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**Pareto-efficient climate  
agreements**

**Abstract:**

Recent contributions show that climate agreements with broad participation can be implemented as weakly renegotiation-proof equilibria in simple models of greenhouse gas abatement where each country has a binary choice between cooperating (i.e., abate emissions) or defecting (no abatement). Here we show that this result carries over to a model where countries have a continuum of emission choices. Indeed, a Pareto-efficient climate agreement can always be implemented as a weakly renegotiation-proof equilibrium, for a sufficiently high discount factor. This means that one need not trade-off a “narrow but deep” treaty with a “broad but shallow” treaty.

**Keywords:** Climate, non-cooperative game-theory, repeated games, weakly renegotiation-proof agreements

**JEL classification:** C72, F53, Q54.

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## 1 Introduction

In simple dynamic models of international environmental public good provision, such as mitigation of climate change, Barrett (1999, 2002) has argued that there is a trade-off between "narrow but deep" and "broad but shallow" treaties: either only a few countries participate each with a large abatement, or many countries participate each with a small abatement.

By applying the Barrett (1999) model, where each country has a binary choice between *cooperating* (i.e., abate emissions) or *defecting* (no abatement), Asheim et al. (2006) show that extended deep participation is feasible. They show that participation can essentially be doubled by letting there be two regional agreements instead of one global agreement.

The analysis of Asheim et al. (2006) exploits the fact that Barrett (1999) considers only strategy profiles with a special structure, namely where there is a subset of participating countries ("signatories") in a treaty, and where a defecting signatory is punished by having *all* other signatories defect in the next period (only). Then if there are too many signatories, these will gain by renegotiating back to cooperation without imposing the punishment, thereby undermining the credibility of the equilibrium. In the setting of a two region world, Asheim et al. (2006) limit the number of punishing countries by letting a defection be punished only by the other signatories in the same region, while the signatories of the other regional treaty continue to cooperate.

It is not obvious that this is the only way to solve the problem posed by Barrett's (1999) model. What if one allows for—in a more general setting—the possibility that only a subset of the signatories within a global treaty punishes a deviant? This possibility is investigated by Froyn and Hovi (2007) within the binary choice model of Barrett (1999). They show that full participation can indeed be implemented as a weakly renegotiation-proof equilibrium.

It is still an open question whether this results carries over to continuum choice model like the one considered by Barrett (2002). To reach a Pareto-efficient agreement in such a setting, one need not only agree on a *broad* treaty with full participation, but also a *deep* treaty where each country's abatement is at an efficient level. We show in this paper that such an efficient broad and deep treaty can always be implemented as a weakly renegotiation-proof equilibrium, for a sufficiently high discount

factor, in a model where the public benefits of emission abatement are linear and private costs of emission abatement are quadratic.

Linear benefits of abatement represent a simplification. Asheim et al. (2006) show that their result holds also when abatement yields non-linear benefits. A task for further research would be to check whether our finding carries over to a less restrictive model with non-linear benefits of abatement and asymmetric countries, as considered in the two-country model in Finus and Rundshagen (1998).

## 2 Main result

Consider a world containing  $n$  countries, with  $N = \{1, \dots, n\}$  denoting the set of all countries. The countries interact in periods (or “stages”)  $0, 1, 2, \dots$ . The countries are identical in all relevant characteristics. In every period, each country  $i$  must choose a non-negative level of abatement  $q_i$  of greenhouse gas emission. Each country  $i$ ’s periodic payoff, relative to the situation where no country abates, is given by

$$\pi_i = b \sum_{j=1}^n q_j - \frac{c}{2}(q_i)^2, \quad (1)$$

where  $b$  is the marginal benefit from abatement (which is a pure public good that benefits each and every country), and  $(c/2)(q_i)^2$  represents the total abatement costs of country  $i$ . We assume that  $b, c > 0$ .

Following Barrett (1999, 2002) and Asheim et al. (2006), we abstract from the future benefits of abatements (which of course is important in the climate change setting), meaning that the situation can be modeled as an infinitely repeated game, with a stage game where the countries simultaneously and independently choose abatement levels, and receive payoffs according to (1).

In the stage game there is a unique Nash equilibrium where each country abates

$$q^1 = \frac{b}{c}.$$

Actually, for each country,  $q^1$  strictly dominates any other action of the stage game and is thus its unique best response independently of what the other countries’ abatement levels are. However, the unique symmetric Pareto-efficient abatement profile entails that each country abates

$$q^n = \frac{nb}{c}.$$

Hence, the Pareto-efficient abatement is  $n$  times the abatement level in the Nash equilibrium of the stage game, cf. Barrett (2002, p. 540). In particular, the  $n$  countries would want to agree on implementing

$$\mathbf{a} = \underbrace{(q^n, \dots, q^n)}_{j \in N}, \underbrace{(q^n, \dots, q^n)}_{j \in N}, \dots,$$

where each country contributes to the Pareto-efficient total abatement in a cost-efficient manner in every period.

In the absence of third-party enforcement, such a Pareto-efficient agreement needs to be self-enforcing, where deviations from this agreement—leading to a short-run benefits for the deviating country—is deterred through the threat of future punishment, which must also be self-enforcing. Here, “self-enforcing” refers to the play of a non-cooperative equilibrium of the infinitely repeated game; the analysis of such equilibria requires the introduction of some game-theoretic formalism.

A history at the beginning of stage  $t$  describes the countries’ abatement levels in periods  $0, \dots, t - 1$ :

$$(q_1(0), \dots, q_n(0)), (q_1(1), \dots, q_n(1)), \dots, (q_1(t - 1), \dots, q_n(t - 1)).$$

A strategy  $s_i$  for country  $i$  is a function which for every history, including the “empty” history at the beginning of state 0, determines an abatement level for player  $i$ . Country  $i$ ’s average discounted payoff in the repeated game is given by

$$(1 - \delta) \sum_{t=0}^{\infty} \delta^t \pi_i(t), \quad (2)$$

where  $\delta \in (0, 1)$  is the discount factor, and  $\pi_i(t)$  is country  $i$ ’s periodic payoff according to (1) in stage  $t$  when the abatement profile is  $(q_1(t), \dots, q_n(t))$ . A strategy profile  $(s_1, \dots, s_n)$  is a subgame-perfect equilibrium if, for every history, there is no country that can increase its discounted payoff by deviating from its strategy, provided that all other players follow their strategies in the continuation of the game. A subgame-perfect equilibrium is weakly renegotiation-proof (Farrell and Maskin, 1989) if there do not exist two histories such that all players strictly prefer the continuation equilibrium in the one to the continuation equilibrium in the other.

We can now state our main result.

**Proposition 1** *For any positive integer  $n \geq 2$  and positive real numbers  $b$  and  $c$ , there is  $\bar{\delta} \in (0, 1)$  such that there exists a weakly renegotiation-proof equilibrium with  $\mathbf{a}$  as the equilibrium path whenever the countries' repeated game payoffs are discounted by  $\delta \in [\bar{\delta}, 1)$ .*

This result follows from a more general result stated in the next section. In the remainder of this section, we describe the structure of the weakly renegotiation-proof equilibria that can be used to implement the Pareto-efficient agreement  $\mathbf{a}$  in a self-enforcing manner.

Following Abreu (1988) we consider simple strategy profiles, consisting of an equilibrium path to be implemented, and  $n$  punishment paths, one for each player. The equilibrium path is followed until a single country deviates, an occurrence that leads to this player's punishment path being initiated in the next period. Also any unilateral deviation from a punishment path leads to the initiation of the (new) deviating country's punishment path. Through these rules, the  $n+1$  paths specify a strategy for each player. Hence, with  $\mathbf{a}$  as the equilibrium path, we need only construct the  $n$  punishment paths and show that the resulting simple strategy profile is indeed a weakly renegotiation-proof equilibrium.

To construct the path,  $\mathbf{p}_i$ , used to punish country  $i$ , consider a function which for any  $n$  determines a subset  $P_i(n) \subset N$  of punishing countries. Let  $P_i(n)$  have the properties that (1)  $i \notin P_i(n)$  and (2) the number of countries in  $P_i(n)$  equals  $n/2$  if  $n$  is even and  $(n+1)/2$  if  $n$  is odd. The interpretation is that each country in  $P_i(n)$  punishes a unilateral deviation by country  $i$  by choosing the abatement level  $q^1$  in the period immediately following country  $i$ 's deviation, while countries in  $N \setminus P_i(n)$  (including country  $i$ ) abates at the Pareto-efficient level  $q^n$ . In the subsequent periods all countries return to the Pareto-efficient abatement level. Hence, the punishment path of country  $i$  is:

$$\mathbf{p}_i = (\underbrace{q^1, \dots, q^1}_{j \in P_i(n)}, \underbrace{q^n, \dots, q^n}_{j \in N \setminus P_i(n)}, \underbrace{q^n, \dots, q^n}_{j \in N}, \underbrace{q^n, \dots, q^n}_{j \in N}, \dots)$$

In the next section we show that, for any positive integer  $n \geq 2$  and positive real numbers  $b$  and  $c$ , the simple strategy profile described by the  $n+1$  paths  $(\mathbf{a}, \mathbf{p}_1, \dots, \mathbf{p}_n)$  is a weakly renegotiation-proof equilibrium, provided that  $\delta$  is sufficiently close to 1. Having  $n/2$  (or  $(n+1)/2$  if  $n$  is odd) countries punishing for one period by choosing their best response  $q^1$  of the stage game is sufficiently many to discipline a potential deviator,

while being sufficiently few to ensure that each punisher in  $P_i(n)$  gains at least as much by reducing its abatement as it loses by the fact that the other countries in  $P_i(n)$  abate less.

### 3 Participation and punishment

In this section we consider a class of strategy profiles in the repeated games described in Section 2, and establish as Proposition 2 under what parameter values members of this class are weakly renegotiation-proof equilibria. Since the strategy profiles used to establish existence in Proposition 1 are members of this class, this result follows as a corollary to Proposition 2.

Fix the set of countries  $N = \{1, \dots, n\}$ . Let  $M = \{i_1, \dots, i_m\} (\subseteq N)$  be the signatories to a treaty, with  $m$  members (where  $0 < m \leq n$ ). The treaty specifies that the agreement  $\mathbf{a}^s$  be implemented, where

$$\mathbf{a}^s = (\underbrace{q^s, \dots, q^s}_{j \in M}, \underbrace{q^1, \dots, q^1}_{j \in N \setminus M}), (\underbrace{q^s, \dots, q^s}_{j \in M}, \underbrace{q^1, \dots, q^1}_{j \in N \setminus M}), \dots,$$

and  $q^s = sb/c$  with  $s > 1$ . Hence, the signatories of the treaty abate  $s$  times the level that constitutes the individual country's best response, while the non-signatories choose the best response level.

Since each signatory is not playing a best response of the stage game, a deviation from the agreement by a signatory must be prevented by the threat of future punishment. To construct the path,  $\mathbf{p}_i^s$ , used to punish country  $i \in M$ , consider a set  $P_i \subset M$  of punishing countries, satisfying  $i \notin P_i$ . We assume that  $|P_i|$ , the number of countries in  $P_i$ , is the same for all  $i \in M$ , while of course the identities of the countries may not (and can not) be the same. Write  $k = |P_i|$ . Each country in  $P_i$  punishes a unilateral deviation by country  $i$  by choosing the abatement level  $q^p = pb/c$  in the period immediately following country  $i$ 's deviation, where  $p \geq 0$ . The other signatories including country  $i$  (i.e., the countries in  $M \setminus P_i$ ) abate at the agreed upon level  $q^s$ . In the subsequent periods all signatories return to the agreed upon level  $q^s$ . All non-signatories (i.e.,  $j \in N \setminus M$ ) continue to play their best response  $q^1$  throughout. Hence,

the punishment path of country  $i \in M$  is:

$$\mathbf{p}_i^s = (\underbrace{(q^p, \dots, q^p)}_{j \in P_i}, \underbrace{(q^s, \dots, q^s)}_{j \in M \setminus P_i}, \underbrace{(q^1, \dots, q^1)}_{j \in N \setminus M}),$$

$$(\underbrace{(q^s, \dots, q^s)}_{j \in M}, \underbrace{(q^1, \dots, q^1)}_{j \in N \setminus M}), (\underbrace{(q^s, \dots, q^s)}_{j \in M}, \underbrace{(q^1, \dots, q^1)}_{j \in N \setminus M}), \dots$$

Since each non-signatory is playing a best response of the stage game, a deviation from the agreement by a non-signatory requires no punishment. Hence, even if a non-signatory unilaterally deviates from  $\mathbf{a}^s$  or  $\mathbf{p}_i^s$  for some  $i \in M$ , the path in question is simply continued, meaning that any such unilateral deviation is followed by  $\mathbf{a}^s$ . Hence, formally, the punishment path of country  $i \in N \setminus M$  equals  $\mathbf{a}^s$ .

The simple strategy profile determined by the  $n + 1$  paths

$$(\mathbf{a}^s, \mathbf{p}_{i_1}^s, \dots, \mathbf{p}_{i_m}^s, \underbrace{\mathbf{a}^s, \dots, \mathbf{a}^s}_{j \in N \setminus M}) \quad (3)$$

corresponds to what Froyn and Hovi (2007) refer to as “Penance  $k$ ”. In Proposition 2 we establish under what conditions this strategy profile is a weakly renegotiation-proof equilibrium.

Since the identities of the signatories and the punishing countries do not matter, as all countries are identical, the conditions of Proposition 2 depend on only the parameters  $\delta$  (the discount factor),  $n$  (the total number of countries),  $m$  (the broadness of the treaty; i.e., the number of signatories),  $k$  (the number of punishing countries),  $s$  (the depth of the treaty), and  $p$  (the severeness of the punishment). In fact,  $\delta$ ,  $k$ ,  $s$  and  $p$  are sufficient to decide whether the simple strategy profile determined by the paths in (3) is weakly renegotiation-proof, while  $n$  and  $m$  do not matter.

**Proposition 2** *The simple strategy profile determined by (3) with  $s > 1$  and  $p \geq 0$  is a weakly renegotiation-proof equilibrium for  $\delta \in (0, 1)$  if and only if  $k$ ,  $s$  and  $p$  satisfy  $s > p$  and*

$$\frac{1}{2\delta} \cdot \frac{(\max\{s - 1, |p - 1|\})^2}{s - p} \leq k \leq \frac{1}{2}(s + p). \quad (4)$$

In the kind of weakly renegotiation-proof equilibria used to establish existence in Proposition 1, we have that

$$s = n \quad \text{and} \quad p = 1.$$



Then expression (4) simplifies to

$$\frac{1}{\delta}(n-1) \leq 2k \leq n+1.$$

If  $n$  is even and  $k = n/2$ , then the right inequality is satisfied, and the left inequality is satisfied if

$$\delta \geq \bar{\delta} := \frac{n-1}{n}.$$

If  $n$  is odd and  $k = (n+1)/2$ , then the right inequality is satisfied, and the left inequality is satisfied if

$$\delta \geq \bar{\delta} := \frac{n-1}{n+1}.$$

This shows that Proposition 1 follows as a corollary to Proposition 2.

It remains to prove Proposition 2, which we do in the next section.

## 4 Proof of Proposition 2

In this section we characterize weak renegotiation-proofness for the simple strategy profile determined by (3) when  $s > 1$  and  $p \geq 0$ .

We first find through Proposition 3 the condition that ensures that this strategy profile is a subgame-perfect equilibrium and then proceed to provide through Proposition 4 the condition that ensures that such a subgame-perfect equilibrium is weakly renegotiation-proof. Proposition 2 is a direct consequence of Propositions 3 and 4.

### 4.1 Subgame-perfectness

Let  $\alpha^s$  denote the average discounted payoff of each signatory when  $\mathbf{a}^s$  is followed. Likewise, let  $\pi_i^s$  denote the average discounted payoff of a signatory  $i$  when  $\mathbf{p}_i^s$  is followed.

**Lemma 1** *The punishment inflicted on country  $i$  through  $\mathbf{p}_i^s$  relative to following the agreement  $\mathbf{a}^s$  equals*

$$\alpha^s - \pi_i^s = (1 - \delta)(s - p)k \frac{b^2}{c}.$$

**Proof.** By inserting  $\mathbf{a}^s$  into (1) and (2), we obtain

$$\alpha^s = bms \frac{b}{c} + b(n - m) \frac{b}{c} - \frac{c}{2} \left( \frac{b}{c} \right)^2. \quad (5)$$

By inserting  $\mathbf{p}_i^s$  into (1) and (2), we obtain

$$\pi_i^s = (1-\delta) \left( b(m-k)s\frac{b}{c} + bkp\frac{b}{c} + b(n-m)\frac{b}{c} - \frac{c}{2} \left( s\frac{b}{c} \right)^2 \right) + \delta\alpha^s.$$

The lemma is obtained by subtracting  $\pi_i^s$  from  $\alpha^s$ . ■

Lemma 1 gives the size of the future punishment inflicted on a signatory when it deviates from the simple strategy profile determined by (3). This must be compared to the short-term gain that a signatory can reap by deviating from the abatement prescribed by this strategy profile. The size of this short-term gain is provided by the following lemma.

**Lemma 2** *Assume that the simple strategy profile determined by (3) prescribes the abatement  $rb/c$ , with  $r \geq 0$ , for country  $i$ . Then the maximal short-term gain that country  $i$  can reap through a unilateral deviation equals*

$$\frac{(r-1)^2}{2} \cdot \frac{b^2}{c}.$$

**Proof.** The short-term gain of a unilateral deviation by country  $i$  does not depend on the fixed behavior of the other countries. Furthermore, independently of  $r$  and the behavior of the other countries, country  $i$  maximizes its short-term payoff by choosing  $q_i = b/c$ . Hence,

$$\left[ b\frac{b}{c} - \frac{c}{2} \left( \frac{b}{c} \right)^2 \right] - \left[ br\frac{b}{c} - \frac{c}{2} \left( r\frac{b}{c} \right)^2 \right] = \frac{(r-1)^2}{2} \cdot \frac{b^2}{c}$$

is the maximal short-term gain that country  $i$  can reap through a unilateral deviation. ■

We can now characterize subgame-perfectness for the set of strategy profiles considered.

**Proposition 3** *The simple strategy profile determined by (3) with  $s > 1$  and  $p \geq 0$  is a subgame-perfect equilibrium for  $\delta \in (0, 1)$  if and only if  $k$ ,  $s$  and  $p$  satisfy  $s > p$  and*

$$\frac{1}{2\delta} \cdot \frac{(\max\{s-1, |p-1|\})^2}{s-p} \leq k. \quad (6)$$

**Proof.** *If part.* Let  $k$ ,  $s$  and  $p$  satisfy  $s > 1$ ,  $p \geq 0$ ,  $s > p$  and (6). We only need to check for one-period deviations, since it follows from

the theory of repeated games with discounting (Abreu, 1988, p. 390) that a player cannot gain by a multi-period deviation if he cannot gain by some one-period deviation.

Throughout, non-signatories are prescribed by the strategy profile to choose  $b/c$  as their abatement level. Hence, even though any one-period deviation by a non-signatory is not punished, it follows from Lemma 2 that they have no incentive to deviate.

Signatories are prescribed to choose  $sb/c$  along  $\mathbf{a}^s$ . It follows from Lemmas 1 and 2 that there is no profitable deviation if

$$(1 - \delta) \frac{(s - 1)^2}{2} \cdot \frac{b^2}{c} \leq \delta(1 - \delta)(s - p)k \frac{b^2}{c}, \quad (7)$$

which can be rewritten as

$$\frac{1}{2\delta} \cdot \frac{(s - 1)^2}{s - p} \leq k. \quad (8)$$

The signatories (including country  $i$  itself) not inflicting punishment on country  $i$  in the first stage of  $\mathbf{p}_i^s$  and all signatories in later stages of  $\mathbf{p}_i^s$  are also prescribed to choose  $sb/c$ , followed by  $\mathbf{p}_j^s$  if there is a unilateral deviation by a signatory  $j$  and by  $\mathbf{a}^s$  if there is no such deviation. Hence, also in these cases there is no profitable deviation if (8) is satisfied.

Finally, the signatories inflicting punishment on country  $i$  are prescribed to choose  $pb/c$  in the first stage of  $\mathbf{p}_i^s$ , followed by  $\mathbf{p}_j^s$  if there is a unilateral deviation by a signatory  $j$  and by  $\mathbf{a}^s$  if there is no such deviation. By Lemmas 1 and 2, there is no profitable deviation if

$$(1 - \delta) \frac{(p - 1)^2}{2} \cdot \frac{b^2}{c} \leq \delta(1 - \delta)(s - p)k \frac{b^2}{c}, \quad (9)$$

which can be rewritten as

$$\frac{1}{2\delta} \cdot \frac{(p - 1)^2}{s - p} \leq k. \quad (10)$$

Since  $s > 1$ , inequalities (8) and (10) are equivalent to inequality (6).

*Only-if part.* Suppose  $s \leq p$ . Since  $s > 1$ , it follows from (7) that there is a profitable deviation from  $\mathbf{a}^s$ .

Assume that  $s > p$ . Suppose that (8) is not satisfied. Then it follows from (7) that there is a profitable deviation from  $\mathbf{a}^s$ . Suppose that (10) is not satisfied. Then it follows from (9) that there is a profitable deviation from the first stage of each punishment path  $\mathbf{p}_i^s$ .

Since (8) and (10) are equivalent to (6), we have that  $s > p$  and (6) are necessary conditions for the subgame-perfectness of the simple strategy profile determined by (3) with  $s > 1$  and  $p \geq 0$ .

## 4.2 Weak renegotiation-proofness

Let  $\beta_i^s$  denote the average discounted payoff of each of the other signatories inflicting punishment on country  $i$  when  $\mathbf{p}_i^s$  is implemented.

**Proposition 4** *Assume that the simple strategy profile determined by (3) with  $s > 1$  and  $p \geq 0$  is a subgame-perfect equilibrium for  $\delta \in (0, 1)$ . Then this strategy profile is weakly renegotiation-proof if and only if  $k$ ,  $s$  and  $p$  satisfy*

$$k \leq \frac{1}{2}(s + p). \quad (11)$$

**Proof.** By the definition of weak renegotiation-proofness, we must determine when there do not exist two continuation equilibria such that all players strictly prefer the one to the other. Given the structure of the simple strategy profile determined by (3), there exist  $m + 1$  different continuation equilibria, implementing the play of  $\mathbf{a}^s$  and  $\mathbf{p}_i^j$  for all  $j \in M$ .

Since the strategy profile is subgame-perfect, it follows from Proposition 3 that  $s > p$ , implying that  $\alpha^s > \pi_i^s$ . Furthermore, all non-signatories as well as all signatories not inflicting punishment strictly prefer  $\mathbf{a}^s$  to any punishment path  $\mathbf{p}_i^s$ . If  $\alpha^s > \beta_i^s$ , then all countries, including the punishing signatories, strictly prefer the continuation equilibrium in the “empty” history to the continuation equilibrium following a unilateral deviation by country  $i$ . If  $\alpha^s \leq \beta_i^s$ , then the continuation equilibrium following a unilateral deviation by country  $i$  is a best continuation equilibrium for each signatory inflicting punishment on country  $i$  and a worst continuation equilibrium for country  $i$  itself, implying that all players never strictly prefer one continuation equilibrium to another.

Hence, there do not exist two continuation equilibria such that all countries strictly prefer the one to the other if and only if

$$\beta_i^s - \alpha^s \geq 0. \quad (12)$$

By inserting  $\mathbf{p}_i^s$  into (1) and (2), it follows that

$$\beta_i^s = (1 - \delta) \left( b(m - k)s \frac{b}{c} + bkp \frac{b}{c} + b(n - m) \frac{b}{c} - \frac{c}{2} \left( \frac{b}{c} \right)^2 \right) + \delta \alpha^s.$$

By comparing with (5) we obtain

$$\beta_i^s - \alpha^s = (1 - \delta)(s - p) \left( \frac{1}{2}(s + p) - k \right) \frac{b^2}{c},$$

implying that (12) is equivalent to (11).

## 5 Illustration

In this concluding section we illustrate how a weakly renegotiation-proof equilibrium implementing an efficient climate agreement  $\mathbf{a}$  may look like, by applying the result of Proposition 1 to a two-region world, each with  $n/2$  countries. Refer to the regions as A and B. Since the total number of countries is even with two equally sized regions, we may satisfy the requirements on the subset  $P_i(n)$  of countries punishing a deviating country  $i$  (namely (1)  $i \notin P_i(n)$  and (2) the number of countries in  $P(n)$  equals  $n/2$ ) by having a unilateral deviation by a country in region A be punished by all countries in region B and vice versa.

This means that a unilateral deviation by a country in region A triggers a one-period reduction in abatement by all countries in region B; this inflicts an equally hard punishment on all countries in A. On the other hand, since  $n/2 < (n+1)/2$ , it follows from Proposition 4 that all countries in region B strictly benefit by carrying out the punishment.

This arrangement can be contrasted with the regional agreements considered by Asheim et al. (2006), where a unilateral deviation by a country in region A triggers a one-period reduction in abatement by all the other countries in region A. Hence, the countries that benefit during the one-period punishment phase are in the same region as the deviating country, while the countries in region B are harmed twice: *both* by the initial unilateral deviation of a country in region A *and* by the subsequent punishment by the other countries in region A.

We want to point out that the alternative proposed in the present paper has the appealing feature of inflicting the punishments within the region which is to blame for the temporary break-down of cooperation, and rewarding the innocent countries of the other region.

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