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Market Power in Hydro-Thermal Systems with Marginal Cost Bidding

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***Abstract*—Traditionally, electricity markets have been designed with the intention of disabling producer side market power or prohibiting exercising it. Nonetheless it can be assumed that players participating in pool markets and aiming to maximize their individual benefits might depart from the optimum in terms of total system welfare. To recognize and analyze such behavior, system operators have a wide range of methods available. In the here presented paper, one of those methods - deriving a supply function equilibrium - is used and nested in a traditional discontinuous Nash game. The result is a case study that shows that marginal cost bidding thermal producers have an incentive to collaborate on scheduling in order to cause similar effects to tacit collusion.**

***Index Terms*—hydro power, thermal power, market power, nash equilibrium**

I. Introduction

Literature shows various examples in which supply functions were applied to analyze market power in electricity markets. [1] models the effects of consumer pricing schemes on market power in electricity spot markets, concluding that variable end consumer billing has collusion-reducing effects compared to fixed rates. [2] use supply functions individual to each participant to cope with the downsides of Cournot competition, namely impossibility of demand curves without any elasticity and the resulting demand distortion for extreme price scenarios. [3] nests the supply function equilibrium in a *prisoners game*, aiming to explain price spikes created through collusion. It concludes that suppliers have incentives to withhold and selectively place supply in order to increase profits. [4] considers individual welfare maximization as well as an aggregated supply function in order to find market power in an optimal power flow setup and uses a similar stepwise convergence algorithm as [5] to derive a *Nash equilibrium*. [6] establishes affine supply functions that are turned piece-wise by incorporation of capacity constraints in order to analyze various games with different generation unit and firm(i.e. player) setups. The approach presented in the following sections will introduce the coupling of time periods as well as active scheduling decisions to a supply function market clearing. No existing literature on combining those concepts in a single model is known to the authors.

II. Supply Function Market Clearing Model

Considering a pool market, every clearing period t participating generation units i will submit bids consisting of both price $b^p_{i,t}[\$]$ and quantity $b^q_{i,t}[MW/t]$. The price curves symbolize the *individual supply function* for each generation unit. After receiving the bids, a market operator here assumed to be clearing for a *uniform price* in a *pool market* will create a supply curve, denoting the period supply curve as a $S_t(d_t)[\$]$, a function depending on the periods' demand d_t:

$$\begin{aligned} S_t(d_t) = \quad & \min_{q_{i,t}} \sum_i b^p_{i,t}(q_{i,t}) q_{i,t} \\ \text{s.t.} \quad & q_{i,t} \leq b^q_{i,t} \qquad \forall i \\ & d_t = \sum_i q_{i,t} \\ & q_{i,t} \in \mathbb{R}^+ \qquad \forall i \end{aligned} \tag{1}$$

As long as the objective function is convex and the quantity bids $b^q_{i,t}$ are enough to fulfill the demand clearing (i.e. $\sum_i b^q_{i,t} \geq d_t \forall t$) constraint, this *market supply curve* will yield a finite global result. Assuming quadratic cost functions $C_i(q_{i,t})$ and generators bidding at *marginal cost* ($MC_i(q_{i,t})$) level yields the following formulation for the price bidding function:

$$\begin{aligned} C_i(q_{i,t}) &= a_i + b_i q_{i,t} + c_i q^2_{i,t} \\ b^p_{i,t}(q_{i,t}) = MC_i(q_{i,t}) &= \frac{\partial C_i(q_{i,t})}{\partial q_{i,t}} = b_i + 2c_i q_{i,t} \end{aligned} \tag{2}$$

These price bids as well as the size of the offered quantity are assumed to be a result of an interal optimization process of each player j which owns a number of generation units. For simplicity's sake we set the number of potential bids per player equal to the number of generation units $i \in j$. It will also be assumed, that these bidding curves $b^q_{i,t}(q_{i,t})$ are predefined and will not be altered depending on factors such as amount of players and will stay time-consistent disregarding external influences such as the fuel prices. As the analyzed time frame will be of short term, this approximation can be considered valid.

Furthermore, affine demand functions are assumed:

$$D_t(d_t) = \alpha_t - \beta_t d_t \tag{3}$$

Generally, demand in electricity markets offers low elasticity, requiring this curve to be sufficiently steep.

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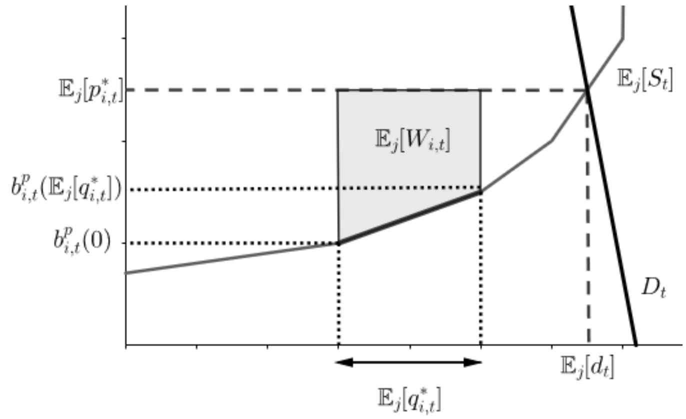

Fig. 1. Generator Surplus for Unit $i \in I_j$

The market clearing price p^*_t can be formulated as the intersection of demand and supply curves at a clearing quantity d^*_t:

$$p^*_t = D_t(d^*_t) = S_t(d^*_t) \tag{4}$$

As the here presented model follows a *pool based auction with uniform pricing*, every player will have its supplied quantity $\sum_{i\in j} q_{i,t}$ remunerated at this market clearing price.

III. Hydro-Thermal Model

Storage technology allows binding in the dimension of time, giving storage facility operators(in the here presented case: hydro power plants $i \in I^{\text{hy}}$) the chance to transfer quantity $r_{i,t}$ from a time stage t to the next. In addition, due to maximum uptimes and cooling down periods, thermal plants can be considered to not operate during the entire duration $t = 1, ..., T$. In a stateless 1-period model scheduling decisions would be implemented through altering the pool of generation units $i \in I^{\text{th}}$. In the case of storage however, time periods are connected and thus scheduling variables $s^{n_i}_{i,t}$ that determine the on/off-states ($b^q_{i,t}$/0) of the thermal plants are required to ensure fluctuating player pools over the time frame. This results in the following formulation for the *hydro-thermal supply function*:

$$\begin{aligned} S_t(d_t, r_{i,t}, s^{n_i}_{i,t}) = \min_{q_{i,t}} & \sum_i b^p_{i,t}(q_{i,t})q_{i,t} \\ \text{s.t. } q_{i,t} &\leq b^q_{i,t} + r_{i,t-1} - r_{i,t} && \forall i \in I^{\text{hy}} \\ q_{i,t} &\leq s^{n_i}_{i,t} && \forall i \in I^{\text{th}} \\ d_t &= \sum_i q_{i,t} \\ q_{i,t} &\in \mathbb{R}^+ && \forall i \end{aligned} \tag{5}$$

A player j would aim to maximize its supply side surplus W_i as seen in figure 1 for all of its units $i \in I_j$. This requires initial assumptions on market clearing price ($\mathbb{E}_j[p^*_t]$) and on procured quantity ($\mathbb{E}_j[q^*_{i,t}]$). Every producer would then alter the stored hydropower inventory $r_{i,t}[MW/t]$ indirectly (through adjusting generation) for all of its storage units in addition to choosing a production schedule $[s^{n_i}_{i,1}, ..., s^{n_i}_{i,T}]$ for its thermal units. It can be considered that the player defines a limited set of predefined schedules s which leads to a reformulation in the form of:

$$s = \begin{bmatrix} \begin{bmatrix} [s^1_{1,1}, ..., s^1_{1,T}] \\ ... \\ [s^1_{I^{\text{th}},1}, ..., s^1_{I^{\text{th}},T}] \end{bmatrix} \\ ... \\ \begin{bmatrix} [s^{N_i}_{1,1}, ..., s^{N_i}_{1,T}] \\ ... \\ [s^{N_i}_{I^{\text{th}},1}, ..., s^{N_i}_{I^{\text{th}},T}] \end{bmatrix} \end{bmatrix} \tag{6}$$
$$s^{n_i}_{i,t} = \{0, b^q_{i,t}\} \qquad \forall i \in I^{\text{th}}, t$$

Scheduling of thermal units is thereby conducted through making index n_i, which refers to the prospective schedule of generation unit i, a decision variable and thus reformulating the players' previous decision problem over the whole time stage as:

$$\begin{aligned} W_j &= \max_{r_{i,t}, n_i} \sum_t \sum_{i\in j} \mathbb{E}_j[W_{i,t}] \\ \mathbb{E}_j[W_{i,t}] &= \begin{matrix} \left(\mathbb{E}_j[p^*_t] - b^p_{i,t}(\mathbb{E}_j[q^*_{i,t}])\right) \cdot \mathbb{E}_j[q^*_{i,t}] \\ + \frac{b^p_{i,t}(\mathbb{E}_j[q^*_{i,t}]) - b^p_{i,t}(0)}{2} \cdot \mathbb{E}_j[q^*_{i,t}] \end{matrix} && \forall i \in I_j, t \\ \text{s.t. } \; S_t(d_t, r_{i,t}, s^{n_i}_{i,t}) &= S_t(d^*_t, r_{i,t}, s^{n_i}_{i,t}) && \forall t \\ r_{i,t} &\leq \bar{r}_i && \forall i \in \{I^{\text{hy}} \cap I_j\}, t \\ r_{i,t} &\in \mathbb{R}^+ && \forall i \in \{I^{\text{hy}} \cap I_j\}, t \\ n_i &\in \mathbb{Z}^+ && \forall i \in \{I^{\text{th}} \cap I_j\}, t \end{aligned} \tag{7}$$

Reservoir storage from one period to the next has an imposed upper capacity of $\bar{r}_i[MW]$.

Defining the set of strategies that a player chooses depending on the assumption on other players strategies as $\langle r_{i,t}, n_i \forall i \in I_j; \mathbb{E}[r_{i,t}], \mathbb{E}[n_i] \forall i \notin I_j \rangle$, the resulting *Nash-Equilibrium* can be formulated as:

$$W_j = \max_{\langle r^*_{i,t}, n^*_i \forall i \in I_{j_2}; \mathbb{E}[r_{i,t}], \mathbb{E}[n_i] \forall i \notin I_{j_2} \rangle} W_{j_2} \quad \forall j, j_2 \neq j \tag{8}$$

In words, a Nash equilibrium is reached if no player has an incentive to store more/less water or change the schedule for the thermal units.

The assumption of complete and symmetric information gives the possibility to solve for this equilibrium:

$$\begin{aligned} p^*_t &= \mathbb{E}_j[p^*_t] = \mathbb{E}_{j_2}[p^*_t] && \forall j, j_2 \neq j \\ q^*_{i,t} &= \mathbb{E}_j[q^*_{i,t}] = \mathbb{E}_{j_2}[q^*_{i,t}] && \forall j, j_2 \neq j \end{aligned} \tag{9}$$

Symmetric information in this case refers to the players having knowledge of each others individual supply functions/bidding curves. As mentioned above, marginal cost bidding was assumed and due to the availability of a large range of historical data within power markets, player knowledge about competitors' marginal cost functions can be considered valid.

Assuming a fixed schedule for all thermal units allows this welfare game to be solved for its equilibrium point. Market clearing condition (4) presents a sub-problem in the welfare-maximization of every player. Taking advantage of the fact that every producer has an incentive to shift generation from time step $t-1$ to t as long as both the capacity constraint for maximum reservoir inventory is not breached and there are generator side surplus gains to be made for this producer,

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time stage transfer of inventory (i.e. arbitrage) will happen. This can be solved by selecting an adequately small step size r^{step} and applying a convergence algorithm:

0) `initialize` $r^{\text{original}} = \begin{bmatrix}[0,..,0]\\ ...\\ [0,..,0]\end{bmatrix}$
1) `calculate` (4) $\forall t$ `to establish sorted set` $\tau = \{\tau_1, ..., \tau_T | p^*_{\tau_1} \geq ... \geq p^*_{\tau_T}\}$ `where` $\tau_t \in \{1, ..., T\}$ `and` $\tau_1 \neq ... \neq \tau_T$
2) `remove` $\max(\tau_t \in \tau)$
3) `if last element in` τ `is equal to` $\max(\tau)$ `:` `remove it and` **`back to 3)`**
4) `if` $\tau = \{\emptyset\}$ `:` **`finished`** `- i.e. converged to equilibrium as shown in` (8)
5) `select first element` τ_t `from` τ
6) `solve objective function of` (7) `to receive:` W_j^{original} `for` r^{original} `and` W_j^{new} `for`
$$r_j^{\text{new}} = \begin{bmatrix} r_{i,t}^{\text{original}} & \forall i \in \{I^{\text{hy}} \cap I_j\}, t \neq \tau_t, \\ r_{i,t}^{\text{original}} & \forall i \in \{I^{\text{hy}} \cup I_j\}, t \neq \tau_t, \\ \min\{r_{i,t}^{\text{original}} + r^{\text{step}}, \bar{r_i}\} & \forall i \in \{I^{\text{hy}} \cup I_j\}, t = \tau_t \end{bmatrix}$$
7) `for each` j `where` $W_j^{\text{new}} > W_j^{\text{original}}$ `:` `set` $r_{i,t}^{\text{original}} := r_{j,i,t}^{\text{new}} \forall i \in \{I^{\text{hy}} \cup I_j\}$ `if` r^{original} `is unchanged: remove` τ_t `from` τ `and` **`back to 4)`**
8) `and` **`back to 1)`**

The algorithm increases the held inventory continuously by step size r^{step} until no player can increase their individual welfare by increasing the inventory, which corresponds with the definition of a Nash equilibrium. Similar convergence algorithms were used to derive equilibrium points in bidding problems for power-flow based problems [4] and within hydro-storage convergence algorithms [5]. As the here presented algorithm solves the market clearing problem via supply curve matching, the convergence algorithm is solely concerned with matching the held inventory.

As mentioned before, this can be only conducted by assuming fixed n_i for all players. Thus, the algorithm has to be brute-forced for all possible iterations that the schedules allow, in other words for all combinations of $s_i^{n_i}$ that s allows.

IV. Case Study

The presented cases will analyze the game setup shown in table I: six generation units owned by three players (one hydro player, one thermal player, one mixed) compete for a (nearly) inelastic market demand ranging between 16 to $20MW/t$ depending on the period. Such demand curve shifts can stem from a variety of factors such as fluctuating base-load caused by renewable generation or consumer behavior.

Case 1, as shown in table II proposes the base case with no available storage capacity and all thermal units running continuously in every period.

The results found in figure 2 show the impact of a lack of period price balancing effects that holding inventory provides: producers owning storage units would have an incentive to increase their surplus by shifting generation into successive periods and thus skimming the price peaks. As there is no storage capacity, high deviations in price are a result of the demand changes.

Case 2 assumes the possibility of schedule changes - the two thermal units $i = 4, 5$ might be shut down in period 3 by choosing another schedule (i.e. setting $n_i = 2$ respectively). This can result in 4 different states, depending on which schedule is chosen by which player.

Those states are shown in form of a decision matrix in table III. This shows that individually, player $j = 2$ has no incentive to shut unit $i = 4$ down, combined with the thermal

TABLE I
GAME SETUP

Generator j	i	type	$b^q_{i,t} \forall t$	$b^p_{i,t}(q_{i,t}) \forall t$
1	1	hydro	$4MW/t$	$0 + 0 \cdot 2 \cdot q_{i,t}$
	2	hydro	$4MW/t$	$0 + 0 \cdot 2 \cdot q_{i,t}$
2	3	hydro	$4MW/t$	$0 + 0 \cdot 2 \cdot q_{i,t}$
	4	thermal	$5MW/t$	$1 + 0.02 \cdot 2 \cdot q_{i,t}$
3	5	thermal	$5MW/t$	$1.5 + 0.02 \cdot 2 \cdot q_{i,t}$
	6	thermal	$8MW/t$	$3 + 0.02 \cdot 2 \cdot q_{i,t}$

period t	Customer Demand
1	$D_t(d_t) = 150000 - 10000 \cdot d_t$
2	$D_t(d_t) = 160000 - 10000 \cdot d_t$
3	$D_t(d_t) = 180000 - 10000 \cdot d_t$
4	$D_t(d_t) = 170000 - 10000 \cdot d_t$

TABLE II
IMPLEMENTED CASES

case #1	$\bar{r_1} = 0$	$s_4 = [[1,1,1,1]]$
	$\bar{r_2} = 0$	$s_5 = [[1,1,1,1]]$
	$\bar{r_3} = 0$	$s_6 = [[1,1,1,1]]$
case #2	$\bar{r_1} = 0$	$s_4 = \begin{bmatrix} s_4^1 = [1,1,1,1] \\ s_4^2 = [1,1,\mathbf{0},\mathbf{0}] \end{bmatrix}$
	$\bar{r_2} = 0$	$s_5 = \begin{bmatrix} s_5^1 = [1,1,1,1] \\ s_5^2 = [1,1,\mathbf{0},\mathbf{0}] \end{bmatrix}$
	$\bar{r_3} = 0$	$s_6 = [[1,1,1,1]]$
case #3	$\bar{r_1} = 1.5$	$s_4 = \begin{bmatrix} s_4^1 = [1,1,1,1] \\ s_4^2 = [1,1,\mathbf{0},\mathbf{0}] \end{bmatrix}$
	$\bar{r_2} = 1.5$	$s_5 = \begin{bmatrix} s_5^1 = [1,1,1,1] \\ s_5^2 = [1,1,\mathbf{0},\mathbf{0}] \end{bmatrix}$
	$\bar{r_3} = 1.5$	$s_6 = [[1,1,1,1]]$

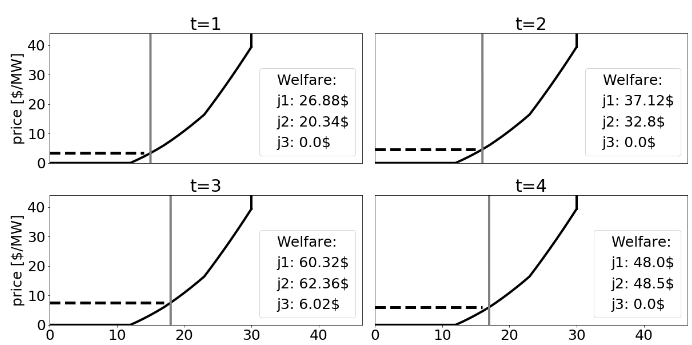

Fig. 2. Result Case 1

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TABLE III
DECISION MATRIX CASE 2

		$j=3$, $n_5=1$	$j=3$, $n_5=2$
$j=2$	$n_4=1$	$W_1=172.32$ $W_2=164.0$ $W_3=6.02$	$W_1=184.32$ $W_2=177.5$ $W_3=6.02$
	$n_4=2$	$W_1=215.52$ $W_2=128.9$ $W_3=87.42$	$W_1=347.52$ $W_2=194.9$ $W_3=162.42$

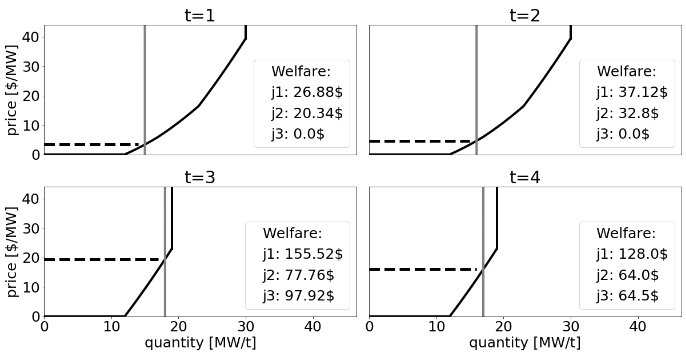

Fig. 3. Result Case 2, with $n_4^*=2, n_5^*=2$

player committing to a shutdown, both are however able to increase their surplus. Those effects are displayed in figure 3. Thus, curtailing available supply and thus increasing the price to maximize individual producer surplus can be enabled through cooperation, resulting in the equilibrium defined by $n_4^*=2, n_5^*=2$. This phenomenon referred to as *opportunistic tacit collusion* by [1] and [3] seems to be amplified by collaboration amongst colluding players. As [7] illustrates - a supply side increase in surplus would come at amplified expenses on the demand side, resulting in a loss of total welfare. Thus, avoiding such coordination is in the interest of a welfare maximizing system operator. However, the result of case 1 - $n_4^*=1, n_5^*=1$ - still fulfills the definition of the Nash equilibrium shown in (8). Thus, the standard case with no shutdowns still offers an equilibrium situation, that could similarly be achieved due to the indifference of player $j=3$ between shutdown or not in case player $j=2$ chooses no shutdown.

Case 3 removes this indifference by adding hydro storage capacity to the respective units. The resulting scheduling decision matrix is shown in figure IV. There now only

TABLE IV
DECISION MATRIX CASE 3

		$j=3$, $n_5=1$	$j=3$, $n_5=2$
$j=2$	$n_4=1$	$W_1=169.556$ $W_2=159.469$ $W_3=2.625$	$W_1=178.775$ $W_2=175.19$ $W_3=12.04$
	$n_4=2$	$W_1=208.806$ $W_2=141.872$ $W_3=62.349$	$W_1=322.882$ $W_2=209.049$ $W_3=117.546$

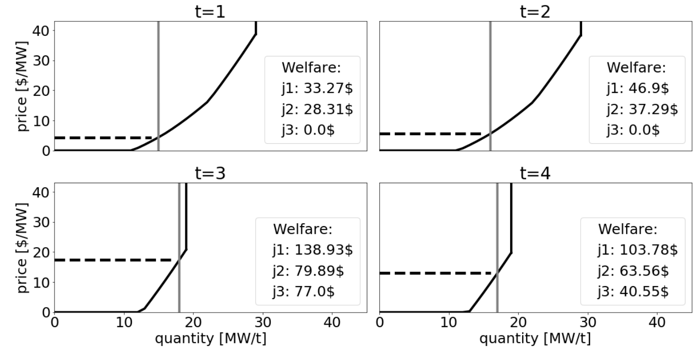

Fig. 4. Result Case 3, with $n_4^*=2, n_5^*=2$

exists a single Nash equilibrium where both players owning thermal units collaborate in order to increase supply side surplus. Total transfers from period to period accumulate to $0.9MW, 1.5MW, 1MW$ respectively. The reason for this (seemingly) low transfer is the small pool of players and the resulting high impact of transfer decisions. Players will aim to maximize their relation of $\sum_{i\in I_j} q_{i,t}^*$ to $\frac{\partial S_t(d_t)}{\partial d_t}$ (Appendix B analyzes this). In the specific case, the hydro power operator does not have a strong incentive for peak skimming as it would hurt this balance, especially since the mixed hydro-thermal operator actively shifts units (with full capacity of $1.5MW$) from period 2 into period 3, to actively create peaks. Appendix A introduces the game from case 3, all hydro power units taken from their respective firms and distributed equally amongst a larger pool of smaller firms. It clearly shows a loss of the ”strategic value” of hydro power for the mixed generator.

V. CONCLUSION

This paper presented a novel aspect of electrical systems: the possibility of exercising market power whilst bidding marginal through colluding on unit commitment. A supply function was applied on a hydro-thermal system spanning over several time periods and individual player (generation firm) surpluses were calculated. Two forms of exercise of market power were noted in thermal/hydro-thermal operators: tacit collusion through alignment of schedules of thermal units, creating price peaks through withholding low cost units; and active creation of price peaks through strategic supply shifts over periods. It has to be noted, however, that the here presented concepts of influencing the market require a large impact of single generation firms. It shows however, that monopolists are still able to bid at marginal cost, which is traditionally considered as system welfare-maximizing, and still exercise market power, resulting in welfare-losses on customer side. This however requires information on scheduling decisions of other (thermal) players. As shared knowledge of the game participants’ schedules might not be complete, future research on this topic is proposed.

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APPENDIX A

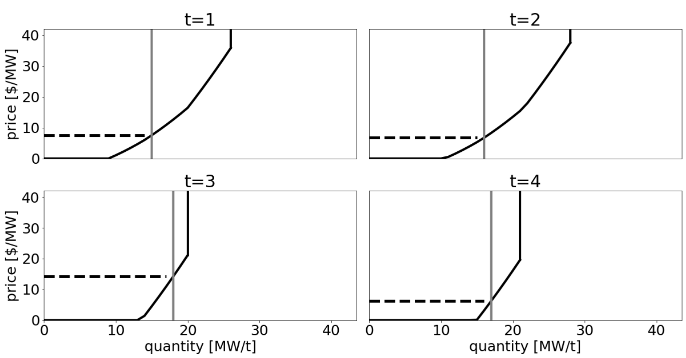

Fig. 5. Result Case 3, with 52 players

Figure 5 shows the results from figure 4 where the hydro power units are taken from the participants and split up into 50 firms with equal capacities. All of the smaller players thus inherit less market power, resulting in them bidding more competitively with reservoir transfers of $4MW, 4.5MW, 4MW$.

APPENDIX B

Assuming a hydro power producer considers shifting $r_{i,t}MW$ from period t to $t+1$, the resulting period surplus changes for unit i are:

$$\begin{aligned} \Delta W_{i,t} &= \frac{\partial S_t(b^q_{i,t}-r_{i,t}, \sum_{i_2\neq i} b^q_{i_2,t})}{\partial d_t} q_{i,t}(b^q_{i,t}-r_{i,t}) \\ \Delta W_{i,t+1} &= \frac{\partial S_{t+1}(b^q_{i,t+1}-r_{i,t+1}, \sum_{i_2\neq i} b^q_{i_2,t})}{\partial d_t} q_{i,t+1}(b^q_{i,t+1}+r_{i,t+1}) \end{aligned} \quad (10)$$

Further considering all of the transferred capacity will be acquired by the market results in:

$$\begin{aligned} \Delta W_{i,t} &= -\frac{\partial S_t(b^q_{i,t}-r_{i,t}, \sum_{i_2\neq i} b^q_{i_2,t})}{\partial d_t} r_{i,t} \\ \Delta W_{i,t+1} &= +\frac{\partial S_{t+1}(b^q_{i,t+1}-r_{i,t+1}, \sum_{i_2\neq i} b^q_{i_2,t})}{\partial d_t} r_{i,t+1} \end{aligned} \quad (11)$$

Thus, in case there is sufficient demand for the shifted capacity, only the resulting slopes of the demand functions in relation to the chosen step size will define if a period shift is conducted. Thus, players generally do not have an incentive to shift capacity if the steepness in period $t+1$ does not outweight period t.