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Motivated Prospects of Upward Mobility

Master's Thesis

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Abstract

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The prospect of upward mobility (POUM) hypothesis conjectures that the reason why the poor do not expropriate the rich and sometimes seem to vote against their self-interest is that they expect to move upward in the income ladder and fear that the higher redistribution may negatively affect them in the future. This thesis explicitly models the beliefs agents have about their future income and studies how and when these beliefs can be overly optimistic resulting in low redistribution.

The model of motivated prospects of upward mobility is built on a model proposed by Minozzi (2013) and differs from it in that this work adopts a cognitive technology of belief distortion from Bénabou and Tirole (2002). In the model, agents collectively choose a linear tax rate under uncertainty about their exogenous future incomes. In addition to the utility from consumption, agents derive utility from the anticipation of their future consumption. This incentivizes them to distort their beliefs. Given the technology for belief distortion, the motivated prospects of upward mobility emerge endogenously as a result of agents' choices between anticipation and consumption.

When belief formation and voting are strategic, the poor will form overly optimistic beliefs and vote for low taxes if the value of anticipation is high enough and if their optimism does not cause too drastic a change in tax policy. If belief formation and voting are non-strategic, the poor will always indulge in optimism and may even vote against their own best interest. A striking result is that if the incomes of the rich increase as the transfers to the poor stagnate, the poor may demand less redistribution. That is, contrary to the classic benchmark model of Meltzer and Richard (1981), an increase in inequality does not necessarily lead to an increase in demand for redistribution. It is also shown, how Minozzi's (2013) model is a special case of fully naive inference and that Minozzi's results are not robust to Bayesian rational agents.

Lastly, a dichotomy between naive and sophisticated cognitive technologies for endogenous belief distortion in the literature of psychological economics is identified, and a general model of motivated beliefs which brings these various cognitive technologies together and shows how they relate is proposed.

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Hypoteesi ylöspäin suuntautuvan tuloliikkuvuuden mahdollisuudesta tarjoaa selityksen sille, miksi vähätuloinen äänestäjien enemmistö ei äänestä korkeita veroja, vaikka se olisi heidän etujensa mukaista. Hypoteesin mukaan vähätuloiset äänestäjät uskovat heidän tulojensa kasvavan ja pelkäävät että korkea tulovero tulevaisuudessa on heille itselleen vahingollinen. Tämä tutkielma mallintaa agenttien uskomukset heidän tulevaisuuden tuloistaan ja tutkii, kuinka nämä uskomukset voivat olla ylioptimistisia johtaen vähäiseen tulonjakoon.

Tutkielman malli motivoiduista ylöspäin suuntautuvan tuloliikkuvuuden näkymistä rakentuu Minozzin (2013) ehdottaman mallin päälle, mutta eroaa kyseisestä mallista hyödyntäes-sään Bénaboun ja Tirolen (2002) ehdottamaa uskomusten vääristämisen kognitiivista teknologiaa. Mallissa agentit valitsevat kollektiivisesti lineaarisen tuloveron ollessaan epävarmoja tulevaisuuden eksogeenisista tuloistaan. Kulutuksen lisäksi agenteille tuo hyötyä heidän tulevaisuuden kulutuksensa ennakointi, mikä luo heille kannustimen vääristää heidän uskomuksiaan. Motivoidut uskomukset ylöspäin suuntautuvan tuloliikkuvuuden mahdollisuuksista syntyvät endogeenisesti agenttien valitessa kulutuksen ja kulutuksen ennakoinnin tuovan hyödyn väliltä kognitiivisen teknologian asettamien rajoitteiden vallitessa.

Kun uskomusten muodostaminen ja äänestäminen ovat strategista, vähätuloisten äänestäjien uskomukset ovat ylioptimistisia, ja he äänestävät alhaista tulojen uudellenjakoa, jos kulutuksen ennakoinnista tuleva hyöty on tarpeeksi suuri ja jos heidän optimisminsa ei aiheuta liian suurta muutosta veropolitiikassa. Jos uskomusten muodostaminen ja äänestäminen eivät ole strategista, vähätuloisten äänestäjien uskomukset ovat aina ylioptimistisia, ja he voivat jopa äänestää omien etujensa vastaisesti. Yllättävä tulos on, että jos suurituloisten äänestäjien tulot nousevat siten, että tulonsiirrot vähätuloisille äänestäjille pysyvät ennallaan, vähätuloisten äänestäjien kysyntä tulojen uudelleenjaolle voi laskea. Vastoin klassisen Meltzerin ja Richardin (1981) esittämän mallin tuloksia, tämän tutkielman mallissa tuloerojen kasvu voi siis vähentää tulojen uudelleenjaon kysyntää. Tutkielma näyttää myös, kuinka Minozzin (2013) malli on naiivin päättelyn erikoistapaus ja että Minozzin tulokset eivät kestä bayesiläistä rationaalisuutta.

Lopuksi, tutkielma tunnistaa kahtiajaon naiivien ja sofistikoituneiden kognitiivisten teknologioiden välillä psykologisen taloustieteen kirjallisuudessa ja kehittää endogeenisesti motivoitujen uskomusten yleisen mallin. Tämän mallin avulla tutkielma näyttää, kuinka nämä erilaiset kognitiiviset teknologiat liittyvät toisiinsa.

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Notation

Sections 3 and 4

Exogenous variables and parameters

δ	Discount factor.
q	Share of likely rich agents.
s	"Savoring" parameter, measures the importance of anticipatory utility.
$\underline{\tau}$	Lower bound of the allowed tax rates.
$\overline{\tau}$	Upper bound of the allowed tax rates.
y_H	Expected period 2 income of the likely rich agents, mean of F_H.
y_L	Expected period 2 income of the likely poor agents, mean of F_L.
χ	Naivete parameter.

Endogenous variables

c_i	Period 2 consumption of agent i.
$F_{t,i}$	Period t information of agent i.
λ	Awareness rate of the likely poor.
$\underline{\lambda}$	Optimal awareness rate of the likely poor given $\lambda \in \left[0, \frac{1}{2(1-q)}\right)$.
$\overline{\lambda}$	Optimal awareness rate of the likely poor given $\lambda \in \left[\frac{1}{2(1-q)}, 1\right]$.
λ^*	Optimal awareness rate of the likely poor.
s^*	Threshold level in importance of anticipation in case $\tau \in [0,1]$.
s^{**}	Threshold level in importance of anticipation in case $\tau \in [\underline{\tau}, \overline{\tau}]$.
s^{***}	Threshold level in importance of anticipation above which the likely rich are worse off with low taxes.
σ_i	Signal agent i receives in period 0.
$\hat{\sigma}_i$	Signal agent i memorizes (sends) in period 0 and recalls (receives) in period 1.
τ	Income tax rate.
τ^*	Equilibrium tax rate.
τ_i^*	Optimal tax rate for agent i.
y_i	Income of agent i.
$\bar{y}$	Average income.

Others

F_L	Income distribution of the likely poor in period 2.
F_H	Income distribution of the likely rich in period 2.
$\mathcal{F}$	Set of possible income distributions in period 2.
$g(\cdot)$	Probability mass function of the signal agent receives in period 0.
Σ_i	Signal set of agent i.
i	Agent index.
$\iota_{gross}(\lambda \vert \chi)$	Expectation of expected period 2 gross income of the likely poor in period 1 from the point of view of period 0 as a function of λ given χ.
$\iota_{net}(\lambda, \tau)$	Expectation of expected period 2 consumption of the likely poor in period 1 from the point of view of period 0 as a function of λ given τ.
$r(\lambda_i \vert \chi)$	The reliability of received signal as a function of period 0 strategy given χ.
$u_{t,i}(\cdot)$	Intertemporal utility of agent i in period t.
$U_{0,i}^{\lambda'}$	Expected utility of the likely poor in period 0 given choice of $\lambda = \lambda'$.
$U_{0,i}(\lambda)$	Expected utility of the likely poor in period 0 as a function of λ.

Section 5

Exogenous variables and parameters

δ	Discount factor.
θ_H	High type in the simple game of motivated beliefs.
θ_L	Low type in the simple game of motivated beliefs.
s	"Savoring" parameter, measures the importance of anticipatory utility.
q	Probability of being of type θ_H in the simple game of motivated beliefs.
χ	Naivete parameter.

Endogenous variables

β_i	Behavioral strategy of Self 1 of agent i.
β_i^*	Optimal behavioral strategy of Self 1 of agent i.
γ	Probability of choosing $\hat{\sigma} = \theta_L$ given signal $\sigma = \theta_H$ in the simple game of motivated beliefs.
$F_{0,i}$	Information of Self 0 of agent i in period 0.
$F_{1,i}$	Information of Self 1 of agent i in period 1.
θ_i	Type of agent i.
λ	Probability of choosing $\hat{\sigma} = \theta_L$ given signal $\sigma = \theta_L$ in the simple game of motivated beliefs.
$\hat{\sigma}_i$	Pure action of Self 0 of agent i.
ϕ_i	Mixed strategy of Self 0 of agent i.
ϕ_i^*	Optimal mixed strategy of Self 0 of agent i.

Others

A	Set of actions.
$A(h)$	Set of actions available in history h.
$\mathcal{A}(h)$	Set of lotteries over the set $A(h)$.
β	Profile of strategies of Self 1s.
B_i	Set of behavioral strategies available to Self 1 of agent i.
Γ	General extensive form game.
Γ'	An extensive form game with perfect information and chance moves.
$\Gamma(\sigma_i)$	A game with heterogeneous beliefs where the players are Self 1s of each agent and agent i's beliefs over θ are given by the distribution $G(\theta\|\sigma_i)$.
$g_i(\sigma)$	Probability of Self 0 of agent i receiving signal σ.
$G(\cdot)$	Joint probability distribution of types.
$G(\cdot\|\sigma)$	Distribution over θ conditional on the information of signal σ.
$\mathcal{F}_{0,i}$	Set of possible posterior distributions for Self 0 of agent i of θ in period 0.
H	Set of histories.
θ	Vector of types.
Θ	Set of possible types.
I	Information partition.
N	Set of agents.
$p_i(\cdot\|\hat{\sigma},\chi)$	Probability weights of nonrecalled signals implied by Bayes Rule.
P	Player assignment function.
$r(\cdot\|\hat{\sigma},\chi)$	Belief system that assigns a probability to each original signal when recalling signal $\hat{\sigma}$.
r	Profile of belief systems.
σ	Profile of signals Self 0s receive.
σ_i	Signal function of agent i, signal Self 0 of agent i receives.
Σ_i	Signal set of agent i.
$\hat{\sigma}$	Profile of signals Self 1s recall.
$u(\beta,\theta)$	Utility function representing preferences over terminal histories in Γ.
$u_{1,i}(\beta,\theta)$	Utility function representing Self 1's preferences over terminal histories in the continuation game.
$u_{0,i}(\beta,\theta)$	Utility function representing Self 0's preferences over terminal histories in the pre-game.
$U_{0,i}(\phi,\sigma)$	Expected utility of Self 0 as a function of strategy profile of Self 0s and given information implied by signal σ.
ϕ	Profile of strategies of Self 0s.
Φ_i	Set of mixed strategies of Self 0 of agent i.
$\mathcal{P}(\cdot)$	Power set.
Ω	Set of possible states of the world.

1 Introduction

Fere libenter homines id quod volunt credunt. (Men readily believe what they want to believe.)

— Julius Caesar*, Commentarii de Bello Gallico (Commentaries on the Gallic War)*

The prospect of upward mobility (POUM) hypothesis conjectures that the reason why the poor do not expropriate the rich and sometimes seem to vote against their self-interest is that they expect to move upward in the income ladder and fear that the higher redistribution may negatively affect them in the future. This work attempts to formalize the POUM hypothesis by explicitly modeling the voters' beliefs about their prospective incomes. Under certain conditions, enough of the poor believe that they will be rich in the future and the electorate chooses low redistribution.

Previously, the POUM hypothesis has been formalized by Bénabou and Ok (2001). They show that under favorable income dynamics, it is possible that more than half of the voters have an above average expected future income. As a result, more than half of the voters prefer low distribution and vote accordingly. While, according to empirical evidence, both perceived upward mobility (Ravallion and Lokshin, 2000; Cojoracu, 2014) and actual upward mobility (Alesina and La Ferrara, 2004; Alesina and Giuliano, 2009; Checchi and Filippin, 2003; Bénabou and Ok, 2001) seem to decrease voters' demand for redistribution, it also seems that perceived mobility and actual mobility do not necessarily correlate (Fischer, 2009; Alesina, Glaeser, and Sacerdote, 2001; Gottschalk and Spolaore, 2001). The puzzle then, and what the model in Bénabou and Ok (2001) fails to explain is why prospects of upward mobility decrease the demand for redistribution even in the absence of actual upward mobility. For instance, in the US, the perceived upward mobility is higher than in Europe, producing a higher POUM effect while there does not seem to be much difference in actual upward mobility across the Atlantic (Alesina, Glaeser and Sacerdote, 2001; Gottschalk and Spolaore, 2001). In addition, as noted by Alesina and Giuliano (2009) and Minozzi (2013), the assumptions underlying the model of Bénabou and Ok (2001) are restrictive and empirically implausible. Therefore, Alesina and Giuliano (2009) suggests that a more plausible mechanism for the POUM effect could be over-optimism. This suggestion is supported by a vast literature in experimental psychology on overconfidence (Weinstein, 1980; Moore and Healy 2008; Alicke and Govorun, 2005).[1] Cruces, Perez-Truglia, and Tetaz (2012) find evidence on how the relatively poor tend to overestimate their current position in the income distribution. It would not be unexpected to observe a similar pattern when it comes to their future incomes.

A formalization of the POUM hypothesis which lets voters have overly optimistic beliefs about their future incomes is provided by Minozzi (2013). In Minozzi's model, citizens vote on future redistribution under uncertainty over their future incomes. When expecting their future consumption, they enjoy anticipation which incentivizes them to hold optimistic beliefs. The weakness of this model is, however, in its naive technology of belief distortion, which allows citizens to effectively decide what to believe and leaves them with no doubts of whether their beliefs truly represent the reality. This might be too simplistic an assumption and potentially misses important mechanisms of belief distortion as argued by Bénabou and Tirole (2002).

The present work attempts to address these problems in the previously proposed models. The basic structure of our model is similar to Minozzi's (2013) model: When voting for a tax rate according to which the future incomes will be redistributed, agents have uncertainty over their future incomes. After voting and before the realization and redistribution of their incomes, they anticipate their future consumption. This anticipation creates an incentive to form overly optimistic beliefs. The departure of the current work from Minozzi's (2013) model is most notably in the technology that agents use to distort their beliefs. The cognitive technology for belief distortion in the current work is adopted and adapted from Bénabou and Tirole (2002) and generalized such that we are able to analyze a whole continuum of cognitive technologies varying in the constraints they impose on belief distortion. The conditions for the POUM effect are derived for each of these cognitive technologies, and it is shown that for a set of cognitive technologies the poor prefer optimism and low taxes over realism and high taxes. Furthermore, in addition to strategic belief formation and voting, we consider sincere belief formation and voting as well, and show that when the voters do not think that their beliefs and voting have a significant effect on the tax policy, they always indulge in optimism and may end up making nonoptimal decisions for themselves.

[1] See also references in Weinberg (2009).

The literature on psychological economics has proposed a number of ways of modeling biases in beliefs as an optimization problem where the trade-off between the benefits and costs of holding biased beliefs is balanced (Akerlof and Dickens, 1984; Bénabou and Tirole, 2002, 2016; Bénabou, 2015; Minozzi, 2013; Brunnermeier and Parker, 2005). The general cognitive technology for belief distortion presented in this work allows us to see how these different cognitive technologies in the literature which have not so far been analyzed together, relate to each other in a simple way: They represent the polar specifications of the strategic sophistication of agents and the dependence of beliefs on prior beliefs and the objective reality.

This work aims to make the following contributions: (1) As a contribution to the literature of political economy and redistribution, it formalizes the POUM effect by explicitly modeling the overly optimistic beliefs of voters and analyzes the conditions that are required for the POUM effect to occur. It also shows how the results of a similar model in Minozzi (2013) are not robust to a more realistic cognitive technology which does not allow the voters to simply choose their beliefs, but takes account the constraints of belief distortion. (2) As a contribution to the literature of psychological economics, it brings together the various technologies for belief distortion in the partly independent strands of literature and shows how they relate. Also, a general cognitive technology which allows us to consider a range of assumptions about the strategic sophistication of agents and the constraints of biased beliefs is provided.

The rest of the work is organized as follows. In section 2, we briefly position the current work into the existing literature in political economy and psychological economics. Section 3 presents the model and derives the conditions for the POUM effect. Also, Minozzi's POUM model is derived as a special case, and its shortcomings are addressed. Section 4 extends the analysis of the model by studying the comparative statistics of changes in the underlying income distribution, presents some welfare analysis and considers the case of nonstrategic belief formation and voting. In section 5, the modeling concepts and analytic tools used to model motivated endogeneous beliefs are reviewed, a more general version of the cognitive technology is presented, and the connection between naive and sophisticated technologies is made. Section 6 concludes. All proofs of the lemmas and propositions are collected in the appendix.

2 Relations to the literature

2.1 Political Economy and Redistribution

If the rational choice model with narrowly defined utility together with the Median Voter Theorem cannot be corroborated by empirical observations, either of these underlying assumptions, rational choice or median voter's power, must be wrong. It might either be the case that modeling voters as income maximizing agents does not capture all the relevant aspects of their decision making or that the outcome that the electoral system provides does not reflect the preferences of the median voter. Reasons for the latter could be, for instance, unequal political participation (Bénabou, 2000; Mahler, 2008), the political influence of the rich (Gilens, 2005), campaign contributions (Baremboim and Karabarbounis, 2008; Campante, 2007; Rodriguez, 2004), economic inequality (Lupu and Pontusson, 2011; Solt, 2008), electoral systems (Iversen and Soskice, 2006; Cukierman and Spiegel, 2003; Austen-Smith, 2000), and interest groups (Dixit and Londregran, 1998).

In this work, the policy outcome is assumed to be the median voter's bliss point and the focus, therefore, is on the former of these possible caveats. Hence, this work can be positioned into the strand of literature initiated by Romer (1975) and Meltzer and Richard (1981), which aims to explain the extent of redistribution in democratic societies by studying what determines the voters' demand for redistributive policies. To ensure the existence of political equilibrium, this literature mostly focuses on unidimensional policy choices, usually choices over a linear tax rate with lump-sum transfers. With this simplification, the policy preferences of voters are single-crossing, and the median voter theorem applies. The remaining question then, and the interest of this literature is how does the median voter decide on her vote.

The obvious starting point is the voter's current income, but preferences so narrowly defined have been unsatisfactory in explaining real-world tax policies (Bénabou, 1996; Borck, 2007; Luebker, 2014). Other factors explaining the demand for redistribution proposed in this literature are, for instance, efficiency costs of taxation (Meltzer and Richard, 1981), different individual (Piketty, 1995; Giuliano and Spilimbergo, 2008) and cultural (Corneo and Gruner, 2002; Alesina and Glaeser, 2004) histories and experiences, social preferences such as altruism, inequality aversion and fairness considerations (Alesina and Glaeser, 2004; Fong, 2001; Alesina and Angeletos, 2005a; Alesina et al. 2012), structure and organization of the family (Todd, 1985; Esping Andersen, 1999; Alesina and Giuliano, 2007), and social mobility (Piketty, 1995; Hirschman and Rothschild, 1973; Bénabou and Ok, 2001).[2] In addition to increasing the scope of preferences, the literature has also studied the role of beliefs (Piketty 1995, Alesina and Angeletos, 2005a) and biased beliefs (Minozzi, 2013; Bénabou and Tirole, 2006; Bénabou, 2008). Given this rich set of explanations for the extent of redistribution, a parsimonious model seems unlikely, and a single effect should be interpreted as a part of the story, complementing and rivaling the other explanations. The part of the story we focus from now on in this work is the POUM effect.

First, social mobility, broadly speaking, refers to both upward and downward mobility. The premise is that instead of current income, the policy preferences depend on future income. When voters are worried that their incomes might decrease relative to others, they could use redistribution as insurance against downward mobility. This would increase the demand for redistribution. The POUM, on the other hand, focuses on the possibility of upward mobility, which has the opposite effect: When the voters expect their incomes to increase relative to others, they vote for less redistribution.

However, social mobility is also often connected to the roles of luck, circumstances, and effort in determining income. If the voters perceive that the effort one exerts determines one's prospects, then they can believe in a mobile society, but if they believe that the circumstances have a major role in determining one's prospects, then they believe in immobile society. Piketty (1995) studies how the interaction of social mobility and beliefs about determinants of income affects voting. In the present work, incomes are exogenous and, in the spirit of the POUM hypothesis, beliefs about social mobility refer solely to beliefs about future incomes.

The first characterization of the POUM effect is perhaps Hirschman's (1973) "tunnel effect" in which people's demand for redistribution decreases when they see the incomes of relatable people in their environment increase. They expect that their turn will follow soon and they, therefore, tolerate more inequality.

The first formalization of the POUM effect was provided by Bénabou and Ok (2001).

[2] A review on the preferences for redistribution is provided by Alesina and Giuliano (2009).

Their approach is to maintain rational expectations and show that favorable income dynamics can make more than half of the voters to expect above-average incomes. The agents vote for a redistribution policy, which will be in place for a predetermined time, and expect their incomes to evolve according to a stochastic transition function. The deterministic part of this transition function is concave, which allows a majority of voters to believe that they will receive an above average income in future. The stochastic part consists of skewed income shocks, which ensure that the skewness of the original income distribution is preserved. The combination of skewed shocks and concave prospects lets the expected incomes and realized incomes diverge and makes the POUM effect possible with invariant income distribution and rational expectations.

Minozzi (2013) develops an "Endogenous Beliefs Model" and proposes an explanation for the POUM effect by abandoning rational expectations and letting voters form overly optimistic prospects about their future income. Minozzi's model relies on a game theoretic multi-self approach, where each citizen has without their knowledge an "agent", who controls their beliefs and optimizes the trade-off between optimistic beliefs and nonoptimal actions. Citizens receive an anticipatory flow utility in period 1 and a flow utility called *outcome utility* in period 2, when they receive their stochastic and exogenous incomes. The agent's objective function for belief formation consists of these two sources of utility. In choosing the optimal beliefs by solving the trade-off between anticipatory and outcome utility, the agent knows the prior prospects of the citizen and how the tax policy is dependent on the chosen beliefs. If the poor citizens value anticipation enough, they will end up with optimistic beliefs and vote for low redistribution.

The POUM effect also emerges in the model of Bénabou and Tirole (2006). In their model agents compensate for their imperfect willpower by motivating themselves with overly optimistic beliefs about their productive ability. When they believe themselves to be abler than others, they prefer less redistribution. Although their model, as the present work, derives the POUM effect by letting agents hold overly optimistic beliefs, their work differs from the current in its mechanism for the belief distortion. Specifically, what incentivizes the agents to hold biased beliefs differs. However, these different incentives are not mutually exclusive, and probably both are at work. The explanation for the POUM effect in Bénabou and Tirole (2006) should, therefore, be seen as a complementary to the current work.

2.2 Psychological Economics and Motivated Beliefs

Psychological economics attempts to draw inspiration from the field of psychology and build models that better represent the cognitive processes of decision makers aiming to close the apparent gap between the observed behavior of people and the behavior postulated by the rational choice theory. The rational choice theory is, however, the primary method of analysis in economics and the work in psychological economics, rather than abandoning this theory, proceeds by widening its scope.[3] The current work broadens the rational choice theory to accommodate psychological factors in two ways. First, we widen the scope of preferences to include anticipation of future consumption. Second, we let agents make optimal decisions about their beliefs.

Anticipatory utility is perhaps little used but certainly not a new idea in the literature of economics: "*When calculating the rate at which future benefit is discounted, we must be careful to make allowance for the pleasures of expectation*", writes Alfred Marshall in his *Principles of Economics* published in 1891 (p. 178, quoted in Löwenstein (1987)). Our mind is both an information processing machine by which we make our decisions and a consuming organ deriving satisfaction from our emotions, as Thomas Schelling (1987) put it. That is, we use our beliefs to predict the consequences of our actions, but we also consume them. Due to this latter function of beliefs, we derive utility or incur disutility simply by believing certain things. As experiments have shown, this consumption value of beliefs has consequences for our information processing (Kunda, 1990; Averill and Rosenn 1972; Lerman et al. 1998) and our behavior (Cook and Barnes 1964; Löwenstein 1987).

Anticipatory utility is modeled usually by letting the utility function have a term which is a linear (Minozzi, 2013; Bénabou, 2008, 2013; Brunnermeier and Parker, 2005) or a general (Caplin and Leahy, 2001; Köszegi, 2010; Bernheim and Thomadsen, 2005) function of expectation of a later period utility flow. In Akerlof and Dickens (1982), agents incur psychic costs of fear modeled as a "fear cost function" which depends on the perceived probability of an accident in their hazardous job.

In addition to preferences, the second important element of decisions in an uncertain

[3] On psychological economics, see, for instance, Rabin (2002) and Tirole (2002).

world is beliefs. Hence, to understand decisions, it is crucial to understand beliefs. The departure from rational expectations is motivated by vast literature in psychology (Weinstein, 1980; Moore and Healy, 2008; Alicke and Govorun, 2005) and behavioral economics (DeBondt and Thaler, 1995; Skala, 2008). In addition to challenging the objectivity of beliefs, the literature in psychology directs us towards the alternative options: Biases in beliefs are not random, they seem to be incentivized and partly determined by desires (Kunda, 1990; Braman and Nelson, 2007; Redlawsk, 2002; Taber and Lodge, 2006). This literature of motivated reasoning asserts that human information processing, memories, and beliefs are affected by our motivations. In addition to *accuracy goals*, reasoning can be motivated by *directional goals*, that is, by desires and preferences.

The literature on motivated reasoning has inspired models of biased beliefs where the beliefs are a result of optimizing the trade-off between accuracy goals and directional goals. Anticipatory utility is one way to model such a directional goal for reasoning, but a complete model also requires the means for belief distortion. We call a *cognitive technology* a framework which provides the agents with the ways and constraints of distorting their beliefs. There are roughly two kinds of cognitive technologies used in the literature. In the first of these which we will call *naive cognitive technologies*, the beliefs can be simply chosen, and they do not need to depend on the prior beliefs or the objective probability distributions of reality. For instance, Minozzi (2013), Brunnermeier and Parker (2005), and Akerlof and Dickens (1982) use a naive cognitive technology. We call the second kind of cognitive technology a *sophisticated cognitive technology*. If the cognitive technology is sophisticated, agents realize that they have incentives to bias their beliefs and assess their beliefs accordingly. Also, the emerging beliefs are influenced by the prior beliefs and are anchored to the reality. This second type of cognitive technology is used in Bénabou and Tirole (2002, 2006), Bénabou (2008, 2013), and Kopczuk and Slemrod (2005), and reviewed in Bénabou (2015) and Bénabou and Tirole (2016). The names for these two types of cognitive technologies follow from their different assumptions on the agents' degree of Bayesian sophistication.

The present work attempts to close the gap between these two strands of literature and show how these two types of cognitive technologies represent the extreme cases of one general framework. A cognitive technology that best represents the information processing of a human being is likely to be found in between these extremes. The framework of this work allows us to study the implications of our model under the whole continuum of cognitive technologies varying from the completely naive technology to the completely sophisticated.

Minozzi (2013) calls the nonstandard beliefs that emerge in his model *endogenous* beliefs whereas Bénabou (2015) refer to these beliefs as *motivated* beliefs.[4] In this work, these terms are used interchangeably. However, the term *motivated* beliefs is more informative. After all, all beliefs that are determined within a model, can be called endogenous. For instance, in this sense, the usual rational expectations are endogenous beliefs as well.

To sum up, a model containing belief distortion has two crucial elements. First, agents must have an incentive to hold biased beliefs. Using the language of Bénabou and Tirole (2002), this can be called the demand for distorted beliefs. In the current work, agents are incentivized to have biased beliefs by letting them derive utility from their high hopes. Second, agents must be able to influence their beliefs. This can be called the supply of distorted beliefs. The current work considers the whole continuum of cognitive technologies from the complete naive to the fully sophisticated, that make the belief distortion possible. Given the incentives and the technology of belief formation, biased subjective beliefs emerge as a result of optimization. This optimization involves trading-off the benefits of holding biased beliefs against the costs of inferior decisions due to inaccurate information and is subject to the constraints of the cognitive technology. The emergence of non-standard beliefs as a result of optimization and purposeful actions distinguishes the motivated beliefs framework from the mechanical failures of rationality or bounded rationality, which leave the motivations of actions intact and only impose constraints on reasoning (Bénabou and Tirole, 2016).

[4]Brunnermeier and Parker (2005) call them optimal beliefs.

Period 0	Period 1	Period 2
Receive signals σ Choose λ	Recall $\hat{\sigma}$ and form beliefs Vote for redistribution Anticipation	Incomes realize Redistribution Consumption

Figure 1: Timeline

3 The Model

3.1 The Economy and the Timing of the Model

The economy consists of a unitary continuum $i \in [0, 1]$ of risk-neutral agents who collectively decide on a tax policy under uncertainty about their exogenous future incomes. In period 0, agents receive a signal conveying information about their prospective future incomes. In period 0, they also engage in various conscious and unconscious psychological processes of belief distortion, reality denial, and information avoidance which determine the signal they will remember in period 1.[5] In the beginning of period 1, agents recall a signal and form beliefs about their future incomes based on what they recall. Then they vote for redistribution. They get to know the policy outcome immediately after the vote, and in the rest of period 1 they experience anticipatory utility as they anticipate their consumption which occurs in period 2 right after the incomes have realized and redistributed. The timeline is given in Figure 1.

3.2 Information and Beliefs

In period 0, each agent receives a noisy signal $\sigma_i \in \mathcal{F} = \{F_L, F_H\}$ conveying information about their future incomes. These signals are identical and independent draws from the

[5]Agents have imperfect recall in the sense that they forget information. The underlying game theoretical construct to model this inconsistency is to model agents consisting of two players, their two temporal selves. See section 5. Also, the parallel interpretation throughout the paper is that the parents have influence over what their offsprings belief when the offsprings are making voting decisions.

following probability mass function:

$$g(\sigma) = \begin{cases} q & \text{if} \quad \sigma = F_H \\ 1-q & \text{if} \quad \sigma = F_L \end{cases}, \tag{1}$$

where F_H and F_L are probability distributions over the future income levels such that $\int_y ydF_H(y) > \int_y ydF_L(y)$ and $y \geq 0$.[6] Using the language of Minozzi (2013), we call the agents who receive signal $\sigma = F_H$ the likely rich and the agents who receive signal $\sigma = F_L$ the likely poor. With a large number of agents, a fraction q of the population is likely rich and a fraction $1-q$ likely poor. Furthermore, we assume that the likely poor agents constitute a majority, that is, we assume $q < \frac{1}{2}$. As agents are risk-neutral, a sufficient statistics for the analysis are the means of distributions F_H and F_L: $y_H = \int_y ydF_H(y)$ and $y_L = \int_y ydF_L(y)$, the incomes that the likely rich and the likely poor, respectively, expect to earn in period 2. In the following, we refer to these distributions by their means and let the signal set be $\{y_L, y_H\}$.[7]

The possibility for belief distortion arises in the period 0 actions. After receiving a signal, each agent decides which of the two signals she will recall in period 1. As we will see, a likely poor agent has an incentive not to recall her true prospects. On the other hand, we make a sensible assumption, that the likely rich agents will always choose to remember the signal they received and they, therefore, have no interesting decision to analyze. After all, if they underestimate their income, they lose anticipatory utility.[8] We

[6]Here the signals are independent for simplicity and to induce some heterogeneity in the resulting income distribution. In general, the signals may be correlated. The special case of perfectly correlated types and signals can be used if the unknown variable is more common to agents in the sense that it reflects some general workings of the economy, like return to effort as in Bénabou and Tirole (2006), government efficiency as in Bénabou (2008) or expected value of a joint project as in Bénabou (2013).

[7]We use a simplifying shortcut here. The underlying formal process, of course, is that Nature draws a state of the world, which determines the incomes of each agent. Agents receive some information about the state of the world via a signal determined by a signal function which lets them know a set of states of the world. Using the prior belief and the signal they then form a posterior belief. The posterior belief is, therefore, a function of the signal and fixed prior beliefs, so it is straightforward to associate a signal with a posterior belief and let the outputs of the signal function be the posterior beliefs agents have immediately after receiving the signal. Moreover, as the signal is a deterministic function of the state of the world, which Nature draws, we can simply let the received signal have the given distribution. See section 5 for a more technical and general presentation of the cognitive technology.

[8]This seems a very plausible conjecture but technically this is not that simple. Depending on the off-equilibrium path beliefs, an agent sending a low signal might end up with higher beliefs than when sending a high signal. In the appendix, we make an assumption about these off-equilibrium path beliefs to exclude this peculiar theoretical possibility.

focus mainly on the more interesting decisions of the likely poor agents. Formally, in period 0, a likely poor agent i chooses a recall or an awareness rate $\lambda_i \in [0,1]$ defined as

$$\lambda_i \equiv Pr[\hat{\sigma}_i = y_L | \sigma_i = y_L], \tag{2}$$

where $\hat{\sigma}_i$ denotes both the signal agent i recalls in period 1 and the action she chooses in period 0.[9]

In period 1, agent i's information is based on a recalled signal $\hat{\sigma}_i \in \{y_L, y_H\}$. The memory of agents is probabilistic and their actions in period 0 determine the probability of each recollection. With probability λ_i, a likely poor agent will correctly recall $\hat{\sigma}_i = y_L$ and with probability $1-\lambda_i$, she will recall $\hat{\sigma}_i = y_H$. By assumption, the likely rich agents always recall $\hat{\sigma}_i = y_H$. Of course, we are not claiming that people literally choose exact probabilities for the occurrences of their future memories. The choices in period 0 should be interpreted as all sorts of unconscious and conscious processes and actions that affect the availability of certain recollections. In equilibrium, agents act as if they were choosing optimal recall rates.

However, agents may not be completely in control of their beliefs. They may know that they have a tendency to forget bad news and remember good news. Therefore, they may not fully trust their recollections. If an agent i recalls $\hat{\sigma}_i = y_H$ in the second period, she will assign a reliability $r(\lambda_i)$ to this signal:

$$r(\lambda_i|\chi) = Pr[\sigma_i = y_H|\hat{\sigma}_i = y_H] = \frac{q}{q + \chi(1-q)(1-\lambda_i)}, \tag{3}$$

where λ_i is given by the period 0 strategy of agent i. χ is the naivete parameter measuring the degree of Bayesian sophistication. $\chi = 1$ corresponds to the full Bayesian rationality which is usually assumed in the applications of game theory.[10] In the other extreme, $\chi = 0$, and the reliability of received signal is always 1. This means that in period 1, agents will completely trust their recollections and that in period 0, they are completely in control of their beliefs in period 1. The role of χ will be analyzed extensively later.

[9] In the jargon of game theory, in period 0, an agent i plays a mixed strategy $\begin{pmatrix} y_L & y_H \\ \lambda_i & 1-\lambda_i \end{pmatrix}$.

[10] Bayesian rationality refers to the use of Bayes rule in updating beliefs.

Note that the reliability in (3) is defined only for the signal $\hat{\sigma}_i = y_H$. By assumption, only the likely poor might send a signal $\hat{\sigma} = y_L$, so the reliability of this signal is always 1.

With probability $1 - \lambda_i$, a likely poor agent recalls $\hat{\sigma}_i = y_H$ and is an *optimist*. In period 1, she expects a gross income

$$E[y_i|F_{1,i}] = r(\lambda_i)y_H + (1 - r(\lambda_i))y_L, \tag{4}$$

which is a linear combination of the expected incomes of the two different types weighted by the reliability. $F_{1,i}$ is the information of agent i in period 1. Note how a decrease in λ_i increases the probability of being an optimist and, as we will see, the expected anticipatory utility. However, the effect is nonlinear for $\chi > 0$ since the reliability decreases as λ_i increases. The more likely it is that a likely poor agent i memorizes a false signal, the less reliable signal $\hat{\sigma}_i = y_H$ becomes. The more agents try to distort their beliefs, the more cautious they are when they are forming their beliefs.

With probability λ_i, a likely poor agent recalls $\hat{\sigma}_i = y_L$ and is a *realist*. As the reliability of signal $\hat{\sigma}_i = y_L$ is always 1, in period 1, she expects a gross income

$$E[y_i|F_{1,i}] = y_L. \tag{5}$$

The likely rich will recall $\hat{\sigma}_i = y_H$, and as they also do not know whether they truly are likely rich or likely poor, their expected income will coincide with the expected income of optimistic likely poor.

3.3 Preferences

In period 2, agents receive an exogenous income, pay taxes, and consume their disposable income. The government's budget is balanced, and all tax revenue collected via a linear income tax is transferred in equal lump-sums to agents. There is no wastage in the redistribution. Agents derive utility linearly from their consumption:

$$u_{2,i}(c_i) = c_i(\sigma_i, \tau) = (1 - \tau)y_i + \tau\bar{y}, \tag{6}$$

where c_i denotes consumption, τ is the income tax rate, and $\bar{y}$ is the average income:

$$\bar{y} = qy_H + (1-q)y_L. \tag{7}$$

In period 1, agents do not yet know their income, but given their beliefs, they form expectations and experience a flow utility due to anticipation. The intertemporal preferences of agents from the perspective of period 1 are given by

$$u_{1,i}(\hat{\sigma}_i, \tau) = sE[u_{2,i}|F_{1,i}] + \delta E[u_{2,i}|F_{1,i}] = (s+\delta)E[(1-\tau)y_i + \tau\bar{y}|F_{1,i}], \tag{8}$$

where the expectations are conditioned on the period 1 information $F_{1,i}$, $\delta \in [0,1]$ is the standard discount factor and $s \geq 0$ is the "savoring" parameter which measures the importance of anticipation. The anticipatory utility is proportional to agent's expectations. The higher expectations she has, the more utility she derives. This gives agents an incentive to distort their beliefs. Setting $s = 0$ yields the standard case with no anticipatory utility and therefore no incentive to distort beliefs. The discount factor and the savoring parameter are common to all agents.

The intertemporal utility from the period 0 perspective is

$$\begin{aligned} u_{0,i}(\sigma_i, \hat{\sigma}_i, \tau) &= \delta E[sE[u_{2,i}|F_{1,i}]|F_{0,i}] + \delta^2 E[u_{2,i}|F_{0,i}] \\ &= \delta s E[(1-\tau)y_i + \tau\bar{y}|F_{1,i}] + \delta^2 E[(1-\tau)y_i + \tau\bar{y}|F_{0,i}]. \end{aligned} \tag{9}$$

The expected period 1 flow utility depends on the information in period 1 and the expected period 2 flow utility depends on the information in the period 0.[11] That is, in period 0, agents know the true objective expectation of their incomes in period 2, but they also know that they will receive higher utility in the period 1 if their beliefs in period 1 are biased upwards. The trade-off, which the optimal period 0 actions optimize, can be seen clearly here. Agents gain more utility if they have high hopes, but as we will see, with high hopes they will vote for low taxation, which then lowers their consumption in the last period.

[11] Note that since information is lost between periods 0 and 1 and $F_{1,i}$ contains less information than $F_{0,i}$ the law of iterated expectations does not hold and $E[sE[u_{2,i}|F_{1,i}]|F_{0,i}] \neq sE[u_{2,i}|F_{0,i}]$, but the smaller information set wins and $E[sE[u_{2,i}|F_{1,i}]|F_{0,i}] = sE[u_{2,i}|F_{1,i}]$.

3.4 The Polity and Voting Decisions

The agents vote for tax rate $\tau \in [\underline{\tau}, \overline{\tau}]$ in the beginning of period 1. Their policy preferences are given by (8), and they depend on the subjective beliefs they have in period 1. Maximization with respect to the tax rate leads to the following voting rule:[12]

$$\tau_i^* = \begin{cases} \underline{\tau} & \text{if} \quad E[y_i|F_{1,i}] \geq \bar{y} \\ \overline{\tau} & \text{if} \quad E[y_i|F_{1,i}] < \bar{y} \end{cases}, \tag{10}$$

where τ_i^* is the preferred tax rate of agent i. If an agent expects in period 1 to earn an above average income in the period 2, she will vote for the minimum redistribution, and if she expects to earn a below average income, she will vote for the maximum redistribution. This parallels the classic result of Meltzer and Richard (1981). The linearity of the policy preferences leads to corner solutions, which simplifies the analysis here. In reality, there are, of course, additional considerations that restrict the tax policies between complete equalization and complete laissez-faire. As we will see, setting $\overline{\tau} < 1$ and $\underline{\tau} > 0$ allows us to exogenously restrict the set of feasible tax policies.

As the policy preferences given by (8) are single-peaked, the Median Voter Theorem (Black, 1948; Downs, 1957) applies and the tax policy will be the tax rate preferred by the median voter. With two groups of voters, the median voter's opinion will be the opinion of the majority.

If agents could not manipulate their expectations or if they did not have any incentives to distort their beliefs (e.g., $s = 0$), they would vote according to their objective prospects, and the unique equilibrium would be the likely poor voting for high taxes and the likely rich voting for low taxes. The median voter would be among the likely poor, and the policy in the unique equilibrium would be high taxes. We will see how the possibility of subjective beliefs that differ from the objective standard allow additional equilibria with other policy outcomes.

Throughout the analysis, we focus on the symmetric decisions within the two groups

[12] We assume that an indifferent agent votes for low taxes. This assumption turns out to be quite crucial as it determines the tax policy in the low tax equilibrium of the model in the case of $\chi = 1$. We could, however, suppose, that there is an arbitrarily small amount of wastage involved in taxation, or that the voters deviate an arbitrarily small amount from the full Bayesian rationality, which both would solve the indifference for low taxes.

of voters. All of the likely rich choose $\hat{\sigma} = y_H$ and all of the likely poor choose the same λ. An optimist will always vote for $\tau = \underline{\tau}$ as seen from (10) and (4) and noting that $r(\lambda) \geq q$ for all $\lambda \in [0, 1]$. A realist will always vote for $\tau = \overline{\tau}$ by (5). Also, the likely rich will always vote for $\tau = \underline{\tau}$, similarly to the the optimistic likely poor. Putting all this together, the policy outcome can be derived as a function of λ. The total share of agents expecting above average income is $q + (1 - q)(1 - \lambda)$. The policy outcome τ^* depends on whether this share exceeds $\frac{1}{2}$ or not:

$$\tau^* = \begin{cases} \underline{\tau} & \text{if} \quad \lambda < \frac{1}{2(1-q)} \\ \overline{\tau} & \text{if} \quad \lambda \geq \frac{1}{2(1-q)} \end{cases}. \tag{11}$$

In line with Minozzi's (2013) model, we first let the agents vote strategically.[13] That is, they take account that their vote might be pivotal. As will be shown later, if agents voted sincerely, the trivial outcome would be everyone maximizing the anticipatory utility. In contrast to models of Bénabou and Tirole (2006) and Bénabou (2008), where voting is sincere, here the possibility of losing income due to less redistribution is the only thing that restricts the optimism of voters. This lets us focus on the trade-off between anticipation and redistribution. Sincere voting is studied in section 4.3.

3.5 Conditions for the POUM Effect, $\tau \in [0, 1]$

To gain some intuition and to analyze an interesting special case, we first set $\overline{\tau} = 1$ and $\underline{\tau} = 0$. The more general and more realistic case of $\overline{\tau} < 1$ and $\underline{\tau} > 0$ is analyzed in the next section.

Now that we know the voting decisions in period 1, we turn to the choice of λ in period 0. Due to the discontinuity of the policy outcome, the likely poor really have only two options to choose from. They either form optimal beliefs among those which support high taxation or optimal beliefs among those which support low taxation. We now derive the conditions under which the likely poor choose optimism and low taxation over realism

[13] Or rather we let agents form their beliefs strategically taking account how it affects the policy outcome. Technically speaking the voting here is sincere but agents can affect their policy preferences via their beliefs. The assumption that the policy outcome is $\overline{\tau}$ in case of $\lambda = \frac{1}{2(1-q)}$ ensures that an optimal choice of λ exists for all $s > 0$.

and high taxation. In other words, we derive the conditions under which the prospects of upward mobility of the likely poor are so high, that a low tax regime is supported.

Let $\overline{\lambda}$ be the optimal recall rate given $\lambda \geq \frac{1}{2(1-q)}$ and $\underline{\lambda}$ the optimal recall rate given $\lambda < \frac{1}{2(1-q)}$. If the likely poor choose $\overline{\lambda}$, the tax rate will be $\tau^* = 1$. The expected utility then is

$$U_{0,i}^{\overline{\lambda}} = \overline{\lambda} u_{0,i}(y_L, y_L, 1) + (1 - \overline{\lambda}) u_{0,i}(y_L, y_H, 1) = \delta s \bar{y} + \delta^2 \bar{y}. \tag{12}$$

Whether they end up being optimists or realists does not matter since in both cases they expect the redistribution to equalize all incomes. If they, on the other hand, choose $\underline{\lambda}$, the tax rate will be $\tau^* = 0$. The expected utility is

$$\begin{aligned} U_{0,i}^{\underline{\lambda}} &= \underline{\lambda} u_{0,i}(y_L, y_L, 0) + (1 - \underline{\lambda}) u_{0,i}(y_L, y_H, 0) \\ &= \underline{\lambda}[\delta s y_L + \delta^2 y_L] + (1 - \underline{\lambda}) \left[\delta s[r(\underline{\lambda}) y_H + (1 - r(\underline{\lambda})) y_L] + \delta^2 y_L\right]. \end{aligned} \tag{13}$$

With probability $\underline{\lambda}$, a likely poor agent recalls $\hat{\sigma}_i = y_L$ and forms realistic beliefs, and with probability $1 - \underline{\lambda}$, a likely poor agent recalls $\hat{\sigma}_i = y_H$ and forms optimistic beliefs weighted by the reliability of the signal. In both cases she still ends up consuming y_L in period 2.

The comparison of (12) and (13) tells us if the likely poor would rather choose high anticipatory utility in period 1 and low taxation with low consumption in period 2 over low anticipatory utility and high taxation with high consumption. The difference between the utilities resulting from these two choices, which we call the *incentive to optimism*, can be written as:

$$U_{0,i}^{\underline{\lambda}} - U_{0,i}^{\overline{\lambda}} = -\delta^2(\bar{y} - y_L) + s\delta[\underline{\lambda} y_L + (1 - \underline{\lambda})[r(\underline{\lambda}) y_H + (1 - r(\underline{\lambda})) y_L] - \bar{y}]. \tag{14}$$

The first term tells what a likely poor agent loses in income and consumption if the tax rate is $\tau^* = 0$ instead of $\tau^* = 1$. The second term tells what she expects to gain in anticipatory utility if she chooses $\underline{\lambda}$ instead of $\overline{\lambda}$. The likely poor are better off in the low tax regime if the incentive to optimism is positive. That is, if $U_{0,i}^{\underline{\lambda}} - U_{0,i}^{\overline{\lambda}} > 0$ the likely poor agents choose $\lambda = \underline{\lambda}$.

Lemma 1 (Awareness choices of the likely poor, $\tau \in [0, 1]$**).** *When* $\tau \in [0, 1]$*, the* *likely poor choose* $\lambda = \underline{\lambda} = 0$ *if*

$$s > s^*(\chi) \equiv \delta \frac{q + \chi(1-q)}{(1-\chi)(1-q)}. \tag{15}$$

Otherwise they choose $\lambda = \overline{\lambda} \in [\frac{1}{2(1-q)}, 1]$.

We have defined s^* to be a threshold such that if $s > s^*$, then agents value anticipation enough for the gain in anticipatory utility to outweigh the loss of income, and the likely poor will be optimistic enough to vote for a low tax rate. If, on the other hand, $s < s^*$, then the anticipation is not enough to compensate for the lost income and the likely poor will remain realistic enough to vote for a high tax rate.

Lemma 2 (Politico-economic equilibria, $\tau \in [0, 1]$**).** *A politico-economic equilibrium is a 4-tuple* $(y_H, \lambda^*, r(\lambda^*|\chi), \tau^*)$.[14]

(i) If $s > s^*$*, there is an equilibrium in which the likely poor choose* $\lambda^* = \underline{\lambda} = 0$*, the likely rich choose* $\hat{\sigma} = y_H$*, and the policy outcome is* $\tau^* = 0$.

(ii) If $s < s^*$*, there are equilibria in which the likely poor choose* $\lambda^* = \overline{\lambda} \in [\frac{1}{2(1-q)}, 1]$*, the likely rich choose* $\hat{\sigma} = y_H$*, and the policy outcome is* $\tau^* = 1$.

The POUM effect occurs in the equilibrium (i), so the condition for the possibility of the POUM effect is equivalent to the condition of the equilibrium (i).

Proposition 1 (The condition for the POUM effect, $\tau \in [0, 1]$**).** *When* $\tau \in [0, 1]$*, the condition for the POUM effect is* $U_{0,i}^{\underline{\lambda}} - U_{0,i}^{\overline{\lambda}} > 0 \iff s > s^*$.

The prospects of upward mobility lead to low taxes if agents value anticipatory utility enough. How much is enough depends on the threshold s^*. The higher s^* is, the less likely the POUM effect is, and conversely, the lower s^* is, the more likely we will observe low taxation. This threshold varies with the parameters of the model. First, the POUM effect becomes more likely with discounting. Myopic preferences put more weight on

[14] There is actually a third type of equilibrium, where all agents choose $\hat{\sigma}_i = y_H$ and the policy outcome is $\tau^* = 0$ even if $s < s^*$. There would be no unilateral incentive to deviate. This equilibrium would be the unique equilibrium if we assumed sincere voting.

anticipation which occurs before consumption.[15] Second, the effects of changes in the income distribution are left for section 4.1. Third, the threshold depends on the degree of Bayesian sophistication χ, which we study more closely now.

Consider first the special case of completely naive inference. Setting $\chi = 0$, we get

$$s^*(0) = \delta \frac{q}{1-q}. \tag{16}$$

This special case corresponds to Minozzi's (2013) model.[16] If, on the other hand, we let agents' inference approach Bayesian rationality, we find:

$$\lim_{\chi \to 1} s^*(\chi) = \infty. \tag{17}$$

The threshold required for the POUM effect to occur approaches infinity as the inference of agents approaches full Bayesian rationality. This means that with full Bayesian rationality the importance of anticipation s can never be above s^* and it can never be optimal for the likely poor to form beliefs that support low taxes as the policy outcome. That is, on contrary to the special case of Minozzi's (2013) model, where $\chi = 0$, if we acknowledge that the people cannot simply choose their beliefs and let $\chi > 0$, the threshold s^* increases dramatically in χ and in the extreme case of full Bayesian rationality, the POUM effect can never occur.

Figure 2 tracks the threshold s^* as a function of χ. To give some concreteness to the results here, we note from the period 0 utility in (9) that if $s = \delta$, then agents value anticipatory utility as much as consumption. The dashed line in Figure 2, denoted by δ, depicts this value of s. For the threshold values $s^* > \delta$, the anticipation of consumption must bring more utility to the agents than the consumption itself to make the POUM effect possible. We see that s^* is below δ only for very small values of χ.

To see why fully Bayesian likely poor agents can never be better off with low taxes, consider again the incentive to optimism given in (14). Plugging in the optimal recall rate

[15] Interestingly, in the model of Bénabou and Ok (2001), discounting makes the POUM effect less likely. This result in their model is, however, derived in a multiperiod setting and is not directly comparable.

[16] Minozzi's model which abstracts from discounting derives $\delta^* = \frac{n-m}{m}$, where δ^* is the threshold of the savoring parameter, n is the (finite) number of agents, and m is the number of the likely poor.

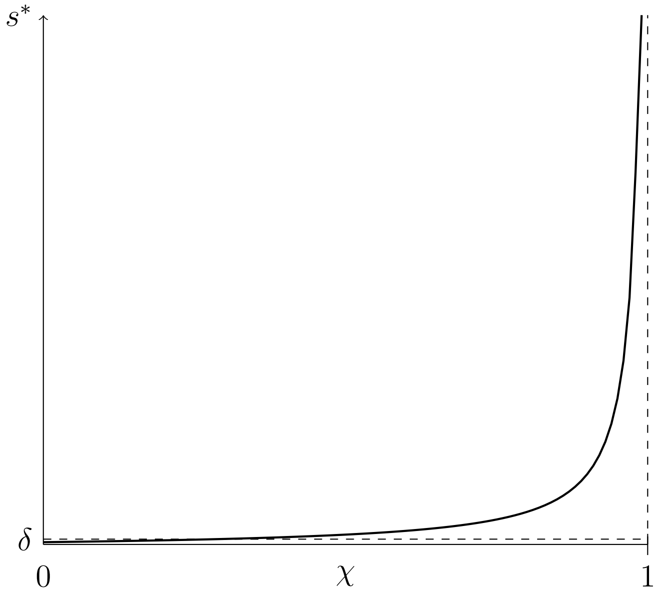

Figure 2: s^* as a function of χ

$\underline{\lambda} = 0$, the incentive to optimism can be written as

$$U_{0,i}^{\underline{\lambda}} - U_{0,i}^{\overline{\lambda}} = -\delta^2(\bar{y} - y_L) + s\delta[r(0|\chi) - q]\Delta y, \tag{18}$$

where $\Delta y \equiv y_H - y_L$. The second term in the right hand side is the gain in anticipation if an agent chooses $\underline{\lambda}$ over $\overline{\lambda}$. Noting that $r(0|\chi) \to 1$ as $\chi \to 0$ and $r(0|\chi) \to q$ as $\chi \to 1$, it is easy to see how the value of the second term goes to zero as $\chi \to 1$ and why it does not when $\chi = 0$. The incentive to optimism is at its maximum when $\chi = 0$ and as agents' inference approaches full Bayesian rationality the utility gain from anticipation vanishes.

The reliability which the agents use to weight the information of their recollection plays a crucial role here. For $\chi = 1$, the reliability $r(\lambda|\chi)$ is an increasing function of λ. The more realistic the likely poor are, the more reliable signal $\hat{\sigma}_i = y_H$ is. On the other hand, when the likely poor systematically memorize and recall $\hat{\sigma}_i = y_H$, they know that no matter what is their true signal, they recall $\hat{\sigma}_i = y_H$. In this case, the signal does not carry any information anymore, and agents form their beliefs relying on the prior distribution, $r(0|\chi) = q$. However, when the degree of Bayesian sophistication decreases, the reliability becomes less and less dependent on λ, and the optimistic poor put more and more weight on their pleasant recollection. When $\chi = 0$, the reliability is independent of λ and no matter how optimistic the likely poor are, they always fully trust their recollections.

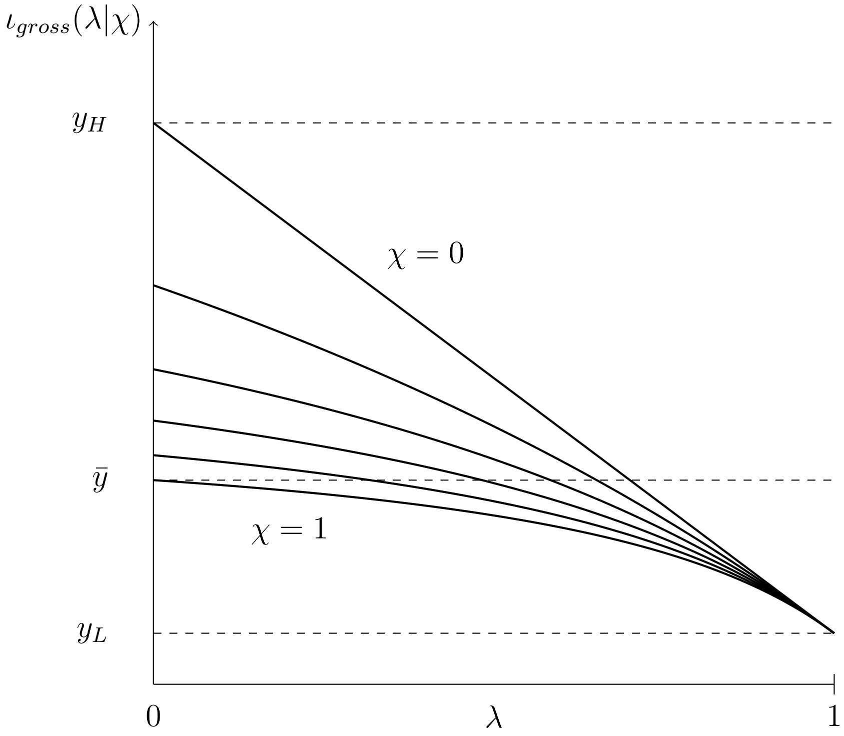

Figure 3: $\iota_{gross}(\lambda|\chi)$ for different values of χ

It is instructive to see how the period-0 expectation of expected period-2 income in period 1, and expected anticipatory utility which is proportional to the expected income, varies with λ and χ. For this, we shortly abstract from taxation to see how the choice of λ and the sophistication of agents' inference interact in forming the belief about future gross income. The expectation of expected gross income of a likely poor agent in period 1 from the point of view of period 0 as a function of λ is[17]

$$\iota_{gross}(\lambda|\chi) \equiv E[E[y_i|F_{1,i}]|\lambda, \chi, F_{0,i}] = (1-\lambda)[r(\lambda)y_H + (1-r(\lambda))y_L] + \lambda y_L. \tag{19}$$

This function is plotted in Figure 3 for different values of χ. The lowest curve corresponds to the case $\chi = 1$. As agents put more and more weight on signal $\hat{\sigma}_i = y_H$ in their period 0 strategy, that is, as they become more and more likely to remember $\hat{\sigma}_i = y_H$, the expected income approaches the average income. In the case of $\lambda = 0$, each of the likely poor and each of the likely rich always recall signal $\hat{\sigma}_i = y_H$. As everyone is pooling on the same signal, receiving this signal does not give any information, and agents rely

[17] See the discussion in section 3.2

on the prior when assessing their future income. In the case of full Bayesian rationality, it is therefore not possible for agents to achieve above average expectations. As they expect average income in the high tax regime, they cannot possibly improve their utility by voting for low taxes.

On the contrary, when $\chi < 1$, agents can achieve above average expectations, and they, therefore, can have a gain in anticipatory utility to trade off against the lost income in the low tax regime. For agents with $\chi < 1$, a decrease in λ does not affect the reliability of the signal as much as it affects for the Bayesian rational agents. In the limiting case of $\chi = 0$, represented by the linear curve in Figure 3, the reliability is independent of λ, and all agents can believe to be of type y_H. The expectations of naive agents are not as constrained as the expectations of Bayesian agents and the more naive the agents are, the less constrained their beliefs are. The naive agents can, therefore, achieve higher hopes and higher anticipatory utility than their Bayesian counterparts.

What values of χ are feasible then? Do people have the introspection to realize that they might have a self-serving tendency to remember positive news and forget bad news or are they always able to deceive themselves into believing what fits them best? Minozzi (2013) justifies his assumption of full naivete by arguing that the belief formation is an automatic and unconscious process and therefore the players cannot recall the process itself and are therefore ignorant of it occurring. They then completely trust their recollections, since they have forgotten the action of their past self or rather since they never even knew about the action of their unconscious self. On the other hand, Bénabou and Tirole (2002) argues that if a person consistently memorizes good news and ignores bad news, she will likely become aware of this tendency and will therefore not fully rely on her recollections. So even if the belief formation is an automatic, unconscious process and people cannot, therefore, recall it happening, they, by learning from their past mistakes, will internalize the existence of this process and start adjusting their reliance on their memories accordingly.

Framed in other words, the implausible consequence of assuming $\chi = 0$ is that people are able to choose their beliefs without them in any way depending on the objective reality. To be clear, the beliefs supplied by a naive cognitive technology are usually restricted to the support of the outcome and can be further constrained to a subset of the support.[18] Also, a naive cognitive technology does take the reality into account, when the beliefs are traded against their adverse consequences. However, in principle, it does not need to. Naive cognitive technologies are also nevertheless insensitive to the distribution of outcomes. When $\chi = 0$, an agent can believe to be likely rich no matter how small the prior probability of being rich is given that this prior probability is positive. Even if the belief formation mechanism is an automatic and unconscious process, it seems implausible that this process does not need in any way to take account the information that the reality inevitably provides, and that people can simply choose their beliefs. Indeed, Kunda (1990) strongly argues that people do not seem to be completely free to believe what they want to believe. According to him, people can bias their beliefs only to the extent that they can justify their new beliefs. The main mechanism for the justification of the new beliefs is a biased memory search which implies that prior beliefs do play a role in determining the new beliefs. Also, according to evidence, changes in beliefs seem to be constrained by pre-existing beliefs (Kunda, 1990). Therefore, a belief formation technology with some Bayesian sophistication, which anchors the beliefs to the prior distribution, and therefore to the reality in our model, would seem more plausible a representation of these psychological processes than a belief formation technology with none Bayesian sophistication.

However, assuming $\chi = 1$ is rather extreme as well and $\chi \in (0, 1)$ would most likely best reflect the reasoning of real people. Bénabou and Tirole (2002) presents the model in the context of people distorting their beliefs to motivate themselves when facing time inconsistency problems. This might be a context where people get enough feedback to learn about their unconscious information processing. In the context of the present work, where people form beliefs about their future incomes and vote for redistribution the feedback mechanism may not facilitate this learning. The actions taken are long-lasting, there are not that many chances of learning, and the real-life mechanism with which votes transform to redistributive policies is noisy and complicated. It might, therefore, be plausible that the sophistication in the belief formation process depends on the context and that, indeed, in the context of forming beliefs about future income, people might be

[18] See the footnote 2 in the appendix to Minozzi (2013).

less sophisticated as in the context of motivating oneself in the everyday activities.

In the case of full Bayesian rationality, the wishful beliefs of the likely poor are bounded above to the average income, which is what they expect to receive if $\tau^* = 1$ as well. They cannot, therefore, increase their anticipatory utility by distorting their beliefs. However, what if they could not expect the incomes to be fully equalized under the high tax policy. Then they might be able to increase their anticipatory utility by distorting their beliefs even if they still ended up with expectations of average income. The case of $\overline{\tau} = 1$ and $\underline{\tau} = 0$ is maybe a bit too unrealistic a simplification and we therefore turn now to the general case of $\tau \in [\underline{\tau}, \overline{\tau}]$.

3.6 Conditions for the POUM Effect, $\tau \in [\underline{\tau}, \overline{\tau}]$

The weakness of the previous setting is that if agents vote for full expropriation, they know that their period 2 incomes will be the average income. Therefore, no matter what they believe, they will expect average income. On the other hand, if the likely poor choose optimism, they will lose all redistribution, which is a very high cost for optimism. To address these problems, we now consider the general case of our model and impose lower and upper limits on the tax rate. That is, we now set $\tau \in [\underline{\tau}, \overline{\tau}]$, and require $\underline{\tau} < \overline{\tau}$ so that the set of tax policies is always nonempty.

If we set $\overline{\tau} < 1$, the anticipated consumption and the consumption of the likely poor realists will now be below average in the high tax regime. This makes the high tax equilibrium less attractive compared to the case of $\overline{\tau} = 1$. The increase in payoff when choosing optimism over realism is therefore now greater, and the condition for the POUM effect should become looser.

At the other extreme, a full laissez-faire policy is not a completely innocuous simplification either. The likely poor have to trade optimism against losing all redistribution. Imposing a lower limit for redistribution makes this trade-off less drastic. If $\underline{\tau} > 0$, there will be some taxation in the low tax regime as well, and the consequences of optimism are less severe for the likely poor. By setting $\underline{\tau} > 0$, we make the decrease in period 2 consumption of the likely poor smaller in case they choose $\underline{\lambda}$ over $\overline{\lambda}$. Again, this should make optimism more attractive and the POUM effect more likely. Of course, an increase in $\underline{\tau}$ decreases the anticipatory utility of the optimists, but the effect in period 2 consumption seems to dominate for the likely poor.

To put these effects together, by restricting the set of available tax policies, we make the POUM effect more feasible in two ways. First, by decreasing the attractiveness of realism by having lower taxes and, therefore, lower anticipation and consumption in the high tax regime. Second, by increasing the attractiveness of optimism by having higher taxes in the low tax regime and therefore higher consumption but possibly lower anticipation. The effects on the period 2 consumption and realists' anticipation seem to dominate the effect on optimists' anticipation so that the smaller is the range of allowed tax policies, the more likely the POUM effect occurs.

As before, the apparently continuous choice reduces to a binary choice, and the likely poor choose between $\overline{\lambda}$ and $\underline{\lambda}$ knowing that choosing the former leads to high taxation and choosing the latter leads to low taxation. If they choose the former, the tax rate will be $\tau^* = \overline{\tau}$ and their expected payoffs are

$$\begin{aligned} U_{0,i}^{\overline{\lambda}} &= \overline{\lambda} u_{0,i}(y_L, y_L, \overline{\tau}) + (1-\overline{\lambda}) u_{0,i}(y_L, y_H, \overline{\tau}) \\ &= \overline{\lambda}\big[\delta s[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}] + \delta^2[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}]\big] \\ &+ (1-\overline{\lambda})\big[\delta s[(1-\overline{\tau})[r(\overline{\lambda})y_H + (1-r(\overline{\lambda}))y_L] + \overline{\tau}\bar{y}] + \delta^2[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}]\big]. \end{aligned} \tag{20}$$

This differs from (12) in that the tax does not fully equalize the incomes. When all incomes are not equalized, different expectations lead to different amounts of anticipation. This allows the anticipatory utility of optimists and realists to diverge also in the high tax regime. Note especially how a realist derives anticipatory utility from an expectation of below average income.

If the likely poor agents choose the latter, the tax rate will be $\tau^* = \underline{\tau}$ and their expected payoffs are

$$\begin{aligned} U_{0,i}^{\underline{\lambda}} &= \underline{\lambda} u_{0,i}(y_L, y_L, \underline{\tau}) + (1-\underline{\lambda}) u_{0,i}(y_L, y_H, \underline{\tau}) \\ &= \underline{\lambda}\big[\delta s[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}] + \delta^2[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}]\big] \\ &+ (1-\underline{\lambda})\big[\delta s[(1-\underline{\tau})[r(\underline{\lambda})y_H + (1-r(\underline{\lambda}))y_L] + \underline{\tau}\bar{y}] + \delta^2[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}]\big]. \end{aligned} \tag{21}$$

With probability $\underline{\lambda}$, a likely poor agent ends up being a realist and anticipates low consumption. With probability $1 - \underline{\lambda}$, she ends up being an optimist and anticipates high consumption. In both cases the period 2 consumption is low. However, in comparison to (13), the period 2 consumption is now higher, and the consequences of optimism are now less severe for the likely poor.

The incentive to optimism is

$$U_{0,i}^{\underline{\lambda}} - U_{0,i}^{\overline{\lambda}} = -\delta^2(\overline{\tau} - \underline{\tau})(\bar{y} - y_L) + \delta s\big[(1-\underline{\lambda})(1-\underline{\tau})r(\underline{\lambda})\Delta y + (1-\underline{\tau})y_L + \underline{\tau}\bar{y}\big] - \delta s\big[(1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda})\Delta y + (1-\overline{\tau})y_L + \overline{\tau}\bar{y}\big]. \tag{22}$$

To not clutter the page with notation, the incentive to optimism is written in a still interpretable, but different and a more compact form than (14). The first term tells the loss of income due to less redistribution. The second term measures the expected anticipatory utility if the the likely poor choose $\lambda = \underline{\lambda}$. With probability $1 - \underline{\lambda}$ there is an increase of $(1-\underline{\tau})r(\underline{\lambda})\Delta y$ from the "base level" of $(1-\underline{\tau})y_L + \underline{\tau}\bar{y}$ in anticipatory utility. The third term similarly measures the expected anticipatory utility if the likely poor choose $\lambda = \overline{\lambda}$. If the incentive to denial is positive, the likely poor will prefer to be optimists and choose $\lambda = \underline{\lambda}$.

Lemma 3 (Awareness choices of the likely poor, $\tau \in [\underline{\tau}, \overline{\tau}]$). *When $\tau \in [\underline{\tau}, \overline{\tau}]$, the likely poor choose $\lambda = \underline{\lambda} = 0$ if*

$$s > s^{**}(\chi) \equiv \frac{\delta(\overline{\tau} - \underline{\tau})q}{(1-\underline{\lambda})(1-\underline{\tau})r(\underline{\lambda}) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau} - \underline{\tau})q}, \tag{23}$$

Otherwise they choose $\lambda = \overline{\lambda} = \frac{1}{2(1-q)}$.

As before, whether the savoring parameter is above or below the threshold s^{**}, the likely poor will either prefer high anticipation with low redistribution or low anticipation with high redistribution. The choice of the likely poor determines the tax rate.

Lemma 4 (Politico-economic equilibria, $\tau \in [\underline{\tau}, \overline{\tau}]$). *A politico-economic equilibrium is a 4-tuple $(y_H, \lambda^*, r(\lambda^*|\chi), \tau^*)$.*[19]

[19]There is actually a third type of equilibrium, where all agents choose $\hat{\sigma} = y_H$ and the policy outcome is $\tau = 0$ even if $s < s^{**}$ as there would be no unilateral incentive to deviate.

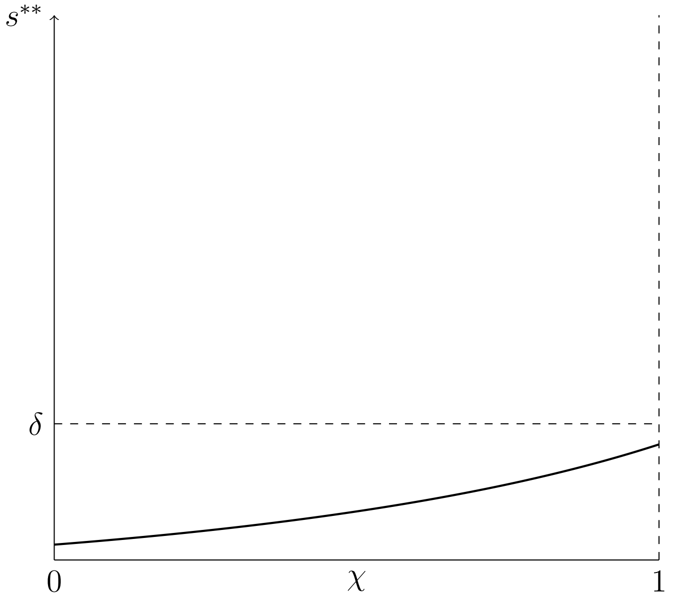

Figure 4: s^{**} as a function of χ

(i) If $s > s^{**}$, there is an equilibrium in which the likely poor choose $\lambda^* = \underline{\lambda} = 0$, the likely rich choose $\hat{\sigma} = y_H$, and the policy outcome is $\tau^* = \underline{\tau}$.

(ii) If $s < s^{**}$, there is an equilibrium in which the likely poor choose $\lambda^* = \overline{\lambda} = \frac{1}{2(1-q)}$, the likely rich choose $\hat{\sigma} = y_H$, and the policy outcome is $\tau^* = \overline{\tau}$.

As before, the POUM effect occurs in the equilibrium (i) and the conditions for the POUM effect are the same as the conditions for this equilibrium.

Proposition 2 (The condition for the POUM effect, $\tau \in [\underline{\tau}, \overline{\tau}]$). *The condition for the POUM effect is $U_{0,i}^{\underline{\lambda}} - U_{0,i}^{\overline{\lambda}} > 0 \iff s > s^{**}$.*

Interestingly, s^{**} is now finite for all $\chi \in [0,1]$. In contrast to the setting in the previous section, the POUM effect becomes possible even if the agents are fully Bayesian information processors. Figure 4 depicts s^{**} as a function of χ. We see that the threshold s^{**} does not increase in χ as explosively as s^* does. As before, to ease the interpretation, the dashed line depicts the values of s for which the agents derive as much utility from the anticipation of consumption as from consumption itself. The parameter values for the allowed tax policies used in Figure 4 are $\underline{\tau} = 0.25$ and $\overline{\tau} = 0.45$, and they represent roughly the total tax revenues as a percentage of gross domestic product in US and in Nordic Countries and, respectively (OECD, 2008).[20] These values and countries are chosen to represent the extremes of taxation among the developed countries and serve only as an example. The hypothetical extremes of tax policies are probably larger than currently existing extremes. As we will see, the bounds of allowed tax policies have a clear effect on s^{**}.

The following proposition makes formal the effect of parameter χ which can be seen in Figure 4.

Proposition 3 (Effect of change in the degree of Bayesian sophistication). *The partial derivative of s^{**} with respect to χ is positive, that is, $\frac{\partial s^{**}}{\partial \chi} > 0$ for all parameter values. The more sophisticated the cognitive technology is, the less likely is the POUM effect.*

Even if the POUM effect is now possible for all $\chi \in [0,1]$, it can still be questioned whether it is feasible for all $\chi \in [0,1]$. Again, the agents may have to value anticipation more than consumption to prefer low taxes if the range of the feasible tax rates is big enough. To see this, consider the threshold value s^{**} when $\chi = 1$:

$$s^{**}(1) = \delta \frac{(\overline{\tau} - \underline{\tau})(1-q)}{(1-\overline{\tau})q}. \tag{24}$$

Now $s^{**} > \delta$, for all pairs $(\underline{\tau}, \overline{\tau})$, such that $\overline{\tau} > (1-q)\underline{\tau} + q$. We could argue that within a jurisdiction, the range of feasible tax rates is small enough and hence, the POUM effect is feasible also for a sophisticated cognitive technology. On the other hand, as discussed, fully Bayesian sophistication may not be the correct specification in the belief distortion technology to represent people's beliefs about their future incomes and their voting behavior. Certainly, the set of values of χ for which the POUM effect is feasible has now increased in comparison to the case in the previous section.

To understand how the likelihood of the POUM effect depends on the maximum and minimum taxes, consider first what happens when we set an upper limit on the tax rate. The upper limit of the tax is relevant when the likely poor choose $\lambda = \overline{\lambda}$ since then the resulting policy is high taxes. Imposing a restriction on how much of the income can be redistributed we make the prospects of choosing $\lambda = \overline{\lambda}$ worse. Consider the effects on the

[20] $q = 0.3$ and δ is normalized to 1. Note that the curve is independent of the values of y_L and y_H.

period 2 consumption and period 1 anticipatory utility separately. First, a decrease in the upper limit of the tax rate decreases the period 2 consumption of the likely poor in the high tax regime, which makes voting for high taxes less rewarding. Second, for those of the likely poor who end up being realists, the lower consumption in period 2 implies lower anticipation in period 1. Those of the likely poor who end up being optimists will expect above-average incomes, and they will, therefore, gain in anticipatory utility as the upper limit of the tax decreases. However, it can be shown that this latter effect is dominated and the effect on ex-ante expected anticipation stays negative.[21] That is, when imposing an upper limit for the tax rate, both anticipation and consumption prospects of choosing $\lambda = \overline{\lambda}$ deteriorate. Proposition 4 formalizes this total effect of the upper limit of the tax rate.

Proposition 4 (Effect of upper limit of tax rate on the conditions for POUM). *The partial derivative of s^{**} with respect to $\overline{\tau}$ is positive, that is, $\frac{\partial s^{**}}{\partial \overline{\tau}} > 0$ for all parameter values. The POUM effect becomes more likely as $\overline{\tau}$ decreases.*

The prospects of choosing $\lambda = \underline{\lambda}$, on the other hand, are now better. The likely poor choosing $\lambda = \underline{\lambda}$ leads to low taxes, so here the lower limit of the tax rate is interesting. Again, there is an effect on the period 2 consumption and on the period 1 anticipation. First, even if the likely poor vote for low taxation, redistribution does not vanish altogether. Since they are trading their optimism against redistribution, the cost of optimism is now lower. The reduction in their period 2 consumption is not as big as with the possibility of complete laissez-laire. This makes choosing high anticipation and low taxes more attractive. Second, when choosing $\lambda = \underline{\lambda}$, all of the likely poor end up being optimists. If they then anticipate above average income, that is, if $\chi < 1$, then an increase in the lower limit of the tax rate will decrease their anticipatory utility. The less sophisticated the agents are, the more they expect to earn, and the higher is the decrease in their anticipation. The effect on anticipatory utility is opposite to the effect on consumption. The effect on consumption, however, seems to dominate. Proposition 5 formalizes this.

Proposition 5 (Effect of lower limit of tax rate on the conditions for POUM). *The partial derivative of s^{**} with respect to $\underline{\tau}$ is negative, that is, $\frac{\partial s^{**}}{\partial \underline{\tau}} < 0$ for all parameter*

[21] $\frac{\partial}{\partial \overline{\tau}} \iota_{net}(\overline{\lambda}, \overline{\tau}) = [q - (1 - \overline{\lambda}) r(\overline{\lambda})] \Delta y > 0$, where $\iota_{net}(\cdot)$ is defined below.

values. The POUM effect becomes more likely as $\underline{\tau}$ increases.

To summarize these effects, the utility from choosing $\lambda = \overline{\lambda}$ decreases when we impose the upper limit for the tax rate and the utility from choosing $\lambda = \underline{\lambda}$ increases with the lower limit. This means that the utility cap between choosing $\lambda = \overline{\lambda}$ and $\lambda = \underline{\lambda}$ increases as the range of allowed tax policies decreases. This utility cap is, by definition, the incentive to optimism. Increase in the incentive to optimism then leads to less stringent conditions for the POUM effect.

To gain further intuition on the conditions for the POUM effect, write s^{**} as

$$s^{**} = \frac{\delta(\overline{\tau} - \underline{\tau})(\bar{y} - y_L)}{\iota_{net}(\underline{\lambda}, \underline{\tau}) - \iota_{net}(\overline{\lambda}, \overline{\tau})} \tag{25}$$

where

$$\iota_{net}(\lambda, \tau) \equiv \lambda[(1-\tau)y_L + \tau\bar{y}] + (1-\lambda)[(1-\tau)(r(\lambda)y_H + (1-r(\lambda))y_L) + \tau\bar{y}] \tag{26}$$

is the ex ante expectation of the expected consumption of the likely poor in period 1 given the choice of λ and the resulting tax policy τ, and where $\underline{\lambda} = 0$, and $\overline{\lambda} = \frac{1}{2(1-q)}$.[22] The nominator of (25) represents the difference in period 2 consumptions in the two different tax regimes. Clearly, when $\overline{\tau}$ decreases or $\underline{\tau}$ increases, this difference becomes smaller. As discussed, when this difference becomes smaller the loss in the period 2 consumption when choosing $\lambda = \underline{\lambda}$ over $\lambda = \overline{\lambda}$ decreases. If the nominator decreases, s^{**} decreases proportionally and the POUM effect becomes more likely. The denominator of (25) is proportional to the difference in expected anticipatory utility of the likely poor between their choices of low or high recall rate. When this difference increases, the likely poor have more to gain in anticipation and belief distortion becomes more attractive. If the denominator increases, s^{**} decreases and the POUM effect becomes more likely.

In choosing their awareness rate, the likely poor agents make a trade-off between anticipatory utility and consumption. Imposing the limits on possible tax rates, we alter this trade-off such that they have less to lose in consumption. The stakes of wrong

[22] From this expression it is simple to derive Minozzi's (2013) result in another form. By setting $\underline{\tau} = 0$, $\overline{\tau} = 1$, $\delta = 1$, and $\chi = 0$, we get $s^{**} = \frac{\bar{y} - y_L}{y_H - \bar{y}}$. Minozzi's (2013) condition for the POUM effect is $\delta > \delta^* = \frac{\bar{y} - y_p}{y_r - \bar{y}}$, where y_p is the income of the likely poor, y_r income of the likely rich, and δ^* the threshold in the savoring parameter δ.

decisions due to biased beliefs are now smaller, and optimism is, therefore, more attractive.

4 Further Analysis

4.1 Effects of Changes in Income Distribution

As already seen, given the value that agents put on anticipation s, the threshold s^{**} determines whether the POUM effect occurs. The comparative statistics of s^{**}, therefore, reveal how the conditions for the POUM effect vary as the parameters of the model change. In this section we consider the effects of changes in y_L, y_H, $\bar{y}$, and q.

Following Minozzi's (2013) analysis, we first study the changes in y_L and y_H holding the average income constant. Proposition 6 collects these results.

Proposition 6 (Effects of changes in y_L and y_H holding $\bar{y}$ constant). *Holding the average income constant, the threshold s^{**} decreases in y_L and y_H, that is, the POUM effect becomes more feasible when y_L or y_H increase.*[23]

If the incomes of the likely rich increase such that the average income stays constant, the conditions for the POUM effect become looser. Similarly, if the incomes of the likely poor increase such that the average income stays constant, the conditions for the POUM effect become again looser. Why we insist on holding the average income constant is that it makes the effects interesting. The average income $\bar{y}$ is a function of both y_L and y_H and taking this into account gives us $\frac{\partial s^{**}}{\partial y_H} = \frac{\partial s^{**}}{\partial y_L} = 0$ as can easily be seen by noting that s^{**} in (23) is independent of both y_H and y_L. So by letting the average income adapt to the changes in the incomes of the likely poor or the likely rich, the condition for the POUM effect would not change.

Holding the average income constant might feel artificial, but looking at the incentive to optimism given in (22) gives us an idea, what does the partial derivatives holding the average income constant mean here.[24] For the agents, changes in the average income mean changes in the transfers they receive, whereas changes in either y_L or y_H mean changes in the expectations of their pre-tax income. That is, holding average income constant means

[23]These effects are the same for s^*

[24]Unfortunately, Minozzi (2013) does not justify this choice in his comparative analysis.

holding the tax revenue and transfers constant, whereas increases in the high and low levels of income mean increased expectations of gross income. Increased prospects of gross income, when the transfers are expected to stagnate make optimism more rewarding. This kind of change in the income distribution could occur, for instance, if the tax is regressive such that the increase in the incomes of the likely rich does not lead to a proportional increase in the tax revenue. We could also interpret the income levels y_L and y_H more loosely as what the likely poor perceive these income levels to be. The perceived income of the likely rich could change without affecting the tax revenue, for instance, if the incomes in other jurisdictions change and the likely poor observe this or if the consumption habits of the likely rich change towards more conspicuous consumption. *"In 1972, a storm of protests from blue-collar workers greeted Senator McGovern's proposal for confiscatory estate taxes. They apparently wanted some big prizes maintained in the game. The silent majority did not want the yacht clubs closed forever to their children and grandchildren while those who had already become members kept sailing along."* writes Okun (2015, page 47).

Similarly, the change in the average income has no effect as such, $\frac{\partial s^{**}}{\partial \bar{y}} = 0$, but holding y_L and y_H constant and letting $\bar{y}$ change gives us

Proposition 7 (**Effect of change in $\bar{y}$ holding y_L and y_H constant**)**.** *Holding y_L and y_H constant, the threshold s^{**} increases in $\bar{y}$, that is, the POUM effect becomes less feasible when the average income increases.*

The case of holding y_L and y_H constant and letting $\bar{y}$ change mirrors the previous discussion. If the likely poor expect increased transfers but the prospects of gross income stay the same, then realism becomes more attractive.

The changes in the fraction of the likely rich produce slightly more complicated effects mainly because the reliability is a function of q, and the optimal recall rate $\overline{\lambda}$ varies with q. We therefore only characterize the effects. Consider a change in the income distribution where the proportion of the likely rich becomes smaller. A decrease in q has three effects. First, it decreases the average income and the tax revenue and, therefore, makes realism less attractive. Second, it decreases $\overline{\lambda}$, the optimal choice if the likely poor opt for high taxes. When there are fewer likely rich agents voting for low taxes, it allows the likely poor to be more optimistic even if they opt for high redistribution. This makes realism more attractive. Third, as q and, hence, $\overline{\lambda}$ decrease, they both contribute to decreasing the reliability of the signal $\hat{\sigma}_i = y_H$ and, therefore, make the anticipated income lower and optimism less attractive.

All effects that work via the reliability of recalled signal depend crucially on χ. Hence, for low values of χ, the reliability does not depend that much on the prior distribution or $\overline{\lambda}$ and the first effect dominates. In this case, POUM effect becomes more likely as the prospects of choosing $\lambda = \overline{\lambda}$ are now worse. For high values of χ, the reliability is highly dependent on the prior and $\overline{\lambda}$ and the second and third effect dominate. In this case, a decrease in q makes POUM less likely. For intermediate values of χ, the relative dominance of these effects varies, and the total effect is nonmonotonic.

4.2 Welfare Analysis

In the simple model of the current paper, utilities are linear in period 2 consumption meaning that the aggregate utility is not sensitive to the distribution of consumption. Therefore, the aggregate utility is trivially maximized by maximizing the anticipation, no matter what the distribution of the consumption ends up being. Hence, the aggregate utility as a measure of welfare is not very informative. This section, therefore, after a brief discussion on the distribution and the aggregation of consumption and anticipation focuses on the welfare of the likely poor and the likely rich separately.

The utility of each agent in the economy consists of two components: the utility from anticipation and the utility from consumption. Thanks to the additivity of these utilities, we can study the aggregate levels of these two components separately. Furthermore, as the utilities with respect to consumption and anticipation are both linear, we say that the welfare consists of aggregate consumption and aggregate anticipation.

As the redistribution does not produce any wastage, the aggregate consumption stays constant at the average consumption throughout the analysis. Due to the linearity of utility with respect to consumption, the average utility derived from consumption remains constant as well. Only the distribution of the consumption and the utility from consumption between the likely poor and the likely rich varies depending on the chosen tax policy. The higher is the tax rate, the more equally the aggregate consumption is distributed among the likely rich and the likely poor.

The more novel component of welfare is the aggregate anticipation, which is the sum of anticipation of those agents who recalled $\hat{\sigma}_i = y_L$ and of those who recalled $\hat{\sigma}_i = y_H$. A fraction $(1-q)\lambda$ of agents recalls $\hat{\sigma}_i = y_L$ and they anticipate a gross income of y_L. A fraction $q + (1-q)(1-\lambda)$ of agents recalls $\hat{\sigma}_i = y_H$ and the anticipate a gross income of $r(\lambda)y_H + (1-r(\lambda))y_L$. Note especially that those who truly belong to the likely rich anticipate the same gross income as those of the likely poor who recall signal $\hat{\sigma}_i = y_H$. The aggregate anticipatory utility derived from the anticipation of gross income is

$$(1-q)\lambda s y_L + [(1-q)(1-\lambda) + q]s[r(\lambda)y_H + (1-r(\lambda))y_L]. \tag{27}$$

The aggregate anticipation depends on the constraints of the cognitive technology and the awareness choices of the likely poor. For $\chi = 1$, the aggregate anticipatory utility is constant at $s\bar{y}$. Bayesian rationality imposes a constraint on beliefs such that on average, agents expect average income. Therefore, for the special case of $\chi = 1$, the aggregate anticipation is similar to the aggregate consumption in the sense that only the distribution of the anticipation varies. As the Bayesian constraint is relaxed and values of $\chi < 1$ are allowed, the aggregate anticipation can exceed the anticipation of average income, and it is no more independent of λ. In this case, the aggregate anticipation is maximized at $\lambda = 0$.

The counterintuitive consequence of the assessment of the reliability of recollections is that for all $\chi > 0$, the likely rich will underestimate their future income. If all of the likely poor choose to memorize the signal $\hat{\sigma}_i = y_H$, then all agents, the likely rich and the likely poor, will recall this signal in period 1. When the likely rich are assessing the reliability of their recollection, they know that no matter which signal an agent receives in period 0, they will recall $\hat{\sigma}_i = y_H$. In the case of full Bayesian rationality, this means that the signal is uninformative and the likely rich use the prior to form their expectations and, therefore, underestimate their future income.[25] If, on the other hand, the likely poor

[25] Interestingly, Cruces, Perez-Truglia, and Tetaz (2012) find evidence, that in addition to the poor overestimating their position in the income distribution, the rich tend to underestimate theirs. However, their proposed mechanism is different: Agents estimate the overall income distribution by extrapolating from the incomes of their reference group. If the reference group does not well represent the overall income distribution, the estimates will be biased. Also, underconfidence is a well-documented phenomenon in

choose to memorize the signal they received, then the likely rich, after recalling $\hat{\sigma}_i = y_H$ know that the only way to recall this signal is to be likely rich. In this case, they put a reliability of 1 to their recollection and form accurate expectations.

This dependence of the anticipation of the rich on the awareness choice of the likely poor can be thought of as a negative externality. As λ decreases, the likely poor are more and more optimistic and the likely rich more and more pessimistic. When the likely poor engage in optimism, they redistribute anticipation. If $\chi = 1$, and the likely poor choose $\lambda = 0$, they equalize all anticipation. In this case, the average anticipation is constant, and the gain in anticipatory utility of the likely poor is exactly offset by the loss in the anticipatory utility of the likely rich. The strength of externality and the redistributive effect increases in χ. For completely naive agents, the reliability of recollection is independent of λ, and there is no externality.

This externality should, however, not be thought of as a causal relationship between the cognitive processes of different agents, but as an *externality across information states*, as Bénabou and Tirole (2002, page 907) put it. The likely rich do not underestimate their prospects because the likely poor overestimate theirs, but because they know that had they themselves been likely poor, they might still have memorized the signal $\hat{\sigma}_i = y_H$. The negative externality for the likely rich is, therefore, caused by their own information processing strategy, that is, by their own hypothetical action in an alternative history.

If the likely poor choose the low tax equilibrium with high expectations, they are obviously better off in this equilibrium. The pessimism of the rich, however, raises the rather surprising question of whether the likely rich are worse or better off in the low tax equilibrium. In the standard case, where the agents do not derive utility from anticipation, the rich have higher consumption when paying low taxes and are obviously better off in the low tax equilibrium. When we take the anticipation into the analysis, the rich still have higher period 2 consumption in the low tax equilibrium, but the negative externality due to the optimism of the poor in this equilibrium erodes their anticipation in period 1. We now see, which of these effects dominates.

In the low tax equilibrium, the utility of the likely rich from the viewpoint of period

the literature of psychology and tends to concern those with the best prospects. See, for instance, Moore and Healy (2008).

0 is

$$u_{0,i}(y_H, y_H, \underline{\tau}) = \delta s \left[(1-\underline{\tau})[r(\underline{\lambda})y_H + (1-r(\underline{\lambda})y_L] + \underline{\tau}\bar{y}\right] + \delta^2[(1-\underline{\tau})y_H + \underline{\tau}\bar{y}], \quad (28)$$

and in the high tax equilibrium the utility of the likely rich is

$$u_{0,i}(y_H, y_H, \overline{\tau}) = \delta s \left[(1-\overline{\tau})[r(\overline{\lambda})y_H + (1-r(\overline{\lambda})y_L] + \overline{\tau}\bar{y}\right] + \delta^2[(1-\overline{\tau})y_H + \overline{\tau}\bar{y}]. \quad (29)$$

Again, whether the anticipation effect dominates depends on the importance of anticipation. The likely poor choosing optimism and low taxes makes the likely rich worse off if (29) is greater than (28). If $(1-\underline{\tau})r(\underline{\lambda}) - (1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau}-\underline{\tau})q > 0$, the condition for this reads:

$$s < \frac{-\delta(\overline{\tau}-\underline{\tau})(1-q)}{(1-\underline{\tau})r(\underline{\lambda}) - (1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau}-\underline{\tau})q}. \quad (30)$$

Since the denominator is positive and the nominator negative, the right-hand side of (30) is negative. As $s \geq 0$, the condition is never satisfied, and the likely rich are always better off in the low tax equilibrium. If, on the other hand, $(1-\underline{\tau})r(\underline{\lambda}) - (1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau}-\underline{\tau})q < 0$, the condition for the likely rich to be worse off in the low tax equilibrium reads:

$$s > \frac{\delta(\overline{\tau}-\underline{\tau})(1-q)}{(\overline{\tau}-\underline{\tau})q + (1-\overline{\tau})r(\overline{\lambda}) - (1-\underline{\tau})r(\underline{\lambda})} \equiv s^{***}(\chi). \quad (31)$$

Obviously, whether the likely rich are worse off in the low tax equilibrium is an interesting question only when the low tax equilibrium is possible. Figure 5 depicts s^{**} and s^{***} as a function of χ. As we have seen, the low tax equilibrium occurs if $s > s^{**}$. By definition of s^{***}, the rich are worse off in the low tax equilibrium if $s > s^{***}$. For $\chi = 1$ the thresholds s^{**} and s^{***} coincide. Therefore, only for the fully Bayesian agents the optimism of the likely poor necessarily makes the rich worse off. For $\chi < 1$ this is not necessarily the case.

Proposition 8 (The welfare of the likely rich). *Whether the likely rich are worse off in the low tax equilibrium depends on the degree of the Bayesian sophistication and the value of anticipation.*

(i) For $\chi = 1$, the likely rich are worse off in the low tax equilibrium than in the high

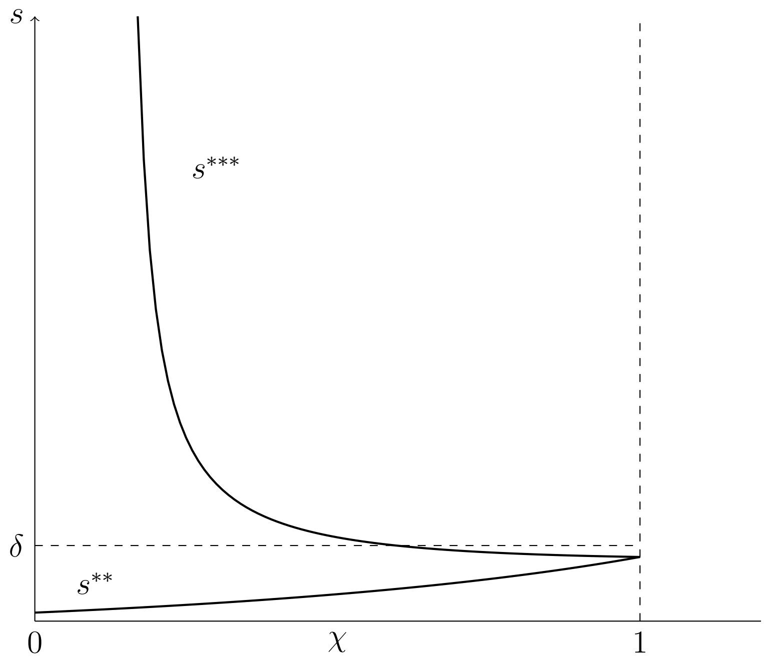

Figure 5: s^{**} and s^{***} as a function of χ

tax equilibrium.

(ii) For $\chi < 1$, the likely rich are worse off in the low tax equilibrium only if $s > s^{***}$.

In Figure 5, below the lower curve, the POUM effect does not occur. Between the two curves, the POUM effect occurs and it makes the likely rich better off. Above the upper curve, the POUM effect occurs, and it makes the likely rich worse off.

Interestingly, an implication of the model is that fully Bayesian likely rich are worse off with low taxes if the value of anticipation is high enough for likely poor to choose optimism and low taxes. Again, however, completely sophisticated cognitive technology might be only theoretical interest. The threshold value s^{***} goes up fairly rapidly for $\chi < 1$, which makes this result less relevant.

4.3 Sincere Voting

The beliefs are most likely to be distorted by desires if the individual cost of holding biased beliefs is small, as is the case in voting if the probability of being pivotal is very small (Bénabou and Tirole, 2016). An alternative assumption about the voting behavior of agents is that they do not consider themselves to be pivotal in the determination of the tax policy and, therefore, form their beliefs without taking account how it affects their policy preferences and voting.

In the model of the current work, agents trade their optimism against redistribution. If we let the agents ignore this trade-off by assuming sincere voting, the only thing restricting the optimism of agents are the constraints of the cognitive technology. Therefore, taking τ^* as given, the dominating action for the likely poor is to choose $\lambda = 0$ for all $s > 0$: The lower λ they choose, the higher anticipatory utility they can expect. The loss of income and consumption in period 2 due to less redistribution does not enter the trade-off since the agents do not think they can in any way influence the policy outcome. In the unique equilibrium all agents recall $\hat{\sigma} = y_H$, they expect at least average income, and the tax policy is $\tau^* = \underline{\tau}$. This is curiously the equilibrium even if the likely poor do not value anticipation very much and are worse off in the equilibrium than if they had all been realists and voted for high taxes.

Interestingly, another way to motivate sincere voting is to derive it as a limiting case of our benchmark model. When the range of the feasible tax rates goes to zero, the threshold s^{**} goes to zero as well: $\overline{\tau} - \underline{\tau} = 0$ implies $s^{**}(\chi) = 0$, and choosing $\lambda = \underline{\lambda}$ over $\lambda = \overline{\lambda}$ is optimal for all $s > 0$. When the upper and lower bounds of the tax policy coincide, the likely poor cannot affect the tax rate by voting, and it is optimal for them to indulge in optimism.

For the clarity of exposition, we consider the case $\tau \in [0, 1]$.[26] The likely poor take τ as given and choose λ to maximize

$$\begin{aligned} U_{0,i}(\lambda) &\equiv \lambda u_{0,i}(y_L, y_L, \tau) + (1-\lambda) u_{0,i}(y_L, y_H, \tau) \\ &= (1-\lambda)\left[\delta s[(1-\tau)[r(\lambda) y_H + (1-r(\lambda)) y_L] + \tau \bar{y}] + \delta^2[(1-\tau) y_L + \tau \bar{y}]\right] \\ &\quad + \lambda\left[(\delta s + \delta^2)[(1-\tau) y_L + \tau \bar{y}]\right]. \end{aligned} \tag{32}$$

The best response independently of the choices of others is $\lambda = 0$. This implies an equilibrium tax rate of $\tau^* = 0$.

Proposition 9 (**Politico-economic equilibrium, sincere voting**). *If the likely poor do not condition their belief and voting choices on the tax policy, then, for all $s > 0$, there*

[26]The case $\tau \in [\underline{\tau}, \overline{\tau}]$ is similar.

is a unique equilibrium, where $\lambda^* = 0$*, the likely rich recall* $\hat{\sigma}_i = y_H$*, and* $\tau^* = 0$*.*

The utility of a representative likely poor agent is

$$U_{0,i}(0) = \delta s[r(0)y_H + (1 - r(0))y_L] + \delta^2 y_L, \tag{33}$$

whereas if the likely poor would have coordinated choosing $\lambda \in [\frac{1}{2(1-q)}, 1]$, a representative likely poor agent would have enjoyed utility

$$U_{0,i}(\lambda) = \delta s\bar{y} + \delta^2 \bar{y} \quad \forall \lambda \in [\frac{1}{2(1-q)}, 1]. \tag{34}$$

From Lemma 1, we know that if $s < s^*$, (34) is greater than (33).

Proposition 10 (Welfare of the likely poor). *If* $s < s^*$*, the likely poor are worse off in the low tax equilibrium, than if they had coordinated on voting for high taxes.*

A free-riding problem emerges among the likely poor: for each, it is individually rational to indulge in optimism, but with coordinated actions they could increase their payoffs. This case is similar to the public goods game, where the individually rational agents do not contribute even if they would all be better off by contributing. Here the public good is the redistribution, and the cost of contribution is lower anticipatory utility. However, the likely poor coordinating on realism to support high taxes is not necessarily a Pareto improvement when considering the whole electorate, as providing a public good in a public good game is. As seen in section 4.2, the likely rich are worse off in the high tax equilibrium if $s < s^{***}$ and $\chi < 1$. However, if $\chi = 1$, then unique equilibrium is Pareto-inferior and the likely poor coordinating on realism would be a Pareto improvement.

In contrast to the case of strategic belief formation, which admittedly is a strong requirement on the behavior of the voters, sincere voting always leads to the POUM effect. When the likely poor do not think that their own beliefs will influence the policy outcome, they maximize their utility by maximizing their optimism.

5 The General Cognitive Technology

5.1 Motivations

To clearly see the connection between naive and sophisticated cognitive technologies of belief distortion, this section presents a general version of the cognitive technology used in the current work. It is shown how the ostensibly different cognitive technologies are special cases of this general model and how the models in Bénabou and Tirole (2002, 2006), Bénabou (2008, 2013) and Kopczuk and Slemrod (2005) relate to other models of biased endogenous beliefs in the literature, such as to the Endogenous Beliefs Model (EBM) in Minozzi (2013) and to the classic reference in biased beliefs by Akerlof and Dickens (1984).

The challenge in bringing these various models together is that in the sophisticated cognitive technologies of the literature, Nature's signal is binary and the action space of Self 0 is the mixed strategies over the two signals, whereas in the naive cognitive technologies signal set is larger or even continuous and mixed strategies are not considered. Also, in the models of a naive cognitive technology, the equilibrium can be characterized using Subgame Perfect Equilibrium, whereas the models with a sophisticated cognitive technology rely on Perfect Bayesian Equilibrium. In addition, especially the EBM of Minozzi (2013) is defined for games with perfect information and chance moves to facilitate the use of Subgame Perfect Equilibrium and backward induction as a solution concept. Since a game with uncertainty and perfect information may be confusing, in the presentation of the general cognitive technology here, we stick to the convention of letting Nature move first and only once and depart from this convention only when deriving the EBM from the general model.

The general cognitive technology presented here is adapted from Bénabou and Tirole (2002), where the supply of distorted beliefs is modeled as a two-player dynamic game of incomplete information within each agent. The model of Bénabou and Tirole (2002) is generalized by letting the signal set be arbitrarily large, but discrete.[27] From this general model, we then see that the only important difference between naive and sophisticated

[27]Applications of naive cognitive technologies usually have continuous signal sets, but since the formalization of EBM in Minozzi (2013) is finite, we stick to finite games as well.

cognitive technologies is the degree of Bayesian sophistication parametrized by χ.

To simplify the exposition and to focus on the differences between naive and sophisticated cognitive technologies we assume that memory repression or rehearsal is costless throughout the analysis. That is, agents do not incur any costs when memorizing and forgetting signals and the only costs of belief distortion are in the form of nonoptimal decisions. This assumption is also reasonable since memory repression costs, while proposed as a possible extension in Brunnermeier and Parker (2005), are otherwise absent in the naive cognitive technologies of the literature. In the language of game theory, signaling is costless, and the resulting game could be characterized as a cheap talk game.

The second motivation for the presentation of the cognitive technology in the form here is to emphasize its portability. In a setting where an economy and interactions of agents can be modeled as a game of imperfect or incomplete information, the actual game can be extended to include a pre-game, where the beliefs that players hold in the actual game are determined. Hence, whenever a game has imperfect or incomplete information and incentives to hold biased beliefs, there is potentially a role for belief distortion. The cognitive technology is presented here such that it is flexible enough to be used in different settings involving uncertainty and beliefs. Hence, this chapter aims to show how to extend any game to study whether the possible belief distortion has interesting consequences for the outcomes of the game.

As a third motivation, this section provides the game theoretical structure of the POUM model proposed in the present work and shows in a more detailed way how this model relates to the POUM model proposed in Minozzi (2013).

A complete model of endogenous belief distortion consists of supply of distorted beliefs which answers to the question of how the agents can bias their beliefs, and demand for distorted beliefs which answers to the question of why the agents would bias their beliefs. The aim of the current chapter is to focus on the supply of distorted beliefs, but for completeness, in the next section, we shortly discuss the demand for distorted beliefs.

5.2 Incentives to Bias Beliefs

In a framework where the belief distortion arises endogenously from the optimal actions of agents, the mere possibility to bias beliefs is not enough for an interesting model. Conventionally, an agent is always weakly better off when having more accurate information. The agent cannot be worse off since information can always be ignored, but the more accurate information can help the agent to evaluate the expected consequences of her actions better. Obviously then, holding distorted beliefs can expose the agent to costly mistakes in maximizing the expected utility. Even if we suppose that agents can choose the beliefs they hold, that is, if we suppose that the supply of distorted beliefs exists, the utility maximizing agent will choose to hold the beliefs that most closely correspond to the objective reality if there is no incentive to bias beliefs. However, in addition to the accuracy goals in reasoning, the literature in psychology has demonstrated that directional goals are important as well (Kunda, 1990). These directional goals refer to other uses of beliefs in addition to their informational value and suggest that beliefs have value to people beyond to that of being a weighting function in the computation of the expected utility. The literature on motivated beliefs uses this insight from psychology to motivate agents to form biased beliefs by building various incentives to for biased beliefs into their utility functions.

One such incentive is given by the affective or consumption value of the beliefs. Here we can make a distinction between beliefs that as such please us such as believing in the predictability and safety of our surroundings, and anticipatory emotions which we feel when expecting the consequences of our actions and which have been extensively discussed above (Bénabou and Tirole, 2016, page 143).

Beliefs might also have instrumental or functional value, for instance, if agents find it useful to distort their beliefs when facing time inconsistency problems. Smokers trying to control their smoking might want their future selves to believe smoking to be dangerous and to choose accordingly (Carrillo and Mariotti, 2000) or people suffering from insufficient motivation when exerting effort might use rosy views about the future payoffs to motivate themselves (Bénabou and Tirole, 2002, 2006). Generally, an agent who suffers from time inconsistency problems could use her beliefs as a commitment device. By manipulating the beliefs of her future incarnations, she has some control over their decisions.

Distorted beliefs work as a commitment device in crisis bargaining game of Minozzi (2013) as well. Parties can improve their positions in the bargaining by optimistically believing that they will be victorious in the conflict that results if the bargaining leads to failure. The beliefs are common knowledge and affect the maximum levels of willingness to pay to avoid conflict and, therefore, to the division of surplus. Even if the players think that the beliefs of their opponents are biased, they know that their opponents will nevertheless act according to these beliefs.

Over-confidence about own abilities might reduce the signaling cost when an agent tries to convince others of her good qualities (Bénabou and Tirole, 2002, page 877; Schwardmann and van der Weele, 2016). Trivers (2011) proposes that this signaling value of beliefs is the primary reason for self-deception: We deceive ourselves mainly in order to deceive others.

5.3 Multiple Selves and Imperfect Recall

Modeling agents consisting of multiple selves has become a standard tool in understanding inconsistent behaviors. Most notably, time inconsistencies and self-control problems indicate that people have conflicting preferences in different points of time. This can be modeled by letting agents have multiple temporal selves with different preferences, as, for instance, in Strotz (1956), Thaler and Shefrin (1981), Laibson (1997) and O'Donoghue and Rabin (1999). Jamison and Wegener (2009) go even further by proposing, that multiple selves need not only be viewed as a modeling device, but people seem to actually use the same brain systems to think about themselves in the future as they use to think about other people.

In contrary to changes in preferences, changes in beliefs and information do not necessarily lead to such inconsistencies that would require the use of the multiple-self approach. Learning and updating beliefs can be modeled without multiple selves. However, in addition to acquiring information people sometimes lose information as well. Forgetting is not something a rational agent would do, and modeling forgetting has not been common in economics since Selten (1975) rejected models with imperfect recall as misspecified. Modeling forgetting while preserving perfect recall can, however, be done with multiple selves. Letting the selves be distinct players and giving them different information, we can model the beliefs and information of agent as a game of incomplete information between her temporal selves. This implies that the agent can have imperfect recall and can forget, but the analysis of the model proceeds at the level of her temporal selves which allows us to use the standard tools of game theory, including perfect recall.

For the purposes of this chapter, we have to specify two forms of imperfect recall. First, forgetting, that is, an information set containing histories that are incompatible with previously held information, is one way to violate perfect recall. The other way relevant here is imperfect inference. A player may remember her past information sets, but still fail to use this information to consistently infer in which element of her current information set she is acting (Rubinstein and Piccione, 1997, page 6). All the cognitive technologies with $\chi < 1$ violate perfect recall since the beliefs are not updated using Bayes rule. In our analysis, we use the multiple-self approach to circumvent the first type of imperfect recall but the second form of imperfect recall will be present whenever $\chi < 1$.

5.4 The Cognitive Technology

5.4.1 Players

The economy consists of a set N of agents. The conventional concrete actions of agents in this economy are captured by a general game Γ, where there is uncertainty over the outcomes. Naturally, this uncertainty induces agents to form beliefs so that they can compare the expected payoffs of their actions. We extend this game by embedding it as a continuation game to a pre-game of intrapersonal strategic communication, which crucially changes the belief formation process and allows biased beliefs in Γ.

To do this, we model each agent as two players. Using the language of Bénabou and Tirole (2002), we call these players Self 0 and Self 1 of an agent. Self 1s actions are the conventional concrete actions that are available to agents in game Γ. Self 0s act in the pre-game and their actions are the more subtle cognitive processes that affect the information of Self 1s. To fix vocabulary, we call *players* all those entities in the economy that make decisions, that is, Self 0s and Self 1s of all agents. Each *agent*, on the other hand, consists of two players, Self 0 and Self 1 of that agent.[28] Agents are the entities we would observe acting in the economy. If the total number of agents is $|N|$, then the total

[28]The EBM model in Minozzi (2013) calls these the player and the belief-forming agent of the player. Multiple-self approach also parallels with Selten's (1975) *agent-normal form* where each information set is given an agent that represents the player to whom the information set belongs and who has the same payoffs as this player. We reserve the word *player* to refer to a rational decision maker in a game.

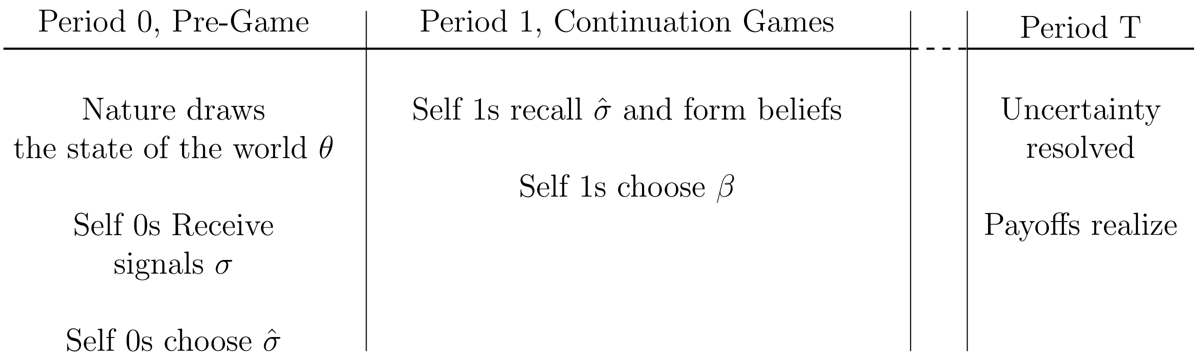

Figure 6: Timeline

number of players is $2|N|$. Giving different names to the temporal selves of agents clarifies the approach taken here, where each agent is modeled as two separate players, and both of the players have perfect recall in the sense of remembering all their past actions and their past information. An agent can, however, forget her past actions, since Self 1 may have less information than Self 0. Nevertheless, as will become clear, we allow Self 1s to have imperfect recall in the sense of non-standard inference.

Agents interact with other agents in an economy where their payoffs are potentially interconnected. Therefore, the selves of the agents act strategically taking account the actions of not only the other self of the same agent but also the selves of all other agents. All these interactions are captured by a dynamic game of imperfect information.

5.4.2 Pre-Game

A crucial component of a model of beliefs is some uncertainty over the outcomes giving room for the formation of beliefs. Here this uncertainty is in the form of a random variable $\theta_i \in \Theta_i$ on whose realization the utility of agent i depends. We call this random variable θ_i the *type* of agent i. Θ_i is the set of possible types of agent i. As usual, incomplete information is modeled as imperfect information, and a player called Nature determines the realizations of random variables.

We grant the first move to Nature. Nature chooses the state of the world by drawing a vector of types θ from a joint probability distribution of types $G(\theta)$. The support of this distribution and the set of possible states of the world is $\Omega = \times_{i \in N} \Theta_i$. None of the players observe the realization of the state of the world directly. Instead, for all i, Self 0 of agent i receives a signal determined by a signal function $\sigma_i : \Omega \mapsto \Sigma_i$, which maps the set of states of the world to the signal set Σ_i.[29] The signal set is a subset of the set of all possible subsets of the set of states of the world, $\Sigma_i \subset \mathcal{P}(\Omega)$, where $\mathcal{P}$ denotes power set. Self 1s do not observe the signal.[30] In general, the signals are noisy, and Self 0s do not get to know the exact state of the world, but a set of the states of the world of which the realized state of the world is an element. Their information after receiving the signal is the prior distribution conditioned on the received signal. Let $G(\theta|\sigma_i) = F_{0,i} \in \mathcal{F}_{0,i}$, where $\mathcal{F}_{0,i}$ is the set of all possible beliefs about θ Self 0 of agent i can end up with. Given the prior distribution $G(\theta)$, the form of the signal function σ_i determines $\mathcal{F}_{0,i}$. Self 0 of agent i, given a signal, updates her beliefs using the Bayes rule. That the Self 0 rationally uses all the available information, represents the idea that the objective reality plays a role in the belief formation process. By knowing the true signal, Self 1 knows the consequences of biased beliefs and can, therefore, address the trade-off between the benefits of distorted beliefs and the costs of inaccurate information.

Self 1 of agent i does not observe the signal σ_i, but she might get information about the state of the world by recalling (receiving) a signal $\hat{\sigma}_i$, which Self 0 has memorized (sent). The action of Self 0 is to choose which signal Self 1 will recall. Let $\hat{\sigma}_i$ denote a pure action of Self 0 of agent i. The set of pure actions of Self 0 is the same as her signal set. In general, Self 0 of agent i plays a mixed strategy ϕ_i, which assigns a probability distribution over Σ_i for each possibly received signal. Let the set of mixed strategies available to agent i be Φ_i. The probabilities assigned to each possible signal to Self 1 can be interpreted as recall or awareness rates. The memory of an agent is probabilistic: If the Self 0 receives the signal σ_i, the probability of remembering signal $\hat{\sigma}_i{}'$ is $\phi_i(\hat{\sigma}_i{}'|\sigma_i)$. Let $\sigma = (\sigma_i)_{i\in N}$ and $\hat{\sigma} = (\hat{\sigma}_i)_{i\in N}$ be the profiles of signals Self 0s receive and signals the Self 1s recall, respectively, and let $\phi = (\phi_i)_{i\in N}$ be the profile of Self 0s' strategies.

5.4.3 Continuation Game

We now describe a general game of imperfect information and show how biased beliefs can be modeled in it by embedding the general game as a continuation game for the pre-game

[29]The signal function here corresponds to the definition of partitional information function in Osborne and Rubinstein (1994, page 68).

[30]More formally, we could define separate uninformative constant signal functions to Self 1s.

described in the previous section. This general game can describe any interesting setting where there is some incentive for agents to hold biased beliefs.

Let the economy without belief distortion be described by a general extensive form game with imperfect information $\Gamma = (N, H, A, P, \{u_i(\beta, \theta)\}_{i \in N}, I, G)$. N is the set of agents, H is the set of histories including the terminal histories and the initial history, A is the set of actions, and $A_i(h)$ is the set of actions available to agent i in history h. P is the player assignment function, which assigns a set of players from N to each history that is not terminal or initial. The initial history is assigned to Nature. G is the probability distribution over Ω, and I is the information partition, which for each player, partitions the histories the player function has assigned to them into information sets. The utilities $\{u_i(\beta, \theta)\}_{i \in N}$ satisfy vNM-assumptions and depend on the realization of θ from G and the profile of strategies of Self 1s $\beta = (\beta_i)_{i \in N}$.

Let $\mathcal{A}_i(h)$ denote the lotteries over the set $A_i(h)$. A behavioral strategy β_i of Self 1 of agent i assigns a lottery from $\mathcal{A}_i(h)$ to each history h where agent i is assigned to act. Let B_i denote the set of behavioral strategies available to agent i.

To allow agents have distorted beliefs in Γ, we use Γ to create a continuation game for the pre-game described in the previous section. In the continuation game following the pre-game, agents recall signals $\hat{\sigma}$, but they have uncertainty over their original signals. We model this uncertainty over the signals by using Γ to define a set of games for each agent, each game corresponding to one possible original signal such that in each of these games there is no uncertainty about the signal Self 0 received. Denote these games $\Gamma(\sigma_i)$, where σ_i is the original signal sent by Nature. These games differ in what agents can expect θ to be in them and for each agent i there is such game for each element in Σ_i. Self 1 of agent i then does not know which of the games $\{\Gamma(\sigma_i) | \sigma_i \in \Sigma_i\}$ best describe the reality and in which she is playing. In principle, the information set of each player in the initial history of the continuation game has as many elements as Ω has. However, to make clear how the belief distortion concerns the beliefs about the signals Self 0s receive, we partition the information sets such that the initial histories of the games $\{\Gamma(\sigma_i) | \sigma_i \in \Sigma_i\}$ become the elements of the initial history of the continuation game. The initial histories of the games $\{\Gamma(\sigma_i) | \sigma_i \in \Sigma_i\}$ are elements in the partitioned information set in the initial history of the continuation game, but, in general, itself nontrivial information sets as well. This dichotomy of uncertainty in the initial history of the continuation game helps us separate the information about signals which is affected by belief distortion and the information within the signals which is not affected by belief distortion. We now form the set of games $\{\Gamma(\sigma_i)|\sigma_i \in \Sigma_i\}$ from Γ.

First, we stick with the convention of letting Nature move only once and at the initial history of the whole game. This means that in the initial history of each game $\Gamma(\sigma_i)$, the state of the world is already determined, but the players, in general, do not know it. Therefore, the player assignment function P now assigns, instead of Nature, a player or players from N to the initial history. [31] Second, players' beliefs over θ in game $\Gamma(\sigma_i)$ are given by the conditional distribution $G(\theta|\sigma_i)$. Third, the information partitions I in games $\{\Gamma(\sigma_i)|\sigma_i \in \Sigma_i\}$ depend on the signal function. Whatever uncertainty the signal σ_i does not resolve, is still left in $\Gamma(\sigma_i)$. If the signal function was perfectly informative, that is, if the elements of the range of the signal function were singletons, then information in games $\{\Gamma(\sigma_i)|\sigma_i \in \Sigma_i\}$ would be perfect. Fourth, the set of players N describes now the set of Self 1s. Fifth, the initial history of $\Gamma(\sigma_i)$ is $(\emptyset, (\sigma_i, \sigma_{-i}), \hat{\sigma})$. Sixth, we denote the beliefs of others as $\{F_{1,j}\}_{j \in N, j \neq i}$. These beliefs are determined by Self 1s' inferences about the original signal and will be discussed in the next section. The beliefs of others are not informative other than in predicting the actions of others. This "agree to disagree" assumption means that agents view the beliefs of others as mistaken, but allows them to take account how these beliefs, even if mistaken, affect the behavior of others.[32] Therefore, note for now, that these beliefs do not depend on what Self 1 of agent i beliefs the original signal to be. Seventh, we add a subscript 1 to the payoff functions to denote that they now represent the preferences of Self 1. N, A, H, P, $\{u_{1,i}(\theta, \beta)\}_{i \in N}$, and the beliefs of others $\{F_{1,j}(\sigma_j)\}_{j \in N, j \neq i}$ are the same in each $\Gamma(\sigma_i) \in \{\Gamma(\sigma_i)|\sigma_i \in \Sigma_i\}$ and these games can now be written as $\Gamma(\sigma_i) = (N, H, A, P, \{u_{1,i}(, \beta, \theta)\}_{i \in N}, \{F_{1,j}\}_{j \in N, j \neq i}, G(\theta|\sigma_i), I)$ where $\sigma_i \in \Sigma_i$. That is, $\Gamma(\sigma_i)$ is a game where the probability distribution over θ is given by the prior distribution and σ_i.

The continuation game following the pre-game, consists of a set of games $\{\Gamma(\sigma_i)|\sigma_i \in$

[31] An equivalent way to construct the game would be to let Nature move twice: once in the initial history of the pre-game and once in the initial history of the continuation game. The first move determines the signal or a subset of the states of the world and the second move the exact state of the world.

[32] See Minozzi (2013) page 577, Brunnermeier and Parker (2005) page 1094, and Aumann (1976).

$\Sigma_i\}$ for each agent i. Agents do not know in which of these games they are playing and use their recalled signal and their knowledge about the prior distribution of signals to form beliefs.

5.4.4 Beliefs

Beliefs are modeled as probability distributions over the elements in the information sets. Here, the interesting beliefs are the beliefs Self 1s form about the signals Self 0s received. Each agent starts the continuation game following the pre-game in the partitioned information set that has as many elements as the signal set has. These elements are the initial histories of the games $\{\Gamma(\sigma_i)|\sigma_i \in \Sigma_i\}$. When recalling a signal, given the strategy of Self 0, Self 1 of agent i forms a belief system r_i. This belief assigns a probability for each possible signal Self 0 could have received, and equivalently, to each game $\Gamma(\sigma_i)$ in which Self 1 could be playing.

We define the probability weight Self 1 assigns to the game corresponding the signal she recalled and the probability weights Self 1 assigns to the games which correspond to the nonrecalled signals separately. Consider first the recalled signal. Following Bénabou and Tirole (2002), the probability of the game corresponding to the recalled signal is given by the reliability of the recalled signal defined as

$$\begin{aligned} r_i(\hat{\sigma}_i'|\hat{\sigma}_i', \chi) &= Pr(\sigma_i = \hat{\sigma}_i'|\hat{\sigma}_i = \hat{\sigma}_i') \\ &= \frac{g_i(\hat{\sigma}_i')\phi_i(\hat{\sigma}_i'|\hat{\sigma}_i')}{g_i(\hat{\sigma}_i')\phi_i(\hat{\sigma}_i'|\hat{\sigma}_i') + \chi \sum\limits_{\substack{\sigma_i \in \Sigma_i \\ \sigma_i \neq \hat{\sigma}_i'}} g_i(\sigma_i)\phi_i(\hat{\sigma}_i'|\sigma_i)}, \end{aligned} \tag{35}$$

where $\hat{\sigma}_i'$ is the signal Self 1 of agent i recalls. That is, when recalling signal $\hat{\sigma}_i'$, Self 1 of agent i assigns probability $r_i(\hat{\sigma}_i'|\hat{\sigma}_i', \chi)$ to her playing in game $\Gamma(\hat{\sigma}_i')$. Here $g_i(\sigma_i)$ denotes the probability of Self 0 of agent i receiving signal σ_i. [33] $\chi \in [0, 1]$ is again the naivete parameter measuring the degree of Bayesian sophistication.

In the case of a binary signal set, the belief system simply assigns the remaining probability to the possibility that the true signal is the one not recalled. Here, we want to

[33]Sticking with the previous definition, formally more correct way would be to denote the probability of receiving signal σ' as $g_i(\sigma_i^{-1}(\sigma'))$.

allow larger signal set which complicates things slightly since the remaining probability has to be divided between many nonrecalled signals. Therefore, for the current purposes, a simple solution for this problem is proposed, and we define the probability weights of the signals that were not recalled in the following way. Let $\hat{\sigma}_i'$ still be the recalled signal. For all $\hat{\sigma}_i'' \neq \hat{\sigma}_i'$, the probability weights of nonrecalled signals implied by Bayes rule are

$$\begin{aligned} p_i(\hat{\sigma}_i''|\hat{\sigma}_i', \chi) &= Pr(\sigma_i = \hat{\sigma}_i''|\hat{\sigma}_i = \hat{\sigma}_i') \\ &= \frac{g_i(\hat{\sigma}_i'')\phi_i(\hat{\sigma}_i'|\hat{\sigma}_i'')}{g_i(\hat{\sigma}_i'')\phi_i(\hat{\sigma}_i'|\hat{\sigma}_i'') + \sum\limits_{\substack{\sigma_i \in \Sigma_i \\ \sigma_i \neq \hat{\sigma}_i''}} g_i(\sigma_i)\phi_i(\hat{\sigma}_i'|\sigma_i)} \quad \forall \hat{\sigma}_i'' \in \Sigma_i. \end{aligned} \tag{36}$$

Now, if $\chi < 1$ the recalled signal gets more weight than what the Bayes rule would imply. Hence, we rescale the probability weights of the nonrecalled signals to ensure that the probabilities sum up to unity. The total amount of probability weight left for the signals that were not recalled is $1 - r_i(\hat{\sigma}_i'|\hat{\sigma}_i', \chi)$, and the total probability weight that these signals have according to Bayes rule is $\sum_{\hat{\sigma}_i'' \in \Sigma_i, \hat{\sigma}_i'' \neq \hat{\sigma}_i'} p_i(\hat{\sigma}_i''|\hat{\sigma}_i', \chi)$. The ratio of these is used to rescale the beliefs implied by Bayes rule in (36). The probability weights that the belief r_i assigns to the nonrecalled signals are given as follows:

$$r_i(\hat{\sigma}_i''|\hat{\sigma}_i', \chi) = p_i(\hat{\sigma}_i''|\hat{\sigma}_i', \chi) \frac{1 - r_i(\hat{\sigma}_i'|\hat{\sigma}_i', \chi)}{\sum\limits_{\substack{\sigma_i'' \in \Sigma_i \\ \sigma_i'' \neq \sigma_i'}} p_i(\hat{\sigma}_i''|\hat{\sigma}_i', \chi)} \quad \forall \hat{\sigma}_i'' \in \Sigma_i \setminus \{\hat{\sigma}_i'\}. \tag{37}$$

It is straightforward to see that

$$\sum_{\substack{\sigma_i'' \in \Sigma_i \\ \sigma_i'' \neq \sigma_i'}} r_i(\hat{\sigma}_i''|\hat{\sigma}_i', \chi) + r_i(\hat{\sigma}_i'|\hat{\sigma}_i', \chi) = 1. \tag{38}$$

A complete belief system is a function that assigns a probability distribution over the initial histories of games $\{\Gamma(\sigma_i)|\sigma_i \in \Sigma_i\}$ to each information set: for all $\hat{\sigma}_i' \in \Sigma_i$

$$r_i(\cdot|\hat{\sigma}_i', \chi) = \Big(r_i(\hat{\sigma}_i'|\hat{\sigma}_i', \chi), \big(r_i(\hat{\sigma}_i''|\hat{\sigma}_i', \chi) \big)_{\hat{\sigma}_i'' \in \Sigma_i \setminus \{\hat{\sigma}_i'\}} \Big) \tag{39}$$

where the first element of the pair is the reliability of recalled signal defined in (35) and the second element gives the probability weights of the nonrecalled signals and is defined in (36) and (37). Let $r = (r_i)_{i \in N}$ be the profile of belief systems. Note that the belief system defines beliefs only to the information sets that are reached with positive probability.

The belief system r describes only the beliefs of Self 1s over the set of possible original signals. The uncertainty that the signal σ_i did not reveal to Self 0 is still part of the game $\Gamma(\sigma_i)$. After receiving signal $\hat{\sigma}_i'$, agent i's Self 1's beliefs over θ are now given by the compound distribution of her belief r_i and the set of distributions $\{G(\theta|\sigma_i)|\sigma_i \in \Sigma_i\}$ in the games $\{\Gamma(\sigma_i)|\sigma_i \in \Sigma_i\}$. We denote this compound distribution and the information of Self 1 of agent i over θ in the initial history of the continuation game by $F_{1,i}$.

These beliefs depend on the strategy of Self 0 and the naivete parameter χ. First, the higher is the probability of recollection that a strategy assigns to an untrue signal, the less weight the corresponding game tends to get in Self 1's beliefs. This is where the forward induction of agents makes it possible for them to doubt memories which they recall more often than what the prior distribution would imply. Note also that the belief system depends on the agents' own strategies, not the strategies of others. This parallels the discussion in section 4.2 on how the belief externalities are externalities across information states or alternative histories, and not across agents.

Second, the naivete parameter controls the sophistication of the inference of agents, and, hence, their ability to this forward induction. $\chi = 1$ is the standard case of full Bayesian rationality, and this is the case which we have been calling a sophisticated cognitive technology. In the other extreme of $\chi = 0$, the cognitive technology is naive, and the reliability is always unity independent of the strategy of Self 0. The literature has mainly focused on these extreme cases, full Bayesian rationality in Bénabou and Tirole (2002, 2006), Bénabou (2008, 2013), and Kopczuk and Slemrod (2005) and full naivete in Minozzi (2013), Brunnermeier and Parker (2005), and Akerlof and Dickens (1984). [34] Here we also allow the intermediate cases of $\chi \in (0, 1)$. The role of the naivete parameter is discussed below.

[34] An exception is Bénabou and Tirole (2000) where naivete parameter values $\chi \in (0, 1)$ are briefly studied.

5.4.5 Strategies and the Equilibrium

Let $\beta_i^*(\hat{\sigma})$ denote the optimal play of Self 1 of agent i given the recollections $\hat{\sigma}$ and implied beliefs $F_{1,i}$ and the beliefs of others $\{F_{1,j}\}_{j \in N, j \neq i}$ in the continuation game. The best response of Self 1 depends on the signals and beliefs of others since they affect the actions of other agents' Self 1s. The strategy of Self 0 enters the objective function of Self 1 via the information $F_{1,i}$. First, the best response of Self 1 depends on her information and, second, the information may affect Self 1's utility more directly, as is the case with anticipatory utility.

The indirect utility of Self 1 as a function of the pure actions of Self 0s is $u_{1,i}(\beta^*(\hat{\sigma}), \theta)$. The payoff of Self 0 of agent i when Self 0s are playing pure strategies $\hat{\sigma}$ can be written as the discounted indirect utility of Self 1:

$$u_{0,i}(\hat{\sigma}, \theta) = \delta u_{1,i}(\beta^*(\hat{\sigma}), \theta), \tag{40}$$

where δ is the standard discount factor.

Let $U_{0,i}(\phi, \sigma)$ be the expected utility of Self 0 as a function of strategy profile of Self 0s and given information $F_{0,i}$ which is implied by signal σ_i. [35] Self 0's optimal strategy ϕ_i^* satisfies for all $\sigma_i \in \Sigma_i$

$$U_{0,i}(\phi_i^*, \phi_{-i}^*, \sigma) \geq U_{0,i}(\phi_i, \phi_{-i}^*, \sigma) \quad \forall \phi_i \in \Phi_i. \tag{41}$$

In the equilibrium of the game, Self 0s of each agent choose the optimal awareness strategy given the optimal strategies and belief formation of Self 1s in the continuation game.

Self 1s' optimal strategies β^* are sequentially rational given their beliefs. The notion of sequential rationality only requires that the actions are optimal given the beliefs, and, therefore, applies here. On the contrary, for $\chi < 1$, the beliefs are not updated using Bayes rule, and they, therefore, are not consistent with the optimal strategies. Perfect Bayesian Equilibrium requires consistency and can be applied only to the case $\chi = 1$. The cases $\chi < 1$ cannot be characterized by Perfect Bayesian Equilibrium. We, therefore, define an equilibrium, which can be thought of as a generalization of Perfect Bayesian

[35] $U_{0,i}(\phi, \sigma)$ is the expectation of (40) where the expectation is taken over an outcome distribution determined by $G(\theta|\sigma_i)$ and the strategy profiles ϕ and β^*.

Equilibrium and which allows the imperfect Bayesian reasoning of Self 1.

Definition 1 (The Cognitive Equilibrium). *The triplet* (ϕ^*, β^*, r) *is an equilibrium if and only if*

(i) for each agent i*, Self 0's strategy* ϕ_i^* *satisfies (41) for all* $\sigma_i \in \Sigma_i$*,*

(ii) for each agent i*, Self 1's strategy* β_i^* *is sequentially rational for all* $\hat{\sigma} \in \times_{i \in N} \Sigma_i$*,*

(iii) Self 1s form beliefs according to (39).

The behavior of agents is characterized by the equilibrium of the game between their temporal selves. The cognitive equilibrium determines the motivated beliefs given the supply of beliefs, that is, the cognitive technology, and the demand for beliefs, that is, the incentives to hold biased beliefs in game Γ. The equilibrium strategies of Self 1s β^* determine the agents' actions influenced by their motivated beliefs and the equilibrium strategies of Self 0s ϕ^* represent agents' cognitive processes.

These equilibrium strategies and beliefs depend on χ. The parameter values $\chi < 1$ should not be thought of as arbitrary deviations from the Bayesian standard but as different specifications of constraints of the cognitive technology. Technically, χ measures the sophistication of Bayesian inference of agents, but in our context, it also measures how much the motivated beliefs depend on the reality and the strategy of Self 0. The higher is χ, the more constrained the beliefs are and the more they depend on reality. Also, the higher is χ, the more aware agents are that they might have an incentive and a tendency to have self-serving biases.

Kunda (1990) argues that an important mechanism for motivated beliefs is a biased memory search. When forming beliefs, people consult their memory. The possible directional goals, however, bias this memory search and memories that support the desired beliefs are more easily recalled than memories that would contradict them. One way to interpret the parameter χ is that it measures the degree of bias in the memory search. The lower is χ, the more biased the memory search is, and the more the resulting beliefs can deviate from reality.

The defined cognitive equilibrium allows us to see an interesting connection between what we have called sophisticated and naive cognitive technologies. For sophisticated

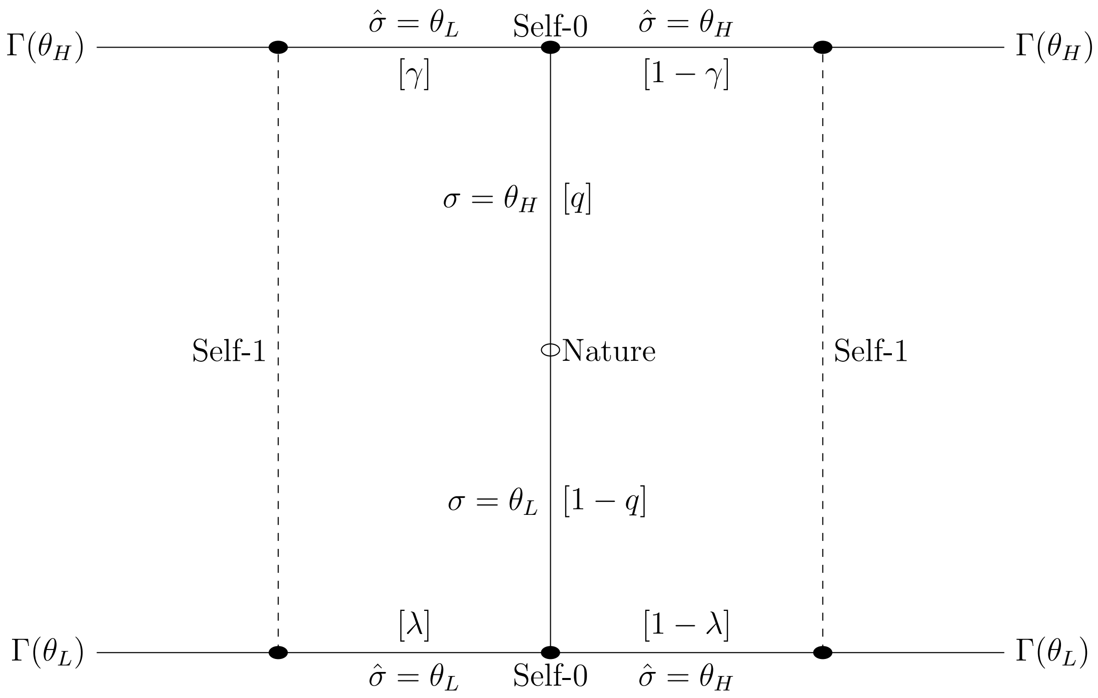

Figure 7: A simple game of motivated beliefs

cognitive technologies, that is, for $\chi = 1$, the cognitive equilibrium is Perfect Bayesian Equilibrium. However, as χ decreases, Self 1s put more and more weight on their recalled signal in their information set. For naive cognitive technologies $\chi = 0$, and agents behave as if they had perfect information regarding the signal Self 0 received. When agents behave as if they were in singleton information sets, the cognitive equilibrium can be characterized as a Subgame Perfect Equilibrium.

To see this and the role χ plays clearly, consider an illustrative example, a simple game of motivated beliefs, with the often used case of a binary signal and suppose the economy consists of only one agent. Let $\Theta = \{\theta_L, \theta_H\}$ such that $\theta_H > \theta_L$ and let the signal be determined by the probability mass function

$$g(\theta) = \begin{cases} q & \text{if} \quad \theta = \theta_H \\ 1-q & \text{if} \quad \theta = \theta_L \end{cases}. \tag{42}$$

Let the signal function be $\sigma(\theta) = \theta$, that is, Self 0 of agent gets to know her type exactly and the resulting beliefs can be represented by degenerate distributions: $F_0 = \theta_H$ or $F_0 = \theta_L$ depending on which signal the agent received. The signal set is $\Sigma = \{\theta_L, \theta_H\}$.

This specification can be depicted graphically and is shown in Figure 7. With a risk-neutral agent, an alternative setting that would lead to the same specification would be to let the signals be noisy such that the resulting beliefs would have expectations of θ_H and θ_L as is the case in the POUM model of this work.

Self 0's strategy is to choose the recall rates λ and γ, where λ and γ are the probabilities of types $\sigma = \theta_L$ and $\sigma = \theta_H$ playing $\hat{\sigma} = \theta_L$, respectively, in Self 0's strategy. Self 0's strategy can be described as a pair (λ, γ).

Suppose Self 0 receives a signal $\sigma = \theta_L$ and remembers the correct signal with probability λ. If Self 1 recalls signal $\hat{\sigma} = \theta_H$, she is in the lower right node in the initial history of $\Gamma(\theta_L)$ in Figure 7. However, as she does not observe the signal Self 0 receives and as she knows that she may not have recalled the correct signal she does not know for sure whether she is in the lower right node in the initial history of $\Gamma(\theta_L)$ or in the upper right node in the initial history of $\Gamma(\theta_H)$. She forms a belief according to (39).

$$r(\theta_H|\theta_H, \chi) = \frac{q(1-\gamma)}{q(1-\gamma) + \chi(1-q)(1-\lambda)}. \tag{43}$$

And since there are now only two signals:

$$r(\theta_L|\theta_H, \chi) = 1 - r(\theta_H|\theta_H, \chi). \tag{44}$$

Self 1 did not observe which signal Self 0 received, so she does not know for certainty in which game, $\Gamma(\theta_H)$ or $\Gamma(\theta_L)$ she is playing. She bases her actions on the expectation of θ:

$$E[\theta] = r(\theta_H|\theta_H, \chi)\theta_H + (1 - r(\theta_H|\theta_H, \chi))\theta_L. \tag{45}$$

The expectation over θ given in (45) is now the mean of a compound distribution of the belief system and the degenerate distributions over θ in games $\Gamma(\theta_L)$ and $\Gamma(\theta_H)$.

For $\chi = 1$ Perfect Bayesian Equilibrium characterizes the solution of the game. However, as the degree of the Bayesian sophistication χ decreases, $r(\theta_H|\theta_H, \chi)$ increases and Self 1 is more and more certain that she is truly playing in the game which the recalled signal indicates. In the limiting case of $\chi = 0$ the reliability of the recalled signal is unity and the expectation is $E[\theta] = \theta_H$. In this case of fully naive cognitive technology, she puts all the probability weight on a single element in her information set and behaves as if the information set was a singleton: If Self 0 receives $\sigma = \theta_L$, but sends a signal $\hat{\sigma} = \theta_H$, Self 1 will behave in the game $\Gamma(\theta_H)$ as if she had perfect knowledge that the game starts in the history $(\sigma = \theta_H, \hat{\sigma} = \theta_H)$. Note as well, that in this specification the signal function reveals the exact type of the agent and the games $\Gamma(\theta_L)$ and $\Gamma(\theta_H)$ have perfect information. Hence, this behavior of players, as if they had a perfect knowledge of all previous moves in the game, makes their beliefs trivial and allows the use of backward induction as a solution concept and makes Subgame Perfect Equilibrium enough to characterize the equilibria.

This illustration is, naturally, generalizable to the more general case discussed earlier. In the case of completely naive cognitive technology, Self 1 will believe any recollection, and Self 0 can, therefore, effectively decide what Self 1 believes. This is especially apparent when the action space of Self 0 is large. A continuous signal set and resulting continuous action space of Self 0, combined with complete naivete makes it possible for the Self 0 to continuously optimize the belief Self 1 holds as is the case in Minozzi (2013) and Akerlof and Dickens (1984).

5.5 Endogenous Beliefs Model

To connect this general cognitive technology with the Endogenous Beliefs Model (EBM) proposed in Minozzi (2013), we now derive EBM as a special case of our model. First, EBM relies on a naive cognitive technology and therefore corresponds to the case of $\chi = 0$. Second, the EBM is defined for games with perfect information and chance moves. We, therefore, slightly modify our framework by making Nature move last.

Let there be a finite set $N = \{1, ..., n\}$ of players, each comprising of their Self 0 and Self 1. Self 1 acts in a general game $\Gamma = (N, H, A, P, \{u_i(\beta, \theta)\}_{i \in N}, G, I)$. In EBM, an anticipatory utility term incentivizes the agents to hold biased beliefs. The game lasts two periods: in period 2, θ realizes and agents gain utility $u_{2,i}(\beta, \theta)$, and in period 1 they gain anticipatory utility $sE[u_{2,i}(\beta, \theta)|F_{1,i}]$ when expecting their period 2 utility. s denotes the "savoring" parameter and measures the value of anticipation. As EBM abstracts from discounting, the payoffs in Γ are defined as

$$u_i(\beta, \theta) = sE[u_{2,i}(\beta, \theta)|F_{1,i}] + u_{2,i}(\beta, \theta). \tag{46}$$

We transform this game of imperfect information to an equivalent game of perfect information with chance moves $\Gamma' = (N, H', A, P', \{u_i(\beta, \theta)\}_{i\in N}, G)$, that is, we let Nature move last. This simplifies the information structure of the game as all information sets become singletons. The game Γ' is now solvable by backward induction.

The games we use to describe the uncertainty over the Self 0s' signals in the continuation game are now $\Gamma'(\sigma_i) = (N, H', A, P', \{u_{1,i}(\beta, \theta)\}_{i\in N}, \{F_{1,j}\}_{j\in N, j\neq i}, G(\theta|\sigma_i))$, where $\sigma_i \in \Sigma_i$. Agent's utility in Γ becomes Self 1's utility in these games:

$$u_{1,i}(\beta, \theta) = sE[u_{2,i}(\beta, \theta)|F_{1,i}] + u_{2,i}(\beta, \theta). \tag{47}$$

Nature starts the game. The action of Nature is a realization from the joint probability distribution of types $G(\theta)$. Information in EBM is symmetric, so we let the signal function be common to all agents. Together with the prior distribution, a signal determines the posterior distribution $G(\theta|\sigma) = F_0 \in \mathcal{F}$. The belief represented by F_0 is a joint distribution over the types of which marginal distribution with respect to θ_i represents the prospects of agent i. Using a shortcut, we can say that Nature starts the game by determining a distribution $F_0 \in \mathcal{F}$ over outcomes as is the case in the EBM. Hence, with a slight abuse of notation, we let $\Sigma = \mathcal{F}$. Self 1s do not observe the signal and do not have access to F_0. Self 0 of agent i chooses, as before, an awareness strategy $\phi_i(\cdot|F_0)$, which, in general, is a probability distribution over $\mathcal{F}$ for each possibly received signal $F_0 \in \mathcal{F}$. Denote a pure action of Self 0 of agent i as F_i.

Plugging $\chi = 0$ into (39) gives us the belief system for a completely naive cognitive technology: for all $F'_i \in \mathcal{F}$,

$$\begin{aligned} r_i(F'_i|F'_i, \chi) &= Pr(\sigma_i = F'_i|\hat{\sigma}_i = F'_i) = 1 \\ r_i(F''_i|F'_i, \chi) &= Pr(\sigma_i = F''_i|\hat{\sigma}_i = F'_i) = 0 \quad \forall F''_i \in \mathcal{F} \setminus \{F'_i\}, \end{aligned} \tag{48}$$

where F'_i is the recalled signal. That is, for any recalled signal, Self 1 assigns a probability

1 for the event that the recalled signal truly represents the state of the world. The beliefs of agent i in the continuation game are again given by the compound distribution of the belief system r_i over the original signal and the set of possible distributions $\mathcal{F}$ over θ. Now, since the belief system assigns all probability to the recalled signal, when recalling signal $\hat{\sigma}_i = F_i$, the belief $F_{1,i}$ over θ in the continuation game is simply F_i.

Self 1s' beliefs regarding the signals are now trivial, and Self 1s behave as if there was no uncertainty over the original signal. Also, if an agent recalls a signal F_i she plays in $\Gamma'(F_i)$ as if it was a subgame. In addition, as discussed, the information sets in a game with perfect information and chance moves are singletons. All this makes it possible to solve the game using backward induction. Let $\beta_i^*(F_i, F_{-i})$ be the subgame perfect play of Self 1 of agent i in the subgame $\Gamma'(F_i)$.[36]

Self 0 knows that her anticipatory feelings depend on her beliefs F_i, but that her period 2 outcomes depend on the objective reality F_0. The expected utility of Self 0 is

$$U_{0,i}(F_i, F_{-i}, F_0) = sE[u_{2,i}(\beta^*(F_i, F_{-i}), \theta)|F_i] + E[u_{2,i}(\beta^*(F_i, F_{-i}), \theta)|F_0]. \tag{49}$$

If we restrict the signal choice to pure strategies, as is usually done in the games with naive cognitive technology and as is done in EBM, the Self 0s' problem can be written as

$$F_i^* = \underset{F_i \in \mathcal{F}}{\arg\max} \left\{ sE[u_{2,i}(\beta^*(F_i, F_{-i}), \theta)|F_i] + E[u_{2,i}(\beta^*(F_i, F_{-i}), \theta)|F_0] \right\}. \tag{50}$$

Knowing how Self 1 will behave given her beliefs, Self 0 effectively chooses the beliefs of Self 1.[37] In the equilibrium, Self 0 chooses Self 1's beliefs to optimize the trade-off between the period 1 anticipatory utility which depends on Self 1's beliefs and the period 2 utility which depends on the true distribution of θ.

Definition 2 (The Cognitive Equilibrium in EBM). *The triplet* (ϕ^*, β^*, r) *is an equilibrium in EBM if and only if*

(i) for each agent i, Self 0's strategy ϕ_i^* *solves (50) for all* $F_0 \in \mathcal{F}$*,*

(ii) for each agent i, Self 1's strategy β_i^* *is subgame perfect in* $\Gamma'(F_i)$ *for all* $F_i \in \mathcal{F}$*,*

[36]Strictly speaking, the equilibrium is not subgame perfect and Γ' is not a subgame, but as the agents are behaving as if they were, we use these terms here.

[37]The freedom in choosing beliefs could be restricted by restricting the signal set $\mathcal{F}$.

(iii) Self 1s form beliefs according to (48).

As before, the equilibrium of the game between the two temporal selves of an agent determines the beliefs and behavior of the agent.

5.6 Discussion

The advantage of using a naive cognitive technology is that it allows the use of backward induction and Subgame Perfect Equilibrium and, therefore, makes models with signal sets containing more than two elements very tractable. In the applications of sophisticated cognitive technologies, the signals are usually binary since increasing the signal set quickly renders the model too complicated for an analytical solution. Therefore, as usual in modeling, the choice of the sophistication of the cognitive technology has to be made between realism and simplicity. A completely naive cognitive technology may be a very crude representation of human reasoning, but in many cases, it allows a very tractable solution. Using a more sophisticated cognitive technology makes the model more complicated, but might lead to different results. As can be seen from the comparison of the POUM model in the current work to the POUM model in Minozzi (2013), a naive cognitive technology may overestimate the degree of bias and therefore its impact. Also, some interesting phenomena such as self-traps and self-doubt that can be explained with a sophisticated cognitive technology cannot occur with a naive cognitive technology as Bénabou and Tirole (2002) argue.

6 Conclusion

Over-optimism seems to be an important mechanism for the POUM hypothesis. We have formalized this mechanism by modeling the means and reasons for belief distortion and derived the conditions in which the poor majority of voters distort their beliefs enough to prefer low taxes in the time of voting. The poor do not expropriate the rich because they themselves believe to be rich someday, and they value these beliefs.

These motivated prospects of upward mobility emerge endogenously as a result of agents' choices between anticipation and consumption. The crucial factors in these choices are the value of anticipation and the relative differences in anticipation and consumption between the potential equilibria.

First, the more the likely poor expect to gain in anticipation when forming biased beliefs, the more biased these beliefs will be. Specifically, if the incomes or perceived incomes of the rich increase while transfers stagnate, the poor will be more likely to indulge in optimism and vote for low taxes. Hence, the striking result is that contrary to the benchmark model of Meltzer and Richard (1981), where the increase in inequality always increases the demand for redistribution, in our model, an increase in inequality can decrease the demand for redistribution.

Second, the less the likely poor expect to lose in consumption when forming biased beliefs, the more biased these beliefs will be. How much the likely poor can expect to lose in consumption depends on the potential tax rates in different equilibria. Hence, the smaller is the difference in the potential policy outcomes, the more likely the POUM effect is. Specifically, if the voters do not think that their vote has an impact in determining the policy outcome, that is, if they do not act strategically, they always form the most optimistic beliefs possible and, therefore, vote for low taxes. If the value of anticipation is low, individually and collectively rational choices diverge, and the poor voters are trapped in a bad equilibrium. By coordinating in voting for higher taxes, they could achieve higher welfare. In this case, the likely poor vote against their own self-interest.

The feasibility of the POUM effect also depends crucially on the specification of the cognitive technology, namely, on the naivete parameter χ. The less constraining the cognitive technology is, the more voters can bias their beliefs. Therefore the POUM effect becomes more feasible as an explanation for the limited size of the government in democracies when we specify the cognitive technology with small values of χ. This can be clearly seen when comparing the results of Minozzi's (2013) POUM model with our results. In Minozzi's model agents are naive and can effectively choose their beliefs without the restrictions of prior beliefs or reality. When making a more conventional assumption about the voters forward-looking behavior and setting $\chi = 1$ corresponding to the standard Bayesian rationality in belief updating, the poor voters cannot bias their beliefs enough for the POUM effect to occur. This result, however, hinges on the simple specification with linear policy preferences and a policy choice between complete equalization and complete laissez-faire. By exogenously restricting the possible tax policies, we show that the POUM effect can be an important factor in voting behavior even if we endow the voters with a more realistic cognitive technology than in Minozzi (2013).

This distinction between naivete and sophistication represents a broader dichotomy of cognitive technologies for belief distortion in the literature. We have shown the somewhat hidden, but simple relation between the naive and sophisticated cognitive technologies to be the different specifications about the strategic sophistication of agents and how the biased beliefs depend on reality. Formally this distinction comes down to the specification of the naivete parameter χ. How the various cognitive technologies of motivated beliefs relate can be seen with the help of the general cognitive technology developed in the current work. It also makes clear the trade-off in tractability of the model when choosing between different technologies of belief distortion. A naive cognitive technology may be very tractable even with a large signal set, whereas, with sophisticated cognitive technology analytical modeling is essentially restricted to a binary signal set. On the other hand, a naive cognitive technology may allow too much freedom in the choice of beliefs and therefore overestimate the impact of biased beliefs. Also, in some cases, a naive cognitive technology may miss some important Pareto-inferior equilibria as argued by Bénabou and Tirole (2002).

Motivated directional beliefs are not simply new bounds of rationality. They have benefits, but carry costs, and can be objects of optimization. (Bénabou and Tirole 2016). They are therefore well fitted for an analysis built on the rational choice model. Whenever there is uncertainty, there must also be beliefs, and in most contexts, people are not motivated solely by accuracy goals when forming beliefs. The functional, affective and strategic values of beliefs compete with the accuracy goals whenever there is uncertainty, and, therefore, the potential applications of motivated beliefs are probably almost as numerous as are applications of beliefs.

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Appendix: Proofs of Lemmas and Propositions

Proof of Lemma 1 . Solve first the optimal recall rate given the constraint $\lambda < \frac{1}{2(1-q)}$. Note that here we are looking for the argument of the maximum in a right-open set. However, as we will see, the argument of the maximum is the lower and closed bound of the set and, hence, the maximum exists.

$$\begin{aligned}\underline{\lambda} &= \underset{\lambda\in\left[0,\frac{1}{2(1-q)}\right)}{\arg\max}\left\{\lambda[\delta s y_L + \delta^2 y_L] + (1-\lambda)[\delta s[r(\lambda)y_H + (1-r(\lambda))y_L] + \delta^2 y_L]\right\} \\ &= \underset{\lambda\in\left[0,\frac{1}{2(1-q)}\right)}{\arg\max}\left\{(1-\lambda)r(\lambda)\right\} \\ &= \underset{\lambda\in\left[0,\frac{1}{2(1-q)}\right)}{\arg\max}\left\{\frac{(1-\lambda)q}{q+\chi(1-q)(1-\lambda)}\right\} \end{aligned} \tag{51}$$

The derivative of the argument can be written as

$$\frac{d}{d\lambda}\left(\frac{(1-\lambda)q}{q+\chi(1-q)(1-\lambda)}\right) = \frac{[\chi(1-q)-q]^2 - [\chi(1-q)]^2}{[q+\chi(1-q)(1-\lambda)]^2} < 0 \tag{52}$$

and is always negative, since $[\chi(1-q)-q]^2 < [\chi(1-q)]^2$. The optimal recall rate is therefore the lower bound of the constraint, that is, $\underline{\lambda} = 0$.

The utility, given that the agents chooses $\lambda < \frac{1}{2(1-q)}$, in (12) is independent of the choice of λ. The best response is the interval $\overline{\lambda} \in [\frac{1}{2(1-q)}, 1]$. Plugging $\underline{\lambda} = 0$ into (14) and solving for s yields (15).

Proof of Lemma 2. If $s > s^*$ the likely poor will choose the awareness rate $\lambda = 0$ and will not want to deviate by Lemma 1. In this equilibrium, no one never chooses $\hat{\sigma}_i = y_L$, so the information set following this action is on off-equilibrium path and the beliefs in the information set following $\hat{\sigma} = y_L$ can't be defined using Bayer rule or its variations. If we define $p \equiv Pr[\sigma_i = y_H|\hat{\sigma}_i = y_L]$ and require $p \leq q$, we rule out the possibility of players strategically memorizing a low signal in order to end up with higher expectations. As the profitability of a deviation depends on whether the agents are able to increase their anticipatory utility by deviating, with these off-equilibrium path beliefs the likely rich have no incentive to deviate either. Given the strategies of the likely rich and the likely poor, the policy outcome as function of λ given in (11) implies $\tau^* = 0$.

If $s < s^*$, the likely poor choose the awareness rate $\lambda \in [\frac{1}{2(1-q)}, 1]$ and will not want to deviate by Lemma 1. Given the strategies of the likely poor and the likely rich, the belief in the information set following $\hat{\sigma} = y_L$ is $Pr[\sigma = y_H|\hat{\sigma} = y_L] = 1$. Therefore by deviating, a likely rich agent would end up believing to be likely poor and lose anticipatory utility. Hence, the likely rich have no incentive to deviate. The policy outcome as function of λ given in (11) in this case implies $\tau * = 1$.

Proof of Proposition 1. By Lemma 2 there is an equilibrium with low taxes if $U^{\underline{\lambda}}_{0,i} - U^{\overline{\lambda}}_{0,i} > 0 \iff s > s^*$.

Proof of Lemma 3. Solve first the optimal recall rate given the constraint $\lambda < \frac{1}{2(1-q)}$. Note that here we are looking for the argument of the maximum in a right-open set. However, as we will see, the argument of the maximum is the lower and closed bound of the set and, hence, the maximum exists.

$$\begin{aligned}
\underline{\lambda} = & \underset{\lambda \in \left[0, \frac{1}{2(1-q)}\right)}{\arg\max} \Big\{ \lambda \left[\delta s[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}] + \delta^2[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}]\right] \\
& + (1-\lambda)\left[\delta s[(1-\underline{\tau})[r(\lambda)y_H + (1-r(\lambda))y_L] + \delta^2[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}]\right] \Big\} \\
= & \underset{\lambda \in \left[0, \frac{1}{2(1-q)}\right)}{\arg\max} \left\{(1-\lambda)r(\lambda)\right\} \\
= & \underset{\lambda \in \left[0, \frac{1}{2(1-q)}\right)}{\arg\max} \left\{ \frac{(1-\lambda)q}{q + \chi(1-q)(1-\lambda)} \right\}
\end{aligned} \tag{53}$$

The derivative of the argument can be written as

$$\frac{d}{d\lambda}\left(\frac{(1-\lambda)q}{q+\chi(1-q)(1-\lambda)}\right) = \frac{[\chi(1-q)-q]^2 - [\chi(1-q)]^2}{[q+\chi(1-q)(1-\lambda)]^2} < 0 \tag{54}$$

and is always negative. The optimal recall rate is therefore given by the lower bound of the constraint, $\underline{\lambda} = 0$.

Solve the optimal recall rate given the constraint $\lambda \geq \frac{1}{2(1-q)}$.

$$\begin{aligned}
\overline{\lambda} = & \underset{\lambda \in \left[\frac{1}{2(1-q)}, 1\right]}{\arg\max} \Big\{ \lambda \left[\delta s[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}] + \delta^2[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}]\right] \\
& + (1-\lambda)\left[\delta s[(1-\overline{\tau})[r(\lambda)y_H + (1-r(\lambda))y_L] + \delta^2[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}]\right] \Big\} \\
= & \underset{\lambda \in \left[\frac{1}{2(1-q)}, 1\right]}{\arg\max} \left\{(1-\lambda)r(\lambda)\right\} \\
= & \underset{\lambda \in \left[\frac{1}{2(1-q)}, 1\right]}{\arg\max} \left\{ \frac{(1-\lambda)q}{q + \chi(1-q)(1-\lambda)} \right\}
\end{aligned} \tag{55}$$

The derivative of the argument can be written as

$$\frac{d}{d\lambda}\left(\frac{(1-\lambda)q}{q+\chi(1-q)(1-\lambda)}\right)=\frac{[\chi(1-q)-q]^2-[\chi(1-q)]^2}{[q+\chi(1-q)(1-\lambda)]^2}<0 \tag{56}$$

and as before, is always negative. The optimal recall rate is therefore given by the lower bound of the constraint, $\overline{\lambda}=\frac{1}{2(1-q)}$. Plugging in the optimal recall rates $\underline{\lambda}$ and $\overline{\lambda}$ and solving for s yields (23).

Proof of Lemma 4. If $s>s^{**}$ the likely poor will choose the awareness rate $\lambda=0$ and will not want to deviate by Lemma 3. In this equilibrium, no one never chooses $\hat{\sigma}_i=y_L$, so the information set following this action is on off-equilibrium path and the beliefs in the information set following $\hat{\sigma}_i=y_L$ can't be defined using Bayes rule or the variation of the Bayes rule presented in this work. If we define $p\equiv Pr[\sigma_i=y_H|\hat{\sigma}_i=y_L]$ and require $p\leq q$, we rule out the possibility of players strategically memorizing a low signal in order to end up with higher expectations. As the profitability of a deviation depends on whether the agents are able to increase their anticipatory utility by deviating, with these off-equilibrium path beliefs the likely rich have no incentive to deviate either. Given the strategies of the likely rich and the likely poor, the policy outcome as function of λ given in (11) implies $\tau^*=\underline{\tau}$.

If $s<s^{**}$, the likely poor choose the awareness rate $\lambda=\frac{1}{2(1-q)}$ and will not want to deviate by Lemma 3. Given the strategies of the likely poor and the likely rich, the belief in the information set following $\hat{\sigma}=y_L$ is $Pr[\sigma=y_H|\hat{\sigma}=y_L]=1$. Therefore by deviating, a likely rich agent would end up believing to be likely poor and lose anticipatory utility. The likely rich have no incentive to deviate. The policy outcome as function of λ given in (11) in this case implies $\tau^*=\overline{\tau}$.

Proof of Proposition 2. By Lemma 4, there is an equilibrium with low taxes if $U^{\underline{\lambda}}_{0,i}-U^{\overline{\lambda}}_{0,i}>0\iff s>s^{**}$.

Proof of Proposition 3.

$$\frac{\partial s^{**}}{\partial\chi}=\frac{-\delta(\overline{\tau}-\underline{\tau})q\big[(1-\underline{\tau})\frac{\partial r(0)}{\partial\chi}-(1-\overline{\tau})(1-\overline{\lambda})\frac{\partial r(\overline{\lambda})}{\partial\chi}\big]}{[(1-\underline{\tau})r(0)-(1-\overline{\tau})(1-\overline{\lambda})r(\overline{\lambda})-(\overline{\tau}-\underline{\tau})q]^2}, \tag{57}$$

where

$$(1-\underline{\tau})\frac{\partial r(0)}{\partial \chi} - (1-\overline{\tau})(1-\overline{\lambda})\frac{\partial r(\overline{\lambda})}{\partial \chi}$$
$$= (1-\overline{\tau})\frac{q(1-q)(1-\overline{\lambda})^2}{[q+\chi(1-q)(1-\overline{\lambda})]^2} - (1-\underline{\tau})\frac{q(1-q)}{[q+\chi(1-q)]^2} < 0 \tag{58}$$

since

$$(1-\overline{\tau})\frac{q(1-q)(1-\overline{\lambda})^2}{[q+\chi(1-q)(1-\overline{\lambda})]^2} < (1-\underline{\tau})\frac{q(1-q)}{[q+\chi(1-q)]^2}$$
$$\iff (1-\overline{\tau})[q^2(1-\overline{\lambda})^2 + 2q\chi(1-q)(1-\overline{\lambda})^2 + \chi(1-q)^2(1-\overline{\lambda})^2]$$
$$< (1-\underline{\tau})[q^2 + 2\chi q(1-q)(1-\overline{\lambda}) + \chi^2(1-q)^2(1-\overline{\lambda})^2] \tag{59}$$

which holds since

$$q^2(1-\overline{\lambda})^2 + 2q\chi(1-q)(1-\overline{\lambda})^2 < q^2 + 2\chi q(1-q)(1-\overline{\lambda}) \tag{60}$$

and $1-\overline{\tau} < 1-\underline{\tau}$. Therefore $\frac{\partial s^{**}}{\partial \chi} > 0$.

Proof of Proposition 4.

$$\frac{\partial s^{**}}{\partial \overline{\tau}} = \frac{\delta \Delta y(\bar{y}-y_L)\big[(1-\underline{\tau})[r(\underline{\lambda}) - (1-\overline{\lambda})r(\overline{\lambda})]\big]}{[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau}-\underline{\tau})q]^2(\Delta y)^2} \tag{61}$$

where

$$r(\underline{\lambda}) - (1-\overline{\lambda})r(\overline{\lambda}) = \frac{q^2}{[q+\chi(1-q)]2(1-q)[q+\chi\frac{1}{2}(1-2q)]} > 0. \tag{62}$$

Therefore $\frac{\partial s^{**}}{\partial \overline{\tau}} > 0$.

Proof of Proposition 5.

$$\frac{\partial s^{**}}{\partial \underline{\tau}} = -\frac{\delta \Delta y(\bar{y}-y_L)\big[(1-\overline{\tau})[r(\underline{\lambda}) - (1-\overline{\lambda})r(\overline{\lambda})]\big]}{[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau}-\underline{\tau})q]^2(\Delta y)^2} \tag{63}$$

where

$$r(\underline{\lambda}) - (1-\overline{\lambda})r(\overline{\lambda}) = \frac{q^2}{[q+\chi(1-q)]2(1-q)[q+\chi\frac{1}{2}(1-2q)]} > 0. \tag{64}$$

Therefore $\frac{\partial s^{**}}{\partial \underline{\tau}} < 0$.

We establish a result that is useful in determining the sign of the partial derivatives of s^{**}.

Lemma 5. $(1-\underline{\tau})r(0)-(1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda})>0$

Proof of Lemma 5.

$$\begin{aligned}&(1-\underline{\tau})r(0)-(1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda})\\&=\frac{[2(1-q)(q+\chi\frac{1}{2}(1-2q))(1-\underline{\tau})-(q+\chi(1-q))(1-2q)(1-\overline{\tau})]q}{(q+\chi(1-q))(2(1-q)(q+\chi\frac{1}{2}(1-2q))}.\end{aligned}\tag{65}$$

Define

$$a\equiv 2(1-q)(q+\chi\frac{1}{2}(1-2q)),\tag{66}$$

$$b\equiv q+\chi(1-q))(1-2q),\tag{67}$$

and write the numerator of (65) as

$$\begin{aligned}&[a(1-\underline{\tau})-b(1-\overline{\tau}]q\\\iff&[a-b-(a\underline{\tau}-b\overline{\tau})]q.\end{aligned}\tag{68}$$

The numerator of (65) are positive if

$$\begin{aligned}&a\underline{\tau}-b\overline{\tau}>a-b\\\iff&a(1-\underline{\tau})>b(1-\overline{\tau})\end{aligned}\tag{69}$$

which holds since $a-b=q>0$ and $\overline{\tau}>\underline{\tau}$ implies $1-\underline{\tau}>1-\overline{\tau}$. The denominator of (65) is positive for all $q\in[0,\frac{1}{2}]$. Since both the denumerator and the numerator of (65) are positive, the expression is positive and this establishes the result.

Proof of Proposition 6. Write s^{**} as.

$$s^{**}=\frac{\delta(\overline{\tau}-\underline{\tau})(\bar{y}-y_L)}{\iota_{net}(\underline{\lambda},\underline{\tau})-\iota_{net}(\overline{\lambda},\overline{\tau})}\tag{70}$$

where

$$\iota_{net}(\lambda,\tau):=\lambda[(1-\tau)y_L+\tau\bar{y}]+(1-\lambda)[(1-\tau)(r(\lambda)y_H+(1-r(\lambda))y_L)+\tau\bar{y}]\tag{71}$$

is the ex ante expectation of the expected net income of the likely poor in period 1 given λ and τ and $\underline{\lambda} = 0$, and $\overline{\lambda} = \frac{1}{2(1-q)}$. Compute the partial derivative with respect to y_H holding the average income $\bar{y}$ constant.

$$\frac{\partial s^{**}}{\partial y_H} = -\frac{\delta(\overline{\tau} - \underline{\tau})(\bar{y} - y_L)[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda})]}{[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau} - \underline{\tau})q]^2(\Delta y)^2} < 0. \tag{72}$$

By lemma 5, the derivative is negative for all parameter values, which implies that s^{**} decreases in y_H, when $\bar{y}$ is hold constant.[38] Compute the partial derivative with respect to y_L holding the average income $\bar{y}$ constant.

$$\frac{\partial s^{**}}{\partial y_L} = -\frac{\delta(\overline{\tau} - \underline{\tau})(y_H - \bar{y})[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda})]}{[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau} - \underline{\tau})q]^2(\Delta y)^2} < 0 \tag{73}$$

By lemma 5, the derivative is negative for all parameter values, which implies that s^{**} decreases in y_L, when $\bar{y}$ is hold constant. [39]

Proof of Proposition 7. Write s^{**} as.

$$s^{**} = \frac{\delta(\overline{\tau} - \underline{\tau})(\bar{y} - y_L)}{\iota_{net}(\underline{\lambda}, \underline{\tau}) - \iota_{net}(\overline{\lambda}, \overline{\tau})} \tag{74}$$

where

$$\iota_{net}(\lambda, \tau) := \lambda[(1-\tau)y_L + \tau\bar{y}] + (1-\lambda)[(1-\tau)(r(\lambda)y_H + (1-r(\lambda))y_L) + \tau\bar{y}] \tag{75}$$

is the ex ante expectation of the expected net income of the likely poor in period 1 given λ and τ and $\underline{\lambda} = 0$, and $\overline{\lambda} = \frac{1}{2(1-q)}$. Compute the partial derivative with respect to $\bar{y}$ holding the average income y_L and y_H constant.

$$\frac{\partial s^{**}}{\partial \bar{y}} = \frac{\delta(\overline{\tau} - \underline{\tau})[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda})(y_H - y_L)]}{[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau} - \underline{\tau})q]^2(\Delta y)^2} > 0 \tag{76}$$

By lemma 5, the derivative is positive for all parameter values, which implies that s^{**} increases in $\bar{y}$, when y_L and y_H are hold constant.[40]

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[38]By letting $\overline{\tau} = 1$, $\underline{\tau} = 0$, $\chi = 0$, and $\delta = 1$, we get $\frac{\partial s^{**}}{\partial y_H} = -\frac{\bar{y} - y_L}{(y_H - \bar{y})^2}$, which is the result in Minozzi (2013).

[39]By letting $\overline{\tau} = 1$, $\underline{\tau} = 0$, $\chi = 0$, and $\delta = 1$, we get $\frac{\partial s^{**}}{\partial y_L} = -\frac{1}{(y_H - \bar{y})}$, which is the result in Minozzi (2013).

[40]By letting $\overline{\tau} = 1$, $\underline{\tau} = 0$, $\chi = 0$, and $\delta = 1$, we get $\frac{\partial s^{**}}{\partial \bar{y}} = \frac{y_H - y_L}{(y_H - \bar{y})^2}$, which is the result in Minozzi (2013).

Proof of Proposition 8. First, consider part (*i*). If $s^{**} \geq s^{***}$, then always when there is a low tax equilibrium, the likely rich are worse off in it. So the condition for the low tax equilibrium implies that the likely rich are worse off in the low tax equilibrium if and only if $s^{**}(\chi) \geq s^{***}(\chi)$. Now, it is easy to see that this condition is satisfied for $\chi = 1$ since $s^{**}(1) = s^{***}(1)$. This establishes part (*i*). Consider now part (*ii*). Show first that $s^{**}(\chi) < s^{***}(\chi)\ \forall \chi \in [0, 1)$.

$$\begin{aligned} & s^{**}(\chi) < s^{***}(\chi) \\ \iff & -(1-\underline{\tau})r(0) + \frac{1}{2}(1-\overline{\tau})r\left(\frac{1}{2(1-q)}\right) + q(\overline{\tau}-\underline{\tau}) < 0. \end{aligned} \tag{77}$$

Now show that the left-hand side of (77) is increasing in χ. The derivative of the left-hand side of (77) with respect to χ is

$$\begin{aligned} & (1-\underline{\tau})\frac{q(1-q)}{[q+\chi(1-q)]^2} - \frac{1}{4}(1-\overline{\tau})\frac{q(1-2q)}{[q+\frac{1}{2}\chi(1-2q)]^2} \\ > & (1-\underline{\tau})\frac{q(1-q)}{[q+\chi(1-q)]^2} - \frac{1}{4}(1-\underline{\tau})\frac{q(1-q)}{[q+\frac{1}{2}\chi(1-2q)]^2} \\ = & (1-\underline{\tau})q(1-q)\left[\frac{1}{[q+\chi(1-q)]^2} - \frac{1}{[2q+\chi(1-2q)]^2}\right], \end{aligned} \tag{78}$$

where (78) is positive for all $\chi \in [0, 1)$ since

$$\begin{aligned} & [q+\chi(1-q)]^2 < [2q+\chi(1-2q)]^2 \\ \iff & [q+\chi(1-q)]^2 < [q+\chi(1-q)+(1-\chi)q]^2 \end{aligned} \tag{79}$$

for all $\chi \in [0, 1)$. Hence, the derivative of the left-hand side of (77) with respect to χ is positive and the left-hand side of (77) is increasing in χ. Since $s^{**}(1) = s^{***}(1)$, the left-hand side of (77) is zero when $\chi = 1$. Since the left-hand side of (77) is increasing in χ, it has to be negative for $\chi \in [0, 1)$. This establishes that $s^{**}(\chi) < s^{***}(\chi)\ \forall \chi \in [0, 1)$.

Now since $s^{**}(\chi) < s^{***}(\chi)\ \forall \chi \in [0, 1)$, an existence of a low tax equilibrium does not necessarily mean that $s > s^{***}$ and the likely rich are worse off only if $s > s^{***}$. This establishes part (*ii*).

Proof of Proposition 9. Denote the optimal choice of the likely poor by λ^*.

$$\lambda^* = \underset{\lambda\in[0,1]}{\arg\max} \Big\{(1-\lambda)\left[\delta s[(1-\tau)[r(\lambda)y_H + (1-r(\lambda))y_L] + \tau\bar{y}] + \delta^2[(1-\tau)y_L + \tau\bar{y}]\right]$$
$$+ \lambda\left[(\delta s + \delta^2)[(1-\tau)y_L + \tau\bar{y}]\right]\Big\}$$
$$= \underset{\lambda\in[0,1]}{\arg\max} \{(1-\lambda)r(\lambda)\}$$
$$= \underset{\lambda\in[0,1]}{\arg\max} \left\{\frac{(1-\lambda)q}{q+\chi(1-q)(1-\lambda)}\right\} \tag{80}$$

The derivative of the argument can be written as

$$\frac{d}{d\lambda}\left(\frac{(1-\lambda)q}{q+\chi(1-q)(1-\lambda)}\right) = \frac{[\chi(1-q)-q]^2 - [\chi(1-q)]^2}{[q+\chi(1-q)(1-\lambda)]^2} < 0 \tag{81}$$

and is always negative. The optimal recall rate is therefore given by the lower bound of the constraint, $\lambda^* = 0$. Since the maximum is unique, the choice $\lambda = \lambda^*$ stictly dominates all other choices of λ and, hence, the unique equilibrium is all the likely poor choosing λ^*.

Proof of Proposition 10. By Lemma 1, if $s < s^*$, (34) is greater than (33).