Discussion Papers No. 495, February 2007
Statistics Norway, Research Department

Knut Einar Rosendahl

Incentives and quota prices in an emission trading scheme with updating

Abstract:
Emission trading schemes where allocations are based on updated baseline emissions give firms less incentives to reduce emissions. Nevertheless, according to Böhringer and Lange (2005a), such allocation schemes are cost-effective if the system is closed and allocation rules are equal across firms. In this paper we show that the cost-effective solution may be infeasible if the marginal abatement costs grow too fast. Moreover, if a price cap or banking/borrowing are introduced, the abatement profile is no longer the same as in the case with lump sum allocation. In addition, we show that with allocation based on updated emissions, the quota price will always exceed the marginal abatement costs. Numerical simulations indicate that the quota price most likely will be several times higher than the marginal abatement costs, unless a significant share of allowances are either auctioned or lump sum distributed.

Keywords: Emission trading, Allocation of quotas, Quota prices.

JEL classification: H21, Q28.

Acknowledgement: I am grateful for valuable comments from Mads Greaker and Cathrine Hagem on an earlier draft.

Address: Knut Einar Rosendahl, Statistics Norway, Research Department.
E-mail: knut.einar.rosendahl@ssb.no

Discussion Papers comprise research papers intended for international journals or books. A preprint of a Discussion Paper may be longer and more elaborate than a standard journal article, as it may include intermediate calculations and background material etc.

Abstracts with downloadable Discussion Papers
in PDF are available on the Internet:
http://www.ssb.no
http://ideas.repec.org/s/ssb/dispap.html

For printed Discussion Papers contact:

Statistics Norway
Sales- and subscription service
NO-2225 Kongsvinger

Telephone: +47 62 88 55 00
Telefax: +47 62 88 55 95
E-mail: Salg-abonnement@ssb.no

1 Introduction

A competitive emission trading market is a cost-effective way of reducing emissions, as long as emission allowances are either auctioned or distributed in a lump sum manner.[1] This well-known result dates back to the 1970's (cf. Montgomery, 1972). The design of real-world emission trading schemes shows, however, that few allowances are auctioned. Moreover, it is difficult to allocate allowances in a lump sum manner over a longer time horizon without creating perverse distributional effects. Thus, other allocation mechanisms are introduced. Within the EU Emission Trading Scheme (EU ETS) (EU, 2003, 2005; Watanabe and Robinson, 2005) allocation for the first two periods (2005-7 and 2008-12) is mostly based on recent historic emission levels, but special rules apply for e.g. new entrants and firms' closure. The allocation rules for future periods are not determined yet, and open to speculation. The SO_2 trading program in the US has mostly lump sum allocation based on grandfathering, but it also includes allocation rules that have created "an additional set of incentives" (Considinea and Larson, 2006).

The prime motive for avoiding auctioning or lump sum allocation is to prevent a detoriation of competitiveness versus countries outside the trading system. In the very recent economic literature there has been some analyses (and discussion) of different kinds of allocation rules. The aim is to find a mechanism that achieves a high degree of both competitiveness and cost-effectiveness. Böhringer and Lange (2005a) actually show that within a closed emission trading system, an allocation rule where allowances are based on emissions k years ago (i.e., the baseyear is continually updated), is cost-effective even though firms take into account that reduced emissions today mean less allowances in the future. The reason is simply that all firms face the same rule, and with a fixed total emission level and equal expectations about the future quota price, the current price is bid up and all firms adjust abatement until marginal abatement costs equal the current quota price *minus* expected benefits from future allowances. In an open system, e.g. the EU ETS linked to external allowances like the Clean Development Mechanism (CDM), the updating system is no longer cost-effective. Böhringer and Lange further show that an output-based system cannot be cost-effective in either an open or closed system.

[1] One example of lump sum allocation is pure grandfathering, where allocations in all periods are entirely based on historic emissions before the system is initiated, and where allocations are independent of e.g. firms' closure or new entrants.

Åhman et al. (2007) propose a ten-year updating rule, saying that allowances should be proportional to activity levels ten years ago. They claim that a ten years lag would significantly weaken the unwanted effects of future allocation on current behaviour, due to discounting the value of future allowances. Keats Martinez and Neuhoff (2005) argue that an updating scheme (based on emissions) can distort the allowance price, which would lead to inefficiencies if different sectors or regions are faced with different allocation rules or discount rates. A simple two-period example is used to demonstrate this claim. They also derive an expression for the quota price in the case with constant emission target and abatement costs. Burtraw et al. (2005, 2006) present numerical simulation results of different allocation rules for the outcome of a regional emission trading market for electricity producers in nine states of the U.S. They find the emission price to be about twice as high with updating compared to auctioning or pure grandfathering. Their updating rule is based on generation levels two years ago. The social costs of this system are three times higher than with the two alternative systems (see also Palmer and Burtraw, 2005).

The aim of the current paper is to examine more deeply the dynamic effects of an allocation scheme based on updated emission levels, within a closed emission trading system. We point to several factors that may alter Böhringer and Lange's (2005a) conclusion that such a system is cost-effective. First, we show that the quota price may become infinitely large, i.e., rendering the cost-effective solution infeasible, if the marginal abatement costs grow too fast. Second, an overallocation of quotas will increase emissions beyond its business as usual level. Third, introducing a (binding) price cap is shown to give less abatement in such a system compared to a system with e.g. auctioning. Fourth, if the system allows banking and borrowing, the system is no longer dynamically cost-effective if allocation is based on updated emission levels. Too much abatement is delayed until later periods.

The analysis is based on an extension of the analytical framework set up by Böhringer and Lange (2005a). The paper examines, both theoretically and numerically, the importance of policy variables such as the allocation rate (i.e., number of allowances based on previous emissions), the time lag between emission and allowance (cf. the ten-year rule in Åhman et al., 2007), the emission target over time, and the time horizon of the system.

In most realistic cases, the quota price turns out to be finite. However, the numerical simulations suggest that it will be several times higher than the marginal abatement costs in most realistic cases. This divergence may give a false impression of the abatement effort (and abatement costs) taking place in the market, and may increase the risk of inefficiencies.[2] The analyses suggest that increasing the amount of auctioning and/or the time lag between emission and allowances help to reduce the gap between price and marginal costs. However, in a growing economy with a fixed or gradually tighter emission target, it is difficult to avoid a quota price that is several times higher than the marginal abatement costs without auctioning a majority of the allowances.

Few papers have analysed the dynamic effects on emission trading markets of implementing allocation rules based on updating, and only Böhringer and Lange (2005a) have done theoretical analyses of such rules as far as we know. The contribution of this paper is therefore to broaden the analytical insight of such allocation schemes, focusing on cost-effectiveness and price effects in the emission trading market.

Other papers have examined the effects of allocation rules based on *current* emissions or output (benchmarking),[3] typically in a static framework. Fischer (2001) concludes that output-based allocation rules shift abatement from output contraction towards emission rate reductions. Böhringer and Lange (2005b) find that output-based allocations are less costly than emission-based allocations with respect to maintaining output and employment in energy-intensive industries. Mæstad (2007) analyses how capital leakage may affect optimal allocation rules based on benchmarking . When it comes to numerical analyses, Jensen and Rasmussen (2000) find within a (dynamic) CGE model for Denmark that output-based allocation leads to higher costs than auctioning or pure grandfathering, but also to less sector adjustments. Kuik and Mulder (2004) find similar results for the Netherlands. Böhringer et al. (1998) also obtain significant efficiency costs of output-based allocation schemes in simulations of a multiregional and multisectoral CGE model for the EU.

Few existing emission trading schemes have so far introduced allocation rules based on updating. According to Burtraw et al. (2005), the NO_x emission trading scheme in 19 eastern states of the U.S. uses updating for

[2]In addition to price cap and banking/borrowing, which are specifically analysed in the paper, inefficiencies may also arise if allocation rules are not fully harmonised, if it is possible to buy some allowances outside the system, or if discount rates or expectations about future quota prices vary across firms.

[3]This corresponds to the rules for new entrants in the EU ETS (output-based allocation).

some portion of the allowances. When it comes to the EU ETS, the allocation rules for future periods (beyond 2012) are not yet clear, but Keats Martinez and Neuhoff (2005) speculate that "market participants can adapt their behaviour today in an effort to influence the future allocation". Different alternatives are discussed,[4] and increased knowledge of the characteristics of different schemes is therefore vital.

Price caps and banking/borrowing are partly used in some trading schemes, at least if we consider penalties without "make good" provisions as price caps (cf. Ellis and Tirpak, 2006). The penalties for non-compliance in the EU ETS are combined with "make good" provisions in the next period. Banking and borrowing are allowed within periods, and limited banking from the first to the second period is permitted in a few countries. The SO_2 trading program in the US has allowed banking between periods (Considinea and Larson, 2006).

The next section sets up the theoretical model, and analyses theoretically under what conditions the cost-effective solution is feasible. It also examines the ratio between the quota price and the marginal abatement costs. The effects of a price cap or banking and borrowing are analysed, too. In Section 3 numerical simulations of the quota price are presented, and Section 4 concludes.

2 Theoretical analyses

Consider the following partial equilibrium model, based on Böhringer and Lange (2005a). Firm i's cost function is given by $c^{i,t}(q^{i,t}, e^{i,t})$, where $q^{i,t}$ denotes production and $e^{i,t}$ emission in time t. Let $\widehat{e}^{i,t}$ denote the optimal level of emission in the case with no environmental policy. The following standard assumptions apply: $c_q^{i,t} > 0$; $c_e^{i,t} < 0$ (for $e^{i,t} < \widehat{e}^{i,t}$); $c_{qq}^{i,t}$, $c_{ee}^{i,t}$, $(-c_{eq}^{i,t}) \geq 0$; and $c_{qq}^{i,t} \cdot c_{ee}^{i,t} - (c_{eq}^{i,t})^2 > 0$. The firms' output is sold at a price $p^{i,t}$. When an emission trading scheme is introduced, the firms must hold allowances corresponding to their emissions $e^{i,t}$ and (if permitted) their net banking or savings of allowances $s^{i,t}$ in each period. We assume that emission allowances are allocated for free to the firms, linearly based on previous emission levels:

[4] For instance, Grubb et al. (2005) argue against updating rules in future periods of EU ETS, and favour a combination of auctioning and benchmarking.

$$\overline{e}^{i,t} = \lambda_0^{i,t} + \sum_{l=1}^{k} \lambda_e^{t,t-l} e^{i,t-l}. \tag{1}$$

The allocation rule we examine consists of a firm-specific lump-sum allocation $\lambda_0^{i,t}$, and allocation rates $\lambda_e^{t,t-l}$ related to emissions in time $t-l$. The allocation rates are equal across firms. It is obvious that this allocation rule, assuming $\lambda_e^{t,t-l} > 0$ for at least some l, implies that the abatement effort in time $t-l$ not only depends on the emission price in time $t-l$, but also on future emission prices. The reason is simply that the current emission level in a firm affects its future allocation of allowances, which are valuable. In our analyses $\lambda_0^{i,t}$ may also be interpreted as auctioned allowances, and we will make use of this interpretation in some relevant cases.[5]

In this paper we only consider a closed emission trading system, meaning that the firms are not allowed to trade allowances outside the system. In the following subsection we assume that the government chooses allocation rates so that a desired total emission level is achieved in each period. Subsequently, we assume that the government introduces a price cap for the quota price so as to avoid excessive costs for the emitting firms. In the final subsection we assume that the government is concerned with the accumulated emissions over time, and allow for banking and borrowing of allowances.

2.1 Base case: No price cap or banking/borrowing

2.1.1 Cost-effective solution

We first assume that the government's environmental objective is to keep total emissions below a certain treshold level $\overline{E}^t$ in each period from $t = 0$ to $t = T$ (where T may be infinity). For the moment, we further assume that the emission target is strictly binding, so that $c_e^{i,t} < 0$. A cost-effective solution is then given by the following optimisation problem:[6]

[5] This follows from Montgomery's (1972) finding referred to above, and that we only consider a partial equilibrium model for the emission trading market. As demonstrated by e.g. Goulder et al. (1999) and Parry et al. (1999), auctioned allowances outperform lump sum allocations in a general equilibrium framework, if the auction revenues are used to cut distortionary taxes.

[6] Strictly speaking, the optimisation problem is a bit more complex, and depends on the product markets affected. Still, the first order conditions are the same.

$$\begin{gathered}\underset{q^{i,t},e^{i,t}}{Max}\sum_{i;t=0}^{t=T}\left[\tfrac{1}{(1+\delta)^t}\left(p^{i,t}q^{i,t}-c^{i,t}(q^{i,t},e^{i,t})\right)\right]\\ s.t.\sum_i e^{i,t}=\overline{E}^t.\end{gathered} \tag{2}$$

This optimisation is taken entirely from Böhringer and Lange (2005a), except that we have introduced a discount rate δ, which we will show is crucial to include. First order conditions are then straightforward and well-known:

$$p^{i,t}=c_q^{i,t}, \text{ and} \tag{3}$$

$$-c_e^{i,t}=-c_e^{j,t}=\sigma_t^*, \tag{4}$$

where σ_t^* is the shadow price of emission.

2.1.2 Market solution

Next, consider the firms' maximisation problem, where we assume competitive markets in both output and emission markets, with σ_t being the quota price:

$$\underset{q^{i,t},e^{i,t}}{Max}\sum_{t=0}^{T}\left[\frac{1}{(1+\delta)^t}\left(p^{i,t}q^{i,t}-c^{i,t}(q^{i,t},e^{i,t})-\sigma_t\left(e^{i,t}-\left[\lambda_0^{i,t}+\sum_{l=1}^{k}\lambda_e^{t,t-l}e^{i,t-l}\right]\right)\right)\right]. \tag{5}$$

The firms' first order conditions are then given by equation (3) and:

$$\sigma_t-\sum_{l=1}^{k}\left(\frac{1}{(1+\delta)^l}\sigma_{t+l}\lambda_e^{t+l,t}\right)=-c_e^{i,t}. \tag{6}$$

Before we compare the market outcome with the cost-effective solution, we notice from equation (6) that the quota price will always exceed the marginal abatement costs (MAC), as long as the allocation rate $\lambda_e^{t+l,t}$ is strictly positive for some l. We return to this issue later.

Nevertheless, despite the divergence between the quota price and MAC, the market solution may be cost-effective (as already pointed out by Böhringer and Lange (2005a) in their Proposition 1). The left-hand side of equation (6) is equal across firms, and hence equation (4) is also fulfilled. Consequently, a cost-effective solution can be obtained by allocation schemes in line with equation (1), i.e., a combination of a lump sum transfer (or auction) and allowances proportional to emissions in earlier periods $t-l$ ($l=1,..,k$). That is, although firms take into account that current emissions affect future allocations, the closed system means that total emissions in each period are fixed and the allocation scheme gives equal incentives to all firms with respect to emission reductions. Böhringer and Lange further state that "any policy which ensures that the allocation rates are identical across firms is efficient."

However, what Böhringer and Lange do not consider is that the cost-effective solution may not be feasible if the quota price becomes infinitely large. In order to examine this, we focus on allocation schemes that only depends on emissions in one previous year (k years ago), i.e., $\lambda_e^{t+k,t}=\lambda_e^{t+k}$. In this case equation (6) simplifies to:

$$\sigma_t=\frac{1}{(1+\delta)^k}\sigma_{t+k}\lambda_e^{t+k}-c_e^{i,t}. \tag{7}$$

By expressing this equation for $t=0,k,2k,..,T$, combining them, and using $\sigma_T=-c_e^{i,T}$ for the last period T, we get the following expression for the relative difference between the initial quota price and MAC:

$$\frac{\sigma_0}{-c_e^{i,0}}=1+\frac{1}{-c_e^{i,0}}\sum_{j=1}^{\frac{T}{k}}\left\{\left(-c_e^{i,jk}\right)\frac{1}{(1+\delta)^{jk}}\left(\prod_{l=1}^{j}\lambda_e^{lk}\right)\right\}. \tag{8}$$

First, we see that σ_0 is finite as long as T is finite, implying that a cost-effective solution is feasible. Second, assuming an infinite system, what happens when we let $T\rightarrow\infty$? In order to simplify matter somewhat, let us focus on allocation schemes where the allocation rate is constant, i.e., $\lambda_e^t=\lambda_e$. Equation (8) then simplifies to:

$$\frac{\sigma_0}{-c_e^{i,0}}=1+\frac{1}{-c_e^{i,0}}\sum_{j=1}^{\frac{T}{k}}\left\{\left(-c_e^{i,jk}\right)\left(\frac{\lambda_e}{(1+\delta)^k}\right)^j\right\}. \tag{9}$$

Based on this equation we state the following proposition:

Proposition 1. *In a closed, infinite emissions trading scheme with a constant allocation rate* λ_e*, the cost-effective solution is feasible (infeasible)* *if the marginal abatement costs increase slower than (at least as fast as) the rate* $\gamma = \left(\frac{(1+\delta)^k}{\lambda_e}\right)^{\frac{1}{k}} - 1$ *in the long run.*

A closed, finite emissions trading scheme with allocation rule (1) always leads to a cost-effective solution.

Proof: See Appendix A.

Although the proposition gives explicit conditions for when a closed, infinite emission trading scheme has a feasible or infeasible cost-effective solution, it is difficult to tell more precisely in what cases one of the two conditions is fulfilled without specifying more about either the cost function, the output market and/or the emission target. Thus, in the next subsection we will discuss the likelihood of having (in)feasible cost-effective solutions.

But first, based on equation (8), we can also prove the following two propositions:

Proposition 2. *In a closed emissions trading scheme, the quota price* σ_t *exceeds the marginal abatement costs* $-c_e^{i,t}$ *if the allocation rate* λ_e^{t+k} *is strictly positive and the emission target in time* $t+k$ *is strictly binding, so that* $-c_e^{i,t+k} > 0$.

Proof: The proof follows directly from equation (8).□

Proposition 3. *In a closed emissions trading scheme, the relative difference between the quota price and marginal abatement costs increases with i) the future allocation rates* λ_e^{t+k}*, ii) the discount factor* $\frac{1}{1+\delta}$*, iii) the future emission reduction (*$E_{BaU}^t - \overline{E}^t$*), and iv) the time horizon of the trading scheme* T*.*

Proof: Note that there is a one-to-one correspondence between the emission target and MAC in each period in this model, as long as the number of firms is exogenous (since $-c_e^{i,t+k} > 0$ are equal across firms). Then the proof follows straightforward from equation (8).□

Hence, the quota price will exceed MAC unless all allocations are based on lump sum principles (or auctioned). A higher allocation rate increases the value of current emissions, and therefore a higher quota price is required before an abatement effort is undertaken. A lower discount rate also increases the value of current emissions, and thus abatement effort is depressed. With stricter emission target in the future and thus higher marginal abatement costs, the future quota price will rise, increasing the value of future allowances. Finally, with a long lifetime of the system, the quota price will exceed MAC longer into the future, and thus increasing the quota price also in the near future.

The effect of increasing k, i.e., the number of years between emission and allocation of allowances, is in fact ambiguous. In most cases a higher k will reduce the quota price because the discounted value of future allowances is reduced. However, if the quota price rises faster than the discount rate in some period, the conclusion is turned around. An example of this is shown in the simulations.

So far, we have assumed that the emission target is strictly binding. Assume now that the target is set above the total business as usual emissions, i.e., the sum of $\widehat{e}^{i,t}$ over all firms. This could happen for instance if the government does not have correct information about projected emission levels on the company level, or if unexpected events reduce the need for emission quotas (e.g., mild weather or downturn in economic activity). Assuming that the target becomes binding in the future, we then have the following proposition:

Proposition 4. *In a closed emissions trading scheme, an overallocation of quotas in a period t (i.e., $\sum_i \widehat{e}^{i,t} \leq \overline{E}^t$) will increase total emissions beyond the business as usual level.*

Proof: Assume the contrary, which implies that total emissions are below the target. Then the price of quotas must be zero (since supply surpasses demand and no banking is allowed). In equation (6), the second term is negative and independent of the outcome in period t. Thus, the right hand side of the equation must be negative, which is only possible for $e^{i,t} > \widehat{e}^{i,t}$. However, this contradicts the initial assumption.$\Box$

Note that total emissions may be lower than the total allocation, if the marginal costs of *increasing* emissions beyond $\widehat{e}^{i,t}$ are strictly positive. However, according to the proposition, emissions will always exceed the business as usual level.[7]

In the rest of this subsection we want to explore more closely, based on Proposition 1, whether a feasible solution exists. Moreover, the discussion of feasibility is strongly connected to the discussion of relative difference between the quota price and marginal abatement costs. That is, if the cost-effective solution is nearly infeasible, the quota price is presumably very high (compared to MAC).

We start with the simplest case of all, i.e., where the emission and output

[7] This result depends on the assumption of twice differentiable cost functions, which implies that the marginal costs of emission passes through zero at the business as usual level.

markets are in a constant steady-state equilibrium.

Conjecture 5. *In a closed, infinite emissions trading scheme with constant emission target $\overline{E}$, cost functions c^i and output prices p^i over time, a cost-effective solution is infeasible if and only if i) the allocation is based entirely on previous emissions (i.e., $\lambda_0^{i,t} = 0$ and $\lambda_e = 1$), and ii) the discount rate is equal to zero.*

Proof: See Appendix A.

Although a zero discount rate is a fairly unlikely assumption, this very simple example contradicts the statement in Böhringer and Lange (2005a) that any policy that ensures equal allocation rates across firms are efficient.

In the more general case, we see from Proposition 1 that the question of feasibility hinges on the development of MAC over time. The same applies to the relative difference between the quota price and MAC. Thus, let us first look at how $-c_e^{i,t}$ changes from year to year:

$$\Delta(-c_e^{i,t}(q^{i,t}, e^{i,t})) = -c_e^{i,t+1}(q^{i,t+1}, e^{i,t+1}) - (-c_e^{i,t}(q^{i,t}, e^{i,t})) \approx -c_{eq}^{i,t}\Delta q^{i,t} - c_{ee}^{i,t}\Delta e^{i,t}. \tag{10}$$

In order to say anything useful about $\Delta q^{i,t}$, we restrict our attention to quota markets where m identical firms produce identical goods. However, we distinguish between a closed product market (i.e., consumers can only buy from firms within the emission trading scheme) and an open product market with a fixed output price $\overline{p}^t$. In the former case, output is given by the demand function $Q^t(p^t) = \sum_i q^{i,t} = mq^{i,t}$, assuming $Q^t(p^t) = Q(p^t)f(t)$.

In the closed product market case we have:

$$\Delta(-c_e^{i,t}(q^{i,t}, e^{i,t})) \approx -c_{eq}^{i,t}\Delta q^{i,t} - c_{ee}^{i,t}\Delta e^{i,t} \approx -c_{eq}^{i,t}\left(\frac{Q'f}{m}\Delta p + \frac{Qf'}{m}\Delta t\right) - c_{ee}^{i,t}\Delta e^{i,t}. \tag{11}$$

We see that the change in MAC is affected in three ways: First, a reduction in the emission target over time increases $-c_e^{i,t}$ due to $c_{ee}^{i,t} \geq 0$. Second, increased demand over time ($f' > 0$) due to e.g. economic growth also increases $-c_e^{i,t}$ due to $-c_{eq}^{i,t} \geq 0$. Third, both reduced emission target and increased demand (for given price) will *decrease* $-c_e^{i,t}$ through depressed demand due to higher output prices (since $-c_{eq}^{i,t} \geq 0$), partly compensating the direct effects.

In the open product market case, if $\overline{p}^t = \overline{p}$, then marginal costs ($c_q^{i,t}$) must also be constant over time. Thus, $\Delta q^{i,t} \approx -\frac{c_{eq}^{i,t}}{c_{qq}^{i,t}}\Delta e^{i,t}$, and we get:

$$\Delta(-c_e^{i,t}(q^{i,t}, e^{i,t})) \approx -c_{eq}^{i,t}\Delta q^{i,t} - c_{ee}^{i,t}\Delta e^{i,t} \approx \left(\frac{\left(-c_{eq}^{i,t}\right)^2}{c_{qq}^{i,t}} - c_{ee}^{i,t} \right) \Delta e^{i,t}. \quad (12)$$

Here we see that if the emission target is constant over time, so is MAC. Otherwise, the sign of (12) is in fact ambiguous. On the one hand, for a fixed output, lower emissions increase $-c_e^{i,t}$ due to $c_{ee}^{i,t} \geq 0$. On the other hand, lower $e^{i,t}$ increases $c_q^{i,t}$, and so output must decrease to compensate, reducing $-c_e^{i,t}$ (both due to $-c_{eq}^{i,t} \geq 0$).

In Appendix B we derive a specific condition for when the cost-effective solution is (in)feasible, given a certain group of cost functions. The condition highlights the importance of economic growth and stricter emission target over time, which both normally extends the distance to the baseline emission level, thus increasing MAC.[8]

2.2 Price cap

So far we have assumed that there is no upper price limit in the emission trading system. However, price caps may be introduced in order to prevent too high abatement costs for the firms.

As shown in Proposition 2, the quota price will always exceed MAC as long as the allocation rate λ_e^{t+k} is strictly positive (and future emission targets are binding). Consequently, the probability of reaching the price cap is higher with allocation rules given by (1) than with only lump sum allocation (or auction).

Let the price cap be given by $\widehat{\sigma}_t$. Assume that, in the case with no price cap, there is a time period $\mathbf{t}$ of at least k years for which we have $\sigma_t > \widehat{\sigma}_t$ for $t \in \mathbf{t}$. Then we have the following proposition:

Proposition 6. *Consider a closed emissions trading scheme with allocation rates λ_e^{t+k} and a price cap $\widehat{\sigma}_t$, and let $\sigma_{C,t}$ and $\sigma_{NC,t}$ denote the quota prices in the case with and without price cap, respectively. Then, $\sigma_{C,t} < \sigma_{NC,t}$ if there exists a current or future time period $\mathbf{t}$ of at least k years for which $\sigma_{NC,t} > \widehat{\sigma}_t$.*

Proof: See Appendix A.

[8] This is at least true in a closed product market with $f' > 0$ and strictly negative cross derivatives $c_{eq}^{i,t}$ (remember that $-c_{eq}^{i,t} \geq 0$).

The proposition states that the price cap not only suppresses the quota price when it is binding, but in fact in all previous years. Moreover, the quota price may become lower than the price cap in parts of the time period **t**, i.e., when the price exceeds the (hypothetical) price cap in the case without such cap. On the other hand, there must be at least some year(s) when the price cap is binding ex-post, so that the emission target is not fulfilled and MAC is different from the case with no price cap. Otherwise, equation (8) would lead to a contradiction. Thus, we state the following proposition:

Proposition 7. *Consider a closed emissions trading scheme with allocation rates* λ_e^{t+k} *and a price cap* $\widehat{\sigma}_t$*, and let* $\sigma_{NC,t}$ *denote the quota price in the case without price cap. Then, total emissions will be higher in some year(s) than in the case with only lump sum allocation if* $\sigma_{NC,t} > \widehat{\sigma}_t$ *for at least one* t*.*

Proof: If the price cap is not binding in the case with only lump sum allocation, the proposition follows from the arguments above, using equation (8). If the price cap is also binding in the case with $\lambda_e^{t+k} = 0$, then in the latter case MAC is set equal to the price cap. However, with $\lambda_e^{t+k} > 0$ the quota price (i.e., the price cap) still exceeds MAC. Consequently, MAC is lower with $\lambda_e^{t+k} > 0$, and thus emissions are lower with only lump sum allocation.□

According to Proposition 7, the outcome of an emission trading scheme with allocation rules based on (1) may depend on the allocation rate λ_e^{t+k} if a price cap is introduced. More precisely, the higher the allocation rate is, the higher is the probability that the emission target is not achieved (for a given price cap).

2.3 Banking and borrowing

In the preceding subsections we have assumed that the government has a specific emission target for each period. Now we assume that the emission target relates to accumulated emissions over the entire time horizon of the emission trading scheme. Thus, we have:

$$\begin{gathered} \underset{q^{i,t},e^{i,t}}{Max} \sum_{i;t=0}^{t=T} \left[\tfrac{1}{(1+\delta)^t} \left(p^{i,t} q^{i,t} - c^{i,t}(q^{i,t}, e^{i,t}) \right) \right] \\ s.t. \sum_{i,t} e^{i,t} = \overline{E}. \end{gathered} \tag{13}$$

Equation (3) carries over from the original optimisation problem. However, equation (4) is replaced by:

$$-c_e^{i,t} = (1+\delta)^{t-s}(-c_e^{j,s}) = \sigma_t^*. \tag{14}$$

Note that for $t = s$ we have equation (4) as a special case. In addition, equation (14) states that MAC must rise over time with the discount rate.

We further assume that firms are allowed to bank and borrow allowances. Their maximisation problem is now:

$$\underset{q^{i,t},e^{i,t}}{Max} \sum_{t=0}^{T} \left[\tfrac{1}{(1+\delta)^t} \left(p^{i,t} q^{i,t} - c^{i,t}(q^{i,t}, e^{i,t}) - \sigma_t \left(e^{i,t} + s^{i,t} - \left[\lambda_0^{i,t} + \lambda_e^t e^{i,t-k} \right] \right) \right) \right]$$
$$s.t. \sum_{t=0}^{T} s^{i,t} = 0. \tag{15}$$

The firms' first order conditions are then given by equations (3) and (7), and by:

$$\sigma^t = (1+\delta)^{t-s}\sigma^s. \tag{16}$$

That is, the quota price will increase with the discount rate in the market solution. Combining equations (7) and (16) implies:

$$\frac{\sigma^t}{-c_e^{i,t}} = \frac{1}{1-\lambda_e^{t+k}}. \tag{17}$$

We see that the relationship between the quota price and MAC is much simpler in the case with banking and borrowing. For instance, with an allocation rate of 95 per cent, the quota price will be 20 times higher than MAC. If the allocation rate is set to unity in some year(s), there will be no emission reductions k year(s) before. The discount rate no longer affects the relative difference between the quota price and MAC because the quota price (and hence the value of future allowances) rises with the discount rate.

What about the effects on cost-effectiveness? From equations (16) and (17) we can derive:

$$-c_e^{i,t} = (1+\delta)^{t-s}(-c_e^{i,s})\frac{1-\lambda_e^{t+k}}{1-\lambda_e^{s+k}}. \tag{18}$$

Thus, if the allocation rate λ_e^{t+k} is constant over time, we see that equation (14) holds, i.e., the emission trading scheme is dynamically cost-effective (unless $\lambda_e^{t+k} = 1$, in which case there would be no abatement and hence no cost-effective solution). A special case of constant allocation rate is $\lambda_e^{t+k} = 0$, which means that only lump sum allocation (or auction) is used. On the other hand, if the allocation rate varies over time, the conclusion no longer holds.

Proposition 8. *Consider a closed emissions trading scheme with banking and borrowing. The trading scheme is dynamically cost-effective if and only if the allocation rate λ_e^{t+k} is constant over time and less than one. Furthermore, if the allocation rate is reduced (increased) over time, the marginal abatement costs increase too fast (slowly).*

Proof: The proof follows directly from comparing equations (14) and (18).□

Gradual reductions in the allocation rates seem more realistic than the opposite, as the government may want to act carefully in the beginning of a new scheme. Thus, an emission trading scheme with allocation rules based on (1) and banking/borrowing allowed, would typically lead to too little abatement in the early periods compared to a dynamic, cost-effective solution.

This is at least true in the last periods of a finite emission trading scheme, as $\lambda_e^{t+k} = 0$ for $t > T - k$. Thus, as long as $\lambda_e^{t+k} > 0$ for at least one year, a finite trading scheme can not be cost-effective and too much abatement is delayed to the last years. We state this in the following conjecture:

Conjecture 9. *Consider a closed emissions trading scheme with banking and borrowing, and a finite time horizon. The trading scheme is not dynamically cost-effective as long as the allocation rate λ_e^{t+k} is strictly positive for at least one t. Moreover, too much abatement is delayed until the last k years of the trading scheme.*

Proof: The proof follows directly from Proposition 8.□

With banking and borrowing the government does not control the total amount of allowances if the allocation rules for all periods are announced from the beginning. The reason is that total emissions in a period are no longer fixed, and these emissions affect the number of total allowances k years later. The analyses of market solution have assumed that the allocation rate λ_e^{t+k} for all t is known from the outset. However, the government can adjust the lump sum allocation ex-post if they want to control the total number of allowances each year (unless borrowing of allowances exceed the planned lump sum allocation).

An interesting consequence of equation (17) is that replacing auctions by free allocation based on previous emissions may not reduce the firms' costs of buying allowances. This is at least true if the trading scheme is cost-effective, which requires that the scheme lasts forever and that the allocation rate is constant over time (cf. Proposition 8). In this case, combining equations (15) and (17), discounted net costs of buying allowances are given by:

$$
\begin{aligned}
& \sum_{t=0}^{\infty} \frac{1}{(1+\delta)^t} \left[\sigma_t \left(e^{i,t} + s^{i,t} - \lambda_e e^{i,t-k}\right)\right] \\
= & \sum_{t=0}^{\infty} \frac{1}{(1+\delta)^t} \left[-c_e^{i,t} \frac{1}{1-\lambda_e} \left(e^{i,t} - \lambda_e e^{i,t} + \lambda_e e^{i,t} - \lambda_e e^{i,t-k}\right) + \sigma_t s^{i,t}\right] \\
= & \sum_{t=0}^{\infty} \frac{1}{(1+\delta)^t} \left[-c_e^{i,t} e^{i,t}\right] + \sum_{t=0}^{\infty} \frac{1}{(1+\delta)^t} \left[\frac{-c_e^{i,t}\lambda_e}{1-\lambda_e} \left(e^{i,t} - e^{i,t-k}\right)\right]. \qquad (19)
\end{aligned}
$$

Here we have used that the discounted value of net savings is zero. The first term is identical to the discounted net costs of buying allowances in the case with only auctioning (since the quota price is then equal to MAC). The sign of the second term depends on the development of emission over time, which again depends on economic growth, the cost function and the discount rate etc. The expression indicates, however, that replacing auctions with free allocaction based on previous emissions will not significantly reduce the firms' costs of buying allowances. In a finite trading system, however, this conclusion may not hold, since the allocation rate is no longer constant over time and MAC is not the same. Still, the savings may turn out to be much smaller than anticipated, due to the relationship in (17).

An infinite trading system with banking and borrowing seems rather unrealistic, as the firms may borrow allowances for an infinitely long period of time (only banking may be more acceptable).[9] Hence, Conjecture 9 becomes highly relevant.

[9] In fact, unless MAC goes to infinity when $t \to \infty$, abatemet will constantly be zero as it is always profitable to borrow allowances.

3 Numerical analyses

In Section 2.1 we discussed conditions for a feasible cost-effective solution in the case with no price cap or banking/borrowing. We believe that in most realistic cases these conditions will be met, and a feasible solution will exist. Nevertheless, the quota price may differ significantly from the marginal abatement cost, and in this section we want to look more into the size of $\sigma_t/(-c_e^{i,t})$ within a numerical framework. We will also look into the effects on emissions in the case with price cap or banking/borrowing.

3.1 Model specification

We will choose cost and demand functions that imply an upper limit to the marginal abatement costs, even if emission goes to zero or time goes to infinity. Thus, our choice of model specification is rather conservative, and cost-effective solutions will always be feasible (without price cap or banking/borrowing) as long as we have a positive discount rate (cf. Proposition 1).

We will apply the following simple cost function for the firms:

$$c^{i,t}(q^{i,t}, e^{i,t}) = C_0(q^{i,t})^2 \left(C_1 + \left(\overline{r}^i - \frac{e^{i,t}}{q^{i,t}} \right)^2 \right). \tag{20}$$

It is straightforward to show that this cost function satisfies the conditions put forward in the beginning of Section 2 as long as $\frac{e^{i,t}}{q^{i,t}} \leq \overline{r}^i$, where $\overline{r}^i$ can be interpreted as the unconstrained emission rate for firm i. That is, it is not only costly to *reduce* emission below this rate, but also to *increase* it, so that $c_e^{i,t} = 2C_0 \left(e^{i,t} - \overline{r}^i q^{i,t} \right) > (<) 0$ for $\frac{e^{i,t}}{q^{i,t}} > (<) \overline{r}^i$. With no constraint on emissions, we see that $c^{i,t}(q^{i,t}, e^{i,t})$ is a simple quadratic cost function with marginal production costs equal to $2C_0C_1q^{i,t}$.

Second, in order to simplify the analyses, we assume a closed product market and a linear inverse demand function: $p^t = A - B^0(1-\mu)^t Q^t$, where $Q^t = \sum_i q^{i,t}$, and μ is the annual growth rate of demand (for a given price). We further assume that there are m identical firms. By calculating $c_q^{i,t}$, inserting into equation (3), together with the inverse demand function, we can derive an expression for $q^{i,t}$ as a function of $e^{i,t}$. Putting this expression into the MAC function gives:

$$-c_e^{i,t} = \frac{2C_0 A\overline{r}^i + 4\left(C_0\right)^2\left(\overline{r}^i\right)^2 e^{i,t}}{2C_0 C_1 + 2C_0\left(\overline{r}^i\right)^2 + mB^0(1-\mu)^t} - 2e^{i,t}C_0. \tag{21}$$

We see that MAC is a linear function of $e^{i,t}$, also when the effects on $q^{i,t}$ are taken into account. Thus, $(-c_e^{i,t})$ does not approach infinity when the emission constraint goes to zero. Moreover, for a given emission level $e^{i,t}$, MAC increases over time. However, when $t \to \infty$, $(-c_e^{i,t})$ does not approach infinity; MAC is bounded above for a given emission level. This confirms the statement in the beginning of this subsection.

In order to calibrate the model, we normalise each firm's initial production and emission level in the business as usual (BaU) solution to unity, and the corresponding product price to 100. The choke price A is set to 5 times the BaU price, and demand is assumed to grow by 2 percent per year ($\mu = 0.02$). The parameter C_1, which implicitly defines the relative importance of abatement costs compared to production costs, is calibrated so that a 10.6 per cent cut in emissions implies that MAC is equal to €13.9 per ton CO_2. The latter figure is taken from Böhringer et al. (2005) and their cost-effective policy simulation (NAP_Opt) of CO_2 emission reductions in the EU. The discount rate is set to 7 per cent unless otherwise stated. The remaining parameters (except the policy parameters) either follow directly from the above mentioned numbers, or have no influence on the results (e.g., the number of firms, as long as the number is fixed). Table 1 sums up the chosen parameter values.

We consider different policy scenarios below. In all scenarios the initial emission target is set to 5 per cent *above* the business as usual level, so as to demonstrate the effect of overallocation. In a few years, however, the business as usual emissions exceed the emission target in all scenarios.

Table 1. Parameter values in simulation model

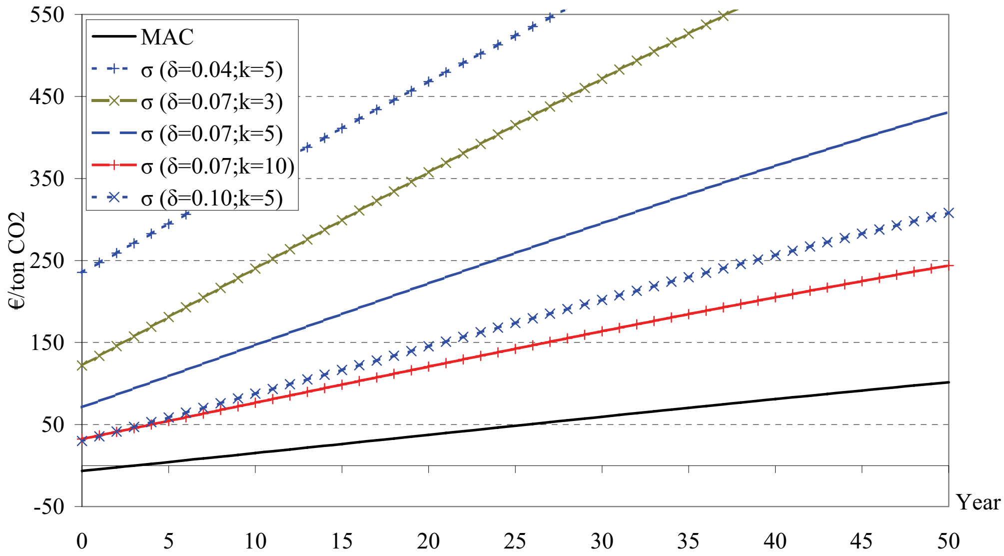
Parameter	C_0	C_1	m	$\overline{r}^i$	A	B_0	μ	δ
Value	88.9	0.56	100	1	500	4	0.02	0.07

3.2 No price cap or banking/borrowing

In this subsection we assume that there is no price cap, and no banking/borrowing is allowed. Hence, the system will be cost-effective, given the assumed cost function.

Let us start with a simple scenario in which the emission target is held constant over time. Because of economic growth ($\mu = 0.02$), MAC will increase over time. Assume for the time being that all allowances are allocated for free based on updated emission levels, i.e., $\lambda_e = 1$. Figure 1 shows the development over time of MAC and the quota price under different assumptions about the time lag (k). The effects of different discount rates (δ) are also displayed here.

Figure 1. Quota price (σ) and MAC with different combinations of time lag (k) and discount rate (δ).



The solid curve at the bottom of the figure shows the marginal abatement costs, which are slightly negative in the first four years. This is due to the overallocation of quotas, cf. Proposition 4. That is, on the margin the benefits from future allowances are set equal to the costs of increasing emissions (beyond the business as usual level). In year 50 MAC is just above €100 per ton CO_2. The steady increase in MAC is of course related to the combination of fixed emission target and growing demand. The quota price also rises over time in all scenarios, but at a much higher level. Even with a time lag of 10 years, the initial quota price is €32. The reason is that with gradually higher MAC, the quota price is also rising over time, and thus the value of future allowances is high compared to current abatement costs. Firms will therefore restrain from emission reductions initially unless the price of emissions is high.

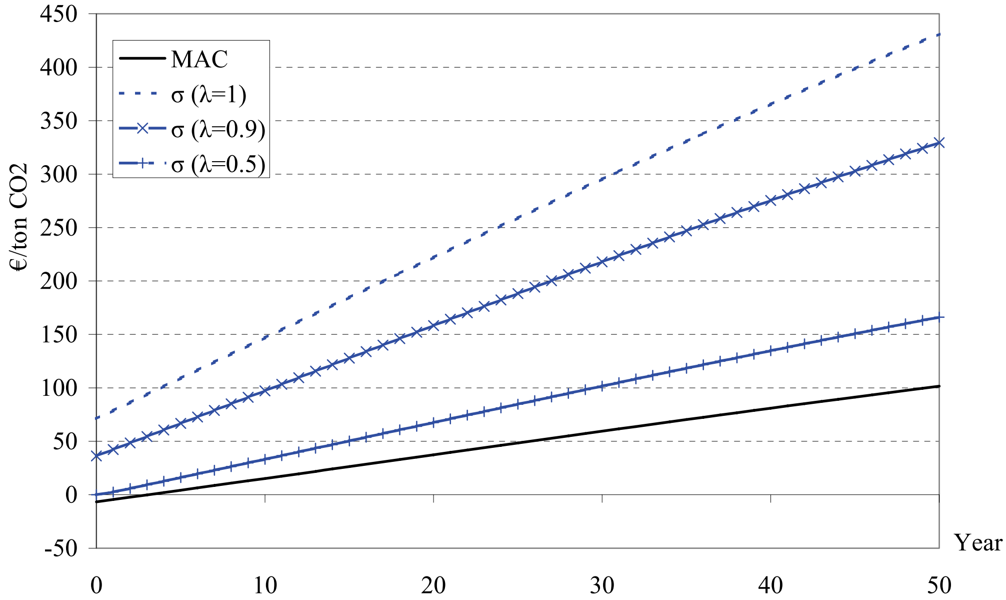
After a couple of decades, the quota price is still several times higher than MAC. For instance, in year 20 the ratio varies between 3 and 13 in the five scenarios displayed in the figure. Thus, the divergence between price and MAC is not just an initial phenomenon.

The figure also demonstrates the huge importance of both the discount rate and the time lag between emission and allowances. As opposed to the discount rate, the time lag is in control by the government. If the time lag is increased from 3 to 10 years, the price drops by around two thirds in most years in Figure 1.

The analyses so far show that by increasing the time lag between emission and allowance, the government can significantly reduce the divergence between quota price and MAC, but the price will still be several times higher than MAC if all allowances are allocated for free. Let us now examine the effects of introducing some auctioning of quotas, and look at different combinations of auction and free allocation (λ_e). Note that the sum of these must be equal to one as long as we assume constant emission target over time. Figure 2 shows the results of assuming a time lag of 5 years (and discount rate of 7 per cent).

The MAC curve and the dotted curve with $\lambda_e = 1$ are the same as in Figure 1. The figure shows that even with a quite small auction rate of 10 percent ($\lambda_e = 0.9$), the quota price drops noticeably (by 25-50 percent). This may seem a bit strange, as the value of future allowances are reduced by merely 10 percent for a given quota price. However, this also reduces the absolute growth in the quota price, thus reducing the shadow price of emissions further. Consequently, the final impact on the quota price is significant, but still the price is several times higher than MAC.

Figure 2. Quota price (σ) and MAC with different combinations of allocation rate (λ_e) ($k = 5$).



If the auction rate is raised to 50 percent ($\lambda_e = 0.5$), the quota price really starts to approach MAC. The initial price is actually zero,[10] and in year 10 the price is merely two times higher than MAC. Interestingly, in this case the time lag has very little influence on the quota price, and initially the quota price is actually slightly *higher* when the time lag is higher. The explanation is that in this case the quota price rises more rapidly than the discount rate over the first 15 years. Therefore, the discounted value of future allowances are worth more if the time lag is large.

An interesting result in Figure 2 is that firms' payments for allowances are initially *higher* with partial auctioning ($\lambda_e = 0.9$ or 0.5) than with 100 per cent auctioning. That is, the higher emission price dominates the reduced payment due to free allowances. However, after respectively 6 and 12 years (for $\lambda_e = 0.9$ and 0.5), the payments get lower than with only auctions. Moreover, in all cases the discounted value of future allowances are highest with only auctioning, but the cost savings from free allowances are moderate (unless $\lambda_e \approx 1$). In the cases presented in Figure 2, 10 and 50 per cent

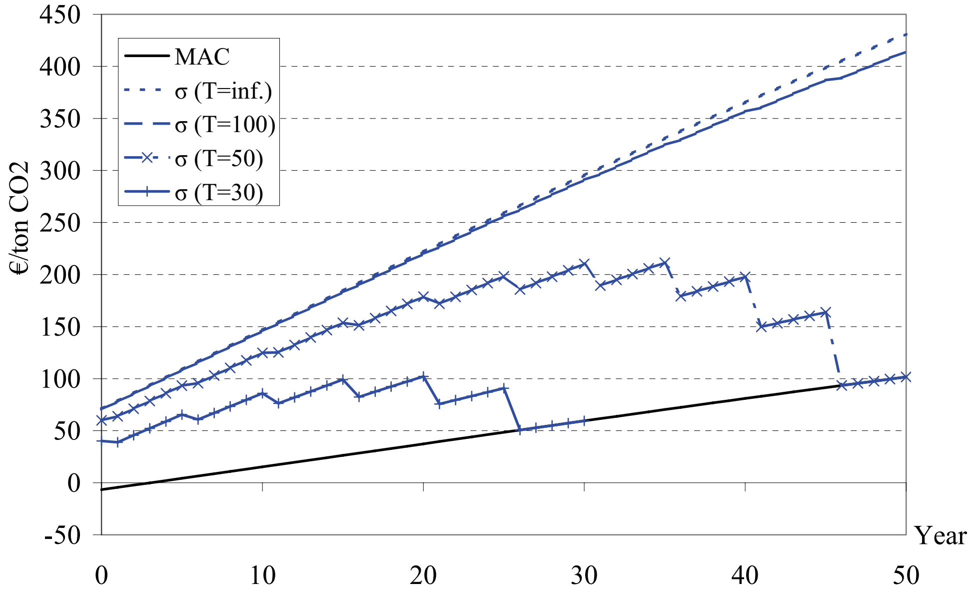
[10]This implies that MAC is less negative in this scenario than displayed in the figure, but still below zero (cf. Proposition 4).

auctioning reduce firms' payments by respectively two thirds and 18 per cent (compared to only auctioning).

So far we have assumed that the quota system lasts forever. More realistically, perhaps, a quota system will last for some decades. Due to discounting, the results presented above will not be much altered as long as the system is expected to last more than 50 years beyond the last year presented in the figures, i.e., more than 100 years in total for Figures 1-2. What if the system is supposed to last for a shorter time period? This is shown in Figure 3, assuming no auction and 5 years time lag.

According to the figure, a system that will last for one century will have almost the same quota prices over the first five decades as an infinite system. If the system only lasts 50 years, the quota price is somewhat reduced already from year 0. We see that the quota price equals MAC in the last five years, as emissions in these years do not lead to any future allowances. This five years period has implications for the preceding five years period, which has further implications on the period before etc., explaining the seemingly volatile price evolvement with a finite quota system. Note that within each five years period, the quota price is increasing. This is consistent with increasing MAC, and backward induction of the increasing quota price in the next period. With a system that lasts for only 30 years, the quota price drops significantly in all years. Still, even in year 25, i.e., 5 years before the system terminates, the quota price is almost two times higher than MAC. We therefore conclude that not only long-lasting quota systems, but also systems lasting for a few decades, will have major divergence between the quota price and MAC.

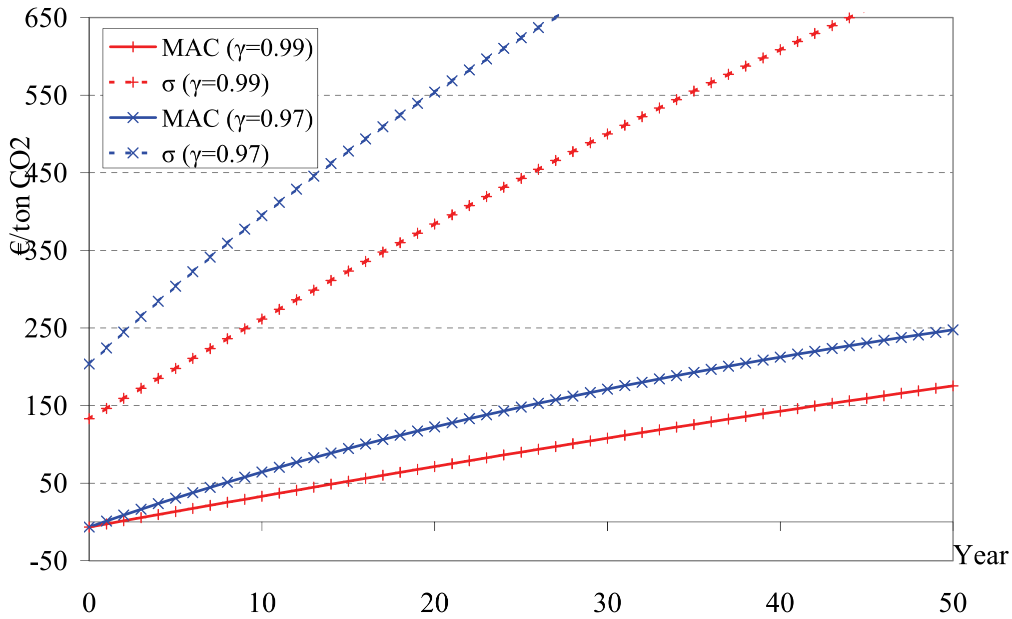
Figure 3. Quota price (σ) and MAC with different duration (T) of the quota system (no auction and $k = 5$).



Let us now assume that the emission target is reduced over time. MAC will then increase both due to stricter target and increased demand. Thus, we will expect the quota price to deviate even more from MAC. On the other hand, with unchanged share of lump sum or auctioned permits, the allocation rate will have to be lower and therefore reduce the number of allocations in the future periods. This may *reduce* the deviation between the quota price and MAC (cf. Proposition 3).

Figure 4 shows the quota prices and MAC in two emission reduction scenarios ($\gamma = 0.99; \gamma = 0.97$). In both scenarios $\lambda_e = \gamma$, i.e., there are no lump sum or auctioned permits, and $k = 5$ (as in Figures 2 and 3).

Figure 4. Quota price (σ) and MAC with different emission reduction factors (no auction and $k = 5$).



The marginal abatement costs start of course at the same level in year 0, but then increase much faster when γ is reduced. The quota price starts at higher levels when γ is reduced. Except for the first few years, however, the ratio is quite similar for $\gamma = 0.97$, 0.99 and 1.

If we reduce the emission reduction factor γ further, the quota price also rises further, but only to a certain point. The maximum level of σ_0 is actually achieved at around $\gamma = 0.90$ ($\sigma_0 = 257$). With even stronger emission reductions, the initial quota price drops. The explanation is that reduced future emission levels also imply lower future allocations, and thus reduced value of having high emission levels today (for a given quota price path).

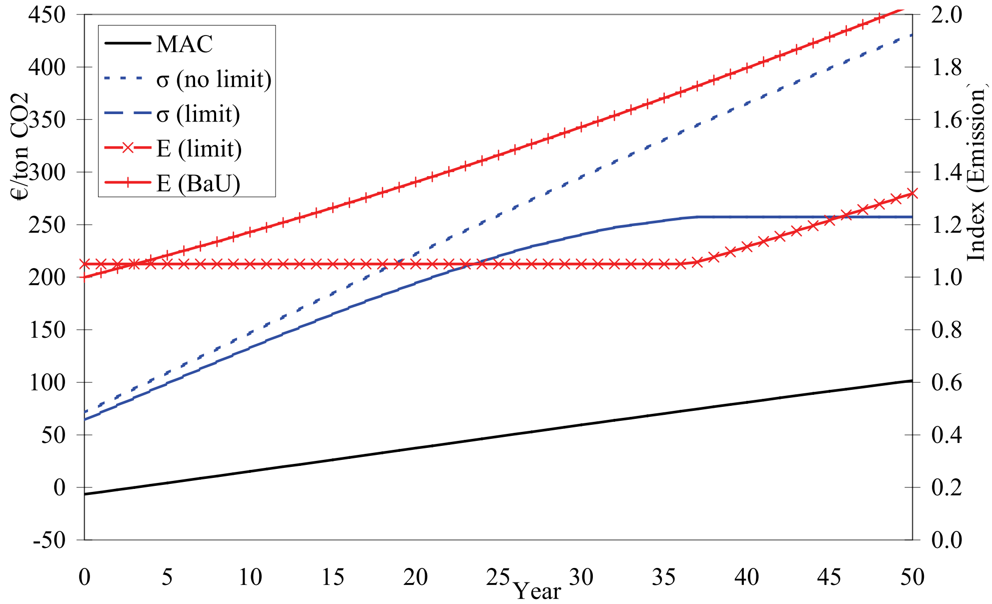
With partial auctioning, we get similar conclusions as above, i.e., the quota price is significantly reduced. Moreover, the effects of reducing the emission target over time are much the same as with only emission-based allocations (Figure 4).

3.3 Price cap

With very high quota price levels compared to MAC, a price cap could be introduced. In this case we know from Proposition 7 that if the price cap is binding, total abatement level is reduced in some years compared to an auction-based allocation. Moreover, according to Proposition 6 the quota price is reduced in all years before and including the year(s) when the price cap is binding.

Let us introduce a price cap $\widehat{\sigma} = 257$ in the scenario with constant emission target (but economic growth), cf. Figure 1. This cap equals the maximum level of MAC when demand becomes infinitely large ($t \to \infty$). We focus on emission-based allocations, i.e., $\lambda_e = 1$, and assume $k = 5$.

Figure 5. Quota price (σ), MAC and emissions (E) with price cap ($\lambda_e = 1$ and $k = 5$).



In the unconstrained price scenario, the quota price passes $\widehat{\sigma}$ after 25 years, but it takes 37 years before the price cap is reached in the constrained scenario. The reason is that when future prices are constrained, the value of future allocations is reduced, and the incentives to reduce current emissions increase. Thus, the initial quota price falls, cf. Proposition 6.

As mentioned above, emission starts to increase when the price cap is reached, despite the fixed emission target. As shown in the figure, emission increases significantly after the first 37 years with price below the cap. After 50 years the emissions are one fourth above the emission target, even though MAC is only 40 per cent of the price cap at this time. With auctioning combined with the same price cap, the cap would never become binding, and the emission target would always be met.

3.4 Banking and borrowing

Finally, consider the case with unconstrained banking and borrowing within an emission trading scheme that lasts 50 years (as mentioned before, unconstrained borrowing seems unlikely within an infinite scheme). From Conjecture 9 we immediately know that the trading scheme will not be dynamically cost-effective. The accumulated emission constraint is assumed to be the same as the one in Figure 3 (with $T = 50$), i.e., 50 times the annual emission target applied in Figures 1-3. Furthermore, with $k = 5$, we assume that λ_e is linearly reduced from 0.9 in year 5 to zero in year 50. From Proposition 8 we then know that MAC increases too fast (i.e., emissions are reduced too fast), so that initial abatement is too low.

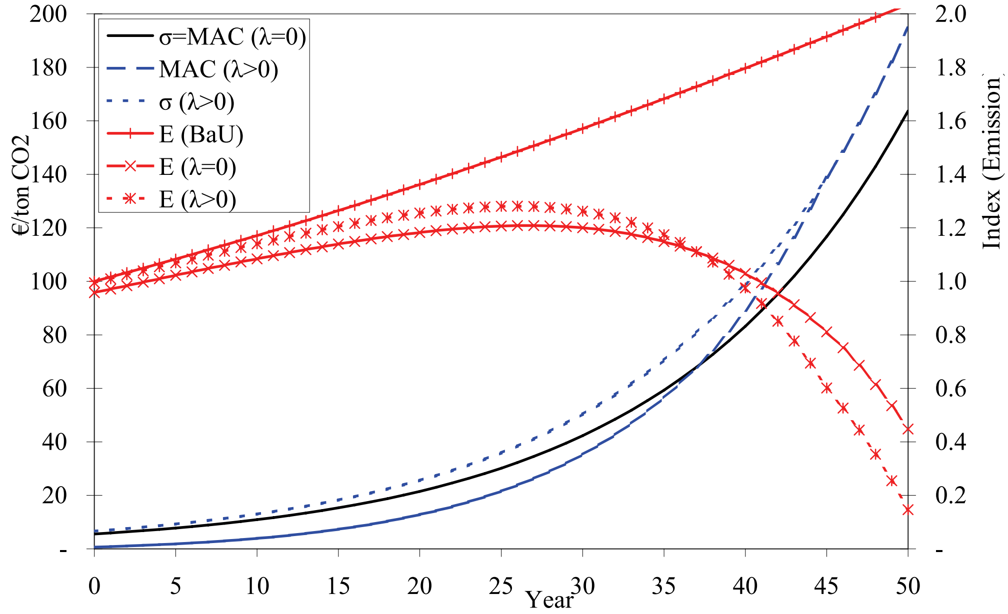
Figure 6 shows the effects on emissions, quota prices and MAC in this case ($\lambda_e > 0$), and in the case with only auctioning ($\lambda_e = 0$). Notice that MAC starts much lower in the former case (but no longer below zero), and rises much faster (as predicted). The quota price is constantly 20 per cent higher with $\lambda_e > 0$ compared to $\lambda_e = 0$ (both rises with the discount rate).

With access to banking, there is no longer any rationale for negative abatement levels initially. Nevertheless, in the initial period of the system, emissions are much more reduced in the case with auctioning than in the case with allocation based on previous emissions. For instance, in the first 15 years abatement is more than two times higher in the former case.

When banking and borrowing are allowed, total discounted abatement costs over the 50 years increase by 6 per cent in this emission trading scheme (compared to the cost-effective scheme). As a comparison, with fixed and constant emission target each year (as in Figure 3), the costs increase by 14 per cent.[11]

[11] Abatement costs are calculated as the difference in the sum of consumer and producer surplus between the emission reduction scenarios and the BaU scenario.

Figure 6. Quota price (σ), MAC and emissions (E) with banking and borrowing.



4 Conclusions

Allocation of free allowances within emission trading schemes is difficult without either distorting firms' decisions, or leading to adverse distributional effects. Nevertheless, Böhringer and Lange (2005a) showed that allocation of free allowances based on updated emissions could lead to a cost-effective solution, as long as the trading scheme is closed. This finding would seem intriguing to policy makers afraid of introducing auctioning due to e.g. competitive considerations or lobbying.

The current paper analyses the dynamic effects of such allocation rules in more detail, pointing to several factors that may alter the conclusion by Böhringer and Lange. First, an infinite emission trading scheme based on updated emissions may in fact be infeasible if the marginal abatement costs in the long run grow faster than a certain rate (Proposition 1). This rate is determined by the discount rate, the allocation rate and the time lag between emission and allocation. The reason is that the price of allowances (quota price) becomes infinitely large.

Second, an overallocation of quotas will increase emissions beyond its business as usual level (Proposition 4), as higher emission levels give more valuable allowances in the future.

Third, if a price cap is introduced in order to avoid too high quota prices, total emissions become higher when allowances are based on updated emissions than in the case with auction (or lump sum allocation). This conclusion holds as long as the quota price would have exceeded the price cap in the case with no such cap (Proposition 7). Distribution of abatement effort will still be cost-effective, but the level of abatement now depends on the allocation mechanism.

Fourth, if banking and borrowing are introduced, the trading scheme will no longer be dynamically cost-effective if allocation of allowances are (partly) based on updated emissions in some year(s). The only exception to this conclusion is that the trading scheme lasts forever (which seems unlikely given unconstrained borrowing), and has a constant allocation rate (Proposition 8). Moreover, if the allocation rate falls over time, e.g., due to a gradual shift from free allocation to auctioning, too little abatement will take place initially.

Although the case for infeasible solution does not seem very likely (cf. the first factor above), the price of allowances may well become very large compared to the marginal abatement costs, especially in a growing economy with a constant or stricter emission target. There are several important factors that determine the ratio between the quota price and MAC, and some of these are determined by the policy makers. An increase in the allocation rate, the duration of the system and future emission reductions, all lead to a higher gap between the quota price and MAC (Proposition 3). A lower time lag between emission and corresponding allocation also increases the gap in most cases.

Numerical simulations indicate that the price of allowances will be several times higher than the marginal abatement costs, unless a majority of the allowances are auctioned. This applies even with time lags of 10 years, and with trading schemes that last only a few decades. Sensitivity analyses (not reported) clearly support these numerical findings. With overallocation, price cap or banking and borrowing, the emission path is also significantly altered when auctioning is replaced by allocation based on updated emissions.

High prices of allowances may give policy makers a false impression of the costs of abatement. Moreover, risk of inefficiencies increase if the system is not as pure and simple as in the model presented in Section 2.1. We have already shown the effects of introducing overallocation, price caps or banking/borrowing, and similar inefficiencies would occur if allocation rules differ across emitters, or if there is some access to external quotas. Also different discount rates among firms, or different expectations about future allocation rules or quota prices, would lead to inefficiencies.

Emission trading schemes with free allocation seems to be the most favoured policiy instrument when it comes to reducing emissions to air. Moreover, lump sum allocation is only partly used. More research on the effects of different allocation mechanisms is therefore needed, both as a guide to design appropriate trading schemes, and to help understand the market for emission allowances.

Acknowledgement.
I am grateful for valuable comments from Mads Greaker and Cathrine Hagem on an earlier draft.

5 References

Böhringer, C., M. Ferris and T.F. Rutherford (1998): Alternative CO_2 abatement strategies for the European Union. In: Braden, J., Proost, S. (Eds.), *Climate Change, Transport and Environmental Policy*, Edward Elgar, Cheltenham, pp. 16–47.

Böhringer,C., T. Hoffmann, A. Lange, A. Löschel and U. Moslener (2005): Assessing Emission Regulation in Europe: An Interactive Simulation Approach, *The Energy Journal* **26** (4), 1-22.

Böhringer, C. and A. Lange (2005a): On the design of optimal grandfathering schemes for emission allowances, *European Economic Review* **49**, 2041–2055.

Böhringer, C. and A. Lange (2005b): Economic Implications of Alternative Allocation Schemes for Emission Allowances, *Scandinavian Journal of Economics* **107**, 563-581.

Burtraw, D., K. Palmer and D. Kahn (2005): Allocation of CO_2 Emissions Allowances in the Regional Greenhouse Gas Cap-and-Trade Program, Discussion Paper 05–25, Resources for the Future, Washington DC.

Burtraw, D., D. Kahn and K. Palmer (2006): CO_2 Allowance Allocation in the Regional Greenhouse Gas Initiative and the Effect on Electricity Investors, *The Electricity Journal* **19** (2), 79-90.

Considinea, T.J. and D.F. Larson (2006): The environment as a factor of production, *Journal of Environmental Economics and Management* **52**, 645-662.

Ellis, J. and D. Tirpak (2006): Linking GHG Emission Trading Schemes and Markets, OECD/IEA, October 2006.

EU (2003): Directive 2003/87/EC of the European Parliament and of the Council of 13 October 2003 establishing a scheme for greenhouse gas emission allowance trading within the Community and amending Council Directive 96/61/EC, European Commission, Brussels.

EU (2005): Further guidance on allocation plans for the 2008 to 2012 trading period of the EU Emission Trading Scheme, European Commission, Brussels.

Fischer, C. (2001): Rebating environmental policy revenues: Output-based allocations and tradable performance standards, Discussion Paper 01-22, Resources for the Future, Washington, DC.

Goulder, L.H., I.W.H. Parry, R.C. Williams and D. Burtraw (1999): The cost-effectiveness of alternative instruments for environmental protection in a second-best setting, *Journal of Public Economics* **72**, 329–360.

Grubb, M., C. Azar and U.M. Persson (2005): Allowance allocation in the European emission trading system: a commentary, *Climate Policy* **5**, 127-36.

Jensen, J. and T. N. Rasmussen (2000): Allocation of CO_2 Emission Allowances: A General Equilibrium Analysis of Policy Instruments, *Journal of Environmental Economics and Management* **40**, 111–136.

Keats Martinez, K. and K. Neuhoff (2005): Allocation of carbon emission certificates in the power sector: how generators profit from grandfathered rights, *Climate Policy* **5**, 61-78.

Kuik, O. and M. Mulder (2004): Emissions trading and competitiveness: pros and cons of relative and absolute schemes, *Energy Policy* **32**, 737-745.

Montgomery, W.D. (1972): Markets in Licenses and Efficient Pollution Control Programs, *Journal of Economic Theory* **5**, 395-418.

Mæstad, O. (2007): Allocation of emission permits with leakage through capital markets, *Resource and Energy Economics* **29**, 40–57.

Palmer, K. and D. Burtraw (2005): Cost-effectiveness of renewable electricity policies, *Energy Economics* **27**, 873-894.

Parry, I.W.H., R.C. Williams III and L.H. Goulder (1999): When can carbon abatement policies increase welfare? The fundamental role of distorted factor markets, *Journal of Environmental Economics and Management* **37**, 52–84.

Watanabe, R. and G. Robinson (2005): The European Union Emission Trading Scheme (EU ETS), *Climate Policy* **5**, 10-14.

Åhman, M., D. Burtraw, J. Kruger and L. Zetterberg (2007): A Ten-Year Rule to guide the allocation of EU emission allowances, *Energy Policy* **35**, 1718-1730.

A Appendix A

Here we present the proofs of Propositions 1 and 6 and Conjecture 5.

Proof of Proposition 1:

As explained above, a cost-effective solution is feasible if and only if the quota price is finite, as equations (3) and (4) are then fulfilled.

The last part of the proposition follows directly from equation (8), as mentioned above.

Let $-c_e^{i,jk} \leqslant K\left(1+\hat{\gamma}\right)^{jk}$ where $\hat{\gamma} < \gamma$. Note that $\left(\frac{\lambda_e}{(1+\delta)^k}\right) = (1+\gamma)^{-k}$. Inserting this into (9) gives $\frac{\sigma_0}{-c_e^{i,0}} \leqslant 1 + \frac{1}{-c_e^{i,0}}\sum_{j=1}^{\frac{T}{k}}\left\{K\left(1+\hat{\gamma}\right)^{jk}\left(1+\gamma\right)^{-jk}\right\} = 1 + \frac{1}{-c_e^{i,0}}\sum_{j=1}^{\frac{T}{k}}\left\{K\left(\frac{1+\hat{\gamma}}{1+\gamma}\right)^{jk}\right\}$. Since $\hat{\gamma} < \gamma$, this is a geometric sum with a geometric factor less than unity, implying that the sum has a finite value when $N \to \infty$. Consequently, the cost-effective solution is feasible.

Let $-c_e^{i,jk} \geq K\left(1+\gamma\right)^{jk}$. We now have $\frac{\sigma_0}{-c_e^{i,0}} \geq 1 + \frac{1}{-c_e^{i,0}}\sum_{j=1}^{\frac{T}{k}}\left\{K\left(1+\gamma\right)^{jk}\left(1+\gamma\right)^{-jk}\right\} = 1 + \frac{1}{-c_e^{i,0}}\sum_{j=1}^{\frac{T}{k}}K = \left(\frac{N}{k}+1\right)K$. Thus, $\sigma_0 \to \infty$ when $T \to \infty$. Consequently, the cost-effective solution is infeasible.□

Proof of Conjecture 5:

With constant emission target, cost function and output prices, we must have $c_e^{i,t} = c_e^i$, i.e., MAC is constant, too. Moreover, since total emission is constant over time, for all t we must have either $\lambda_0^{i,t} > 0$, or $\lambda_e = 1$.

If $\lambda_e = 1$ and $\delta = 0$, then $\gamma = 0$, and from Proposition 1 we have that the cost-effective solution is infeasible (since MAC grows by the rate zero). If instead $\lambda_e < 1$ or $\delta > 0$, then $\gamma > 0$, and by Proposition 1 the cost-effective solution is feasible.□

Proof of Proposition 6:

The proof is easily shown by backward induction. Assume that $\sigma_{C,t+1}<\sigma_{NC,t+1}$ holds for some $t+1 \leq t_2$, where t_2 is the last year in time period $\mathbf{t}$. Define $\overline{\sigma}_t = -c_e^{i,t} + \sum_{l=1}^{k} \left(\frac{1}{(1+\delta)^l} \sigma_{C,t+l} \lambda_e^{t+l,t} \right) < -c_e^{i,t} + \sum_{l=1}^{k} \left(\frac{1}{(1+\delta)^l} \sigma_{NC,t+l} \lambda_e^{t+l,t} \right) = \sigma_{NC,t}$ (cf. (6)). If the price cap is not binding, we have from equation (6) that $\sigma_{C,t} = \overline{\sigma}_t < \sigma_{NC,t}$. If the price cap is binding, we have $\sigma_{C,t} < \overline{\sigma}_t < \sigma_{NC,t}$. Since the condition obviously holds for $t \in \mathbf{t}$, the proof is completed.□

B Appendix B

In this appendix we derive a specific condition for when the cost-effective solution is (in)feasible. We restrict our analysis to cost functions with the following characteristics: *i)* $c_e^{i,t}(q^{i,t}, e^{i,t}) = \widehat{c}_e^i(e^{i,t})g(t)$, with $g'(t) \geq 0$ and $\lim_{t\to\infty} g(t+1) = (1+\mu)g(t)$ (i.e., $g(t)$ grows by a factor μ in the long run);[12] and *ii)* $\lim_{e^i\to 0} \widehat{c}_e^i(ke^i) = k^{-\varsigma}\widehat{c}_e^i(e^i)$ (i.e., $\widehat{c}_e^i$ is homogenous of degree $-\varsigma$ for e^i near zero).[13] Moreover, we assume that $\overline{E}^{t+1} = \gamma\overline{E}^t$ ($\gamma \leq 1$), i.e., the emission target is either unchanged or reduced by a constant rate per year. Note that μ affects the second term in (11), whereas γ and ς affect the last terms in (11) and (12). All parameters indirectly affect the first term in (11) and (12).

Then we have the following conjecture:

Conjecture B1. *Consider a closed, infinite emissions trading scheme and cost functions with characteristics i) and ii). A cost-effective solution is infeasible if and only if the allocation rate λ_e and the time lag k are set so that $\lambda_e \geq \left(\frac{1+\delta}{(1+\mu)\gamma^{-\varsigma}}\right)^k$, where μ, γ and ς are defined above.*

[12] One obvious example is a separable cost function, in which case $g(t) \equiv 1$. Another example is the case with $c^{i,t}(q^{i,t}, e^{i,t}) = \frac{q^{i,t}}{e^{i,t}}$ and iso-elastic demand function.

[13] If e.g. $\widehat{c}_e^i(e^i)$ is iso-elastic with exponent -ς, then the MAC function is homogenous of degree -ς.

Proof:
In the long run $-c_e^{i,t}(q^{i,t}, e^{i,t})$ increases by the rate:
$\frac{(-\widehat{c}_e^i(e^{i,t+1})g(t+1))-(-\widehat{c}_e^i(e^{i,t})g(t))}{(-\widehat{c}_e^i(e^{i,t})g(t))} = \frac{(-\widehat{c}_e^i(\gamma e^{i,t})(1+\mu)g(t))-(-\widehat{c}_e^i(e^{i,t})g(t))}{(-\widehat{c}_e^i(e^{i,t})g(t))}$
$= \frac{(-\gamma^{-\varsigma}\widehat{c}_e^i(e^{i,t})(1+\mu)g(t))-(-\widehat{c}_e^i(e^{i,t})g(t))}{(-\widehat{c}_e^i(e^{i,t})g(t))} = (1+\mu)\gamma^{-\varsigma} - 1.$
If $\lambda_e \geq \left(\frac{1+\delta}{(1+\mu)\gamma^{-\varsigma}}\right)^k$, then from Proposition 1 we get that:
$\gamma \leq \left(\frac{(1+\delta)^k}{\left(\frac{1+\delta}{(1+\mu)\gamma^{-\varsigma}}\right)^k}\right)^{\frac{1}{k}} - 1 = (1+\mu)\,\gamma^{-\varsigma} - 1,$
and thus the cost-effective solution is infeasible.
If on the other hand $\lambda_e < \left(\frac{1+\delta}{(1+\mu)\gamma^{-\varsigma}}\right)^k$, then $\gamma > (1+\mu)\,\gamma^{-\varsigma} - 1$ and the cost-effective solution is feasible.□

The conjecture states for instance that a cost-effective solution is infeasible if the emission target is constant over time ($\gamma = 1$), and the allocation rate is set at $\lambda_e \geq \left(\frac{1+\delta}{1+\mu}\right)^k$. That is, if the MAC function increases by a rate equal to or higher than the discount rate, the solution is infeasible unless a certain amount of lump sum allocation (or auction) is introduced. If μ is slightly lower than δ, the cost-effective solution is feasible, but the quota price will become very high compared to MAC unless the allocation rate is set sufficiently below unity.

If the government wants to reduce the emission target over time ($\gamma < 1$), and don't want to use lump sum allocation (or auction), we see from Conjecture B1 that the emission reduction factor γ must exceed $\gamma^* = 1/\frac{(1+\delta)}{(1+\mu)}^{\left(\frac{1}{\varsigma-1}\right)}$. For instance, with $\delta = 0.07$, $\varsigma = 3$ and $\mu = 0.02$, we must have $\gamma > 0.976$, i.e., the emission target cannot be reduced by more than 2.4 per cent per yer. Again, if γ is set only marginally above 0.976, the quota price will become very high.

Finally, an interesting observation is that if the right-hand side of the expression in Conjecture B1 is less than one, then the allocation rate λ_e must be lower the *higher* k is. The reason is simply that the discounted value of future allowances will increase over time, and the longer into the future current emissions affect allowances, the more incentives fims have to increase emissions today.