzustand s mit x_s, gilt L = (x_1, x_2, …,

x_S). Beim Erwartungsnutzen existiert nun eine reellwertige Nutzenfunktion u, so dass die Lotterie L mit

(6) $V(L) = \Sigma_s u(x_s)p_s$

bewertet wird. Bei der originalen Prospect Theorie gibt es drei zentrale Unterschiede zum Erwartungsnutzen. Erstens wird eine Editing Phase eingeführt, in der die Lotterien vom Entscheidungsträger vor der Bewertung transformiert werden. Eine Operation ist das Abtrennen von sicheren Gewinnen und Verlusten. Die Editing Phase ist wenig formalisiert und spielt für die Arbeiten in den Kapiteln 5-7 kaum eine Rolle. Daher wird an dieser Stelle nicht näher darauf eingegangen. Zweitens wird die Nutzenfunktion u durch eine Wertfunktion v ersetzt, die referenzpunktabhängig ist. Wertfunktionen werden im Gegensatz zu Nutzenfunktionen unter Sicherheit ermittelt. Referenzpunktabhängigkeit besagt, dass Entscheidungsträger nicht das Endvermögen bewerten sondern Gewinne und Verluste relativ zu einem Referenzpunkt r. Dabei wird in der Prospect Theorie von „diminishing sensitivity" (eine marginale Erhöhung eines Gewinnes oder Verlustes hat eine umso stärkere Auswirkung, je näher man sich am Referenzpunkt befindet) und Verlustaversion (ein gegebener Verlust hat eine stärkere Auswirkung auf die Attraktivität einer Lotterie als ein gleich hoher Gewinn) ausgegangen. Drittens gehen die Wahrscheinlichkeiten im Gegensatz zum Erwartungsnutzen nicht linear in die Bewertung der Lotterien ein sondern werden durch eine Wahrscheinlichkeitsgewichtungsfunktion transformiert. Diese Funktion $f: [0, 1] \rightarrow [0, 1]$ ist streng monoton steigend mit $f(0) = 0$ und $f(1) = 1$. Dabei wird davon ausgegangen, dass kleine Wahrscheinlichkeiten übergewichtet werden (d.h. $f(p) > p$), während große Wahrscheinlichkeiten untergewichtet werden. Somit erfolgt die Bewertung einer Lotterie in der originalen Prospect Theorie insgesamt wie folgt:

(7) $V(L) = \Sigma_s v(x_s - r)f(p_s)$.

Ein zentrales Problem der originalen Prospect Theory ist es, dass sie Verstöße gegen die stochastische Dominanz impliziert, die sehr unrealistisch sind. Nehmen wir bspw. an, dass $f(0{,}25) > 0{,}25$ gilt. Dann würde eine Lotterie die mit einer Wahrscheinlichkeit von jeweils 25% zu den Auszahlungen $(x - |\varepsilon_1|, x - |\varepsilon_2|, x - |\varepsilon_3|, x - |\varepsilon_4|)$ führt, für hinreichend kleine Werte von ε_i gegenüber der sicheren Zahlung von x vorgezogen werden, solange die Wertfunktion wie angenommen Stetigkeit erfüllt.

Aus diesem Grund wurde die kumulative Prospect Theorie (KPT) entwickelt, bei der im Gegensatz zur originalen Prospect Theorie dekumulierte Wahrscheinlichkeiten transformiert werden. Die KPT basiert auf den rangplatzabhängigen Nutzen, der derartig transformierte Wahrscheinlichkeiten in die Erwartungsnutzentheorie integriert hat. Beim rangplatzabhängigen Nutzen werden die Auszahlungen einer Lotterie zunächst in eine absteigende Rangfolge gebracht, d.h. es gilt $x_r > x_t$ für $t > r$. Nun erfolgt die Bewertung der Lotterien wie folgt:

(8) $\quad V(L) = \Sigma_s\, u(x_s)\pi_s(L)$ mit $\pi_s(L) = f(\Sigma_{r\leq s}\, p_r) - f(\Sigma_{r<s}\, p_r)$.

Es gilt also $\pi_1(L) = f(p_1)$, $\pi_2(L) = f(p_1 + p_2) - f(p_1)$, etc. Diese Transformation dekumulierter Wahrscheinlichkeiten impliziert keine Verstöße gegen die stochastische Dominanz, da sich die Wahrscheinlichkeitsgewichte $\pi_s(L)$ zu eins summieren. Ein wichtiger Faktor beim rangplatzabhängigen Nutzen ist die Krümmung der Gewichtungsfunktion f, die wiederum streng monoton steigend ist und $f(0) = 0$ und $f(1) = 1$ erfüllt. Wenn f konvex ist, werden die Wahrscheinlichkeiten der schlechtesten Konsequenzen übergewichtet und die der besten Konsequenzen untergewichtet. Neuere experimentelle Studien legen nahe, dass f invers-S-förmig ist, d.h. erst konkav und dann konvex. Dies impliziert, dass die Wahrscheinlichkeit der schlechtesten und besten Konsequenzen übergewichtet und die der mittleren Konsequenzen untergewichtet werden.

Bei der KPT wird im Vergleich zum rangplatzabhängigen Nutzen die Nutzenfunktion wiederum durch eine referenzpunktabhängige Wertfunktion ersetzt. Zudem wird davon ausgegangen, dass die Wahrscheinlichkeitsgewichte abhängig vom Vorzeichen der betrachteten Konsequenz sein können, d.h. die Wahrscheinlichkeitsgewichtung von Gewinnen und Verlusten kann unterschiedlich sein. Wie beim rangplatzabhängigen Nutzen werden die Konsequenzen einer Lotterie in eine absteigende Rangfolge gebracht. Für eine Lotterie mit k Gewinnen, d.h. $x_k \geq 0 \geq x_{k+1}$ gilt dann:

(9) $\quad V(L) = \Sigma_s v(x_s - r)\pi_s(L)$ mit $\pi_s(L) = \begin{cases} f^+(\sum_{r\leq s} p_r) - f^+(\sum_{r<s} p_r) & \text{falls } s \leq k \\ f^-(\sum_{r\geq s} p_r) - f^-(\sum_{r>s} p_r) & \text{falls } s > k. \end{cases}$

Es gibt also zwei unterschiedliche Gewichtungsfunktion, eine für Verluste (f^-) und eine für Gewinne (f^+). Zudem werden im Verlustbereich kumulierte, im Gewinnbereich dekumulierte Wahrscheinlichkeiten transformiert.

Kapitel 5 befasst sich mi der Frage, unter welchen Bedingungen die KPT Risikoaversion erfüllt. Risikoaversion wird dabei als Aversion gegenüber sogenannten mean-preserving spreads definiert. Ein mean-preserving spread einer Lotterie wird dadurch konstruiert, dass man die eine schlechte Konsequenz verschlechtert und eine gute verbessert, ohne dabei den monetären Erwartungswert der Lotterie zu verändern. Bei Risikoaversion muss eine mean-preserving spread nutzenmindernd wirken. Es ist bekannt, dass in der Erwartungsnutzentheorie eine konkave Nutzenfunktion zu Risikoaversion führt, während beim rangplatzabhängigen Nutzen die gleiche Bedingung erfüllt sein und zusätzlich eine konvexe Gewichtungsfunktion vorliegen muss. Kapitel 5 zeigt, dass Risikoaversion bei der KPT etwas komplexer ist: Die Wertfunktion muss sowohl im Gewinn- als auch im Verlustbereich konkav sein. Zudem muss die Gewichtungsfunktion f^+ konvex sein, während f^- konkav sein muss, da im Verlustbereich ja kumulierte Wahrscheinlichkeiten transformiert werden. Schließlich gibt es noch eine weitere Bedingung, die notwendig ist, da die Wertfunktion an der Stelle null einen Knick aufweisen kann. Diese Bedingung legt fest, dass die Steigung der Wertfunktion abhängig von den Gewichtungsfunktionen bei einer Annäherung von oben an null nur zu einem gewissen Grad steiler sein darf als bei einer Annäherung von unten an null.

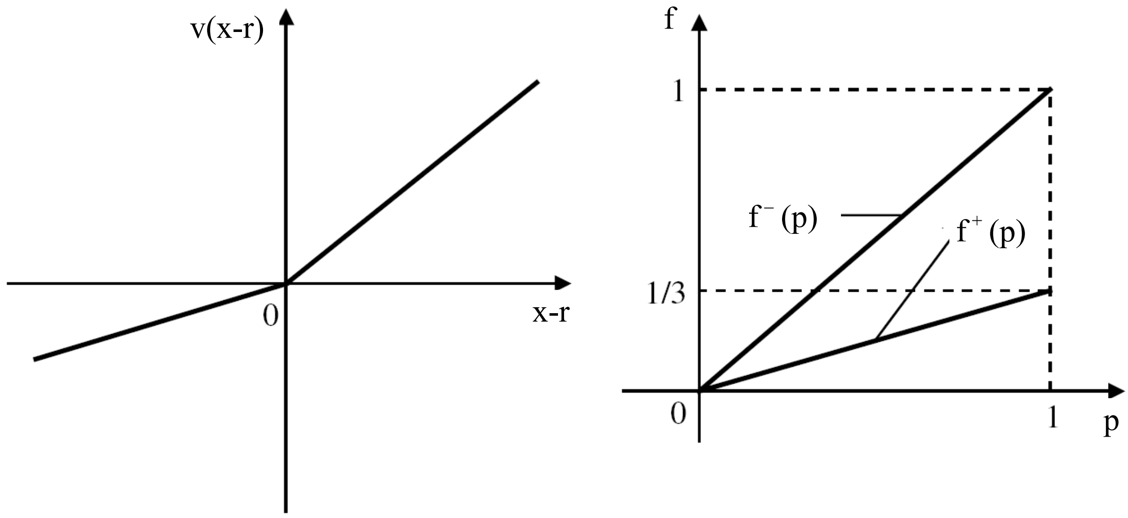

Abbildung 1: Risikoaversion bei einer konvexen Wertfunktion

Abbildung 1 verdeutlicht, dass diese Bedingung dennoch mit einer konvexen Werfunktion vereinbar ist. Dies ist jedoch nur der Fall, wenn die Gewichtungsfunktion f^+ bei eins eine Sprungstelle hat, wie in der rechten Graphik in Abbildung 1 gezeigt. Dennoch ist dieses Resultat überraschend, da Risikoaversion in allen anderen Theorien unabdingbar mit einer konkaven Nutzen- bzw. Wertfunktion verbunden ist.

Kapitel 6 befasst sich ebenfalls mit der KPT und untersucht Bedingungen, unter welchen die Wertfunktion, wie in der linken Graphik von Abbildung 1 dargestellt, stückweise linear ist mit einem möglichen Knick am Nullpunkt. Das Modell, das als lineare KPT, bzw. LKPT bezeichnet werden kann, ist zum einen aus empirischer Sicht interessant, da es beginnend mit Edwards (1955) zahlreiche experimentelle Studien gibt, die Evidenz für lineare Wert- bzw. Nutzenfunktionen finden. Zum anderen ist eine lineare Wertfunktion auch aus theoretischer Sicht interessant. Während beim Erwartungsnutzen die Risikoeinstellung allein durch die Krümmung der Nutzenfunktion determiniert wird, spielt in der LKPT die Krümmung der Gewichtungsfunktionen die dominante Rolle. Zudem ist die LKPT vergleichsweise einfach und eignet sich daher gut für theoretische Anwendungen, die zum Teil auch bereits erfolgt sind. Das zentrale Ergebnis von Kapitel 6 ist, dass sich die LKPT im Vergleich zur KPT aus sehr einfachen Präferenzbedingungen herleiten lässt. Ausgehend von der Axiomatik des Erwartungsnutzens muss das Unabhängigkeitsaxiom nur durch ein einfaches neues Axiom ersetzt werden. Dieses neue Axiom wird mit „independence of common increments“ bezeichnet. Dabei werden zwei Lotterien betrachtet, die auf einem identischen Wahrscheinlichkeitsvektor bzw. auf einer identischen Menge von Umweltzuständen basieren. Die Präferenz zwischen beiden Lotterien darf sich gemäß independence of common increments nicht ändern, wenn bei beiden zu einer ranggleichen Konsequenz ein beliebiger Betrag hinzuaddiert wird, wobei sich durch die Addition weder der Rang noch das Vorzeichen der Konsequenzen ändern dürfen.

Kapitel 7 entwickelt ein Modell der KPT, in dem der Referenzpunkt im Gegensatz zu vorherigen Varianten vom Umweltzustand abhängen kann. Die Wertfunktion in Gleichung (9) ist dann durch $v(x_s - r_s)$ gegeben. Da als Referenzpunkt meist das Anfangsvermögen des Entscheidungsträgers gewählt wird, erlaubt das Modell somit, ein stochastisches Anfangsvermögen zu betrachten. Dies ist unter anderem bei der Analyse des Preference Reversal Phänomens notwendig. Um zu beurteilen, inwieweit dieses Phänomen durch einen zustandsabhängigen Referenzpunkt erklärt werden kann, bedient sich Kapitel 7 einer einfachen Parametrisierung des Modells. Für $z = x - r$ wird die Wertfunktion wie folgt gewählt:

$$(10) \qquad v(z) = \begin{cases} z^{\alpha} & \text{falls } z \geq 0 \\ -\lambda |z|^{\alpha} & \text{falls } z < 0, \end{cases}$$

wobei sowohl α als auch λ strikt positiv sind. Diese parametrische Form wurde bereits in zahlreichen empirischen und theoretischen Analysen verwendet. Der Parameter λ determiniert den Knick am Ursprung. Für $\lambda > 1$ ist die Wertfunktion im Verlustbereich steiler als im Gewinnbereich, weshalb man dann von Verlustaversion spricht. Der Parameter α bestimmt dagegen die Krümmung der Wertfunktion, die im Gewinnbereich konkav und im Verlustbereich konvex ist.

Um die Anzahl der Parameter möglichst gering zu halten, wird von einer einheitlichen Gewichtungsfunktion der Form

(11) $f(p) = p^{\beta} / (p^{\beta} + [1 - p]^{\beta})^{1/\beta}$

mit $\beta > 0$ ausgegangen. Auch diese parametrische Form wurde bereits in anderen Studien verwendet. Für $\beta = 1$ gilt $f(p) = p$, d.h. es findet keine Wahrscheinlichkeitsgewichtung statt. Liegt β dagegen zwischen 0,4 und 1 ist die Gewichtungsfunktion im Einklang mit der jüngeren empirischen Evidenz invers-S-förmig.

Zur Analyse der Preference Reversals (PRs) wird von zwei Lotterien ausgegangen, dem sogenannten P-Bet und dem \$-Bet. Um diese Lotterien einfach zu halten, wird angenommen, dass man beim P-Bet mit der Wahrscheinlichkeit p den Betrag x gewinnt, beim \$-Bet mit der Wahrscheinlichkeit q den Betrag y. Es gilt $p > q$ und $y > x$ sowie $px = qy$. Dies bedeutet, dass beide Lotterien den gleichen monetären Erwartungswert haben, der \$-Bet aber riskanter als der P-Bet ist. Die Literatur zu PRs hat nun beobachtet, dass viele Personen bei einer direkten Auswahl den P-Bet gegenüber dem \$-Bet präferieren. Fragt man sie jedoch nach dem minimalen Verkaufspreis (z) für beide Lotterien, gegen sie einen höheren Preis für den \$-Bet an. Diese Inkonsistent wird als Standard PR bezeichnet, im umgekehrten Fall (d.h. der \$-Bet wird gewählt, es gilt aber $z_{\$} < z_{P}$) spricht man vom Non-Standard PR. Non-Standard PRs wurden jedoch sehr selten beobachtet. Abbildung 2 zeigt, für welche Parameterkonstellationen das Modell in Kapitel 7 Standard und Non-Standard PRs vorhersagt. Dabei wird von $p = 0{,}8$, $q = 0{,}2$ und $\beta = 1$ ausgegangen. Standard PRs ergeben sich in einem Bereich, in dem $\alpha < 1$ und $\lambda > 1$ gilt, für Non-Standard PRs gilt das Gegenteil, d.h. diese können nur bei $\alpha > 1$ und $\lambda < 1$ auftreten. Da nahezu alle empirischen Studien, die die in (10) charakterisierte funktionale Form der Wertfunktion verwenden, Werte im Bereich $\alpha < 1$ und $\lambda > 1$ vorhersagen, kann man die Schlussfolgerung ziehen, dass die Erklärung von PRs durch zustandsabhängige Referenzpunkte empirisch plausibel ist.

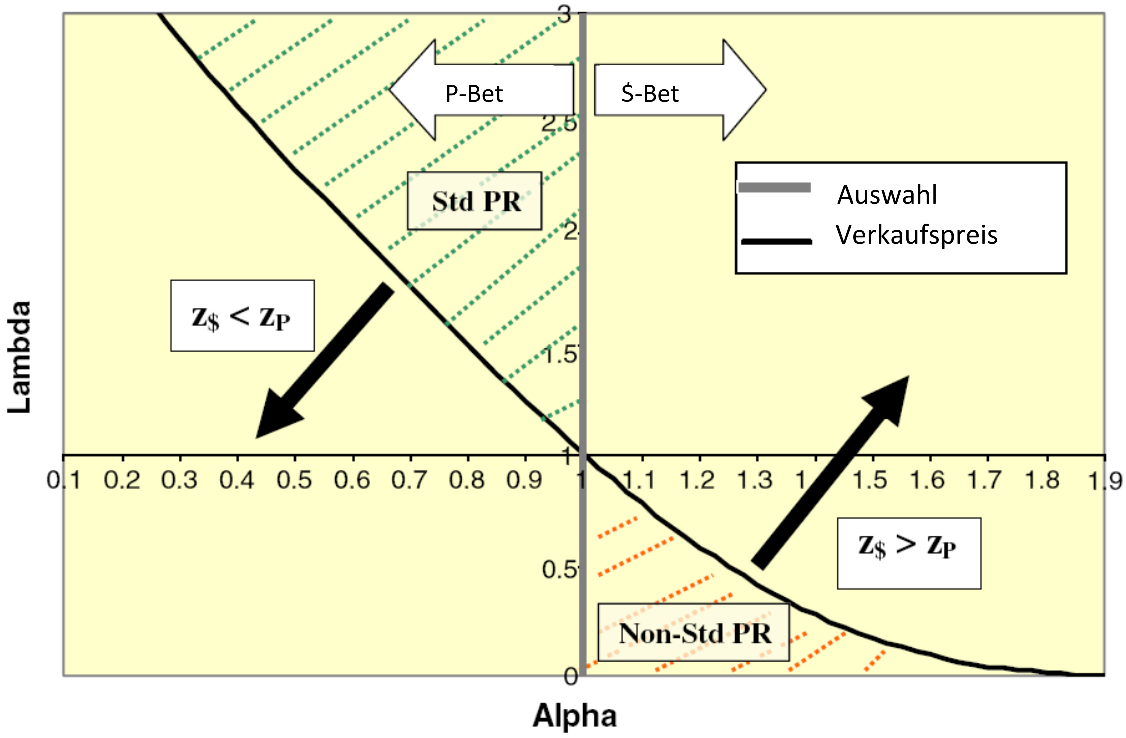

Abbildung 2: Preference Reversals ohne Wahrscheinlichkeitsgewichtung

Da $\beta = 1$ angenommen wurde, kann das PR Phänomen also ohne Wahrscheinlichkeitsgewichtung erklärt werden. Abbildung 3 zeigt den Einfluss der Wahrscheinlichkeitsgewichtung auf die in Abbildung 2 dargestellten Bereiche. Es ist ersichtlich, dass sich sowohl die Grenze, ab der der P-Bet gewählt wird, als auch die Grenze, ab der $z_\$ > z_P$ gilt, nach links, d.h. in den Bereich kleinerer Wert von α verschiebt. Die Schlussfolgerungen bleiben jedoch auch bei der Berücksichtigung von Wahrscheinlichkeitsgewichtung unverändert, da Standard-PRs auch in Abbildung 3 bei empirisch plausiblen Parameterwerten auftreten, während Non-Standard PRs nur für $\lambda < 1$ impliziert werden.

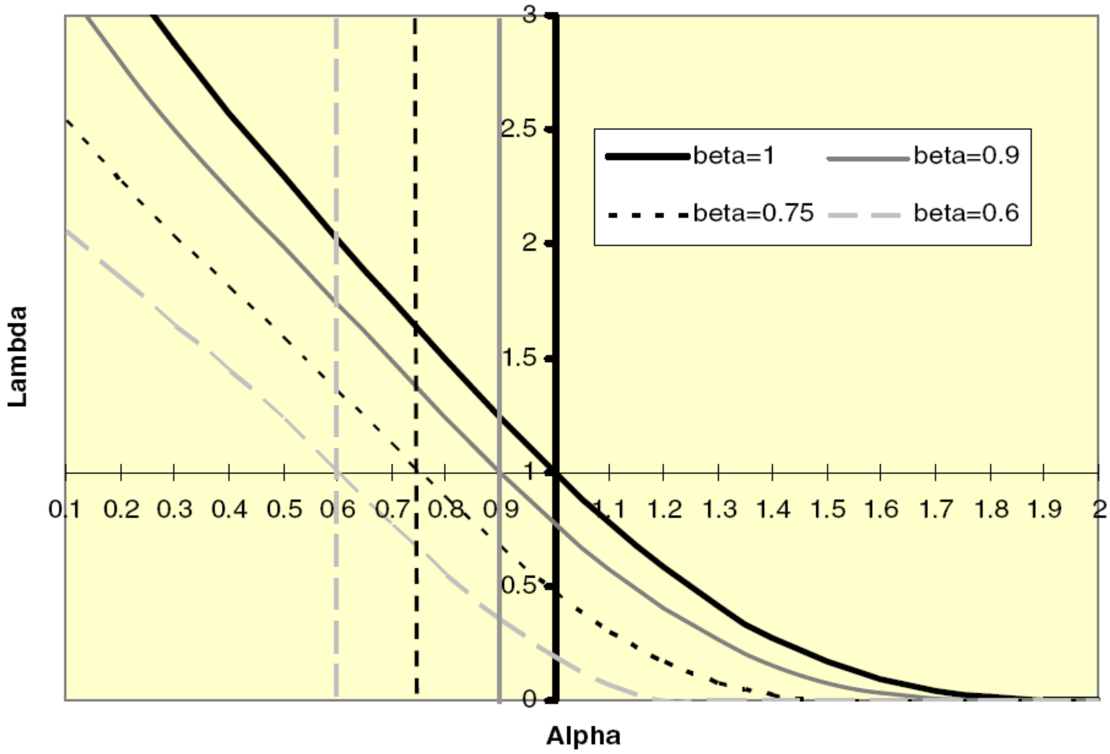

Abbildung 3: Preference Reversals mit Wahrscheinlichkeitsgewichtung

Literatur

Allais, M. (1953). Le Comportement de l'Homme Rationnel devant le Risque: Critique des Postulats et Axiomes de l'Ecole Américaine. *Econometrica 21*, 503–546.

Busemeyer, J. R., & Townsend, J. T. (1993). Decision Field Theory: A dynamic cognition approach to decision making. *Psychological Review 100*, 432-459.

Edwards,W. (1955). The prediction of decisions among bets. *Journal of Experimental Psychology 50*, 201–214.

Kahneman, D., & Tversky, A. (1979). Prospect theory: an analysis of decision under risk. *Econometrica 47*, 263–291.

Lichtenstein, S., & Slovic, P. (1971). Reversals of preferences between bids and choices in gambling decisions. *Journal of Experimental Psychology 89*, 46–55.

Lopes, L.L. (1995). Algebra and process in the modeling of risky choice. *The Psychology of Learning and Motivation 32*, 177–220.

Luce, R. D. (1959). *Individual choice behavior*. New York: Wiley.

Preston, M.G., & Baratta, P. (1948). An experimental study of the auction value of an

uncertain outcome. *American Journal of Psychology 61*, 183–193.
Thurstone, L. L. (1927). A law of comparative judgment. *Psychological Review 34*, 273-286
Tversky, A. (1969). Intransitivity of preferences. *Psychological Review 76*, 31-48.
Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: cumulative representation of uncertainty. *Journal of Risk and Uncertainty 5*, 297–323.

Kapitel 2

Testing Expected Utility Plus Noise

Anmerkungen:

- Dieses Kapitel basiert auf gemeinsamen Experimenten mit Tibor Neugebauer, Luxembourg School of Finance, University of Luxembourg
- Dieses Kapitel ist bereits veröffentlicht als Schmidt, U. and T. Neugebauer, Testing Expected Utility in the Presence of Errors, *Economic Journal 117* (2007), 470-485.

1 Introduction

The common consequence effect and the common ratio effect, both introduced by Allais (1953), are the most prominent and most investigated violations of expected utility (EU). They have motivated the development of alternative theories of choice under risk which are able to accommodate the observed behavioural patterns. Nowadays a large number of alternative theories exist (Starmer, 2000; Sugden, 2004; and Schmidt, 2004, for surveys), and naturally the question arises which theory can explain observed behaviour best.

Many studies have addressed this question; most prominent seem to be those of Harless and Camerer (1994) and Hey and Orme (1994). Since EU and its alternatives are deterministic theories, but observed choices are stochastic both papers integrated an error term into their estimations. This fact has aroused interest in a general discussion of the role of errors in decision making under risk. The discussion can be disentangled into two issues. The first issue concerns the modelling of the stochastic component (Harless and Camerer 1994; Hey and Orme 1994; Camerer and Ho 1994; Hey 1995; Loomes and Sugden 1995) and corresponding experimental tests (Carbone 1997; Ballinger and Wilcox 1997; Loomes and Sugden 1998; Carbone and Hey 2000; Buschena and Zilberman 2000; Loomes, Moffat and Sugden 2002). The second issue concerns the performance of EU and its alternatives for given specifications of the error term. In this context, Hey (1995), building upon the results of Hey and Orme (1994), arrives at the following conclusion:

> "It may be the case that these further explorations may alter the conclusion to which I am increasingly being drawn: that one can explain experimental analyses of decision making under risk better (and simpler) as EU plus noise – rather than through some higher level functional – as long as one specifies the noise appropriately."

This conclusion is reinforced by the results of Buschena and Zilberman (2000) which show that, under heteroscedastic error terms, the alternative theories do not offer a “statistically significant improvement in predictive power over EU”.

These conclusions are obviously in conflict with the high violation rates of EU observed in the common consequence and common ratio effects, and lead us to the following question: can EU - plus an appropriate error term - be the correct representation of preferences although there exist choice problems for which most subjects violate EU? Answering this question with yes and assuming that EU is the correct model obviously implies that the observed violations of EU are due to errors. The goal of the present paper is to analyse whether this is true.

For this purpose we have designed and run an experiment, in which subjects have to respond to identical binary choice problems on three different days. If a subject makes the same choice on all three days we say the observed choice is repetition-consistent, otherwise we say the observed choice is repetition-inconsistent. We propose a simple model based on the assumption that individuals have a deterministic preference ordering over lotteries which can be represented by EU. According to the model, choice must not always be consistent with this preference ordering, because with some probability individuals commit errors. If the error probability is not too high and the observed choice is repetition-consistent, it is very likely that this choice reflects the true preference of the individual. Under the assumption that EU plus error term is the correct representation of true preferences, we show that violations of EU are less likely when choices are repetition-consistent than when choices are repetition-inconsistent. On the contrary, if true preferences are in conflict with EU for a given choice problem, violations of EU are more likely if choice is repetition-consistent than when it is repetition-inconsistent. Therefore, our experimental data allow us to test EU plus error term against any alternative model that has, in the absence of errors, different predictions than EU for the considered choice problems. Since we observe a lower violation rate of EU in repetition-consistent choice than in repetition-inconsistent choice we conclude that EU plus error term describes the data better than these alternatives. The experimental design is presented in the next section. Section 3 discusses our theoretical framework and explains our hypothesis in more detail. The results are presented in Section 4, and Section 5 contains some concluding remarks.

2 Experimental Design

The experiment was conducted at the Centre of Experimental Economics at the University of York with 24 participants. Each participant had to attend five separate sessions, A, B, C, D, and E, on five different days. After a subject had completed all five sessions, one question of one session was randomly selected and played out for real. The average payment to the subjects was £34.17 with £80 being the highest and £0 being the lowest payment.

In each of the five sessions subjects were presented the same 30 lottery pairs, 28 risky ones and two ambiguous ones (which are not analysed in this paper). All risky lotteries were composed of the four consequences £0, £10, £30, £40. The probabilities of these consequences are recorded in Appendix 1 for all 28 lottery pairs. Note that in each pair the left lottery is safer than the right lottery, though in the experiment the left-right juxtaposition was randomised. The lotteries were presented as segmented circles on the computer screen. If a subject received a particular lottery as a reward he or she had to spin a wheel on the corresponding circle. The amount won was then determined by the segment of the circle in which the arrow on the wheel stopped.

The lottery pairs in Appendix 1 contain 28 common consequence or common ratio effects, or combinations of both effects (see bottom of the table in the appendix). A common consequence or common ratio effect involves two lottery pairs such that an EU maximiser either chooses the safe lottery in both pairs or the risky lottery in both pairs. In contrast, it is commonly observed in experiments that many subjects choose the safe lottery in one pair and the risky lottery in the other pair, a choice pattern violating EU. Since sessions D and E elicited certainty equivalents, our analysis will only rely on the data of sessions A, B, and C. In these sessions the single lottery pairs appeared in randomised order on screen and subjects had to indicate whether they prefer the left lottery or the right lottery or whether they are indifferent. After pressing the corresponding key the choice had to be confirmed by pressing the return key. If a question of sessions A, B, or C was selected for the reward of a subject, she or he could simply play out the chosen lottery. In the case of indifference, one lottery of the pair was chosen by the experimenter.

3 The Hypothesis

Our theoretical framework is based on the theory of errors developed by Hey (1995). In this theory individuals are assumed to have deterministic preferences between lotteries which can be represented by a functional V where V(S, R) > 0 (< 0) indicates that lottery S is strictly preferred (strictly not preferred) to lottery R. However, individuals sometimes make errors such that the actual choice may not correspond to the given preference relation. Formally, there is a stochastic error term ε such that in practice the value of V(S, R) + ε determines the choice between S and R. More precisely, the individual will choose S (R) if and only if V(S, R) + ε > 0 (< 0). It is assumed that ε has a median of zero which implies that the actual choice is contrary to the true preferences with a probability of less than 50%. For simplicity one could additionally assume that ε is symmetrically distributed around a mean of zero; such an assumption is, however, not necessary for the present analysis. We do not assume that ε is identical for all lottery pairs, which implies that the constant error model of Harless and Camerer (1994) can be constructed as a special case of our framework. In the following, a lottery S will be represented by a vector $S = (x_1, s_1; x_2, s_2; \ldots; x_n, s_n)$ indicating that consequence x_i has probability s_i. Contrary to Hey (1995), we assume that preferences are always in accordance with EU. Hence, there exists a von Neumann-Morgenstern utility function u such that V(S, R) equals the difference between the EU of S and the EU of R, i.e. $V(S, R) = \Sigma_i u(x_i)(s_i - r_i)$.

Recall from the preceding section that the common consequence effect and the common ratio effect involve two lottery pairs (S, R) and (S^*, R^*) such that each EU maximiser prefers the safe lottery S over the risky lottery R in the first lottery pair if and only if she or he prefers S^* to R^* in the second one. In terms of our model this gives $V(S, R) \geq 0$ if and only if $V(S^*, R^*) \geq 0$ for all functions u. In the experiment we have six observations for each problem: the three choices from the first lottery pair in sessions A, B, and C and the three choices from the second lottery pair in sessions A, B, and C. In the following, these observations will be represented by a vector where the first three entries report the choices from the first lottery pair in sessions A, B, and C respectively, while the last three entries report the choices from the second lottery pair in sessions A, B, and C respectively. Hence, for instance (S, R, S, R^*, R^*, S^*) indicates that a subject chose S over R in sessions A and C, R over S in session B, R^* over S^* in sessions A and B, and S^* over R^* in session C. In the following, we say that a given problem satisfies *repetition-consistency* if the first three entries are identical and the last three entries are identical for this problem, i.e. we have (S, S, S, S^*, S^*, S^*), (R, R, R, R^*, R^*, R^*), (S, S, S, R^*, R^*, R^*), or (R, R, R, S^*, S^*, S^*). In other words, repetition-consistency for a given choice problem means that the three choices for the first lottery pair are identical and the three choices for the second lottery pair are identical, but it does not necessarily mean that choices are in accordance with EU. Conversely, repetition-consistency is violated if the individual made contradictory choices in the single sessions for at least one of the two lottery pairs, which we call *repetition-inconsistency*. Besides repetition-consistency, the second important concept is given by *choice-errors*. A choice-error occurs if one given choice in any session and for any lottery pair deviates from true preferences, for example, a subject chose R over S although $V(S, R) > 0$. Note that the probability of a choice-error is always less than 50% since the median of ε has been assumed to be equal to zero. Assuming that EU is the correct model, a violation of EU can obviously not occur in the absence of a choice-error. But it can occur without a violation of repetition-consistency, if the same choice-error is committed for one lottery pair three times in row.

Let us assume that according to true preferences, S is better than R and, consequently, S^* is better than R^* since true preferences are assumed to be consistent with EU. Moreover, assume that the probability of a choice-error is α when choosing between S and R and β when choosing between S^* and R^*. This implies that (S, S, S, S^*, S^*, S^*) is observed with a probability of $(1-\alpha)^3(1-\beta)^3$, (R, R, R, R^*, R^*, R^*) with a probability of $\alpha^3\beta^3$, (S, S, S, R^*, R^*, R^*) with a probability of $(1-\alpha)^3\beta^3$ and (R, R, R, S^*, S^*, S^*) with a probability of $\alpha^3(1-\beta)^3$. Since these are the only four choice patterns satisfying repetition-consistency, this easily allows us (see Appendix 2) to calculate the probability of a violation of EU in the case of repetition-consistency, which will be referred to as p_{rc}. Analogously, we calculate in the appendix the probability of a violation of EU in the case of repetition-inconsistencies, which is called p_{ri}.

It turns out that according to our model, violations of EU have a higher probability in the case of repetition-inconsistency than in the case of repetition-consistency, i.e. for any given $\alpha, \beta > 0$ we have $p_{ri} - p_{rc} > 0$. Precisely this implication of the model will be tested in the experiment. Table 1 reports this difference for varying values of α and β. Suppose, for instance, that $\alpha = \beta = 0.2$, then the probability of a violation of EU equals 0.03 in case of repetition-consistency (see Table A3 in Appendix 2), while it equals 0.43 in case of repetition-inconsistency (see Table A4 in Appendix 2). This yields a difference of

40 percentage points. Table 1 shows that, in accordance with our hypothesis, this difference between p_{ri} and p_{rc} is always positive if, as implied by our model, α and β are less than 0.5.

Table 1
The difference between p_{ri} and p_{rc}

	α										
	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	-	0.35	0.37	0.38	0.38	0.38	0.36	0.31	0.24	0.13	0
0.05	0.35	0.36	0.37	0.38	0.39	0.38	0.36	0.32	0.24	0.13	0
0.10	0.37	0.37	0.38	0.39	0.39	0.39	0.36	0.32	0.24	0.13	0
0.15	0.38	0.38	0.39	0.39	0.4	0.39	0.37	0.32	0.24	0.13	0
0.20	0.38	0.39	0.39	0.4	0.4	0.39	0.36	0.32	0.24	0.13	0
β 0.25	0.38	0.38	0.39	0.39	0.39	0.38	0.35	0.31	0.23	0.12	0
0.30	0.36	0.36	0.36	0.37	0.36	0.35	0.33	0.29	0.21	0.12	0
0.35	0.31	0.32	0.32	0.32	0.32	0.31	0.29	0.25	0.18	0.1	0
0.40	0.24	0.24	0.24	0.24	0.24	0.23	0.21	0.18	0.14	0.07	0
0.45	0.13	0.13	0.13	0.13	0.13	0.12	0.12	0.1	0.07	0.04	0
0.50	0	0	0	0	0	0	0	0	0	0	0

So far we have verified that in our model, for *S* preferred to *R*, violations of EU have a higher probability in the case of repetition-inconsistency than in the case of repetition-consistency. Let us call this result Hypothesis 1. Recall that ε and therefore also α and β were not assumed to be identical for all choice problems. Moreover, note that in the analysis so far the values of α and β were completely arbitrary, i.e. Hypothesis 1 is valid for all values of α and β less than 0.5. This implies that Hypothesis 1 is valid within our framework for all choice problems and all subjects as long as *S* is preferred to *R*. What happens if *R* is preferred to *S* (and therefore also *R** to *S**)? It turns out (see appendix) that in this case the probabilities of violations of EU, p_{rc} and p_{ri}, are completely identical to the case *S* preferred to *R*, i.e. the hypothesis is also valid for this case. Overall, we can therefore conclude that, independently of particular preferences, our model of EU plus error term implies that violations of EU have, for all subjects and all choice problems, a higher probability in cases of repetition-inconsistency. This result will be tested by our experiment. However, our design can go one step further since it does not only allow a test of the implications of EU plus error term, but it also allows, in some sense, a discrimination between EU and non-expected utility (NEU). Consider any arbitrary NEU model which implies, in the absence of choice-errors, a violation of EU for a particular choice problem, i.e. according to true preferences, S is preferred to R and R^* is preferred to S^* (or alternatively, R is preferred to S and S^* is preferred to R^*). As we show in the appendix, such NEU model has the opposite implication than EU in our framework, i.e. violations of EU have a lower probability in cases of repetition-inconsistency. Thus, our test does not only allow one to analyse the implications of EU but also to compare the implications of EU and NEU, at least for NEU preferences which yield choice patterns for the considered choice problem contradicting the choice patterns implied by EU.

Before presenting the experimental results, let us clarify some points of the data analysis. Note that in the case of repetition-inconsistency we may observe violations of EU in some sessions while in others we do not. Suppose that we have for a given subject and a given choice problem the response pattern (S, S, R, S^*, R^*, R^*). Here, we have a violation of EU in session B while the choices in sessions A and C are consistent with EU. Consequently, this pattern will be regarded as a 33.33% violation of EU in the statistical analysis of the next section. Analogously, (S, S, S, S^*, R^*, R^*) has to be regarded as a 66.67% violation, (S, S, S, R^*, R^*, R^*) as a 100% violation, and (S, S, S, S^*, S^*, S^*) as a 0% violation. Finally, it should be explained how indifference has been treated in the analysis. If for a given lottery pair and a given subject there was one indifference in the three sessions this indifference has been treated as a missing observation. If there were two or three cases of indifference, the complete choice problem for this subject has been removed from the analysis. This procedure allowed us to exclude violations of EU which simply result from “imprecise” preference.

4 Results

4.1 Overall Data

Let us first give an overview of our results and consider the complete sample of all subjects and all lottery pairs (observed choice behaviour for every single lottery pair is reported in Appendix 1). In this sample the overall violation rate of EU is 23.44%. Restricting attention to cases of repetition-consistency, this violation rate decreases to 12.45%; whereas, it increases to 45.03% in the cases of repetition-inconsistency. This is consistent with the hypothesis of this paper and provides support for EU, at least within a framework which takes choice-errors into account.

4.2 Between-subject Analysis

Table 2 records for each choice problem the average violation rate of EU for repetition-inconsistent choice (second column) and for repetition-consistent choice (third column). Additionally, the second and third columns report in brackets the number of subjects in the respective category, i.e. the number of independent observations. Where the number of subjects for a given choice problem does not add up to 24, some subjects were excluded because they stated indifference at least twice in one involved lottery pair (see end of the preceding section). The last column of Table 2 reports the difference between these violation rates. A positive difference indicates a higher violation rate for repetition-inconsistent choice than for repetition-consistent choice, while a negative difference indicates the opposite. The size of the difference is tested by a two-tailed Wilcoxon rank sum test. Significant test results are also reported in the last column of Table 2 with asterisks (right-hand side); one character indicates a significance level of 10%, two characters of 5%, three characters of 1%, and no character indicates insignificance at 10%. At a significance level of 5%, the test results suggest that the exclusion of repetition-inconsistent choice would lead to a significant reduction of the violation rate for 20 of the 28 choice problems and in no choice problem to a significant increase. In other words, our hypothesis is supported for 20 choice problems while the contrary is not supported for any choice problem. In sum, the between-subject analysis confirms the inferences from the overall data for both the common consequence and the common ratio effect.

4.3 Within-subject Analysis

Let us now turn to the within-subject analysis. Table 3 records the average violation rate of EU for repetition-consistent choice problems (left-hand side of second column) and for the repetition-inconsistent choice problems (left-hand side of third column) for each subject; the number of observations are reported in brackets. Where these observations do not add up to 28, some choice problems had to be excluded since the subject indicated indifference at least twice in one of the involved lottery pairs. The difference in the violation rates between repetition-inconsistent and repetition-consistent choice is recorded in the last column of Table 3. A positive difference indicates a higher violation rate for repetition-inconsistent choice than for repetition-consistent choice, a negative difference indicates the opposite, and a missing observation indicates subjects whose choices were never repetition-inconsistent. Table 3 shows that the exclusion of repetition-inconsistent choice leads to a reduction of the violation rate for 20 out of 21 subjects without missing observations. The reduction of violations is significant at 1% when one moves from repetition-inconsistent to repetition-consistent choice as a Wilcoxon signed ranks test shows (see last row of the table). Therefore, the within-subject analysis also supports our hypothesis.

4.4 Analysis of Choice-Errors

We can also use our data in order to get some insights into the occurrence of choice-errors. More precisely, we will set the observed violation rates in the case of repetition-consistency equal to p_{rc} and the observed violation rate in the case of repetition- inconsistency equal to p_{ri}. Doing this enables us to calculate α and β in both cases with the help of equations (1) and (2) in the appendix. In other words, we can derive the implication of our data for the frequency of choice-errors. Additionally, we can check whether the derived values of α and β are similar for the cases of repetition-inconsistency and those of repetition-consistency or whether there are systematic differences which could be regarded as evidence against our model.

To ensure identification it is necessary to rely on a constant error model in this analysis, i.e. we assume that the probability of a choice-error is identical for a given subject in each lottery pair, which implies $\alpha = \beta$. Without this assumption one could only derive a continuum of admissible α and β combinations for each observation. However, we do not assume that the probability of errors is identical for different subjects.

Let us first mention that our model implies some restrictions on the values of p_{rc} and p_{ri} if α and β are assumed to be greater than zero and less than 0.5. Tables A3-A6 in the appendix show that we must have $0 \leq p_{rc} \leq 0.5$ and $0.33 \leq p_{ri} \leq 0.5$ in the case of EU as well as $0.5 \leq p_{rc} \leq 1$ and $0.5 \leq p_{ri} \leq 0.67$ in the case of a NEU model which implies a violation of EU for the considered choice problem. (The numbers 0.33 in the case of EU and 0.67 in the case of NEU cannot be taken directly from the tables since they result from strictly positive values of α and β less than 0.01). Comparing these numbers with the observed violation rates for the single subjects in Table 3 yields the following results: (i) in the case of repetition-consistency all violation rates are consistent with the predictions of EU whereas no violation rate is consistent with NEU; (ii) in the case of repetition-inconsistency 15 out of 21 violation rates without missing observations are consistent with the predictions of EU whereas only 7 are consistent with NEU.

Table 2

Violation rates for each choice problem across all subjects

Choice problem	Repetition-Inconsistency [#]	Repetition-Consistency [#]	Difference	Significance level [a)]
1, 2	69.05% [7]	18.18% [11]	51%	**
1, 3	53.33% [5]	56.25% [16]	-3%	
1, 4	47.92% [8]	26.67% [15]	21%	*
2, 3	26.67% [5]	25.00% [12]	2%	
2, 4	43.75% [8]	33.33% [9]	10%	
3, 4	63.89% [6]	26.67% [15]	37%	*
5, 6	33.33% [6]	5.56% [18]	28%	***
5, 7	30.56% [6]	0.00% [16]	31%	***
6, 7	39.58% [8]	7.14% [14]	32%	***
8, 9	50.00% [10]	55.56% [9]	-6%	
8,10	48.61% [12]	60.00% [10]	-11%	
9,10	50.00% [10]	11.11% [9]	39%	***
11,12	53.33% [10]	0.00% [11]	53%	***
13,14	47.22% [6]	0.00% [14]	47%	***
15,16	45.83% [4]	0.00% [15]	46%	***
17,18	22.22% [6]	5.56% [18]	17%	**
17,19	27.78% [6]	0.00% [18]	28%	***
17,20	29.17% [8]	0.00% [15]	29%	***
18,19	46.67% [5]	5.26% [19]	41%	**
18,20	47.62% [7]	6.25% [16]	41%	***
19,20	33.33% [4]	0.00% [19]	33%	**
21,22	37.50% [8]	7.69% [13]	30%	***
21,23	41.67% [8]	15.38% [13]	26%	**
22,23	28.57% [7]	15.38% [13]	13%	
24,25	38.89% [6]	6.25% [16]	33%	***
26,27	48.81% [14]	0.00% [8]	49%	***
26,28	54.17% [8]	14.29% [14]	40%	***
27,28	58.97% [13]	0.00% [9]	59%	***

a) *** 1%, ** 5%, * 10 % (significance level, two-tailed Wilcoxon rank sum test).

Table 3

Violation rates and choice-error approximations for each subject

	Repetition-Inconsistency		Repetition-Consistency		Difference
Subject ID	Violation rate [#]	Approxi-mation α, β	Violation rate [#]	Approxi-mation α, β	
#1	66.67% [4]	-	22.22% [18]	34.49%	44%
#2	- [0]	-	0.00% [8]	0.00%	-
#3	53.13% [16]	-	0.00% [11]	0.00%	53%
#4	42.59% [9]	19.65%	26.32% [19]	36.29%	16%
#5	44.05% [14]	23.21%	21.43% [14]	34.12%	23%
#6	50.00% [6]	50.00%	18.18% [22]	32.56%	32%
#7	26.67% [15]	-	33.33% [12]	39.20%	-7%
#8	44.44% [21]	24.20%	0.00% [7]	0.00%	44%
#9	38.19% [24]	9.93%	0.00% [2]	0.00%	38%
#10	38.10% [7]	9.74%	28.57% [21]	37.23%	10%
#11	- [0]	-	7.14% [14]	25.24%	-
#12	60.00% [5]	-	29.41% [17]	37.58%	31%
#13	44.44% [3]	24.20%	0.00% [25]	0.00%	44%
#14	54.17% [8]	-	0.00% [20]	0.00%	54%
#15	50.00% [6]	50.00%	0.00% [20]	0.00%	50%
#16	41.67% [2]	17.52%	9.09% [11]	26.94%	33%
#17	57.14% [21]	-	0.00% [7]	0.00%	57%
#18	45.83% [4]	27.97%	29.17% [24]	37.48%	17%
#19	33.33% [8]	0.00%	29.41% [17]	37.58%	4%
#20	- [0]	-	0.00% [26]	0.00%	-
#21	33.33% [2]	0.00%	0.00% [26]	0.00%	33%
#22	33.33% [3]	0.00%	0.00% [23]	0.00%	33%
#23	42.86% [14]	20.29%	33.33% [12]	39.20%	10%
#24	45.61% [19]	27.35%	11.11% [9]	28.44%	35%
Average	45.03% [8.79]	20.27%	12.45% [16.04]	18.60%	33%***

*** 1% significant (two-tailed Wilcoxon signed ranks test); - missing observation

Assuming EU, we have now calculated the implied relative frequencies of choice-errors (i.e. $\alpha = \beta$) for the subjects with consistent violation rates. The results are stated on the right-hand sides of the second and third columns of Table 3. Although there exist substantial differences for these values in the cases of repetition-inconsistency and those of repetition-consistency for a number of subjects, there does not exist a systematic pattern and the average values (see last row of Table 3) are quite close (0.1860 and 0.2027). Moreover, the difference between these average values is insignificant at any reasonable significance level as a Wilcoxon signed ranks test indicates (p = .861).

Overall, our observed error rate of about 20% is not in contrast with previous studies (comparable numbers have been reported by Harless and Camerer, 1994 and Birnbaum and Bahra, 2005) which could be interpreted a further support of our results, at least under the restrictions of a constant error model.

5 Conclusions

This paper presented an experimental analysis of individual choice under risk in the presence of errors. Overall, our results show that the main implication of representing preferences by EU plus error term is supported by our data. Finally, we should comment on one issue which has often been put forward as an argument against representing preferences by EU plus error term. Recall that violations of EU in common consequence and common ratio effects are not random deviations but appear to be highly systematic. For instance, in the classical common ratio problem formed by lottery pairs one and two (see Table A1, i.e. S offers a 100% chance of £30; R offers a 80% chance of £40 and a 20% chance of £0; S^* offers a 25% chance of £30 and a 75% chance of £0; R^* offers a 20% chance of £40 and a 80% chance of £0) the usual pattern of violations is the choice of S and R^*; whereas, the choice of R and S^* is very rarely observed (this is also true for our data). Such highly systematic violations seem, at first glance, to contradict a model such as ours that attributes violations of EU to errors. However, there is not necessarily a contradiction. Instead, an obvious reason for systematic violations may be an asymmetric error term. But also with symmetric error terms, systematic violations of EU can be explained in the present model. Suppose we have u(£0) = 0, u(£30) = 30, and u(£40) = 35. This yields for the common ratio problem $V(S, R) = 30 - 0.8 \cdot 35 = 2$ and $V(S^*, R^*) = 0.5$. If the error term is uniformly distributed between -1 and 1, we can never observe the choice of R and S^* whereas the choice of S and R^* is observed in 25% of the cases. Consequently, our model is consistent with systematic deviations of EU even in the presence of a symmetric error term. We should, however, add that the main goal of the present paper was not to analyse specific hypotheses concerning the error term but to investigate whether the fundamental implications of EU plus error term are broadly confirmed or not.

Appendix 1: The Lottery Pairs

Table A1

The Lottery Pairs

	Safe Lottery				Risky Lottery				Overall Choice (in %)		
No.	£0	£10	£30	£40	£0	£10	£30	£40	Safe	Indifferent	Risky
1	0.00	0.00	1.00	0.00	0.20	0.00	0.00	0.80	69.4	0.0	30.6
2	0.75	0.00	0.25	0.00	0.80	0.00	0.00	0.20	25.0	20.8	54.2
3	0.30	0.60	0.10	0.00	0.32	0.60	0.00	0.08	8.3	16.7	75.0
4	0.00	0.60	0.10	0.30	0.02	0.60	0.00	0.38	40.3	8.3	51.4
5	0.00	1.00	0.00	0.00	0.70	0.00	0.00	0.30	65.3	0.0	34.7
6	0.00	0.50	0.50	0.00	0.35	0.00	0.50	0.15	66.7	0.0	33.3
7	0.50	0.50	0.00	0.00	0.85	0.00	0.00	0.15	52.8	8.3	38.9
8	0.00	0.00	0.70	0.30	0.15	0.00	0.00	0.85	65.3	1.4	33.3
9	0.80	0.00	0.14	0.06	0.83	0.00	0.00	0.17	27.8	19.4	52.8
10	0.20	0.00	0.74	0.06	0.23	0.00	0.60	0.17	18.1	5.6	76.4
11	0.00	0.20	0.80	0.00	0.00	0.50	0.00	0.50	68.1	4.2	27.8
12	0.50	0.10	0.40	0.00	0.50	0.25	0.00	0.25	66.7	13.9	19.4
13	0.00	0.20	0.60	0.20	0.20	0.00	0.40	0.40	65.3	16.7	18.1
14	0.00	0.10	0.30	0.60	0.10	0.00	0.20	0.70	69.4	13.9	16.7
15	0.20	0.80	0.00	0.00	0.80	0.00	0.00	0.20	68.1	19.4	12.5
16	0.10	0.40	0.50	0.00	0.40	0.00	0.50	0.10	72.2	18.1	9.7
17	0.00	0.40	0.60	0.00	0.40	0.00	0.00	0.60	59.7	1.4	38.9
18	0.50	0.20	0.30	0.00	0.70	0.00	0.00	0.30	70.8	1.4	27.8
19	0.00	0.20	0.30	0.50	0.20	0.00	0.00	0.80	56.9	2.8	40.3
20	0.00	0.20	0.70	0.10	0.20	0.00	0.40	0.40	62.5	2.8	34.7
21	0.00	0.00	0.50	0.50	0.10	0.00	0.00	0.90	56.9	1.4	41.7
22	0.50	0.00	0.50	0.00	0.60	0.00	0.00	0.40	43.1	12.5	44.4
23	0.25	0.50	0.25	0.00	0.30	0.50	0.00	0.20	26.4	16.7	56.9
24	0.00	0.50	0.00	0.50	0.20	0.20	0.00	0.60	63.9	0.0	36.1
25	0.50	0.25	0.00	0.25	0.60	0.10	0.00	0.30	56.9	8.3	34.7
26	0.00	0.25	0.50	0.25	0.00	0.35	0.00	0.65	11.1	9.7	79.2
27	0.00	0.00	0.75	0.25	0.00	0.10	0.25	0.65	29.2	5.6	65.3
28	0.25	0.25	0.50	0.00	0.25	0.35	0.00	0.40	20.8	9.7	69.4

Note: common consequence effects: (3,4), (6,7), (9,10), (18,19), (18,20), (19,20), (21,22), (26,27), (26,28), (27,28); common ratio effects: (1,2), (5,7), (8,9), (11,12), (17,18), (24,25); combinations of common ratio and common consequence effect: (1,3), (1,4), (2,3), (2,4), (5,6), (8,10), (13,14), (15,16), (17,19), (17,20), (21,23), (22,23).

Appendix 2: The Hypothesis

In the following, we derive the hypothesis of the paper assuming that true preferences satisfy EU. Therefore, we first calculate for one given choice problem the probabilities of a violation of EU (i) for cases of repetition-inconsistency and (ii) for cases of repetition-consistency. Initially, we assume that subjects prefer S over R according to true preferences and consequently S^* over R^*. (Later we will consider the opposite preferences). Table A2 records all possible response patterns of subjects and the resulting relative frequency of a violation of EU. In the fifth row, for instance, the subject chooses S and S^* in session A (consistent with EU) but violates EU in sessions B and C by choosing S and R^*. Consequently, we observe a violation of EU in two out of three cases. Recalling that S is preferred to R according to true preferences (and therefore also S^* to R^*) we obtain the probability of each choice pattern stated in the third column (recall that the subject chooses R by mistake with a probability of α and R^* with a probability of β). Finally, the fourth column indicates whether the response pattern contains a repetition-inconsistency (i.e. choice between S and R or S^* and R^* is not identical in all three sessions) or not.

From Table A2 we can now calculate the desired probabilities. The probability of a violation of EU in cases of repetition-consistency is given by

$$p_{rc} = \frac{(1-\alpha)^3\beta^3 + \alpha^3(1-\beta)^3}{(1-\alpha)^3(1-\beta)^3 + \alpha^3\beta^3 + (1-\alpha)^3\beta^3 + \alpha^3(1-\beta)^3}. \tag{1}$$

The resulting values of this probability for varying values of α and β are reported in Table A3.

Table A2

Possible Response Patterns

Observation	Relative frequency of a violation of EU	Probability of observation	Repetition-Inconsistency
S, S, S, S^*, S^*, S^*	0	$(1-\alpha)^3 (1-\beta)^3$	No
S, S, S, S^*, S^*, R^*		$(1-\alpha)^3 (1-\beta)^2 \beta$	Yes
S, S, S, S^*, R^*, S^*	1/3		
S, S, S, R^*, S^*, S^*			
S, S, S, S^*, R^*, R^*		$(1-\alpha)^3 (1-\beta) \beta^2$	Yes
S, S, S, R^*, S^*, R^*	2/3		
S, S, S, R^*, R^*, S^*			
S, S, S, R^*, R^*, R^*	1	$(1-\alpha)^3 \beta^3$	No
S, S, R, S^*, S^*, S^*	1/3	$(1-\alpha)^2 \alpha (1-\beta)^3$	Yes
S, S, R, S^*, S^*, R^*	0	$(1-\alpha)^2 \alpha (1-\beta)^2 \beta$	Yes
S, S, R, S^*, R^*, S^*	2/3		
S, S, R, R^*, S^*, S^*	2/3		
S, S, R, S^*, R^*, R^*	1/3	$(1-\alpha)^2 \alpha (1-\beta) \beta^2$	Yes
S, S, R, R^*, S^*, R^*	1/3		
S, S, R, R^*, R^*, S^*	1		
S, S, R, R^*, R^*, R^*	2/3	$(1-\alpha)^2 \alpha \beta^3$	Yes
S, R, S, S^*, S^*, S^*	1/3	$(1-\alpha)^2 \alpha (1-\beta)^3$	Yes
S, R, S, S^*, S^*, R^*	2/3	$(1-\alpha)^2 \alpha (1-\beta)^2 \beta$	Yes
S, R, S, S^*, R^*, S^*	0		
S, R, S, R^*, S^*, S^*	2/3		
S, R, S, S^*, R^*, R^*	1/3	$(1-\alpha)^2 \alpha (1-\beta) \beta^2$	Yes
S, R, S, R^*, S^*, R^*	1		
S, R, S, R^*, R^*, S^*	1/3		
S, R, S, R^*, R^*, R^*	2/3	$(1-\alpha)^2 \alpha \beta^3$	Yes
R, S, S, S^*, S^*, S^*	1/3	$(1-\alpha)^2 \alpha (1-\beta)^3$	Yes
R, S, S, S^*, S^*, R^*	2/3	$(1-\alpha)^2 \alpha (1-\beta)^2 \beta$	Yes
R, S, S, S^*, R^*, S^*	2/3		
R, S, S, R^*, S^*, S^*	0		
R, S, S, S^*, R^*, R^*	1	$(1-\alpha)^2 \alpha (1-\beta) \beta^2$	Yes
R, S, S, R^*, S^*, R^*	1/3		
R, S, S, R^*, R^*, S^*	1/3		
R, S, S, R^*, R^*, R^*	2/3	$(1-\alpha)^2 \alpha \beta^3$	Yes
S, R, R, S^*, S^*, S^*	2/3	$(1-\alpha) \alpha^2 (1\text{-}\beta)^3$	Yes
S, R, R, S^*, S^*, R^*	1/3	$(1-\alpha) \alpha^2 (1-\beta)^2 \beta$	Yes
S, R, R, S^*, R^*, S^*	1/3		
S, R, R, R^*, S^*, S^*	1		
S, R, R, S^*, R^*, R^*	0	$(1-\alpha) \alpha^2 (1-\beta) \beta^2$	Yes
S, R, R, R^*, S^*, R^*	2/3		
S, R, R, R^*, R^*, S^*	2/3		
S, R, R, R^*, R^*, R^*	1/3	$(1-\alpha) \alpha^2 \beta^3$	Yes
R, S, R, S^*, S^*, S^*	2/3	$(1-\alpha) \alpha^2 (1-\beta)^3$	Yes

R, S, R, S^*, S^*, R^* R, S, R, S^*, R^*, S^* R, S, R, R^*, S^*, S^*	1/3 1 1/3	$(1-\alpha)\ \alpha^2\ (1-\beta)^2\ \beta$	Yes
R, S, R, S^*, R^*, R^* R, S, R, R^*, S^*, R^* R, S, R, R^*, R^*, S^*	2/3 0 2/3	$(1-\alpha)\ \alpha^2\ (1-\beta)\ \beta^2$	Yes
R, S, R, R^*, R^*, R^*	1/3	$(1-\alpha)\ \alpha^2\ \beta^3$	Yes
R, R, S, S^*, S^*, S^*	2/3	$(1-\alpha)\ \alpha^2\ (1-\beta)^3$	Yes
R, R, S, S^*, S^*, R^* R, R, S, S^*, R^*, S^* R, R, S, R^*, S^*, S^*	1 1/3 1/3	$(1-\alpha)\ \alpha^2\ (1-\beta)^2\ \beta$	Yes
R, R, S, S^*, R^*, R^* R, R, S, R^*, S^*, R^* R, R, S, R^*, R^*, S^*	2/3 2/3 0	$(1-\alpha)\ \alpha^2\ (1-\beta)\ \beta^2$	Yes
R, R, S, R^*, R^*, R^*	1/3	$(1-\alpha)\ \alpha^2\ \beta^3$	Yes
R, R, R, S^*, S^*, S^*	1	$\alpha^3\ (1-\beta)^3$	No
R, R, R, S^*, S^*, R^* R, R, R, S^*, R^*, S^* R, R, R, R^*, S^*, S^*	2/3 2/3 2/3	$\alpha^3\ (1-\beta)^2\ \beta$	Yes
R, R, R, S^*, R^*, R^* R, R, R, R^*, S^*, R^* R, R, R, R^*, R^*, S^*	1/3 1/3 1/3	$\alpha^3\ (1-\beta)\ \beta^2$	Yes
R, R, R, R^*, R^*, R^*	0	$\alpha^3\ \beta^3$	No

Table A3

Probability of violations of EU in the case of repetition-consistency (p_{rc})

		α										
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
	0.00	0	0	0	0.01	0.02	0.04	0.07	0.14	0.23	0.35	0.5
	0.05	0	0	0	0.01	0.02	0.04	0.07	0.14	0.23	0.35	0.5
	0.10	0	0	0	0.01	0.02	0.04	0.07	0.14	0.23	0.35	0.5
	0.15	0.01	0.01	0.01	0.01	0.02	0.04	0.08	0.14	0.23	0.36	0.5
	0.20	0.02	0.02	0.02	0.02	0.03	0.05	0.09	0.15	0.24	0.36	0.5
β	0.25	0.04	0.04	0.04	0.04	0.05	0.07	0.1	0.16	0.25	0.36	0.5
	0.30	0.07	0.07	0.07	0.08	0.09	0.1	0.14	0.19	0.27	0.38	0.5
	0.35	0.14	0.14	0.14	0.14	0.15	0.16	0.19	0.23	0.3	0.39	0.5
	0.40	0.23	0.23	0.23	0.23	0.24	0.25	0.27	0.3	0.35	0.42	0.5
	0.45	0.35	0.35	0.35	0.36	0.36	0.36	0.38	0.39	0.42	0.46	0.5
	0.50	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

From Table A2 we can also calculate the probability of violations of EU in cases of repetition-inconsistency. This probability is given by

$$p_e \quad = \quad \frac{f(\alpha;\beta)}{1-[(1-\alpha)^3(1-\beta)^3+\alpha^3\beta^3+(1-\alpha)^3\beta^3+\alpha^3(1-\beta)^3]} \qquad (2)$$

where

$$\begin{aligned} f(\alpha;\beta) = \quad & (1-\alpha)^3(1-\beta)^2\beta + 2(1-\alpha)^3(1-\beta)^2 + (1-\alpha)^2\alpha(1-\beta)^3 \\ & + 4(1-\alpha)^2\alpha(1-\beta)^2\beta + 5(1-\alpha)^2\alpha(1-\beta)\beta^2 + 2(1-\alpha)^2\alpha\beta^3 \\ & + 2(1-\alpha)\alpha^2(1-\beta)^3 + 5(1-\alpha)\alpha^2(1-\beta)^2\beta + 4(1-\alpha)\alpha^2(1-\beta)\beta^2 \\ & + (1-\alpha)\alpha^2\beta^3 + 2\alpha^3(1-\beta)^2\beta + \alpha^3(1-\beta)\beta^2 \qquad (3) \end{aligned}$$

and the resulting values of this probability for varying values of α and β are reported in Table A4.

We took the inequality $p_{rc} \geq p_{ri}$ and verified with the "Maple" software that, under the restriction $0 < \alpha, \beta < 0.5$, there do not exist values of α and β such that this inequality is satisfied. Additionally, we calculated the difference between p_{ri} and p_{rc} for varying values of α and β. We considered a rather high number of values of α and β between zero and 0.5 by starting from $\alpha = \beta = 0$ and increasing α and β in 0.0001 steps. A shortened overview of our calculations is given in Table 1 in Section 4 which shows that the difference is always positive, in accordance with our hypothesis.

So far we have shown that our hypothesis is true for one subject preferring S to R and S^* to R^* in a given choice problem. Since the difference between p_{ri} and p_{rc} is, however, positive for all admissible values of α and β we can conclude that the hypothesis is true for all subjects and all choice problems as long as S and S^* are preferred.

Table A4

Probability of violations of EU in the case of repetition-inconsistency (p_{ri})

		α										
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
	0.00	-	0.35	0.37	0.38	0.4	0.42	0.43	0.45	0.47	0.48	0.5
	0.05	0.35	0.36	0.37	0.39	0.4	0.42	0.43	0.45	0.47	0.48	0.5
	0.10	0.37	0.37	0.38	0.39	0.41	0.42	0.44	0.45	0.47	0.48	0.5
	0.15	0.38	0.39	0.39	0.41	0.42	0.43	0.44	0.46	0.47	0.49	0.5
	0.20	0.4	0.4	0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.49	0.5
β	0.25	0.42	0.42	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.5
	0.30	0.43	0.43	0.44	0.44	0.45	0.46	0.47	0.47	0.48	0.49	0.5
	0.35	0.45	0.45	0.45	0.46	0.46	0.47	0.47	0.48	0.49	0.49	0.5
	0.40	0.47	0.47	0.47	0.47	0.47	0.48	0.48	0.49	0.49	0.5	0.5
	0.45	0.48	0.48	0.48	0.49	0.49	0.49	0.49	0.49	0.5	0.5	0.5
	0.50	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Recall that the probabilities of observing a choice of *S*, *R*, *S**, or *R** are $1 - \alpha$, α, $1 - \beta$, and β, respectively, given our initial assumption that *S* and *S** are preferred. Now suppose that, according to true preferences, *R* is preferred to *S* and *R** to *S**. In this case, the probabilities of observing a choice of *S*, *R*, *S**, or *R** are α, $1 - \alpha$, β, and $1 - \beta$, respectively. Consequently, all values of α in Table A2 and also in the equations of p_{rc} and p_{ri} have to be replaced by $1 - \alpha$, while all values of β have to be replaced by $1 - \beta$. However, it turns out that p_{rc} and p_{ri} remain unchanged if all values of α are replaced by $1 - \alpha$ and all values of β are replaced by $1 - \beta$. Since p_{rc} and p_{ri} remain unchanged, the difference between p_{ri} and p_{rc} will again be positive for all admissible values of α and β, which means that the hypothesis is also valid if *R* and *R** are preferred.

Let us now consider NEU preferences which violate EU for the considered choice problem. Recall again that the probabilities of observing a choice of *S*, *R*, *S**, or *R** are

$1 - \alpha$, α, $1 - \beta$, and β, respectively, if S and S^* are preferred. Now suppose that a subject violates EU by preferring S and R^* (or R and S^*). In this case, the probabilities of observing a choice of S, R, S^*, or R^* are $1 - \alpha$, α, β, and $1 - \beta$, respectively (or α, $1 - \alpha$, $1 - \beta$, and β, respectively, if R and S^* are preferred). This means that in Table A2 and in the equations of p_{rc} and p_{ri} either β has to be replaced by $1 - \beta$ or α has to be replaced by $1 - \alpha$. For both cases we calculated p_{rc} (see Table A5), and p_{ri} (see Table A6) and it turned out that both cases yield identical values of p_{ri} and identical values of p_{rc}. We also calculated the difference between both probabilities for a high number of values of α and β by varying α and β in 0.0001 steps. A shortened overview of the calculations is presented in Table A7. It turns out that the difference is always negative, i.e. we have the opposite implication than in the case of EU.

Table A5
p_{rc} in the case of NEU

		α										
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
	0.00	1	1	1	0.99	0.98	0.96	0.93	0.86	0.77	0.65	0.5
	0.05	1	1	1	0.99	0.98	0.96	0.93	0.86	0.77	0.65	0.5
	0.10	1	1	1	0.99	0.98	0.96	0.93	0.86	0.77	0.65	0.5
	0.15	0.99	0.99	0.99	0.99	0.98	0.96	0.92	0.86	0.77	0.64	0.5
	0.20	0.98	0.98	0.98	0.98	0.97	0.95	0.91	0.85	0.76	0.64	0.5
β	0.25	0.96	0.96	0.96	0.96	0.95	0.93	0.9	0.84	0.75	0.64	0.5
	0.30	0.93	0.93	0.93	0.92	0.91	0.9	0.86	0.81	0.73	0.62	0.5
	0.35	0.86	0.86	0.86	0.86	0.85	0.84	0.81	0.77	0.7	0.61	0.5
	0.40	0.77	0.77	0.77	0.77	0.76	0.75	0.73	0.7	0.65	0.58	0.5
	0.45	0.65	0.65	0.65	0.64	0.64	0.64	0.62	0.61	0.58	0.54	0.5
	0.50	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table A6

p_{ri} in the case of NEU

		α										
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
	0.00	-	0.65	0.63	0.62	0.6	0.58	0.57	0.55	0.53	0.52	0.5
	0.05	0.65	0.64	0.63	0.61	0.6	0.58	0.57	0.55	0.53	0.52	0.5
	0.10	0.63	0.63	0.62	0.61	0.59	0.58	0.56	0.55	0.53	0.52	0.5
	0.15	0.62	0.61	0.61	0.59	0.58	0.57	0.56	0.54	0.53	0.51	0.5
	0.20	0.6	0.6	0.59	0.58	0.57	0.56	0.55	0.54	0.53	0.51	0.5
β	0.25	0.58	0.58	0.58	0.57	0.56	0.55	0.54	0.53	0.52	0.51	0.5
	0.30	0.57	0.57	0.56	0.56	0.55	0.54	0.53	0.53	0.52	0.51	0.5
	0.35	0.55	0.55	0.55	0.54	0.54	0.53	0.53	0.52	0.51	0.51	0.5
	0.40	0.53	0.53	0.53	0.53	0.53	0.52	0.52	0.51	0.51	0.5	0.5
	0.45	0.52	0.52	0.52	0.51	0.51	0.51	0.51	0.51	0.5	0.5	0.5
	0.50	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table A7

The difference between p_{ri} and p_{rc} in the case of NEU

		α										
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
	0.00	-	-0.35	-0.37	-0.38	-0.38	-0.38	-0.36	-0.31	-0.24	-0.13	0
	0.05	-0.35	-0.36	-0.37	-0.38	-0.39	-0.38	-0.36	-0.32	-0.24	-0.13	0
	0.10	-0.37	-0.37	-0.38	-0.39	-0.39	-0.39	-0.36	-0.32	-0.24	-0.13	0
	0.15	-0.38	-0.38	-0.39	-0.39	-0.4	-0.39	-0.37	-0.32	-0.24	-0.13	0
	0.20	-0.38	-0.39	-0.39	-0.4	-0.4	-0.39	-0.36	-0.32	-0.24	-0.13	0
β	0.25	-0.38	-0.38	-0.39	-0.39	-0.39	-0.38	-0.35	-0.31	-0.23	-0.12	0
	0.30	-0.36	-0.36	-0.36	-0.37	-0.36	-0.35	-0.33	-0.29	-0.21	-0.12	0
	0.35	-0.31	-0.32	-0.32	-0.32	-0.32	-0.31	-0.29	-0.25	-0.18	-0.1	0
	0.40	-0.24	-0.24	-0.24	-0.24	-0.24	-0.23	-0.21	-0.18	-0.14	-0.07	0
	0.45	-0.13	-0.13	-0.13	-0.13	-0.13	-0.12	-0.12	-0.1	-0.07	-0.04	0
	0.50	0	0	0	0	0	0	0	0	0	0	0

References

Allais, M. (1953). ‘Le comportement de l'homme rationnel devant le risque, critique des postulates et axiomes de l'Ecole Américaine’, *Econometrica*, vol. 21, pp. 503-546.

Ballinger, T. P. and Wilcox, N. (1997). ‘Decisions, error and heterogenity’, ECONOMIC JOURNAL, vol. 107, pp. 1090-1105.

Birnbaum, M. and Bahra, J. (2005). ‘Gain-loss separability and coalescing in risky decision making’, *mimeo*.

Buschena, D., and Zilberman, D. (2000). ‘Generalized expected utility, heteroscedastic error, and path dependence in risky choice’, *Journal of Risk and Uncertainty*, vol. 20, pp. 67-88.

Carbone, E. (1997). ‘Investigation of stochastic preference theory using experimental data’, *Economics Letters*, vol. 57, pp. 305-312.

Carbone, E. and Hey, J. D. (2000). ‘Which error story is best?’, *Journal of Risk and Uncertainty*, vol. 20, pp. 161-176.

Camerer, C.F. and Ho, H. T. (1994). ‘Violations of the betweenness axiom and nonlinearity in probability’, *Journal of Risk and Uncertainty*, vol. 8, pp. 167-196.

Harless, D.W. and Camerer, C. F. (1994). ‘The predictive utility of generalized expected utility theories’, *Econometrica*, vol. 62, pp. 1251-1289.

Hey, J.D. (1995). ‘Experimental investigations of errors in decision making under risk’, *European Economic Review*, vol. 39, pp. 633-640.

Hey, J.D. and Orme, C. (1994). ‘Investigating generalizations of expected utility theory using experimental data’, *Econometrica*, vol. 62, pp. 1291-1326.

Loomes, G., Moffatt, P. G. and Sugden, R. (2002). ‘A microeconometric test of alternative stochastic theories of risky choice,’ *Journal of Risk and Uncertainty*, vol. 24**,** pp. 103-130.

Loomes, G. and Sugden, R. (1995). ‘Incorporating a stochastic element into decision theories’, *European Economic Review*, vol. 39, pp. 641-648.

Loomes G., and Sugden, R. (1998). ‘Testing alternative stochastic specifications for risky choice’, *Economica*, vol. 65, pp. 581-598.

Schmidt, U. (2004). ‘Alternatives to expected utility: some formal theories’, in (S. Barberà, P.J. Hammond, and C. Seidl, eds.), *Handbook of Utility Theory Vol. II*, chapter 15, Boston: Kluwer.

Starmer, C. (2000). ‘Developments in non-expected utility theory: the hunt for a descriptive theory of choice under risk’, *Journal of Economic Literature*, vol. 38, pp. 332-382.

Sugden, R. (2004). ‘Alternatives to expected utility: foundations,’ in (S. Barberà, P.J. Hammond, and C. Seidl, eds.), *Handbook of Utility Theory Vol. II*, chapter 14, Boston: Kluwer.

Kapitel 3

Testing Violations of Independence Conditions in the Presence of Errors and Splitting Effects

Anmerkungen:

- Dieses Kapitel basiert auf gemeinsamen Experimenten mit Michael H. Birnbaum, Department of Psychology, California State University Fullerton, USA und Miriam Schneider, CAU Kiel
- Ein entsprechender Aufsatz ist zur Publikation eingereicht.

1 Introduction

There exist many studies reporting evidence that expected utility (EU) theory fails to provide an accurate description of peoples´ behavior in several choice problems under risk. One main problem is that choices violate the crucial independence axiom in a systematic way as shown by the famous paradoxes of Allais (1953). These violations have motivated the development of numerous alternative theories (e.g. rank-dependent utility, disappointment and regret models, prospect theory, etc.) which aim to provide a more realistic accommodation of actual choice behavior (see Sugden, 2004, Schmidt, 2004, or Abdellaoui, 2009 for recent surveys). Most of these theories rely on independence conditions that are weakened variants of the independence axiom of EU. Experimental investigations of these weakened independence conditions revealed violation rates rather similar to those reported for the independence axiom of EU (Wakker, Erev, and Weber, 1994; Wu, 1994; Birnbaum and Chavez, 1997; Birnbaum, 2005, 2008).

Other studies have shown that people are not perfectly consistent when choosing between risky lotteries (see e.g. Camerer, 1989; Starmer and Sugden, 1989; Harless and Camerer, 1994; Hey and Orme, 1994); that is, in repeated choice problems they choose one option in the first repetition but the other option in the second one. Such "errors" imply that choices involve a stochastic component. To take into account a stochastic component is also necessary for econometric evaluations of the descriptive performance of EU and its alternatives. Nowadays one very intensively discussed questions in decision theory is how to model this stochastic component adequately (recent papers among many others are Gul & Pesendorfer, 2006; Blavatskyy, 2007, 2008; Conte, Hey, and Moffat, 2007; Hey, Morone, and Schmidt, 2007; Wilcox, 2009; Harrison and Rutström, 2009; etc.)

An interesting question in this context is whether the empirical performance of EU improves if we model the stochastic component properly. For instance already in 1995 John Hey concluded: "It may be the case that these further explorations may alter the conclusion to which I am increasingly being drawn: that one can explain experimental analyses of decision making under risk better (and simpler) as EU plus noise – rather than through some higher level functional – as long as one specifies the noise appropriately" (Hey, 1995; see also

Buschena and Zilberman, 2000).[1] If this is really the case then some of the reported violations of EU might be at least partly caused by errors instead of being intrinsic violations. There exist several recent studies showing that this may indeed be true, see Blavatskyy (2006) for violations of betweenness, Sopher and Gigliotti (1993), Regenwetter and Stober (2006), Birnbaum and Schmidt (2008), (2009) for violations of transitivity, and Schmidt and Hey (2004), Butler and Loomes (2007) for preference reversals. In the present paper we will analyze whether reported violations of the independence axiom of EU and violations of weaker independence conditions may be caused by errors. In the next section we show that even EU with an error component could easily generate the systematic pattern of violations of the independence axiom observed in experimental research.

A study by Schmidt and Neugebauer (2007) provided some evidence in favor of this model. The study considers only choice problems where subjects chose the same option three times in row. Provided that the probability of errors is not too high, these choices should reflect “true preferences” since it is rather improbable that a subject makes the same error three times in row. It turns out that in these choice problems the incidence of violations of independence decreases substantially. Related evidence has been reported by Butler and Loomes (2009). Also in their study, violations of independence can be well attributed to randomness of behavior. The goal of the present paper is to provide a more systematic analysis to estimate the incidence of “true” violations as opposed to those that might be attributed to “error”. We perform a repeated choice experiment and fit an error model that is neutral with respect to violations of any independence condition. This model allows us to discriminate precisely which portion of violations can be attributed to errors and which part should be considered as “real” violations. Note that such an analysis is not possible with a model of EU plus error term (as used by Schmidt and Neugebauer, 2007) since this model presupposes that true preferences can be represented by EU and, thus, satisfy the independence axiom, coalescing, and transitivity. The model we use assumes only that there is a true choice probability and an error rate that can be different for each choice but it does not assume independence or coalescing (two implications of EU we aim to test).

A further systematic deviation from EU (and in fact also from most of alternatives to EU like rank-dependent utility, rank- and sign-dependent utility, and cumulative prospect

[1] In this context also the studies of Harrison and Rutström (2008, 2009) seem to be relevant which criticize the low statistical power of many tests of EU and find evidence that the behavior of roughly 50% of subjects is better described by EU than by prospect theory.

theory) is provided by splitting effects (also called violations of coalescing). A splitting effect (violation of coalescing) occurs if splitting an event with a given consequence into two separate events systematically influences choice behavior. There exists robust evidence that splitting an event with a good (bad) consequence increases (decreases) the attractiveness of a lottery in comparison to other lotteries (Starmer and Sugden 1993; Birnbaum and Navarette, 1998 Humphrey 1995, 2001).[2] While Birnbaum and Navarette employed splitting effects in order to generate substantial violations of first-order stochastic dominance, the papers of Humphrey show that splitting effects contributed to previously reported violations of transitivity (see also Birnbaum & Schmidt, 2008). It may well be the case that splitting effects may also contribute to violations of independence conditions. We will analyze this question while controlling for errors at the same time.

This paper is organized as follows. The next section presents our error model and discusses in general the issue of testing independence in the presence of errors. Section 3 is devoted to our experimental design while section 4 reports the results. Discussion and concluding observations appear in the last section.

2 Errors and Violations of Independence

In this section we will discuss the possible role of errors for generating systematic violations of independence and introduce our error model. Consider a simple variant of the common ratio effect taken from Birnbaum (2001).

Choice 1: Which do you choose?	
R: .99 to win \$0	*S:* .98 to win \$0
.01 to win \$46	.02 to win \$23
Choice 2: Which do you choose?	
R': .50 to win \$0	*S':* \$23 for sure
.50 to win \$46	

Figure 1: A Common ratio effect

[2] For similar evidence of splitting effects in other contexts than choice under uncertainty see e.g. Weber, Eisenführ and von Winterfeldt (1988) and Bateman et al. (1997).

According to EU theory, a person should prefer R over S if and only if that person prefers R' over S' because for a utility function u with the normalization $u(0) = 0$, $u(23) > (<) 0.5u(46)$ implies $0.02u(23) > (<) 0.01u(46)$. There are four possible response patterns in this experiment, RR', RS', SR', and SS', where e.g. RS' represents preference for R in the first choice and S' in the second choice. The response patterns RR' and SS' are consistent with EU while the other two patterns violate the independence axiom of EU. Suppose we obtain data as follows from 100 participants:

	R'	S'
R	51	23
S	11	15

Table 1: A response pattern

In this case 23 people switched from R to S', whereas only 11 reversed preferences in the opposite pattern. The conventional statistical test (test of correlated proportions) is significant, $z = 2.06$ which is usually taken as evidence that EU theory is not correct. This particular result is also called "certainty effect" in reference to the fact that people more often choose the "sure thing" in the second choice. Can this result have occurred by random errors? Note that in principle systematic deviations from independence can also be explained by EU plus a random error term. In this case a subject chooses R over S if $EU(R) - EU(S) + \varepsilon > 0$ where ε is a normally distributed random variable with $E(\varepsilon) = 0$. Suppose $EU(R) > EU(S)$ and note that $EU(R') - EU(S') = 50(EU(R) - EU(S))$. This shows that errors of this particular form may much more easily influence the choice between R and S than the choice between R' and S' implying that we observe much more erroneous SR' than RS' patterns. Since this error model, however, assumes that true preferences can be represented by EU, it does not allow to test whether true preference are in fact satisfying independence. Therefore, we employ a more general error model which is described next.

Suppose that each person has a "true" preference pattern, which may be one of the four possible response combinations. Let $p_{RR'}$, $p_{RS'}$, $p_{SR'}$, and $p_{SS'}$ represent the "true" probabilities of the four preference patterns. These probabilities may be interpreted as the relative frequency of subjects for which true preferences correspond to the given pattern. However, due to errors subjects´ choices may deviate from true preferences. Let e represent the probability of an error in reporting one's true preference for the choice between R and S. Analogously, e' is the probability of an error for the choice between R' and S'. Is it possible that, given the data in Table 1, all subjects adhere to EU? In other words, are the data in Table 1 compatible with $p_{RS'} = p_{SR'} = 0$? The answer is "yes," despite the significant difference in the two types of violations.

In our model, the probability that a person shows the observed preference pattern RS' is given as follows:

(1) $P(RS') = p_{RR'}(1 - e)e' + p_{RS'}(1 - e)(1 - e') + p_{SR'}ee' + p_{SS'}e(1 - e')$

In this expression, $P(RS')$ is the probability of observing this preference pattern. This probability is the sum of four terms, each representing the probability of having one of the "true" patterns and having the appropriate pattern of errors and correct responses to produce each observed data pattern. For example, the person who truly has the RR' pattern could produce the RS' pattern by correctly reporting the first choice and making an "error" on the second choice. There are three other equations like (1), each showing the probability of an observed data pattern given the model.

Given only the data of Table 1, this model is under-determined. There are four response frequencies to fit. These have three degrees of freedom, because they sum to the number of participants. The four "true" probabilities must sum to 1, and there are two "error" probabilities. Thus, we have three degrees of freedom in the data and five parameters to estimate. That means that many solutions are possible. Two solutions that fit the data perfectly are shown in the table below:

Parameter	Model 1: EU fits	Model 2: EU does not hold
$p_{RR'}$	*0.80*	*0.67*
$p_{RS'}$	*0.00*	*0.17*
$p_{SR'}$	*0.00*	*0.00*
$p_{SS'}$	*0.20*	*0.16*
e	*0.10*	*0.15*
e'	*0.30*	*0.15*

Table 2: Fitting the data

This table shows that we can "save" EU in this case by allowing that people might have unequal errors in their responses. So given the error model and our data in Table 1 it is not possible to conclude that EU can be refuted. In order to reach a firmer conclusion we need a way to estimate parameters that do not assume that error rates are necessarily equal or that EU is correct. Put another way, we need to enrich the structure of the data so that we can determine the model. We will do this by adding replications of each choice problem in the experimental design.

Consider the case of one choice problem presented twice, for example, Choice 1 above. There are four response patterns possible, *RR*, *RS*, *SR*, and *SS*. The probability that a person will show the *RR* pattern is given as follows:

(2) $P(RR) = p(1 - e)(1 - e) + (1 - p)e^2,$

where p is the true probability of preferring R and e is the error rate on this choice.

By adding replications to both choices in the test of EU, we have now four choices with 16 (4 × 4) possible response patterns, which have 15 degrees of freedom. But we still have only 5 parameters to estimate from the data, two error terms and four probabilities of the four "true" response patterns. (Because the four probabilities sum to 1, only three degrees of freedom are used in this estimation). The general model (which allows all four probabilities to be non-zero) is now over-determined, with 10 degrees of freedom to test the general error model. EU theory is then a special case of this general model in which two of the true probabilities are fixed to zero. In sum, without replications, two theories are perfectly compatible with these data, one of which assumed EU is true. However, with replications we can estimate the error terms and test the applicability of EU model.

This study will include experiments in which there are four replications. With two choices and four replications, there are 256 possible response patterns (4^4 = 256). Because many of the patterns will be observed with zero frequency, we use the *G*-statistic to measure the badness of fit:

(3) $G = 2\sum f_i ln(f_i/q_i),$

where f_i is the observed frequency and q_i is the predicted frequency of a particular response pattern. The parameters are then selected to minimize this statistic, which theoretically has a chi-square distribution. The difference in G between a fit of the model that allows all four patterns to have non-zero probabilities and the special case in which $p_{RS'} = p_{SR'} = 0$ is chi-square distributed with 2 degrees of freedom. This test allows us to conclude whether observed deviations from EU are significant or whether they might be caused by errors in the response of subjects.

3 Experimental Design

The experiment was conducted at the University of Kiel with 54 subjects, mostly economics and business administration students (all undergraduates). Altogether there were six sessions each consisting of nine subjects and lasting about 90 minutes. Subjects received a 5 Euro show-up fee and had to respond to 176 pairwise choice questions which were arranged in four booklets of 44 choices each. After a subject finished all four booklets one of her choices was randomly chosen and played out for real. The average payment was 19.14 Euro for 90 minutes, i.e. 12.76 Euro per hour, which exceeds the usual wage of students (about 8 Euro per hour) considerably.

Lotteries were presented as in Figure 2 and subjects had to circle their choice. Prizes were always ordered form lowest to highest. Explanation and playing out of lotteries involved a container containing numbered tickets from one to 100. Suppose a subject could for instance play out lottery A in Figure 2. Then she would win 20 Euro when drawing a ticket from 1 to 50, 30 Euro for a ticket between 51 and 80, and 40 Euro for a ticket between 81 and 100. All this was explained in the instructions which were give to the students in printed form and read out aloud. At the end of instructions, subjects had to answer four transparent dominance questions which were controlled by the experimenter before proceeding.

A:	*50% to win 20 Euro*	*B:*	*33% to win 10 Euro*
	30% to win 30 Euro		*34% to win 15 Euro*
	20% to win 40 Euro		*33% to win 60 Euro*

Figure 2: Presentation of lotteries

Lotteries in the booklets were presented in a pseudo-random order. The ordering of lotteries was different in each booklet and no choice problem was followed by another testing the same independence property. Only after finishing one booklet a subject received the next one. Moreover, for half of the subjects each booklet contained only coalesced or only split choice problems whereas for the other half split and coalesced choice problems were intermixed in each booklet. Our stimuli involved 11 tests of independence conditions, nine of which were investigated in both, coalesced and split forms. All these 20 tests were replicated four times with counterbalanced left-right positioning. Additionally, in order to test the attentiveness of subjects, each booklet included two transparent stochastic dominance questions, one based on outcome monotonicity and one on event monotonicity.

Our tests of independence conditions and the involved lottery pairs are presented in Table 3. Each lottery pair consists of a safe lottery *S* (in which you can win prize s_i with probability p_i) and a risky lottery *R* for which possible prizes and probabilities are denoted by r_i and q_i respectively. We took the lotteries from previous studies which reported high violation rates but adjusted outcomes in order to get an average expected value of about 12 Euro. Table 3 shows only the coalesced forms of the lottery pairs. For the tests of independence conditions in split variants we used the canonical split form of these pairs. In the canonical split form of a pairwise choice, both lotteries are split so that there are equal probabilities on corresponding ranked branches and the number of branches is equal in both gambles and minimal. A presentation of the lottery pairs employed in the split tests can be found in the appendix. Note that each pairwise choice problem presented in Table 3 has a unique canonical split form.

The first six tests in Table 3 are four common consequence effects (CCE1-4) and two common ratio effects (CRE1 and 2). Such tests have been widely used for testing the independence axiom of EU; the paradoxes of Allais are special variants of a CCE and a CRE. CCEs can be formally described by $S = (x, p_1; s_2, p_2; s_3, p_3)$, $R = (x, q_1; r_2, q_2; r_3, q_3)$, $S' = (x, p_1 - \alpha; s_2, p_2; s_3, p_3; x', \alpha)$, and $R' = (x, q_1 - \alpha; r_2, q_2; r_3, q_3; x', \alpha)$, i.e. S' and R' are constructed from *S* and *R* by shifting probability mass (α) from the common consequence x to a different common consequence x'. Consequently, an EU maximizer will prefer *S* over *R* if and only if she will prefer S' over R'. Note that in Table 3 the first row of a choice problem always characterizes the lotteries *S* and *R* and the second one the lotteries S' and R'. For CCE1 we have for instance $x = 0, p_1 = 0.8, p_2 = 0.2, s_2 = 19, p_3 = 0$ for S, $q_1 = 0.90, q_2 = 0.10, r_2 = 44, q_3 = 0$ for *R* and S' and R' are constructed by setting $\alpha = 0.4$ and $x' = 44$.

Property	No.	p_1 s_1	p_2 s_2	p_3 s_3	q_1 r_1	q_2 r_2	q_3 r_3
CCE1	5	*0.80* 0	*0.20* 19		*0.90* 0	*0.10* 44	
	13	*0.40* 0	*0.20* 19	*0.40* 44	*0.50* 0	*0.50* 44	
CCE2	1	*0.89* 0	*0,11* 16		*0,90* 0	*0,10* 32	
	2	*1,00* 16			*0,01* 0	*0,89* 16	*0,10* 32
CCE3	5	*0,80* 0	*0,20* 19		*0,90* 0	*0,10* 44	
	6	*1,00* 19			*0,10* 0	*0,80* 19	*0,10* 44
CCE4	9	*0,70* 0	*0,30* 21		*0,80* 0	*0,10* 21	*0,10* 42
	10	*0,70* 0	*0,20* 21	*0,10* 42	*0,80* 0	*0,20* 42	
CRE1	15	*0,98* 0	*0,02* 23		*0,99* 0	*0,01* 46	
	16	*1,00* 23			*0,50* 0	*0,50* 46	
CRE2	20	*0,80* 0	*0,20* 28		*0,86* 0	*0,14* 44	
	19	*0,40* 0	*0,60* 28		*0,58* 0	*0,42* 44	
UTI	29	*0,73* 0	*0,02* 15	*0,25* 60	*0,74* 0	*0,01* 33	*0,25* 60
	30	*0,73* 0	*0,02* 15	*0,25* 33	*0,74* 0	*0,26* 33	
LTI	33	*0,75* 1	*0,23* 34	*0,02* 36	*0,75* 1	*0,24* 33	*0,01* 60
	34	*0,75* 33	*0,23* 34	*0,02* 36	*0,99* 33	*0,01* 60	
UCI	37	*0,20* 9	*0,20* 10	*0,60* 24	*0,20* 3	*0,20* 21	*0,60* 24
	38	*0,40* 9	*0,60* 21		*0,20* 3	*0,80* 21	
LDI	23	*0,60* 1	*0,20* 18	*0,20* 19	*0,60* 1	*0,20* 2	*0,20* 32
	24	*0,10* 1	*0,45* 18	*0,45* 19	*0,10* 1	*0,45* 2	*0,45* 32
UDI	25	*0,20* 6	*0,20* 7	*0,60* 20	*0,20* 1	*0,20* 19	*0,60* 20
	26	*0,45* 6	*0,45* 7	*0,10* 20	*0,45* 1	*0,45* 19	*0,10* 20

Table note: The first lottery pair of a choice problem always characterizes the lotteries *S* and *R* and the second one the lotteries *S′* and *R′*.

Table 3: The lottery pairs

The lotteries in the four CCEs of our experiment are taken from Starmer (1992) who observed high violation rates for these lotteries. The typical pattern of violations in CCE1-4 is that people prefer R over S but S' over R'. The same is true for the two CREs (CRE1 and 2) presented in Table 3. A CRE can be formally described by $S = (x, 1 - \beta(1 - p_1); s_2, \beta p_2)$, $R = (x, 1 - \beta(1 - q_1); r_2, \beta q_2)$, $S' = (x, p_1; s_2, p_2)$, and $R' = (x, q_1; r_2, q_2)$, i.e. S and R are constructed from S' and R' by multiplying all probabilities by β and assigning the remaining probability $1 - \beta$ to the common consequence x. EU implies again that people choose either the risky or the safe lottery in both choice problems. In CRE1 (taken from Birnbaum, 2001) and CRE2 (taken from Starmer and Sugden, 1989), however, substantial violations of EU have been observed with many people choosing R and S'.

The remaining five independence properties in Table 3 are weakened variants of the independence axiom of EU which were employed to derive alternative theories. We focus on variants which are implied by rank-dependent utility (Quiggin, 1981, 1982; Luce, 1991, 2000; Luce and Fishburn, 1991; Luce and Marley, 2005), cumulative prospect theory (Starmer and Sugden, 1989; Tversky and Kahneman, 1992; Wakker and Tversky, 1993), and configural weight models (Birnbaum and McIntosh, 1996). A central property in this context is tail independence (TI) which was introduced by Green and Jullien (1988) using the term ordinal independence. Formally, TI demands that $S = (x_1, p_1; \ldots; x_i, p_i; x_{i+1}, p_{i+1}; \ldots; x_n, p_n) \succcurlyeq R = (x_1, p_1; \ldots; x_i, p_i; x_{i+1}, q_{i+1}; \ldots; x_n, q_n)$ if and only if $S' = (x_1, q_1; \ldots; x_i, q_i; x_{i+1}, p_{i+1}; \ldots; x_n, p_n) \succcurlyeq R' = (x_1, q_1; \ldots; x_i, q_i; x_{i+1}, q_{i+1}; \ldots; x_n, q_n)$ where $x_1 \geq x_2 \geq \ldots \geq x_n$. This means that if two lotteries share a common tail (i.e. identical probabilities of receiving any outcome better than x_{i+1}), then the preference between these lotteries must not change if this tail is replaced by a different common tail. Note that in the definition above the upper tail is the common tail and thus the condition is called upper tail independence (UTI). TI, however, also demands that preferences must not change if lower common tails are exchanged which will be called lower tail independence (LTI). TI is a very general property which is implied by many models including all variants of rank-dependent utility (RDU) as well as cumulative prospect theory (CPT). Therefore, rejecting TI would provide serious evidence against all these models. In his experiments, Wu (1994) observed violation rates of UTI of up to 50%. Similar evidence has been reported by Birnbaum (2001) and Wakker, Erev, and Weber (1994) where the latter paper tests comonotonic independence, the analogue to TI in choice under uncertainty. Our study tries to find out whether the reported violations of TI may be due to splitting effects and/or errors. The lotteries we use for the test of UTI are taken from Wu (1994). LTI has, as far as we know, not been tested before. Our construction of lotteries in the test of LTI is similar to that used in the test of UTI.

Another property implied by CPT and the common versions of RDU is upper cumulative independence (UCI), which demands that decision weights depend only on cumulative probabilities. Formally, UCI demands that If S = $(s_1, p_1; s_2, p_2; \alpha, p_3) \prec R = (r_1, p_1; \gamma, p_2; \alpha, p_3)$ then $S' = (s_1, p_1 + p_2; \gamma, p_3) \prec R' = (r_1, p_1; \gamma, p_2 + p_3)$, where $\alpha > \gamma > s_2 > s_1 > r_1$. Substantial violations of UCI have been reported by Birnbaum and Navarette (1998) and Birnbaum, Patton, and Lott (1999). Our lottery pairs are taken from the latter paper which observed violation rates of 40.1% for these pairs, where the typical violating pattern is *RS'*.

The final property we test is distribution independence (DI). Whereas configural weight models and original prospect theory imply that DI holds, it should be violated according to RDU and CPT, at least if the weighting function is inverse-S shaped as commonly suggested by empirical research (Camerer and Ho, 1994; Wu and Gonzalez, 1996; Tversky and Fox, 1995; Gonzalez and Wu, 1999; Abdellaoui, 2000; Bleichrodt and Pinto, 2000; Kilka and Weber, 2001; Abdellaoui, Vossmann, and Weber, 2005). For three-outcome lotteries, DI demands that S = $(s_1, \beta; s_2, \beta; \alpha, 1 - 2\beta) \succcurlyeq R = (r_1, \beta; r_2, \beta; \alpha, 1 - 2\beta)$ if and only if $S' = (s_1, \delta; s_2, \delta; \alpha, 1 - 2\delta) \succcurlyeq R' = (r_1, \delta; ; r_2, \delta; \alpha, 1 - 2\delta)$ where α is either the highest or the lowest outcome in both lotteries. If α is the highest outcome, the condition is called upper distribution independence (UDI), otherwise lower distribution independence (LDI). The lotteries used in our tests of UDI and LDI are taken from Birnbaum (2005). The evidence reported in that paper and in Birnbaum and Chavez (1997) indicates that one should observe either no violations or violations contrary to CPT with inverse-S weighting function.

4 Results

We will first comment on our results with respect to first-order stochastic dominance. In each booklet there were two transparent dominance questions. With four booklets we have altogether eight tests of transparent dominance per person. Out of our *54* subjects five violated dominance once and one subject twice (7 out of 8 x 54 = 432 is 1.6%). We conclude from this result that our subjects were sufficiently attentive and motivated. This conclusion is supported by the fact that all our estimated error rates (Table 4) are between *2%* and *22%* (mean *11.4%*) which is in line with estimations in comparable studies.

Table 4 gives an overview of our results on our tests of the single independence conditions. For all conditions listed in the first column we report the estimated probabilities (or relative frequencies) of the four possible response patterns to be reflecting true preferences in columns two to five. A subscript, *S* in the first column indicates that this test of the respective independence condition was performed by presenting both choices in canonical split form. In columns six and seven we report the error rates in the choices between *S* and *R (e)* as well as between *S′* and *R′ (e′)*. The final column presents the statistics of a chi-square test between the fit of a general model – i.e. a model that allows all four response patterns to have non-zero probabilities – and the fit of a model which satisfies the respective independence condition – i.e. a model corresponding to the special case in which $p_{RS'} = p_{SR'} = 0$ must hold. One asterisk (two asterisks) in this column indicate that we can reject the null of $p_{RS'} = p_{SR'} = 0$ in favor of the general model at a significance level of *5% (1%)*.

Property	p_{SS}	p_{SR}	p_{RS}	p_{RR}	e	e'	Test
CCE1	*0.44*	*0.02*	***0.30***	*0.24*	*0.15*	*0.11*	***20.36*****
CCE1$_S$	*0.52*	***0.20***	*0.00*	*0.28*	*0.13*	*0.16*	***12.77*****
CCE2	*0.02*	*0.00*	*0.10*	*0.88*	*0.02*	*0.08*	***15.33*****
CCE2$_S$	*0.09*	*0.03*	*0.05*	*0.84*	*0.07*	*0.07*	***7.61****
CCE3	*0.25*	***0.21***	*0.16*	*0.39*	*0.16*	*0.12*	***18.69*****
CCE3$_S$	*0.52*	***0.24***	*0.00*	*0.25*	*0.13*	*0.16*	***12.63*****
CCE4	*0.67*	*0.01*	***0.29***	*0.02*	*0.14*	*0.09*	***21.96*****
CCE4$_S$	*0.80*	*0.01*	*0.02*	*0.17*	*0.14*	*0.12*	*0.82*
CRE1	*0.25*	*0.00*	***0.64***	*0.11*	*0.11*	*0.07*	***44.64*****
CRE1$_S$	*0.44*	*0.00*	***0.46***	*0.10*	*0.15*	*0.05*	***27.21*****
CRE2	*0.57*	*0.00*	***0.20***	*0.23*	*0.14*	*0.11*	***18.00*****
CRE2$_S$	*0.84*	*0.02*	*0.01*	*0.12*	*0.17*	*0.12*	*0.45*
UTI	*0.06*	*0.01*	***0.52***	*0.40*	*0.13*	*0.18*	***18.76*****
UTI$_S$	*0.17*	*0.01*	*0.00*	*0.82*	*0.14*	*0.18*	*0.05*
LTI	*0.04*	*0.00*	*0.14*	*0.82*	*0.05*	*0.15*	*3.96*
LTI$_S$	*0.04*	*0.00*	*0.01*	*0.95*	*0.06*	*0.08*	*0.24*
UCI	*0.14*	*0.08*	*0.13*	*0.66*	*0.13*	*0.22*	*3.88*
UCI$_S$	*0.13*	*0.09*	*0.02*	*0.76*	*0.13*	*0.09*	*5.07*
LDI	*0.94*	*0.00*	*0.00*	*0.06*	*0.02*	*0.05*	*0.00*
UDI	*0.16*	*0.02*	*0.03*	*0.79*	*0.09*	*0.10*	*1.80*

Table note: * denotes a significance level of 5%, ** a significance level of 1%.

Table 4: Tests of independence conditions

We will first analyze the tests of the independence axiom of EU. This axiom demands $p_{RS'} = p_{SR'} = 0$ whereas empirical research on CCEs and CREs has reported systematical violations, most frequently of the RS' pattern. Using as in previous research only coalesced lotteries, we can confirm this result in our analysis: In all our six tests (CCE1-4, CRE1-2), the independence axiom is rejected in favor of the more general model. Moreover, most violations are given by responses of the pattern RS' (estimated probabilities range from *10%* to *64%*, mean *28%*) whereas the opposite pattern SR' occurs very rarely (apart from CCE3 estimated probabilities never exceed *3%*). The only exception is CCE3 where the estimated probability of pattern SR' slightly exceeds that of RS'. In summary, our first result indicates that the typical evidence on violations of the independence axiom of EU can be also observed in a framework controlling for errors; therefore, these violations can be regarded as true violations.

A quite different picture arises when the same test are performed with lotteries presented in split form. From our six tests, two ($CCE4_S$ and $CRE2_S$) are not significant (i.e. EU cannot be rejected) and two ($CCE1_S$ and $CCE3_S$) are significant but precisely in the opposite direction as observed in previous tests that used coalesced lotteries (including ours). From the two remaining tests, only one (CRE1) shows the same pattern in both split and coalesced forms as in previous research whereas for $CCE2_S$ only very low violation rates (i.e. 3% and 5% for both violating patterns) are estimated. We can, therefore, conclude that splitting effects have a substantial influence on tests of the independence axiom of EU, both for CCEs and CREs. Presenting lotteries in their canonical split forms does not at all generate the clear pattern of violations reported in previous studies and also found in our results with coalesced lotteries. So the question arises whether the previous evidence should be indeed regarded as evidence against the independence axiom or whether an interpretation in terms of violations of coalescing seems to be more appropriate.

Next, consider the tests of the weaker independence conditions. For UTI we can observe a substantial and systematic violation as the estimated probability of the violating pattern RS' amounts to *52%*. This picture is entirely in line with the high violation rates observed by Wu (1994) and similar evidence reported by Birnbaum (2001) and Wakker, Erev, and Weber (1994). We can conclude that violations of TI are not caused by errors but reflect true preferences. This is a serious challenge for CPT and the whole class of rank-dependent models which all imply that TI holds. It is, however, astonishing that this clear evidence of violations of TI virtually disappears when we present lotteries in their canonical split form. The estimated frequency of the *RS′* pattern decreases from *52%* in the coalesced test to *0%* in the split test while the frequency of the opposite violation *SR′* amounts to only *1%* in both cases. Therefore, also violations of TI in our and previous studies seem to be mainly caused by violations of coalescing.

Our new test of LTI did not generate significant violations. The same is true for UCI where splitting has only a small impact. Comparing this result to the high violation rates of UCI observed in previous papers (Birnbaum and Navarette, 1998; Birnbaum, Patton, and Lott 1999) with lotteries identical to those in our coalesced test, the difference of results may be due to large error rates in these tests. The estimated error rates in our coalesced test of UCI are the highest of all our choices.

Our tests of DI yield results similar to previous tests by Birnbaum and Chavez (1997) and Birnbaum (2005). Significant violations of DI predicted by CPT are also not observed in our tests, which fails to confirm the predictions of the inverse-S weighting function commonly proposed for rank-dependent models.

Problems	p_{SS}	p_{SR}	p_{RS}	p_{RR}	e	e'	**Test**
1-3	*0.02*	*0.00*	*0.07*	*0.90*	*0.02*	*0.07*	***7.89****
5-7	*0.47*	*0.00*	***0.28***	*0.26*	*0.15*	*0.13*	***17.42*****
9-11	*0.70*	*0.00*	*0.06*	*0.24*	*0.14*	*0.14*	*1.78*
10-12	*0.82*	*0.14*	*0.00*	*0.04*	*0.09*	*0.12*	***7.83****
13-14	*0.52*	***0.22***	*0.00*	*0.26*	*0.11*	*0.16*	***18.52*****
15-17	*0.29*	*0.00*	*0.13*	*0.57*	*0.11*	*0.15*	***8.04****
19-21	*0.76*	*0.03*	*0.07*	*0.14*	*0.11*	*0.12*	*4.56*
20-22	*0.56*	*0.00*	***0.23***	*0.20*	*0.14*	*0.16*	***17.31*****
29-31	*0.06*	*0.00*	*0.13*	*0.80*	*0.13*	*0.13*	***10.66*****
30-32	*0.18*	***0.40***	*0.00*	*0.42*	*0.18*	*0.18*	***24.72*****
33-35	*0.02*	*0.02*	*0.01*	*0.95*	*0.05*	*0.07*	*3.51*
34-36	*0.05*	*0.13*	*0.01*	*0.81*	*0.15*	*0.08*	*3.76*
38-39	*0.13*	*0.15*	*0.01*	*0.71*	*0.22*	*0.08*	*3.85*
41-42	*0.35*	*0.04*	***0.35***	*0.27*	*0.20*	*0.14*	***15.19*****

Table 5: Splitting effects

Since our results show a substantial influence of splitting effects on testing independence properties we also provide a direct analysis of these effects in Table 5. As far as we know, splitting effects have not been analyzed before in a framework controlling for errors. Table 5 compares choices in a given lottery pair in coalesced form with choices in the same pair in split form. If no splitting effects occur, choice should be identical in both problems. This means that a subject either chooses the risky lottery in both problems or the safe lottery in both problems. In contrast, Table 5 shows that many people choose differently in the coalesced and split problems even when we control for errors where e (e') is the estimated error rate of the choice problem stated first (second) in the first column of the table. The last column shows again the chi-square tests comparing the fit of a model which satisfies coalescing and thus implies $p_{RS'} = p_{SR'} = 0$, and the fit of a general model that allows for splitting effects and thus for non-zero probabilities of all four possible response patterns. It turns out that in nine out of 14 tests the null $p_{RS'} = p_{SR'} = 0$ of has to be rejected in favor of the general model allowing for splitting effects. Five of these (bold font) exceed 20%.

5 Conclusions

Two major conclusions can be drawn from our study: first, violations of independence and coalescing are most likely not a mere consequence of choice errors. Even if we control for errors, EU can be significantly rejected in favor of a more general model in many of our tests. Similarly, previously reported violations of coalescing are also robust with respect to our control for errors. Second, splitting effects have a strong impact on tests of independence conditions; if all lotteries are presented in split form, violations of independence become either insignificant or unsystematic. Given this evidence, violations of independence could be simply regarded as framing effects, resulting from coalesced presentation of lotteries. This interpretation presupposes that choice between split lotteries reflects true preference whereas coalescing leads to distortions. This view seems to be debatable and leads to the question whether coalesced or split presentation of lotteries induces more biased behavior. We think this question indicates the need for further research on the impact of presenting lotteries on choice behavior. One interesting aspect is the question which way of presenting lotteries could minimize violations of coalescing. Another important question is whether alternatives to EU improve the empirical performance of EU also if all lotteries are presented in split form. Our results suggest that this might be not the case. Finally, more attention should be devoted to theories which allow for violations of coalescing. Models which imply splitting effects include original prospect theory, prospective reference theory, and configural weight theory, their comparative performance in explaining theses effects has, however, not yet been studied systematically.

References

Abdellaoui, M. (2000). Parameter-free elicitation of utility and probability weighting functions. *Management Science*, 46:1497-1512.

Abdellaoui, M. (2009). Rank-dependent utility. In: Anand, P., Pattanaik, P. K., and Puppe, C. (eds.), *The Handbook of Rational and Social Choice*, 90-112. Oxford University Press, Oxford, New York.

Abdellaoui, M., Vossmann, F., and Weber, M. (2005). Choice-based elicitation and decomposition of decision weights for gains and losses under uncertainty. *Management Science*, 51: 1384-1399.

Allais, M. (1953). Le comportement de l'homme rationnel devant le risqué, critique des postulates et axioms de l'ecole americaine. *Econometrica*, 21:503-546.

Bateman, I., Munro, A., Rhodes, B., Starmer, C., and Sugden, R. (1997). Does part-whole bias exist? An experimental investigation. *Economic Journal*, 107: 322-332.

Birnbaum, M. H. (2001). Decision and choice: Paradoxes of choice. In Smelser, N. J. and Baltes, P. B. (eds.), *International Encyclopedia of the Social and Behavioral Sciences.* Elsevier Oxford.

Birnbaum, M. H. (2005). Three tests of independence that differentiate models of risky decision making. *Management Science*, 51:1346-1358.

Birnbaum, M. H. (2008). New paradoxes of risky decision making. *Psychological Review*, 115: 463-501.

Birnbaum, M. H. and Chavez, A. (1997). Tests of theories of decision making: Violations of branch independence and distribution independence. *Organizational Behavior and Human Decision Processes*, 71:161-194.

Birnbaum, M. H. and McIntosh, W. R. (1996). Violations of branch independence in choices between gambles. *Organizational Behavior and Human Decision Processes*, 67:91-110.

Birnbaum, M. H. and Navarrete, J. B. (1998). Testing descriptive utility theories: Violations of stochastic dominance and cumulative independence. *Journal of Risk and Uncertainty*, 17:17-49.

Birnbaum, M. H., Patton, J. N., and Lott, M. K. (1999). Evidence against rank-dependent utility theories: Tests of cumulative independence, interval independence, stochastic dominance, and transitivity. *Organizational Behavior and Human Decision Processes*, 77:44-83.

Birnbaum, M. H. and Schmidt, U. (2008). An experimental investigation of violations of transitivity in choice under uncertainty. *Journal of Risk and Uncertainty*, 37:77-91.

Birnbaum, M. H. and Schmidt, U. (2009). Testing transitivity in choice under risk. *Theory and Decision*, forthcoming.

Blavatskyy, P. (2006). Violations of betweenness or random errors?. *Economics Letters,* 91: 34-38

Blavatskyy, P. (2007). Stochastic expected utility theory. *Journal of Risk and Uncertainty*, 34:259-286.

Blavatskyy, P. (2008). Stochastic utility theorem. Journal of Mathematical Economics, 44:1049-1056.

Bleichrodt, H. and Pinto, J. L. (2000). A parameter-free elicitation of the probability weighting function in medical decision analysis. *Management Science*, 46: 1485-1496.

Buschena, D. E. and Zilberman, D. (2000). Generalized expected utility, heteroscedastic error, and path dependence in risky choice. Journal of Risk and Uncertainty, 20:67-88.

Butler, D. J. and Loomes, G. C. (2007). Imprecision as an account of the preference reversal phenomenon. *American Economic Review*, 97:277-297.

Butler, D. J. and Loomes, G. C. (2009). Imprecision as an Account of Violations of Independence. Working paper, University of East Anglia.

Camerer, C. F. (1989). An experimental test of several generalized utility theories. *Journal of Risk and Uncertainty*, 2:61-104.

Camerer, C. F. and Ho, T. (1994). Violations of the betweenness axiom and nonlinearity in probability. *Journal of Risk and Uncertainty*, 8:167-196.

Conte, A., Hey, J. D., and Moffat, P. G. (2007). Mixture models of choice under risk. Discussion Paper No. 2007/06, University of York, Department of Economics.

Gonzalez, R. and Wu, G. (1999). On the form of the probability weighting function. *Cognitive Psychology*, 38: 129-166.

Green, J. R. and Jullien, B. (1988). Ordinal independence in nonlinear utility theory. Journal of Risk and Uncertainty, 1:355-387.

Gul, F. and Pesendorfer, W. (2006). Random expected utility. *Econometrica*, 74:121-146.

Harrison, G. W. and Rutström, E. (2008). Risk aversion in the laboratory. In: Cox, J. C. and Harrison, G. W. (eds.), *Risk Aversion in Experiments.* Emerald, Bingley (UK).

Harrison, G. W. and Rutström, E. (2009). Expected utility and prospect theory: One wedding and decent funeral. *Experimental Economics*, forthcoming.

Harrless, D. and Camerer, C. F. (1994). The predictive utility of generalized expected utility theories. *Econometrica*, 62:1251-1289.

Hey, J. D. (1995). Experimental investigations of errors in decision making under risk. *European Economic Review*, 39:633-640.

Hey, J. D., Morone, A., and Schmidt, U. (2007). Noise and bias in eliciting preferences. Discussion Paper No. 2007/04, University of York, Department of Economics.

Hey, J. D. and Orme, C. (1994). Investigating generalizations of expected utility theory using experimental data. *Econometrica*, 62:1291-1326.

Humphrey, S. J. (1995). Regret aversion or event-splitting effects? More evidence under risk and uncertainty. *Journal of Risk and Uncertainty*, 11:263-274.

Humphrey, S. J. (2001). Non-transitive choice: Event-splitting effects or framing effects?, *Economica,* 68: 77-96.

Kilka, M. and Weber, M. (2001). What determines the shape of the probability weighting function under uncertainty. *Management Science*, 47: 1712-1726.

Luce, R. D. (1991). Rank- and sign-dependent linear utility models for binary gambles. Journal of Economic Theory, 53:75-100.

Luce, R. D. (1992). *Utility of Gains and Losses: Measurement-Theoretic and Experimental Approaches*. Lawrence Erlbaum Asccociation, Mahwah, NJ.

Luce, R. D. and Fishburn, P. C. (1991). Rank- and sign-dependent linear utility models for finite first-order gambles. *Journal of Risk and Uncertainty,* 4:29-59.

Luce, R. D. and Marley, A. A. J. (2005). Ranked additive utility representations of gambles: old and new axiomatizations. *Journal of Risk and Uncertainty*, 30: 21-62.

Quiggin, J. (1981). Risk perception and risk aversion among Australian farmers. *Australian Journal of Agricultural Economics*, 25: 160-169.

Quiggin, J. (1982). A theory of anticipated utility. *Journal of Economic Behavior and Organization*, 3:323-343.

Regenwetter, M. and Stober, C. (2006). Testing transitivity by testing the triangle inequality instead of weak stochastic transitivity. Edwards Bayesian Research Conference, Fullerton, CA, January 2006.

Schmidt, U. (2004). Alternatives to expected utility: Formal theories. In: Barberà, S., Hammond, P.J., and Seidl, C. (eds), *Handbook of Utility Theory*, volume II: Extensions, 757-838. Kluwer Academic Publishers, Dordrecht.

Schmidt, U. and Hey, J. D. (2004). Are preference reversals errors? An experimental investigation. *Journal of Risk and Uncertainty*, 29:207-218.

Schmidt, U. and Neugebauer, T. (2007). Testing Expected Utility in the Presence of Errors. *Economic Journal*, 117:470-485.

Sopher, B. and Gigliotti, G. (1993). Intransitive cycles: Rational choice or random errors? An answer based on estimation of error rates with experimental data. *Theory and Decision*, 35:311-336.

Starmer, C. (1992). Testing new theories of choice under uncertainty using the common consequence effect. *The Review of Economic Studies*, 59:813-830.

Starmer, C. and Sugden, R. (1989). Violations of the independence axiom in common ratio problems: An experimental test of some competing hypothesis. *Annals of Operational Research*, 19:79-102.

Starmer, C. and Sugden, R. (1993). Testing for juxtaposition and event-splitting effects. *Journal of Risk and Uncertainty*, 6:235-254.

Sugden, R. (2004). Alternatives to expected utility: Foundations. In: Barberà, S., Hammond, P.J., and Seidl, C. (eds) *Handbook of Utility Theory*, volume II: Extensions, 685-755. Kluwer Academic Publishers, Dordrecht.

Tversky, A. and Fox, C. R. (1995). Ambiguity aversion and comparative ignorance. *The Quarterly Journal of Economics*, 110:585-603.

Tversky, A. and Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5:297-323.

Wakker, P., Erev, I., and Weber, E. (1994). Comonotonic independence: The critical test between classical and rank-dependent utility theories. *Journal of Risk and Uncertainty*, 9:195-230.

Wakker, P. and Tversky, A. (1993). An axiomatization of cumulative prospect theory. *Journal of Risk and Uncertainty*, 7:147-175.

Weber, M., Eisenführ, F., and von Winterfeldt, D. (1988). The effect of splitting attributes in multiattribute utility measurement. *Management Science*, 34: 431-445.

Wilcox, N. T. (2009). 'Stochastically more risk averse:' A contextual theory of stochastic discrete choice under risk, *Journal of Econometrics*, forthcoming.

Wu, G. (1994). An empirical test of ordinal independence. *Journal of Risk and Uncertainty*, 9:39-60.

Wu, G. and Gonzalez, R. (1996). Curvature of the probability weighting function. *Management Science*, 42:1676-1690.

Appendix

Problem	p_1 s_1	p_2 s_2	p_3 s_3	p_4 s_4	q_1 r_1	q_2 r_2	q_3 r_3	q_4 r_4
$CCE1_S$	*0,80*	*0,10*	*0,10*		*0,80*	*0,10*	*0,10*	
	0	19	19		0	0	44	
	0,40	*0,10*	*0,10*	*0,40*	*0,40*	*0,10*	*0,10*	*0,40*
	0	19	19	44	0	0	44	44
$CCE2_S$	*0,89*	*0,01*	*0,10*		*0,89*	*0,01*	*0,10*	
	0	16	16		0	0	32	
	0,01	*0,89*	*0,10*		*0,01*	*0,89*	*0,10*	
	16	16	16		0	16	32	
$CCE3_S$	*0,80*	*0,10*	*0,10*		*0,80*	*0,10*	*0,10*	
	0	19	19		0	0	44	
	0,10	*0,80*	*0,10*		*0,10*	*0,80*	*0,10*	
	19	19	19		0	19	44	
$CCE4_S$	*0,70*	*0,10*	*0,10*	*0,10*	*0,70*	*0,10*	*0,10*	*0,10*
	0	21	21	21	0	0	21	42
	0,70	*0,10*	*0,10*	*0,10*	*0,70*	*0,10*	*0,10*	*0,10*
	0	21	21	42	0	0	42	42
$CRE1_S$	*0,98*	*0,01*	*0,01*		*0,98*	*0,01*	*0,01*	
	0	23	23		0	0	46	
	0,50	*0,50*			*0,50*	*0,50*		
	23	23			0	46		
$CRE2_S$	*0,80*	*0,06*	*0,14*		*0,80*	*0,06*	*0,14*	
	0	28	28		0	0	45	
	0,40	*0,18*	*0,42*		*0,40*	*0,18*	*0,42*	
	0	28	28		0	0	45	
UTI_S	*0,73*	*0,01*	*0,01*	*0,25*	*0,73*	*0,01*	*0,01*	*0,25*
	0	15	15	60	0	0	33	60
	0,73	*0,01*	*0,01*	*0,25*	*0,73*	*0,01*	*0,01*	*0,25*
	0	15	15	33	0	0	33	33
LTI_S	*0,75*	*0,23*	*0,01*	*0,01*	*0,75*	*0,23*	*0,01*	*0,01*
	1	34	36	36	1	33	33	60
	0,75	*0,23*	*0,01*	*0,01*	*0,75*	*0,23*	*0,01*	*0,01*
	33	34	36	36	33	33	33	60
UCI_S	*0,20*	*0,20*	*0,60*		*0,20*	*0,20*	*0,60*	
	9	10	24		3	21	24	
	0,20	*0,20*	*0,60*		*0,20*	*0,20*	*0,60*	
	9	9	21		3	21	21	

Table note: The first lottery pair of a choice problem always characterizes the lotteries *S* and *R* and the second one the lotteries *S′* and *R′*.

Table A1: The lottery pairs in canonical split form

Kapitel 4

A Theory of Imprecise Probability Perception

1 Introduction

Experimental studies of repeated decision making under risk demonstrate that individual choice patterns are stochastic in nature (e.g. Camerer (1989), Starmer and Sugden (1989), Wu (1994), Hey and Orme (1994), Ballinger and Wilcox (1997), Hey (2001)). Stochastic choice patterns can result from deliberate randomization by individuals with quasi-concave preferences (Machina (1985), Chew et al. (1991)). However, experimental tests have rejected this hypothesis (e.g. Hey and Carbone (1995)). Individuals can also choose in a stochastic manner if they have multiple preference relations on the set of risky lotteries that are represented in a random utility model (Loomes and Sugden (1995), Gul and Pesendorfer (2006)). However, Sopher and Narramore (2000) find that variation in individual decisions is not systematic in a statistical sense, which strongly supports random error rather than random utility model.

Another possible explanation for the revealed stochastic choice patterns is that individuals make mistakes due to inattantiveness, carelessness, insufficient motivation, fatigue etc. Various models of stochastic choice have been proposed in the literature to capture this degree of randomness in the observed choices (Luce and Suppes (1965), Harless and Camerer (1994), Hey and Orme (1994), Loomes et al. (2002), Buschena and Zilberman (2000), Blavatskyy (2007)). A distinctive feature of all these models of random errors is that the stochastic component is added on the utility scale. However, random errors can also enter into a decision problem through the distortion of probabilities or outcomes. The former possibility is considered in this paper.

In our model the given probabilities can be either objective or subjective ones. In the case of objective probabilities we assume that individuals are inexperienced in dealing with probability information such that there is some variability in their perception. In the case of subjective probabilities we presuppose that subject can come up with subjective probabilities for the single events but they are rather doubtful whether they are correct such that there is again some variability in the final probability information on which decisions are based.

Our model is related to other models in which single (rather than cumulative) probabilities are transformed like original prospect theory (Kahneman & Tversky, 1979), prospective reference theory (Viscusi, 1989) or the models of Handa (1977) and Karmarkar (1978). In contrast to these models however, choice is stochastic in our model and our model predicts that only small and large probabilities are systematically biased.

2 The Model

We consider a finite set of outcomes $X = (x_1, x_2, ..., x_n)$ which need not to be monetary consequences. Objects of choice are lotteries or probability distributions over X. Each lottery p is identified by a vector $(p_1, p_2, ..., p_n)$ where p_i is the probability of x_i. Obviously we have $0 \leq p_i \leq 1$ and $\sum_i p_i = 1$. The set of all lotteries is denoted by P. Each subject has a preference relation $\succcurlyeq$ over P which is complete and transitive and satisfies the independence and continuity axioms. Consequently, there exists a vector of von Neumann-Morgenstern utilities $(u_1, u_2, ..., u_n)$ such that $p \succcurlyeq q \Leftrightarrow EU(p) \equiv \sum_i u_i p_i \geq \sum_i u_i q_i \equiv EU(q)$.

As argued in the introduction, we assume that random errors may occur in the perception of probabilities. More precisely, for every $p \in P$ there exists a vector of random errors $(\varepsilon(p_1), \varepsilon(p_2), ..., \varepsilon(p_n))$ such that $p_i + \varepsilon(p_i)$ is the perceived probability of x_i in lottery p. Each error $\varepsilon(p_i)$ is drawn from an identical distribution. For convenience, we will often write ε_i instead of $\varepsilon(p_i)$ if it is clear which lottery is referred to.

While true preferences can be represented by EU with objective probabilities, the actual choice between two lotteries is determined by EU with perceived probabilites. More formally, we have that $p \succcurlyeq_c q \Leftrightarrow EU_c(p) \equiv \sum_i u_i(p_i + \varepsilon_i) \geq \sum_i u_i(q_i + \varepsilon_i) \equiv EU_c(q)$ where $p \succcurlyeq_c q$ indicates that p is chosen over q in a binary choice. Although subjects perceive objective probabilities with error, they realize that perceived probabilities should satisfy the basic properties of probability distributions, i.e. $0 \leq p_i + \varepsilon_i \leq 1$ and $\sum_i (p_i + \varepsilon_i) = 1$.

Moreover, we assume that $\varepsilon_i = 0$ if $p_i = 0$. Note that $\sum_i (p_i + \varepsilon_i) = 1$ implies $\varepsilon_i = 0$ if $p_i = 1$. It also implies that $\sum_i \varepsilon_i = 0$ since $\sum_i p_i = 1$. This restriction means that errors are not drawn independently. Without loss of generality, we can set $\varepsilon_1 = 1- \sum_{i>1} \varepsilon_i$. For convenience we assume in the following that x_1 is the worst outcome, i.e. $x_i \succcurlyeq x_1$ for all i. Now we get

(1) $$EU_c(p) = EU(p) + \sum_{i>1} \varepsilon_i(u_i - u_1).$$

In priciple, errors are drawn from a symmetric distribution with zero mean and support $[-\alpha, \alpha]$ with $\alpha \geq 0$. Let us denote the cumulative distribution function (cdf) of this distribution by F* with $F^*: [-\alpha, \alpha] \rightarrow [0, 1]$ and assume that F* is strictly increasing, i.e. there is no non-degenerate subinterval of $[-\alpha, \alpha]$ with zero probability of an error in this interval. However, if $p_i < \alpha$ or $p_i > 1 - \alpha$ distribution of errors has to be truncated due to the restriction $0 \leq p_i + \varepsilon_i \leq 1$. So in fact the distribution of errors for a given p_i is characterized by the cdf F defined by

(2) $$F(\delta) = \frac{F^*(\delta) - F^*(\max\{-p_i, -\alpha\})}{F^*(\max\{1-p_i, \alpha\}) - F^*(\max\{-p_i, -\alpha\})}.$$

To understand the logic behind equation (2), we can consider the reasonable assumption that $\alpha < 0.5$, i.e. a probability of 50% will be never perceived as an impossibility or a certainty. In the case we get

(3) $$F(\delta) = \begin{cases} \dfrac{F^*(\delta) - F^*(-p_i)}{1 - F^*(-p_i)} & \text{if } p_i < \alpha \\ F^*(\delta) & \alpha \le p_i \le 1-\alpha \\ \dfrac{F^*(\delta)}{F^*(1-p_i)} & \text{if } p_i > 1-\alpha. \end{cases}$$

However, the assumption $\alpha < 0.5$ is not needed for the following results.

Proposition 1:
If errors have cdf defined by (2) then
(i) $E(\varepsilon_i) > 0$ if $p_i < \alpha$ and $E(\varepsilon_i) < 0$ if $p_i > 1-\alpha$
(ii) $E(\varepsilon_i) > E(\varepsilon_j)$ if either $p_i < p_j < \alpha$ or $p_j > p_i > 1-\alpha$.

Proof: See appendix

Proposition 1 tells us that if probabilities are sufficiently small, random errors have an upward bias whereas the opposite is true for sufficiently large probabilities. This can be intuitively explained as follows: for a low probability of say 0.2α the most extreme negative error is -0.2α whereas the most extreme positive error is given by α, so positive errors are more likely to occur and expected error becomes positive. Analogously, for large probabilities positve errors have smaller support than negative ones such that now negative errors are more likely and expected error is negative.

Notice that our model of random errors in probabilities can be interpreted as a behavioral foundation for simple non-linear probability weighting introduced in the prospect theory (Kahneman and Tversky (1979)). Mistakes in the perception of the probability information naturally lead to the overweighting of small probabilities and underweighting of large probabilities.

3 Behavioral implications

3.1 The fourfold pattern of risk attitudes and reflection effect

Tversky and Kahneman (1992) found for two-outcome lotteries with one prize being zero that subjects exhibit risk seeking behavior when lotteries yield a gain with low probability or a loss with high probability and at the same time subjects exhibit risk averse behavior when lotteries yield a gain with high probability or a loss with low probability. This mixed risk attitude became known as the fourforld pattern of risk attitudes. When n=2 equation (1) becomes $EU_c(p) = EU(p) + \varepsilon_2(u_2 - u_1)$. Since $u_2 - u_1 > 0$ then $E(EU_c(p)) > EU(p)$ if $E(\varepsilon_2) > 0$ and $E(EU_c(p)) < EU(p)$ if $E(\varepsilon_2) < 0$. If a subject has risk neutral preferences then $EU(p)=E(p)$ and she is more likely to exhibit risk seeking behavior when $E(\varepsilon_2) > 0$. Proposition 1 shows that $E(\varepsilon_2) > 0$ when $p_2 < \alpha$. Recall that p_2 is probability of the highest outcomes, i.e. it is the probability of the gain for the gain lotteries and it is the probability of zero for the loss lotteries. In other words, risk seeking behavior is observed for gains of low probability and losses of high probability. Analogously, we have risk averion if the probability of the gain is high or the probability of the loss is low. Similarly, our model explains the reflection effect of Kahneman and Tversky (1979) which says that risk attitudes tend to reverse when positive outcomes are replaced with negative ones.

3.2 Event-splitting effects and violations of stochastic dominance

Event-splitting effects are a robust phenomenon showing that splitting an event with an given outcome has systematic impact of choice behavior (cf. Humphrey (1995, 1999, 2001)). More specifically, splitting the probability of a good outcome generally increases the desirability of a lottery whereas the opposite is true if the probability of a bad outcome is split. Note that in our model $2E(p_i/2 + \varepsilon_i) \geq E(p_i + \varepsilon_i)$ for all p_i which means that splitting the probability of an outcome increases the perceived perceived probability of this outcome. Consequently, if we split the probability of a good outcome, the resulting lottery will be prefereed to a lottery in which the probability of a bad outcome is split. This means that our model can explain the violations of stochastic dominance Birnbaum (2005) has observed in an experimental design which precisly utilizes this pattern of event-splitting.

3.3 Common consequence and common ratio effects

The common consequence and common ratio effect of Allais (1953) are the most famous violations of the independence axiom of EU. An example of the common ratio effect is given by the following two lottery pairs: p = (0, 1, 0) and q = (0.2, 0, 0.8) as well as p* =

(0.75, 0.25, 0), and q* = (0.8, 0, 0.2) where X = (0, 3000, 4000). The typical observed pattern is that people choose p over q and q* over p*. Let us assume for convenience that EU(p) = EU(q) and consequently EU(p*) = EU(q*) and note that $EU_c(p)$ = EU(p). Equation (1) implies that $E(EU_c(q)) < EU(q)$ for $\alpha > 0.2$ since in this case $E(\varepsilon_3) < 0$. Thus, subjects are more likely to choose p over q. Moreover, for $\alpha > 0.2$ we have $E(EU_c(q^*)) > EU(q^*)$ since $E(\varepsilon_3) > 0$ now. For $\alpha < 0.25$ $E(EU_c(p^*)) = EU(p^*)$, so due to EU(p*) = EU(q*) people will choose q* more frequently. This conclusion also holds for $\alpha > 0.25$ because of property (ii) in Proposition 1.

The common consequence effect can be described by the lotteries of the classical Allais paradox where p = (0, 1, 0), q = (0.01, 0.89, 0.1), p* = (0.89, 0.11, 0), and q* = (0,9, 0, 0.1) with x = (0, 1 million, 5 million). Let us again assume that EU(p) = EU(q) and consequently EU(p*) = EU(q*). Again, $EU_c(p)$ = EU(p). According to (1), $E(EU_c(q)) = EU(q) + E(\varepsilon_2)(u_2 - u_1) + E(\varepsilon_3)(u_3 - u_1)$. Note that $E(\varepsilon_2) < 0$ and $E(\varepsilon_3) > 0$ for $\alpha > 0.11$. So it may well be that $E(EU_c(q)) < EU(q) = EU_c(p)$. Now $E(EU_c(p^*)) = EU(p^*) + E(\varepsilon_2)(u_2 - u_1)$ and $E(EU_c(q^*)) = EU(q^*) + E(\varepsilon_3)(u_3 - u_1)$. Note that $E(\varepsilon_3) > E(\varepsilon_2)$ since 0.1 < 0.11 and $\alpha > 0.11$. Moreover, $u_3 - u_1 > u_2 - u_1$ so $E(EU_c(q^*)) > E(EU_c(p^*))$.

3.4 Violations of betweenness

Violations of betweenness can be illustrated with the following example due to Prelec (1991) and Camerer and Ho (1994). Consider a lottery pair p = (0.66, 0.34, 0) and q = (0.83, 0, 0.17) as well as a probability mixture m=16/17p+1/17q = (0.67, 0.32, 0.01) where X = (0, 17000, 26000). The typical observed pattern is that people choose p over q and m over p. According to equation (1) p is chosen over q when $EU(p) + \varepsilon(p_2)(u_2 - u_1) > EU(q) + \varepsilon(q_3)(u_3 - u_1)$ and m is chosen over p when $(EU(q) - EU(p))/17 + \varepsilon(m_2)(u_2 - u_1) + \varepsilon(m_3)(u_3 - u_1) > \varepsilon(p_2)(u_2 - u_1)$. Since $q_3<p_2$ and $m_2<p_2$ proposition 1 implies that $E(\varepsilon(q_3))\geq E(\varepsilon(p_2))$ and $E(\varepsilon(m_2))\geq E(\varepsilon(p_2))$. Moreover, $E(\varepsilon(m_2))>0$ when $\alpha > 0.01$. So it may well be that people choose p over q more frequently and at the same time they choose m over p more frequently, which is consistent with experimental evidence.

4 Conclusion

This paper presents a new model of stochastic choice under risk or uncertainty where individuals assess probability information with random errors. Although individuals make perception errors, they realize that the perceived probabilities should be between zero and one and they should also add up to one. This assumption implies that random errors have an upward bias for small probabilities and a downward bias—for large probabilities. Therefore, the proposed model can explain many well-known behavioural anomalies such as the fourfold pattern of risk attitudes, the reflection effect, event-splitting effects, violations of stochastic dominance, common consequence and common ratio effects and violations of betweenness.

References

Allais, M. (1953) “Le Comportement de l’Homme Rationnel devant le Risque: Critique des Postulates et Axiomes de l’École Américaine” *Econometrica* **21**, 503-546.

Ballinger, P. and N. Wilcox (1997) “Decisions, error and heterogeneity” *Economic Journal* **107**, 1090-1105.

Birnbaum, M. H. (2005). A comparison of five models that predict violations of first-order stochastic dominance in risky decision making. *Journal of Risk and Uncertainty, 31,* 263-287.

Blavatskyy, Pavlo (2007) “Stochastic Expected Utility Theory” *Journal or Risk and Uncertainty* **34**: 259-286

Buschena, David and David Zilberman. (2000). “Generalized Expected Utility, Heteroscedastic Error, and Path Dependence in Risky Choice” *Journal of Risk and Uncertainty* **20**, 67-88

Camerer, C. (1989) “An experimental test of several generalized utility theories.” *Journal of Risk and Uncertainty* **2**, 61-104

Camerer, C. and Ho, T. (1994) “Violations of the Betweenness Axiom and Nonlinearity in Probability” *Journal of Risk and Uncertainty* **8**, 167-196

Chew, S., L. Epstein and U. Segal (1991) “Mixture symmetry and quadratic utility” *Econometrica* **59**, 139-163

Gul, F. and Pesendorfer, W. (2006) “Random Expected Utility” *Econometrica* **71(1)**, 121-146

Handa, J. (1977) “Risk, Probabilities, and a New Theory of Cardinal Utility“ *Journal of Political Economy* **85**, 97-122

Harless, D. and C. Camerer (1994) The predictive utility of generalized expected utility theories, *Econometrica* **62**, 1251-1289

Hey, J. (2001) “Does repetition improve consistency?” *Experimental economics* **4**, 5-54

Hey, J. and Carbone, E. (1995) "Stochastic Choice with Deterministic Preferences: An Experimental Investigation" *Economics Letters* **47**, 161-167

Hey, J.D. and C. Orme (1994) Investigating generalisations of expected utility theory using experimental data, *Econometrica* **62**, 1291-1326

Humphrey, S. (1995) “Regret aversion or event-splitting effects? more evidence under risk and uncertainty“ *Journal of Risk and Uncertainty* **11**(3): 263-274

Humphrey, S. (1999). Probability learning, event-splitting effects, and the economic theory of choice. *Theory and Decision, 46,* 51-78.

Humphrey, S. (2001). Are event-splitting effects actually boundary effects? *Journal of Risk and Uncertainty, 22, 7*9-93.

Kahneman, D. and A. Tversky (1979) "Prospect Theory: an Analysis of Decision Under Risk" *Econometrica* **47**, 263-291.

Karmarkar, U. (1978) "Subjectively weighted utility: a descriptive extension of the expected utility model", *Organizational Behavior and Human Performance* **21**, pp. 61–72

Loomes, Graham, Peter Moffatt and Robert Sugden (2002) "A Microeconomic Test of Alternative Stochastic Theories of Risky Choice" *Journal of Risk and Uncertainty* **24**(2): 103-130

Luce, R. D. and Suppes, P. (1965) "Preference, utility, and subjective probability" in R. D. Luce, R. R. Bush & E. Galanter (eds.), *Handbook of mathematical psychology,* Vol. III, 249–410, Wiley, New York NY

Machina, M. (1985) "Stochastic Choice Functions Generated from Deterministic Preferences over Lotteries" *Economic Journal* **95**, 575-594

Prelec, D. (1990) "A 'pseudo-endowment' effect, and its implications for some recent nonexpected utility models" *Journal of Risk and Uncertainty* **3**, 247-259

Sopher, B. and M. Narramore (2000) "Stochastic choice and consistency in decision making under risk: an experimental study" *Theory and Decision* **48**, 323-350

Starmer, Ch. and Sugden, R. (1989) "Probability and juxtaposition effects: An experimental investigation of the common ratio effect." *Journal of Risk and Uncertainty* **2**, 159-178

Tversky, A. and D. Kahneman (1992) "Advances in Prospect Theory: Cumulative Representation of Uncertainty" *Journal of Risk and Uncertainty* **5**, 297-323.

Viscusi, W. K. (1989) "Prospective Reference Theory: Toward an Explanation of the Paradoxes" *Journal of Risk and Uncertainty* **2**, 235-264.

Wu, G. (1994) "An Empirical Test of Ordinal Independence" *Journal of Risk and Uncertainty* **9**, 39-60.

Appendix

Proof of Proposition 1.

(i) If $p_i < \alpha$ then we can use equation (2) to write $E(\varepsilon_i)$ as $E(\varepsilon_i) = \int_{-p_i}^{p_i+\alpha} x dF(x) =$

$$= \frac{\int_{-p_i}^{p_i+\alpha} x dF^*(x)}{F^*(\max\{1-p_i,\alpha\}) - F^*(-p_i)} = \frac{\int_{-p_i}^{2p_i} x dF^*(x) + \int_{2p_i}^{p_i+\alpha} x dF^*(x)}{F^*(\max\{1-p_i,\alpha\}) - F^*(-p_i)}$$. Note that $\int_{-p_i}^{2p_i} x dF^*(x) = 0$

because, by definition, F* is a cdf of a distribution that is symmetric around zero. Since

$\int_{2p_i}^{p_i+\alpha} xdF^*(x) > 0$ and $F^*(\max\{1-p_i,\alpha\}) > F^*(-p_i)$ because F* is strictly increasing, we have $E(\varepsilon_i) > 0$. By analogy we can also show that $E(\varepsilon_i) < 0$ if $p_i > 1-\alpha$.

(ii) Let us consider the case when $p_i < p_j < \alpha$. Using the result from part (i), we can write that $E(\varepsilon_i) > E(\varepsilon_j)$ if $\int_{2p_i}^{p_i+\alpha} xdF^*(x) > \int_{2p_j}^{p_j+\alpha} xdF^*(x)$. Note that $\int_{2p_i}^{p_i+\alpha} xdF^*(x) = \int_{2p_i}^{p_i+p_j} xdF^*(x) + $ $+ \int_{p_i+p_j}^{p_i+\alpha} xdF^*(x) > \int_{p_i+p_j}^{p_i+\alpha} xdF^*(x) > \int_{2p_j}^{p_j+\alpha} xdF^*(x)$. The latter inequality holds because $p_i < p_j$ and function F* is strictly increasing. Therefore, $E(\varepsilon_i) > E(\varepsilon_j)$ if $p_i < p_j < \alpha$. Similarly, we can show that $E(\varepsilon_i) > E(\varepsilon_j)$ if $p_j > p_i > 1-\alpha$. *Q.E.D.*

Kapitel 5

Risk Aversion in Cumulative Prospect Theory

Anmerkungen:

- Dieses Kapitel basiert auf gemeinsamen Arbeiten mit Horst Zank, School of Economics, University of Manchester, UK
- Dieses Kapitels ist bereits veröffentlicht in: U. Schmidt and H. Zank, Risk Aversion in Cumulative Prospect Theory, *Management Science 54* (2008), 208-216.

1 Introduction

Cumulative prospect theory (CPT) has emerged as one of the most prominent alternatives to expected utility (EU). It is widely used in empirical research and, building upon prospect theory (Kahneman and Tversky 1979) and the work of Starmer and Sugden (1989), various axiomatic characterizations of CPT have been proposed (Tversky and Kahneman 1992, Wakker and Tversky 1993, Chateauneuf and Wakker 1999, Zank 2001, Wakker and Zank 2002, Schmidt 2002, Schmidt and Zank 2006). It belongs to a larger family of theories that includes rank-dependent models as well as rank- and sign-dependent models (Quiggin 1981, 1982, Yaari 1989, Schmeidler 1989, Luce 1991, Luce and Fishburn 1991, Luce 2000).

While the rank-dependent utility (RDU) model introduced by Quiggin (1981, 1982) generalizes EU by introducing a weighting function which transforms decumulative probabilities, CPT as well as the gain-loss version of RDU (Luce 1991, Luce and Fishburn 1991, Luce 2000, Luce and Marley 2005), which will be referred to as rank- and sign-dependent utility (RSDU), additionally allow for reference-dependence (utility is defined on deviations from a reference point, i.e., on gains and losses, and not on final wealth positions) and sign-dependence (there exist two separate weighting functions, one transforming probabilities of gains and one transforming probabilities of losses, which do not need to coincide). Due to reference-dependence a decision maker can exhibit loss aversion, which is commonly interpreted as the (marginal) disutility of a given loss being larger than the (marginal) utility of a gain of equal size.[1] Sign-dependence permits separate analyses of probabilistic risk attitudes for gains and for losses.

The additional flexibility gained by reference- and sign-dependence indicates that CPT is able to explain a variety of phenomena that are incompatible with EU. But CPT also has clear descriptive limitations. For example, as with all rank-dependent theories, the model rests on the principle of comonotonic independence (Schmeidler 1989). While earlier studies indicated mixed evidence regarding the descriptive validity of this principle (e.g., Wakker, Erev and Weber 1994), Birnbaum (2004, 2005) and Birnbaum and Navarette (1998) have identified a number of behavioral patterns that refute the rank-dependent formulation. These shortcomings include violations of nontransparent stochastic dominance and coalescing. See also the review of Marley and Luce (2005), who concluded that these violations rule out CPT as a descriptive model.

There are also other aspects of behavior that are problematic for CPT. In Wu, Zhang and Abdellaoui (2005) and L'Haridon (2006) it was demonstrated that the predictions of the original prospect theory model of Kahneman and Tversky (1979) are empirically superior to those of CPT unless one accounts for a certainty effect. Wu and Markle (2004) question the assumption of gain-loss separability underlying CPT (i.e., that the valuation of a mixed prospect is the sum of the separate valuations of the gain part and the loss part of the prospect). Recently, Birnbaum and Bahra (2007) reported systematic violations of gain-loss separability.

In this paper we provide a theoretical account of risk aversion and second-order sto-

[1] The view that loss attitude is captured solely by utility has been questioned in Schmidt and Zank (2005) as being in direct contrast to risk attitude which is usually defined as a behavioral property. They propose a behavioral condition of loss aversion which has been tested in Brooks and Zank (2005).

chastic dominance under CPT. If the cumulative distribution F_P of a prospect P weakly exceeds the cumulative distribution, F_Q, of a second prospect Q, with at least one strict inequality (i.e., $F_P(x) \geq F_Q(x)$ for all outcomes x and $F_P(y) \neq F_Q(y)$ for some outcome y), then Q first-order stochastically dominates P. Second-order stochastic dominance requires more: Q second-order stochastically dominates P if $\int_{-\infty}^{x}[F_P(t) - F_Q(t)]dt \geq 0$ for all x with a strict inequality for some y. Second-order stochastic dominance implies first-order dominance but the reversed implication does not hold in general. The notion of risk aversion used in this paper is equivalent to the notion of second-order stochastic dominance if CPT is assumed. This relationship is elaborated below. We follow the traditional approach of assuming a general model for decision under risk to which behavioral risk properties are added. This is not typically done so for CPT. Hence, to start with, we need to clarify two points. First, we need to specify the general version of CPT that we assume, and second, we need to clarify which behavioral property of risk aversion we adopt. We do this under two separate headings.

General Utility and Probability Weighting. One observes that in the literature the CPT-model is often referred to with specific restrictions for the shape of utility and probability weighting functions, so that a complete account of risk attitudes is obtained. Besides loss aversion, one aspect of risk attitudes refers to the observed behavior for gains and separately observed behavior for losses. Utility for gains is usually found to be concave in line with the hypothesis of diminishing sensitivity (Kahneman and Tversky 1979). For losses there are fewer empirical studies and the evidence is less clear-cut, however, often a convex shape for utility or a linear utility is suggested (e.g., Fennema and van Assen 1999, Abdellaoui, Bleichrodt and Paraschiv 2007). Although there are individual differences, there is consistent evidence that probabilities are transformed according to an inverse S-shaped weighting function, irrespective of whether the probabilities refer to gains or to losses (e.g., Preston and Baratta 1948, Wu and Gonzalez 1996, Abdellaoui 2000, Bleichrodt and Pinto 2000, Abdellaoui, Vossmann and Weber 2005). Combining all these features, Tversky and Kahneman (1992) put forward the hypothesis of the "four-fold pattern of risk attitude," with the implications of risk seeking over small probability gains and high-probability losses, and risk aversion for high-probability gains and small-probability losses.

In this paper we drop all restrictions on the shape of the utility and weighting functions as is done in RSDU, and adopt a very general form of CPT. We want to derive utility curvature and the shape of probability weighting functions from additional preference conditions concerning risk behavior. So, the assumptions on utility, which is defined on deviations from the reference point, are those of continuity and strict monotonicity. The weighting functions are also strictly increasing but are not assumed continuous. Note therefore, that, with respect to the first point raised above, we make no additional assumptions beyond those needed to derive CPT from preference axioms (see Chateauneuf and Wakker 1999).

Risk Behavior. Next we clarify the behavioral property of risk aversion that we adopt in this paper. One way to define risk aversion is in the weak sense. Weak risk aversion holds if the expected value of a prospect for certain is always preferred to the prospect itself. Another possibility is to define risk aversion in the strong sense. According to Rothschild and Stiglitz (1970), an individual exhibits strong risk aversion if she or he always dislikes mean-preserving spreads in risk. To give a simple example of a mean-preserving spread, suppose a prospect yields with some positive probability, the outcome y. Then, an elementary mean preserving spread of that prospect is a new prospect where the outcome y is split up into two distinct outcomes, say x and z, with $x > y > z$ and corresponding likelihoods such that the expected value (or mean) of the original prospect is preserved. Such an elementary mean-preserving spread leads to an increase in risk and variance. A mean-preserving spread results from a finite sequence of elementary mean-preserving spreads. In the framework adopted in this paper, aversion to elementary mean-preserving spreads implies aversion to any mean-preserving spread, and hence strong risk aversion.[2]

At this point it is worth noting that, due to the monotonicity constraints on utility and weighting functions adopted in this paper, strong risk aversion comes down to second-order stochastic dominance. In this sense, and summarizing the two points outlined before, we provide a theoretical analysis of strong risk aversion (or second-order stochastic dominance) for general CPT-preferences. Recall, that for other well-known theories of decision under risk such analyses have already been provided. For example it is well-known that under EU the curvature of the utility function captures all information concerning risk attitudes, so that under EU weak and strong risk aversion lead to the same implications (i.e., utility is concave).

Because the modern decision models capture more general preferences than can be accommodated under EU, a distinction between the different forms of risk aversion has emerged as meaningful. An important result towards this finding has been provided by

[2]Since mean-preserving spreads involve splitting of consequences, empirical tests of strong risk aversion may be problematic if coalescing is violated. The studies of Birnbaum (2004, 2005) and Birnbaum and Navarette (1998) indicate that splitting good consequences tends to increase the attractiveness of a lottery while splitting bad consequences tends to decrease the attractiveness. In empirical studies it may be difficult to disentangle these effects from the effects of strong risk aversion.

Chew, Karni and Safra (1987). They showed that strong risk aversion is satisfied within the RDU model if and only if the utility function is concave and the weighting function is convex. That result has been a major step in understanding the relationship between risk perception modelled through the curvature of the utility function and risk perception modelled through the curvature of the probability weighting function.

It is important to recall that nearly all theoretical applications of RDU and EU assume strong risk aversion. Although RDU is a very popular decision model, it does not perform well empirically because more flexibility is needed to accommodate for reference- and sign-dependence in addition to rank-dependence, as is done in the model of Luce and Fishburn (1991) and in CPT. Given the central role of strong risk aversion for the analysis of risk attitudes and the central role of the latter models for empirical research, it is important to have a clear understanding of the consequences of strong risk aversion in those theories. Because CPT, the model that we concentrate on in this paper, incorporates two weighting functions and a reference-dependent utility, the consequences of strong risk aversion for the curvature of those functions are not obvious. Further, it is unclear what relationship holds between strong risk aversion and loss aversion. Providing an answer to these questions is the main goal of this paper.

It is well-known that if we drop reference- and sign-dependence, then CPT reduces to RDU. If we consider prospects that involve only gains (only losses), then the CPT expression reduces to an RDU expression with utility defined over gains (losses). In these cases, we should therefore expect similar results as those derived by Chew, Karni and Safra (1987). Indeed, our results confirm that the conditions for strong risk aversion under CPT and RDU coincide in those cases, i.e., utility is concave for gains and also concave for losses and the weighting function for gains (losses) is convex (concave). Consequently, the implications of strong risk aversion for RDU and CPT are at least partly in conflict with the empirical results on the shape of the utility and weighting functions discussed above. Note that, in contrast to previous studies, we derive our results without any prior assumption of differentiability for the utility function or the weighting functions. While this finding shows that the assumption of a differentiable utility is immaterial for the result of Chew, Karni and Safra (1987), it seems natural not to assume differentiability in the context of CPT because a non-differentiable utility at the reference point is characteristic for the latter model. In our proofs differentiability[3] is derived entirely from preference conditions, in particular monotonicity and strong risk aversion.

That our results apply to more general preferences can also be inferred from the fact that we allow for discontinuous weighting functions in agreement with most axiomatizations of general CPT-preferences. Results of empirical studies indicate that discontinuities of the weighting functions seem plausible for probabilities of 0 and 1. There is also theoretical interest in models that have discontinuous weighting functions at 0 and 1 but which are are continuous elsewhere (e.g., Bell 1985, Chateauneuf, Eichberger and Grant 2005, and see also Birnbaum and Stegner 1981). In our approach, continuity of the weighting functions, except at 1 for the gain weighting function and at 0 for the loss weighting function, is obtained as a consequence of convexity (concavity) together with monotonicity of the gain (loss) weighting function.

Our results show that dispensing of continuity for the weighting functions has consequences for the overall shape of utility. Utility does not need to be concave over the entire

[3]Our conditions do not imply full differentiability of the utility function over the entire domain. Utility will be differentiable allmost everywhere, that is, there can be a countable set of outcomes where utility is not differentiable.

domain because at the reference point non-concavity may occur. So, in extreme cases strong risk aversion and convex utility (that is, one that is linear for losses and linear but steeper for gains) may coexist under CPT. From a theoretical point of view, this result is noteworthy since Chateauneuf and Cohen (1994) tried to derive the coexistence of risk aversion and convex utility under RDU. They have shown that RDU is compatible with convex utility only in the presence of weak risk aversion, while strong risk aversion forces utility to be concave everywhere. Our results here confirm their point more generally because we dispense of continuity of the weighting functions and of differentiability assumptions for utility.

A further consequence of the discontinuous weighting functions points to a particular relationship between strong risk aversion and loss aversion. Strong risk aversion is compatible with gain seeking (i.e., the opposite of loss aversion) while utility is concave if and only if loss aversion holds. Because of the implied concavity of utility for gains and concavity for losses and the convexity/concavity constraints for the weighting functions, the relationship between strong risk aversion and loss aversion is characterized by the ratio of the left and right derivative of the utility function at the reference point together with a ratio of gain and loss decision weights. Theoretical arguments have motivated Köbberling and Wakker (2005) to propose the former utility ratio as index of loss aversion. Our results support their measure since in our framework this index of loss aversion arises naturally and entirely from preference conditions. The ratio between the gain and loss decision weights has not appeared in the literature before and we will employ it to define an index of probabilistic loss aversion. This index, which we discuss in more detail later in the paper, appears to be related to the distinct weighting of probabilities induced by sign-dependence. When this latter index is at least 1, as is the case for RDU-preferences, then loss aversion must hold under strong risk aversion. As a consequence an overall concave utility is then obtained.

The paper is organized as follows. In the next section we present the CPT-model and derive our main results. Section 3 considers the consequences of strong risk aversion for the popular variant of CPT with power utility. All proofs are presented in the Appendix.

2 Cumulative Prospect Theory and Risk Aversion

In this section we recall cumulative prospect theory for decision under risk. A preference relation, $\succcurlyeq$, over prospects is assumed. A *prospect* is a finite probability distribution over the set of monetary outcomes (here identified with the set of real numbers, $\mathbb{R}$). It is represented by $P := (p_1, x_1; \dots; p_n, x_n)$ meaning that probability p_j is assigned to outcome x_j, for $j = 1, \dots, n$. The probabilities p_j are nonnegative and sum to one. With this notation we implicitly assume that outcomes are ranked in decreasing order, i.e., $x_1 \geq \cdots \geq x_n$. The reference point or *status quo* is 0. Outcomes are interpreted as changes from the status quo. Positive outcomes are *gains* and negative outcomes are *losses*.

Cumulative Prospect Theory (*CPT*) holds if prospects are evaluated by the following functional:

$$CPT(p_1, x_1; \dots; p_n, x_n) = \sum_{j=1}^{n} \pi_j U(x_j),$$

with utility U and decision weights $\pi_j, j = 1, \dots, n$, explained next.

Utility assigns to each outcome a real value and is strictly increasing and continuous. For convenience $U(0) = 0$ is set, which constrains the otherwise cardinal utility (i.e., unique up to positive scale and location) to a be a ratio scale (i.e., unique up to positive scale). Differentiability of the utility function is not required. In particular, all CPT models in the literature allow for non-differentiability at 0 in order to model loss aversion.

The decision weights are generated by probability weighting functions w^+ and w^-, which map the interval $[0,1]$ into itself, are strictly increasing, and assign 0 to 0 and 1 to 1. Discontinuous weighting functions are explicitly allowed. For $(p_1, x_1; \ldots ; p_n, x_n)$, with $k \in \{0, \ldots, n\}$ such that $x_1 \geq \cdots \geq x_k \geq 0 > x_{k+1} \geq \cdots \geq x_n$ ($k = 0$ means there are only losses, and $k = n$ that there are no losses), the decision weights are defined as

$$\pi_j = \begin{cases} w^+(p_1 + \cdots + p_j) - w^+(p_1 + \cdots + p_{j-1}), & \text{if } j \leq k, \\ w^-(p_j + \cdots + p_n) - w^-(p_{j+1} + \cdots + p_n), & \text{if } j > k, \end{cases}$$

where we follow the common convention that $\sum_{i=j}^{j-1} p_i = 0$. Under CPT the weighting functions are uniquely determined.

The CPT functional presented above takes a rank-dependent utility (RDU) form if the gain weighting function equals the dual of the loss weighting function, i.e., if $w^+(p) = 1 - w^-(1-p) =: w(p)$ for all $p \in [0,1]$. This excludes sign-dependence. In general RDU-preferences have been modeled without reference-dependence, and then utility is defined over final wealth positions instead of changes in wealth as done for CPT and in this paper. Nevertheless, the results that we derive below have important implications for general RDU as well.

The special case of RDU-preferences when rank-dependence is irrelevant gives expected utility (EU). Recall that under expected utility, where there is no flexibility in modeling distortions in probabilities (i.e., $w(p) = p$ for all $p \in [0,1]$), one can only attempt to capture the phenomenon of risk aversion in terms of constraints, usually concavity, of the utility function. Since the flow of alternatives to expected utility theory different notions of risk aversion have been proposed and analyzed. The most prominent one, introduced by Rothschild and Stiglitz (1970), defines risk aversion as aversion to mean-preserving spreads, and is referred to as *strong risk aversion*. Formally, $\succcurlyeq$ exhibits strong risk aversion if for all prospects $P = (p_1, x_1; \dots; p_n, x_n)$ and $\delta > 0$ it follows that

$$P \succcurlyeq (p_1, x_1; \dots; p_i, x_i + \frac{\delta}{p_i}; \dots; p_j, x_j - \frac{\delta}{p_j}; \dots; p_n, x_n),$$

whenever $p_i, p_j > 0$. Note that, due to our notation, δ must be chosen such that the rank-ordering of outcomes is maintained. The implications of strong risk aversion for CPT are as follows:

THEOREM 1 *Suppose that cumulative prospect theory holds. Then the following two statements are equivalent:*

1. *Strong risk aversion holds.*

2. *The gain weighting function w^+ is convex and the loss weighting function w^- is concave. The utility function is concave for losses and also concave for gains. Further, the following relationship is satisfied:*

$$[w^-(q+s) - w^-(s)][U(y) - U(y - \frac{\delta}{q})] \geq [w^+(r+p) - w^+(r)][U(x + \frac{\delta}{p}) - U(x)] \tag{1}$$

for all $x \geq 0 \geq y$, all probabilities p, q, r, s such that $r + p + q + s \leq 1$, $p, q > 0$, and all $\delta > 0$. □

Before proceeding we present some important implications of Statement 2 of Theorem 1, in particular, we reformulate inequality (1) using the properties of utility and the weighting functions. This is done in several stages below.

First, we note that, as a consequence of strict monotonicity and convexity, the gain weighting function w^+ is continuous on the half-open interval $[0, 1[$, but in general may have a jump at 1. Therefore, the left and right derivatives, $w^{+\prime}(p_-)$ respectively $w^{+\prime}(p_+)$, at any probability $0 < p < 1$ exist and are positive (here p_+ and p_- indicate that p is approached from right and left, respectively), and also the right derivative at 0 exists. Similarly, the loss weighting function is continuous on $]0, 1]$, but may have a jump at 0, its left and right derivatives at any probability $0 < p < 1$ exist and are positive, and also its left derivative at 1 exists.

Second, as utility is concave for losses and concave for gains, it follows that left and right derivatives of utility at any outcome unequal to 0 exist and are positive. Further, the left and right derivatives of utility at 0 exists. That the right derivative at 0, $U'(0_+)$, is bounded follows from inequality (1).

Next we discuss the implications of the previously mentioned concavity and convexity restrictions for inequality (1), which we have rewritten in the equivalent form:

$$\frac{U(y) - U(y - \frac{\delta}{q})}{U(x + \frac{\delta}{p}) - U(x)} \geq \frac{w^+(r+p) - w^+(r)}{w^-(q+s) - w^-(s)}, \tag{2}$$

for all $x \geq 0 \geq y$, all probabilities p, q, r, s such that $r + p + q + s \leq 1$, $p, q > 0$, and all $\delta > 0$. Concavity implies that the difference $[U(y) - U(y - \frac{\delta}{q})]$ increases as y becomes smaller, hence, is smallest at $y = 0$. Similarly, $[U(x + \frac{\delta}{p}) - U(x)]$ is largest at $x = 0$. Taking limits when δ approaches 0, and using the fact that the left and right derivative of utility at 0 are bounded, implies that

$$\frac{U'(0_-)}{U'(0_+)} \geq \frac{[w^+(r+p) - w^+(r)]/p}{[w^-(q+s) - w^-(s)]/q}, \tag{3}$$

for all probabilities p, q, r, s such that $r+p+q+s \leq 1$, $p, q > 0$. Because utility is concave (except that at 0 it can be non-concave), this latter inequality is equivalent to inequality (2). Note also that the ratio $U'(0_-)/U'(0_+)$ is positive because one can always find some probabilities p, q, r, s with $r + p + q + s \leq 1$, $p, q > 0$, such that the ratio on the right in (3) is positive. This follows directly from strict monotonicity of the weighting functions. Note that the latter term is bounded from above because $U'(0_-)/U'(0_+)$ is finite-valued.

Next we look at the consequences of convexity/concavity of the weighting functions. In contrast to the results for utility it is not possible to identify a probability where the ratio on the right in (3) is maximal. Convexity of w^+ implies that for any probability $r > 0$ we have $[w^+(r-\varepsilon) - w^+(r)]/\varepsilon \leq w^{+\prime}(r_-)$, and concavity of w^- implies that for any probability $s > 0$ we have $[w^-(s+\varepsilon) - w^-(s)]/\varepsilon \geq w^{-\prime}(s_+)$, for small $\varepsilon > 0$. Hence, (3) is equivalent to:

$$\frac{U'(0_-)}{U'(0_+)} \geq \frac{w^{+\prime}((r+p)_-)}{w^{-\prime}((s+q)_+)},$$

for all positive probabilities r, p, q and s such that $r + p + s + q \leq 1$.[4] If $r + p + s + q < 1$, then $w^{+\prime}((r+p)_-)$ being non-decreasing (convexity) and $w^{-\prime}((s+q)_+)$ being non-increasing (concavity) imply that $w^{+\prime}((r+p)_-)/w^{-\prime}((s+q)_+)$ can potentially be improved by increasing either r or s. Therefore, we obtain the equivalent relation

$$\frac{U'(0_-)}{U'(0_+)} \geq \sup_{r \in]0,1[} \frac{w^{+\prime}(r_-)}{w^{-\prime}(1-r_-)}. \tag{4}$$

This shows that in Theorem 1 we can replace inequality (1) with inequality (4).

Next, we discuss the expression in inequality (4). It is interesting to note that the ratio $U'(0_-)/U'(0_+)$ provides information on loss aversion. Given the concavity constraints on utility and because in this paper loss aversion is interpreted as utility being steeper for losses than for corresponding gains (i.e., $U(-z) - U(-z-\delta) > U(z+\delta) - U(z)$ for all $z \geq 0, \delta > 0$), loss aversion does only hold if $U'(0_-)/U'(0_+) \geq 1$. This measure

[4]Whenever we use the notation $w^{+\prime}((r+p)_-)/w^{-\prime}(s+q)_+)$ we implicitly assume that this ratio of derivatives is well defined.

of loss aversion was already proposed by Benartzi and Thaler (1995) and formalized by Köbberling and Wakker (2005). The latter authors denote the ratio $U'(0_-)/U'(0_+)$ as index of loss aversion and employ it in order to compare the degree of loss aversion between different individuals. In the present paper this measure of loss aversion arises in a natural way, derived solely from preference conditions.

Requiring $U'(0_-)/U'(0_+) \geq 1$ in Theorem 1 gives the following corollary.

COROLLARY 2 *Assume that CPT holds and that strong risk aversion is satisfied. Then, the utility function is concave if and only if loss aversion holds.* $\square$

The following example illustrates that in general loss aversion need not hold.

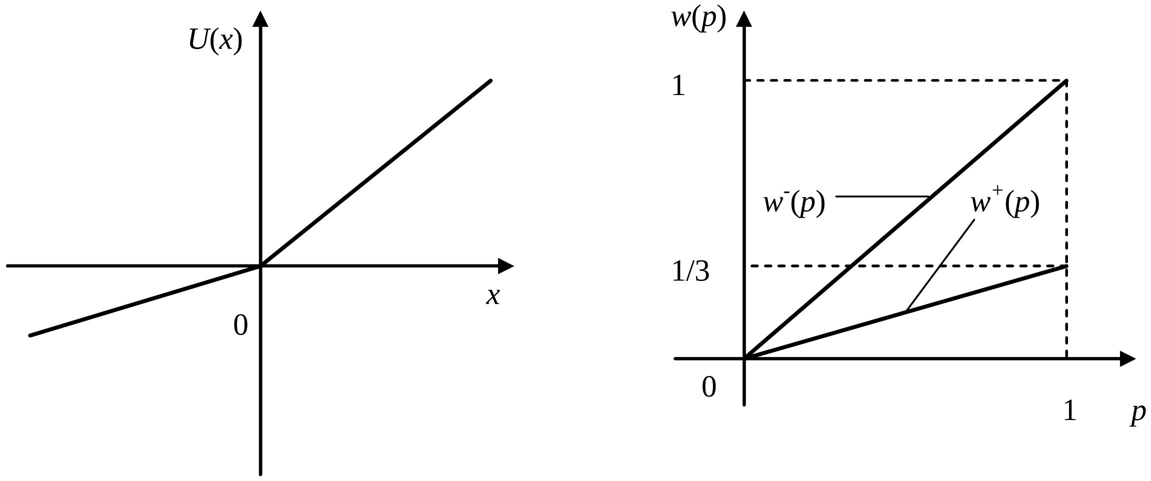

Figure 1: Convex Utility and Strong Risk Aversion under CPT.

EXAMPLE 1: Suppose CPT holds with $U(x) = x$ for $x \geq 0$; $U(y) = y/2$ for $y < 0$; $w^-(p) = p$ for $p \in [0,1]$; and $w^+(p) = p/3$ for $0 \leq p < 1$ and $w^+(1) = 1$ (Figure 1 illustrates these functions). Then, all conditions in statement 2 of Theorem 1 are satisfied but utility is strictly convex at 0. This may occur also more generally if the weighting function for gains is everywhere flatter than the weighting function for losses.

Recall that, when CPT evaluates a prospect where the smallest gain x has probability $0 < \varepsilon < 1$, $U(x)$ is multiplied by the decision weight $w^+(r) - w^+(r - \varepsilon)$, where r indicates the probability of getting an outcome at least as good as x. Replacing x with a loss y such that the rank-order of all other outcomes is maintained leads to a prospect where in the CPT evaluation $U(y)$ is multiplied by the decision weight $w^-(1-r+\varepsilon)-w^-(1-r)$. Note that in Example 1 the decision weight of the loss y always dominates the decision weight of the gain x for any probability $0<\varepsilon<1$. This can be viewed as *probabilistic loss aversion,* analogously to the concept of probabilistic risk aversion (Wakker 1994). Formally, probabilistic loss aversion can be defined as $w^-(1-r+\varepsilon)-w^-(1-r) > w^+(r)-w^+(r-\varepsilon)$ for all $0<\varepsilon<r\leq 1$. Here this is equivalent to $w^{-\prime}(1-r_-)/w^{+\prime}(r_-)>1$ for all $0\leq r\leq 1$. Therefore, this ratio can be interpreted as degree of probabilistic loss aversion at cumulative probability r. Analogously, $w^{+\prime}(r_-)/w^{-\prime}(1-r_-)$ can be interpreted as degree of probabilistic gain seeking. Given this interpretation, our results imply that strong risk aversion holds if the degree of loss aversion is not less than the supremum of probabilistic gain seeking.

Two important special cases which imply loss aversion and therefore that utility is overall concave are mentioned next. A first case is that of RDU. Recall that in this paper this is the case of CPT with $w^+(p) = 1-w^-(1-p) =: w(p)$ for all $p\in[0,1]$. This case is important because it shows that the results of Chew, Karni and Safra (1987) and Chateauneuf and Cohen (1994) follow directly from Theorem 1, the latter being formulated under less restrictive conditions for utility and weighting function than those earlier results. Obviously, when RDU holds with convex weighting function w, then we have

$$\sup_{r\in]0,1[}\frac{w^{+\prime}(r_-)}{w^{-\prime}(1-r_-)}=\sup_{r\in]0,1[}\frac{w'(r_-)}{w'(r_+)}\geq 1.$$

Consequently, we have the following corollary of Theorem 1:

Corollary 3 *Assume that RDU holds. Then strong risk aversion is satisfied if and only if the utility function is concave and the weighting function is convex.* □

A second special case that implies loss aversion is when both weighting functions are assumed continuous on the entire probability interval $[0, 1]$, that is jumps at 0 for w^- or at 1 for w^+ are excluded. In that case $\sup_{r\in(0,1)}[w^{+\prime}(r_-)/w^{-\prime}(1-r_-)]$ will be at least 1, and, therefore, utility must be concave for all outcomes. The arguments used to prove this become more transparent if one compares the gain weighting function with the dual of the loss weighting function, $\tilde{w}^-(p) := 1 - w^-(1-p)$, which is convex and continuous on $[0, 1]$. Because of continuity and convexity on $[0, 1]$ there exists some $p \in]0, 1[$ with $\tilde{w}^{-\prime}(p_+) \leq w^{+\prime}(p_-)$, or equivalently $w^{-\prime}(1-p_-) \leq w^{+\prime}(p_-)$. This then implies that $\sup_{r\in(0,1)}[w^{+\prime}(r_-)/w^{-\prime}(1-r_-)] \geq 1$, hence, loss aversion.

The preceding analysis shows that, while under RDU assuming a continuous or discontinuous weighting function is irrelevant for the shape of the utility function, for CPT the assumption of continuous weighting functions is crucial and forces utility to be concave. We conclude this section with the following result.

Corollary 4 *Suppose that CPT holds and that strong risk aversion is satisfied. Further, assume that the weighting functions are continuous on* $[0, 1]$*. Then, gain seeking behavior is excluded, i.e., the utility function is concave.* □

3 Parametric Utility

The interest in a particular parametric form for utility has lead to corresponding derivations of CPT. We conclude this paper with a discussion on the implications of strong risk aversion for CPT-preferences with power utility, that is, when

$$U(x) = \begin{cases} \sigma^+ x^\alpha, & \text{with } \sigma^+ > 0, \alpha > 0, \text{ for all } x \geq 0, \\ -\sigma^- |x|^\beta, & \text{with } \sigma^- > 0, \beta > 0, \text{ for all } x < 0. \end{cases}$$

This utility has been proposed by Tversky and Kahneman (1992) and is the most used parametric form in empirical and theoretical applications (many references are given in Wakker and Zank 2002). However, this variant of CPT does not fit well with strong risk aversion. To illustrate this, note that utility being concave for both gains and losses implies $\beta \geq 1 \geq \alpha$. As mentioned in the analysis following Theorem 1, a further implication of strong risk aversion is that the right derivative of utility at 0 is bounded. Hence, $\alpha = 1$ follows. Similarly, because $U'(0_-)/U'(0_+) > 0$ (recall the remarks following inequality (3)) $\beta = 1$ follows. Consequently, utility must be linear for gains and linear for losses, and the resulting utility index of loss aversion is given by $\lambda = \sigma^-/\sigma^+$.

The preceding analysis shows that strong risk aversion and CPT exclude power utility. More generally, the ratio $U'(0_-)/U'(0_+)$ is not well defined for power utility under CPT unless the two powers are equal. Related to this point are also the arguments put forward by Köbberling and Wakker (2005, Section 7), who showed that power utility is problematic for the index of loss aversion. They suggested an alternative parametric form for utility under CPT, namely a two-sided exponential utility (see Zank 2001 for a preference foundation). Also Luce (2000) gives an argument for exponential utility.

4 Appendix: Proofs

Proof of Theorem 1: In the theorem it is assumed that CPT holds. Let us first assume Statement 1 and derive Statement 2. Suppose strong risk aversion holds. In what follows we prove that if the outcomes of a prospect are all gains or the status quo then utility must be concave and the weighting function w^+ convex. Then a similar result is derived for the case that all outcomes of a prospect are losses or the status quo: utility is again concave and the weighting function w^- is also concave. In both cases we cannot rely on results from the literature as our assumptions are weaker: we do not assume differentiability of the utility function, rather we derive a certain degree of differentiability from preference conditions. Further we do not assume continuity of the weighting functions. The final step in the derivation of Statement 2 is to consider mixed prospects, that is, a prospect containing a gain and a loss each with positive probability.

First we consider the prospect $P := (p_1, x_1; \ldots; p_i, x_i; p_{i+1}, x_{i+1}; \ldots; p_n, x_n)$ with outcomes x_i, x_{i+1} of the same sign that have positive probabilities. We consider an elementary mean-preserving spread of P

$$(p_1, x_1; \ldots; p_i, x_i + \frac{\delta}{p_i}; p_{i+1}, x_{i+1} - \frac{\delta}{p_{i+1}}; \ldots; p_n, x_n)$$

for $0 < \delta$ such that the rank and sign of the outcomes $x_i + \delta/p_i$, $x_{i+1} - \delta/p_{i+1}$ agree with the rank and sign of x_i, x_{i+1}, respectively. Strong risk aversion implies that

$$(p_1, x_1; \ldots; p_i, x_i + \frac{\delta}{p_i}; p_{i+1}, x_{i+1} - \frac{\delta}{p_{i+1}}; \ldots; p_n, x_n) \preccurlyeq P$$

for all such $0 < \delta$. After elimination of common terms, substitution of CPT gives

$$\pi_i[U(x_i + \frac{\delta}{p_i}) - U(x_i)] \leq \pi_{i+1}[U(x_{i+1}) - U(x_{i+1} - \frac{\delta}{p_{i+1}})].$$

In particular, for all $p \in]0, 1/2]$, by choosing $p_i = p_{i+1} = p$, we get

$$\pi_i[U(x_i + \delta) - U(x_i)] \leq \pi_{i+1}[U(x_{i+1}) - U(x_{i+1} - \delta)],$$

for all $x_i \geq x_{i+1}$ and all appropriate $\delta > 0$, or equivalently,

$$\frac{\pi_i}{\pi_{i+1}} \leq \frac{[U(x_{i+1}) - U(x_{i+1} - \delta)]}{[U(x_i + \delta) - U(x_i)]}. \tag{5}$$

Observe that the left hand side of this inequality is independent of x_i, x_{i+1}, and δ, and similarly the right hand side is independent of $p \in]0, 1/2]$. These facts will be exploited in what follows.

Step 1: Outcomes in P are gains or they are equal to the status quo. Suppose that there exists $x_i \geq x_{i+1} > 0$ and $\delta > 0$ such that $U(x_{i+1}) - U(x_{i+1} - \delta) \geq U(x_i + \delta) - U(x_i)$. Note that U is strictly increasing, hence these utility differences are all positive. It then follows from (5) that

$$\frac{\pi_i}{\pi_{i+1}} \leq 1.$$

Suppose now that there do not exist $x_i \geq x_{i+1} > 0$ and $\delta > 0$ such that $U(x_{i+1}) - U(x_{i+1} - \delta) \geq U(x_i + \delta) - U(x_i)$. Then for all $x_i \geq x_{i+1} > 0$ and all small enough $\delta > 0$ it follows that $U(x_{i+1}) - U(x_{i+1} - \delta) < U(x_i + \delta) - U(x_i)$. This implies that U is a strictly convex function on the domain of gains. Hence, U is differentiable everywhere except on a countable subset of $I\!R_+$. Further, left and right derivatives exist at any outcome in $I\!R_{++}$ and the right derivative at the status quo exists. Moreover, all derivatives are positive. We can therefore choose $x_i \geq x_{i+1} > 0$ such that $U'(x_i)$ and $U'(x_{i+1})$ exist. It follows that

$$\begin{aligned}
\lim_{\delta \to 0} \frac{U(x_{i+1}) - U(x_{i+1} - \delta)}{U(x_i + \delta) - U(x_i)} &= \lim_{\delta \to 0} \frac{[U(x_{i+1}) - U(x_{i+1} - \delta)]/\delta}{[U(x_i + \delta) - U(x_i)]/\delta} \\
&= \frac{U'(x_{i+1})}{U'(x_i)}.
\end{aligned}$$

Further, by letting x_{i+1} converge to x_i we get

$$\lim_{x_{i+1} \to x_i} \lim_{\delta \to 0} \frac{U(x_{i+1}) - U(x_{i+1} - \delta)}{U(x_i + \delta) - U(x_i)} = \lim_{x_{i+1} \to x_i} \frac{U'(x_{i+1})}{U'(x_i)} = 1.$$

Finally, from formula (5),

$$\frac{\pi_i}{\pi_{i+1}} \leq 1$$

has been derived for this case. Hence, more generally, $\pi_i/\pi_{i+1} \leq 1$ must hold for any $p \in]0, 1/2]$. Substitution of the weighting function w^+ for the decision weights in the latter inequality gives

$$w^+(\sum_{j=1}^{i-1} p_j + p) - w^+(\sum_{j=1}^{i-1} p_j) \leq w^+(\sum_{j=1}^{i-1} p_j + 2p) - w^+(\sum_{j=1}^{i-1} p_j + p),$$

for all $p \in]0, 1/2]$, or equivalently that the weighting function w^+ is convex.

Convexity and monotonicity are sufficient to show that the weighting function w^+ is continuous on $[0, 1[$. Recall that a monotonic continuous function is differentiable almost everywhere in its domain. In addition the weighting function w^+ is convex, and this means w^+ is differentiable except on a countable subset of $[0, 1]$, and that the left and right derivatives at each point in $]0, 1[$ exists as well as the right derivative at 0. Discontinuity of the weighting function at 1 cannot be excluded, hence, the left derivative of the weighting function at 1 may not be defined (that is, this left derivative may be infinite).

The final part of this step is to show that the utility function is in fact concave. Recall that for all $x_i \geq x_{i+1} > 0$, all small enough $\delta > 0$, and all $p \in]0, 1/2]$,

$$\frac{\pi_i}{\pi_{i+1}} \leq \frac{[U(x_{i+1}) - U(x_{i+1} - \delta)]}{[U(x_i + \delta) - U(x_i)]}$$

holds. Moreover, the right hand side is independent of p. Hence,

$$\sup_{p \in]0,1[} \frac{\pi_i}{\pi_{i+1}} \leq \frac{[U(x_{i+1}) - U(x_{i+1} - \delta)]}{[U(x_i + \delta) - U(x_i)]}.$$

Note that $\sup_{p \in]0,1[} \pi_i/\pi_{i+1} = 1$ because w^+ is convex and therefore differentiable at some $p \in]0, 1[$. Hence,

$$1 \leq \frac{[U(x_{i+1}) - U(x_{i+1} - \delta)]}{[U(x_i + \delta) - U(x_i)]}$$

for all $x_i \geq x_{i+1} > 0$ and all small enough $\delta > 0$, implying concavity of U. Consequently, U is differentiable on $I\!R_+$, except on a countable subset. We conclude that the left and right derivatives of the utility function at each point $x > 0$ exist and in particular the right derivative at the status quo is well defined. This completes the proof of Step 1.

Step 2: Outcomes in P are losses or they are equal to the status quo. Similar arguments to those used in Step 1 apply to U on $I\!R_-$, and to w^-, respectively. Therefore, we can conclude that the utility function is concave for losses and that the weighting function w^- is concave. The latter follows from the fact that

$$\frac{\pi_i}{\pi_{i+1}} \leq 1$$

is equivalent to

$$w^-(2p + \sum_{j=i+2}^{n} p_j) - w^-(p + \sum_{j=i+2}^{n} p_j) \leq w^-(p + \sum_{j=i+2}^{n} p_j) - w^-(\sum_{j=i+2}^{n} p_j),$$

for all $p \in]0, 1/2]$.

Moreover, w^- is continuous on $]0, 1]$ and differentiable except on a countable subset of $]0, 1[$. The left and right derivative of w^- exists at each point in $]0, 1[$, and also the left derivative of w^- at 1 is well defined. The right derivative at 0 may not be defined. Similarly to the previous step, we conclude that the utility function U is differentiable on $I\!R_-$, except on a countable set, and that the left and right derivative exists at each $x < 0$. In addition, because in inequality (5) the right hand side term is bounded from below, the left derivative of U at 0 is well defined (but in general it may not agree with the right derivative established in step 1 of this proof, so we may have a kink at 0).

Step 3: Some outcomes in P can be gains and some can be losses. This step will focus entirely on the derivation of the inequality in Statement 2 of the theorem as the remaining statements have been established. Suppose we have a prospect

$P = (p_1, x_1; \ldots; p_k, x_k; p_{k+1}, x_{k+1}; \ldots; p_n, x_n)$, where $x_{k-1} > x_k \geq 0 \geq x_{k+1} > x_{k+2}$ and $p_k, p_{k+1} > 0$. Then strong risk aversion implies

$$(p_1, x_1; \ldots; p_k, x_k + \frac{\delta}{p_k}; p_{k+1}, x_{k+1} - \frac{\delta}{p_{k+1}}; \ldots; p_n, x_n) \preccurlyeq P,$$

for all small enough $\delta > 0$ such that the rank of $x_k + \delta/p_k$, $x_{k+1} - \delta/p_{k+1}$ is k, respectively $k+1$. Substitution of CPT gives

$$\pi_k[U(x_k + \frac{\delta}{p_k}) - U(x_k)] \leq \pi_{k+1}[U(x_{k+1}) - U(x_{k+1} - \frac{\delta}{p_{k+1}})].$$

Therefore, for any such probabilities $p_k, p_{k+1} > 0$ and any appropriately small $\delta > 0$ the following must be satisfied:

$$\frac{U(x_k + \frac{\delta}{p_k}) - U(x_k)}{U(x_{k+1}) - U(x_{k+1} - \frac{\delta}{p_{k+1}})} \leq \frac{w^-(\sum_{j=k+1}^n p_j) - w^-(\sum_{j=k+2}^n p_j)}{w^+(\sum_{j=1}^k p_j) - w^+(\sum_{j=1}^{k-1} p_j)}.$$

This inequality needs to be satisfied for all probabilities $p_1, \ldots, p_n$, all $x_k \geq 0 \geq x_{k+1}$ and any small enough $\delta > 0$. Hence, inequality (1) follows. This concludes the proof of Statement 2.

Let us now assume that Statement 2 holds. Take any prospect $P = (p_1, x_1; \ldots; p_n, x_n)$ and let $1 \leq i < j \leq n$ and $p_i, p_j > 0$. Define the prospect

$$P(\delta) := (p_1, x_1; \ldots; p_i, x_i + \frac{\delta}{p_i}; \ldots; p_j, x_j - \frac{\delta}{p_j}; \ldots; p_n, x_n)$$

for $\delta > 0$ such that the rank-ordering of the outcomes is maintained.

If $x_i, x_j, x_i + \delta/p_i$, and $x_j - \delta/p_j$ are of the same sign, then the curvature of the corresponding weighting function and the concavity of the utility on the corresponding domain imply $P(\delta) \preccurlyeq P$. Hence, strong risk aversion has been derived for the case that spreads involve only outcomes of the same sign.

Consider now the case that $x_i + \frac{\delta}{p_i} > 0 > x_j - \frac{\delta}{p_j}$. We have to show that in this case $P(\delta) \preccurlyeq P$ also holds. Using repeatedly the fact that strong risk aversion holds if spreads involve outcomes of the same sign, we obtain that

$$P(\delta) \preccurlyeq (p_1, x_1; \ldots ; p_i, x_i; \ldots ; p_k, x_k + \frac{\delta}{p_k}; p_{k+1}, x_{k+1} - \frac{\delta}{p_{k+1}}; \ldots ; p_j, x_j; \ldots ; p_n, x_n).$$

Hence, using inequality (1) and transitivity we get $P(\delta) \preccurlyeq P$.

Summarizing the above cases we conclude that strong risk aversion holds, hence Statement 1 of the theorem. This concludes the proof of Theorem 1. □

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References

Abdellaoui, Mohammed (2000), "Parameter-free Elicitation of Utility and Probability Weighting Functions," Management Science 46, 1497–1512.

Abdellaoui, Mohammed, Frank Vossmann and Martin Weber (2005), "Choice-based Elicitation and Decomposition of Decision Weights for Gains and Losses under Uncertainty," Management Science 51, 1384–1399.

Abdellaoui, Mohammed, Han Bleichrodt and Corina Paraschiv (2007), "Measuring Loss Aversion Under Prospect Theory: A Parameter-Free Approach," forthcoming in Management Science.

Bell, David E. (1985), "Disappointment in Decision Making under Uncertainty," Operations Research 33, 1–27.

Benartzi, Shlomo and Richard H. Thaler (1995), "Myopic Loss Aversion and the Equity Premium Puzzle," Quarterly Journal of Economics 110, 73–92.

Birnbaum, Michael H. (2004), "Causes of Allais Common Consequence Paradoxes: An Experimental Dissection," Journal of Mathematical Psychology 48, 87–106.

Birnbaum, Michael H. (2005), "Three New Tests of Independence That Differentiate Models of Risky Decision Making," Management Science 51, 1346–1358.

Birnbaum, Michael H. and Jeffrey P. Bahra (2007), "Gain-Loss Separability and Coalescing in Risky Decision Making.," Management Science 53, 1016–1028.

Birnbaum, Michael H. and Juan B. Navarrete (1998), "Testing Descriptive Utility Theories: Violations of Stochastic Dominance and Cumulative Independence," Journal of Risk and Uncertainty 17, 49–78.

Birnbaum, Michael H. and Steven E. Stegner (1981), "Measuring the Importance of Cues in Judgment for Individuals: Subjective Theories of IQ as a Function of Heredity and Environment," Journal of Experimental Social Psychology 17, 159–182.

Bleichrodt, Han and Jose Luis Pinto (2000), "A Parameter-Free Elicitation of the Probability weighting Function in Medical Decision Analysis," Management Science 46, 1485–1496.

Brooks, Peter and Horst Zank (2005), "Loss Averse Behavior," Journal of Risk and Uncertainty 31, 301–325.

Chateauneuf, Alain and Michèle Cohen (1994), "Risk Seeking with Diminishing Marginal Utility in a Non-Expected Utility Model," Journal of Risk and Uncertainty 9, 77–91.

Chateauneuf, Alain and Peter P. Wakker (1999), "An Axiomatization of Cumulative Prospect Theory for Decision under Risk," Journal of Risk and Uncertainty 18, 137– 145.

Chateauneuf, Alain, Jürgen Eichberger, and Simon Grant (2005), "Choice under Uncertainty with the Best and Worst in Mind: NEO-Additive Capacities," CERMSEM, CEM, University of Paris I.

Chew, Soo Hong, Edi Karni, and Zvi Safra (1987), "Risk Aversion in the Theory of Expected Utility with Rank Dependent Probabilities," Journal of Economic Theory 42, 370–381.

Fennema, Hein and Marcel van Assen (1999), "Measuring the Utility for Losses by Means of the Tradeoff Method," Journal of Risk and Uncertainty 17, 277–295.

Kahneman, Daniel and Amos Tversky (1979), "Prospect Theory: An Analysis of Decision under Risk," Econometrica 47, 263–291.

Köbberling, Veronika and Peter P. Wakker (2005), "An Index of Loss Aversion," Journal of Economic Theory 122, 119–131.

L'Haridon, Olivier (2006), "Behavior in the Loss Domain: An Experiment Using the Probability Tradeoff Consistency Condition," GRID, ENS, Cachan, France.

Luce, R. Duncan (1991), "Rank- and Sign-Dependent Linear Utility Models for Binary Gambles," Journal of Economic Theory 53, 75-100.

Luce, R. Duncan (2000), Utility of Gains and Losses: Measurement-Theoretical and Experimental Approaches, Lawrence Erlbaum Associates, New Jersey.

Luce, R. Duncan and Peter C. Fishburn (1991), "Rank- and Sign-Dependent Linear Utility Models for Finite First-Order Gambles," Journal of Risk and Uncertainty 4, 29–59.

Luce, R. Duncan and Anthony A.J. Marley (2005), "Ranked Additive Utility Representations of Gambles: Old and New Axiomatizations," Journal of Risk and Uncertainty 30, 21–62.

Marley, Anthony A.J. and Duncan R. Luce (2005), "Independence Properties vis-à-vis Several Utility Representations," Theory and Decision 58, 77–143.

Preston, Malcolm G. and Philip Baratta (1948), "An Experimental Study of the Auction Value of an Uncertain Outcome," American Journal of Psychology 61, 183–193.

Quiggin, John (1981), "Risk Perception and Risk Aversion among Australian Farmers," Australian Journal of Agricultural Economics 25, 160–169.

Quiggin, John (1982), "A Theory of Anticipated Utility," Journal of Economic Behaviour and Organization 3, 323–343.

Rothschild, Michael and Joseph E. Stiglitz (1970), "Increasing Risk: I. A Definition," Journal of Economic Theory 2, 225–243.

Schmeidler, David (1989), "Subjective Probability and Expected Utility without Additivity," Econometrica 57, 571–587.

Schmidt, Ulrich (2002), "Reference-Dependence in Cumulative Prospect Theory," Journal of Mathematical Psychology, forthcoming.

Schmidt, Ulrich and Horst Zank (2005), “What is Loss Aversion?” Journal of Risk and Uncertainty 30, 157–167.

Schmidt, Ulrich and Horst Zank (2006), “An Axiomatization of Linear Cumulative Prospect Theory with Applications to Portfolio Selection and Insurance Demand,” forthcoming in Decisions in Economics and Finance.

Starmer, Chris and Robert Sugden (1989), “Probability and Juxtaposition Effects: An Experimental Investigation of the Common Ratio Effect,” Journal of Risk and Uncertainty 2, 159–178.

Tversky, Amos and Daniel Kahneman (1992), “Advances in Prospect Theory: Cumulative Representation of Uncertainty,” Journal of Risk and Uncertainty 5, 297–323.

Wakker, Peter P. (1994) “Separating Marginal Utility and Probabilistic Risk Aversion,” Theory and Decision 36, 1–44.

Wakker, Peter P., Ido Erev and Elke U. Weber (1994), “Comonotonic Independence: The Critical Test between Classical and Rank-Dependent Utility Theories,” Journal of Risk and Uncertainty 9, 195–230.

Wakker, Peter P. and Amos Tversky (1993), “An Axiomatization of Cumulative Prospect Theory,” Journal of Risk and Uncertainty 7, 147–176.

Wakker, Peter P. and Horst Zank (2002), “A Simple Preference-Foundation of Cumulative Prospect Theory with Power Utility,” European Economic Review 46, 1253–1271.

Wu, George and Richard Gonzalez (1996), "Curvature of the Probability Weighting Function," Management Science 42, 1676–1690.

Wu, George and Alex Markle (2004), "An Empirical Test of Gain-Loss Separability in Prospect Theory," Working Paper, University of Chicago.

Wu, George, Jiao Zhang, and Mohammed Abdellaoui (2005), "Testing Prospect Theories Using Probability Tradeoff Consistency ," Journal of Risk and Uncertainty 30, 107–131.

Yaari, Menahem E. (1987), "The Dual Theory of Choice under Risk," Econometrica 55, 95-115.

Zank, Horst (2001), "Cumulative Prospect Theory for Parametric and Multiattribute Utilities," Mathematics of Operations Research 26, No. 1, 67–81.

Kapitel 6

A Simple Model of Cumulative Prospect Theory

<u>Anmerkungen:</u>

- Dieses Kapitel basiert auf gemeinsamen Arbeiten mit Horst Zank, School of Economics, University of Manchester, UK
- Dieses Kapitels ist bereits veröffentlicht in: U. Schmidt and H. Zank, A Simple Model of Cumulative Prospect Theory, *Journal of Mathematical Economics 45* (2009), 308-319.

1 Introduction

Empirical research has shown that expected utility theory (EU) fails to provide a good description of individual behavior in situations of uncertainty. Examples are the famous paradoxes of Allais (1953) and Ellsberg (1961). This evidence has motivated the development of alternative theories (the so-called non-expected utility theories), which allow for the exhibition of "paradoxical behavior." Building upon its predecessor prospect theory (Kahneman and Tversky, 1979), cumulative prospect theory (CPT) has nowadays become one of the most prominent of these alternatives (Starmer 2000).

A new criticism of EU was put forward by Rabin (2000) and Rabin and Thaler (2001). Following earlier work by Hansson (1988), these authors showed that reasonable degrees of risk aversion over small and modest stakes imply unreasonable high degrees of risk aversion over large stakes in the EU framework. For instance an EU-maximizer, who initially rejects a 50-50 bet of losing \$10 and winning \$11 at any wealth level, would also reject any 50-50 bet of losing \$100 and winning \$$x$ for an arbitrarily large value of x. Since this high degree of risk aversion seems to be irrational, Rabin (2000) concluded that EU is only a good representation of risk neutral behavior, which means that utility has to be linear. Neilson (2001) has shown that this criticism on EU carries over to rank-dependent utility (RDU), a further alternative to EU and a precursor of CPT. More precisely, in the rank-dependent utility framework the utility function should also be linear because a concave utility implies, as for EU, unreasonable high degrees of risk aversion over large stakes.

What drives these two results is the possibility to represent choice behavior by a mathematical functional where attitudes towards uncertainty can entirely be separated from attitudes towards outcomes. Indeed, under EU, the decision weight by which the utility of an outcome is multiplied, equals the probability of occurrence of the corresponding outcome or associated event. Obviously, this probability is unrelated to the magnitude of the outcome. Under RDU the decision weight reveals information about the probability and further about the rank of an outcome compared to other possible payoffs. Again no dependence of the decision weight and the magnitude of outcomes exists. For CPT, in addition to the rank of an outcome, the sign of the outcome relative to the status quo is influencing the decision weights, but beyond that no dependence on the magnitude of outcomes holds.

It appears therefore that in order to avoid the Rabin paradoxical behavior, but at the same time maintain the independence of decision weights and the magnitude of outcomes, utility in the above described models has to be linear. Considering this strong implication, the goal of the present paper is to investigate linear utility for decision under uncertainty by providing an axiomatic analysis of CPT.

Because CPT combines three desirable features (rank-dependence, reference-dependence, and sign-dependence) with theoretical tractability, it is currently seen as the most promising decision model (see Starmer 2000, page 370). The model was first proposed by Starmer and Sugden (1989). Later, axiomatizations of CPT have been provided by Luce and Fishburn (1991), Tversky and Kahneman (1992), Wakker and Tversky (1993), Chateauneuf and Wakker (1999), and Schmidt (2003). This paper provides a new axiomatization of CPT with a piecewise linear utility function. More precisely, utility is linear for gains and linear for losses with a possible kink at the status quo. If loss aversion is satisfied, utility is steeper on the domain of losses than it is on the domain of gains.

Linear utility has a long tradition in theoretical and empirical research. An axiomatic foundation of subjective expected utility with linear utility was provided by de Finetti (1931). Preston and Baratta (1948) used a linear utility model in order to estimate probability distortions. Edwards (1955) reports about a series of experiments which support our model. He finds evidence for sign-dependent probability distortions and also for linear utility. Many other studies observed linear utility for losses (Hershey and Schoemaker 1980, Schneider and Lopes 1986, Cohen, Jaffray, and Said 1987, Weber and Bottom 1989, Lopes and Oden 1999). Generally, for small stakes the evidence suggests that utility is linear (Lopes 1995, Fox, Rogers, and Tversky 1996, Kilka and Weber 2001).

Handa (1977) axiomatized a model of subjective expected value, which was implicitly used by Preston and Baratta (1948) and already discussed in Edwards (1955). A model for decision under risk that combines linear utility and distorted probabilities is the dual theory (DT) of Yaari (1987). Such a model has been analyzed in Safra and Segal (1998) for a restricted class of weighting functions. For decision under uncertainty the model of Chateauneuf (1991) provides a Choquet expected utility form with linear utility. A similar model is presented in de Waegenaere and Wakker (2001). A further study by Diecidue and Wakker (2002) provides an axiomatization of linear utility and decision weights in the framework of de Finetti. In all of these studies, reference-dependence and therefore also loss aversion is excluded.

Also, linear utility has often been employed in economic applications. Some examples are firm behavior under risk (Demers and Demers 1990), insurance demand (Doherty and Eeckhoudt 1995, Schmidt 1996), insurance pricing (Wang 1995, 1996, Wang, Young and Panjer 1997), agency theory (Schmidt 1999a), the equity premium puzzle (Epstein and Zin 1990) and efficient risk-sharing (Schmidt 1999b). We are convinced that CPT with linear utility may generate new insights in such theoretical applications. Interestingly some applications of our model to investment behavior have already been conducted (Barberis, Huang, and Santos 2001, Barberis and Huang 2001, Roger 2003). Also, van der Hoek and Sherris (2001) propose a risk measure for portfolio choice and insurance decisions based on DT and choose different weighting functions for gains and losses. Therefore, our model can serve as a theoretical basis for their risk measure through the additional freedom gained by reference- and sign-dependence.

Our paper is organized as follows. Some preliminary notation is introduced in the next section. Then, in Section 3, a first representation theorem is presented for the case of a finite state space. These results are then extended to more general state spaces in Sections 4 and 5. Concluding remarks are presented in Section 6. The appendix contains proofs of the main theorems.

2 Notation

Let S be a (finite or infinite) set of states of the world. Subsets of S will be denoted by $A, B, \dots$; the complement of A (with respect to S) is denoted by A^c. The state space is endowed with an *algebra* $\mathcal{A}$ of subsets of S. Therefore, (i) $S \in \mathcal{A}$, (ii) if $A \in \mathcal{A}$ then $A^c \in \mathcal{A}$, and (iii) if $A, B \in \mathcal{A}$ then $A \cup B \in \mathcal{A}$. Subsets of S which are contained in $\mathcal{A}$ are called *events*. A (finite) *partition* $\{A_1, \dots, A_n\}$ of S is a collection of disjoint events, the union of which equals S.

The set of outcomes is $I\!R$, indicating money. Members of the outcome set are denoted by $x, y, z, \dots$. An act $f : S \to I\!R$, $s \mapsto f(s)$ assigns to each state an outcome. We assume throughout that acts are *bounded* (i.e., for any act f there exists $z \in I\!R$ such that $|f(s)| \leqslant z$ for all states s) and *measurable* (i.e., the inverse of each interval is an event).

The set of all acts is denoted by $\mathcal{F}$. An important subset of $\mathcal{F}$ is the set of simple acts, $\mathcal{F}^s$. Simple acts take only finitely many values. Therefore, for $f \in \mathcal{F}^s$ there exists a partition $\{A_1, \ldots, A_n\}$ such that $f = \sum_{i=1}^n x_i 1_{A_i}$, where 1_{A_i} is the indicator function of event A_i. It is understood that the act f assigns outcome x_i for states $s \in A_i$, $i = 1, \ldots, n$.

We use the notation $f_A g$ for an act that agrees with act f on event A and with act g on the complement A^c. Also, we use $h_i f$ instead of $h_{\{s_i\}} f$ for some state $s_i \in S$. Sometimes we identify constant acts with the corresponding outcome. We may thus write $f_A x$ for an act agreeing with f on A and giving outcome x for states $s \in A^c$; similarly we use $x_A f$.

We assume a preference relation $\succcurlyeq$ on the set of acts. As usually, the statement $f \succcurlyeq g$ means that act f is weakly preferred to act g. The symbols $\succ$ and $\sim$ denote strict preference and indifference, respectively. Sometimes we write $f \preccurlyeq g$ $(f \prec g)$ instead of $g \succcurlyeq f$ $(g \succ f)$. The preference relation $\succcurlyeq$ is a *weak order* if it is *complete* ($f \succcurlyeq g$ or $g \succcurlyeq f$ for any acts f, g) and *transitive* ($f \succcurlyeq g$ and $g \succcurlyeq h$ implies $f \succcurlyeq h$). A functional $V : \mathcal{F} \to I\!R$ represents the preference relation $\succcurlyeq$ if for all $f, g \in \mathcal{F}$ we have $f \succcurlyeq g \Leftrightarrow V(f) \geqslant V(g)$. Obviously, if a representing functional exists, then the preference relation is a weak order.

A classical example of a representing functional is Savage's (1954) subjective expected utility (SEU). *Subjective expected utility* holds if a preference relation can be represented by the functional

$$SEU(f) = \int_S U(f(s))dP(s),$$

where $U : I\!R \to I\!R$ is the utility function and P is an (additive) probability measure on $\mathcal{A}$. Utility is cardinal (i.e., it is unique up to scale and location) and the probability measure is unique. For a simple act $f = \sum_{i=1}^n x_i 1_{A_i}$ the above integral reduces to

$$SEU(f) = \sum_{i=1}^n U(x_i)P(A_i).$$

Since its introduction by Savage, many preference conditions describing SEU have been offered (e.g. Anscombe and Aumann, 1963; Wakker, 1984, 1989; d 'Aspremont and Gevers, 1990; Gul, 1992). Savage's framework in which SEU has been derived is now accepted as a natural way of modelling decision under uncertainty, and we have adopted that setup here.

A further example of a representing functional is Choquet expected utility (CEU). This functional has been introduced by Schmeidler (1989, first version 1982) and generalized by Gilboa (1987). It extends SEU by allowing the probability measure to be non-additive. This so-called *capacity* v satisfies $v(S) = 1, v(\emptyset) = 0$, and $v(A) \geqslant v(B)$ if $A \supset B$ and $A, B \in \mathcal{A}$. A capacity v is *strictly monotonic* if $v(A) > v(B)$ for $A \supsetneqq B$ and $A, B \in \mathcal{A}$.

Choquet expected utility holds if the preference relation can be represented by the functional

$$CEU(f) = \int_{\mathbb{R}_+} v(\{s \in S | U(f(s)) \geqslant \tau\}) d\tau + \int_{\mathbb{R}_-} [v(\{s \in S | U(f(s)) \geqslant \tau\}) - 1] d\tau.$$

Utility is cardinal (similar to SEU) and the capacity is unique. For a simple act $f = \sum_{i=1}^{n} x_i 1_{A_i}$, such that $U(x_i) \geqslant U(x_{i+1})$ for $i = 1, \ldots, n-1$, CEU can be written as

$$CEU(f) = \sum_{i=1}^{n} U(x_i)[v(\cup_{j=1}^{i} A_j) - v(\cup_{j=1}^{i-1} A_j)].$$

Derivations of CEU have further been provided by Wakker (1989), Nakamura (1990), and Chew and Karni (1994). CEU-forms with linear utility are presented in Chateauneuf (1991), de Waegenaere and Wakker (2001), and Diecidue and Wakker (2002).

In terms of the underlying preference conditions the difference between these two representing functionals, SEU and CEU, is captured in the strength of the *sure-thing-principle*: $f_A h \succcurlyeq g_A h \Leftrightarrow f_A h' \succcurlyeq g_A h'$ for all involved acts. For SEU the full force of this principle is required, whereas for CEU the principle is required only for acts which are pairwise *comonotonic* (f, g are comonotonic if there exists no states s, s' such that $f(s) > f(s')$ and $g(s) < g(s')$). The weakened version of the sure thing principle has been called comonotonic independence in Chew and Wakker (1996). We will use the term *comonotonic sure thing principle* here.

The central model in this paper is cumulative prospect theory (CPT). Under CPT a key role is assigned to the reference-point. It is common to assume that the reference point is the status quo outcome and in axiomatic work this reference-point is assumed exogenously. Empirically it is difficult to identify the exact location of the reference point because framing and other biases can influence its location (see also the early discussion in Kahneman and Tverky 1979, 1984 and more recently in Novemsky and Kahneman 2005).

We also follow the standard and assume that the reference-point is exogenously given. For simplicity of exposition, we assume that the reference-point is the zero outcome. Outcomes are interpreted as deviations from the reference-point, hence as gains or losses. For a given act f we define the gain-part f^+ as the act "f with all losses $f(s) < 0$ replaced by zero" and the loss part f^- as the act "f with all gains $f(s) > 0$ replaced by zero." The act f can then be viewed as the statewise sum of f^+ and f^-. *Cumulative Prospect Theory* holds if the representing functional for $\succcurlyeq$ has the form

$$CPT(f) = CEU^+(f^+) + CEU^-(f^-),$$

where CEU^+ is a CEU-form depending on a capacity v^+, and CEU^- is a CEU-form depending on a capacity v^-. The capacities are uniquely determined under CPT, and the utility is a ratio scale (i.e., unique up to scale) as it is fixed at the status quo: $U(0) = 0$. If in the above equation we use instead of the capacity v^-, the dual capacity

$\hat{v}^-(\cdot) := 1 - v^-(S\backslash\cdot)$, then we can write

$$CPT(f) = \int_{I\!R_+} v^+(\{s \in S | U(f(s)) \geqslant \tau\})d\tau + \int_{I\!R_-} [\hat{v}^-(\{s \in S | U(f(s)) \leqslant \tau\})]d\tau.$$

For a simple act $f = \sum_{i=1}^n x_i 1_{A_i}$, such that $U(x_i) \geqslant U(x_{i+1})$ for $i = 1, \ldots, n-1$, and for some $k \in \{0, \ldots, n\}$ indicating the number of gain outcomes of the act f (i.e., $U(x_k) \geqslant 0 > U(x_{k+1})$), CPT can be written as

$$\begin{aligned} CPT(f) &= \sum_{i=1}^{k} U(x_i)[v^+(\cup_{j=1}^{i} A_j) - v^+(\cup_{j=1}^{i-1} A_j)] \\ &\quad + \sum_{i=k+1}^{n} U(x_i)[v^-(\cup_{j=1}^{i} A_j) - v^-(\cup_{j=1}^{i-1} A_j)]. \end{aligned}$$

The latter functional has been introduced by Starmer and Sugden (1989).

Axiomatizations with general utility have appeared in Luce and Fishburn (1991), Luce (1991), Tversky and Kahneman (1992), and Wakker and Tversky (1993). Derivations of CPT with specific forms for the utility function (linear/exponential, power, and variants of multiattribute utility) have been provided in Zank (2001) generalizing Wakker and Zank (2002). All these functional forms, including SEU and CEU, are special cases of the cumulative utility functional presented in Chew and Wakker (1996). Thus, like CEU, CPT satisfies the comonotonic sure-thing principle. The difference between the two theories rests in the axiom that allows for a separation of utility from capacities. This additional axiom[1] is stronger under CEU than under CPT, which can be inferred from the observation that CEU is the special case of CPT when $v^+ = v^-$.

In the remainder of the paper we concentrate on a special case of CPT, where utility

[1]Intuitively, such an axiom requires, in addition to the comonotonic sure-thing principle, a form of weak comparative probability axiom (Gilboa 1987), as entailed in the comonotonic tradeoff consistency of Wakker and Tversky (1993) and Köbberling and Wakker (2003).

is linear. More precisely, the utility function will have the form

$$U(x) = \begin{cases} x, & \text{if } x \geqslant 0, \\ \lambda x, & \text{if } x \leqslant 0, \end{cases}$$

where the loss aversion parameter λ is positive.

Preference conditions are proposed to characterize CPT with linear utility, which we refer to as LCPT. The new condition, called independence of common increments, entails sign-dependence and the comonotonic sure thing principle, and moreover it implies linearity of utility on the gain domain and separately on the loss domain. The preference conditions are introduced in the next section for the finite states case. These results are then extended in Section 4 to simple acts on general state spaces, and finally in Section 5 some technical conditions are employed for the case of general acts. Possible extensions are presented in Section 6.

3 Finite State Spaces

Assume that the state space S is finite. That is, $S = \{s_1, \ldots, s_n\}$ for a natural number $n \geqslant 3$, and $\mathcal{A} = 2^S$. Acts $f = (f(s_1), \ldots, f(s_n))$ can be identified with the Cartesian product space $I\!R^n$. Hence, in this section we refer to acts as vectors $(f_1, \ldots, f_n)$ (f_i is short notation for $f(s_i)$). An act f is *rank-ordered* if its outcomes are ordered from best to worst: $f_1 \geqslant \cdots \geqslant f_n$. For each act f there exists a permutation ρ of $\{1, \ldots, n\}$ such that $f_{\rho(1)} \geqslant \cdots \geqslant f_{\rho(n)}$, i.e. such that the outcomes are *rank-ordered with respect to* ρ. For each permutation ρ of $\{1, \ldots, n\}$ the set $I\!R^n_\rho$ consists of those acts which are rank-ordered with respect to ρ. For example, if $\rho = id$ (i.e. $\rho(i) = i$ for all i), then $I\!R^n_{id}$ is the set of rank-ordered acts. Acts from a rank-ordered set $I\!R^n_\rho$ are obviously comonotonic.

The preference relation $\succcurlyeq$ on $I\!R^n$ satisfies *monotonicity* if $f \succ g$ whenever $f_i \geqslant g_i$ for all states s_i with a strict inequality for at least one state. By employing this condition we exclude null states, that is, states where the preference is independent of the magnitude of outcomes. Formally, a state s_i is *null* if $x_i f \sim y_i f$ for all acts f and all outcomes x, y.

The continuity condition defined here is continuity with respect to the Euclidean topology on $I\!R^n$: $\succcurlyeq$ satisfies *continuity* if for any act f the sets $\{g \in I\!R^n | g \succcurlyeq f\}$ and $\{g \in I\!R^n | g \preccurlyeq f\}$ are closed subsets of $I\!R^n$.

We now introduce the main condition in the paper. *Independence of common increments* holds if for any two acts $(f_1, \ldots, f_n)$ and $(g_1, \ldots, g_n)$ and $x \in I\!R$ we have

$$\begin{aligned} (f_1, \ldots, f_i, \ldots, f_n) &\succcurlyeq (g_1, \ldots, g_i, \ldots, g_n) \Rightarrow \\ (f_1, \ldots, f_i + x, \ldots, f_n) &\succcurlyeq (g_1, \ldots, g_i + x, \ldots, g_n), \end{aligned}$$

whenever $f_i, f_i + x, g_i, g_i + x$ are of the same sign (that is, either they are all gains or they are all losses), and all involved acts are pairwise comonotonic (that is, they are all from the same set of rank-ordered acts $I\!R^n_\rho$).

Independence of common increments says that a common absolute change of an outcome of the same rank does not reverse the preference between two acts as long as this change is not too large to affect the rank or the sign of the outcomes. For x small enough, repeated application of this principle on acts containing only gains (or only losses) yields $(f_1 + x, \ldots, f_n + x) \succcurlyeq (g_1 + x, \ldots, g_n + x)$, indicating that it implies a weakened variant of the concept of constant absolute risk aversion (CARA), which could be called sign-dependent CARA. The restrictions on x mentioned above are crucial for the difference to CARA. The principle, however, is stronger than sign-dependent CARA used in Zank (2001) for the derivation of CPT with linear/exponential utility, as the exponential form is excluded.

One can show that repeated application of independence of common increments implies local additivity on sets of pairwise comonotonic acts having the same number of gain outcomes. Therefore, there exist outcomes $x_1, \ldots, x_n$ such that

$$
\begin{aligned}
(f_1, \ldots, f_i, \ldots, f_n) &\succcurlyeq (g_1, \ldots, g_i, \ldots, g_n) \Rightarrow \\
(f_1 + x_1, \ldots, f_n + x_n) &\succcurlyeq (g_1 + x_1, \ldots, g_n + x_n)
\end{aligned}
$$

if $f_k, g_k \geqslant 0 > f_{k+1}, g_{k+1}$ and all acts are pairwise comonotonic. This shows that the property comes close to additivity on rank ordered sets. Such a condition has been used by Weymark (1981) to derive the generalized Gini welfare functions. The condition has been termed comonotonic additivity in de Waegenaere and Wakker (2001) and Diecidue and Wakker (2002). Our condition here is weaker because of its reference- and sign-dependent nature. If we would drop the sign- and the rank-dependence restrictions we would get additivity on general sets. That and monotonicity are equivalent to the non-existence of a Dutch book, a condition used by de Finetti (1931) to derive subjective expected utility with linear utility. This demonstrates that the only features we have added to additivity are rank-dependence, reference-dependence, and sign-dependence, the basic characteristics of CPT.

In the following we show that independence of common increments is a necessary condition for CPT with linear utility, and that it implies the comonotonic sure thing principle. We present the results and corresponding proofs in the main text to further clarify the nature of this principle.

LEMMA 1 *If LCPT holds for $\succcurlyeq$ on $I\!R^n$ then independence of common increments is satisfied.*

PROOF: We prove the lemma for the case that acts are rank-ordered. The remaining cases are similar. Hence, suppose that $(f_1, \ldots, f_i, \ldots, f_n) \succcurlyeq (g_1, \ldots, g_i, \ldots, g_n)$ with $f, g \in I\!R^n_{id}$. Assume that there exists x such that $(f_1, \ldots, f_i+x, \ldots, f_n), (g_1, \ldots, g_i+x, \ldots, g_n) \in I\!R^n_{id}$ and that f_i, f_i+x, g_i, g_i+x have the same sign, say they are gains . Then, substituting LCPT we get

$$\begin{aligned}
& \sum_{j=1}^{k} f_j[v^+(\{s_1, \ldots, s_j\}) - v^+(\{s_1, \ldots, s_{j-1}\})] \\
& + \sum_{j=k+1}^{n} \lambda f_j[v^-(\{s_1, \ldots, s_j\}) - v^-(\{s_1, \ldots, s_{j-1}\})] \\
\geqslant \; & \sum_{j=1}^{k'} g_j[v^+(\{s_1, \ldots, s_j\}) - v^+(\{s_1, \ldots, s_{j-1}\})] \\
& + \sum_{j=k'+1}^{n} \lambda g_j[v^-(\{s_1, \ldots, s_j\}) - v^-(\{s_1, \ldots, s_{j-1}\})].
\end{aligned}$$

Adding on both sides of the inequality above $x[v^+(\{s_1, \ldots, s_i\}) - v^+(\{s_1, \ldots, s_{i-1}\})]$ gives the desired result. Note that in the above summands k and k' may differ, showing that act f may contain a different number of outcomes which are gains than act g. In the case that $f_i, f_i + x, g_i, g_i + x$ are all losses we add $\lambda x[v^-(\{s_1, \ldots, s_i\}) - v^-(\{s_1, \ldots, s_{i-1}\})]$ on both sides of the inequality. Hence, independence of common increments holds. $\square$

In earlier derivations of CPT complex independence condition have been used. In Tversky and Kahneman (1992), and Wakker and Tversky (1993) the conditions is termed sign-comonotonic tradeoff consistency. Luce and Fishburn (1991) and Luce (1991) use a condition called compound gamble and joint receipt. Our principle does not immediately imply these conditions, however, it does this in the presence of the remaining preference conditions as is shown in Theorem 3 below. In the following lemma we show that the comonotonic sure thing principle is implied by independence of common increments.

Lemma 2 *Assume that $\succcurlyeq$ on $I\!R^n$ is a weak order satisfying continuity. Then independence of common increments implies the comonotonic sure thing principle.*

Proof: We prove the lemma assuming that all acts are from $I\!R^n_{id}$. For acts from $I\!R^n_\rho$, where ρ is an arbitrary permutation of $\{1,\ldots,n\}$, the proof is complicated only by the tedious indexing of outcomes, otherwise results are derived in a similar fashion. Suppose $f, g \in I\!R^n_{id}$, such that $f_i = g_i = h_i$, and

$$h_i f \succcurlyeq h_i g (\Leftrightarrow f \succcurlyeq g).$$

Clearly, $\min\{f_{i-1}, g_{i-1}\} \geqslant h_i$ if $i \in \{2,\ldots,n\}$ and $h_i \geqslant \max\{f_{i+1}, g_{i+1}\}$ if $i \in \{1,\ldots,n-1\}$. We show that the outcome h_i can be replaced with any h'_i satisfying $\min\{f_{i-1}, g_{i-1}\} \geqslant h'_i$ if $i \in \{2,\ldots,n\}$ and $h'_i \geqslant \max\{f_{i+1}, g_{i+1}\}$ if $i \in \{1,\ldots,n-1\}$ without changing the above preference. Suppose, that $i \in \{2,\ldots,n\}$ and that $0 > \min\{f_{i-1}, g_{i-1}\}$ $(\geqslant h'_i)$. Then applying independence of common increments with $x = h'_i - h_i$ gives

$$h_i f \succcurlyeq h_i g \Leftrightarrow h'_i f \succcurlyeq h'_i g.$$

Similarly, if $i \in \{1,\ldots,n-1\}$ and we have $(h'_i \geqslant)$ $\max\{f_{i+1}, g_{i+1}\} > 0$, then applying independence of common increments with $x = h'_i - h_i$ gives

$$h_i f \succcurlyeq h_i g \Leftrightarrow h'_i f \succcurlyeq h'_i g.$$

Suppose now that for $i \in \{2,\ldots,n-1\}$ we have $\min\{f_{i-1}, g_{i-1}\} \geqslant 0 \geqslant \max\{f_{i+1}, g_{i+1}\}$ (or for $i = 1$ we have $0 \geqslant \max\{f_{i+1}, g_{i+1}\}$, or for $i = n$ we have $\min\{f_{i-1}, g_{i-1}\} \geqslant 0$). Then, using continuity we find that independence of common increments can first be applied with $x = -h_i$ implying

$$h_i f \succcurlyeq h_i g \Leftrightarrow 0_i f \succcurlyeq 0_i g,$$

and a second application of the principle with $x' = h_i'$ gives

$$0_i f \succcurlyeq 0_i g \Leftrightarrow h_i' f \succcurlyeq h_i' g.$$

This shows, that "state-wise" the comonotonic sure thing principle holds (also called comonotonic coordinate independence in Wakker, 1989), and by appropriate successive application of it one can show that the comonotonic sure thing principle holds for general events A in addition to single states $\{s_i\}$. This completes the proof of the lemma. $\square$

We can now present the main result of this section.

THEOREM 3 *Suppose that* $\succcurlyeq$ *is a preference relation on* $I\!R^n$*, for* $n \geqslant 3$*. Then the following two statements are equivalent:*

(i) LCPT holds with strictly monotonic capacities v^+, v^-*.*

(ii) The preference relation $\succcurlyeq$ *is a monotonic continuous weak order satisfying independence of common increments.*

The LCPT-utility function U *is a ratio scale with uniquely determined* λ*, and the capacities* v^+, v^- *are uniquely determined.* $\square$

The proof of this theorem is presented in the appendix. Further remarks on how the preference conditions can be weakened are postponed until Section 6.

4 Simple Acts

In the previous section we have introduced preference conditions characterizing LCPT in the case that the state space is finite. These results can be extended for more general state spaces, and this is the purpose of the present section. As a first step we reformulate our preference conditions to accommodate general state spaces. In this section S, the set of states of the world, can be finite or infinite. We focus on acts $f \in \mathcal{F}^s$ of the form $f = \sum_{i=1}^n x_i 1_{A_i}$, for a partition $\{A_1, \ldots, A_n\}$. When there is no confusion we avoid the explicit mentioning of the particular partition.

The preference relation $\succcurlyeq$ on $\mathcal{F}^s$ satisfies *monotonicity* if $x_A f \succ y_A f$ whenever $x > y$ and the event A is non-null. The definition of a null event is the natural extension of the definition of a null state: an event B is *null* if $x_B f \sim y_B f$ for all acts f and all outcomes x, y.

In this section continuity is also defined with respect to the Euclidean topology: $\succcurlyeq$ satisfies *simple-continuity* if for any simple act $f = \sum_{i=1}^n x_i 1_{A_i}$ the sets $\{(y_1, \ldots, y_n) \in \mathbb{R}^n | \sum_{i=1}^n y_i 1_{A_i} \succcurlyeq f\}$ and $\{(y_1, \ldots, y_n) \in \mathbb{R}^n | \sum_{i=1}^n y_i 1_{A_i} \preccurlyeq f\}$ are closed subsets of $\mathbb{R}^n$.

Independence of common increments holds if for any two simple acts f and g which can be represented using the same partition $\{A_1, \ldots, A_n\}$, any event A_i from this partition, and any outcome $x \in \mathbb{R}$ we have

$$\begin{aligned} y_{A_i} f &\succcurlyeq z_{A_i} g \Rightarrow \\ (y+x)_{A_i} f &\succcurlyeq (z+x)_{A_i} g, \end{aligned}$$

whenever $y, z, y+x, z+x$ are of the same sign (that is either they are all gains or they are all losses), and all involved acts are pairwise comonotonic.

We can now formulate the main result of this section:

THEOREM 4 *Suppose there exist at least three disjoint non-null events. Then, the following two statements are equivalent for a preference relation $\succcurlyeq$ on the set of simple acts $\mathcal{F}^s$:*

(i) LCPT holds,

(ii) the preference relation $\succcurlyeq$ is a simple-continuous monotonic weak order that satisfies independence of common increments.

The LCPT-utility function U is a ratio scale with uniquely determined λ, and the capacities v^+, v^- are uniquely determined. □

The proof of Theorem 4 is presented in the Appendix.

5 General Result

In the previous section we have introduced the axioms describing LCPT for a preference relation on the set of simple acts. In this section we extend the functional derived in Theorem 4 to the set of all acts, $\mathcal{F}$. To do this it is not necessary to extend all properties of $\succcurlyeq$ on $\mathcal{F}^s$ to hold on the entire set of acts $\mathcal{F}$. It turns out that independence of common increments and monotonicity need to hold only for $\succcurlyeq$ on $\mathcal{F}^s$ if we employ an appropriate continuity condition. The idea behind this is to exploit the specific structure of the set of acts $\mathcal{F}$.

The distance between two acts f, g, measured in the *supnorm* is defined as $\sup_{s\in S}|f(s)-g(s)|$. We say that $\succcurlyeq$ is *supnorm-continuous* if for each act f the sets $\{g \in \mathcal{F}|g \succcurlyeq f\}$ and $\{g \in \mathcal{F}|g \preccurlyeq f\}$ are closed sets under the supnorm. That supnorm-continuity is not very restrictive follows from the fact that, when restricted to $I\!R^n$, it is equivalent to Euclidean continuity (and, thus, also equivalent to simple continuity on $\mathcal{F}^s$), and further by the fact that it is equivalent to continuity of the utility function under SEU, CEU, and general CPT (see Observation 2 in Chew and Wakker 1996).

The next lemma shows that weak ordering and supnorm-continuity ensures the existence of a *certainty equivalent* for each act, i.e., a constant act $x(f)$ such that $x(f) \sim f$. Then we exploit the well-known fact that $\mathcal{F}^s$ is a supnorm-dense subspace of $\mathcal{F}$: the existence of a certainty equivalent $x(f)$ to each act f allows us to define $LCPT(f)$ as the value of $LCPT(x(f))$ established in Theorem 4, and therefore the extension of LCPT from $\mathcal{F}^s$ to $\mathcal{F}$ is established.

LEMMA 5 *Suppose there exist at least three disjoint non-null events. Further, assume that $\succcurlyeq$ on $\mathcal{F}$ is a weak order that satisfies supnorm-continuity, and that LCPT holds on $\mathcal{F}^s$. Then each act f has a certainty equivalent $x(f)$.* $\square$

The proof of the lemma is given in the appendix.

Take now any act f. Recall that f is bounded, such that there exist $x, y \in I\!R$ with $x \geqslant f(s) \geqslant y$ for all states $s \in S$. It is now easy to generate simple acts f^l, g^l (bounded by x, y from above and below, respectively) which converge in the supnorm, respectively, from above and below to f. This holds similarly for the corresponding certainty equivalents, so that the definition $LCPT(f) = LCPT(x(f))$ makes sense. With this definition also the uniqueness results established in Theorem 4 carry over to LCPT on $\mathcal{F}$. Actually this argument would hold true on any subset $\mathcal{F}'$ of acts containing all simple acts, i.e., $\mathcal{F} \supseteq \mathcal{F}' \supseteq \mathcal{F}^s$. The theorem below summarizes the previous analysis in the main result of this section:

THEOREM 6 *Suppose there exist at least three disjoint non-null events. Then, the following two statements are equivalent for a preference relation $\succcurlyeq$ on the set of acts $\mathcal{F}$:*

(i) LCPT holds,

(ii) the preference relation $\succcurlyeq$ is a supnorm-continuous weak order on $\mathcal{F}$ that satisfies monotonicity and independence of common increments on $\mathcal{F}^s$.

The LCPT-utility function U is a ratio scale with uniquely determined λ, and the capacities v^+, v^- are uniquely determined. □

6 Concluding Remarks

The extensions described in this section are focusing on the results in Section 3. In Theorem 3 the continuity condition can be dropped if the existence of a certainty equivalent for each act is ensured. The technique to prove the result would be similar to the one used in Diecidue and Wakker (2002) by employing results of Aczel (1966). We have indicated that independence of common increments implies locally the comonotonic additivity used in Diecidue and Wakker. The proof here would be more complicated as one has to deal with the sign dependent nature of the independence principle, which initially implies local comonotonic additivity on rank-ordered subsets of $I\!R^n$ in which precisely k states have gain outcomes ($k = 0, \ldots, n$). On these subsets LCPT would hold and one needs to fit together the different LCPT-functionals in order to derive LCPT on all of $I\!R^n$. Once the result for finite spaces is established, the results for simple acts and those for general acts can similarly be derived without any continuity assumption. Again the existence of a certainty equivalent to each act is required.

Instead of dropping continuity one could, from a technical point of view, dispense of monotonicity. The absence of monotonicity would make the exposition of the results considerably more complicated. A minor complication is that rank-ordering of outcomes then needs to be defined with respect to the restriction of the preference to constant acts.

Further, as stated in Section 3 one advantage of having monotonicity was to ensure that all states are essential or non-null. In the absence of monotonicity one would need to assume explicitly that there are at least three non-null states in Theorem 3 which would add complexitiy to the theorem's proof . A further advantage of having monotonicity is that utility is increasing, hence representing LCPT functionals that agree on $I\!R^n_{id,+}$ with a functional

$$(f_1, f_2, \ldots, f_n) \mapsto \frac{1}{2}f_1 - \frac{1}{3}f_2 + \frac{1}{6}f_3,$$

where in a state (here s_2) an increase in an outcome leads to a decrease in utility, are avoided. Also, recall that we have introduced capacities as nonadditive but monotonic extensions of probability measures. Therefore, the marginal impact of an event for a capacity is nonnegative. By dispensing of monotonicity the marginal impact of an event may be negative, and this feature seems unreasonable from a measurement theory point of view as capacities are traditionally seen as non-additive extensions of probability measures.

Let us now focus on the principle of independence of common increments, which can be formulated more appealing from an empirical point of view. Many studies suggest that individuals pay comparably more attention to extreme outcomes (e.g. Lopes 1987, Gilboa 1988, Jaffray 1988, Cohen 1992), that is to worst and best outcomes, compared to outcomes of intermediate rank. Suppose that S is a finite state space. For act f, outcome x, and event A denote $(f+x)_A f$ the act assigning $f_i + x$ for $s_i \in A$ and f_i for $s_i \notin A$. Independence of common increments implies that for any two acts f and g and $x \in I\!R$ we have

$$\begin{aligned} f &\succcurlyeq g \Rightarrow \\ (f+x)_A f &\succcurlyeq (g+x)_A g, \end{aligned}$$

whenever $f_i, f_i + x, g_i, g_i + x$ are of the same sign (that is either they are all gains or they are all losses) for $s_i \in A$, all involved acts are from the same set of rank-ordered acts $I\!R^n_\rho$, and, moreover, $A = \{s_{\rho(1)}, \ldots, s_{\rho(m)}\}$ or $A = \{s_{\rho(l)}, \ldots, s_{\rho(n)}\}$ for some $m, l \in \{1, \ldots, n\}$.

This version of the principle, which could be called *independence of common increments at tails*, says that a preference between two acts remains unchanged if the best outcomes or the worst outcomes are increased or decreased by the same common outcome, if the original and the modified outcomes are all of the same sign, and all involved acts are pairwise comonotonic. To relate this condition to the earlier version in Section 3 note that the new condition actually is equivalent to independence of common increments, as the next lemma shows.

LEMMA 7 *Suppose $\succcurlyeq$ is a preference relation on $I\!R^n$. Then independence of common increments is equivalent to independence of common increments at tails.* $\square$

Given the result in this lemma, Theorem 3 holds if we replace independence of common increments by independence of common increments at tails.

A final comment refers to our assumption that acts are bounded. We have restricted our analysis throughout the paper to such acts. Our result in Section 5 can be extended to unbounded acts by using a technique similar to the definition of integrals. Such techniques are discussed for example in Wakker (1993).

7 Appendix

PROOF OF THEOREM 3: First, statement (i) is assumed, and statement (ii) is concluded. Suppose LCPT holds for $\succcurlyeq$ on $I\!R^n$ with strictly monotonic capacities. Weak ordering is immediate from the existence of the representing functional for $\succcurlyeq$. Monotonicity holds because utility is increasing and the capacities are strictly monotonic. Continuity of utility implies continuity of $\succcurlyeq$. Independence of common increments holds by Lemma 1. This completes the proof of statement (ii).

Next, statement (ii) is assumed and statement (i) is derived. The proof consists of several intermediate results. First, it is shown that on the set of rank-ordered acts $I\!R^n_{id}$ the preference relation is represented by the additive function described in Lemma 8 below. Then (Lemma 9), it is shown that the additive function in Lemma 8 is a restriction of a LCPT functional. Lemma 10 indicates that similar results can be derived for $\succcurlyeq$ on $I\!R^n_\rho$, for any permutation ρ of $\{1, \ldots, n\}$. Then, it is shown that the different LCPT restrictions fit together into a general functional, such that LCPT holds for $\succcurlyeq$ on $I\!R^n$.

Lemma 8 *The preference relation $\succcurlyeq$ on $I\!R^n_{id}$ is represented by the additive functional:*

$$(f_1, \ldots, f_n) \mapsto \sum_{i=1}^{n} V_i(f_i),$$

with continuous strictly increasing functions $V_1, \ldots, V_n : I\!R \to I\!R$, which are uniquely determined satisfying $V_i(0) = 0$ for all i and $\sum_{i=1}^n V_i(1) = 1$.

Proof: The proof follows by combining different existing results. First note that by Lemma 2 the comonotonic sure thing principle holds. Then the result follows from Corollary 3.6 in Wakker (1993). There, an additive functional representation is derived with cardinal functions V_i. By fixing $V_i(0) = 0$ for all i and $\sum_{i=1}^n V_i(1) = 1$, the statement in our lemma is derived. $\triangle$

Let now $k \in \{0, \ldots, n\}$ be arbitrarily fixed. We concentrate on acts $f \in I\!R^n_{id}$, having k gain outcomes, i.e., $f_k \geqslant 0 > f_{k+1}$. Independence of common increments is satisfied, so that for $x \in I\!R$

$$f \sim g \Rightarrow (f_i + x)_i f \sim (g_i + x)_i g,$$

whenever $f_i, f_i + x, g_i, g_i + x$ are of the same sign. Substitution of the additive functional derived in Lemma 8 gives

$$V_i(f_i) - V_i(g_i) = V_i(f_i + x) - V_i(g_i + x),$$

which implies, first locally and then by continuity globally, linearity of V_i on $I\!R_+$ if $i \leqslant k$ and on $I\!R_-$ if $i \geqslant k+1$. As k was chosen arbitrarily we conclude that the functions V_i are linear for gains and linear for losses. Continuity, monotonicity, and the fact that $V_i(0) = 0$ implies that the V_i's are of the form

$$V_i(x) = \begin{cases} \alpha_i^+ x, & \text{if } x \geqslant 0, \\ \beta_i x, & \text{if } x \leqslant 0, \end{cases}$$

with $\alpha_i^+ > 0, \beta_i > 0$ for all $i = 1, \ldots, n$. Moreover, $\sum_{i=1}^n V_i(1) = 1$ implies $\sum_{i=1}^n \alpha_i^+ = 1$, so that we can refer to the α_i^+'s as decision weights for gain outcomes. Let now $\sum_{i=1}^n V_i(-1) = -\lambda$ with λ positive. Then we can define $\alpha_i^- := \beta_i/\lambda$ as the decision weights corresponding to loss outcomes (they are nonnegative and their sum equals 1). Let us summarize:

LEMMA 9 *There exist positive decision weights α_i^+ and positive decision weights α_i^- such that the preference relation $\succcurlyeq$ on $I\!R^n_{id}$ is represented by the additive functional:*

$$(f_1, \ldots, f_n) \mapsto \sum_{i=1}^n V_i(f_i),$$

with functions V_i of the form

$$V_i(x) = \begin{cases} \alpha_i^+ x, & \text{if } x \geqslant 0, \\ \lambda \alpha_i^- x, & \text{if } x \leqslant 0, \end{cases}$$

for a positive λ. △

In the preceding analysis we have restricted attention to acts $f \in I\!R^n_{id}$. It is easy to show that for acts $f \in I\!R^n_\rho$, where ρ is an arbitrary permutation of $\{1, \ldots, n\}$, similar results can be derived. The proof is complicated only by the more complex indexing of outcomes. We can conclude the following statement:

LEMMA 10 *There exist positive decision weights $\alpha^+_{i,\rho}$ and positive decision weights $\alpha^-_{i,\rho}$ such that the preference relation $\succcurlyeq$ on $I\!R^n_\rho$ is represented by the additive functional:*

$$LCPT_\rho(f_1, \ldots, f_n) \mapsto \sum_{i=1}^{n} V_{i,\rho}(f_{\rho(i)}),$$

with functions $V_{i,\rho}$ of the form

$$V_i(x) = \begin{cases} \alpha^+_{i,\rho} x, & \text{if } x \geqslant 0, \\ \lambda_\rho \alpha^-_{i,\rho} x, & \text{if } x \leqslant 0, \end{cases}$$

for a positive λ_ρ. $\triangle$

It remains to show that the decision weights $\alpha^+_{i,\rho}$ and $\alpha^-_{i,\rho}$, and the loss aversion parameters λ_ρ are all independent of the permutation ρ.

First we show that λ_ρ is independent of ρ. If for a permutation ρ of $\{1, \ldots, n\}$ the set $I\!R^n_\rho \bigcap I\!R^n_{id}$ contains nonconstant acts, then $LCPT_\rho$ and $LCPT_{id}$ jointly represent the preference relation $\succcurlyeq$ on the intersection $I\!R^n_\rho \bigcap I\!R^n_{id}$. If for a permutation ρ of $\{1, \ldots, n\}$ the set $I\!R^n_\rho \bigcap I\!R^n_{id}$ contains only constant acts, then, as $n \geqslant 3$, it follows that there exists a permutation σ of $\{1, \ldots, n\}$ such that the set $I\!R^n_\rho \bigcap I\!R^n_\sigma$ contains nonconstant acts and also $I\!R^n_\sigma \bigcap I\!R^n_{id}$ contains nonconstant acts. As the LCPT-representations are uniquely determined, it follows that the loss aversion parameters λ_ρ are independent of ρ, hence equal to λ. Obviously, the different LCPT-representations agree on the subset of constant acts which are commonly contained in each set $I\!R^n_\rho$. Using continuity one can show that each act f has a *certainty equivalent*, i.e., a constant act $x(f) \sim f$. It then follows for acts $f \in I\!R^n_\rho, g \in I\!R^n_\sigma$ that

$$f \succcurlyeq g \Leftrightarrow x(f) \succcurlyeq x(g)$$

implying

$$LCPT_\rho(f) = LCPT(x(f)) \geqslant LCPT(x(g)) = LCPT_\sigma(g).$$

This shows that the different functionals $LCPT_\rho$, agree on common domain and are restrictions of one general LCPT functional representing $\succcurlyeq$ on $I\!R^n$.

Next we determine the capacities v^+ and v^-. Let A be any event. We consider the act 1_A0 assigning outcome 1 for state $s \in A$ and 0 for $s \notin A$. By setting $v^+(A) := LCPT(1_A0)$ we define a capacity on S for gains, satisfying

$$\alpha^+_{i,\rho} = v^+(\{s_{\rho(1)}, \ldots, s_{\rho(i)}\}) - v^+(\{s_{\rho(1)}, \ldots, s_{\rho(i-1)}\})$$

for $i = 1, \ldots, n$ and any permutation ρ. This definition makes sense because for different premutations ρ, σ, the respective LCPT representations agree on common domain, hence the corresponding decision weights are equal. Because the decision weights are unique the same is true for the capacity v^+. Moreover, as all decision weights are positive, it follows that the capacity is strictly monotonic.

By setting $v^-(A) := LCPT(-1_A0)/\lambda$ we define a second capacity on S now for losses, satisfying

$$\alpha^-_{i,\rho} = v^-(\{s_{\rho(1)}, \ldots, s_{\rho(i)}\}) - v^-(\{s_{\rho(1)}, \ldots, s_{\rho(i-1)}\})$$

for $i = 1, \ldots, n$ and any permutation ρ. The capacity for losses v^- is also unique, and strictly monotonic.

We can now conclude that $\succcurlyeq$ on $I\!R^n$ is represented by an LCPT functional with unique capacities v^+, v^-, and unique loss aversion parameter λ. This concludes the proof of Theorem 3. $\square$

Proof of Theorem 4: That statement (i) implies statement (ii) is immediate from the definition of LCPT. We assume statement (ii) and prove statement (i). For a partition $\{A_1, \ldots, A_n\}$ with at least 3 non-null events, let $\mathcal{F}^s_{\{A_1,\ldots,A_n\}}$ denote the set of simple acts of the form $\sum_{i=1}^n x_i 1_{A_i}$, for outcomes x_i. Because any simple act belongs to some set $\mathcal{F}^s_{\{A_1,\ldots,A_n\}}$ where the partition $\{A_1, \ldots, A_n\}$ contains at least 3 non-null events, we can restrict the proof to simple acts defined over such partitions only. Obviously, a set $\mathcal{F}^s_{\{A_1,\ldots,A_n\}}$ can be identified with $I\!R^n$ (or $I\!R^m$ if precisely $m \geqslant 3$ events in the partition $\{A_1, \ldots, A_n\}$ are non-null), and further the restriction of $\succcurlyeq$ to $\mathcal{F}^s_{\{A_1,\ldots,A_n\}}$ is a weak order that satisfies monotonicity, continuity, and independence of common increments. Hence, statement (ii) of Theorem 3 holds and we conclude that LCPT holds on $\mathcal{F}^s_{\{A_1,\ldots,A_n\}}$ with unique capacities $v^+_{\{A_1,\ldots,A_n\}}, v^-_{\{A_1,\ldots,A_n\}}$. Note that these – in fact restrictions of – capacities may not be strictly monotonic if some events in the partition $\{A_1, \ldots, A_n\}$ are null. If some A_j is null, then the capacities $v^+_{\{A_1,\ldots,A_n\}}, v^-_{\{A_1,\ldots,A_n\}}$ are uniquely extended such that $v^+_{\{A_1,\ldots,A_n\}}(A \cup A_j) = v^+_{\{A_1,\ldots,A_n\}}(A), v^-_{\{A_1,\ldots,A_n\}}(A \cup A_j) = v^-_{\{A_1,\ldots,A_n\}}(A)$ for any event A.

The above arguments can be repeated for any fixed partition of S containing at least 3 non-null events, and it remains to show that these different LCPT-functionals are restrictions of a general LCPT functional representing $\succcurlyeq$ on $\mathcal{F}^s$.

It is well known that given two arbitrary simple acts $f = \sum_{i=1}^n x_i 1_{A_i}$, and $g = \sum_{j=1}^m y_j 1_{B_j}$, there exists a partition $\{C_{i,j}\}_{i=1,j=1}^{n,m}$, which is a common refinement of both $\{A_1, \ldots, A_n\}$, and $\{B_1, \ldots, B_m\}$, such that f, g can be represented as simple acts with respect to the same partition. One can for example define $C_{i,j}$ as the intersection of the events A_i and B_j. Then, $f = \sum_{i=1}^n \sum_{j=1}^m x_i 1_{C_{i,j}}$, and $g = \sum_{i=1}^n \sum_{j=1}^m y_j 1_{C_{i,j}}$. Suppose that

$LCPT_{\{A_1,\ldots,A_n\}}$ represents $\succcurlyeq$ on $\mathcal{F}^s_{\{A_1,\ldots,A_n\}}$ and $LCPT_{\{B_1,\ldots,B_m\}}$ represents $\succcurlyeq$ on $\mathcal{F}^s_{\{B_1,\ldots,B_m\}}$. Let further $LCPT_{\{C_{i,j}\}_{i=1,j=1}^{n,m}}$ represent $\succcurlyeq$ on $\mathcal{F}^s_{\{C_{i,j}\}_{i=1,j=1}^{n,m}}$. As $\mathcal{F}^s_{\{A_1,\ldots,A_n\}}, \mathcal{F}^s_{\{B_1,\ldots,B_m\}}$ are both included in $\mathcal{F}^s_{\{C_{i,j}\}_{i=1,j=1}^{n,m}}$ it follows that $LCPT_{\{C_{i,j}\}_{i=1,j=1}^{n,m}}$ represents $\succcurlyeq$ on $\mathcal{F}^s_{\{A_1,\ldots,A_n\}}$ and on $\mathcal{F}^s_{\{B_1,\ldots,B_m\}}$. Uniqueness results imply that the utility in $LCPT_{\{C_{i,j}\}_{i=1,j=1}^{n,m}}$ can be chosen equal to the utility in $LCPT_{\{A_1,\ldots,A_n\}}$ and that in $LCPT_{\{B_1,\ldots,B_m\}}$, and further that the capacities $v^+_{\{A_1,\ldots,A_n\}}, v^-_{\{A_1,\ldots,A_n\}}$ and $v^+_{\{B_1,\ldots,B_m\}}, v^-_{\{B_1,\ldots,B_m\}}$ are restrictions of the unique capacities $v^+_{\{C_{i,j}\}_{i=1,j=1}^{n,m}}, v^-_{\{C_{i,j}\}_{i=1,j=1}^{n,m}}$ to the corresponding partitions. Because for any finite partition $\{A_1,\ldots,A_n\}$ of S all sets of acts $\mathcal{F}^s_{\{A_1,\ldots,A_n\}}$ contain the constant acts it follows that all the different LCPT-representations agree on the subset of constant acts, and thus have a common LCPT-utility function, U, with unique loss aversion parameter λ. Further, continuity implies that each simple act $f = \sum_{i=1}^n x_i 1_{A_i}$ has a certainty equivalent $x(f)$. We can then define a general functional V on $\mathcal{F}^s$ by

$$V(f) := U(x(f)) = LCPT_{\{A_1,\ldots,A_n\}}(f),$$

and observe that for any simple acts $f = \sum_{i=1}^n x_i 1_{A_i}$ and $g = \sum_{j=1}^m y_j 1_{B_j}$ we have

$$f \succcurlyeq g \Leftrightarrow x(f) \succcurlyeq x(g)$$

which is equivalent to

$$LCPT_{\{A_1,\ldots,A_n\}}(f) \geqslant LCPT_{\{B_1,\ldots,B_m\}}(g) \Leftrightarrow V(f) \geqslant V(g),$$

hence, this general functional V represents the preference on $\mathcal{F}^s$.

We now determine a gain capacity v^+ and a loss capacity v^- using the previously derived LCPT-utility, U, with unique loss aversion parameter λ. For any event A we define $v^+(A) = U(x(1_A 0))/U(1_S)$ and $v^-(A) = U(-1_A 0)/[\lambda U(1_S)]$. Because the LCPT-utility is a ratio scale it follows hat these capacities are uniquely determined. Further, because $V(f) = LCPT_{\{A_1,\ldots,A_n\}}(f)$ for any simple act f, these general capacities agree with the corresponding capacities $v^+_{\{A_1,\ldots,A_n\}}, v^-_{\{A_1,\ldots,A_n\}}$ on their common domain of events. Thus, our general representing functional V is in fact a LCPT-functional representing $\succcurlyeq$ on $\mathcal{F}^s$. Hence statement (i) of the theorem follows.

By the very construction of V, it follows that the uniqueness results established for the preference on $\mathcal{F}^s_{\{A_1,\ldots,A_n\}}$ also hold for V. This concludes the proof of the theorem. $\square$

Proof of Lemma 5: Let $f \in \mathcal{F}$. Because all acts are bounded there exist $x, y \in I\!R$ such that $x \geqslant f(s) \geqslant y$ for all states s. We can construct (similar to the classical derivation of the Lebesgue integral) two sequences of simple acts f^l, g^l converging in the supnorm from above and below, respectively, to f. These sequences can be chosen such that

$$x \geqslant f^l(s) \geqslant f^{l+1}(s) \geqslant f(s) \geqslant g^{l+1}(s) \geqslant g^l(s) \geqslant y$$

holds for each state s. As LCPT holds on $\mathcal{F}^s$ it follows that

$$LCPT(x) \geqslant LCPT(f^l) \geqslant LCPT(g^l) \geqslant LCPT(y)$$

and hence

$$x \succcurlyeq f^l \succcurlyeq g^l \succcurlyeq y$$

for all l. By supnorm-continuity and the fact that the two sequences f^l, g^l and, thus, the corresponding certainty equivalents $x(f^l), x(g^l)$, converge to f it follows that $x \succcurlyeq f \succcurlyeq y$. This means that the sets constant acts $\{z : z \succcurlyeq f\}$ and $\{z : z \preccurlyeq f\}$ are nonempty, and because of continuity of $\succcurlyeq$ on the set of constant acts, these closed sets must have a nonempty intersection. They contain at least one element $x(f) \sim f$ with

$$LCPT(x(f)) = \begin{cases} x(f), & \text{if } x(f) \geqslant 0, \\ \lambda x(f), & \text{if } x(f) \leqslant 0. \end{cases}$$

Therefore, because LCPT is strictly increasing over constant acts, we can also conclude that in the intersection of these two sets there exists a unique element $x(f) \sim f$, the certainty equivalent of f. This concludes the proof of the lemma. $\square$

PROOF OF LEMMA 7: First we assume that independence of common increments holds. Let $f, g \in I\!R^n_\rho$, such that $f \succcurlyeq g$. Suppose that $A = \{s_{\rho(1)}, \ldots, s_{\rho(m)}\}$ for some $1 \leqslant m \leqslant \min\{k, k'\}$, where k, k' denote the number of gain outcomes of f, g, respectively. We have to show that for any $x \in I\!R$ such that $(f+x)_A f, (g+x)_A g \in I\!R^n_\rho$ it follows that $(f+x)_A f \succcurlyeq (g+x)_A g$. For $x = 0$ there is nothing to show. If $x > 0$, then independence of common increments can repeatedly be applied to $s_{\rho(1)}$, then to $s_{\rho(2)}$, etc., until $s_{\rho(m)}$, such that, by induction, we get:

$$f \succcurlyeq g \Rightarrow (f+x)_A f \succcurlyeq (g+x)_A g.$$

If $x < 0$, independence of common extremes can repeatedly be applied starting with $s_{\rho(m)}$ then to $s_{\rho(m-1)}$, etc., until $s_{\rho(1)}$, such that, by induction, we get $f \succcurlyeq g \Rightarrow (f+x)_A f \succcurlyeq (g+x)_A g$. Note here, that $f_{\rho(m)} + x, g_{\rho(m)} + x$ must be positive in order to apply independence of common increments.

The proof follows similarly for $A = \{s_{\rho(l)}, \ldots, s_{\rho(n)}\}$ for some $\max\{k, k'\} < l$. Therefore, independence of common increments at tails holds.

For the reversed implication, assume that independence of common increments at tails holds. Let $i \in \{1, \ldots, n\}$. We have to show that for any $x \in I\!R$ such that $f, g, (f+x)_{\rho(i)} f, (g+x)_{\rho(i)} g \in I\!R^n_\rho$ and $f_{\rho(i)}, f_{\rho(i)} + x, g_{\rho(i)}, g_{\rho(i)} + x$ are of the same sign it follows that

$$f \succcurlyeq g \Rightarrow (f+x)_{\rho(i)} f \succcurlyeq (g+x)_{\rho(i)} g.$$

If $f_{\rho(i)}, g_{\rho(i)}$ are gains then we apply independence of common increments at tails first with

x and $A = \{s_{\rho(1)}, \ldots, s_{\rho(i)}\}$ and then with $-x$ and $A' = \{s_{\rho(1)}, \ldots, s_{\rho(i-1)}\}$ and get

$$\begin{aligned} f &\succcurlyeq g \Rightarrow (f+x)_A f \succcurlyeq (g+x)_A g \\ &\Rightarrow (f-x)_{A'}(f+x)_A f \succcurlyeq (g-x)_{A'}(g+x)_A g \\ &\Leftrightarrow (f+x)_{\rho(i)} f \succcurlyeq (g+x)_{\rho(i)} g. \end{aligned}$$

Note that in order to maintain comonotonicity of $f, g, (f+x)_{\rho(i)}f, (g+x)_{\rho(i)}g$, the outcome x must be chosen such that $f_{\rho(i-1)} \geqslant f_{\rho(i)}+x$ and $g_{\rho(i-1)} \geqslant g_{\rho(i)}+x$. Under these conditions the above applications of independence of common increments at tails are well defined.

If $f_{\rho(i)}, g_{\rho(i)}$ are losses then we apply independence of common increments at tails first with x and $A = \{s_{\rho(i)}, \ldots, s_{\rho(n)}\}$ and then with $-x$ and $A' = \{s_{\rho(i+1)}, \ldots, s_{\rho(n)}\}$. It follows that independence of common increments holds, as i, and ρ were arbitrary. This completes the proof of the lemma. $\square$

References

Aczél, János, (1966), "Lectures on Functional Equations and Their Applications." Academic Press, New York.

Allais, Maurice (1953), "Le Comportement de l'Homme Rationnel devant le Risque: Critique des Postulats et Axiomes de l'Ecole Américaine," Econometrica 21, 503–546.

Anscombe, Frank J. and Robert J. Aumann (1963), "A Definition of Subjective Probability," Annals of Mathematical Statistics 34, 199–205.

Barberis, Nicholas and Ming Huang (2001), “Mental Accounting, Loss Aversion, and Individual Stock Returns,” Journal of Finance 56, 1247–1292.

Barberis, Nicholas, Ming Huang, and Tano Santos (2001), “Prospect Theory and Asset Prices,” The Quaterly Journal of Economics 56, 1247–1292.

Chateauneuf, Alain (1991), “On the Use of Capacities in Modeling Uncertainty Aversion and Risk Aversion,” Journal of Mathematical Economics 20, 343–369.

Chateauneuf, Alain and Peter P. Wakker (1999), “An Axiomatization of Cumulative Prospect Theory for Decision under Risk,” Journal of Risk and Uncertainty 18, 137–145.

Chew, Soo Hong and Edi Karni (1994), “Choquet Expected Utility with a Finite State Space: Commutativity and Act- Independence,” Journal of Economic Theory 62, 469–479.

Chew, Soo Hong and Peter P. Wakker (1996), “The Comonotonic Sure-Thing Principle,” Journal of Risk and Uncertainty 12, 5–27.

Cohen, Michèle (1992), “Security Level, Potential Level, Expected Utility: A Three-criteria Decision Model under Risk”, Theory and Decision 33, 101–134.

Cohen, Michèle, Jean-Yves Jaffray, and Tanios Said (1987), “Experimental Comparisons of Individual Behavior under Risk and under Uncertainty for Gains and for Losses,” Organizational Behavior and Human Decision Processes 39, 1–22.

d’Aspremont, Claude and Louis Gevers (1990), “Invariance, Neutrality and Weakly Continuous Expected Utility.” In Jean-Jaskold Gabszewicz, Jean-François Richard, and

Laurence A. Wolsey (Eds.), Economic Decision-Making: Games, Econometrics and Optimisation: Contributions in honour of Jacques H. Dreze, 87–100, North-Holland, Amsterdam.

de Finetti, Bruno (1931), “Sul Significato Soggettivo della Probabilità,” Fundamentà Mathematicae 17, 298–329. Translated into English as “On the Subjective Meaning of Probability,” in Paola Monari and Daniela Cocchi (Eds., 1993) “Probabilità e Induzione,” Clueb, Bologna, 291–321.

Demers, Fanny S., and Michel Demers (1990), “Price Uncertainty, the Competitive Firm and the Dual Theory of Choice under Risk”, European Economic Review 34, 1181–1199.

De Waegenaere, Anja and Peter P. Wakker (2001), “Nonmonotonic Choquet Integrals,” Journal of Mathematical Economics 36, 45–60.

Diecidue, Enrico and Peter P. Wakker (2002), “Dutch Books: Avoiding Strategic and Dynamic Complications, and a Comonotonic Extension,” Mathematical Social Sciences 43, 135–149.

Doherty, Neil A., and Louis Eeckhoudt (1995), “Optimal Insurance without Expected Utility: The Dual Theory and the Linearity of Insurance Contracts”, Journal of Risk and Uncertainty 10, 157–179.

Edwards, Ward (1955), “The Prediction of Decisions Among Bets,” Journal of Experimental Psychology 50, 201–214.

Ellsberg, Daniel (1961), “Risk, Ambiguity and the Savage Axioms,” Quarterly Journal of Economics 75, 643–669.

Epstein, Larry G. and Stanley E. Zin (1990), " 'First-Order' Risk Aversion and the Equity Premium Puzzle," Journal of Monetary Economics 26, 387–407.

Fox, Craig R., Brett A. Rogers, and Amos Tversky (1996), "Options Traders Exhibit Subadditive Decision Weights," Journal of Risk and Uncertainty 13, 5–17.

Gilboa, Itzhak (1987), "Expected Utility with Purely Subjective Non-Additive Probabilities," Journal of Mathematical Economics 16, 65–88.

Gilboa, Itzhak (1988), "A Combination of Expected Utility and Maxmin Decision Criteria", Journal of Mathematical Psychology 32, 405–420.

Gul, Faruk (1992), "Savage's Theorem with a Finite Number of States," Journal of Economic Theory 57, 99–110.

Handa, Jagdish (1977), "Risk, Probabilities, and a New Theory of Cardinal Utility," Journal of Political Economy 85, 97–122.

Hansson, Bengt (1988), "Risk Aversion as a Problem of Conjoint Measurement." In Peter Gärdenfors and Nils-Eric Sahlin (Eds.), Decision, Probability, and Utility; Selected Readings, 136–158, Cambridge University Press, Cambridge.

Hershey, John C. and Paul J.H. Schoemaker (1980), "Prospect Theory's Reflection Hypothesis: A Critical Examination," Organizational Behavior and Human Performance 25, 395–418.

Jaffray, Jean-Yves (1988), "Choice under Risk and the Security Factor: An Axiomatic Model", Theory and Decision 24, 169–200.

Kahneman, Daniel and Amos Tversky (1979), "Prospect Theory: An Analysis of Decision under Risk," Econometrica 47, 263–291.

Kahneman, Daniel and Amos Tversky (1984), "Choices, Values, and Frames," American Psychologist 39, 341–350.

Kilka, Michael and Martin Weber (2001), "What Determines the Shape of the Probability Weighting Function under Uncertainty," Management Science 47, 1712–1726.

Köbberling, Veronika and Peter P. Wakker (2003), "Preference Foundations for Nonexpected Utility: A Generalized and Simplified Technique," Mathematics of Operations Research 28, 395–423.

Lopes, Lola L. (1987), "Between Hope and Fear: The Psychology of Risk. In Leonard Berkowitz (Ed.), Advances in Experimental and Social Psychology (pp. 255–295), San Diego, Academic Press.

Lopes, Lola L. (1995), "Algebra and Process in the Modeling of Risky Choice," The Psychology of Learning and Motivation 32, 177–220.

Lopes, Lola L. and Gregg C. Oden (1999), "The Role of Aspiration Level in Risky Choice: A Comparison of Cumulative Prospect Theory and SP/A Theory," Journal of Mathematical Psychology 43, 286–313.

Luce, R. Duncan (1991), "Rank- and-Sign Dependent Linear Utility Models for Binary Gambles," Journal of Economic Theory 53, 75–100.

Luce, R. Duncan and Peter C. Fishburn (1991), "Rank- and Sign-Dependent Linear Utility Models for Finite First-Order Gambles," Journal of Risk and Uncertainty 4, 29–59.

Nakamura, Yutaka (1990), "Subjective Expected Utility with Non-Additive Probabilities on Finite State Spaces," Journal of Economic Theory 51, 346–366.

Neilson, William (2001) "Calibration results for rank-dependent expected utility," Economics Bulletin 4, 1–5.

Novemsky, Nathan and Daniel Kahneman (2005), "The Boundaries of Loss Aversion," Journal of Marketing Research 42, 119–128.

Preston, Malcolm G. and Philip Baratta (1948), "An Experimental Study of the Auction Value of an Uncertain Outcome," American Journal of Psychology 61, 183–193.

Rabin, Matthew (2000), "Risk Aversion and Expected-utility Theory: A Calibration Theorem," Econometrica 68, 1281–1292.

Rabin, Matthew and Richard H. Thaler (2001), "Anomalies: Risk Aversion," Journal of Economic Perspectives 15, 219–232.

Roger, Patrick (2003), "Is Time-Diversification Efficient for a Loss-Averse Investor? " Mimeo, University Louis Pasteur, Strasbourg.

Safra, Zvi and Uzi Segal (1998), "Constant Risk Aversion," Journal of Economic Theory 83, 19–42.

Savage, Leonard J. (1954), "The Foundations of Statistics." Wiley, New York.

Schmeidler, David (1989), "Subjective Probability and Expected Utility without Additivity," Econometrica 57, 571–587.

Schmidt, Ulrich (1996), "Demand for Coinsurance and Bilateral Risk-Sharing with Rank-Dependent Utility," Risk Decision and Policy 1, 217–228.

Schmidt, Ulrich (1999a), "Moral Hazard and First-Order Risk Aversion," Journal of Economics, Supplement 8, S. 167–179.

Schmidt, Ulrich (1999b), "Efficient Risk-Sharing and the Dual Theory of Choice Under Risk," Journal of Risk and Insurance 66, S. 597–608.

Schmidt, Ulrich (2003), "Reference-Dependence in Cumulative Prospect Theory," Journal of Mathematical Psychology 47, 122–131.

Schneider, Sandra L. and Lola L. Lopes (1986), "Reflection in Preferences under Risk: Who and When May Suggest Why," Journal of Experimental Psychology: Human Perception and Performance 12, 535–548.

Starmer, Chris (2000), "Developments in Non- Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk," Journal of Economic Literature 38, 332–382.

Starmer, Chris and Robert Sugden (1989), "Violations of the Independence Axiom in Common Ratio Problems: An Experimental Test of some Competing Hypotheses," Annals of Operations Research 19, 79–102.

Tversky, Amos and Daniel Kahneman (1992), "Advances in Prospect Theory: Cumulative Representation of Uncertainty," Journal of Risk and Uncertainty 5, 297–323.

van der Hoek, John and Michael Sherris (2001), "A Class of Non-Expected Utility Risk Measures and Implications for Asset Allocations", Insurance: Mathematics and

Economics 28, 69–82

Wakker, Peter P. (1984), “Cardinal Coordinate Independence for Expected Utility,” Journal of Mathematical Psychology 28, 110–117.

Wakker, Peter P. (1989), “Additive Representations of Preferences, A New Foundation of Decision Analysis.” Kluwer Academic Publishers, Dordrecht.

Wakker, Peter P. (1993), “Additive Representations on Rank-Ordered Sets II. The Topological Approach,” Journal of Mathematical Economics 22, 1–26.

Wakker, Peter P. and Amos Tversky (1993), “An Axiomatization of Cumulative Prospect Theory,” Journal of Risk and Uncertainty 7, 147–176.

Wakker, Peter P. and Horst Zank (2002), “A Simple Preference-Foundation of Cumulative Prospect Theory with Power Utility,” European Economic Review 46, 1253–1271.

Wang, Shaun S. (1995), “Insurance Pricing and Increased Limits Ratemaking by Proportional Hazard Transforms,” Insurance: Mathematics and Economics 17, 43–54.

Wang, Shaun S. (1996), “Premium Calculation by Transforming the Layer Premium Density,” Astin Bulletin 26, 71–92.

Wang, Shaun S., Virginia R. Young, and Harry H. Panjer (1997), “Axiomatic Characterization of Insurance Prices,” Insurance: Mathematics and Economics 21, 173–183.

Weber, Elke U. and William P. Bottom (1989), ”Axiomatic Measures of Perceived risk: Some Tests and Extensions,” Journal of Behavioral Decision Making 2, 113–131.

Weymark, John A. (1981), "Generalized Gini Inequality Indices," Mathematical Social Sciences 1, 409–430.

Yaari, Menahem E. (1987), "The Dual Theory of Choice under Risk," Econometrica 55, 95–115.

Zank, Horst (2001), "Cumulative Prospect Theory for Parametric and Multiattribute Utilities," Mathematics of Operations Research 26, 67–81.

Kapitel 7

Third-Generation Prospect Theory

<u>Anmerkungen:</u>

- Dieses Kapitel basiert auf gemeinsamen Experimenten mit Chris Starmer, School of Economics, University of Nottingham, UK and Rober Sugden, Department of Economics, University of East Anglia, UK.
- Dieses Kapitels ist bereits veröffentlicht in: U., Schmidt, C. Starmer, and R. Sugden, Third-Generation Prospect Theory, Journal of Risk and Uncertainty 36 (2008), 203-223.

1. Introduction

In this paper we present a new theory of decision under risk: *third-generation prospect theory* (PT^3 for short). The motivation for the theory is empirical: our model is presented as a descriptive theory intended to outperform the current 'best buys' in the literature. PT^3 has three key features: *reference dependence, decision weights* and *uncertain reference points* (i.e. reference points that can be lotteries). The first two features are the common characteristics of different versions of prospect theory including the original version (Kahneman and Tversky, 1979) and the later *second-generation* versions featuring cumulative decision weights (e.g. Starmer and Sugden, 1989; Luce and Fishburn 1991; Tversky and Kahneman, 1992; Wakker and Tversky 1993). Variants of second-generation prospect theory are increasingly widely applied in both theoretical and empirical work (recent examples are Wu, Zhang and Abdellaoui, 2005; Baucells and Heukamp, 2006; Davies and Satchell 2004; Trepel, Fox and Poldrack, 2005) and some have argued that such theories may be serious contenders for replacing expected utility theory at least for specific purposes (see Camerer, 1989). No doubt this is partly because there is considerable empirical support for both reference-dependence and decision weights (see Starmer, 2000).

While second-generation prospect theory has been relatively successful in organising a range of experimental and field data, it naturally has some descriptive limitations (see for example, Birnbaum and Bahara, 2006), and of particular interest here is the fact that no variant to date has been able to explain an apparently robust and especially troubling failure of expected utility theory: the so-called *preference reversal phenomenon* (PR for short). By allowing reference points to be lotteries, our variant is also able to accommodate PR. This explanation of PR is highly parsimonious in the sense that, relative to other variants, it requires no extra parameters.

We should emphasise that our purpose is not merely to present another possible preference based explanation of PR; we seek to evaluate whether our explanation is empirically plausible. To investigate this, we explore the incidence and pattern of PR predicted by our theory given alternative parameterisations. We find that the standard patterns of PR are predicted for typical parameterisations of prospect theory already established in the empirical literature. Consequently we suggest that our model constitutes a best buy theory: it offers the predictive power of previous variants of prospect theory and adds to that an explanation of PR. The latter comes 'free of charge' since it involves no extra parameters and no re-parameterisation.

2. Existing Explanations of Preference Reversal

PR is one of the most notorious anomalies in individual decision making, but despite the large volume of literature it has generated, no satisfactory preference-based account of it has thus far been produced (see Cubitt, Munro and Starmer, 2004). The classic instances of PR involve decisions relating to pairs of gambles. In the simplest cases, gambles are binary lotteries with just one positive outcome (the *prize*); the other outcome is zero. One of the lotteries, usually called the 'P bet', gives the better chance of winning a prize while the other, the '$ bet' – has the larger prize. In a typical experiment investigating PR, agents' preference orderings over pairs of such bets are elicited in two ways: in a pairwise choice task, and by comparing willingness-to-accept (WTA) valuations of lotteries elicited separately for P and $ bets. PR is a widely observed tendency for agents to reveal a preference for the P bet in choice but the $ bet in valuation. We will call this pattern *standard PR*. Such inconsistencies between choice and valuation might arise through chance or error. But the opposite inconsistency, in which the $ bet is chosen but the P bet is given a higher value (*non-standard PR*), is much less frequently observed. It is this asymmetry between the two types of reversal which constitutes the puzzle of PR.

In the psychology literature it has been common to interpret PR as evidence that preferences do not satisfy procedural invariance but, instead, depend upon the method used to elicit them. On this view, if preferences are to be invoked at all in explaining PR, those preferences must be context-sensitive: that is, they must allow *different* preferences to govern decisions in choice and valuation tasks. We have no quarrel with the claim that in general behaviour is context sensitive and that specific forms of context sensitivity, such as the scale compatibility effect or the prominence effect, contribute to a full explanation of PR (Slovic, Griffin and Tversky, 1990). Our interest lies in exploring whether stable and context-independent features of agents' preferences also play an important explanatory role.

We will treat the use of WTA valuations as one of the defining characteristics of a PR experiment. In fact, there have been surprisingly few experiments in which willingness-to-pay (WTP) valuations of P and $ bets have been used. Such experiments have produced mixed results, but asymmetric PR is generally less pronounced than in WTA experiments, and sometimes is not present at all. It seems that WTP treatments tend to reduce the frequency of standard reversals and to increase the frequency of non-standard ones (Lichtenstein and Slovic, 1971; Knez and Smith, 1987; Casey, 1991). These findings are compatible with the hypothesis that PR is the product of several causal mechanisms, at least one of which is in some way linked to WTA valuations. We suggest that our model captures a mechanism of the latter kind. In it, loss aversion imparts a tendency for PR that is specific to the case of WTA valuations.

Economists have suggested several models of context-free preferences as possible accounts of PR. All of them relax at least one of the axioms of expected utility theory. One subset of them retains transitivity and relaxes the independence and/or reduction axioms (Holt, 1986; Karni and Safra, 1987; Segal, 1988). Recent studies, however, continue to generate strong PR in experimental designs implementing controls for the explanations postulated in these theories (Tversky, Slovic and Kahneman, 1990; Cubitt, Munro and Starmer, 2004). Another possible explanation is that PR arises as a consequence of context-free, but non-transitive preferences. Persistent non-transitive cycles of choice analogous to PR have been observed in experimental studies (Loomes, Starmer and Sugden, 1989, 1991; Humphrey 2001), but the only preference theory that has been put forward to explain such behaviour is regret theory (Bell, 1982; Loomes and Sugden, 1983), which has failed other tests (Starmer and Sugden 1998). Also the accommodation of PR by choice errors is empirically not convincing (Schmidt and Hey, 2004).

A new preference-based explanation for PR is provided by Sugden's (2003) model of reference-dependent subjective expected utility (RDSEU). This model predicts PR when preferences are loss averse. The key novel feature of the theory is that, in contrast to previous reference-dependent theories, agents' reference points need not be constant but may be state-dependent, i.e. the reference point may be given by an act or lottery[1]. But although this model has the merit of explaining PR, it has a serious weakness as a potential 'best buy' for general use. Specifically, because it is linear in probabilities, it cannot accommodate other well-documented departures from expected utility theory, such as the Allais Paradox. Our model generalises Sugden's theory to allow non-linear probability weighting. The resulting model, PT^3, has the explanatory capacity of other variants of prospect theory, plus the added ability to explain PR.

3. Theory

In this section we introduce PT^3. In this theory, preferences are defined over (Savage) acts. Consider a finite state space S, consisting of the states s_i, $i = 1, \ldots, n$, and a set of consequences X given by an interval of the real line. Each state s_i has a probability $\pi_i \geq 0$, with $\sum_i \pi_i = 1$. ‘Probability’ may be interpreted either subjectively or objectively; for convenience in this paper, we will use the objective interpretation. F is the set of all acts. A particular act $f \in F$ is a function from S to X, i.e. an act f specifies for each state s_i the resulting consequence $f(s_i) \in X$.

A key feature of our model is that preferences over acts are reference dependent. We formalise this following the approach of RDSEU. For any two acts f and g, $f \succsim_h g$ means that f is weakly preferred to g viewed from act h, the *reference act*. For present purposes the reference act can be interpreted as the status quo position. While reference dependence is one defining characteristic of prospect theory, in first and second-generation variants the point of reference is always a sure outcome (or in the current context, h is restricted to be a constant act). We relax this restriction by adopting a key innovation of RDSEU.

Sugden’s axiom system implies maximisation of the function:

(1) $$V(f, h) = \Sigma_i v(f(s_i), h(s_i))\pi_i$$

In this expression, $v(f(s_i), h(s_i))$ is a *relative value* function. It can be interpreted as the desirability of the consequence of act f in state s_i relative to the consequence of a reference act h in the same state. This function is increasing in its first argument; $v(f(s_i), h(s_i)) = 0$ when $f(s_i) = h(s_i)$. The function V(f, h) is the expectation of relative value. It assigns a real value to any act $f \in F$ viewed from any reference act $h \in F$ (i.e. $V: F \times F \rightarrow \mathbb{R}$). It is a preference representation in the sense that, for all f, h, g in F, $f \succsim_h g \Leftrightarrow V(f, h) \geq V(g, h)$.

Notice, however, that the preference representation in (1) is linear in probabilties. PT^3 relaxes this restriction of RDSEU by generalising (1) to:

(2) $$V(f, h) = \Sigma_i v(f(s_i), h(s_i))W(s_i ; f, h)$$

where $W(s_i; f, h)$ is the *decision weight* assigned to state s_i when f is being evaluated from h. In principle, decision weights could be determined by a simple transformation of state probabilities (i.e. $W(s_i; f, h) = w(\pi_i)$) as in Handa (1977). In the contemporary literature on prospect theory it has become conventional to construct decision weights cumulatively using a rank-dependent transformation (Quiggin, 1982; Tversky and Kahneman, 1992). One of the key theoretical rationales for the cumulative construction is that, unlike the first-generation approach, it results in monotonic preferences. In PT^3 we retain the rank-dependent approach, but reconfigure it to work with statewise reference dependence.[2]

In order to construct cumulative weights for a given f, h pair, states must be ordered according to the 'attractiveness' of f's consequences in each state. This is because, in a cumulative construction, the weight attached to a given state depends not only on the probability of that state but also on the position of its consequence in the ranking of all consequences associated with f. In PT^3 the attractiveness of the consequence in each state of a given act depends on the corresponding consequence of the reference act. Hence, in general, the ordering of consequences must be constructed separately for each f, h pair.

Consider any f, h pair. Relative to that pair, there is a *weak gain* in a state s_i if $v(f(s_i), h(s_i)) \geq 0$, and a *strict loss* if $v(f(s_i), h(s_i)) < 0$. Let m^+ be the number of states in which there are weak gains and let $m^- = n - m^+$, be the number of states in which there are strict losses. We re-assign subscripts so that, for all subscripts i, j, we have $i > j$ if and only if $v(f(s_i), h(s_i)) \geq v(f(s_j), h(s_j))$, and so that the states with weak gains are indexed $m^+, \ldots, 1$ and the states with strict losses are indexed $-1, \ldots, -m^-$.[3]

Cumulative decision weights are then defined as follows:

$$W(s_i; f, h) =$$

$$(3)\qquad \begin{cases} w^+(\pi_i) & \text{if } i = m^+, \\ w^+(\sum_{(j \geq i)} \pi_j) - w^+(\sum_{(j > i)} \pi_j) & \text{if } 1 \leq i \leq m^+ - 1, \\ w^-(\sum_{(j \leq i)} \pi_j) - w^-(\sum_{(j < i)} \pi_j) & \text{if } -m^- + 1 \leq i \leq -1, \\ w^-(\pi_i) & \text{if } i = -m^-, \end{cases}$$

where w^+ and w^- are, respectively, probability weighting functions for the gain and loss domains (w^+, w^- are strictly increasing mappings from [0, 1] onto [0, 1]).

PT^3 straightforwardly captures several models as special cases. RDSEU is the special case in which decision weights are untransformed state probabilities (i.e. $w^+(\pi_i) = w^-(\pi_i) = \pi_i$ for all i). Cumulative (or second-generation) prospect theory is the special case in which the relative value function takes the form $v(f(s_i), h(s_i)) = u(f(s_i) - h(s_i))$, where u(.) is a 'value' function, and in which reference acts are constrained to be certainties (i.e. $h(s_i) = h(s_j)$ for all i, j). Expected utility theory is the special case in which decision weights are untransformed state probabilities, as in RDSEU, and relative value is independent of the reference outcome (i.e. $v(f(s_i), h(s_i)) = u(f(s_i))$ where u(.) is a von Neumann-Morgenstern utility function).

4. Preference Reversal in PT^3: The General Case

Consider two acts with the basic structure of P-and \$-bets. Specifically, let f^P represent an act giving an increment of wealth x with probability p and a zero increment otherwise, and let $f^\$$ be an act giving an increment of wealth y with probability q and a zero increment otherwise, with $y > x > 0$ and $1 > p > q > 0$. Standard PR is observed when (i) f^P is revealed preferred to $f^\$$ in a straight choice between the two gambles and (ii) $f^\$$ has a higher WTA valuation than f^P. Given PT^3, the condition for (i) is straightforward. As a normalisation, we define consequences as increments or decrements of wealth relative to the agent's wealth (treated as a certainty) prior to the PR experiment. Taking the agent's reference act to be her pre-experiment wealth, we may write:[4]

(4) $$f^P \succsim_h f^\$ \Leftrightarrow w^+(p)\, v(x, 0) - w^+(q)\, v(y, 0) \geq 0.$$

Now consider (ii). Given PT^3, we can define willingness to accept (WTA) as follows. Consider an agent selling a P-bet. Her situation is depicted as follows:

	P	1-p
h^P	X	0
k^P	WTA^P	WTA^P

In this case, the agent's reference act, denoted h^P, is the P bet. Her WTA valuation of this bet, denoted WTA^P, is the increment of wealth such that she is indifferent between retaining h^P or giving up h^P in exchange for the certainty of that increment. Hence, we define WTA^P as the sure payoff of some constant act k^P defined such that $V(k^P, h^P) = 0$. With $WTA^\$$ defined in an analogous way, the values of WTA^P and $WTA^\$$ are then determined, respectively, by the solutions to equations (5) and (6):

(5) $$w^-(p)\, v(WTA^P, x) + w^+(1 - p)\, v(WTA^P, 0) = 0$$

(6) $$w^-(q)\, v(WTA^\$, y) + w^+(1 - q)\, v(WTA^\$, 0) = 0.$$

Standard preference reversal is implied by the model when $w^+(p)\, v(x, 0) > w^+(q)\, v(y, 0)$ and $WTA^\$ > WTA^P$. The fact that none of the terms in expression (4) features in either of expressions (5) or (6) provides a clue to the fact that, under certain conditions, PT^3 predicts standard PR. In fact, as we demonstrate below, both standard and non-standard PR can occur as a consequence of either loss aversion or probability weighting or both. However, our objective is to do significantly more than show that our model can accommodate PR in principle. Our aim is to explore whether PT^3 provides an empirically convincing account of observed instances of PR. To this end, we undertake calibration exercises designed to assess the empirical plausibility of our model's explanation of PR.

5. A Parameterised Form of PT^3

For the purpose of the calibrations it is necessary to adopt specific functional forms for our general model. In selecting these we are guided by three criteria. First, we seek a model flexible enough to allow us to investigate how the predicted incidence of PR varies with three key aspects of the agent's preferences: *attitudes to consequences*, *attitudes to probability*, and *attitudes to gain and loss*. Second, subject to that constraint, we seek to use the simplest model possible – that is, a model with just one parameter for each of the three attitudes we consider. Third, for comparability with existing evidence, we use wherever possible the functional forms that are most common in previously published research. By constraining ourselves to simple and widely used functional forms, we make the calibrations tougher and more meaningful tests of our model's explanation of PR.

In order to operationalise the model, we need to specify the form of reference dependence. We impose the restriction that the relative value function takes the form $v(f(s_i), h(s_i)) = u(z)$, where $z = (f(s_i) - h(s_i))$. When h is a constant act, this special case of statewise reference dependence is then equivalent to that built into earlier generations of prospect theory; u(.) is the counterpart of the value function in those theories.

Next we specify the class of value functions to be used in the calibration exercise. We adopt the power function which has been widely used in recent empirical literature (see Starmer 2000). Specifically,

$$(7) \qquad u(z) = \begin{cases} z^{\alpha} \text{ if } z \geq 0 \\ \\ -\lambda\,|z|^{\alpha} \text{ if } z < 0. \end{cases}$$

The parameters α and λ are required to be strictly positive. The first of these parameters controls the curvature of the value function. If $\alpha < 1$, this function is concave in the domain of gains and convex in the domain of losses (the property of *diminishing sensitivity*). Diminishing sensitivity imparts a tendency for risk aversion with respect to gains and risk-loving with respect to losses. While the empirical literature has suggested some differences in the exponents of the value function between the domains of gains and losses, in the interests of parsimony we will we apply the same exponent in both domains. The parameter λ controls attitudes to gain and loss. With $\lambda = 1$ there is loss neutrality. For λ values above unity, there is loss aversion: losses are weighted more heavily than gains. For values below unity, the opposite is the case.

We model decision weights via a single-parameter probability weighting function. Again, for reasons of parsimony we impose the restriction of identical weighting functions for gains and losses (i.e. $w^{+}(\pi) = w^{-}(\pi)$). Hence for the purpose of the calibration exercise the probability weighting function is denoted simply by $w(\pi)$; it takes the form

(8) $$w(\pi) = \pi^{\beta} / (\pi^{\beta} + (1 - \pi)^{\beta})^{1/\beta}$$

with $\beta > 0$. This type of weighting function has been discussed by Tversky and Kahneman (1992) and Prelec (1998); variants of it have been widely used in the empirical literature. With $\beta = 1$, decision weights are linear (i.e. $w(\pi) = \pi$) but with $0.4 \leq \beta < 1$ the function generates an inverse-S pattern of weights with over-weighting (under-weighting) of probabilities below (above) some critical probability π^*. Inverse-S weighting has been reported across a wide range of empirical studies (Wu and Gonzalez, 1996, 1999; Abdellaoui, 2000; Bleichrodt and Pinto, 2000; Abdellaoui, Vossmann and Weber, 2005).

That completes the specification of the generic model to be used in the calibrations. We will refer to this specification as *parameterised* PT^3.

Notice that, when applied to cases in which reference acts are certainties, parameterised PT^3 can also be interpreted as a parameterisation of cumulative prospect theory. In fact, models of this kind have already been estimated using experimental data (e.g. Tversky and Kahaneman, 1992; Loomes, Moffatt and Sugden, 2002). But because cumulative prospect theory does not allow reference acts to be lotteries, these estimations have not used data from PR experiments. Thus, parameter values from these estimations are applicable to our model, while it remains a genuine test of that model to ask whether, given those parameter values, it predicts observed patterns of PR.

We take the following to be relatively well-established stylised facts concerning the median values of the three parameters for experimental subjects. First, many studies suggest the existence of loss aversion, while its opposite is almost unknown; values of the loss aversion parameter in the range $1 \leq \lambda \leq 2.5$ would capture a reasonably wide range of evidence. Studies fitting variants of prospect theory with power utility almost invariably find diminishing sensitivity. Although some studies have found values of α as low as 0.22 (Loomes, Moffatt and Sugden, 2002, note 17), values in the range $0.5 \leq \alpha \leq 1$ are typical. Inverse-S probability weighting, while not universal, is a very common finding; it would be reasonable to expect values of β in the range $0.5 \leq \beta \leq 1$. These ranges of values will be the focus for evaluating the predictions of our model.

6. Parameterised PT^3 and Preference Reversal

For simplicity, we restrict attention to P and $ bets which give either a positive payoff or zero. This case has been widely studied in the empirical literature. A feature of the power utility function is that model predictions are unchanged if all outcomes are multiplied by any positive constant. Exploiting this property, we may normalise the expected value of the P-bet to unity by setting its payoff $x = 1/p$. Given this normalisation, we can characterise any pair of P and $ bets by a three-parameter vector (p, q, r), where p is the probability of winning the prize in the P bet, q is the corresponding probability for the $ bet, and r is the expected value of the $ bet as a ratio of the expected value of the P bet (implying that the positive payoff of the $ bet is $y = r/q$). Notice that the condition $y > x$ (i.e. the $ bet has the higher prize) implies $rp > q$.

Substituting the functional form (7) into (4), the agent's choice between the two bets is determined by:

(9) $\quad f^P \succsim_h f^\$ \Leftrightarrow w(p)/ w(q) \geq (pr/ q)^\alpha$,

or, equivalently:

(10) $\quad f^P \succsim_h f^\$ \Leftrightarrow \alpha \leq \log[w(p)/ w(q)] / \log(rp/ q)$.

(For the moment, it is more convenient not to substitute in the parameterisation of the probability weighting function.) The following property of the model is an immediate implication of (10):

Property 1: The choice between P and \$ is independent of the value of λ. For any given value of β, there is a critical value of α at which the two bets are indifferent. At lower values of α, P is chosen; at higher values, \$ is chosen.

This property reflects the fact that, in the choice task, all consequences are positive or zero. Because the negative domain of the value function is not relevant for this task, diminishing sensitivity (i.e. $\alpha < 1$) plays essentially the same role in PT^3 as diminishing marginal utility does in expected utility theory: the lower the value of α, the greater the attractiveness of the safer P bet relative to the riskier \$ bet.

Substituting (7) and (8) into (5) and (6) and then rearranging, we arrive at the following formulae for the valuations of the two bets:

(11) $WTA^P = (1/p) / [((1-p)/p)^{\beta/\alpha} (1/\lambda)^{1/\alpha} + 1]$ and

(12) $WTA^\$ = (r/q) / [((1-q)/q)^{\beta/\alpha} (1/\lambda)^{1/\alpha} + 1]$.

Thus:

(13) $WTA^P/ WTA^\$ = q[((1-q)/q)^{\beta/\alpha} (1/\lambda)^{1/\alpha} + 1] / rp[((1-p)/p)^{\beta/\alpha} (1/\lambda)^{1/\alpha} + 1]$.

Examination of (13) yields:

Property 2: As λ increases, the value of $WTA^P/ WTA^\$$ falls; in the limit, as $\lambda \to \infty$, this value tends to q/rp, where $q/rp < 1$.

In other words, increases in the loss aversion parameter λ increase $WTA^\$$ relative to WTA^P; at sufficiently high values of λ, we have $WTA^\$ > WTA^P$. Intuitively, this is because the act of selling a bet carries the risk of losing the prize of that bet in the state in which the bet wins; since the \$ bet has the higher prize, the potential for loss in selling it is greater. Thus, loss aversion induces reluctance to sell low-probability high-prize bets.

Properties 1 and 2 are enough to give a preliminary sense of some of the combinations of parameter values that will induce PR. In order for the P bet to be selected in the choice task, α must be lower than some critical value. Given any such value of α, the \$ bet will have the higher WTA valuation if the value of λ is sufficiently high. Thus, standard PR is induced by the combination of sufficiently low α and sufficiently high λ.

It is convenient to explore the implications of the model graphically in (α, λ) space. This space is divided into quadrants by the lines $\alpha = 1$ and $\lambda = 1$. The stylised facts presented in Section 4 suggest that we should focus on the north-west quadrant, in which the value function is either linear or exhibits diminishing sensitivity (i.e. $\alpha \leq 1$) and in which there is either loss neutrality or loss aversion (i.e. $\lambda \geq 1$). We call this the *empirically plausible quadrant*.

For any given pair of bets and any given value of the decision weight parameter β, (α, λ) space can be divided into four regions by identifying two boundaries. One boundary – *the choice boundary* – identifies the locus of (α, λ) pairs along which the P and \$ bets are indifferent in choice (i.e. $f^P \sim_h f^\$$). We know from Property 1 that the choice boundary is a vertical line; P is chosen to the left of this line and \$ is chosen to the right. A second boundary – *the valuation boundary* – is the locus of (α, λ) pairs along which the P and \$ bets have equal WTA valuations. We know from Property 2 that P has the higher valuation below this boundary and \$ has the higher valuation above it.

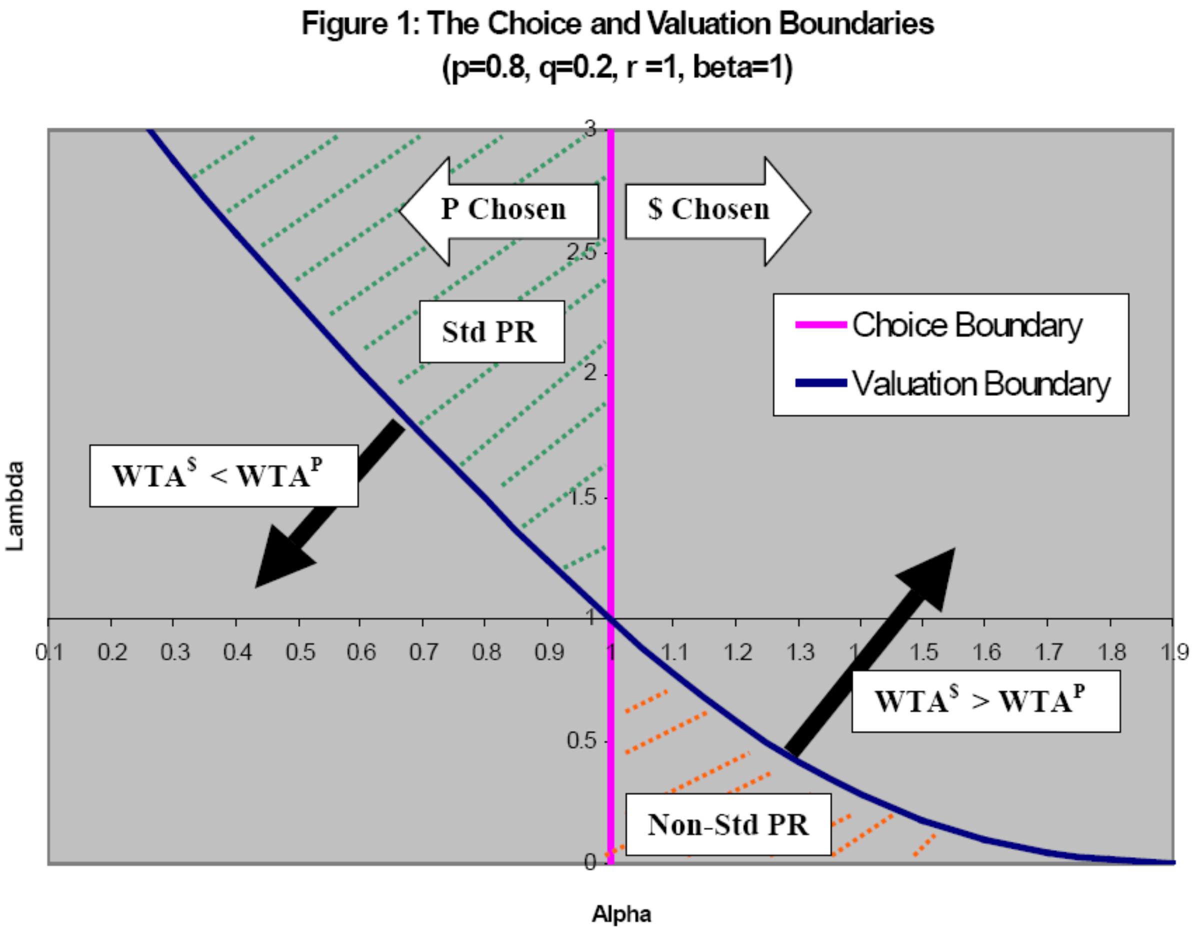


Figure 1 plots these two boundaries for a typical pair of bets, defined by (p, q, r) = (0.8, 0.2, 1), with $\beta = 1$. This particular combination of parameters will be called the *benchmark case*. Standard PR occurs in the region above the valuation boundary and to the left of the choice boundary; non-standard PR occurs in the region below the valuation boundary and to the right of the choice boundary. The two boundaries intersect at (1, 1). (This reflects the fact that, when $\alpha = 1$, $\lambda = 1$ and $\beta = 1$, our model reduces to the maximisation of expected value; since the two bets have equal expected value, they are equally preferred and have equal valuations.) In this benchmark case, the empirically plausible quadrant is made up of two sub-regions, separated by the valuation boundary. Above this boundary there is standard PR. Below it, the P bet is both preferred in the straight choice and valued more highly. Thus, our model predicts the classic asymmetry between standard and non-standard reversals: the former occur at parameter values within the empirically plausible quadrant, while the latter do not.

We now investigate the implications of moving away from the benchmark case. We begin by deriving some further general properties of parameterised PT^3 in relation to PR. First, we note that the benchmark case has the mathematically convenient property that $p = 1 - q$; pairs of bets with this property will be called *symmetrical*.

The following property of the valuation boundary can be derived by substituting $\lambda = 1$ and $\alpha = \beta$ into (11) and (12):

> *Property 3:* If $\alpha = \beta$ and $\lambda = 1$, then $WTA^P = 1$ and $WTA^\$ = r$, i.e. the valuation of each bet is equal to its expected value.

Notice that Property 3 implies that, if $r = 1$, the valuation boundary passes through the point $(\beta, 1)$ in (α, λ) space. With rather more manipulation (see Appendix), it is also possible to prove:

> *Property 4*: If the P and \$ bets are symmetrical and if $\lambda = 1$ and $\beta = 1$, $WTA^P = WTA^\$$ when $\alpha = \log(p/ q) / \log(rp/ q)$.

Given that the probability weighting function is specified by (8), symmetry has the convenient implication that $w(p) / w(q) = (p/ q)^\beta$. Substituting this equality into (10), we arrive at:

> *Property 5:* If the P and \$ bets are symmetrical, the choice boundary is the vertical line $\alpha = \beta \log(p/ q) / \log(rp/ q)$.

Notice that Property 5 implies that if $r = 1$, the choice boundary is $\alpha = \beta$. The following additional properties are derived in the Appendix:

Property 6: If $\beta < 1$ and $r = 1$, the choice boundary lies to the right of (respectively passes through, lies to the left of) the point $(\beta, 1)$ if $p + q$ is greater than (equal to, less than) 1.

Property 7: As $p, q \rightarrow 0$, $WTA^P / WTA^\$ \rightarrow (1/r)(q/p)^{1-\beta/\alpha}$. Thus, if $r = 1$, for any given α, β with $\alpha < \beta$ (respectively $\alpha = \beta$, $\alpha < \beta$), $WTA^P > WTA^\$$ (respectively $WTA^P = WTA^\$$, $WTA^P < WTA^\$$) for values of p and q sufficiently close to zero.

Property 8: If $q = 0.5$, $WTA^\$ > r \Leftrightarrow \lambda > 1$.

Property 7 tells us that, if $r = 1$, then as p and q approach zero, the valuation boundary approaches the vertical line $\alpha = \beta$. Since $WTA^P \rightarrow 1$ as $p \rightarrow 1$, it is an implication of Property 8 that, if $r = 1$ and $q = 0.5$, the valuation boundary converges to the horizontal line $\lambda = 1$ as p approaches unity.

Using Properties 1 to 8, we investigate configurations of choice and valuation boundaries as we move away from the benchmark case.

First, we maintain the benchmark assumptions $r = 1$ and $\beta = 1$, and consider the effects of variations in the values of p and q. As in the benchmark case, the choice and valuation boundaries intersect at (1, 1). We know from Property 7 that as p and q approach zero, the valuation boundary converges to the vertical line $\alpha = 1$, which is also the choice boundary; thus, in this limit, the region of standard preference reversal disappears. Now consider another limiting case, defined by $q = 0.5$ and $p \rightarrow 1$. We know from Property 8 that the valuation boundary converges to the horizontal line $\lambda = 1$; in this limit, the region of standard preference reversal takes up the whole of the empirically plausible quadrant. Figures 2a, 2b, 3a and 3b show the position of the valuation boundary for different values of p and q between these two limiting cases. Figures 2a and 2b show the effects of varying p, holding q constant at 0.1 (Figure 2a) and 0.4 (Figure 2b). Figures 3a and 3b show the effects of varying q, holding p constant at 0.9 (Figure 3a) and 0.7 (Figure 3b). Together, these figures reveal a tendency for the region of standard PR to expand as a consequence of increases in either p or q. In all cases, however, standard PR occurs in some part of the empirically plausible quadrant while non-standard PR occurs only outside this quadrant.

Figure 2a: Variation in P bet
(q=0.1, r=1, beta=1)

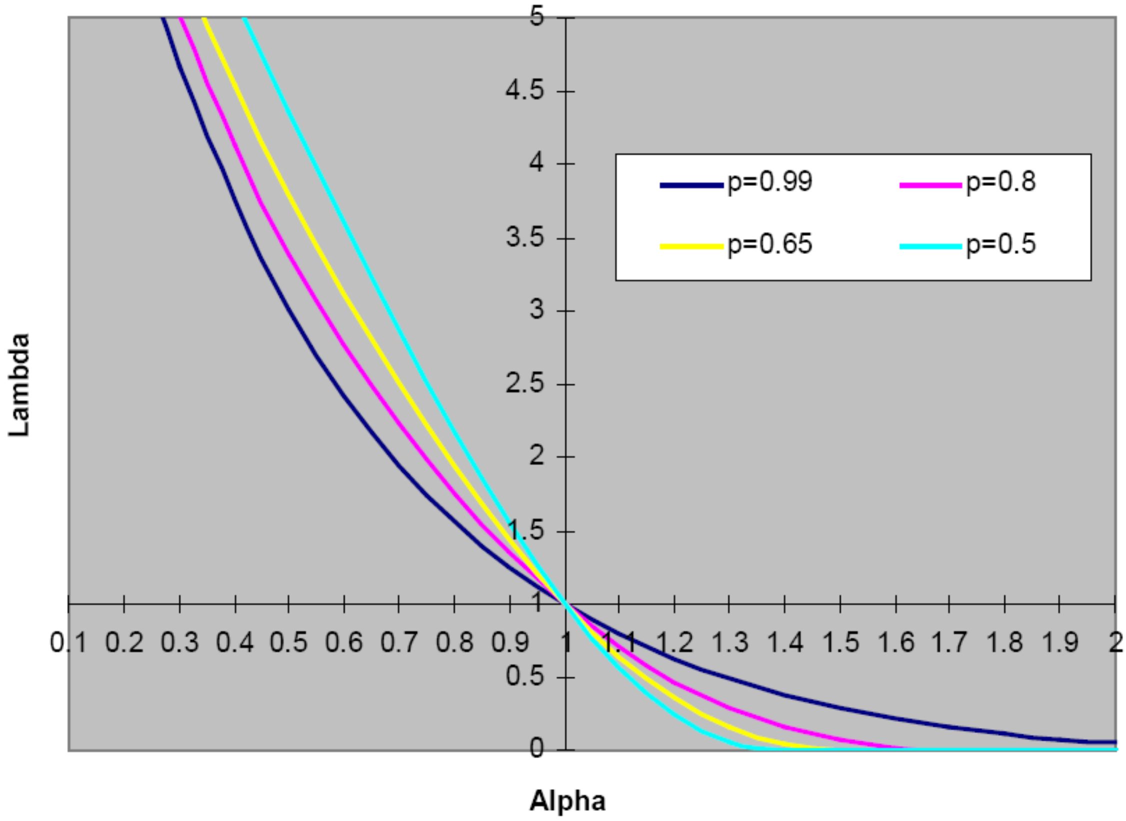


Figure 2b: Variation in P bet
(q=0.4, r=1, beta=1)

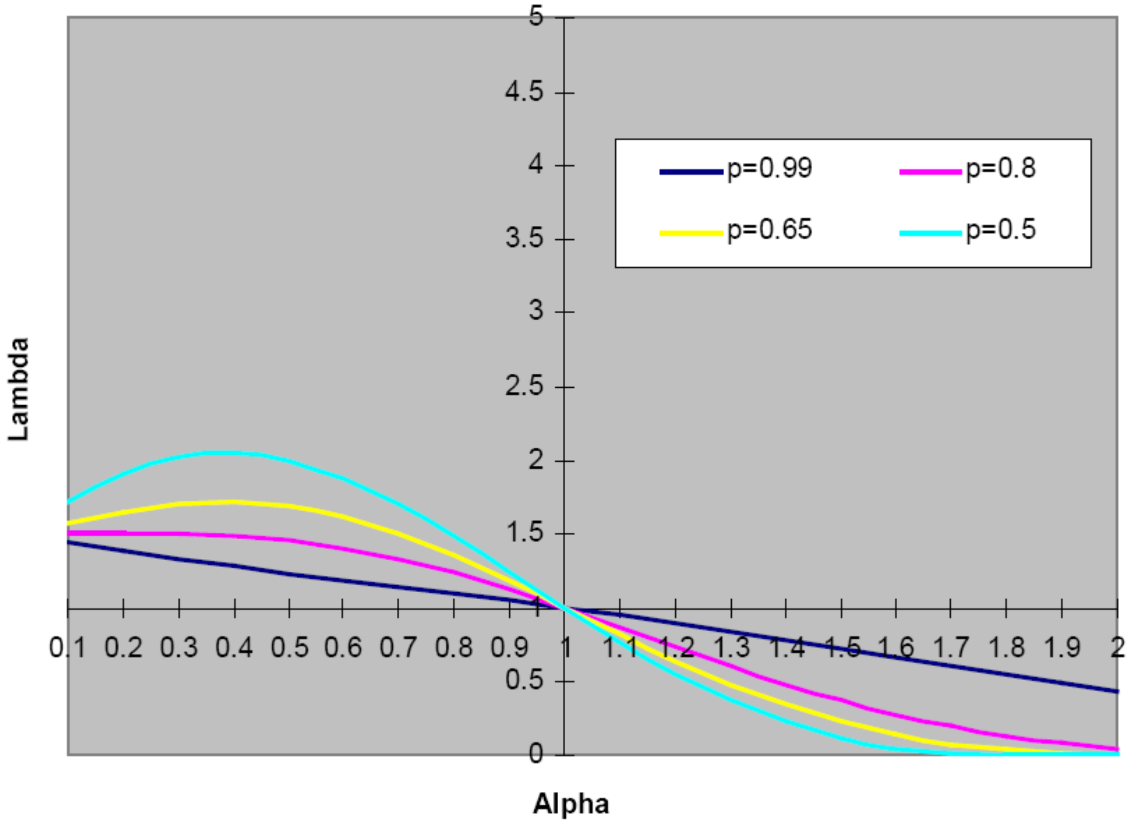


Figure 3a: Variation in $ bet
(p=0.9, r=1, beta=1)

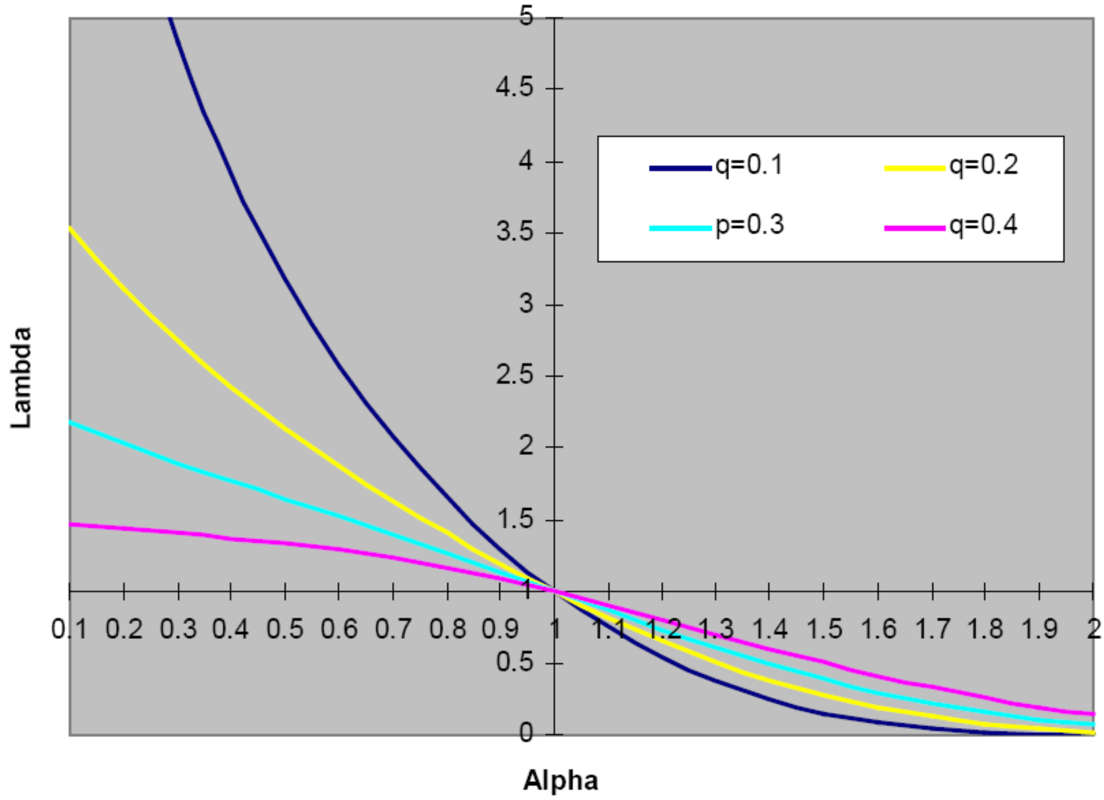
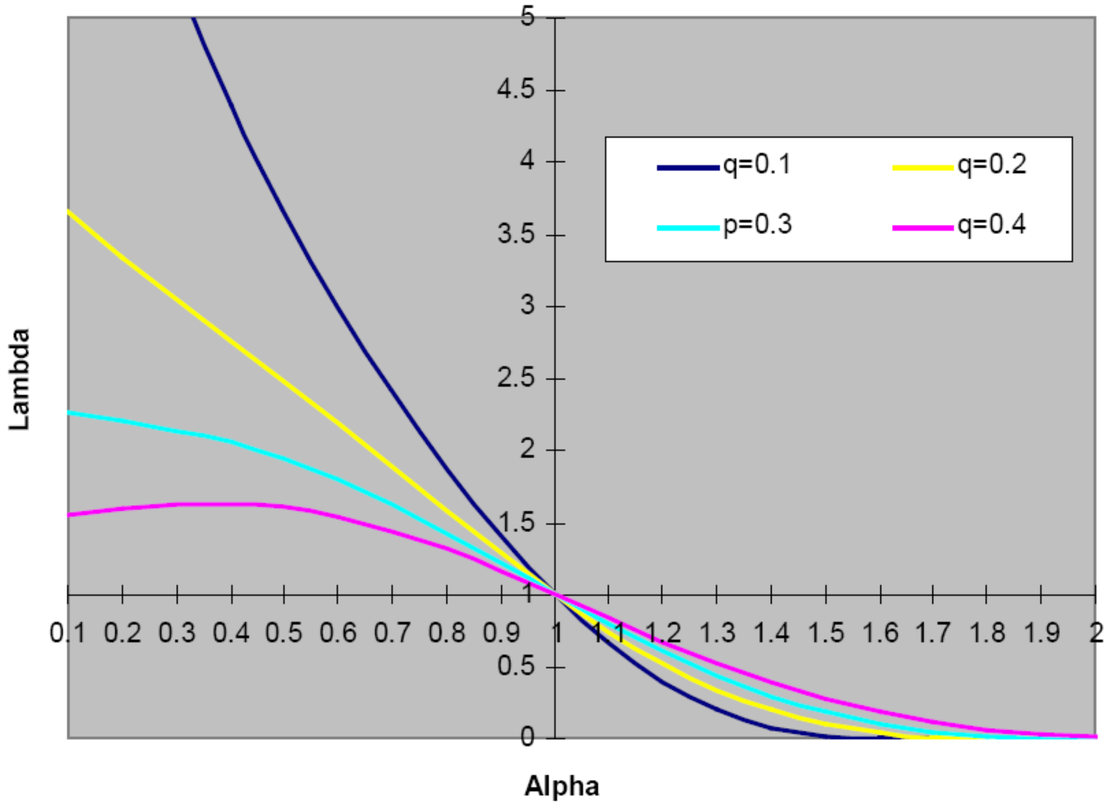
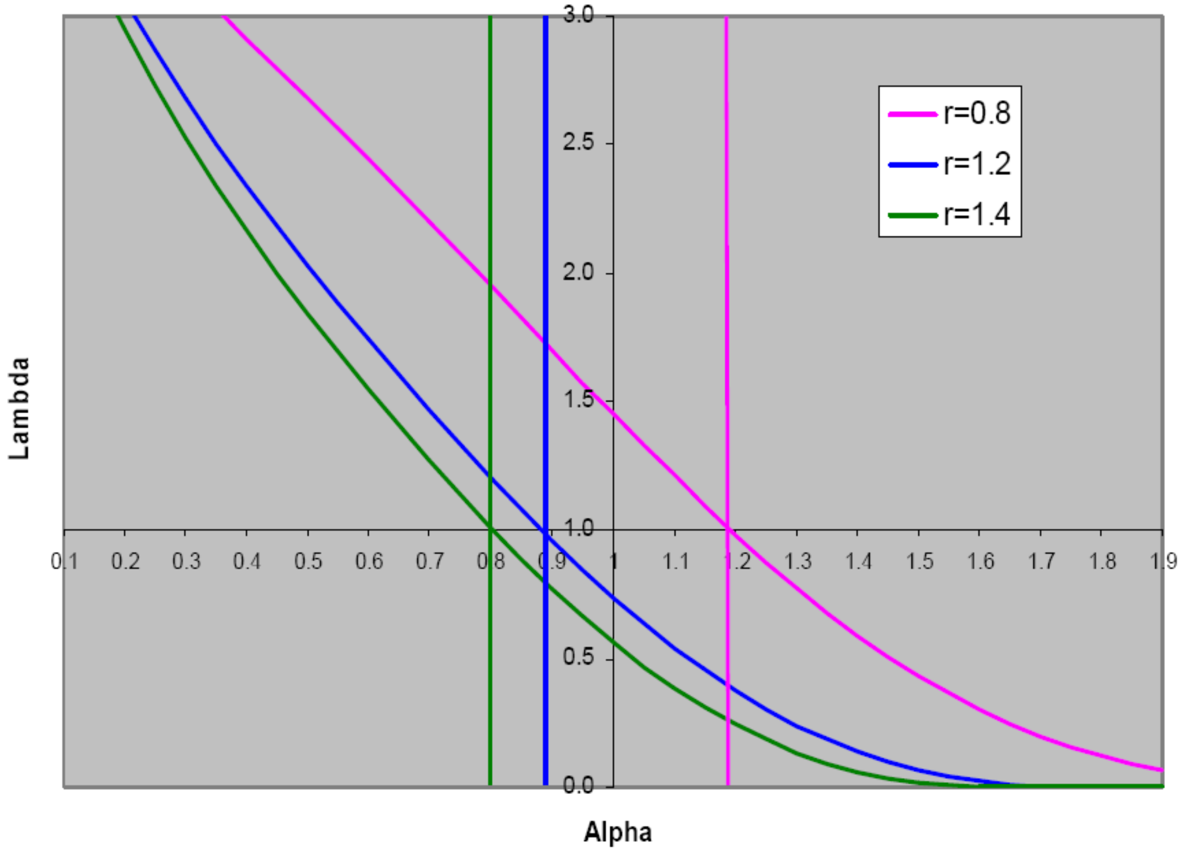


Figure 3b: Variation in $ bet
(p=0.7, r=1, beta=1)



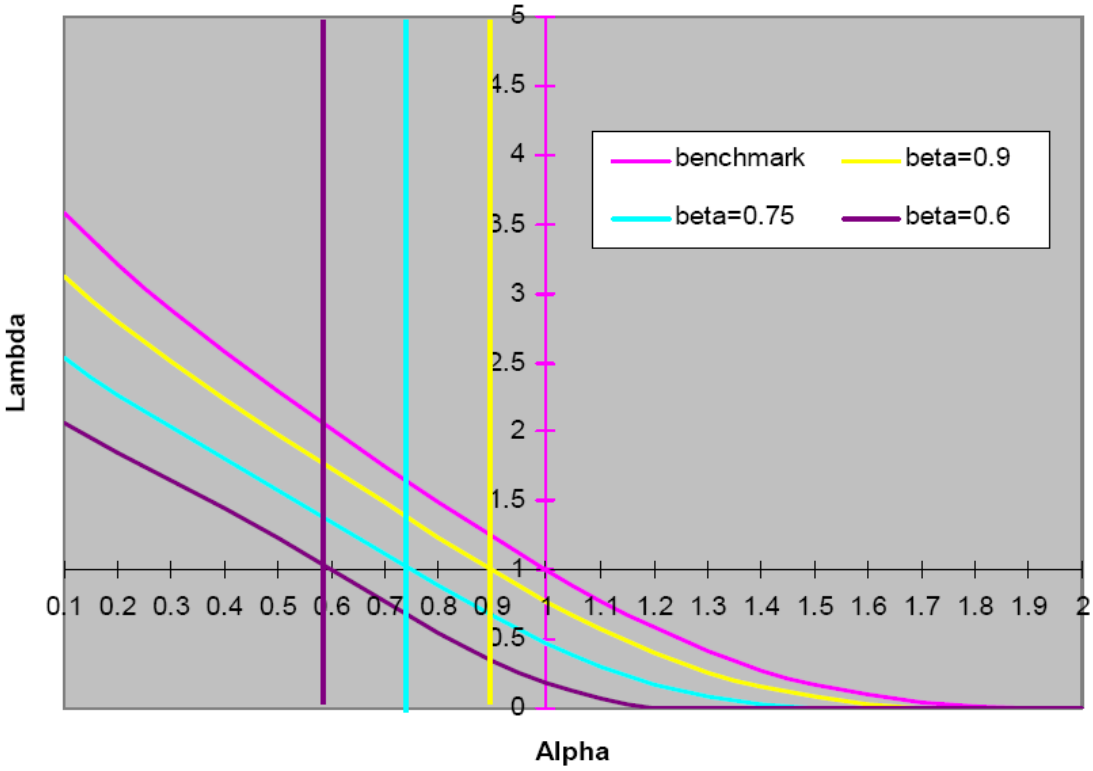
Next, maintaining the benchmark assumptions of symmetry and $\beta = 1$, we consider the effect of variations in the value of r. It follows from Properties 4 and 5 that the choice and valuation boundaries intersect at $(\alpha^*, 1)$, where $\alpha^* = \log(p/q) / \log(rp/q)$. As r increases, α^* falls, expanding the regions in which the \$ bet is favoured. The intuition for this is straightforward: an increase in r increases the \$-bet prize relative to the P-bet prize, and so makes the \$-bet relatively more attractive. Notice that if $r > 1$, the empirically plausible quadrant is made up of *three* sub-regions. Below the valuation boundary, the P bet is favoured in both choice and valuation. To the right of the choice boundary, the \$ bet is favoured in both choice and valuation. Above the valuation boundary and to the left of the choice boundary, there is standard PR. Non-standard PR occurs only outside the empirically plausible quadrant. Figure 4 shows the configurations of choice and valuation boundaries for $r = 0.8$, $r = 1.2$ and $r = 1.4$ when the other parameters take their benchmark values (i.e. $p = 0.8$, $q = 0.2$, $\beta = 1$).

Figure 4: variation in r
(p=0.8, q=0.2, beta = 1)



In the cases we have considered so far, our model has consistently predicted the classic asymmetry between standard and non-standard PR in the empirically plausible quadrant. However, it has failed to predict another stylised fact about PR experiments: that, even when the two bets have equal expected value, a significant proportion of subjects not only value the $ bet more highly but also choose it in preference to the P bet. Because of the role of diminishing sensitivity in choice, our model predicts that P will be chosen whenever r = 1, α < 1, and β = 1. To show that this is not a problem for our approach, we note that the benchmark assumption β = 1 is an extreme case – the case in which the probability weighting function is linear. We now consider the implications of assuming lower values of β, that is, an inverse-S function.

Figure 5: Variation in beta (p=0.8, q=0.2, r=1)



So, maintaining the benchmark assumptions of symmetry and r = 1, we investigate the effect of variations in β. It follows from Properties 3 and 5 that the choice and valuation boundaries intersect at (β, 1). Figure 5 plots these boundaries for the benchmark pair of bets (0.8, 0.2, 1) for three empirically plausible values of β, namely 0.9, 0.75 and 0.6. Essentially, the effect of reducing the value of β is to shift both boundaries to the left, expanding the regions in which the $ bet is favoured. The intuition for this is that, as the value of β falls, small probabilities (such as 0.2, the probability that the $ bet wins) are increasingly overweighted while large probabilities (such as 0.8, the probability that the P bet wins) are increasingly underweighted. The resulting configurations of choice and valuation boundaries are similar to those generated by setting r > 1. Again, the empirically plausible quadrant of

(α, λ) space is made up of three sub-regions. In one, the P bet is favoured in both choice and valuation; in another, the $ bet is favoured in both choice and valuation; in the third, there is standard PR.

All the diagrams we have presented so far have the common feature that standard PR occurs only when $\lambda > 1$, and non-standard PR occurs only when $\lambda < 1$. The reader should not infer from this that loss aversion is essential if PT^3 is to predict PR. To the contrary, both standard and non-standard PR are compatible with $\lambda = 1$ for some pairs of bets. That this is the case follows from Properties 3 and 6. Let $\beta < 1$, and $r = 1$. Property 3 tells us that the valuation boundary passes through the point (β, 1). Let α^* be the value of α at which the choice boundary crosses the line $\lambda = 1$. Property 6 tells us that if $p + q > 1$, then $\alpha^* > \alpha$. In other words, for values of α in the range $\beta < \alpha < \alpha^*$, standard PR occurs with $\lambda = 1$. Conversely, if $p + q < 1$, then $\alpha^* < \alpha$; for values of α in the range $\alpha^* < \alpha < \beta$, non-standard PR occurs with $\lambda = 1$. Figures 6a and 6b illustrate these possibilities for, respectively, the pairs of bets (0.8, 0.4, 1) and (0.6, 0.2, 1) with $\beta = 0.7$.

In principle, then, our model can predict PR in the absence of loss aversion. It can also predict PR in cases in which the model differs from expected utility theory *only* in respect of loss aversion. (Consider the case in which $p = 0.8$, $q = 0.2$, $r = 0.8$ and $\beta = 1$, shown in Figure 4. Notice that standard PR occurs at some points in (α, λ) space at which $\alpha = 1$ and $\lambda > 1$.) But these cases depend on special assumptions about the characteristics of the two bets. In contrast, the classic PR phenomenon occurs across a wide range of values of p, q and r. In particular, it occurs with $p + q < 1$, and it occurs with $r = 1$. It is a merit of our model that it explains PR across the range in which it has been observed.

The conclusion we wish to emphasise is this. Our model predicts the stylised facts of PR experiments on the assumption that subjects' values of the parameters α, λ and β are distributed over ranges that correspond with estimates derived from non-PR experiments – namely, values of α somewhat less than 1, values of λ somewhat greater than 1, and values of β somewhat less than 1. Loss aversion, diminishing sensitivity and inverse-S probability weighting are all implicated in our explanation of PR.

Figure 6a: bets (0.8, 0.4, 1) with beta = 0.7

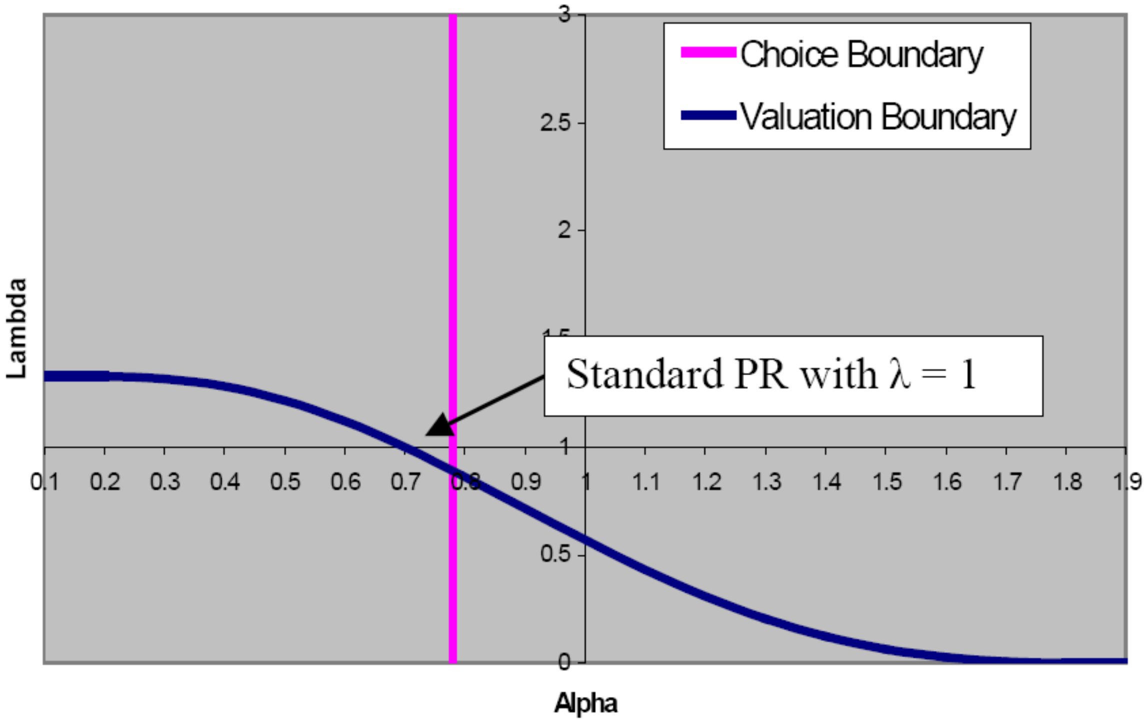
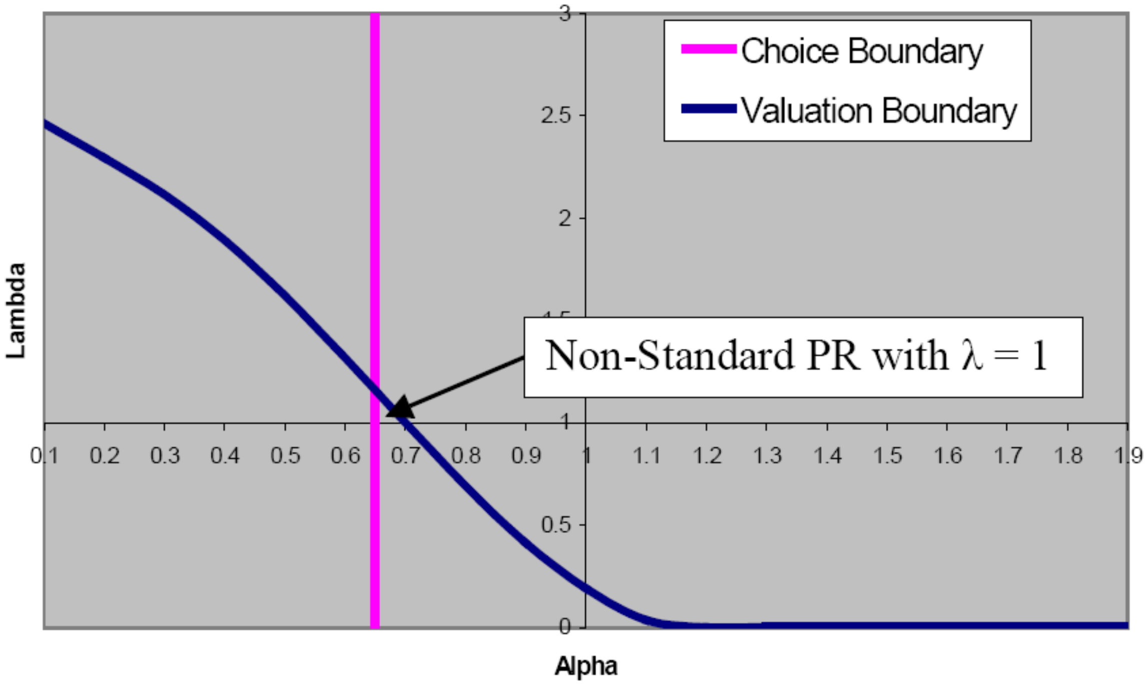


Figure 6b: bets (0.6, 0.2, 1) with beta = 0.7



7. Conclusions

In recent decades economists and psychologists have made considerable advances in understanding risky choice behaviour, prompted by anomalies relative to expected utility theory such as the Allais paradoxes. These developments have led economists to identify important new explanatory factors in decision making such as loss aversion and probability weighting. Moroever, theorists have found sophisticated ways of representing these factors in compact and tractable preference models. Explaining preference reversal (PR), however, has been a persistent stumbling block. While PR has long been recognised as an important departure from standard theory, and many theorists have attempted to provide preference-based explanations, we contend that no previous preference model has achieved this in an empirically satisfactory way.

We have presented a new model of risk preference: third generation prospect theory (PT^3). Our theory retains the empirically grounded features of prospect theory (loss aversion, diminishing sensitivity and non-linear probability weighting), but extends the model by allowing reference points to be lotteries. The resulting model retains all the predictive power of previous variants of prospect theory, but in addition provides a framework for determining the money valuation that an agent places on a lottery. Exploiting this feature of the model, we have shown that PR is consistent with PT^3. More significantly, when PT^3 is made operational by using simple functional forms with parameter values derived from existing experimental evidence, it predicts observed patterns of PR across a wide range of specifications of P and $ bets, consistent with the range in which PR has in fact been observed.

We do not claim that PT^3 provides a *complete* explanation of PR. We recognise that psychologists have proposed credible non-preference mechanisms of context-sensitive choice and valuation behaviour that are consistent with observations of PR. Predictions based on those mechanisms have been tested and confirmed in experimental tasks other than PR and, in some cases, outside the domain of theories of choice under uncertainty. This evidence clearly suggests that non-preference mechanisms contribute to PR. We assert only that PT^3 has a similar claim to be a model of mechanisms which contribute to that phenomenon. It too is based on psychologically credible hypotheses – loss aversion, diminishing sensitivity, the overweighting of small probabilities and the underweighting of large ones. It too is consistent with observations of PR. It too has been tested and confirmed in experimental tasks other than PR – namely, pairwise choices between lotteries involving gains and losses. If one accepts prospect theory as an explanation of observed regularities in choice among lotteries, it seems reasonable to conclude that the mechanisms modelled by PT^3 play a significant role in the explanation of PR.

More generally, we offer PT^3 as a flexible and parsimonious model of choice under uncertainty which organises a large body of experimental evidence. We hope that it will find fruitful applications in future work.

Appendix: Derivations of Properties 4, 6, 7 and 8

Property 4: Substituting $\lambda = 1$ and $\beta = 1$ into (11) and (12) gives $WTA^P = (1/p) / [((1-p)/p)^{1/\alpha} + 1] = (1/p) / [((1-p)^{1/\alpha} + p^{1/\alpha})/p^{1/\alpha}]$ and $WTA^\$ = (r/q) / [((1-q)/q)^{1/\alpha} + 1] = (r/q) / [((1-q)^{1/\alpha} + q^{1/\alpha})/q^{1/\alpha}]$. But by symmetry, $(1-p)^{1/\alpha} + p^{1/\alpha} = (1-q)^{1/\alpha} + q^{1/\alpha}$. Thus $WTA^P/WTA^\$ = (q/rp)(p/q)^{1/\alpha}$ and so $WTA^P = WTA^\$ \Leftrightarrow rp/q = (p/q)^{1/\alpha}$, i.e. $WTA^P = WTA^\$ \Leftrightarrow \alpha = \log(p/q)/\log(rp/q)$.

Property 6: Substituting $\alpha = \beta$, $r = 1$ and (8) into (9) and rearranging, we arrive at $f^P \succsim_h f^\$ \Leftrightarrow [q^\beta + (1-q)^\beta]^{1/\beta} \geq [p^\beta + (1-p)^\beta]^{1/\beta}$. Since $0 < p, q < 1$ and $\beta \leq 1$, this implies $f^P \succsim_h f^\$ \Leftrightarrow q^\beta + (1-q)^\beta \geq p^\beta + (1-p)^\beta$. If $\beta < 1$, the function $\phi(\pi) = \pi^\beta + (1-\pi)^\beta$ has an inverse U-shape, symmetrical around a maximum at $\pi = 0.5$. Thus, the P bet is chosen (respectively: the two bets are indifferent, the \$ bet is chosen) if q is closer than (respectively: equally close as, less close than) p to 0.5, i.e. if $p + q$ is greater than (respectively: equal to, less than) 1.

Property 7: From (13), as $p, q \to 0$, $WTA^P / WTA^\$ \to (1/r)\, [q^{1-\beta/\alpha} (1/\lambda)^{1/\alpha} + q] / [p^{1-\beta/\alpha} (1/\lambda)^{1/\alpha} + p]$. Since $0 < p, q < 1$ and $1 - \beta/\alpha < 1$, $(p^{1-\beta/\alpha}/ p) \to \infty$ as $p \to 0$, and $(q^{1-\beta/\alpha}/ q) \to \infty$ as $q \to 0$. Thus, $WTA^P / WTA^\$ \to (1/r)\, [q^{1-\beta/\alpha} (1/\lambda)^{1/\alpha}]/ [q^{1-\beta/\alpha} (1/\lambda)^{1/\alpha}]$, i.e. $WTA^P / WTA^\$ \to (1/r)\, (q/p)^{1-\beta/\alpha}$.

Property 8: Substituting $q = 0.5$ into (12) gives $WTA^\$ = 2r/ [(1/\lambda)^{1/\alpha} + 1]$. Thus $WTA^\$ > r \Leftrightarrow (1/\lambda)^{1/\alpha} < 1$. Since $\alpha > 0$, $(1/\lambda)^{1/\alpha} < 1 \Leftrightarrow \lambda > 1$.

References

Abdellaoui, M. (2000), Parameter-Free Elicitation of Utilities and Probability Weighting Functions, *Management Science* 46, 1497-1512.

Abdellaoui, M., F. Vossmann, and M. Weber (2005), Choice-based Elicitation and Decomposition of Decision Weights for Gains and Losses under Uncertainty, *Management Science* 51, 1384-1399.

Baucells, M., and F.H. Heukamp (2006), Stochastic Dominance and Cumulative Prospect Theory, *Management Science,* forthcoming.

Bell, D. (1982), Regret in Decision Making under Uncertainty, *Operations Research* 20, 961–81.

Birnbaum, M.H., and J.P. Bahra (2006), Gain-Loss Separability and Coalescing in Risky Decision Making, *Management Science*, forthcoming.

Bleichrodt, H., andJ.L. Pinto (2000), A Parameter-Free Elicitation of the Probability Weighting Function in Medical Decision Analysis, *Management Science* 46, 1485–1496.

Camerer, C. (1989), An Experimental Test of Several Generalized Utility Theories, *Journal of Risk and Uncertainty* 2, 61-104.

Casey, J. (1991), Reversal of the Preference Reversal Phenomenon*, Organizational Behavior and Human Decision Processes* 48, 224-251.

Cubitt, R.P., A. Munro, and C. Starmer (2004), Testing Explanations of Preference Reversal, *Economic Journal* 114, 709-726.

Davies, G.B., and Satchell, S.E. (2004), Continuous Cumulative Prospect Theory and Individual Asset Allocation, Cambridge Working Papers in Economics (CWPE 0467).

Handa, J. (1977), Risk, Probabilities, and a New Theory of Cardinal Utility*, Journal of Political Economy* 85, 97-122.

Holt, C.A. (1986), Preference Reversals and the Independence Axiom*, American Economic Review* 76, 508-515.

Humphrey, S. (2001), Non-Transitive Choice: Event-Splitting Effects or Framing Effects?, *Economica*, 68, 77-96.

Kahneman, D., and A. Tversky (1979), Prospect Theory: an Analysis of Decision under Risk, *Econometrica* 47, 263-291.

Karni, E., and Z. Safra (1987), Preference Reversals and the Observability of Preferences by Experimental Methods, *Econometrica* 55, 675-685.

Knez, M. and V. Smith (1987), Hypothetical Valuations and Preference Reversals in the Context of Asset Trading, in Roth, A. (ed.), Laboratory Experimentation in Economics: Six Points of View, Cambridge, Cambridge University Press, pp. 131-154.

Koszegi, B., and M. Rabin (2004), A Model of Reference-Dependent Preferences, Working Paper.

Lichtenstein, S. and P. Slovic (1971), Reversals of Preferences between Bids and Choices in Gambling Decisions, *Journal of Experimental Psychology* 89, 46-55.

Loomes, G., P. Moffatt and R. Sugden (2002), A Microeconometric Test of Alternative Stochastic Theories of Risky Choice, *Journal of Risk and Uncertainty*, 24, 103-130.

Loomes, G., C. Starmer, and R. Sugden (1989), Preference Reversal: Information Processing or Rational Non-Transitive Choice?, *Economic Journal* 99, Supplement, 140-151.

Loomes, G., C. Starmer, and R. Sudgen (1991), Observing Violations of Transitivity by Experimental Methods, *Econometrica* 59, 425-439.

Loomes, G., and R. Sugden (1983), A Rationale for Preference Reversal, *American Economic Review* 73, 428-432.

Luce, D., and P.C. Fishburn (1991), Rank- and Sign-Dependent Linear Utility Models for Finite First-Order Gambles, *Journal of Risk and Uncertainty* 4, 29-59.

Prelec, D. (1998), The Probability Weighting Function, *Econometrica* 66, 497-527.

Quiggin, J. (1982), A Theory of Anticipated Utility, *Journal of Economic Behavior and Organization*, 3, 323-343.

Schmidt, U., and .D. Hey (2004), Are Preference Reversals Errors? An Experimental Investigation, *Journal of Risk and Uncertainty* 29, 207-218.

Segal, U. (1988), Does the Preference Reversal Phenomenon Necessarily Contradict the Independence Axiom?, *American Economic Review* 78, 233-236.

Slovic, P., D. Griffin, and A. Tversky (1990), Compatibility Effects in Judgment and Choice, in Hogarth, R., (ed.) Insights in Decision-Making, Chicago, University of Chicago Press, pp. 5-27.

Starmer, C. (2000), Developments in Non-expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk, *Journal of Economic Literature* 38, 332-382.

Starmer, C., and R. Sugden (1989), Probability and Juxtaposition Effects: An Experimental Investigation of the Common Ratio Effect, *Journal of Risk and Uncertainty* 2, 159-178.

Starmer, C., and R. Sugden (1998), Testing Alternative Explanations of Cyclical Choices, *Economica* 65, 347-361.

Sugden, R. (2003), Reference-dependent Subjective Expected Utility, *Journal of Economic Theory* 111, 172-191.

Trepel, C., C.R. Fox, and R.A. Poldrack (2005), Prospect Theory on the Brain? Toward a Cognitive Neuroscience of Decision under Risk, *Cognitive Brain Research* 23, 34-50.

Tversky, A., and D. Kahneman (1992), Advances in Prospect Theory: Cumulative Representation of Uncertainty, *Journal of Risk and Uncertainty* 5, 297-323.

Tversky, A., P. Slovic, and D. Kahneman (1990), The Causes of Preference Reversal, *American Economic Review* 80, 204-217.

Wakker, P.P., and A. Tversky (1993), An Axiomatization of Cumulative Prospect Theory, *Journal of Risk and Uncertainty* 7, 147-176.

Wu, G., and R. Gonzalez (1996), Curvature of the Probability Weighting Function, *Management Science* 42, 1676-1690.

Wu, G., and R. Gonzalez (1999), Nonlinear Decision Weights in Choice under Uncertainty, *Management Science* 45, 74-85.

Wu, G., J. Zhang, and M. Abdellaoui (2005), Testing Prospect Theory using Probability Tradeoff Consistency, *Journal of Risk and Uncertainty* 30, 107-131.

Notes

[1] A related theory is proposed by Koszegi and Rabin (2004) who present a version of prospect theory in which reference points are expectations about outcomes. Koszegi and Rabin define preferences over prospects (i.e. probability distributions over outcomes) rather than acts. In terms of our framework, this approach can be thought of as assuming that the act being evaluated is stochastically independent of the reference act.

[2] Our analysis of PR does not depend on the cumulative transformation of probabilities. The acts that we analyse have no more than one strictly positive consequence and no more than one strictly negative one. For such acts, the cumulative transformation is observationally equivalent to Handa's simple transformation.

[3] If there are distinct states s_i, s_j such that $v(f(s_i), h(s_i)) = v(f(s_j), h(s_j))$, there may be more than one way of re-assigning subscripts consistently with these conditions. However, all permissible re-assignments generate the same value of $V(f, h)$.

[4] In addition, $f^P \sim_h f^\$ \Leftrightarrow w^+(p)\, v(x, 0) - w^+(q)\, v(y, 0) = 0$. From now on, to avoid cluttering the exposition, we will not state conditions for indifference explicitly. In all cases, the condition for indifference can be constructed from the condition for weak preference by substituting an equality for a weak inequality.

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