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## Results from an extended life cycle model

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**Abstract:**
An extended life cycle model is used to investigate how variation in the level of expected pensions influences non-pension wealth accumulation. We try to explain why the offset effects between pension wealth and private savings are not one to one by accounting for different risks and market imperfections, which includes uninsured risk on earnings, mortality risk, borrowing constraints and bequest motive. The model is calibrated on Norwegian household data from 1992 to 2005. Based on the calibrated model, simulations are performed to explore consequences of introducing these factors. The result shows that by simply accounting these risks and constraints, we can explain most of the departure between empirical findings and theoretical prediction.

**Keywords:** Life cycle model, Offset effect, pension wealth, private savings

**JEL classification:** D91

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**Address:** Zhiyang Jia, Statistics Norway, Research Department. E-mail: jia@ssb.no

Weizhen Zhu, The Ragnar Frisch Centre for Economic Research

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## Sammendrag

Ifølge tradisjonell økonomisk teori om sparing (livsløpshypotesen) forventer en at en krones økning i pensjonsrettigheter skal føre til en like stor reduksjon i privat sparing. I empiriske analyser av dette fenomenet finner en imidlertid ofte en mye mindre effekt. Vi prøver å forklare forskjellen ved hjelp en utvidet livsløpsmodell, hvor vi inkluderer effektene av usikkerhet i inntekt og levetid, imperfeksjoner i kapitalmarkedet og at individet kan ha ønsker om å etterlate seg arv. Analysen viser at ved å inkludere disse faktorene, kan en forklare det meste av avviket mellom empiriske funn og teoretisk prediksjon ifølge den tradisjonelle livsløpsmodellen.

# 1 Introduction

The impact of public pension scheme on private savings is an important issue in the discussion of pension reforms. While acknowledging the fact that the main objective of the public pension system is to provide financial security for the elderly through the retirement period, economists and policy makers are also worried that the public pension systems have contributed to reduce savings in the economy and thus hinder future economic growth, especially in the light of the ageing problem in many developed economies. Policies aiming to promote private savings for retirement are implemented or planned in many countries. One popular proposal is to supplement or replace the unfunded defined benefit system by funded defined contribution system or tax-favored savings account, which explicitly or implicitly reduces the generosity of the pension system and make the potential retirees more exposed to investment risks. Many argue that this will not only induce higher private savings but also lower the financial burdens of the public pension system. However, how individuals will respond to these reforms is still an open question. There is always a danger that such a reform will undermine the financial security of some retirees, since they may fail to adjust their behavior in response to the new pension system.

The life cycle hypothesis of consumption plays a crucial role in the discussion of pension design, since it serves as the main workhorse for understanding the consumption and savings behavior, see for example Auerbach and Kotlikoff (1995), Blanchard and Frischer (1989) and Lindbeck and Persson (2003). In the simplest life cycle model, it is assumed that market is complete and individuals save only to smooth their consumptions over life cycle. Consumption is simply a function of life time wealth. It does not matter whether the income is in terms of wage or pension. When an increase in the future pension benefit is financed by increased taxes on wages and does not change the life time wealth, the increased pension wealth will be completely offset by a reduction of private savings. This feature is typically labeled as "the perfect offset[1]" and has strong implications for pension system designs. However, this complete market life cycle model rests on very strong assumptions. First, income and mortality risk are ignored. When there are uninsurable risks, risk averse individuals will save more to prepare for future unlucky periods. This leads to the so called "precautionary saving motive". Skinner (1987) argued that precautionary savings against uncertain income comprise a large fraction of aggregate savings. We know that if individuals save also for reasons other than retirement, the degree of substitutability between pension wealth and private savings will be affected, which will change the offset effects. Furthermore, capital market is assumed to be perfect and there is no credit rationing. In reality, physical collateral such as private housing is typically required for large loans, while unsecured loans are typically bearing very high interest rate. On the other hand, public pensions are annuities

[1] In the case when changes in pension wealth imply also changes in life time wealth, there will not be one to one relationship between pension wealth and private savings even under the complete market life cycle model. We follow Gale (1998) and apply a bias correct parameter for these cases. See section 5 for detail.

which can not be withdrawn before retirement. For low income households whose borrowing constraints are binding, an increase of pension wealth may have no effect on savings. Finally, other important variables which may influence individuals' savings behavior, such as labor supply behaviors and factor prices are assumed to be exogenous and fixed. However, these factors may respond to changes in pension wealth and lead to additional impact on savings. For example, individuals may adjust the timing of their withdrawal from labor market as results of changes in pension wealth, which leads to the so called "induced retirement effect" and complicates the connection between pension wealth and private savings.

Given these theoretical ambiguities, considerable efforts have been used to empirically determine the effect of pension wealth on saving. However, the quantitative results differ widely in the literature. Among the earliest works on the effect of pensions on other savings, Cagan (1965) and Katona (1965) obtained a positive effect of pension wealth on other saving; Feldstein (1974), Feldstein (1982) and Feldstein and Pellechio (1979) found that social security wealth crowded out other wealth substantially; and Boyle and Murray (1979) found that public pension plans had no visible effect on household savings behavior, using Canadian data. In recent studies, Euwals (2000) found an ambiguous impact of the public part of the Dutch pension system on savings, but a negative impact of occupational pensions. Attanasio and Brugiavini (2003) found evidence that saving rates increased as a result of a reduction in pension wealth by exploiting the Italian pension reform of 1992. The diversity of the results may be a result of a diversity in actual pension system and underlying social economic factors such as individual attitude towards savings and demographic circumstances across different countries or different time. More importantly, the identification strategies which differ in empirical studies contribute strongly to this. In fact, there are still many challenges remaining in the empirical analysis of offset effect, see for example discussion in CBO (1998). The main results in the literature point to an offset effect which is much smaller than 1, typically in the range of 20-60%.

As discussed earlier, many factors can explain the gap between the empirical findings and the prediction from the simple complete market life cycle model. However, the quantitive contributions and relative importance of these factors have not yet been studied in the literature. From a policy point of view, with this type of knowledge specific measures can then be taken to increase effectiveness and reduce the undesirable side effect of pension policy changes. In this paper, we build up and calibrate an extended stochastic dynamic model of household savings behavior which accounts for bequest motive, liquidity constraints, mortality risk and income risk. Extending the complete market life cycle model is certainly not new. Earlier contributions include Yaari (1965), Skinner (1985) on mortality risk, Deaton (1991) on liquidity constraints. Zeldes (1989) and Carroll (1992,1997) look at the level of consumption and saving when labor income is risky and there is imperfect insurance. More recently, Rodepeter and Winter (1998), Domeij and Klein (2002) and Love (2006) combine these features together and extend the basic life cycle model further to investigate optimal household savings path.

What is new in this paper is to use this type of model to look at how different risks/constraints influence the offset effect between pension wealth and non pension wealth. Based on the calibrated model, simulations are performed to explore consequences of introducing different factors. The existence of precautionary savings motive, due to for example income risks, is typically considered to be one of the main factors which leads to a lower offset effect, since the public pension is no longer a perfect substitute of the private savings. However, our results shows the opposite: households who face income risk would actually reduce more private savings than those who do not, if both experience the same increase in pension wealth. The reason is that pension wealth is considered to be a much less risky income source. An increase in pension wealth, will lower overall income risk and thus increase consumption for those with higher initial income risks. This pattern is illustrated by both a simple analytical example and more sophisticated simulations. We also find the influence of income risk is very small given our model estimates. Mortality risk works in the opposite direction as income risk. All else being equal, households facing mortality risks reduce less private savings in the presense of a pension wealth increase. Thus, we cannot say in general that the existence of precautionary savings will increase or dampen the offset effects between pension wealth and private savings. The other two factors: borrowing constraint and bequest motive have a similar effect as the mortality risk has. That is, both will dampen the offset effect. When all these factors are considered, middle aged households (45 to 50 year olds) reduce their private savings by around 45% of the increased pension wealth. Our exercise shows that it is important to take these factors into account when we investigate the offset effect between the pension wealth and private wealth, and eventually study the effect of pension system to savings. Although the extended model still cannot explain fully the departure between empirical findings documented Hernæs and Zhu (2007) (23% for middle education level) and Jia and Zhu (2009b) (35% for middle education level with financial wealth at the median.), it shows that by simply accounting for these risks and constraints, we can explain most of the departure between empirical findings and theoretical prediction.

The rest of this paper is organized as follows: Section 2 provides an analytical illustration for offset effect under income risk only. Section 3 presents our extended dynamic model of household savings behavior. The model is calibrated in Section 4. Section 5 discusses our definition of offset effect, and analyzes the simulation results. The final section concludes.

# 2 Offset effect under income risk: an analytical illustration

We start our discussion with a simple analytical example to show the possible effects of precautionary saving motive on the offset effect of pension wealth on private savings.

Suppose in an economy, one agent lives for three periods. He works in the first two periods and receives wage income $y_t$, $t = 1, 2$. The wage income $y_t$ is random, and is realized at the beginning of each period. The individual retires at the end of period 2 and gets pension income $p$ in period 3, which is a function of life time earnings $p = f(y_1 + y_2)$. There is only one risk free asset available in the market with real rate of return $r$. We assume that the period utility is $u(c_t) = \log(c_t)$, where $c_t$ is consumption at time $t$. At any given time $t$, The individual maximizes the remaining life time utility $\sum_{s\geq t} u(c_s)(1+\delta)^{t-s}$, subject to the budget constraints. There is no bequest motivated savings in this model. The model can be solved with backward induction as follows:

When $t = 3$, the agent's value function is:

$$\begin{aligned} V_3(a_3, p) &= \max_{c_3} \{\log(c_3)\} \\ \text{subject to} &: \; c_3 = (1+r)a_3 + p, \quad a_4 \geq 0, \end{aligned}$$

$a_t$ is the asset at the end of period $t-1$. For notation simplicity, we rewrite $\beta = \frac{1}{1+\delta}$, and $R = (1+r)$.

It is obvious that the optimal choice for the individual at period 3 is to consume all he has, thus

$$V_3(a_3,\ p) = \log(Ra_3 + p)$$

When $t = 2$, the value function is:

$$\begin{aligned} V_2(a_2, y_2) &= \max_{c_2} \{\log(c_2) + \beta E_2 [V_3(a_3,\ p)]\} \\ \text{subject to} &: \; c_2 + a_3 = Ra_2 + y_2 \end{aligned}$$

Maximization gives:

$$\begin{aligned} c_2 &= \frac{R^2 a_2 + Ry_2 + p}{(1+\beta)R} \\ a_3 &= \frac{\beta(R^2 a_2 + Ry_2) - p}{(1+\beta)R} \end{aligned}$$

$$V_2(a_{2,}y_2) = (1+\beta)\log(R^2 a_2 + Ry_2 + p) + \phi(\beta, R)$$

where $\phi(\beta, R) = -\log[(1+\beta)R] + \beta \log(\beta/(1+\beta))$.

Note that at the beginning of period 2, wage income $y_2$ is realized so no income risk remains. Therefore, the optimal solutions $c_2(a_2, y_2)$ and $c_3(a_3, p)$ are of the same form for the two cases: without and with income risk. The difference occurs in the first period.

When $t = 1$, the value function is:

$$\begin{aligned} V_1(y_1) &= \max_{c_1} \{\log(c_1) + \beta E_1 [V_2(a_2, y_2)]\} \\ \text{subject to} &: \; c_1 + a_2 = y_1 \end{aligned}$$

For illustration purpose, we assume that the second period wage income $y_2$ can take only two values $\{E[y_2] - \sigma_y, E[y_2] + \sigma_y\}$ ( $\sigma_y \geq 0$ ) with equal probability 0.5. Denote the corresponding pension benefit at period 3 as $\{E[p] - \sigma_p, E[p] + \sigma_p\}$ ( $\sigma_p \geq 0$ and $\sigma_p = 0$ if $\sigma_y = 0$) respectively. For notation simplicity, we write $\sigma^2 = (\sigma_p + R\sigma_y)^2$. $\sigma^2$ measures the size of the future income risk, including both labor income at period 2 and pension income at period 3. Denote the present value of expected life time income as $L$, i.e.

$$L = \frac{R^2 y_1 + RE[y_2] + E[p]}{R^2}.$$

With some simple but tedious calculations, we get

$$c_1^* = \frac{L + B(E[p], \sigma^2)]}{(1 + \beta + \beta^2)}, \tag{1}$$

$$a_2^* = y_1 - c_1^*, \tag{2}$$

where

$$B(E[p], \sigma^2) = \frac{\beta(1+\beta)}{2} L - \sqrt{(\frac{\beta(1+\beta)}{2})^2 L^2 + \frac{(\beta^2 + \beta + 1)}{R^2} \sigma^2}.$$

It is easy to show that $B(E[p], \sigma^2) \leq 0$, and equality holds if and only if $\sigma^2 = 0$.

When there is no uncertainty, we have $\sigma_y = 0$, so $\sigma^2 = 0$ and $B$ reduces to zero. The optimal consumption at period 1 will simply be $L/(1 + \beta + \beta^2)$. In contrast, the consumption under income risk is always smaller than the consumption without income risk, since $B(E[p], \sigma^2) < 0$ when $\sigma^2 > 0$. Thus, "prudent" individuals lower their consumption to prepare for possible income fall in future. This pattern is labeled as "precautionary saving" in the literature. Another useful observation is that the higher the uncertainty of income is, the lower the consumption and the higher the savings are.

The effect of the income risk on the offset effect of pension wealth on private wealth is not as straightforward. We see from equation (1) and (2) that the optimal savings at period 1, $a_2$ depends on the expected pension wealth $E[p]$. Now suppose the agent gets information that the pension will increase by $\Delta E[p]$. We look at the change of non-pension savings at the end of period 1, $\Delta a_2^*$. It follows from (1) and (2):

$$\Delta a_2^* = -\Delta c_1^* = \frac{-1}{(1 + \beta + \beta^2)} (\Delta E[p]/R^2 + B(E[p] + \Delta E[p], \sigma^2)] - B(E[p], \sigma^2)]) \tag{3}$$

When there is no income risk, we have $\Delta a_2^* = \frac{-1}{(1+\beta+\beta^2)}(\Delta E[p]/R^2) < 0$. So increased pension wealth will induce a reduction in the nonpension wealth with magnitude

$$|\Delta a_2^*| = \frac{(\Delta E[p]/R^2)}{(1+\beta+\beta^2)}.$$

For the case when income risk is present, we note that given that $\Delta E[p]$ is positive, $B(E[p]+\Delta E[p],\sigma^2)]- B(E[p],\sigma^2)]$ is strictly increasing w.r.t $\sigma^2$ and equal to 0 when $\sigma^2 = 0$. Thus, the private wealth reduction ( $|\Delta a_2^*|$) increases with future income risk $\sigma^2$ in this simple setup. This implies that for the same pension wealth increase, the individuals facing income risk actually offset more than those who face no income risk. They increase their consumption more, since the increased pension income will not only reduce the savings for retirement but also the "precautionary" saving due to the income uncertainty. However, their consumption after the increase is still lower than the case when there is no income uncertainty. This is consistent with the argument that "at a given wealth and income level, the marginal propensity to consumption to consume out of wealth is higher under uncertainty" (Zeldes, 1992). What we can see more is that the offset effect increases with the increase of the variance of the income risk in this simple model.

However, generalizations of above conclusions to more complicated income process, different preference setups, how would it work together with other factors such as mortality risk are very difficult since typically there is no close form solution for the optimal consumption and savings. In what follows we will set up a simulation model to look at this problem in more general settings.

## 3 The model

We set up a model that describes the savings behavior of married couples (households) over the life cycle. A household has two members: a husband and a wife. They enter into labor market at age 30 and work until age of 67. For simplicity, we assume that the husband and the wife are at the same age, thus they will retire at the same year.

We allow both the husband and the wife to face an uncertain life span but can survive no longer than 99. When one household member dies, the survivor remains in the household gets all the asset and living alone afterwards. For simplicity, we rule out divorces or remarriages. We call a household is *alive* when there is at least one member alive. Thus, a household's life span is defined by the maximum of the two household members' age of death.

We assume that markets are incomplete. There is no insurances available against earnings and mortality risk. The household can only trade one-period risk-free bonds with real interest rate $r$.

During the working years, each individual supplies one unit of labor and receive wage income $y_t$ at each period. Labor earnings can be decomposed into two part, the deterministic part $g(x_t)$ and the random part $v_t$. $x_t$ is a vector of individual characteristics. $v_t$ is a random variable which represents productivity shock. Detailed information on the income evolving process can be found below in section (4.1). In this paper, for computational simplicity, it is assumed that only husband's income is random. After retirement, individuals receive yearly pension income $p_t$ which is a function of income history $\{y_t\}_{t=30,..,66}$. There is no uncertainty on pension income level once the whole income path is realized.

## 3.1 Preference and the household's problem

The periodic utility function for husband and wife is denoted as $U(c_t)$, where $c_t$ is the household consumption. At period $t$, the husband and the wife solve for the consumption level $c_t$ that maximizes the corresponding remaining life time utility:

$$\max_{\{c_t \geq 0\}} u(c_t) + E\sum\nolimits_{i=t+1}^{T} \beta^{i-t} u(c_i). \tag{4}$$

Where $0 < \beta < 1$ is the subjective discount factor. Despite the couples value the household consumption exactly the same way, they will generally not agree on the consumption and savings decisions when there is mortality risk. Typically, women want to save more than men do since they have lower pension entitlements and expect to live longer. Following the collective approach of household behavior, we model the household's decision as an outcome of a cooperative bargaining process. We assume that the households maximize a weighted sum of the husband's and the wife's expected future utility with a bargaining parameter $\lambda$. If $\lambda = 1$, the household behaves as if the husband has exclusive decision-making control. In this paper, $\lambda$ is assumed to be constant over time and is not affected by the realization of income risks. Similar settings can be found in Cubeddu and Rios-Rull (1997) and Domeij and Klein (2002).

The household members are assumed to have the *same* constant relative risk aversion (CRRA) preference:

$$u(c) = \frac{(c/\eta)^{1-\gamma} - 1}{1-\gamma}, \ \gamma > 0.$$

where $\eta$ is the household equivalence scale which is defined as $\sqrt{N}$, where $N$ is the number of household members. $1/\gamma$ is the inter-temporal substitution elasticity between consumption in any two periods, i.e., it measures the willingness to substitute consumption between different periods. The smaller $\gamma$ (the larger $1/\gamma$) the more willing is the household to substitute consumption over time. One of the properties of the CRRA function is that $u'''(c) > 0$, implying a positive motive for precautionary saving. This feature is very useful since we want to study the effect of risks in the life cycle model.

If one member in the household dies, the surviving one will get the remaining asset. If the household dies, the remaining asset $a$ is regarded as a bequest, which gives utility

$$u_b(a) = \theta \frac{a^{1-\gamma} - 1}{1-\gamma}.$$

$\theta$ measures the importance of bequest motive. When $\theta = 0$, there is no bequest motive.

The budget constraint (in the working period) is given by

$$\begin{aligned} c + a_{t+1} &= Ra_t + y_t^m + y_t^f - T(a_t, y_t^m, y_t^f, r), \\ a_{t+1} &\geq -\underline{a}_{t+1}. \end{aligned}$$

Where $T$ is a tax function. There are taxes on both labor income and capital gain, but not on wealth. The unit of tax calculation is household. We assume that the real interest rate and the tax function are given exogenously. $\underline{a}_{t+1}$ is the maximum amount that the household can borrow at age $t$. We assume that $\underline{a}_{100} = 0$, so no one can borrow at age 99, since they will die with probability one next period. For all the other periods, we assume that the borrowing limits are the same and equal to $\underline{a}$.

# 4 Calibration and Solution method

## 4.1 Earnings Process

We assume that individuals' earnings over time follow an exogenously given stochastic process, which can be decomposed into a deterministic part and a random part. Denote $y_{it}$ as the income which contributes to pensions for individual $i$ of age $t$. We have:

$$\tilde{y}_{it} = \ln(y_{it}) = w_{it} + v_{it} \tag{5}$$

where $w_{it}$ is the deterministic component of age-specific income which can be interpreted as permanent income, attributable to age, education levels, age dummies and year dummies. The component $v_{it}$ is the random component of an agent's income.

There are several models in the literature available for the study of income dynamics over time. Jia and Zhu (2009a) analyzes the dynamic structure of income of male adults in Norway for the period 1992 to 2005, and found that an extended "profile heterogeneity model" fits the data well. Here, we apply a simpler version of the model used there and assume that $v_{it}$ has the structure:

$$v_{it} = \alpha_i + \beta_i e_{it} + z_{it}, \tag{6}$$

where $\alpha_i, \beta_i$ are time-invariant individual specific effects, which are obtained at the start of working life. $e_{it}$ denotes the potential labor market experiences. $z_{it}$ is period specific

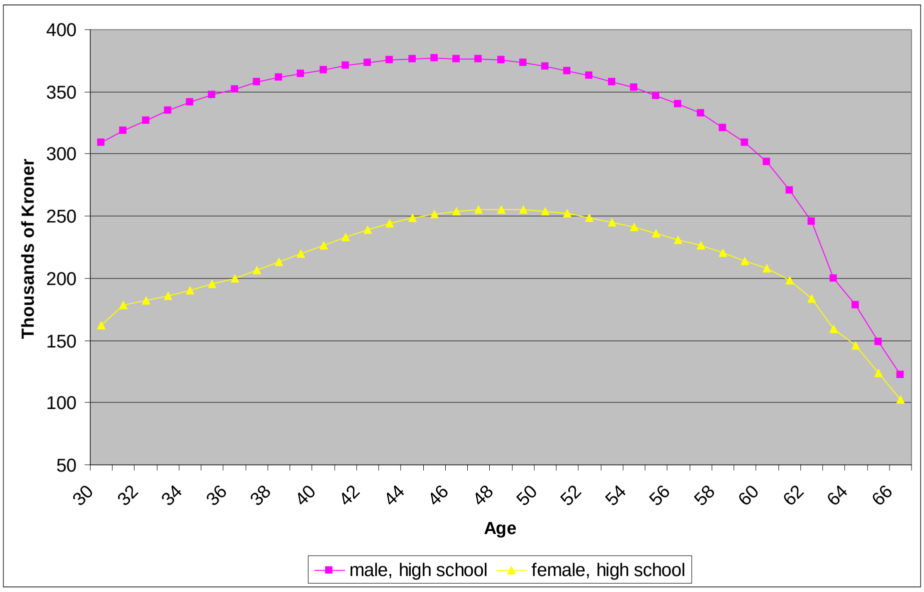


Figure 1: Age income profile, high school graduates

earnings shock which is independent of $\alpha_i$ and $\beta_i$. We assume that $z_{it}$ is realized at the beginning of each period over the life cycle. It follows $AR(1)$ process:

$$z_{it} = \rho z_{it-1} + \delta_{it} \tag{7}$$

$\rho$ is the degree of persistence in the income shocks. Furthermore, we assume: $\alpha_i \sim i.i.d.$ $N(0, \sigma_\alpha^2)$, $\beta_i \sim i.i.d.$ $N(0, \sigma_\beta^2)$, $\delta_{it} \sim i.i.d.$ $N(0, \sigma_\delta^2)$.

Based on the Norwegian registered income tax files covering the period 1992-2005, we estimate the above model in two steps. In the first step, we run a regression using equation (5). We then proceed to estimate the covariance parameters using a Minimum Distance Estimator on the predicted residuals.

Separated regressions are run for married male and females of age 30 to 66. Figure (1) displays the estimated deterministic part of age-earnings profiles for male and female with high school education.

As we mentioned earlier, we will only look at the income risk for husbands for computational simplicity. Income for wives are assumed to be exogenously given and equal to the deterministic part of income regression. Table (1) reports the coefficient estimates for equation (6) and (7) for males. What we can see is that the individual specific unobserved effects $\alpha_i$ and $\beta_i$ account for the majority of the unexplained crosssection income variations.

We then approximate the AR(1) process by a five-state Markov process using the method by Tauchen (1985). Flodén (2008) examined the accuracy of methods used to approximate AR(1) processes with discrete Markov chains and found this method behaves quite well.

| | estimate | t-value |
|---|---|---|
| var($\alpha$) | 1.1411 | 84.0 |
| var($\beta$) | 0.0014 | 81.8 |
| cov($\alpha, \beta$) | -0.0373 | -78.3 |
| $\rho$ | 0.4363 | 123.2 |
| var($\delta$) | 0.1021 | 215.3 |

Table 1: Estimates of the AR(1) income process, Norwegian adult males

$$\begin{pmatrix} 0.1307 & 0.6164 & 0.2459 & 0.0071 & 0.0000 \\ 0.0285 & 0.4259 & 0.4985 & 0.0468 & 0.0003 \\ 0.0037 & 0.1820 & 0.6288 & 0.1820 & 0.0037 \\ 0.0003 & 0.0468 & 0.4985 & 0.4259 & 0.0285 \\ 0.0000 & 0.0071 & 0.2459 & 0.6164 & 0.1307 \end{pmatrix}$$

Table 2: Probability Transition Matrix for the AR(1) Approximation

The approximation process is characterized by a vector of support points:

$$z = \{-1.142, -0.571, 0.000, 0.571, 1.142\},$$

and a probability transition matrix given in Table (2).

## 4.2 The public pension scheme

In the calibration and main simulations, we use a somewhat stylized version of the Norwegian old age pension scheme provided by the National Insurance System. It is a defined benefit system and has two components: a basic amount and an earnings related component. See for example, Jia (2005) for a more detailed description of the Norwegian pension system. Other types of pensions, such as occupational pensions are ignored in this study.

The pension income can be seen as a function of the working-life accumulated income. The relationship between average life time earnings and pension income is illustrated in Figure (2).

## 4.3 Surviving probability

The households enter the economy at age 30, retire at age 67 and are dead by age 100. The survival probabilities for each household member depend on age and sex, and are taken from the life tables in 2004 from Statistics Norway. For the case without life-time uncertainty, we assume both members of households die at age 80, which is approximately the average life expectancy in Norway.

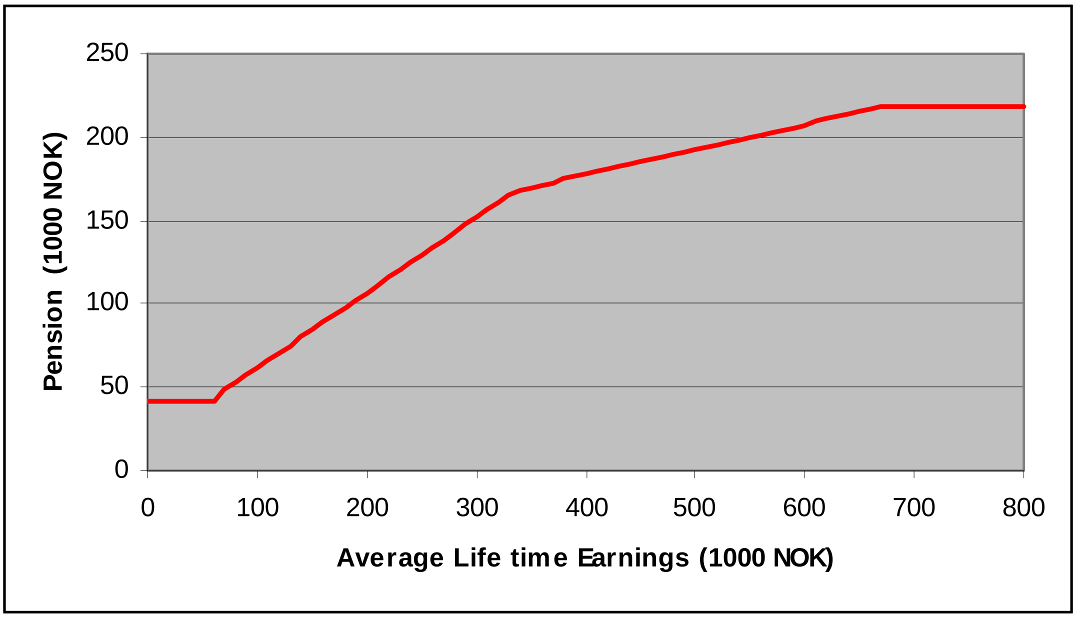


Figure 2: A Stylized Norwegian Pension System

| Variables | | Calibrated Value |
|---|---|---|
| subject discount factor | $\beta$ | 0.9512 |
| borrowing constraints | $\underline{a}$ | -220,000 NOK |
| parameter for bequest | $\theta$ | 3.2 |

Table 3: Calibration Result

## 4.4 Preference and life cycle-parameters

Preference and life cycle parameters are important to our simulation results. However, a full estimation of the model is numerically difficult. Thus, we choose a simpler method, where we set the values for some parameters and calibrate the rest. We set the coefficient of relative risk aversion $\gamma$ to 2, which is used in many studies in the literature, see for example, Domeij and Klein (2002). The risk free rate of return is set to 4%, close to an average real interest rate in Norway over past 30 years. The bargaining power parameter is set to 0.5.

The parameters remain to be determined are: the subjective discount factor $\beta$, the preference parameter represents the bequest motive, $\theta$ and the borrowing limit $\underline{a}$. Based on the Norwegian household data from 1992 to 2005, we calibrate the model using three statistics: the ratio of average wealth to average income among 30 to 66-year-old households; the ratio of working households with positive wealth; and the average bequests. The observed value for these three statistics are 1.915, 0.81, 525,000 NOK[2] respectively.

The calibration results are given in Table (3).

[2] 1 Euro is approximately 7.8 NOK in 2005.

| State variables | | properties |
|---|---|---|
| Age | $t$ | Deterministic(Exogenous) |
| Income shock to husband | $z^h$ | Income process(Exogenous) |
| Accumulated pension claim, husband | $APC$ | Exogenous(updating through income) |
| Household mortality status | $S$ | Endogenous |
| Asset | $a$ | Endogenous |
| Control variable | | |
| Consumption | $c$ | |

Table 4: State variables and control variable

### 4.5 Numerical solution to the households' problem

As the period utility function is CRRA, the analytical solution for the maximization problem (4) does not exist. We solve this problem by numerical methods.

The state variables are summarized in Table 4. In order to numerically solve this problem, some discretizations are needed for the continuous state variables.

We have already discretized the income shocks, $z$. The pension claim after retirement is simplified a bit such that it is related only to the average income over the working years. We use a simple variable to keep track of the shock history over the years. For age $t$ , we define a variable $APC$ as the ratio of average realized income to the average deterministic income:

$$APC_{it} = \frac{\sum_{s=1}^{t} \exp(y_{is})}{\sum_{s=1}^{t} \exp(w_{is})},$$

with updating rule:

$$APC_{it+1} = (APC_{it} * \sum_{s=1}^{t} \exp(w_{is}) + y_{it+1}) / \sum_{s=1}^{t+1} \exp(w_{is}).$$

We then discretize it equally by 100 grid points within the range $[\exp(-1.15), \exp(1.15)]$[3].

We use 200 equally spaced grid points for asset. The range of the wealth at the end of each period is set to be $[\underline{a}, 6,000,000]$, where $\underline{a}$ is the borrowing limit for the household.

To solve the problem efficiently, we use the method of endogenous grid points proposed by Carroll (2006). To do this, we introduce a variable $\mu_t$ which Carroll (2006) called "cash on hand" in period $t$. The idea is to generate the optimal pair $(\mu_t, c_t)$ given other state variables' value while the grid on $\mu_t$ is not predetermined but endogenously generated from the end of period asset $a_{t+1}$. This greatly decreases the computational burden since "expectations are never computed for any grid-point not used in the final interpolating function" (Carroll (2006), p315). The detailed backward induction setup is given in Appendix A.

[3]The boundaries is derived from the $AR(1)$ approximation results.

# 5 The offset effect

The main purpose of this study is to look at to what extent the pension wealth reduces savings when we extend the basic life cycle consumption models to allow for different risks, imperfect credit market and bequest motivations. During the working life, the backward induction solution for the optimal private wealth gives the relationship between the present value of cash wages earned up to date, the present value of expected lifetime earnings, and the present value of future pension benefits. That is, we can write wealth as a function of present value of expected pension wealth $Ep_i^t$ and other state variables $X_i^t$:

$$a_i^{t+1} = f(X_i^t, Ep_i^t).$$

Under the complete market life cycle model, this function is linear and offset effects can be defined straightforwardly as the parameter in front of the expected pension wealth $Ep_i^t$ and it is equal to the partial derivative $\partial a_i^{t+1}/\partial Ep_i^t$. In the case of the extended models, this is no longer true. The standard practice is to estimate a linear approximation of this function. However, the specification of this linear approximation plays very important roles and there is no consensus in the literature. Fortunately, we can simulate partial derivative $\partial a_i^{t+1}/\partial Ep_i^t$ easily here. We only need to obtain the optimal consumption policy functions at each age under two different pension systems where one is simply a small uniform shift of the other, and apply a simple finite difference approximation to calculate the derivatives.

In the literature, the term "perfect offset" is linked to the prediction from the simplest life cycle model where people save solely to smooth the consumption, since in this model consumption in each period depends only on the present value of total income (wages and pensions), but not on the allocation of compensation between them. Typically, it is understood as the situation in which there is a dollar-for-dollar reduction in accumulated private savings with accumulated pension wealth. It is thus desirable to have a definition of offset that gives a value of -1 in this case to be consistent with other studies in the literature.

If we simply use the the partial derivative $\partial a_i^{t+1}/\partial Ep_i^t$, the offset effect for the analytical example given in section 2 when $\sigma^2 = 0$ is equal to

$$\frac{-1}{(1+\beta+\beta^2)},$$

which is not $-1$. This "bias" has been pointed out by Gale (1998) and he suggested to adjust the parameter by a factor $1/Q(t)$ when time is continuous. In the case where time is discrete, it can be shown that

$$Q(t) = \left(\sum_{s=1}^{t}(1+r)^{t-s+s/\gamma}\beta^{s/\gamma}\right) / \left(\sum_{i=1}^{T}\beta^{i/\gamma}(1+r)^{i/\gamma-i}\right). \tag{8}$$

Detailed calculation can be found in Appendix B[4]. In our analytical example, we have $\gamma = 1$, $T = 3$ and $t = 1$, using (8) we have

$$Q(1) = 1/(1 + \beta + \beta^2).$$

Appling this factor to the estimated partial derivative will correctly give us the perfect offset "$-1$".

In the light of above discussion, in the rest of this paper, we use the following offset parameter definition:

$$\kappa = \frac{\partial a_i^{t+1}}{\partial Ep_i} Q(t)^{-1} \tag{9}$$

Note that although it is a little counter-intuitive, it is absolutely possible for $\kappa$ to be smaller than $-1$. Recall that in the analytical example when there is income risk, the reduction is actually bigger than the situation when there is no income risk, which leads to a $\kappa$ smaller than $-1$. On the other hand, a positive $\kappa$ is also possible. Positive offset implies that increased pension wealth actually increases other savings, which can be a result of so called "the induced retirement effect". So although we typically find estimated offset between $-1$ and 0 in the literature, the true value domain has no such restriction.

## 5.1 Simulation Results

We run several simulations to look at the consequences of including the income risk, mortality risk, borrowing constraints and bequest motive in the complete market life cycle model. The "bias" corrected offset parameter over life cycle is calculated for each case. We try to shed some lights on the roles of each component.

In theory, bequest motive will increase the private savings since it is similar to the case where the planning horizon is increased but there is zero income for the extended periods. The bequest motive will dampen the offset effect simply because the increased pension wealth has to be distributed to the bequest as well.

The effect of borrowing constraint on savings and the offset parameter is relatively straightforward as well. When households are subject to a specific borrowing constraints, they are not able to increase their debts, although it is optimal for them to do so. In this case, we will observe no offset at all. Figure (3) shows the estimated offsets over the working life for three different cases of credit rationing, 220,000 NOK, 500,000 NOK and

[4] This adjustment parameter is attained under the assumption that the pension wealth increase is recognized before the working life starts. Some may consider this assumption somewhat restrictive. However, similar adjust factors can be calculated under the assumption that the pension wealth increase is recognized at any age $t$ during the working life. Under the standard life cycle model, as long as we keep the adjust factor and the simulation of the partial derivatives $\partial a_i^{t+1}/\partial Ep_i^t$ consistent, the value of $\kappa$ will always give -1. Under the extended model, the results may differ. However, this is no right or wrong using either of these two methods. We stick to the first one since we want to compare the results we get to those in Hernæs and Zhu (2007).

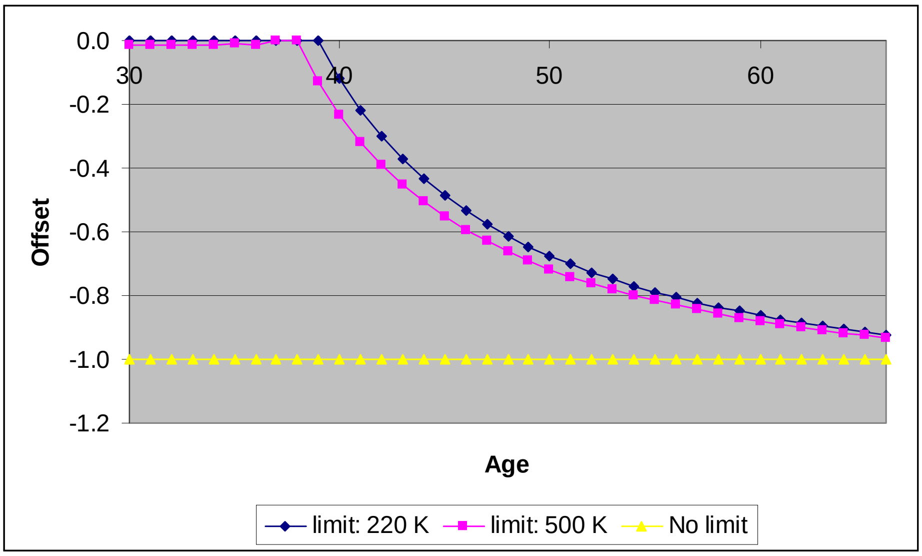


Figure 3: Offset effects: Different borrowing limits.

no limit, when the basic model is extended only with borrowing constraint. It shows that the borrowing constraint have a quite strong effect on the offset estimates. As we expected, the tighter the constraint is, the lower the offset.

Now we look at income risk. Figure (4) shows a single simulation of savings for a household with high school education when there is only income risk. The savings profile over the working life exhibits the so called "hump shape". The household borrows in the early years of working life, then saving starts to rise and reaches its peak around end of 50s. The saving fluctuates over the working life with household income, while consumption is relatively stable over the time. These features of life cycle model with stochastic income have been well-known in the literature. Figure (5) shows the average offset effect estimate of 10, 000 simulations. We see that the offset parameter is slightly lower than -1 (the perfect offset case) for all the working period. This is consistent to what is shown in the analytical example. Income risk tends to increase the offset effect of pension wealth on private savings, even with a different preference and much more complicated income process. It increases towards -1 as the household gets older and income risks over the life are realized. Another conjecture we had when we discuss the analytical example is that the higher the volatility of the income risk is, the lower the offset will be. To test this idea, we double the variance of the AR(1) error terms $z_{it} = \rho z_{it} + \delta_{it}$. Simulation result is presented in Figure (5) as well. It confirms our conjecture.

However, the estimated offset in this case is very close to -1, which suggests that income risk does not seem to influence the offset behavior much – given our estimated income risk. It can be argued that our estimated income risk is far too small, which leads to the small effect on offset. However, according to our estimated income process, there is around 18% chance that the wage income will decrease by 45% in next period. This risk is certainly

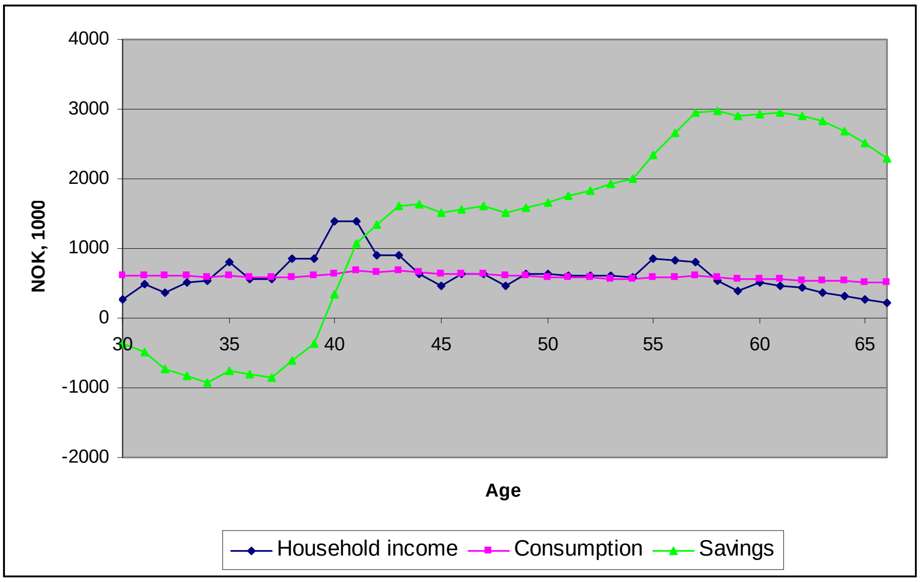


Figure 4: Age Savings profile with only income risk, a single simulation

not small.

Now we look at the influence of mortality risk. Note that we assume that there is no insurance available. It is not possible for any single individual household to borrow. However, when both members of household are alive, mortality risk does not automatically imply non-negative restriction on asset,since we have assumed that at most one member can die at any given year (other than 100). There are three effects which work on the different direction on savings. Firstly, when mortality risk is introduced, households have a longer planning horizon which has also longer period with lower income (years after retirement), so it may be desirable to save more. Secondly, as mentioned above, mortality risk prevents single household to borrow. This also will increase savings. Thirdly, the introduction of mortality risk gives lower discount factor (the discount factor times the probability of surviving), which tends to lower the savings. Figure (6) shows the age-savings profiles for household whose members are both alive at 66, when there is only mortality risk present. It shows that given our chosen parameters, the first two effects dominate over the whole working life.

Including the mortality risk lowers the offset considerably compared with the prediction from the complete market life cycle model. See Figure (7). In the same figure, the average offset effect profile under the full extended model is also presented. That is the case where all the four factors are considered. The offset effect is nearly zero at the start of working life. It increases as age increases. The average offset for age 45-50 is found to be around 45%.

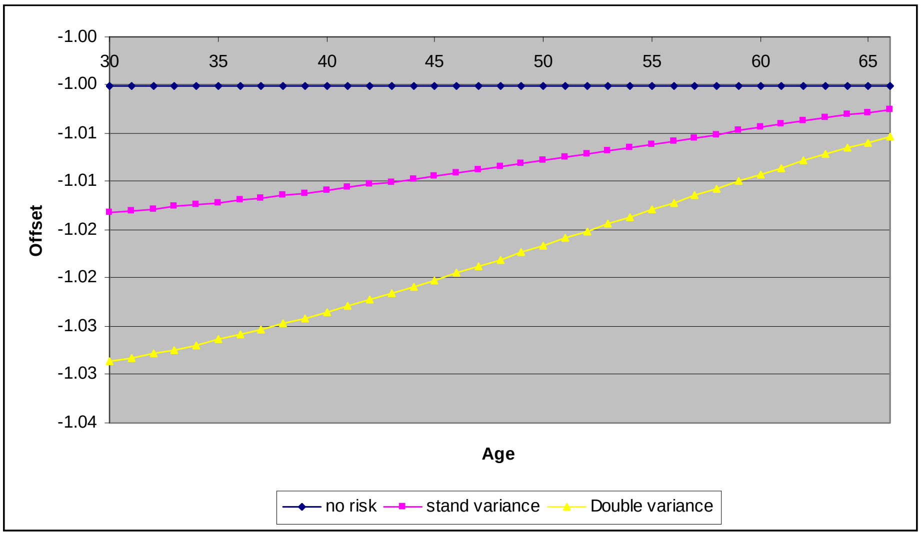


Figure 5: Offset Effect: different income risk.

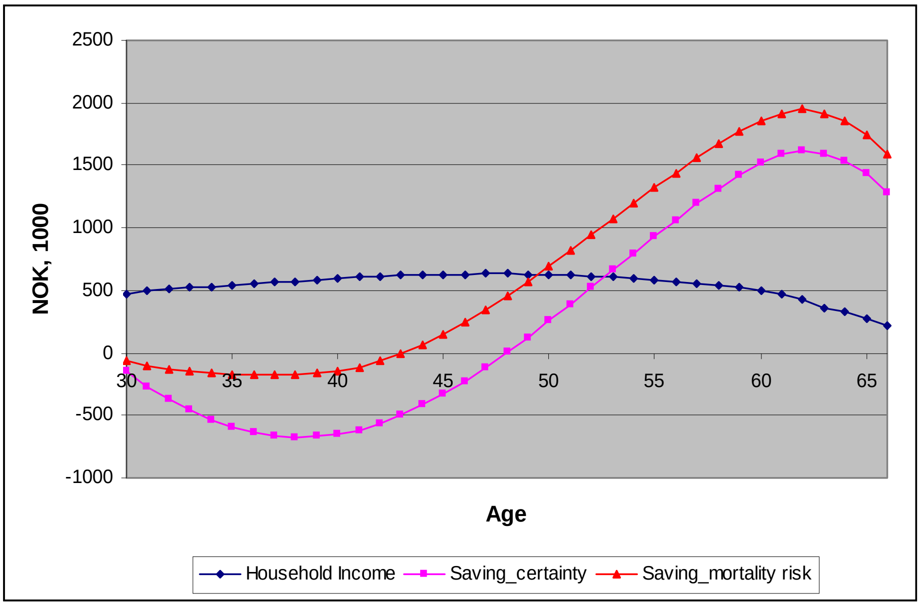


Figure 6: Savings profile, with and without mortality risk.

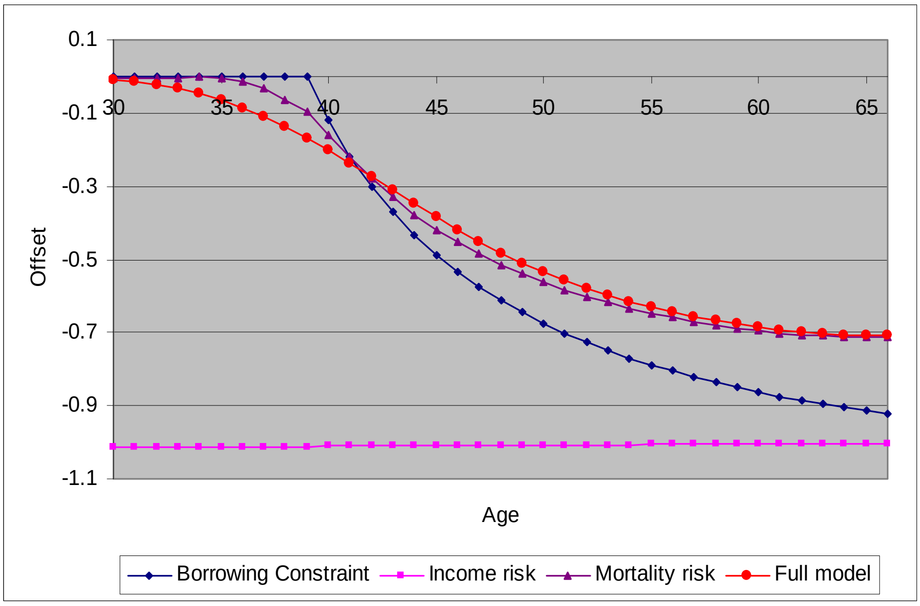


Figure 7: Offset Effect over the Working Life

## 5.2 Further discussions

In this section, we discuss other issues which may influence the results, such as the bargaining power within the household, other pension schemes, and the size of the income risk etc.

### 5.2.1 Bargaining Power within the household

In the above simulations, we have assumed that the household members have equal saying on household consumption and savings decisions over the life cycle. As mentioned earlier, women live longer and have lower income and pensions. Compared with men, they have extra incentives to save more. Therefore, the choice of bargaining power parameter will influence the savings behavior as well as the offset parameters over the life cycle. Figure (8) depicts savings behavior over the working life for two values of bargaining power parameter: husband has lower bargaining power, $\lambda = 0.2$, the same power as the wife, $\lambda = 0.5$. As we expected, the household's saving increases as the husband's bargaining power declines. The differences of the accumulated wealth around 60 years old between equal weight households ($\lambda = 0.5$)and wife dominated households ($\lambda = 0.2$) is around 200, 000 NOK. With regards to the offset parameter, wife dominated households are less willing to substitute private wealth with increased pension wealth. The differences are sizable (Figure (9)).

### 5.2.2 A lump-sum pension system

While preferences and saving motives are important to answer the question how a pension system affects savings behavior, the pension system's own design is also crucial. The current Norwegian pension system is essentially a defined benefit, earnings related system. Here,

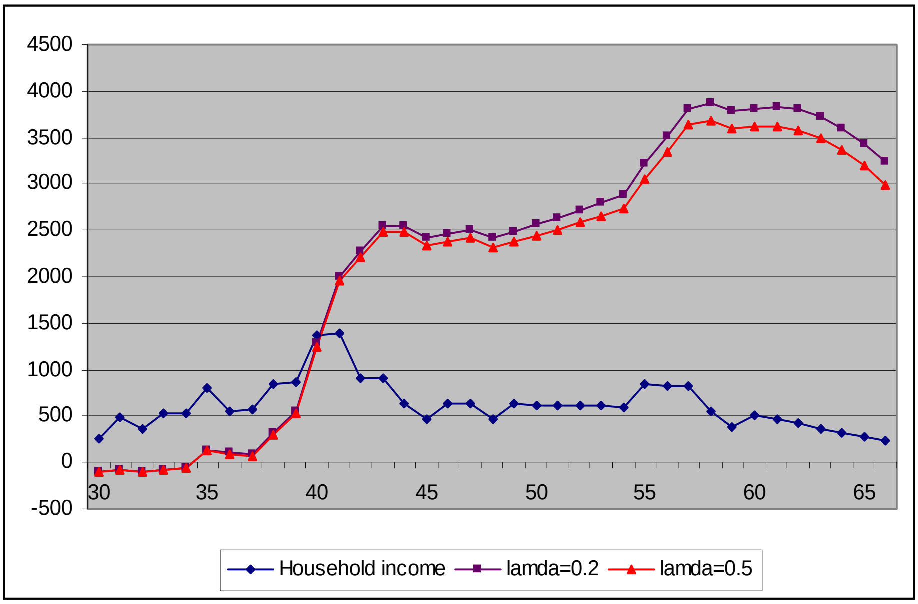


Figure 8: Savings profile: different bargaining power within household

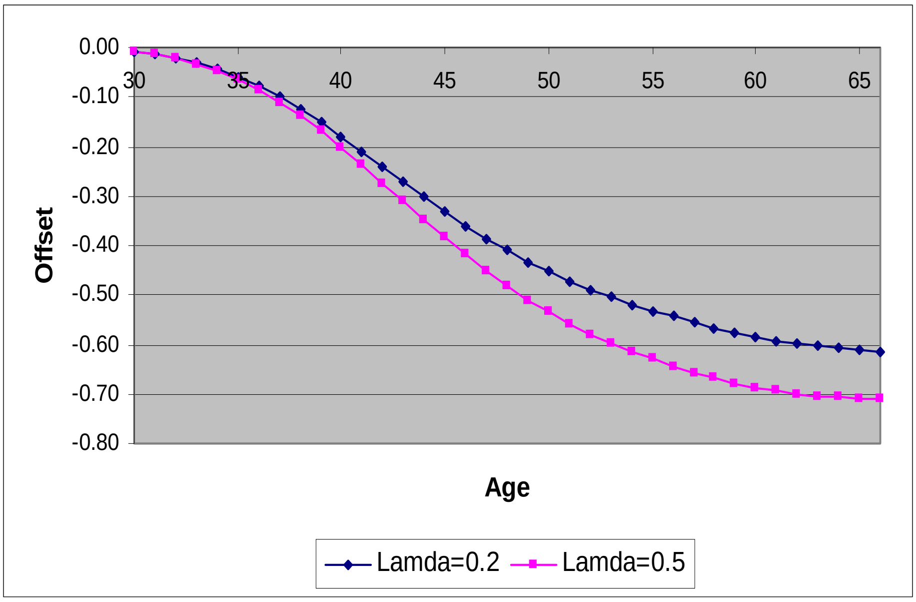


Figure 9: Offset Effects: different bagarning power within household.

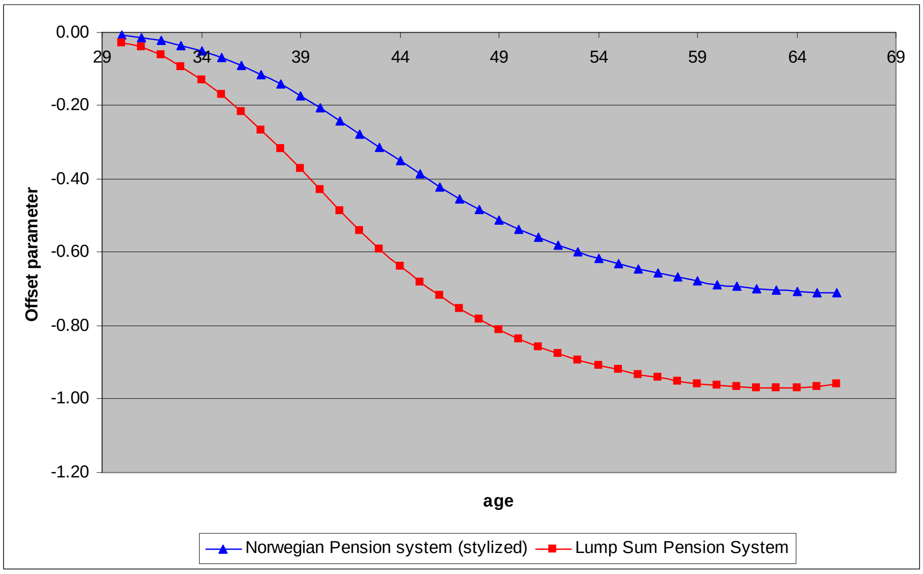


Figure 10: Offset effects under two different pension schemes.

we consider a lump-sum pension system where benefits do not depend on past earnings. A similar pension system is also discussed by Domeij and Klein (2002). The benefit level is set at a value which is just enough to cover the basic living needs, i.e. one basic amount (56,000 NOK in 2003). In short, this pension system serves mainly as income security for the elderly.

Figure (10) shows the offset estimates under these two pension schemes. Households are more willing to substitute private wealth with pension wealth under the lump-sum pension system. One explanation is that when pension income doesn't depend on past earnings, there is actually less risks facing the households. So their behaviors are more similar to the case that the complete market life cycle model predicts.

### 5.2.3 Rich households and poor households.

An important empirical pattern found both in Hernæs and Zhu (2007) and Jia and Zhu (2009b), among other empirical studies on offset effects, is that rich households typically are more willing to substitute private wealth with pension wealth. It leads to a higher absolute value of the estimated offset parameter for the richer. In the complete market life cycle model, this will not happen. No matter rich or poor, the offset will be always -1. Many explanations are proposed to explain this pattern. For example, some argue that compared with rich households, low income households may have limited liquidity and face more credit rationing, which makes the offset much smaller. Others note the correlation between earnings and education. They argue that higher educated households are more knowledgable on financial planning, pension rules etc, which leads to behaviors more similar to a life cycle model predicts.

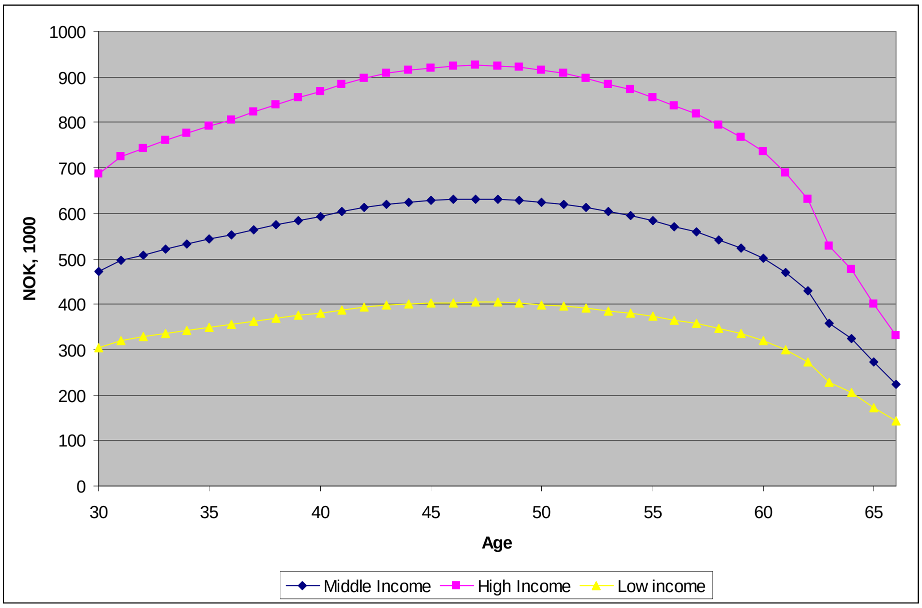


Figure 11: Income trends: rich, middle and poor household

In our simulations below, we simply construct three different income trends: high, middle and low. The stochastic part of the income process is the same AR(1) presented in section (4.1). All households face the same credit constraints and have the same discounting factor. The income trends are plotted in Figure (11) and the offset estimates are presented in Figure (12). The simulation results show a clear pattern: higher income households offset more. This suggests that although the explanations listed above are possible and reasonable, the differences on the offset effects between rich and poor households can be generated without these factors present.

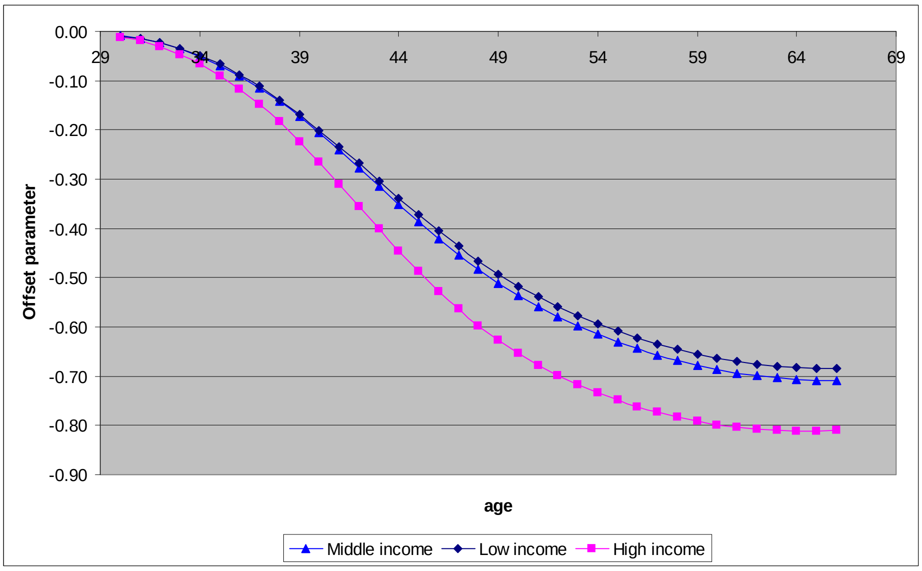


Figure 12: Offset parameters: Rich and Poor Households

We also have done some sensitivity tests on rate of return $r$, the inter-temporal substitution elasticity $\gamma$. The quantities of the simulated offsets change with these parameters. But the main picture remains.

# 6 Conclusion

In this paper, we analyze an extended model of life cycle savings decision. It adds income risk, mortality risk, borrowing constraints and bequest motivation to the complete market life cycle model. The model is then used to investigate how these factors influence the substitution between private and pension wealth over the working years. The simulation results suggest that when households are under influences of these above mentioned factors, they tend to reduce less private savings with the same pension wealth change, compared with the case when they are not. The average offset for age 45-50 is found to be around 45%, which is still somewhat higher than Hernæs and Zhu (2007) and Jia and Zhu (2009b) found in their studies. However, our result shows that by simply accounting these risks and constraints, we can explain most of the departure between empirical findings and theoretical prediction. It seems that borrowing constraints and mortality risk plays more important roles. The interaction between the different factors discussed in this paper are quite complicated. Once either mortality risk or borrowing constraints are taken into account, adding in the other factors will not change the offset effect much. Back to the policy question raised earlier in this paper, this seems to imply that popular policy proposals such as removing the capital market imperfection or increasing job market security will not contribute much to increase the responses to pension reforms.

This study also shows that there may be considerable differences on the behavior responses to pension wealth shift even when there is no difference on "taste" of savings. The offset effects depend on many factors, such as life time expected labor earnings, bargaining power within the household and income shock volatilities. Since these factors are often unobserved to econometricians, how to resolve these issues in the empirical investigations of the relationship between private savings and public pension wealth remains a difficult challenge.

## References


Attanasio, O. P., and A. Brugiavini (2003): "Social Security and Household Saving," *Quarterly Journal of Economics*, CXVIII, 1075–1119.

Auerbach, A., and L. J. Kotlikoff (1995): *Macroeconomics: an intergrated approach.* South-Western college publishing.

Blanchard, O. J., and S. Frischer (1989): *Lectures on Macroeconomics.* MIT Press.

Boyle, P., and J. Murray (1979): "Social Security Wealth and Private Saving in Canada," *Canadian Journal of Economics*, 12, 456–468.

Cagan, P. (1965): "The Effect of Pension Plans on Aggregate Saving," Columbia University Press, New York.

Carroll, C. D. (1992): "The Buffer Stock Theory of Savings: Some Macroeconomic Evidence," *Brookings Papers on Economic Activity*, 2, 61–135.

——— (1997): "Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis," *Quarterly Journal of Economics*, 112, 1–55.

Carroll, C. D. (2006): "The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems," *Economics leters*, 91(3), 312–320.

CBO (1998): "Social Security and Private Saving: A Review of the Empirical Evidence," Congressional Budget Office, Memorandum.

Cubeddu, L., and J.-V. Rios-Rull (1997): "Marital Risk and Capital Accumulation," Discussion paper, Federal Reserve Bank of Minneapolis Research Department Staff Report 235.

Deaton, A. (1991): "Savings and Liquidity Constraints," *Econometric*, 59(5), 1221–1248.

Domeij, D., and P. Klein (2002): "Public Pensions: To What Extent Do They Account for Swedish Wealth Inequality," *Review of Economics Dynamics*, 5, 503–534.

Euwals, R. (2000): "Do Mandatory Pensions Decrease Household Savings? Evidence for the Netherlands," *De Economist*, 148(5), 643–670.

Feldstein, M. (1974): "Social Security, Induced Retirement, and Aggregate Capital Formation," *The Journal of Political Economy*, 82, 905–926.

——— (1982): "Social Security and Private Saving: Reply," *The Journal of Political Economy*, 90, 630–642.

Feldstein, M., and A. Pellechio (1979): "Social Security and Household Wealth Accumulation: New Microeconometric Evidence," *The Review of Economics and Statistics*, 61, 361–368.

Flodén, M. (2008): "A Note on the Accuracy of Markov-chain Approximations to Highly Persistent AR(1) Processes," *Economics Letters*, 99, 516–520.

Gale, W. G. (1998): "The Effects of Pensions on Household Wealth: A Reevaluation of Theory and Evidence," *Journal of Political Economy*, 106, 706–723.

Hernæs, E., and W. Zhu (2007): "Pension Entitlements and Wealth Accumulation," Memorandom, No 12/2007, Department of Economics, University of Oslo.

Jia, Z. (2005): "Retirement Behavior of Working Couples in Norway: A Dynamic Programming Approach," Discussion paper No. 405, Research Department, Statistics Norway.

Jia, Z., and w. Zhu (2009a): "The Dynamic Structure of Male income in Norway, 1992-2005," Unpublished manuscript, Statistics Norway.

Jia, Z., and W. Zhu (2009b): "Heterogeneity in the Offset Effect of the Pension Wealth on Other Private Wealth," Unpublished manuscript, Frisch Centre, 2009.

Katona, G. (1965): "Private Pensions and Individual Savings," Discussion paper, Ann Arbor: University Michigan, Survey Research Center, Instistute for Social Research.

Lindbeck, A., and M. Persson (2003): "The Gains from Pension Reform," *Journal of Economic Literature*, 41, 74–112.

Love, D. (2006): "Buffer Stock Saving in Retirement Accounts," *Journal of Monetary Economics*, 53, 1473–1492.

Rodepeter, R., and J. Winter (1998): "Savings Decisions Under Life-time and Earnings Uncertainty," SonderForschungsBereich 504 working paper, No. 58.

Skinner, J. (1985): "Variable Lifespan and the Intertemporal Elasticity of Consumption," *Review of Economics and Statistics*, 67, 616–623.

Skinner, J. (1987): "Risky Income,Life cycle Consumption and Precautionary Savings," Discussion paper, NBER Working Paper Series, working paper No. 2336.

Tauchen, G. (1985): "Finite State Markov Chain Approximation to Univariate and Vector Auto regressions," *Economics Letters*, 20, 177–181.

Yaari, M. (1965): "Uncertain Lifetime, Life Insurance and the Theory of the Cconsumer," *Review of Economic Studies*, 32, 137–150.

Zeldes, S. P. (1989): "Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence," *Quarterly Journal of Economics*, 104, 275–298.

——— (1992): "Comment to "The Bud'er Stock Theory of Savings: Some Macroeconomic Evidence," *Brookings Papers on Economic Activity*, 2, 141–149.

## Appendix A: Backward Induction

We illustrate the backward induction here by exploiting the case with certain life span for a single individual.

The backward induction starts at the end of life span $H$. $P$ is the pension income, which is fixed for given income history. However, we will still treat it as a state variable for illustration purpose.

At period $H$, the value function is defined as:

$$W_H(A_H, P) = \max_c \{u(c) + \theta U_b(A_{H+1})\}, \tag{10}$$
$$\text{s.t.} c + A_{H+1} = (1+r)A_H + P = m_H.$$

where $m_H$ is the cash on hand at time $H$.

We start with a set of grid points for $A_{H+1}$. We solve out the optimal consumption for each grid point of $A_{H+1}$, using the first order condition:

$$u'(c_H^*) = \theta U_b(A_{H+1}).$$

Based on the pair $\{A_{H+1}, c_H\}$, and make use of the relationship

$$m_H + c^* = A_{H+1}.$$

We can generate the grid for cash on hand $m_H$ and the optimal consumption $c^*(m_H)$ on those grid points. The value function $V_H(m_H, P)$ [5]follows immediately.

At period $t = H - 1$,

$$W(A_t, P) = \max_{c_t} \{u(c_t) + \beta W_{t+1}(A_H, P)\} \tag{11}$$
$$\text{s.t.} c + A_t = (1+r)A_{t-1} + P = m_t$$

First order condition gives us:

$$\begin{aligned} u'(c_t^*) &= \beta \frac{\partial W_{t+1}}{\partial A_{t+1}}|_{A_{t+1}^* = (1+r)A_t - c_t^*} \\ &= \beta(1+r)\frac{\partial V_{t+1}}{\partial m_{t+1}}|_{m_{t+1} = (1+r)A_{t+1}^* + P} \end{aligned} \tag{12}$$

[5]We have the relationship $V_H(m_H, P) = V_H((1+r)A_H + P) = W_H(A_H, P)$

Note that, we can rewrite the optimization problem as the following:

$$V_t(m_t, P) = \max_{A_{t+1}} \{u(m_t - A_{t+1}) + \beta W_{t+1}(A_{t+1}, P)\}$$

This implies that (the envelope theorem)

$$\frac{\partial V_t(m_t)}{\partial m_t} = u'(m_t - A^*_{t+1}) = u'(c^*_t(m_t)) \tag{13}$$

Then we can have the relationship

$$u'(c^*_t) = \beta(1+r)\frac{\partial V_{t+1}(m_{t+1}, P)}{\partial m_{t+1}} = \beta(1+r)u'(c^*_{t+1}(m_{t+1})) \tag{14}$$

For any given value of next period asset $A_{t+1,}$ the cash on hand $m_{t+1}$ is generated. Since $c^*_{t+1}(m_{t+1})$ is already known, we can calculate $c^*_t$ using (14). Applying again the cash on hand definition:

$$m_t + c^*_t = A_{t+1}.$$

we now have the optimal consumption $c^*_t(m_t)$ at a grid of$\{m_t\}$.

When there is mortality risk and income risk, (14) becomes

$$u'(c^*_t) = \beta(1+r)E(\frac{\partial V_{t+1}(m_{t+1}, P)}{\partial m_{t+1}}) = \beta(1+r)E(u'(c^*_{t+1}(m_{t+1}))) \tag{15}$$

The main idea is exactly the same.

## Appendix B: Bias correction parameter.

The Euler equation at year t can be written as:

$$u'(c_t) = [\beta R]\, u'(c_{t+1})$$

*i.e.*

$$c_t^{-\gamma} = \beta R c_{t+1}^{-\gamma}$$

we have:

$$c_{t+1} = \beta R^{1/\gamma} c_t \tag{16}$$

From age $t$ to the end of life, the budget constraints will be:

$$\sum_{j=t}^{T} \frac{1}{R^{j-t}} c_j = R a_t + \sum_{j=t}^{R-1} \frac{1}{R^{j-t}} y_j + \sum_{j=R}^{T} \frac{1}{R^{j-t}} p_j$$

use (16), it can be rewrite as:

$$c_t \cdot (\sum_{j=t}^{T} \frac{1}{R^{j-t}} [\beta R]^{\frac{j-t}{\gamma}} - 1) = a_{t+1} + \sum_{j=t+1}^{R-1} \frac{1}{R^{j-t}} y_j + \sum_{j=R}^{T} \frac{1}{R^{j-t}} p_j$$

Define $p^t = \sum_{j=R}^{T} R^{t-j} p_j$, which is the accumulated pension wealth evaluated at age $t$, we have:

$$\frac{\partial a_{t+1}}{\partial p^t} = \left[ \frac{\partial c_t}{\partial p^t} (\sum_{j=t}^{T} \frac{1}{R^{j-t}} [\beta R]^{\frac{j-t}{\gamma}} - 1) - 1 \right] \tag{17}$$

The period consumption can be written as:

$$c_t = \left[ \left( \sum_{j=1}^{R-1} R^{-j} y_j \right) + \left( \sum_{j=R}^{T} R^{-j} p_j \right) \right] [\beta R]^{t/\gamma} / \sum_{j=1}^{T} R^{j/\gamma - j} \beta^{j/\gamma} \tag{18}$$

which is:

$$c_t = Y + p^t \phi_t \tag{19}$$

where $\phi_t$ is: $\phi_t = R^{-t} [\beta R]^{j/\gamma} / \sum_{j=1}^{T} R^{j/\gamma - j} \beta^{j/\gamma}$.

Define $\omega_t = \sum_{j=t}^{T} \frac{1}{R^{j-t}} [\beta R]^{\frac{j-t}{\gamma}}$, use (19), (17) becomes:

$$\begin{aligned} \frac{\partial a_t}{\partial p^t} &= [\phi_t(\omega_t - 1) - 1] & (20) \\ &= -\left( \sum_{j=1}^{t} R^{j/\gamma - j} \beta^{j/\gamma} \right) / \left( \sum_{j=1}^{T} \beta^{j/\gamma} R^{j/\gamma - j} \right). & (21) \end{aligned}$$

Thus

$$Q(t) = \left( \sum_{j=1}^{t} R^{j/\gamma - j} \beta^{j/\gamma} \right) / \left( \sum_{j=1}^{T} \beta^{j/\gamma} R^{j/\gamma - j} \right).$$

**B** *Returadresse*:
Statistisk sentralbyrå
NO-2225 Kongsvinger


**Statistics Norway**

*Oslo*:
PO Box 8131 Dept
NO-0033 Oslo
Telephone: + 47 21 09 00 00
Telefax: + 47 21 09 00 40

*Kongsvinger*:
NO-2225 Kongsvinger
Telephone: + 47 62 88 50 00
Telefax: + 47 62 88 50 30

E-mail: ssb@ssb.no
Internet: www.ssb.no



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