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# Assessing Smoothing Effects of Wind Power around Trondheim via Koopman Mode Decomposition

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**Abstract.** To cope with a large-scale introduction of renewables into power systems, it is important to understand the reduction of variability in the aggregated generation or the so-called smoothing effect. Knowledge of the degree of smoothing is used for assessing the potential impact of intermittent generation on the power system operation. Here, smoothing effects of aggregated wind power are assessed around Trondheim, Norway, by applying a recently proposed smoothing index based on the so-called Koopman Mode Decomposition (KMD). The method is shown to effectively decompose complex time-series of wind power outputs into a finite number of modes, each of which oscillates with a single frequency for all locations (or hypothetical wind farms). It is shown that the method is able to reconstruct the original power outputs well by only a small number of modes that retain the variability of the original time-series, and is able to provide a relevant quantification of the smoothing effects for each individual frequency (or mode).

## 1. Introduction

To cope with a large-scale introduction of renewables into power systems, it is important to understand the reduction of variability in the aggregated generation or the so-called *smoothing effect*. Knowledge of the degree of smoothing is used for assessing the potential impact of intermittent generation on the power system operation. There is a long history of research on the smoothing effects: see literature review in [1] and references therein. The work presented here aims to assess smoothing effects of aggregated wind power around Trondheim, Norway, using hourly-measured data on wind speed.

This assessment is conducted by applying a recently-proposed smoothing index [1] based on the so-called Koopman Mode Decomposition (KMD). KMD is a novel concept of nonlinear time-series analysis based on spectral properties of the Koopman operator for nonlinear dynamical systems [2, 3, 4, 5]. A finite truncation of KMD, which is often called Dynamic Mode Decomposition, is shown to effectively decompose time-series of wind power outputs into a finite number of modes, each of which oscillates with a single frequency for hypothetical Wind-Farm (WF) sites. It is shown that KMD is able to reconstruct the original power outputs well by only a small number of modes that retain the variability of the original time-series, and is able to provide a relevant quantification of the smoothing effects for each individual frequency (or mode), that is, with clear time-scale separation.



In this proceeding, we report a quantification result on the smoothing effects of wind power for three different combinations of locations for the three hypothetical WF sites around Trondheim, Norway. The data on wind power are derived by converting 92-day long time-series of hourly wind speed data with the standard power curve. By applying the KMD-based smoothing index to the data on wind power, it is clarified how the geographical choice of locations as well as frequencies influences the smoothing effect of wind power.

## 2. Quantifying Smoothing Effects via Koopman Mode Decomposition [1]

Consider  $N + 1$  vector-valued temporal snapshots of wind power, which are measured or simulated, in the per unit (p.u.) system (that is, power/(rated power)) collected at  $m$  locations:  $\{\mathbf{P}_0, \dots, \mathbf{P}_N\}$ ,  $\mathbf{P}_k \in \mathbb{R}^m$ , where  $k$  denotes a specific time-instance. The sampled data are then decomposed into a finite sum via KMD:

$$\left. \begin{aligned} \mathbf{P}_k &= \sum_{i=1}^N \tilde{\lambda}_i^k \tilde{\mathbf{v}}_i, \quad k = 0, \dots, N-1, \\ \mathbf{P}_N &= \sum_{i=1}^N \tilde{\lambda}_i^N \tilde{\mathbf{v}}_i + \mathbf{r}. \end{aligned} \right\} \quad (1)$$

Above, they are computed via the Arnoldi-type algorithm [2] that outputs  $N$  pairs of the so-called Ritz-values  $\tilde{\lambda}_i \in \mathbb{C}$  and Ritz-vectors  $\tilde{\mathbf{v}}_i \in \mathbb{C}^m$ . The vector  $\mathbf{r}$  is called the residual in [2], and if it is assumed to be *zero*, (1) becomes

$$\mathbf{P}_k = \sum_{i=1}^N \tilde{\lambda}_i^k \tilde{\mathbf{v}}_i, \quad k = 0, \dots, N. \quad (2)$$

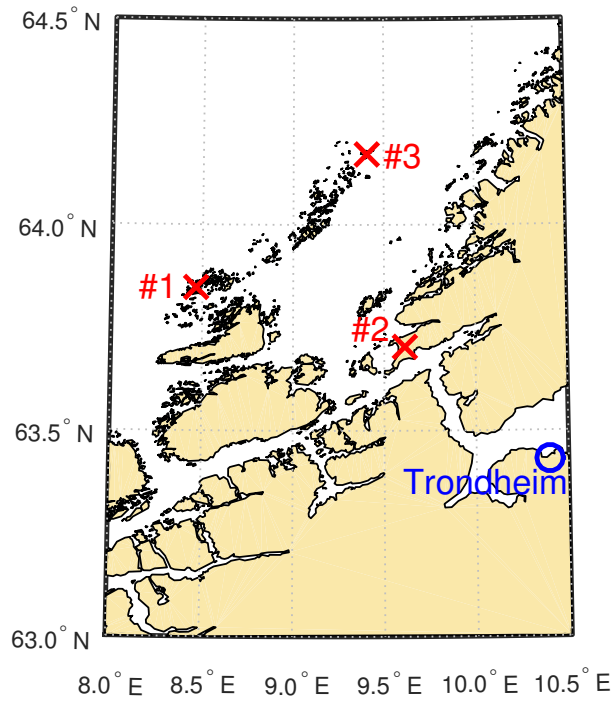
Modal frequencies are calculated according to  $f_i = \text{Im}(\ln(\tilde{\lambda}_i))/(2\pi T_s)$ , where  $T_s$  is the sampling period. The vector  $\tilde{\mathbf{v}}_i = \mathbf{A}_i \angle \boldsymbol{\alpha}_i := [A_{i1} \angle \alpha_{i1}, A_{i2} \angle \alpha_{i2}, \dots, A_{im} \angle \alpha_{im}]^\top$  ( $\top$  denotes the transpose operation) is here called the *Koopman Mode* (KM) oscillating with the frequency  $f_i$  and contains the magnitudes (moduluses)  $A_{ij}$  and phases (arguments)  $\alpha_{ij}$  of power fluctuations corresponding to the  $m$  measurements; e.g. outputs of  $m$  WFs. To identify lightly damped or undamped oscillations with large magnitude, all  $N$  KMs are sorted by  $(\tilde{\lambda}_i)^N \|\tilde{\mathbf{v}}_i\|$ , where  $\|\cdot\|$  denotes the euclidean norm, and higher ranked ones are called *dominant* KMs, which are here used to evaluate smoothing effects.

KMD will be applied to wind powers at  $m$  locations that represent hypothetical WFs. The total (aggregated) power  $P_{\text{tot},k}$  can be expressed using (2) as

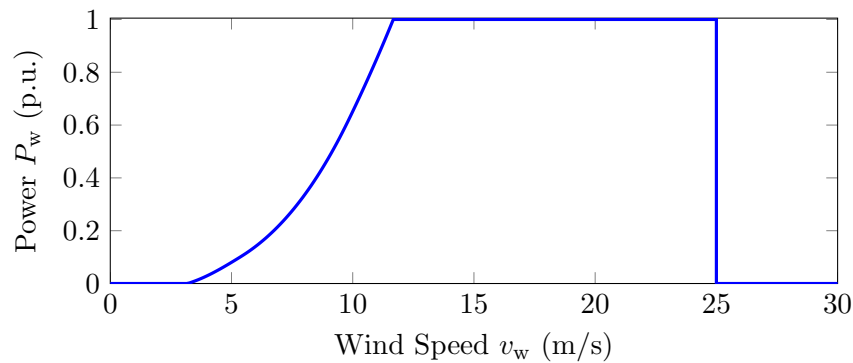
$$P_{\text{tot},k} = \mathbf{P}_k^\top \mathbf{1}_m = \sum_{i=1}^N \tilde{\lambda}_i^k \sum_{j=1}^m [\tilde{\mathbf{v}}_i]_j =: \sum_{i=1}^N \tilde{\lambda}_i^k \bar{v}_i, \quad (3)$$

where  $\mathbf{P}_k \in \mathbb{R}^m$  contains the measured powers at time  $k$ ,  $\mathbf{1}_m$  is the  $m$ -length vector of ones,  $[\tilde{\mathbf{v}}_i]_j$  is the  $j$ -th component of  $\tilde{\mathbf{v}}_i$ , and  $\bar{v}_i \in \mathbb{C}$  is the scalar KM of the total power. That is, a spectral decomposition of the total power is achieved by applying KMD to individual outputs. The following index is used here to quantify the smoothing effects:

$$c_i = \frac{1}{m(m-1)} \sum_{j=1}^m \sum_{\substack{l=1 \\ l \neq j}}^m \hat{A}_{ij} \hat{A}_{il} \cos(\Phi_{jl}^i), \quad (4)$$



**Figure 1.** Approximate locations of hypothetical wind farms around Trondheim indicated by the red crosses

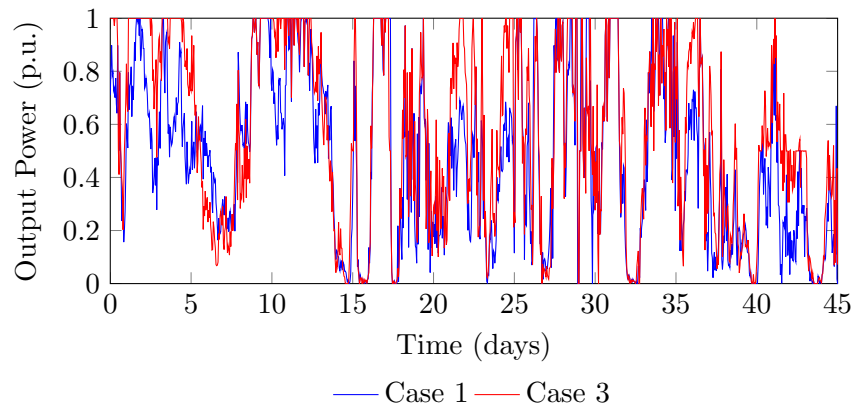


**Figure 2.** Power curve utilized in this work

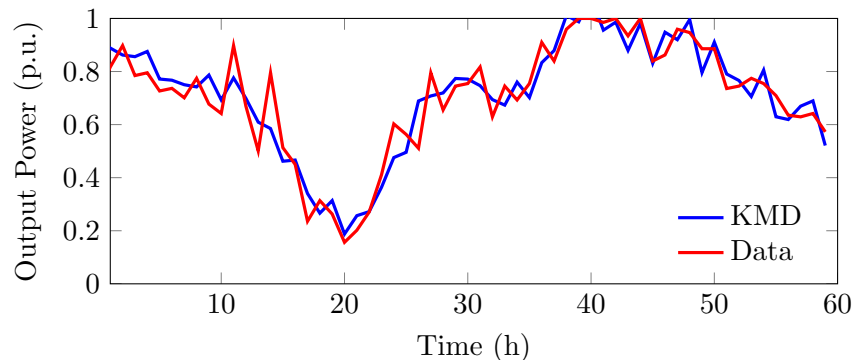
where  $\Phi_{jl}^i := \alpha_{ij} - \alpha_{il}$ , and  $\hat{A}_{ij} := A_{ij}/\max(\mathbf{A}_i)$ , i.e. components of  $\mathbf{A}_i$  normalized by the largest component. The subscript  $i$  refers to the  $i$ -th *dominant* KM oscillating with the frequency  $f_i$ . We call this the *averaged smoothing index via KMD*, and it can be regarded as a generalization of a previously proposed index [6] which was shown to be effective for quantifying smoothing effects in Japan.

### 3. Smoothing Assessment around Trondheim

The considered hypothetical WF sites around Trondheim are shown in Figure 1. The distances between #1–#2, #1–#3, and #2–#3 are about 58 km, 58 km, and 53 km, respectively. Here, 92-day long time-series of hourly wind speeds are used, and converted to wind power via the power curve depicted in Figure 2. The coefficient necessary for the power curve is determined



**Figure 3.** Examples of the time-series of summed wind powers for Cases 1 and 3



**Figure 4.** Examples of the summed wind power and reconstructed time-series by KMD

**Table 1.** Variances of total powers  $P$  and of reconstructed time-series via KMD  $\tilde{P}$

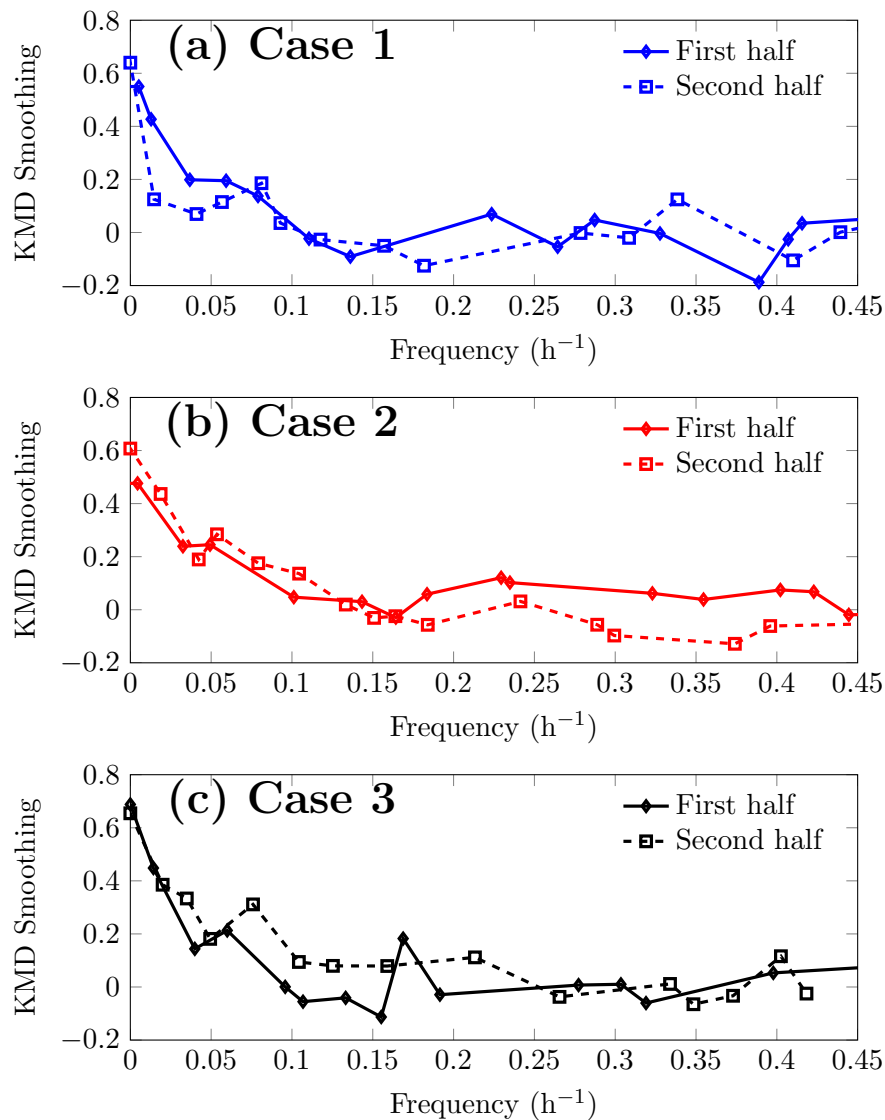
Case	$P_{\text{tot,first}}$	$P_{\text{tot,last}}$	$\tilde{P}_{\text{tot,first}}$	$\tilde{P}_{\text{tot,last}}$
Case 1	0.10	0.08	0.12	0.06
Case 2	0.11	0.09	0.12	0.08
Case 3	0.12	0.10	0.12	0.09

with theory in [7], and the equations used here are given in [1]. We consider the following three different combinations of locations in the case study on smoothing effects:

- Case 1: Locations #1 and #2
- Case 2: Locations #1 and #3
- Case 3: Locations #2 and #3

The mean power outputs of the full 90-day period are 0.415, 0.500, and 0.508 p.u. In the following, we look at half of the intervals separately, denoted by ‘first’ and ‘last’, where each one corresponds to a 46-day long time-series. Examples of the time-series of the normalized aggregated outputs for Case 1 and 3 are given in Figure 3 for a 45-day period (almost whole the first half of the time-series), where the slightly higher output is derived for Case 3.

KMD is now applied to the wind power time-series, and a 60-h interval is shown in Figure 4 together with the reconstructed output by KMD (2) with about 15 complex-conjugate oscillatory



**Figure 5.** Results on smoothing index based on KMD

KM pairs. The variances of the reconstructed time-series with KMD  $\tilde{P}$  and the original time-series of powers  $P$  are given in Table 1. The results show that the variability is retained in the spectral decomposition, and that the variability is similar or slightly less for the second half of the time-series.

The smoothing results of applying the index to powers for all cases are shown in Figure 5. A lower value of the index indicates lower average coherence, implying more smoothing. The areas under the curves quantify the achieved smoothing, and the smoothing results are integrated and given in Table 2. For example, for the second half of the time-series, the results indicate that ‘Case 1’ has more favorable smoothing than ‘Case 3.’ This is in agreement with the variances of the total powers for the different cases in Table 1, where the total power for ‘Case 1’ has a lower variance than ‘Case 3.’

**Table 2.** Results of integrating the curves in Fig. 5.

	Case 1	Case 2	Case 3
First half	0.012	0.041	0.022
Second half	0.006	0.017	0.043

#### 4. Conclusion

In this paper we conducted the assessment of smoothing effects of wind power around Trondheim by applying the KMD (Koopman Mode Decomposition). The assessment was based on time-series data on wind power generated from measured data on wind speeds around Trondheim. This shows that KMD is able to reconstruct the original power outputs well by only a small number of modes and to provide a relevant data-driven quantification of the smoothing effects for each individual frequency of mode.

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#### References

- [1] Raak F, Susuki Y, Tsuboki K, Kato M and Hikihara T 2017 *Nonlinear Theory and Its Applications, IEICE* **8** 342–357
- [2] Rowley C W, Mezić I, Bagheri S, Schlatter P and Henningson D S 2009 *Journal of Fluid Mechanics* **641** 115–127
- [3] Budišić M, Mohr R and Mezić I 2012 *Chaos: An Interdisciplinary Journal of Nonlinear Science* **22** 047510
- [4] Mezić I 2013 *Annual Review of Fluid Mechanics* **45** 357–378
- [5] Susuki Y, Mezić I, Raak F and Hikihara T 2016 *Nonlinear Theory and Its Applications, IEICE* **7** 430–459
- [6] Nanahara T, Asari M, Sato T, Yamaguchi K, Shibata M and Maejima T 2004 *Wind Energy* **7** 75–85
- [7] Ackermann T 2012 *Wind Power in Power Systems* (John Wiley & Sons, Ltd.)