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Discussion Papers No. 422, May 2005
Statistics Norway, Research Department

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Identifying Structural Breaks in Cointegrated VAR Models

Abstract:
The paper describes a procedure for decomposing the deterministic terms in cointegrated VAR models into growth rate parameters and cointegration mean parameters. These parameters express long-run properties of the model. For example, the growth rate parameters tell us how much to expect (unconditionally) the variables in the system to grow from one period to the next, representing the underlying (steady state) growth in the variables.

The procedure can be used for analysing structural breaks when the deterministic terms include shift dummies and broken trends. By decomposing the coefficients into interpretable components, different types of structural breaks can be identified. Both shifts in intercepts and shifts in growth rates, or combinations of these, can be tested for. The ability to distinguish between different types of structural breaks makes the procedure superior compared to alternative procedures. Furthermore, the procedure utilizes the information more efficiently than alternative procedures. Finally, interpretable coefficients of different types of structural breaks can be identified.

Keywords: Johansen procedure, cointegrated VAR, structural breaks, growth rates, cointegration mean levels.

JEL classification: C32, C51, C52.

Acknowledgement: Thanks to Søren Johansen, Ragnar Nymoen, Terje Skjerpen and Anders Rygh Swensen for valuable comments on various versions of the paper. An earlier version of the paper was presented at the Econometric Society European Meeting (ESEM) conference in Madrid (August 20 - 24, 2004).

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1 Introduction

In analysing a dynamic economic model we are often interested in identifying and testing its long-run properties. The cointegrating vectors are examples of long run relationships between different variables. However, also the underlying growth rates (i.e. steady state growth rates) can be identified in cointegrated vector autoregressive (VAR) models. Hungnes (2002) shows how these growth rates can be estimated within a full information maximum likelihood framework, as well as how to test for restrictions on these growth rates.

The growth rates tell us how much to expect (unconditionally) the variables in the system to grow from one period to the next. If the system is used for forecasting, the vector of growth rates will be very important in providing good forecasts. In fact, as the forecasting horizon approaches infinity, the forecast will rely on this vector only.

Structural breaks often imply changes in the growth rates of the variables. With many and frequent structural breaks in time series integrated of order 1, it will normally be best to estimate the system as if the variables in it were integrated of order 2. With less frequent structural identification is possible.

Structural breaks have been discussed intensively in the context of univariate autoregressive time series. Perron (1989) suggests three models: Model A, a 'crash model', with change in intercept but where the slope of the linear trend is unchanged; Model B, a 'changing growth model', allows a change in the slope of trend function without any sudden change in the level at the time of the break; and Model C, where both intercept and slope are changed at the time of the break. Johansen et al. (2000) present a generalization of model C in a multivariate framework, and allow for testing hypotheses corresponding to Model A.

In this paper all deterministic terms in a cointegrated VAR model are decomposed into interpretable counterparts. The corresponding coefficients describe the long run (steady state) growth rates in the variables, and possibly shifts in level and growth rates (the latter depending on the type of deterministic variables that are included in the system). Combined with the coefficients of the cointegrating vectors, they also describe levels and trends (and possibly shifts in these) in the cointegrating vectors. The decomposition therefore allows us to test all three types of structural breaks suggested in Perron (1989). The paper presents a model C for a multivariate framework where we allow for testing hypotheses corresponding to both A and B. In addition the method presented here makes it possible to identify the growth rates and the size of the different types of shifts.

Johansen et al. (2000) show how the traditional cointegration analysis can be used in order to identify some types of structural breaks. They show that within their framework they can identify (and test for) shifts in the trends in the cointegrating vectors, but not in the levels of the cointegrating vectors. In order to use traditional cointegration analysis one needs to disregard observations following immediately after structural breaks by including impulse dummies. The number of impulse dummies after the break corresponds to the number of lags in the system, and the inclusion of these dummies implies a reduction in the effective sample.

An alternative could be to use a two step approach, where the coefficients of the deterministic part are estimated in the first step and a traditional cointegration analysis could be conducted on the de-trended time series in a second step. Saikkonen and Lütkepohl (2000) suggest such a two-step approach. However, they only consider testing the cointegration rank, and do not consider how to impose restrictions on the system in order to test for different types of structural breaks.

The estimation approach suggested here therefore has three important advantages compared to the alternatives. First, it allows testing for all the different types of structural breaks. Second, it utilizes all the information better by not disregarding observations after a break point. Third, it identifies interpretable coefficients of the different types of structural shifts. On the other hand, a disadvantage with the approach suggested here, is that it involves a more complicated maximizing problem. However, a program, GRaM (see Hungnes, 2005), has been developed to estimate these systems.

The estimation approach is illustrated by applying two data sets. In the first illustration the same data set as in Johansen et al. (2000) is used, and it is shown that the approach in the present paper can handle more types of breaks. In the second illustration a money demand system for Germany covering the period of the (re-) unification is analysed. The data set is used to test for different types of structural breaks.

The estimation approach in the present paper does not consider identification of the cointegrating rank. To determine the cointegrating rank in data series with structural breaks, the procedures in Johansen et al. (2000) or Saikkonen and Lütkepohl (2000) can be applied.

Only situations where the break points are known are considered in this paper. Lütkepohl et al. (2004) suggest an approach for identifying the break point and cointegrating rank if the break points are unknown (but the number of breaks is known).

The paper is organized as follows: In Section 2 the model is formulated. Section 3 presents the estimation problem. In Section 4 the estimation procedure is used to identify structural breaks on two different data sets. Section 5 concludes and describes other situations where this procedure can be informative.

Throughout the paper we define the orthogonal complement of the full column rank matrix A as $A_\perp$ such that $A'_\perp A = 0$ and $(A, A_\perp)$ has full rank. (The orthogonal complement of a nonsingular matrix is 0, and the orthogonal complement of a zero matrix is an identity matrix of a suitable dimension.) Furthermore, for a matrix A with dimension $n \times m$ ($m \leq n$), we define $\overline{A} = A\left(A'A\right)^{-1}$.

2 Model formulation

2.1 Conventional formulation of cointegrated VAR

Let Y_t be an n-dimensional vector of variables that are integrated of order one at most. α and β are matrices of dimension $n \times r$ (where r is the number of cointegrating vectors) and $\beta' Y_t$ is an $r \times 1$ vector where all elements are $I(0)$. Furthermore, Γ_i ($i = 1, 2, ..., p-1$) are $n \times n$ matrices of coefficients, where p is the number of lags. Δ is the difference operator. D_t^* is a vector of deterministic variables. The errors ε_t are assumed to be Gaussian white noise ($\varepsilon_t \sim NID\left(0, \Omega\right)$).

$$\Delta Y_t = \alpha \left(\beta' Y_{t-1}\right) + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \delta D_t^* + \varepsilon_t, \ \ t = 1, 2, ..., T. \tag{1}$$

It is common to distinguish between deterministic variables that are restricted to lie in the cointegration space and those which are not. Let $\delta D_t^* = \delta_0 D_{0,t}^* + \delta_1 D_{1,t}^*$, where $D_{0,t}^*$ includes the deterministic variables restricted to lie in the cointegrating space (i.e such that $\delta_0 = \alpha \overline{\alpha}' \delta_0$ or equivalently $\alpha'_\perp \delta_0 = 0$). Disregarding different types of dummies (such as impulse dummies, shift dummies and seasonal dummies), the most common two specifications for these deterministic variables are $\left(D_{0,t}^*, D_{1,t}^*\right) = (1, \emptyset)$ (i.e. restricted constant, excluding a linear drift in Y_t, labelled H_c) and $\left(D_{0,t}^*, D_{1,t}^*\right) = (t, 1)$ (i.e. restricted linear trend, excluding a quadratic trend in Y_t, labelled H_l). If, in practice there are trends in the data H_l is recommended, and in systems without trends H_c is recommended.

Let us assume that the process in (1) is generated by hypothesis H_l. The system grows at the unconditional rate $E\left[\Delta Y_t\right] = \gamma$ with long run (cointegration) mean levels $E\left[\beta' \left(Y_t - \gamma\right)\right] = \mu$. We can re-parameterize the system as

$$\Delta Y_t - \gamma = \alpha \left(\beta' Y_{t-1} - \mu - \rho \left(t-1\right)\right) + \sum_{i=1}^{p-1} \Gamma_i \left(\Delta Y_{t-i} - \gamma\right) + \varepsilon_t, \tag{2}$$

where

$$\rho \equiv \beta' \gamma \tag{3}$$

is the vector of trend coefficients in the cointegrating vectors. For the system to be stable, the following restriction must hold:

Condition 2.1 *Assume that $n-r$ of the roots of the characteristic polynomial*

$$A(z) = (1-z) I_n - \alpha\beta' z - \sum_{i=1}^{p-1} \Gamma_i (1-z) z^i$$

are equal to 1 and the remaining roots are outside the complex unit circle.

2.2 Alternative formulation of cointegrated VAR

Here we will present the cointegrated system slightly differently from Equation (1). There are two reasons for changing the representation. First, it will be easier to interpret. Second, it will be easier to formulate structural breaks in the system.

Let D_t be a vector of q deterministic variables, such as trend and seasonally dummies. The system can then be written as

$$\Delta Y_t - \gamma \Delta D_t = \alpha \left(\beta' \left(Y_{t-1} - \gamma D_{t-1}\right) - \mu\right) + \sum_{i=1}^{p-1} \Gamma_i \left[\Delta Y_{t-i} - \gamma \Delta D_{t-i}\right] + \varepsilon_t, \tag{4}$$

where γ is now an $n \times q$ matrix of coefficients.

If $D_t = t$, the system in (4) is equal to the system in (2). This is the case with linear trend in the variables, i.e. H_l.

In the case where there are no trends in the variables, D_t vanishes from (4), and the system can be written as

$$\Delta Y_t = \alpha \left(\beta' Y_{t-1} - \mu\right) + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \varepsilon_t. \tag{5}$$

In either case there is a one-to-one correspondence between the system written in the conventional way, as in (1), and in the alternative way, as in (4) or (5). If the system in (1) is estimated with e.g. $\left(D^*_{0,t}, D^*_{1,t}\right) = (t, 1)$, we can always identify the coefficients of (4).

Also, when seasonal dummies are included, (1) and (4) are statistically equivalent.

Generally, however, when other deterministic variables are included in D_t there is no such one-to-one relationship between the formulations in (1) and (4).

An alternative way to write the system, is to write the system where the deterministic components are removed. Let Y^d be defined as Y with the deterministic components removed, i.e.

$$Y_t^d = Y_t - \gamma D_t$$

with D_t as the vector of deterministic variables and γ as the corresponding matrix of coefficients. Hence, the system can alternatively be written as

$$\Delta Y_t^d = \alpha \left(\beta' Y_{t-1}^d - \mu \right) + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i}^d + \varepsilon_t. \tag{6}$$

We have the following theorem:

Theorem 2.1 (Granger's representation theorem with deterministic variables) *Under Condition 2.1, Y_t in* (4) *has the moving average representation*

$$Y_t = C \sum_{i=1}^{t} \varepsilon_i + \iota + \gamma D_t + B_t, \tag{7}$$

where $C = \beta_\perp \left(\alpha'_\perp \Gamma \beta_\perp \right)^{-1} \alpha'_\perp$ with $\Gamma = I_n - \sum_{i=1}^{p-1} \Gamma_i$. The process B_t is stationary with zero expectation. The level coefficients ι depends on initial values in such a way that

$$\mu = \beta' \iota. \tag{8}$$

Proof. By using $Y_t^d = Y_t - \gamma D_t$ (i.e. the system in (6)), the proof follows from the proof of Theorem 4.2. in Johansen (1995). ■

By formulating the system as in (4) we achieve that the representation of the process in (7) is valid in the whole sample. This is an advantage of this approach compared to Johansen et al. (2000) where such a representation does not exist in the periods after a break. (This is the reason why Johansen et al. (2000) exclude these observation points by including impulse dummies.)

2.3 Structural breaks

If there are structural breaks in the time series, there might be both level shifts and trend shifts. A shift dummy picks up the level shift in a time series. A shift dummy is a dummy equal to zero up till a specified period and unity afterwards. Both the shift dummy and the corresponding broken trend are included in the vector of deterministic variables, D_t.

A broken trend picks up the trend shift. The broken trend is constructed as the accumulated value of the corresponding shift dummy. Therefore, the accumulated shift dummy has no level shift. The advantage of defining the broken trend without a shift in the levels, is that it becomes much easier to identify the different types of structural breaks.

We first consider different types of structural breaks in the cointegration space. The coefficient matrix $\rho = \beta'\gamma$ contains information about these breaks. If the (vector of) coefficients in ρ corresponding to the shift dummy are significantly different from zero, this implies a change of the intercepts in the cointegrating space. Similarly, significant coefficients of the accumulated shift dummy (i.e. the broken trend) implies a shift in the slope of the trend in the cointegrating vectors. Therefore, according to the definitions in Perron (1989), the cointegration space follows a 'crash model' (Model A) if the coefficients of the shift dummy are significant whereas the coefficients of the broken trend are insignificant. On the other hand, the cointegrating vectors behave as a 'changing growth model' (Model B) if the coefficients of the shift dummy are insignificant whereas the coefficients of the broken trend are significant. If the coefficients of both the level-shift and trend-shift dummies are significant, the cointegrating space behaves as Model C.

The different types of structural breaks in the time series can be identified in similar ways by examining the coefficient matrix γ. The time series follows a 'crash model'

(A) if the corresponding coefficients of the shift dummy are significant whereas the corresponding coefficients of the broken trend are insignificant. Correspondingly, the time series follows a 'changing growth model' (B) if the corresponding coefficients of the shift dummy are insignificant whereas the corresponding coefficients for the broken trend are significant.

If the time series follows a 'crash model' (A), it is impossible to construct a linear relationship of the time series of type B. Therefore, the coefficients of the broken trend in the cointegrating space must be zero as well. Similarly, if the time series follows a 'changing growth model' (B) none of the cointegrating vectors can be of type A.

However, the reverse implication does not apply. If there are no trend or level shifts in the cointegrating vectors, this does not imply that there are no shifts in the time series. The time series may still have trend and level shifts. If so, we say that the cointegrating vectors also are co-breaking vectors.

The concept of co-breaking was introduced by Hendry and Mizon (1998). If deterministic breaks in a system of equations can be removed by taking linear combinations of the variables, the variables are said to co-break. Not many analysis on co-breaking have been done. An important reason is that one needs at least as many breaks as variables in the system. If not, there will always exists at least one linear combination of the variables where the deterministic breaks can be removed. Hendry and Mizon (1998) label such situations 'spurious co-breaking'.

Here we do not test if there are *any* linear combination that remove the deterministic breaks. We only test if the cointegrating vectors also are co-breaking vectors. To be precise, let $sp\left(B'\right)$ denote the co-breaking space (as $sp\left(\beta'\right)$ denotes the cointegrating space). The hypothesis we are testing is if $sp\left(\beta'\right) \in sp\left(B'\right)$.

3 Estimation

3.1 Restrictions

There are different methods for imposing restrictions on the cointegrating vectors. Here we will only consider restrictions on the cointegrating space. Let $\beta^* = \left(\beta', -\mu\right)'$ and $X_t^* = (X_t', 1)'$, such that restrictions on the cointegration mean levels can also be imposed. These restrictions on the cointegrating space can be written as

$$R_\beta' \beta^* = 0, \tag{9}$$

where each column in R_β represents a restriction on β^*. Equivalently, we can write

$$\beta^* = H_\beta \cdot \phi_\beta, \tag{10}$$

where $H_\beta = \left(R_\beta\right)_\perp$. We may refer to ϕ_β as the 'free parameters' in β^* under the imposed restrictions. However, this is not entirely correct. The cointegration space, and therefore ϕ_β, is only unique up to a normalization and rotation of the cointegration space. We may therefore introduce the normalization $\phi_\beta = (I_r, \phi_b)$.

Next, we look at how to impose restrictions on the $n \times q$ matrix γ (where q is the number of deterministic variables). The restrictions we want to impose on γ can be written as

$$R_\gamma' \gamma' = 0. \tag{11}$$

These restrictions can be restrictions on both level and trend shifts. The restrictions on γ can alternatively be written as

$$\gamma' = H_\gamma \cdot \phi_\gamma, \tag{12}$$

where $H_\gamma = (R_\gamma)_\perp$ and ϕ_γ are the 'free parameters' in γ.

As discussed above, restrictions on γ imply restrictions on ρ. However, if we want to test for different types of structural breaks in the cointegrating space, we have to impose restrictions on ρ directly. Let these restrictions be written as

$$R_\rho' \rho' = 0 \Leftrightarrow R_\rho' \left[\gamma' J \beta^*\right] = 0, \tag{13}$$

where $J = (I_n, 0_{n\times 1})$. Since $\rho = \beta'\gamma = \beta^{*\prime} J\gamma$, restrictions on ρ therefore imply restrictions on the product of β and γ.[1] (These restrictions may be transformed into restrictions on β^* or γ, see Section 3.3.)

3.2 The estimation problem

Next, we consider how to estimate the system. First, suppose we knew ϕ_b and ϕ_γ (and therefore β^* and γ). Then the remaining coefficients could be estimated by applying OLS. Let $l\left(\alpha\left(\phi_b, \phi_\gamma\right), H_\beta\phi_\beta, H_\gamma\phi_\gamma, \Gamma_1\left(\phi_b, \phi_\gamma\right), ..., \Gamma_{p-1}\left(\phi_b, \phi_\gamma\right), \Omega\left(\phi_b, \phi_\gamma\right)\right)$ be the corresponding log-likelihood value.

Problem 3.1 *The maximum likelihood estimates of β^*, γ and ρ can be derived from* (10), (12) *and $\rho = \beta'\gamma$ respectively, where ϕ_γ and ϕ_b are given by the solution of the following maximization problem*

$$\max_{\phi_\gamma, \phi_b} \left\{ l\left(\alpha\left(\phi_b, \phi_\gamma\right), H_\beta\phi_\beta, H_\gamma\phi_\gamma, \Gamma_1\left(\phi_b, \phi_\gamma\right), ..., \Gamma_{p-1}\left(\phi_b, \phi_\gamma\right), \Omega\left(\phi_b, \phi_\gamma\right)\right) \right.$$
$$\left. \textit{subject to } R_\rho' \left[H_\gamma \phi_\gamma J H_\beta \phi_b\right] = 0 \right\}.$$

The estimation problem described above is solved using GRaM (an acronym for 'Growth *Ra*tes and cointegration *M*eans'), which is programmed in Ox Professional 3.3 (see Doornik (2001)). Since it utilizes OxPack, the program is interactive and easy to use.[2]

[1] If there are restrictions on ρ, ϕ_b and ϕ_γ are not the free parameters in β and γ. This is because the restrictions imposed on ρ imply restrictions between ϕ_b and ϕ_γ.

[2] The program is still under development, and is expected to be made downloadable from my homepage http://folk.ssb.no/hhu in April 2005. The program requires Ox Professional 3.3 (or later versions),

3.3 Alternative formulations of the estimation problem

To apply Problem 3.1 we must use an algorithm that allows for maximizing under restrictions. However, many maximizing algorithms, such as BFGS (Broyden, Fletcher, Goldfarb and Shanno) and SA (simulated annealing), does not allow for such restrictions. An alternative is to transform the restrictions on ρ into restrictions on β or γ. (The restrictions might have to be updated for each iteration).[3]

If $rank\left(R_{\beta}\right) + rank\left(R_{\rho}\right) \leq n - r$ (i.e. the total number of restrictions on β and ρ does not exceed the number of variables minus the number of cointegrating vectors), the maximization problem could be simplified. Suppose we know ϕ_{γ} (and therefore γ), we could construct the variables $Y_t^d = Y_t - \gamma D_t$ (i.e. 'de-trended' variables in (6)), and estimate the remaining coefficients as suggested in e.g. Johansen (1995, Chap. 7.2.1), where the restrictions imposed on β now are the joint set of R'_{β} and $R'_{\rho}\gamma'$. The joint restrictions could be written as

$$\begin{pmatrix} R'_{\beta} \\ R'_{\rho}\gamma' J \end{pmatrix} \beta^* = 0,$$

and the maximization problem could be written as

$$\max_{\phi_{\gamma}} \left\{ l\left(\alpha\left(\phi_{\gamma}\right), \beta^*\left(\phi_{\gamma}\right), H_{\gamma}\phi_{\gamma}, \Gamma_1\left(\phi_{\gamma}\right), ..., \Gamma_{p-1}\left(\phi_{\gamma}\right), \Omega\left(\phi_{\gamma}\right)\right)\right\}, \tag{14}$$

where $H_{\gamma} = \left(R_{\beta}, J'\gamma R_{\rho}\right)_{\perp}$. This alternative formulation can be used in most of the empirical applications in this paper.

If there are many restrictions on β and γ, i.e. $rank\left(R_{\beta}\right) + rank\left(R_{\rho}\right) > n - r$, the restrictions on ρ can be transformed into restrictions on γ. The joint restrictions on γ

since it applies the function MaxSQP. MaxSQP implements a sequential quadratic programming technique to maximize a non-linear function subject to non-linear constraints.

[3] If BFGS or SA is chosen as maximizing algorithm in GRaM, the program identifies the appropriate formulation based on the number of restrictions. (If SQP is chosen, GRaM applies Problem 3.1.)

is then[4]

$$\begin{pmatrix} I_n \otimes R_\gamma' \\ \beta \otimes R_\rho' \end{pmatrix} vec\gamma' = 0.$$

By applying this form of the restriction on γ, the maximizing problem is equal to that in Problem 3.1, but without the constraint (since the constraint is already imposed on γ). (In this situation we use $H_\gamma = \left(I \otimes R_\gamma, \beta \otimes R_\rho\right)_\perp$ with $vec\gamma' = H_\gamma \cdot \phi_\gamma$ and $H_\beta = \left(R_\gamma\right)_\perp$ with $\beta = H_\beta \cdot \phi_\beta$.)

3.4 Distribution of the likelihood tests

The distribution of most of the likelihood ratio (LR) tests that apply are shown in the literature to be χ^2-distributed. The LR test for the restrictions on β^*, as formulated in (9), are known to be asymptotically χ^2-distributed with $r \cdot rank\left(R_\beta\right)$ degrees of freedom, see e.g. (Johansen, 1995, Section 7.2.1). Johansen et al. (2000) shows that (at least a subset of) the restrictions on γ, as formulated in (11), are asymptotically χ^2-distributed with $n \cdot rank\left(R_\gamma\right)$ degrees of freedom.[5] The restrictions on β^* and γ are independent, so the total numbers of degrees of freedom is just the sum (i.e. $r \cdot rank\left(R_\beta\right) + n \cdot rank\left(R_\gamma\right)$).

Restrictions on ρ can be reformulated into restrictions on β^* or γ; so if restrictions on β^* and γ can be tested based on a χ^2-distribution, restrictions on ρ can be tested based on the same distribution as well. The appropriate degrees of freedom can also be found by transforming these restrictions into restrictions on β^* or γ.

[4]Here, $\otimes$ is the operator for the Kronecker product and $vecA$ indicates that all columns in A are stacked in one row vector.

[5]Johansen et al. (2000) only considers shifts in the trend.

4 Empirical illustrations

4.1 Uncovered interest parity and the Italian/German exchange rate

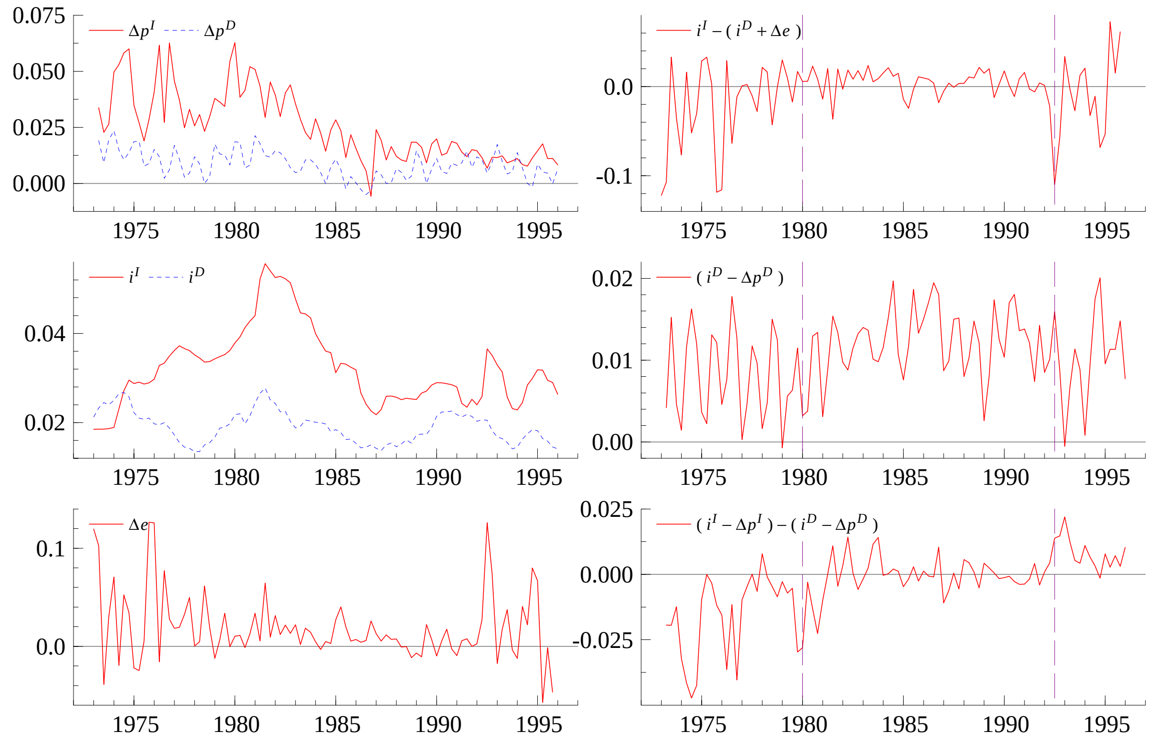
Johansen et al. (2000) apply their method to analyse the uncovered interest parity (UIP) hypothesis between Germany and Italy. We analyse the same data using our method. The data used in the analysis are first differences of log consumer price indices for Italy and Germany (Δp_t^I, Δp_t^D); the first difference of log nominal exchange rate between Italian Lira and German Mark (Δe_{t+1}) representing the rational expectation to future exchange rates; and nominal interest rates on long-term treasury bonds in both countries (i_t^I, i_t^D). The data, $Y_t' = \left(\Delta p_t^I, \Delta p_t^D, \Delta e_{t+1}, i_t^I, i_t^D\right)$, is plotted in the left part of Figure 1.[6] Johansen et al. (2000) introduce two break points; in 1980q1 and 1992q3. The former corresponds to the creation of the EMS (but is also supposed to capture the oil price shock and the modification on the US monetary policy), and the latter corresponds to the exit of Italy from the EMS and the reunification of Germany. The vector of deterministic variables is therefore given by $D_t = (t, D1980q1_t, D1992q3_t, cum\,(D1980q1_t)\,, cum\,(D1992q3_t))$. Here $D1980q1_t$ is a step dummy equal to zero before $1980q1$ and unity from $1980q1$. $D1992q3_t$ is defined similarly, and $cum\,(D1980q1_t)$ and $cum\,(D1992q3_t)$ are the variables for the corresponding broken trends.

In Figure 1 we see that there are trends in inflation and interest rates. Therefore, we use the model H_l. The estimation period is 1973q4 - 1995q4, and the number of lags is set to 2 ($p = 2$).

The cointegration rank test results are reported in Table 1. The significance probabilities are computed according to Johansen et al. (2000). They are based on an approximation using a Gamma distribution (suggested by Doornik, 1998) in the presence of structural breaks. The reported significance probabilities suggest a rank of two.

[6] The data are available at http://www.blackwellpublishers.co.uk/ectj/dataset5.htm. Source: Johansen et al. (2000).

Figure 1: Italian/German exchange rate data



The time series in the left part: inflation in Italy (Δp^I) and Germany (Δp^D); interest rates in Italy (i^I) and Germany (i^D); and log difference of LIT/DM exchange rate (Δe). In the right part the three components we expect to be stationary are plotted.

Johansen et al. (2000), analysing the same data set but handling the breaks differently, find support of three cointegrating vectors. To be able to compare our results with theirs, we continue the analysis with three cointegrating vectors.[7]

Following Johansen et al. (2000), we suggest the three stationary linear combinations reported below, cf. the right part of Figure 1. They correspond to the UIP hy-

[7]However, the probability values are only asymptotically valid. Johansen (2002) shows that there can be significant discrepancies between the true critical values and the asymptotical values.

Table 1: Cointegration rank test

Rank:	loglik	Hypothesis	Trace	p-value
$r=5$	1865.07			
$r=4$	1860.48	$r\leq 4$	9.19	0.83
$r=3$	1852.97	$r\leq 3$	24.22	0.87
$r=2$	1832.76	$r\leq 2$	64.62	0.14
$r=1$	1792.99	$r\leq 1$	144.16	0.00
$r=0$	1748.71	$r=0$	232.74	0.00

pothesis, the German real interest rate, and the real interest rate differential:

$$\begin{aligned} y_{1t} &= i_t^I - \left(i_t^D + \Delta e_{t+1}\right), \\ y_{2t} &= \left(i_t^D - \Delta p_t^D\right), \\ y_{3t} &= \left(i_t^I - \Delta p_t^I\right) - \left(i_t^D - \Delta p_t^D\right). \end{aligned}$$

These suggested stationary components imply that the cointegration space should span the space of the matrix reported below:

$$\beta' \in sp\begin{pmatrix} 0 & 0 & -1 & 1 & -1 \\ 0 & -1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & -1 \end{pmatrix}.$$

The restrictions imposed on the cointegration space when the vector of variables is augmented with an intercept (i.e. $\beta^{*\prime} = \left(\beta', -\mu\right)$) is therefore

$$R'_\beta = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & : & 0 \\ 1 & 0 & 1 & 1 & 0 & : & 0 \end{pmatrix},$$

where the left 2×5 matrix (which involve the restrictions on β) is orthogonal to the cointegration space, and the right vector with zeros corresponds to that we have not imposed any restrictions on the intercept in the cointegration space.

The restrictions implied by the suggested cointegration space are not accepted at a 5 per cent level, see Table 2. In the following we test different types of breaks both

Table 2: Restrictions on the cointegration space

Test	loglik	LR	d.f.	p-value
No restrictions	1852.97			
Restrictions on coint. space	1844.74	16.45	6	0.012

Table 3: Test results for structural breaks in cointegrating vectors

Test	loglik	LR	d.f.	p-value
No restrictions	1852.97			
No trend in first period	1848.68	8.58	3	0.035
No trend in second period	1845.56	14.81	3	0.002
No trend in third period	1834.72	36.49	3	0.000
No trend-break from 1st to 2nd	1847.72	10.50	3	0.015
No trend-break from 2nd to 3rd	1834.74	36.46	3	0.000
No level-break from 1st to 2nd	1845.39	15.15	3	0.002
No level-break from 2nd to 3rd	1838.20	29.53	3	0.000
Test	loglik	LR	d.f.	p-value
Restricted cointegration space	1844.74			
No trend in first period	1838.15	13.19	3	0.004
No trend in second period	1843.33	2.83	3	0.419
No trend in third period	1826.34	36.81	3	0.000

with and without the suggested restrictions on the cointegrated space.

In the upper part of Table 3 different test of breaks in trends and levels in the cointegration space are tested when the restrictions on the cointegration space are not imposed. Some of the tests are repeated in the bottom part of Table 3 when the cointegration restrictions are imposed.

The first three hypotheses in Table 3 correspond to the hypotheses in Table 9 in Johansen et al. (2000). For each of the three subperiods (1973q4 - 1979q4, 1980q1 - 1992q2, 1992q3 - 1995q4) the hypothesis of no trend is rejected at a 5 per cent level. The reported significance probabilities are somewhat smaller than the corresponding values reported in Johansen et al. (2000). The discrepancies stem from the fact that they include impulse dummies in two periods after the breaks.[8]

In the next two lines of Table 3 hypotheses of no break between the different peri-

[8]In our framework we can produce test results that are approximately equal to those obtained in Johansen et al. (2000) by extending the vector of deterministic variables with the impulse dummies $(I1980q2_t,\ I1980q3_t,\ I1992q4_t,\ I1993q1_t)$.

Table 4: Test results for structural breaks in variables

Test	loglik	LR	d.f.	p-value
No restrictions	1852.97			
No trend in first period	1846.47	13.00	5	0.023
No trend in second period	1845.34	15.25	5	0.009
No trend in third period	1834.06	37.81	5	0.000
Test	loglik	LR	d.f.	p-value
Restricted cointegration space	1844.74			
No trend in first period	1836.72	16.04	5	0.007
No trend in second period	1843.12	3.25	5	0.662
No trend in third period	1825.79	37.91	5	0.000

ods are tested. Both hypotheses are rejected. These hypotheses could also be tested by the method in Johansen et al. (2000).

However, tests of level-breaks can not be tested by the method suggested in Johansen et al. (2000). The test results reported in Table 3 show that such hypotheses are rejected, using the procedure introduced in the current paper.

The three first restrictions are re-tested when the restrictions on the cointegration space are imposed, see the last part of Table 3.[9] [10] Note that, at this stage of the testing, the restrictions that there are no trends in the cointegration space are rejected for the first and third period. For the second period, however, the restrictions are now far from being rejected. This is consistent with the impression obtained when looking at the right part of Figure 1: in the second period (1980q1-1992q2) there are no trends in the suggested stationary relationships.

We can also test hypotheses about breaks in the trends of the variables. In Table 4 results from these tests are reported. Again we test both with and without the restricted cointegration space.

From the upper part of Table 4 we see that the hypotheses that there is no trend

[9]We analyse the system also with these restrictions on the cointegrating space imposed even though they were rejected (see Table 2), because they were included in the analysis of Johansen et al. (2000). They found that these restrictions in the cointegration space could not be rejected when combined with additional restrictions on the trends in the cointegrating space.

[10]Here $rank\left(R_\beta\right) = 2$ and $rank\left(R_\rho\right) = 1$. Therefore; $rank\left(R_\beta\right) + rank\left(R_\rho\right) = 3 \geq n - r = 2$, and the alternative formulation of the maximizing problem in (14) can not be used. Therefore, one of the two alternative formulation must be used.

can be rejected for all the three periods, when no restrictions are imposed on the cointegration space. When restrictions are imposed on the cointegration space, however, we can not reject the hypothesis that there is no trend in the variables in the second period.

4.2 Money demand in unified Germany

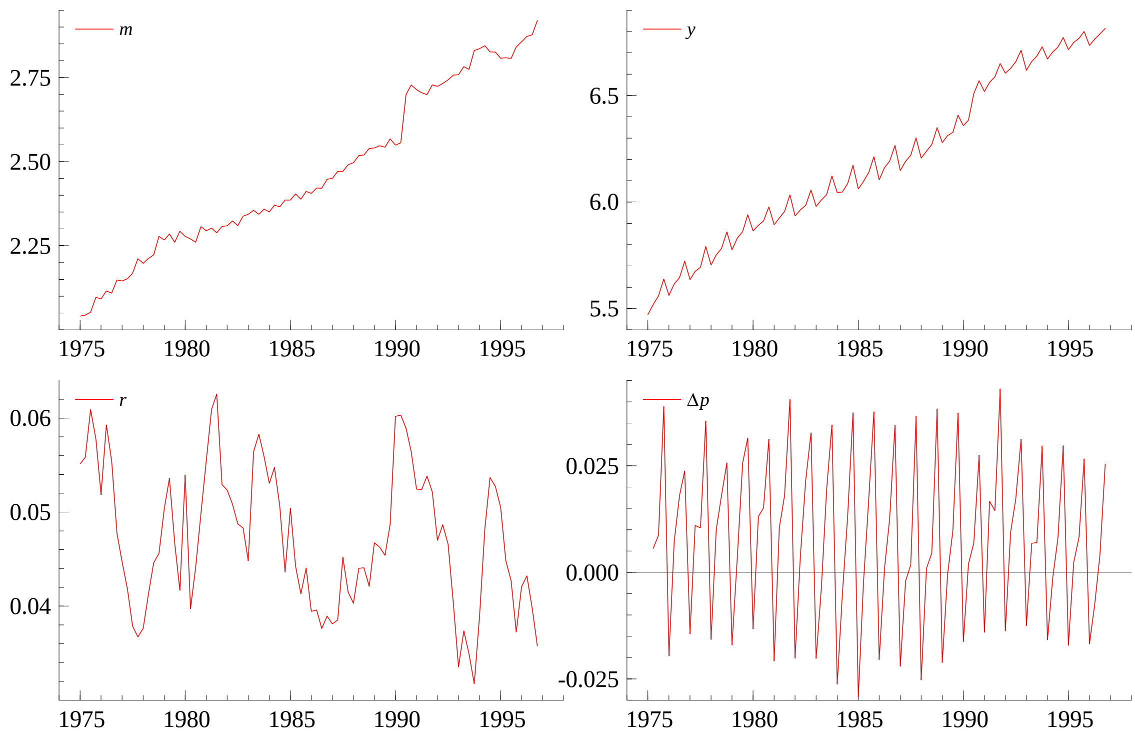
The unification of Germany in 1990 lead to dramatic shifts in German time series. Lütkepohl and Wolters (1998) and Saikkonen and Lütkepohl (2000) use a data set covering the unification to estimate a model for money demand in Germany. They use four variables; (log of) real money M3 (m), (log of) real GNP (y), an opportunity cost of money (r) and inflation (Dp). The opportunity cost of money is defined as the difference between a long term interest rate and the own rate on M3. The data run from 1975q1 to 1996q4.[11] The official date of unification is October 3, 1990. However, the monetary unification took place July 1, 1990.

In the data series for money and income there is a significant shift in level from 1990q3, cf. Figure 2. For the opportunity cost of money there is no obvious shift in level or trend. To take account for the shift in level for money and income we include a shift dummy in our empirical analysis. This shift dummy is zero until 1990q2 and one thereafter. In order to test for the different types of structural shifts suggested by Perron (1989) we also include a broken trend, defined as the cumulate of the shift dummy. Due to significant seasonal pattern, especially in income and inflation, we also include (centered) seasonal dummies.

The cointegration rank test results are reported in Table 5, where the significance probabilities are computed according to Johansen et al. (2000). The reported values suggest a rank of two, which is the same cointegration rank as Saikkonen and Lütkepohl (2000) found.

[11] The data are available at: ftp://141.20.100.2/pub/econometrics/germanm3.zip. Source: Lütkepohl and Wolters (1998).

Figure 2: German money demand data



The money stock (m), (real) income/gdp (y), real interest rate (r) and inflation quarter/quarter (Δp) in Germany (West Germany until 1990q2 and unified Germany thereafter).

Table 6 reports the test for homogeneity between money and income in the two cointegrating vectors. Homogeneity is clearly rejected.

Next, we test the different types of structural breaks, both in the cointegrating space and for the variables. The test of no trend-break in the cointegrating space can not be rejected, since the restriction implies only a slight reduction in the log-likelihood (cf. Table 7). However, the hypothesis of no level-break is rejected at the conventional

Table 5: Cointegration rank test

Rank:	loglik	Hypothesis	Trace	p-value
$r = 4$	1237.15	$r \leq 4$		
$r = 3$	1232.21	$r \leq 3$	9.86	0.41
$r = 2$	1223.65	$r \leq 2$	26.99	0.26
$r = 1$	1208.66	$r \leq 1$	56.97	0.04
$r = 0$	1178.72	$r = 0$	116.85	0.00

Table 6: Test of restrictions on the cointegrating space

Test	loglik	LR	d.f.	p-value
No restrictions	1223.65			
Restrictions on cointgration space	1217.00	13.30	2	0.001

Table 7: Tests of structural breaks

Test	loglik	LR	d.f.	p-value
No restrictions	1223.65			
No trend-break in cointgrating space	1223.03	1.23	2	0.540
No level-break in cointgrating space	1218.82	9.67	2	0.008
No break in cointegrating space	1217.90	11.49	4	0.022
No trend-break in variables	1221.56	4.19	4	0.381
No level-break in variables	1160.16	126.97	4	0.000
No break in variables	1152.76	141.78	8	0.000
No break in cointegrating space & no trend-break in variables	1217.07	13.16	6	0.040

levels of significance (since the significance probability is only 0.8 per cent). The joint hypothesis, implying no breaks in the cointegration space has a significance probability of 2.2 per cent. If we apply a 5 per cent significance level we reject these restrictions, but with a 1 per cent significance level it can formally not be rejected.

Also for the variables we can not reject the hypotheses of no trend-break, but reject the hypotheses of no level-break. Not surprisingly, also the joint hypothesis of no structural breaks is clearly rejected.

In Table 7 also the results of a joint combined test of no breaks in the cointegration space and no trend-break in the variables is reported. This has a significance probability of 4 per cent, and whether we reject the hypothesis or not depends on the critical level we choose to use.

5 Conclusions

In this paper I have shown how to decompose deterministic parts in cointegrated VAR models into interpretable long run counterparts. The inference procedure includes hypothesis testing on the corresponding coefficients. The procedure is applied on two data sets in which structural breaks are an important feature, and we show how to test for different types of structural breaks in these two concrete settings.

It is important to note that structural breaks are only one type of problem this method can be used to analyse. A decomposition of the deterministic terms into interpretable counterparts will often be important even though there are no structural breaks. For example, Hungnes (2002) used a simplified version of the procedure to estimate a cointegrated VAR model where some variables were allowed to grow and some were not. As mentioned in the introduction, the growth rates are important in forecasting, and identifying the growth rates is important in order to judge the forecasting ability of a model.

Another application for the procedure is to test for "zero-mean" convergence. "Zero-mean" convergence implies that the difference between two variables (say; output per capita in two countries) is stationary with zero mean. To test if the mean is zero, the deterministic terms must be decomposed.

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