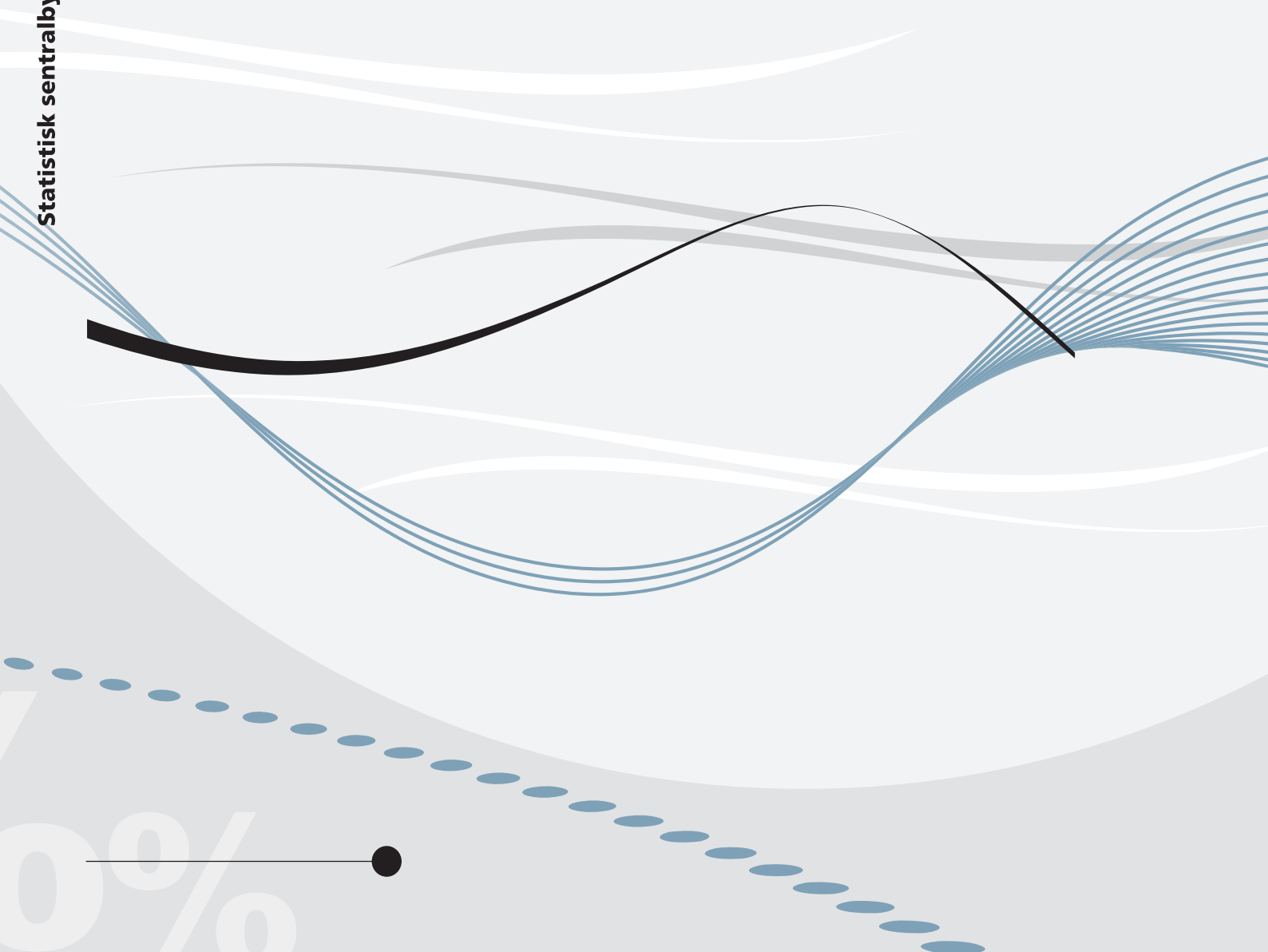




*Pål Boug*

## **Exact and inexact decompositions of international price indices**





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## Exact and inexact decompositions of international price indices

**Abstract:**

Decompositions of international price indices are usually *inexact* in the sense that the underlying aggregator formula is *not* exactly reproduced. In this paper, we compare analytically the exact and inexact decompositions of international price indices, paying particular attention to the bias in aggregate inflation occurring from using the first order Taylor series approximation and not the quadratic approximation lemma to a geometric average of price levels. Our calculations, using the Norwegian clothing industry as an illustration, reveal that the bias in aggregate inflation over the sample period of 1997–2016 is quite substantial and as high as 0.5 percentage points in some years. We therefore conclude that the quadratic approximation lemma should be used in practise to exactly reproduce the underlying aggregator formula.

**Keywords:** International price indices, exact and inexact decompositions, first and second order Taylor series approximation, quadratic approximation lemma, bias in aggregate inflation

**JEL classification:** C43, E31, F14

**Acknowledgements:** I am grateful to seminar participants at Statistics Norway, Walter Erwin Diewert in particular, for helpful discussions, and Thomas von Brasch, Ådne Cappelen, Terje Skjerpen and Anders Rygh Swensen for comments and suggestions on earlier drafts. I am also grateful to Andreas Benedictow for assistance in providing the data used in the empirical illustration. The usual disclaimer applies.

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ISSN 1892-753X (electronic)

## Sammendrag

Dekomponering av internasjonale prisindekser er typisk *ikke* eksakt i den forstand at underliggende aggregeringsformel ikke er reproduisert eksakt. I denne artikkelen, sammenligner vi analytisk eksakt og ikke eksakt dekomponering av internasjonale prisindekser med spesiell oppmerksomhet til feilen som oppstår i aggregert inflasjon når første ordens Taylor approksimasjon og ikke kvadratisk approksimasjonslemma brukes til dekomponering av et geometrisk snitt av prisnivåer. Våre beregninger basert på årlige data fra klesindustrien i Norge viser at feilen i aggregert inflasjon er nokså betydelig i perioden 1997-2016 og så høy som 0,5 prosentpoeng i enkelte år. Vi konkluderer derfor med at kvadratisk approksimasjonslemma bør brukes i praksis for å kunne reproducere den underliggende aggregeringsformelen eksakt.

# 1 Introduction

Classical index number theory generally advocates the use of superlative price index number formulae, including the Fisher, Walsh and Törnqvist price indices; see for example ILO *et al.* (2004). These yield good approximations of the true inflationary effects of international trade given the central assumptions that the importing countries are free to choose between all goods and services and that changes in country composition of imports follow from *changes* in relative price levels between countries. The observed import patterns have, however, changed over time as a result of a gradual liberalization of international trade along with large initial price *level* differences between exporting countries. Thus, aggregating international prices by means of classical index number formulae may deliver significant biased estimates of the true price index by failing to capture the deflationary effects of imports increasingly originating from low price level countries, China in particular.

In this paper, we argue in line with Benedictow and Boug (2017) that these deflationary effects are closely related to what the Boskin Commission calls the outlet substitution bias, which occurs in classical index number formulae due to the failure of adequately accounting for situations where discount outlets capture market shares from high-cost retailers; see Boskin *et al.* (1996). Drawing on Diewert (1998) and White (2000), who point out that a reasonable concept for the true price index in such situations is the average price paid by consumers over all outlets, we employ a geometric average of price levels for a commodity group of interest as the underlying aggregator formula so as also to capture the deflationary effects of changes in country composition of imports following from trade liberalization.<sup>1</sup> Some studies also seek to include the deflationary effects of the observed shifts of imports towards low price level countries by

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<sup>1</sup>The concept of average prices when there is price variation for the same quality of good or service is also recommended in the System of National Accounts 2008; see for example European Commission *et al.* (2009, p. 303). Using geometric or arithmetic averages of price levels for a good or service of interest is one simple and handy method of constructing international price indices. Certainly, there is a vast literature on various methods for constructing price indices, both across space and time, for international comparison purposes; see for example Balk (2008, chapter 7) for an overview. Recently, Brasch (2017) has generalised the standard economic import price index to allow for barriers to trade in the form of quantity constraints.

employing either a geometric or an arithmetic average of price levels; see for example Røstøen (2004), Nickell (2005), ECB (2006), Pain *et al.* (2006), Kamin *et al.* (2006), Wheeler (2008), MacCoille (2008) and Thomas and Marquez (2009). However, because a first order Taylor series approximation is used, the decompositions of international price indices in Nickell (2005) among others are *inexact* in the sense that the underlying aggregator formula is *not* exactly reproduced.

We are therefore motivated to compare analytically the exact and inexact decompositions of international price indices, paying particular attention to the bias in aggregate inflation occurring from using the first order Taylor series approximation and not the quadratic approximation lemma by Diewert (1976) to a geometric average of price levels. It turns out that the bias in aggregate inflation vanishes only in the special cases when inflation rates are equal across exporting countries and/or when no switching of imports occurs from high price level to low price level countries or vice versa. Accordingly, the bias may be significant in practice when applying a first order Taylor series approximation to a geometric average of price levels.

As an empirical illustration, we estimate the bias in aggregate inflation using yearly data from the Norwegian clothing industry, which has experienced massive trade liberalization and increasing imports from China and other low price level countries since the Uruguay Round Agreement starting in the mid 1980s. Our calculations reveal that the bias in aggregate inflation over the sample period of 1997 – 2016 is quite substantial and as high as 0.5 percentage points in some years when using a first order Taylor series approximation. We therefore argue that the quadratic approximation lemma should be applied in practise when decomposing a geometric average of price levels.

The rest of the paper is organized as follows: Section 2 compares analytically the exact and inexact decompositions to the commonly used aggregator formula. Section 3 presents the empirical illustration. Section 4 provides a conclusion.

## 2 Analytical comparison

As pointed out by Diewert (2002), it is well known that a second order Taylor series approximation to a quadratic function, evaluated at two points, will exactly reproduce the quadratic function. It is not so well known, however, that the arithmetic average of two first order Taylor series approximations evaluated at two points will also reproduce a quadratic function exactly, a result called the quadratic approximation lemma by Diewert (1976). We utilise these properties in our context, as reference for comparing the exact and inexact decompositions, by first writing the geometric average of price levels used by Nickell (2005) among others as a quadratic function on the form

$$(1) \quad F(S_t, p_t) = \sum_{n=1}^N S_{nt} p_{nt},$$

where  $(S_{1t}, \dots, S_{Nt}) \equiv S_t$  is a set of  $N$  value shares of imports of a commodity group of interest in period  $t$ ,  $0 \leq S_{nt} \leq 1$  and  $\sum_{n=1}^N S_{nt} = 1, \forall t$ , and  $(p_{1t}, \dots, p_{Nt}) \equiv p_t$  is a set of  $N$  (logarithmic) price levels of a commodity group of interest in period  $t$ .<sup>2</sup>

The second order Taylor series approximation to  $F(S_t, p_t)$  evaluated around period  $t - 1$  is

$$(2) \quad \begin{aligned} \Delta F(S_t, p_t) &= \sum_{n=1}^N F_{S_n}(S_{t-1}, p_{t-1}) \Delta S_{nt} + \sum_{n=1}^N F_{p_n}(S_{t-1}, p_{t-1}) \Delta p_{nt} \\ &+ \sum_{n=1}^N F_{S_n p_n}(S_{t-1}, p_{t-1}) \Delta S_{nt} \Delta p_{nt}, \end{aligned}$$

where  $\Delta$  denotes the difference operator,  $F_{S_n}(S_{t-1}, p_{t-1})$  and  $F_{p_n}(S_{t-1}, p_{t-1})$  are the first order partial derivatives of  $F(S_t, p_t)$  with respect to  $S_n$  and  $p_n$ , respectively, evaluated at period  $t - 1$ , and  $F_{S_n p_n}(S_{t-1}, p_{t-1})$  are the second order partial derivatives of  $F(S_t, p_t)$  with respect to  $S_n$  and  $p_n$ , evaluated at period  $t - 1$ .<sup>3</sup>

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<sup>2</sup>Our analytical framework below builds on Diewert (2002). Whereas Diewert (2002) considers a quadratic function  $F(z_1, \dots, z_N)$  consisting of one set of  $N$  variables defined as  $(z_1, \dots, z_N) \equiv z$ , we consider two sets of  $N$  variables in (1). In what follows, lower case letters indicate natural logarithms of a variable.

<sup>3</sup>The two expressions for the other second order partial derivatives,  $F_{S_n S_n}(S_{t-1}, p_{t-1})$  and  $F_{p_n p_n}(S_{t-1}, p_{t-1})$ , are both equal to zero for all  $n$ .



Similarly, the second order Taylor series approximation to  $F(S_t, p_t)$  evaluated around period  $t$  is

$$(3) \quad \Delta F(S_t, p_t) = \sum_{n=1}^N F_{S_n}(S_t, p_t) \Delta S_{nt} + \sum_{n=1}^N F_{p_n}(S_t, p_t) \Delta p_{nt} - \sum_{n=1}^N F_{S_n p_n}(S_t, p_t) \Delta S_{nt} \Delta p_{nt}.$$

Now, we can apply the quadratic approximation lemma by taking the arithmetic average of the first order Taylor series approximations inherent in (2) and (3) to obtain

$$(4) \quad \Delta F(S_t, p_t) = \sum_{n=1}^N (1/2) [F_{S_n}(S_{t-1}, p_{t-1}) + F_{S_n}(S_t, p_t)] \Delta S_{nt} + \sum_{n=1}^N (1/2) [F_{p_n}(S_{t-1}, p_{t-1}) + F_{p_n}(S_t, p_t)] \Delta p_{nt}.$$

Since (2), (3) and (4) are equivalent and yield *exact* decompositions of (1), it follows that the expression  $\sum_{n=1}^N F_{S_n p_n}(S_{t-1}, p_{t-1}) \Delta S_{nt} \Delta p_{nt}$  from (2) and the expression  $\sum_{n=1}^N F_{S_n p_n}(S_t, p_t) \Delta S_{nt} \Delta p_{nt}$  from (3) define the bias in aggregate inflation, but with opposite signs. We can simplify the expressions for the bias in absolute value,  $B_t$ , as

$$(5) \quad B_t = \left| \sum_{n=1}^N \Delta S_{nt} \Delta p_{nt} \right|,$$

because  $F_{S_n p_n}(S_{t-1}, p_{t-1}) = F_{S_n p_n}(S_t, p_t) = 1$ . Hence, the bias from using *inexact* decompositions to (1) is equal to a weighted sum of underlying country specific inflation rates with the respective changes in the value shares of imports as weights.<sup>4</sup> As such,  $B_t = 0$  only in the special cases when the inflation rates are equal across exporting countries and/or no switching of imports occurs from high price level to low price level countries or vice versa.

To compare the exact and inexact decompositions in more detail, we assume one low price level and one high price level country, apply (4) to (1) and write the *exact*

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<sup>4</sup>Note that  $\Delta S_{nt} = S_{nt} - S_{nt-1}$  and that  $\Delta p_{nt} = p_{nt} - p_{nt-1}$ , which is, due to the use of natural logarithms, approximately equal to the inflation rate given by  $(P_{nt} - P_{nt-1})/P_{nt-1}$ .

decomposition of aggregate inflation,  $\Delta p_t$ , as

$$(6) \quad \Delta p_t = \overline{S}_{1t} \Delta p_{1t} + (1 - \overline{S}_{1t}) \Delta p_{2t} + \Delta S_{1t} (\overline{p}_{1t} - \overline{p}_{2t}),$$

where  $\Delta p_{1t}$  and  $\Delta p_{2t}$  are the inflation rates in the low price level and the high price level country, respectively, in period  $t$ ,  $\overline{p}_{1t}$  and  $\overline{p}_{2t}$  are the average price levels of period  $t$  and  $t - 1$  in the low price level and the high price level country, respectively, and  $\overline{S}_{1t}$  is the low price level country's average value share of imports of period  $t$  and  $t - 1$ .<sup>5</sup> The first two terms on the right hand side of (6) correspond to aggregate inflation when the Törnqvist price index is used as the underlying aggregator formula. The last term,  $\Delta S_{1t} (\overline{p}_{1t} - \overline{p}_{2t})$ , constitutes the deflationary effects of the shifts of imports from the high price level to the low price level country due to lowering of trade barriers. The greater the change in the import share and the greater the difference in relative price levels, the greater the deflationary effects in  $\Delta p_t$ .

Note that the deflationary effects are zero only in the special cases when the import share is constant ( $\Delta S_{1t} = 0$ ), and/or when the composition of trade changes between countries with identical price levels ( $\overline{p}_{1t} - \overline{p}_{2t} = 0$ ). It is therefore likely that the Törnqvist price index, or any other classical index number formula for that matter, fails to account for the deflationary effects in (6). Suppose that the low price level country has relatively high inflation for a particular tradable good and that barriers to trade are reduced. As a result, imports from the low price level country increase at the expense of imports from the high price level country. Using the Törnqvist price index as an import price index will thus only capture the higher inflation and not the lower price level due to the shift in imports. Hence, the Törnqvist price index does not represent the *true* inflationary effects of imports in this case.

Applying the first order Taylor series approximations from (2) and (3) to (1) instead of the quadratic approximation lemma, the comparable *inexact* decompositions

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<sup>5</sup>Equation (6) is also shown in a different context in Benedictow and Boug (2017). To derive (6), we have utilised that  $\overline{S}_{2t} = 1 - \overline{S}_{1t}$  and that  $\Delta S_{2t} = -\Delta S_{1t}$ .

of  $\Delta p_t$  evaluated at period  $t - 1$  and  $t$  become

$$(7) \quad \Delta p_t \approx S_{1t-1} \Delta p_{1t} + (1 - S_{1t-1}) \Delta p_{2t} + \Delta S_{1t} (p_{1t-1} - p_{2t-1})$$

and

$$(8) \quad \Delta p_t \approx S_{1t} \Delta p_{1t} + (1 - S_{1t}) \Delta p_{2t} + \Delta S_{1t} (p_{1t} - p_{2t}),$$

respectively. The first two terms on the right hand side of (7) and (8) now correspond to aggregate inflation when the geometric Laspeyres and the geometric Paasche price indices are used as the underlying aggregator formula. The deflationary effects in (7) and (8) are also somewhat different from those in (6) as relative price levels in period  $t - 1$  and  $t$  are not the same as the relative arithmetic mean of price levels in period  $t$ . It follows from (5) in the case of one low price level and one high price level country that the bias in aggregate inflation when using the first order Taylor series approximations and not the quadratic approximation lemma to (1) is

$$(9) \quad B_t = | \Delta S_{1t} (\Delta p_{1t} - \Delta p_{2t}) |.$$

Because (8) is used by Nickell (2005) among others, it is implicitly assumed that  $B_t = 0$  or negligible in existing analyses of the impact of imports from emerging countries on inflation in developed countries.<sup>6</sup> Having established the analytical framework for comparing the exact and inexact decompositions of international price indices based on (1), we now turn to the empirical illustration to shed light on the potential significance of the bias in aggregate inflation in practise.

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<sup>6</sup>See equation (1) in Nickell (2005).

### 3 Empirical illustration

As noted in the introduction, we use yearly data from the Norwegian clothing industry over the sample period of 1997 – 2016. Our empirical illustration is motivated by the fact that the Norwegian clothing industry has undergone massive trade liberalisation since the Uruguay Round Agreement starting in the mid 1980s, which has increased the imports of clothing from China and other low price level countries at the expense of imports from high price level countries, the euro area in particular.<sup>7</sup> The significant shift in trade pattern over the last three decades or so has contributed to reduced purchasing prices for Norwegian importers of clothing, and thereby also the consumer prices on clothing.

Contrary to Benedictow and Boug (2017), who employ the *exact* decomposition in (6) directly with available data on export price indices and purchasing power parity (PPP) adjusted relative price levels between countries, Nickell (2005) among others operationalize (1) by replacing  $p_{nt}$  with price levels *relative* to the price level in a numeraire country. It can be verified that the bias in (9) is *independent* of this operationalization, but that (6) and (8) now become

$$(10) \quad \Delta p_t = \overline{S_{1t}} \Delta p_{1t} + (1 - \overline{S_{1t}}) \Delta p_{2t} + \Delta S_{1t} (\overline{p_{1t}} - \overline{p_{2t}}) - \Delta p_{Numt}$$

and

$$(11) \quad \Delta p_t \approx S_{1t} \Delta p_{1t} + (1 - S_{1t}) \Delta p_{2t} + \Delta S_{1t} (p_{1t} - p_{2t}) - \Delta p_{Numt},$$

respectively, where  $\Delta p_{Numt}$  is the inflation rate in the numeraire country in period  $t$ . Our calculations of the bias in aggregate inflation are thus based on (10) and (11) rather than (6) and (8). Although the cross-country distribution of the deflationary effects is sensitive to the choice of numeraire country, the size of the aggregate deflationary effects

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<sup>7</sup>See Høegh-Omdal and Wilhelmsen (2002) for a summary of the trade policy liberalization of the Norwegian clothing industry.

is not when more than two countries are involved in the calculations.<sup>8</sup>

The underlying data are price indices (measured in local currencies) for the main exporters of clothing to Norway: the Euro area (*ea*), Denmark (*dk*), Sweden (*se*), United Kingdom (*uk*), Turkey (*tr*), China (*cn*), Hong Kong (*hk*), Vietnam (*vn*), Bangladesh (*ba*) and India (*in*).<sup>9</sup> Together these countries covered about 85 per cent of Norwegian imports of clothing throughout the sample period.<sup>10</sup> Data on prices of clothing are only available for China from 1997, defining the starting point of the sample period. The price indices of clothing are converted to a common currency, the Norwegian krone (NOK), by means of bilateral exchange rates, and the import weights are defined as the value shares of imports of clothing from the countries listed above. Price indices and exchange rates are acquired from Macrobond and values of imports are acquired from foreign trade statistics at Statistics Norway. Finally, relative price levels between countries in period  $t$ , following Benedictow and Boug (2017), are calculated by means of the formula

$$(12) \quad \frac{P_{nt}}{P_{eat}} = \frac{P_{n2011}}{P_{ea2011}} \frac{I_{nt}}{I_{eat}},$$

where  $\frac{P_{n2011}}{P_{ea2011}}$  are PPP adjusted relative price levels of clothing in 2011 with the Euro area (*ea*) as the numeraire country, based on the OECD statistics and the international comparison program by the World Bank,<sup>11</sup> and  $I_{nt} = \frac{P_{nt}}{P_{n2011}}$  and  $I_{eat} = \frac{P_{eat}}{P_{ea2011}}$  are price indices of clothing in period  $t$  with 2011 as the base year,  $\forall t$ . Table 1 summarizes the statistics for prices and weights, used in the calculations, over the sample period.

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<sup>8</sup>Using a high price level country as the numeraire country will increase the size of the deflationary effects from a low price level country with a rising import share, whereas using a low price level country as the numeraire country will increase the size of the deflationary effects from a high price level country with a falling import share. That said, as shown in Benedictow and Boug (2017), the evolution of the deflationary effects in (10) and (11) can be decomposed into the relative price levels in the base period and the relative inflation rates in period  $t$  between the low price level and the high price level country. Hence, higher inflation over time in the low price level country with a rising import share will dampen the deflationary effects from the base period over time and vice versa.

<sup>9</sup>We simplify matters by treating the Euro area as one country. Note that export prices for Vietnam, Bangladesh and India are proxied by consumer prices due to lack of price data on clothing for these countries.

<sup>10</sup>The remaining exports of clothing to Norway come from countries with relatively small import shares.

<sup>11</sup>See OECD (2011) and WB (2015).

Table 1: Summary of statistics. The Norwegian clothing industry<sup>1</sup>

Country( $n$ ) <sup>2</sup>	Prices		Weights <sup>5</sup>		
	$P_{n2011}/P_{ea2011}$ <sup>3</sup>	$\Delta p_n$ <sup>4</sup>	1997	2016	$\Delta S_n$ <sup>6</sup>
Sweden( $se$ )	1.25	1.8	4	2	-0.1
Denmark( $dk$ )	1.24	2.8	10	1	-0.5
Euro area( $ea$ )	1.00	1.7	41	16	-1.3
United Kingdom( $uk$ )	0.79	1.6	8	1	-0.4
Turkey( $tr$ )	0.64	1.0	3	7	0.2
China( $cn$ )	0.58	2.9	24	51	1.4
Hong Kong( $hk$ )	0.47	1.6	5	0	-0.3
Bangladesh( $ba$ )	0.30	4.7	1	9	0.4
Vietnam( $vn$ )	0.26	4.4	1	8	0.3
India( $in$ )	0.22	4.4	2	4	0.1

<sup>1</sup> See the Appendix for data definitions and sources. <sup>2</sup> Together these countries covered close to 85 per cent of Norwegian imports of clothing throughout the sample period of 1997 – 2016,  $n \equiv (se, dk, ea, uk, tr, cn, hk, ba, vn, in)$ . <sup>3</sup> PPP adjusted relative price levels for clothing in 2011 with the Euro area ( $ea$ ) as the numeraire country; see OECD (2011) and WB (2015). <sup>4</sup> Average annual export price inflation of clothing, measured in the Norwegian currency (NOK), per cent. <sup>5</sup> Value shares of imports of clothing, per cent, do not sum to unity due to rounding errors. <sup>6</sup> Average annual change in value shares of imports of clothing, percentage points.

Using the Euro area as the numeraire country implies that  $\frac{P_{ea2011}}{P_{ea2011}}$  equals unity. The relative price levels on clothing in 2011 are thus easy to interpret. For instance, the price level in India was around 20 per cent as high as in the Euro area in 2011. The corresponding figure for Sweden was around 125 per cent. Accordingly, United Kingdom, Turkey, China, Hong Kong, Bangladesh, Vietnam and India stand out as low price level countries and Sweden and Denmark as high price level countries. It is further evident that the average annual export price inflation has varied considerably across the countries. Relatively high inflation in most of the low price level countries throughout the sample period implies significant catch up effects in the export price levels. After China abandoned the USD peg in 2005, leading to a substantial appreciation of the yuan against the USD, Chinese export prices increased rapidly. The import shares have also changed markedly across the countries. Most importantly, the share of imports from China has increased by 27 percentage points from a level around 25 per cent in 1997, mainly at the expense of the share of imports from the Euro area. The Chinese import share accelerated from 2001 when China joined the WTO, but peaked around 2012 at 55 per cent. The shares of imports from most of the other low price level countries

have also increased significantly throughout the sample period, mainly at the expense of the shares of imports from the high price level countries.<sup>12</sup> To sum up, the significant differences in the inflation rates and the changing import shares across the exporting countries illustrate how a first order Taylor series approximation to (1) imposes a likely bias in aggregate inflation.

Figure 1 shows the exact and inexact decompositions to (1) based on (10) and (11) together with the Törnqvist price index<sup>13</sup> and the bias in aggregate inflation based on (9). A particularly high aggregate deflation is evident in 2003, which is mainly attributable to high rates of deflation in the low price level countries and NOK appreciation of more than 10 per cent that year. Likewise, the aggregate inflation of more than 11 per cent in 2015 is mainly explained by high rates of inflation in the low price level countries in addition to NOK depreciation of close to 30 per cent in the wake of the huge drop in the oil price in 2014. The discrepancy between aggregate inflation calculated by (10) and the Törnqvist price index is rather significant in many years. For instance, the discrepancy is as high as 4.5 percentage points in 2000 as the Törnqvist price index does not take into account the deflationary effects from the switch in imports towards low price level countries. The deflationary effects, which are dominated by China, pull down aggregate inflation by an annual average of 2.1 percentage points over the sample period. As a result, the *total* effects based on (10) and the inflationary effects alone based on the Törnqvist price index contribute to aggregate inflation by an annual average of -0.9 and 1.2 percentage points, respectively, from 1997 to 2016.

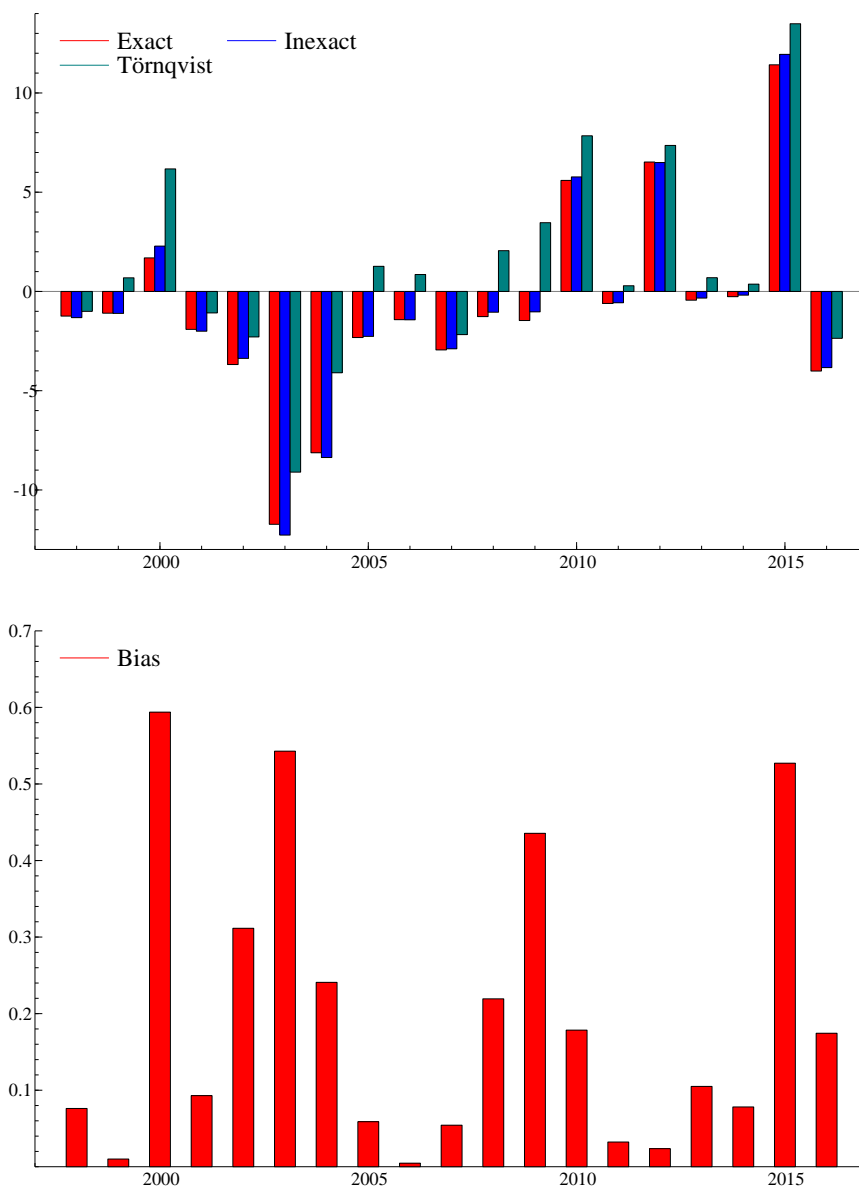
Our calculations also reveal that the bias in aggregate inflation over the sample period is quite substantial and as high as 0.5 percentage points in some years when using a first order Taylor series approximation and not the quadratic approximation lemma to (1). The magnitude of the yearly bias in aggregate inflation may have important

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<sup>12</sup>The imports of clothing from the United Kingdom have fallen considerably, consistent with the export price level approaching the export price level of the Euro area towards the end of the sample period. That the imports of clothing from Hong Kong, despite a relatively low price level country, have diminished may be explained by other reasons than price, for instance changing preferences among the Norwegian consumers of clothing.

<sup>13</sup>Note that  $\Delta p_{eat}$ , the export price inflation in the Euro area, is subtracted from the Törnqvist price index to make it comparable with  $\Delta p_t$  in (10).

Figure 1: Exact and inexact decompositions of  $\Delta p_t$ , Törnqvist price index and bias



*Note:* Data from the Norwegian clothing industry. The exact decomposition and the Törnqvist price index are based on (10), the inexact decomposition is based on (11) and the bias in aggregate inflation is based on (9). Upper panel in per cent and lower panel in percentage points.



implications for the estimation of pricing-to-market models of import prices of clothing and for the inflation targeting central bank in the conduct of monetary policy.

## 4 Conclusions

In this paper, we have compared analytically the exact and inexact decompositions of international price indices based on a geometric average of price levels and derived an expression for the bias in aggregate inflation arising from applying the first order Taylor series approximation and not the quadratic approximation lemma. We have shown that the bias in aggregate inflation is zero only in the special cases when inflation rates are equal across exporting countries and/or when no switching of imports occurs from high price level to low price level countries or vice versa. Hence, the bias may be significant in practise when using a first order Taylor series approximation to a geometric average of price levels as the import patterns indeed have changed dramatically over time following massive trade liberalization in many countries.

Our empirical illustration, using yearly data from the Norwegian clothing industry over the sample period of 1997 – 2016, revealed that the bias in aggregate inflation is quite substantial and as high as 0.5 percentage points in some years. We therefore conclude that the quadratic approximation lemma should be applied in practise when decomposing international price indices based on a geometric average of price levels. Admittedly though, as the deflationary effects of the shifts of imports towards low price level countries are driven by trade liberalization and price level differences between countries rather than by changes in relative prices, the ratio of a geometric average (like any other average) of price levels must deviate from classical index number theory and violate the identity axiom.<sup>14</sup> Otherwise, neglecting the price level differences between countries by using a classical index number formula leads, as we have seen, to an even more significant bias in aggregate inflation than the bias arising from applying the first order Taylor series approximation to a geometric average of price levels.

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<sup>14</sup>The identity axiom says that a price index should not change if underlying prices do not change, see for example ILO *et al.* (2004).

## References

- [1] Balk, B.M. (2008): Price and quantity index numbers. Models for measuring aggregate change and difference, Cambridge University Press, New York.
- [2] Benedictow, A. and P. Boug (2017): Calculating the real return on a sovereign wealth fund, *Canadian Journal of Economics* 50(2), 571-594.
- [3] Boskin, M.J., E. Dulberger, R. Gordon, Z. Griliches and D. Jorgenson (1996): Toward a more accurate measure of the cost of living, final report to the US Senate Finance Committee, Washington, D.C.
- [4] Brasch, T. von (2017): The import price index with trade barriers: theory and evidence. *Empirical Economics* 52(1), 123-141.
- [5] Diewert, W.E. (1976): Exact and superlative index numbers, *Journal of Econometrics* 4(2), 115-145.
- [6] Diewert, W.E. (1998): Index number issues in the consumer price index, *Journal of Economic Perspectives* 12(1), 47-58.
- [7] Diewert, W.E. (2002): The quadratic approximation lemma and decompositions of superlative indexes, *Journal of Economic and Social Measurement* 28(1-2), 63-88.
- [8] ECB (2006): Effects of the rising trade integration of low-cost countries on euro area import prices, *ECB Monthly Bulletin*, August, Box 6, 56.
- [9] European Commission, IMF, OECD, UN and World Bank (2009): System of national accounts 2008, <http://unstats.un.org/unsd/nationalaccount/sna2008.asp>
- [10] Høegh-Omdal, K. and B.R. Wilhelmsen (2002): The effects of trade liberalisation on clothing prices and on overall consumer price inflation, *Economic Bulletin* Q4, The Norwegian Central Bank, 134-139.
- [11] ILO, IMF, OECD, Eurostat, UNECE and World Bank (2004): Producer price index manual: theory and practice, International Monetary Fund, Washington DC.

- [12] Kamin, S.B., M. Marazzi and S.W Schindler (2006): The impact of Chinese exports on global import prices, *Review of International Economics* 14(2), 179-201.
- [13] MacCoille, C. (2008): The impact of low-cost economies on UK import prices, *Bank of England Quarterly Bulletin*, Vol. 48, No. 1, 58-65.
- [14] Nickell, S. (2005): Why has inflation been so low since 1999?, *Bank of England Quarterly Bulletin*, Spring, 92-107.
- [15] OECD (2011): 2011 PPP benchmark results, Table 1.11, OECD statistics, <http://stats.oecd.org/>.
- [16] Pain, N., I. Koske and M. Sollie (2006): Globalisation and inflation in the OECD Economies, Working Papers No. 524, OECD.
- [17] Røstøen, J. (2004): External price impulses to imported consumer goods, *Economic Bulletin Q3*, The Norwegian Central Bank, 96-102.
- [18] Thomas, C.P. and J. Marquez (2009): Measurement matters for modelling US import prices, *International Journal of Finance and Economics* 14(2), 120-138.
- [19] WB (2015): Purchasing power parities and the real size of world economies: a comprehensive report of the 2011 international comparison program, Table 2.9, The World Bank, <http://siteresources.worldbank.org/ICPEXT/Resources/ICP-2011-report.pdf>
- [20] Wheeler, T. (2008): Has trade with China affected UK inflation? Discussion Paper No. 22, External MPC Unit, Bank of England.
- [21] White, A.G. (2000): Outlet types and the Canadian consumer price index, *Canadian Journal of Economics* 33(2), 488-505.

## Appendix

$I_{dkt}$ : Domestic supply price index of apparel and accessories except knitwear from  $t = 1997, \dots, 2000$ , producer price index of textiles and leather products from  $t = 2000, \dots, 2005$  and producer price index of wearing apparel for foreign markets from  $t = 2005, \dots, 2016$ , measured in local currency (DKK), 2011=1. Source: Macrobond.

$I_{set}$ : Export price index of textiles and wearing apparel from  $t = 1997, \dots, 2016$ , measured in local currency (SEK), 2011=1. Source: Macrobond.

$I_{ukt}$ : Producer price index of wearing apparel from  $t = 1997, 1998$  and export price index of clothing and footwear from  $t = 1998, \dots, 2016$ , measured in local currency (GBP), 2011=1. Source: Macrobond.

$I_{eat}$ : Producer price index of textiles, leather and wearing apparel from  $t = 1997, \dots, 2016$ , measured in local currency (EUR), 2011=1. Source: Macrobond.

$I_{trt}$ : Producer price index of textiles and wearing apparel from  $t = 1997, \dots, 2016$ , measured in local currency (TRY), 2011=1. Source: Macrobond.

$I_{cnt}$ : Producer price index of clothing from  $t = 1997, \dots, 2016$ , measured in local currency (CNY), 2011=1. Source: Macrobond.

$I_{hkt}$ : Consumer price index (total) from  $t = 1997, \dots, 2005$  and producer price index of wearing apparel from  $t = 2005, \dots, 2016$ , measured in local currency (HKD), 2011=1. Source: Macrobond.

$I_{vnt}$ : Consumer price index (total) from  $t = 1997, \dots, 2016$ , measured in local currency (VND), 2011=1. Source: Macrobond.

$I_{bat}$ : Consumer price index (total) from  $t = 1997, \dots, 2016$ , measured in local currency (BDT), 2011=1. Source: Macrobond.

$I_{int}$ : Consumer price index (total) from  $t = 1997, \dots, 2016$ , measured in local currency (INR), 2011=1. Source: Macrobond.

$S_{nt}$ : Value share of imports from country  $n$  in Norwegian imports of clothing in period  $t$ ,  $n \equiv (se, dk, ea, uk, tr, cn, hk, ba, vn, in)$ . Source: Foreign trade statistics, Statistics Norway.

*Bilateral exchange rates*:  $\frac{USD}{DKK}$ ,  $\frac{USD}{SEK}$ ,  $\frac{USD}{GBP}$ ,  $\frac{USD}{EUR}$ ,  $\frac{USD}{TRY}$ ,  $\frac{USD}{CNY}$ ,  $\frac{USD}{HKD}$ ,  $\frac{USD}{VND}$ ,  $\frac{USD}{BDT}$  and  $\frac{USD}{INR}$  are used to convert the prices of clothing measured in local currencies into USD.  $\frac{NOK}{USD}$  is then used to convert the country specific prices in USD into NOK. Source: Macrobond.

$\frac{P_{n2011}}{P_{ea2011}}$ : Relative price levels of clothing between country  $n$  and the numeraire country  $ea$ , the Euro area, in 2011,  $n \equiv (se, dk, ea, uk, tr, cn, hk, ba, vn, in)$ . Source: OECD (2011) and WB (2015).

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ISSN: 1892-753X



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