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BAYESIAN NETWORKS IN ADDITIVE MANUFACTURING AND RELIABILITY ENGINEERING

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ABSTRACT

Azarakhsh Hamedi: Bayesian networks in additive manufacturing and reliability engineering
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A Bayesian network (BN) is a powerful tool to represent the quantitative and qualitative features of a system in an intuitive yet sophisticated manner. The qualitative aspect is represented with a directed acyclic graph (DAG), depicting dependency relations between the random variables of the system. In a DAG, the variables of the system are shown with a set of nodes and the dependencies between them are shown with a directed edge. A DAG in the Bayesian network can be a causal graph under certain circumstances. The quantitative aspect is the local conditional probabilities associated with each variable, which is a factorization of the joint probability distribution of the variables in the system based on the dependency relation represented in the DAG.

In this study, the benefits of using BNs in reliability engineering and additive manufacturing is investigated. In the case of reliability engineering, there are several methods to create predictive models for reliability features of a system. Predicting the possibility and the time of a possible failure is one of the important tasks in the reliability engineering principle. The quality of the corrective maintenance after each failure is affecting consecutive failure times. If a maintenance task after each failure involves replacing all the components of an equipment, called perfect maintenance, it is considered that the equipment is restored to an “as good as new” (AGAN) condition, and based on that, the consecutive failure times are considered independent. Not only in most of the cases the maintenance is not perfect, but the environment of the equipment and the usage patterns have a significant effect on the consecutive failure times. In this study, this effect is investigated by using Bayesian network structural learning algorithms to learn a BN based on the failure data of an industrial water pump.

In additive manufacturing (AM) field, manufacturing systems are normally a complex combination of multiple components. This complex nature and the associated uncertainties in design and manufacturing parameters in additive manufacturing promotes the need for models that can handle uncertainties and are efficient in calculations. Moreover, the lack of AM knowledge in practitioners is one of the main obstacles for democratizing it. In this study, a method is developed for creating Bayesian network models for AM systems that includes experts’ and domain knowledge.

To form the structure of the model, causal graphs obtained through dimensional analysis conceptual modeling (DACM) framework is used as the DAG for a Bayesian network after some modifications. DACM is a framework for extracting the causal graph and the governing equations between the variables of a complex system. The experts’ knowledge is extracted through a probability assessment process, called the analytical hierarchy process (AHP) and encoded into local probability tables associated with the independent variables of the model. To complete the model, a sampling technique is used along with the governing equations between the intermediate and output variables to obtain the rest of the probability tables.

Such models can be used in many use cases, namely domain knowledge representation, defect prognosis and diagnosis and design space exploration. The qualitative aspect of the model is obtained from the physical phenomena in the system and the quantitative aspect is obtained from the experts’ knowledge, therefore the model can interactively represent the domain and the experts’ knowledge. In prognosis tasks, the probability distribution for the values that an output variable can take is calculated based on the values chosen for the input variables. In diagnosis tasks, the designer can investigate the reason for having a specific value in an output variable among the inputs. Finally, the model can be used to perform design space exploration. The model reduces the design space into a discretized and interactive Bayesian network space which is very convenient for design space exploration.

Keywords: additive manufacturing, Bayesian networks, causal models, reliability engineering

PREFACE

This work is a result of an 8-months research in the Additive Manufacturing research group in the Automation Technology and Mechanical Engineering (ATME) (former Mechanical engineering and industrial systems, MEI) laboratory in Tampere University (Former Tampere University of Technology).

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LIST OF SYMBOLS AND ABBREVIATIONS

ABS	Acrylonitrile Butadiene Styrene
ABAO	As Bad as Old
AGAN	As Good as New
AHP	Analytical Hierarchy Process
AI	Artificial Intelligence
AIC	Akaike Information Criterion
AM	Additive Manufacturing
ASM	Aggressive Space Mapping
ASTM	American Society for Testing and Materials
BDc	Bayesian Dirichlet Criterion
BIC	Bayesian Information Criterion
CAD	Computer Aided Design
CBM	Condition Based Maintenance
CEN	European Committee for Standardization
CF	Censored Failure
CFPR	Carbon Fibre Reinforced Plastic
CFR	Constant Failure Rate
CM	Corrective Maintenance
CI	Consistency Index
CPT	Conditional Probability Table
CR	Consistency Ratio
CTF	Contingency Table Fit
DA	Dimensional Analysis
DACM	Dimensional Analysis Conceptual Modelling
DAG	Directed Acyclic Graph
DBN	Dynamic Bayesian Network
DC	Direct Current
DED	Direct Energy Deposition
DFR	Descending Failure Rate
DFX	Design for X
DSE	Design Space Exploration
EBM	Electron Beam Melting
EM	Expectation Maximization
FDM	Fused Deposition Modelling
FE	Finite Element
FFF	Fused Filament Fabrication
FMECA	Failure Modes and Effective Critically Analysis
FPBN	Fault Predicting Bayesian Network
FTA	Fault Tree Analysis
HMM	Hidden Markov Model
ICME	Integrated Computational Materials Engineering
IFR	Increasing Failure Rate
IPD	Interaction Preserving Discretizations
ISO	International Organization for Standardization
JPD	Joint Probability Distribution
KL	Kullback-Leibler
MAR	Missing at Random
MCAR	Missing Completely at Random
MDL	Minimum Description Length
MEAM	Material Extrusion Additive Manufacturing
MI	Mutual Information
MML	Minimum Message Length

MNAR	Missing Not at Random
MPT	Marginal Probability Tables
NIST	National Institute of Standardization
NMAR	Not Missing at Random
PHM	Prognosis and Health Management
PLA	Poly Lactic Acid
PM	Preventative Maintenance
RBD	Reliability Block Diagram
RCM	Reliability Centred Maintenance
RI	Random Consistency Index
RUL	Remaining Useful Lifetime
SADT	Structured Analysis and Design Technique
SC	Structural Coefficient
SLA	Stereo Lithography
SLM	Selective Laser Melting
SLS	Selective Laser Sintering
SSM	State Space Models
TTF	Time to Failure
TTTF	Total Time to Failure
VP	Vat Photo-Polymerization
WAAM	Wire and Arc Additive Manufacturing

$\boldsymbol{\alpha}$	Thermal expansion
σ	Thermal constraint
δ	Curling defect
$\mathbf{F}$	Force
m	Mass
a	Acceleration
θ	Temperature
p	Pressure
ρ	Material Density
q	Heat Energy Input
k	Coefficient of conduction

1. INTRODUCTION

Limited resources and fierce competition in the market encourages the manufacturers to adopt new manufacturing technologies which are more flexible, more predictable, more agile and needs less preparation time (d'Aveni, 2015). This urges manufacturers to use more complex manufacturing equipment and processes. Handling this complexity requires more knowledge and sophisticated methods and models.

One of the consequences of using more complex systems and models is facing uncertainty in the system (de Rocquigny, Devictor, & Tarantola, 2008). Uncertainty in manufacturing systems may have different source and it is classified by Nannapaneni et al. (2016) three categories. Uncertainty may be because of the quality of the data, e.g. inadequate, missing, erroneous data. Another category of uncertainty occurs because of assumptions and approximations in the models used. These two types of uncertainty happened because of lack of knowledge and called epistemic uncertainty. The third category of uncertainty is because of natural varieties in the manufacturing process and called statistical or aleatory uncertainty.

Uncertainty shows itself in industrial practice in different situations. As Rocquigny et al. (2008) discuss, uncertainty may occur because of variability or error in measurements in variables, having an expected value for a variable, having confidence intervals for some variables, variables relating to the risk percentage, having probability of exciding a threshold or having ranges for variables in the design phase. Some areas such reliability of the equipment are in direct relation with uncertainties in the system (O'Connor & Kleyner, 2012).

On the other hand, one of the obstacles to using new complex equipment and systems in manufacturing is the lack of expert's knowledge of using those processes among designers and practitioners. Creative design and manufacturing with new technologies like additive manufacturing need special tools, knowledge, and expertise which is sparse due to the recentness of these technologies (Gardan, 2014). New concepts such as design for X (DFX) combines the state of the art models and the expert's knowledge of manufacturing equipment and processes to provide interactive tools for designers. Such systems enable designers to maximize their creativity in the early design stage while the system can show the result of their choices in real-time with some degree of uncertainty (Laverne, Segonds, D'Antonio, & Le Coq, 2017).

One of the of the approaches to creating a model that can handle the uncertainty includes experts' knowledge, provides an interactive interface and is efficient in computation is using Bayesian networks. There are several other approaches such as fuzzy logic, neural networks, and rule-based expert systems but none of them can handle all of the mentioned criteria at the same time (McNaught & Chan, 2011).

The need for bidirectional inferable models, i.e. Semantic (up-down evidence reading) and perceptual (down-up evidence reading) inferable models, leads to initial deployment of the Bayesian networks. A Bayesian network is a graphical probabilistic model that represents a qualitative and a quantitative relationship between a set of random variables. The qualitative part is described using directed acyclic graphs (DAG) to show the dependencies between random variables and the quantitative part is the probabilistic relationships between those variables. The quantitative part is based on local probability distributions between the random variables and it represents a particular factorization of the joint probability distribution of the variables based on the relations specified through the DAG (Pearl, 2004).

In this representation, each random variable is represented with a node or vertex in the graph and a directed arc, also called an arrow or an arc, from node A to B shows that node B is dependent to A and A is possibly a cause for the node B. Since there can be many different factorizations for a joint distribution, there can be as many BNs for the same distribution. A fully connected network is the best realization of the joint probability distribution in the form of a Bayesian network. The missing arcs between nodes is a valuable information in a BN. They represent the conditional independence between the random variables and help representing the joint distribution in a more compact form (Judae Pearl, 1988).

The other benefit of using a Bayesian network is that using it, it is possible to encode the expert's knowledge into a model. The experts' knowledge can be extracted in the form of the dependency relations between variables, i.e. the structure of the network, or the quality of interactions between variables, i.e. the probability tables (Williamson, 2001).

After creating the model, using the DAG and the probability tables, it is possible to perform Bayesian inference between the variables of the model. The inference process calculates the effect of changes in the probability distribution of one or several nodes on the probability distributions of the other nodes. Several inference algorithms have been developed which can perform this task efficiently. This enables the Bayesian networks to be not only fast and efficient in calculations, but also have an interactive nature (Guo & Hsu, 2002).

1.1 Research Objective and Scope

This study tries to investigate the possibility of using a systematic approach to create models for the uncertainty associated problems in the industrial domain using Bayesian networks.

There are two general approaches for creating Bayesian network models for a problem in a system, namely the knowledge elicitation approach and the machine learning approach. BNs can be obtained in a subjective manner by eliciting experts knowledge and the domain knowledge for the dependency of the variables and the probability distributions (Koller, Friedman, Getoor, & Taskar, 2007). Multiple methods have been developed to obtaining expert's and domain knowledge for the structure of the Bayesian networks (K.W. Przytula & Thompson, 2002; Richardson & Domingos, 2003) and the corresponding probability distributions (Nunes et al., 2018).

The other method of creating a Bayesian network model is to use the available data in the domain and obtain a Bayesian network using machine learning algorithms. Although several methods have been developed to perform the machine learning (Daly, Shen, & Aitken, 2009) which are quite effective and efficient, the main problem is acquiring suitable data and preparations of the data to be used in the machine learning process (sections 3.1.5-3.1.7).

Therefore, the first objective of this research is to develop a systematic method to create interactive Bayesian network models for complex systems in order to predict the results of the choice of design and manufacturing parameters in the early stage design phase.

The second objective of this research is to create a predictive Bayesian network model for an industrial problem using the data and machine learning to get familiar with the challenges and develop a systematic approach for similar problems.

The methods that are created and gathered in this study are implemented in two industrial case studies. A problem in an additive manufacturing system is chosen to be modelled with a BN model using experts' and domain's knowledge and an equipment reliability case study is chosen to be modelled with BNs using machine learning and data.

1.2 Case Studies

One of the emerging technologies for which helps to address the needs in today’s fast pacing manufacturing is additive manufacturing (AM). AM is the process of manufacturing of parts by adding materials is layer by layer directly from their digital blueprints (d’Aveni, 2015). There are several obstacles in integrating AM technologies into production systems, one of which is the high degree of complexity in AM systems. Such systems are created by adjoining several complex subsystems and this makes it very difficult to create a holistic model for them to predict and assure the quality of the manufactured parts (Kathryn et al., 2016).

The other reason for the complexity of AM processes is the sheer number of input variables and that a big portion of the processes within the system are not identified. Therefore, finding the right parameters for the system to reach desirable dimensional, mechanical and metallurgical characteristics is a multi-criteria problem (Stavropoulos & Foteinopoulos, 2018). Moreover, the other major challenges in democratizing AM is lack of knowledge and expertise of AM among designers and practitioners (Lindqvist, Piili, & Salminen, 2016), therefore the models should be able to contain and represent experts’ knowledge in the field.

One of the challenges in manufacturing using AM is the defects in the manufactured parts. Additive manufacturing process causes a number of defects in the parts and since the process is complicated and fast pacing, it is hard to create exact models for them (Mindt, Desmaison, Megahed, Peralta, & Neumann, 2017). Moreover, choices in design and manufacturing parameters have a significant effect on the extent of these defects. Therefore, there is a significant amount of uncertainty associated with the variables of the system (Béraud, Vignat, Villeneuve, & Dendievel, 2014).

The other case study in this thesis is addressing the failure prediction in reliability engineering principle. Failure in manufacturing equipment imposes costs to the production. These costs can be the cost of downtime, excess maintenance, lost production, equipment repair, equipment replacement, and safety risks. These can affect companies in short or mid-terms and it can even lead to loss of business in the long term. Taking advanced maintenance policies can reduce cost and risk significantly. Manufacturers can take advantage of preventive or planned maintenance by creating predictive models of the failures from the historical data of their equipment components (Letot, Equeter, Dutoit, & Dehombreux, 2017). Using such models and considering the current situation of the machinery, the optimal time of maintenance of the system can be predicted and costly failure can be prevented.

Modelling the mechanism of failure is necessary to perform preventative and predictive maintenance. The well-known parametric models can describe the systems with good accuracy, but they lack the ability to adopt the changes in multiple variables of the model at the same time (Langseth, 1998).

The quality of each maintenance procedure, which is taking place after each failure, is affecting the expected time to the next failure. The maintenance quality can be ranging from perfect, i.e. bringing the equipment back to the “as good as new” condition, to a maintenance that makes the equipment’s health even worse than before the maintenance. The other factor which is important in the reliability of the equipment is the usage of the equipment and the environment of the operation (Carlo & Arleo, 2017). All these factors should be considered when a model is created for the reliability of equipment.

One of the major issues is the quality of the data in real-world cases. In the field of prediction and health management in reliability engineering, the data for the health condition of the equipment is very hard to find, partly due to the privacy policy of the companies and partly due to the nature of such systems. Field systems are typically not properly instrumented and the process of collecting data is time-consuming and expensive (Saxena, Goebel, Simon, & Eklund, 2008). The data used in this study is a single variable dataset of the failure times of industrial water pumps.

The other problem with the health condition data in the industrial domain is being subject to missingness and censoring. Missingness occurs when a data point is being failed to record 3.1.7). Censoring is a condition specific to failure data and it is basically the data which becomes invisible due to reasons such as ending the study or occurring before the study begins etc. (section 2.3.1).

1.3 Problem Definition

As mentioned before, this study tries to investigate the modelling process using BNs with two approaches in the industrial domain. The objectives of this research are implemented on the problems of the case studies. Therefore, the research questions of this study are:

How to model the probability of occurrence of a defect in an additive manufacturing process or:

- How to use the benefits of Bayesian networks in creating interactive models for curling defect problem in the additive manufacturing process which contains experts’ knowledge in the field of AM?

And the expected result in this field is:

- To create a methodology for creating Bayesian network models for problems in complex systems in a systematic manner.

And the research questions in reliability engineering are:

- How to use machine learning in Bayesian networks for predicting failure times historical time to failure data?

Expected result for the case study in reliability engineering is:

- To extract a predictive model from data to estimate the next time to failure.

1.4 Methodology

To answer the first research question, this study tries to use dimensional analysis conceptual modelling (DACM) framework to obtain the structure of the Bayesian network and analytical hierarchy process (AHP) and a sampling technique to obtain the probability distributions.

DACM is proposing a series of methods to simplify, organize and simulate the behaviour of a system in the form of cause-effect relationships using qualitative information about that system. The result is a directed graph containing the causal relationships between the variables of the system and the governing equations between those variables (Coatanéa, Roca, Mokhtarian, Mokammel, & Ikkala, 2016).

AHP is initially developed as a method to derive priorities for different criteria in a multi-criteria decision problem. AHP decomposes the criterion for decision problem into sub-criteria and acquires the expert's preferences on those sub-criteria by performing two by two comparisons between them and finally synthesises a weight for each of them using special mathematical machinery (Saaty & Vargas, 2012).

To address the second research question, this study also tries to exploit the machine learning approach for obtaining a Bayesian network model for an industrial system using data. Several challenges are associated with the quality of the data in most of the industrial cases.

This study attempts to encounter the problems which are normally associated with datasets available in the field of reliability engineering in a systematic manner. Then a model for the problem is created using a machine learning technique and the model is validated against the dataset.

1.5 Thesis Outline

This thesis is formed in five sections. After the introduction in this section, in the second section, the relevant background about Bayesian networks, additive manufacturing and reliability engineering is described briefly. This section creates the context of the case studies and shows the importance and the need for performing this study.

In the third section, the theoretical aspects of the methods used in the study are described in detail. The parts of Bayesian networks theory that are used in the study, the AHP and DACM methods which are used in developing the additive manufacturing case study and the methods developed for two case studies as well as the state of the art methods are described in this section.

The fourth section is dedicated to the details about the implementation of developed methodologies in the case studies. All steps are described in details and the resulting model is presented

In the fifth section, first, a brief discussion about the result of the case studies is presented and finally, the conclusion of the study is discussed.

2. BACKGROUND

This section provides background about the Bayesian networks and the case studies of this thesis. First, the Bayesian network is introduced briefly and then some information regarding additive manufacturing systems and prognosis and health management is provided in order to set the context and show the importance of the research. The focus of the section is mostly on the case studies and the detail of the Bayesian network is described in section 3.1 in detail.

2.1 Bayesian networks

A Bayesian network (BN) is a graphical probabilistic model, which represents the qualitative and quantitative relationships of a set of random variables a single model. A directed acyclic graph (DAG), which is also called the structure of the BN, is illustrating the dependency the random variables. The random variables are shown with nodes and the dependencies between them are shown with a directed edge. In Bayesian networks the qualitative part, the DAG can be considered as a causal graph under certain circumstances (see section Backgrounds3.1.1).

The quantitative part of a BN is the conditional probability distributions of the set of random variables which their dependency relations are shown in the DAG. Having the DAG, the joint distribution of the random variables can be factorized into a multiplication of conditional probability distributions. This enables BNs to provide a compact representation of the joint probability distributions. In this representation, each random variable is represented with a node (vertices) and a directed arc (also called arrow of arc) from node A to B shows that node B is conditioned on node A in that particular factorization of that joint distribution. Since there can be many different factorizations for joint distribution, there can be as many BNs for the same distribution. The valuable information in a BN is the missing arcs between nodes. They are representing the conditional independence of random variables in that particular variable set (Ghahramani, 2001; McNaught & Chan, 2011).

The variables in a BN can be continuous, categorical, discrete valued or a combination of them. if the variables are continuing variables, the numerical values and their probability distribution functions are used and If they are categorical, intervals or discreet, they are described with categories or states and conditional probability tables (CPTs) (see section Backgrounds3.1.1).

Bayesian networks allow us to use the information from a subset of variables in the system to predict the behaviour of any other subset of variables in that system and make rational decisions according to that (Munteanu & Bendou, 2001).

The structure of a Bayesian network can be obtained using two general approaches. The first approach is trying to use machine learning techniques to learn the structure from the data recorded about the system previously. Therefore, the resulting network approximates the joint probability of that dataset. Williamson (2001) calls these networks as abstract structures. The other approach is to have an interpretation of the Bayesian network in which the graph is representing a causal representation of the system and it may be subjective or objective. In the subjective case, the relation between two nodes, which is represented by a directed arc, is a direct causal relationship. In the objective case, this relation is the belief of an agent about the causal relationship between the variables of the system (nodes).

Advantages and uses of using Bayesian networks

Heckerman (1995) counted a few advantages of using Bayesian networks as follows. First, handling incomplete data is a natural feature of Bayesian networks. Most of the other data analysis methods, e.g. regression and classification are prone to magnificent errors in case system variables are highly anti-correlated and for example, one of them is unobserved. Bayesian networks can encode statistical dependencies between variables, so they can handle incomplete data.

Secondly, using Bayesian networks, one can learn the causal relationship between variables in that domain. This can include valuable information about a system and the result can be utilized in other analysis methods. Moreover, using the causal network, it is possible to perform interventions and investigate the predicted results.

The third advantage is that Bayesian networks model domain knowledge and the data at the same time. Therefore, using the causal relationships in the Bayesian networks and Bayesian and non-Bayesian statistical tools makes a sophisticated package for data analysis.

Bayesian networks are used in several domains such as medical diagnosis, map learning, natural language processing, image processing, computational biology, civil infrastructure networks, epidemiology, etc. (Koller & Friedman, 2013).

McNaught and Chan (2011) named a few uses of Bayesian networks in the industry as follows. Bayesian networks have been vastly used in fault diagnostics and failure prediction in manufacturing due to the uncertain nature of events in those principles. BNs are also used for reliability and risk assessment of manufacturing processes. The cause-effect modelling in BNs is a good tool to be used for manufacturing process scheduling under uncertainty. Also, BNs have been used in the field of predictive maintenance to determine the optimal time for a maintenance task to be performed. In a more general perspective, a set of Bayesian network models for different aspects of a factory have been combined to maximize the productivity of the factory. In a similar approach, BNs have been used as recommender systems to the customers of a customized manufacturing system to choose the best combinations.

2.2 Additive manufacturing

Additive Manufacturing (AM) is the process of joining material, layer by layer, Line by line or piece by piece, in order to fabricate a product directly from its digital 3D model. The term additive is used in opposition to subtractive manufacturing in which a product is created by subtracting material from a material block (ISO/ASTM, 2015).

As Yunlong and Yaoyao (2015) stated, additive manufacturing has three main advantages to previous methods. First, the production of highly complex parts can be done in a single process and the manufacturing cost will not increase with the complexity. Secondly, multi-material parts with complex material combinations can be produced easily with this method. And finally, manufacturing preparation time can be significantly decreasing since parts can be manufactured directly from their digital 3D models.

Initial use cases of AM was rapid prototyping for architects and designers (Ngo, Kashani, Imbalzano, Nguyen, & Hui, 2018). But nowadays, AM has several use-cases in the fields such as aeronautical, maritime, turbomachinery, biomedical, spare parts manufacturing, modification of manufactured parts and restoration of broken parts. In the aerospace industry, AM enables engineers to create optimized components with low weight, reduce the manufacturing lead-times and improve but-to-fly ratios (Ding, Shen, Pan, & Cuiuri, 2016). Maritime use cases are including but not limited to afloat manufacturing of spare parts and maintenance of equipment (Strickland, 2016). Complex multi-part components in turbomachinery such as disk-blades and burners can be manufactured as a single part using AM (Klocke et al., 2014). In the field of biomedical applications, AM facilitated creating customized implants, biodegradable implants, etc. (Bartolo et al., 2012).

A wide range of materials is used in additive manufacturing, and new materials are added to this range continuously. A non-exhaustive list of these materials includes concrete, ceramics, polymers and metals and composites. Concrete is mostly used in building houses using additive manufacturing (Wu, Wang, & Wang, 2016). Among the polymers, acrylonitrile butadiene styrene (ABS) and polylactic acid (PLA) are the most frequently used materials (Gonzalez-Gutierrez et al., 2018). In metal AM, Titanium alloys such as Ti6AL4V, steel alloys such as SS316 or H13, Aluminium alloys such Al-Si-Mg, super alloys such as IN625 and many other alloys are used (Frazier, 2014). Poor mechanical properties of polymers lead to the use of Carbon Fibre Reinforced Plastic (CFRP) in additive manufacturing (Ning, Cong, Qiu, Wei, & Wang, 2015).

In terms of the available standards, American Society for Testing and Materials (ASTM) committee F42 is one of the most active parties in defining standards for Additive manufacturing materials, parts and processes (ASTM, 2018). The European Committee for Standardization (CEN) is also an active organization in the standardization of AM through several actions and projects (CEN-CENELEC, 2018). International Organization for Standardization (ISO) has the ISO/TC 261 committee working on AM, many of them with collaboration with ASME F42 committee (ISO, 2018). The other entity which is active in this field is the National Institute of Standards and Technology (NIST) in the United States (NIST, 2018a). NIST is running multiple projects for supporting standardization of real-time control of additive manufacturing systems, quality assurance AM systems, system integration for AM, and characteristics of additive manufacturing materials (NIST, 2018b). Monzón et al. (2015) reviewed the efforts on developing and implementing standards for AM until 2015.

Although AM brings many advantages to manufacturing, there are some shortcomings as well. Cost of manufacturing with AM relatively high compared to mass production, production is very material and equipment –agnostic and the assuring reliability of the manufactured part is always a big challenge. Although there have been massive investments in the standardization of AM, the process is quite difficult and time-consuming (Jurrens & Energetics Incorporated, 2013; Pellegrino, Makila, McQueen, & Taylor, 2016).

The process of printing a part using AM starts with a digital model 3D of the object. The second step is to add support structures to the part, so that overhanging parts can be printed. Then the model should be cut into slices using slicer software, which replicates the layers which are going to be manufactured (Kathryn et al., 2016).

2.2.1 Additive manufacturing technologies

In 1986, Charles W. Hull has patented the first method of additive manufacturing called stereolithography or SLA (Charles W. Hull, 1986). Since then, there has been a significant amount of research on the topic. Nowadays, there are several techniques in additive manufacturing namely material extrusion, powder fusion, material jetting, binder jetting, direct energy deposition and sheet lamination etc. In the rest of this subsections, a short description of four of these techniques is provided.

Stereo Lithography (SLA)

In Stereo Lithography (SLA) or Vat Photo-polymerization (VP) a photosensitive liquid monomer, polymer or resin is cured or solidified using a controlled source of ultraviolet light, electron beam or laser. The light applied with the shape of each slice to polymerize the liquid into a solid layer. Then the platform moves downwards to make space for a new layer of liquid of the solidified layer. The process continues until the whole object is shaped layer by layer as shown in Figure 1 (Wong & Hernandez, 2012).

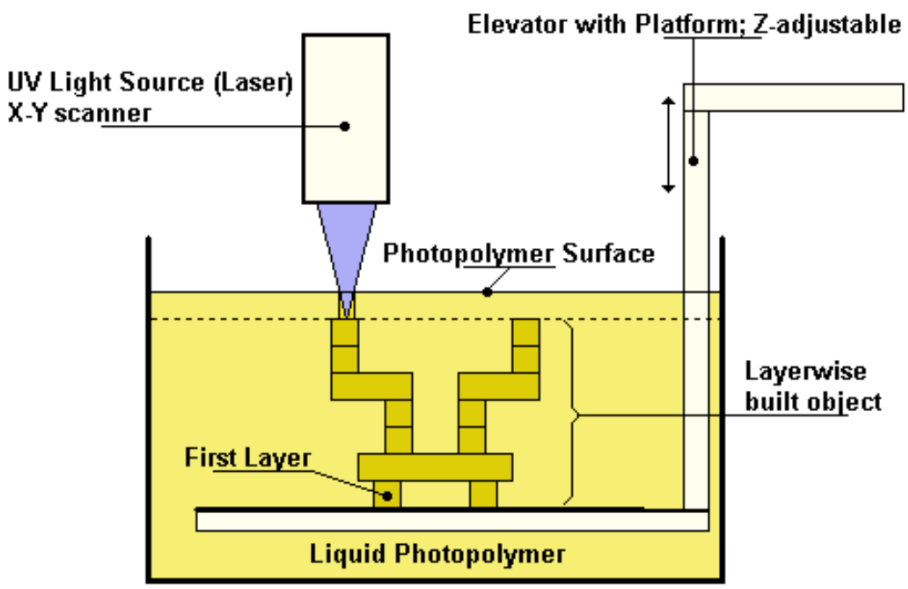

Figure 1. *SLM process* (Proform, 2018)

Part manufactured by SLM can be post-processed with light curing, to reach to better mechanical properties, and surface enhancement. SLM can be used for manufacturing ceramic parts by adding ceramic particles or using polymer-driven ceramifiable monomers (Ngo et al., 2018).

Possible defects in SLA are shrinkage, curling defect and distortions due to removing the part from the platform. Shrinkage is the direct effect of forming polymers from monomers. The curling defect in SLA happens as a result of shrinkage between the layers. And finally, the process of removing the manufactured part from the platform may cause further distortions in the part due to the liberation of internal forces cumulated between layers (Bugeda, Cervera, Lombera, & Onate, 1995).

Material Extrusion

Material Extrusion AM (MEAM) is the process of softening the material and passing it through a nuzzle and deposit is layer by layer in order to manufacture a 3D part. A MEAM machine usually consists of a two-axis (x and y) CNC manipulator which moves the extruder and a platform which moves in z-axis which moves the manufacturing part downwards to be ready for printing the next layer. The material can be in the form of solid filaments, powders or powder plus bounder liquid and the softening process is normally done by heating. The extrusion process can be done by either plunges, screws or wheels as shown in Figure 2 (Gonzalez-Gutierrez et al., 2018).

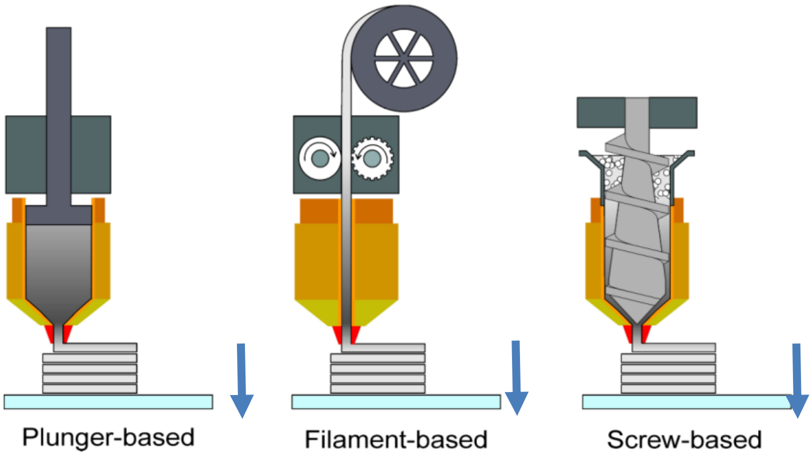

Figure 2. *Material Extrusion Additive Manufacturing* (Gonzalez-Gutierrez et al., 2018)

This technique can be used for manufacturing with metals, polymers, ceramics and composites. In case the material is used as the form of filaments, the process is called Fused Filament Fabrication (FFF) or Fused Deposition Modelling (FDM). FDM is the most common method of AM. FDM machines are available from around one hundred Euros up to several thousand Euros, from desktop home versions up to industrial production versions. The other reason is that the process of manufacturing is safe and simple and the filaments have a good variety of materials (Gonzalez-Gutierrez et al., 2018).

Cooling profile of FDM manufactured parts have a direct relationship with distortions and porosity in them. The mechanical properties of the parts are affected by the bond between the layers of the manufactured part, which is, in turn, is affected by the temperature of the extruder and the temperature of the last layer of the part (Stavropoulos & Foteinopoulos, 2018). Poor surface finish and mechanical properties are the main flaws of this technology. Using fibre reinforced filaments can be a solution for the latter problem (Ngo et al., 2018).

Powder Fusion

In powder fusion AM, a thin layer of fine powder which is spread and packed on the top of a descendant platform is fused together using pressure, heat or a binder. The source of the heat can be a laser beam or an electron beam. The fusion process can take place at two levels. In Selective Laser Sintering (SLS) the powder particles are not getting fully melted but they fuse together in molecular level. In a Selective Laser Melting (SLM) or Electron Beam Melting (EBM), the powder particles are melt and the fusion happens in a liquid phase, shown in Figure 3 (Stavropoulos & Foteinopoulos, 2018).

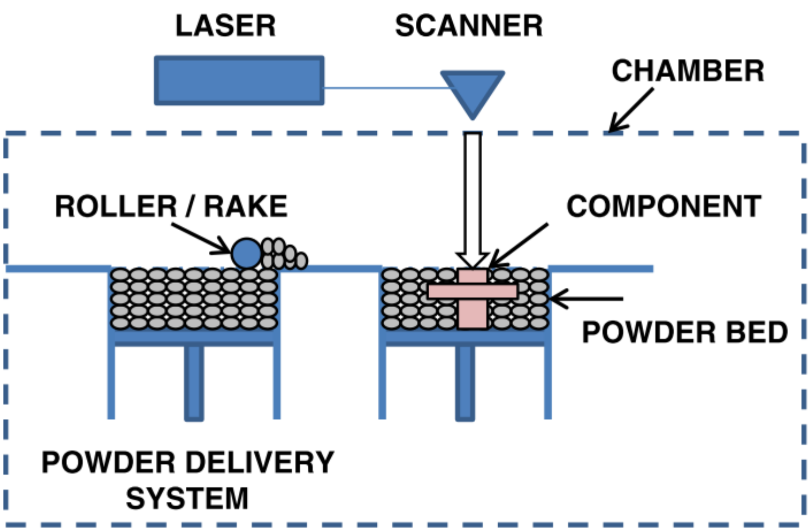

Figure 3. *Powder bed fusion* (Frazier, 2014)

The quality of parts is highly dependent on the powder shape, size, material and distribution. The other effective parameter is the chemistry and rheology of the binder, in the binder based processes, and the amount and flow of heat energy input to the system in the heat based processes. The heat sintering and melting process cause high residual stress in the manufactured parts. These stresses are the source of several defects in the parts, such as deformations, curling defect, lack of thickness, etc. Therefore, thermal and thermo-mechanical modelling of the process is of utmost importance for optimizing the manufacturing process for minimizing the defects (Ngo et al., 2018; Stavropoulos & Foteinopoulos, 2018). In the next subsection, a detailed description of defects in the SLM process is provided.

Post-processing procedures that usually take place in powder bed techniques are coating, sintering and infiltration. Superior resolution, good surface quality and good mechanical properties of the parts manufactured with powder bed techniques make them one of the most favourable techniques, especially in metal AM (Ngo et al., 2018).

Direct Energy Deposition (DED)

The reason for calling this technique direct energy depositions (DED) that here the energy is guided and focused to a narrow region and the material is deposited and melted simultaneously in the same region. There are several variations for these methods and this technology is mainly used for metal AM. The form of the material feed can be powder or filament and the energy source can be laser, electron beam, or electric arc (Stavropoulos & Foteinopoulos, 2018). Figure 4 is showing a simplified schematic of an electron beam DED.

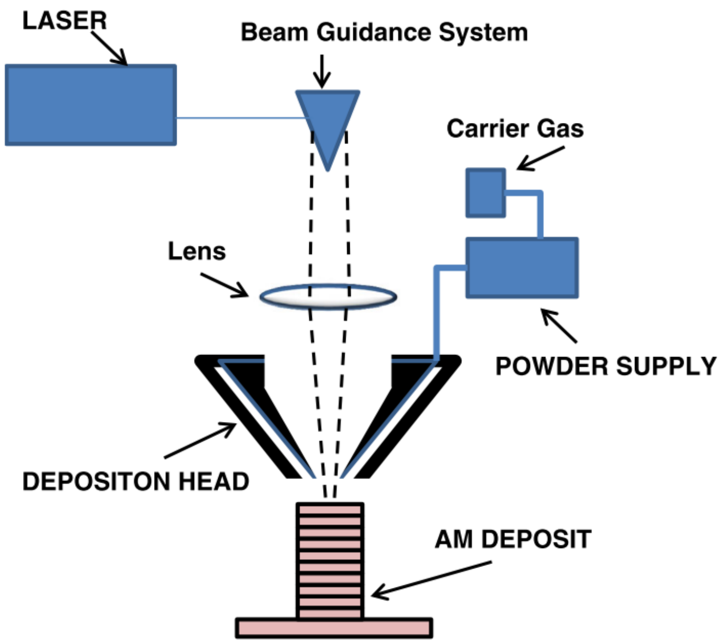

Figure 4. *A generic powder and electron beam DED system* (Frazier, 2014)

If a DED process uses metal wire filaments and electric arcs, it is called Wire and Arc additive manufacturing (WAAM) (Figure 5). While the powder bed based AM techniques are focused on fine details of the parts, WAAM systems are able to build larger parts (in the scale of $5.8m \times 1.2m \times 1.2m$) with higher deposition rates (3 to 10.63 kilograms per hour) (Ding et al., 2016). DED manufacturing systems normally consist of a robotic arm and a turning table, therefore they normally have a minimum of five degrees of freedom. Therefore, it is possible to manufacture parts which are difficult to manufacture with the other technologies. DED is also used for modifying parts and repairing cracks in metal parts (Pinkerton, Wang, & Li, 2008).

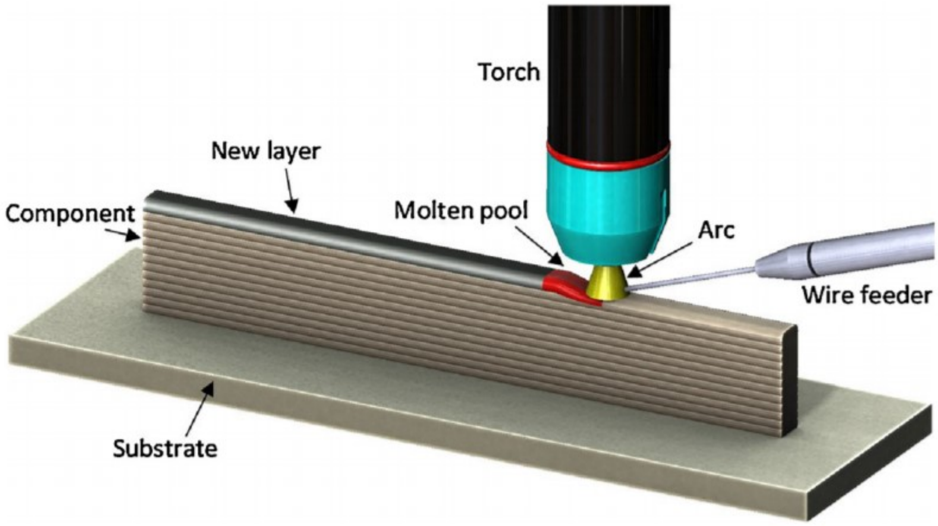

Figure 5. *Wire and Arc Additive manufacturing* (McAndrew et al., 2018)

2.2.2 Defects in Additive Manufacturing

The shape, strength and the size of an AM manufactured part is depending on 1- the raw material used, 2- the manufacturing equipment such as precision of equipment and equipment characteristics, and 3- the process parameters, powder bed temperature, manufacturing environment temperature, such as energy input, nozzle temperature, traverse speed, welding torch angle etc. (Kathryn et al., 2016).

Defects in additive manufacturing can be classified into two levels. Defects can cumulate during the manufacturing process and affect the geometry of the part. These defects are normally a result of residual stresses in the workpiece due to the thermal cycle in the manufacturing process, plastic strains caused by shrinkage and constraints of clamping. Distortions may stop the building process if they become magnificent enough (Mindt et al., 2017).

The other group are defects such as surface roughness, porosity, cracks, splatters and denudation can be described as microscopic defects. For a detailed description of different defects and the factors affecting it in Taheri et al.'s (2017) work.

Defects in Powder Bed Fusion

In the PBF process with SLM, lack of thickness and curling defect are the two common geometry related defects. Curling defect occurs on the overhang surfaces where parts are not supported with a support structure as shown in Figure 6. The heat conduction rate of an unsupported overhanging part can be up to one hundred times less than a solid material supported part. Employing excessive heat energy, e.g. high laser power, in the layer of an unsupported overhanging part leads to a magnificent thermal constraint on that layer. If this constraint exceeds the strength of the material, a plastic deformation happens. Cumulating these relatively small deformations in multiple layers leads to a curl in the overhanging part (D. Wang, Yang, Yi, & Su, 2013).

Figure 6. *Curling defect in overhanging parts* (Tounsi & Vignat, 2017)

Curling defect is not purely dependent on the geometry of the part, but also on the choice of the support structure (Tounsi & Vignat, 2017), and process parameter settings (Béraud et al., 2014). Toward reducing this defect, as shown in Figure 7, the support structures are used to dissipate excessive heat and to resist distortion by increasing the inertia of the part. While using more dense support structure seems beneficial to minimize the curling effect, it increases the manufacturing time and material cost (Mokhtarian, Coatanéa, Paris, Mbow, Pourroy, Marin, Vihinen, et al., 2018).

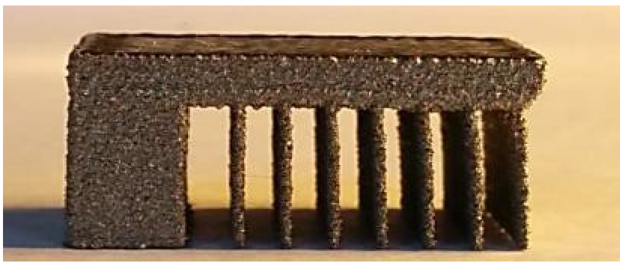

Figure 7. *An overhanging part with a support structure (Tounsi & Vignat, 2017)*

2.3 Reliability in machinery

Reliability, as De Carlo (2013) defines and discusses, is “the probability that a component (or an entire system) will perform its function for a specific period of time when operating in its design environment". Based on this definition, reliability is a measure for judging that the component is working or not, and the exact environmental and usage conditions should be defined. A broader definition describes reliability as the science to analyse, predict, prevent and mitigates the failures in the time domain.

Failure, or hard failure, is an inoperable state or an event in a system, in which the system or any part of it is not working as specified previously (Dudenhoeffer, 1994). Faults, or soft failures, on the other hand, are the defects which are happening and may or may not cause a failure in the system. Therefore, as shown in Figure 8, failures can be the result of a long term process in which an initial defect escalate among the time and reaches a critical condition that causes the failure in the machine (Lee et al., 2014).

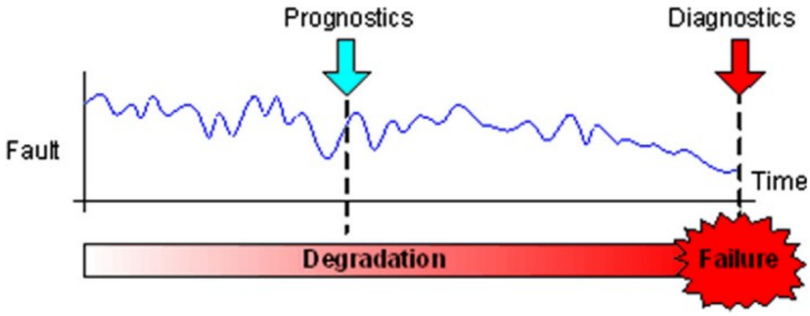

Figure 8. *Perception of degradation, diagnostics and prognostics in health management* (Lee et al., 2014)

Failures can be further classified in repairable or non-repairable. In repairable failures, the system can return to its operational state with repair or replacement of a minimal number of system part in a short time. Non-repairable failures are the ones that need the system to be completely replaced or require an extensive overhaul to restore the system (Dudenhoeffer, 1994).

Prognosis and health management

Prognosis and health management (PHM) is an umbrella term which covers many activities in order to maintain the health of a system by diagnosing the faults and taking appropriate decisions based on the prognosis of possible failures. The aim of PHM is to reduce the downtime of the machinery and preventing associated costs.

To create a PHM system, the faults within the system should be identified and the causes for it should be diagnosed. Moreover, the health of a system can be prognosed based on the history of the system and its current situation. The health management discipline assesses the impact of failures and minimizes the possible costs and losses by carrying on timely and appropriate maintenance actions based on the output of diagnostics and prognostics (Lee et al., 2014).

To be more precise, diagnosis is detecting the failure mode within a system or among the subsystems. It analyses the nature of a problem and provides the means to isolate it. On the other hand, prognosis tries to indicate the time to the next failure time and remaining useful lifetime of the system until a complete failure occurs. Prognostics continuously uses the indication of degradations in the system and considers the time factor to make the most accurate predictions (Lee et al., 2014).

Fault Diagnosis

To create a fault diagnosis system the essential components are a data collection subsystem to record events and sensor data, a signal processing subsystem to transform sensor data into information and detect faults and a database or knowledge representation system to determine the source of the fault. The knowledge representation subsystem can be implemented using databases, ontologies, physics models, black box models, or Bayesian networks (Lee et al., 2014).

Bayesian networks have been used as a sophisticated tool for creating knowledge representation models for diagnostics in the industrial domain. The possibility of representing uncertainty in the system, expressiveness of BNs, possibility of including expert's knowledge in the model, modularity and forward and backward simulation are some of the advantages of using BNs in diagnostics. The BN structure can be obtained using expert's knowledge regarding cause and effects of a failure in the system, mapping already existing models such as fault trees into BNs or using structural learning algorithms which learn the structure from data. A recent literature review on uses of BNs in diagnostics can be found in an article by Cai et al. (2017) work.

Failure Prognosis

On the other hand, prognosis ties to model the degradation of a component and predict the time that a fault or a failure occurs in it. Several methodologies have been developed to create the model and perform the prediction, and described by first hitting time process, remaining useful lifetime (RUL) evaluation, etc. (Letot et al., 2017).

Degradation modes can be classified into normal models, which is estimating the reliability of a model in normal conditions, and accelerated models, which try to estimate the degradation in normal condition given the data obtained in a condition that the time or stress on the component is accelerated (Letot et al., 2017).

Bayesian networks are very well suited for prognosis uses in reliability engineering. Variables which are influencing degradation in equipment, variables related to the operating environment and usage variables are uncertain variables which may have complicated interrelations. The ability to represent dependencies and conditional independencies between variables, efficient calculation scheme, compact representation and interactive interface of Bayesian networks make them a sophisticated tool in fault prognostics (Langseth & Portinale, 2007).

Maintenance

As Letot et al. (2017) describe, maintenance is the act of performing periodic tasks in order to ensure that the functionality of the components is available until the next scheduled maintenance period.

Several maintenance policies and method have been developed so far, namely corrective maintenance (CM), preventative maintenance (PM), reliability centred maintenance (RCM) and Condition-based maintenance (CBM) etc. Corrective maintenance is the situation in which the equipment is maintained after a failure happens and its purpose is to put the equipment back to the functional state (Peysson, Ouladsine, Noura, Leger, & Allemand, 2008).

As Lee et al. describe (2014), Preventative maintenance (PM) uses the mean time between failures as a reference for scheduling maintenance for machinery. The strong assumption upon static and deterministic conditions limits this type of maintenance and this method cannot be used under dynamic conditions. PM increases the availability of the system compared to CM and decreases cost up to a tenth the costs of CM (Carlo & Arleo, 2017), but it is still not optimal for the costs and the time of maintenance. Moreover, the failure history of a system is not the only factor that is effective in predicting the failure time.

On the other hand, for dynamic systems which the future behaviour is not predictable based on the historical observations and the domain knowledge, reliability centred maintenance (RCM) is more suitable. RCM uses statistical tools such as failure modes and effective critically analysis (FMECA) to predict the probability of having expected reliability in a certain period by identifying the failure modes and estimate the time before those failure modes may happen. Nevertheless, RCM is prone to fail if the changes in the dynamics of the system are magnificent.

Condition-based maintenance (CBM) consists of two major activities, data acquisition and condition monitoring. This method is mainly used when the system conditions are deterministic, stationary or static and the sensor outputs are a good indicator of the system health.

The other concept in maintenance is the quality of the maintenance. As an alternative to the classical maintenance quality classification described in the literature such as EN 13306:2010, i.e. corrective maintenance and preventative maintenance, a newer classification suggests that a maintenance activity can be perfect, imperfect, minimal, worse or worst, based on the restoration of the equipment after maintenance (Carlo & Arleo, 2017).

De Carlo and Arleo (2017) described these five types of maintenance as follows. A maintenance procedure is called perfect maintenance, when it restores the equipment to an “as good as new” (AGAN) condition. AGAN is a condition in which the maintained equipment would have the same failure rate and lifetime distribution as new equipment and generally is achieved by replacement of all the components with a new one.

Imperfect maintenance renders the equipment to a younger condition, but not to an AGAN condition. The maintained equipment failure rate and lifetime distribution lay somewhere between its premaintenance condition and AGAN condition.

Minimal maintenance restores the equipment just to an “as bad as old” (ABAO) condition, in which, the failure rate and lifetime distribution of the equipment are similar to equipment which has the same age and never failed yet. Minimal repair is done by only replacing faulty components of the equipment. Figure 9 depicts the effect of these three types of maintenance on the failure rate of equipment.

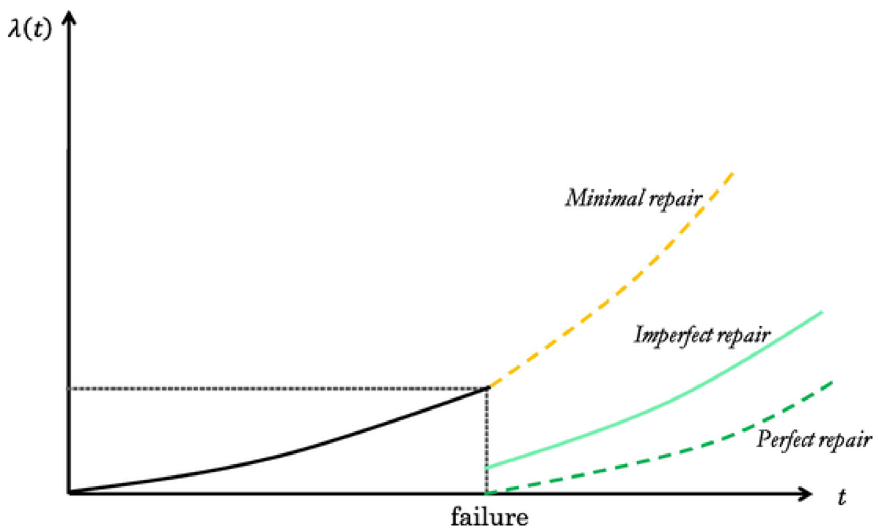

Figure 9. *Perfect, imperfect and minimal maintenance and their effect on the failure rate* (Carlo & Arleo, 2017)

Worse maintenance is when the maintenance accidentally causes the equipment to fall into a worse operating condition in terms of failure rate and the lifetime of it. Finally, worst maintenance is the conditions that worse maintenance is accompanied by creating a new failure of breaks in the system.

2.3.1 Characteristics of failure data

The data collected for failures in systems are normally a time series. The data normally consists of readings of several sensors in the system, time stamps for start time, events in the system, maintenance times and failure times (NASA, 2007). Among the sensor data, oil quality and vibration data describe the performance of the machine very well and have been traditionally used for diagnosis purposes. There are several other useful sensor data including, but not limited to, temperature, acoustic emissions, ultrasonic, etc. The data from several sensors and other sources can be fused together to achieve superior descriptive qualities (Lee et al., 2014).

The process of detection and prediction of failure can be divided into two periods. The first period is the observation interval in which some variables in the system are observed. The second period is the prediction time in which the system is predicting a failure in the future time (Kelleher, Namee, & D'Arcy, 2015). The variables for which the data is recorded in the process of observation can be divided into two groups. Covariates are the variables which represent the characteristics and the environment of the mechanical equipment and response variables are describing the survival times of the equipment (Langseth, 1998).

One of the most important characteristics of the failure data is that this type of data contains censored observations. As Miller et al. (1998) described, data may have four types of censoring. Type one is when the failure in equipment has been observed for a period and the observation is stopped or finished. Then for the equipment which has not failed in that period, there is no failure data recorded, even though it may fail any time after the recording stopped. The second type of censoring is when it is decided to stop recording the failure times after a certain number of failures happened.

The third type of censoring in data happens mostly in medical applications and it's when the data collection becomes impossible at a random time at the middle of the study. It happens, for example, when the follow up becomes impossible due to patients' conditions, the patient drops out, etc. It is important to note that for random censoring, a crucial assumption is that the patients are randomly chosen and their type three censored times and their possible failure (decease) time are assumed to be independent.

Finally, the other type of censoring is interval censoring. For example, if before the beginning of the observation, some of the equipment has already experienced failures and there is no record for that it is called left-censored data. If the failures are happening after the data recording stopped, it is called right-censored data and it is similar to type one censoring.

This study tries to provide a brief review of the classical and current methods for fault diagnosis and failure prognosis in section 3.5.1. Afterwards, the methods for creating failure predicting models from single-valued TTF data is reviewed in section 3.5.2. Then a Bayesian network based model for predicting the TTF values and censored TTF values is developed based on a single-valued dataset in section 4.2.

3. METHODOLOGY

This study takes advantage of several methods from different principles to create meaningful models for the systems using experts' knowledge and data. Bayesian belief networks use several methods from statistics and computer science to obtain the Bayesian network structure from the data, estimate the parameters of the network, perform inference between nodes, etc.

The structure of the Bayesian networks for a problem in a system can also be obtained using the governing equations of the system, domain knowledge, experts' knowledge and literature. This process can be carried out systematically using dimensional analysis conceptual modelling (DACM) framework which gathers multiple methods from several domains to produce causal graphs between variables of the system and acquire the governing equations between them. The causal graph can be translated into a Bayesian network structure and the governing equations can be used to obtain some of the network's parameters. To extract experts' knowledge and use them as parameters for the rest of the nodes, Analytical Hierarchy Process (AHP) from multicriteria decision-making domain is used in this study.

This section also reviews the classical and well-known methods in topics of the case studies. The methods for modelling complex systems in additive manufacturing are reviewed and then a detailed description of the method developed in this study is provided. In the reliability engineering case study, the classical methods in fault diagnosis and failure prognosis in the field of equipment health management is reviewed. Then the method for creating a predictive model from time to failure datasets with a single value is described.

The rest of this section is formatted as follows. In the first subsection, a detailed description of Bayesian networks and related knowledge and methods that are used in this study is reviewed in section 3.1. Then, in sections 3.2 and 3.3, a brief description of the aspects of the AHP and DACM that are used in this study is provided. And finally, the methods used in the case studies reviewed and the developed methods are described in detail in sections 3.4 and 3.5.

3.1 Bayesian networks

3.1.1 Backgrounds

To set the ground for a description of the properties and processes in the Bayesian network, it is needed to review the basis of the Bayesian networks theory. In this sub-section, a brief review of Bayesian probabilities, independence between random variables, directed acyclic graphs and causal graphs, the principle of the common cause, Markov causal condition and faithfulness condition, the formal definition of a Bayesian network, d-separation and i-maps is provided.

Probabilistic event and probability distributions

A sample space $\Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$ for a random procedure is the set of outcomes ω_i, possible for that random procedure. An event E, which is the phenomenon of interest in probability study, can be defined as a subset of the set Ω. Events in this sense can only have a true/false character. Then, a probability distribution is a function from events space to the space of the real numbers in the range $[0,1]$ and $\mathrm{P} : \mathbb{P}\,(\Omega) \rightarrow [0,1]$, in which $\mathbb{P}\,(\Omega)$ is called the power set of Ω (Daly et al., 2009).

Since events are subsets of outcomes set, it is possible to use set operations to define the probability of occurrence of two events A and B as $P(A \cap B)$. Therefore, the conditional probability of occurrence of A, given that event B is occurred is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{1}$$

In which $P(B)$ must be strictly positive. Equation (1) implies that the probability of occurrence of evet A, given that event B is occurred is equal to the joint probability of A and B divided by the probability of B. Then intuitively by changing the place of A and B it can be stated that

$$P(A|B)P(B) = P(A \cap B) = P(B \cap A) = P(B|A)P(A) \tag{2}$$

And by rearranging the equation a convenient formula is forming as

$$P(A|B) = \frac{P(A)\,P(B|A)}{P(B)} \tag{3}$$

which is known as the Bayes' formula. $P(A)$ is called prior probability, a priori, or unconditional probability of the event A. It means the probability of happening of the event A without considering any information about event B. It is also called antecthe edent set of propositions and may lead to consequences when the inference rules are applied to them. $P(A|B)$ is called posteriori probability and is the conditional probability of A given B. $P(B|A)$ is the called likelihood of occurrence of event B given event A has occurred. $P(B)$ is acting as a normalization constant and it is the conditional probability of variable B (Daly et al., 2009).

If for two events A and B

$$P(A|B) = P(A) \; and \; P(B|A) = P(B) \tag{4}$$

Then the events A and B are independent. Moreover, the events A and B are conditionally independent if we have a third variable C such that

$$P(A|B \cap C) = P(A|C) \; and \; P(B|A \cap C) = P(B|C) \tag{5}$$

In which two equation imply each other if the probability of events A, B, C are strictly positive (Daly et al., 2009; Ghahramani, 2001)

Random variable X can be defined as a function from the sample space of Ω to a measurable space M, which is the space of measurable quantities of the variable X. When the statement $P(X = "a\ measure")$ is made, reading as the probability of random variable X being equal to “a measure”. X to be equal to “a measure is an event, say event A. In fact, the intention is to calculate the probability of the event A and it can be described as $P(A) = \{\omega | \omega \in \Omega, X(\omega) = "a\ measure"\}$. This long notation is not normally used and instead the first expression is commonly used.

A joint distribution, e.g. $P(X, Y)$, is a multidimensional version of the probability distribution. Similar to single dimensional version, it is possible to calculate the probability of an event by specifying values to the random variables in the joint probability distribution.

The conditional probability rule and the Bayes rule can be rewritten using the notation of Random variables. From the definition of conditional probability in equation (1) the following equation can be obtained

$$P(X, Y) = P(X)P(Y|X) \tag{6}$$

Which is called the chain rule of conditional probabilities. This formula can be extended to multiple variables in the form of this equation

$$P(X_1, X_2, \ldots, X_n) = P(X_1)P(X_2|X_1) \ldots P(X_n|X_1, \ldots, X_{n-1}) \tag{7}$$

This equation implies that the joint probability of n random variables can be expressed in terms of, for example, the probability of the first one, the probability of the second one given the first, and so on. The order of this expression is not important, and the result remains the same with any order of combinations. A more general context, this can be as a factorization of the joint probability distribution. A factor is a function from a set of random variables, say D to the set $\mathbb{R}$. D is called the scope of the factor. A factor with nonnegative entries is nonnegative itself (Koller & Friedman, 2013, p. 24,104).

To calculate the probability distribution of one of the variables of a joint distribution, i.e. marginalize it, the probabilities of all other random variables in the joint distribution can be summed up.

$$P(X) = \sum_{y \in M(Y)} P(X, Y = y) \tag{8}$$

In which $M(Y)$ is the measure space or the domain of random variable Y.

Independence and conditional independence between variables

Two random variables X and Y are called independent, or marginally independent, if there exist a distribution P in which the following equation holds

$$P(X|Y) = P(X) \; and \; P(Y|X) = P(Y) \tag{9}$$

With $P(x)$ and $P(Y)$ are both positive. The independence between random variables X and Y is shown by $(X \perp Y)$. From the equation (9) and using the chain rule an equivalent definition is that a distribution P satisfies $(X \perp Y)$ if and only if $P(X, Y) = P(X)P(Y)$ (Koller & Friedman, 2013, p. 24).

Now, if X, Y, Z are sets of random variables in a probability distribution P and satisfy $(X \perp Y \mid Z)$ for all the values of all the variables in them, X and Y are conditionally independent given Z. The variables in the Z are called observed variables. Similar to the second definition of marginal independence, it can be stated that the distribution P satisfies $(X \perp Y \mid Z)$ if and only if $P(X, Y|Z) = P(X|Z)P(Y|Z)$.

Conditional independence holds five main properties, namely symmetry, decomposition, weak union contraction and intersection. Symmetry denotes that if X and Y are independent given Z, then symmetrically Y and X are independent given Z or $(X \perp Y \mid Z) \Rightarrow (Y \perp X \mid Z)$. Decomposition states that is X is independent of W and Y given Z, then X and Y are independent themselves or $(X \perp Y, W \mid Z) \Rightarrow (X \perp Y \mid Z, W)$. Weak union says that if X is independent of Y and W given Z then X and Y are independent, given W and Z or

$(X \perp Y, W \mid Z) \Rightarrow (X \perp Y \mid Z, W)$. To know more details about the rest of properties please refer to (Koller & Friedman, 2013, p. 25)

The importance of conditional independence is that by finding them in a joint distribution, the space needed for saving and representing the data increases dramatically, and the representation can be more interpretable for humans. For example, an n dimensional joint distribution of binary variables needs $2^n - 1$ storage spots. Now if it is represented in the following factorized form

$$P(X_1, X_2, \ldots, X_n) = P(X_1|X_2, X_3, \ldots, X_n)P(X_2, \ldots, X_{n-1}) \quad (10)$$

And we know that X_1 is independent of $X_3, \ldots, X_n$ given X_2, then the joint probability can be represented as

$$P(X_1, X_2, \ldots, X_n) = P(X_1|X_2)P(X_2, \ldots, X_{n-1}) \quad (11)$$

Which is a more compact and representation.

Directed Acyclic graph and causal graph

A graph is defined as a set of vertices or nodes and a set of edges or arcs which are connecting those vertices to each other. A directed graph is a graph that its edges have a direction associated with them. A directed acyclic graph (DAG) is a directed graph, in which there is no sequence of edges that loop around a cycle. This means that it is not possible to return to start from a node and return to the same node by following the direction of the arcs. If we ignore the direction of the arcs it is possible that we have loops in the graph (Daly et al., 2009, p. 102).

In a directed graph, node A is a parent for node B and node B is a child for node A if there is a directed arc from A to B. The Decedents of a node are the children of that and the children of those children and so on. A directed path is a series of nodes starting with A and ending to B in which each node in the series is the child of the previous node. An undirected path is a series of nodes in which each node is a child of a parent of the previous node (Daly et al., 2009, p. 102).

A causal graph is a directed acyclic graph in which there exists a directed arc from node A to node B only if there is a direct causal relationship between the node A is the node B. Two nodes have a direct causal relationship if their causal relation does not pass through any other nodes. A causal path is a directed path that represents a sequence of causal relationships.

The principle of the common cause

As Williamson describes in the second chapter of the book Foundation of Bayesianism (2001), The principle of the common cause is the link between the probabilistic dependency and the causality. As a definition, suppose that two variables are probabilistically dependent and neither is casing the other one, then two conditions are occurring:

1- They have one or more common causes, which is called existence condition and
2- They are conditionally independent given those common causes which are called screening condition.

This theory is the base ground for statistical experimentation.

This theory has at least two counterexamples, one for each condition. For the existence condition, the variables can be accidentally correlated meaning there may be no suitable obvious common cause for those variables. To solve the problem in such situations, two strategies are mainly used, namely causal extension and setting restrictions. Causal extension tries to extend the intuitive concept of causality and extend the causality to a hidden or latent or unmeasured common cause. This strategy has at least two flaws, first it is difficult to find the latent common cause and second, extending the causality concept from its intuitive character may lead causality to lose its meaning (Williamson, 2001, pp. 85–87).

Setting restrictions strategy is performed in two forms, correlation restrictions or causal restrictions, where the former is speaking about the type of correlation two variables have and the latter is speaking about that the nature of the variables should support the causal relationship (Williamson, 2001, pp. 87–88).

For the screening condition, there may be some extra-causal constraints, such as overlap in definition or logical, mathematical and physical laws, which leads to a probabilistic correlation for the variables. For more details on these counterexamples and the strategies for dealing with them and the difficulties associated with these strategies, please consult Williamson (2001).

In the case of a Bayesian network, which is a representation of causal relations ships using the causal graph and probabilistic independence relationships represented by CPT and MPTs, the relation between causation and probability should be further explored and a solution should be found to avoid the problems in the relation between causation and association. One of the solutions is Markov causal condition.

Markov causal condition and the faithfulness condition

Assume that there exists a causal graph G, with a set of vertices V and a set of edges E, and a probability distribution P over the vertices V which is generated by the causal relationships represented using the graph G. Therefore, the set V represents both the random variables of the system and the nodes in the causal graph between them.

In this condition, for the node A, the parent nodes are the direct causes and the children nodes are the direct effects. Markov causal condition says that, conditioned on all direct causes of the node A, the node A is independent, probabilistically independent, of all variables in the set V which are not direct causes or effects of the node A. In other words, if $Parents(A)$ are the set of parent nodes for the node A in the graph G, then the causal Markov condition is defined by Hausman and Woodward (1999) as:

“For all distinct variables A and B in the variable set V, if A does not cause B, then”

$$P(A|B \,\&\, Parents(A)) = P(A|Parents(A)) \tag{12}$$

The converse of the Markov causal condition is called the Faithfulness condition and it is described as that a distribution P over the variables in the set V satisfies no independence relationships beyond those represented by the graph G (Uhler, Raskutti, Bühlmann, & Yu, 2012).

The combination of the Markov and the Faithfulness conditions imply that “A causes B if and only if A and B are probabilistically dependent conditional on the set of all the direct causes of A in a probability distribution generated by the given causal structure among the variables in V ”. Moreover, causal Markov condition implies that if two nodes A and B do not have any causal relationships and have no common ancestors, they are independent conditional on an empty set, i.e. they are unconditionally independent (Hausman & Woodward, 1999).

A Bayesian network

Now that all the building blocks of what is called a Bayesian network are described, it can be defined as follows.

A Bayesian network is a pair of a graph G and an associated probability distribution P, (G, P), in which the graph is created by a set of vertices V and edges E and it satisfies the Markov causal condition with the joint probability distribution P over vertices V.

The joint probability distribution P can be rewritten into a product of conditional distributions based on the causal relationships given by the causal graph. Conversely, a joint probability distribution of a set of variables can be obtained by multiplication of conditional probability distribution (Koller & Friedman, 2013).

D-Separation

D-separation is a graph-based conditional independence test which can be obtained from the Markov causal condition. As Ghahramani (2001) describes, having the node setts A, B and C as disjoint subsets of the set V, A and B are conditionally independent if there is a set C which d-separates them. This means that for every undirected path between a node in A and a node in B, there is a node D such that

1. D has converging arrows and D itself and its descendants are not in C
2. D does not have a converging arrow and D is in C

Perfect-map or I-map

Having the Markov causal condition, d-separation is a sufficient condition for conditional independencies in P. Moreover, if a graph G is found which replicates the conditional independencies in P, this graph is called the faithful graph to P. If a graph G and a probability distribution of the nodes of the graph, P, are satisfying the combination of these two statements as shown in the equation (13), then G is an I-map or a perfect-map of P (Daly et al., 2009, p. 102).

$$A \perp_G B \mid C \iff A \perp_P B \mid C \tag{13}$$

In an I-map, the arcs in the graph are directly modelling the dependencies between the nodes and the dependencies between nodes will result in having a direct arc between the nodes. In this process, one of the either Markov causal condition or faithfulness condition is assumed to be applying, meaning that “an effect is independent of its non-effects, given its direct causes and that the conditional independencies in the graph are equivalent to those in its probability distribution” (Daly et al., 2009, p. 103).

Probability tables and network parameters

The probability distributions used in this study are discrete probabilities, although in practice they can be discrete or continuous. The probability distribution of each node is called the local probability distribution. The local probability distributions are marginal for the root node (the nodes with no parents) and conditional for the nodes which have parents. The conditional probability for each node given its parents are presented in conditional probability tables (CPT) and the marginal probability distributions are presented in marginal probability tables (MPT). The values in the probability tables are called the

network's parameters of the network and can be obtained by experts' knowledge elicitation or learnt from the data alongside learning the structure.
The Joint distribution of the system can be presented in a compact way using the Bayesian network and the Global Semantics of Bayesian networks is the product of such conditional distributions for all the network.

$$P(x_1, x_2, \dots x_n) = \prod_i P(x_i | pa_i) \tag{14}$$

Assuming the number of parents for each node are bounded, the number of parameters needed is growing just linear and can be calculated as $(N_{node} - 1) \times \prod(N_{parents})$ in which N is the number of states for a node. Local semantics in Bayesian Networks states that each node is independent of its nondependent nodes (Markov condition or assumption). By choosing the direct causes of a node as parents of that node, the local conditional independence conditions will be satisfied and therefore the local semantics are useful in constructing Bayesian Networks (Conrady & Jouffe, 2007).

Software for BNs

Several opensource and commercial software packages are developed for representation, machine learning and inference in Bayesian networks. A list of available software packages, their type of licence, their pricing, their platform and their abilities are provided in a list in Appendix C.

3.1.2 Association Measures

One way to discover associations between variables in a dataset is by using information theory-based and probabilistic measures. Entropy, Kullback-Leibler Divergence, Mutual Information, Pearson correlation, Spearman rank-order, Phi and Point biserial are a few of them. In the rest of this subsection a description of Entropy, Kullback-Leibler Divergence, Mutual Information, Pearson correlation which will be used later in this manuscript.

Entropy, Kullback-Leibler Divergence and Mutual Information

Entropy is a formal quantification of uncertainty. It shows how even the probability distribution of a random variable is. In other words, entropy is the measure of information one can get, on average, from each value of the distributed variable in the domain. One of the interpretations of entropy can be calculated using Shannon's formula:

$$H(X) = E_{P(X)}[-\log P(X)] \ = -\sum_{i} p(x_i) \ln p(x_i) \tag{15}$$

In which $E_{P(X)}[\ ]$ is the expected value with respect to the distribution of random variable X. This formula calculates the number of bits needed to describe the random variable X. Since the probability distribution of a random variable is non-negative, the value of entropy is non-negative too. The lower range of entropy value of a discrete variable can be zero and it happens when the discrete random variable has no uncertainty, i.e. the probability of one of the values in the random variable is equal to 1 and for the rest of values it is equal to 0. This implies the situation that we are certain about the outcome of the random event. On the other hand, if the distribution of probabilities of a random variable is uniform, the value of entropy will grow to its maximum. This situation is called complete uncertainty in which the entropy value is a function of the number of states of the variable (Conrady & Jouffe, 2007).

In the case of a dataset with multiple random variables, another interpretation of entropy can be the measure of structuredness and regularities in the data (Yao, 2003). A more structured dataset tends to have lower entropy. For any two variables X and Y in a joint probability distribution, entropy is defined as

$$H(X,Y) = E_{P(X,Y)}[-\log P(X,Y)] \ = -\sum_{i}\sum_{j} p(x_i, y_j) \ln p(x_i, y_j) \tag{16}$$

The degree of deviation of two probability distributions can be measured by calculating the relative entropy of two distributions. This measure is also known as Kullback-Leibler (KL) divergence or I-divergence and can be calculated as

$$D(P||Q) = \ E_{P(X)}\left[\frac{P(X)}{Q(X)}\right] = \sum_{i} p(x_i) \ln \frac{p(x_i)}{q(x_i)} \tag{17}$$

In which $P(X)$ and $Q(X)$ are probability distributions and P is absolutely continuous with respect to Q, i.e. $P(x) \to 0$ if $Q(x) \to 0$. KL divergence is a non-negative with a minimum value of 0 in case $P(X) \ = \ Q(X)$. The maximum value is obviously for the case that $P(X)$ is maximum (equals to 1) while $Q(X)$ has its lowest value. The other attribute of this measure is that it is not symmetric, meaning $D(P||Q) \neq D(Q||P)$.

Observation of other predictive random variables can increase the amount of information and consequently the entropy value increases. The entropy of a random variable, X, given the observations of another random variable, Y, is called conditional entropy and can be calculated as

$$H(X|Y) = -\sum_{i,j} p(x_i, y_j) \log \frac{p(x_i, y_j)}{p(y_j)} = -\sum_{i,j} p(x_i, y_j) \log p(x_i|y_j) \tag{18}$$

The value of conditional entropy is non-negative and non-symmetric, which the later means $H(X;Y) \neq H(Y,X)$. It can also be expressed as

$$H(X|Y) = H(X,Y) - H(Y) \tag{19}$$

The difference between the marginal entropy of a variable of choice, X, and the conditional entropy of the same variable given the observations of another random variable, Y, is called entropy reduction or mutual information between x and y. Mutual information can show us what will be the benefit of observing a particular random variable in predicting the variable of choice. In this way, we can find out which variable has the most predictive importance.

$$I(X|Y) = H(X) - H(X|Y) = \sum_i \sum_j P(x_i) P(x_i|y_j) \log_2 \left(\frac{P(x_i|y_j)}{P(x_i)} \right) \tag{20}$$

It can also be expressed using conditional entropy and entropy of X and Y

$$I(X;Y) = H(X) + H(Y) - H(X,Y) \tag{21}$$

The other interpretation of mutual information can be obtained with KL divergence and the degree of independence of two variables. Mutual information can be described as KL divergence of the joint probability distribution of X and Y, i.e. $P(X,Y)$, with a probability distribution if random variables X and Y are independent and their joint probability distribution is obtained by multiplying the marginal distribution of X and Y, meaning $Q(X,Y) = P(X) \times P(Y)$. In this way, the real probability distribution of X and Y is compared with a situation with an assumption of independence of X and Y.

$$D(P(X,Y)||Q(X,Y)) = D(P(X,Y)||P(X) \times P(Y)) = E_{P(X,Y)}\left[\frac{P(X,Y)}{P(X) \times P(Y)}\right]$$
$$= \sum_i \sum_j P(x_i, y_j) \log_2 \frac{P(x_i, y_j)}{P(x_i) \times P(y_j)}$$
$$= \sum_i \sum_j P(x_i)P(x_i|y_j) \log_2 \left(\frac{P(x_i|y_j)}{P(x_i)}\right) \qquad (22)$$

Mutual information in a non-negative and symmetric value.

According to Yao (2003), conditional entropy and mutual information can be used to determine one-way associations between variables. If two variables X and Y have a functional association, i.e. they have a deterministic relationship with each other that implies $P(X|Y)$ is either 1 or 0, these equations will hold:

$$H(X|Y) = 0 \qquad (23)$$

$$H(X,Y) = H(Y) \qquad (24)$$

$$I(X;Y) = H(X) \qquad (25)$$

A functional dependency is the strongest one-way association between variables. The value of mutual information is in its maximum and the conditional entropy value is minimum. On the contrary, probabilistic independence between two variables X and Y implies these equalities:

$$H(X|Y) = H(X) \qquad (26)$$

$$H(Y|X) = H(Y) \qquad (27)$$

$$H(X,Y) = H(X) + H(Y) \qquad (28)$$

$$I(X;Y) = 0 \qquad (29)$$

Two random variables are associated if they are not independent. For two independent variables, the value of mutual information is minimum, and the condition entropy reaches its maximum. Moreover, the joint uncertainty about X and Y is the sum of the uncertainty of each of them.

Pearson Product–Moment Correlation

Pearson correlation shows how linear is the relation between the variables. Pearson correlation coefficient is normally shown by r and the formula for two variables is

$$r = \frac{\sum_{i=1}^{n}(\frac{x_i - \bar{x}}{s_x})(\frac{y_i - \bar{y}}{s_y})}{n-1} \tag{30}$$

In which n is the sample size, $\bar{x}$ (or $\bar{x}$) is the sample mean for variable x (or y) s_x (or s_y) is the sample standard deviation for variable x (or y) and can be calculated as

$$s_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} x_i - \bar{x}} \tag{31}$$

Pearson correlation can have values between -1 and 1, where values close to zero show a weak linear relationship. Values close to 1 show a strong forward correlation and values close to -1 shows a strong reverse correlation between variables under study (Boslaugh & Watters, 2008).

3.1.3 Varieties of Bayesian networks

There are several varieties of Bayesian networks, each of which are created to meet certain needs within the scientific and engineering community. Here as an example, three of these varieties, namely naïve Bayesian networks, dynamic Bayesian networks and influence diagrams are described briefly. For further information regarding the varieties please consult Koller & Friedman's (2013)

Naïve Bayesian networks

A Naïve Bayesian network is a network with only one parent. It assumes a target random variable that is the effect of all other random variables in V, and those cause variables are conditionally independent of each other. This means, to use this method, it should be assumed that none of the causes has any dependency with others. Despite its oversimplified structure and unrealistic assumption, the performance of naïve Bayesian networks for use cases like classification is surprisingly good (Judae Pearl, 1988; Langley, Iba, & Thompson, 1992).

Hidden Markov Models and Dynamic Bayesian networks

A Hidden Markov Model (HMM) is a dynamic model which describes the probability distribution for a sequence of observations. Although the observations can be of any type of data which can represent a probability distribution, we assume in our study that the values are sampled over equally spaced time intervals and discrete (or discretised) (Ghahramani, 2001).

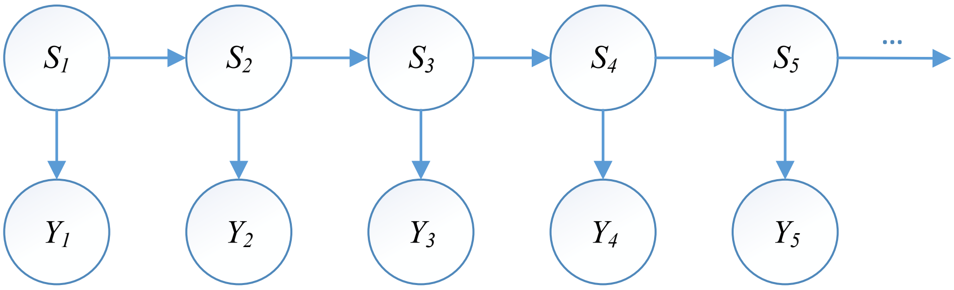

Figure 10. *A simple Hidden Markov Model*

As Ghahramani (2001) described, an HMM should have three properties. The first one is that in an HMM, hidden variables are discrete. It should have some states which are not observable (hidden) and it should satisfy the Markov property - that is given the value of the previous state, the current state should be independent of all the states prior to the previous states. The output of the system is satisfying the Markov output property, i.e. given the current input, the output is independent of states and observations of all other time.

$$P(S_{1:T}, Y_{1:T}) = P(S_1)P(Y_1|S_1) \prod_{t=2,\dots,T} P(S_t|S_{t-1})P(Y_t|S_t) \tag{32}$$

Where $S_{1:T}$ is the states of the system and $Y_{1:T}$ are the observations of the system. To calculate any probability distribution in any state, it is enough to know the initial state $P(S_1)$ and the state transition matrix $P(S_t|S_{t-1})$, and the output model defining $P(Y_t|S_t)$.

In a state space linear-Gaussian model, each real-valued observation Y_t in each time step is generated by a k-dimensional state variable X_t, which is a first order Markov process such that:

$$P(X_{1:T}, Y_{1:T}) = P(X_1)P(Y_1|X_1) \prod_{t=2,\dots,T} P(X_t|X_{t-1})P(Y_t|X_t) \tag{33}$$

This model is a factorization of the joint probability distribution and can be interpreted as a Bayesian network which is similar to HMM's.

The difference between this model and HMM is that S is replace with the hidden variable X and the state transition matrix $P(X_t|X_{t-1})$ is normally decomposed to a function that calculates the mean value of the X_t and a zero mean random noise

$$X_t = f_t(X_{t-1}) + w_t \tag{34}$$

This is called the transient function. In a similar manner, the observation probability is decomposed to

$$Y_t = g_t(Y_{t-1}) + v_t \tag{35}$$

With assuming a Gaussian Noise for both equations and linear functions for g and f, the linear-Gaussian state space model would be

$$X_t = AX_{t-1} + w_t \tag{36}$$

$$Y_t = AY_{t-1} + v_t \tag{37}$$

Where A is the state transition matrix and C is the observation matrix. The interesting point with HMMs and state space models (SSM) is that they provide a closed system with their state transition probabilities/matrixes, inputs, and outputs. This means in an HMM, a discrete K-valued input matrix will map to a discrete K-valued matrix through a $K \times K$ transition matrix. Similarly, a Gaussian distributed hidden state in an SSM, after a linear transformation and adding a Gaussian noise, will result in another Gaussian distributed hidden state.

To model time-dependent systems using one can use a Dynamic Bayesian Network (DBN). The system state at time t is described by a set of variables X_t and the sensory data E_t describes the observations of the system at the same time. A model for the sensor can be a conditional probability distribution of the observable variables given the state variables, i.e. $P(E_t|X_t)$. Furthermore, the states at time t is related to state at time $t+1$ with the transitional model $P(x_{t+1}|x_t)$. The other concept is keeping the track of the world, meaning computing the current conditional probability given all previous observations $P(x_t|e_1, e_2, \dots, e_t)$ (Conrady & Jouffe, 2007).

Dynamic Bayesian Networks are a generalization of Kalman Filters (Kalman, 1960) and Hidden Markov Models. The representation will more compact and more interpretable in a DBN. Each node in an HMM represents a state of the system and in a DBN they represent the dimensions of the system.

Influence diagrams

Bayesian networks can be used not only for showing the probability of certain states happening but with some modifications, they can be used as a decision aid tool. If two additional type of node, namely decision nodes and utility nodes, are added to a Bayesian network, the network can provide the means to make a powerful tool to make decisions under uncertainty. A decision node is representing the possible choices in the system and the utility nodes represent the value of a particular event in the system. The resulting network is called influence diagrams, decision graphs or decision networks (Daly et al., 2009, p. 106).

Causal Bayesian Networks and Causal Discovery

Bayesian networks can have many interpretations depending on the view of their constructors and their usages. The basic interpretation is that in BNs, arcs are representing the probabilistic dependencies between random variables and their combination with the conditional probability tables.

In another view, BNs are representing the causal relations between variables. A Causal Network is a Bayesian Network in which the parents are direct causes of each node. This may be used to predict the result of any intervention (change on purpose not observing the evidence) in any nodes. In a causal network, if we change the probability of happening of each of node, with a certain answer, i.e. changing the probability of happening of a state to 100% or 0%, the arrow from that node to its parent node can be removed. Modelling a system as a causal model will lead to a composition of stable mechanism which can be reconfigured locally with local changes in the model according to the interventions (Conrady & Jouffe, 2007, Chapter 10).

Causal discovery is the act of finding the cause and effect relationship between variables using raw data. In many occasions, it is taught from experiments that the dependency between variables can be in some specific causal directionality and not others. If these results are used in a systematic way, they can be used to infer causal relations from the raw data. The strength of the dependencies of one structure can be used to determine the most compatible structure (Conrady & Jouffe, 2007, Chapter 10).

One main source of information to obtain causal effects is random experiments. For most use cases, random experiments are needed to distinguish the effects of a system. For example, imagine the effect of a new medicine is going to be tested. An experiment should be made with two groups of patients who are randomly taking the medicine or placebo. Then the dependence of the result can be tested, and causal relation can be discovered. The problem with the first approach is that the experiments can be costly, time-consuming or even impossible. The other source can be observational data, e.g. the data from other similar procedures or big data, and then using machine learning tools in Bayesian networks to find the conditional dependency relations between variables.

For more detail please consult Conrady and Jouffe (2007) and Heckerman (Heckerman, Meek, & Cooper, 2006)

3.1.4 Evidential reasoning

Having evidence for some of the nodes in the network, it is possible to calculate the probability of any proposition with the aid of the given conditional probabilities in the network. These calculated probabilities can often be used to describe the structure of the model itself. There are three types of reasoning in Bayesian networks. Diagnostic Reasoning is going directly from effect to cause. It can be shown as a conditional probability of the cause given the effect. The second type is prognosis, in which the reasoning starts from a cause, and predict the effect. Finally, the last type is Inter-Causal reasoning. Once we condition on a common effect using an observation of its value, it is possible to compute the probabilistic relation of one cause to the other cause via that effect (Conrady & Jouffe, 2007, Chapter 4).

Nature of the evidence

The evidence can be of two different natures. *Hard Evidence* is the piece of information about the value of a node or the value of one of the states of the node. On the contrary to hard evidence for inference, evidence can be in the *numerical* or *probabilistic* form. This evidence can be some assumptions about possible conditions of a domain (Conrady & Jouffe, 2007, Chapter 4).

The difficulty of the reasoning task

"Reasoning in Bayesian networks subsumes the satisfiability problem in propositional logic and, hence, is NP-hard." (Pearl, 2004). The process of reasoning in a BN is performed through inference algorithms. The inference is to find the marginal probability distribution of a node, after performing changes in the distribution of the other nodes in the network, e.g. changing in probability distributions of a node based on the new evidence. For small size Bayesian networks, it is possible to marginalize a node by summing over all possible states of all other nodes in the graph, i.e.

$$P(x_n) = \sum_{x_1}\sum_{x_2} \dots \sum_{x_{n-1}} p(x_1, x_2, \dots, x_n) \tag{38}$$

But the number of summations will grow exponentially with the number of nodes in the network. To reduce the complexity of reasoning, several algorithms are developed to calculate the marginal probability with a lower complexity (Guo & Hsu, 2002). Some of these algorithms are performing the inference in the exact form while the others are using some heuristics and stochastic methods to approximate the inference problems.

In this study, Belief propagation algorithm (sum-product algorithm) and junction three algorithm which is the mostly used exact algorithm is described in the rest of this sub-section (Koller & Friedman, 2013).

Belief Propagation Algorithm or Sum-Product Algorithm

Belief Propagation Algorithm or Sum-Product Algorithm is based on the fact that the marginal probability can be calculated by summing over all possible states of all other nodes in the graph. These algorithms solve the inference problem in a linear time complexity (Yedidia, Freeman, & Weiss, 2001).

For undirected graphs with no loops which are singly connected, belief propagation algorithm can be used for performing exact and approximate inferences. As Ghahramani (2001) described, this method propagates the information received by observing some evidence in the network. It updates the marginal probabilities of all variables through a local message passing protocol. Since this method is for singly connected graphs, the node N for which we have new evidence separates the graph to two sets. The set, $S^+(N)$ is containing N and its parents and the other nodes connected to its parents. The other set, $S^-(N)$ is N's children and the other nodes connected to N through its children.

The message to be passed from the node N to its children is the changes in the probabilities of each state of the node N given the evidence observed in the $S^+(N)$ set, therefore if N has K different states, the message is a K dimensional vector. The message from N to each of its parents is the probability of the evidence observed in the set $S^-(N) \cup \{N\}$ given each state of that parent.

The marginal probability of each node is proportional to the product of the message received from its parents, weighted by the conditional probability of the node given its parents and the message received from its children.

$$P(N|S) \propto [\sum_{\{p1,\ldots pk\}} P(N|p_1, \ldots, p_k) \prod_{i=1}^{k} P(p_i|e^+(p_i))] \prod_{j=1}^{l} P(c_j, e^-(c_i)|n) \tag{39}$$

For a more detailed description of this algorithm, the reader is referred to (Mooij, 2008) and (Yedidia et al., 2001).

Junction Tree or Clique Tree Algorithm

Junction tree algorithm can be used in case there exist more than one undirected path between two nodes in the graph, i.e. multiply connected networks (Ghahramani, 2001).

This algorithm consists of seven steps (Kahle, Savitsky, Schnelle, & Cevher, 2008) and starts with moralizing the network, meaning transforming the directed graph to an undirected graph. This is done by adding an undirected edge between the parent nodes and then changing directed edges to undirected edges by adding an edge in the reverse direction. The second step is to triangulate the graph. This means for any cycle in the undirected graph, if the number of nodes is bigger than three, we should add an edge to a pair of non-consecutive nodes in that cycle, which is called a chord. Now, in the third step, we can form the junction tree. Junction trees are tree graphs created using a hypergraph formed from the cliques of the triangulated graph in the last step. A hypergraph is a set of all nonempty subsets of a graph.

A junction tree should have another property which is called running intersection property or junction property, which is that the intersection of any two nodes in a path in a junction tree should be contained in every node in that path. In the fourth step, the conditional probability distribution (CPT) tables are used to assign potentials for each clique; the potential is the joint probability distribution of that clique.

Since the main reason for forming the junction trees is to apply a message-passing algorithm, in the fifth step, the algorithm defines a root node to start the message passing procedure from it. After setting the root node, the algorithm uses one of the messages passing algorithms in graphical models to pass the changes in the nodes all the way to the leaf node and backward. Therefore, the message-passing step, step six, consists of two messages and the junction tree guarantees the convergence of the algorithm. In the seventh step, we use the result of the last step, which is the modified joint distribution of each clique to calculate the marginalized distribution of node of choice.

A measure of conflict in evidential reasoning

Adding new evidence to a node in a network is not always decreasing the uncertainty. To detect and measure the "conflict" in the evidence, it is possible to compare the joint probability distribution (JPD) in the network before and after using the evidence. To do so, the entropy of the JPD of a fully unconnected model, the straw model, of the system is used as a reference. If the entropy of the network after using the evidence is bigger than the entropy of the straw model, the evidence is conflicting (Conrady & Jouffe, 2007).

The global conflict value for the current set of evidence with n observations can be calculated with:

$$GC(E) = \log_2 \left(\frac{\prod_{i=1}^{n} P(e_i|e_1, \dots, e_{i-1})}{\prod_{i=1}^{n} P(e_i)} \right) \tag{40}$$

Bayes factor has a hypothetical piece of evidence that has not been observed used is equal to:

$$BF(E,h) = \log_2 \frac{P(h|E)}{P(h)} \tag{41}$$

And the local conflict or local consistency can be calculated by summing global conflict and Bayes factor values.

$$LC(E,h) = GC(E) + BF(E,h) = \log_2 \left(\frac{\prod_{i=1}^{n} P(e_i|e_1, \dots, e_{i-1}) P(h|E)}{\prod_{i=1}^{n} P(e_i)\, P(h)} \right) \tag{42}$$

3.1.5 Machine learning in Bayesian networks

Both the network parameters (MTPs and CPTs) and network structure of a Bayesian network can be learnt from the data using machine learning algorithms. In the rest of this subsection, first the method of learning parameters in this study is reviewed and then the structural learning algorithms are briefly described.

Learning the parameters

For learning a Bayesian network's parameters there can be two approaches. In the first case, the conditional probability tables, $P(x_i|pa_i)$ for a qualitatively described Bayesian network structure can be estimated using the maximum likelihood method from the dataset associated with the network. On the other hand, a pure Bayesian approach includes designing a network using expert knowledge and hyper parameter nodes. In this case the data act as the piece of evidence to perform Bayesian updating meaning updating the distributions of the hyper parameters. The procedure of updating uses gradient based or expectation maximization based approaches with is similar to neural networks (Daly et al., 2009, pp. 112–115; Koller et al., 2007, pp. 42–47).

In the case of this study, where the network variables are discrete, a much simpler method is used to estimate the parameters. This method, called the counting method, counts the number of occurrences of the data point in the dataset for each state. For the MPTs, it is enough to count the number of data points for each estate of that variable and for CPTs, this number should be counted for each state, considering the combination of the stats of their parent node. For more detail on the process, please consult Conrady & Jouffe (2007, pp. 99–105).

Structural learning

To learn a Bayesian network's structure, at least three groups of methods are available. The first group are score-based approaches which utilize a metric to determine the quality of candidate networks given the observed data. The metric is normally defined as the likelihood of the data given the network and it trades off the complexity of the network versus the degree of fit to the given data. These group of algorithms are less sensitive to the quality of data. The second group is constraint-based algorithms, in which, the conditional independence between variables in the dataset is used to determine the best structure. Statistical tests identify marginal and conditional dependencies and based on that links are added or removed between nodes (Conrady & Jouffe, 2007; Munteanu & Bendou, 2001). The third group is the dynamic programming approaches in which using score-based dynamic programming techniques, optimal models for a small set of variables can be obtained and the models can be combined if necessary (Daly et al., 2009, sec. 4.11).

Score-based algorithms

The score-based algorithms are working based on a searching approach. They normally start with an empty set of arcs between variables and move on with searching in the neighbourhood for a structure that describes the joint distribution better. A neighbourhood is the set of graphs, which are different in only one atomic graphical element, e.g. deletion or addition of one arc or change of direction in only one arc. The score can be measured, for example, according to the likelihood of the structure being true, given the data. Then a heuristic search algorithm is used to find the network that maximized the score of the network (Scutari, 2010).

The process of the search is a hard task and heuristic algorithms including greedy search, genetic and evolutionary algorithms, simulated annealing, particle swarm optimization is used to reduce the complexity. The other approach is to search is the space of equivalent classes, which is the algorithm that is used in this study and is described later in this subsection (Daly et al., 2009, sec. 4.5-4.7).

Choosing the right scoring function for the learning process is an important criterion. The highest match between the dataset and the model is always for a fully connected network, which has the maximum number of parameters and in most of the cases complicated and useless. Therefore, the score for learning process should consider at least two measures at the same time, namely the complexity of the model and the goodness of fit to the data. So, most of the scores are rewarding for a better match to data and penalizing the complexity. Bayesian Dirichlet criterion (BDc), Bayesian information criterion (BIC), Akaike information criterion (AIC), minimum description length (MDL), minimum message length (MML) are a few of these scoring criteria. The other important property of these scores is that they are decomposable, meaning they can be calculated locally for each node and then combined to have a holistic score for the model (Daly et al., 2009, p. 112).

As an example, a brief description of one of the criteria used in this study, the MDL criterion, is provided as follows. Minimum Description Length (MDL) is a principle which is based on the idea that the best way for capturing features in the data is to construct a model which represents the data in the shortest description possible for both the data and the model (Rissanen, 2006). Therefore, MDL a two-component score, consisting of the number of bits required for representing a model and the data given that model. The best solution has the lowest value for MDL. In terms of the Bayesian network, the model is the network and the probability tables. The other component, in our case, is the log-likelihood of the data, given the model, which has an inverse relationship with the Bayesian network model given the observations data.

In the case of this study, MDL is calculated as:

$$MDL\,(B, D) = \alpha DL(B) + DL(D|B) \tag{43}$$

In which α is a constant called structural coefficient, $DL(B)$ is the number of bits to represent the Bayesian network B, and $DL(D|B)$ is the likelihood (number of bits to represent) of dataset D given the Bayesian network B. The minimum value for the first component occurs when we have a set of fully unconnected nodes. On the other hand, the minimum value for the second component occurs when all nodes in the network are connected, i.e. fully connected network. The criteria is to minimize the sum and to minimize the sum, the best trade-off between these two should be found (Conrady & Jouffe, 2007).

Constraint-based methods

The second group of structural learning algorithms are working based on Bayesian rules to find the structure and called constraint-based algorithms. These algorithms analyse the probabilistic relationships between variables with conditional independence tests and based on that, create networks that satisfied d-separation conditions. These algorithms normally use a statistical test such as χ^2 and G tests to determine dependencies between variables (Daly et al., 2009, sec. 4.8).

The generic procedure has three steps 1- finding an undirected graph, called a skeleton, to represent the independencies with some search method, 2- setting the V-structures, i.e. structures with converging directions and 3- setting the direction of the other arcs to satisfy the cyclic property of the graph (Conrady & Jouffe, 2007).

The EQ method

Finding the best network in the search space of possible networks is an NP-hard computational task. Heuristic search algorithms that are normally used can easily trap in local minima. Munteanu and Bendou (2001) developed the EQ framework to solve this problem and it uses the space of essential graphs of an equivalent class to search for a suitable graph.

Two Bayesian Networks are Equivalent if they represent the same joint probability distribution. In a more formal way, the Bayesian network B and B' are equivalent for a set of variables V, in a joint probability distribution, if for each parameter θ of B, we have a parameter θ' for B' such that

$$P(V/S, \theta) = P(V/S', \theta') \tag{44}$$

Verma and Pearl (1991) defined the condition for two BNs to be in the same equivalence class as having the same skeleton and the save V-structures. In a DAG, the skeleton is the graph with undirected edges and a V-structure is a configuration like this:$A \rightarrow B \leftarrow C$.

In a Bayesian Network from an equivalence class, an essential edge is an edge, which is present in all the BNs of that class. Therefore, all the edges in V-structures of an equivalence class are essential. However, there can be other essential edges in an equivalence class. An Essential Graph of a Bayesian Network is a partially directed acyclic graph that its edges are similar to that graph and the essential edges are directed (Garrido, 2008). Therefore, the essential graphs set is a subset of the Equivalent class in which in addition to V-structures, the essential edges are directed too.

The graphs belonging to an equivalent class have the same performance score, e.g. MDL or BDc, and this can lead to a problem in the learning procedure (Munteanu & Bendou, 2001). A search algorithm, which uses such scores of the BN to find the best graph, may choose the wrong graph, with a similar score with the correct graph in the same equivalence class, in an intermediate point of the search. Extension of the search procedure based on the wrong intermediate graph will lead to finding a graph with wrong causal relations.

The EQ framework proposes a local scoring scheme to solve this problem. In each essential graph, transformation algorithms create non-empty subsets of graphs, called instantiable graphs, by making small changes in the structures in the essential graphs set. A small transformation means suppression or addition of only one single edge. On the other hand, the score of a Bayesian network can be decomposed as the sum of the local scores calculated from a node and its parents. Using this fact, they calculated the change in the score of the whole BN after performing the small transformation. Since each of the instantiable graphs is belonging to the same equivalence class and the score of all the graphs in an equivalence class is the same, we can use the calculated score for each transformed graph as the score of the whole equivalence class. The score can be calculated as:

$$\Delta S(G', G) = S(G') - S(G) = S\left(A \middle| Pa_{G'}(A)\right) - S(A|Pa_G(A)) \quad (45)$$

In which, G is the current essential graph and G' is the instantiable graph after small transformation; $S(G)$ is the score of the whole BN, $S(A|Pa(A))$ is the score of a node given its parents. Using this score, the best instantiable graph will be used to continue the learning procedure.

The procedure of learning the structure then can be summarized as follows (Munteanu & Bendou, 2001):

1. Setting constraints in order to avoid making non-instantiable graphs from essential graphs in the transformation process and creating rules according to these constraints
2. Making operators for creating new instantiable graphs by adding or removing edges and V structures according to the rules and calculating the change in the local score for each of them
3. Creating the essential graph corresponding to the created instances and
4. Calculating the score for the found equivalence class and compare to others to find the best structure

It is also possible to include the prior expert knowledge to the process of learning the structure of a network. Forbidding relations, fixing portions of the structure, or using prior distributions over the network parameters are the techniques that help to accurately learn the network's structure with the fewer amount of data (Conrady & Jouffe, 2007).

Structural Coefficient (α)

Structural Coefficient (SC) is a tool for controlling the complexity of the structure of a network. This value can determine the significance threshold for the learning process. In the case of this study, α is used in calculating MDL score which is used in the EQ learning process. SC controls the internal number of observations, N', where the equation for it is:

$$N' = N/SC \tag{46}$$

With N as the number of samples in the dataset.

An SC value equal to one (1) helps to prevent overfitting of the model to the training data if a large amount of data is available for machine learning. For datasets with relatively few numbers of data points, this number should be decreased to increase the number of observations. An SC value equal to zero will result in all the relationships between variables become significant and the trained network will become a fully connected network. In case the dataset is too big, the value should be increased to train using a sample of the training data (Conrady & Jouffe, 2007).

3.1.6 Validation of the Bayesian network

Contingency Table Fit (CTF)

One of the measures to check the fitness of the Bayesian network model with the data set is to check the contingency table fit (CTF) value. A fully connected Bayesian network is always the best representation of the contingency table, i.e. a network in which none of the conditional independencies between random variables is considered in the structure of the network. On the other hand, an unconnected network assumes that there is no dependency between variables and is the worst representation of the contingency table. By comparing the networks structured in the machine learning with a fully connected network and an unconnected network networks, it is possible to measure the descriptive power of the network for any dataset (Conrady & Jouffe, 2007).

In the case of Bayesian networks, it is possible to use an information theory based metric, e.g. entropy, to measure the fit. A detailed description of the entropy and conditional entropy is provided in section 3.1.2. Using the entropy, conditional entropy and the conditional dependencies of the variables in the Bayesian network, it is possible to calculate the entropy for the whole network. The CTF value then can be calculated by comparing the entropy of the current network with the entropy of a fully connected network and the entropy of an unconnected network (Bayesia, 2018):

$$CTF = \frac{H(B) - H(unconnected)}{H(fully\ connected) - H(unconnected)} \tag{47}$$

The CTF value in the case of this study can be between 0 and 1, in which 0 means the worst fit and 1 means the best fit.

Running the learning process for multiple times

To find a suitable value for SC, it is possible to train the network with a range of SC values and calculate the CTF value for each of them. By plotting the CTF value versus structural coefficient, it is possible to judge that how adding complexity will affect the precision of the result, based on the SC value. If increasing SC value does not result in better CTF, then probably the model is overfitting to the data. The elbow in the plot, i.e. where increasing SC is not resulting in better predictions, is the best SC value that can be used.

3.1.7 Pre-processing of data for Bayesian networks

Most of the well-known algorithms developed in Bayesian networks environment, including the algorithms used in this study, are based on discrete valued random variables. If the data is continuous, it should be discretized before it can be used.

Discretization of continuous variables

Discretization is to transform continuous data to a set of finite non-overlapping intervals (Muhlenbach & Rakotomalala, 2005). Machine learning algorithms for learning the structure of a Bayesian network and performing inference in BNs are mostly developed for discrete variables space. Although there are structural learning algorithms for continuous variables, they lack either in expressiveness or in interpretability. Therefore, discretization of a continuous variable is inevitable and yet in most of the cases, it leads to a better result. Discretization is the process of transforming quantitative data into nominal qualitative data. This process leads to a loss of information in the data and finding an optimal discretization is an NP-complete task (Mabrouk & Gonzales, 2010).

To find a suitable discretization method, researchers proposed several heuristic based methods (Kotsiantis & Kanellopoulos, 2006) and taxonomy based methods (García, Luengo, Sáez, López, & Herrera, 2013). Discretizers can be classified according to multiple criteria, namely supervised and unsupervised or univariate and multivariate techniques (Bakar, Othman, & Shuib, 2009).

In supervised discretization, all variables are discretized to have the best representation of with respect to one of the variables in the dataset while unsupervised methods are discretizing the data without any prior knowledge about the associations hidden in the data. Supervised discretization is considered when the aim is to make a classification for a target variable (Muhlenbach & Rakotomalala, 2005).

Univariate discretizers consider only one variable in the dataset while multivariate discretizers consider the interaction between multiple variates simultaneously to find the best intervals. These interactions contain hidden information about other variables which is the primary criteria if the data is going to be used for investigating the dependencies and causal relations in the dataset using, for example, Bayesian network structure learning (S. Monti & Cooper, 1998; Stefano Monti & Cooper, 1998).

Since there are a significant number of techniques available for discretization, a selected number of most frequently used ones is reviewed in the rest of this subsection. Interested readers are referred to (Bakar et al., 2009; Dougherty, Kohavi, & Sahami, 1995; García et al., 2013) for more information.

Equal distance or equal interval width is an unsupervised method that makes bins with equal repetition in the range of a variable. For a continuous variable with a minimum value x_{min} and a maximum value x_{max} and k equally sized bins, the bin width will be calculated as

$$\delta = \frac{x_{min} - x_{max}}{k} \tag{48}$$

And then these bins are a size to calculate thresholds $x_{min} + i\delta$, $k = 1, 2, \ldots , k - 1$. This method is sensitive to outliers and can return bins with no data point with unclean data. To avoid this problem, the normalized equal distance algorithm can be used to remove outliers before equal partitioning.

The equal frequency method creates bins with an equal number of observations and results in a uniform distribution in the bins. Therefore, for a continuous variable with N values, the number of values in each bin is N/k.

The other algorithm is called K-Means which discretise the variables using the clustering idea. The variables cluster around a k number of values which are the mean value of the variables in that clusters (Joiţa, 1995).In case the distribution and the variable domain of the variables are unknown, it is best to use this algorithm.

Discretization in Bayesian network learning

In some studies, univariate supervised algorithms used for multivariate discretization for Bayesian networks. In these algorithms, each node is considered as a target value and the univariate discretization is performed for all those variables.

For learning the structure of Bayesian networks, one of the first efforts has been done by (Friedman & Goldszmidt, 1996) and (S Monti & Cooper, 1999; Stefano Monti & Cooper, 1998). They both tried to perform the discretization in the framework of structural learning in Bayesian networks. They combined discretization and learning in the search based structural learning process to find the best discretization for each network they find in the search space.

In another effort, Nguyen et al. (2014) introduced Interaction Preserving Discretizations (IPD) in which they form micro-bins for each variable and then try to merge the bins using the entropy-based score to identify the optimal discretization for each variable. Mabrouk et al. (2010) have shown that entropy-based discretization result is suboptimal and developed an algorithm which performs discretization for BNs based on clustering scheme which has outperformed the previous methods. In their method, first, they approximate the joint distribution of continuous variables with a mixture of non-truncated Gaussian distributions and then use the EM method to determine the number of cut point and the mean values and the variances of the Gaussian distributions. The cut points will produce some intervals in continuous variables, which are now modelled by a summation of Gaussian distributions. The parts of the Gaussian distributions that are left outside of each interval is considered as a loss of information; so, in the second step, they tried to minimize this loss.

As mentioned before, for training Bayesian networks, it is important to preserve the dependencies between variables in the discretization process. Therefore, many approaches tried to perform the discretization based on the structure of the BN. This leads to the development of multiple Bayesian Discretization-Learning algorithms which performs the discretization alongside with the search for the best structure for the BN (Friedman & Goldszmidt, 1996; Mabrouk & Gonzales, 2010; Stefano Monti & Cooper, 1998).

In the case of this study, a univariate genetic algorithm optimization based method is used for discretization. The algorithm is called R2-GenOpt which maximizes the R^2 between the continuous variable ant its corresponding discrete variable. It uses the genetic algorithm to find the optimal discrete representation of the continuous values of a variable. The algorithm is also able to find the optimal number of the states for the discrete variable. The algorithm is developed by S.A.S (Bayesialab-S.A.S, 2019).

Missing Values Processing

Having missing values is a common phenomenon in datasets. Missing can happen because of a failure in the recording system, human mistakes, errors in sensors or a non-response in a survey. When dealing with a large amount of data, the intuitive approach is to remove the records with missing data. This approach is also called likewise deletion or case-wise deletion. It has been shown (Koller & Friedman, 2013) that this approach can lead to a magnificent amount of bias if it is not done carefully.

Conrady and Jouffe (2007) provided a very good description of different types of missing values and the mechanisms behind the missingness. The rest of this subsection is written based on their book. Missing value can be classified into four types:

1. Missing Completely at random (MCAR)

In this class, the missing mechanism is totally independent of other variables. For this class, it is possible to confidently remove the data points with missing values without affecting the distribution of the data. The problem is that it is not possible to confirm that the missingness is in this class.

2. Missing at random (MAR)

The missing mechanism in the MAR class is dependent on observed variables. In this case, it is not possible to remove the data points with missing values because it will change the distribution of the variable with the missing values with respect to the other variables.

3. Missing Not at Random (MNAR) or Not missing at random (NMAR)

In this situation, the missing mechanism is depending on hidden or unobserved causes. The result of deleting missing values here is similar to MAR class.

4. Filtered Values

Filtered values are the values which are not missing at all, they are the value which was not existing in the first place. Most often, these values are not possible if some other variables take some specific values. The missing values in this class are similar to missing values in the MAR class. The difference is that the filtered value should not be processed as a missing value, but it should be considered as a special type of observation.

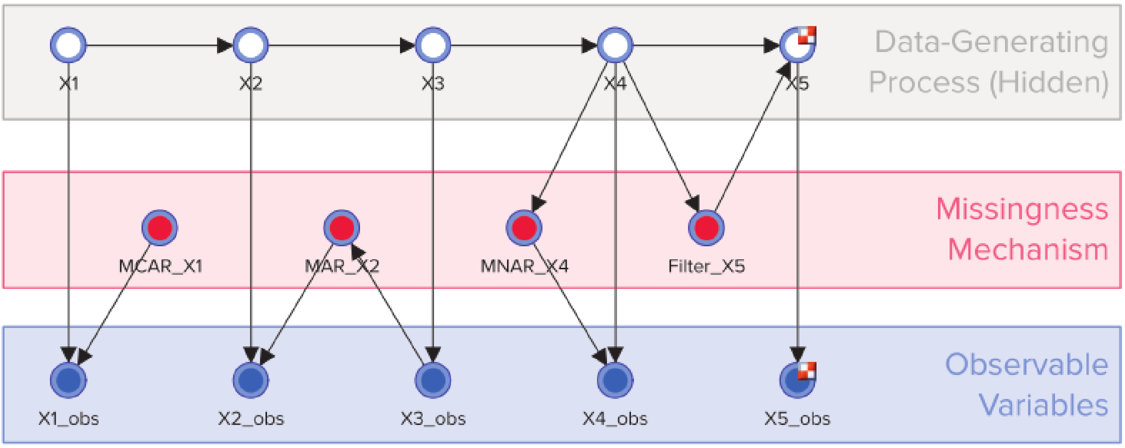

Figure 11. *Missing value mechanisms* (Conrady & Jouffe, 2007)

In addition to potentially changing the distribution of the variables, the likewise deletion approach can reduce the number of data points. As mentioned before, this method is only suitable when we know the missing values are in MCAR type or the number of missing values is relatively small.

The other approach to process the missing values is to replace them with some fixed values. The replacing value can be coming from an expert's knowledge or, for example, can be the mean or mode of the variable with the missing value. Replacing the missing values with the mean or modal value will add the number of instances for those values and will lead to magnificent changes in the distribution of the data. Therefore, this method is not recommended in general.

The third approach can be replacing the missing values with some inferring method. Static imputation is the first method in this group, in which, the missing values are replaced with a random draw from the non-missing values of the same variable. Using this method, the resulting distribution of the variables will be similar to likewise deletion, but with no decrease in the amount of data and not additional bias. The second method in this group is the structural expectation maximization method. This method replaces the missing variables according to the network structure.

Expectation Maximization method

Expectation maximization (EM) is a method to replace the missing values using the available values and the parameters calculated using them. This method consists of two steps. The first step uses a maximum likelihood estimator to find an expected value for the missing value, based on the parameters calculated from the other incomplete data. Since our data is in discrete form, parameters can be estimated by counting the number of datapoint in each state of the variable. The estimator replaces the missing values with a number in the range between 0 and 1. In the second step, a new set of parameters are calculated using the real data and the estimated values. At this point, one iteration of the EM algorithm is finished. Using the new parameters, a new set of likelihoods will be calculated for the missing values and the procedure will be continued. In each iteration, the likelihood value for the network will be compared with the previous iteration, and if it does not change, it means that the algorithm reached local minima. The method guarantees reaching a local minimum (Koller & Friedman, 2013).

3.1.8 General concerns about using Bayesian methods

There are concerns about using Bayesian statistics in the scientific community. The major concern is that Bayesian statistics are considering statisticians subjective knowledge as the beginning point for creating models. In case that the amount available data is limited, the final posterior model will look like the prior knowledge and if the amount of data is magnificent, the posterior model will look like the data (Swiler, 2006).

The other concern about Bayesian networks is the approaches to obtain the Bayesian network. As mentioned before, there are two general approaches to learn Bayesian networks, namely a machine learning approach and the interpreted approach. The problem with the machine learning approach is that in most of the real world cases, the amount of the data is not sufficient for the learning algorithms to obtain a reliable network. Moreover, the quality of the data is not reliable. In the interpreted approach, the main problem is with the reliability of the independence assumptions that can be made. For example, it may be difficult to elicit the knowledge of an expert (Williamson, 2001). That is why systematic solutions for the interpreted approach is needed.

3.2 Analytical Hierarchy Process

In contrast to classical probabilities which is the actual probability of a physical happening, Bayesian probability is the statisticians' degree of belief in a happening (Heckerman, 2008). In this scene, to measure a Bayesian probability, there is no need to perform repeated trails. One question which comes to mind is, how and on what scale one can measure the degree of belief in some happening?

There can be many different probability assessment methods to answer this question and be used to the marginal and conditional probability tables for a Bayesian network.

Probability assessment method normally relays on the knowledge of experts in the domain under study. The issue with these methods can be the degree of sensitivity of a system to the precision of the assessments. In most decisions making tasks, the decisions are not sensitive to small deviations in assessed probabilities. Nevertheless, sensitivity analysis methods are the well-established methods to investigate if extra precision is needed (Heckerman et al., 1995).

The other problem in probability assessment can be due to the means a question is phrased. An unsuitable question can cause the expert not to be able to reflect their true beliefs and lead to lack of accuracy in the assessment. For that, the Analytical Hierarchy Process (AHP) method which is used in the multi-criteria decision-making domain can be used to collect information from the experts more accurately.

AHP was initially developed to derive priorities in multi-criteria decision problems (Saaty & Vargas, 2012). In general, AHP has three principles, namely decomposition, measurement of preferences and priority synthesis. The workflow of the process starts with defining the goal of the study, which is the description of the problem under study. Then the criteria and the sub-criteria that the decision must be evaluated should be defined. After decomposing the criteria, a pairwise comparison between elements

The steps for an AHP process is described by Saaty and Vergas (2012) as follows:

1. Defining the objective or goal of the study and identifying the domain of the study is done in this step. The objective is the question which should be answered by the multi-criteria decision-making technique. The objectives can be broken down to sub-objectives if possible.

2. The structure of the problem should be decomposed and the criterion, the sub-criterion should be identified and the alternatives of the decision making. Criteria are the means which should be satisfied in order to reach the objectives and sub-objectives. The domain of the study should be investigated to find the important criteria, stack holders and actors in the domain. The criteria can have negative (cost) or positive (benefit) impact on the objective. Then the possible solution alternatives of the problem should be identified. Each alternative is affected by a combination of criterion with different orders of magnitude. Then the hierarchical structure of the problem should be formed. A sample hierarchy for a problem with one goal, 6 criteria and 3 alternatives are shown in Figure 12.

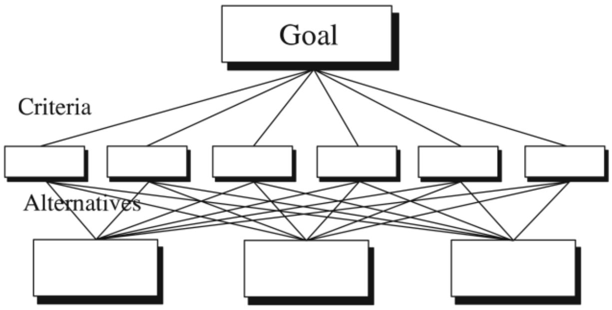

Figure 12. *A tree level hierarchy* (Saaty & Vargas, 2012)

3. In this step, the matrix of pairwise comparisons is created. The criteria should be compared with each other and, after that, the sub-criteria should be compared pairwise. This is a relative comparison in which the criteria are compared in pairs according to a common attribute. These comparisons are based on a set of fundamental scales which is described in Table 1.

Table 1. Fundamental scales for AHP pairwise comparison (Saaty & Vargas, 2012)

Intensity of importance	**Definition**	**Explanation**
1	Equal Importance	Two activities contribute equally to the objective
2	Weak to	Experience and judgment **slightly** favour one activity over another
3	Moderate importance	
4	Moderate plus to	Experience and judgment **strongly** favour one activity over another
5	Strong importance	
6	Strong plus to	An activity is favoured **very strongly** over another; its dominance demonstrated in practice
7	Very strong or demonstrated importance	
8	Very, very strong to	The evidence favouring one activity over another is of the **highest possible** order of affirmation
9	Extreme importance	
1-1/2-1/3-1/4-1/5-1/6-1/7-1/8-1/9	These are the reverse values for the numbers above for the case the relation is reversed.	These show the reverse relationship between two compared activities.

The fundamental scales are meant to show the fraction one criteria in more important comparing to another criterion. Using the fundamental scales, one can create the matrix of comparison as follows

$$A = \begin{array}{c|cccc} C & C_1 & C_2 & \dots & C_n \\ C_1 & & & & \\ C_2 & & \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1j} \\ a_{21} & a_{22} & \dots & a_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & \dots & a_{ij} \end{bmatrix} & & \\ \vdots & & & & \\ C_n & & & & \end{array} \tag{49}$$

In which each element, a_{ij} is the result of the comparison C_n between two criterions. The comparison matrices are always positive and reciprocal, meaning for any i and j, $a_{ij} = {}^{1}/_{a_{ji}}$.

4. In the next step, the weights and consistency ratios should be calculated. The comparison between variables, and consequently, the values of the comparison matrix should be checked for consistency. This means the comparison between criterion i and k, should be predictable with a comparison between criterion i and j and a comparison between the criterion j and k. This implies a relation like $a_{ik} = a_{ij} \times a_{jk}$. This happens if the matrix of comparison is in the ideal form

$$A' = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \dots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & & \frac{w_2}{w_n} \\ \vdots & & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \dots & \frac{w_n}{w_n} \end{bmatrix} \tag{50}$$

In which, $W = (w_1, w_2, \dots, w_n)$ are the real weights for each criterion. If we want to calculate W from the matrix A above, we can multiply it from right by W

$$\begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \dots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & & \frac{w_2}{w_n} \\ \vdots & & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \dots & \frac{w_n}{w_n} \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_4 \end{bmatrix} = n \times \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_4 \end{bmatrix} \tag{51}$$

In the real-world analysis, the experts' opinion a_{ij} may not be exactly equal to the ideal matrix values $\frac{w_p}{w_k}$. Then the solution for finding the weights will change to $A \times W =$

$\lambda_{max} \times W$ where λ_{max} is the largest Eigen value of the matrix A which is a perturbed version of the matrix A'.

According to Saaty and Vargas (2012), the exact solution for obtaining weights matrix from the matrix of comparisons is to raise the matrix of comparisons to high power and then summing over the rows and normalize the results. They also proposed two methods for approximating the weights. The first one is to normalize the geometric means of each row, i.e. calculating $\sqrt[j]{(a_{m1}.a_{m2}.\dots.a_{mj})}$, $m = 0, \dots i$ for all rows of matrix A and then averaging the resulting values of all rows (Tomashevskii, 2014). The second approximate way is to normalize the elements of each column and then averaging over each row. In this method, first, we calculate the sum of each column and then divide each element of the matrix by that, i.e. $m_{ij} = \frac{a_{ij}}{\sum_{i=0}^{n} a_{ij}}$. Then the weights can be calculated by averaging over each row of the resulting matrix of pervious step, i.e. $w_n = \frac{\sum_{j=1}^{n} m_{ij}}{n}$.

Now that we can calculate the approximate values for weights the only question is how consistent the matrix of comparisons is. If the matrix A is consistent, the value of λ_{max} would be equal to n and otherwise $\lambda_{max} \geq n$. λ_{max} can be easily calculated by adding the columns of A and multiplying the resulting vector by the weights vector.

If $\lambda_{max} \neq n$, we need to have a measure of inconsistency, to validate the matrix of comparison. This can be measured by calculating the ratio between the variance of error incurred in estimating A, the consistency Index (CI) and the ratio of error incurred in a reciprocal comparison matrix with randomly chosen values, the Random Consistency Index (RI). The value of CI is calculated from $C.I. = (\lambda_{max} - n)/(n-1)$ and the values for R.I. can be obtained from Table 2.

Table 2. Table of random consistency index (Saaty & Vargas, 2012)

N	1	2	3	4	5	6	7	8	9	10
Random Consistency Index (R.I)	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

Using the table above, a consistency ration (CR) can be calculated as $CR = CI/RI$. If the value of CR is lower than 10%, the inconsistency is acceptable, and in case it is more than that, the matrix of comparisons should be revised.

5. The alternative solution can be evaluated based on the criteria and the weights calculated in the previous step. Each alternative can be scored based on the combination and the value of the criterion it has and the calculated weights for each criterion. Then the scores can be the basis for an absolute comparison between the alternatives.

3.3 Dimensional Analysis Conceptual Modelling

Dimensional Analysis Conceptual Modelling (DACM) is proposing a mechanism to organize, simplify and simulate the behaviour of a system in the form of cause-effect relationships using qualitative information about that system. In their work, Coatanéa and his colleagues (2016) used Dimensional analysis theory to find causal relationships between the phenomena happening in a system.

Dimensional analysis

Dimensional analysis (DA) is originally used to find the relationship among the variables in a system based on the dimensions of these variables. One of the theories used in DA is the principle of dimensional homogeneity. Having an equation like

$$y = \sum_i y_i x_i \tag{52}$$

To be a physical relation, all the $a_i x_i$ must have the same dimension as y (Bhashkar & Nigam, 1990). As an example, the principle of dimensional homogeneity constraint the variables of both sides of the equation $F = ma$ to have the same dimensionality. Therefore, the dimension of Force, F must be the multiplication of the dimensions of Mass, m (M) and Acceleration a ($L \times T^{-2}$) and that is $M \times L \times T^{-2}$.

Π-theorem

The other theory that is used in the dimensional analysis is the Π-theorem introduced by Vaschy-Buckingham (1914). If a physical system is described by a mathematical equation, it can be written as:

$$F(Q_1, Q_2, \dots, Q_n, r', r'', \dots) = o. \tag{53}$$

In which $Q_1, Q_2, \dots, Q_n$ are the variables of the system which are of n distinct kinds and $r', r'', \dots$ are a set of ratios between the variables involved in the equation. The ratios can be for example the ratio between the variables describing the dimensions of a physical object, which can be fixed, e.g. in an equilateral triangle, or not. Now, if the ratios do not change during the phenomenon described with the equation, and all the required system variables are considered in the equation, the equation is a complete representation of the relations among the variables of the system. Therefore, the equation is reduced to:

$$F(Q_1, Q_2, \dots, Q_n) = o. \quad (54)$$

Such an equation is called a complete equation and the coefficients of it are dimensionless numbers. This means they are not dependent on the fundamental units which the variables Q are described with, but they are depending on some fixed iterations of Q which characterize the system and differentiates it from other systems.

As an example, to describe the area surrounded by a curved line with every point of it in a constant distance with one central point, e.g. the surface of a circle, this equation can be used:

$$\frac{S}{r^2} = constant \quad (55)$$

In which S is the surface and r is the distance between the curve line and the central point, e.g. radius of the circle. If the value of the constant is equal to approximately 3.1415, i.e. the π number, the distance between the points of the curve line to a central point is constant, i.e. the shape of the curved line will be a circle. The constant will remain equal to π as long as the shape is a circle and vice versa.

Another example can be the relation between absolute temperature (θ), specific volume (v), and pressure (p) of a gas in a closed container.

$$\frac{pv}{\theta} = constant \quad (56)$$

Here the constant is not dimensionless, and it depends on the units chosen for p, θ and v, even for a given gas. Further exploration in such systems shows that the equation can be written as:

$$\frac{pv}{R\theta} = N \quad (57)$$

In which R is a value that is fixed for any given gas with fixed p, θ and v, but changes with the type of gas. R is a quantity that can be measured by a unit derived from the units of p, θ and v, and if we do so, N will be a dimensionless constant and the equation is a complete equation.

Every complete equation with the form of equation (53) can be written in the form

$$\prod_0 = f(\prod_1, \prod_2, \dots, \prod_n) \tag{58}$$

In which Π_i are the dimensionless products. Moreover, a dimensionless number can be of the form

$$\pi_k = y_i . x_j^{\alpha_{ij}} . x_l^{\alpha_{il}} . x_m^{\alpha_{im}} \tag{59}$$

In which x_i are the repeating variables, y_i are the performance variables and α_{ij} is the exponents for the repeating variables.

Bond graphs

Bond graphs are used for providing a graphical description of the dynamic behaviour of a physical system (Broenink, 1999). As shown in Figure 13, the theory of bond graphs introduces 3 types of fundamental variables, overall system variables, Power variables and State variables.

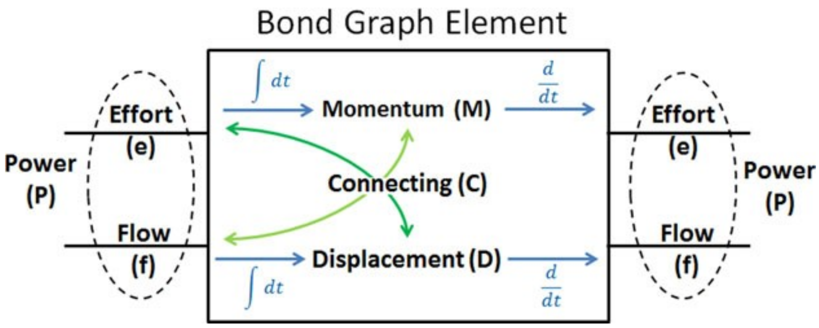

Figure 13. *Fundamental variables and their interconnections in the bond graph context* (Mokhtarian, Coatanéa, & Paris, 2017)

The overall system variables including energy and efficiency rate in the block. The power variables can be in the form of effort or flow. As an example, an electrical voltage is an effort and an electrical current is a flow. As shown in Figure 13, inputs effort and flow will be transformed into outputs effort and flow, through the state variables and the mathematical relationships between them. In state variables, displacement is the outcome of the integration of flow over time and the momentum is the result of the integration of effort over time. Coatanéa et al. (2016) added a third variable to the state variables called connecting variable, which describes the material, component-specific properties, geometric dimensions, tolerances, etc.. The output power variables are generated from the differentiation of a combination of the state variables.

DACM

DACM combined the principles in bond graphs and dimensional analysis to create a causal model of a system and then find the possible conflicts in it. The process starts with indicating the boundaries for the model. Then the functional model of the system is created. Next, variables of the system are assigned to the functional model. At this stage, applying DACM's causal rules and colour patterns leads to a coloured causal graph between variables of the system. Using this causal graph and dimensional analysis, the governing equations of the system can be extracted. The causal graph and the behavioural equations can be used further for qualitative and quantitative simulations.

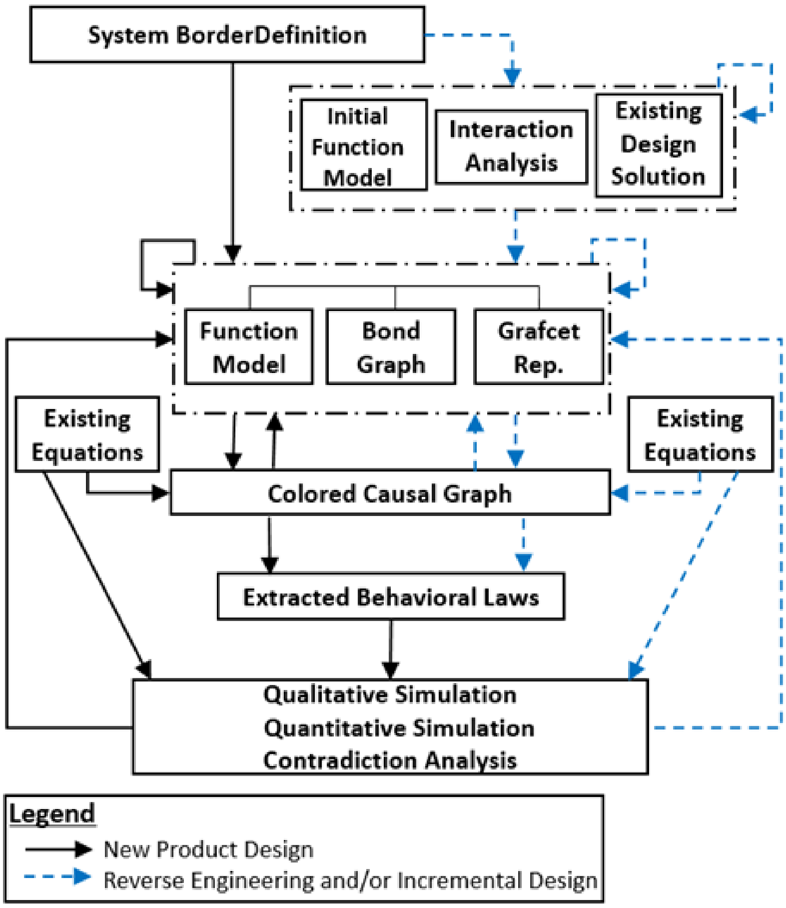

Figure 14. *DACM modelling approach* (Mokhtarian, Coatanéa, Paris, Mbow, Pourroy, Marin, & Ellman, 2018)

Figure 14 depicts the sequence of steps for creating a model and the theories that are integrated into the framework for each step. Steps in DACM

1- Indicating the model's objective and borders

A model can address the phenomena in a system in any scope and any degree of granularity. Since models are created to address a problem within a system and not the whole system, rationales and boundaries should be set at the beginning of the modelling process. These borders are chosen based on the problem at hand and the expert's knowledge.

2- Functional modelling

The overall functionality of the model is decomposed into a chain of functions that are in interaction with each other. Functions are boxes containing verbs of actions that are connected to each other in the sequence of occurrence. DACM uses the generic functional model of bond graph theory for the causal ruled is bond graph is already validated and also it can take advantage of the analogy among the energy domains (Paynter & Briggs, 1961). Moreover, DACM uses the set of functional vocabulary introduced by Hirtz et al. (2002) to reduce the variability in modelling and use the systematic approach provided by them. Table 3 shows the mapping between functional vocabulary and generic functional blocks

Table 3. Functional mapping for models transformation to generic functions blocks (Coatanéa et al., 2016)

Possible name of functions to describe the organs	**Functional basis vocabulary**	**Generic functional blocks**
To transform effort into flow or flow into effort **To resist** effort or flow	To Magnitude	**To Magnitude (Resistor: R)**
To transform flow into displacement **To store** displacement **To transform** displacement into effort **To provide** effort	To Magnitude To Provision	**To Provision (Capacitor: C)**
To transform effort into momentum **To store** momentum **To transform** momentum into flow **To provide** flow	To Magnitude To Provision	**To Provision (Inertia: I)**
To transform input effort into output effort of another magnitude **To transform** input flow into the output flow of another magnitude	To Signal To Magnitude To Convert	**To Convert (Transformer: TF)**
To transform input effort into the output flow of another magnitude **To transform** input flow into output effort into output effort of another magnitude	To Convert	**To Convert (Gyrator: GY)**

To connect the efforts of different magnitudes when flows are similar **To connect** the flow of different magnitudes when efforts are similar	To Branch To Channel To Connect To Support	**To Connect (Flow Junction: JF) (Effort Junction: JE)**
To provide a constant effort **To provide** a constant flow	To Provision	**To Provision (Source of Effort: SE) (Source of flow: SF)**

3- Assigning system variables to the functional structure

After forming the functional model in step 2, a set of fundamental categories of variables used in bond graph theory is assigned to the functional model. Table 4 shows a list of these variables and their categories. State variables are allocated to the boxes of functional model and power variables are allocated to the arrows.

Table 4. The fundamental category of variables in bond graph theory *(Mokhtarian, Coatanéa, Paris, Mbow, Pourroy, Marin, & Ellman, 2018)*

Primary Category of Variable	Secondary Category of Variable
Overall System variables	**Energy (En)**
	Efficiency rate (η)
Power Variables (P)	**Generalized Effort (E)**
	Generalized Effort (F)
State Variables	**Generalized Displacement (D)**
	Generalized Momentum (M)
	Connecting Variables (C)

4- Develop a causal ordering of variables

In this step, the cause-effect relationships among the variables are defined in the form of a causal graph. Colour should be assigned to the variables placed in the functional model and their colours should be chosen as below:

- The variables which are imposed on the system by the environment or decided to be fixed in the design process are called exogenous variables and coloured in black.
- The variables which have some degree of freedom, do not depend on other variables and can be chosen in the design process are called independent variables and coloured in green.

- The variables which are dependent on other variables and are hard to control are called dependent variables and they can be selected during the design process.
- The last group are the Performance variables. These are the variables that designers try to minimize, maximize or set a target value for them and are important to evaluate the overall performance of the system. These variables are coloured in red.

Using the order of functions in the functional model the order of appearance of variables in it and the rules in bond graphs theory, the causal relationships between variables are extracted in the form of a causal graph. Mokhtarian et al. (2017) developed an iterative algorithm, called causal ordering algorithm, to develop a causal graph from the functional model created in the last stem, in a systematic manner (Figure 15).

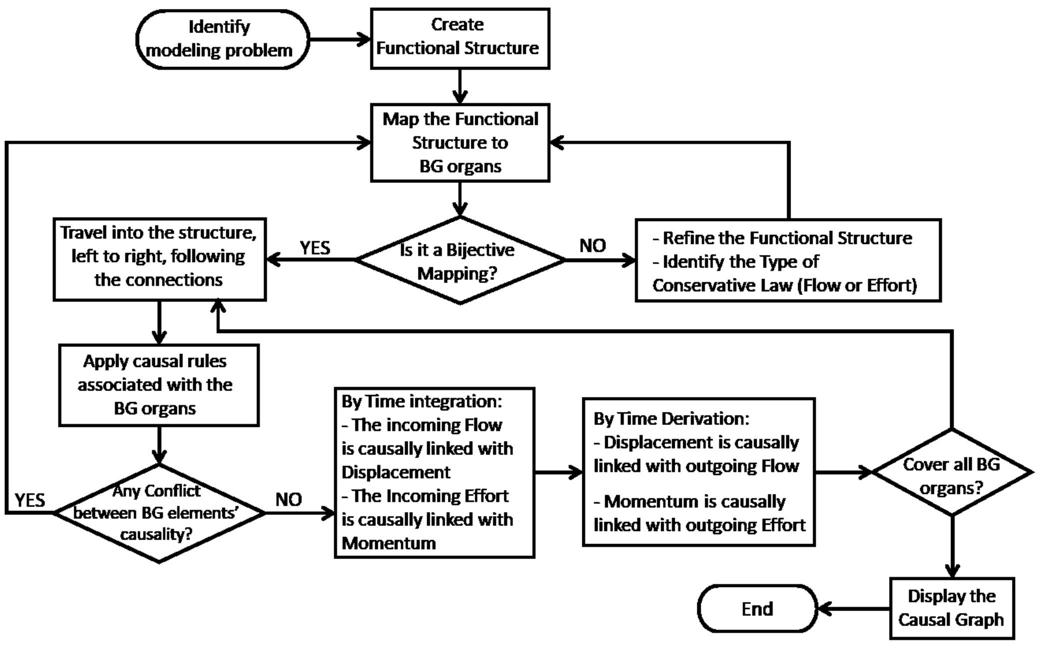

Figure 15. *Causal ordering algorithm* (Mokhtarian et al., 2017)

5- Construct the model's behavioural equations

Using the causal relationships in the previous step and the combination of rules in dimensional analysis and Π-theorem, the governing laws of the system can be generated. This process is also automated through the algorithm developed by Mokhtarian et al. (Mokhtarian et al., 2017).

3.4 Methodology in Additive manufacturing

This section starts with providing a background on the methods used for modelling additive manufacturing systems, as an example of a complex system. Then the methodology developed for this study is described in section 3.4.2.

3.4.1 Background

Previous efforts for modelling additive manufacturing processes are focused on using microscopic finite element (FE) models. For example, Masoomi et al. (2017) modelled the thermal procedures including heat transfer and phase changes in a Laser PBF process using a three dimensional FE analysis. In their study, they used the model to investigate the effect of thermal processes in microstructure characteristics of the manufactured parts. For more examples of such studies, the interested reader can check (Chua, Lee, & Ahn, 2018; Ding, Pan, Cuiuri, & Li, 2014; Fu & Guo, 2014; Schoinochoritis, Chantzis, & Salonitis, 2017)

Some researchers considered a mesoscopic scope for their models. They used the physics of melting pull formation and the flow of the melted materials and the process of solidification to predict the result of manufacturing. Moreover, they managed to simulate and predict the process behind microscopic defects such as splatter, pores and denudation in the manufacturing process. For example, Khairallah et al. (2016) created a fine-scope model which assumes the metal powder is made of randomly distributed particles. They demonstrated the effect of Marangoni convection and recoil pressure on the formation of pores in AM of 316 stainless steel.

The other approach is the experimental approach in which the conditions that result in specific controlled process characteristics are found through experiments (Ghouse et al., 2017). The major challenge with this approach is that experiments are expensive and time-consuming. Moreover, additive manufacturing processes are not material-agnostic and machine-agnostic, meaning that changing the material or using the same material in another machine will lead to a new process which needs to be optimized again (Tapia, Khairallah, Matthews, King, & Elwany, 2018).

Surrogate models are also utilized to optimize processes in additive manufacturing. Surrogate models or response surface models are approximations of an exact model that are computationally efficient and can provide accurate enough results based on a few simulation results (Viana, Gogu, & Haftka, 2010). Tapia et al. (2018) developed a modelling framework to create Gaussian process based surrogate models. In their study, they used their framework to create surrogate models out of the physics-based models available in the literature for process planning in laser powder bed fusion AM.

There have been studies to combine microscopic, macroscopic and mesoscopic models and use them to create holistic models. Mindt et al. (2017) proposed such a model for integrated computational materials engineering (ICME) use cases, in which a holistic model is used in order to predict the quality of the manufactured parts.

3.4.2 DACM to Bayesian networks

The method used in this study is an extension of the DACM framework. One of the outcomes of the DACM framework is a causal model between the variables of the system. DACM also extracts the governing equations between variables in the system. Since the causal graph is a directed acyclic graph, it can be a starting point to create a Bayesian network model for the system.

As mentioned before, the benefits of creating a Bayesian model are 1- expert's knowledge can be included in model 2- it can include the uncertainties of the system in the model 3- the model can be used interactively for exploring the variables space 4- Bayesian inference makes it possible to understand the interaction between variables 5- the network can be used in diagnostic path to find out the reason for having a specific value in the output nodes 6- the network can be used in prognosis path to see the effect of any combination of input nodes on the output nodes 7- available Bayesian inference engines provide fast and efficient means to observe the result of changes in variables on the other variables.

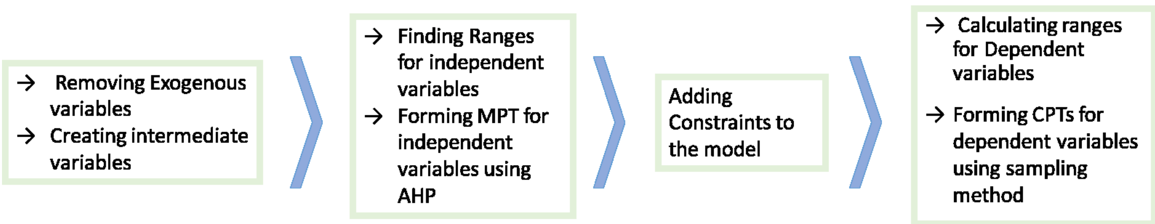

Figure 16. *The workflow for creating a Bayesian network using DACM Framework's outputs*

As mentioned in section 3.1, a Bayesian network has two aspects. A DAG which is the qualitative relation between the variables. The other aspects are the quantitative value, which is the value for variables and the marginal and conditional probability tables related to each of them. The rest of this section describes a methodology for obtaining both aspects of a BN model using DACM framework. The workflow is as follows

1- **Creating and modifying the causal graph:**

A causal graph is created to address the problem within the system using the DACM framework. The Causal graph is relating the DACM's independent variables and exogenous variables to the dependent variables and the performance variables. Figure 17 shows a sample Causal graph created using the DACM Framework.

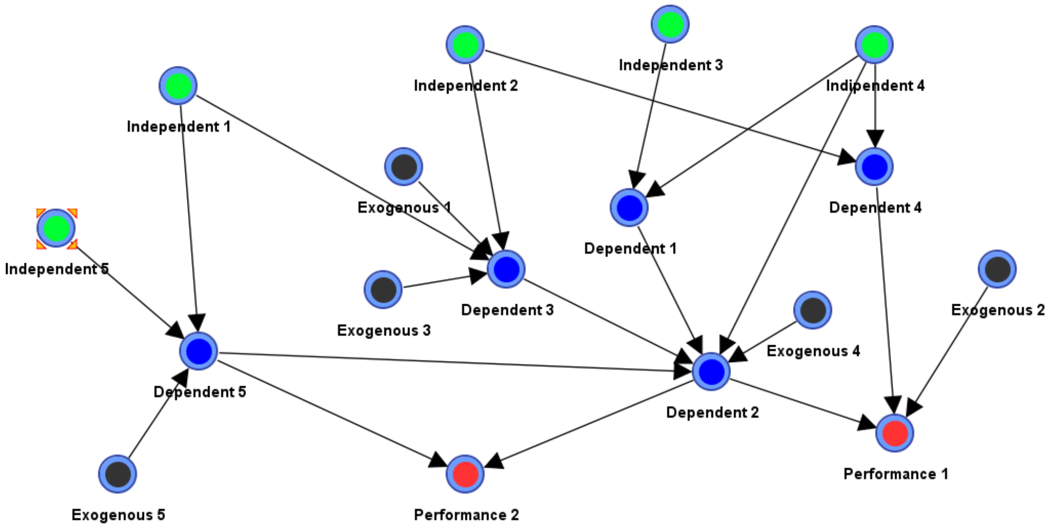

Figure 17. *Sample Causal Graph Created with DACM Framework*

This causal graph should be modified to be used as the DAG for a Bayesian network. The first change is to remove the exogenous variables from the causal graph. Nodes in a BN are discrete or continues random variables which can have a number of states or a continuous domain.

Exogenous variables are constant values, which are used to describe the relationship between variables in governing equations. Since the nodes in a BN are random variables which are not constant, exogenous variables should be removed from the graph. The effect of these variables will not be eliminated in the system because they already exist in governing equations between variables. The resulting graph is shown in Figure 18.

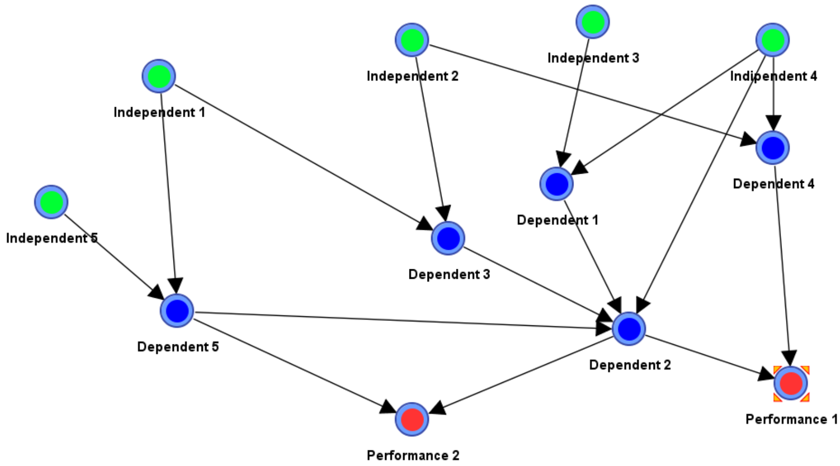

Figure 18. *The network after removing Exogenous variables*

The second change is to add a set of intermediate nodes in graphs to avoid ending up with dependent nodes with too many inputs arcs. Having a node with multiple input arcs can lead to a huge CPT and make inference hard, if not impossible. For example, a node with four states and ten parents with three states each will have a CPT with $4 \times 3^{10} = 236{,}196$ states.

There is a governing equation, or a Π-equation associated with each dependent node. For a dependent node with too many inputs from other nodes, the governing equation can be separated into smaller chunks and each chunk can be calculated in an intermediate node. Then the values calculated in the intermediate nodes can be used as inputs to that dependent node. In this way, not only the number of inputs is decreasing, but also the calculated values can be reused in other nodes if necessary. The sample graph will change to Figure 19 after adding two intermediate nodes for “Dependent variable 2”.

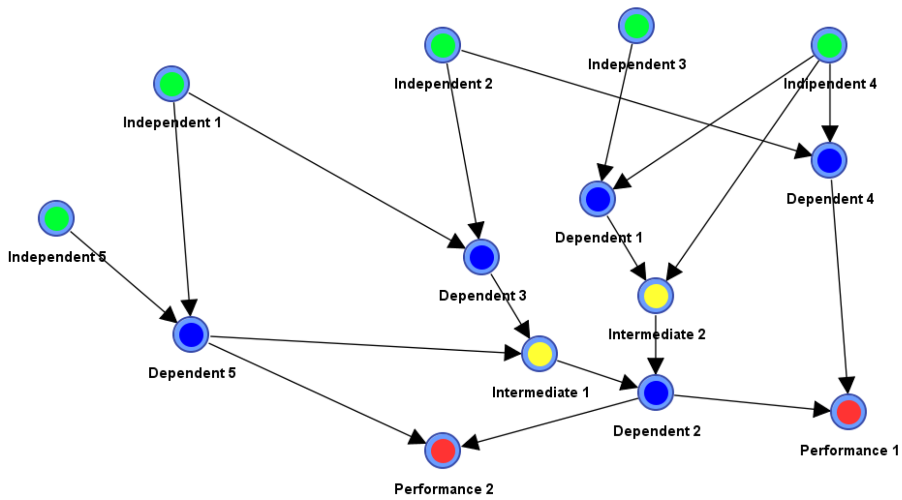

Figure 19. *Sample graph after adding intermediate variables*

Using intermediate nodes has a downside as well. As these nodes are aggregating the information of their parent node, some information can be lost in the process. For example, if an intermediate node has three parents and each of them has three states, the total number of states are $3^3 = 27$. Now if the information from these 27 states is aggregated in an intermediate node with three states which is representing the information of those 27 states, it is possible that a significant amount of data is lost in the process. In cases that the model needs to be very accurate, the intermediate nodes can be avoided. Also using more states for the intermediate node can reduce the information lost.

2- Finding value ranges for independent variables and using AHP to obtain Marginal Probability tables for independent variables

Independent variables can be either continuous or discrete. Regardless of the attribute, these variables have a specific domain of values. For examples in the case of manufacturing a bolt, its dimensions can vary from a few millimetres to a few centimetres. This range for variables can be extracted from the previous work in the literature, data sheets, standards, experts' knowledge, etc. in the domain of each variable, some of the values are more likely to be used or are more suitable in certain situations. This augments a level of uncertainty to the choice of values for each variable in the system. This uncertainty is modelled using a probability distribution over the domain of the variables.

If the Bayesian model is going to work with discrete distributions (as in the case of this study), continuous variables should be discretized, i.e. their domain should be divided into a number of intervals. Although having a good resolution for the discretization is important, the complexity of the model increases considerably with the number of intervals (Cooper, 1990). A rule of thumb may be having a minimum of 3 and maximum of 10 states, based on the context and the complexity of the network.

To obtain the probability tables for independent variables in a BN, one can randomly choose some probability distribution, e.g. some random distribution, or use a method of probability assessment (Shadbolt & Smart, 2015). Using expert's knowledge, it is possible to 1- include extra information about the domain in the model 2- help designers to make informed decisions in the early stage design phase and 3- reach to desired MAP distribution after updating the model with a fewer amount of data.

In this study, AHP is used to collect expert's knowledge for the independent variables. AHP tables are created to obtain the preference of an expert for each interval of each independent variable. For a variable with three intervals, the AHP table will be similar to Table 5.

Table 5. AHP table for a variable with three intervals

Independent variable A	**Interval 1**	**Interval 2**	**Interval 3**	**weights**
Interval 1	1.0000			
Interval 2		1.0000		
Interval 3			1.0000	
	MEV=			
	C.I.=			

After creating all the tables needed, the tables alongside a description can be sent to the expert to be filled. The description instructs how to read the table and how to do the pairwise comparison. A sample of instruction document is available in Appendix A. after collecting the expert's preferences, the mathematical machinery described in section 3.2 is used to calculate a weight for each interval. These weights can be directly used as the probability of choosing an interval by an expert (Saaty & Vargas, 2012, sec. 21) (Saaty & Vargas, 1998).

3- Setting constraints for the model

In this step, avoid impossible combinations of values for independent variables some constraint should be added to the causal graph. For example, in an L-PBF system, the amount of heat input energy, coming from a laser beam, cannot be more than the energy needed for melting the whole part. Therefore, there should be a constraint between the mass of the part being manufactured, and the amount of energy deployed for each layer. Also, a part cannot have any combination of dimensions, e.g. some exceedingly long and narrow beams may not have the tolerance for their own weight force after being manufactured.

Constraints can be in multiple forms. Ratios are one of the forms for setting constraint, in which an upper and lower value for the ratio of two or more variables are considered. For example, for two variables A and B, a threshold for their ratio can be chosen, namely r. Then the values within the range of A and B such that $\frac{A}{B} < r$, should be omitted from the simulation running based on the model.

The second set of constraint can occur when one variable is limiting another variable. For example assume variable A has a domain $(a1, a2)$ and the variable B has the domain $(b1, b2)$. Now imagine these two variables are the diameters of two pipe that should fit in each other, e.g. pipe B should fit in the pipe A. Then the constraint is that any value for the diameter of the pipe B should be smaller than the value for the diameter of the pipe A.

The other type of constraints appears when a variable cannot exist without the existence of certain values for other variables. This happens in situations like when a function within the functional model of a system appears only when another function gets a certain value. For example, in a boiler's output, the steam temperature is only valid when the of the boiler reaches the boiling point.

The constraints can be deployed in the graph with if conditions. Figure 20 shows the model in Figure 19 after addition of some ratio constraints between variables "independent 3" and "independent 4", and "independent 4" and "independent 2".

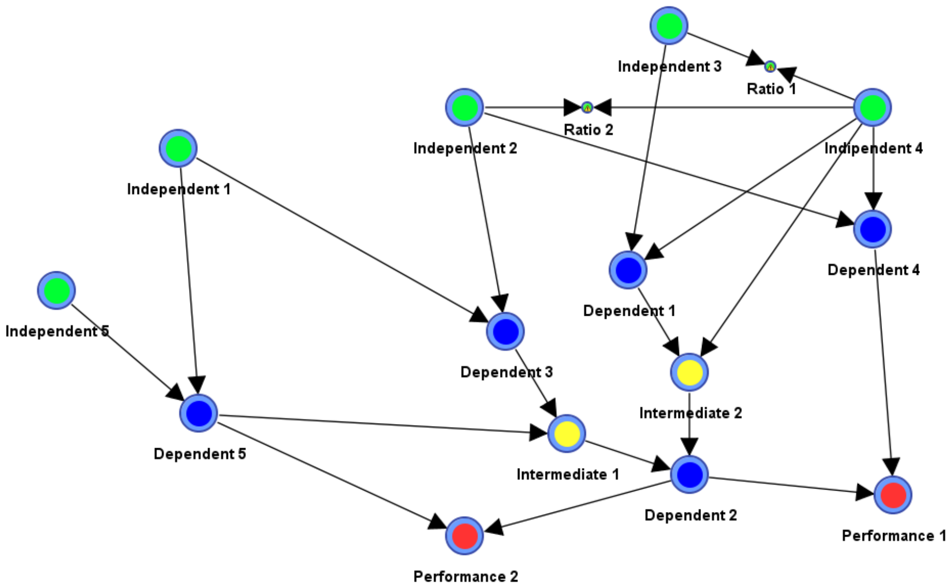

Figure 20. *The graph after adding ratio constraints*

The other kind of constraints which can be added are the ones that can refine the model based on the experts' knowledge. For example, having a magnificent amount of support structures for a part with an overhanging shape will lead to a time-consuming support removal post processing and causes wasting the raw material. Therefore, the preference of manufacturers is to minimize the support structure while benefiting from it, so the possible curling defect remains in a tolerated range. This information can be obtained by consulting with experts' and using AHP to quantify their knowledge and embed them into the graph.

4- Creating conditional probability tables for the rest of the nodes using the sampling technique

Since DACM provides the governing equations for dependent variables, a Monte Carlo like sampling technique is used to obtain probability tables for dependent variables (Niinimaki, 2015). Let's calculate the CPT for the variable "dependent 5". As shown in Figure 21, variable "dependent 5" has two parents, "Independent 1" and "independent 5".

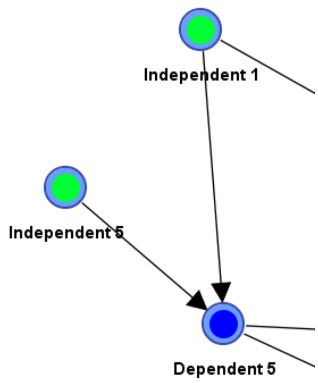

Figure 21. *Dependent 5 and its two parent nodes, independent 5 and independent 1.*

Let's assume the governing equations is:

$$dependent\ 5 = f(exogenous\ 5, independent\ 5, independent\ 1) \tag{60}$$

Note that the variable "exogenous 5" which has been removed from the graph in step 1, shows up in the governing equation (60) again. The sampling technique starts with finding the domain of dependent variables based on the domain of their parents and the governing equation. Using the maximum and minimum values in the domains of the independent variables and the governing equation, the domain of the dependent variable are calculated. Then this range is divided into several intervals which are called states, similar to step 2,. The process is shown in Figure 22.

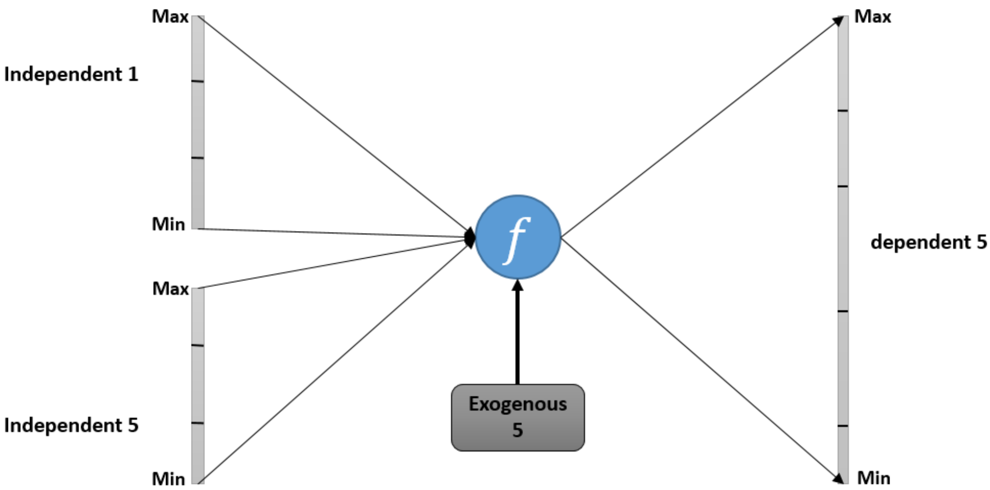

Figure 22. *The process of finding the range of the dependent variable*

The procedure continues with picking random sample values from the domain of each independent variable and calculating the corresponding value within the domain of dependent variable. By repeating the sampling for several times, say thousands of times, several data point in the dependent variables is calculated tables as shown in Figure 23.

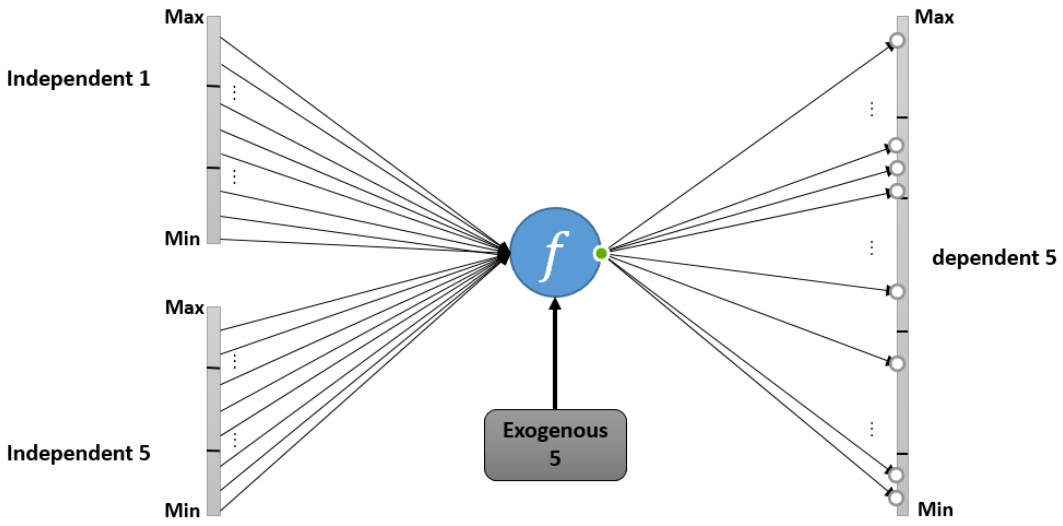

Figure 23. *Sampling method for finding probability distribution of dependent nodes*

The number of datapoints for each state of the dependent variables can be counted to form the conditional probability tables. A sample probability table for the variable "Dependent 5" is shown in Table 6. In this sample, each variable has 3 states, characterized with Low, Average and High attributes.

Table 6. A sample CPT for variable Dependent 5.

Independent 1	**Independent 5**	**Dependent 5**		
		Low	**Average**	**High**
Low	Low	34.8%		
	Average			
	High			
Average	Low			
	Average			
	High			
High	Low			
	Average			
	High			

Suppose that 10000 samples are taken from each of the independent variables, and 2000 of them are from the "Low" state of each independent variable. Then suppose after calculating the dependent variable for each set of samples, 768 of them happened to occur in the first interval of the dependent variable. Then the probability of happening for this interval, given the parent nodes are having value in their "Low" state is $\frac{768}{2000} = 34.8\%$.

3.5 Methodology in Reliability Engineering

Several groups of methods are available for prognosis and diagnosis in the area of prognosis and health management (PHM), for instance, reliability model-based methods and anomaly detection methods for fault diagnostics and statistical methods, and survival analysis for failure prediction.

In the rest of this section, first, a short review of the non-Bayesian and Bayesian methods for modelling faults and predicting failures in mechanical systems is provided. Then, the methods used in this study are briefly described.

3.5.1 Background

Fault diagnosis

Leonhardt and Ayoubi (1997) suggest that a fault diagnostic system consists of two main steps, symptom extraction and diagnostic. Symptom extraction is the act of determining the symptoms of a failure, either using analytical symptom generation by computers or using the heuristics of an expert human operator. Diagnostics is described (ibid) as a binary measure which indicates a system is failed or not, hence it is a mapping from continuous symptom space into a discrete diagnosis space. To perform this mapping, or better said this interpretation, three general methodologies are available, classification methods, inference methods and combinations of these two like neuro-fuzzy systems.

Classification methods, as described by Leonhardt & Ayoubi (1997), use reference patterns for learning and interpret using methods such as statistics, neural networks, geometric methods and fuzzy classifiers. One of the methods in this class is anomaly detection. Anomaly detection focuses on finding abnormal behaviour in time series data of the sensors (Salfner, Lenk, & Malek, 2010). Several statistics based and machine learning based methods have been developed for anomaly detection (Schwabacher, 2005). These methods are used both for diagnosing the reasons of anomalies (Lu, Li, Wu, & Yang, 2009) and for predicting the failure within a fixed period in future (Brotherton, Jahns, Jacobs, & Wroblewski, 2000).

The inference-based methods of diagnostics systems use linguistic rules for performing the interpretation. Among the such this class of diagnostics systems for diagnosing faults in industrial systems, fault tree analysis (FTA) (Patil, Waghmode, Chikali, & Mulla, 2009) and reliability block diagrams (RBD) (Čepin, 2011) are among the well-known methods. To bring an example, and extending it in the rest of the subsection, FTAs are described in more details as follows.

FTA is a graphical tree based top-down method, which is used to represent the logic of the effect of faults and events in the components of a system on that system. The modelling procedure starts with choosing one of the undesired states of a component as the root node, which is called the top event. Then using logical gates, e.g. AND, OR, XOR and the Voting gate, the causes for that state are added to the tree according to their logical relationship to the root node and the other nodes. The intermediate nodes are called cause events and the root nodes are called bottom event (Zhi-qiang Cai, Sun, Si, & Wang, 2010). FTAs can be used in early design stages to get a better understanding of the possible failure situations or it can be used in the operational stage of the system to diagnose the reason of failures in the system. Advantages of using FTAs include (Chelson, 1971):

1- Its structure lead to better organization and a more precise analysis of failures
2- It can be used as an event-oriented reliability model for a system. It is also possible to include probabilistic computations into it.
3- It indicates the dependencies and independencies between variables in the reliability model.

One of the negative aspects of the FTA is being a failure-oriented modelling method, meaning it is checking for failures in the system instead of the success of the system. The other negative aspect is if the analysis is carried to a very detailed level of the systems, the number of variables can be overwhelming. Therefore, the recommendation is to use the FTA for critical systems and use other analysis methods for the other parts.

A newer classification groups the fault diagnostics methods into three classes of model-based, data-driven and signal processing-based approaches (B. Cai et al., 2017). In this classification, model-based methods try to create mathematical models for the industrial systems and use that for diagnosing the problems. Signal processing-based methods use detection theory-based tools to distinguish the roots of the fault by comparing the signals with the signals from a healthy system. Data-driven methods use artificial intelligence (AI) in general to perform the diagnosis. These methods are suitable for complex systems where the other methods cannot be easily used.

Bayesian networks based methods for fault diagnosis

Bayesian networks have gained significant attention in reliability engineering use cases due to their ability for modelling uncertainty since the 1990s (Langseth & Portinale, 2007). There are three general methods to obtain a Bayesian network in reliability use cases, namely machine learning, using expert knowledge, or converting some other model into a Bayesian network.

In cases that the expert's knowledge is limited or hard to acquire but enough data about the system is in hand, using machine learning methods to generate models from the data is the method of choice. Bacha et al. (2015) developed a structure and parameter learning algorithm along with a unique data acquisition system to receive to construct a Bayesian network using real-time data for faults in an industrial direct current (DC) motor. Although they have not used any mathematical modelling and their model is acquired using structure learning for Bayesian networks, the resulting model shows a high degree of accuracy. As another example, Yavuz et al. (2006) developed a structure learning algorithm to generate Bayesian networks for fault diagnostics in aeroplanes. In their method, they developed an algorithm based on the particle swarm technique to obtain a Bayesian network model from data without using any expert knowledge of the domain.

Using experts' knowledge is the method of choice for creating Bayesian networks in reliability use cases (Langseth & Portinale, 2007). As an example, Yontay (2016) has developed a method to analyse the reliability of systems in the early stage design in his doctoral dissertation. In his work, he developed a framework to incorporate different sources of expert's knowledge to create reliability assessment models and combine the knowledge elicited from the experts into a single Bayesian network.

From the group of methods that are transforming a model acquired from other disciplines into a Bayesian network, some example is using modelling frameworks like bond graphs and translating them into Bayesian networks (Lo, Wong, & Rad, 2003), translating and extending FTA with Bayesian networks (Bobbio, Portinale, Minichino, & Ciancamerla, 2001) or translating reliability block diagrams into Bayesian networks (Torres-Toledano & Sucar, 1998). To bring an example of the transformation process, a methodology for transforming an FTA into a fault predicting Bayesian network (FPBN) is described as follows.

fault predicting Bayesian networks (FPBN) are developed to integrate all the information regarding failure prediction and represent it with nodes and arcs in a Bayesian network (Zhiqiang Cai, Sun, Si, & Wang, 2009). Cai et al. (2009) described FPBN with a set $\{X, A, P\}$. X is the set of all variables, which are represented as nodes in the Bayesian network. A is the set of the edges connecting the nodes to each other. P is the set of conditional probability distributions of each node in X, given the parents of that node.

The nodes in this model are further classified is three classes $X = \{M, C, E\}$. M is the set of failure modes, meaning the nodes that describe the actual state of the parts, accessories or subsystems in a system. C is the set of failure causes and is the real cause of a certain failure. The states of these nodes can be obtained by direct information, model inference, reliability calculation and experts' estimation. Finally, E is the set of failure detection nodes, i.e. is the visible states of the sensors or indicators, which can be changed based on the state of other failure modes or failure cause nodes. The values of all nodes are discreet, and they are normally chosen as true and false values.

The direction of the arcs in FPBN has some considerations as well. Only failure cause nodes can affect the failure mode nodes. Moreover, both failure cause and failure mode nodes can affect failure detection nodes. On the other hand, it is difficult to assign causality between failure cause nodes and these relations can be learnt from the data.

To create FPBN based on Fault Tree Analysis, Bobbio et al. (2001) and later on Zhiqiang et al. (2010) developed a method to translate Fault trees to FPBNs directly. To translate the structure, the top node can be considered as a failure mode node. The cause nodes and root nodes can be the set of cause mode node in the FPBN. To complete the model, a set of detection information nodes can be added to the FPBN, which are equal to detection nodes. These nodes are representing the fault detection sensors and systems in the equipment. The direction of the arrows then is chosen based on the same logic used in FPBN. The arrow's direction is from the cause nodes towards the failure mode node and failure detection nodes.

The CPTs of failure cause can be determined with the same method described for FPBNs. On the other hand, for the failure mode nodes and fail detection nodes, the logical relations can be used to obtain the CPTs. CPTs of a node X_1 which is connected to other nodes $\{X_2, X_3, X_4, \ldots\}$ by an AND gate is calculated as follows

$$P(X1|X2, X3, \ldots, Xn) = \begin{cases} 1, (X2 = 1, X3 = 1, \ldots, Xn = 1) \\ 0, else \end{cases} \tag{61}$$

In addition, for the OR gate, the equation is

$$P(X1|X2, X3, \ldots, Xn) = \begin{cases} 0, (X2 = 0, X3 = 0, \ldots, Xn = 0) \\ 1\,, else \end{cases} \tag{62}$$

The relation for the NOT gate is

$$P(X1 = 1|X2) = \begin{cases} 0, X2 = 1 \\ 1, X2 = 0 \end{cases} \tag{63}$$

A voting gate is a decision gate, which will show 1 in the output if a specific number of its input is equal to 1. Therefore, the equation for calculating the CPTs for this gate is

$$P(X1 = 1|X2, X3, \ldots, Xn) = \begin{cases} 1, \sum_{j=2}^{n} B(Xi) \geq k \\ 0, else \end{cases} \tag{64}$$

Failure Prognostics

Several statistical (parametric and nonparametric), data-driven, and model-based methods have been developed to predict the remaining lifetime of the equipment from the historical records of time to failure data. One of the methods is survival analysis which aims to perform such predictions. Methods in survival analysis discipline are in two groups, parametric and non-parametric. Parametric methods consist of estimation methods to find parameters for several classes of probability distributions, regression models, etc. Non-parametric methods consist of test methods and regression methods like Cox's proportional hazard models (Miller et al., 1998).

As mentioned before, survival analysis is a discipline which aims to predict the lifetime of alive creatures, machines, electrical equipment etc. As described by Miller et al. (1998), in general, the survival analysis is formulated as follows. Assume that T is a random variable which represents the lifetime and has the density function of $f(t)$ and distribution function $F(t)$. The survival function of t is defined as:

$$S(t) = 1 - F(t) = P\{T > t\} \tag{65}$$

Which formulates the chance of survival. The hazard function or hazard rate is defined as:

$$\lambda(t) = \frac{f(t)}{1 - F(t)} \tag{66}$$

This function is interpreted as:

$$\lambda(t).dt \cong P\{t < T < t + dt \mid T > t\} = P\left\{\begin{matrix} failure\ in\ the \\ interval\ (t, t + dt) \end{matrix} \middle| \begin{matrix} survived\ past \\ time\ t \end{matrix}\right\} \tag{67}$$

The integral of the survival function over time is:

$$\int_0^t \lambda(u)du = \int_0^t \frac{f(u)}{1 - F(u)} du = -\log\big(1 - F(u)\big)\Big|_{\mathbf{0}}^{\boldsymbol{t}} = -\log S(t) \tag{68}$$

Which after realignment leads to:

$$S(t) = e^{-\int_0^t \lambda(u)du} \tag{69}$$

Which implies that the survival function of T is an exponential function of the hazard rate's integral over past time.

As De Carlo and Arleo (2017) state, hazard function of equipment normally has the shape of a "bathtub" curve, meaning it has a descending part at the beginning, a horizontal part at the middle and raising part at the end of lifetime of equipment. The descending part showing decreasing failure rate (DFR) illustrating the fact that brand-new equipment suffers early failures due to potential manufacturing defects. The middle horizontal part is called the constant failure rate (CFR) and shows the useful lifetime of the equipment. The last raise in the failure rate is called the increasing failure rate (IFR) which is due to the wear-out in the equipment's end of useful life.

Each of these sections can be modelled with a suitable probability distribution based on the specific failure rate in each equipment. The probability distributions mostly used in survival analysis are exponential distributions, gamma distributions, Weibull distributions, Rayleigh, lognormal, Pareto for the DFRs and CFRs and there are also specific distributions to cover IFRs (Miller et al., 1998).

The parameters for these distributions to fit the survival curve can be estimated using maximum likelihood methods or linear combinations or order statistics. Linear and log-linear models are also among the parametric methods for modelling the survival rate of a system (Miller et al., 1998).

Several non-parametric methods have been also developed to predict the survival of the systems. Specific methods are available for the datasets with only one variable, which is relevant for the case of this study. One of the methods for single variable data is life tables in which the time domain is divided into intervals, normally equal distance, and then the information such as number of working equipment at the beginning of each interval, number of failures during each interval, number of lost to follow-ups in each interval and number of withdrawal in each interval and the probability of surviving given being functional in each interval is recorded into a table called the life table. Then these data are used to estimate the mean and variance of the survival times using several algorithms with different degrees of accuracy (Miller et al., 1998).

The other method for single variable data is called product-limit estimator of Kaplan-Meier estimator (Kaplan & Meier, 1958). In Kaplan-Meier estimator, the intervals of the time domain are not equidistance and the survival probability are assumed to be independent of the other survival probabilities. Therefore, the probability of survival in consecutive time intervals is calculated by multiplying the probability of survival in all the intervals up to that interval, i.e.

$$\begin{aligned} S(I_t) = P\{T > I_t\} &= P\{T > I_1\}P\{T > I_2|T > I_1\} \dots P\{T > I_2|T > I_1\} \\ &= P_1.P_2 \dots P_k \\ P_i = P\{T > I_i|T > I_{i-1}\} \end{aligned} \tag{70}$$

$S()$ is the survival rate in the interval I and P in the probability of survival in each interval given that the system is survived in the previous intervals. The estimates are drawn with respect to intervals in time in a step shaped curves called Kaplan-Meier curves (Rich et al., 2010). There are also other non-parametric types of estimators for single variable data such as hazard function estimators and robust estimators for mean and median etc. (Miller et al., 1998).

In cases that the data for multiple interacting variables, covariates, are available, one of the most well-known non-parametric regression methods for estimation of survival times in Cox (1972) regression, which is an extension to the hazard model. Consider a group of independent covariates $x = x_1, \dots x_n$ where $x_i = x_{i1}, \dots x_{in}$. also consider that these covariates are associated with the survival time $T_i = T_1 \dots T_n$ and censoring time $c_i = c_1, \dots c_n$. Cox's proportional hazard model implies that hazard rate is the product of a scalar depending on a set of regression coefficients β and a hazard function $\lambda_0(t)$, such that:

$$\begin{aligned} &\lambda(t;x) = e^{\beta' x} \lambda_0(t) , \\ &\beta = (\beta_1, \dots \beta_p)' \end{aligned} \tag{71}$$

where both regression and hazard function are unknown. The term $e^{\beta' x}$ can be replace by any positive function of $\beta' x$. Then it is proved that the hazard function forms a family of distributions in this form:

$$S(t;x) = \exp\{-\int_0^t \lambda_0(u)du\}^{e^{\beta' x}} \tag{72}$$

Then to find the parameters of this distribution, Cox suggests a conditional likelihood which calculates the likelihood only for the equipment that their failure times are not censored. Using this, and maximum likelihood method, the parameters of this distribution can be calculated.

In this model, $\lambda_0(t)$ is also called the base line hazard function. The coefficients represent the effect of each covariate on the hazard function. A negative coefficient decreases the hazard function and a positive coefficient increases the hazard function. One drawback of the model is that the effect of different covariates is assumed to be constant over the time. For more detailed information please consult Cox (1972) and Miller et al. (1998).

Bayesian networks-based methods for failure prognosis

Most of the alternatives for survival analysis are not able to predict the time to event values and instead they predict the occurrence of events in the systems (Štajduhar, Dalbelo-Bašić, & Bogunović, 2009). One of the best alternatives to survival analysis for predicting failures is the Bayesian networks-based methods. Creating prediction models for failures in a system using Bayesian networks helps in many ways. First, it gives a qualitative insight about what can be changed to improve the lifetime of a system. It also provides the means to predict the failures in similar equipment and perform preventative and predictive maintenance to improve the reliability and availability of the system. Moreover, It can depict the interconnection between covariates in a sophisticated manner and it represents the knowledge in the system very well (Langseth, 1998).

Like Bayesian networks based methods for diagnostics, there are several methods to obtain a Bayesian network in the context of failure prognosis, namely translating other models into a Bayesian network, creating a Bayesian network for fault prognosis from the scratch using expert's knowledge, or machine learning.

Weber et al. (2001) developed a method to use structured analysis and design technique (SADT) (Ross, 1977) to develop failure models based on failure mode, effects, and criticality analysis (FMECA) (USA Department Of Defense, 1980) using functional modelling approaches and then representing them in the form of a Bayesian network for fault prediction. They used the functioning and malfunctioning studies at the design phase to form the structure of the BN and obtained the probability tables using FMECA.

In the case of using expert's knowledge to form a network from the scratch, Medjaher et al. (2009) used the expert's knowledge and domain knowledge to form the structure of a Dynamic BN and then obtained the parameters of the network using FMECA. Another good work in this area is the study by Bartram and Mahadevan (2013) in which they used the expert's knowledge and published reliability data to determine the structure and parameters of a dynamic Bayesian network. Then they used the DBN to perform diagnosis on the system via particle filtering. They also predicted the remaining useful life of the system using the DBN and Monte Carlo sampling.

BN structure learning using machine learning

Langseth (1998) used machine learning to develop a Bayesian network model for analysis of survival times of mechanical equipment. A portion of a dataset called "Offshore Reliability DAta" (OREDA) containing the failure data of 29 equipment and 2921 failure times and 300 censored survival times have been used. Each failure time contains 10 attributes describing the inventory, one describing the severity of the failure and on hold the time to failure. The dataset is divided into two subsets, a training set which includes all the censored data and 70% of the rest of the data, and the test set. The network obtained from the data is used for both qualitative analysis and quantitative analysis. In the qualitative analysis the effect of covariates on the response variables, i.e. "time to fail" and to "severity class", has been investigated. The quantitative analysis was to use the model to predict survival times. The predicted survival times were validated against a test dataset and the result of a Cox regression analysis. The qualitative part does not show a significant improvement from the regression model, but the qualitative aspect provides a sophisticated representation power for the characteristics of the system.

One of the key issues while using machine learning for obtaining BN is the issue of censored values. Most of the studies which tried to create models for prognosis use cases tried to predict the occurrence of an invent, not the time to event values. Therefore, most of the method used for handling censored data for structure learning in BNs are focusing on whether counting a period with no events are an event-free period or not. For example, Langseth (1998) handles the censored data by using all of them in the training set as an event-free period.

Some researchers only considered the data point which has been observed to a certain minimum amount of time. The rest of the datapoint is not used for the model learning process. This approach can create a large amount of bias in the datasets which have a large number of censored values (Delen, Walker, & Kadam, 2005).

The other approach is to separate the censored times into two groups. One group is labelled as event-free if the censored times are not bigger than a T^* value. The rest of the censored times are doubled and used in both event free and event-occurred outcomes and a probability for each of them is assigned based on Kalan-Meier method (Jahanbani Fard, 2015). Štajduhar and Dalbelo-Bašic´(2010) used this approach and extended it with a weighting method for handling censored data and developed a machine learning algorithm for learning Bayesian networks' structure and parameters from the data.

For cases where the time to failure is important, one approach can be to neglect the occurrence with censored failure times. If the data is not heavily censored, i.e. the amount of censored data is about 10% of data points, this approach will not affect the process of learning significantly (Štajduhar et al., 2009).

A methodology to create a prognosis system.

Letot et al. (2017) described the classic methodology to create an adaptive model to predict the failures in brand new equipment. This four-stage methodology corresponds to four different maintenance policies and depicts the relations between the policies and the level of knowledge about the reliability and degradation of the equipment.

The first stage is to run the equipment until it fails. Then corrective maintenance (CM) is being performed to restore the equipment to them as good as new condition. The initial failure times are also obtained from the equipment. In the second stage, based on the failure data acquired in the first stage, parametric models can be created for the failure times and preventative maintenances can be performed on the equipment.

In the third stage, the degradation in the components of the system can be monitored and based on this monitoring data, condition-based maintenance models can be created. The variables to be monitored can be detected using experiments, sensitivity analysis and the return of experience. The quality of the monitored values should be tuned to be suitable for creating the model. The model can be designed based on threshold values for the monitored variables, which surpassing them should lead to performing preventative maintenance.

And finally, predictive models for degradation and adaptive maintenance models can be obtained based on the historical data of degradation in the components. The process of choosing the suitable model for degradation is complex, and normally goodness of fit criterion is used as a guide. After the model is created, the current degradation in components can be inspected and baes on this information, the optimized time for the next preventative maintenance, or even the optimal time for preventative replacement of a component can be predicted.

Handling censored data and missing values

Having censored data is the challenge in most of the real world data sets for reliability engineering related tasks. The general approach for handling censored values is modeling it with a probability distribution and using the maximum likelihood method for estimating the parameters (Millard, Neerchal, & Dixon, 2012). For the values which are missed to record in the dataset, based on the nature of missingness, different methods can be used. The description of the types of missingness and the methods can be seen in section 3.1.7.

3.5.2 The methodology for this study

This study tries to investigate the possibility of using Bayesian networks for modeling failures in the industrial domain. The dataset used in this study is a single variable dataset of time to failure for industrial pumps.

Failures in industrial pumps, in case the available data is only for time to failure, are traditionally modeled with homogenous or non-homogenous counting processes (Dudenhoeffer, 1994). A process $\{N(t), t \geq 0\}$ is a counting process if:

1. $N(t) \geq 0,$
2. $N(T)$ is an integer
3. If $i < j$, then $N(i) < N(j)$
4. For $i < j, N(j) - N(i)$ is the number of events occurring in the interval (I, j)

In case of having a constant failure rate, it is possible to model the failures in a pump with a homogenous Poisson process. A counting process $\{N(t), t \geq 0\}$ is a homogenous Poisson process if with a rate $\lambda > 0$,

1. $N(0) = 0$
2. The number of events occurring in disjoint time intervals is independent. This attribute is also stated as independent increments
3. The number of events in any interval of length $t = t_j - t_i$ is Poisson distributed with mean λ_t:

$$P\left[N(t_j) - N(t_i) = n\right] = e^{-\lambda(t_j - t_i)} \frac{\left(\lambda(t_j - t_i)\right)^n}{n!} \qquad (73)$$
$$t_j < t_i, n = 0,1, \ldots$$

Moreover, the conditional probability that the system will survive until time t_j given that it is operating at the time t_i is

$$R(t_i, t_j) = e^{-\lambda(t_j - t_i)} \qquad (74)$$
$$t_j < t_i$$

A non-homogenous Poisson process is similar to the homogenous Poisson process, but the occurrence function is a function of the age of the system, i.e., $\lambda = f(t)$. Normally pump failure data indicate that the rate of failures increases with the age of the pump.

Methods of this study

Bayesian networks are generally used to model the interaction between several variables. In this study, the data consists of consecutive failure times of several similar pumps. Usually, the maintenance policy for these pumps is corrective maintenance, and the quality of maintenance is close to perfect. Therefore, normally it is confidently assumed that the consecutive failure times are independent of each other.

As mentioned before, the time to failure of a system is not only depending on time, but the environment of operation and the usage pattern is effective on the failure times of the equipment. Therefore, the assumption of having independent consecutive time to failures (TTF) can be doubted, because the usage and environment conditions will not change after corrective maintenance on equipment.

Some of the TTF times in the dataset are censored because the observation of pump failures is stopped at a certain time and the TTFs after that time is not recorded. Since these unobserved times are censored failure times (CF), therefore there might be a relationship between their duration and the recorded failure times

To investigate the effects of these condition on the failure times, this study attempts to create Bayesian network models from TTF and CF data while relaxing the assumption of independence of TTF distributions. A Bayesian learning algorithm is used to find a dependence relationship between consecutive TTFs and CFs, i.e., a causal graph which shows a causal relationship between consecutive failures and CFs. The Bayesian learning algorithm finds the relations between the variables using an association metric. These algorithms are described in section 3.1.5.

The methods used for this case study is mostly described along with the implementation in section 4.2, and a summary is only provided here. To create the model, first, the dataset should be modified. The single variable TTF dataset, which is representing the consecutive TTFs for multiple pumps, is reorganized to separate TTF variables, i.e. time to first failure, time to second failure and so on. This is possible because the pumps are made by the same manufacturer and with same mechanical design. The instance of different pumps can be considered as a unit pump. Having multiple instances for each TTF, a distribution for each of them becomes available. Moreover, the censored TTF times are grouped into separate CFs based on the TTF they are related to.

The algorithms for structural learning and inference used in this study are designed for discrete variables. The time to failure data are continuous variables. Therefore they should be discretized. The methods for discretization are discussed in section 3.1.10. Since the genetic algorithm based method is the optimal method, it is used for discretization.

Then for learning the structure of the Bayesian network, the EQ algorithm is used. The EQ algorithm is described in section 3.1.5. The threshold for the minimum association metrics is set in a low value, so even a small dependency is considered between the variables of the system. The metrics of associations used in EQ is described in section 3.1.5 and the metrics are described in section 3.1.2.

To select the network, contingency table fit used as the metric. Based on the minimum value of acceptable structural coefficient chosen for the structural learning algorithm, the resulting network may be ranging between a fully connected network to completely unconnected network. As described in section 3.1.6, if the acceptable structural coefficient is low, the final network will be close to a fully connected network and vice versa. The detailed description of the process of creating the model is described in section 4.2.

4. IMPLEMENTATION AND RESULTS

This section is describing the process of implementing the methods described in section 3 on the case study problems. Section 4.1 is providing the detail for the additive manufacturing case study, and section 4.2 is dedicated to the reliability engineering case study.

4.1 Additive manufacturing case study

The methodology described in section 3.4 is used to tackle one of the common defects in PBF additive manufacturing of cantilever shaped parts, namely the curing defect, which is described in section 2.2.2. In the case study, a model for the curling defect problem in an L shaped part is created. The model aims to relate the choice design and manufacturing parameters to the performance variables in a conceptual design phase.

4.1.1 The causal model for curling defect

As described in section 2.2.2, the overhanging parts' manufacturing process may result in a curling defect in them. This defect is happening due to thermal constraints on the part in each layer due to magnificent increase in temperature because of the amount of energy input in each layer and fast rate of cooling down due to the high thermal flow of metallic parts (Tounsi & Vignat, 2017).

Mokhtarian et al. (2018) developed a DACM model for this kind of defect. This study uses their model as the starting point and then try to modify the graph to correct some deficiencies. Then using the method described in section 3.4, the graph is translated to a Bayesian network. A brief description of the step by step procedure of creating the causal graph is as follows.

At the first stage, DACM oversees the problem from the functional perspective and attempts to develop a functional model describing the occurrence of the curling defect in the process. The model aims to describe this phenomenon using a simple cantilever deflection model without complicating the problem by going too much into detail. The functional model, shown in Figure 24, is divided into three domains. These three domains are 1- cyclic functions of the AM process, 2- useful functions of the support structure and 3- non-desired functions. Then the behavioral laws are collected from the literature and not created using the DACM algorithms. However, having a functional model, behavioral laws are used as a basis to generate the causal graph.

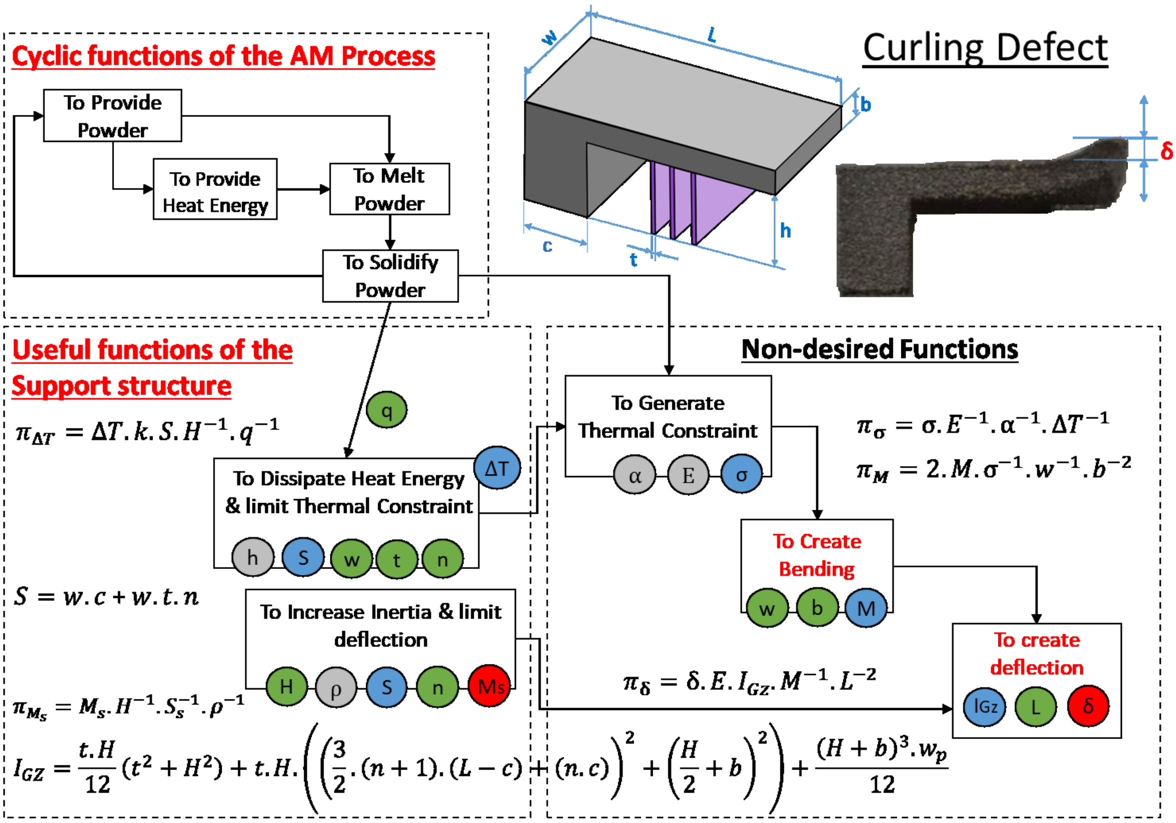

Figure 24. *The functional model for the cantilever part manufactured with curling defect, updated from Mokhtarian et al. (2018)*

The functional model of the support structure includes two functionalities of the supports. The function ‘to dissipate’ heat energy is used to define by the conduction variables, and the function ‘to increase inertia’ contains the variables defining the supports geometry and material density (Mokhtarian, Coatanéa, Paris, Mbow, Pourroy, Marin, Vihinen, et al., 2018).

Two changes are made to the model developed by Mokhtarian et al. (2018) in this study to improve it. First, the heat dissipation due to convection had changed to heat dissipation through conduction, for conduction seems to be more relevant due to the nature of materials in the system. Convections needs a fluid or gas medium to happen and as in this system, metal powder cannot act like any of them. On the other hand, the high thermal conductivity of metal powder can be a good means for heat dissipation.

The other change is in the inertia calculations. The original model, the effect of the inertia created by supports where neglected. The original inertia is formulated as the Eq. (80)

and to consider the effect of the supports on the total inertia, the Eq. (81) should be added to it.

The non-desired functions of the supports are related to the generation of a thermal constraint, which leads to creating the bending moment, and the function ‘to resist’, which acts against the deflection. Table 7 represents the variables with their associated dimensions.

Table 7. Variables for the DACM model of Curling defect

Variables	Symbol	Dimension
Heat Energy input	q	ML-2T-2
Coefficient of conduction	k	MT-3t-1
The temperature difference between layers	ΔT	t
The surface of Heat Exchange	S	L2
Number of supports	n	--
Thickness of supports	t	L
Material Density	ρ	ML-3
The total mass of the supports	Ms	M
The width of the supports	w	L
The height of the supports	H	L
Length of the part	L	L
Thermal constraint	σ	ML-1T-2
Thermal expansion	α	t-1
Elasticity Modulus	E	ML-1T-2
Moment of Inertia	IGZ	L4
Moment induced by thermal constraint	M	ML2T-2

Curling defect	δ	L
The thickness of the beam	b	L
Length of the base	c	L

The models provided by DACM Framework include a causal graph and behavioral equations between variables. The governing equations described in Pi number forms are:

$$\pi_{\Delta T} = \Delta T.k.S.q^{-1}.H^{-1} \quad (75)$$

The formula for ΔT is different from the original formula from Mokhtarian et al.'s paper (2018). In the original paper, the heat dissipation is through heat convection, but in this study, it is changed to heat conduction since it seems more reasonable, as mentioned before. Alongside with the change to heat dissipation, the surface of heat exchange is also changed. In the initial model, it was the vertical surfaces of the supports, because the heat was supposed to be absorbed by the powder around supports. In this study, the surface changed to the vertical cross section of supports and the base, because the heat assumed to be absorbed by the base plate of the machine. The rest of governing equations are as follows.

$$\pi_{M_s} = M_s.H^{-1}.\ S^{-1}.\rho^{-1} \quad (76)$$

$$\pi_{\sigma} = \sigma.\ E^{-1}.\alpha^{-1}.\Delta T^{-1} \quad (77)$$

$$\pi_M = 2.M.\sigma^{-1}.w^{-1}.b^{-2} \quad (78)$$

Since the formula for the thermal constraint is only used for calculating moment induced by thermal constraint, the thermal constraint formula is embedded into the moment induced by thermal constraint formula. Then the formula will change to equation (79).

$$\pi_M = 2.M.E^{-1}.\alpha^{-1}.\Delta T^{-1}.wp^{-1}.b^{-2} \quad (79)$$

$$I_{GZ_{Cantilever}} = \frac{(H+b)^3.w}{12} \quad (80)$$

$$I_{GZ_{Supports}} = \frac{t.H}{12}(t^2 + H^2) + t.H\left(\left(\frac{3}{2}.(n+1).(L-c) + (n.c)\right)^2 + \left(\frac{H}{2} + b\right)^2\right) \quad (81)$$

$$I_{GZ} = I_{GZ_{Cantilever}} + I_{GZ_{Supports}} \quad (82)$$

$$\pi_\delta = \delta.E.I_{Gz}.M^{-1}.L^{-2} \quad (83)$$

The values for Pi numbers of this study are equal to one. Formulas are arranged to calculate the variable the Pi number is made for. Using the governing equations and the functional model the causal graph between the variables of the system can be produced. The causal graph is demonstrated in Figure 25.

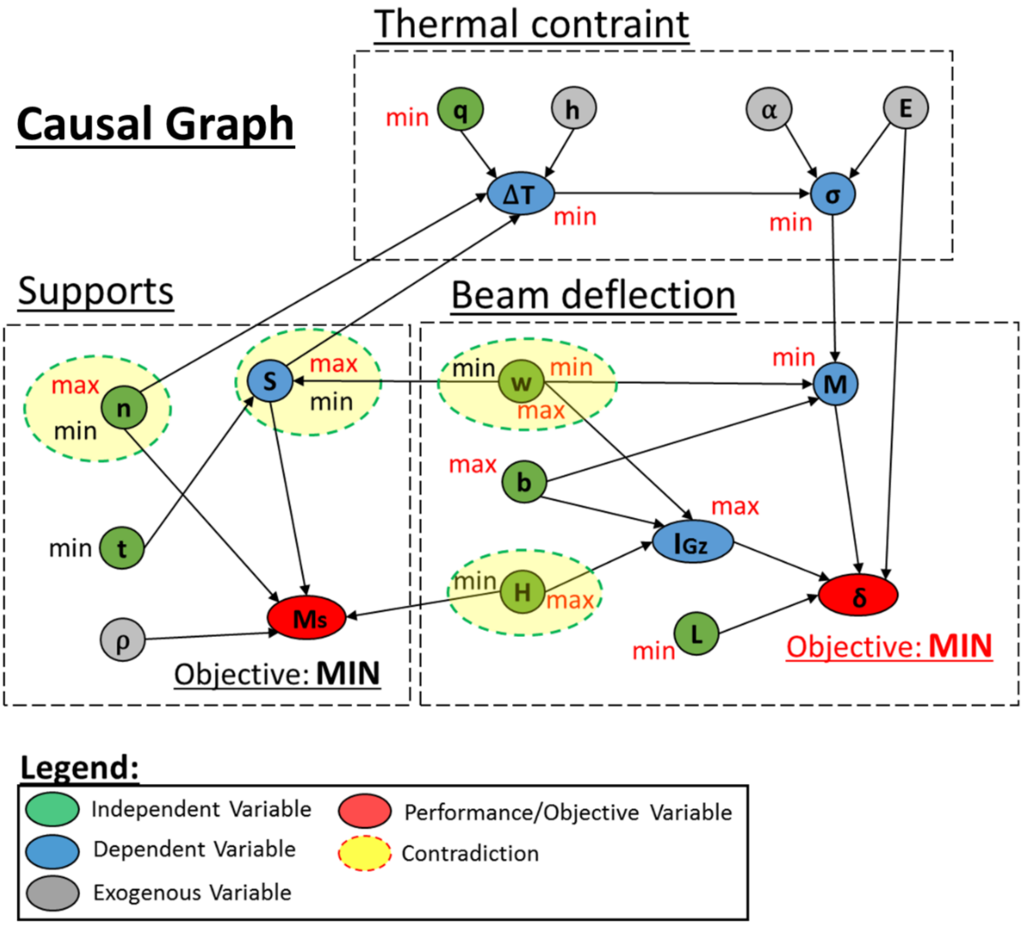

Figure 25. *The causal graph obtained from the functional model and the governing equation Mokhtarian et al. (2018)*

An ideal objective of the current case study is to minimize the curling defect (δ) while minimizing the total mass of the support structure (Ms). The causal graph produced by DACM method needs some modifications before it can be used as a Bayesian network, as mentioned in section 3.4.2.

4.1.2 Translating the causal graph to a Bayesian network.

The causal graph in Figure 25 and the governing equations (75)-(83) are then used as a basis to establish a probabilistic model; a Bayesian Network model. Model is reformed into Figure 26 for better visibility.

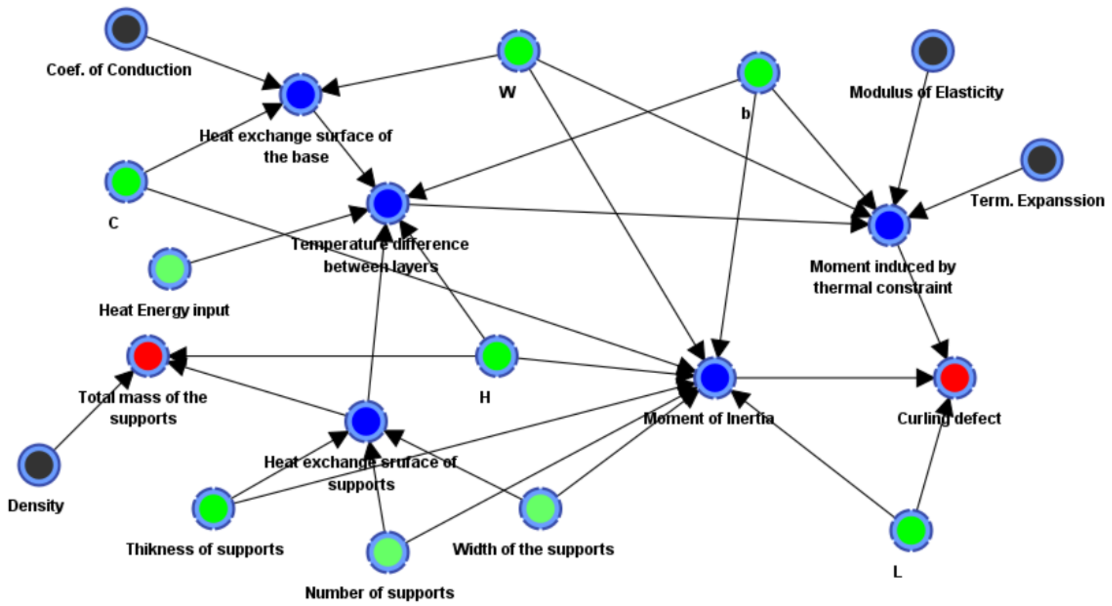

Figure 26. *The causal graph produced by DACM Framework for curling defect in PBF*

The process of transformation, as described in section 4.1.2, consists of four steps shown in Figure 27.

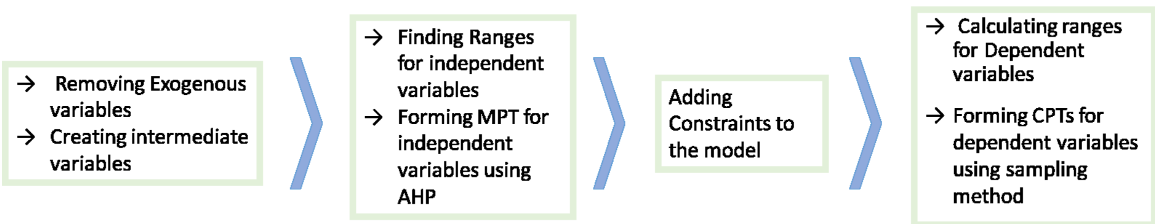

Figure 27. *The workflow for creating a Bayesian network using DACM as described in section 4.1.2*

In the first step of transforming the causal model into a Bayesian network, the exogenous variables are removed from the graph. Moreover, the number of input arcs to the node Moment of inertia is too many, so an intermediate node called “moment of inertia of supports” is created as an intermediate variable. The values of the intermediate variable are then added to the node “moment of inertia”. The resulting graph is shown in Figure 28.

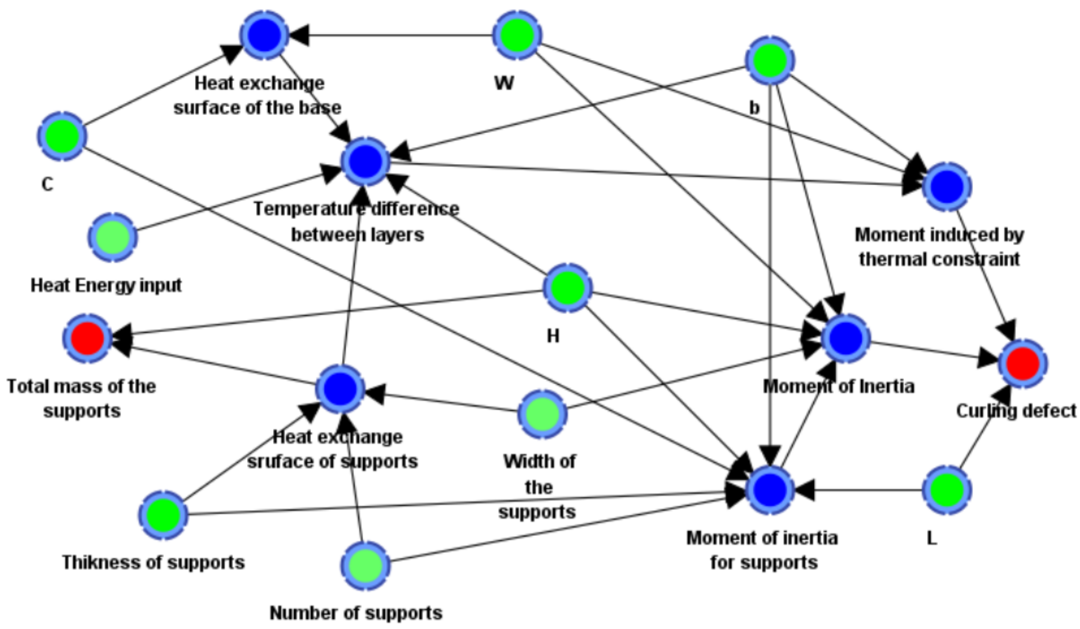

Figure 28. *The graph after removing exogenous variables and adding intermediate variable "Moment of inertia of supports".*

The second modeling step requires to find valid ranges for the independent variables and dividing ranges into intervals. The ranges have been extracted from the case studies used in the literature or the manuals of the machines and datasheets for the materials used in the process. In this study, Titanium alloy Ti6AL4V powder is used for manufacturing the parts. The values for dimensions are coming from Tounsi, and Vignat's study (2017) and the values for the exogenous variables are coming from Yan and Yu (2015) and are shown in Table 8.

Table 8. The values for the exogenous value and independent variables

Variables	Interval Description	Values and Ranges	Symbol	Base Units
Modulus of Elasticity	Fixed Values for Titanium Ti-6Al-4V Alloy	113.8×10^9	E	$g.mm^{-1}.s^2$
Coefficient of Conduction		7.1×10^6	K	$g.mm.k^{-1}.s^{-3}$
Thermal Expansion		8.7×10^6 (AZOM, 2018)	α	K^{-1}
Density		4.43×10^{-3} (AZOM, 2018)	ρ	$g.mm^{-3}$

Heat Energy Input	Low Average High	100-150 150-250 250-400 (Sheldon, 2015)	q	$\mathrm{g \cdot mm^2 \cdot s^{-2}}$
Thickness of supports	Low Average High	0.1-0.3 0.3-0.6 0.6-1	t	mm
Height of the part	Low Average High	1-3 3-6 6-12	H	mm
Length of the base	Low Average High	1-4 4-7 7-10	c	mm
Length of the straight part	Low Average High	5-15 15-25 25-40	L	mm
The thickness of the straight part	Low Average High	1-2 2-4 4-6	b	mm
The width of the part	Low Average High	1-3 3-6 6-12	w_p	mm
The width of the supports	Low Average High	0.3-4.2 4.2-8.1 8.1-12	w	mm
Number of Supports	4 levels	1-5, 5-10, 10-15, 15-20	n	number

It also requires integrating expert's preferences for choosing the intervals for an ideal design and manufacturing condition. Note that the independent variables in the model are the ones that can be modified conveniently to change the performance variables. As mentioned in section 4.1.2, the expert's preferences at this stage are captured using the

Analytical Hierarchy Process (AHP) method. The experts' knowledge is collected through AHP tables. The tables have been sent to experts along with a description on how to form AHP questions and how to fill tables. For instance, the experts are asked about their preference for choosing an interval with higher values over an interval with average values to reduce the curling defect. Experts are answering based on their knowledge on the domain and their experience. A sample of the documents sent to experts is presented in Appendix A.

After experts' preferences collected, a Python script is used to calculate the weights and consistency indexes. The python code is accessible at (Hamedi, 2018). For example, the resulting table for calculating the thickness of supports is shown in Table 9.

Table 9. The weights calculated for the variable thickness of the support using AHP and experts' knowledge.

Thickness of supports	**High (0.1-0.3mm)**	**Medium (0.3-0.6mm)**	**Low (0.6-1mm)**	**weights**
High (0.1-0.3mm)	1.0000	0.2500	0.1429	0.0786
Medium (0.3-0.6mm)		1.0000	0.3333	0.2628
Low (0.6-1mm)			1.0000	0.6586
	MEV=			3.0324
	C.I.=			0.0162

Completed AHP tables for all of the independent variables and calculated weights and consistency indexes are available in Appendix B.

In the third step, the constraints of the system should be added to the model. For this model, two types of constraints are used. Dimensional aspects of the part should have some limitation relative to each other. For example, imagine a part with a small thickness of the beam and small width and an exceedingly long length of the beam. Manufacturing such a shape for an L shape cantilever is normally not desirable. Moreover, for parts with such geometric aspects, the curling defect will be magnificent due to the small surface for heat conduction, and the curling defect can be cumulated in the length of the beam. For solving the curling defect for the part with such dimensionality, a good option is changing the position of the part so that the beam stands vertically.

To set the constraints for dimensional variables, a ratio between the length of the straight part and the other dimensional variables is considered. To prevent dimensional values to get exceedingly large or too small relative to each other, the ratio values between them should be bound from both sides.

The other type of constraint aims to limit the surface of the supports. The sum of cross-section surfaces of the supports cannot exceed the lower surface of the beam. Moreover, to make it easier to remove the supports, the thickness and the number of supports must be way smaller than the lower surface of the beam. The value chosen for the ratios is obtained through consulting with the experts. Table 10 shows the variable ratios chosen and the values for the ratios.

Table 10. Ratio values for dimensional constraints

Ratio name	Variables	Values
Beam's Thickness Ratio	$c_1 = \frac{b}{L}$	$\frac{1}{10} \leq c_1 \leq \frac{2}{10}$
Part's Width Ratio	$c_2 = \frac{w_p}{L}$	$\frac{1}{6} \leq c_2 \leq \frac{1}{2}$
Base Length Ratio	$c_3 = \frac{c}{L}$	$\frac{1}{4} \leq c_3 \leq \frac{1}{2}$
Part's Height Ratio	$c_4 = \frac{H}{L}$	$\frac{1}{3} \leq c_4 \leq 1$
The thickness of supports Ratio	$c_5 = \frac{n.t}{L}$	$\frac{25}{10000} \leq c_1 \leq \frac{1}{2}$

Moreover, the heat input to the system should be proportional to the amount of material in the system. For example, if the amount of heat is way more than the heat needed for melting a part with a certain mass, the whole material in the system will melt, and the process cannot be continued. Therefore, a constraint between the amount of energy and the mass of the part should be considered. This constraint can be set as a function of the mass of the part.

For the nodes like the number of supports and the width of the support, constraints are set to refine the model and include experts' knowledge. There are some disadvantages with using a higher number of supports, including increasing manufacturing time, the difficulty of removing the supports and waste of material in the support structures. Hence, the number of supports should be limited. The width of the supports can also be chosen based on the width of the part. Therefore, the experts' preference had been elicited for the intervals of these variables using AHP tables. The AHP tables are available in Appendix B. After adding all these constraints; the final graph is shown in Figure 29.

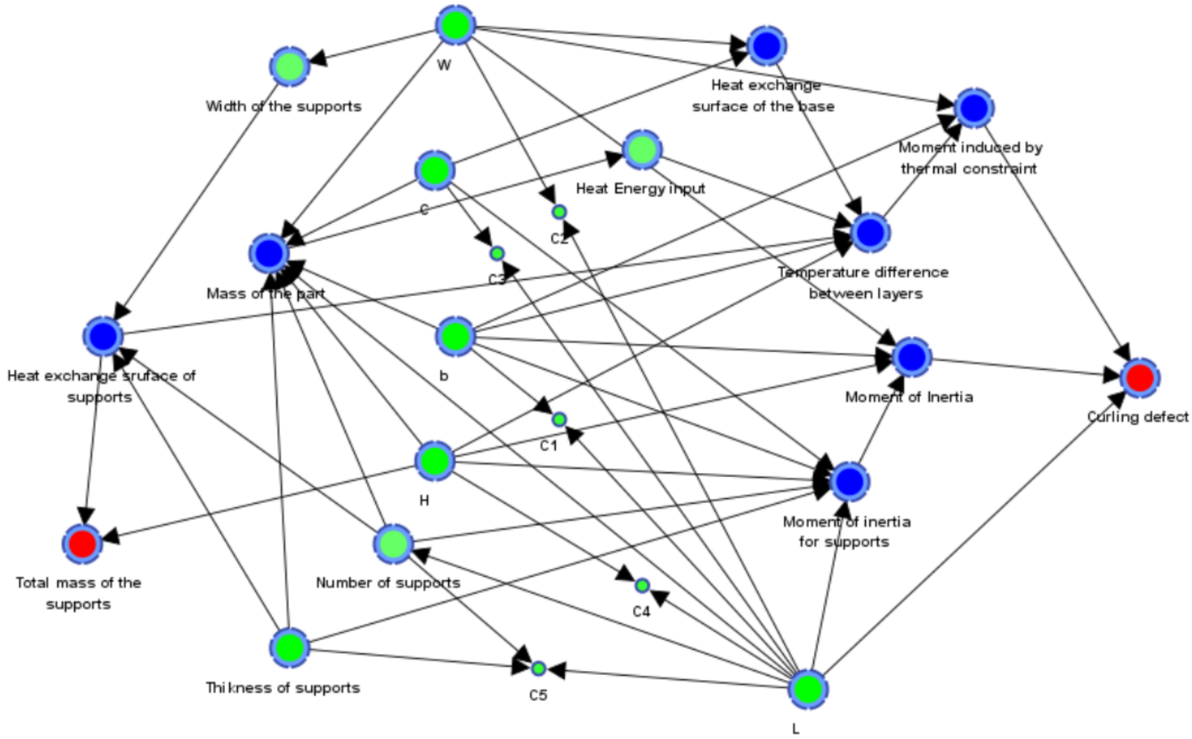

Figure 29. *The final Bayesian network structure for the curling defect case study*

Note that the independent nodes such as "number of supports" and "width of the supports", the color was chosen in the graph is green instead of blue, although they are shown as dependent variables in the graph. That is because although they are not independent, it is still possible to change their values directly.

In the fourth and the last step, the ranges and intervals for the dependent variables and performance variables should be calculated as described in section 3.4.2. The process starts with calculating the range for dependent variables. For example, for the node "Heat Exchange surface of base", first the domain should be calculated, based on the ranges of the variables it depends on, i.e., "Width of the part" and "Length of the base". The equation for calculating the surface is $S_{base} = w_p \times c$ and therefore, the calculated domain for the surface of heat exchange is $1 \times 1 = 1$ and $12 \times 10 = 120$. This domain is then divided into three intervals, Low (1-15), Average (15-30) and High (30-120). The complete table for the domains and the intervals for all the independent and performance variables is shown in the Table 10.

Table 11. Independent variable, their values, and formulas

Variables	Intervals Description	Values and Ranges	Formula/Symbol	Units
Heat Exchange surface of the base	Low Average High	$1 - 15$ $15 - 30$ $30 - 120$	$S_b = w.c$	mm^2
Heat Exchange surface of supports	Low Average High	$0.03 - 10$ $10 - 20$ $20 - 240$	$S_s = w.t.n$	mm^2
Mass of the part	Low Average High	$1.83 - 400$ $400 - 1000$ $1000 - 7200$	$M_p = b.L.w + c.H.w + n.t.H.w$	g
Temperature difference between layers	Low Average High Very High	$9.124 - 500$ $500 - 1000$ $1000 - 2000$ $2000 - 65637$	$\Delta T = \frac{10^6.q.H}{S.k}$	K
Moment of inertia of the supports	Low Average High Very High	$17.33 - 10000$ $10000 - 20000$ $20000 - 50000$ $50000 - 15734173$	Equation (81)	$g.mm^2$
Moment of Inertia	Low Average High	$0.66 - 20$ $20 - 200$ $200 - 5832$	$I_{gz} = \frac{w_p.(H+b)^3}{12}$	$g.mm^2$
Moment induced by thermal constraint	Very low Low Average High Very High	$1.937 \times 10^{10} - 3 \times 10^{10}$ $3 \times 10^{10} - 5 \times 10^{11}$ $5 \times 10^{11} - 8 \times 10^{12}$ $8 \times 10^{12} - 10^{13}$ $10^{13} - 1.404 \times 10^{17}$	$M = \frac{1}{2}.E.\alpha.\Delta T.wp.b^2$	$g.mm^2.s^{-2}$
Total mass of the supports	Very low Low Average High Very High	$1.329 \times 10^{-4} - 0.1$ $0.1 - 0.5$ $0.5 - 1$ $1 - 5$ $5 - 12.758$	$M_s = S_s.H.\rho$	g
Curling defect	Very low Low Average High Very High	$7.2954 \times 10^{-7} - 0.05$ $0.05 - 0.2$ $0.2 - 2$ $2 - 10$ $10 - 2.990 \times 10^6$	$\delta = \frac{M.L^2}{E.I_{gz}}$	mm

Now, using these domains and the intervals of the parents, we can calculate the CPT of the child node using the method described in section 3.4.2. The CPTs for all of the nodes of the model can be found in (Hamedi, 2019).

Validation of the model

After forming the CPTs, the Bayesian network is ready for validation. The validation process can be done in multiple ways including performing experiments, using the data from the other studies or asking experts to validate the model (Geiger, Paz, & Pearl, 2014; Haiqin Wang, 2006; Schietekat, Waal, & Gopaul, 2016). Performing experiments and comparing the results with the prediction of the model may be the optimal method for validating the model. Since the resources for performing experiments is not available for this study, this option is not possible.

The second way is to use the data from the other studies and validate the model against them. The problem with this method is that it is very difficult, if not impossible to find other studies which use a similar set of variables.

The last method is to use experts' knowledge for validating the model. The model in this study has been evaluated by a small group of experts, but no systematic validation process has been done on the model.

4.2 Reliability engineering case study

The purpose of this section of this thesis is to provide a predictive model for failure in the pumps according to their consecutive failure time history; and in the next step, provide a recommender system for predictive maintenance on the pumps.

Based on the type of data and the characteristics of the task, the question above is parted into these questions.

1. How to breakdown the variables in the dataset to a set of meaningful variables?
2. How to model the missing value mechanism in our predictive model? How to process the missing values?
3. How to discretize the continuous variables for the failure times in a way that the dependency between variables is reserved and the discrete variable is easily interpreted?
4. what constraints do we need to set for the learning algorithm, to avoid unwanted causal relations?

5. How to validate the Bayesian network created with the machine learning algorithm?
6. How to use Bayesian reasoning to predict the next failure time?

The rest of this section tries to answer these questions for this case study.

4.2.1 Data preparation

The dataset for this case study contains the failure data of 74 pumps used in a paper an industrial plant in Finland. For each pump, the starting date, the times between the start and each failure, the times between the start and each maintenance, and the cumulated lifetime is recorded. A few entries of the main dataset are shown in Table 12.

Table 12. The dataset for pump failure times

ID No					
1	10/23/1995	11/7/2016			
	0	2718			
	Started	Cumulated lifetime			
2	11/21/1995	1/14/1998	10/6/2000	11/7/2016	
	0	381	511	1948	
	Started	Failure	Failure	Cumulated lifetime	
3	11/24/1995	7/6/2004	11/7/2016		
	0	1231	2954		
	Started	Failure	Cumulated lifetime		
4	11/27/1995	7/31/2000	11/9/2000	2/23/2010	11/7/2016
	0	826	835	2070	3131
	Started	Failure	Failure	Failure	Cumulated lifetime

The data

The failure times are the times in which the pumps failed and stopped functioning. The maintenance policy has been mostly corrective maintenance. The quality of the maintenance has been near perfect, so after each failure, the whole parts of the pumps have been changed, and the pumps assumed to be as good as new (AGAN) after each maintenance.

The censoring in the data is random right censoring. As shown in Figure 30 and Table 12, each pump is started at a random time and monitored until a specific date. The cumulated lifetime is the age of the pump at the time the data collection is ended. This value shows the duration that the pump has been working from the starting time, and it is not failed yet; therefore there is a censored record after this period for a new failure.

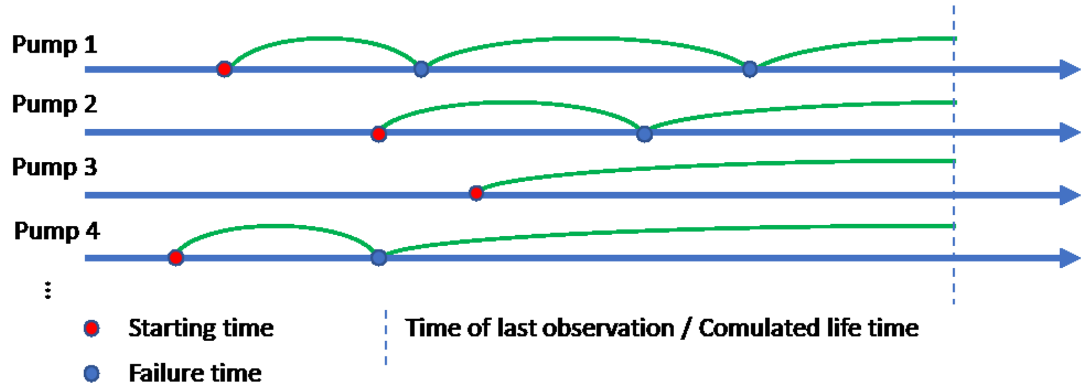

Figure 30. *The process and timeline of data collection*

Figure 30 shows the timeline for the instances of pumps, their start time, their failure times and the last date of data recording. Pumps are installed at separate times and may or may not have failures during the data recording period. Since all of the pumps are manufactured with the same mechanical design and material, the time of their setup does not have any effect on their failure times. Therefore, the starting times of all instances can be aligned by shifting them to the left. Figure 31 shows the timelines after shifting.

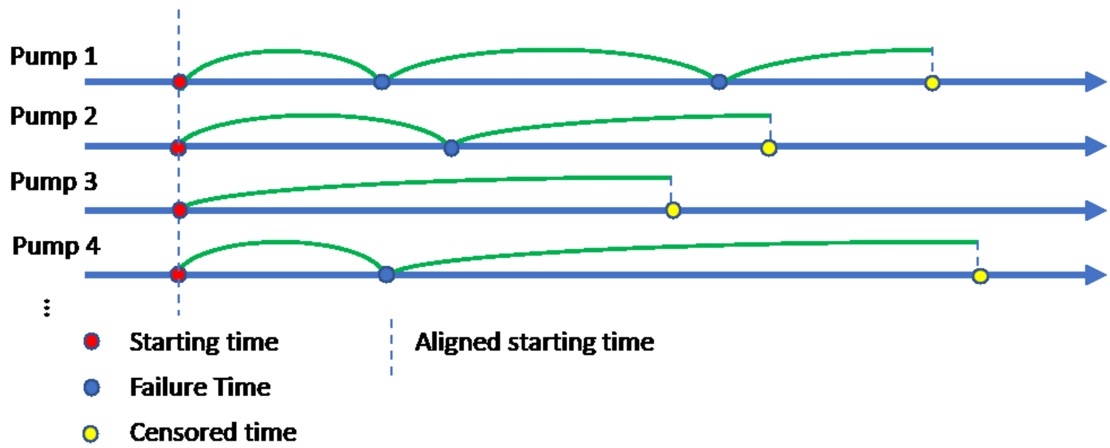

Figure 31. *The pump failure timelines after alignment*

Based on the assumption about the similarity of the pumps, it can be assumed that all the pumps are the instances of one pump. Therefore, the timeline will look like Figure 32.

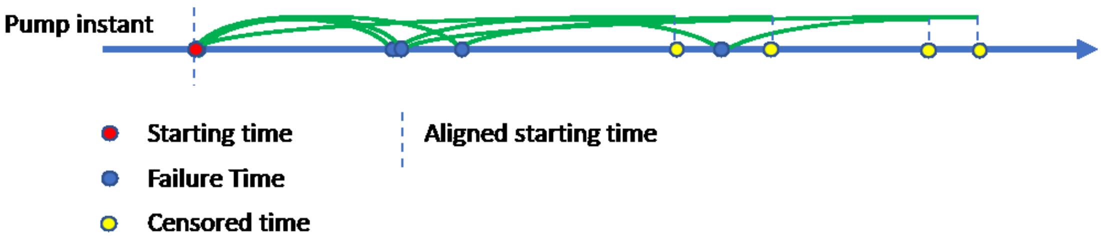

Figure 32. *The failure timeline for an instance of the pump*

Table 13 shows some statistics about the dataset. The total number of data points are 74 instances. The number of instances for each failure, the number of censored values for each failure and the percentage of not censored data points are presented in the table. Percentage of not censored values is calculated by dividing the number of previous failures by the censored values of the current failure.

Table 13. Number of censored and not censored failures

	Instances	Censored	Not Censored %
Number of pumps	74		
Failure 1	70	4	94.59%
Failure 2	57	16	77.14%
Failure 3	30	27	52.63%
Failure 4	12	17	43.33%
Failure 5	8	4	66.67%
Failure 6	2	6	25.00%
Failure 7	1	1	50.00%
Failure 8	1	0	100.00%
Failure 9	0	1	0%

The time from the starting of the pump to each failure time is named total time to failure (TTTF) in this study. The data is rearranged in a way that each failure time of a pump is associated with the corresponding failure number, i.e., the time to failure (TTF) value for each failure occurrence is calculated. The TTF between every two consecutive failures is calculated by subtracting their TTTF values. A simple excel formula has been used to calculate the subtraction. The first 15 rows of the rearranged data are shown in Table 14.

Table 14. Pre-processed dataset

Pump ID	TTF1	TTF2	TTF3	TTF4	TTF5	TTF6	TTF7	TTF8	Cen-sored time	Cumu-lated lifetime
1									2718	2718
2	381	130							1437	1948
3	1231								1723	2954
4	826	27	1217						1061	3131
5	1137	601							916	2654
6	597	1660							231	2488
7	1101	263	767						1	2132
8	567	2	149						2283	3001
9	770	815							909	2494
10	758	276	208	201	594				197	2234
11	897	0							1932	2829
12	2471								193	2664
13	37	N/R	N/R	N/R	238	6	809	166	169	1745
14									2585	2585
15	1057	477							1212	2746

In addition to censoring in the data, there are several datapoints which their values are missing. For the cases that there is a missing value for any failures, and it is not possible to calculate the TTF. Those data points are considered as missing values. An “N/R” value is placed in the data set for the cells with a missing value, as shown in Table 14.

The censored times (CT) for each TTF value are moved to a value called CT#, in which # is corresponding to the number of censored TTF. These values are shown in the columns CT1 to CT9 of Table 15. For each row, the TTF values after the last failure are impossible values, meaning that it is not possible to have a TTF $i + 1$ when there is no TTF i . Therefore, these values are marked with an asterisk ($*$) as not available, so the software can detect and handle them as filtered value, not missing values.

Table 15. Censored failure times for each TTF

ID	CF1	CF2	CF3	CF4	CF5	CF6	CF7	CF8	CF9
1	2718	*	*	*	*	*	*	*	*
2	*	*	1437	*	*	*	*	*	*
3	*	1723	*	*	*	*	*	*	*
4	*	*	*	1061	*	*	*	*	*
5	*	*	916	*	*	*	*	*	*
6	*	*	231	*	*	*	*	*	*

7	*	*	*	1	*	*	*	*	*
8	*	*	*	2283	*	*	*	*	*
9	*	*	909	*	*	*	*	*	*
10	*	*	*	*	*	197	*	*	*
11	*	*	1932	*	*	*	*	*	*
12	*	193	*	*	*	*	*	*	*
13	*	*	*	*	*	*	*	*	169
14	2585	*	*	*	*	*	*	*	*
15	*	*	1212	*	*	*	*	*	*

The number of datapoint for the variable TTF7, TTF8, CF7, and CF9 is only one data point, and there is no data point for the CF8. Therefore, it is not possible to use them for creating the model, and they are omitted from the dataset.

Missing values

The missingness of data for failure times in the dataset has two reasons. Some failure times have not been recorded due to an unknown reason. These are shown with a number 0 in the database. The other missing failure data are missing because they have not happened at all (in case of the first failures) or the previous failure has not happened yet (failure $i+1$ when failure i is not happened). These missingness can be modeled using the censoring concept in survival analysis, but the Bayesian missing value models can give a better perspective and it can be integrated into the structural learning process as well (see section 3.1.7), therefore, the missing value concept is used to address this issue.

Type of missingness should be characterized to determine the method for handling the missing values. For the first group of missing values, the values that are not recorded, it is not possible to classify them in any specific class. For the second group, since the missingness of failure $i+1$ is dependent on missingness of failure i, it is possible to classify them as missing at random (MAR). Assuming that the missingness is MAR, the Structural Expectation Maximization is chosen as the missing value estimation method, which has shown good results in the literature (Friedman, 1998).

4.2.2 Discretization of variables and machine learning of the structure

The type of variables in the data is continued, and the algorithm used in this study is based on discrete variables. Therefore, the variables should be discretized using one of the methods mentioned in 3.1.7. The genetic optimization-based algorithm implemented in the software can preserve dependencies in the variables and can provide the optimal discretization, so it is used to discretize the variables in the dataset. The algorithm finds the best number of states and the optimal intervals for each state to discretize the variable into them. The maximum number of states for a variable can be set, and the variable decides whether using that number or choosing a smaller number of states. Note that the length of the state intervals will not be equal.

To have a good resolution in the model variables, the number of intervals chosen for the discretization algorithm is ten states. Table 16 and Table 17 shows the number and the range of intervals obtained from the discretization algorithm for the TTF variables and CF variables respectively.

Table 16. Intervals of the states of discretized TTF values

TTF #	TTF1	TTF2	TTF3	TTF4	TTF5	TTF6
Interval 1	<=89	<=50	<=46	<=116	<=238	<=6
Interval 2	<=326	<=163	<=149	<=142	<=301	<=815
Interval 3	<=478	<=276	<=239	<=193	<=594	*
Interval 4	<=658	<=407	<=470	<=201	<=1565	
Interval 5	<=897	<=509	<=563	<=363	*	
Interval 6	<=1231	<=640	<=644	<=379		
Interval 7	<=1554	<=815	<=819	*		
Interval 8	<=1751	<=989	<=1170			
Interval 9	<=1988	<=1282	<=1217			
Interval 10	<=2471	<=1673	<=1556			
Interval 11	*	*	*			

Table 17. Intervals of the states of discretized CF values

CF #	CF1	CF2	CF3	CF4	CF5	CF6
Interval 1	<=7	<=4	<=33	<=48	<=1	<=0
Interval 2	<=1850	<=193	<=231	<=180	<=3	<=197
Interval 3	<=2585	<=292	<=342	<=363	<=554	<=380
Interval 4	<=2718	<=308	<=912	<=475	*	<=400
Interval 5	*	<=650	<=1051	<=643		<=577
Interval 6		<=675	<=1285	<=743		*
Interval 7		<=1014	<=1437	<=855		
Interval 8		<=1687	<=1673	<=932		
Interval 9		<=1723	<=1932	<=1065		
Interval 10		<=2000	<=2275	<=2283		
Interval 11		*	*	*		

The intervals marked with an asterisk ($*$) are the intervals which contained the filtered values. The algorithm obtained all ten states for the variables TTF1, TTF2, TTF3 and CF2, CF3, CF4 but for the rest of the variables, the optimal number of states is less than ten.

Model learning

This study relies on machine learning in BNs to find a Bayesian network which describes the relation between TTF variables and CF variable. Finding the best Bayesian network in the search space of all possible BNs is NP-hard (Munteanu & Bendou, 2001). The heuristic search algorithms, which are normally used, can easily trap in local minima. To tackle this problem, Munteanu and Bendou (2001) developed the EQ framework to use the space of essential graphs of an equivalent class to search for a suitable graph. The detailed description of the EQ framework and algorithm can be found in section 3.1.5.

As described in section 3.1.5, there are equivalent classes for a Bayesian network. These classes describe the joint probability of the random variables of the system in the same way, but the causal relationships between variables can be irrational in some of them. It is important to set constraints for the structural learning algorithm to avoid having models with irrational causal relationships.

These constraints can be extracted with the aid of experts. In the case of this study that the number of variables is limited and possible relations between variables are intuitive, the expert knowledge has not been used, and the constraints have been extracted by the author. Since the failures are happening in consecutive order, the failure i can affect the next failure, i.e., failure $i+1$ but it cannot affect the previous failure, i.e., failure $i-1$. These constraints apply to CF variables and between consecutive CF and TFF values and TTF and CF value as well. Therefore, to avoid such relations, arcs between these variables are set as forbidden, so the structural learning algorithm ignores networks with such relations in them.

4.2.3 Network selection and Validation

The structural learning algorithm can find multiple networks based on the threshold set for minimum association metric. In the Bayesialab software, this metric is set using a value called structural coefficient (SC) value. A suitable Bayesian network should be neither too complicated nor too simplified. A complicated BN is a network with a structure close to a fully connected network. Although such a model can represent the joint probability distribution of dataset more accurately, it would be very hard to interpret the network and inference in it.

On the other hand, an overly simplified BN has a structure close to an unconnected network, which is the worst representation of the joint probability distribution of the random variables. To find the middle ground between these extremes, the structural co-efficient analysis tool in the software is used. The tool runs the learning algorithm for multiple times for different CS values and calculates the contingency table fit (CTF) value for the network found in each iteration. CTF describes the quality of representing the joint probability distribution by the model and it is a value between 0 and 1, with 0 stands for the worst fit and 1 stand for the best fit. A description of this metric is provided in section 3.1.6. Figure 33 is showing a portion of the result of running the tool for 60 values of SC in the range of 0.60 and 0.01.

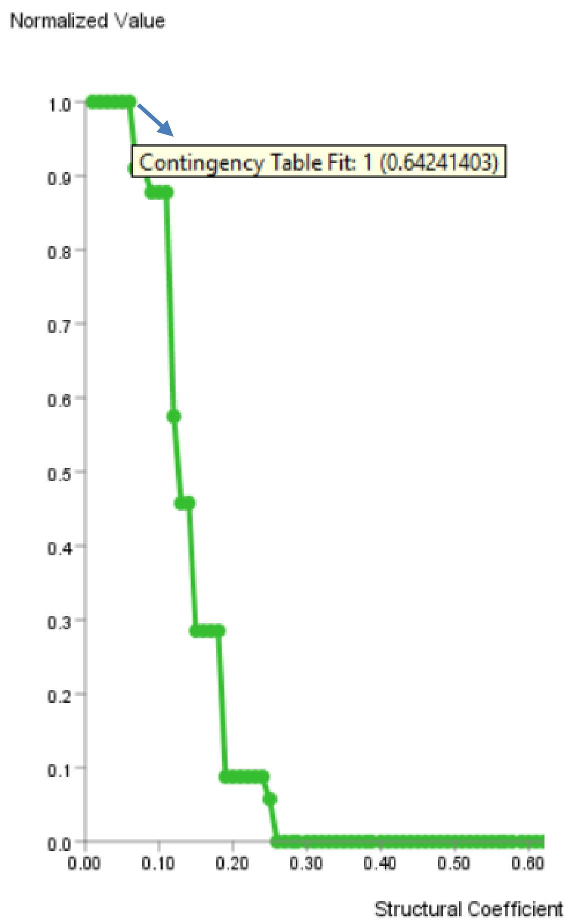

Figure 33. *The SC and CTF curve*

In this Figure, the x-axis is showing the structural coefficient values, and the Y-axis is the normalized value of CTF. As shown in Figure 33, the value of the contingency table fit, shown by a green line, is increasing with the decrease in the value of the structural co-efficient. That is because by lowering the SC value, the resulting network is getting close to a fully connected BN, which is the exact representation of the joint distribution.

The chosen network is created by an SC value of 0.6 and CTF value of 1. The reason for choosing this network is that it is the least complicated network that includes all the variables (except CT1) in the model and has a complete CTF value. The resulting network is shown in Figure 34.

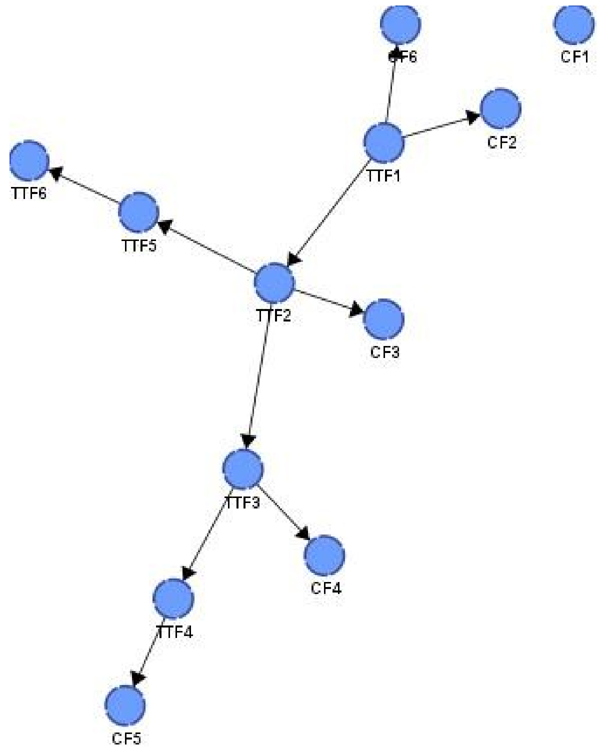

Figure 34. *BN structure learned from the dataset*

The model shows, for example, that there is a dependency between the consecutive TTF values. The TTF1 to 4 are affecting each other sequentially and each of them is affecting the next CF respectively. There are also some relations like the relation found between TTF2 and CF5 which are not following the sequential effect, similar to TTF1 to 4, but it is still logical to have such a relationship. The number of datapoint for the variables like TTF5, TTF6, CF5, and CF6 is quite insufficient to be used for any machine learning method, but the algorithm could find logical relations for them.

The variable CF1 is not included in the model by the algorithm. This result is logical because censored failure 1 cannot be influenced by any other variable in this system. CF1 is the censored instances of TTF1 which is the first influencing variable in the model. Note that the model can be restructured into a tree-shaped graph having the TTF1 value as the top node.

It is also possible to see the marginal probability distributions for each variable using the Bayesialab software, which is shown in Figure 35 and Figure 36 for TTF and CF values respectively.

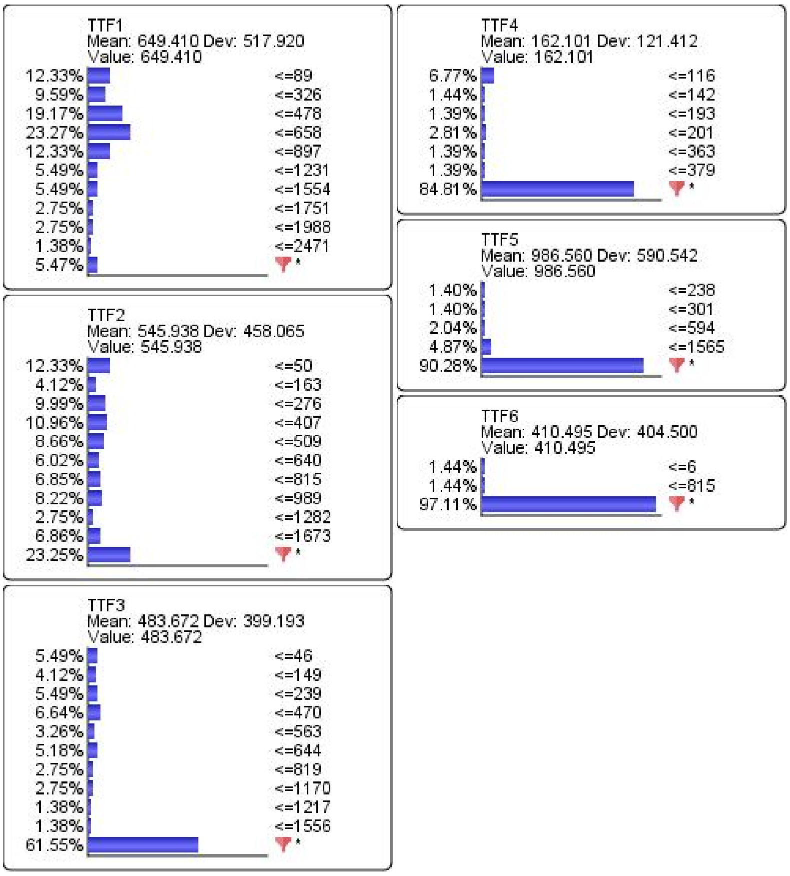

Figure 35. *Monitor screens for TTF variables*

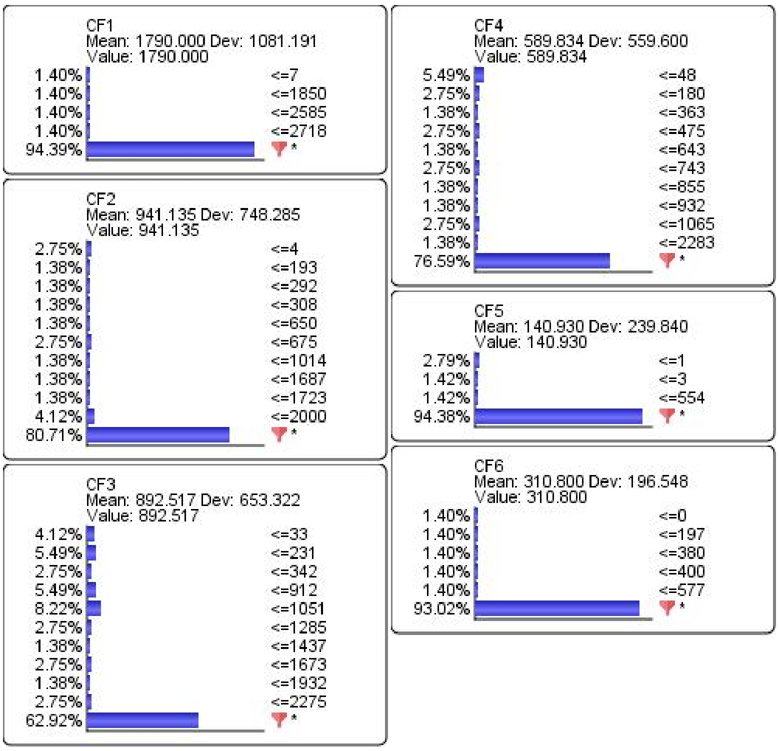

Figure 36. *Monitor screens for CF variables*

The filtered values are shown in the last state with a funnel sign. For the TTF1 and TTF2, shown in Figure 35, the filtered values are only 5.47% and 23.25% of all the data. But as there are fewer data points available for the TTF 3 to 6, the amount of filtered values is increasing accordingly.

Among the CF values, shown in Figure 36, CF3 has the largest number of occurrences. The variable CF 1 is not included in the model by the structural learning algorithm, but the monitor is shown here to demonstrate its marginal distribution. For the rest of the variables, as shown before, the number of unavailable value (filtered values) are significant.

The discussion regarding the results of this model and the shortcomings is provided in section 5.2.

5. DISCUSSION AND CONCLUSION

This section starts with a discussion about the result of two case studies and in the end, the conclusion for this study is provided.

5.1 Additive manufacturing case study

The model for the additive manufacturing case study can be used to 1- represent the knowledge in the domain 2- to diagnose the reason for the curling defect and 3- to explore the design space in the initial conceptual design phase. Each of these use cases is discussed in this the rest of this subsection.

5.1.1 Knowledge representation

The Bayesian network has been used in various cases to represent the knowledge in the corresponding domain. For example Del Águila and Del Sagrado (2012) used BNs to create a decision support system in software engineering domain, Gupta and Pedro (2004) developed a Bayesian network to represent the human common sense in linguistics for humanoid robots to be able to respond to situations and Sedki and Beaufort (2012) developed a method that uses cognitive mapping to produce a Bayesian network for representing the knowledge in a domain.

In the field of Additive manufacturing, Wang et al. (2018) developed a knowledge management system in which they developed a BN to model the knowledge in the domain of AM to help designers in early-stage design steps. Their model consists of an overview model and a detailed information layer, and they used the knowledge and data available in the literature to form the structure of the model and learn the parameters of the model.

The model in this study contains and visualizes the experts' knowledge in the domain trough probability distributions of the independent nodes and the structure of the network. The knowledge about the system under study is also augmented into the model through the causal graph and the constraints.

A general view of the marginal probability tables of each independent and performance node is depicted in Figure 37.

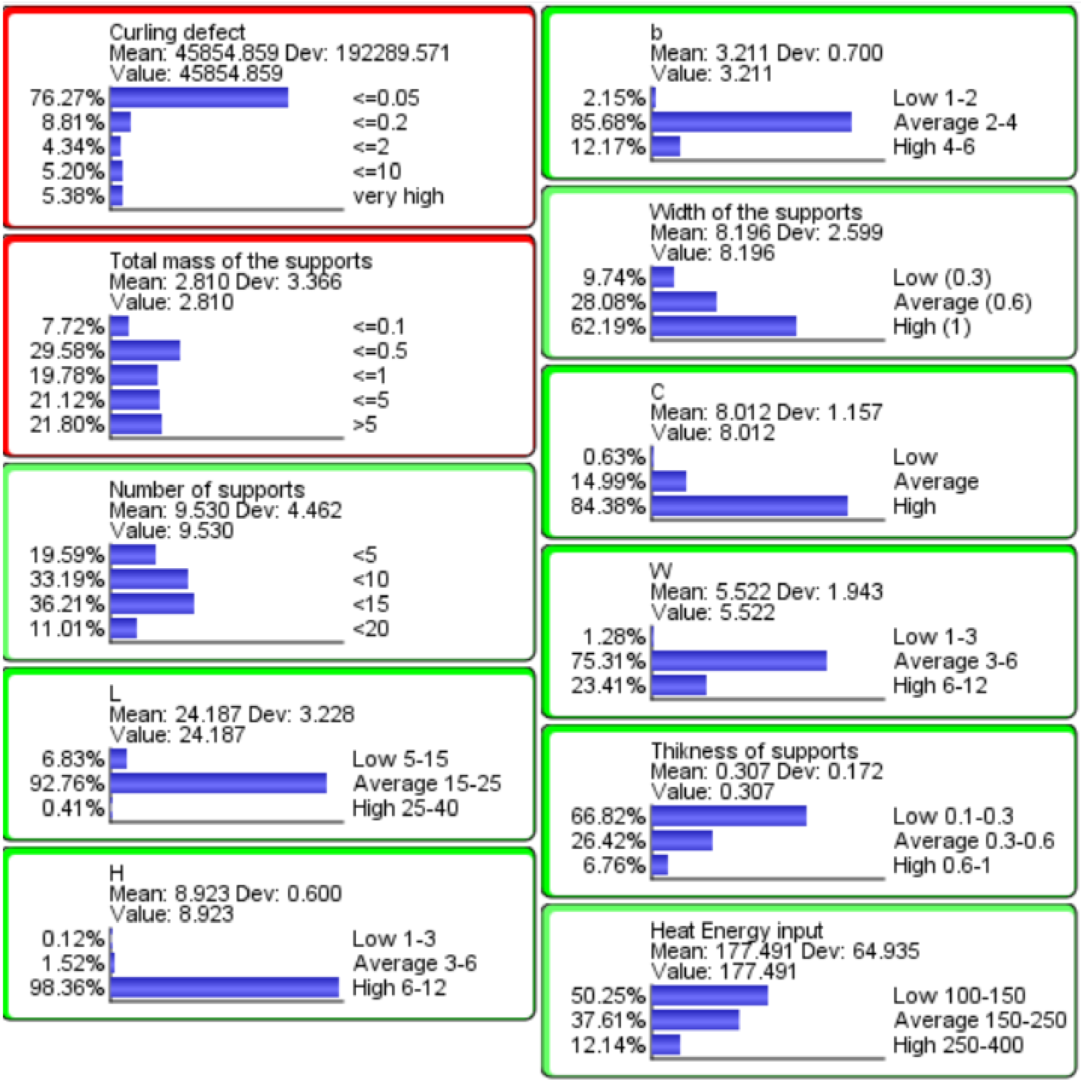

Figure 37. *The probability of occurrence of each interval for each node for independent and performance variables*

Bayesialab software demonstrates the marginal probability table for each variable as a box, which is called a monitor (Bayesialab, 2018). Each box shows a set of information, including the name of the variable on the top of the box, the mean value and the standard deviation of the probability distribution of each node and a “value” which is here it is equal to the mean value of the distributions. The marginal distribution for each interval of the independent variables is displayed with the blue bars, having the probability values on the left side of them and the names of the intervals on the right side.

The first two boxes with red frames on the upper right corner of Figure 37 shows the marginal probability tables for performance variables. For example, given the current setting for the independent variables and the constraints, there is only 5.38% chance that the final part would have a curling defect with more than 10mm. This shows that the experts’ knowledge has been extracted for the variable configuration, at least in theory, in most of the cases will lead to having a very small defect. Although the results are mostly in the safe region, i.e., less than 0.05mm, there is still a need for models to predict the cases which have a higher amount of defect.

The other element which is reflected in the model is that given the expert's knowledge about their preference on intervals and the constraint of the system, the marginal probability for each state of each value is recalculated. For example, for the variable "Width of the part" the probability for the intervals obtained through AHP is shown in Table 18.

Table 18. Probabilities for intervals of "Width of the part" variable obtained with AHP

Width of the part	Probabilities
Low	23.85%
Medium	62.50%
High	13.65%

But after setting constraints, these probabilities have changed to the values shown in Table 19.

Table 19. Probabilities for intervals of "Width of the part" variable after setting constraints

Width of the part	Probabilities
Low	1.28%
Medium	75.31%
High	23.41%

This is due to the removal of the values which create unwanted combinations through constraints set for the model.

The causal graph represents dependencies and independencies between variables of the system and Bayesian inference principles simulate the interactions between variables based on these relations. A simple example is demonstrated as follows, to demonstrate some of the basic interactions between the variables of the model.

Based on the description of the DACM model in the section 4.1.1, an increase in the number of supports has a positive effect, which is reducing the curling defect, and a negative effect, which is increasing the total mass of the supports. So, using the monitors for the variables "Number of Supports", "Total mass of supports" and "Curling defect", this sort of interaction between the nodes can be demonstrated. The default values for marginal probabilities for each of these nodes shown in the top row of Figure 38.

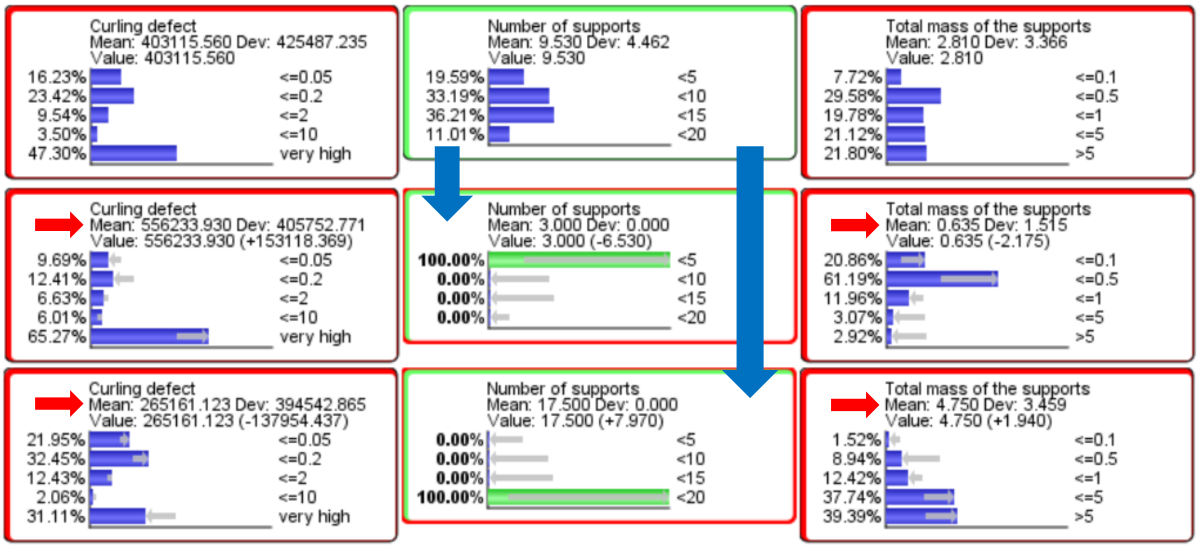

Figure 38. *Changes in Curling defect and Mass of the supports due to changes in the number of supports*

Hard evidence can be set for any of the states of this node in the monitor to show the effect of changing the number of supports. Here, the hard evidence represents choosing one of the intervals of a variable, and by doing so, the software propagates the effect of this evidence through the network and demonstrates the posterior marginal distributions for the other nodes. As an example, if in a small number of supports are chosen in the design phase without any change in the other variables, after calculating the posterior distributions, according to the model we can expect that the value for the curling defect increases by 37.98% in average and the value for the mass of the supports decrease by 77.40% in average, as shown in the second row of Figure 38. This means that if the value for the number of supports is fixed within its first interval, all combinations of all other variables will lead to a probability distribution for the performance variables with the mean values and standard deviations described in the middle row of Figure 38.

On the other hand, if the number of supports increases, this should lead to less curling defect in average and more mass for the supports in average, which is evident in the last row of Figure 38. By choosing the interval with the highest values for the variable “number of supports” in the monitor window and after calculation of posterior distributions, the value of curling defect decrease by 34.22% in average and the value of the mass of supports increases by 69.04% in average for all possible combinations of the other variables.

Changes for marginal probabilities can also be seen in both variables in Figure 38. For example, having the highest values for the variable” number of supports” increases the chance for having a curling defect in the lowest interval, i.e., less than 0.02mm, from

16.23% to 21.95% for all combinations for the other variables. This change can be observed by comparing the value in the first row of Figure 38 with the third row.

5.1.2 Prediction and Diagnosis

One of the frequent uses of BNs in the industry is for fault diagnosis and providing decision support for the practitioners (K Wojtek Przytula, Dash, & Thompson, 2003). The reason for such a vast use is that Bayesian network can reflect the experts' knowledge and uncertainty simultaneously, therefore it can reflect the uncertain nature of fault diagnosis very conveniently and represent experts' knowledge on possible solutions for the problem.

The amount of literature in this domain is significant, but to bring a few studies in the industrial domain, a brief literature review is provided as follows. McNaught and Chan (McNaught & Chan, 2011) developed a fault diagnosis system in manufacturing systems. The system supports the personnel to diagnose the faults during production system testing. Another example of a systematic procedure for creating Bayesian networks for fault diagnosis in industrial systems using bond graph theory is presented in the study of Lo et al. (2003). Li et al. also developed a method to create BNs for diagnosis of faults in manufacturing based on a method called defect factor analysis. In this method, they use the experts' knowledge to form a set of factors which result in defects and their relations and also the probability tables. The method is implemented on diagnosing the defects if machining processes.

In the case of this study, the model can be used for diagnosing the reason for having a specific value in the performance variables. This means that specifying an interval for a performance variable, i.e., hard evidence that a performance variable is in a specific interval, the model can calculate the posterior distribution for all the other variables in the model, i.e., show the most probable combination of other value. The use for this diagnosis is at the early design stage. Having a specific tolerance range for each of the performance variables, the designer can check the most probable values for each interval in design variables and the manufacturing variables.

For example, by setting the value for curling defect to between 2mm to 10mm, the posterior probability for all the design and manufacturing variables are shown in Figure 39.

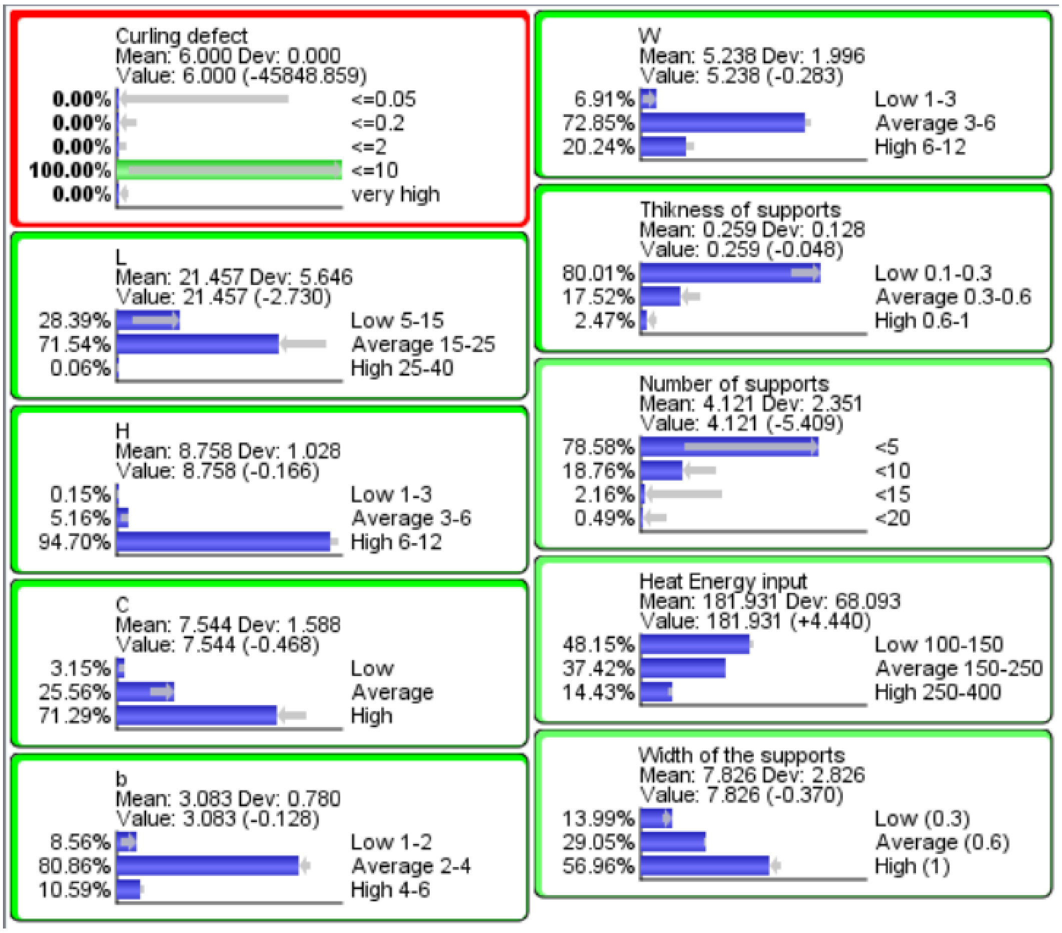

Figure 39. *Diagnosing the reason for having a curling defect between 2mm to 10mm.*

Fault prediction is the other use case of Bayesian networks in the industrial domain. Wang et al. (2017) used Bayesian networks to predict tool abnormalities in the process of manufacturing semiconductors. Their model is predicting the faults based on the current status of sensors in a real-time manner.

The model in this study can be a very useful tool for predicting the effect of choosing a combination of design and manufacturing variables on the performance variables. For example, if a designer knows the intervals for some of the design variables, they can see the probability distribution of the defect with all possible combinations for the other variables in the model. In this way, the designer can see the result of each decision in the process of design and make informed decisions. The model can also be integrated into a computer-aided design (CAD) software to make an interactive design environment.

For example, assume that the height of the part, the thickness of the cantilever, the length of it are known, but the designer wants to know the effect of their choices for the other variables on the amount of defect. The initial combination of variables is shown in Figure

40 with choosing the average values for the thickness of the straight part and the length of the part, and the interval with the highest values for the height of the part.

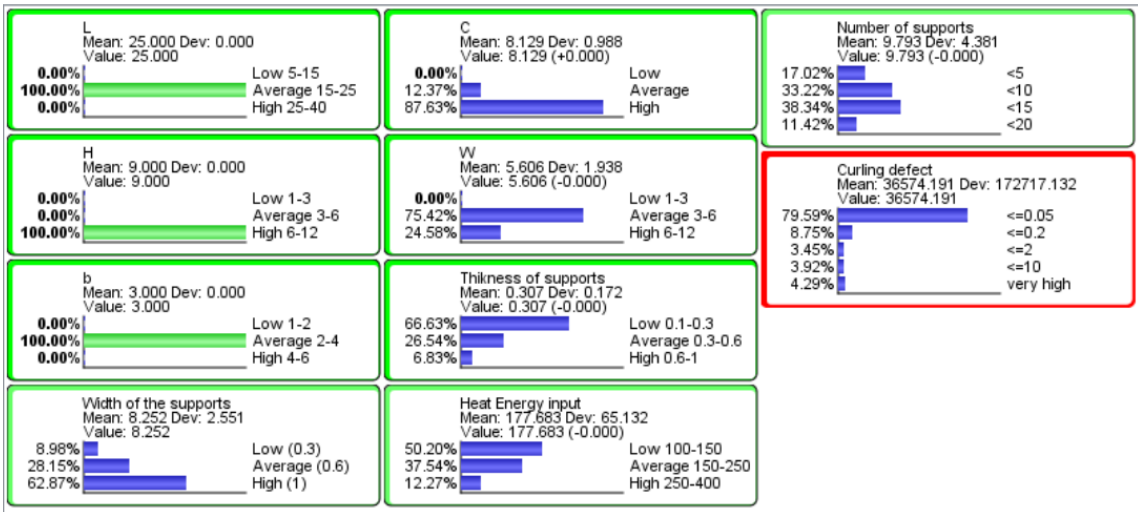

Figure 40. *The monitors for the model with the initial parameter design configuration*

The model suggests that the probability of having less than 0.05mm curling defect with all combinations for all the other variables is 79.59%. This interval is the most desirable interval for the amount of curling defect. There is also an 8.2% chance that the curling defect is more than 2mm.

Now assume that the designer wants to see the effect of choosing design parameters for the supports and check if their effect on the amount of defect. Since the length of the part is in the average range, having a high number of supports and having them with maximum width may help to reduce the defect. Also, since the process of removing supports is costly and time-consuming, the designer prefers to have the support structurs with a small thickness so that they can be removed easily. The resulting probability distribution for the curling defect variable is shown in Figure 41.

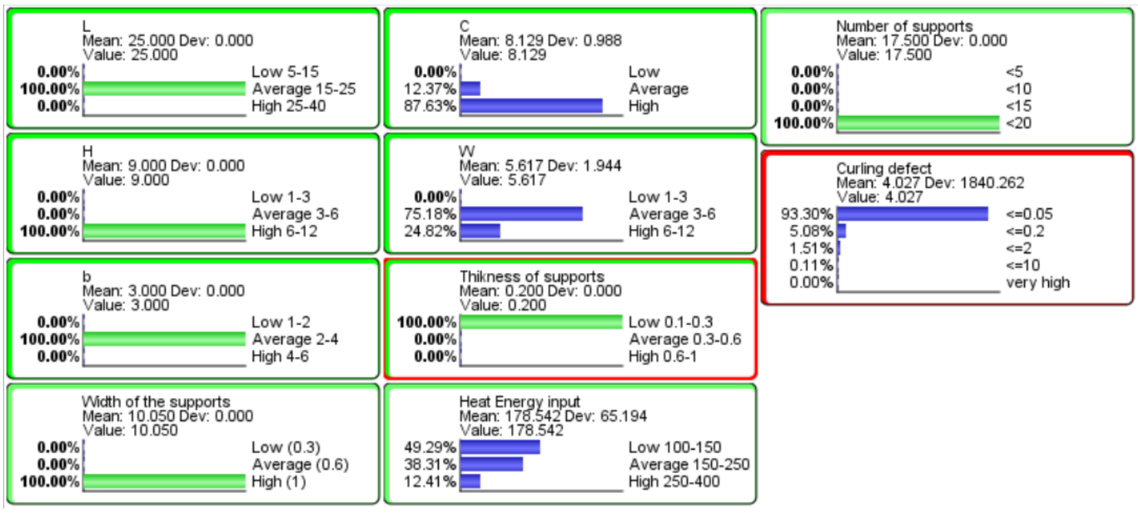

Figure 41. *The monitors for design variables after choosing more design variables*

The posterior distribution is shown in Figure 41. The probability distribution of the curling defect shows that the mean value has changed from 36574.191 to 4.027. This means that all combinations for all the other design variables will lead to values for curling defect that are around 4.027 on average. Moreover, the chance for having more than 2mm defect is almost zero. The designer can continue with the design process, but they can be sure that the amount of defect won't be in very high values.

5.1.3 Design Space Exploration

One of the benefits of extending the causal graph obtained by DACM is that it provides the possibility of exploring in the design space efficiently. Design space exploration (DSE) is the process of exploring the design variables' space to discover sets of suitable combinations of designs alternatives (Sharpe, Morris, Goldsberry, Seepersad, & Haberman, 2017). Design space exploration is the process of discovering and evaluating valid design alternatives before implementing (Kang, Jackson, & Schulte, 2011).

Often the simulation models are complicated and computationally expensive. Therefore, some surrogate modelling is used to map the complicated model into a model which is simpler and is accurate enough. Several methods are developed to perform the mapping including set-based methods (Jawad Qureshi, Dantan, Bruyère, & Bigot, 2014), interval-based methods (Panchal, Fernández, Paredis, Allen, & Mistree, 2007), graph and grid-based methods (Schulz et al., 2017), space mapping methods (J. W. Bandler, Cheng, Hailu, & Nikolova, 2004).

The method introduced and implemented in this study can be classified as a space mapping method. For example, Bandler et al. (2013; 2004) developed the Aggressive Space

Mapping (ASM) method in which they mapped accurate and complicated models into a set of coarse models which are accurate enough but fast to evaluate. The process of ASM can be summarized as (J. Bandler, 2013):

1- Preliminaries: Creating a library of fast, parameterized course models and developing inexpensive means to evaluate them.

2 & 3- In a specific problem: choose a suitable "course" model for the system and extract the parameters. The course model is expected to be capable of meeting the system specifications, both in inputs and outputs. The relationship between these two models can be represented as:

$$x_c = P(x_f) \tag{84}$$

In which x_c and x_f are respectively the vectors representing the course model and the fine model. The function P is expected to be a linear mapping between these two models if they are a good match. Then in the course model is being optimized with a conventional method which results in the solution x_c^*. Finally, the parameters of the fine model can be calculated using the inverse:

$$x_f^* = P^{-1}(x_c^*) \tag{85}$$

Assign the optimized parameters to the fine model and run it. If the specifications are met, you have a sufficiently accurate course model.

4- Further iterations: use the real data from the situation or generated data from the fine model to update the course model with a mapping. This step is called parameter extraction. Then the steps on step 3 can be repeated to exceed the optimization specifications or to some fixed number of iterations.

Similar methods have been developed using Bayesian networks to empower designers. Shahan and Seepersad (2009) developed a method using Bayesian networks for collaborative design problems in distributed design projects. In their method, each designer develops a small Bayesian network that represents the regions of interests in their design space. Then these Bayesian networks are combined to form a global network which shows the interest of each designer. Sharpe et al. (2017) developed Kernel-based Bayesian network classifiers in which they used a Genetic Algorithm method to learn a Bayesian network structure and parameters from a small set of data and then used the BN to explore the design space. Conti and Kaijima (2017) developed a Bayesian network meta-model to enable bidirectional inference in a design analysis system. They used machine learning to learn network structure and parameters from a set of simulated data and then used that to simulate the result of choosing specific design parameters on the outputs or find the most probable parameters to have a specific value in the output. Later on, they developed a method for developing meta-models which are not limited to Bayesian networks and implement it on a case study using Bayesian networks (Conti & Kaijima, 2018). Another example of Bayesian network structure learning algorithms for supporting early stage design support can be found in Matthews' (2007) work.

In this study, the mapping is from the space of interactions between continues variables through accurate mathematical equations into a space of probabilistic interactions between discretized values with a limited range. The benefit of this mapping is that not only the mapped model is easy to evaluate; it is enriched with experts' knowledge.

As an example, the process can be formed as defining a target tolerance for the defect and then trying to find the best combination for the other variables to have minimal material loss. Assuming that defect less than 0.2mm is acceptable, we can set hard evidence for the first two intervals of the "Curling defect" in its monitor. Then the posterior distribution for the other variables is calculated by the software, as shown in Figure 42.

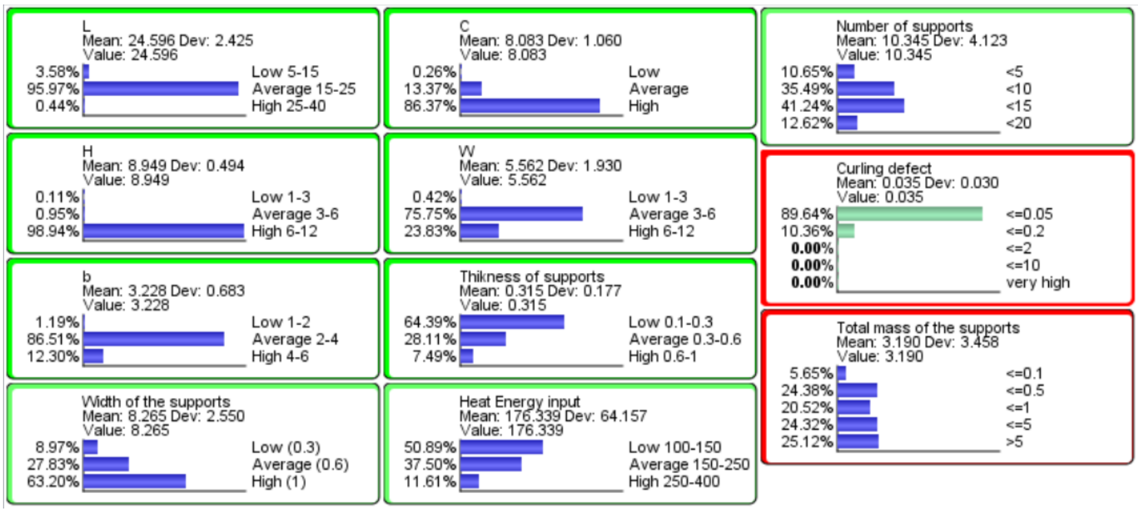

Figure 42. *Posterior distributions after setting a target value of having less than 0.2mm of defect*

The designer then can start exploring this design space to reach the efficient values for all the variables in systematic design space exploration (DSE) method. DOE methods such as Taguchi (Mistree, Lautenschlager, Erikstad, & Allen, 1993) method or Bayesian methods (Nabifar, 2012) can be used to explore the design space of this model. Further discussion of these methods is out of the scope of this study.

5.2 Reliability engineering case study

A model for describing the interrelation between the consecutive TTF values and CF values is presented in section 4.2.3. The initial hypothesis of this study was that although near perfect maintenance is taking place after each failure in the system, consecutive failure times are not independent of each other. This shows that the maintenance procedures are not perfect, or the conditions of the working environment and usage pattern is affecting the failure times. Consecutive failure times and censored failure times are separated into a set of variables and dependency between them is investigated through using a structural learning algorithm. The resulting model of dependencies is shown in Figure 43.

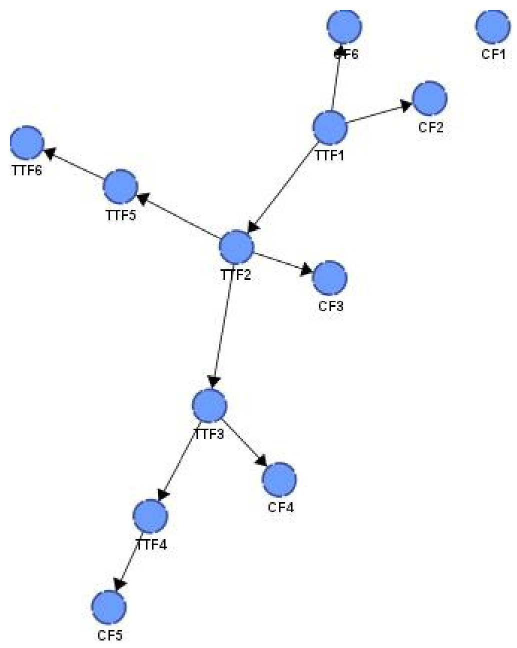

Figure 43. *The BN showing the dependencies between TTF and CF variables*

Table 20 shows some association and independence metrics of the nodes in the model. The mutual information (MI) is an asymmetric measure that shows having some information about one node can help finding out some information about the other node. The minimum amount of MI can be as low as zero, and the maximum of it can be equal to the entropy of the parent node. Pearson correlation is determining the strength of any possible linear relationship between two nodes, which is asymmetrical again. Pearson correlation has a range between 1 and -1. Positive values represent direct linear relation, and negative value show reverse linear relation. A description of these measures and the formula for calculating them is presented in section 3.1.2.

Table 20. Association metrics between nodes in the model

Relationship Analysis			
Parent	**Child**	**Mutual information**	**Pearson's Correlation**
TTF2	**CF3**	0.7918	-0.7356
TTF1	**TTF2**	0.7344	-0.0727
TTF2	**TTF3**	0.5962	-0.5855
TTF3	**CF4**	0.5387	-0.1319
TTF1	**CF2**	0.482	-0.4263
TTF3	**TTF4**	0.2404	-0.0989
TTF2	**TTF5**	0.1524	0.2435
TTF1	**CF6**	0.1286	0.5135
TTF4	**CF5**	0.0788	-0.2364
TTF5	**TTF6**	0.0258	0.2269

As shown in Table 20, the strength of the relation for TTF1−>TTF2, and TTF2−>CF3 are the strongest with the almost similar MI value of 0.7344 and 0.7918. The Pearson's correlation for TTF2−>CF3 is high as well, but for TTF1−>TTF2, the value is close to zero. This might be because the relationship between these two variables is highly non-linear. The next strong relations are TTF2−>TTF3, TTF3−>CF4, TTF1−>CF2 and TTF3−>TTF4. Since the number of data points for the variables TTF5, TTF6, CF5, and CF6 are very scarce, the dependencies found by the algorithm between them and the other parameters are quite weak. This fact is shown in the MI value between these variables and the other nodes in Table 20.

The relation between the TTFs, and between TTFs and CFs are not in a manner to find a trend or a general rule or equation for them, but the model can help to estimate the most probable next time to failure and estimate a distribution for the non-event period based on the history of the failures and working hours of an equipment.

For example, imagine that a pump experience two failure in up to now. The first failure (TTF1) has occurred between 89 to 326 hours of work of the pump and the second failure (TTF2) has occurred between 407 and 509 hours after the first failure.

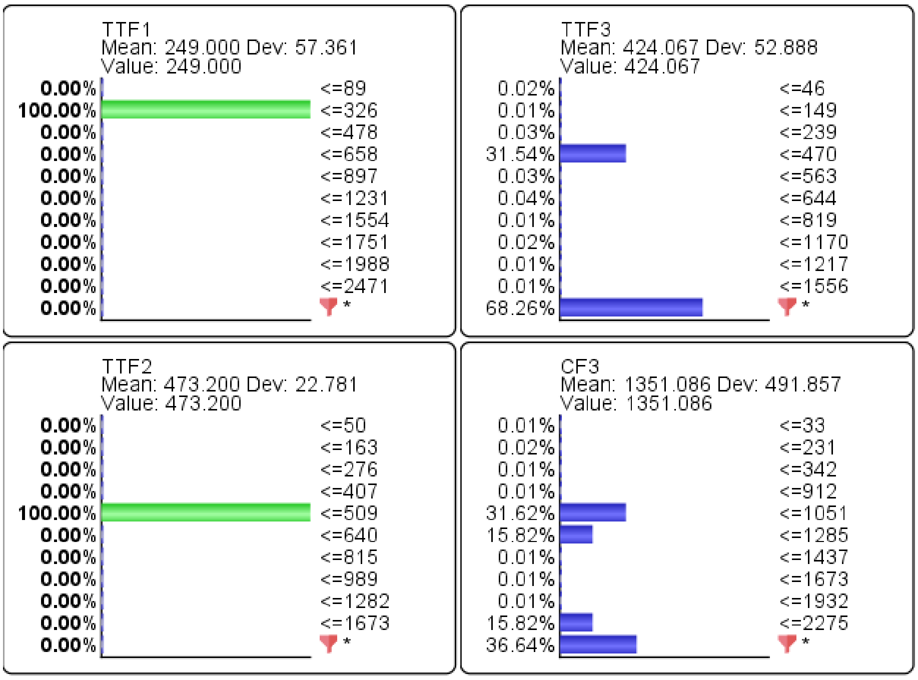

Figure 44. *Monitor windows after setting evidence for the failures*

As shown in Figure 44, after setting the evidence for the variables TTF1 and TTF2, the posterior distribution for the variables TTF3 and CF3 is calculated by the Bayesialab software. The monitors show that having this history from the pump and based on the model, the most probable interval that the next failure may occur in is the interval between 239 and 470 hours. The model suggests that based on the historical data, 31.54% of the failures have happened in this interval. The average value calculated for the TTF3 variable is about 424 hours with a standard deviation of 52.88 hours. This can provide a more accurate measure for the most probable value for TTF3. The model also shows that 60% of the data points in TTF3 for such an arrangement in TTF2 and TTF1 are missing.

The monitor for the censored TTF3 (CF3) variable is also shown in Figure 44. The intervals can give some insight about the probability to have no event in each interval. Also, the mean value of no event hours and its standard deviation is shown on the top of the monitor.

5.3 Conclusion

This study aimed to investigate the possible uses of Bayesian networks in industrial domains. Two approaches for making Bayesian network models have been studied and used in two case studies.

The first research question of the study is answered by developing a method to create Bayesian networks for complex systems. The method is developed by combining and extending a systems' engineering methodology framework, DACM, and a multicriteria decision making method, AHP. The method completed by designating the constraints of the system into the Bayesian network model. Based on this method, a case study for modelling the process behind the curling defect in powder bed fusion systems is developed. The steps for creating the model is shown and the uses of the model have been discussed. It is shown that the resulting model can be used for knowledge representation, diagnosis and prognosis, and design space exploration.

The possibility for using machine learning algorithms for obtaining Bayesian network models is also studied. The second research question of the study is answered by developing a failure prediction Bayesian network model for a single variable dataset. The corrective maintenance after a failure are assumed to be a perfect maintenance. This study attempts to relax this assumption and investigate the relation between consecutive failure times and create a predictive model. The other challenge of this case study was the limited amount of data, missingness of the datapoint and an extensive amount of censored values. All these challenges have been addressed using Bayesian network specific approaches. The resulting model can be used to predict the next time to failure values.

Limitation and suggestions

The method developed for modelling complex systems can be extended by augmenting the graph theory with concepts of ideality and contradiction from TRIZ theory. TRIZ is a problem-solving, analysis and forecasting theory developed by the Russian scientist Genrich Altshuller and his colleagues (Savransky, 2001). Ideality looks to the world without assuming any limitations and create models for this ideal system. Contradiction on the other hand, detects the limitations and flaws of the system and brings the model to a more realistic state.

One of the limitations of this study is that the models are not verified. Both models can be verified against and confirmed using data. In the AM study, experimental data of parts with curling defect could be used, but it has not happened due to lack of resources. Similarly, in the reliability study more data for the failure times was not available.

The model can also get updated with the experimental data. Having the model as the prior and updating it with the experimental data using, for example, Maximum a posteriori method, a posterior model can be obtained. This new model is closer to the real world process and more reliable.

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APPENDIX A: SAMPLE DESCRIPTION FOR HOW TO FILL THE AHP TABLES

This study aims at finding a combination of variables to minimize the ‘curling’ defect and ‘defect in the parts with overhanging surfaces, at the early design stages. The design space for finding the best combination of variables is exponentially growing in dimension as the number of variables increase. Therefore, it is necessary to find a way to reduce the complexity of this task. One way to cope with this problem is to use the experts’ knowledge to find out the suitable values for variables and focus on finding the combination of variables in the design space. We use the Analytical Hierarchy Process (AHP) as the method of knowledge elicitation (Shadbolt & Smart, 2015) in the form of preference of choosing an interval of values for a variable. Then, Bayesian networks can be used to relate these probabilities to each other and help us to find the best combination.

The ranges of the variables are divided into three or four intervals to create the AHP tables. The experts are supposed to compare intervals pairwise and express their preference with a numerical value. Table 21 is an example of the AHP table created for this study. In an AHP table, if an interval in the row is preferred over an interval in the column, a value in the range of one (1) to nine (9) can be chosen to show this preference. The highest preference is shown by nine (9) and the lowest preference (neutral) is shown by one (1). Eventually, experts can select any natural number from one (1) to nine (9). Conversely, if an interval in the row is less preferable than an interval in the column, a value between one (1) and (1/9) can be chosen to show this. One (1) is showing the least disfavor and 1/9 shows the highest disfavour.

The arrows in Table 21 show how one should read the table. It is enough to fill the upper triangle of the matrix and the lower triangle of the matrix is calculated by reversing the values in the upper triangle.

Table 21. AHP table for scan velocity

Scan velocity		**Low**	**Average**	**High**	**weights**
200-500 mm/s	Low	1.0000			
500-900 mm/s	Average		1.0000		
900-1300 mm/s	High			1.0000	
		MEV=			
		C.I.=			

To provide an example of asking questions to fill the Table 21, one can ask a question in this form:

“In an L-PBF process for making a part with the shape A, what is your preference to choose low Scan Velocity over an average scan velocity to obtain a good part?”

If the low value of Scan Velocity is more preferable in the process of manufacturing that specific part shape with L-PBF, a number in the range one (1) to nine (9) can be chosen, say four (4). This number is shown in green in Table 22.

Similarly, if there is a disfavor for the average value of scan velocity over high values of scan velocity, the expert may choose a number in the range of one (1) to 1/9, say 1/3 in this case. Table 22 shows the AHP table with the obtained values.

Table 22. AHP Table with values

Scan velocity		**Low**	**Average**	**High**	**weights**
200-500 mm/s	**Low**	1.0000	4.0000	2.0000	
500-900 mm/s	**Average**		1.0000	1/3.000	
900-1300 mm/s	**High**			1.0000	
		MEV=			
		C.I.=			

The attached AHP tables are designed to collect knowledge about preferable intervals for both manufacturing and design parameters. The tables require to be filled in a way that they represent the best practice in choosing parameters for manufacturing a high-quality part

APPENDIX B: AHP TABLES FOR CURLING DEFECT CASE STUDY

Length of the base	High	Average	Low	weights
High (7-10)	1.0000			0.5816
Average (4-7)		1.0000		0.3090
Low (1-4)			1.0000	0.1095
	MEV=			3.0037
	C.I.=			0.0018

Length of the straight part	High	Average	Low	Weights
High (25-40)	1.0000	0.3333	2.0000	0.2222
Average (15-25)		1.0000	6.0000	0.6667
Low (5-15)			1.0000	0.1111
	MEV=			3.0000
	C.I.=			0.0000

Height of the part	High	Average	Low	weights
High (1-3)	1.0000	3.0000	6.0000	0.6548
Average (3-6)		1.0000	3.0000	0.2499
Low (6-12)			1.0000	0.0953
	MEV=			3.0183
	C.I.=			0.0091

Width of the part	High	Average	Low	weights
High (1-3)	1.0000	0.2500	0.5000	0.1365
Average (3-6)		1.0000	3.0000	0.6250
Low (6-12)			1.0000	0.2385
	MEV=			3.0183
	C.I.=			0.0091

Thickness of the straight part	High	Average	Low	weights
High (1-2)	1.0000	0.3333	0.5000	0.1466
Average (2-4)		1.0000	5.0000	0.6571
Low (4-6)			1.0000	0.1963
	MEV=			3.1632
	C.I.=			0.0816

Thickness of supports	High	Average	Low	weights
High (0.1-0.3)	1	0.25	0.1429	0.0786
Average (0.3-0.6)		1	0.3333	0.2628
Low (0.6-1)			1	0.6586
	MEV=			3.0324
	C.I.=			0.0162

APPENDIX B: AHP TABLES FOR CURLING DEFECT CASE STUDY

Width of the part	Low			
Width of the support	**Low 0.3*W**	**Average 0.6*W**	**High 1*W**	**weights**
Low 0.3*W	1.0000	4.0000	5.0000	0.6738
Average 0.6*W		1.0000	3.0000	0.2255
High 1*W			1.0000	0.1007
	MEV=			3.0858
	C.I.=			0.0429
Width of the part	**Average**			
Width of the support	**Low 0.3*W**	**Average 0.6*W**	**High 1*W**	**weights**
Low 0.3*W	1.0000	0.2500	0.2000	0.0936
Average 0.6*W		1.0000	0.3333	0.2797
High 1*W			1.0000	0.6267
	MEV=			3.0858
	C.I.=			0.0429
Width of the part	**High**			
Width of the support	**Low 0.3*W**	**Average 0.6*W**	**High 1*W**	**weights**
Low 0.3*W	1.0000	0.2000	0.1667	0.0780
Average 0.6*W		1.0000	0.3333	0.2872
High 1*W			1.0000	0.6348
	MEV=			3.0940
	C.I.=			0.0470

Mass of the part	Low			
Heat Energy Input	**Low (100-150)**	**Average (150-250)**	**High (250-400)**	**weights**
Low (100-150)	1.0000	6.0000	9.0000	0.7557
Average (150-250)		1.0000	5.0000	0.1881
High (250-400)			1.0000	0.0562
	MEV=			3.1632
	C.I.=			0.0816
Mass of the part	**Average**			
Heat Energy Input	**Low (100-150)**	**Average (150-250)**	**High (250-400)**	**weights**
Low (100-150)	1.0000	3.0000	5.0000	0.6370
Average (150-250)		1.0000	3.0000	0.2583
High (250-400)			1.0000	0.1047
	MEV=			3.0385
	C.I.=			0.0193

Mass of the part	**High**			
Heat Energy Input	**Low (100-150)**	**Average (150-250)**	**High (250-400)**	**weights**
Low (100-150)	1.0000	0.3333	2.0000	0.2493
Average (150-250)		1.0000	3.0000	0.5936
High (250-400)			1.0000	0.1571
	MEV=			3.0536
	C.I.=			0.0268

Length of the straight part	**Low**				
Number of supports	**>5**	**>10**	**>15**	**>20**	**weights**
>5	1.0000	2.0000	6.0000	9.0000	0.5232
>10		1.0000	5.0000	8.0000	0.3427
>15			1.0000	3.0000	0.0916
>20				1.0000	0.0425
	MEV=				4.0925
	C.I.=				0.0308
Length of the straight part	**Average**				
Number of supports	**>5**	**>10**	**>15**	**>20**	**weights**
>5	1.0000	0.3333	0.5000	2.0000	0.1682
>10		1.0000	0.5000	3.0000	0.3284
>15			1.0000	2.0000	0.3835
>20				1.0000	0.1198
	MEV=				4.2153
	C.I.=				0.0718
Length of the straight part	**High**				
Number of supports	>5	>10	>15	>20	weights
>5	1.0000	0.1667	0.1250	0.1111	0.0359
>10		1.0000	0.2500	0.2000	0.1215
>15			1.0000	0.3333	0.2918
>20				1.0000	0.5507
	MEV=				4.2807
	C.I.=				0.0936

APPENDIX D: SOFTWARE PACKAGES FOR BAYESIAN NETWORKS

Software packages and online services for learning and inference in BNs								
Name	Description	availability	API	Learn-ing	Infer-ence	License	Link	Price
Bayes Server	Commercial low-price tool with good features	Offline	Direct: .NET and JAVA Indirect: Python, R, MATLAB, Excel functions, Apache Spark: through .NET and JAVA API	yes	Yes	commercial	1	Academic: 283€ - 568€ Commercial: 568€-1137€
BayesPy	Python API from AALTO University	Offline	Python	No	Yes	MIT License	2	Free
WEKA	Developed in University of Waikato New Zeeland	Offline	Java	Yes	Yes	-	3	Free
Bayes Fusion	Developed in University of Pittsburgh for BNs and influ-ence diagrams	Offline	C++ wrappers for Java and .NET are available	Yes	Yes	Commercial, Free for academic use	4	Academic: free Commercial: contact
Paul Govan's BN	An R library for Bayesian networks	Offline/ Online	R	Yes	Yes	Apache 2.0 License	5 & 6	Free
MSBNx	Microsoft Bayesian Networks platform	Offline	COM-based API through Ac-tiveX	No	Yes	-	7	-
Bayesialab	Commercial software with good tools	Online/ Of-fline	Java – can export to Java, .NET, JS, Python, R, MATLAB	Yes	Yes	Commercial	8	Academic use 490€, extensions have sep-arate prices
Agena risk	Commercial tool for risk anal-ysis and decision support	Offline	Java	Yes	No	Commercial and Free	9	Free limited version full version upon sub-scription

1- https://www.bayesserver.com/

2- http://www.bayespy.org/index.html

3- https://www.cs.waikato.ac.nz/ml/index.html

4- https://www.bayesfusion.com/

5- https://github.com/paulgovan/BayesianNetwork

6- https://paulgovan.shinyapps.io/BayesianNetwork/

7- https://msbnx.azurewebsites.net/

8- http://www.bayesialab.com/

9- http://www.agenarisk.com/