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SIMPLE PREFILTER DESIGN IN GPC FOR A WIDE CLASS OF INDUSTRIAL PROCESSES

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Abstract: This paper presents a simple design method for improving the robustness of the GPC using the T polynomial as a filter. Although different methods have been proposed in literature, the one presented here not only proposes a choice of the polynomial, but differs from the others in the way that the polynomial is used to filter the predictions. The method is valid for a wide range of processes commonly found in industry, those that can be described by means of a static gain, time constant and dead-time and has special interest when the process has uncertainties in the dead-time. *Copyright © 1999 IFAC*

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1. INTRODUCTION

As is well known, Generalized Predictive Control (GPC) uses a CARIMA model to calculate the predictions along the horizon (Clarke *et al.*, 1987). This model has the following form:

$$A(z^{-1})y(t) = z^{-d}B(z^{-1})u(t-1) + \frac{T(z^{-1})e(t)}{\triangle} \quad (1)$$

$$A(z^{-1}) = 1 + a_1z^{-1} + a_2z^{-2} + ... + a_{na}z^{-na}$$
$$B(z^{-1}) = b_0 + b_1z^{-1} + b_2z^{-2} + ... + b_{nb}z^{-nb}$$
$$T(z^{-1}) = t_0 + t_1z^{-1} + t_2z^{-2} + ... + t_{nt}z^{-nt}$$

$y(t)$ is the plant output, $u(t)$ is the control action, $\triangle = 1 - z^{-1}$ and d is the dead-time of the system.

In this model the T-polynomial represents the stochastic characteristics of the noise and the disturbances. The main objective of the use of $T(z^{-1})$ is to reduce the effect of the noise and disturbances on the predicted output without affecting the nominal setpoint tracking. On the other hand, low frequency disturbances can be removed by the $\triangle$ operator that appears in the prediction.

However, in industrial practice it is difficult to estimate the real characteristics of the noise, so $T(z^{-1})$ is rarely estimated but best treated as a filter. If T is appropriately chosen, the roll-off of the filter can attenuate the prediction error caused by model mismatch, which is particulary important at high frequencies. Notice that dead-time uncertainties are one of the most characteristic high-frequency unmodelled dynamics, and have a dangerous influence on the closed-loop stability since they can deteriorate the system phase margin.

The T polynomial seems to be a good solution, but its correct choice is a problem that has not completely been solved, although its effect on the robustness of the closed loop system has been analyzed in several papers (Clarke *et al.*, 1987; Clarke and Mothadi, 1989; Robinson and Clarke, 1991; Yoon and Clarke, 1995). In these papers the authors deal with the correct tuning of

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the prefilter in order to increase some robustness indices in certain particular cases of GPCs. A systematic two-step design for the GPC taking into account performance and robustness is presented in (Ansay and Wertz, 1997). But despite these efforts, the selection of T is still an open field.

In this paper the T polynomial is used as a filter in order to improve the robustness of the GPC, but in a different way as in the referred works. The proposed method makes the controller coefficients independent of T, making the controller tuning and the robustness improvement independent. Then the tuning of the prefilter is very simple and can be performed taking into account both the disturbance rejection performance and robustness of the closed-loop. This latter property is very important because it permits the tuning of the controller in two steps; first the GPC parameters are chosen to attempt the nominal set-point performance specifications and then the filter is chosen for the disturbance rejection and robust performance conditions. Also the proposed method is simpler than the T tuning method presented in (Ansay and Wertz, 1997).

There are several claims that in practice low-order models coupled with dead-times are sufficient for most purposes (Chien and Fruehauf, 1990). In process control, reduced-order modelling is often employed where a low-order transfer function plus a dead-time is used to represent the dynamic behaviuor of a high-order process (Tan *et al.*, 1996). The method described in this paper has been developed for processes that are described by a simple model consisting of a static gain, time constant and dead-time. As the model is low order, high-frequency uncertainties are easy to be found, being the dead-time uncertainties the most dangerous ones, since they cause a considerable robustness loss (Camacho and Bordons, 1999).

The paper is organized as follows: A closed-loop analysis of the GPC, where an equivalent structure of the controller is derived, is presented in section 2. In section 3 the proposed structure of the prefiltered GPC is presented and the T tuning method is given. Section 4 is dedicated to presenting some simulation results and finally the conclusions and perspectives of the work are presented in section 5.

2. CLOSED-LOOP ANALYSIS OF THE GPC

The GPC algorithm (Clarke *et al.*, 1987) consists of applying a control sequence that minimizes a multistage cost function of the form

$$J(N_1, N_2, N_3) = \sum_{j=N_1}^{N_2} [\hat{y}(t+j \mid t) - w(t+j)]^2 +$$

$$+ \sum_{j=1}^{N_3} \lambda(j)[\triangle u(t+j-1)]^2 \qquad (2)$$

where N_1 and N_2 are the minimum and maximum prediction horizons, N_3 is the control horizon, $\lambda(j)$ is the control weighting sequence, $w(t+j)$ is a future set-point or reference sequence, $\triangle u(t)$ is the incremental control action and $\hat{y}(t+j \mid t)$ is the optimum j-step ahead prediction of the system output on data up to time t computed using the CARIMA model of equation (1). For the analysis of this paper it will be considered that $A(z^{-1}) = 1 + a_1 z^{-1}$ $(na = 1)$, $B(z^{-1}) = b_0$ $(nb = 0)$ and that the process always exhibits a dead-time greater than one sample $(d > 0)$. Without loss of generallity and as the process exhibits a dead-time d, the horizons are chosen as $N_1 = d+1$, $N_2 = d+N$, $N_3 = N$ and $\lambda(j) = \lambda$.

To compute the optimal prediction $\hat{y}(t+j \mid t)$ the following Diophantine equation is used (for the sake of clarity the dependency on z^{-1} is omitted from now on):

$$T = E_j \triangle A + z^{-j} F_j \qquad (3)$$

where the polynomials E_j (degree of $E_j = j-1$) and F_j (degree of $F_j = na = 1$) can be obtained by dividing T by $\tilde{A} = \triangle A$ until the remainder of the division can be factorized as $z^{-j}F_j$.

Using (1) and (3), it follows that:

$$T\hat{y}(t+j \mid t) = E_j B \triangle u(t+j-d-1) +$$
$$+ F_j y(t) + T E_j e(t+j) \qquad (4)$$

Taking the expected value of the previous equation and assuming that the expected values of the future errors are zero, the optimal prediction is given by:

$$T\hat{y}(t+j \mid t) = E_j B \triangle u(t+j-d-1) + F_j y(t) \qquad (5)$$

For this paper it is interesting to separate the computation of the predictions in two sets: from $t+1$ to $t+d$ as:

$$T\hat{y}(t+j \mid t) = E_j B \triangle u(t+j-d-1) + F_j y(t) \qquad (6)$$

with $j = 1 \ldots d$ and from $t+d+1$ to $t+d+N$ as:

$$T\hat{y}(t+d+j \mid t) = E_j B \triangle u(t+j-1) +$$
$$+ F_j \hat{y}(t+d) \qquad (7)$$

with $j = 1 \ldots N$. Note that for the model considered in this paper $F_j = f_{j0} + f_{j1} z^{-1}$.

Solving (7) it is possible to write:

$$\hat{\mathbf{y}}_+ = \mathbf{S}_1 \mathbf{u} + \mathbf{S}_2 \hat{\mathbf{y}}_- \qquad (8)$$

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where $\hat{\mathbf{y}}_{+} = (\hat{y}(t+d+1 \mid t)\ \hat{y}(t+d+2 \mid t) \ldots \hat{y}(t+d+N \mid t))^T$, $\mathbf{u}$ is the vector of future control action increments and $\hat{\mathbf{y}}_{-} = [\hat{y}(t+d \mid t)\ \hat{y}(t+d-1 \mid t)]^T$. $\mathbf{S}_1$ and $\mathbf{S}_2$ are constant matrices of dimension $N \times N$ and $N \times 2$ respectively and their coefficients are functions of the coefficients of the polynomials T, E_j and F_j (Camacho and Bordons, 1999). If the obtained predictions are introduced in the cost function, $J(N)$ is a function of $\hat{\mathbf{y}}_{-}$, $\mathbf{u}$ and the reference sequence. Minimizing $J(N)$ with respect to the future control actions and keeping only $\triangle u(t)$ it is possible to write (Camacho and Bordons, 1999):

$$\triangle u(t) = l_{y1}\hat{y}(t+d \mid t) + l_{y2}\hat{y}(t+d-1 \mid t) + + \sum_{i=1}^{N} k_i w(t+d+i) \tag{9}$$

where l_{yi} and k_i are functions of the model parameters b_0 and a_1 and of the controller parameters N, λ and T. The control scheme is shown in figure 1.

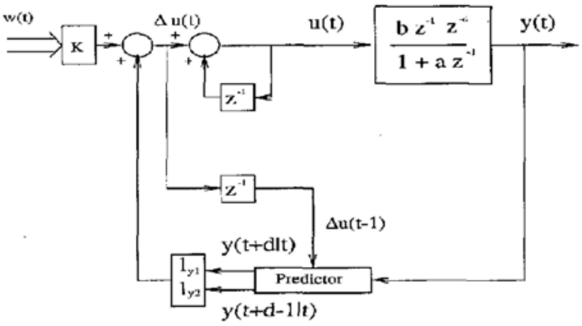

Fig. 1. Control Scheme of the GPC

The predictions in (9) can be computed using the solution of (6) as follows. First consider the solution for $j = 1$:

$$T = E_1 \triangle A + z^{-1} F_1 \tag{10}$$

where $E_1 = 1$ and $F_1 = z(T - \tilde{A})$. Using $\tilde{B} = \triangle B$ the prediction is:

$$T\hat{y}(t+1 \mid t) = \tilde{B}u(t-d) + z(T-\tilde{A})y(t) = = \tilde{B}z^{-d}u(t) + z(T-\tilde{A})y(t) \tag{11}$$

Using (11) the prediction $\hat{y}(t+2 \mid t)$ can be computed as:

$$T\hat{y}(t+2 \mid t) = \tilde{B}z^{-d}u(t+1) + +z(T-\tilde{A})\hat{y}(t+1 \mid t) \tag{12}$$

Multiplying (12) by T and using (11):

$$T^2\hat{y}(t+2 \mid t) = \tilde{B}z^{-d}\left[T + (T-\tilde{A})\right]u(t+1) + +z^2(T-\tilde{A})^2 y(t) \tag{13}$$

Using the same procedure till $t+d$, it can be obtained that:

$$\hat{y}(t+d \mid t) = \tilde{B}\frac{z^{-d}}{T}\left[\frac{1-(1-\tilde{A}/T)^d}{\tilde{A}/T}\right]u(t+d-1) + +z^d(1-\tilde{A}/T)^d y(t) \tag{14}$$

considering the plant model $P(z) = \frac{Bz^{-1-d}}{A}$ and $R(z) = z^d(1-\tilde{A}/T)^d$, it follows that:

$$\hat{y}(t+d \mid t) = P(z^d - R)u(t) + Ry(t) = = Pz^d u(t) + R(y(t) - Pu(t)) \tag{15}$$

Using this equation the final control scheme for the GPC can be drawn as in figure 2 where the relation between the control action and the predictions and references has been expressed as a two degree of freedom primary controller.

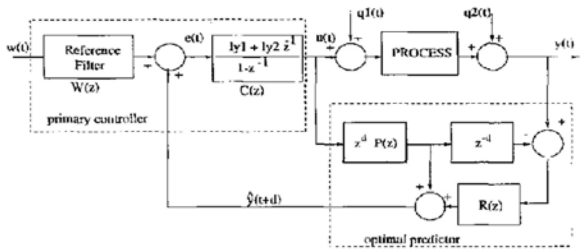

Fig. 2. Classical representation of the GPC

As can be seen only parameter T of the controller affects the computation of the prediction $\hat{y}(t+d \mid t)$ while all parameters (N, λ and T) affect the primary controller. This structure will be used in the next section to analyze the robustness of the GPC.

3. A SIMPLE DESIGN METHOD OF THE T-POLYNOMIAL

The study of the robustness of the GPC is made using the block diagram of figure 2. The plant will be represented by a transfer function P and unstructured uncertainties will be considered, that is: $P = P_n + DP$ with P_n the nominal plant. Also $P = Gz^{-d}$ where G represents the plant without the dead-time and G_n is its nominal value.

$W(z)$ and $C(z)$ represent the two-degree of freedom primary controller in figure 2. To evaluate the closed-loop performance the transfer function between the reference and the output is computed for the nominal case:

$$y(t) = \frac{WCP_n}{1+CG_n}r(t) \tag{16}$$

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that is, as expected, d is eliminated from the characteristic equation. Note that R does not appear in (16).

The norm-bound uncertainty region for $DP(e^{j\omega})$ ($\triangle P(\omega)$) is computed in order to maintain closed-loop stability $\forall \omega \in (0, \pi)$ (Morari and Zafiriou, 1989):

$$\mid DP(e^{j\omega}) \mid \leq \triangle P(\omega) = \frac{\mid 1 + C(e^{j\omega})G_n(e^{j\omega}) \mid}{\mid C(e^{j\omega})R(e^{j\omega}) \mid} \quad (17)$$

As can be seen, after the definition of C and W the filter R could be used to improve the robustness of the controller. As R is a function of T, this polynomial could be chosen in order to obtain a low pass filter in R. In this way the proposed approach shows clearly the relation between the robustness of the GPC and T. However the selection of T is not straightforward because also C and W have $T-$dependence. Note that T is used to compute the predictions from $t+d+1$ to $t+d+N$ and so the final coefficients of the control law (l_i and k_i) depend on the coefficients of T. This is the principal reason for the not trivial methods presented in the literature to design T (Yoon and Clarke, 1995; Ansay and Wertz, 1997). As pointed out in (Yoon and Clarke, 1995) increasing the low pass characteristics of T do not necessary improve the robustness of the GPC. From the block diagram of the GPC shown in figure 2 these properties could be easily seen.

But this problem can be solved using the method proposed in this paper, that is, a design procedure for T that allows the independence between performance and robustness with a very simple method. First note that if T is only used to compute the optimal predictions from $t+1$ to $t+d$ and an unitary polynomial $T = 1$ is used to compute the predictions from $t + d + 1$ to $t + d + N$ the dependence of C and W with T is eliminated since the first d predictions do not appear in the cost function and therefore they do not affect the controller coefficients. On the other hand R is a function of T and could be used to improve the robustness without changing the closed-loop relation (16) which defines the nominal performance of the controller. Now the tuning procedure can be done in two independent steps: (i) compute N and λ for closed loop performance and (ii) compute T in order to define the low pass characteristics of R. This two step design procedure relates the proposed controller with the internal model control IMC (Morari and Zafiriou, 1989) where a stabilizing controller is computed in a first step and in a second step a filter is included to improve the robustness. The same idea is also used in (Normey-Rico *et al.*, 1997) where the robustness of an industrial dead-time compensator controller is increased with a low pass filter.

As has been mentioned, the filter R could be used to improve the robustness of the system at the desired frequency region without modifying the reference to output nominal performance. However, the disturbance rejection performance of the system is affected by R as is the case in others $T-$design methods for the GPC (Yoon and Clarke, 1995; Ansay and Wertz, 1997). But in this controller the tuning of the filter in order to attempt a compromise between robustness and disturbance rejection is straightforward. Note that the closed-loop transfer function between the input disturbance q_1 and the control action u can be used as a messurement of the disturbance rejection qualities of the system. In general, for a good disturbance rejection performance $\mid u/q_1 \mid$ must be close to one for all the frequencies in the desired bandwidth:

$$\left|\frac{u(e^{j\omega})}{q_1(e^{j\omega})}\right| = \left|\frac{C(e^{j\omega})P_n(e^{j\omega})R(e^{j\omega})}{1 + C(e^{j\omega})G_n(e^{j\omega})}\right| = 1 \quad (18)$$

On the other hand, if we compute the multiplicative uncertainty norm boundary of the system, it follows that:

$$\delta P(\omega) = \frac{\triangle P(\omega)}{\mid P_n(e^{j\omega}) \mid} = \frac{\mid 1 + C(e^{j\omega})G_n(e^{j\omega}) \mid}{\mid C(e^{j\omega})P_n(e^{j\omega})R(e^{j\omega}) \mid} = \left|\frac{q_1(e^{j\omega})}{u(e^{j\omega})}\right| \quad (19)$$

That is, the higher disturbance rejection performance gives lower robustness. Notice that similar results can be obtained for the output disturbance $q_2(t)$.

As, in general, the model uncertainties are dominant at high frequencies, R must be chosen in order to increase the value of δP at those frequencies, but maintaining the unitary gain of u/q_1 for the frequencies above ω_0, that is, there is a compromisse between robustness and disturbance rejection.

To compute the polynomial T the expression of R is used: $R(z) = z^d(1 - \tilde{A}/T)^d$. Note that as $\tilde{A}(1) = 0$, $R(1) = 1$ for every $T \neq 0$. For the type of stable plant considered here and in order to eliminate the dependence between R and A the following T is proposed:

$$T = (1 + a_1 z^{-1})(1 - a_t z^{-1}) \qquad a_t > 0 \quad (20)$$

Thus R is given by:

$$R = (\frac{1 - a_t}{1 - a_t z^{-1}})^d \qquad a_t > 0 \quad (21)$$

that is, R is a $d-$order low pass filter. The value of a_t is chosen having an estimation of the

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uncertainties and d automatically compensates the effect of the dead-time in the controller.

As was analyzed in (Camacho and Bordons, 1999), for the type of processes analyzed in this paper, the deterioration in the performance caused by the uncertainties in d is higher for higher values of the nominal dead-time. If no filter is used, a simple error of one sample could cause unstability if the nominal dead-time is 10 or higher. Because of that, the use of T is necessary in real applications when the process exhibits significant dead time. With the normal procedure used in GPC the design of T is not easy and in general high order filters are nedded to stabilize the closed-loop (Yoon and Clarke, 1995). For the method proposed here a simple T is used that could stabilize process with long dead-times and high frequency uncertainties. To show the effect of the proposed T two different dead-times and different errors in d are considered for the same gain and pole of the plant when considering $\lambda = 0.8$ and $N = 15$ that allow for a good nominal setpoint response. In both cases the same $T = A(1-0.8z^{-1})$ is used and the norm-bound uncertainty is compared to the modelling error and to the norm-bound uncertainty of the non-filtered GPC. In figure 3.a the nominal dead-time is $d = 2$ and the real dead-time is $d_p = 3$ and in figure 3.b the nominal dead-time is $d = 7$ and the real dead-time is $d_p = 9$. Note that while for the small dead-time case the GPC with $T = 1$ is stable even in the presence of the error in the estimation of d, in the other case the system is not robust stable. On the other hand the proposed filter with the same tuning gives a stable closed-loop in both cases. Note that the price to pay for increasing robustness is a smaller bandwidth for disturbance rejection as can be seen in figure 4.

4. A SIMULATION CASE STUDY

In order to illustrate properties of the proposed design method of the T polynomial in the GPC, a simulation example is presented that corresponds to the temperature control in a heat exchanger. The model corresponds to a pilot plant at the "Departamento de Ingeniería de Sistemas y Automática de la Universidad de Sevilla" that is described with details in (Normey-Rico *et al.*, 1997). The model of the plant, obtained with the "step test" identification method is given by:

$$P(s) = \frac{k}{1+s\tau}e^{-t_d}$$

where the nominal values of the parameters are: $t_d = 14s, k = 0.11$ and $\tau = 6s$. By performing several identification tests with different operation conditions it was possible to note an uncertainty in the values of the plant parameters. The gain k

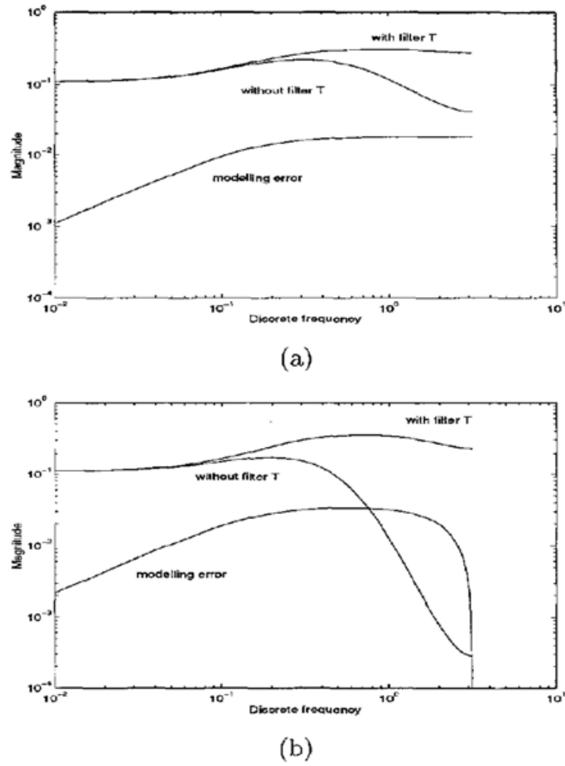

Fig. 3. Modelling error and additive norm-bound uncertainty for the GPC with filter T and without filter T for small dead-time (a) and long dead-time (b).

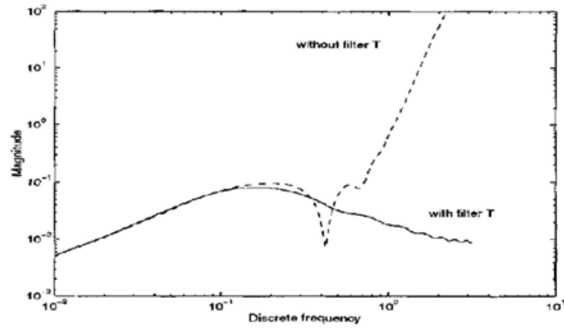

Fig. 4. Modulus of the disturbance to output transfer fuction for the long dead-time case

could vary between 0.08 and 0.13, the dead-time between 11 and 18 seconds and τ between 5.7 and 6.3 seconds.

To show the performance and robustness of the proposed method for the tuning of the $T-$polynomial in the GPC two simulation tests were performed with the same control parameters $N = 15$, $\lambda = 1$ and the sampling time $T_s = 1s$. In the first simulation a GPC with filter $T = 1$ is compared to a GPC with filter $T = A(1-0.8z^{-1})$. The closed loop behaviour of both control systems when parameter uncertainties are considered

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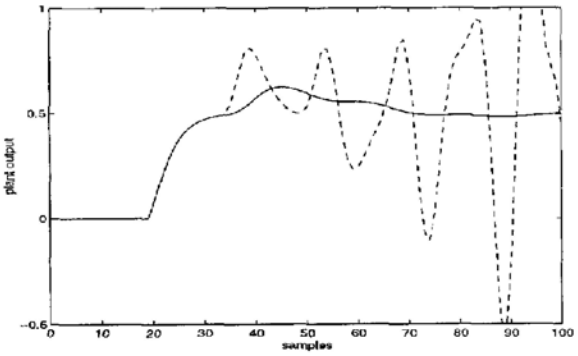
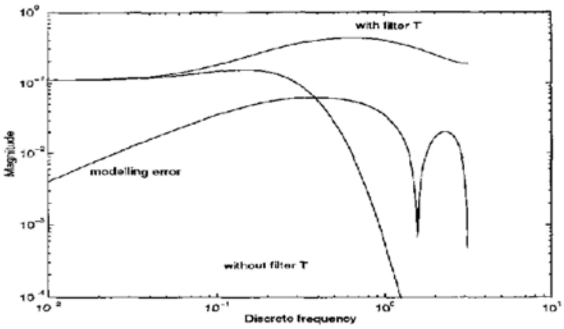
($t_d = 18s, k = 0.12$ and $\tau = 5.7s$) is shown in figure 5.a. Note that the GPC with filter $T = 1$ is unstable. In the test a step change in the reference is performed at $t = 0$ samples. To analyse the improvment in the robustness of the controller the modelling error $\mid DP \mid$ and the norm-bound uncertainty ΔP are shown in figure 5.b. Note that the proposed low order filter allows a robust stable behaviour of the controller.



(a)



(b)

Fig. 5. (a) Behaviour of the GPC with filter T (solid line) and without filter T (dashed line), (b) Modelling error and norm-bound uncertainty for the GPC with and without T

In order to evaluate the disturbance rejection properties of the controller a second simulation is presented in figure 6. In this case at $t = 130$ samples a 0.1 step disturbance is added at the output of the plant and at $t = 210$ samples a 0.3 step disturbance is added at the input of the plant. Note that in both cases the controller has a good performance showing the good compromise between robustness and disturbance rejection.

5. CONCLUSIONS

This work has presented a simple and effective method to design the prefilter in a generalized predictive controller that can be applied to a wide class of industrial processes. The proposed method allows a two-step controller design: first computing the GPC parameters in order to obtain

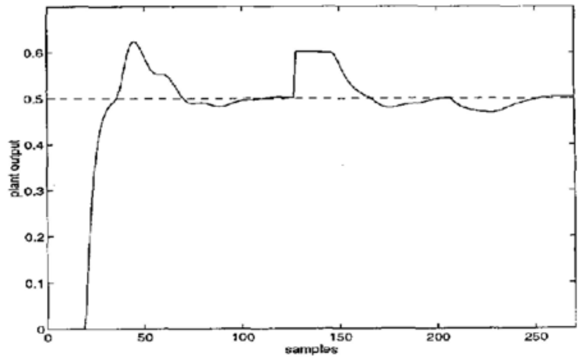


Fig. 6. Behaviour of the GPC with the proposed filter T for set-point and disturbance responces

a desired closed loop performance, and then defining a simple T polynomial in order to attempt some robust and disturbance rejection specifications. Some simulation results confirm the good qualities of the proposed controller and show that the method is simpler than the others proposed in literature.

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