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House Prices, Expectations, and Time-Varying Fundamentals*

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Abstract

We investigate the behavior of the equilibrium price-rent ratio for housing in a simple Lucas-type asset pricing model. We allow for time-varying risk aversion (via external habit formation) and time-varying persistence and volatility in the stochastic process for rent growth, consistent with U.S. data for the period 1960 to 2011. Under fully-rational expectations, the model significantly underpredicts the volatility of the U.S. price-rent ratio for reasonable levels of risk aversion. We demonstrate that the model can approximately match the volatility of the price-rent ratio in the data if near-rational agents continually update their estimates for the mean, persistence and volatility of fundamental rent growth using only recent data (i.e., the past 4 years), or if agents employ a simple moving-average forecast rule that places a large weight on the most recent observation. These two versions of the model can be distinguished by their predictions for the correlation between expected future returns on housing and the price-rent ratio. Only the moving-average model predicts a positive correlation such that agents tend to expect higher future returns when house prices are high relative to fundamentals—a feature that is consistent with survey evidence on the expectations of real-world housing investors.

Keywords: *Asset pricing, Excess volatility, Housing bubbles, Predictability, Time-varying risk premiums, Expected returns.*

JEL Classification: D84, E32, E44, G12, O40, R31.

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1 Introduction

House prices in the United States increased dramatically in the years prior to 2007. During the boom years, many economists and policymakers argued that a bubble did not exist and that numerous fundamental factors were driving the run-up in prices.¹ But in retrospect, many studies now attribute the run-up to a classic bubble driven by over-optimistic projections about future house price growth which, in turn, led to a collapse in lending standards.² Reminiscent of the U.S. stock market mania of the late-1990s, the mid-2000s housing market was characterized by an influx of unsophisticated buyers and record transaction volume. When the optimistic house price projections eventually failed to materialize, the bubble burst, setting off a chain of events that led to a financial and economic crisis. The “Great Recession,” which started in December 2007 and ended in June 2009, was the most severe economic contraction since 1947, as measured by the peak-to-trough decline in real GDP (Lansing 2011).

This paper investigates the influence of agents’ expectations on the behavior of house prices in a simple Lucas-type asset pricing model. We allow for time-varying risk aversion (via external habit formation) and time-varying persistence and volatility in the stochastic process for rent growth, consistent with U.S. data for the period 1960 to 2011. Under fully-rational expectations, the model significantly underpredicts the volatility of the U.S. price-rent ratio for reasonable levels of risk aversion. We demonstrate that the model can approximately match the volatility of the price-rent ratio in the data if near-rational agents continually update their estimates for the mean, persistence and volatility of fundamental rent growth using only recent data (i.e., the past 4 years), or if agents employ a simple moving-average forecast rule that places a large weight on the most recent observation. These two versions of the model can be distinguished by their predictions for the correlation between expected future returns on housing and the price-rent ratio. Only the moving-average model predicts a positive correlation such that agents tend to expect higher future returns when house prices are high relative to fundamentals—a feature that is consistent with survey evidence on the expectations of real-world housing investors.

As part of our quantitative analysis, we apply the Campbell and Shiller (1988) log-linear approximation of the return identity to the housing market. The variance of the log price-rent ratio must equal the sum of the ratio’s covariances with: (1) future rent growth rates, and (2) future realized housing returns. The magnitude of each covariance term is a measure

¹See, for example, McCarthy and Peach (2004) and Himmelberg, et al. (2005). In an October 2004 speech, Fed Chairman Alan Greenspan (2004) argued that there were “significant impediments to speculative trading” in the housing market that served as “an important restraint on the development of price bubbles.” In a July 1, 2005 media interview, Ben Bernanke, then Chairman of the President’s Council of Economic Advisers, asserted that fundamental factors such as strong growth in jobs and incomes, low mortgage rates, demographics, and restricted supply were supporting U.S. house prices. In the same interview, Bernanke stated his view that a substantial nationwide decline in house prices was “a pretty unlikely possibility.” For additional details, see Jurgilas and Lansing (2013).

²For a comprehensive review of events, see the report of the U.S. Financial Crisis Inquiry Commission (2011). Recently, in a review of the Fed’s forecasting record leading up to the crisis, Potter (2011) acknowledges a “misunderstanding of the housing boom. . . [which] downplayed the risk of a substantial fall in house prices.”

of the predictability of future rent growth or future realized returns when the current price-rent ratio is employed as the sole regressor in a forecasting equation. As in the U.S. data, the moving-average model exhibits the property that a higher price-rent ratio in the current period strongly predicts lower realized returns in the future, but the predictive power for future rent growth is very weak. Interestingly, even though a higher price-rent ratio in the data predicts lower *realized* returns, the survey evidence shows that real-world investors fail to take this relationship into account; instead they continue to forecast high future returns following a sustained run-up in the price-rent ratio. Such behavior is consistent with a moving-average forecast rule but is inconsistent with the fully-rational and near-rational versions of our model. We also show that the moving-average model can deliver either a positive or negative regression coefficient on the price-rent ratio when the ratio is used to predict future rent growth. The sign of the regression coefficient is influenced by the value of a utility curvature parameter. Our simulation results can therefore help account for the empirical findings of Engsted and Pedersen (2012), who document significant cross-country and sub-sample instability in the sign of this regression coefficient using housing market data from 18 OECD countries over the period 1970 to 2011.

An additional contribution of the paper is to derive an approximate analytical solution for the fully-rational house price in the case when fundamental rent growth exhibits time-varying persistence and volatility. Our specification for rent growth employs the bilinear times series model originally developed Granger and Andersen (1978) which allows for nonlinear behavior within a continuous state space. Our solution procedure makes use of a change of variables to preserve as much of the model’s nonlinear characteristics as possible.

Standard dynamic stochastic general-equilibrium (DSGE) models with fully-rational expectations have difficulty producing large swings in house prices that resemble the patterns observed in the U.S. and other countries over the past decade. Indeed, it is common for such models to postulate extremely large and persistent exogenous shocks to rational agents’ preferences for housing in an effort to bridge the gap between the model and the data.³ Leaving aside questions about where these preference shocks actually come from and how agents’ responses to them could become coordinated, we demonstrate numerically that an upward shift in the representative agent’s preference for housing raises the mean price-rent ratio under all three expectations regimes. At the same time, the preference shift lowers the average realized return on housing. Under rational expectations, the agent will take this relationship into account when forecasting such that the conditional expected return on housing will move in the opposite direction as the price-rent ratio in response to a shift in housing preferences. Hence, while a fully-rational model with housing preference shocks could potentially match the volatility of the price-rent ratio in the data, such a model would still predict a negative correlation between the expected return on housing and the price-rent ratio—directly at odds with the survey evidence described below.

Kocherlakota (2009) remarks: “The sources of disturbances in macroeconomic models are

³See for example, Iacoviello and Neri (2010), among others.

(to my taste) patently unrealistic... I believe that [macroeconomists] are handicapping themselves by only looking at shocks to fundamentals like preferences and technology. Phenomena like credit market crunches or asset market bubbles rely on self-fulfilling beliefs about what others will do.” These ideas motivate consideration of a model where investors’ subjective forecasts serve as an endogenous source of volatility for house prices.

1.1 Survey Evidence on Investor Expectations

The fundamental value of an asset is typically measured by the present-value of expected future cash or service flows that will accrue to the owner. Service flows from housing are called “imputed rents.” The discount rate used in the present-value calculation is comprised of a risk-free yield and a compensation for perceived risk, i.e., a risk premium. Their sum defines the rate of return that an investor expects to receive to justify purchase of the asset. All else equal, a lower risk premium implies a lower expected return and a lower discount rate in the present-value calculation. Future service flows will be discounted less and the fundamental value will rise.

Cochrane (2009) argues that one cannot easily tell the difference between a bubble and a situation where rational investors have low risk premia, implying lower expected returns on the risky asset. Specifically, he remarks “Crying bubble is empty unless you have an operational procedure for distinguishing them from rationally low risk premiums.” Along similar lines, Favilukis, et al. (2011) argue that the run-up in U.S. house prices relative to rents was largely due to a financial market liberalization that reduced buyers’ perceptions of the riskiness of housing assets. The authors develop a theoretical model where easier lending standards and lower mortgage transaction costs contribute to a substantial rise in house prices relative to rents, but this is not a bubble. Rather, the financial market liberalization allows fully-rational households in the model to better smooth their consumption in the face of unexpected income declines, thus reducing their perceptions of economic risk. Lower risk perceptions induce households to accept a lower rate of return on the purchase of risky assets like houses. A lower expected return leads to an increase in the model’s fundamental price-rent ratio, similar to that observed in the data. In the words of the authors, “a financial market liberalization drives price-rent ratios up because it drives risk premia down. . . . Procyclical increases in [fundamental] price-rent ratios reflect rational expectations of lower future returns.”

In our view, the relaxation of lending standards in the mid-2000s was an endogenous consequence of the house price run-up, not an exogenous fundamental driver of the run-up. Standards were relaxed because lenders (and willing borrowers) expected house price appreciation to continue indefinitely. Empirical evidence supports this view. Within the United States, past house price appreciation in a given area had a significant positive influence on subsequent loan approval rates in the same area (Dell’Ariccia, et al. 2011, Goetzmann, et al. 2012).

Case and Shiller (2004) make the point that “the mere fact of rapid price increases is not in itself conclusive evidence of a bubble. . . .The notion of a bubble is really defined in terms

of people’s thinking about future price increases.” Survey evidence on people’s expectations about future house price appreciation can therefore be a useful tool for diagnosing a bubble. Rational investors with low risk premiums would expect low future returns after a sustained price run-up, whereas irrationally exuberant investors in the midst of a bubble would expect high future returns because they simply extrapolate recent price action into the future. A variety of evidence from both stock and real estate markets (discussed below) shows that real-world investors typically expect high future returns near market peaks, not low future returns.

Shiller (2000) developed a questionnaire to study investor expectations about future stock market returns in Japan and the U.S. during the 1990s. From the data, he constructed an index of “bubble expectations” which reflected the belief that stock prices would continue to rise despite being high relative to fundamentals. He found that the index moved roughly in line with movements in the stock market itself, suggesting that investors tend to extrapolate recent market trends when making predictions about future returns.

Two additional studies by Fischer and Statman (2002) and Vissing-Jorgenson (2004) also find evidence of extrapolative expectations among U.S. stock market investors during the late 1990s and early 2000s. Using survey data, they found that investors who experienced high portfolio returns in the past tended to expect higher returns in the future. Moreover, expected returns reached a maximum just when the stock market itself reached a peak in early 2000.

Recently, in a comprehensive study of the expectations of U.S. stock market investors using survey data from a variety of sources, Greenwood and Shleifer (2013) find that measures of investor expectations about future stock returns are: (1) positively correlated with the price-dividend ratio and past stock returns, and (2) positively correlated with investor inflows into mutual funds. They conclude (p. 30) that “[O]ur evidence rules out rational expectations models in which changes in market valuations are driven by the required returns of a representative investor...Future models of stock market fluctuations should embrace the large fraction of investors whose expectations are extrapolative.” We apply this advice in the present paper to a model of housing market fluctuations.

Using survey data on homebuyers in four metropolitan areas in 2002 and 2003, Case and Shiller (2004) found that about 90 percent of respondents expected house prices to increase over the next several years. More strikingly, when asked about the next ten years, respondents expected future annual price appreciation in the range of 12 to 16 percent per year—implying a tripling or quadrupling of home values over the next decade. Needless to say, these forecasts proved wildly optimistic. In a study of data from the Michigan Survey of Consumers, Piazzesi and Schneider (2009) report that “starting in 2004, more and more households became optimistic after having watched house prices increase for several years.”

Anecdotal evidence further supports the view that U.S. housing investors had high expected returns near the market peak. The June 6, 2005 cover of Fortune magazine was titled “Real Estate Gold Rush—Inside the hot-money world of housing speculators, condo-flippers and get-rich-quick schemers.” One week later, the June 13, 2005 cover of Time magazine was titled

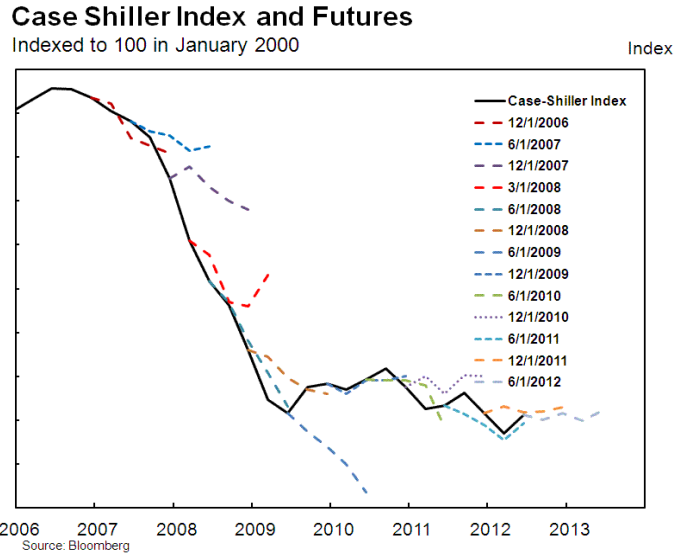


Figure 1: Futures market forecasts for house prices tend to overpredict subsequent actual house prices when prices are falling—a pattern consistent with a moving-average forecast rule.

“Home Sweet Home—Why we’re going gaga over real estate.” Both covers depicted happy and celebrating housing investors—all suggesting a rosy outlook for U.S. real estate.

In surveys during 2006 and 2007, Shiller (2007) found that places with high recent house price growth exhibited high expectations of future price appreciation and that places with slowing house price growth exhibited downward shifts in expected appreciation. Indeed by 2008, in the midst of the housing market bust, Case, Shiller, and Thompson (2012) show that survey respondents in prior boom areas now mostly expected a decline in house prices over the next year. In a review of the time series evidence on housing investor expectations, the authors conclude (p. 17) that “12-month expectations [of future house prices changes] are fairly well described as attenuated versions of lagged actual 12-month price changes.” Overall, the evidence appears to directly contradict the view that declining risk premiums (resulting in low expected returns) were the explanation for the run-up in U.S. house prices relative to rents.

The survey evidence described above shows that there is strong empirical support for considering extrapolative or moving-average type forecast rules, particularly in the housing market. As shown originally by Muth (1960), a moving-average forecast rule with exponentially-declining weights on past data will coincide with rational expectations when the forecast variable evolves as a random walk with permanent and temporary shocks. But even if this is not the case, a moving-average forecast rule can be viewed as boundedly-rational because it economizes on the costs of collecting and processing information. As noted by Nerlove (1983, p. 1255): “Purposeful economic agents have incentives to eliminate errors up to a point justified by the costs of obtaining the information necessary to do so...The most readily available

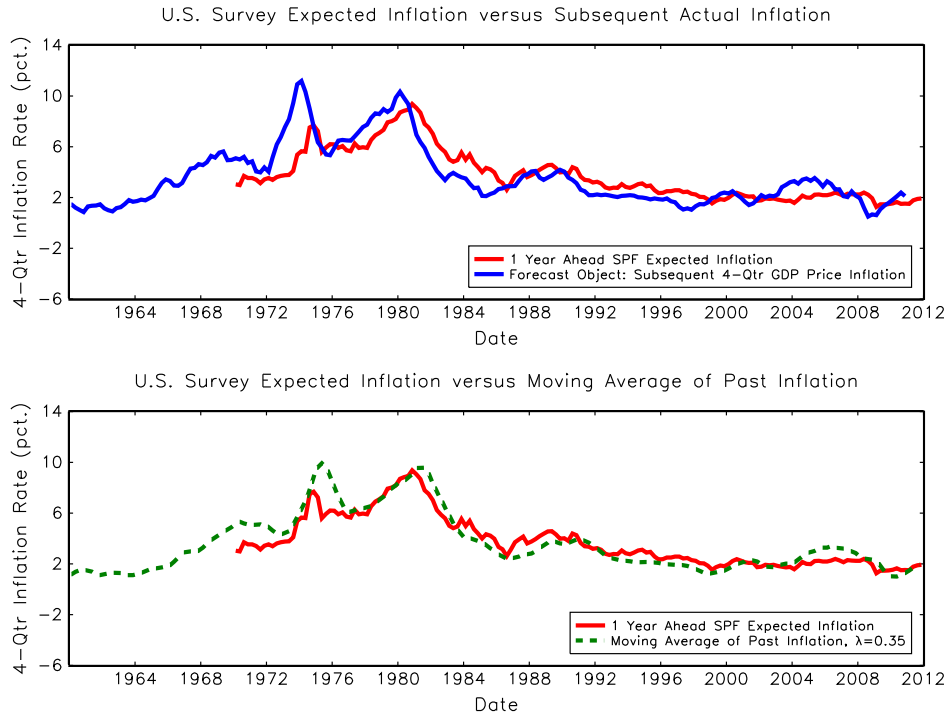


Figure 2: One-year ahead U.S. inflation expectations derived from the Survey of Professional Forecasters (SPF) versus subsequent realized 4-quarter GDP price inflation. The realized inflation series is shifted back so that the vertical distance between the two series represents the forecast error. The professional forecasters tend to systematically underpredict subsequent realized inflation in the sample period prior to 1979 when inflation was rising and systematically overpredict it thereafter when inflation was falling. The survey pattern is well-captured by a moving-average of past observed inflation rates.

and least costly information about the future value of a variable is its past value.”

Figure 1 shows that futures market forecasts for the Case-Shiller house price index (which are only available from 2006 onwards) often exhibit a series of one-sided forecast errors. The futures market tends to overpredict future house prices when prices are falling—a pattern that is consistent with a moving-average forecast rule. Similarly, the top panel of Figure 2 shows that U.S. inflation expectations derived from the Survey of Professional Forecasters tend to systematically underpredict subsequent realized inflation in the sample period prior to 1979 when inflation was rising and systematically overpredict it thereafter when inflation was falling. Rational expectations would not give rise to such a sustained sequence of one-sided forecast errors.⁴ The bottom panel of Figure 2 shows that the survey pattern is well-captured

⁴Numerous studies document evidence of bias and inefficiency in survey forecasts of U.S. inflation. See, for example, Roberts (1997), Mehra (2002), Carroll (2003), and Mankiw, Reis, and Wolfers (2004). More recently, Coibion and Gorodnichenko (2012) find robust evidence against full-information rational expectations in survey forecasts for U.S. inflation and unemployment.

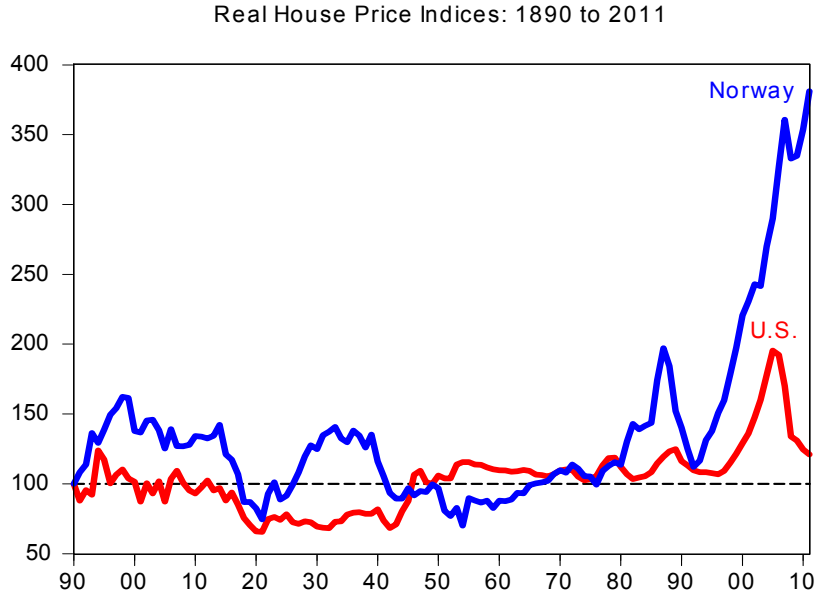


Figure 3: Periods of stagnant real house prices are interspersed with booms and busts. Norway experienced a major housing price boom in the late 1980s followed by a crash in the early 1990s. The earlier boom-bust pattern in Norway is similar in magnitude to the recent boom-bust pattern in U.S. house prices. Real house prices are indexed to 100 in 1890.

by an exponentially-weighted moving-average of past inflation rates, where the weight λ on the most recent inflation observation is 0.35. Interestingly, a weight of 0.35 on the most recent inflation observation is consistent with a Kalman filter forecast in which agents' perceived law of motion for inflation is a random walk plus noise (Lansing 2009).

1.2 Related Literature

Numerous empirical studies starting with Shiller (1981) and LeRoy and Porter (1981) have shown that stock prices appear to exhibit excess volatility when compared to the discounted stream of ex post realized dividends.⁵ Similarly, Campbell, et al. (2009) find that movements in U.S. house price-rent ratios cannot be fully explained by movements in future rent growth.

A large body of research seeks to explain asset price behavior using some type of distorted belief mechanism or misspecified forecast rule in a representative agent framework. Examples along these lines include Barsky and DeLong (1993), Timmerman (1996), Barberis, Shleifer, and Vishney (1998), Lansing (2006, 2010), Adam, Marcet, and Nicolini (2008), Branch and Evans (2010), Fuster, et al. (2012), and Hommes and Zhu (2013), among others.

An empirical study by Chow (1989) finds that an asset pricing model with moving-average

⁵Lansing and LeRoy (2012) provide a recent update on this literature.

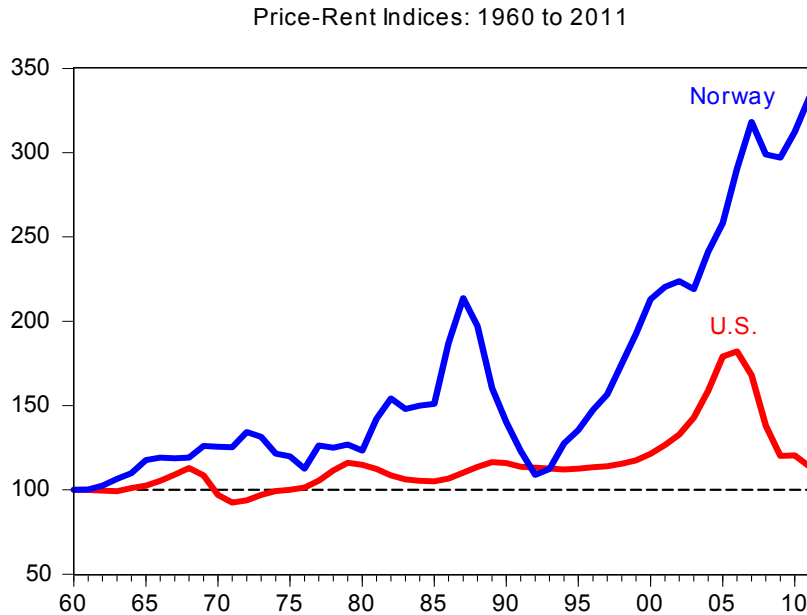


Figure 4: The U.S. price-rent ratio peaked in early 2006 and has since fallen to its pre-boom level. The price-rent ratio for Norway has continued to trend upwards and currently stands about 50 percent above the last major peak achieved two decades ago. Price-rent ratios are indexed to 100 in 1960.

expectations outperforms one with rational expectations in accounting for observed movements in U.S. stock prices and interest rates. Huh and Lansing (2000) show that a model with backward-looking expectations is better able to capture the temporary rise in long-term nominal interest rates observed in U.S. data at the start of the Volcker disinflation in the early-1980s. Some recent research that incorporates moving-average forecast rules or adaptive expectations into otherwise standard models include Sargent (1999, Chapter 6), Evans and Ramey (2006), Lansing (2009), and Huang, et. al (2009), among others. Huang, et al. (2009) state that “adaptive expectations can be an important source of frictions that amplify and propagate technology shocks and seem promising for generating plausible labor market dynamics.”

Using an estimated New Keynesian DSGE model that allows for both rational and moving-average expectations, Levine, et al. (2012) find that the estimated fraction of agents who employ a moving-average forecast rule lies in the range of 0.65 to 0.83. Gelain, et al. (2013) show that the introduction of simple moving-average forecast rules for a subset of agents can significantly magnify the volatility and persistence of house prices and household debt in a standard DSGE model with housing. Granziera and Kozicki (2012) show that a simple Lucas-type asset pricing model with backward-looking, extrapolative-type expectations can roughly

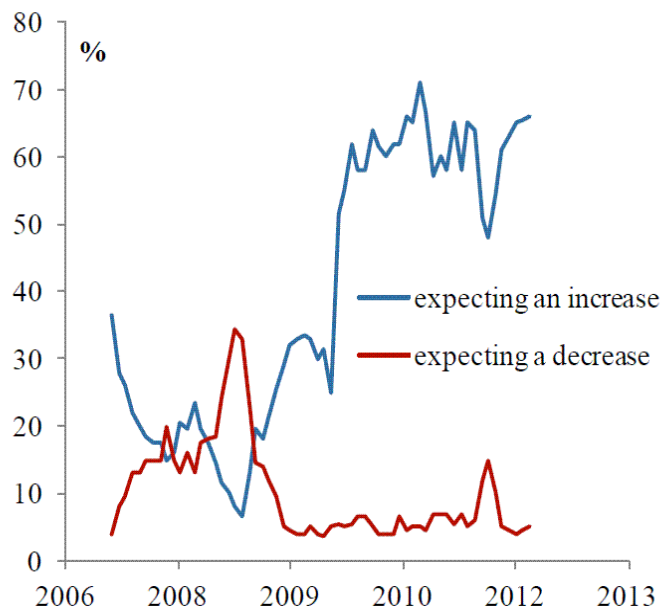


Figure 5: The percentage of Norwegian households who believe that property prices will keep rising has gone up from a low of 10 percent in 2008 to nearly 70 percent in 2012. Norwegians appear to expect high future returns on real estate even after a sustained run-up in the price-ratio. Data source: Financial Supervisory Authority of Norway.

match the run-up in U.S. house prices from 2000 to 2006 as well as the subsequent sharp downturn.

Constant-gain learning algorithms of the type described by Evans and Honkapoja (2001) are similar in many respects to moving-average expectations; both formulations assume that agents apply exponentially-declining weights to past data when constructing forecasts of future variables. Adam, et al. (2012) show that the introduction of constant-gain learning can help account for recent cross-country patterns in house prices and current account dynamics. In contrast to our setup, however, their model assumes the presence of volatile and persistent exogenous shocks to the representative agent’s preference for housing services, a feature that helps their model to fit the data.

2 Housing Market Data

Figure 3 plots real house price indices in the U.S. and Norway from 1890 to 2011. The U.S. data are updated from Shiller (2005) while data for Norway are updated from Eitrheim and Erlandsen (2004, 2005). Both series show that real house prices were relatively stagnant for most of the 20th century. Norway and other Nordic countries experienced a major housing price boom in the late 1980s followed by a crash in the early 1990s. The earlier boom-bust pattern in Norway is similar in magnitude to the recent boom-bust pattern in U.S. house prices

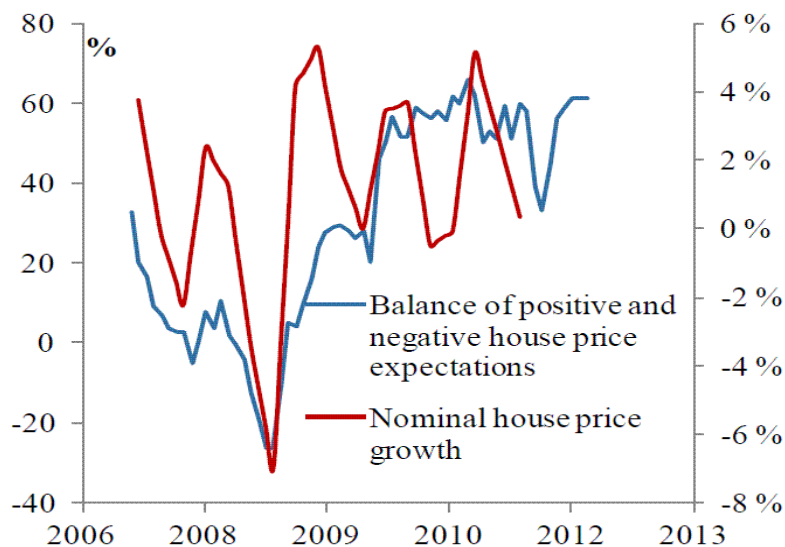


Figure 6: The balance of Norwegian households expecting a house price increase over the next 12 months is strongly correlated with nominal house price growth over the preceding 12 months, suggestive of a moving-average type forecast rule. Data source: Financial Supervisory Authority of Norway and Norges Bank.

(Knutsen 2012). After peaking in 2006, U.S. real house prices have since dropped by nearly 40 percent. Starting in the late 1990s, Norwegian house prices experienced another major boom but so far no bust. On the contrary, real house prices in Norway have continued to rise by nearly 30 percent since 2006.

Figure 4 plots price-rent ratios in the U.S. and Norway from 1960 onwards. The U.S. ratio peaked in early 2006 and has since returned to its pre-boom level. The price-rent ratio for Norway has continued to trend upwards and currently stands about 50 percent above the last major peak achieved two decades ago. Price-income ratios for the two countries display a similar pattern.⁶

Figure 5 plots the results of a survey of Norwegian households about expected house price changes over the next 12 months. The percentage of households who believe that property prices will keep rising has gone up from a low of 10 percent in 2008 to nearly 70 percent in 2012. Comparing Figure 5 to the price-rent ratio for Norway in Figure 4 suggests that Norwegians appear to expect high future returns on housing even after a sustained run-up in the price-rent ratio. This pattern is directly at odds with the idea of rationally low risk premiums. Figure 6 shows that the balance of Norwegian households expecting a house price increase over the subsequent 12 months is strongly correlated with nominal house price growth over the preceding 12 months, suggestive of a moving-average type forecast rule.

Figure 7 provides a detailed look at U.S. house prices and rents using quarterly data

⁶See Jurgilas and Lansing (2013).

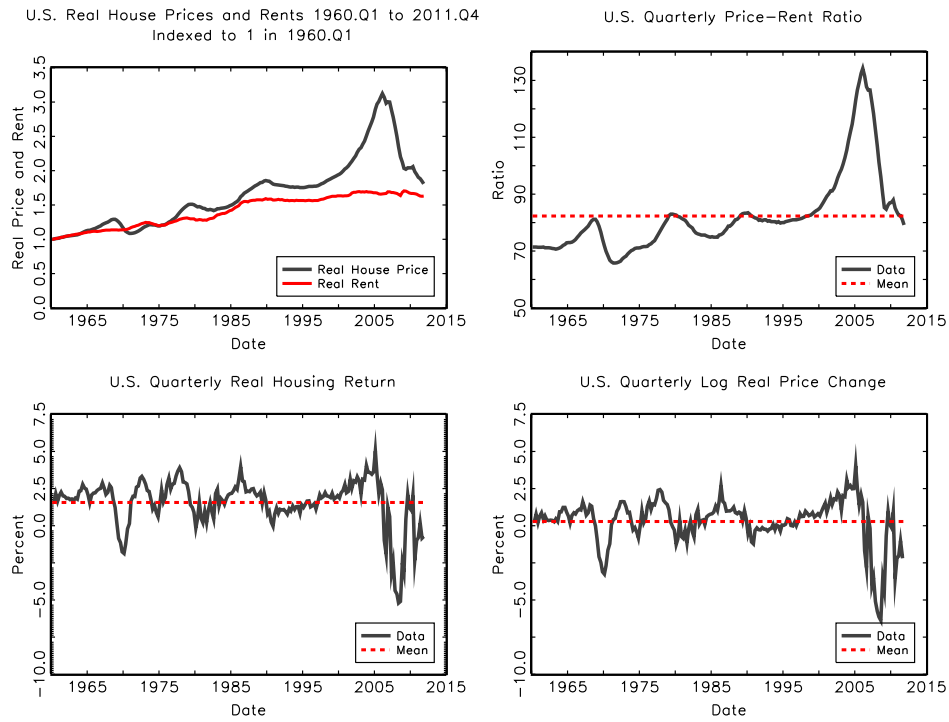


Figure 7: Quarterly housing returns and log price changes were consistently positive and rising during the U.S. housing boom of the mid-2000s.

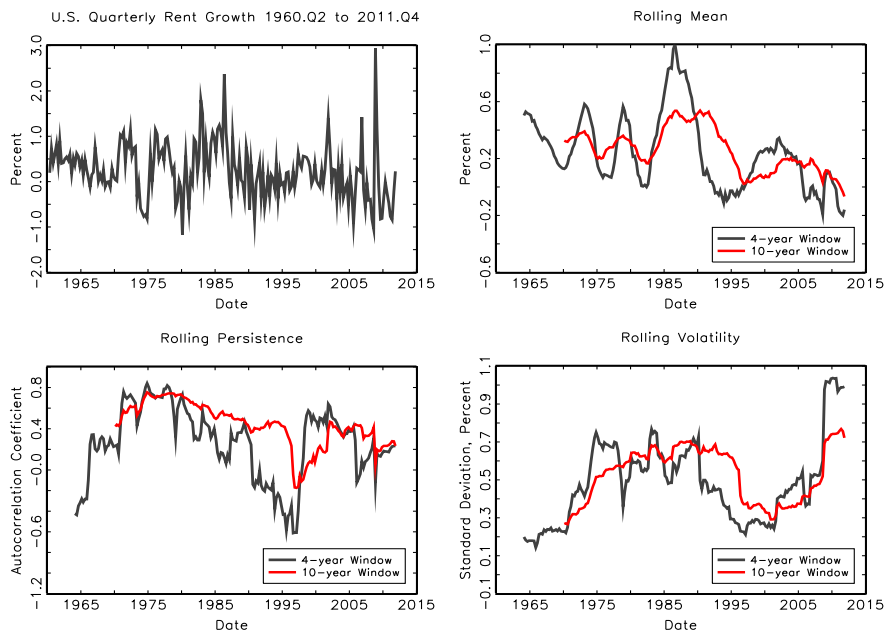


Figure 8: U.S. real rent growth exhibits time-varying mean, persistence and volatility.

from 1960.Q1 to 2011.Q4.⁷ The bottom two panels plot the quarterly real return on housing together with the quarterly real price change (in percent).⁸ Returns and log price changes were consistently positive and rising during the U.S. housing boom of the mid-2000s. After observing such a long string of favorable housing returns, it seems quite natural that lenders and homebuyers would have expected the favorable returns to continue, as confirmed by the survey evidence.

Figure 8 plots quarterly rent growth in the data together with rolling summary statistics for window lengths of 4-years and 10-years. All of the summary statistics exhibit considerable variation. For example, the rolling 10-year autocorrelation coefficient (lower left panel) ranges from a low of -0.17 to a high of 0.75 . Variation of this magnitude can have significant implications for the quantitative predictions of rational asset pricing models, particularly in the presence of habit formation (Otrok, et al. 2002). We therefore allow fundamental rent growth to exhibit time varying persistence and volatility in the theoretical model to be described next.

3 Model

Housing services are priced using a version of the frictionless pure exchange model of Lucas (1978). The representative agent's problem is to choose sequences of c_t and h_t to maximize

$$\widehat{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t^\theta h_t^{1-\theta} - \kappa C_{t-1}^\theta H_{t-1}^{1-\theta} \right)^{1-\alpha} - 1}{1-\alpha}, \quad (1)$$

subject to the budget constraint

$$c_t + p_t h_t = y_t + p_t h_{t-1}, \quad c_t, h_t \geq 0, \quad (2)$$

where c_t is the agent's consumption in period t , h_t is the housing service flow, y_t is income, β is the subjective time discount factor, and α is a curvature parameter that influences the coefficient of relative risk aversion. To allow for time-varying risk aversion, we assume that an individual agent's felicity is measured relative to the lagged per capita consumption basket $C_{t-1}^\theta H_{t-1}^{1-\theta}$, which the agent views as outside of his control.⁹ The parameter $\kappa \geq 0$ governs the importance of the external habit stock.¹⁰ The symbol \widehat{E}_t represents the agent's subjective expectation, conditional on information available at time t , as explained more fully below. Under rational expectations, \widehat{E}_t corresponds to the mathematical expectation operator E_t

⁷Quarterly data for U.S. nominal house prices and nominal rents are from www.lincolnst.edu, using the Case-Shiller-Weiss price data from the year 2000 onwards, as documented in Davis, Lehnert, and Martin (2008). Nominal values are converted to real values using the Consumer Price Index (all items) from the Federal Reserve Bank of St Louis.

⁸Consistent with our model, the quarterly net housing return in period t is defined as $p_t / (p_{t-1} - n_{t-1}) - 1$, where p_t is the real house price and n_t is the quarterly real rent payment.

⁹Maurer and Meier (2008) find strong empirical evidence for "peer-group effects" on individual consumption decisions using panel data on U.S. household expenditures.

¹⁰Otrok, et al. (2002) show that a one-lag habit specification similar to (1) can match the historical mean U.S. equity premium.

evaluated using the objective distribution of shocks, which are assumed known by the rational household. The symbol p_t is the price of housing services in consumption units.

The first-order condition that governs the agent's purchase of housing services is given by

$$p_t = \underbrace{\left(\frac{1-\theta}{\theta}\right) c_t}_{\equiv n_t} + E_t M_{t+1} p_{t+1}, \quad (3)$$

where $n_t \equiv c_t(1-\theta)/\theta$ is the imputed rent (or utility dividend) from housing and M_{t+1} is the stochastic discount factor. Equation (3) reflects the assumption that housing exists in unit net supply such that $h_t = 1$ for all t . Substituting this equilibrium condition into the budget constraint (2) yields, $c_t = y_t$ for all t .¹¹

In equilibrium, the exponential growth rates of rent, consumption, and income are identical. In the data, imputed rent reflects not only the utility dividend, but also the marginal collateral value of the house in the case when the agent's borrowing constraint is binding. We illustrate this point analytically in Appendix A. Although we abstract from directly modeling a borrowing constraint, we can implicitly take the effect on house prices into account by calibrating the effective cash flows in the model to match the stochastic properties of U.S. rent growth.¹² Rent growth in the model is governed by the following law of motion

$$x_{t+1} \equiv \log\left(\frac{n_{t+1}}{n_t}\right) = \bar{x} + (\rho + \psi\varepsilon_t)(x_t - \bar{x}) + \varepsilon_{t+1}, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2), \quad (4)$$

where \bar{x} is the steady state growth rate which differs from the ergodic mean $E(x_t) = \bar{x} + \psi\sigma_\varepsilon^2/(1-\rho)$ when $\psi \neq 0$. Equation (4) is a simple version of the bilinear time series model originally developed by Granger and Andersen (1978) and explored further by Sesay and Subba Rao (1988). By appropriate choice of the parameters \bar{x} , ρ , σ_ε , and ψ , the above specification can match the unconditional moments of U.S. rent growth and deliver time-varying persistence and volatility, consistent with the data.

The equilibrium stochastic discount factor is

$$\begin{aligned} M_{t+1} &= \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\theta\alpha-1+\theta} \left[\frac{1-\kappa(c_{t+1}/c_t)^{-\theta}}{1-\kappa(c_t/c_{t-1})^{-\theta}}\right]^{-\alpha}, \\ &= \beta \exp [(-\theta\alpha-1+\theta)x_{t+1}] \left[\frac{1-\kappa \exp(-\theta x_{t+1})}{1-\kappa \exp(-\theta x_t)}\right]^{-\alpha}, \end{aligned} \quad (5)$$

where we have imposed $h_t = 1$ and made use of the fact that model consumption growth and model rent growth are identical.¹³

¹¹Following Otrok, et al. (2002) we impose an upper bound on the ratio C_{t-1}/c_t to ensure that the utility function (1) is always well defined. The upper bound is never reached in the model simulations.

¹²Piazzesi, et al. (2007) similarly abstract from modeling a borrowing constraint in their analysis of the links between stock returns and housing returns.

¹³Over the period 1960 to 2011, the correlation coefficient between quarterly U.S. rent growth and quarterly U.S. per capita consumption growth (nondurables and services) is 0.47.

Dividing both sides of the first-order condition (3) by the current period rent n_t and substituting in the expression for M_{t+1} yields

$$\frac{p_t}{n_t} = 1 + \beta \widehat{E}_t \exp \{ [\theta(1-\alpha)] x_{t+1} \} \left[\frac{1 - \kappa \exp(-\theta x_{t+1})}{1 - \kappa \exp(-\theta x_t)} \right]^{-\alpha} \frac{p_{t+1}}{n_{t+1}}, \quad (6)$$

which shows that the price-rent ratio in period t depends on the agent's subjective joint forecast of next period's rent growth x_{t+1} and next period's price-rent ratio p_{t+1}/n_{t+1} . It is convenient to transform equation (6) using a nonlinear change of variables to obtain

$$z_t = g(x_t) + \beta \exp \{ [\theta(1-\alpha)] x_t \} \widehat{E}_t z_{t+1}, \quad (7)$$

$$\text{where } z_t \equiv g(x_t) \frac{p_t}{n_t}$$

$$g(x_t) \equiv \frac{\exp \{ [\theta(1-\alpha)] x_t \}}{[1 - \kappa \exp(-\theta x_t)]^\alpha}$$

Under this formulation, z_t represents a composite variable that the agent must forecast. The transformed first-order condition (7) shows that the value of the composite variable z_t in period t depends in part on the agent's subjective forecast of that same variable.

By making use of the definition of z_t , equation (6) can be written as

$$\frac{p_t}{n_t} = 1 + \beta [1 - \kappa \exp(-\theta x_t)]^\alpha \widehat{E}_t z_{t+1}, \quad (8)$$

which shows that the agent's subject forecast $\widehat{E}_t z_{t+1}$ has a direct influence on the dynamics of the equilibrium price-rent ratio p_t/n_t .

Equation (3) can be rearranged to obtain the standard relationship

$$1 = \widehat{E}_t [M_{t+1} R_{t+1}], \quad (9)$$

where $R_{t+1} \equiv p_{t+1}/(p_t - n_t)$ defines the gross return on housing from period t to $t + 1$.

3.1 Rational Expectations

Proposition 1. *An approximate analytical solution for the value of the composite variable $z_t \equiv g(x_t) p_t/n_t$ under rational expectations is given by*

$$z_t = a_0 \exp \{ a_1 [x_t - E(x_t)] + a_2 (v_t - \sigma_\varepsilon^2) \},$$

where $v_t \equiv \varepsilon_t (x_t - \bar{x})$ such that $E(v_t) = \sigma_\varepsilon^2$. The constants $a_0 \equiv \exp \{ E[\log(z_t)] \}$, a_1 , and a_2 are Taylor series coefficients that depend on the preference parameters β , θ , α , κ , and the rent growth parameters \bar{x} , ρ , σ_ε , and ψ .

Proof: See Appendix B.

The approximate solution in Proposition 1 preserves the nonlinear features of the model in several ways. First, rent growth x_t is a state variable that follows a nonlinear law of motion, as given by equation (4). Second, the solution for z_t depends on the state variable $v_t \equiv \varepsilon_t(x_t - \bar{x})$ which also follows a nonlinear law of motion, as derived in Appendix B. Third, given the solution for z_t , the price-rent ratio is determined as a nonlinear function of the state variable x_t , such that $p_t/n_t = z_t/g(x_t)$. In the model simulations, we use the approximate solution in Proposition 1 to construct the conditional forecast $E_t z_{t+1}$ each period using the expression derived in Appendix A. We then substitute the resulting value for $E_t z_{t+1}$ into the nonlinear first-order (7) to obtain the realized value of the composite variable z_t each period.

To compute the rational expected return on housing, we rewrite the gross return as

$$\begin{aligned}
R_{t+1} &= \left[\frac{p_{t+1}/n_{t+1}}{p_t/n_t - 1} \right] \exp(x_{t+1}), \\
&= \left[\frac{[1 - \kappa \exp(-\theta x_{t+1})]^\alpha \exp\{-\theta(1 - \alpha)x_{t+1}\} z_{t+1}}{\beta [1 - \kappa \exp(-\theta x_t)]^\alpha E_t z_{t+1}} \right] \exp(x_{t+1}), \\
&= \left[\frac{1 - \kappa \exp(-\theta x_{t+1})}{1 - \kappa \exp(-\theta x_t)} \right]^\alpha \exp\{[1 - \theta(1 - \alpha)]x_{t+1}\} \frac{z_{t+1}}{\beta E_t z_{t+1}}, \tag{10}
\end{aligned}$$

where we have eliminated p_{t+1}/n_{t+1} using the definitional relationships for z_{t+1} and $g(x_{t+1})$, and we have eliminated p_t/n_t using the transformed first-order condition (8).¹⁴

From equation (10), the conditional expectation of the log return is given by

$$\begin{aligned}
E_t \log(R_{t+1}) &= -\log(\beta) - \alpha \log[1 - \kappa \exp(-\theta x_t)] + E_t \log(z_{t+1}) - \log(E_t z_{t+1}) \\
&\quad + \alpha E_t \log[1 - \kappa \exp(-\theta x_{t+1})] + [1 - \theta(1 - \alpha)] E_t x_{t+1}, \tag{11}
\end{aligned}$$

where the conditional forecasts for terms involving x_{t+1} are computed using the true law of motion for rent growth (4) and the conditional forecasts $E_t z_{t+1}$ and $E_t \log z_{t+1}$ are computed using the approximate rational solution from Proposition 1, as detailed in the appendix. To derive an analytical expression for $E_t \log(R_{t+1})$, we approximate the nonlinear term $\log[1 - \kappa \exp(-\theta x_{t+1})]$ in (11) using the expression $d_0 + d_1[x_{t+1} - E(x_t)]$, where d_0 and d_1 are Taylor series coefficients.

The conditional expectation of the 4-quarter compound return is formulated as

$$E_t \underbrace{[r_{t+1 \rightarrow t+4}]_{\text{4-quarter return}}} = E_t \sum_{j=1}^4 \log(R_{t+j}), \tag{12}$$

where the terms $E_t \log(R_{t+i})$ for $i = 2, 3, 4$ are computed by iterating equation (10) forward, taking logs, linearizing where necessary, and then applying the law of iterated expectations.

¹⁴Our procedure for expressing the rational return on the risky asset in terms of the composite variable z_t follows Lansing and LeRoy (2012).

3.2 Near-Rational Expectations

The rational expectations solution in Proposition 1 is based on strong assumptions about the representative agent’s information set. Specifically, the fully-rational solution assumes that agents know the true stochastic process for rent growth (4) which exhibits stochastic persistence and volatility. An agent with less information may be inclined to view rent growth as being governed by an AR(1) process with shifting parameters—a specification that could also account for the appearance of stochastic persistence and volatility in the observed rent growth data. Along these lines, we consider a near-rational agent who has the following perceived law of motion (PLM) for rent growth.

$$x_{t+1} = \bar{x}_t + \gamma_t(x_t - \bar{x}_t) + \eta_{t+1}, \quad \eta_t \sim N(0, \sigma_{\eta,t}^2), \quad (13)$$

where \bar{x}_t , γ_t , and $\sigma_{\eta,t}$ are time-varying AR(1) parameters.

Following the learning literature, we assume that agents estimate the parameters of the PLM (13) using recent data. In this way, they seek to account for the perceived shifts in the parameters governing rent growth. Specifically, we assume that the near-rational agent infers the parameters of (13) by computing moments over a rolling sample window of length T_w :

$$\bar{x}_t = \text{Mean}(x_j), \quad \text{where } j \in [T_w - 1, t], \quad (14)$$

$$\gamma_t = \text{Corr}(x_j, x_{j-1}), \quad (15)$$

$$\sigma_{\eta,t} = \sqrt{\text{Var}(x_j)(1 - \gamma_t^2)}. \quad (16)$$

The learning mechanism summarized by equations (14) through (16) is a version of the sample autocorrelation learning (SAC) algorithm described by Hommes and Sogner (1998). The advantage of this algorithm is that it endogenously enforces the restriction $\gamma_t \in (-1, 1)$, which ensures that perceived rent growth is always stationary.¹⁵

If the true law of motion for rent growth was governed by an AR(1) process with constant parameters, then the rational expectations solution would take the form shown in Proposition 1, but with $a_2 = 0$. Under “near-rational” expectations, we assume that the representative agent employs the correct perceived form of the rational expectations solution, but the agent continually updates the parameters of the perceived rent growth process (13), which in turn delivers shifting coefficients in the perceived optimal forecast rule.

As shown in Appendix C, the near-rational agent’s conjectured solution for the composite

¹⁵For other applications of the SAC learning algorithm, see Lansing (2009, 2010) and Hommes and Zhu (2013).

variable z_t takes the form

$$z_t \simeq b_{0,t} \exp [b_{1,t} (x_t - \bar{x}_t)], \quad (\text{PLM}) \quad (17)$$

$$\widehat{E}_t z_{t+1} = b_{0,t} \exp \left[b_{1,t} \gamma_t (x_t - \bar{x}_t) + \frac{1}{2} (b_{1,t})^2 \sigma_{\eta,t}^2 \right], \quad (18)$$

where $b_{0,t}$ and $b_{1,t}$ depend on the most recent estimates of the AR(1) parameters \bar{x}_t , γ_t , and $\sigma_{\eta,t}$. We follow the common practice in the learning literature by assuming that the representative agent views the most recent parameter estimates as permanent when computing the subjective forecast $\widehat{E}_t z_{t+1}$.¹⁶

Substituting the subjective forecast (18) into the nonlinear first-order condition (7), yields the following actual law of motion (ALM) for the composite variable z_t :

$$z_t = g(x_t) + \beta b_{0,t} \exp \left\{ [\theta(1 - \alpha)] x_t + b_{1,t} \gamma_t (x_t - \bar{x}_t) + \frac{1}{2} (b_{1,t})^2 \sigma_{\eta,t}^2 \right\}. \quad (19)$$

Following the methodology under rational expectations, the near-rational conditional expectation of the log return is given by

$$\begin{aligned} \widehat{E}_t \log(R_{t+1}) &= -\log(\beta) - \alpha \log[1 - \kappa \exp(-\theta x_t)] + \widehat{E}_t \log(z_{t+1}) - \log(\widehat{E}_t z_{t+1}) \\ &\quad + \alpha \widehat{E}_t \log[1 - \kappa \exp(-\theta x_{t+1})] + [1 - \theta(1 - \alpha)] \widehat{E}_t x_{t+1}, \end{aligned} \quad (20)$$

where the subjective forecasts involving x_{t+1} are computed using the agent's perceived law of motion (13), and the subjective forecasts $\widehat{E}_t z_{t+1}$ and $\widehat{E}_t \log z_{t+1}$ are computed using the agent's conjectured solution (17), as shown in the appendix. To derive an analytical expression for $\widehat{E}_t \log(R_{t+1})$, we approximate the nonlinear term $\log[1 - \kappa \exp(-\theta x_{t+1})]$ in (20) using the expression $d_{0,t} + d_{1,t}(x_{t+1} - \bar{x}_t)$, where $d_{0,t}$ and $d_{1,t}$ are time-varying Taylor series coefficients that shift over time due to the agent's perception that the approximation point \bar{x}_t is shifting.

The near-rational agent's forecast for the 4-quarter compound return is formulated along the lines of equation (12), but now the subjective expectation operator \widehat{E}_t is used in place of the mathematical expectation operator E_t and the agent's perceived laws of motion are used in place of the actual laws of motion when computing the subjective forecasts.

3.3 Moving-Average Expectations

Motivated by the survey evidence on the expectations of real-world investors, we consider a forecast rule that is based on a simple moving-average of past observed values of the forecast variable. Such a forecast requires only a minimal amount of computational and informational

¹⁶Otrok, et al. (2002) employ a similar procedure which they describe (p. 1275) as "a kind of myopic learning." For an earlier example of this type of learning applied to the U.S. stock market, see Barsky and Delong (1993). More recently, Collin-Dufresne, et al. (2012) examine the asset pricing implications of fully-rational learning about the parameters of dividend growth. In their model, agents' rational forecasts take into account the expected future shifts in the estimated parameters via Bayes law.

resources. Specifically, the agent does not need to know or estimate the underlying stochastic process for rent growth. The agent's subjective forecast rule is given by:

$$\begin{aligned}\widehat{E}_t z_{t+1} &= \lambda z_t + (1 - \lambda) \widehat{E}_{t-1} z_t, \quad \lambda \in [0, 1], \\ &= \lambda \left[z_t + (1 - \lambda) z_{t-1} + (1 - \lambda)^2 z_{t-2} + \dots \right].\end{aligned}\tag{21}$$

Substituting $\widehat{E}_t z_{t+1}$ from equation (21) into the transformed first-order condition (7), yields the following actual law of motion for the composite variable z_t :

$$z_t = \frac{g(x_t)}{1 - \beta \lambda \exp\{\theta(1 - \alpha)x_t\}} + \frac{\beta(1 - \lambda) \exp\{\theta(1 - \alpha)x_t\}}{1 - \beta \lambda \exp\{\theta(1 - \alpha)x_t\}} \widehat{E}_{t-1} z_t,\tag{22}$$

where the previous subjective forecast $\widehat{E}_{t-1} z_t$ acts like an endogenous state variable that evolves according to the following law of motion:

$$\widehat{E}_t z_{t+1} = \frac{\lambda g(x_t)}{1 - \beta \lambda \exp\{\theta(1 - \alpha)x_t\}} + \left[\frac{1 - \lambda}{1 - \beta \lambda \exp\{\theta(1 - \alpha)x_t\}} \right] \widehat{E}_{t-1} z_t.\tag{23}$$

We postulate that the agent's subjective forecast for the 4-quarter compound return is constructed in the same way as the forecast for z_{t+1} . Specifically, the subjective return forecast is constructed as a moving-average of past observed 4-quarter returns, where the same weight λ is applied to the most recent return observation.

$$\widehat{E}_t \underbrace{[r_{t+1 \rightarrow t+4}]_{4\text{-quarter return}}} = \lambda [r_{t-4 \rightarrow t}] + (1 - \lambda) \widehat{E}_{t-1} [r_{t-4 \rightarrow t}]\tag{24}$$

4 Calibration

Table 1 shows the baseline parameter values used in the model simulations. We also examine the sensitivity of the results to a range of values for some key parameters, namely α , κ , T_w , and λ .

Table 1. Baseline Parameter Values

Parameter	Value	Description/Target
α	2	Utility curvature parameter.
κ	0.8	Utility habit parameter.
β	0.99	Match mean of U.S. price/quarterly rent ratio = 82.3.
θ	0.87	Match mean of U.S. price/quarterly income ratio = 12.
\bar{x}	0.0083%	Match mean U.S. rent growth.
ρ	0.38	Match autocorrelation of U.S. rent growth.
σ_ε	0.457%	Match Std. Dev. of U.S. rent growth.
ψ	67.4	Match Skewness of U.S. rent growth.
T_w	16 quarters	Match Std. Dev. of U.S. price/quarterly rent = 13.95.
λ	0.93	Match Std. Dev. of U.S. price/quarterly rent = 13.95.

The utility function (1) implies the following expression for the coefficient of relative risk aversion (CRRA_t):

$$\text{CRRA}_t = -\frac{c_t U_{cc}}{U_c} = \frac{\alpha\theta}{1 - \kappa \exp(-\theta x_t)} + 1 - \theta, \quad (25)$$

which yields $\text{CRRA}_t = 8.8$ for the baseline calibration when $x_t = E(x_t)$. During the simulations, the risk aversion coefficient ranges from a low of 7.7 to a high of 9.4. Hence, our baseline calibration keeps the risk aversion coefficient below the maximum level of 10 considered plausible by Mehra and Prescott (1985).¹⁷

The parameters β and θ are chosen simultaneously so that the rational expectations model exhibits a mean price-rent ratio and a mean price-income ratio which are close to the sample means in U.S. data.¹⁸ Data on U.S. price-rent ratios from 1960 to 2011 are from the Lincoln Land Institute (see footnote 6). Data on U.S. price-income ratios for the period 1985 to 2012 are from Gudell (2012). The same values of β and θ are used for the near-rational model and the moving-average model.

We choose the rent growth parameters \bar{x} , ρ , σ_ε , and ψ to match the mean, first-order autocorrelation, standard deviation, and skewness of U.S. rent growth over the period 1960.Q2 to 2011.Q4. The analytical moment formulas for model rent growth are contained in Appendix D. The calibrated value of \bar{x} is considerably smaller than the mean growth rate in the data since the analytical moments imply $E(x_t) = \bar{x} + \psi\sigma_\varepsilon^2 / (1 - \rho)$. Table 2 compares the properties of U.S. rent growth to those produced by a long simulation of the model. In addition to hitting the four targeted statistics, the model does a reasonably good job of replicating the second-order autocorrelation and the kurtosis of U.S. rent growth.

Table 2. Properties of Rent Growth: Data versus Model

Statistic	U.S. Data	
	1960.Q2 to 2011.Q4	Model
<i>Mean</i> (x_t)	0.236%	0.228%
<i>Std Dev</i> (x_t)	0.583%	0.584%
<i>Corr</i> (x_t, x_{t-1})	0.48	0.48
<i>Corr</i> (x_t, x_{t-2})	0.26	0.19
<i>Skew</i> (x_t)	0.64	0.64
<i>Kurt</i> (x_t)	5.1	4.5

Note: Model statistics are computed from a 15,000 period simulation.

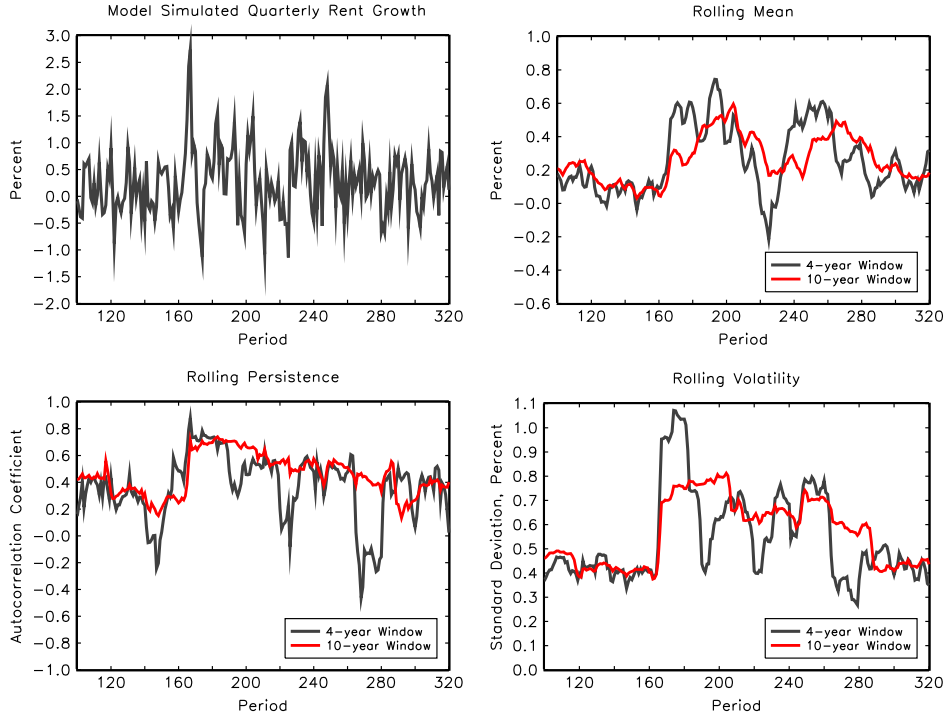


Figure 9: Model rent growth exhibits time-varying mean, persistence and volatility. The near-rational agent’s perception that the AR(1) parameters for rent growth are shifting appears to be justified.

5 Quantitative Results

Figure 9 plots simulated rent growth from the model together with 4-year and 10-year rolling summary statistics. The model statistics exhibit considerable variation over time, similar to the U.S. data statistics plotted earlier in Figure 8. An agent observing such variation would be inclined to believe that the parameters governing the stochastic process for rent growth are indeed changing over time, consistent with the PLM (13).

Figure 10 plots simulated price-rent ratios for the three different expectation regimes. The standard deviation of the model price-rent ratio under rational expectations is 2.95 versus 13.95 in the data. For the other two expectation regimes, the standard deviation of the simulated price-rent ratio is very close to that in the data. This is a direct consequence of our calibration of the forecast rule parameters T_w and λ . Due to the self-referential nature of the transformed first-order condition (7), the form of the agent’s subjective forecast $\hat{E}_t z_{t+1}$

¹⁷In the habit formation model of Campbell and Cochrane (1999), the calibration implies an extremely high coefficient of relative risk aversion—around 80 in the model steady state.

¹⁸Since $n_t/c_t = (1 - \theta)/\theta$ and $y_t/c_t = 1$, we have $p_t/y_t = (1/\theta - 1) p_t/n_t$, which is used to pin down the value of θ . The value of β is pinned down using the analytical expression for the Taylor series coefficient a_0 in Proposition 1.

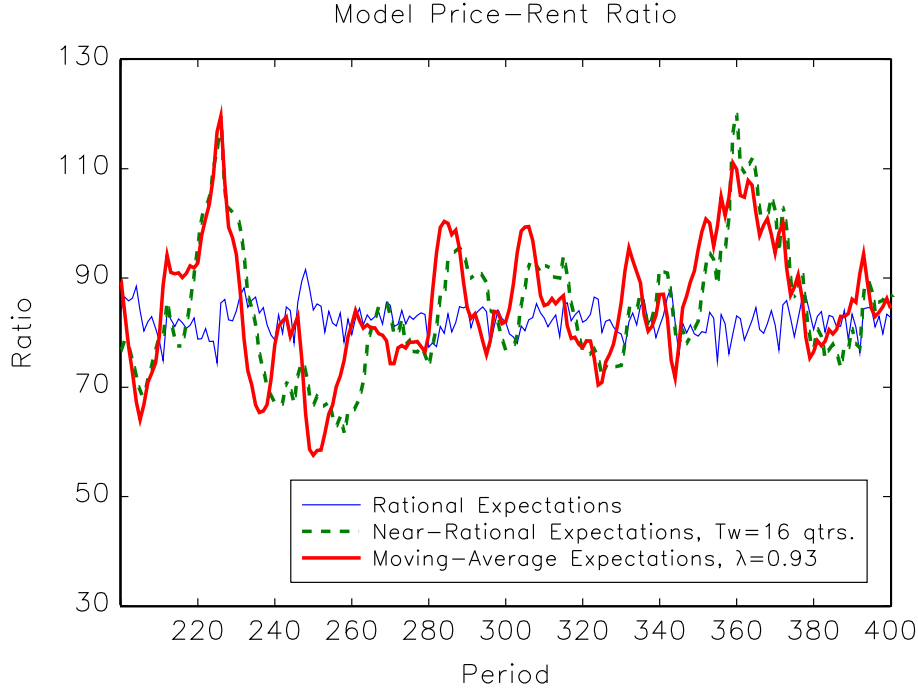


Figure 10: The standard deviation of the model price-rent ratio under rational expectations is 2.95 versus 13.95 in the data. Under the other two expectation regimes, the standard deviation of the model price-rent ratio is very close to that in the data.

influences the dynamics of the object that is being forecasted. Interestingly, the near-rational and the moving-average forecast regimes produce similar patterns for the simulated price-rent ratio. The contemporaneous correlation between the simulated price-rent ratios in the two regimes is 0.84. As we shall see, however, these two models have very different implications for the correlation between the agent’s expected future return on housing and the price-rent ratio. Only the moving-average model predicts a positive correlation between these two variables, consistent with the survey evidence discussed in the introduction.

Table 3. Comparison of Percentage Forecast Errors

Statistic	Model Simulations		
	RE	Near-RE	MAE
$Mean(err_{t+1})$	0.00	0.00	0.00
$\sqrt{Mean(err_{t+1}^2)}$	0.009	0.047	0.067
$Corr(err_{t+1}, err_t)$	0.00	0.39	0.37
$Corr(err_{t+1}, err_{t-1})$	0.00	0.11	0.04

Note: Model statistics are computed from a 15,000 period simulation. RE = rational expectations, Near-RE = near-rational expectations, MAE = moving-average expectations.

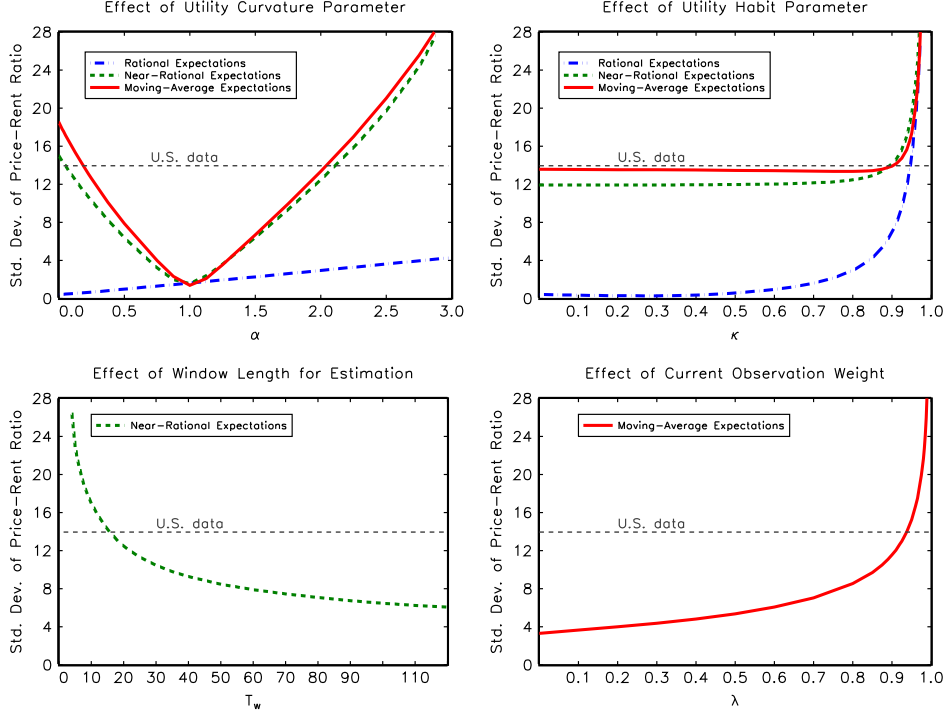


Figure 11: The figure shows how different parameter values affect the volatility of the model price-rent ratio under each of the three expectation regimes. The rational expectations model can match the observed volatility in the data (dashed horizontal line) when $\kappa \simeq 0.945$ which implies a mean coefficient of relative risk aversion $\simeq 30$.

Table 3 summarizes the quantitative properties of the representative agent’s forecast errors under each of the three expectations regimes. The percentage forecast error for the composite variable z_{t+1} is given by

$$err_{t+1} = \log \left(\frac{z_{t+1}}{\widehat{E}_t z_{t+1}} \right). \quad (26)$$

All three expectation regimes deliver unbiased forecasts such that $Mean(err_{t+1}) = 0$. The accuracy of each forecast rule can be measured by the root mean squared percentage error (RMSPE), defined as $\sqrt{Mean(err_{t+1}^2)}$. The rational expectations model exhibits the lowest RMSPE whereas the moving-average model exhibits the highest RMSPE. It is important to recognize, however, that an individual atomistic agent could not do better in the moving-average model by switching to the fundamentals-based forecast rule derived in the rational expectations model. When the actual law of motion for the composite variable z_t is given by (22), the fundamentals-based forecast rule is no longer the most accurate forecast. In particular, using the fundamentals-based forecast rule from the rational expectations model to predict z_{t+1} in the moving-average model delivers a RMSPE of 0.182—considerably higher than the value of 0.067 delivered by the moving-average forecast rule (23). Hence an individual

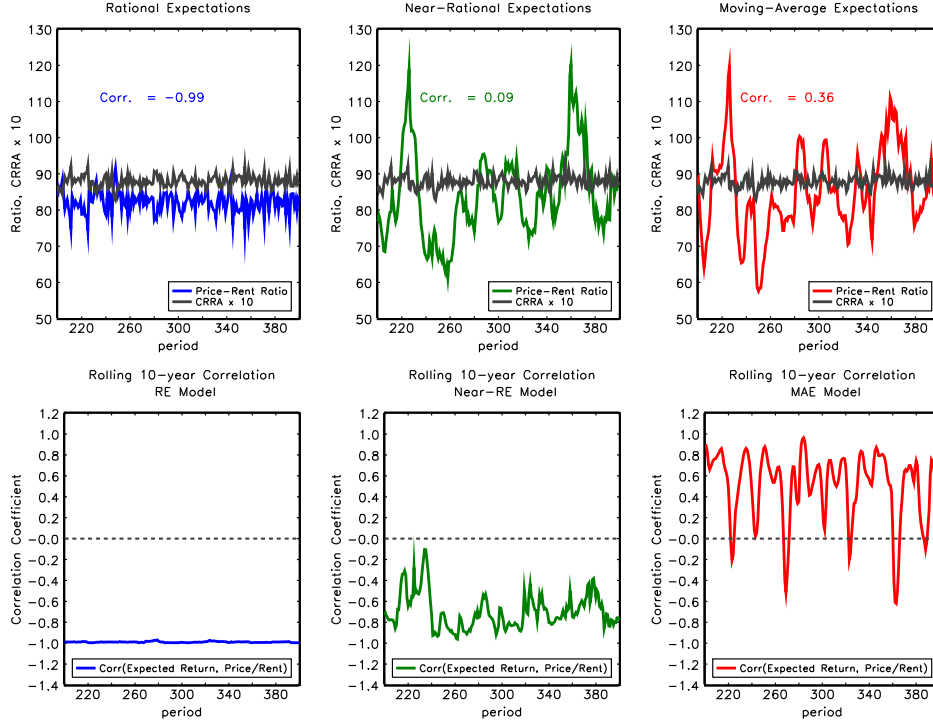


Figure 12: Under rational expectations, the correlation coefficient between the price-rent ratio and the coefficient of relative risk aversion is close to -1 , implying that risk aversion is low when the price-rent ratio is high, and vice versa. The bottom panels show that under rational and near-rational expectations, the rolling correlation coefficient between the expected 4-quarter compound return and the price-rent ratio is strongly negative. However, the rolling correlation coefficient is typically positive under moving-average expectations, implying that agents tend to expect higher future returns when the price-rent ratio is high, consistent with the survey evidence.

atomistic agent can become “locked-in” to the use of a moving-average forecast rule so long as other agents in the economy are using the same forecasting approach.¹⁹

Table 3 also shows that the autocorrelation of the forecast errors in both the near-rational model and moving-average model are reasonably low—less than 0.4. Hence, a large amount of data would be required for the representative agent in either model to reject the null hypothesis of uncorrelated forecast errors, making it difficult for the agent to detect a misspecification of the subjective forecast rule.

Figure 11 shows how some key parameter values affect the volatility of the model price-rent ratio under each expectation regime. The dashed horizontal line in each panel marks the observed standard deviation of 13.95 in U.S. data. The top right panel shows that the rational expectations model can match the observed volatility in the data when $\kappa \simeq 0.945$

¹⁹Lansing (2006) investigates the concept of forecast lock-in using a standard Lucas-type asset pricing model.

which together with the other baseline parameters implies a mean coefficient of relative risk aversion $\simeq 30$. The bottom two panels show that lower values of T_w (near-rational model) or higher values of λ (moving-average model) both serve to magnify the volatility of the simulated price-rent ratio. By construction, the baseline values of $T_w = 16$ quarters and $\lambda = 0.93$ deliver price-rent ratio volatilities that are close to those in the data.

Table 4 compares unconditional moments from the model to the corresponding moments in U.S. data. The near-rational model and the moving-average model are both successful in matching the volatility and persistence of the U.S. price-rent ratio. However, only the moving-average model comes close to matching the strong persistence of U.S. housing returns, delivering an autocorrelation coefficient for returns of 0.61 versus 0.87 in the data. Unfortunately, none of the three expectation regimes can reproduce the strong negative skewness and the large excess kurtosis in U.S. housing returns.

Table 4. Unconditional Moments: Data versus Model

		U.S. Data		Model Simulations	
Statistic		1960.Q2-2011.Q4	RE	Near-RE	MAE
Price Rent	<i>Mean</i>	82.3	82.3	85.0	85.8
	<i>Std. Dev.</i>	13.95	2.95	13.79	13.36
	<i>Corr. Lag 1</i>	0.99	0.45	0.96	0.96
	<i>Skewness</i>	2.01	0.40	0.32	0.00
	<i>Kurtosis</i>	6.94	4.56	3.17	3.04
Housing Return	<i>Mean</i>	1.56%	1.55%	1.55%	1.54%
	<i>Std. Dev.</i>	1.60%	4.22%	4.33%	4.32%
	<i>Corr. Lag 1</i>	0.87	-0.24	0.26	0.61
	<i>Skewness</i>	-1.76	0.30	0.28	-0.16
	<i>Kurtosis</i>	7.27	4.34	3.44	3.37

Note: Model statistics are computed from a 15,000 period simulation. RE = rational expectations, Near-RE = near-rational expectations, MAE = moving-average expectations.

The top panels of Figure 12 show the correlations between the model price-rent ratios and the coefficient of relative risk aversion computed from equation (25). Under rational expectations, the correlation coefficient is close to -1 , implying that risk aversion is low when the price-rent ratio is high, and vice versa. Under near-rational expectations, the correlation coefficient is close to zero at 0.09. Under moving-average expectations, the correlation coefficient is positive at 0.38.

The bottom panels of Figure 12 illustrate a key distinguishing feature of the three expectation regimes, namely, the correlation between the agent's subjective conditional forecast of the 4-quarter compound return $\hat{E}_t [r_{t+1 \rightarrow t+4}]$ and the current price-rent ratio p_t/n_t . We plot rolling correlation coefficients over a 10-year sample window to roughly capture the time duration of the recent boom-bust cycle in the U.S. housing market. Under rational expectations (bottom left panel), the rolling 10-year correlation remains close to -1 , implying that expected

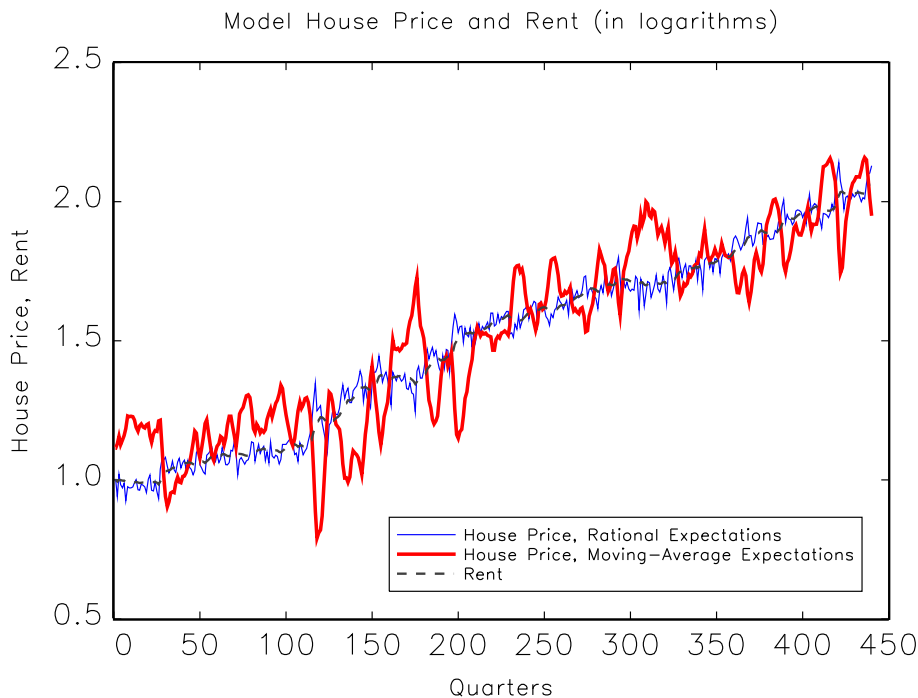


Figure 13: The moving-average model delivers periods of relatively stagnant real house prices interspersed with booms and busts, reminiscent of the long-run house price data plotted in Figure 3.

returns are low when the price-rent ratio is high—a feature that is directly at odds with the survey evidence on the expectations of real-world housing investors. The near-rational model (bottom center panel) suffers from a similar problem—exhibiting a rolling 10-year correlation that is always in negative territory, averaging around -0.7 over the full simulation. However, the moving-average model (bottom right panel) delivers a rolling 10-year correlation that is typically positive, averaging around 0.6 over the full simulation. In this case, agents tend to expect higher future returns when the price-rent ratio is high, consistent with the survey evidence. Although not shown, the rolling 10-year correlation coefficient between the expected 8-quarter compound return and the price-rent ratio in the moving-average model is even higher, averaging close to 0.7 over the full simulation. The corresponding correlation coefficients in the rational and near-rational models remain strongly negative.

Figure 13 plots simulated house prices (in logarithms) under rational expectations and moving-average expectations, together with the common simulated rent series. The moving-average model delivers periods of relatively stagnant real house prices interspersed with booms and busts, reminiscent of the long-run house price data plotted earlier in Figure 3.

Table 5 compares selected correlation coefficients from the model to the corresponding coefficients in U.S. data for two different sample periods, i.e., the full sample from 1960.Q2 to

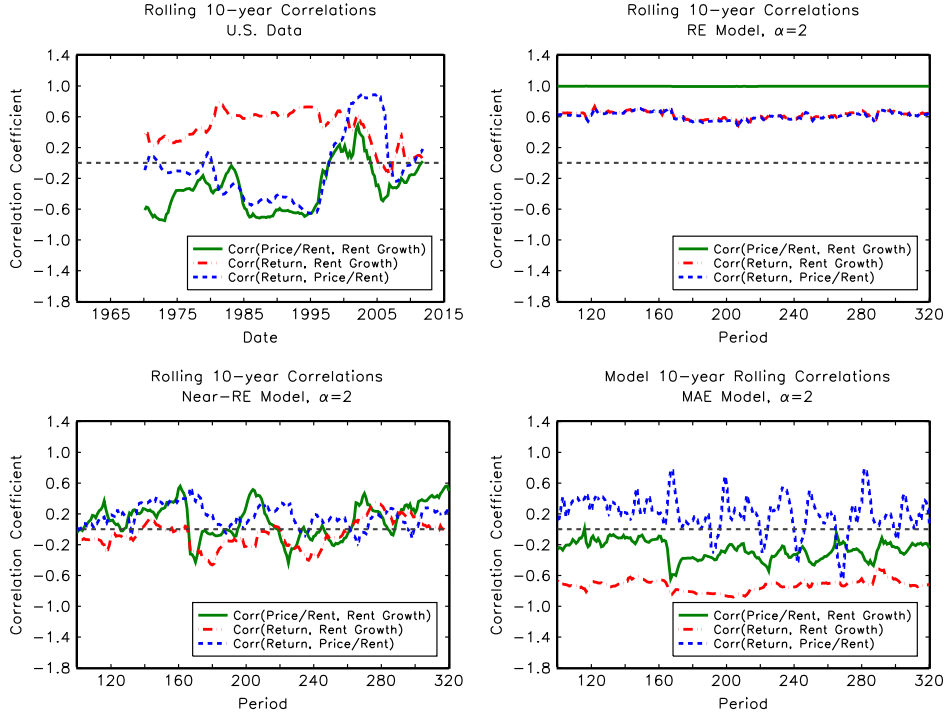


Figure 14: The rolling 10-year correlations between the price-rent ratio, rent growth, and housing returns in U.S. data can be either positive or negative, depending on the sample window. The rational expectations model predicts a consistently strong positive correlation of 0.99 between the price-rent ratio and rent growth.

2011.Q4 and a shorter sample from 2000.Q1 to 2011.Q4 which covers the recent boom-bust cycle in U.S. house prices. Figure 14 plots rolling 10-year correlations between the price-rent ratio p_t/n_t , rent growth x_t , and the housing return R_t . The rolling correlation coefficients in the U.S. data (top left panel) can be either positive or negative, depending on the sample window. For the model, Table 5 shows the results for two different values of the utility curvature parameter α , namely, the baseline value of $\alpha = 2$ and an alternative calibration with $\alpha = 0.5$ which implies $\text{CRRA}_t = 2.3$ when $x_t = E(x_t)$. Figure 14 plots the same set of model correlation coefficients for the baseline value of $\alpha = 2$ using a rolling 10-year sample window.

For both values of α , the rational expectations model delivers a strong positive correlation of 0.99 between the price-rent ratio p_t/n_t and rent growth x_t , in contrast to the negative (-0.27) or near-zero (-0.04) correlation in the data, depending on the sample period. When $\alpha = 2$, the moving-average model comes close to matching two out of the three U.S. data correlations shown in Table 5, namely $\text{Corr}(p_t/n_t, x_t)$ and $\text{Corr}(R_t, p_t/n_t)$. However, the moving-average model misses badly with respect to the correlation between realized housing returns R_t and rent growth x_t . The moving-average model with $\alpha = 2$ predicts $\text{Corr}(R_t, x_t) =$

-0.67, whereas the correlation coefficient in the data, is typically positive or close to zero (top left panel of Figure 14). The sign of $Corr(R_t, x_t)$ in both the near-rational model and the moving-average model is strongly affected by the magnitude of the parameter α , which influences how prices change in response to a shock to rent growth. Table 5 shows that a positive value of $Corr(R_t, x_t)$ can be obtained in both models when $\alpha < 1$. A calibration with $\alpha < 1$ would still allow either model to match the volatility of the price-rent ratio, as shown by the sensitivity results plotted in the top left panel of Figure 11.

Table 5. Correlation Coefficients: Data versus Model

Statistic	Model Simulations							
	U.S. Data		RE		Near-RE		MAE	
	1960.Q2- 2011.Q4	2000.Q1- 2011.Q4	$\alpha = 2$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0.5$
$Corr\left(\frac{p_t}{n_t}, x_t\right)$	-0.27	-0.04	0.99	0.99	-0.09	0.41	-0.37	0.61
$Corr(R_t, x_t)$	0.31	0.10	0.62	0.73	-0.10	0.82	-0.67	0.90
$Corr\left(R_t, \frac{p_t}{n_t}\right)$	-0.16	0.08	0.61	0.75	0.08	0.18	0.05	0.23

Note: Model statistics are computed from a 15,000 period simulation. RE = rational expectations, Near-RE = near-rational expectations, MAE = moving-average expectations.

As mentioned in the introduction, standard DSGE models with housing typically postulate large and persistent exogenous shocks to rational agents' preferences for housing services in an effort to magnify the volatility of house prices generated by the model. Table 6 shows the effects of a permanent upward shift in the agent's preference for housing services h_t relative to consumption c_t . The preference shift is accomplished by reducing the parameter θ in the utility function (1) from the baseline value of 0.87 to a new value of 0.75, holding other parameters constant at the baseline values shown in Table 1. After the preference shift, the relative weight on housing services $1 - \theta$ is roughly doubled from the baseline value of 0.13 to a new value of 0.25. Table 6 shows that an upward shift in the agent's preference for housing raises the mean price-rent ratio under all three expectations regimes. At the same time, the preference shift lowers the mean realized return on housing and lowers the mean value of $CRRR_t$. Under rational expectations, the agent will take this relationship into account when forecasting such that the conditional expected return on housing, $E_t[r_{t+1 \rightarrow t+4}]$ will tend to move in the opposite direction as the price-rent ratio p_t/n_t in response to a persistent shift in housing preferences. Hence, while a fully-rational model with housing preference shocks could potentially match the volatility of the price-rent ratio in the data, such a model would still predict a negative correlation between the expected return on housing and the price-rent ratio, which is contrary to the survey evidence described in the introduction.

Table 6. Effect of a Permanent Increase in Housing Preference

Housing Preference		Model Simulations			
		$1 - \theta$	<i>Means</i>	RE	Near-RE
0.13		p_t/n_t	82.3	85.0	85.8
		$r_{t+1 \rightarrow t+4}$	5.81%	5.79%	5.73%
		CRRR $_t$	8.8	8.8	8.8
0.25		p_t/n_t	84.2	86.4	87.0
		$r_{t+1 \rightarrow t+4}$	5.69%	5.68%	5.64%
		CRRR $_t$	7.7	7.7	7.7

Note: Model statistics are computed from a 15,000 period simulation. RE = rational expectations, Near-RE = near-rational expectations, MAE = moving-average expectations.

5.1 Predictability Regressions

Campbell and Shiller (1988) show that a log-linear approximation of the stock market return identity implies that the variance of the log price-dividend ratio must equal the sum of the ratio's covariances with: (1) future dividend growth rates, and (2) future realized stock market returns. Applying the Campbell-Shiller methodology to the housing market return identity $R_{t+1} \equiv p_{t+1}/(p_t - n_t) = (p_{t+1}/n_{t+1}) \exp(x_{t+1}) / (p_t/n_t - 1)$ yields

$$\begin{aligned} \log(R_{t+1}) &= \log(p_{t+1}/n_{t+1}) - \log(p_t/n_t - 1) + x_{t+1} \\ &\simeq \delta_0 + \log(p_{t+1}/n_{t+1}) - \delta_1 \log(p_t/n_t) + x_{t+1}, \end{aligned} \quad (27)$$

where δ_0 and δ_1 are Taylor series coefficients. Solving for $\log(p_t/n_t)$ yields

$$\log(p_t/n_t) \simeq (\delta_0/\delta_1) + (1/\delta_1) \log(p_{t+1}/n_{t+1}) + (1/\delta_1) x_{t+1} - (1/\delta_1) \log(R_{t+1}), \quad (28)$$

where $\delta_1 = \exp[E \log(p_t/n_t)] / \{\exp[E \log(p_t/n_t)] - 1\} > 1$ such that $1/\delta_1 < 1$. The next step is to iterate equation (28) forward and successively eliminate $\log(p_{t+j}/n_{t+j})$ for $j = 1, 2, 3, \dots$. Applying a transversality condition such that $\lim_{j \rightarrow \infty} (1/\delta_1)^j \log(p_{t+j}/n_{t+j}) = 0$ yields

$$\log(p_t/n_t) \simeq \frac{\delta_0}{\delta_1 - 1} + \sum_{j=1}^{\infty} (1/\delta_1)^j [x_{t+j} - \log(R_{t+j})], \quad (29)$$

which shows that movements in the log price-rent ratio must be accounted for by movements in either future rent growth rates or future log housing returns.

The variables in the approximate return identity (29) can be expressed as deviations from their unconditional means, while the means are consolidated into the constant term. Multiplying both sides of the resulting expression by $\log(p_t/n_t) - E \log(p_t/n_t)$ and then taking the

unconditional expectation of both sides yields

$$\begin{aligned} \text{Var}(p_t/n_t) &= \text{Cov} \left[\log(p_t/n_t), \sum_{j=1}^{\infty} (1/\delta_1)^j x_{t+j} \right] \\ &\quad - \text{Cov} \left[\log(p_t/n_t), \sum_{j=1}^{\infty} (1/\delta_1)^j \log(R_{t+j}) \right]. \end{aligned} \quad (30)$$

Equation (30) states that the variance of the log price-rent ratio must be accounted for by the covariance of the log price-rent ratio with either: (1) future rent growth rates, or (2) future realized housing returns. The magnitude of each covariance term is a measure of the predictability of future rent growth or future realized returns when the current price-rent ratio is employed as the sole regressor in a forecasting equation.

To investigate the predictability implications of our model versus those in the data, we estimate the following regression equations:

$$r_{t+1 \rightarrow t+4} \equiv \sum_{j=1}^4 \log(R_{t+j}) = \text{constant} + \hat{b} \log(p_t/n_t) + u_{t+1}, \quad (31)$$

$$x_{t+1 \rightarrow t+4} \equiv \sum_{j=1}^4 x_{t+j} = \text{constant} + \hat{b} \log(p_t/n_t) + \omega_{t+1}, \quad (32)$$

where u_{t+1} and ω_{t+1} are statistical error terms.

Table 7 reports the results of predictability regressions in the form of equations (31) or (32). In the case of the U.S. stock market, it is well documented that the log price-dividend ratio exhibits strong predictive power for future realized stock returns but weak predictive power for future dividend growth rates (Cochrane 2008, Engsted, et al. 2012). Table 7 shows that analogous results are obtained for the U.S. housing market, particularly in the more recent sample period starting in the year 2000. The R^2 statistic is much larger in the regression that seeks to predict future returns versus the regression that seeks to predict future rent growth. The estimated coefficient \hat{b} is consistently large and negative in the return regression, implying that a higher price-rent ratio predicts lower *realized* returns in the future. In the rent growth regression, however, \hat{b} is negative and significant over the full sample starting in 1960, but positive and insignificant in the more recent sample starting in the year 2000.

All of the models produce a negative and significant value of \hat{b} in the first regression that seeks to predict future returns. The magnitude of the \hat{b} coefficients produced by the near-rational model (-0.278 or -0.252) and the moving-average model (-0.397 or -0.384) are reasonably close to those in the data (-0.182 or -0.286). In contrast, the magnitude of the \hat{b} coefficients produced by the rational expectations model (-0.863 or -0.553) are much higher than those in the data. With a few exceptions, the R^2 statistics in the model regressions are reasonably close to those in the data. The model calibration with $\alpha = 0.5$ delivers somewhat lower R^2 statistics in the return regression since this value implies a less volatile stochastic discount factor and hence less movement in the dependent variable, $r_{t+1 \rightarrow t+4}$.

The intuition for the predictability of realized returns in both the data and the model is straightforward. A high price-rent ratio implies that the ratio is more likely to be above its long-run mean. If the price-rent ratio is stationary, then the ratio will eventually revert to its long-run mean. The inevitable drop in the price-rent ratio over a long horizon produces a lower realized return. Interestingly, even though a higher price-rent ratio in the data predicts lower *realized* returns, the survey evidence shows that real-world investors fail to take this relationship into account; instead they continue to forecast high future returns following a sustained run-up in the price-rent ratio, consistent with a moving-average forecast rule.

In sharp contrast to the U.S. data, the rational expectations model produces a large positive and significant value of \hat{b} in the regression that seeks to predict future rent growth. This is to be expected since time-varying rent growth (together with time-varying risk aversion) is an important fundamental driver of house prices under rational expectations. Similar to the U.S. data, both the near-rational model and the moving-average model produce small estimated coefficients in the rent growth regression. Moreover, the sign of \hat{b} in both models can be either positive or negative, depending on the calibrated value of α . Lower values of α (implying lower risk aversion on average) produce positive values of \hat{b} .

In a recent study using price and rent data from the housing markets of 18 OECD countries over the period 1970 to 2011, Engsted and Pedersen (2012) find evidence of cross-country and sub-sample instability in the estimated coefficients for predictive regressions that take the form of equations (31) or (32). Using data either before or after 1995, they find that the estimated regression coefficients for a given country can be either positive or negative when predicting future rent growth along the lines of equation (32). Table 6 confirms a similar sort of instability in the sign of \hat{b} when attempting to predict future U.S. rent growth using either the full sample of data from 1960 to 2011 or the more recent sample from 2000 to 2011. Engsted and Pedersen (2012) also find that the relative magnitude of the R^2 statistics for the two types of predictive regressions can differ across countries and across time periods for a given country. In line with their overall empirical findings, our simulation results show that, depending on model assumptions, the signs and magnitudes of the estimated regression coefficients and the resulting R^2 statistics can vary substantially, particularly with respect to rent growth predictability.

Table 7. Predictability Regressions: Data versus Model

	Model Simulations							
	U.S. Data		RE		Near-RE		MAE	
	\hat{b}		\hat{b}		\hat{b}		\hat{b}	
Predictive Regression	1960.Q2-2011.Q4	2000.Q1-2011.Q4	$\alpha = 2$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0.5$
$r_{t+1 \rightarrow t+4}$	-0.182	-0.286	-0.863	-0.553	-0.278	-0.252	-0.397	-0.384
	(0.024)	(0.094)	(0.011)	(0.017)	(0.005)	(0.006)	(0.005)	(0.008)
R^2 (%)	21.7	18.4	30.2	6.6	19.4	9.3	27.8	14.3
$x_{t+1 \rightarrow t+4}$	-0.039	0.002	0.118	0.428	-0.001	0.020	-0.009	0.029
	(0.007)	(0.017)	(0.004)	(0.011)	(0.000)	(0.002)	(0.001)	(0.001)
R^2 (%)	12.0	0.0	6.7	9.6	0.0	1.1	0.8	2.7

Notes: Standard errors in parenthesis. Model regressions use data from a 15,000 period simulation. RE = rational expectations, Near-RE = near-rational expectations, MAE = moving-average expectations.

6 Conclusion

Stories involving speculative bubbles can be found throughout history in various countries and asset markets.²⁰ These episodes can have important consequences for the economy as firms and investors respond to the price signals, potentially resulting in capital misallocation.²¹ The typical transitory nature of these run-ups should perhaps be viewed as a long-run victory for fundamental asset pricing theory. Still, it remains a challenge for fundamental theory to explain the ever-present volatility of asset prices within a framework of efficient markets and fully-rational agents.

Like stock prices, real-world house prices exhibit periods of stagnation interspersed with boom-bust cycles. A reasonably-parameterized rational expectations model significantly underpredicts the volatility of the U.S. price-rent ratio, even when allowing for time-varying risk aversion and time-varying stochastic properties of rent growth. We showed that a simple asset pricing model can match the volatility and persistence of the U.S. price-rent ratio, as well as other quantitative and qualitative features of the data if agents in the model employ simple moving-average forecast rules. With such a forecast rule, agents tend to expect higher future returns when house prices are high relative to fundamentals—a feature that is consistent with survey evidence on the expectations of real-world housing investors. The moving average model is also successful in generating data that is broadly consistent with the predictability properties of future realized housing returns and future rent growth in U.S. housing market data.

²⁰See, for example, the collection of papers in Hunter, Kaufman, and Pomerleano (2003).

²¹Lansing (2012) examines the welfare consequences of speculative bubbles in a model where excessive asset price movements can affect the economy's trend growth rate.

A Appendix: Effect of a Borrowing Constraint

Here we show analytically how imputed rent can reflect not only a utility dividend, but also the marginal collateral value of the house in the case when the representative agent's borrowing constraint is binding. Following Campbell and Hercowitz (2009), the representative agent's problem in the presence of a borrowing constraint can be formulated as

$$\max_{c_t, h_t, b_{t+1}} \widehat{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t, \quad (\text{A.1})$$

where the current-period Lagrangian \mathcal{L}_t is given by

$$\begin{aligned} \mathcal{L}_t = & \frac{\left(c_t^\theta h_t^{1-\theta} - \kappa C_{t-1}^\theta H_{t-1}^{1-\theta}\right)^{1-\alpha} - 1}{1-\alpha} + \lambda_t [y_t + b_{t+1} + p_t (h_{t-1} - h_t) - c_t - Rb_t] \\ & + \lambda_t \Gamma_t [\omega p_t h_t - b_{t+1}]. \end{aligned} \quad (\text{A.2})$$

In the above expression, b_t is the stock of mortgage debt at the end of period $t - 1$ and R is the gross real interest rate on the debt. The last term of the Lagrangian reflects the borrowing constraint which says that the agent may only borrow up to a fraction $\omega \geq 0$ of the current housing value $p_t h_t$. When the $\Gamma_t > 0$, the borrowing constraint is binding.

From (A.2), the first-order conditions with respect to h_t and b_{t+1} are given by

$$p_t = \underbrace{\left(\frac{1-\theta}{\theta}\right) c_t + \Gamma_t \omega p_t}_{\equiv n_t} + E_t \underbrace{\frac{\lambda_{t+1}}{\lambda_t}}_{\equiv M_{t+1}} p_{t+1}, \quad (\text{A.3})$$

$$\Gamma_t = 1 - \beta R E_t M_{t+1}, \quad (\text{A.4})$$

where we have imposed the equilibrium condition $h_t = 1$ and divided both sides by λ_t . Comparing equation (A.3) to the original first-order condition (3) shows that the imputed rent n_t now consists of two terms: the standard utility dividend plus the marginal collateral value of the house in the case when the borrowing constraint is binding, i.e., when $\Gamma_t > 0$. Hence, by calibrating the effective cash flows in the model to mimic the stochastic properties of rent growth in the data, we implicitly (but imperfectly) take into account the effect of a binding borrowing constraint on the equilibrium house price.

B Appendix: Approximate Rational Solution

The methodology for computing the rational expectations solution in Proposition 1 follows the procedure in Lansing (2009). First we rewrite the law of motion for rent growth (4) as

follows

$$\begin{aligned}
x_{t+1} - E(x_t) &= \rho [x_t - E(x_t)] + (1 - \rho) \underbrace{[\bar{x} - E(x_t)]}_{= \frac{-\psi \sigma_\varepsilon^2}{1-\rho}} + \varepsilon_{t+1} + \psi \underbrace{\varepsilon_t (x_t - \bar{x})}_{\equiv v_t}, \\
&= \rho [x_t - E(x_t)] + \varepsilon_{t+1} + \psi (v_t - \sigma_\varepsilon^2), \tag{B.1}
\end{aligned}$$

where \bar{x} is the deterministic steady state growth rate, $E(x_t)$ is the ergodic mean growth rate, and we have made use of $E[\varepsilon_t (x_t - \bar{x})] = E(v_t) = \sigma_\varepsilon^2$.

The law of motion for $v_{t+1} - \sigma_\varepsilon^2$ follows directly from the law of motion for rent growth (4) and the rewritten version (B.1):

$$\begin{aligned}
v_{t+1} - \sigma_\varepsilon^2 &= \varepsilon_{t+1} (x_{t+1} - \bar{x}) - \sigma_\varepsilon^2, \\
&= \varepsilon_{t+1} [x_{t+1} - E(x_t)] + \varepsilon_{t+1} \underbrace{[E(x_t) - \bar{x}]}_{= \frac{\psi \sigma_\varepsilon^2}{1-\rho}} - \sigma_\varepsilon^2, \\
&= \varepsilon_{t+1} \left\{ \rho [x_t - E(x_t)] + \psi (v_t - \sigma_\varepsilon^2) + \frac{\psi \sigma_\varepsilon^2}{1-\rho} \right\} + (\varepsilon_{t+1}^2 - \sigma_\varepsilon^2), \tag{B.2}
\end{aligned}$$

Iterating ahead the conjectured law of motion for z_t yields

$$z_{t+1} = a_0 \exp \left\{ a_1 [x_{t+1} - E(x_t)] + a_2 (v_{t+1} - \sigma_\varepsilon^2) \right\}. \tag{B.3}$$

Substituting equations (B.1) and (B.2) into equation (B.3) and then taking the conditional expectation yields

$$E_t z_{t+1} = a_0 \exp \left\{ a_1 \rho [x_t - E(x_t)] + a_1 \psi (v_t - \sigma_\varepsilon^2) + (a_2)^2 \sigma_\varepsilon^4 + \frac{1}{2} w_t^2 \sigma_\varepsilon^2 \right\}, \tag{B.4}$$

$$\text{where } w_t \equiv a_1 + a_2 \rho [x_t - E(x_t)] + a_2 \psi (v_t - \sigma_\varepsilon^2) + a_2 \frac{\psi \sigma_\varepsilon^2}{1-\rho}.$$

In deriving (B.4), we have used the properties of the conditional lognormal distribution which imply

$$\begin{aligned}
E_t \exp(a_2 \varepsilon_{t+1}^2) &= \exp \left\{ E_t (a_2 \varepsilon_{t+1}^2) + \frac{1}{2} \text{Var}_t (a_2 \varepsilon_{t+1}^2) \right\}, \\
&= \exp \left\{ E_t (a_2 \varepsilon_{t+1}^2) + \frac{1}{2} E_t \left[(a_2 \varepsilon_{t+1}^2)^2 \right] - \frac{1}{2} [E_t (a_2 \varepsilon_{t+1}^2)]^2 \right\}, \\
&= \exp \left\{ a_2 \sigma_\varepsilon^2 + \frac{1}{2} (a_2)^2 3 \sigma_\varepsilon^4 - \frac{1}{2} (a_2)^2 \sigma_\varepsilon^4 \right\}, \\
&= \exp \left[a_2 \sigma_\varepsilon^2 + (a_2)^2 \sigma_\varepsilon^4 \right]. \tag{B.5}
\end{aligned}$$

Next we substitute the conditional expectation (B.4) into the transformed first-order condition (7) and then take logarithms to obtain

$$\begin{aligned} \log(z_t) &= F(x_t, v_t) = \log\{g(x_t) + \\ &\quad \beta a_0 \exp\left[(\theta(1-\alpha)x_t + a_1\rho(x_t - E(x_t)) + a_1\psi(v_t - \sigma_\varepsilon^2) + (a_2)^2\sigma_\varepsilon^4 + \frac{1}{2}w_t^2\sigma_\varepsilon^2)\right]\}, \\ &\simeq \log(a_0) + a_1[x_t - E(x_t)] + a_2(v_t - \sigma_\varepsilon^2). \end{aligned} \quad (\text{B.6})$$

The expressions for the Taylor-series coefficients $a_0 = \exp[E \log(z_t)]$, a_1 , and a_2 are derived as follows

$$\log(a_0) = F[E(x_t), E(v_t)], \quad (\text{B.7})$$

$$a_1 = \left. \frac{\partial F}{\partial x_t} \right|_{E(x_t), E(v_t)}, \quad (\text{B.8})$$

$$a_2 = \left. \frac{\partial F}{\partial v_t} \right|_{E(x_t), E(v_t)}, \quad (\text{B.9})$$

where $E(x_t) = \bar{x} + \psi\sigma_\varepsilon^2/(1-\rho)$ and $E(v_t) = \sigma_\varepsilon^2$. Solving the above three equations yields the values for the three undetermined coefficients.

Taking logs of equation (B.3) and then forming the conditional expectation yields

$$\begin{aligned} E_t \log(z_{t+1}) &= \log(a_0) + a_1 E_t[x_{t+1} - E(x_t)] + a_2 E_t(v_{t+1} - \sigma_\varepsilon^2), \\ &= \log(a_0) + a_1\rho[x_t - E(x_t)] + a_1\psi(v_t - \sigma_\varepsilon^2), \end{aligned} \quad (\text{B.10})$$

where $E_t(v_{t+1} - \sigma_\varepsilon^2) = 0$ from equation (B.2). The above expression and equation (B.4) are used to compute the expected log return in equation (11).

C Appendix: Approximate Near-Rational Solution

First we solve for the perceived rational solution in the case when the perceived AR(1) parameters for rent growth are constant such that $\bar{x}_t = \bar{x}$, $\gamma_t = \gamma$, and $\sigma_{\eta,t} = \sigma_\eta$ for all t . The near-rational agent's conjectured law of motion in this case is given by

$$z_t \simeq b_0 \exp[b_1(x_t - \bar{x})]. \quad (\text{C.1})$$

Iterating ahead the conjectured law of motion for z_t and taking the subjective conditional expectation yields

$$\widehat{E}_t z_{t+1} = b_0 \exp\left[b_1\gamma(x_t - \bar{x}) + \frac{1}{2}(b_1)^2\sigma_\eta^2\right]. \quad (\text{C.2})$$

Substituting the conditional forecast (C.2) into the transformed first-order condition (7) and then taking logarithms yields

$$\begin{aligned}\log(z_t) &= F(x_t) = \log \left\{ g(x_t) + \beta b_0 \exp \left[(\theta(1-\alpha))x_t + b_1 \gamma (x_t - \bar{x}) + \frac{1}{2} (b_1)^2 \sigma_\eta^2 \right] \right\} \\ &\simeq \log(b_0) + b_1(x_t - \bar{x}),\end{aligned}\tag{C.3}$$

where $b_0 \equiv \exp[E \log(z)]$ and b_1 are Taylor-series coefficients. The expressions for the Taylor series coefficients are derived as follows

$$\log(b_0) = F(\bar{x}) = \log \left\{ g(\bar{x}) + \beta b_0 \exp \left[(\theta(1-\alpha))\bar{x} + \frac{1}{2} (b_1)^2 \sigma_\eta^2 \right] \right\},\tag{C.4}$$

$$b_1 = F'(\bar{x}) = \frac{1}{b_0} \left\{ g'(\bar{x}) + \beta b_0 [\theta(1-\alpha) + b_1 \gamma] \exp \left[(\theta(1-\alpha))\bar{x} + \frac{1}{2} (b_1)^2 \sigma_\eta^2 \right] \right\},\tag{C.5}$$

which yield a set of two equations that can be solved for the two undetermined coefficients b_0 and b_1 .

For the case when the perceived AR(1) parameters are shifting as in the PLM (13), equations (C.4) and (C.5) can be rewritten as

$$b_{0,t} = \frac{g(\bar{x}_t)}{1 - \beta \exp \left[(\theta(1-\alpha))\bar{x}_t + \frac{1}{2} (b_{1,t})^2 \sigma_{\eta,t}^2 \right]},\tag{C.6}$$

$$b_{1,t} = \frac{g'(\bar{x}_t)}{b_{0,t}} + \beta [\theta(1-\alpha) + b_{1,t} \gamma_t] \exp \left[(\theta(1-\alpha))\bar{x}_t + \frac{1}{2} (b_{1,t})^2 \sigma_{\eta,t}^2 \right],\tag{C.7}$$

where we have substituted in the most recent estimates for the perceived AR(1) parameters, as given by \bar{x}_t , γ_t , and $\sigma_{\eta,t}$. We follow the common practice in the learning literature by assuming that the agent views the most recent parameter estimates as permanent when computing the subjective forecast $\widehat{E}_t z_{t+1}$. Given the most recent estimates for the AR(1) parameters, equations (C.6) and (C.7) are solved simultaneously each period to obtain values for $b_{0,t}$ and $b_{1,t}$ for use in the subjective forecast (18). The subjective forecast is substituted into the transformed first-order condition (7) to obtain the actual law of motion (19).

Iterating ahead the perceived law of motion (17) and then taking logs and forming the subjective conditional expectation yields

$$\begin{aligned}\widehat{E}_t \log(z_{t+1}) &= \widehat{E}_t [\log(b_{0,t+1}) + b_{1,t+1} (x_{t+1} - \bar{x}_{t+1})], \\ &= \log(b_{0,t}) + b_{1,t} \gamma_t (x_t - \bar{x}_t),\end{aligned}\tag{C.8}$$

where once again we assume that the agent views the most recent parameter estimates \bar{x}_t and γ_t as permanent such that the most recent values for $b_{0,t}$ and $b_{1,t}$ are also viewed as permanent. The above expression and $\widehat{E}_t z_{t+1}$ from equation (18) are used to compute the expected log return in equation (20).

D Appendix: Moments of Rent Growth

This section summarizes the formulas for the unconditional moments of rent growth which are used to calibrate the true law of motion (4). From Granger and Andersen (1978) and Sesay and Subba Rao (1988), we have

$$E(x_t - \bar{x}) \equiv M_1 = \frac{\psi\sigma_\varepsilon^2}{1 - \rho} \quad (\text{D.1})$$

$$E[(x_t - \bar{x})^2] \equiv M_2 = \frac{\sigma_\varepsilon^2 [1 + 2\psi^2\sigma_\varepsilon^2 + 4\rho\psi M_1]}{1 - \rho^2 - \psi^2\sigma_\varepsilon^2} \quad (\text{D.2})$$

$$E[(x_t - \bar{x})(x_{t-1} - \bar{x})] \equiv M_{1,1} = \rho M_2 + 2\psi\sigma_\varepsilon^4 M_1 \quad (\text{D.3})$$

$$E[(x_t - \bar{x})^3] \equiv M_3 = \frac{\sigma_\varepsilon^2 [6\psi^3\sigma_\varepsilon^4 + 3(1 + 6\rho\psi^2\sigma_\varepsilon^2)M_1 + 9\psi(\rho^2 + \psi^2\sigma_\varepsilon^2)M_2]}{1 - \rho^3 - 3\rho\psi^2\sigma_\varepsilon^2} \quad (\text{D.4})$$

$$E[(x_t - \bar{x})^4] \equiv M_4 = \frac{\sigma_\varepsilon^2 \{3\sigma_\varepsilon^2(8\psi^4\sigma_\varepsilon^4 - 1) + 96\rho\psi^3\sigma_\varepsilon^4 M_1 + 6[1 + 12\psi^2\sigma_\varepsilon^2(\rho^2 + \psi^2\sigma_\varepsilon^2)]M_2 + 16\rho\psi(\sigma_\varepsilon^2 + 3\psi^2\sigma_\varepsilon^2)M_3\}}{1 - \rho^4 - 6\rho^2\psi^2\sigma_\varepsilon^2 - 3\psi^4\sigma_\varepsilon^4} \quad (\text{D.5})$$

Given the above expressions, the moments of x_t can be computed as follows

$$E(x_t) = \bar{x} + M_1, \quad (\text{D.6})$$

$$Var(x_t) = M_2 - (M_1)^2, \quad (\text{D.7})$$

$$Corr(x_t, x_{t-1}) = \frac{M_{1,1} - (M_1)^2}{Var(x_t)}, \quad (\text{D.8})$$

$$Skew(x_t) = \frac{M_3 - (M_1)^3 - 3M_1 Var(x_t)}{Var(x_t)^{1.5}}, \quad (\text{D.9})$$

$$Kurt(x_t) = \frac{M_4 + (M_1)^4 - 4M_1 [M_3 + (M_1)^3] + 6M_2 (M_1)^2}{Var(x_t)^2}. \quad (\text{D.10})$$

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