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SAIFUL ISLAM

Applicability and Advantages of Implementation of MIMO Techniques in Radar Systems

Master of Science Thesis

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Examiners and Topic are approved by the faculty council meeting of the faculty of Electrical and Computer Engineering on March 06, 2013

Abstract

Tampere University of Technology
Master's Degree Program in Electrical Engineering
SAIFUL ISLAM: **Applicability and Advantages of Implementation of MIMO Techniques in Radar Systems**
Master of Science Thesis, 52 Pages
July 2014
Major: Wireless Communications Circuits and Systems
Examiner(s): Professor Mikko Valkama, M.Sc. Markus Allén
Keywords: Angular diversity, Beamforming, Spatial diversity, Statistical MIMO radar.

High-accuracy object detection using radio frequency signal has become popular field for research since last couple of years. Huge amount of research work are being done in this field now a days. Although radar systems were invented for the purpose of military, they are also used for civil service at present.

MIMO communication systems becomes popular in recent years because of higher capacity, increased coverage and better voice and data quality in telecommunication systems. The overwhelming popularity of MIMO systems draws radar researchers' attention to study the probability of implementing MIMO techniques in radar systems. This trend has been followed in this thesis. The applicability of MIMO in radar systems has been examined along with small simulations outcomes, which ends with analysis of the result and further research probability in this field. Any type of diversity is required for MIMO radar. Some of the probable diversity techniques are discussed with a signal model along with their advantages and disadvantages.

This thesis starts with a brief discussion about radar principle and different types of radar systems, followed by detailed discussion on MIMO technology and their implementation on radar systems. Angular diversity i.e. beamforming is considered, in the simulation part of the thesis, to implement MIMO. Ideal propagation environment is assumed in the simulations in order to keep the focus on the beamforming mechanism itself. Approximately 10 dB signal-to-noise ratio gain is obtained in the simulations using reasonably low number of antennas. The thesis ends up with short discussion on the advantages of MIMO application in radar along with future research possibilities in this arena.

Preface

First of all, I would like to thank my creator, the almighty ALLAH, to give me the opportunity to pursue my higher study in such a modern university, Tampere University of Technology. I have passed very nice and charming time here. I would like to express my gratitude to my thesis supervisor, Professor Mikko Valkama, Department of Electronics and Communications Engineering, Tampere University of Technology, for giving me to do research work on such a nice and interesting topic. Special thanks goes to Markus Allén, for his kind guidance as my thesis guide. I am thankful to Markus for his patience and giving valuable time to me. I appreciate Mr George Vallant, from Cassidian, for providing valuable resources. I would like to thank the Bangladeshi community in Tampere, especially M M Mahbubul Syeed, Zahidul Bhuiyan, Saad, Rubel, Tareq, Khyrul Kabir, Ricky, and Sagor. Thanks to my course mate Pradeep Ganesh for the nice moments that we enjoyed while working together. Gratitude to my parents, M N Nabi and Eanor Begum, and my brothers Shaidul Islam and Moinul Islam for their support.

Finally, I would like to express my love to my lovely wife, Sabrina Akter. You have passed such a painful time, at home, without me. Thanks for your patience and continuous support.

Saiful Islam
July 2014

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Acronyms

3G	3^{rd} Generation
APES	Amplitude and Phase EStimation
AWGN	Additive White Gaussian Noise
CDM	Code Division Multiplexing
CSI	Channel State Information
CW-FM	Continuous Wave – Frequency Modulated
DoA	Direction of Arrival
DoD	Direction of Departure
FDM	Frequency Division Multiplexing
GPR	Ground Penetrating Radar
HSPA	High Speed Packet Access
ISAR	Inverse Synthetic Aperture Radar
LFM	Linear Frequency Modulated
LTE/LTE-A	Long Term Evolution/ Long Term Evolution – Advanced
MIMO	Multiple Input Multiple Output
MTI	Moving Target Indicator
RCS	Radar Cross Section
RFID	Radio Frequency IDentification
SAR	Synthetic Aperture Radar
SDM	Space Division Multiplexing
SLAR	Side Looking Airborne Radar
SNR	Signal to Noise Ratio
STAP	Space Time Adaptive Processing
TDM	Time Division Multiplexing
WLAN	Wireless Local Area Network

1 Introduction

Multiple Input – Multiple Output (MIMO) is a widely used technology in the field of wireless communications. MIMO offers higher speed of data transmission, better quality of throughput, more reliable transmission, enhanced capacity and increased coverage without requiring any additional bandwidth or increased transmit power. It fulfills its target by spreading the same total transmit power over the antennas to achieve an array gain, which improves the spectral efficiency, or to achieve a diversity gain that reduces fading. All these are already proved in communication engineering aspects. Due to huge success of implementation of MIMO in communication research, it has drawn attention to radar system engineers. Radar is a one way radio detection system. On the other hand, communication system is a two way system. As a result, there are some differences while implementing MIMO in Radar systems.

Radar is electronic object detection device that uses radio waves to detect different types of objects e.g. aircrafts, ships, spacecraft, motor vehicles, guided missiles etc. Moreover, radar is useful for estimating range, altitude, direction or speed of objects also. According to the radar principle [38], the transmitter radiates an energy signal to the environment in a certain direction or omnidirectionally. The radiated energy is captured by the object. Some of the energy is absorbed by the object, some portion is refracted and a small portion of the radiated signal is reflected back by the object. The reflected energy is captured by the receiver and the further processing is accomplished i.e. estimating the parameters e.g. range, velocity, direction etc.; in the receiver block [37]. There are two types of signals that are used for radars: pulse signal and continuous wave signal. The pulse signal requires directional radiation i.e. directional antenna is used in pulse type radars. For omnidirectional radiation, continuous wave signal is used. The radio frequency energy transmitted by pulse-modulated radars consists of a series of equally spaced pulses, frequently having durations of about 1 microsecond or less [3], separated by relatively long periods, compared to the transmitted pulse, during which no energy is transmitted.

From the radar point of view, the transmission of signals from transmitter to target, and reflecting back to the receiver are considered as propagation of signals through communication channels. Target position, velocity and other target features determine the channel characteristics which are mathematically described by a channel matrix [15]. The target of implementing MIMO in Radar systems is to reduce the matrix estimation complexity as well as enhancing the detection performance. Adaptation of the principles of communication theory in radar is fruitful as it is possible to use certain results from communication theory and practice. On the contrary, MIMO radar has some drawback as well. The noncoherent combination of orthogonal signal causes significant SNR loss. In [1], it has been shown that the SNR in MIMO can be even 10 dB lower than its phased array counterpart. Another limitation of MIMO radar is the effective use of range-Doppler space. MIMO radar can enlighten only $1/N$ portion of the whole space whereas, phased array radar can focus the whole range Doppler space [1]. Another drawback of MIMO radar is its increased design complexity.

There are several types of diversity that can be used for MIMO radar. Considering the spatial diversity point of view, MIMO radars can be divided into two types [39]; a) Collocated MIMO radar and b) Distributed MIMO radar. In collocated MIMO radar, both the transmitter and receiver are located in the same spatial position. One single antenna is used for both transmission and reception. Temporal switching i.e. Time Division Duplexing is used for switching. As both the transmitter and receiver are located in the same location, it is easier to maintain coherence between them. Actually, Coherent MIMO radar is an upgraded version of phased array radar. In phased array radar, same signal having different phase shifts is used. On the contrary, Coherent MIMO radar employs multiple different signals. In the case of Distributed MIMO radar [40], the transmitters and receivers are located in different geographical locations. Orthogonal signals are transmitted from all the transmitters simultaneously. The receivers also intercept all the signals and all the received signals are processed in a joint processing center. It is difficult to maintain synchronization in distributed MIMO because of the antennas being in different location. Sometimes, it is known as statistical (or Non-coherent) MIMO radar. Distributed MIMO radar can overcome the radar cross section scintillation of targets by correlation processing [15]. Multipath diversity of MIMO communication is introduced by statistical MIMO, into the design of radar. Also multistatic radar systems are benefitted from angular spread if time and phase synchronization is maintained among them during operation.

Angular diversity is also used in MIMO radar. This type of diversity is used in collocated MIMO radar. Beamforming technique is used for implementing angular diversity. In this technique, a narrow electromagnetic beam having highly specific directivity is produced. The detection accuracy and resolution performance depends on the beam angle. If the angle is wide enough, it can detect more objects. On the contrary, resolution performance will be better if the beam angle is narrow [41]. There is another type of diversity which is known as polarization diversity. Signals having same frequency and phase but different polarization are used for polarization diversity [31], [32]. The research outcome of polarimetric MIMO is not yet mature enough.

Diversity enables enhancement of the MIMO radar performances multiple fundamental aspects [39] e.g. 1) Significant improvement in parameter identifiability, 2) Target detection and parameter estimation by applying adaptive arrays directly, and 3) Diversity offers much enhanced flexibility for transmit beampattern design. It has been proved that the target identifiability of target in MIMO radar is increased upto M_t times than that of its phased array counterpart. Here, M_t is the number of transmit antennas. Moreover, the probing signals transmitted via its antennas can be optimized to obtain several transmit beampattern design with superior performance. In general, significant number of advantages can be obtained by implementing MIMO in radar system. Some of them are [15], [16], [39], a) Increment of total power and sensitivity of the whole systems, b)High accuracy of position estimation of a target, c) Increased resolution capability, d) Intersection of the main beams is less than a monostatic system resulting reduction in the power returning from the clutter, and e) Resistance to jamming and increased survivability etc.

The application of Radar is extremely wide at present. Radar was invented to serve the purpose of military. Now-a-days, they are used in marine communication, air traffic controlling, ballistic missile guidance, surveillance, weather forecasting, earth sciences etc.

2 Radar Basics

The elaboration of the term ‘RADAR’ is Radio Detection and Ranging. Radar is a device that employs electromagnetic sensors for detection and location of reflecting objects. It is actually an object detection that uses electromagnetic waves to determine several parameters e.g. range, altitude, direction, speed of the object etc. The usability of modern radars is highly diverse. At the beginning of radar, it was used only for detecting aircraft, enemy ships, spacecraft, guided missiles, weather forecasting etc. But now a days, the application area of radars has been increased to air traffic control, radar astronomy, antimissile systems, air defense, ocean surveillance systems, outer space surveillance, meteorological precipitation monitoring, flight and altimetry control systems, mining and many more.

2.1 Principle of Operation

The operating principle of radar system is based on the reflection principle. According the reflection principle, a target, exploring around the environment, can be detected by using a radio wave. A radio wave is transmitted to the atmosphere. If there is any object in the environment, it will absorb a portion of the signal energy, another portion will be refracted (or passed through) the object body. The remaining portion will be reflected by the object towards the transmitter (source). The receiver can be located along with the transmitter or separated from the transmitter. The receiver receives the reflected signal and after further processing, the necessary parameters of the object are determined. Some important points, which are useful for understanding of radar detection, are given below [38]:

(a) The reflected signal can never have the same intensity as the original has.

(b) The scope of determination of reflecting signals depends on the signal strength and pulse duration of the original signal.
(c) If the target is close the transmitter, short pulse duration will be useful.
(d) A sufficiently long interval between each pulse is required for distant targets.

A block diagram of simple radar is given in figure 2.1.

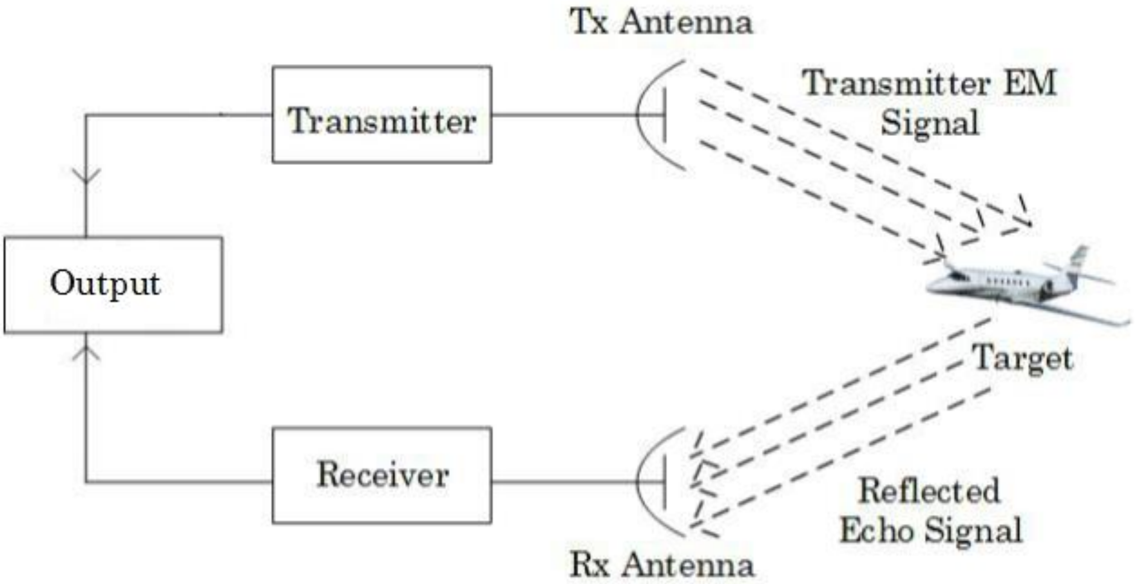


Figure 2.1: Block diagram of a Basic Radar
[Note: The image of the airplane used here with permission from Cessna Inc.]

2.2 Radar Equation

Let us consider an isotropic antenna, transmitting an electromagnetic signal having power P_t. The range of the radar antenna is denoted by R. So, the power density at the distance R will be

$$power\ density = P_t \frac{1}{4\pi R^2} \tag{2.1}$$

In all practical cases, the radiating element can't be isotropic i.e. the antenna will be directive. Now, having the directive gain, G, the power density of the signal at distance R will be

$$power\ density = P_t \frac{G}{4\pi R^2} \tag{2.2}$$

The targets scattering properties are characterized by its Radar Cross Section (RCS), denoted by the symbol σ. It is not necessary for the RCS to be the same as the physical cross section. Therefore, the power intercepted by the target is

$$power\ intercepted\ by\ the\ target = P_t \frac{G\sigma}{4\pi R^2} \tag{2.3}$$

So, the echo signal power reaches that reaches the receiver is

$$power\ density\ at\ the\ receiver = P_t \frac{G\sigma}{4\pi R^2} \frac{1}{4\pi R^2} \tag{2.4}$$

The effective aperture of the receiving antenna is A_e. The received signal power will be [3]

$$P_r = P_t \frac{G\sigma}{4\pi R^2} \frac{1}{4\pi R^2} A_e \tag{2.5}$$

But the effective aperture of an antenna is dependent on the gain, G. The relationship between effective aperture and gain is, $G = \frac{4\pi A_e}{\lambda^2}$. So, expression for the received power will be [3]

$$P_r = P_t \frac{G\sigma}{4\pi R^2} \frac{1}{4\pi R^2} \frac{G\lambda^2}{4\pi} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \tag{2.6}$$

From equation (2.6), we can determine the range of a radar system. The range of radar is the maximum distance from where it can detect a target. In the maximum distance, the signal power will be the minimum that can be intercepted. So, from equation (2.6), the range will be

$$R_{max}^4 = \left(\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 P_{r,min}} \right) \tag{2.7}$$

If the target is located in the far field, the considerations of parameters will be slightly different. Suppose there is a flat, two dimensional target is located in the far field. It will cause refraction through the target and a mirrored reflection [9]. This reflection is in relation to the 'virtual' source from the same distance R behind the target. The whole situation is pictured in figure 2.2.

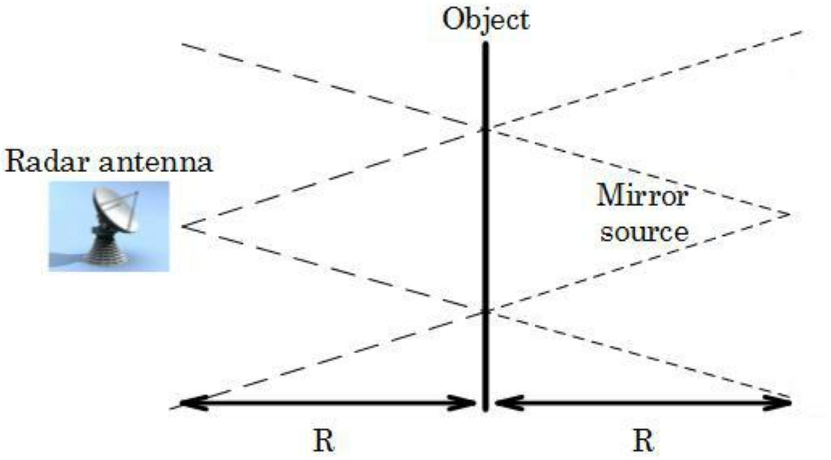


Figure 2.2: Reflection on a flat, far field target[9]

So, for a flat shaped - 2D target, the power density at the receiver will be

$$power\ density\ at\ the\ receiver = \frac{P_t G_t}{4\pi(2R)^2} \tag{2.8}$$

Here, G_t it the transmitter antenna gain. The signal power intercepted by the receiver is

$$P_r = \frac{P_t G_t G_r \lambda^2}{4(4\pi)^2 R^2} \tag{2.9}$$

In (2.9), G_r is the receiver antenna gain. The resulting expression for range is [9]

$$R_{max} = \sqrt{\frac{P_t G_t G_r \lambda^2}{P_{r,min} 4(4\pi)^2}} \tag{2.10}$$

In the above discussion, it is considered that both the transmitter and receiver are located at the same place i.e. the radar system is monostatic. These expressions will be different for bistatic cases. In bistatic radar systems, the transmitter and receiver are located in separate place instead of being located in same place.

There are several reasons behind using bistatic radar [9]. The first and foremost reason is to reduce the antenna coupling so that it can detect even the smallest signals. To protect the receiver from electrical interference can be another reason. The third reason can be keeping the secrecy of the receivers from enemy's visibility. Moreover, if there is more than one receiver, those will be able to detect signals from foreign transmitters. Finally, to make efficient use of single transmitter, multiple receivers are used.

Figure 2.3: Simple illustration of bistatic radar

The expression for bistatic radar range is almost similar as that of monostatic. The difference is in the parameter, R. Distance R is divided into two basic parameters, R_t and R_r, denoting the distance of the target from the transmitter and receiver respectively [9]. The expression for received power is

$$P_r = P_t \frac{G_t G_r \lambda^2}{(4\pi)^3 R_t^2 R_r^2} \sigma_B \tag{2.11}$$

Here, σ_B is the bistatic radar cross section of the target.

2.3 Information content of Radar signal

After receiving an echo signal, it is fed to the signal processor to extract the required target parameters. The extraction of parameters is dependent with the type of algorithm used in the signal processor. The following information content can be obtained from an echo signal, depending upon the principle that is put to use [3]

1. Distance or range, R
2. Changes in distance according to time i.e. velocity, $\frac{dR}{dt}$
3. Changes of azimuth over time, $\frac{d\psi}{dt}$
4. Changes in elevation over time, $\frac{d\vartheta}{dt}$
5. Target size
6. Shape of the target, can be expressed as changes of radar cross section over azimuth, $\frac{d\sigma}{d\psi}$

7. Target polarization
8. Signature or information about the target.

The most important and commonly evaluated parameters are the range, velocity and direction.

2.4 Types of Radar

According to the waveform used, radar can be classified into two basic types. They are

I. Pulse Radar, and
II. Continuous Wave Radar (CW Radar)

Both of the two categories have many sub-categories. Regarding the application or purpose, there are also several types of radars. For example [3],

A) High Resolution Radar
B) Surveillance Radar
C) Moving Target Indicator (MTI) Radar
D) Tracking Radar
E) Imaging Radar
F) Sidelooking Airborne Radar (SLAR)
G) Guidance Radar
H) Doppler Weather Radar
I) Multifunction Radar

In this section, only pulse radar, CW radars and their subclasses will be discussed.

2.4.1 Pulse Radar

Pulse radar is the basic form of most of the radar systems. It radiates a pulse of specific time period towards any specific direction. Once the transmitter sends the pulse train in a certain direction, it waits for a while to whether it may get any echo, of the transmitted pulse, reflected by the target. After a fixed time interval, it sends another pulse train [42].

2.4.2 Continuous Wave (CW) Radar

Instead of using pulse train, continuously propagating waves are used in CW radars [42]. The transmitting and receiving signal energy ratio can be up to 10-20 dB in this type of radars. Moreover, the small receiving signal becomes overlapped by the transmitting signal. CW radar can be of two types,

a) Unmodulated CW Radar, and
b) Modulated CW radar.

Modulated CW radar can also be divided into two types,

i) Sawtooth frequency modulated CW radar, and
ii) Sinusoidal frequency modulated CW radar.

CW radar is less expensive, has less complex architecture than its pulse Doppler counterpart. Moreover, they are typically small in size. However, unmodulated CW radars are unable to measure distance, but modulated CW radars can.

2.4.3 Synthetic Aperture Radar

There are two types of Radar images; a) Circularly Scanning Plan-Position Indicator (PPI), and b) Side looking images [22]. SAR or Synthetic Aperture Radar is a coherent, side looking radar systems that generates a high resolution image of Earth's surface. SAR is mainly used for remote sensing and navigation purposes.

The radiating element of SAR is mounted on a base, e.g. satellite or aircraft. The flight path will be parallel to the longitudinal axis. The direction of the emitted pulses is downward perpendicular to the flight path. The pulses fall in a small narrow area of the Earth's surface, in many directions. As a result, the returning echoes arrive to the SAR receiver in different times [22].

The working principle of SAR is almost similar as Phased Array Radar (discussed in section 4.1.1). The difference between them is the numbers of antenna elements are used. Phased array radar used more than one antenna element. On the other hand, SAR uses single antenna in time division multiplexing system. Any moving vehicle can be a platform, e.g. airplane or helicopter or ship, on which the SAR antenna will be mounted.

The SAR processor keeps all the returned signal stored in the database and after every fixed time period, all the returning signals are combined using signal processing techniques [29]. It is possible to see the terrain image through rain and clouds with synthetic aperture radar. SAR also has special capability to portray special geographical features e.g. Ice, Ocean waves, soil moisture, man-made objects etc. A stable full coherent transmitter along with efficient and powerful SAR processor and accurate information about the object flight path is necessary for obtaining a high resolution SAR image [30].

3 Introduction to MIMO

MIMO implies multiple-input and multiple-output in radio communication system. MIMO implements the use of multiple antennas at both the transmitting and receiving end to improve communication performance. It is one of several forms of smart antenna technology [36]. Now-a-days, MIMO technique has come limelight in communications research as it offers significant increment in data throughput and link range without requiring any additional bandwidth or increased transmit power. For the same reason, it has also drawn the focus of the radar researchers'. The implementation technique of MIMO in radar systems will be discussed in detail in chapter 4. MIMO fulfills its target by spreading the same total transmit power over the antennas to achieve an array gain that improves the spectral efficiency or to achieve a diversity gain that reduces fading. Because of these properties, MIMO is an important part of modern wireless communication standards e.g. LTE and LTE-A, HSPA+ etc.

3.1 Necessity of MIMO

Due to the increasing demand for high data rate and high link quality wireless access, future wireless application may create instability. As the spectrum is limited, it has become a scarce and expensive resource. For the limitation of transmit power due to the regulation, device and system capacity concerns, high quality link access with high data rate become a bottleneck of present communication system. For that purpose, diversity techniques both in transmitter and receiver has been introduced. Among them, space diversity has the superiority as both time and frequency domain processing has limits but space processing is literally unlimited in comparison with them [36].

3.2 Principle of Operation

Let us consider MIMO communication systems with *N* transmit antennas and *M* receive antennas. We can have a pictorial description of the system from figure 3.1. The signal that is transmitted from ith antenna is $s_i(t)$. The received signal in jth antenna of the receiving array is $y_j(t)$.

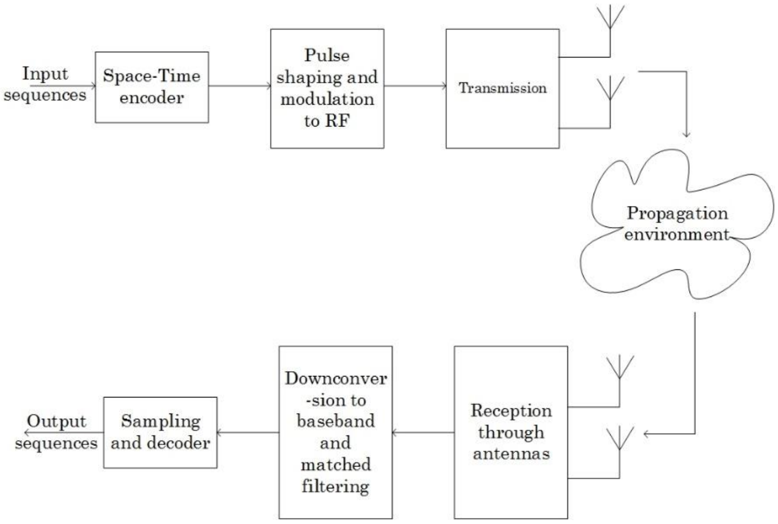


Figure 3.1: Basic MIMO system block diagram

The time varying channel response for between the transmit and receive antenna can be denoted as $h_{i,j}(t,\tau)$. So, the channel response for the MIMO system can be described by the following matrix

$$\boldsymbol{H}(t,\tau) = \begin{bmatrix} h_{1,1}(t,\tau) & h_{1,2}(t,\tau) & \cdots\cdots & \cdots & h_{1,N}(t,\tau) \\ h_{2,1}(t,\tau) & h_{2,2}(t,\tau) & \ddots\ddots & \ddots & h_{2,N}(t,\tau) \\ \vdots & \vdots & & & \vdots \\ h_{M,1}(t,\tau) & h_{M,2}(t,\tau) & \cdots\cdots & \cdots & h_{M,N}(t,\tau) \end{bmatrix} \tag{3.1}$$

The received signal model can be defined as

$$y_j(t) = \sum_{j=1}^{N} s_i(t) \circledast h_{i,j}(t,\tau) + n_i(t) \tag{3.2}$$

Here $\circledast$ implies convolution between the transmitted signal and the channel response and $n_i(t)$ is the noise that added with the transmitted signal, and $i = 1, 2, \dots, M$. The received signal can be expressed in frequency domain as

$$Y_j(f) = \sum_{j=1}^{N} S_i(f) H_{i,j}(f) + N_i(f) \tag{3.3}$$

Here $s_i(t)$ and $y_j(t)$ represents the transmitted and received signal matrices

$$s_i(t) = [s_1(t), s_2(t), \mathrm{s}(t), s_4(t) \dots \mathrm{s}_N(t)] \tag{3.4}$$

$$y_j(t) = [y_1(t), y_2(t), \mathrm{y}(t), \mathrm{y}(t) \dots y_M(t)] \tag{3.5}$$

The main driver for MIMO technology is the fact that MIMO systems have the potential to increase the capacity of a communication system as a linear function of *min(M,N)* without increasing transmit power or expanding bandwidth. This requires that the channel matrix **H** is with full rank [11].

3.3 Multipath Propagation

The idea of radio communications is to transmit signals via electromagnetic wave propagation through a given medium. The signal is supposed to experience various physical phenomena at the time of propagation. These interactions include reflections, diffraction, scattering, absorption etc. The result is that each of the receiving antennas observes multiple realizations of the transmitted signal having individual delays, magnitudes, polarization behavior, as well as directional characteristics. In wireless communications these mechanisms are commonly classified under the term multipath propagation, which is illustrated in Figure 3.2.

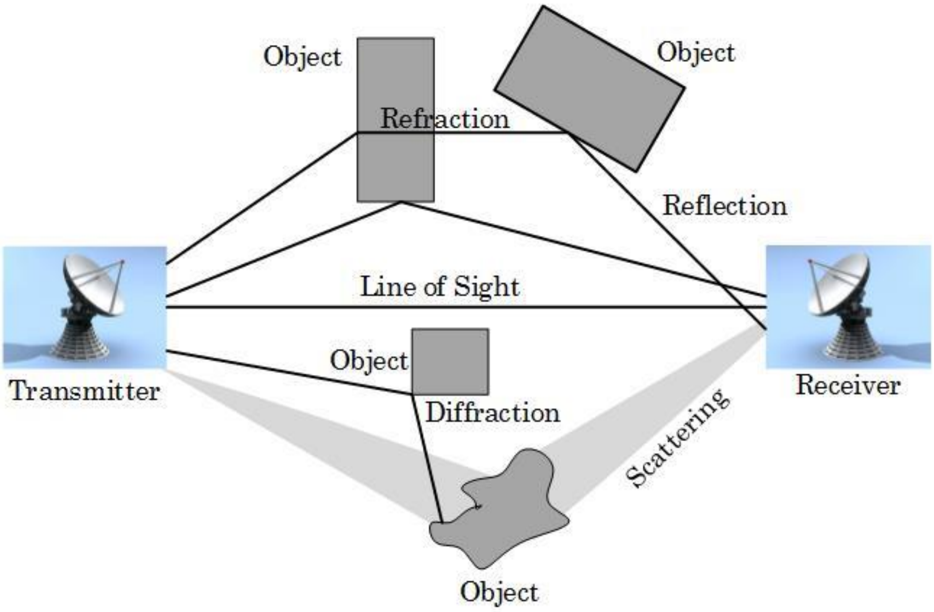


Figure 3.2: Illustration of Multipath Propagation

There is a significant influence of multipath propagation on the received signal due to the constructive and destructive superposition of incoherent signal components leading to fading. Therefore, in conventional radio systems it has been considered as impairment. Furthermore, the information carrying signal has some bandwidth. In a multipath environment the result is that all the frequencies in the signal band reach the receiver along each of the propagation paths, and the observed phase shift of the signal at different frequencies depends on the electric length of the individual signal paths. As a result, the fading of the received signal is frequency dependent due to either destructive or constructive effect from the combination of the individual components. In addition the channel may not be static, i.e., either the transmitter or the receiver (or both) may be mobile, or interacting objects in the environment may be moving. Hence, the channel is also varying over time, which translates to Doppler shift. This kind of dynamic wireless channel is called selective. The time-frequency-space selectivity of the channel is commonly characterized by respective measures of coherence time, coherence bandwidth, and coherence distance. Without going into details, these measures describe the correlation of the channel in each domain, i.e., how rapidly the channel changes in each dimension, respectively.

3.4 MIMO in Fading Channel

The MIMO channel usually varies over frequency. From the channel response matrix, we can say that each coefficients of the channel matrix can be expressed as a linear time varying channel filter

$$h_{M,N}(t) = \sum_{i} a_i(t)\delta\big(\tau - \tau_i(t)\big) \tag{3.6}$$

Here δ implies the dirac delta function and $a_i(t)$ is the complex coefficients for delays τ_i. The frequency domain representation of the coefficients of channel matrix H can be as follows

$$H_{M,N}(t,f) = \int_{-\infty}^{\infty} H_{M,N}(t,\tau)e^{-j2\pi\tau}d\tau = \sum_{i} a_i(t)e^{-j2\pi f\tau_i(t)} \tag{3.7}$$

It is evident from the above equation that MIMO has a frequency dependency under multipath fading environment. Recalling from equation 3.3, the input-output relationship for a MIMO channel is

$$y(t) = \int_{\tau} h(t,\tau)s(t-\tau)d\tau + n(t) \tag{3.8}$$

3.5 Spatial Multiplexing

The basic idea of multiplexing is to divide the whole data stream into several segments and transmit those via different communication channels. If the data stream is distributed into several frequency bands, it is known as frequency division multiplexing (FDM). The distributions, on the basis of time slot and unique codes, are known as time division multiplexing (TDM) and Code division multiplexing (CDM) respectively. The remaining category of multiplexing is based on space division (SDM) or spatial multiplexing. The main concern of this section is to discuss spatial multiplexing principle, mathematical representation, its advantages and disadvantages.

Spatial multiplexing requires multiple antennas which are spatially separated. The bits are transmitted by these spatially separated antennas through different independent data channels. An illustration of spatial multiplexing has been pictured below, in figure 3.3,

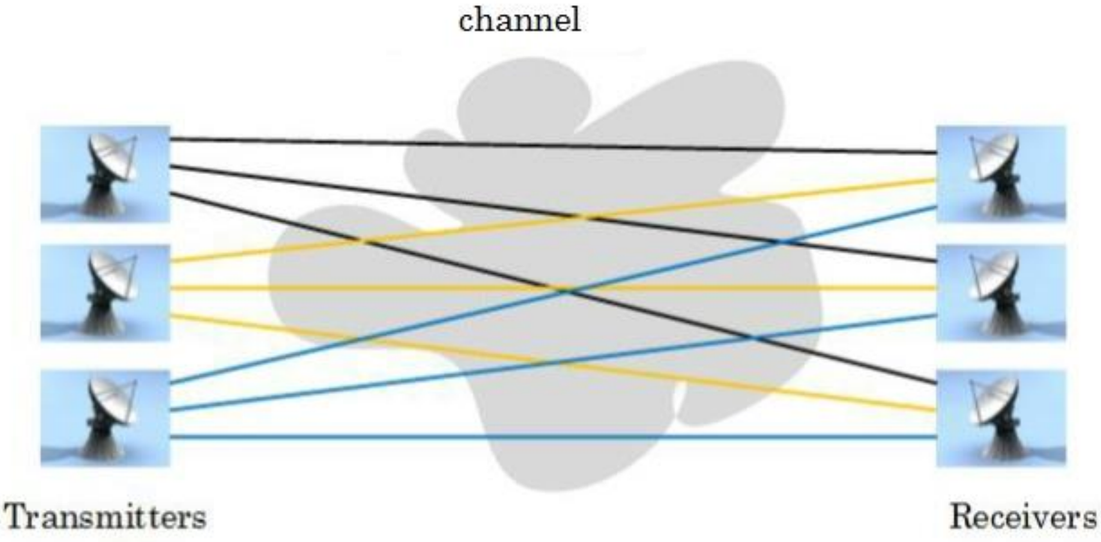


Figure 3.3: Illustration of Spatial Multiplexing system.

The multiplexing order i.e. the number of parallel data streams are dependent upon number of TX and RX antennas are used. The formula for determining maximum multiplexing order is

$$N_s = \min(N_T, N_R) \tag{3.10}$$

where, N_T is number of transmitting antennas and N_R is number of receiving antennas. Spatial multiplexing can increase spectral efficiency. The multiplexing can be performed both in open loop and closed loop approach. Channel state information (CSI) is utilized in closed loop approach [7].

Spatial multiplexing doesn’t require any bandwidth expansion. Moreover, space-time equalization is necessary at the receiving end. It is recommended to have greater number (at least equal) of receiver antennas than the number of transmitter antennas. Being the fading process of the spatial channel independent, it is possible to separate the data streams in the equalizer block [5].

3.6 Applications of MIMO

The application area of MIMO in communication networks is numerous. It increases the spectral efficiency as well as data rate. It improves the signal quality by increasing the signal strength. It can be implemented in DVB-T[34] and DVB-H[35]. In LTE-Advanced, MIMO plays a significant role as it increases the data rate and capacity by employing diversity techniques.

In Wi-Fi-IEEE 802.11n, MIMO is implemented to boost data rates up to 600 Mbps through multiple antennas and signal processing technique. It is possible to employ upto 4×4 MIMO in Wi-Fi WLAN. It is possible to enhance performance both in physical layer and MAC layer (e.g., frame aggregation, block-ACK, space-time coding, power save, green field mode, etc.) [10]. MIMO can play a significant role in the implementation of wireless mesh network and wireless Ad-Hoc networks. It has notable applicability in WiMAX (IEEE 802.16e) and RFID as well.

Last but not the least, MIMO can a good way to implement a smart home network. Cable TV, multimedia devices, computers, phone lines and other music and storage devices can reliably be connected by MIMO. By using the advantages of interpolability, MIMO can affect the characteristics of the installed 802.11 wireless base [10].

The implementation of MIMO in Radar systems is under research at present.

4 Implementation of MIMO in Radar

4.1 Concept of MIMO Radar

Since last couple of years, MIMO radar has become an interesting topic of research. Researchers took the name, MIMO, from the communication aspects. The term, MIMO, implies implementation of multiple antennas at the transmitting end as well as at the receiving end. From the radar point of view, the transmission of signals from transmitter to target, and reflecting back to the receiver are considered as propagation of signals through communication channels. Target position, velocity and other target features determine the channel characteristics which are usually described by a channel matrix. The main target of implementing MIMO in Radar is to reduce the matrix estimation complexity. By adopting the principles of communication theory in radar is fruitful as it is possible to use certain results from communication theory and practice. On the contrary, MIMO radar has some drawbacks e.g. high SNR loss and higher computation complexity.

4.1.1 Phased Array Radar

A phased array is a collection of antennas by which the radiated signal direction can be varied towards a specific angle as well as other undesired direction can be suppressed [12]. Phased array radar is constructed on the basis of the well-known Phase concept. According to the phase concept, the received energy will be the maximum when all the arriving signals are in phase.

Instead of using Mechanical scanning of antenna array, electronic scanning is used in phased array radar. This is because there are some drawbacks of using mechanical scanning in antenna array. For example, mechanical scanning suffers from slow antenna positioning which results less accurate detection. Also it has a low reaction time as well as it introduces some mechanical errors. On the other hand, electronic scanning provides increased data rates so that the target estimation will be more perfect. It has the capability of instantaneous beam positioning which results less reaction time. It is capable of eliminating the errors caused by mechanical movement of antenna arrays. Multimode operation is also possible with the implementation of electronic scanning. Moreover, processing of multiple targets detection is another significant benefit of electronic scanning.

There are three methods for electronic beam steering. The methods are discussed in a brief below:

a. Time delay scanning: It employs time delay to achieve the desired phase relationship. It consists highly flexible frequency utilization which is higher than other methods. Distinct delay networks are installed in front of each radiating elements for implementing time delay scanning. The required phase shift can be executed by the determination of the time delays accurately. The time delay between adjacent array elements required to scan the beam at a certain angle, θ, is given by the following equation [13]

$$t = \frac{d}{c} \sin \theta \tag{4.1}$$

Here, c is the velocity of propagation (speed of light). The whole process of time delay scanning is complex, heavy and costly.

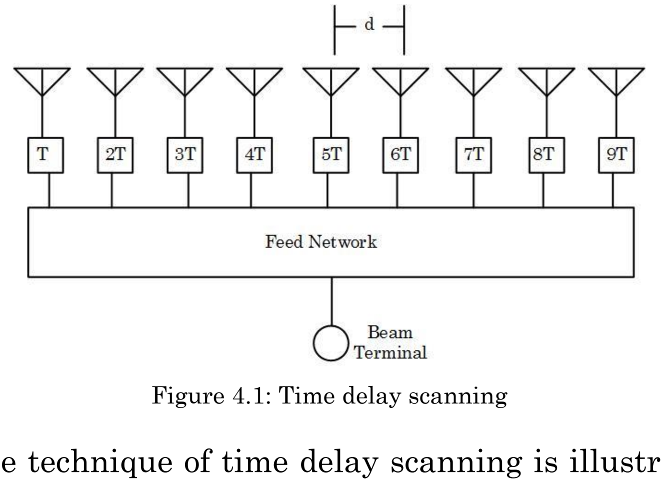


Figure 4.1: Time delay scanning

In figure 4.1, the technique of time delay scanning is illustrated. Here, T is the time delay and d is the distance between two radiating elements.

b. Frequency scanning: Another type of scanning technique that employs frequency instead of phase is available. There is a specific frequency namely, base frequency for all antenna elements. All the antenna elements are in the same phase at base frequency. The phase across the antenna aperture is changed in a linear manner while the frequency is changed, which results the scanning of the beam [52]. This scanning technique consists simpler implementation and less costly maintenance. They have been developed and deployed in the past to provide elevation-angle scanning in combination with mechanical horizontal rotation for 3D radars [3]. A very basic frequency scanning technique is illustrated in figure 4.2.

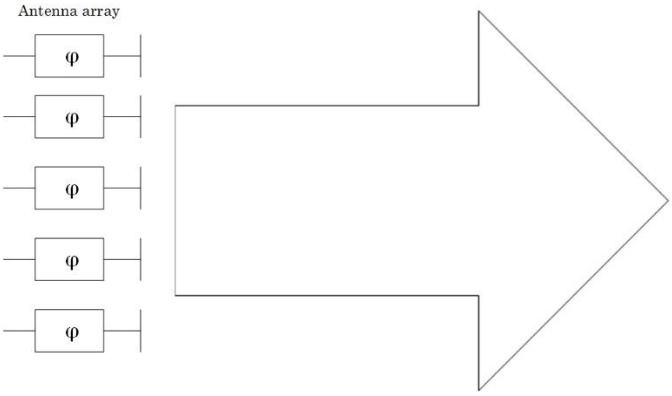


Figure 4.2: Frequency scanning

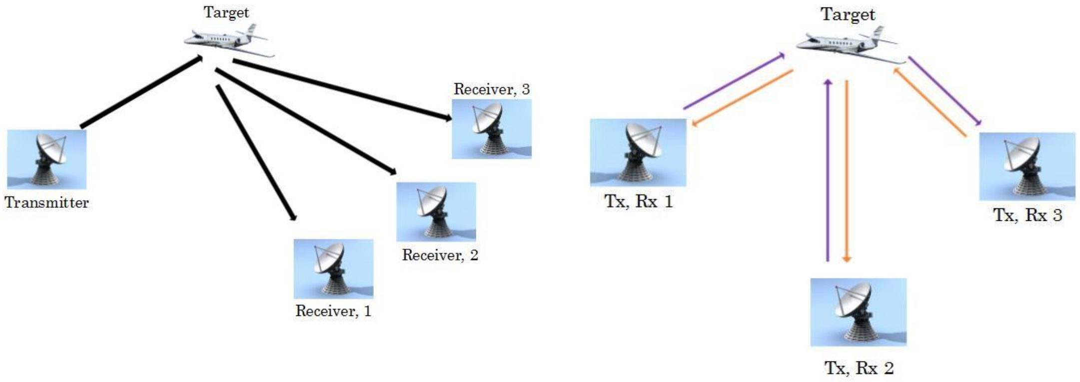
c. Phase scanning: The antenna beam need to be in normal with the phase front for implementing phase scanning. This phase front is regulated in such a way that the beam of the antenna element can be steered [3]. The phase shifters can be adjusted electronically so that they permit rapid scanning. With an inter-element spacing s, the incremental phase shift ψ between adjacent elements for a scan angle θ_0 is

$$\psi = \frac{2\pi}{\lambda} s \sin(\theta_0) \tag{4.2}$$

If the phase shift ψ is frequency independent, the scan angle θ_0 will be frequency-dependent. The implementation cost of this method is less than time delay scanning, but it has more cost than frequency scanning.

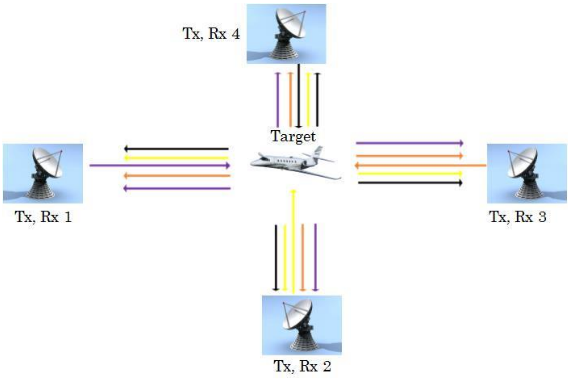
4.1.2 Multistatic Radar

The concept of multistatic radar has been developed on the basis of employing multiple of transmitters and receivers in the whole radar system. The sensitivity of monostatic radar is limited by its power aperture product and location accuracy is limited by its aperture. These limitations can be addressed by using a multiplicity of transmitters and receivers. The basic structure of typical multistatic radar is illustrated in figure 4.3.



a) Multiple bistatic case

b) Multiple monostatic case



c) Fully multistatic case

Figure 4.3: A typical scenario of multistatic radars
[The image of the airplane is used with permission from Cessna Inc.]

Spatial diversity offers much higher quality of information gathered, resulting enhanced detection and classification of targets [14]. Multistatic radars can be constructed by three basic building blocks. A node of multistatic radar system can be a) a transmitter, b) a receiver, or c) both transmitter and receiver. If the node is a transmitter, there are some designer specific parameters, i.e., the designer can adjust those for optimizing the system performances. The parameters are: carrier signal frequency, pulse duration, power of pulse, pulse bandwidth, pulse repetition frequency, type of modulation, polarization of transmission etc. On the other hand, designer specified parameters for receiver are: choice of frequency band, bandwidth, polarization etc. [14]. Each node in a multistatic radar system is capable of doing operation from the following options

1) Multiple monostatic operation, here each node has a collocated transmitter and receiver. The receiver collects reflected signals only from the collocated transmitter.
2) Multiple bistatic operation, here each node collects reflected signals from the transmitting nodes only. The transmitting and receiving nodes are not collocated.
3) Full multistatic operation, here the nodes are spatially distributed and the receivers have freedom of choice of reflected signals.

In multistatic radar systems, each of the radar nodes can operate as independent radar. Each system may process the received signals individually and the other things e.g. detection estimation and parameter estimation can be processed in a separate processing center which is known as fusion center. Reciprocally, the entire received signal can be sent to a separate processing center without any prior processing. The received signals will be processed jointly in the central processing center. For joint processing, a common time and frequency synchronization between transmitter and receiver should be maintained during the operation. It can be a significant drawback of joint processing, to maintain the time and frequency synchronization, in the case of having huge distance between the transmitter and receiver.

Multistatic radar possesses significant number of advantages. Some of them are discussed below

a) The first advantage can be the increment of total power and sensitivity of the whole systems. Multiple transmitter and receiver units can be employed, in a distributed manner, to decrease signal power losses. If the target is detected by spatially separated receiving units, the reflected signals are statistically independent at every receiving units. Received signal fluctuations can be mitigated while they are amalgamated at joint processing center. As a result, the detection performance is enhanced. Observing from different direction simultaneously may help the whole system to improve the probabilities of detecting stealth objects [16].

b) The high accuracy of position estimation of a target is another advantage of multistatic radar systems. Range measurement offers more precise result than angle measurement in monostatic radar systems. This is because of angle measurement being related with antenna beamwidth. If the range of a monostatic radar is increased, subsequently the angle measurement accuracy decreases. Reciprocally, multistatic radar systems obtain different range measurements from spatially distributed receivers. By using special computation techniques e.g. triangulation and exact angle of arrival information, it can enhance the accuracy of the position estimation [16].

c) Increased resolution capability is also an advantage of multistatic radar systems. The detection probability and measurement accuracy, in the presence of additional targets and other interference sources, can be understood by its resolution capability.

d) Distributed radar system offers less intersection between the main beams than that of a monostatic radar system. This less intersection reduces the reflection of power from clutters [16]. This can be another advantage of multistatic radar systems.

e) In a multistatic radar system, all the receivers and transmitters are located in different geographical positions. So, it is difficult for targets to determine the exact positions of the transmitters and receivers. In this way, it is less susceptible to jamming and safer from direct physical attack by missiles or any other means [16].

4.1.3 MIMO Radar

The most general definition of MIMO radar is considered as combination of multiple of transmit and receive antenna using independent waveform for detection purpose. Here, MIMO radar is defined as a radar system having some sort of diversity (either temporal, spectral or spatial) to transmit and receive multiple signals [53]. Although MIMO communication is not a new concept, the research interest of implementing MIMO in radar systems is now increasing. In contrast with beamforming, MIMO radar mainly focuses on exploiting waveforms having orthogonality with each other. Beamforming requires high correlation between signals either at transmitter or receiver by array elements. On the other hand, MIMO radar exploits the independence between the signals at the array elements [15]. Depending upon the post-processing technique and diversity, MIMO radar can be classified mainly in three categories. a) coherent MIMO radar, b) statistical MIMO radar and the most recent c) polarimetric MIMO radar.

4.1.3.1 Coherent MIMO Radar

The term ‘coherence’ implies synchronization between the transmitter and receiver part of a radar system. The characteristics of the transmitted signal should be known to the receiver at the time of post processing. It uses antenna arrays for transmitting and receiving signals. There is no restriction on the arrays about their physical position i.e. they can be collocated or distributed. The array elements can be uniformly spaced or it can be spaced non-uniformly. As maintaining synchronization in distributed radar systems is difficult and complex, collocated multi antenna array technique is mostly used in coherent MIMO radar. A pictorial description of coherent MIMO radar using linear antenna array has been given in figure 4.4.

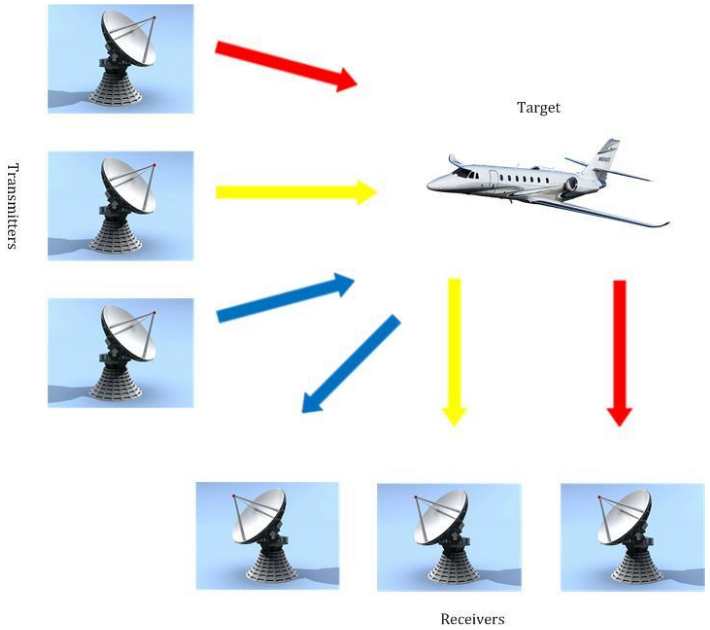


Figure 4.4: Coherent MIMO Radar Configuration
[The image of the airplane is used with permission from Cessna Inc.]

Target localization using coherent MIMO radar exhibits high resolution, but it is subjected to spurious sidelobes. This problem can be mitigated by employing multiple frequency transmission with distributed antennas. High resolution and ambiguous target localization is compatible with coherent MIMO radar. Moreover, ambiguities can be controlled through the number of sensors. To some extent, coherent MIMO radar has similarities with phased array radar because both of the radars use antenna array. But unlike phased array radar, coherent MIMO radar does not transmit same waveform from each array element [16].

Let us consider a MIMO radar system having M_t and M_r number of transmit and receive antennas respectively. The baseband signal from the mth transmitter array element is $x_m(t)$ and the average signal energy of the signal $x_m(t)$ is 1. All the transmitted signals are mutually orthogonal. The target location is $X_o = (x_o, y_o)$ and it is assumed that the target direction from the transmitter and receiver is θ and θ' respectively. The transmitted signal at the target location is

$$x_m^t(t) = x_m\big(t - \tau_{tm}(x_o, y_o)\big) \tag{4.3}$$

Here $m=1, 2, 3, \dots M_t$ and $\tau_{tm}(x_o, y_o)$ represent the time delay between the target and mth transmit antenna. All the transmitted signals are assumed to be narrow band signals. So, the sum of all transmitted signals will be [16]

$$x^t(t) = \sum_{m=1}^{M_t} x_m\big(t - \tau_{tm}(x_o, y_o)\big) = \sum_{m=1}^{M_t} e^{-j2\pi f_o \tau_{tm}(\theta)} x_m(t - \tau_t) \tag{4.4}$$

In the above equation, τ_t represents the common delay between every transmit elements and τ_{tm} represents the delay between the target and mth transmit antenna. The transmit antenna steering vector, $\boldsymbol{a(\theta)}$, and the transmitted signal vector, $\boldsymbol{x(t)}$, can be defined as [16]

$$\boldsymbol{a}(\theta) = \begin{bmatrix} e^{-j2\pi f_o \tau_{t1}(\theta)} \\ e^{-j2\pi f_o \tau_{t2}(\theta)} \\ e^{-j2\pi f_o \tau_{t3}(\theta)} \\ \vdots \\ e^{-j2\pi f_o \tau_{tM_t}(\theta)} \end{bmatrix}, and\ \boldsymbol{x}(t - \tau_t) = \begin{bmatrix} x_1(t - \tau_t) \\ x_2(t - \tau_t) \\ x_3(t - \tau_t) \\ \vdots \\ x_{M_t}(t - \tau_t) \end{bmatrix} \tag{4.5}$$

Now, the signal at the target location $x^t(t)$ can be rewritten in the vector form as

$$\boldsymbol{x^t}(t) = \boldsymbol{a^H}(\theta)\boldsymbol{x}(t - \tau_t) \tag{4.6}$$

The target experiences this signal and reflects it back to the receiver. If $y_k(t)$ is reflected signal at the receiving end, the signal at the kth antenna can be written as [16]

$$y_k(t) = \alpha_k x^t\big(t - \tau_{rk}(x_o, y_o)\big) + w_k(t) \tag{4.7}$$

$\tau_{rk}(x_o, y_o)$ is the time delay between target and the kth receive antenna, and $w_k(t)$ is a zero mean complex random process representing receiver noise and other disturbances. α_k is the complex constant factor proportional to the RCS seen by the kth receive antenna. As the antenna array elements in both transmitter and receiver are closely spaced, we can consider $\alpha_k = \alpha$, $\tau_{rk}(x_o, y_o) = \tau_r$. Now, the received signal can be rewritten as [16]

$$y_k(t) = \alpha e^{-j2\pi f_o \tau_{r1}(\theta')} x(t - \tau_t - \tau_r) + w(t) \tag{4.8}$$

The reflected signal can be rewritten as

$$\boldsymbol{y}(t) = \alpha \boldsymbol{b}(\theta') \boldsymbol{a}^H(\theta) \boldsymbol{x}(t - \tau) + \boldsymbol{w}(t) \tag{4.9}$$

Here, $\boldsymbol{y}(t)$ is the received signal vector, $\boldsymbol{b}(\theta')$ is the receive antenna steering vector and $\boldsymbol{w}(t)$ is the receive interference vector. These vectors can be defined as [16]

$$\boldsymbol{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_{M_r}(t) \end{bmatrix}, \boldsymbol{b}(\theta') = \begin{bmatrix} e^{j2\pi f_o \tilde{\tau}_{r1}(\theta')} \\ e^{j2\pi f_o \tilde{\tau}_{r2}(\theta')} \\ \vdots \\ e^{j2\pi f_o \tilde{\tau}_{rM_r}(\theta')} \end{bmatrix} and\ \boldsymbol{w}(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \\ \vdots \\ w_{M_r}(t) \end{bmatrix} \tag{4.10}$$

Here, $\tilde{\tau}$ is the delay between the target and the receiving element. The channel matrix, $\mathbf{H}$, of dimension $M_r \times M_t$ can be defined as

$$\boldsymbol{H} = \alpha \boldsymbol{b}(\theta') \boldsymbol{a}^H(\theta) \tag{4.11}$$

Now, the receive signal can be rewritten as

$$\boldsymbol{y}(t) = \boldsymbol{H} \boldsymbol{x}(t - \tau) + \boldsymbol{w}(t) \tag{4.12}$$

If a matched filter is applied at the receiving end and the received signal is fed through the filter, which is matched to $x_m(t)$, the output of the matched filter can be expressed in vector form as

$$\bar{\boldsymbol{y}} = \bar{\boldsymbol{\alpha}} + \bar{\boldsymbol{w}} \tag{4.13}$$

$\bar{\boldsymbol{y}}$ is a $M_r \times M_t \times 1$ complex matrix corresponding to the output of the matched filter at every receiver, $\bar{\boldsymbol{w}}$ is a $M_r \times M_t \times 1$dimension complex noise vector and $\bar{\boldsymbol{\alpha}}$ is a complex vector having similar dimension defining the

Kronecker product of both transmit and receive antenna steering vector and the vector α. It can be written as

$$\bar{\boldsymbol{\alpha}} = [\boldsymbol{b}(\theta') \otimes \boldsymbol{a}(\theta)]\alpha \tag{4.14}$$

4.1.3.2 Statistical MIMO Radar

The main idea of using statistical processing in MIMO is to use orthogonal signals at the transmitting end. In contrast with coherent MIMO radar, Noncoherent MIMO radar receiver doesn't have the phase information of the transmitting signal. It uses widely separated antenna arrays while the inter-element spacing between the array elements are large enough for the transmit-receive pair to see the target in a different angle. Multiple look angles help the transmit-receive pair to see various RCS because of the target's complex shape [16]. The inter-element distance should be wide enough so that the received signals from each transmit-receive pair is independent. This is known as spatial diversity. The core idea of using spatial diversity is to maximize the target SNR, by using the diversity gain instead of coherent gain in traditional phased array radar [17].

The first idea of using multiple antennas in radar was proposed by MIT Lincoln laboratory in 2003 [17]. In that model, it was proposed to place both the transmitting and receiving antenna separately so that the entire system could get more spatial diversity gain. They named this radar model as Statistical MIMO radar. Statistical processing is mostly used in distributed antenna system as it is difficult to maintain coherence in distributed systems. Statistical processing helps the system to mitigate the fading effect over communication channels and enhance the system performance. Distributed MIMO radar concept is based on this property of MIMO communications and target RCS's statistical behavior is exploited by this distributed MIMO as well [18]. Distributed MIMO is sometimes known as Statistical MIMO radar [16]. An illustration of non-coherent distributed MIMO has been given below, in fig. 4.5.

The distributed statistical MIMO radar can overcome the angular sparkle of targets by auto-correlations and cross correlation processing [19]. Multipath diversity of MIMO communication is introduced by this distributed statistical MIMO, into the design of radar. Not only statistical MIMO radar but also multistatic radar use the idea of angular diversity and multiple transmit and receive antennas. If time and phase synchronization is maintained properly during operation, it can be beneficial for multistatic radar also. There is one more condition for taking this advantages of multiple widely separated antenna that the received signals should be processed in a joint processing center of multistatic radar. In broader sense, we can say that statistical MIMO radar is the extended version of multistatic radar system [16].

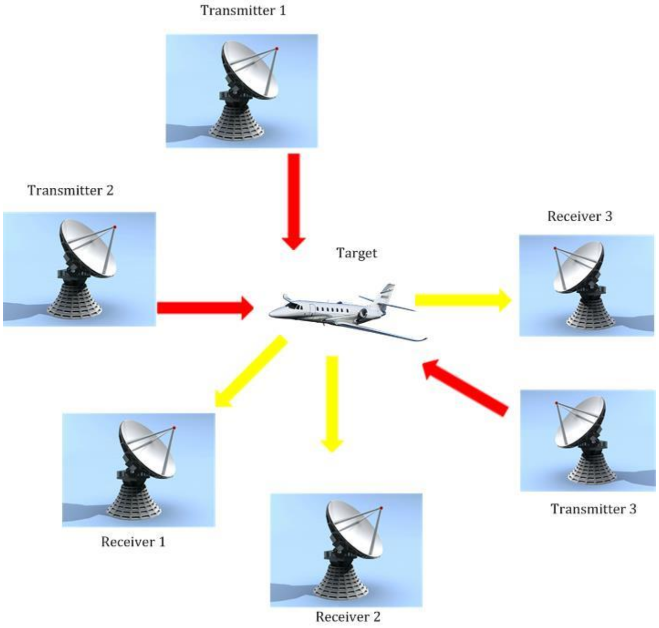


Figure 4.5: Illustration of Noncoherent distributed MIMO radar
[The image of the airplane is used with permission from Cessna Inc.]

Let us consider that there is a statistical MIMO radar system consisting of M_t transmitter and M_r receiving antennas. The transmitters and receivers are widely separated and let us imagine that (x_{tm}, y_{tm}) and (x_{rk}, y_{rk}) coordinates are representing the position of mth transmitter and kth receiver. The whole situation is pictured in figure 4.6.

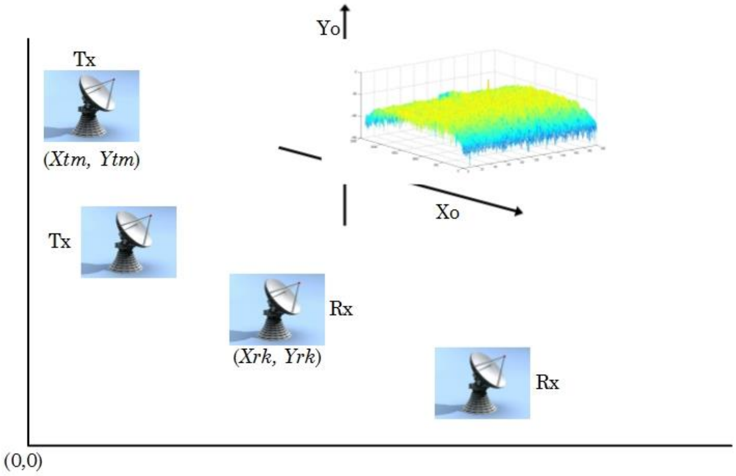


Figure 4.6: Statistical distributed MIMO radar configuration

A stationary complex target is located at $X_o = (x_o, y_o)$. The expression for the narrowband signal is $x_m(t)$, which is transmitted from the mth transmitter element. So, the signal at the target location is

$$x_m^t(t) = x_m\big(t - \tau_{tm}(x_o, y_o)\big) \tag{4.15}$$

The term $\tau_{tm}(x_o, y_o)$ represents the time delay between the target and the mth transmitter element. Now, the baseband equivalent signal received by the kth receiver element is [16]

$$y_k(t) = \sum_{m=1}^{M_t} \alpha_{km} x_m^t \left(t - \tau_{tm}(x_o, y_o) - \tau_{rk}(x_o, y_o)\right) \tag{4.16}$$

Here, $\tau_{rk}(x_o, y_o)$ is the delay between the target and the kth receiver element. And α_{km} is the distribution of the target that can be seen between the mth transmitter and kth receiver element.

The received signal can be expressed in exponential form, which is as follows [16]

$$y_k(t) = \sum_{m=1}^{M_t} \alpha_{km} e^{-j\tau_{tm}(\theta)} e^{-j\tau_{rk}(\theta')} x_m(t - \tau) \tag{4.17}$$

In the equation above,

$$\tau = \tau_{tm}(x_o, y_o) + \tau_{rk}(x_o, y_o) \tag{4.18}$$

$$\tau_{tm}(\theta) = 2\pi f_o\big(\tau_{tm}(x_o, y_o) + \tau_{t1}(x_o, y_o)\big) \tag{4.19}$$

And

$$\tau_{rk}(\theta') = 2\pi f_o\big(\tau_{rk}(x_o, y_o) + \tau_{r1}(x_o, y_o)\big) \tag{4.20}$$

Here, *f*₀ is the operating frequency. The transmit array vector $\boldsymbol{a}(\theta)$ and the transmit signal vector $\boldsymbol{x}(t)$ can expressed as below

$$\boldsymbol{a}(\theta) = \begin{bmatrix} e^{-j\tau_{t1}(\theta)} \\ e^{-j\tau_{t2}(\theta)} \\ \vdots \\ e^{-j\tau_{tM_t}(\theta)} \end{bmatrix}, \boldsymbol{x}(t-\tau) = \begin{bmatrix} x_1(t-\tau) \\ x_2(t-\tau) \\ \vdots \\ x_{M_t}(t-\tau) \end{bmatrix} \tag{4.21}$$

Similarly, the receive array steering vector $\boldsymbol{b}(\theta')$ and received signal vector $\boldsymbol{y}(t)$ can be as follows

$$\boldsymbol{b}(\theta') = \begin{bmatrix} e^{-j\tau_{r1}(\theta')} \\ e^{-j\tau_{r2}(\theta')} \\ \vdots \\ e^{-j\tau_{rM_r}(\theta')} \end{bmatrix}, \boldsymbol{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_{M_r}(t) \end{bmatrix} \tag{4.22}$$

The received signal $\boldsymbol{y}(t)$ can be represented in vector form, which is as follows [16]

$$\boldsymbol{y}(t) = \boldsymbol{b}(\theta')\alpha\boldsymbol{a}(\theta)\boldsymbol{x}(t-\tau) + \boldsymbol{w}(t) \tag{4.23}$$

Here, $\boldsymbol{w}(t)$ represents zero mean complex Gaussian interference i.e. receiver noise, clutter and jamming etc. The distribution matrix of the target,α, having the dimension of $M_r \times M_t$. The matrix α can be expressed as follows

$$\alpha = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1M_t} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2M_t} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{M_r1} & \alpha_{M_r2} & \cdots & \alpha_{M_rM_t} \end{bmatrix} \tag{4.24}$$

In [18], it is mentioned that if at least one of the following four conditions is satisfied, the kmth and lith elements of α will be uncorrelated [18].

$$(a)\; x_{rk} - x_{rl} > \frac{\lambda}{\Delta x} d(R_k, X_o)$$
$$(b)\; y_{rk} - y_{rl} > \frac{\lambda}{\Delta y} d(R_k, X_o)$$
$$(c)\; x_{tm} - x_{tl} > \frac{\lambda}{\Delta x} d(T_m, X_o)$$
$$(d)\; y_{tm} - y_{tl} > \frac{\lambda}{\Delta y} d(T_m, X_o)$$

On the other hand, if the following conditions hold jointly, the kmth and lith elements of α will become fully correlated [18].

$$\begin{gathered}(i)\ x_{rk}-x_{rl}\ll\frac{\lambda}{\Delta x}d(R_k,X_o)\\(ii)\ y_{rk}-y_{rl}\ll\frac{\lambda}{\Delta y}d(R_k,X_o)\\(iii)\ x_{tm}-x_{tl}\ll\frac{\lambda}{\Delta x}d(T_m,X_o)\\(iv)\ y_{tm}-y_{tl}\ll\frac{\lambda}{\Delta y}d(T_m,X_o)\end{gathered}$$

The function of d(R, X) and d(T, X) defining the distance between the target and receiver, and between the target and the transmitter respectively. Lambda (λ) denotes the carrier wavelength. There can be three special cases in accordance with the above conditions. They are

1. Transmit antennas are closely spaced and receive antennas are widely spaced. In this case, the column of α matrix will have identical value and there are M_r number of different RCS values will be obtained. A coherent process gain M_t can be achieved.

2. Transmit antennas are widely spaced and received antennas are closely spaced. In this situation, we will have M_t different RCS values and the each row of $\boldsymbol{\alpha}$ matrix will have similar value. As a result, we will get M_r coherent process gain. The received signal vector can be expressed as
$$\boldsymbol{y}(n)=\boldsymbol{b}(\theta')\boldsymbol{\alpha}^H\boldsymbol{a}(\theta)\boldsymbol{x}(n)+\boldsymbol{w}(n)\tag{4.25}$$

3. The remaining exception can be both transmit and the receive antennas are closely spaced. In that case, the RCS matrix $\boldsymbol{\alpha}$ will fall into a single coefficient α. The target model of statistical MIMO radar will become the target model of coherent MIMO radar. And the received signal model will be equivalent to the model of coherent MIMO radar.

Now, the channel matrix H can be defined as

$$\boldsymbol{H}=\boldsymbol{b}(\theta')\alpha\boldsymbol{a}(\theta)\tag{4.26}$$

Then the received signal model can be written as

$$\boldsymbol{y}(n) = \boldsymbol{H}\boldsymbol{x}(t-\tau) + \boldsymbol{w}(n) \tag{4.27}$$

If the received signal is fed to the input of filter which is matched with $x_m(t)$, and the corresponding output is sampled at the time instant τ, the output of the matched filter will be

$$\bar{\boldsymbol{y}} = \bar{\boldsymbol{\alpha}} + \bar{\boldsymbol{w}} \tag{4.28}$$

Here, $\bar{\boldsymbol{y}}$ is a complex vector corresponding the matched filter output at each receiver, $\bar{\boldsymbol{\alpha}}$ is a complex vector containing the product of transmitting signal and channel response and $\bar{\boldsymbol{w}}$ is also a complex vector representing the noise and interferences.

4.1.3.3 Polarimetric MIMO Radar

There is another type of waveform diversity available, namely, polarization diversity. From the term ‘polarization diversity’, it can be easily understood that waveforms with different polarization is used for target detection. This type of radar is known as polarimetric MIMO radar. Although the concept of polarization diversity is proposed in early 80s, it failed to draw researchers’ attention. Increased computational system complexity and expensive measurement systems may be the key factor for its failure [31].

Now a days, polarization diversity is again in the top of the researchers’ interest. This is because, the reflected signal is converted into a scalar signal for further processing in traditional radar systems. As a result, some information, related with the angle of the reflected signal, are lost. To retrieve these information and target features, the entire reflected signal should be processed as a vector signal which requires polarization diversity [32]. Polarimetric MIMO radar offers improved sensing environment, reduced CFAR, improved parameter identifiability, noise and jamming suppression enhancement and higher resolution [31].

The main drawbacks of polarimetric MIMO radars are increased system complexity and expensive equipment. Also there should be both transmit and receive polarization diversity in the system, as well as coherence between them.

4.2 Spatial Diversity in MIMO Radar

For implementing MIMO in any type of communication system or Radar system, there must be some type of diversity present. It has already been demonstrated in several articles [43], [44], [45] that diversity enables enhancement of the MIMO radar performances multiple fundamental aspects e.g. 1) Parameter identifiability is improved significantly, 2) Target detection and parameter estimation by applying adaptive arrays directly, and 3) Diversity offers much enhanced flexibility for transmit beampattern design. It has been shown in [20] that the target identifiability of target in MIMO radar is increased upto M_t ,here M_t is the number of transmit antennas, times than that of its phased array counterpart. Moreover, the probing signals can be optimized to obtain several transmit beampattern design with superior performance. The theory of antenna diversity (e.g. using more than one transmitting element) has already been proposed and implemented in some areas [18]. In the conventional approach, an array of closely spaced transmitting elements is used to combine in a single beam towards a certain spatial direction, known as beamforming. The detailed discussion on beamforming will take place in later sections.

4.2.1 Collocated MIMO Radar

In the collocated MIMO Radar case, the transmitter elements are closely spaced so that the observed Radar Cross Sections of the target are identical. The spacing between the radiating elements can be uniformly distributed or it can be randomly distributed. The idea behind of implementing MIMO in collocated case is to increase the spatial resolution. In several research articles [46], [47], [48], it has been shown that collocated MIMO can enhance the interference rejection capability, and parameter identifiability. Collocated MIMO also offers higher sensitivity to detection of slowly moving targets. Last but not the least; it has direct applicability of adaptive array techniques [21]. Refer to section 4.1.3.1, the baseband signal at the target location is

$$x^t(n) = \sum_{m=1}^{M_t} e^{-j2\pi f_o \tau_{tm}(\theta)} x_m(n) = \boldsymbol{a}^*(\theta)\boldsymbol{x}(n);\ n = 1,2,\dots,N \quad (4.29)$$

Similarly, the received baseband signal is

$$y(n) = \sum_{k=1}^{K} \beta_k b^c(\theta_k) a^*(\theta_k) x(n) + w(n) \quad (4.30)$$

In the equation above, K is the number of targets that reflected back; β_k is the complex amplitudes, which are proportional to the Radar Cross Sections (RCSs) of the targets; θ_k is the location parameter and $w(n)$ is the sum of all types of noises and interferences. Coherent processing is the most suitable for collocated MIMO radar systems. As the radiating elements are located in the same place, it is also possible to use the same radiating elements for both transmitting and receiving. Time division duplexing is used for that purpose. Now-a-days, Collocated MIMO and Coherent MIMO are used as synonyms. On the contrary, it is difficult to maintain coherence in distributed MIMO. As a result, non-coherent MIMO is mostly used in distributed MIMO radar system.

4.2.2 Distributed MIMO Radar System

In distributed MIMO case, the transmitting antenna units are sparsely distributed in the space. At the receiving end, the received signal is considered as the composition of far field target echo and its multipath scattered signals. It is also known as statistical MIMO or non-coherent MIMO because of employing non-coherent processing at the receiving side. By implementing the autocorrelation and cross-correlation processing, distributed MIMO technique can solve the problem of angular sparkle of targets, which usually caused by complex targets that have more than one scattering center within the resolution cell of radar.

In distributed Radar systems, the whole network consists of several single radar units. Each unit is known as node [19]. The detection ability of the distributed MIMO radar system is estimated through the gain of multipath diversity, and spatial resolution is obtained through the resolution gain. According to [19], the characteristics of a node of a distributed MIMO radar system are

a) An integrated array antenna will be used as transmitting antenna. It is omnidirectional in nature.
b) Uncorrelated signals i.e. orthogonal signals are transmitted by the node radar.
c) Either the signal phase-locked of the distributed multistatic radar system or the accurate synchronic detection function between multiple platforms is important to realize.

Each node radar is a combination of transmitting and array, orthogonal wave designed transmitter, multichannel receiver, multimode communication channel, signal processor and system data fusion terminal etc. The block diagram of main components of a node is given in Figure 4.7.

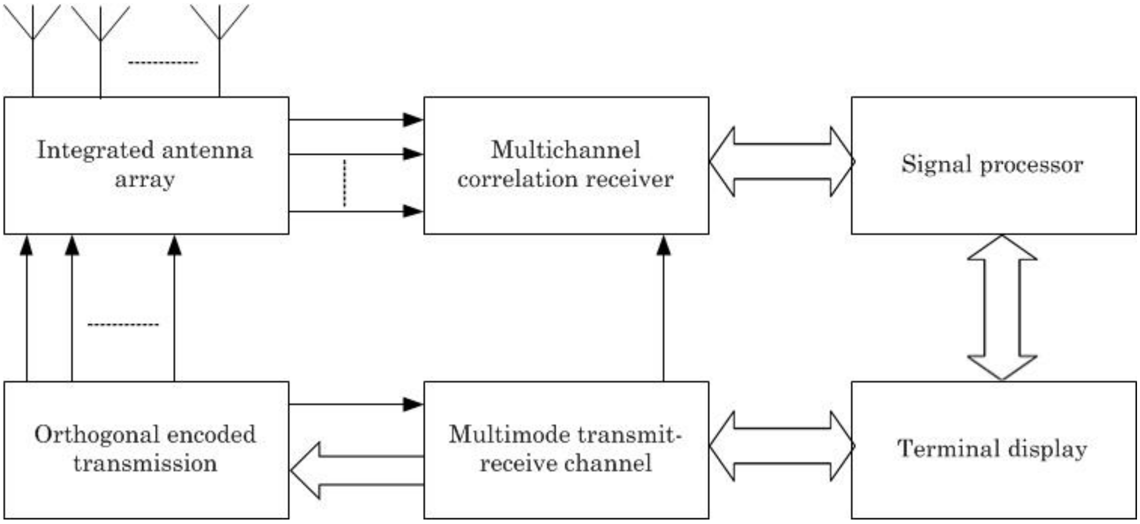


Figure 4.7: The main components of a node[19]

The cross-correlation processing and autocorrelation processing of target echo signals are performed by the multichannel receiver and multimode communication channel respectively.

The essential technologies for implementation of distributed MIMO are integrated array antenna technology, encoded waveform design and multistatic accurate synchronic detection technology [19].

4.3 Classical Beamforming Principle

The basic idea behind classical beamforming is to employ very closely spaced antenna array such that there is a very small time delay difference in different array elements. This is a process in conjunction with array of sensors to provide a versatile form of spatial filtering. In Figure 4.8, a basic beamforming setup has been pictured.

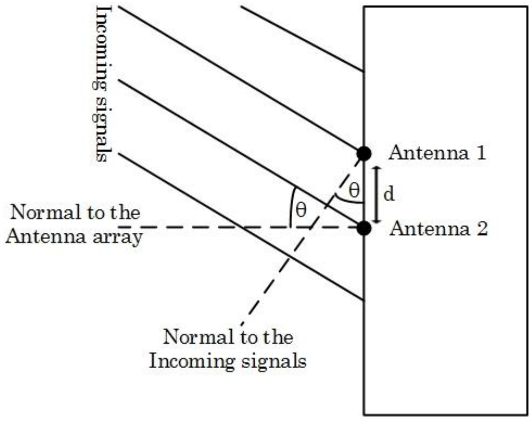


Figure 4.8: Basic beamformer model

In the figure above, d is the spacing between antenna elements and θ is incoming signal angle. The propagation distance difference

$$d_e = d \sin\theta \tag{4.31}$$

And the electrical angle between the array elements is

$$\Phi = 2\pi \frac{dsin\theta}{\lambda} \tag{4.32}$$

According to spatial sampling theorem,$d \leq (\lambda/2)$. For simplification, $d = \lambda/2$ is considered in this case. As a result, the value of phase difference (electrical angle) will be $\Phi = \pi sin\theta$.

The received signal snapshots, from each antenna, are fed to the signal processor where adaptive control algorithm has been employed. The collected snapshots for M array elements are

$$\boldsymbol{u}(k) = [u(1,k) \quad u(2,k) \quad \cdots \quad u(M,k)]^T \tag{4.33}$$

The spatial signal processing for combining all signals are given below, in Figure 4.9.

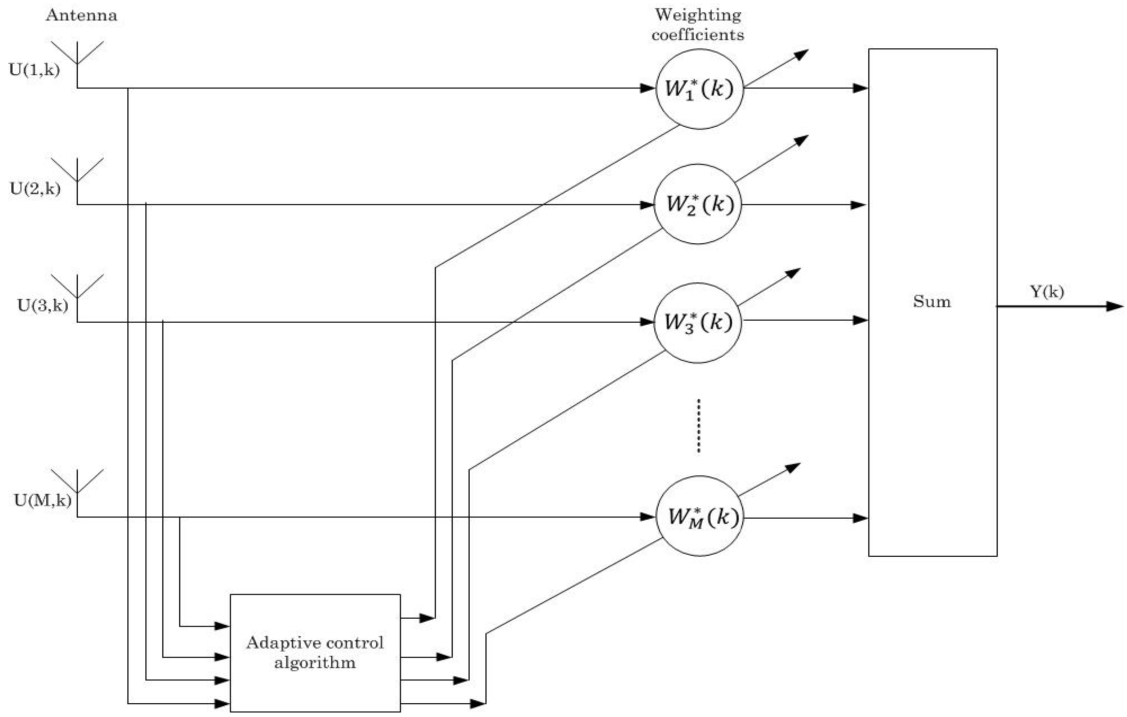


Figure 4.9: Block Diagram of the signal processor

On the other hand

$$\boldsymbol{u}(k) = \boldsymbol{x}(k)\boldsymbol{s}(k) + \boldsymbol{z}(k) \tag{4.34}$$

In 4.35, the vectors ***u(k)***, ***x(k)***, ***s(k)*** and ***z(k)*** are the reflected signal, the transmitted signal, the steering vector and random noise respectively. The steering vector is a function of electrical angle between the antenna elements, namely Φ.

$$\boldsymbol{s}(\Phi) = [1 \quad e^{j\Phi} \quad \dots \quad e^{j(M-1)\Phi}]^T \tag{4.35}$$

For the reducing the computation complexity, the expression of ***u(k)*** can be simplified as follows

$$\boldsymbol{u}(k) = \sum_{l=1}^{L} x_l(k)s(\Phi_l) + \boldsymbol{z}(k) \tag{4.36}$$

The output, ***y(k),*** after signal combination will be

$$\boldsymbol{y}(k) = \boldsymbol{w}^H\boldsymbol{u}(k) = w_1^*u(1,k) + w_2^*u(2,k) + \cdots + w_M^*u(M,k) \tag{4.37}$$

$\boldsymbol{w}^H$ is the complex valued weighting matrix and is adjusted in such a way that the desired signal is amplified and interfering signal is attenuated [33].

5 Advantages of MIMO applications in Radar systems

MIMO application in radar systems offers vast advantages in performances. Implementation of MIMO can improve several performance parameters of radar, depending upon the diversity technique used for MIMO. In this chapter, the improvements of several application areas of MIMO radar are discussed.

5.1 Improvement of Parameter Identifiability

Parameter identifiability is an important parameter for radar systems. The maximum number of targets can be uniquely identified, is known as parameter identifiability. MIMO radars can improve this very basic parameter much better than its phased array counterpart as well as conventional single antenna radar systems. In [23], it has been shown that the number of particularly detected targets in a MIMO radar system is

$$K_{max} \epsilon \left[\frac{2(M_t + M_r) - 5}{3}, \frac{2M_tM_r}{3}\right) \tag{5.1}$$

In (5.1), K_{max} is the maximum number of detectable target, which is related to the number of elements in MIMO steering vector. M_t and M_r is the number of array elements in the transmitter and receiver respectively. The value of K_{max} can be changed by changing of some specific conditions. They are [16]

1. The arrays being linear or nonlinear
2. The spacing between the array elements may be uniform to non-uniform

3. The number of elements being shared with both transmitter and receiver

The smallest value of K_{max} can be obtained while same array elements are used for both transmitter and receiver i.e. the MIMO system will be collocated. This situation can be considered as the worst case. Similarly, the largest number will represent the best case, where the transmitter and receiver are located away from each other as well as not sharing the array elements [23].

On the contrary, phased array radar (using same parameter value but $M_t=1$) will give the value of K_{max} in the range of

$$K_{max} \epsilon \left\lceil \frac{2M_r - 3}{3} \right\rceil \tag{5.2}$$

Comparing (5.1) and (5.2), it can easily be noticed that MIMO radar systems offers M_t times higher parameter identifiability than its phased array counterpart. The improvement is twice even if it is in the worst case.

5.2 Implementation of Adaptive array techniques

Another significant advantages of MIMO radar is that direct application of adaptive localization and detection techniques. It is already known that adaptive techniques offer much better resolution and better interference rejection capabilities than any other detection systems. Space-time adaptive processing (STAP) is a popular technique for MIMO radar. It implies doing concurrent processing of array antenna signals along with multiple pulse coherent waveforms. Instead of using primary statistical properties of echo signal, STAP utilize second order properties of echoes [50]. It offers detection of targets with very low velocity those are usually obscured by jamming or interferences [49]. An example is given in [24], space-time adaptive processing (STAP) requires secondary properties of the echoes to apply adaptive processing. Selection of the better secondary range bins is a difficult thing. On the other hand, the signals, reflected by the target, to a MIMO receiver are linearly independent to each other. So, there is no effect of secondary range bins for applying adaptive array techniques in MIMO radar. It is easily possible to employ adaptive localization and detection techniques in MIMO radar systems without the secondary range bins.

Moreover, it is even possible to use adaptive processing for range compression.

There are several adaptive processing algorithms, which are used for MIMO radar, namely CAPON and APES. CAPON gives higher resolution and APES offers better amplitude estimation [16]. In ref [16], another adaptive processing algorithm is discussed, which is known as Robust CAPON beamforming. This algorithm gives higher accuracy of target location as well as target amplitude.

5.3 Improvement of Detection Performance

The detection performance of MIMO radar, for both collocated and distributed systems, in different target and clutter conditions, is an interesting topic for research at present. There is lots of work in process now. Implementation of MIMO will enhance the detection performance of radar systems, especially in moving target detection, in a significant amount. The receiver forms narrow beams by using the phase shifts, which results scanning of the whole space easily [26].

Distributed MIMO radars enhance the detection performance by taking the advantages of spatial properties of targets' radar cross section (RCS). The RCS of complex targets varies rapidly with regards to antenna look angle. These rapid alterations cause signal fading and degrade radar performance. With sufficiently distributed antennas, the target RCS is illuminated with several signals from different look angles. Every signal consist independent information. Spatial diversity maximizes the RCS properties of the target and improves the radar performance by mitigating these targets' scintillations [51].

5.4 Other Advantages

Estimation of arrival angle can also be a significant advantage obtained by MIMO implementation in radar systems. In simple words, the estimation process is to determine the direction from which the reflected signal comes back. For this purpose, the transmitters are widely distributed and receiver arrays are collocated. The receiver array scans all the incoming signals reflected from the target and measure the SNR. The direction, from which maximum SNR is obtained, is considered as the direction of arrival.

Another notable advantage of MIMO radar is the improved performance of fading mitigation. As joint processing is used in both collocated and distributed MIMO radar, the average SNR is maximized during the processing. This will eliminate the effect of multipath fading. In the case of fading mitigation, MIMO performance is same as that of in communication systems. Statistical or distributed MIMO shows better performance against multipath fading than collocated MIMO radar.

MIMO radar may offer several others advantages e.g. higher image resolution, better detection probability, reduction of false alarm and many more. In [27], the authors have shown that MIMO radars have lower false alarm rate than that of its phased array counterpart. Rajdip and Neelakanthan have shown in [28] that MIMO radar has better efficiency in multiple target detection, even if the targets are closely located, than phased array radars.

6 Simulation Results and Discussion

It is very difficult to realize practical systems, especially in communication engineering areas. Radar systems is not an exception also. Moreover, it is one of the most costly systems to realize practically. For that purpose, we always have to make a simulation model to observe how well the system is performing. Similarly, simulating the radar systems environment was also required for this project.

6.1 Simulation Model Description

The simulation results of MIMO application in radar systems is discussed in this chapter. There are several techniques of implementing MIMO, namely, collocated MIMO and distributed MIMO. Collocated MIMO *aka* angular diversity is implemented, by means of receive beamforming, in this simulation. There are no transmitter or receiver impairments in the simulations and the channel consists only of additive white Gaussian noise. The whole simulation is carried out at base-band level. To illustrate MIMO in radar systems, the following parameters are considered:

Table 6.1: Parameters for simulation

Parameter	Value
Number of Pulses, NoP	200
Signal bandwidth	10 MHz
Pulse width	10 μs
Signal to Noise Ratio, SNR	10 dB
Sampling frequency	120 MHz
Normalized Doppler shift of the target	0.1
Number receiving antenna	8

Pulse radar system is simulated for this project. Also, only a single target is considered here.

6.2 Obtained Simulation Results

At the beginning, generation of a complex signal is required. A train of 200 pulses has been considered. The transmitted signal was an LFM (Linear Frequency Modulation) signal. The bandwidth of the signal is 10 MHz. The generated transmitted signal is illustrated in Figure 6.1.

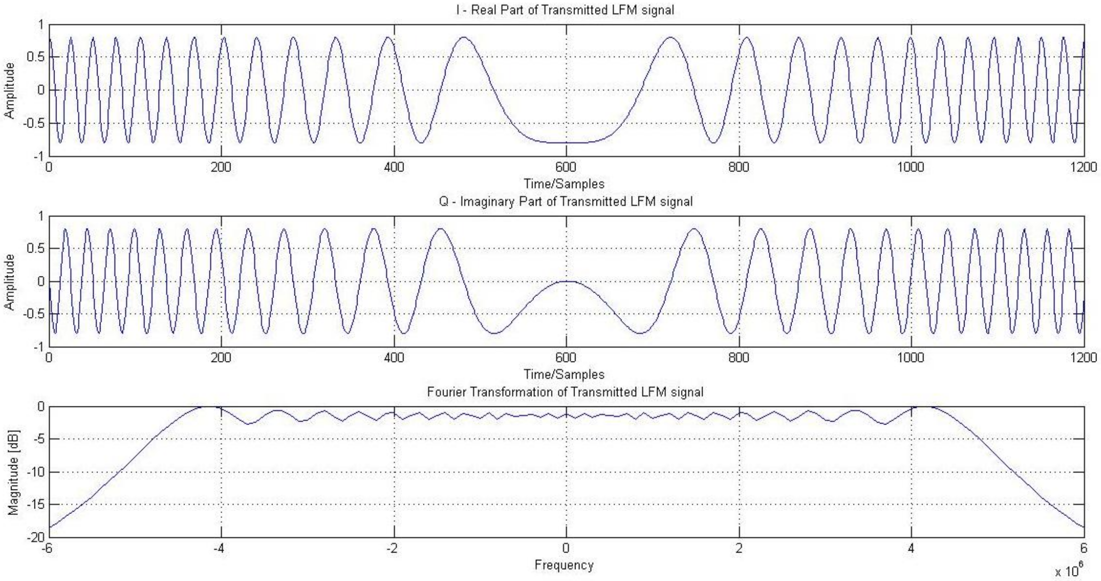


Figure 6.1: Transmitted LFM Signal

The transmitted signal has been reflected by the target and reached at the receiver. During this travel time of the signal, it experiences several type of noises and interferences. Additive White Gaussian Noise (AWGN) has been considered for all types of noise and interferences. There are 8 different receiving elements at the receiver end. The reflected signal is fed through all the receiving elements. Conventional beamforming technique has been implemented in this case. The received signal vector is scaled with a steering vector during the beamforming process. The scaled signal vector is fed to a matched filter. A 3-D plot of range Doppler matrix has been given in figure 6.2.

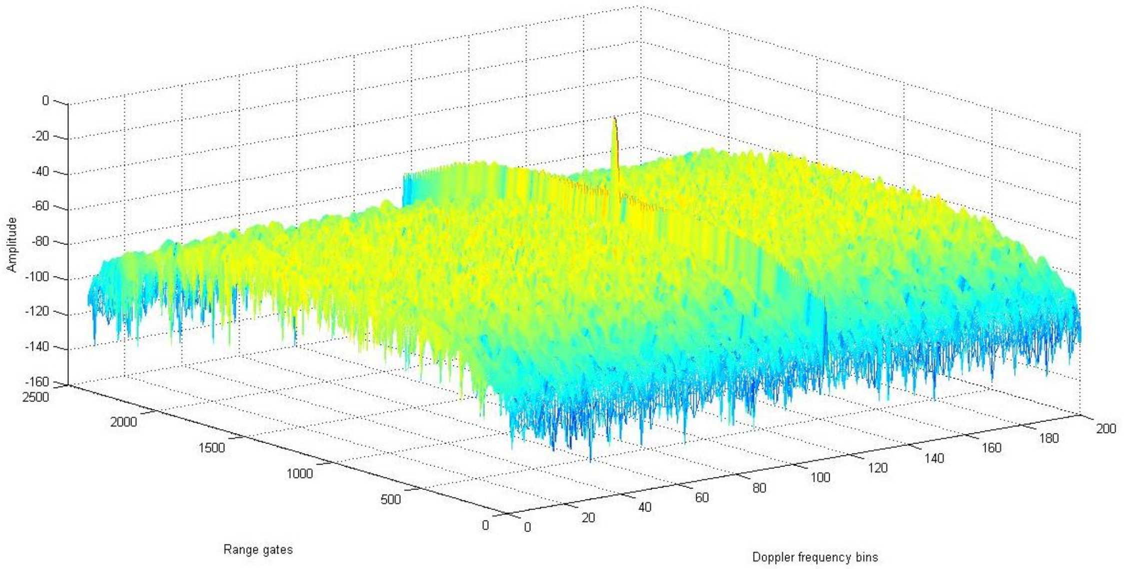


Figure 6.2: Range Doppler matrix of the received signal after matched filter

2500 range gates and 200 Doppler frequency bins are examined for the simulation. Plots for the range Doppler matrix, amplitude response of the Doppler spectrum at range gate number 1250 and amplitude response of all the range gates at certain Doppler region (normalized Doppler frequency 0.1, in this case) has been shown in Figure 6.3.

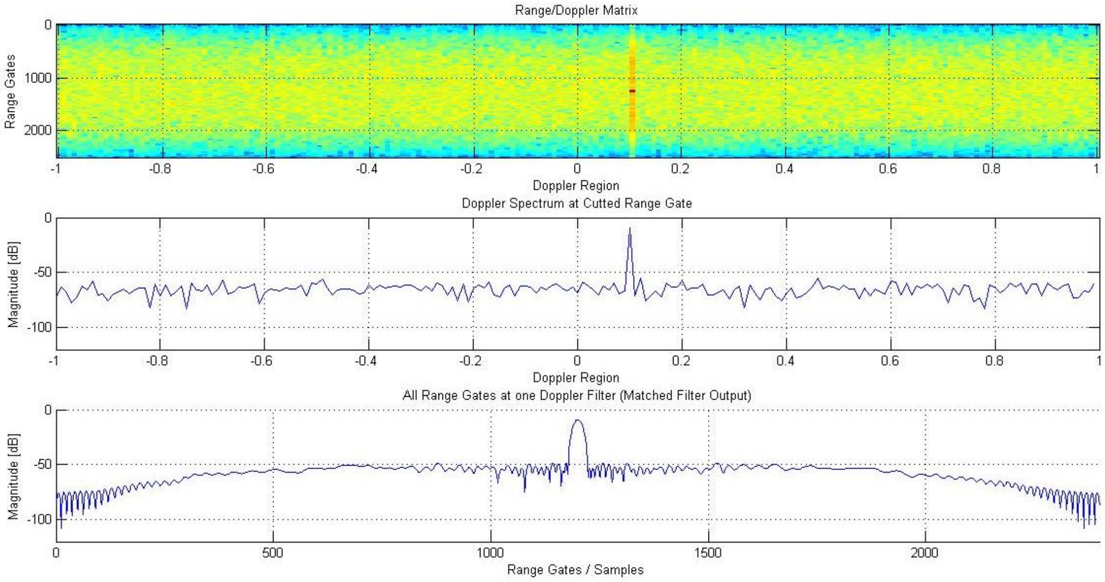


Figure 6.3: 1) Range Doppler Matrix of the received signal; 2) Amplitude response of the Doppler spectrum; and 3) Amplitude response of the range gates at Doppler region

From the received signal after Doppler processing, noise powers are estimated. This has been done so that the Signal-to-Noise ratio (SNR) can be calculated. The simulation has been done for both single antenna system and multi antenna system. For multi antenna system, eight antenna elements have been considered. Figure 6.4 depicts the obtained SNR for both single antenna and multi antenna cases.

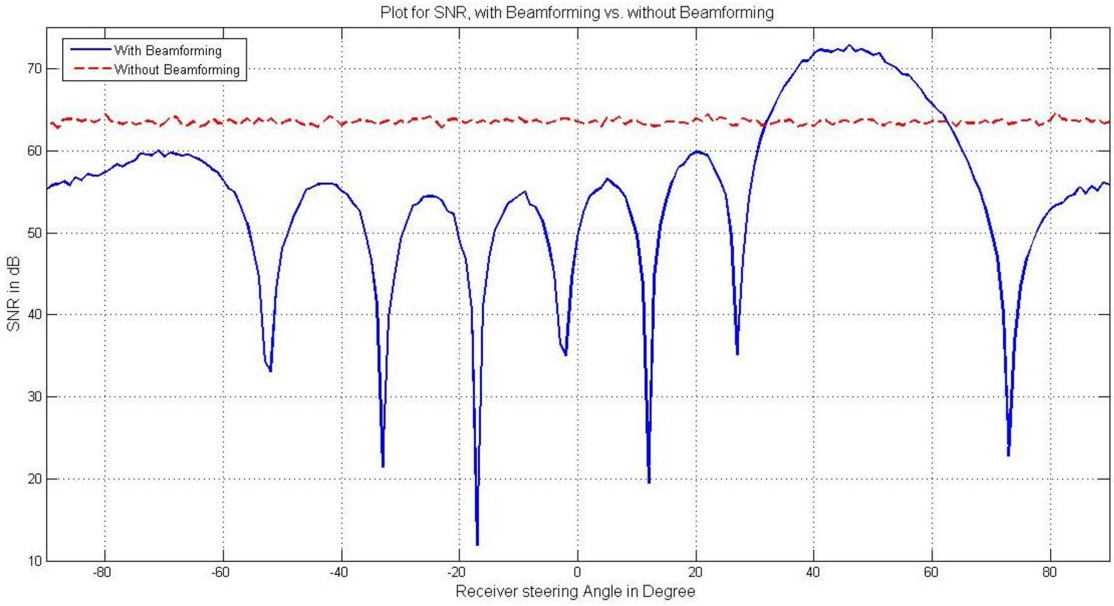


Figure 6.4: Obtained SNR vs Antenna steering angle; with and without beamforming

The receiver look angle is steered between -90 to 90 degrees. The direction of arrival of the received signal is 45 degrees.

6.3 Discussion on Obtained Results

Analyzing the above figures, it can be seen that there is a flat SNR, in every receiver look angle, for single antenna (without beamforming) case. On the contrary, the SNR value varies as a function of receiver look angle when the beamforming is employed. From Figure 6.4, the maximum value of obtained SNR with angular diversity is 73 dB and this value is at the receiver look angle of 45 degrees. At the look angle 45 degrees, the value of SNR, without beamforming, is about 63 dB. So, it results that a 10 dB higher SNR value can be obtained by implementing receive beamforming with eight antennas instead of a single antenna system. This simulation gives maximum SNR value at 45 degrees antenna look angle. This is because the direction of arrival of the received signal is also 45 degrees.

So, it can be concluded from the above discussions and simulation results that by implementing beamforming or even MIMO in radar systems, it is possible to get a higher SNR value. Higher SNR value helps to detect target with higher resolution, more accurate Direction of Arrival (DoA), higher detection probability as well as to reduce false alarm rate. In a word, MIMO in radar systems will enhance the performance of the system in several aspects.

7 Conclusion

The possibilities of implementing Multiple-Input Multiple-Output (MIMO) techniques has drawn the interest of researcher since last few years. Very little amount of research work has been accomplished till now. So, there is a strong possibility of potential future research in this track. In this project, all the simulations are performed in ideal condition i.e. no physical distortion, no 3^{rd} order distortion, no unwanted noise and interference has been considered. It can be a good topic of research to examine behavior of MIMO radar in practical conditions. The waveform optimization techniques, Self-interference mitigation, and MIMO radar system issues are also important research theme for radar.

The performance enhancement of MIMO radar has been discussed theoretically in this thesis. So, it can also be interesting to investigate how the performance parameters are enhanced if MIMO is implemented. As signal processing is used for MIMO joint processing, checking the applicability of existing signal processing algorithm, modification and development of new algorithms are also good area for future research in radar engineering.

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