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Rate of Penetration Optimization using Moving Horizon Estimation

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Abstract

Increase of drilling safety and reduction of drilling operation costs, especially improvement of drilling efficiency, are two important considerations in the oil and gas industry. The rate of penetration (ROP, alternatively called as drilling speed) is a critical drilling parameter to evaluate and improve drilling safety and efficiency. ROP estimation has an important role in drilling optimization as well as interpretation of all stages of the well life cycle. In this paper, we use a moving horizon estimation (MHE) method to estimate ROP as well as other drilling parameters. In the MHE formulation the states are estimated by a forward simulation with a pre-estimating observer. Moreover, it considers the constraints of states/outputs in the MHE problem. It is shown that the estimation error is with input-to-state stability. Furthermore, the ROP optimization (to achieve minimum drilling cost/drilling energy) concerning with the efficient hole cleaning condition and downhole environmental stability is presented. The performance of the methodology is demonstrated by one case study.

Keywords: Moving horizon estimation, drilling optimization, rate of penetration.

1 Introduction

The oil and gas industry needs to reduce the operation cost and operate safely during drilling operations. Drilling optimization can mean different things, for instance it can be avoidance of drilling problems (poor hole cleaning, kick/lost circulation, pack-off ect.) or it means drilling as efficiently as possible (maximizing the drilling speed). In this paper, the main motivation is to improve drilling efficiency while maintaining good drilling operational environment in consideration of implementing control and optimization strategy.

Drilling parameters heavily affect drilling performances. If they are not adjusted properly, they will make the operation less efficient. Weight on bit (WOB), rotary speed (RPM), flow rate, bit hydraulics and more importantly the type of bits used, are the most important drilling factors affecting rate of penetration (ROP, alternatively drilling speed) and the drilling costs. Real time drilling parameters' manipulation forms the basis for an important methodology that considers past drilling data, predicts drilling trend and gives advice on optimum drilling parameters in order to save drilling costs and reduce the probability of encountering problems. Lots of studies have been performed for determining relationships between ROP and related drilling parameters. The main challenge is that the existing models might not be very accurate in predicting ROP. Some post-analyses possibly provide good prediction compared with historical data, but models have less ability to look ahead to future ROP. In this paper, an MHE method proposed by Sui and Johansen (2014) for the ROP estimation is employed. The reason of using MHE observer is that it can provide a high degree of robustness in the presence of modeling uncertainties since it is based on a batch of the most recent information/measurements. Moreover, the constraints of states and parameters can be

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naturally considered in the MHE problem, which may lead to the more accurate ROP estimation. Sui and Johansen (2014) proposed a novel MHE observer where the states are estimated by a forward simulation with a pre-estimating observer. Compared with standard MHE approaches, it has additional degrees of freedom to optimize the noise and disturbance filtering through the pre-estimator, see more discussions in Sui and Johansen (2014).

Hole cleaning efficiency is the ability of drilling fluids to transport and suspend drilled cuttings. Cuttings transport and hole cleaning efficiency are the prime concern and remain a vital challenge when planning and drilling wells. Several factors can influence hole cleaning efficiency, such as wellbore deviation and the percentage of time spent drilling, sliding or circulating, RPM, WOB, ROP and flow rate, etc. A cutting concentration C_a is the key parameter of evaluating cuttings transport. It is recommended that the concentration of annular cuttings should be kept below some limit by volume ($C_a < 6\% - 8\%$) for trouble-free drilling. Drilling pressure margin defines the operational pressure boundaries during drilling. In the oil industry, it primarily focuses on fluid pressures under various conditions. The downhole pressure should be managed within the drilling pressure range in order to prevent potential drilling problems, such as a well kick or lost circulation, possibly resulting in serious events, like blowouts.

In general the ROP optimization means that the drilling operational parameters, like WOB and RPM are manipulated to drill the present formation most efficiently. In this paper the optimization problem is formulated with respect to different drilling requirements, for instance minimizing drilling cost or mechanical specific energy while keeping efficient hole cleaning condition ($C_a < 6\% - 8\%$) and managing downhole pressure within drilling pressure margin, which is achieved by manipulating WOB, RPM and circulation rate. Case study illustrates a good behavior of manipulating parameters like WOB and RPM in order to achieve the ROP optimization, that gives helpful decision support, improve drilling efficiency, and make safe drilling.

2 Moving horizon estimator

In the following, a linear discrete-time system is considered,

$$x_{t+1} = Ax_t + Bu_t + \xi_t, \tag{1}$$
$$y_t = Cx_t + \eta_t, \tag{2}$$

where $x_t \in X \subseteq R^{n_x}, u_t \in U \subseteq R^{n_u}$ and $y_t \in R^{n_y}$ are the state, the input and the measurement respectively. $\xi_t \in R^{n_x}$ is an unknown state disturbance, $\eta_t \in R^{n_y}$ is a measurement noise vector and disturbances ξ_t, η_t are known only to the extent that they lie, respectively, in the polyhedral sets Ξ and Σ. It is assumed that:

(A1) the pair (A, C) is observable.

(A2) X is a polyhedral set, and contains the origin in its interior.

(A3) $x_t \in X$ for all $t \geq 0$.

The idea of MHE is to estimate the current states by solving a least squares optimization problem, which penalizes the deviation between the measurements and predicted outputs and possibly the distance from the estimated state and a priori information state. The basic strategy is to estimate the state using a moving window of data, such that the size of the data set used for estimation is fixed by looking at only a subset of the available information. At time t, the information vector is defined as

$$I_t = col(y_{t-N}, \ldots, y_t, u_{t-N}, \ldots, u_{t-1}), \tag{3}$$

where N is the window length or horizon. The problem consists in estimating, at any time $t = N, N+1, \ldots$, the state vectors $x_{t-N}, \ldots, x_t$, on the basis of the a priori estimate $\bar{x}_{t-N,t}$ and I_t. The MHE problem proposed by Sui and Johansen (2014) is formulated, as follows,

$$J(\hat{x}_{t-N,t}; \bar{x}_{t-N,t}, I_t) = \|y^t_{t-N} - \hat{y}^{t,t}_{t-N,t}\|^2_\Pi + \|\hat{x}_{t-N,t} - \bar{x}_{t-N,t}\|^2_M \tag{4a}$$

subject to

$$\hat{x}_{i+1,t} = A\hat{x}_{i,t} + Bu_i + L(y_i - \hat{y}_{i,t}), i = t-N, \ldots, t-1, \tag{4b}$$
$$\hat{y}_{i,t} = C\hat{x}_{i,t}, \quad i = t-N, \ldots, t, \tag{4c}$$
$$\hat{x}_{i,t} \in X, \quad i = t-N, \ldots, t, \tag{4d}$$

where $\Pi > 0, M > 0$ are weight matrixes and $L \in R^{n_x \times n_y}$ which satisfies $\rho(\Phi) < 1$ ($\Phi := A - LC$) and

$$y^t_{t-N} = \begin{bmatrix} y_{t-N} \\ y_{t-N+1} \\ \cdots \\ y_t \end{bmatrix}, \quad \hat{y}^{t,t}_{t-N,t} = \begin{bmatrix} \hat{y}_{t-N,t} \\ \hat{y}_{t-N+1,t} \\ \cdots \\ \hat{y}_{t,t} \end{bmatrix}. \tag{5}$$

The optimal solution of (4) is defined by $\hat{x}^o_{t-N,t}$ and it yields the sequence of the state estimates $\hat{x}^o_{i,t}, i = t-N, \ldots, t$ from (4b). It is assumed that the a priori estimate is determined from $\hat{x}^o_{t-N-1,t-1}$, that is

$$\bar{x}_{t-N,t} = A\hat{x}^o_{t-N-1,t-1} + Bu_{t-N-1} + L(y_{t-N-1} - \hat{y}^o_{t-N-1,t-1}), \tag{6a}$$
$$\hat{y}^o_{t-N-1,t-1} = C\hat{x}^o_{t-N-1,t-1}. \tag{6b}$$

The estimation error is defined as

$$e_{t-N} = x_{t-N} - \hat{x}^o_{t-N,t}. \tag{7}$$

Theorem 1(Sui and Johansen, 2014) Suppose that Assumptions (A1)-(A3) hold. There always exist weight matrices $\Pi > 0$ and $M > 0$ such that the error e_t is input-to-state stable (ISS). Moreover, when $\xi_t = 0$ and $\eta_t = 0, t = 0, 1, \ldots, e_t$ is exponentially stable.

Proposition 1(Sui and Johansen, 2014) Suppose that Assumptions (A1)-(A3) hold. If the weight matrices M, Π satisfy

$$\Phi^T M \Phi - M \leq -Q_1, \tag{8a}$$
$$M - F_N^T \Pi F_N \leq -Q_2, \tag{8b}$$
$$M = M^T > 0, \tag{8c}$$
$$\Pi > 0, \tag{8d}$$

for some small $Q_1 > 0, Q_2 > 0$, then the estimated error e_t is ISS, where

$$F_N = \begin{bmatrix} C \\ C\Phi \\ \vdots \\ C\Phi^N \end{bmatrix}.$$

In the paper, M is chosen as a symmetric matrix or $M = M^T$. Then we have

$$M > \Phi^T M \Phi, \tag{9}$$

such that the inequality (8a) holds. The above inequality (9) is a linear matrix inequality (LMI) (Boyd et al., 1998), which can be efficiently solved with some existing toolboxes. Assuming all variables are reasonably scaled, we propose to choose the matrix Π such that

$$\Pi = \Pi_1^T \Pi_1, \tag{10}$$

and Π_1 is chosen to satisfy

$$\Pi_1 F_N = \sqrt{\bar{\alpha}} I_{n_x}, \tag{11}$$

where $\bar{\alpha} > 0$ is a scalar tuning parameter and I_{nx} is a n_x dimensional identity matrix. Since the system is observable, it leads to

$$\Pi_1 = \sqrt{\bar{\alpha}} F_N^+, \tag{12}$$

where $F_N^+ = (F_N^T F_N)^{-1} F_N^T$ is the pseudo-inverse. According to (8b), $\bar{\alpha}$ should be chosen such that $\bar{\alpha} I_{n_x} > M$. Since the positive tuning parameter $\bar{\alpha}$ is scalar, good tuning performance may depend on appropriate scaling of the state and output variables and the associated dynamic.

Remark 1 The proposed MHE method (4) is applied in the paper to estimate ROP and other drilling parameters, where the estimation error can be proven to be input-to-state stable. Comparisons of the proposed MHE observers with other observers, like Luenberger observers and Kalman filters are given in Sui and Johansen (2014). Besides the proposed MHE method, other type of MHE estimation methods can also be applied to the problem to estimate ROP.

3 ROP model

ROP is an important drilling parameter for both drilling cost and efficiency. ROP is defined as the slope of the measured depth evaluated over a short time. It gives a snapshot perspective of how a particular formation is being drilled or how the drilling system is functioning under specific operational conditions. The mathematical expression of ROP is shown below:

$$\frac{dh}{dt} = R_r, \tag{13}$$

where h is the measured depth. Several ROP models were proposed in the recent 30 years. The simplest contains only a few parameters, while as many as twenty variables have been identified for the complex rock/bit interaction. In the study (Bourgoyne et al., 1984; Beck et al., 1995; Rupert et al., 1981), it is convenient to divide the factors which affect the ROP into the list: formation characteristics, mechanical factors (e.g. WOB, bit type and RPM), hydraulic factors, drilling fluid properties. Some models are derived from extensive laboratory investigations, and work well under controlled conditions. It is difficult, however, to extrapolate the results to field conditions due to the lack of data.

The drilling process is very complex. There exists nearly no model that could accurately describe the drilling rate under all conditions. Two factors seem to have a major impact, namely the cuttings cleaning process and the drillability of the rock. To approach this problem, Warren Warren (1987) arrived at the following equation for the drilling rate, which is

$$R_r = \left(K \frac{D^3}{N_r W^2} + \frac{b}{N_r D} + c \frac{D \rho \mu}{\rho_w F} \right)^{-1}. \tag{14}$$

All parameters shown in this section are given in Table 1. The first term on the right hand side gives the maximum drilling rate. The second term relates to the mechanical efficiency of the drill bit, like tooth embedment, and the third term relates to the efficiency of the drilled cuttings transport. The hydraulic jet impact force, F, is given with the equation:

$$F = 0.06183 \rho Q v (1 - A_v^{-0.122}), \tag{15}$$

Para.	Description	Unit
h	Measured depth	m
R_r	Rate of penetration	m/hr
K	Rock drillability	kPa
D	Diameter of drill bit	m
N_r	Rotary speed	rad/s
W	Weight on bit	Newton
b	Dimensionless constant	–
c	Dimensionless constant	–
ρ	Mud density	kg/m^3
ρ_w	Water density	kg/m^3
μ	Plastic viscosity	$kg/(ms)$
F	Jet impact force	Newton
Q	Mud flow rate	m^3/s
v	Jet nozzle velocity	m/s
d	Jet nozzle diameter	m
n	Dimensionless constant	–
ρ_o	Density of formation	kg/m^3
h_b	Fractional tooth wear	m
τ_H	Formation abrasiveness constant	hr
H_1, H_2	Bit coefficients	–
$(\frac{W}{D})_m$	Bit constant	–
ℓ_b	Coefficient	–

Table 1: Model parameters in Section 3.

where the factor A_v is the ratio between the nozzle velocity and the return velocity of the drilling fluid. For the case of three equal nozzles, this expression becomes:

$$A_v = \frac{D^2}{20d^2}. \tag{16}$$

In (14), the model did not take differential pressure into account, so the differential pressure term from Bourgoyne and Young's model (Bourgoyne et al., 1984) is added into the ROP model, which is:

$$e^{nh(\rho-\rho_o)}. \tag{17}$$

This term is directly multiplied to the drilling rate equation (14). Another critical consideration in this paper is the worn condition of drill bits, which naturally affects ROP. A composite tooth wear equation of roller-cone bits can be obtained by combining the relations approximating the effect of tooth geometry, bit weight and rotary speed on the rate of tooth wear (Bourgoyne and Yong, 1974), which is given by

$$\dot{h}_b = \frac{1}{\tau_H}(\frac{N_r}{60})^{H_1}\left(\frac{(\frac{W}{D})_m - 4}{(\frac{W}{D})_m - (\frac{W}{178D})}\right)\left(\frac{1+\frac{H_2}{2}}{1+H_2 h_b}\right). \tag{18}$$

To take the bit worn condition into account, the term

$$1 + \ell_b h_b \tag{19}$$

is multiplied to ROP model (14). Coefficient K represents the rock strength meaning the relative drillability of a rock under perfect cleaning conditions, which tends to vary during the drilling activity. The poor selection of drilling parameters might lead to degraded estimation performance. Here it is assumed that

$$\dot{K} = 0.$$

The three equations, (14), (17) and (19) constitute the basic ROP model. In summary, the drilling system can be formulated in the state space representation

$$\dot{x} = f(x, u), \tag{20}$$
$$y = g(x), \tag{21}$$

where the state x, input u and output y are given as

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} h \\ K \\ h_b \end{bmatrix}, u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} N_r \\ W \\ Q \end{bmatrix}, y = h. \tag{22}$$

Then the model becomes

$$\dot{x}_1 = \left(\frac{D^3 x_2}{u_1 u_2^2} + \frac{b}{u_1 D} + \frac{cD\mu}{0.06183\rho_w u_3 v(1-A_v^{-0.122})}\right)^{-1} \times e^{nx_1(\rho-\rho_o)}(1+\ell_b x_3), \tag{23}$$

$$\dot{x}_2 = 0, \tag{24}$$

$$\dot{x}_3 = \frac{1}{\tau_H}\left((\frac{u_1}{60})^{H_1}\frac{(\frac{W}{D})_m - 4}{(\frac{W}{D})_m - (\frac{u_2}{178D})}\right)\left(\frac{1+\frac{H_2}{2}}{1+H_2 x_3}\right), \tag{25}$$

$$y = x_1. \tag{26}$$

The nonlinear ROP model (23)-(26) can be linearized around a solution (x^0, u^0) which satisfies

$$\dot{x}^0 = f(x^0, u^0). \tag{27}$$

The perturbations in x, u and y can be defined as

$$x = x^0 + \Delta x, \tag{28a}$$
$$u = u^0 + \Delta u, \tag{28b}$$
$$y = y^0 + \Delta y. \tag{28c}$$

Then a linearized model is shown below

$$\Delta\dot{x} = A(x^0, u^0)\Delta x + B(x^0, u^0)\Delta u + \nu \tag{29a}$$
$$\Delta y = C(x^0, u^0)\Delta x + \kappa, \tag{29b}$$

where ν is added as unknown state disturbances and κ is added as a measurement noise; A, B, C can be expressed as

$$A(x^0,u^0) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & 0 & 0 \\ 0 & 0 & a_{33} \end{bmatrix},$$
$$B(x^0,u^0) = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ 0 & 0 & 0 \\ b_{31} & b_{32} & 0 \end{bmatrix},$$
$$C(x^0,u^0) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$

where $x^0 = (x_1^0, x_2^0, x_3^0)^T$, $u^0 = (u_1^0, u_2^0, u_3^0)^T$ and

$$\lambda = \frac{D^3 x_2^0}{u_1^0 {u_2^0}^2} + \frac{b}{u_1^0 D} + \frac{cD\mu}{0.06183\rho_w u_3^0 v(1-A_v^{-0.122})},$$
$$\gamma = e^{nx_1^0(\rho-\rho_o)}, \quad \vartheta = (1+\ell_b x_3^0),$$
$$\varpi = \frac{D\mu}{0.06183\rho_w v(1-A_v^{-0.122})},$$
$$a_{11} = \frac{\gamma\vartheta}{\lambda}, \quad a_{12} = \frac{-\gamma\vartheta D^3}{\lambda^2 u_1^0 {u_2^0}^2}, \quad a_{13} = \frac{\gamma\ell_b}{\lambda},$$
$$a_{33} = \frac{-H_2}{\tau_H}(\frac{u_1^0}{60})^{H_1}\left(\frac{(\frac{W}{D})_m - 4}{(\frac{W}{D})_m - (\frac{u_2^0}{178D})}\right)\frac{1+\frac{H_2}{2}}{(1+H_2 x_3^0)^2},$$
$$b_{11} = \frac{\gamma\vartheta}{\lambda^2}\left(\frac{D^3 x_2^0}{{u_1^0}^2 {u_2^0}^2} + \frac{b}{{u_1^0}^2 D}\right),$$
$$b_{12} = \frac{2\gamma\vartheta D^3 x_2^0}{\lambda^2 {u_1^0}^2 {u_2^0}^3}, \quad b_{13} = \frac{2c\gamma\vartheta\varpi}{\lambda^2 {u_3^0}^2},$$
$$b_{31} = \frac{H_1}{\tau_H}\left((\frac{u_1^0}{60})^{H_1+1}\frac{(\frac{W}{D})_m - 4}{(\frac{W}{D})_m - (\frac{u_2^0}{178D})}\right)\left(\frac{1+\frac{H_2}{2}}{1+H_2 x_3^0}\right),$$
$$b_{32} = \frac{1}{178D\tau_H}(\frac{u_1^0}{60})^{H_1+1}\frac{(\frac{W}{D})_m - 4}{\left((\frac{W}{D})_m - (\frac{u_2^0}{178D})\right)^2}\left(\frac{1+\frac{H_2}{2}}{1+H_2 x_3^0}\right).$$

This linear model (29) can be easily converted to the discrete-time expression shown in (1)-(2). At time t, given measurements $I_t = col(h_{t-N}, \dots, h_t, Nr_{t-N}, W_{t-N}, Q_{t-N}, \dots, Nr_{t-1}, W_{t-1}, Q_{t-1})$, solving the MHE problem (4), it can estimate R_r, K and h_b. The next sections will focus on the ROP optimization while maintaining good drilling operational environments.

4 Hole cleaning

Good hole cleaning refers to the efficient removal of drilling cuttings during drilling operations. For this condition to hold, many factors must be in place, such as cuttings size and density, hole size and angle, ROP, flow rate, cutting transport ratio and mud properties.

Para.	Description	Unit
ϕ	Formation porosity	-
d_a	Wellbore diameter	inch
d_d	Drillstring outside diameter	inch
E_t	Cutting transport ratio	-
v_m	Mud velocity	ft/min
v_s	Cuttings slip velocity	ft/min
$\bar{v}_s$	Uncorrected equivalent slip velocity	ft/min
C_{ang}	Correction factor for inclination	-
C_{size}	Correction factor for size	-
C_{mw}	Correction factor for mud weight	-
μ_a	Apparent viscosity	centi-poise
μ	Plastic viscosity	centi-poise
ρ	Mud weight	lbm/gal
$D_{cuttings}$	Cuttings diameter	inch
Y_p	Yield point	$lbf/100ft^2$
τ_c	Limit tolerance of hole cleaning	-
v_c	Cuttings velocity	ft/min
θ_{ang}	Inclination angle	degree

Table 2: Drilling parameters used in Section 4

These parameters heavily affect the removal of cuttings from the hole. A cutting concentration C_a is one of the important parameters to evaluate the hole cleaning behavior. It is calculated as, see Hyd (2006)

$$C_a = \frac{R_r(1-\phi)A_w}{E_t Q}, \tag{30}$$

where $A_w = \frac{\pi(d_a^2 - d_d^2)}{4}$ and the parameters used in this section are given in Table 2. The cuttings transport ratio, E_t, can be calculated as, see Hyd (2006)

$$E_t = \frac{v_m - v_s}{v_m}, \tag{31}$$

where the mud velocity v_m is determined by the mud flow rate Q and the cross area of wellbore, A_w, or

$$v_m = \frac{Q}{A_w}. \tag{32}$$

The cuttings slip velocity v_s is defined as a flow velocity difference between cuttings and drilling fluid. It is related to inclination, cutting size, and mud weight. Its calculation is taken from Rudi Rubiandini (1999); Ranjbar (2010), where v_s is described as follows:

$$v_s = \bar{v}_s C_{ang} C_{size} C_{mw}. \tag{33}$$

In (33), the uncorrected equivalent slip velocity $\bar{v}_s$ is influenced by the drilling fluid property, cuttings transport velocity and wellbore geometry. It is calculated based on experimental data shown as follows

$$\bar{v}_s = 0.00516\mu_a + 3.006 \quad \text{if} \quad \mu_a < 53, \tag{34}$$
$$\bar{v}_s = 0.02554(\mu_a - 53) + 3.28 \quad \text{if} \quad \mu_a > 53, \tag{35}$$

where μ_a is the apparent viscosity and calculated by

$$\mu_a = \mu + \frac{5Y_p(d_a - d_d)}{v_s + v_c}. \tag{36}$$

In (33), the correction factor for inclination is calculated by the following expression:

$$C_{ang} = 0.0342\theta_{ang} - 0.000233\theta_{ang}^2 - 0.213. \tag{37}$$

The cuttings size correction factor is expressed by:

$$C_{size} = -1.04D_{cuttings} + 1.286. \tag{38}$$

The mud weight correction factor is expressed by:

$$C_{mw} = 1 - 0.0333(\rho - 8.7) \quad \text{if} \quad \rho > 8.7,$$
$$C_{mw} = 1 \quad \text{if} \quad \rho < 8.7.$$

In general, the cutting concentration C_a by volume will result in hole cleaning problems, like mud rings and/or wellbore pack-off occurring if $C_a > 6\% - 8\%$. Normally C_a should be less than some given boundary, or

$$C_a \leq \tau_c, \tag{39}$$

where τ_c is determined with respect to specified formations/enviroments. Combining with equation (30) and (39), in order to achieve the efficient hole cleaning condition, it requires

$$\frac{R_r(1-\phi)A_w}{E_t Q} \leq \tau_c.$$

Together with (31)-(33), the above nonlinear inequality can be simply expressed as follows

$$R_r \leq \chi(Q, \tau_c, \beta), \tag{40}$$

where $\chi(\cdot)$ is a nonlinear function of Q, τ_c and $\beta = [\phi, d_a, d_d, D_{cuttings}, \rho, \theta_{ang}, Y_p, v_c, \mu, \mu_a]$. From (40), it is easy to know that ROP should be within some range to keep the hole cleaning efficiency, otherwise the accumulation of cuttings might hinder the drilling speed, even result in the serious drilling risk, like packoff or stuck pipe.

5 Wellbore pressure margin

To ensure safe and stable drilling operation, bottom hole pressure should be kept within some safe margin between pore and fracture pressure gradients. Exceeding the fracture pressure P_{frac} will fracture the rock formation, and there is a high risk of an underground blowout. If the pressure in the well is lower then the formation pressure P_{pore}, it can lead to an unstable hole, where the walls fall onto the drill pipe. This can lead to a stuck pipe, or a twist-off, which is breaking the drill pipe.

Bottom hole pressure consists of two components, the hydrostatic pressure P_h and the dynamic fluid pressure loss P_{Loss}. The hydrostatic pressure P_h is calculated by

$$P_h = \rho g h \cos(\theta_{ang}), \tag{41}$$

where g is the gravitational acceleration constant. Frictional pressure loss is a function of several factors, such as flow rate, wellbore geometry and drill string configuration, fluid rheological behavior, flow regime and fluid properties. It could be described by the mathematical expression (42),

$$P_{Loss} = \frac{f \rho h v_m^2}{Re}, \tag{42}$$

where f is the friction factor and Re is Reynolds number. Friction factor depends on Re and the roughness of the pipe ϵ. Roughness of the pipe represents the pipe wall irregularities. The Reynolds number, Re, gives a measurement of the ratio of inertial forces to viscous forces. Over the years, the Reynolds number is the most important parameter to define the regime of a drilling fluid flow (laminar, transient, or turbulent). With the correct units, Re is defined as

$$Re = \frac{d_a v_m \rho}{\mu}. \tag{43}$$

For flow of a Newtonian fluid the flow is considered laminar if the Reynolds number is less than 2000, transitional from 2000 to 3000, and the turbulent for Reynolds numbers greater than 3000. The friction factor for laminar flow is related to Re by the following equation

$$f = \frac{16}{Re} \quad \text{for} \quad Re \leq 2000. \tag{44}$$

The friction factor for fully developed turbulent flow is described by

$$\frac{1}{\sqrt{f}} = -1.8\log\left((\frac{\epsilon}{3.7d_a})^{1.11} + \frac{6.9}{Re} \right) \quad \text{for} \quad Re \geq 3000. \tag{45}$$

During the transition phase, the friction factor is highly uncertain. One approach is to use a linear function

$$f = a_f Re + b_f \quad \text{for} \quad 2000 < Re < 3000 \tag{46}$$

Para.	Description
J_1	Drilling cost
C_b	Cost of bit
C_r	Rig cost
C_m	Downhole motor cost
t_d	Drilling time
t_t	Trip time
t_c	Connection time
ΔD	Formation interval drilled

Table 3: Drilling parameters used in Section 6

to approximate the friction factor in this phase. The bottom hole pressure P_{bhp} is then calculated by

$$P_{bhp} = P_h + P_{Loss}. \tag{47}$$

Therefore, to make the well safe and reduce the drilling risks, the bottom hole pressure should be within the P_{frac} and P_{pore}, i.e.

$$P_{pore} < P_{bhp} < P_{frac}. \tag{48}$$

6 Drilling optimization

In the real-time drilling process, although high ROP is desired, ROP optimization should be restricted to drilling safety and efficiency. The drilling speed should not seriously affect the borehole environment, maintain the high hole cleaning efficiency and keep bottom hole pressure within safe pressure window in order to avoid drilling risks, such as kick/lost circulation. Furthermore, the high ROP may damage the drill bit and create borehole instability. Main parameters which should be at most appropriate candidates for the ROP optimization, are WOB, RPM and flow rate. In the paper, we will focus on the optimal approach to regulate drilling parameters (N_r, W, Q) to drill the present formation most efficiently.

6.1 Minimum drilling cost per foot

In order to study the optimization of the drilling process it is critical to identify the objective function of the problem. The drilling is usually paid on a per-day basis. Thus, the major concern when optimizing the drilling process is naturally to reduce the drilling cost. The calculation of cost per foot is conducted by the cost equation expressed as Bourgoyne et al. (1984)

$$J_1 = \frac{t_d C_m + C_r(t_d + t_c + t_t) + C_b}{\Delta D}. \tag{49}$$

The total time of leasing the drilling rig consists of the drilling time, the time spent making pipe connections, and the total trip time. t_d may be expressed as the formation interval drilled, divided by ROP

$$t_d = \frac{\Delta D}{R_r}. \tag{50}$$

The total time spent making connections to the drill string is a function of both formation interval and the constant connection time (t_c^0). It is calculated as

$$t_c = \frac{t_c^0 \Delta D}{27}. \tag{51}$$

The total trip in/out time can be expressed as

$$t_t = t_t^0 \left(\frac{\Delta D}{ROP t_d^0} + \frac{\Delta D}{1000} \right). \tag{52}$$

Then from (49), the cost function becomes

$$J_1 = \frac{C_m}{R_r} + C_r \left(\frac{1}{R_r} + \frac{t_c^0}{27} + \frac{t_t^0}{ROP t_d^0} + \frac{1}{1000} \right) + \frac{C_b}{\Delta D}. \tag{53}$$

It is easy to know that the drilling cost, J_1 is inversely proportional to R_r. Its optimization criterion of interest becomes to seek for the maximum drilling rate.

6.2 Minimum mechanical specific energy

The concept of Mechanical Specific Energy (MSE) was introduced by Teale in 1965 Teale (1965). Teale defined MSE as the mechanical energy and the efficiency of bits used to remove a unit volume of rock. In Teale, the MSE model was conducted based on scientific experimental results and shown as

$$MSE = \frac{4W}{\pi D^2} + \frac{480 N_r T}{D^2 R_r}, \tag{54}$$

where T is the surface torque. From (54), MSE is a function of WOB, RPM, ROP, torque and bit diameter. Such relationship is a guideline to manipulate drilling operational parameters, such as WOB and RPM to optimize drilling performance in such a way that the process has maximum efficiency. Define

$$J_2 = MSE. \tag{55}$$

In other words, the optimal criterion is to minimize MSE or J_2 by regulating WOB and RPM.

6.3 Predefined ROP trajectory

In section 6.1, the optimization criterion to minimize the drilling time can be easily converted to maximizing the drilling rate ROP, see equation (53). Alternatively ROP trajectory can be pre-defined with respect to designed drilling plan or can be real-time given based on drillers' experiences under different drilling circumstances. Therefore the optimization criteria becomes to manipulate WOB, RPM and flow rate to have realtime ROP approach to the defined ROP trajectory as close as possible. Then the cost function can be formulated as

$$J_3 = R_r - R_{sp}, \tag{56}$$

where R_{sp} is the expected setpoint given by operators.

6.4 ROP optimization

From the above discussion, we know that ROP optimization is restricted to hole cleaning conditions and safety operations. In Section 4, the efficient hole cleaning condition is provided, see (40). Together with the non-negative limit of ROP, the boundary of ROP can be considered as

$$0 \leq R_r \leq \chi(Q, \tau_c, \beta). \tag{57}$$

The boundary of the bottom hole pressure to ensure the safe and stable drilling operation should be given as

$$P_{pore} < P_{bhp} < P_{frac},$$

or

$$P_{pore} < P_h + \frac{f\rho h v_m^2}{Re} < P_{frac}.$$

The limits of WOB and RPM are also assumed as follows

$$W_\ell \geq W \ \geq W_u, \tag{58}$$

$$N_{r\ell} \geq N_r \geq N_{ru}, \tag{59}$$

where W_ℓ and W_u are the given boundary of WOB; $N_{r\ell}$ and N_{ru} are the boundary of RPM. Therefore, the constraints of ROP , WOB and RPM, the bottom hole pressure are added into the following optimization problem. The formulation can be written as

$$\min_{W,Q,N_r} J_{1(2,3)} \tag{60a}$$

subject to

$$R_r = \left(K\frac{D^3}{N_r W^2} + \frac{b}{N_r D} + c\frac{D\rho\mu}{\rho_w F}\right)^{-1} \times e^{nh(\rho-\rho_o)}(1+\ell_b h_b), \tag{60b}$$

$$W_u \geq W \ \geq W_\ell, \tag{60c}$$

$$N_{ru} \geq N_r \geq N_{r\ell}, \tag{60d}$$

$$\chi(Q, \tau_c, \beta) \geq R_r \geq 0, \tag{60e}$$

$$P_h + \frac{f\rho h v_m^2}{Re} > P_{pore}, \tag{60f}$$

$$P_h + \frac{f\rho h v_m^2}{Re} < P_{frac}. \tag{60g}$$

Para.	Value	Para.	Value
D	0.4445	ρ_w	1000
b	8×10^{-4}	c	0.002
μ	0.02	v	66.5
d	0.019	n	-4.87×10^{-6}
ρ_o	890	N	7
H_1	1.84	H_2	6
$(\frac{W}{D})$	8	τ_H	30

Table 4: Values of drilling parameters in Section 7

Then, the ROP optimization can be summarized as below:

Algorithm

1. At time t, the information data set with N length, or $I_t = col(h_{t-N}, \ldots, h_t, Nr_{t-N}, W_{t-N}, Q_{t-N}, \ldots, Nr_{t-1}, W_{t-1}, Q_{t-1})$ is obtained.
2. Solve the MHE problem (4) to estimate R_r, K and h_b.
3. Based on the estimated values, the ROP optimization problem (60) is solved.
4. The optimal solution W, N_r, Q then is applied to the system.
5. Set $t = t + 1$, go to step 1.

7 Case study

In the simulation we select TOMLAB npsol algorithm using a BFGS Quasi-Newton method to solve the nonlinear optimization problem.

7.1 ROP estimation

Table 4 shows values of constant drilling parameters used in ROP estimation. The trajectory of measured depth h is shown in Figure 1. The total simulation time is $2800sec$ ($46.7mins$). From the figure, it is easy to see that during drilling, the depth is increased from 4274m to 4300m. Then the average ROP is easily calculated around $33.4m/hr$. Choosing MHE horizon $N = 7$, and obtaining the measurement sequence $I_t = col(h_{t-N}, \ldots, h_t, Nr_{t-N}, W_{t-N}, Q_{t-N}, \ldots, Nr_{t-1}, W_{t-1}, Q_{t-1})$, the MHE problem (4) can be solved to obtain the estimated h, h_b and K. Then instantaneous ROP can be easily calculated from equation (14) regarding the estimated h, h_b and K. The trajectory of instantaneous ROP is shown in Figure 2. The fractional tooth wear is shown in Figure 3. From Figure 2, we know that MHE observer has a good ability to estimate instantaneous ROP, especially predict the trend of ROP. Estimated ROP from the model is varying with the time and the drilling

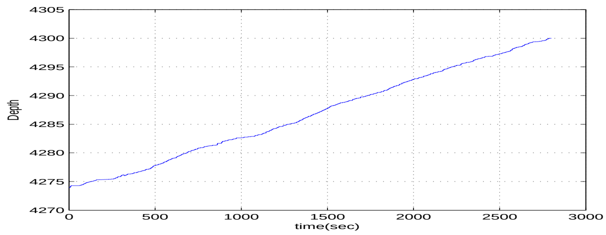

Figure 1: Measured depth used in the case study

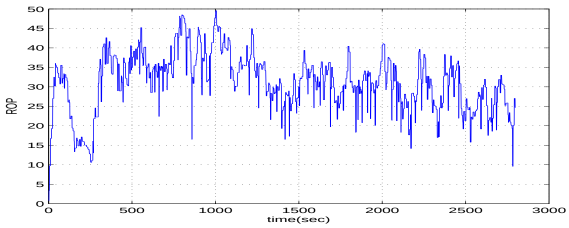

Figure 2: Instantaneous ROP calculated in the case study

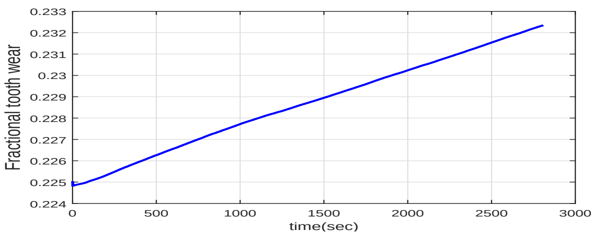

Figure 3: Fractional tooth wear calculated in the case study

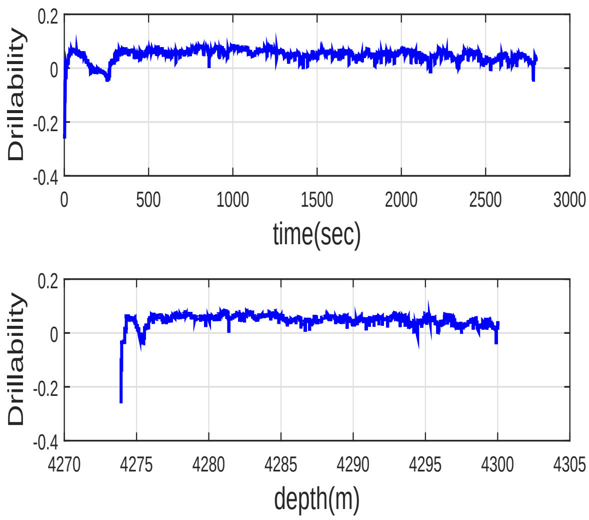

Figure 4: Drillability calculated in the case study

operational parameters, like WOB, RPM and mud properties. For instance, during time $(160sec, 260sec)$ the depth is not increased too much, which is obviously illustrated by ROP values shown in Figure 2. Figure 4 shows the drillability (both time-based values and depth-based values) derived from the model. The drillability is a petrophysical parameter which defines the drilling resistance of the rock. This parameter is useful to determine formation characteristics. In Aadnøy (2010) several field examples are shown where a clay diaper and also a high pressure reservoir are found using drillability data. The drillability is also the only data obtained at the drill bit face, while other logs are recorded from distance away from the drillbit. The use of the drillability should therefore be further developed.

7.2 ROP optimization

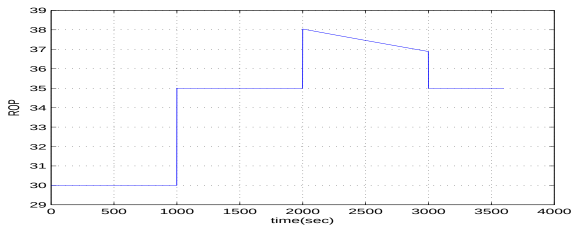

Figure 5: ROP trajectory.

In the case study, the optimization criteria is chosen to minimize the cost function J_3. Suppose the desired ROP trajectory is given as

$$R_{sp}(t) = \begin{cases} 30 & 0 \leq t < 1000 \\ 35 & 1000 \leq t < 2000 \\ 40 & 2000 \leq t < 3000 \\ 35 & t \geq 3000 \end{cases}. \tag{61}$$

In the optimization process, the hole cleaning efficiency and wellbore pressure stability issues are considered. The upper bound of RPM is set 21rad/s and the upper bound of WOB is 149KNewton. Figure 5 shows the trajectories of ROP. Figures 6-7 show the corresponding control variables, WOB and RPM which are solved by problem (60). During the time $t \in [0, 2000s)$, the system has a good performance to manage ROP to approach the setpoint by regulating WOB and RPM. From the ROP model, we know that with the increase of the measured depth h, either WOB or RPM, or both of them should be increased to keep the constant ROP, which is shown in Figures 6-7. At $t = 2000s$, WOB and RPM go to their upper bound, where ROP arrives at its highest value. Then with the time increasing, although WOB and RPM are kept at the maximum operational values, ROP values are decreasing due to the increase of the measured depth h. Then after $t = 3000s$, the system can approach the predefined ROP trajectory closely again since it has capacity to regulate WOB and RPM.

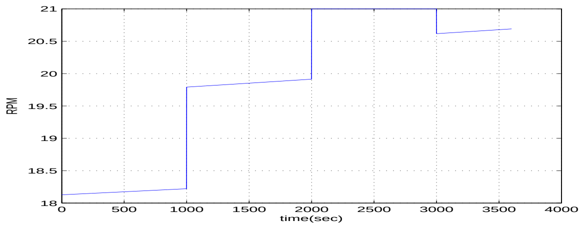

Figure 6: Manipulated RPM.

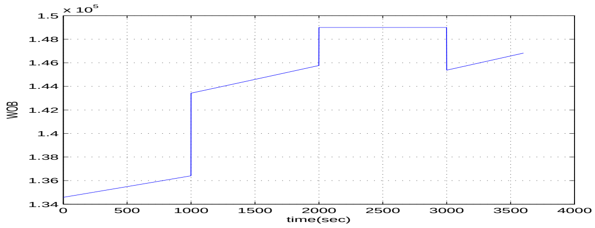

Figure 7: Manipulated WOB.

8 Conclusions

This paper presents a modified ROP model with a state-space expression. In addition to the mechanical drilling parameters, hole cleaning situation and wellbore pressure stability. Constraints are included in the ROP optimization. The case gives a good example of optimizing well safety and cost with this optimization approach. Main advantages of the approach are:

- Consider the effect of several drilling parameters on ROP.
- Optimally adjust the drilling parameters in order to improve the performance of the drilling.
- Realize the automation of drilling.

In the future the more accurate ROP model will be considered by using wired drill pipe data and the responding ROP optimization and correction will be further taken into account.

References

Rheology and Hydraulics of Oil - Well Drilling Fluids. 2006.

Aadnøy, B. S. *Modern well design-second edition.* CRC Press, Taylor and Francis Group, London, UK. ISBN 978-0-415-88467-9, 2010.

Beck, F. E., Arco, A., Powell, J. W., and Zamora, M. The effect of rheology on rate of penetration. In *SPE/IADC Drilling Conference, 28 February-2 March, Amsterdam, Netherlands.* 1995. doi:10.2118/29368-MS.

Bourgoyne, A. T., Keith, K. M., Chenevert, M. E., and Yong, F. S. *Applied Drilling Engineering.* SPE Textbook series., 1984.

Bourgoyne, A. T. and Yong, F. S. *A multiple regression approach to optimal drilling and abnormal pressure detection.* Soc. Pet. Eng. J., 1974. doi:10.2118/4238-PA.

Boyd, S. P., Ghaoui, L. E., Feron, E., and Balakrishnan, V. Linear matrix inequalities in system and control theory. *SIAM*, 1998. doi:10.1137/1.9781611970777.

Ranjbar, R. *Cuttings transport in inclined and horizontal wellbore.* Master thesis of University of Stavanger, 2010.

Rudi Rubiandini, R. S. Equation for estimating mud minimum rate for cuttings transport in an inclined-until horizontal well. In *SPE/IADC Middle East Drilling Technology Conference.* 1999. doi:10.2118/57541-MS.

Rupert, J. P., Prado, C. W., and Blatted, S. R. The effect of weight material type and mud formulation on penetration rate using invert oil system. In *SPE Annual Technical Conference and Exhibition, 4-7 October, San Antonio, Texas.* 1981. doi:10.2118/10102-MS.

Sui, D. and Johansen, T. A. Linear constrained moving horizon estimator with pre-estimating observer. *Systems and Control Letters*, 2014. 67:40–45. doi:10.1016/j.sysconle.2014.02.003.

Teale, R. *The concept of specific energy in rock drilling.* International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts, 1965. doi:10.1016/0148-9062(65)90022-7.

Warren, T. M. Penetration rate performance of roller cone bits. In *SPE Drilling Engineering, SPE-13259-PA.* 1987. doi:10.2118/13259-PA.