Rødseth K L. A note on input congestion.

Economics Letters. 120 (3), 2013, 599-602.
0165-1765

The article has been published in final form by Elsevier at

http://dx.doi.org/10.1016/j.econlet.2013.06.021

The final publication is available in: Economics Letters. 2013, 120 (2), 599-602.
https://doi.org/10.1016/j.econlet.2013.06.021

A Note on Input Congestion

Abstract : The notion of effective space is introduced and input congestion is explained by economic activities' exhaustion of effective space. In this setting, I show that profit maximization is inconsistent with input congestion at the *firm level*, but not necessarily with input congestion at the *industry level*, when effective space is shared among producers.

Keywords: *Congestion; Firm and industry technologies*; *Externalities*

JEL classification: *D24*; *D62*; *Q32*

1. Introduction

Input congestion is present when there are negative returns to inputs in production, i.e. when employment of additional units of inputs obstructs the output. The concept originates from Färe and Svensson (1980) who related it to the law of diminishing returns by Turgot. In the classical treatment of this law, space (land) is a fixed factor on which variable factors cause congestion (overcrowding). In line with the law, Färe and Svensson (1980) evaluated congestion when some inputs are fixed while others are variable.

Input congestion has become an important topic in the Data Envelopment Analysis (DEA) literature. Unlike Färe and Svensson (1980), the DEA literature has not emphasized the role of fixed inputs in congestion. See Cherchye et al. (2001) for a critical discussion on congestion in DEA.

The current paper explains input congestion in a way similar to the law of diminishing returns, but instead of considering space as an essential input it considers *effective space* as an essential input. The concept of effective space concerns the *quality* of space which, contrary to space itself, diminishes as a result of human activities. Real life examples are numerous: plowing contributes to degradation of land for cultivation; the quality of grazing land is negatively related to the number of animals; the quality of a road, both in terms of decay and of average speed, depends upon traffic; feed spills from aquaculture reduce the water quality and contribute to fish diseases.

Production analysis is generally concerned with inputs which the entrepreneur exercises effective control over (Chambers, 1988). Effective space may not conform to this requirement and is therefore usually not accounted for. In the following, I consider "congestion models" (e.g. in the DEA literature) that do not incorporate effective space to be reduced forms (Murty et al., 2012) of the "true technology" that incorporates effective space. In this setting, I show that a production function exhibiting *free disposability* of inputs allows detecting input congestion when increases in economic activity come at the expense of effective space. I find that profit maximization is inconsistent with input congestion at the *firm level*, but not necessarily with congestion at the *industry level*, when effective space is shared among producers.

2. Congestion measurement by the reduced form technology

Consider an industry consisting of $k=(1,2)$ firms. Each firm is represented by a (reduced form) production function $\tilde{f}^k\left(x^k\right)$ that converts an input $x^k \in \Re_+$ into an output $y^k \in \Re_+$. I assume that $\tilde{f}^k$ is differentiable and introduce the axiom of free disposability of inputs:

$$\partial y^k / \partial x^k = \partial \tilde{f}^k\left(x^k\right) / \partial x^k \geq 0, \qquad k=(1,2) \tag{1}$$

Equation 1 rules out input congestion as the output is assumed not to decrease in the input. Congestion, on the other hand, arises when the marginal product is negative. Formally, for firm *k*:

$$\begin{aligned} \partial y^k / \partial x^k &= \partial \tilde{f}^k \left(x^k \right) / \partial x^k \geq 0 \text{ if } x^k \leq \mu^k \\ \partial y^k / \partial x^k &= \partial \tilde{f}^k \left(x^k \right) / \partial x^k < 0 \text{ if } x^k > \mu^k \end{aligned} \tag{2}$$

Notice that input congestion is considered at the firm level. Equation 2 resembles Färe and Svenssons' (1980) concept of monotone output-limitational (MOL) congestion, which states that the technology is congested if it fails to satisfy the free disposability assumption. It is also related to Briec and Kerstens' (2006) S-disposability axiom that treats free disposability of inputs as a local technology property.

3. A new look at input congestion

Recall that equation 2 defines input congestion without relating it to fixed inputs. Hence, it is in line with current treatments on input congestion in DEA. In the following, I aim at assessing the underlying determinants of equation 2 by establishing a more comprehensive model to study the dynamics of input congestion.

Denote space by $b \in \Re_+$ and effective space by b^k, where $b^1 = b^2$ (i.e. effective space is the same for both producers). Effective space is a function of the *quantity* of space available and the firms' employment of the marketable input, $x^1 + x^2$. The comprehensive technology for producer *k* is defined by:

$$\begin{matrix} y^k = f^k\left(x^k, b^k\right) \\ b^k = g\left(x^1 + x^2, b\right) \end{matrix} \Leftrightarrow \quad y^k = f^k\left(x^k, g\left(x^1 + x^2, b\right)\right), \qquad k = (1,2) \tag{3}$$

I assume that f^k is everywhere twice-continuously differentiable; finite, non-negative, real valued, and single valued for all non-negative and finite input vectors; zero when the input vector is the zero vector. The two functions f^k and g are assumed to satisfy axioms (i)-(v):

(i) Free disposability of inputs	$\partial f^k / \partial x^k \geq 0, \ \ \partial f^k / \partial b^k \geq 0, \qquad k = (1,2)$
(ii) Concavity	$\partial^2 f^k / \partial x^{k^2} \leq 0, \ \ \partial^2 f^k / \partial b^{k^2} \leq 0, \quad k = (1,2)$
(iii) Quality degradations	$\partial g / \partial x^1 = \ \partial g / \partial x^2 < 0$
(iv) Linearity	$\partial^2 g / \partial x^{1^2} = \ \partial^2 g / \partial x^{2^2} = 0$
(v) Quality increases in space	$\partial g / \partial b \geq 0$

The two first axioms are standard in production theory. Axioms (iii)-(v) imply that the quality of space degrades linearly[1] with economic activity and increases in (unspoiled) space. In addition, I assume that effective space is an abundant factor when only one of the two producers operates (and maximizes profits). Let x^{l*} be the profit maximizing input vector for one of the two firms and define:

[1] Linearity is assumed for convenience as it simplifies expositions. The results in the paper may also be derived under convexity, i.e. under decreasing exhaustion of effective space.

(vi) Abundance $\lim_{x^k \to 0} \frac{\partial f^k\left(x^k, g\left(x^k + x^{l*}, b\right)\right)}{\partial g} = 0, \quad l \neq k, \quad l, k = (1,2)$

I derive the marginal product of x^k by taking the first-order derivative of equation 3:

$$\frac{\partial y^k}{\partial x^k} = \underbrace{\frac{\partial f^k}{\partial x^k}}_{\geq 0} + \underbrace{\frac{\partial f^k}{\partial g}\frac{\partial g}{\partial x^k}}_{\leq 0} \begin{matrix} > \\ = \\ < \end{matrix} 0, \qquad k = (1,2) \tag{4}$$

The first term (the *direct* effect) in equation 4 represents the marginal productivity of x^k in the production of y^k, which is positive by axiom (i). The second term (the *indirect* effect) represents reductions in y^k due to exhaustion of effective space. It is the product of $\partial f^k / \partial g$, the marginal productivity of effective space in the production of y^k, and $\partial g / \partial x^k$, the exhaustion of effective space by a marginal increase in economic activity. According to equation 4, congestion occurs when the indirect effect *dominates* the direct effect. Note that axiom (vi) is sufficient (but not necessary) to secure that there is no congestion when x^k approaches zero, since the indirect effect is zero by axiom (vi) whereas $\partial f^k / \partial x^k$ is greater or equal to zero by axiom (i). By relating equation 4 to equation 2, congestion in the reduced form technology from section 2 can be explained in terms of exhaustion of effective space.

Next, I derive the second-order derivative of equation 3 by taking the derivative of equation 4 and applying axioms (i)-(iv):

$$\frac{\partial^2 y^k}{\partial x^{k^2}} = \underbrace{\frac{\partial^2 f^k}{\partial x^{k^2}}}_{\le 0} + \underbrace{\frac{\partial^2 f^k}{\partial g^2}\left(\frac{\partial g}{\partial x^k}\right)^2}_{\le 0}, \qquad k=(1,2) \tag{5}$$

Equations 4 and 5 imply that the *direct effect* is increasing concave, while the *indirect effect* is decreasing concave. These curvature properties secure the point of congestion - if any - is global, since the indirect effect dominates the direct effect from this point on. However, they are not sufficient for securing that congestion takes place as x^k approaches infinity. The reason is that effective space becomes exhausted (zero) when x^k is sufficiently large. If the direct effect dominates the indirect effect at this point there is no congestion. Congestion may, however, be secured by imposing the Inada (1963) condition that the marginal productivity of effective space approaches infinity as effective space approaches zero. Alternatively, effective space can be treated as an essential input; see Shephard (1970). The latter approach will imply a severe form of congestion, namely that no production can take place when effective space is exhausted. This is similar to output-prohibitive (OP) congestion in the terminology of Färe and Svensson (1980).

Assume now that the two producers are profit maximizers and face the same prices, where $p \in \Re_+$ denotes the output price and $w \in \Re_+$ denotes the input price. If (quantitative) space is (quasi)fixed, the profit maximization problem for producer *k* is:

$$\pi^k\left(p,w,x^l,b\right) = \max_{x^k} pf^k\left(x^k, g\left(x^k + x^l, b\right)\right) - wx^k, \qquad l \ne k, \quad l,k=(1,2) \tag{6}$$

with first-order condition:

$$\frac{\partial \pi^k}{\partial x^k} = p\left(\frac{\partial f^k}{\partial x^k} + \frac{\partial f^k}{\partial g}\frac{\partial g}{\partial x^k}\right) = w, \qquad k = (1,2) \tag{7}$$

The value of the marginal productivity of x^k equals the factor price in optimum. Since the two prices are non-negative, equation 7 cannot hold with equality when the marginal productivity is negative. That is, profit maximization cannot be consistent with input congestion at the firm level.

3.1 Congestion at the industry level

Define the industry production, y^I, as the sum of the firms' production functions:

$$y^I = f^1\left(x^1, g\left(x^1 + x^2, b\right)\right) + f^2\left(x^2, g\left(x^1 + x^2, b\right)\right) \tag{8}$$

Taking first order derivatives yields:

$$\frac{\partial y^I}{\partial x^k} = \underset{\geq 0}{\frac{\partial f^k}{\partial x^k}} + \underbrace{\frac{\partial f^k}{\partial g}\frac{\partial g}{\partial x^k}}_{\leq 0} + \underbrace{\frac{\partial f^l}{\partial g}\frac{\partial g}{\partial x^k}}_{\leq 0} \ \substack{> \\ = \\ <} \ 0, \quad l \neq k, \quad l,k = (1,2) \tag{9}$$

The marginal productivity $\partial y^I / \partial x^k$ encompasses both the negative impact of reduced effective space for producer *l*'s output and the direct and indirect effects for producer *k* (equation 4). This means that the marginal contribution of x^k to the industry output is smaller or equal to its contribution to firm *k*'s output, i.e. $\partial y^I / \partial x^k \leq \partial y^k / \partial x^k$. This insight leads to two important observations: **(Observation 1)** Congestion may occur at the industry level in cases where it does not occur at the firm level, and **(Observation 2)** profit maximization at the firm level may lead to congestion at the industry level.

I turn to graphical analysis for explaining observations 1 and 2. Figure 1 consists of two panels, where the upper panel depicts the *reduced form* (i.e. effective space is implicitly considered) production functions for firm *k* and the overall industry, while the lower panel depicts the direct and indirect effects of a marginal increase in x^k for the firm and industry outputs. x^l is assumed to be exogenous, which means that the industry output equals $f^l\left(x^l,g\left(x^l,b\right)\right)$ when $x^k=0$, and that producer *l*'s output declines relative to $f^l\left(x^l,g\left(x^l,b\right)\right)$ when x^k is employed and causes degradation of effective space. For convenience, effective space is not exhausted (zero) for any amount of x^k in the figure.

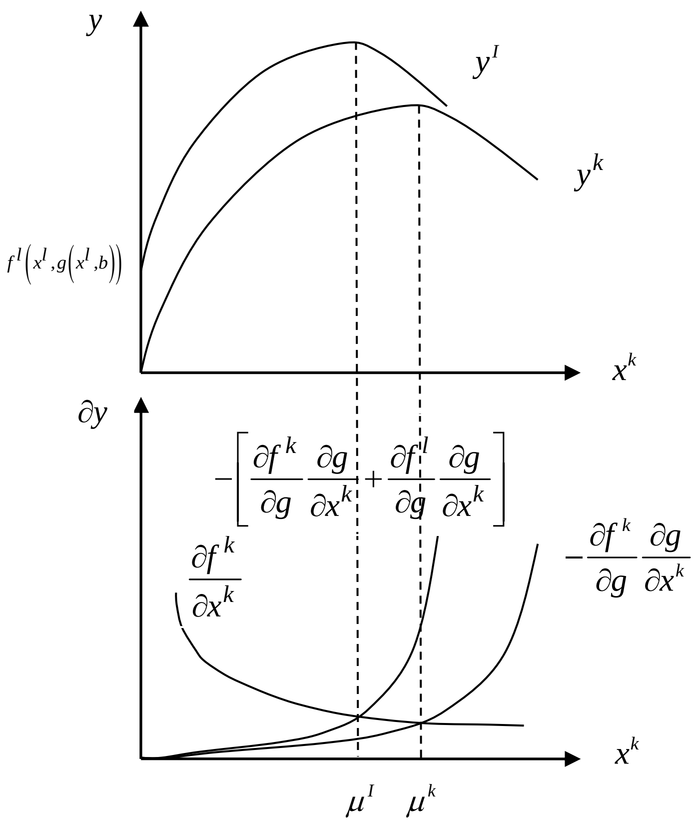

Figure 1: *Congestion at the firm and industry level*

Congestion occurs when the slopes of the reduced form production functions in the upper panel of figure 1 are negative. The determinants of the slopes are depicted in the lower panel, in accordance with equations 4 and 9. The slope of firm *k*'s reduced form production function is positive when $x^k < \mu^k$ (i.e. when the direct effect dominates the indirect effect; see the lower panel of figure 1), zero when $x^k = \mu^k$ (i.e. when the direct effect equals the indirect effect), and negative when $x^k > \mu^k$ (i.e. when the indirect effect dominates the direct effect). The slope of the industry reduced form production function takes the negative impact of reduced effective space for producer *l* into account and is positive when $x^k < \mu^I$, zero when $x^k = \mu^I$, and negative when $x^k > \mu^I$. Because of the negative impact of x^k on producer *l*'s output, $\partial y^I / \partial x^k \leq \partial y^k / \partial x^k$ which means that $\partial y^k / \partial x^k \geq 0$ at the point of industry congestion (where $\partial y^I / \partial x^k = 0$), and thus $\mu^I \leq \mu^k$. If x^k is chosen such that $\mu^I < x^k \leq \mu^k$, there is congestion at the industry level but not at the firm level (**Observation 1**). Note that this result is contingent on effective space not being exhausted (i.e. that a point of congestion occurs) and that x^k influences producer l's output negatively, i.e. $\left(\partial f^l / \partial g\right)\left(\partial g / \partial x^k\right) < 0$.

According to equation 7, profit maximization is only consistent with employment of x^k such that $\partial y^k / \partial x^k \geq 0$, or stated differently, with $x^k \leq \mu^k$. If $\mu^I < x^{k*} \leq \mu^k$ is the profit maximizing input bundle, then profit maximization leads to congestion at the industry level, although congestion does not occur at the firm level (**Observation 2**). This case can be illustrated by a simple numerical example. Let the relative price, *w*/*p*, be equal to 2. If both firms are profit maximizers, then - in accordance with equation 7 - the marginal products $\partial y^k / \partial x^k$, $k = (1, 2)$, equal 2. Assume for example that:

$$\frac{\partial y^1}{\partial x^1} = \frac{\partial f^1}{\partial x^1} + \frac{\partial f^1}{\partial g}\frac{\partial g}{\partial x^1} = 5 + (-3) = 2$$
$$\frac{\partial y^2}{\partial x^2} = \frac{\partial f^2}{\partial x^2} + \frac{\partial f^2}{\partial g}\frac{\partial g}{\partial x^2} = 6 + (-4) = 2 \qquad (10)$$

Applying equation 9 and axiom (iii), I derive the corresponding marginal products at the industry level:

$$\frac{\partial y^I}{\partial x^1} = \frac{\partial y^1}{\partial x^1} + \frac{\partial f^2}{\partial g}\frac{\partial g}{\partial x^1} = \frac{\partial y^1}{\partial x^1} + \frac{\partial f^2}{\partial g}\frac{\partial g}{\partial x^2} = 2 + (-4) = -2$$
$$\frac{\partial y^I}{\partial x^2} = \frac{\partial y^2}{\partial x^2} + \frac{\partial f^1}{\partial g}\frac{\partial g}{\partial x^2} = \frac{\partial y^2}{\partial x^2} + \frac{\partial f^1}{\partial g}\frac{\partial g}{\partial x^1} = 2 + (-3) = -1 \qquad (11)$$

For the selected values, input congestion takes place at the industry level but not at the firm level.

4. **Summary and conclusions**

This paper has introduced the concept of effective space and related it to input congestion. In contrast to other research on input congestion, I consider congestion at the industry level *in addition* to congestion at the firm level. My results show that profit maximization is inconsistent with congestion at the firm level, but that it does not rule out congestion at the industry level when effective space is shared among producers. Clearly, these results relate input congestion to the *tragedy of the commons* [see Hardin (1968)], a widely accepted explanation for overexploitation of resources that are commonly held. Intuitively, my model suggests that policy instruments which increase the costs of inputs that contribute to degradation of effective space so that their factor prices reflect external costs are optimal for dealing with the *tragedy*. For example, an optimal tax $t^* = -p\left(\partial f^l / \partial g\right)\left(\partial g / \partial x^k\right)$ levied on

x^k secures that the first order condition for firm k's profit maximization problem (equation 7) coincides with the corresponding first order condition for profit maximum at the industry level.

The relationship between firm efficiency and industry efficiency has recently been addressed; see e.g. Färe and Zelenyuk (2003) and Kuosmanen et al. (2013; 2010). The main results in the aggregation literature are in line with my results by showing that "*coordination and efficient allocation of resources across individual firms is of critical importance for the efficiency of an aggregate entity* (e.g. the industry),.., *but* (that) *the coordination does not play a role at the firm level*" (Kuosmanen et al., 2013, p. 1569). Using Cobb-Douglas technologies, Kuosmanen et al. (2013; 2010) provide numerical examples which illustrate that inefficiency may prevail at the industry level in cases where all firms in the industry operate efficiently at the firm level. My paper complements their findings by providing a parallel result for input congestion.

Several studies have considered industry efficiency using DEA [e.g. Kuosmanen et al. (2013; 2010)]. There are to my knowledge no similar treatments on input congestion in the DEA literature, which has primarily been concerned with input congestion at the firm level. This paper illustrates that input congestion at the industry level is likely to be more economically important (and sound) than input congestion at the firm level. Hence, future research on input congestion in DEA should consider the possibility to extend the scope of the DEA aggregation literature to take input congestion into account.

5. Acknowledgement

The author thanks Eirik Romstad and one anonymous referee for their valuable comments. The usual disclaimer applies.

6. References

Briec, W., Kerstens, K., 2006. S-disposability assumption and duality between technology and cost function. Lille Economie & Management.

Chambers, R.G., 1988. Applied production analysis: a dual approach. Cambridge University Press, Cambridge.

Cherchye, L., Kuosmanen, T., Post, T., 2001. Alternative treatments of congestion in DEA: A rejoinder to Cooper, Gu, and Li. Eur J Oper Res 132, 75-80.

Färe, R., Svensson, L., 1980. Congestion of production factors. Econometrica 48, 1745-1753.

Färe, R., Zelenyuk, V., 2003. On aggregate Farrell efficiencies. Eur J Oper Res 146, 615-620.

Hardin, G., 1968. The tragedy of the commons. Science 162, 1243-1248.

Inada, K.-I., 1963. On a two-sector model of economic growth: comments and a generalization. Rev Econ Stud 30, 119-127.

Kuosmanen, N., Kuosmanen, T., Sipiläinen, T., 2013. Consistent aggregation of generalized sustainable values from the firm level to sectoral, regional or industry levels. Sustainability 2013, 1568-1576.

Kuosmanen, T., Kortelainen, M., Sipiläinen, T., Cherchye, L., 2010. Firm and industry level profit efficiency analysis using absolute and uniform shadow prices. Eur J Oper Res 202, 584-594.

Murty, S., Robert Russell, R., Levkoff, S.B., 2012. On modeling pollution-generating technologies. J Environ Econ Manage 64, 117-135.

Shephard, R.W., 1970. Proof of the law of diminishing returns. Z Nationalokonomie 30, 7-34.