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ECONOMETRIC ANALYSIS OF LABOR SUPPLY IN A LIFE CYCLE CONTEXT WITH UNCERTAINTY

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CONTENTS

ABSTRACT

The paper considers a recent econometric approach for analysing labor supply in a life cycle context. The model we present extends MaCurdy's (1982) model in that the wage rate is assumed to depend on previous labor market experience. Our data gives, however, no support to this hypothesis. The empirical results reported should be interpreted with caution because observations on hours worked may be seriously biased and the wage rates are only observed in one period.

1. INTRODUCTION*

This paper focuses on life cycle analysis of labor supply. The labor supply function is defined as the amount of time per year the individual wants to spend in the labor market at a given wage rate. For the observing econometrician the problem consists in estimating the parameters of the corresponding probability distribution of the supply function.

Most work on labor supply is confined to a static setting. However, it is reasonable to assume that individuals make their current labor market decisions subject to past experience, plans and expectations about future income, family, education and employment career. Therefore, the coefficients of the labor supply function estimated within the one period framework confuses the response of those wage changes arising from movements along a given lifetime wage profile with parametric shifts in the wage profile resulting from differences between individuals. Consequently, the parameters of the static model have no clear behavioral interpretation in the context of a life cycle framework. The life cycle approach implies that we are able to distinguish between factors that determine an individual's dynamic behavior and factors that determine differences in behavior between individuals.

The theoretical approach relies on recent work on labor supply by Heckman and MaCurdy [1980], and MaCurdy [1982].

The econometric model developed here endogeneizes the individual market wage by assuming that it might depend on past labor market experience. The model is also shown to be consistent with optimal behavior under uncertainty with respect to job opportunities and future wages. Two measures of labor supply, namely annual participation probabilities and desired hours of work, are derived from assumptions about individual preferences and lifetime income constraints in an environment of uncertainty. Accordingly, our model extends the Heckman/MaCurdy approach since they assume that wages are exogenous. When the market wage depends on experience, a rational individual does not directly compare the current wage rate with the shadow price of leisure. Instead he compares the current wage plus the expected increase in income anticipated from previous investment in working experience (virtual wage) with the current shadow price of leisure.

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The empirical results presented are based upon interviews of a sample of married Norwegian women in 1980 about their employment, schooling and family history from 1970-1980. Unfortunately, the quality of the data is poor and the estimated parameters must therefore be interpreted with caution.

The paper is organized as follows. The following section is an overview of essential theoretical assumptions and limitations. In section 3 the formal theory of life cycle labor supply with endogenous wages is presented. Here the consumer is supposed to be perfectly certain about future preferences, wages and job possibilities and to have freedom to choose the desired hours of work. Section 4 introduces uncertainty into the model, and in section 5 the econometric specification is discussed. In section 6 a stepwise estimation procedure is proposed. Section 7 contains the empirical results.

2. OVERVIEW OF THEORETICAL ASSUMPTIONS

As mentioned above, individuals' supply decisions in one period depend on past experience and expectations about future job opportunities, income, education, and family composition. Individual's labor market behavior is characterized by the fact that the labor market is only one among several fields in which they may offer work. For example, the women's role in society and in the family does still imply a number of commitments outside the labor market which vary through the life cycle. Factors influencing female supply propensities and behavior change in both strength and type over life.

In the present analysis, a woman's preferences are assumed to coincide with those of the household. We assume that she adjusts consumption and leisure in each period according to a smooth, strictly concave, increasing utility index. However, a serious limitation is that credit constraints and the effect of taxes are neglected.

Previous work on life cycle labor supply have treated the wage rate as exogenous. For instance, Heckman and MaCurdy (1980) specify a wage function independent of past labor market experience while acknowledging that this is unsatisfactory. Here, we adopt a wage specification where the current wage depends on previous labor market experience.

Our analysis also attempts to introduce uncertainty (cf. MaCurdy, 1982). This is motivated by the fact that future wages, interest rates and job opportunities are uncertain. In the presence of uncertainty we assume that the individual maximizes expected lifetime utility; conditional upon past (and current) assets, interest rates and employment history.

Following MaCurdy (1982), the consumption and hours of work functions depend on current wage, a life cycle component, λ_t, and a vector of taste shifters. The variable λ_t is the marginal utility of wealth in (current) period t. This variable summarizes past and future information relevant to the woman's current choices. In an environment of perfect certainty, λ_t is a deterministic function of t. However, in the presence of uncertainty, λ_t becomes a random process, since the woman may revise her plans and set a new value on current wealth as new unanticipated information arrives. Although our model extends MaCurdy's work in that wages are allowed to depend on previous labor market experience, the decomposition into a life cycle component and current variables prevails. However, the current wage variable must be replaced by a "virtual" wage variable. The virtual wage is defined here as the current wage, plus the income increase anticipated as a result of past and current investment in work. The estimation of the consumption and hours of work equation is complicated by the fact that neither the marginal utility of wealth, λ_t, nor the virtual wage are observed. However, due to the chosen form of the wage function the virtual wage can be expressed by λ_t, current and past income, interest rates and an individual-specific constant.

The optimal savings allocation rule implies that the expected value of λ_{t+1} as of period t depends only on λ_t. This property enables us to obtain an estimate of the mean of the process $\{\lambda_t\}$. As indicated above, λ_t is a function of past income and future expected wages. This means that in order to predict the effect of shifts in anticipated future wages, it is necessary to specify λ_t as a function of these variables and to estimate the parameters of this function. The parameters of the λ_t function are also needed to explain variation in labor supply across individuals. (If data on consumption were observed, the analysis could be considerably simplified, see MaCurdy, 1983.) For the sake of expository simplicity, suppose the supply equation has the form

$$\log h(t) = \alpha \log \tilde{W}(t) + \beta \log \lambda_t + \gamma_t x(t) + \varepsilon(t),$$

where h(t) is hours of work supplied in period t, $\tilde{W}(t)$ is the virtual wage, x(t) is a vector of taste shifters and $\varepsilon(t)$ is a random term. The coefficient α is interpreted as the <u>intertemporal substitution elasticity</u> of labor supply. Recall that the intertemporal substitution elasticity measures the impact of wage increases from one period to another on the labor supply in the current period relative to the supply in the other period. It is implicitly understood that λ_t is given, which means that the elasticity only measures the substitution effect, if no unanticipated event occurs. Thus, the α-parameter accounts for the impact on labor supply of evolutionary wage changes, i.e. of wage changes that follows an expected profile over the cycle. It is important to emphasize that the α-coefficient does not account for the effect of parametric changes in the wage profile, i.e. for wage changes that alters the level/or the trend of the wage profile. The reason for this is that when a parametric wage change occurs, the individual revises his plans and sets a new value for λ_t. Thus, as mentioned above, we must know λ_t as a function of past income and future expected wages in order to obtain the total effect of wage changes. This is a demanding task which is not dealt with in the present report.

3. A DYNAMIC MODEL OF LABOR SUPPLY WITH EXPERIENCE DEPENDENT MARKET WAGE

Assume that the individual's utility function at time t (age) is a strongly additively separable, strictly concave function of the form

$$U_t(L(t),\ C(t)) = f_t(C(t)) + g_t(L(t))$$

where $C(t)$ denotes consumption of consumer goods and $L(t)$ denotes leisure consumed at age t.

Assume first that the consumer has full knowledge about future interest rates and wages, and that the market is cleared at all periods. Moreover, assume that the consumer can freely borrow and lend at an interest rate $r(t)$. The horizon is finite with length T and the total number of hours available each year is M. If ϱ is the rate of time preference the lifetime utility function is assumed additive separable in time and given by

$$(3.1) \quad \sum_{k=0}^{T} (1 + \varrho)^{-k} U_k(L(k), C(k)).$$

As mentioned above, there is considerable empirical evidence that current market wages depend upon past labor market experience (see Mincer, 1972). In the present study we have therefore adopted a market wage specification that incorporates past employment experience. Let W(t) denote the woman's market wage in period t. Assume that

$$W(t) = w(t, \underset{\sim}{h}(t-1))$$

where h(t) = M-L(t) is hours of work in year t and $\underset{\sim}{h}(t) = (h(t), h(t-1), ..)$. For analytical convenience we assume that w is differentiable with respect to the components of $\underset{\sim}{h}(t)$. The wealth constraint equation is given by

$$(3.2) \quad B(0) + \sum_{k=0}^{T} R(k)h(k)w(k, \underset{\sim}{h}(k-1)) - \sum_{k=0}^{T} R(k)C(k) = 0$$

where B(0) is the initial wealth, and

$$R(k) = \prod_{j=0}^{k} \frac{1}{1+r(j)}$$

is the discount factor that converts real income in period t into its equivalent in period zero. Here, goods prices are normalized to one.

Now consider the consumers optimal choice problem, i.e. the maximization of the lifetime utility function subject to the budget constraint. The Langrangian of this problem is given by

$$(3.3) \quad \Omega = \sum_{k=0}^{T} ((1+\varrho)^{-k}[f_k(C(k)) + g_k(L(k))] + \mu(k)h(k))$$

$$+ \mu(B(0) + \sum_{k=0}^{T} [R(k)h(k)W(k) - R(k)C(k)])$$

where μ and $\mu(k) \geq 0$ are Langrange multipliers that correspond to the budget constraint and the constraints $h(k) \geq 0$. Since $\underset{\sim}{h}(k)$ enters in the individual's wage function the budget constraint becomes nonlinear and we must examine the problem whether there exists a unique solution.

Assuming the $w(k, \underset{\sim}{h}(k-1))$ is a concave function in $\underset{\sim}{h}(k-1)$ it follows that

$$\sum_k h(k)w(k,\underset{\sim}{h}(k-1))R(k)$$

is a concave function in h(k), k = 0,1,...,T. This is so because a sum of concave functions is concave. Hence

$$\sum_{k=0}^{T} R(k)h(k)w(k,\underset{\sim}{h}(k-1)) - \sum_{k=0}^{T} R(k)C(k)$$

is a concave function in h(k) and C(k), k=0, 1,..., T. Moreover, $f_k(C(k))$ and $g_k(M-h(k))$ are strictly concave functions so that the Langrangian (Ω) is strictly concave. Thus a unique maximum of (3.3) is guaranteed.

The first order conditions yield

$$(3.4)\quad f_t'(\bar{C}(t)) = \lambda_t$$

$$(3.5)\quad g_t'(\bar{L}(t)) = \tilde{W}_t\lambda_t$$

where (3.5) only holds when

$$g_t'(M) \leq W_t^*\lambda_t$$

and $\bar{L}(t) = M$ otherwise. Here

$$\lambda_t = \mu R(t)\,(1+\varrho)^t$$

$$(3.6a)\quad \tilde{W}_t = W(t) + \sum_{k>t} \frac{R(k)}{R(t)}\,\bar{h}_t(k)\,\frac{\partial W_t(k)}{\partial h(t)},$$

$$(3.6b)\quad W_t^* = W(t) + \sum_{k>t} \frac{R(k)}{R(t)}\,\bar{h}_t(k)\,\frac{\partial W_t(h)}{\partial h(t)}\bigg|_{h(t)=0}$$

and $\bar{L}(t)$ and $\bar{C}(t)$ are the optimal values of leisure and consumption, respectively. The function $\tilde{W}_t$ can be interpreted as the current wage plus the discounted future income increase. Hereafter $\tilde{W}_t$ will be called the virtual wage function where it is understood that the optimal values of hours of work are inserted. Thus, we realize that if $\tilde{W}_t$ were observable, then the analysis would be completely analogous to the case with experience independent wages. The Lagrange multiplier μ is the marginal utility of wealth in period zero and it is determined by (3.2), (3.4) and (3.5). It is a function in wages, interest rates and individual characteristics at all ages but cannot be expressed in closed form.

If we compare an agent A with experience dependent wages, to agent B with experience independent wages, we see that at the same level of current wage rate A may work while B may not work. This is so because agent A is investing in future returns by working today while agent B perceives no such effect from current behavior.

Heckman and MaCurdy (1980) interpret μ as a permanent income component in analogy with Friedman's and Mincer's permanent income hypothesis. According to Friedman (1957), labor supply is a function of "permanent" wage rates and "permanent" income. Mincer (1962) relaxes this assumption and postulates that labor supply responds also to "transitory" wages and income. Since μ summarizes the effects of wages and income outside the current period and since it adjusts the effect of current virtual wage on labor supply according to income in other periods, it naturally adopts a permanent income measure interpretation. Because this term is constant throughout the life cycle it can be estimated for each individual provided panel data are available.

Below we state without proof the qualitative properties of the model. We have

$$\frac{\partial \bar{L}(t)}{\partial W(t)} \lesseqgtr 0 \quad \text{for fixed } \mu,$$

$$\frac{\partial \bar{L}(t)}{\partial \mu} < 0, \quad \frac{\partial \bar{C}(t)}{\partial \mu} < 0, \quad \frac{\partial \mu}{\partial W(t)} \lesseqgtr 0.$$

These properties follow easily from the properties of the utility function.

The inequalities above mean that in an environment of perfect certainty a wage growth over the life cycle for an individual implies that hours of work increase, or in case the person does not work, the probability of working increases. This is so because the factor μ remains constant throughout the life cycle. This type of changes is denoted evolutionary wage changes because they refer to movements along a given wage profile. If we consider different individuals with different wage profiles, the picture is more complicated because a shift in the wage profile induces a shift in the factor μ.

Since μ decreases and $\tilde{W}_t$ increases as a function of W(t) the total impact on labor supply is ambiguous and cannot be settled a priori. Furthermore, observe that the μ function depends solely on wages in periods in which the consumer works. This follows from the budget constraint equation, since the terms for which hours of work equal zero vanish.

The measure of lifetime labor supply are defined by

$$N_1 = \sum_{k=0}^{T} \bar{h}(k)$$

and

$$N_2 = \sum_{k=0}^{T} \varkappa\,(\bar{h}(k))$$

where $\varkappa(x) = 1$ for $x > 0$ and zero otherwise. The first measure (N_1) is the total number of hours the woman wishes to work while the second (N_2) is the total number of periods she wishes to work. The following example discussed in Heckman and MaCurdy (1980) illustrates the impact of wage changes on these two measures.

Let

$$W(t) = \begin{cases} \omega(t) & \text{for } 0 \le t \le t_1, \quad t_2 \le t \le T \\ \omega(t) + c & \text{for } t_1 \le t \le t_2 \end{cases}$$

where $\omega(t)$ is a function of time and c is a non-negative constant that is independent of experience. This means that future wage increase is not affected by c.

Now suppose that the consumer does not work in $[t_1, t_2]$. Then an increase in c will increase the probability of working in $[t_1, t_2]$. Since this increase is not large enough to cause the woman to work, this increase has no effect on participation at other ages since μ is independent of virtual wages in periods in which the woman does not work. If the woman works in $[t_1, t_2]$ then we know that μ is a decreasing function in the wage in period t. Hence, an increase in c decreases the marginal value of wealth. Since μ decreases, the reservation wage at ages outside $[t_1, t_2]$ increases so that the corresponding participation probability at those ages decreases. Thus, for wage changes in $[t_1, t_2]$ there is no change in N_1 and N_2 provided the woman does not work in $[t_1, t_2]$. Conversely, if the woman works in $[t_1, t_2]$, an increase in c implies a decrease in μ so that the reservation wage at other ages increases. This means that the entry date of participation increases and the exit date decreases and hours of work outside $[t_1, t_2]$ decreases. The effect on hours of work in $[t_1, t_2]$ can, however, not be settled a priori. Therefore N_2 decreases while N_1 may increase or decrease.

4. UNCERTAINTY

In the uncertainty case the consumer's problem is to determine optimal behavior in the presence of uncertain future preferences, wage rates, interest rates and job possibilities. There is also uncertainty about future job opportunities, wages and interest rates. Wages and interest rates may fluctuate in an unexpected manner and cause the consumer to revise her plans. For non-employed persons there may be considerable uncertainty about the chances on the labor market. Persons employed in certain industries may likewise face the risk of being laid off.

We conceive the process of obtaining employment as a two step event. First the individual decides whether or not to search for jobs. Second the firms select workers from those who search. As an individual decision criterion we apply the expected utility paradigm. Individuals maximize expected lifetime utility conditional on past wages, past and current interest rates and preferences. Since uncertainty implies that the individual may revise her plans, or may be unable to realize them, the predicted hours of work, $h_t(s)$, in period s as evaluated in period t $(s>t)$ may not coincide with realized hours of work in period s.

Specifically, consider an agent in period t that is not working. By maximization of the conditional expected utility (subject to a subjective probability structure) in period t she finds $h_t(t)$. If $h_t(t) = 0$ she decides not to search in period t. Conversely, if $h_t(t) > 0$ she will search. The actual hours of work in period t may not be positive because she may not succeed in obtaining a job.

We make the following assumptions about the search behavior: The cost of search is low due to small regional labor markets. This means that unless the subjective probability of getting a job is low, search will take place.

For the sake of analytical simplicity we suppose that the cost of search is "almost" neglegible so that a non-working woman that wants work searches unless the subjective probability of getting a job equals zero. Thus the socalled "discouraged" workers are those who want work (at the given market wage) but have subjective probability of getting a job equal to zero. Furthermore, the woman is assumed to know the current wage rate she would get if she gets a job. Since the cost of search is negligible search behavior is therefore not affected by uncertainty about current opportunities unless the subjective probability of getting a job vanishes.

Now let us proceed to the formal analysis. The woman is assumed to maximize expected utility conditional on past consumption of leisure and goods, and given current and past wages. In the uncertainty case we find it convenient to express the budget constraint (3.2) in terms of the current decision variables.

At age t the woman faces the budget constraints

$$C(t) = B(t) + h(t)W(t) - S(t), \quad h(t) \geq 0 \tag{4.1}$$

where B(t) is the wealth at the beginning of period t and S(t) is the wealth at the end of period t. We have

$$B(t+1) = (1+r(t+1))S(t)$$

i.e., wealth at the beginning of period t+1 is equal to the wealth at the end of period t plus the interest in period t The terminal condition is $B(T) \geq 0$.

In order to simplify the optimizing problem we shall assume that the wage function has the form

$$W_t = D_t(\underset{\sim}{h}_t)\pi_t$$

where

$$\underset{\sim}{h}_t = (h(t-1), h(t-2), \ldots),$$

$D_t(\underset{\sim}{h}_t)$ is a function that is known to the individual and π_t is a stochastic term that reflects the womans uncertainty about her future wages. We suppose that π_t does not depend on $D_t(\underset{\sim}{h}_t)$.

Let $\bar{L}(k)$, $\bar{h}(k)$, $\bar{C}(k)$ and $\bar{S}(k)$ denote the demand functions, respectively, conditional on all information up to age k. The Lagrangian for the corresponding utility maximization problem as of period t is given by

$$(4.2)\quad \Omega_t = U_t(L(t), C(t)) + \lambda_t(h(t)W(t) + B(t) - S(t) - C(t)) + \nu_t h(t) + \tilde{E}_t\{\sum_{k=t+1}^{T}(1+\varrho)^{t-k}U_k(L(k), h(k)W(k) + (1+r(k))S(k-1) - S(k))\}$$

where λ_t and $\nu_k \geq 0$ are Lagrange multipliers and $\tilde{E}_t$ denotes the expectation operator conditional on information prior to t+1.. From (4.2) we get the following first order conditions:

$$\frac{\partial\Omega_t}{\partial C(t)} = 0 \Rightarrow$$

$$(4.3)\quad f_t'(\bar{C}(t)) = \lambda_t.$$

$$\frac{\partial\Omega_t}{\partial L(t)} = 0 \Rightarrow$$

$$(4.4)\quad g_t'(\bar{L}(t)) = \lambda_t W(t) + \tilde{E}_t\{\sum_{k=t+1}^{T}(1+\varrho)^{t-k}f_k'(\bar{C}(k))\bar{h}(k)\frac{\partial W(k)}{\partial h(t)}\} + \nu_t.$$

$$\frac{\partial\Omega_t}{\partial S(t)} = 0 \Rightarrow$$

$$(4.5)\quad \lambda_t = \tilde{E}_t\{(\frac{1+r(t+1)}{1+\varrho})f_{t+1}'(\bar{C}_{t+1})\}.$$

Combining (4.3) and (4.5) yields

$$(4.6)\quad \lambda_t = \tilde{E}_t\{\frac{1+r(t+1)}{1+\varrho}\cdot\lambda_{t+1}\}.$$

By inserting (4.3) into (4.4) we get

$$(4.7)\quad g_t(\bar{L}(t)) = \lambda_t W(t) + \tilde{E}_t\{\sum_{k=t+1}^{T}(1+\varrho)^{t-k}\lambda_k \bar{h}(k)\frac{\partial W(k)}{\partial h(t)}\} + \nu_t$$

where $\nu_t > 0$ when $h(t) = 0$ and $\nu_t = 0$ otherwise.
Now eq. (4.6) implies that we may write

$$(4.8)\quad \lambda_{t+1} = \frac{1+\varrho}{1+r(t+1)}\lambda_t \varepsilon_{t+1}$$

where $\tilde{E}_t \varepsilon_{t+1} = 1$. Hence

$$(4.9)\quad (1+\varrho)^{t-k}\lambda_k = \frac{R(k)}{R(t)}\zeta_k(t)$$

where

$$\zeta_k(t) = \varepsilon_k \varepsilon_{k+1} \cdots \varepsilon_{t+1} \qquad \tilde{E}_t \zeta_k(t) = 1.$$

As a consequence (4.7) may be written

$$(4.10)\quad g_t(\bar{L}(t)) = \lambda_t \tilde{W}_t(\bar{L}(t)) + \nu_t$$

where

$$(4.11)\quad \tilde{W}_t(\bar{L}(t)) = W(t) + \tilde{E}_t\{\sum_{k=t+1}^{T}\frac{R(k)}{R(t)}\zeta_k(t)\bar{h}(k)\frac{\partial W(k)}{\partial h(t)}\}.$$

Here, $W(k)$ is the wage at age k given that the choices of hours of work up to time $k-1$ has been made optimally. The function $\tilde{W}_t$ is the expected value of future discounted marginal earnings plus present wage where the discount factor is modified by $\zeta_k(t)$. This variable accounts for the effect of unanticipated shocks in the marginal value of wealth.

From (4.10) we see that when

$$(4.12)\quad g_t(M) < \lambda_t \tilde{W}_t(M) \equiv \lambda_t W_t^*$$

the woman decides to work. In that case hours of work is determined by

$$(4.13)\quad g_t(\bar{L}(t)) = \lambda_t \tilde{W}_t(\bar{L}(t)).$$

Thus, we have obtained the familiar decision criterion except for the fact that the function $\tilde{W}_t$ is not observable to the analyst.

5. THE ECONOMETRIC SPECIFICATION

As indicated above the econometric implementation of the above model is complicated by the fact that neither the function D_t nor λ_t are observable. In the following we shall only consider the supply function given that the woman works. In the present section we assume that logW(t) is linear in experience where experience is measured by

$$H(t) = \sum_{k\leq t} \log\left(\frac{\bar{L}(k)}{M}\right).$$

The wage function is assumed to have the form

$$(5.1)\quad \log W(t) = X_2(t)b + \eta H(t-1) + \varepsilon(t)$$

where $X_2(t)$ is a vector of exogenous variables that affect the wage and $\varepsilon(t)$ is a zero mean disturbance term. Assumption (5.1) implies that

$$(5.2)\quad \tilde{W}_t(\bar{L}(t)) = W(t) + \eta\tilde{E}_t\left\{\sum_{k=t+1} \frac{R(k)}{R(t)}\zeta_k(t)\bar{h}(k)W(k)/\bar{L}(t)\right\}.$$

The problem now is to predict

$$\tilde{E}_t\left\{\sum_{k=1+t}^{T} \frac{R(k)}{R(t)}\zeta_k(t)\bar{h}(k)W(k)/\bar{L}(t)\right\}.$$

If we are willing to assume that

$$(5.3)\quad E_t\tilde{E}_t\left\{\sum_{k=1+t}^{T} R(k)\zeta_k(t)\bar{h}(k)W(k)\right\} \approx E_t\left\{\sum_{k=1+t}^{T} R(k)\bar{h}(k)W(k)\right\}$$

then one possibility is to specify $\sum_{k>t} \bar{h}(k)W(k)R(k-t)/\bar{L}(t)$ as

$$\sum_{k>t} \bar{h}(k)W(k)R(k-t)/\bar{L}(t) = m_0 + X(t)m + \varkappa(t) \tag{5.4}$$

where m_0 is an individual specific parameter, $\varkappa(t)$ is an error term (possibly autocorrelated) and $X(t)$ is a vector of all the exogenous variables (of age t) that enter the model. When the parameters m_0 and m have been estimated we may predict the mean of $\tilde{W}_t$ by

$$E\tilde{W}_t = EW(t) + \eta(m_0 + X(t)m). \tag{5.5}$$

Note that if the error process, $\varepsilon(t)$, in the wage equation (5.1) is autocorrelated then the experience variable $H(t-1)$ may be correlated with $\varepsilon(t)$. This is so because if $\varepsilon(s)$, $s < t$ changes then labor supply in period s changes so that $h(s)$ and thereby $H(t-1)$ is affected. But $\varepsilon(s)$ also influences $\varepsilon(t)$ since they are correlated.

Let

$$S(t, L(t)) = \frac{g_t'(L(t))}{\lambda_t}.$$

The function $S(t,L(t))$ is the shadow price of leisure in period t. Like in the static case it is the ratio between the marginal (expected) value of leisure and the marginal value of wealth. However, the marginal value of wealth is now a function of past virtual wages and of the distribution of future wages. The important fact is that once λ_t is given we only need to know the virtual wage of the current period in order to determine current hours of work because the effect of wages and preferences in other periods are summarized in λ_t.

Note that the formulation (5.2) rules out the case where the investment capital is lost as a result of turnover or interruption of the employment career.

As functional form for the marginal preference for leisure we assume

$$(5.6) \qquad g_t'(L(t)) = A(t)\left[\frac{L(t)}{M}\right]^{\alpha-1} u_t, \qquad \alpha < 1,$$

where A(t) is a taste shifter that depends on variables affecting the taste for leisure, u_t is an error term that is non-random to the woman but random to the analyst. The error process is non-negative and $\log u_t$ has zero mean. Let $X_1(t)'$ denote the vector of variables that affect the preferences and assume that logA(t) is linear in an unknown parameter vector c,

$$(5.7) \qquad \log A(t) = X_1(t)c .$$

Also let

$$(5.8) \qquad \mu_t = \lambda_t(1+\varrho)^{-t}/R(t).$$

By (4.6) the process $\{\mu_t\}$ is a martingale which means that $\tilde{E}_1\mu_t = \mu_1$. Hence, it may not be unreasonable to assume that $E\log\mu_t \approx \log\mu_1$ so that the shadow price function can be represented as

$$(5.9) \qquad \log S(t,L(t)) = a + (\alpha-1)\log(\frac{L(t)}{M}) + X_1(t)c + t\log(1+\varrho)$$
$$+ \log R(t) + \varepsilon_1(t)$$

where

$$\varepsilon_1(t) = \log u_t - \log\mu_t + \log\mu_1$$

and

$$a = -\log\mu_1 .$$

If we assume that the interest rate r(t) is approximately constant we may write

$$t\log(1+\varrho) + \log R(t) \approx t\log(\frac{1+\varrho}{1+r}) \equiv tk$$

so that by (5.3), (5.4) and (5.9)

$$(5.10) \qquad (1-\alpha)Y(t) = a + E\log\tilde{W}_t - X_1(t)c - kt + \varepsilon_2(t) - \varepsilon_1(t)$$

when the woman works where

$$\varepsilon_2(t) = \log\tilde{W}_t - E\log\tilde{W}_t$$

and

$$Y(t) = -\log(\frac{L(t)}{M}).$$

Using the approximation

$$E\log\tilde{W}_t \approx \log E\tilde{W}_t$$

(5.10) can be written as

$$(5.11) \quad (1-\alpha)\log Y(t) = a + \log E\tilde{W}_t - X_1(t)c - kt + e(t)$$

where

$$e(t) = \varepsilon_2(t) - \varepsilon_1(t).$$

It is likely that $\{e(t)\}$ is autocorrelated and we have assumed that this error process follows a first order autoregressive process, i.e.

$$(5.12) \quad e(t) = \zeta e(t-1) + \delta(t).$$

Here $\{\delta(t)\}$ is a white noise process.

6. A STEPWISE ESTIMATION PROCEDURE

The parameter, a, in equation (5.11) is individual specific according to theory. With panel data it is possible to estimate these individual specific parameters.

Step 1: In step one the wage and the earnings equations are estimated. As noted above we ignore the selectivity bias problem which stems from the fact that the subsample of those who work is not a random sample (see Heckman and MaCurdy, 1980).

For woman i the wage and the earnings equation are given by

$$(6.1) \quad \log W_i(t) = b_{0i} + X_{2i}(t)b + \eta H_i(t-1) + \varepsilon_i(t)$$

and

$$(6.2) \quad \sum_{k>t} h_i(k)W_i(k)/L_i(t) = m_{0i} + X_i(t)m + \varkappa_i(t)$$

where $\{\varepsilon_i(t)\}$ and $\{\varkappa_i(t)\}$ are error processes that may be autocorrelated. As mentioned above autocorrelation in $\{\varepsilon_i(t)\}$ implies that $\varepsilon_i(t)$ and $H_i(t-1)$ are correlated. Here we assume that $\{\varepsilon_i(t)\}$ is a white noise process which allows us to ignore this problem. When the parameters of (6.1) and (6.2) have been estimated then the virtual wage may be predicted by

$$(6.3) \quad \log E\tilde{W}_i(t) = \hat{b}_{0i} + X_{2i}(t)\hat{b} + \hat{\eta}H_i(t-1) + \hat{\eta}(\hat{m}_{0i} + X_i(t)\hat{m})$$

where "^" indicates estimates.

Step 2: In step two we estimate the structural labor supply equation (5.11). Combining (5.11) and (5.12) and subtracting means give

$$(6.4) \quad V_i(t) = \zeta V_i(t-1) + \frac{1}{1-\alpha}Z_{2i}(t) - \frac{\zeta}{1-\alpha}Z_{2i}(t-1) + Z_{1i}(t)f - \zeta Z_{1i}(t-1)f + \delta_i^*(t)$$

where

$V_i(t) = Y_i(t) - \bar{Y}_i, Z_{1i}(t) = (t_i - \bar{t}_i, X_{1i}(t) - \bar{X}_{1i}), (1-\alpha)\delta_i^*(t) = \delta_i(t) - \bar{\delta}_i,$

$\bar{Y}_i = \sum_t Y_i(t)/T, \bar{t}_i = \sum_t t/T, \bar{X}_{1i} = \sum_t \bar{X}_{1i}(t)/T, \bar{\delta}_i = \sum_t \delta_i(t)/T$

and the parameter vector f is given by

$$f_1 = -\frac{1}{1-\alpha}, \quad f = -\frac{kc_j}{1-\alpha} \quad \text{for } j>1.$$

Eq. (6.4) can be estimated by using regression programs with the intercept constrained to zero and the parameter vector associated with $(-Z_{1i}(t-1), -Z_{2i}(t-1))$ constrained to be the product of the parameter vector associated with $(Z_{1i}(t), Z_{2i}(t))$ and the parameter associated with $V_i(t-1)$.

Thus, all the parameters ζ, $1-\alpha$, k and c can be estimated from (6.4). The individual intercept, a_i, which has the interpretation as the negative logarithm of the marginal wealth in period one, can now be estimated from (5.11).

7. EMPIRICAL RESULTS

The data consists of 380 married women, age 25-38 in 1970. These women where interviewed in 1980 about their labor market, educational and family career from 1970 until 1980. (See NOS, 1980.) The annual hours of work variable is constructed as follows: The average hours worked per week is multiplied by numbers of weeks (minus vacations) per year. This means that this variable may be seriously downward biased because weeks worked may differ from number of weeks per year. The woman is "employed" in a year if she works at least three months that year. As independent variables that influence the preferences we use "number of children" and "age of youngest child". As independent variables that affect the market wage we use "years of education (after elementary school)", and labor market experience, as defined in (5.1). We also use a regional labor market tightness indicator that measures the deviance in the vacancies-to unemployed ratio from the corresponding national level.

A serious limitation with our data is that income and wage are only obtained for one year (1980). This implies that the wage rate at other periods has to be predicted by a wage equation estimated from data for 1980. As discussed below, this may lead to biased estimates of the parameters of the wage equation which in turn will produce biased wage predictions.

As outlined in Section 6, estimation is performed in two steps. In step one the wage equation is estimated. These estimates are displayed in table one. Table one shows that the tightness indicator and the experience variable have no effect on the wage rate. Hence, we have $W(t) \approx \tilde{W}_t$. In order to test for possible selectivity bias we have estimated a non-structural logit function for the participation probability and used the logarithm of the estimated participation probability as an additional independent variable. See Dagsvik (1986) for a justification of this method.

The table shows that the coefficient that controls for selectivity bias is not significantly different from zero.

The parameter estimates of the wage equation is likely to be biased because the error term in the wage equation may be correlated with the independent variables. One reason for that is that an individual specific intercept, which would be possible to estimate if we had wage data for several periods, would probably be correlated with the independent variables. Since we cannot estimate the individual specific intercepts, the error term contains a component that stem from the individual differences in the intercepts. Second, as explained in section 5, the experience variable, H(t-1), may be correlated with the error term $\varepsilon(t)$.

Table 1. Parameter estimates of the wage equation

	Estimates	Standard error
Intercept	3.607	2.32
Years of education	$5.1 \cdot 10^{-2}$	$0.59 \cdot 10^{-2}$
Labor market experience	$-3.1 \cdot 10^{-6}$	$17.5 \cdot 10^{-6}$
Tightness indicator	$-5.4 \cdot 10^{-5}$	$22 \cdot 10^{-5}$
Log participation probability	$-5.3 \cdot 10^{-2}$	$8.8 \cdot 10^{-2}$
Residual standard deviation	$4.9 \cdot 10^{-2}$	
R^2 - wage equation	0.27	

Step two is slightly different from the procedure suggested in section 6 in that we estimate a reduced form hours of work equation. Since $W(t) \approx \tilde{W}(t)$ we may for computational convenience substitute ElogW(t) with a linear combination of "years of education" and the tightness indicator. The results are given in table two.

Even though the coefficient of the tightness indicator is not significantly different from zero in the wage equation it may have a direct impact on the hours of work equation because of regional variations in rationing and institutional constraints on working hours.

Recall that the dependent variable is

$$V_i(t) = Y_i(t) - \bar{Y}_i \quad \text{where}$$

$$Y_i(t) = - \log \left(1 - \frac{h_i(t)}{M}\right)$$

and M is 150 times number of weeks in a year. Thus $V_i(t)$ is positively related to hours of work, $h_i(t)$. We see that all the coefficients have the expected sign and are significantly different from zero (at 5 % level).

Table 2. Parameter estimates of the reduced form hours of work equation

Variables	Estimates	Standard error
Age	$4.2 \cdot 10^{-3}$	$0.9 \cdot 10^{-3}$
Number of children	$-1.1 \cdot 10^{-2}$	$0.3 \cdot 10^{-2}$
Age of youngest child	$2.6 \cdot 10^{-3}$	$0.6 \cdot 10^{-3}$
Years of education	$3.2 \cdot 10^{-3}$	$0.7 \cdot 10^{-3}$
Tightness indicator	$7 \cdot 10^{-5}$	$3 \cdot 10^{-5}$
Autocorrelation coeff. (ζ)	0.60	0.01
Multiple correlation (R_1^2)	0.41	
Multiple correlation (R_2^2)	0.14	

Also the autocorrelation of the error term is significantly different from zero. Measured by the squared multiple correlation coefficient (R_1^2) these variables explain 41 per cent of the variance in the $V_i(t)$ variable. Recall that we have controlled for individual differences by allowing the intercept be individual specific. Therefore R_1^2 measures how much of the temporal evolution that is explained by the variables above.

However, R_1^2 measures the effect of the explanatory variables when the lagged dependent variable is included. The correlation coefficient R_2^2 measures the goodness of fit of the equation

$$V_i(t) = \tilde{Z}_i(t)r + \tilde{e}_i(t)$$

where the error term $\tilde{e}_i(t)$ is defined by

$$\tilde{e}_i(t) = e_i(t) - \sum_t e_i(t)/T$$

and $\tilde{Z}_i(t)$ consists of the variables in table 2. It is easily veryfied that

$$R_2^2 = \frac{R_1^2 - \zeta^2}{\zeta^2} .$$

The goodness of fit measured by R_2^2 demonstrates that 14 per cent of the variation in hours of work (over the life cycle) is explained by the independent variables.

If only the subsample of those who work were used the corresponding estimate is 25 per cent. (These results are not reported here.)

By combining the estimates of table 1 and table 2 it is possible to obtain an estimate of $1/(\alpha-1)$ which is the intertemporal substitution elasticity of leisure. By dividing the years-of-schooling coefficients of table 2 by the corresponding coefficient of table 1 we get $1/(\hat{\alpha}-1) = -6.3 \cdot 10^{-2}$ with standard error about 0.02. This means that intertemporal substitution elasticity of labor supply is 0.88 for a woman who works 10 hours per week and 0.25 for a woman who works 30 hours per week.

It is likely that this estimate is downward biased because the estimate of the coefficient of the schooling variable in the wage equation may be upward biased. This is explained as follows: Write the wage equation for woman i

$$(7.1) \quad \log W_i(t) = b_{oi} + bE_i(t) + u_i(t)$$

where $E_i(t)$ is the level of education of year t, $u_i(t)$ is the error term and b_{oi} is an individual specific intercept that accounts for permanent individual differences in productivity that is not captured by $E_i(t)$. For expository simplicity we have suppressed the remaining explanatory variables. Evidently (7.1) may be rewritten

$$(7.2) \quad \log W_i(t) = \bar{b}_o + bE_i(t) + u_i^*(t)$$

where $\bar{b}_o$ is the mean intercept and

$$u_i^*(t) = u_i(t) + b_{oi} - \bar{b}_o .$$

Now, it is likely that b_{oi} is positively correlated with $E_i(t)$ because level of education depends on motivation and ability which also affects the permanent level of productivity. As a consequence, the error term $u_i^*(t)$ will be positively correlated with $E_i(t)$ which implies that the least squares estimate on the basis of (7.2) will produce an upward biased estimate of b. If, however, we had wage observations for several periods for each woman then it would be possible to estimate and control for the individual intercepts in (7.1).

We also note that since the coefficient of the experience variable in the wage equation is not significantly different from zero there is no (significant) variation over the life cycle in predicted wage after the education career is completed. This fact suggests also that the estimate of the intertemporal substitution elasticity may be poor simply because there is not much variation in the wage data.

REFERENCES

Central Bureau of Statistics (1980): Women's work 1980. NOS, B242.

Dagsvik, J. (1986): Stochastic Properties of the Market Wage and the Labor Supply Functions. A Theoretical Justification. Manuscript, Central Bureau of Statistics, Oslo. (Forthcoming.)

Friedman, M. (1957): A Theory of Consumption. Princeton Univ. Press, Princeton.

Galambos, J. (1978): The Asymptotic Theory of Extreme Order Statistics. Wiley, New York.

Heckman, J.J. (1979): Sample Selection Bias as a Specification Error. Econometrica, 47, 153-162.

Heckman, J.J. and MaCurdy, T.E. (1980): A Life Cycle Model of Female Labor Supply. Rev. Econ. Studies, 47, 47-74.

MaCurdy, T.E. (1982): Interpreting Empirical Models of Labor Supply in an Intertemporal Framework with Uncertainty. Working Paper No. 23, Stanford University.

MaCurdy, T.E. (1983): A simple scheme for estimating an intertemporal model of labor supply and consumption in the presence of taxes and uncertainty. Int. Econ. Rev.

Mincer, J. (1962): Labor Force Participation of Married Women: A Study of Labor Supply. In H.G. Lewis (Eds.). Aspect of Labor Economies. Princeton Univ. Press.

Mincer, J. (1972): Schooling, Experience, and Earnings. Mimeograph, National Bureau of Economic Research, New York.

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