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Assessing Hydropower Operational Profitability Considering Energy and Reserve Markets

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Abstract: We analyze the operational profitability of a hydropower system selling both energy and reserve capacity in a competitive market setting. A mathematical model based on stochastic dynamic programming is used to compute the water values for the system considering different power plant configurations. The uncertainties in inflow and both energy and reserve capacity prices are considered through a discrete Markov chain. Subsequently, the system operation is simulated based on the obtained water values to assess system performance and expected revenues from the two markets.
The model is applied in a case study for a Norwegian hydropower producer, showing how the power plant operation changes and profitability increases when considering sale of reserve capacity. We emphasize on how the water values are influenced by the opportunity to sell reserve capacity, and assess how the representation of non-convex relationships in the water value computations as well as simulation influence the profitability.

1 Introduction

The increasing share of non-dispatchable renewable energy resources in Europe calls for efficient and reliable arrangements for balancing services. For large-scale energy storage operators, such as hydropower producers, this development may change the market products typically being delivered. Today, the producers' primary source of remuneration is from selling energy in the day-ahead market. However, the flexibility of hydropower enables active contribution in reserve capacity and balancing markets as well, which in turn will challenge the way hydropower reservoirs traditionally have been operated.

For a hydropower producer, the expected marginal value of water, hereafter referred to as the *water value*, express the opportunity cost of water. Having the possibility to utilize the facilities for both energy generation and reserve capacity may impact the strategic evaluation and scheduling of resources. These products will to some extent rely on the same resource (water), but their resource deployment will differ. Consequently, the consideration of both products will impact the water values, and in turn the expected operational profitability of a hydropower project. The term *operational profitability* is used throughout the text to emphasize that we do not consider investment costs for the project, only profitability from operation.

The early works on operational planning (scheduling) of hydropower reservoirs used the principles of stochastic dynamic programming (SDP) [1–3]. SDP decomposes the multi-stage planning problem into a sequence of single-stage sub-problems that can be solved by backward induction. The method allows explicit representation of uncertainty (typically in inflow) and the representation of nonlinear and non-convex relationships. Non-convexities typically occur when representing the head-dependent relationship between power output and water discharge, and the unit commitment of generators. The major drawback of the SDP method is that the state variables (typically reservoir volumes) need to be discretized, so the overall problem size becomes computationally intractable when considering systems with many reservoirs. Thus, SDP-based scheduling models applied to multi-reservoir systems often rely on aggregation-disaggregation techniques, such as in [4–6]. Numerous other methodological approaches have been applied to the scheduling problem, see e.g. [7, 8] for an overview. The stochastic dual dynamic programming (SDDP) introduced in [9] has become particularly popular. The SDDP method allows solving the scheduling problem without discretizing the state variables, and is therefore computationally tractable for systems with multiple reservoirs. Although frequently addressed in recent literature, see e.g. [10–14] the SDDP method does not easily facilitate non-convexities.

Several approaches incorporating treatment of energy, reserve capacity and balancing markets in long- and medium-term hydropower scheduling methods have recently been presented. Some authors have decomposed the scheduling problem into intra- and inter-stage problems, as discussed in [15], where the inter-stage problem will take care of the longer-term and strategic decisions, e.g. how much water to use in a given week, while intra-stage decisions concern the detailed operation using a much finer time-resolution. Based on this scheme [16] proposed a method for stochastic medium-term hydropower scheduling considering participation in both the day-ahead and secondary reserve markets. Inter-stage decisions regarding operation of seasonal reservoirs are found by use of SDP, and the shorter term intra-stage decisions, e.g., related to sales of spinning reserves, are found by solving a multi-stage mixed-integer problem. A different method for incorporating sales of spinning reserves in a medium-term hydropower scheduling model based on linear programming was presented in [17]. It extends the hybrid SDP/SDDP algorithm in [18, 19] by allowing sequential sales of reserve capacity and energy, treating both prices as stochastic. However, as documented in [20], the approximation error introduced when linearizing non-convex system characteristics in the system simulation can be substantial, particularly when considering sales of reserve capacity.

This work concerns the assessment of operational profitability for a hydropower plant considering sales of energy and spinning reserve capacity. In Section 2 we present an SDP model suited for computing water values in this market context. Although the

Nomenclature

Index Sets

$\mathcal{B}$	Set of reservation blocks within the week
$\mathcal{G}$	Set of hydropower generators
$\mathcal{K}$	Set of time steps within the week
$\mathcal{K}_b$	Set of time steps associated with block b
$\mathcal{S}_t$	Set of state variables
$\mathcal{S}_{E,t}$	Set of stochastic state variables
$\mathcal{S}_{P,t}$	Set of endogenous state variables

Decision Variables

α_{t+1}	Future expected profit, €
γ, δ	SOS-2 variables
c_b	Reserved capacity, MW
p_{gk}	Generated energy, MWh/h
q^D_{gk}	Discharge, m^3/s
q^S_k	Spillage, m^3/s
u_{gk}	Status indicator for generator (on/off)
v_k	Reservoir volume, Mm^3
x_t	Decision variables
y_{gk}	Incurred start-up cost, €

Stochastic Variables

λ^C	Weekly average reserve capacity price, €/MW
λ^E	Weekly average energy price, €/MWh
I	Sum weekly inflow to reservoir, Mm^3

Parameters

β_g	Start-up cost, €
η_{gm}	Energy equivalent, MW/m^3/s
κ_i	Water value, €/Mm^3
$\mathbb{P}(\ldots)$	Transition probability matrix
$\Phi(\ldots)$	Water value matrix, €/Mm^3
τ_b	Duration of reservation block b, hours
τ_k	Duration of time step k, hours
$\tilde{\tau}_k$	Relative duration of time step k, fraction
ζ^C_b	Reserve capacity price scaling coefficient
ζ^E_k	Energy price scaling coefficient
F	Conversion factor, from m^3/s to Mm^3/h
H	Head for given initial reservoir volume, m
H^0	Reference head, m
K	Last time step in week
M_g	Number of discharge points
N_E	Number of nodes per week in Markov model
N_P	Number of discrete reservoir volumes
$P(Q_{gm})$	Generated energy with discharge Q_{gm}, MWh/h
P^{max}_g,P^{min}_g	Max./Min. capacity, MW
Q^{min}	Minimum river flow, m^3/s
Q_{gm}	Discharge in point m, m^3/s
T	Number of weeks in planning horizon
V^{max},V^{min}	Max./Min. reservoir volume, Mm^3
V_n	Reservoir volume at point n, Mm^3

SDP method has limited capability to address multi-reservoir systems, it allows a detailed representation of the non-convex system characteristics, which are of particular importance when considering sales of spinning reserve capacity. In the presented model we represent non-linearities using a mixed-integer linear programming (MILP) formulation, where non-convex functional relationships are approximated to piecewise linear functions. The model allows representation of uncertainty in inflows and market prices. System operation is simulated using the water values obtained from the SDP model. Consequently, by computing the water values and simulating system operation without convexifying non-convex relationships, we add to the existing literature by assessing the approximation error in purely linear models such as [17]. We believe that the presented model is well suited for detailed multi-market studies of hydropower systems with one or a few reservoirs and with inherit non-convexities in the model formulation. Moreover, it can be used to benchmark the scheduling policies from state-of-the-art linear models, and to quantify the approximation errors in those.

In Section 3 we apply the model to a hydropower system in South-Western Norway to assess how investments in different generator technologies are expected to generate profit for the operator when considering both the day-ahead energy and primary reserve capacity markets.

2 Scheduling Model

We consider a single reservoir connected to a multi-generator power station suited for delivering energy and reserve capacity to separate markets. It is assumed that the reservoir operator (hydropower producer) is a risk-neutral price-taker in both markets. This assumption is fair for producers in the liberalized Nordic market. Note that the model can easily be extended to treat multiple reservoirs, but the computational burden will increase exponentially due to the discretization of state variables.

2.1 Model Overview

We consider a planning period of one year comprising decision stages of one week. The overall decision problem is then to find an operating strategy that maximizes the expected profit for the entire planning period while respecting all relevant constraints. The decision problem can be formulated as a multi-stage stochastic optimization problem, and the expectation is to be taken over the stochastic variables. We assume that the probability distributions of the stochastic variables can be discretized, and that problem can be decomposed into weekly decision stages. Consequently, the realizations of the stochastic variables for an entire week are known at the beginning of that week. The use of weekly decision stages is the standard approach when computing water values for producers in the Nordic market. For further discussion of decision stages and a visualization of the corresponding scenario tree, please see [21].

In the decomposition to an SDP problem, we define a set of system states $\mathcal{S}_t$ that comprises all information passed from one decision stage $t-1$ to the next t. A subset $\mathcal{S}_{P,t} \subset \mathcal{S}_t$ of these state variables are endogenous to the optimization problem and will be described in Section 2.3. The stochastic variables being realized at the beginning of the current week t can be defined as state variables $\mathcal{S}_{E,t} \subset \mathcal{S}_t$, as described further in Section 2.2. Assuming we are in state $\{s_t^e, s_t^p\} \in \mathcal{S}_t$ at the beginning of decision stage t, the decomposed decision problem in (1) is a function of the current week's profit J_t resulting from the immediate decisions x_t, and the future expected profit.

$$\alpha_t(s_t^p, s_t^e) = \max_{x_t} \left\{ J_t(x_t, s_t^p, s_t^e) + \mathbb{E}\Big[\alpha_{t+1}(s_{t+1}^p, s_{t+1}^e)|s_t^e\Big] \right\} \quad (1)$$

2.2 Stochastic Variables

Three stochastic variables are considered in this work; the sum weekly inflow to the reservoir, and the weekly average energy and reserve capacity prices. We assume that the exogenous random variables are Markovian, so that (1) represents a Markov decision process. The Markovian property allows the conditional probability distribution of future states to only depend on the current state. Thus, we are able to capture the weekly correlations in the stochastic variables up to the first lag. The expected future value function is computed by taking the probability weighted average over the N_E discrete values in $\mathcal{S}_{E,t+1}$. The process of defining the number of discrete values (nodes) to represent each stochastic variable is in essence a trade-off between the relative importance of the stochastic variable and computation time, see e.g. [22]. One the one hand, one would like to have many nodes to obtain high accuracy, but on the other we know that the computational time increases exponentially with the number of states. The process of defining the N_E nodes in our case study is further discussed in Section 3.

$$\alpha_t(s_t^p, s_t^e) = \max_{x_t} \Bigg\{ J_t(x_t, s_t^p, s_{t+1}^e) + \sum_{s_{t+1}^e \in N_E} \mathbb{P}(s_{t+1}^e|s_t^e) \cdot \alpha_{t+1}(s_{t+1}^p, s_{t+1}^e) \Bigg\} \quad (2)$$

In general we assume that the stochastic variables are auto- and cross-correlated between weeks, and we compute a discrete Markov chain to represent these correlations. Similar approaches were presented in [23, 24].

2.3 Decision Problem

The decomposed weekly decision problem for week t is formulated as a MILP problem described by (3)-(19). For this problem the realization of the current week's stochastic variables in s_t^e are known. These are the sum weekly inflow I_t, the weekly average energy price λ_t^E and the weekly average reserve capacity price λ_t^C. The initial state is defined by the initial storage and the stochastic variables: $s_t = \left(v_0, I_t, \lambda_t^E, \lambda_t^C\right)$. Note that for brevity of mathematical formulation, the week index is only used to indicate change of week.

$$\alpha_t(s_t^p, s_t^e) = \max \Bigg[\sum_{k \in \mathcal{K}} \tau_k \zeta_k^E \lambda^E \sum_{g \in \mathcal{G}} p_{gk} + \sum_{b \in \mathcal{B}} \tau_b \zeta_b^C \lambda^C c_b - \sum_{k \in \mathcal{K}} \sum_{g \in \mathcal{G}} y_{gk} + \alpha_{t+1}(v_K, s_{t+1}^e) \Bigg] \quad (3)$$

$$\alpha_{t+1}(v_K, s_{t+1}^e) = \sum_{n=1}^{N_P} \gamma_n \cdot \alpha_{t+1}(V_n, s_{t+1}^e) \quad (4)$$

$$v_K = \sum_{n=1}^{N_P} \gamma_n V_n \quad (5)$$

$$\sum_{n=1}^{N_P} \gamma_n = 1 \quad (6)$$

$$v_k + F\tau_k \sum_{g \in \mathcal{G}} q_{gk}^D + F\tau_k q_k^S = v_{k-1} + \tilde{\tau}_k I \quad ,\forall k \quad (7)$$

$$p_{gk} = \sum_{m=1}^{M_g} \delta_{gm} \cdot P(Q_{gm}) \quad ,\forall g,k \quad (8)$$

$$q^D_{gk} = \sum_{m=1}^{M_g} \delta_{gm} Q_{gm} \quad ,\forall g, k \tag{9}$$

$$\sum_{m=1}^{M_g} \delta_{gm} = 1 \quad ,\forall g \tag{10}$$

$$p_{gk} \geq P^{min}_g u_{gk} \quad ,\forall g, k \tag{11}$$

$$p_{gk} \leq P^{max}_g u_{gk} \quad ,\forall g, k \tag{12}$$

$$\sum_{g \in \mathcal{G}} p_{gk} - \sum_{g \in \mathcal{G}} P^{min}_g u_{gk} \geq c_b \quad ,\forall b, k \in \mathcal{K}_b \tag{13}$$

$$\sum_{g \in \mathcal{G}} P^{max}_g u_{gk} - \sum_{g \in \mathcal{G}} p_{gk} \geq c_b \quad ,\forall b, k \in \mathcal{K}_b \tag{14}$$

$$\sum_{g \in \mathcal{G}} q^D_{gk} + q^S_k \geq Q^{min} \quad ,\forall k \tag{15}$$

$$y_{gk} \geq \beta_g \cdot \big(u_{gk} - u_{g,k-1}\big) \quad ,\forall g, k \tag{16}$$

$$u_{gK} - u_{g0} = 0 \quad ,\forall g \tag{17}$$

$$V^{min} \leq v_k \leq V^{max} \quad ,\forall k \tag{18}$$

$$0 \leq c_b \leq C^{max} \quad ,\forall b \tag{19}$$

$$p, y, q^D, q^S \in \mathbb{R}^+ \quad ,\alpha_{t+1} \in \mathbb{R}$$

$$u \in \{0, 1\} \quad ,\gamma \quad SOS2, \delta \quad SOS2$$

The amount of energy sold to the day-ahead market and reserve capacity to the reserve market is optimized in (3) while considering start-up costs and the expected future profit.

Rather than fixing v_K and treat it in the intra-week SDP algorithm, we let the model treat it as a decision variable. This technique was discussed in [25] and was referred to as re-optimization. We discretize the reservoir volume into $n = 1..N_P$ points, each with a corresponding volume V_n, and let $V_1 = V^{min}$ and $V_{N_P} = V^{max}$ and apply (20). Discrete values for the future expected profit $\alpha_{t+1}(V_n, s^e_{t+1})$ are found in (21). These values are computed in the SDP algorithm and serve as parameters in the weekly decision problem, as will be described in Section 2.4.

$$V_{n-1} \leq V_n \leq V_{n+1} \quad ,n = 2..N_P - 1 \tag{20}$$

$$\alpha_{t+1}(V_n, s^e_{t+1}) = \alpha_{t+1}(V_1, s^e_{t+1}) + \sum_{i=1}^{n-1} \kappa_i \cdot (V_{i+1} - V_i) \quad ,n = 2..N_P \tag{21}$$

In (4) we require $\alpha_{t+1}(v_K, s^e_{t+1})$ to be a linear combination of the discrete values in (21), where the variables γ_n indicate the use of each point n. The final reservoir volume v_K is found in (5). The function $\alpha_{t+1}(v_K, s^e_{t+1})$ is not necessarily convex, thus we model it as a special ordered set of type 2 (SOS-2), by adding (6) and requiring that at most two γ_n could be non-zero, and that these must be adjacent. Reservoir balances in (7) keep track of the reservoir volume in each time step k. Note that when $k = 1$, v_0 enters as a initial state on the right-hand side in (7).

Hydropower generation is modeled as a piece-wise linear function of discharge in (8)-(10). The formulation allows a non-convex function where head effects are approximated. We discretize q^D through each generator g into $m = 1..M_g$ points, each with a corresponding discharge Q_{gm}, according to (22). A generation of $P(Q_{gm})$ can be found in (23), and we require p_{gk} to be a linear combination of these values in (8). Note that $P(Q_{gm}) = 0$ for $m = 1$.

$$Q_{g,m-1} \leq Q_{gm} \leq Q_{g,m+1} \quad ,\forall g, m = 2..M_g - 1 \tag{22}$$

$$P(Q_{gm}) = \sum_{i=1}^{m-1} \frac{\eta_{gi} H}{H^0} \cdot (Q_{g,i+1} - Q_{g,i}) \quad ,\forall g, m = 2..M_g \tag{23}$$

The variables δ_{gm} indicate the use of each point m for a generator. The generation p_{gk} is not necessarily a concave function of discharge, thus we model it as a SOS-2, by adding (10) and requiring that at most two δ_{gm} could be non-zero for each generator, and that these should be adjacent. The minimum and maximum generation limits are enforced by (11) and (12), respectively.

The reserve capacity c_b is used for both down-regulation in (13) and up-regulation in (14). In line with the current market structure for reserve capacity in Norway, reserve capacity is sold in blocks b covering a set of time steps $\mathcal{K}_b$, e.g. all weekdays from midnight to 8:00 am. The reserve capacity sales result in a reserve capacity requirement which is tied to the entire power station and limits the opportunities in the day-ahead market. In this work we treat the reserve capacity as a symmetric product, which is in line with the primary reserve market in Norway, but this requirement can easily be relaxed by introducing separate variables for down- and up-regulation. Note that constraints ensuring that there is always enough water to activate the sold reserve capacity could be included, as was done in [17, 26], but this was not considered of significant importance in the presented case study.

The total amount of water discharged and spilled to the downstream river should meet a minimum river-flow requirement in (15). This constraint is motivated by environmental considerations in the case study.

A start-up cost β_g is incurred whenever a generator goes from a non-spinning to a spinning state. Incurred start-up costs are given by the variable y_{gk} in (16). The time-coupling in (16) indicates that each unit's initial spinning state u_{g0} at the beginning of the week should enter the state space. In our experience this detail has little impact on the computed water values. Thus it was simplified in (17), requiring that the initial spinning state should be equal to the spinning state at the end of the week u_{gK}. The reservoir volume and the reserve capacity are limited in (18) and (19), respectively.

2.4 Solution Strategy

The SDP solution strategy is a combination of optimization and backward induction, see Algorithm 1. We loop over the N_P discrete initial reservoir volumes in line 5 and the N_E number of nodes in the Markov model in line 7 to solve the weekly decision problem in line 10. For each node s^e_t in the Markov model, the expected future profit $\alpha_t(V_n, s^e_t)$ is computed in line 13. The water values are computed in line 15.

The water values at the end-of horizon are initialized to zero in the first iteration. We consider a planning horizon of one year and assume that the generation system and the expected market prices and inflow do not change after that year. Under these static conditions, we use the water values found for the beginning of the first week ($t = 0$) in iteration j as the updated water values for the end of the last week ($t = T$) in iteration $j + 1$. The algorithm is solved repeatedly until the expected water values in the water value matrix Φ stabilize according to a pre-defined tolerance ϵ in line 20. Different convergence criteria can be defined, as discussed e.g. in [22], but this is not further discussed here. Note that the computation of water values in line 15 is not strictly necessary, as one could store the expected future profit instead of the water values and avoid (21). However, we prefer the direct formulation for clarity of presentation and to preserve adequate scaling of the optimization problems.

Algorithm 1 SDP algorithm

$j \leftarrow 0, \Delta \leftarrow \infty, \Phi^j \leftarrow 0$
while $\Delta > \epsilon$ **do**
  $j \leftarrow j + 1$
  for $t = T - 1..0$ **do**
    for $n = 1..N_P$ **do**
      $H \leftarrow H(n)$
      for $s^e_{t+1} = 1..N_E$ **do**
        $\{\lambda^E_{t+1}, \lambda^C_{t+1}, I_{t+1}\} \leftarrow StochVar(s^e_{t+1})$
        $\kappa_i \leftarrow \Phi^j(i, s^e_{t+1}, t+1), i = 1..N_P - 1$
        $\alpha_{t+1}(V_n, s^e_{t+1}) \leftarrow Optimize\ (3) - (19)$
      end for
      for $s^e_t = 1..N_E$ **do**
        $\alpha_t(V_n, s^e_t) \leftarrow \sum_{s^e_{t+1}=1}^{N_E} \mathbb{P}(s^e_{t+1}|s^e_t) \cdot \alpha_{t+1}(V_n, s^e_{t+1})$
        if $n > 1$ **then**
          $\Phi^j(V_{n-1}, s^e_t, t) \leftarrow \frac{\alpha_t(s^e_t, n) - \alpha_t(s^e_t, n-1)}{V_n - V_{n-1}}$
        end if
      end for
    end for
  end for
  $\Delta \leftarrow \sum_{t=0}^{T} \sum_{s^e_t=1}^{N_E} \sum_{n=1}^{N_P-1} |\Phi^j(V_n, s^e_t, t) - \Phi^{j-1}(V_n, s^e_t, t)|$
  $\Phi^{j+1}(V_n, s^e, T) \leftarrow \Phi^j(V_n, s^e, 0), \forall n, s^e$
end while

3 Case Study

We applied the presented model to the Maudal watercourse, located in the south-western part of Norway. The system comprises a single reservoir with 63 Mm^3 storage capacity and a power station, as illustrated in Fig. 1. The system receives both storable and non-storable inflow, and the expected annual values are indicated in Fig. 1. The non-storable inflow enters downstream of the generators, but can be subtracted from the minimum flow requirement for the downstream river.

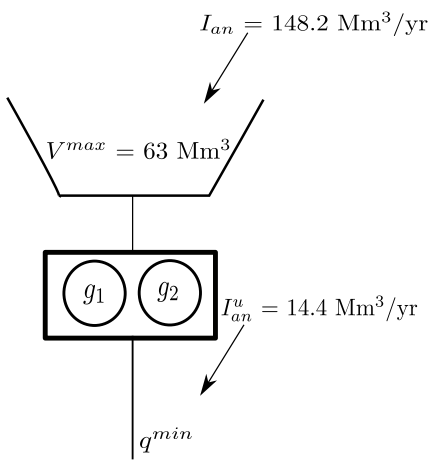

Fig. 1: Watercourse topology and technical data.

The operator considers reinvesting in generators, and is evaluating a set of different possibilities. One of the key uncertainties impacting this investment decision is how much profit one should expect from selling reserve capacity in the future. Two different investment alternatives are compared in the following, cases A and B, both with two generators and a total installed capacity of 24.8 MW. Technical data for the different generators are shown in Table 1, including the number of discrete discharge points (M_g), maximum (P^{max}) and minimum (P^{min}) generation and the generator type. The Francis generator in case A has a higher maximum efficiency, but does not have the ability of the Pelton turbine to run at low output to deliver reserve capacity. The maximum allowed sales of symmetric reserve capacity is 8.87 and 10.75 MW for case A and B, respectively. We assume start-up costs of 100 € and 50 € for the largest (g_1) and smallest (g_2) generator, respectively. These values are similar to those reported in [27].

Table 1 Simulated cases.

Case	unit no.	M_g	P^{max}	P^{min}	Type
A	g_1	10	21.11	6.57	Francis
A	g_2	12	3.71	0.49	Pelton
B	g_1	13	16.06	2.34	Pelton
B	g_2	13	8.79	0.97	Pelton

The two cases were analysed in two different modes. First, in the *E-mode*, we consider sales of energy only by letting $C^{max} = 0$ in (19). Second, in the *E+C mode*, we allow selling in both the energy and reserve capacity markets. The water values for the two cases operated in the two different modes were obtained by using the model presented in Section 2 and then used to simulate the hydropower plant operation in a sequential optimization process.

The reservoir level was discretized in N_P = 21 equidistant points, which was considered appropriate for this particular reservoir. The process of finding the discretized stochastic variables is described below. The Hourly energy prices were sampled from an ARIMA(2,0,3) model fitted to day-ahead prices, while reserve capacity prices were sampled from a GARCH(1,1) model with ARMA(1,1) as the mean process fitted to primary reserve prices. Both models were fitted with historical data from 2013-2015 for Maudal's price area, NO2 in NordPool. No significant correlation between day-ahead and reserve capacity prices was observed in the historical data, therefore independent models for the two price processes was chosen. From 1000 samples of each price a discrete price model comprising 15 price nodes per stage (5 energy and 3 reserve capacity prices) was identified by following the approach discussed in [17]. No significant correlation between inflow and price was detected in the historical record, which justifies our use of a separate inflow model. 1000 inflow samples generated from a VAR(1) model described in [28] were used to create 5 inflow nodes per stage.

The model was implemented in the `AMPL` modeling language [29] using the CPLEX optimization solver [30], version 12.6. We used an absolute MIPGAP of $1e-5$. All tests were carried out on an Intel Core i7-4940MX processor with 3.30 GHz and 32 GB RAM. In each time stage, the state space is described by $N_P \times N_E$ = 21×5×15 = 1575 discrete states. The decomposed weekly decision problem for case D for a given week comprise 1075 variables (443 binary) and 800 constraints. The SDP algorithm typically converges in 7-10 iterations, and one iteration took on average approximately 4 hours when using the MILP formulation. We emphasize that the model has not been optimized for computational performance, and expect that speed-up can be obtained e.g. by adjusting the default SOS2-structure, see [31, 32] for further details.

The results from this case study are presented in the following subsections. First, the computed water values are analysed in Section 3.1. Subsequently, we present simulation results obtained by using the computed water values for dispatch in Section 3.2.

3.1 Water Values

Fig. 2 show the water value surfaces for the median price and inflow node for the two modes for case B. The E+C mode (grey surface) provides higher water values than the E mode (black surface) most of the time. In general the water values decrease faster with increasing reservoir level in the E+C mode. In certain periods of the year, the water values are higher at high reservoir levels in the E mode. The pronounced increase in mode E+C in Fig. 2 for the summer season (around week 25-30) is primarily due to expectation of high prices in the primary reserve capacity market for that season.

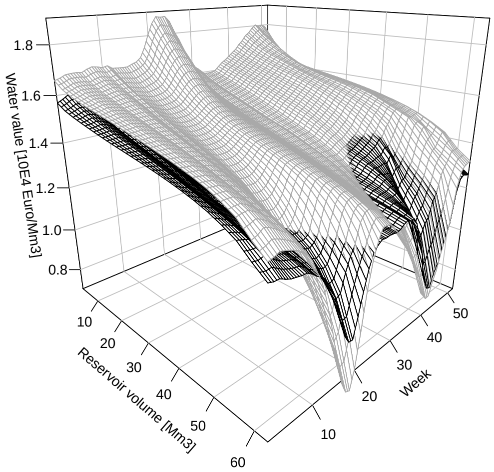

Fig. 2: Water values for case B in the E+C (grey) and E (black) mode.

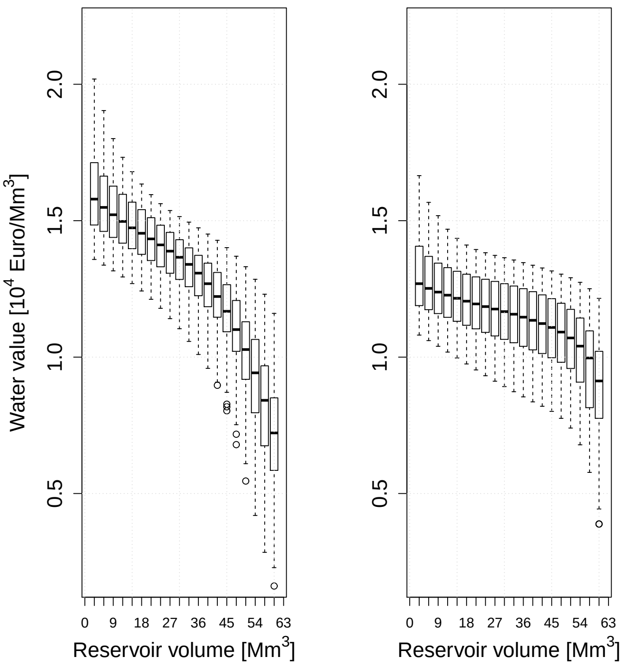

Fig. 3: Water values for case B for week 15. Values for the E+C mode (left) and the E mode (right) are shown.

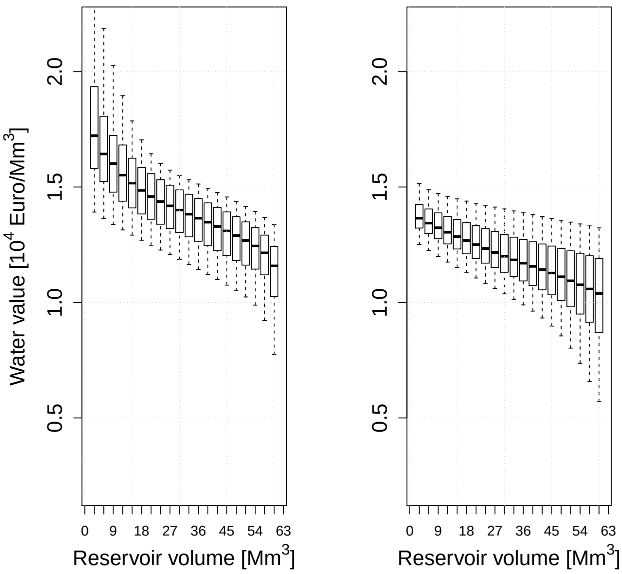

Fig. 4: Water values for case B for week 25. Values for the E+C mode (left) and the E mode (right) are shown.

In order to explain the shape of the water value surface in Fig. 2, we study two specific weeks in the Figs. 3 and 4. Water values for weeks 15 and 25 for case B are presented in Figs. 3 and 4, for both modes. The bottom and top of the boxes indicate the lower and upper quartiles, respectively. Note that, in addition to uncertainty in energy price and inflow, mode E+C also faces uncertainty in reserve capacity prices. The steeper shape in the E+C mode in both Fig. 3 and 4 indicate that the risk of spillage impacts the water values more in the E+C mode. In the E+C mode sales of up-regulation reserves is expected in the future, which in turn will limit the generators ability to discharge, and consequently increase the risk of spillage. This effect is evident in Fig. 3. Furthermore, both Fig. 3 and Fig. 4 indicate that the water value is likely to be higher in the E+C mode at low reservoir levels. This is also evident in Fig. 2. In certain parts of the year it will be optimal to have generators spinning at energy prices below the water value for the purpose of delivering reserves. Thus, water is needed both to sell energy and reserve capacity, and consequently the value of the scarce resource is higher in the E+C mode. The consequence of a low reservoir in week 25 is obviously more pronounced in the E+C mode, since the producer may not be able to keep the units spinning. This impact is similar to what will typically be found when considering minimum flows [24].

3.2 Simulation Results

Simulations were run to study the system performance and operational profitability for the two cases in each mode. A set of 1000 scenarios, each with weekly prices and inflows for one year, were sampled initially from the established Markov model and used in the simulations. The simulations were arranged as weekly decisions in sequence using the model formulation defined in Section 2.3 and the water values obtained from the SDP model.

In order to test the importance of accurate representation of non-convexities, the simulations were run three times using the same scenario set. In the first run, both the SDP model and the simulations were run using the proposed MILP formulation in 2.3. In the second run we relaxed the weekly decision problem in (3)-(19) to an LP problem to resemble the degree of detail typically used in medium- and long-term hydropower scheduling models. Finally, in the third run we relaxed the weekly decision problem in the SDP model and simulated using a MILP formulation. This combination resembles how practical hydropower scheduling is often carried out, simulating the system using a more detailed system representation than what is used when computing the water values. The expected profits are presented in Table 2. All values are in percentage of the expected profit in the E mode for the respective case in run 1.

Table 2 Expected Profits. All values are in percentage of the expected profit in the E mode for the respective case in run 1.

Run	Case	Mode	SDP	Sim.	En. Sales	Cap. Sales	Total
1	A	E	MILP	MILP	100.00	0.00	100.00
1	A	E+C	MILP	MILP	94.16	13.17	107.33
1	B	E	MILP	MILP	100.00	0.00	100.00
1	B	E+C	MILP	MILP	92.48	18.30	110.78
2	A	E	LP	LP	100.33	00.00	100.33
2	A	E+C	LP	LP	95.43	17.13	112.55
2	B	E	LP	LP	100.73	0.00	100.73
2	B	E+C	LP	LP	93.03	20.02	113.05
3	A	E	LP	MILP	99.76	00.00	99.76
3	A	E+C	LP	MILP	94.02	12.28	106.30
3	B	E	LP	MILP	99.79	0.00	99.79
3	B	E+C	LP	MILP	92.70	17.44	110.14

In all three runs and in both cases the total profit increases when the opportunity to sell reserve capacity is introduced. This is expected since mode E in essence is a constrained version of mode E+C. Table 2 also shows that the expected profit increases by 5.22 and 2.27 percentage points for case A and B, respectively, in the E+C mode when comparing run 2 and run 1. The increase in expected profit when relaxing the problem formulation is primarily due to the added flexibility of selling reserves without strictly respecting the minimum power output requirement and the non-convex production function. Note that this flexibility is highly dependent on characteristics of the hydropower system being studied, see e.g. [20] for a similar test on a different system. Comparing run 3 with run 1, reduction in expected total profit of 1.03 and 0.64 percentage points is observed for case A and B, respectively, in the E+C mode. This difference serves to quantify the approximation error when linearizing non-convexities in the SDP water value computation. As expected, we obtain a better result when the simulation and the water values are based on the same model formulation. This improvement comes at a significant increase in computational effort due to the vast amount of weekly decision problems to be solved in the SDP model and the added complexity of solving MILP problems over LP problems.

Fig. 5 shows reservoir trajectory percentiles for case B in the E and E+C mode as stapled and solid-drawn lines, respectively. The E+C mode follows a higher trajectory in the late winter period (weeks 10-15, before snow melt) to store enough water for the summer season. Due to sales of reserve capacity in the E+C mode, water is used more aggressively to keep the generators spinning during the summer season, giving a lower trajectory in autumn and early winter.

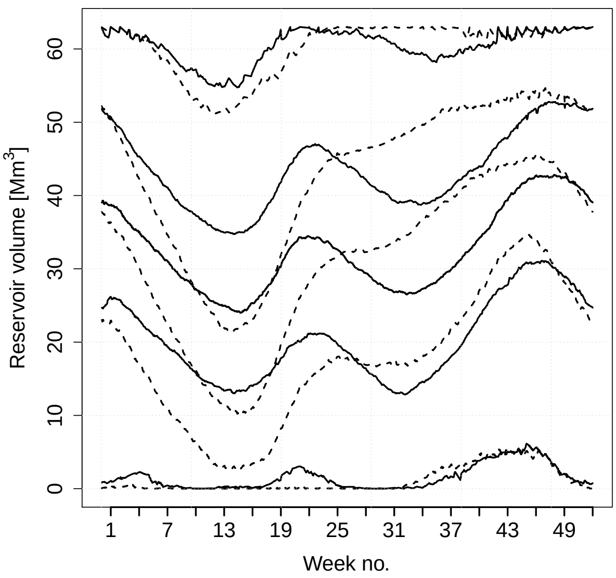

Fig. 5: Reservoir trajectories for case B, mode E (stapled) and E+C (solid-drawn). 0, 25, 50, 75 and 100 percentiles.

Fig. 6 shows the duration curve of generation for each generator for case A operating in E (stapled) and E+C (solid-drawn) modes. Both units are operating in a larger percentage of time in the E+C mode. Generator g_1 operates approximately 30 % of the time at around 15.44 MW in the E+C mode. This corresponds to the situation where the system sells the maximum rate of reserve capacity (8.87 MW). In this situation g_2 is running on minimum output, leaving 3.22 MW (3.71 MW-0.49 MW) for up-regulation, while g_1 provides 8.87 MW down-regulation and 5.65 MW up-regulation.

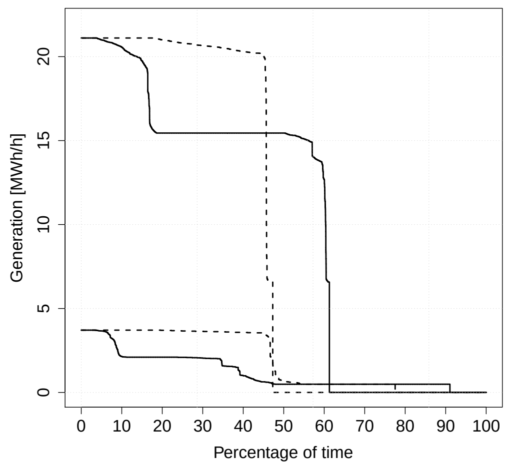

Fig. 6: Duration curves for the two generators in case A. E mode as stapled lines, E+C mode as solid-drawn.

Fig. 7 shows the duration curve of generation for each generator for case B operating in E (stapled) and E+C (solid-drawn) modes. As for case A, both units are operating in a larger percentage of the time in the E+C mode. Moreover, generator g_2 produces less energy in the E+C than in the E mode and is primarily run at minimum output to support g_1 in delivering up-regulation reserves.

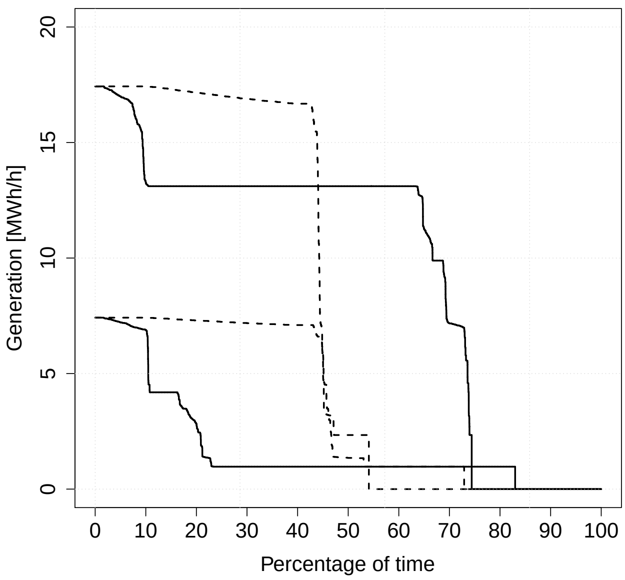

Fig. 7: Duration curves for generation, case B. E mode as stapled lines, E+C mode as solid-drawn.

4 Conclusion

Accurate water values are of great importance when analysing the operational profitability of new investments in or upgrading of existing hydropower stations. We present a model for optimal scheduling of both energy and reserve capacity under uncertainty in market prices and inflow. The model allows exact representation of non-convexities such as head dependent production functions and generator unit commitment.

The model was applied in a case study for a Norwegian hydropower producer, showing how the operational profitability of future investments in the generation system depends on whether and how reserve capacity sales is considered. The operational pattern changes to less seasonal shifting of energy when a reserve capacity market is introduced. Moreover, the water values show a more pronounced variation for different reservoir fillings when considering two markets, being more exposed to both the risk of missing high-price periods and spillage.

Models based on linear programming are traditionally used for long-term hydropower scheduling. The presented model allows representation of non-convex system characteristics and is well suited for systems with a few reservoirs. Thus, it can be used to benchmark the scheduling policies from linear models, and to quantify the approximation errors. This was demonstrated in the case study, showing that the linear approximation significantly overestimated the operational profitability.

In general, our findings point out the importance of taking reserve capacity markets into account when upgrading and modernizing existing systems. The long lifetime of such investments implies a significant risk if future market opportunities are not taken into account.

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5 References

1 J. D. C. Little, "The use of storage water in a hydroelectric system," *Journal of the Operations Research Society of America*, vol. 3, no. 2, pp. 187 – 197, 1955.

2 J. Lindqvist, "Operation of a hydrothermal electric system: A multistage decision process," *AIEE Transactions–pt. III (Power Apparatus and Systems)*, vol. 81, no. 3, pp. 1–7, April 1962.

3 S. Stage and Y. Larsson, "Incremental cost of water power," *Transactions of the American Institute of Electrical Engineers*, vol. 80, no. 3, pp. 361–364, 1961.

4 A. Turgeon and R. Charbonneau, "An aggregation-disaggregation approach to long-term reservoir management," *Water Resources Research*, vol. 34, no. 12, pp. 3585–3594, 1998.

5 O. Wolfgang, A. Haugstad, B. Mo, A. Gjelsvik, I. Wangensteen, and G. Doorman, "Hydro reservoir handling in Norway before and after deregulation," *Energy*, vol. 34, no. 10, pp. 1642–1651, 2009.

6 T. A. A. Neto, M. V. F. Pereira, and J. Kelman, "A risk-constrained stochastic dynamic programming approach to the operation planning of hydrothermal systems," *IEEE Transactions on Power Aparatus and Systems*, vol. PAS-104, no. 2, pp. 273 – 279, 1985.

7 J. W. Labadie, "Optimal operation of multireservoir systems: State-of-the-art review," *Journal of Water Resources Planning and Management*, vol. 130, no. 2, pp. 93–111, 2004.

8 G. Steeger, L. A. Barosso, and S. Rebennack, "Optimal bidding strategies for hydro-electric producers: A literature survey," *IEEE Transactions on Power Systems*, vol. 29, no. 4, pp. 1758–1766, 2014.

9 M. V. F. Pereira and L. M. V. G. Pinto, "Multi-stage stochastic optimization applied to energy planning," *Mathematical Programming*, vol. 52, pp. 359–375, 1991.

10 B. Flach, L. Barroso, and M. Pereira, "Long-term optimal allocation of hydro generation for a price-maker company in a competitive market: latest developments and a stochastic dual dynamic programming approach," *IET Generation, Transmission & Distribution*, vol. 4, no. 2, pp. 299–314, 2010.

11 Q. Goor, R. Kelman, and A. Tilmant, "Optimal multipurpose-multireservoir operation model with variable productivity of hydropower plants," *Journal of Water Resources Planning and Management*, vol. 137, no. 3, pp. 258–267, 2011.

12 S. Cerisola, J. M. Latorre, and A. Ramos, "Stochastic dual dynamic programming applied to nonconvex hydrothermal models," *European Journal of Operational Research*, vol. 218, pp. 687–897, 2012.

13 H. Abgottspon, K. Njálsson, M. A. Bucher, and G. Andersson, "Risk-averse medium-term hydro optimization considering provision of spinning reserves," in *International Conference on Probabilistic Methods Applied to Power Systems (PMAPS)*, Durham, England, 2014.

14 G. Steeger and S.Rebennack, "Dynamic convexification within nested benders decomposition using lagrangian relaxation: An application to the strategic bidding problem," *European Journal of Operational Research*, vol. 257, no. 2, pp. 669–686, 2017.

15 N. Löhndorf, D. Wozabal, and S. Minner, "Optimizing trading decisions for hydro storage systems using approximate dual dynamic programming," *Operations Research*, vol. 61, no. 4, pp. 810 – 823, 2013.

16 H. Abgottspon and G. Andersson, "Medium-term optimization of pumped hydro storage with stochastic intrastage subproblems," in *Proc. 18th Power System Computation Conference*, Wroclaw, Poland, 2014.

17 A. Helseth, M. Fodstad, and B. Mo, "Optimal medium-term hydropower scheduling considering energy and reserve capacity markets," *IEEE Transactions on Sustainable Energy*, vol. 7, no. 3, pp. 934–942, 2016.

18 A. Gjelsvik, M. M. Belsnes, and A. Haugstad, "An algorithm for stochastic medium-term hydrothermal scheduling under spot price uncertainty," in *Proc. 13th Power System Computation Conference*, Trondheim, Norway, 1999.

19 A. Gjelsvik, B. Mo, and A. Haugstad, *Handbook of Power Systems I*. Springer, 2010, ch. Long- and medium-term operations planning and stochastic modelling in hydro-dominated power systems based on stochastic dual dynamic programming, pp. 33–55.

20 M. N. Hjelmeland, A. Helseth, and M. Korpås, "A case study on medium-term hydropower scheduling with sales of capacity," *Energy Procedia*, vol. 87, pp. 124–131, 2016.

21 A. Helseth, A. Gjelsvik, B. Mo, and U. Linnet, "A model for optimal scheduling of hydro thermal systems including pumped-storage and wind power," *IET Generation, Transmission & Distribution*, vol. 7, no. 12, pp. 1426 – 1434, 2013.

22 K. D. W. Nandalal and J. J. Bogardi, *Dynamic Programming Based Operation of Reservoirs, Applicability and Limits*. Cambridge University Press, 2007.

23 R. Rajaraman, L. Kirsch, F. L. Alvarado, and C. Clarck, *The Next Generation of Electric Power Unit Commitment Models*. Boston: Kluwer Academic Publishers, 2001, vol. 36, ch. 6.

24 I. Guisandez, J. I. Perez-Diaz, and J. R. Wilhelmi, "The influence of environmental constraints on the water value," *Energies*, vol. 9, no. 6, pp. 1–21, 2016.

25 J. A. Tejada-Guibert, S. Johnson, and J. R. Stedinger, "Comparison of two approaches for implementing multireservoir operating policies derived using stochastic dynamic programming," *Water Reseources Research*, vol. 29, no. 12, pp. 369–3980, 1993.

26 M. Chazarra, J. Garciá-González, J. I. Pérez-Díaz, and M. Arteseros, "Stochastic optimization model for the weekly scheduling of a hydropower system in day-ahead and secondary regulation reserve markets," *Electric Power Systems Research*, vol. 130, pp. 67–77, 2016.

27 O. Nilsson and D. Sjelvgren, "Hydro unit start-up costs and their impact on the short term scheduling strategies of swedish power producers," *IEEE Transactions on Power Systems*, vol. 12, no. 1, pp. 38–44, 1997.

28 S. Martino, I. H. Haff, A. Løland, and B. Mo, "Statistical properties of historical inflow series for long-term models," Norwegian Computing Center, Tech. Rep. SAMBA/50/13, 2014.

29 R. Fourer, D. M. Gay, and B. W. Kerninghan, *AMPL: A Modeling Language for Mathematical Programming*, 2nd ed. Duxburry, 2003.

30 "IBM ILOG CPLEX optimizer," http://www-01.ibm.com/software/.

31 M. Padberg, "Approximating separable nonlinear functions via mixed zero-one programs," *Operations Research Letters*, vol. 27, no. 1, pp. 1–5, 2000.

32 S. Rebennack, “Computing tight bounds via piecewise linear functions through the example of circle cutting problems,” *Mathematical Methods of Operations Research*, vol. 84, no. 1, pp. 3–57, 2016.