Norwegian School of Economics

Bergen, Fall 2018

The Role of Beta Strategies in Other Asset Pricing Anomalies

Mats Engedal Bostad and Pål Kjellevold

Supervisor: Francisco Santos

Master Thesis, Finance

NORWEGIAN SCHOOL OF ECONOMICS

Acknowledgements

We want to express our sincere gratitude to our supervisor, Francisco Santos. His teaching in the courses FIE 400 - Investments and especially FIE 451 - Applied Asset Management, has played a significant role in the motivation of our research subject. His invaluable guidance through the past year made this thesis possible. Additionally, we would like to thank Heine Didriksen at the IT-department at NHH for providing us with sufficient computational power to handle our calculations.

Bergen, December 2018

Mats Engedal Bostad

Pål Kjellevold

Abstract

This thesis is based on the findings of Liu (2018), and therefore considers long-short, zero-cost portfolios based on documented asset pricing anomalies. These include momentum, composite equity issuance, return volatility, and idiosyncratic volatility. Consistent with the observations in Liu (2018), we find that the relevant long-short portfolios embed significantly negative realized betas and therefore load in the low-beta anomaly. Neutralization of this exposure decreases the economic magnitude and statistical significance of their abnormal returns. In order to demonstrate this, we follow the methodology of Liu (2018) and propose a modification to one of the beta mitigation techniques. Also, we contribute with other methods, documented in the existing literature, that are designed either to reduce the beta imbalance or to account for the portfolios' exposure to the beta anomaly. Furthermore, we contribute by testing all methods of beta mitigation for alternative pre-formation beta estimation techniques, in order to investigate if these affect the explanatory power of the beta anomaly. Consistent with the findings of Liu (2018), we find that the mitigation of the inherent beta imbalance in the long-short anomaly portfolios either decreases or removes these strategies' abnormal returns. The magnitudes of these reductions vary by choice of beta neutralization method and pre-formation beta estimation technique.

Contents

List of Tables

List of Figures

1 Introduction

Jensen, Black, and Scholes (1972) made the original empirical finding that stocks with low systematic risk outperform stocks with high systematic risk, in terms of risk-adjusted returns. This observation, the beta anomaly, is a widely documented failure of the Capital Asset Pricing Model (CAPM). Liu (2018) finds that a broad section of long-short, zero-cost anomaly portfolios are loading in the beta anomaly. This is because they embed negative and significant realized CAPM betas. Mitigation of this exposure reduces the economic magnitude and statistical significance of their abnormal returns (Liu, 2018).

We replicate the methodology of Liu (2018) in order to examine these results. Thus, we test if the beta anomaly holds explanatory power over the abnormal returns to long-short anomaly portfolios. In particular, we test anomaly strategies formed on characteristics including momentum (MOM), composite equity issuance (CEI), return volatility (VOL) and idiosyncratic volatility (IVOL). We contribute with alternative methods for taking the beta anomaly into account, which is documented in the existing literature. These include the application of leverage, double sorts, and regression tests. Furthermore, we propose a modification to one of the beta-mitigation techniques in Liu (2018). Additionally, we contribute by testing if the choice of pre-formation beta estimation technique impacts the explanatory power of the beta anomaly. In order to do this, we form long-short anomaly portfolios on the basis of three different beta estimation methods that are proposed in the existing asset pricing literature.

The first pre-formation beta estimation technique we utilize is the same as in Liu (2018) and includes simple rolling CAPM regressions of daily stock returns. Second, we borrow the methodology proposed by Frazzini and Pedersen (2014), which is based on individual estimations of stocks' correlation with the market and their volatilities. Lastly, we exploit the methodology of Fama and French (1992), where we estimate pre-formation stock betas on a portfolio basis. We denote these techniques β^{SR}, β^{BAB}, and β^{FF} respectively. We form anomaly strategies from these estimates and therefore study a total of 12 long-short anomaly portfolios.

The beta imbalance of each anomaly portfolio stems from an overrepresentation or overweight of low-beta stocks in the long leg and high-beta stocks in the short leg. In order to correct this beta imbalance, the first technique we borrow from Liu (2018) includes an elimination of low-beta stocks in the long leg and high-beta stocks in the short leg. This results in realized portfolio betas that are non-different from zero for 9 out of the 12 studied anomaly strategies. The reductions in abnormal returns vary from 27% to 69%, and pre-formation beta estimation technique β^{BAB} appears to explain this effect most efficiently among the three.

The following three beta-mitigation methods involve neutralization of the portfolio betas through modifications of individual stock weights in each portfolio leg. We argue that the method of weighting by beta ranks is inefficient because it considers a weighting scheme that is too extreme in comparison with the original value-weighted portfolios. The weight-shifting method involves shifting weight from low-beta stocks to high-beta stocks in the long leg of each anomaly portfolio. Symmetrically, weight is shifted from high-beta stocks onto low-beta stocks in the short leg. This method is a definite improvement over the beta-rank weighting method. The modified weight-shifting method includes a distribution of the subtracted weight that is proportional to stocks' size, whereas the original method utilizes an equal distribution. We show that our modification improves beta mitigation efficiency and argue that it makes it more comparable to the original long-short portfolios. Overall, the results from the modified weight-shifting method are similar to those acquired from the elimination method. This relates both to reductions in beta and abnormal returns. Forming portfolios on β^{BAB} results in the highest explanatory power for the beta anomaly.

We contribute with the fifth technique for neutralizing the anomaly portfolios' realized betas. This method is borrowed from Frazzini and Pedersen (2014) and includes the application of leverage to own portfolio legs. We lever the long leg and de-lever the short leg, such that both legs have a realized beta of 1. We fund the difference at the risk-free rate. The leverage technique is the most efficient method for neutralizing beta as it results in completely market neutral portfolios for all of the 12 strategies. The leverage technique provides evidence that the choice of pre-formation beta estimation technique has little to no impact on the explanatory power of the beta anomaly. Reductions in abnormal returns range from 30% to 40%.

A double sort is a standard tool used to study how one characteristic vary while holding the other constant (Fama & French, 1992). Consistent with the observation of Liu (2018), we find that even though we sort each strategy on beta and anomaly characteristic, the beta quintiles still exhibit significant variation. Liu (2018) argues that this contaminates an interpretation of such double sorts. We do however argue that a graphical presentation of combinations of extreme quintile anomaly strategies clarifies the relationship between realized beta and abnormal returns. This contribution suggests that abnormal returns are significantly reduced as portfolio betas are neutralized, and that pre-formation beta estimation techniques β^{BAB} and β^{FF} provides the most explanatory power to the beta anomaly.

Our last contribution involves CAPM regressions of anomaly strategies' returns where we include the BAB-factor as an explanatory variable. Because the BAB-factor proxies for the beta anomaly, we show that each of the original value-weighted anomaly portfolios is loading in the beta anomaly. Furthermore, the abnormal returns to each strategy are significantly reduced when we introduce the BAB-factor. These regression tests indicate that the choice of pre-formation beta estimation technique has little impact on the explanatory power of low-beta. Additional tests include the same regression specifications for anomaly strategies after beta-mitigation techniques have been applied. The reductions in abnormal returns are of a smaller economic magnitude when the BAB-factor is introduced compared to the first tests. This suggests that the beta-mitigation techniques work as intended, with varying effectiveness across beta-mitigation and pre-formation beta estimation techniques.

We conclude that their exposure to the beta anomaly can explain a *part* of the abnormal return to each anomaly strategy. The explanatory power varies both on the basis of beta mitigation method and on the choice of pre-formation beta estimation technique.

This thesis proceeds as follows. Section 2 presents literature that relates to our findings. Section 3 presents the data and methodology that we use in order to perform our empirical analysis. Section 4 discusses our main findings, and section 5 concludes.

2 Literature Review

This section sheds light on existing literature that relates our findings, which is motivated by the work of Liu (2018). We will, therefore, begin with a presentation of his most prominent finding. This includes the observation that the low-beta anomaly holds explanatory power over the abnormal returns to a broad section of other asset pricing anomalies. In our extension of his work, the quantification of systematic risk, and estimation techniques thereof are paramount. Thus, we will provide a presentation of the cross-sectional relationship between risk and return, including the beta anomaly. Asset pricing anomalies are results of the many failures of the CAPM to fully account for the positive relationship between risk and return. We study a selection of the anomaly characteristics that are analyzed in the work of Liu (2018). These include momentum, composite equity issuance, return volatility, and idiosyncratic volatility. As a result, we will end this section by presenting literature that documents abnormal returns from forming portfolios based on these anomaly characteristics.

Liu (2018) considers the formation of monthly rebalancing, long-short portfolios based on twelve documented asset pricing anomalies. The common characteristic of his entire section of strategies is that they all have realized portfolio CAPM betas that are significantly negative, which implies a positive exposure to the beta anomaly. Furthermore, neutralizing the anomaly portfolios' betas with the goal of mitigating the exposure to the beta anomaly decreases the abnormal returns to these strategies. This does not provide any reassurance to advocates of the efficient market hypothesis. If this finding is true, then explanations for the low beta anomaly would appear to be of increased importance. Solving the low beta puzzle would necessarily also imply a solution to a broad section of other asset pricing puzzles.

2.1 The Cross-sectional Relationship Between Risk and Return

Proceeding the resurgence of modern portfolio theory following Markowitz (1952), the positive relationship between risk and return has been widely accepted by the academic field of finance and economics. The discovery of this relationship led to the hypothesis of risk-based preferences in expected returns and the simultaneous discovery of the Capital Asset Pricing Model (CAPM) by Sharpe (1964), Lintner (1965) and Mossin (1966). This model has been subject to recurring scrutiny since its first publication in the 1960s. Jensen et al. (1972) made the original, empirical observation that the slope of the security market line is flatter than the CAPM predicts. Rebutting these findings, "Roll's Critique" argues that the act of performing empirical tests of the CAPM is infeasible because one can never know the true constituents of the market portfolio (Roll, 1977). Regardless of the dispute between advocates of the efficient market hypothesis and those on the other side of the fence, the CAPM is still widely taught and practiced in academia and the industry alike.

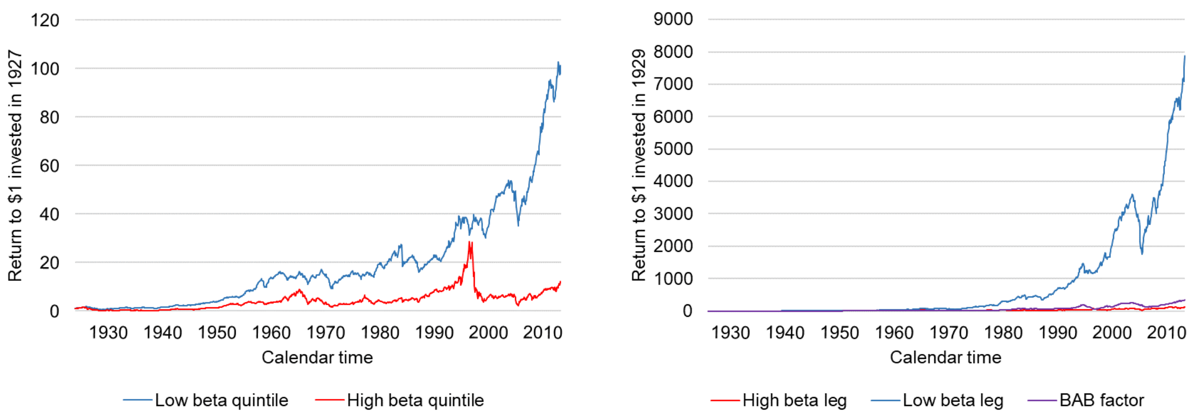
Because of the inherent flaws that are related to an empirical implementation of the CAPM, there exists no exact method to estimate the systematic risk of a stock. As a response, different beta estimation techniques have been contributed to the financial literature. A preliminary approach includes rolling regressions of excess stock returns onto the market excess return. This simple method is utilized in Liu (2018). Because empirical tests have found individual stock betas to be imprecise, Fama and French (1992) employ a different approach where betas are estimated on a portfolio level. In a more recent publication, Frazzini and Pedersen (2014) emphasize the fact that stocks' correlations with the market portfolio move slower than their volatilities. In order to take this into account, they propose a beta estimation technique in which volatilities and correlations are estimated individually.

The empirical observations of Jensen et al. (1972) included in particular that the compensation for holding stocks with low systematic risk relative to high systematic risk is higher than the CAPM predicts. A self-financing trading strategy that is long low-beta stocks and short high-beta stocks will earn abnormal returns as a result. Figure 1 displays two proxies for the beta anomaly. Figure 1 (a) shows the cumulative returns to extreme quintiles of beta estimated from simple rolling CAPM regressions. It illustrates that the low-beta quintile experiences superior returns compared to the high-beta quintile. Figure 1 (b) illustrates the cumulative return to the BAB-factor proposed by Frazzini and Pedersen (2014). Naturally, they construct this strategy from the beta estimation technique they propose themselves. The BAB-factor is a monthly rebalancing, zero-cost portfolio that is long low-beta assets and short high-beta assets. In order to create a market neutral strategy, the low-beta leg is levered such that the realized beta of the long leg is equal to 1. Similarly, the short leg is de-levered such that the realized beta of the short leg also is equal to 1. The difference between each leg is funded by borrowing at the risk-free rate.

Figure 1: The beta anomaly

Both figures display the cumulative returns to proxies for the beta anomaly. The sample period is 1927 to 2016. The starting year is 1927 for Figure (a), which is a replication of the beta anomaly portfolio in Liu (2018). Stocks are sorted into quintiles in every month based on their pre-formation beta estimate. The return to each portfolio leg in every month is the value-weighted return to each extreme beta quintile. The starting year of Figure (b) is 1929, which is a replication of the BAB-factor that is proposed by Frazzini and Pedersen (2014). Each month, stocks are sorted into one out of two portfolio legs. If the pre-ranking beta of a stock is less than the cross-sectional median, it is assigned the low-beta leg. Otherwise, the stock is assigned the high beta leg. The return to each leg in every month is the value-weighted return. Subsequently, the low-beta leg is leveraged in every month such that the realized beta is equal to 1. Simultaneously, the high-beta leg is de-levered such that the portfolio beta is equal to 1. The difference is funded by borrowing at the risk-free rate.

(a) Long-short extreme beta-quintiles (b) The BAB-factor



2.2 Asset Pricing Anomalies

Failures of the CAPM regarding a full account of the relationship between risk and return does not only relate to the characteristic of a stock's systematic risk. The landscape of current financial research provides extensive documentation of alternative anomaly characteristics. Upon forming long-short, zero-cost portfolios based on these characteristics, abnormal returns can be achieved that are robust both to the CAPM as well as alternative asset pricing models like the Fama French models.

2.2.1 Momentum

Jegadeesh and Titman (1993) were the first to show that abnormal returns can be acquired through a trading strategy that buys past winners and sells past losers. Using formation periods that vary from 3 to 12 months, they allocate stocks to deciles based on their cumulative return in the respective formation periods. They do then proceed to form portfolios where they buy deciles with the highest preceding cumulative return and sell stocks in the deciles with the lowest cumulative return. They do also consider different holding periods for the momentum portfolios before they eventually close their position or rebalance. Using the same variation of holding periods, they study a total of 16 momentum portfolios. Jegadeesh and Titman (1993) conclude that the profitability of their portfolios does not arise as a result of being exposed to systematic risk. This conclusion is based on a decomposition of momentum profits into different sources and the development of different tests.

One of these tests includes an estimation of post-ranking betas for each decile in a momentum strategy with a formation- and holding period of 6 months. Their findings suggest that the deciles including stocks with high past returns have lower systematic risk than the deciles of stocks with the lowest cumulative return. This results in a negative realized beta of -0.08 for a strategy that is long-short the extreme deciles. Jegadeesh and Titman (1993) do not provide information on the statistical significance of this estimate, even though it is rather close to zero concerning economic magnitude. Their sample period from 1965 to 1989 and choice of formation and holding periods are however varying factors. A negative post-ranking beta for the momentum portfolio is also observed in Liu (2018).

2.2.2 Composite Equity Issues

Daniel and Titman (2006) contribute to the asset pricing anomaly literature with a characteristic they name composite equity issuance. They introduce the construction of this measure by dividing the information that impacts stock prices into two components. Tangible information is contained in financial statements and includes for instance book value, earnings, cash flow, and sales growth. Intangible information is private and may include changes in expectations of future cash flows or discount rates.

The decomposition of information is made with the intent of dividing total returns into tangible and intangible returns. Thus, the tangible return of a stock is the part of the return that can be explained by accounting variables, and intangible return is the part of the total stock return that remains unexplained by accounting measures. In order to estimate these return components, they use cross-sectional regressions where the log change of return in a given period is regressed on accounting variables. The proxy for the intangible return is therefore defined as the error of these regressions.

The findings of Daniel and Titman (2006) suggest that accounting variables can explain about 60% of past returns. It is however surprising that they find no significant relation between tangible returns and future returns, while the intangible return is strongly negatively related to future returns. In order to investigate this relationship further, they introduce the measure of composite equity issuance as an additional explanatory variable in the preliminary regressions. It serves to capture parts of the intangible returns that are not revealed in the first regression tests. Composite equity issuance is built on the premise that managers time the equity markets based on private information. It is calculated as the log change in market capitalization minus the cumulative stock return for a given period. Thus, it measures the amount of equity that firms issue or retire in exchange for cash or services. Actions that extract cash from the firm, such as dividends or repurchases of shares, reduce composite equity issuance. On the other hand, share-based acquisitions or stock option plans which retain cash in the firm increase composite equity issuance.

Daniel and Titman (2006) find that firms with higher past intangible returns have higher market betas. Analogous to this finding, is that market betas decrease when intangible returns are low. Additionally, multiple regression tests show that the composite equity issuance variable is significantly, negatively related to future stock returns. Liu (2018) demonstrates that long-short anomaly strategies that buy firms with low composite equity issuance, and sell firms with high composite equity issuance are significantly positively related to future returns and have negative realized portfolio betas. These observations are therefore consistent with the findings of Daniel and Titman (2006).

2.2.3 Return Volatility and Idiosyncratic Volatility

Ang, Hodrick, Xing, and Zhang (2006) show that common risk factors included in either the CAPM or the asset pricing models proposed by Fama and French are unable to account for the abnormal returns to strategies formed on total return volatility or idiosyncratic volatility. Ang et al. (2006) were the first to analyze the returns to portfolios where stocks are sorted into quintiles based on these volatility measures. This method is borrowed in Liu (2018), and ultimately also exploited in this thesis.

In order to estimate idiosyncratic volatility, or firm-specific volatility, Ang et al. (2006) consider the root mean squared error of a Fama and French 3-factor regression[1] with individual stock returns as the dependent variable. Return volatility, which is the total volatility of a stock, is calculated as the standard deviation of individual stock returns without any control for systematic risk factors.

Ang et al. (2006) focus on the formation of portfolios with monthly rebalancing and a formation period of one month. They use CAPM and Fama French 3-factor regressions in order to display the robustness of these anomalies. In addition, they control for a broad section of cross-sectional effects that the existing literature has identified as proxies for risk factors or anomalies. These include size, book-to-market, leverage, liquidity, volume, turnover, bid-ask spread, coskewness, dispersion in analyst forecasts and momentum effects. In the light of this thesis, it is therefore disappointing that they choose to leave out the estimated beta coefficients in these regression tests. However, J. Liu, Stambaugh, and Yuan (2018) find that beta is positively correlated with idiosyncratic volatility in the cross-section.

The findings of Ang et al. (2006) provide evidence that buying extreme quintiles of high volatility and selling quintiles with low volatility yields significantly negative abnormal returns. The opposite strategy would therefore yield significantly positive abnormal returns. Also, the positive relationship between beta and idiosyncratic volatility that is observed in Liu et al. (2018) is consistent with the beta imbalance observed in the long-short volatility strategies in Liu (2018).

In the next section, we will expand upon the data and methodology that is used in order to perform our empirical analysis. This includes the estimation of pre-formation betas and construction of long-short anomaly strategies.

[1] Ang et al. (2006) do also note that estimating firm-specific risk relative to the CAPM yields very similar results.

3 Data and Methodology

The sample used in this analysis includes all common stocks listed on the NYSE, AMEX and NASDAQ covering the period 1927 to 2016, and is collected from the Chicago Center for Research in Security Prices (CRSP). In order to adjust returns in the event of a delisting, a variable is created that assumes the delisting return if the return on the stock is missing, and otherwise takes on the value of the non-missing return. The CRSP value-weighted index serves as a proxy for the market return and the one-month T-bill rate collected from Kenneth French's Data Library is used as the risk-free rate. Subsequently, all stocks that have available observations on return, price, and number of shares outstanding are used to calculate pre-formation betas and form long-short portfolios based on anomaly characteristics. Finally, all stocks that have at least one available estimate of beta and anomaly characteristic in month t are included in the sample.

3.1 Beta Estimates

Beta estimates are of primary interest in this analysis. Thus, three different beta estimation techniques are exploited. The first approach follows the original study of Liu (2018) and involves simple rolling CAPM regressions. The second technique follows the method outlined in Frazzini and Pedersen (2014) and involves individual computations of volatility and stocks' correlation with the market portfolio. The last technique follows the approach of Fama and French (1992), where betas are estimated on a portfolio basis.

3.1.1 Betas Estimated from Simple Rolling Regressions

A stock's CAPM beta is estimated using its daily excess return in the past twelve months, and regressing it onto the market excess return in the same period where a minimum of 150 non-missing observations are required. In order to adjust for non-synchronous trading, the sum of coefficients method following Dimson (1979) is applied, where the specification

$$r_{i,t} - r_{f,t} = \hat{\alpha}_i + \sum_{l=0}^{5} \hat{\beta}_{i,t-l}(r_{m,t-l} - r_{f,t-l}) + \hat{\epsilon}_{i,t}, \tag{1}$$

is estimated in the rolling windows. In this specification, $r_{i,t}$ denotes the return to stock i, $r_{f,t}$ is the risk-free rate and $r_{m,t}$ is the return of the market on day t. Consequently, each stock's beta estimate in every month t is calculated as

$$\hat{\beta}_{i,t} = \sum_{l=0}^{5} \hat{\beta}_{i,t-l}. \tag{2}$$

In order to reduce the influence of outliers, stocks are sorted into percentiles based on beta estimates in every month t, where the 1^{st} and the 100^{th} percentiles are removed. Thus, two percent of stocks are removed from the sample. We proceed by referring to this pre-formation beta estimation technique as β^{SR}, which is the exact technique that is utilized in Liu (2018).

3.1.2 Betas Estimated from Separate Calculations of Volatilities and Correlations

An alternative approach to estimating a stock's systematic risk is to follow the method proposed in Frazzini and Pedersen (2014). A stock's CAPM beta is estimated in the specification

$$\hat{\beta}_i^{TS} = \hat{\rho}\frac{\hat{\sigma}_i}{\hat{\sigma}_m}, \tag{3}$$

where $\hat{\sigma}_i$ and $\hat{\sigma}_m$ are estimated volatilities for stock i and the market m in the same period and $\hat{\rho}$ is their correlation coefficient. Volatilities are estimated using 1-day log returns in a rolling window of twelve months where a minimum of 120 observations is required. The correlation between stock i and the market is calculated using overlapping 3-day log-returns[2] to account for non-synchronous trading, which only affects correlations (Frazzini & Pedersen, 2014). The rolling window includes five years of 3-day overlapping log-returns requiring at least 750 non-missing observations. We use daily data, rather than monthly data, as the accuracy of covariance estimation improves with sample frequency (Merton, 1980). In order to reduce the impact of outliers, the time series estimate of betas are shrunk towards the

[2]The 3-day overlapping log-return of stock i on day t is computed as: $r_{i,t}^{3d} = \sum_{k=0}^{2} ln(1 + r_{t+k}^{i})$.

cross-sectional mean:

$$\hat{\beta}_{i,t} = w_i \hat{\beta}_i^{TS} + (1 - w_i)\hat{\beta}^{XS}. \tag{4}$$

In equation (4), w_i is the asset-specific and time-varying Bayesian shrinkage factor[3], $\hat{\beta}_i^{TS}$ is the time series estimate of beta for security i and $\hat{\beta}^{XS}$ is the cross-sectional mean. We proceed by denoting this pre-formation beta estimation method β^{BAB} for the remainder of the analysis.

3.1.3 Betas Estimated on a Portfolio Basis

The third alternative beta estimation technique follows from Fama and French (1992). All stocks listed on the NYSE are sorted by size in every month[4], determined by market capitalization, in order to create NYSE decile breakpoints. Stocks listed on the NYSE, AMEX, and NASDAQ that satisfy the CRSP requirements noted in the introduction of section 3 are then allocated to one out of ten size portfolios based on the NYSE decile breakpoints.

Proceeding the allocation of securities based on size, each size portfolio is subdivided into ten portfolios based on stocks' pre-formation CAPM beta estimates. This yields a total of 100 portfolios in every month t. Stock i's pre-formation beta estimate is computed using monthly excess returns and regressing it onto the market excess return in the same period. A rolling window specification of five years with a minimum of 24 observations is employed

$$r_{i,t} - r_{f,t} = \hat{\alpha}_i + \hat{\beta}_{i,t}(r_{m,t} - r_{f,t}) + \hat{\beta}_{i,t-1}(r_{m,t-1} - r_{f,t-1}) + \hat{\epsilon}_{i,t}, \tag{5}$$

where $r_{i,t}$ denotes the return of stock i, $r_{f,t}$ is the risk-free rate and $r_{m,t}$ is the return of the market portfolio in month t. Consequently, the sum of coefficients method is applied to

[3]The asset-specific, time-varying Vasicek (1973) Bayesian shrinkage factor is estimated in the specification: $w_i = 1 - \sigma^2_{i,TS}/(\sigma^2_{i,TS} + \sigma^2_{XS})$, where $\sigma^2_{i,TS}$ denotes the variance of the estimated pre-ranking betas for security i and σ^2_{XS} is the cross-sectional variance of estimated pre-ranking betas. The shrinkage factor has a cross-sectional mean of 0.649.

[4]Fama and French (1992) form size portfolios in June of each year because they also employ accounting data to compute stocks' book-to-market ratio, leverage, and earnings-to-price ratio. Forming size portfolios in every month allows securities to change portfolios more often. As a result, stocks will receive new beta estimates more frequently.

attain the pre-formation beta of stock i in month t (Dimson, 1979).[5]

After each stock has been assigned to one of the 100 portfolios in month t, the post-formation, value-weighted return of each portfolio is computed. We use the entire sample of portfolios returns to estimate the post-formation betas in the following CAPM specification

$$r_{p,t} - r_{f,t} = \hat{\alpha}_i + \hat{\beta}_{p,t}(r_{m,t} - r_{f,t}) + \hat{\epsilon}_{p,t}, \tag{6}$$

where $r_{p,t}$ is the value-weighted return to portfolio p, $r_{f,t}$ is the risk-free rate and $r_{m,t}$ is the market return in month t. This results in a total of 100 beta estimates for the entire time series. Finally, each stock is assigned one of the 100 post-formation beta estimates in month t based on which portfolio it constituted in that corresponding month. We denote the beta estimation technique following Fama and French (1992) as β^{FF}.

3.1.4 On the Methods of Estimating Betas

Included in Table 1, are the summary statistics of beta estimates from the three beta estimation techniques. We observe that the total number of estimates vary across each technique. The differences in observations mainly appear due to the different number of observations on stock returns that are required in each technique. The last row of Panel A in Table 1 presents evidence that the method used to mitigate the effect of outliers has less of an impact on the standard deviation of beta estimates for β^{SR} than the alternative techniques. Not surprisingly, shrinking beta estimates towards the cross-sectional mean (β^{BAB}) and estimating betas on a portfolio basis (β^{FF}) reduce the standard deviation of estimates more than removing one percent of extreme estimates (β^{SR}).

Panel B presents a correlation matrix of the various beta estimates based on 2.54 million observations.[6] Findings show that estimates vary ($\rho < 1$), and therefore suggest that the choice of beta estimation technique will impact the explanatory power of the beta anomaly.

[5]The sum of coefficients method following Dimson (1979) in the case of monthly excess returns with a lag of one month is taken as: $\hat{\beta}_{i,t} = \hat{\beta}_{i,t} + \hat{\beta}_{i,t-1}$.

[6]In order for a stock to be included in the calculation, it is required to have an available beta estimate for each of the three beta estimation techniques.

Table 1: Summary statistics of beta estimates

Reported in this table are the summary statistics of each pre-formation beta estimation technique. The sample period is 1927 to 2016. Number of observations in Panel B is the total number of beta estimates for the entire sample. Weighted mean is the cross-sectional mean of beta estimates, and volatility displays their standard deviation. In Panel C, the correlation matrix is based on a total of 2,540,524 beta estimates.

Panel A: Pre-formation beta estimation technique			
	β^{SR}	β^{BAB}	β^{FF}
Panel B: Summary statistics			
Number of observations	3 162 937	2 601 245	2 801 347
Weighted mean	1.040	0.993	1.048
Volatility	0.778	0.392	0.345
Panel C: Correlation matrix			
	β^{SR}	β^{BAB}	β^{FF}
β^{SR}	1	0.537	0.294
β^{BAB}	0.537	1	0.276
β^{FF}	0.294	0.276	1

3.2 The Beta Anomaly

We construct two proxies for the low-beta anomaly. The first includes a replication of the method applied in Liu (2018), where we create monthly rebalancing, beta anomaly portfolios that are long-short extreme quintiles of beta. Because we have three alternative beta estimates for our sample, we create such strategies for all of them. Other than the different beta estimates from each technique, the following procedure is the same for all three low beta strategies. Each month, stocks are assigned into quintiles based on an ascending sort of their most recent beta estimates. We proceed by being long the bottom quintile (low beta) and short the top quintile (high beta). Finally, we compute the monthly, value-weighted portfolio return using the one-month lagged market capitalization of each stock. We repeat these procedures for each beta estimation technique. The result is three different beta-sorted portfolios, one for each beta estimation method. Because construction and return pattern of these strategies are very similar, we treat these three beta strategies as one proxy for the beta anomaly.

Our alternative proxy for the beta anomaly is the BAB-factor, which initially was contributed by Frazzini and Pedersen (2014). They do naturally employ their own beta estimation technique, which we borrow in this analysis (β^{BAB}). Every month t, each stock i is sorted in ascending order based on their estimated beta. Subsequently, the stocks are assigned to one out of two portfolios. Stocks that have a beta estimate that is lower than the time-varying median is assigned to the low-beta portfolio. Similarly, stocks that have beta estimates that are higher than the time-varying median is assigned the high-beta portfolio. The portfolios are rebalanced every month. On any portfolio formation date in month t, let z be a $n \times 1$ vector of all beta ranks. Also, define 1_n as a $n \times 1$ vector of ones. Following the calculations that are shown by Frazzini and Pedersen (2014), we estimate the average rank $\bar{z}$ as

$$z = \begin{bmatrix} z_1 \\ z_2 \\ \dots \\ z_n \end{bmatrix} \qquad 1_n = \begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \end{bmatrix} \qquad \bar{z} = \frac{1_n \; z}{n} \tag{7}$$

In order to construct weights for each portfolio leg that sum up to 1, we use the normalizing factor k and the weights provided by Frazzini and Pedersen (2014) in the following specification

$$k = \frac{2}{1_n \, |z - \bar{z}|} \qquad \begin{cases} W_L = k(z - \bar{z})^+ \\ W_H = k(z - \bar{z})^- \end{cases} \tag{8}$$

Note that in equation (8), x^+ indicates the positive elements of a vector x, while x^- indicates the negative elements of the same vector (Frazzini & Pedersen, 2014). In order to re-scale the portfolio such that the overall portfolio beta is equal to zero, we follow the exact procedure of Frazzini and Pedersen (2014) and estimate the following specification:

$$r_t^{BAB} = \frac{1}{\beta_{t-1}^L}(r_t^L - r_{f,t}) - \frac{1}{\beta_{t-1}^H}(r_t^H - r_{f,t}) \tag{9}$$

Subscript L denotes the low-beta portfolio, H indicates the high-beta portfolio and $r_{f,t}$ is the risk-free rate in month t. The above equation is interpreted as leveraging of the low-beta leg and de-leveraging of the high-beta leg, such that both legs have a realized beta of 1. This ensures that the BAB-portfolio is market neutral, and traded using a zero cost strategy. On average, our replicated BAB-portfolio is long \$1.52 of low-beta stocks and short \$0.68 of high-beta stocks. Thus, \$0.84 is borrowed at the risk-free rate.

3.3 Long-Short Anomaly Portfolios

The calculations of anomaly characteristics in this analysis follow the work of Liu (2018), and therefore considers monthly rebalancing long-short portfolios. Each month, stocks are sorted into quintiles based on an anomaly characteristic where the strategy is long the quintile with the desired characteristic and short the corresponding undesired characteristic.

The momentum (MOM) of stock i in month t, is estimated with a formation period of six months with a one-month gap between the end of the formation period and the portfolio formation date (Liu, 2018). We use rolling windows and calculate stocks' cumulative return in every month t as in

$$MOM_{i,t} = \left(\prod_{t-7}^{t-1} (1 + r_{i,t}) \right)^{\frac{1}{6}} - 1, \tag{10}$$

where each stock i is ranked in an ascending manner based on MOM. Subsequently, every stock is assigned a quintile in month t based on their past cumulative return. The return to the long-short momentum strategy is then taken as the value-weighted return to the top quintile (winners) minus the value-weighted return to the bottom quintile (losers).

The composite equity issuance (CEI) of stock i is calculated as the log-change in market capitalization in the past twelve months minus the cumulative stock return in the same period (Daniel & Titman, 2006).

$$CEI_{i,t} = log \left(\frac{ME_{i,t}}{ME_{i,t-12}} \right) - \left(\left(\prod_{t-12}^{t} (1 + r_{i,t}) \right)^{\frac{1}{12}} - 1 \right) \tag{11}$$

Subsequently, stocks are sorted into quintiles based on CEI on an ascending basis. The return to the long-short composite equity issuance strategy is computed as the value-weighted return to the bottom quintile (low issuance activity) minus the value-weighted return to the top quintile (high issuance activity).

Return volatility (VOL) is estimated as the standard deviation of stocks' daily excess return in the past two months where a minimum of 20 observations on returns are required.

$$VOL_{i,t} = \sqrt{\frac{\sum_{t-2}^{t}(r_{i,t} - \bar{r})^2}{n-1}} \tag{12}$$

Stocks are then ranked in an ascending manner and assigned a quintile such that the top quintile includes stocks with high return volatility and the bottom quintile holds stocks with low return volatility. The return to the long-short return volatility strategy is defined as the value-weighted return to the bottom quintile (low return volatility) minus the value-weighted return to the top quintile (high return volatility).

The idiosyncratic volatility (IVOL) of a stock is estimated as the root mean squared error from a CAPM regression of the stock's excess return in the past two months requiring at least 20 observations.

$$\begin{aligned} r_{i,t} &= \hat{\alpha}_i + \hat{\beta}_{i,t}\, r_{m,t} + \hat{\epsilon}_{i,t} \\ IVOL_{i,t} &= \sqrt{var(\hat{\epsilon}_{i,t})}\,, \end{aligned} \tag{13}$$

We proceed to sort stocks in an ascending manner and assign them to quintiles, such that the bottom quintile holds stocks with low idiosyncratic volatility and the top quintile includes stocks with high idiosyncratic volatility. The return to the long-short idiosyncratic volatility strategy is subsequently taken as the value-weighted return to the bottom quintile (low idiosyncratic volatility) minus the value-weighted return top quintile (high idiosyncratic volatility).

Following the methodology of Liu (2018), Table 2 presents the summary statistics of the long-short anomaly strategies. We contribute with a presentation of the dynamics of each anomaly portfolio formed on all alternative beta estimation methods. Note that the starting year of each time series of portfolios' returns varies based on the respective formation periods of anomaly characteristics and beta estimation techniques. The second row of panel B shows that the simple average of monthly portfolio returns seems to increase with sample size. Anomaly portfolios formed on β^{SR} always have the highest average monthly returns, while portfolios formed on β^{BAB} always show the lowest monthly return.

Panel C displays each anomaly portfolio's loading in the beta anomaly. In order to estimate this relationship, we follow the approach of Liu (2018) and calculate

$$r_{p,t}^{\beta} - r_{f,t} = \hat{\alpha}_i + \hat{\gamma}(r_{b,t}^{\beta} - r_{f,t}) + \hat{\epsilon}_{i,t}, \tag{14}$$

where $r_{p,t}^{\beta}$ denotes the return to the long-short anomaly portfolio that is constructed from stocks which have an available beta estimate from beta estimation technique β in month t. $r_{b,t}^{\beta}$ is the monthly return to the portfolio formed on the same beta estimation technique β, which is long-short extreme quintiles of beta. $r_{f,t}$ is the risk-free rate in month t. As a result, γ is the loading factor of the anomaly portfolios with this proxy for the beta anomaly when we do not control for any other risk factors. Based on the coefficients and corresponding t-statistics, all anomaly strategies are loading the beta anomaly. This result holds for all pre-formation beta estimation techniques.

Panel D presents the realized CAPM betas for the anomaly portfolios. The first row shows the estimated beta for the long leg of each anomaly portfolio, while the third row shows that of the short leg. Post-formation beta estimates are very similar for each anomaly across beta estimation techniques. Thus, results in this panel indicate that which method is used to estimate pre-formation betas has little impact on the realized beta once returns are aggregated to the portfolio level. The column 'Long-short' shows the beta imbalance in each anomaly strategy.

Consistent with the findings of Liu (2018), we reveal an inherent beta imbalance in each anomaly strategy in Table 2. The post-formation beta of the long (short) leg of each anomaly portfolio is less (larger) than 1. The result is therefore negative and significant realized betas for each of the anomaly strategies. The magnitude of coefficients in Panel C is consistent with the realized long-short portfolio betas. When post-formation betas are increasingly negative, there is an increase in the strategies loading factor with the beta anomaly.

Table 2: Summary statistics of anomaly portfolios

Reported in this table are the summary statistics of the long-short anomaly portfolios. The sample period is 1927 to 2016. Monthly returns are reported in percents. Return volatility is the standard deviation of the time-series of portfolio returns in percent. Mean (min, max) holdings is the average (minimum, maximum) number of stocks in a portfolio in a month. γ is estimated in the specification $r^{\beta}_{p,t} - r_{f,t} = \hat{\alpha}_i + \hat{\gamma}(r^{\beta}_{b,t} - r_{f,t}) + \hat{\epsilon}_{p,t}$, where $r^{\beta}_{p,t}$ denotes the return to a long-short, zero-cost portfolio p in month t, and $r^{\beta}_{b,t}$ denotes the return to the beta-anomaly portfolio based on the corresponding beta estimation technique in Panel A. Panel D reports the realized portfolio betas for each anomaly strategy. The t-statistics are adjusted for heteroscedasticity using Newey and West (1987) standard errors.

	MOM			CEI			VOL			IVOL		
Panel A: Pre-formation beta estimation technique												
	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}
Panel B: Summary												
Starting Year	1927	1929	1929	1928	1929	1929	1927	1929	1929	1927	1929	1929
Monthly Return	0.528	0.348	0.395	0.221	0.200	0.218	0.466	0.368	0.416	0.524	0.356	0.407
Return Volatility	6.38	6.32	6.35	3.91	3.87	3.90	7.51	7.50	7.61	7.12	7.16	7.23
Mean Holdings	588.55	495.17	530.68	565.42	494.36	529.31	588.38	495.07	530.63	588.38	495.07	530.63
Min Holdings	101	94	91	99	94	91	101	94	91	101	94	91
Max Holdings	1379	1029	1142	1303	1029	1141	1379	1028	1142	1379	1028	1142
Panel C: Loading in the beta anomaly												
γ	0.364	0.442	0.453	0.377	0.416	0.390	0.751	0.818	0.883	0.609	0.620	0.776
t	[4.26]	[4.60]	[5.15]	[9.69]	[7.92]	[8.54]	[14.05]	[13.37]	[15.40]	[10.76]	[10.31]	[12.09]
Panel D: Realized betas												
Long	0.983	0.966	0.973	0.903	0.916	0.908	0.761	0.757	0.761	0.855	0.852	0.853
t	[20.24]	[20.21]	[19.91]	[34.37]	[31.38]	[33.68]	[53.78]	[51.11]	[52.40]	[80.38]	[71.96]	[72.15]
Short	1.410	1.400	1.404	1.326	1.319	1.324	1.500	1,463	1.477	1.404	1.372	1.383
t	[26.15]	[24.67]	[25.26]	[21.65]	[20.53]	[20.93]	[23.95]	[22.69]	[22.40]	[24.30]	[23.30]	[23.02]
Long-short	-0.428	-0.434	-0.431	-0.422	-0.403	-0.416	-0.739	-0.706	-0.716	-0.548	-0.519	-0.530
t	[-4.34]	[-4.30]	[-4.28]	[-6.65]	[-5.91]	[-6.21]	[-10.43]	[-9.75]	[-9.69]	[-8.39]	[-7.75]	[-7.78]

4 Empirical Analysis

The complete revision history of Liu (2018) includes three techniques that are designed to mitigate the beta imbalance. One of these was omitted in a revision on October 29, 2018. There are two reasons for including the omitted technique in this thesis. First, it provides intuition for how the imbalance effectively can be mitigated. Second, it sheds light on the advantages of the two alternative methods. As a result, we follow each of the three techniques that are proposed and successfully replicate the main results of Liu (2018).

Our contribution to the findings of Liu (2018) is twofold. The first aspect relates to the use of alternative pre-formation beta estimation techniques. We investigate if the choice of such impacts the explanatory power of low-beta strategies on other asset pricing anomalies. The second part of our contribution includes four alternative methods for correcting the beta imbalance. First, we contribute with a modification of one of the techniques that is proposed in Liu (2018). Second, we create double sorts on anomaly characteristics and beta in the spirit of Fama and French (1992). The third technique includes the application of leverage to individual portfolio legs, as proposed in Frazzini and Pedersen (2014). As a fourth and final technique, we perform CAPM regression tests of anomaly portfolios' returns where we add the BAB-factor as an explanatory variable.

We measure the efficiency of each beta mitigation technique by their ability to neutralize the realized beta of each anomaly strategy. Furthermore, this efficiency is conditional on how comparable the modified portfolios are to the original value-weighted portfolios. The corresponding reductions in abnormal returns are treated as results of the application of these methods. Consequently, the reductions in abnormal returns do not count towards the efficiency of each beta mitigation technique.

4.1 Correcting the Beta Imbalance: Elimination of Stocks

The evident beta imbalance in each anomaly strategy can be viewed as an overrepresentation of low-beta stocks in the long leg, and high beta stocks in the short leg. One of the methods utilized in Liu (2018) therefore involves the elimination of portfolio constituents that cause the beta imbalance. We follow this approach and replicate Table 3, where we contribute by testing the results for alternative beta estimates. Every month, low-beta stocks are removed in the long leg and high-beta stocks are removed in the short leg of each anomaly portfolio. The percentage of stocks that are removed for each portfolio vary from 35% to 70% based on the respective anomalies and pre-formation beta estimation techniques. The amount of stocks that is removed in each of the 12 strategies is however fixed for every month and equal for both portfolio legs. Furthermore, the number of eliminated stocks is chosen with the intention to achieve realized portfolio betas as close to zero as possible. The upper bound of eliminated stocks in percentage of the original portfolios is set to 70%, such that the modified portfolios still include some of the stocks with the original anomaly characteristics. Naturally, we aim to keep as many of the stocks as possible, conditional on a satisfactory realized beta.

Panel C of Table 3 provides evidence that the method of elimination reduces the anomaly portfolios' exposure to the beta anomaly. The first row displays the realized portfolio beta for the value-weighted anomaly portfolios, and the third row presents the realized beta after stocks have been eliminated. All portfolios formed on momentum and composite equity issuance show betas that are non-different from zero once the elimination method is applied. Furthermore, the elimination method works good for idiosyncratic volatility strategies, as none of the realized betas are significant at the 5% level. The realized beta for the strategies formed on return volatility is closer to zero after elimination, but they remain negative. The reason is that return volatility displays the most substantial beta imbalance of all the anomalies. It is therefore intuitive that a large percentage of stocks must be removed in order to achieve a realized beta of zero. However, the upper bound of 70% for the elimination process is binding, and therefore we do not achieve realized betas of zero for return volatility.

A comparison across pre-formation beta estimation techniques, reveals that strategies formed on β^{BAB} experience the most efficient neutralization of realized betas. This is both due to the magnitude of reductions in realized betas and the number of stocks that are necessary to eliminate in order to achieve market neutral strategies.

Panel D presents the corresponding abnormal returns to the CAPM beta estimates in Panel C. Between the original value-weighted strategies and post-elimination, reductions in abnormal returns range from 27% to 69%. The overall trend is that reductions are largest for anomaly portfolios where stocks are eliminated on the basis of beta estimates from estimation

Table 3: CAPM estimates for long-short anomaly portfolios after elimination

Reported in this table are the CAPM estimates of long-short anomaly portfolios and the corresponding t-statistics. The sample period is 1927 to 2016. In each month, value-weighted anomaly portfolios are formed from univariate sorts into quintiles of all NYSE, AMEX and NASDAQ stocks. The monthly anomaly portfolio returns are defined as the difference between value-weighted returns of extreme quintiles. Alpha estimates are denoted in percent. α_{vw} (β_{vw}) is the CAPM alpha (beta) estimate of the value-weighted long-short portfolios. α_{el} (β_{el}) is the CAPM alpha (beta) estimate of the long-short portfolios where stocks are eliminated. Δ_α (Δ_β) is the difference between α_{vw} (β_{vw}) and α_{el} (β_{el}) in percent. The t-statistics are corrected for heteroscedasticity using Newey and West (1987) standard errors.

	MOM			CEI			VOL			IVOL		
Panel A: Pre-formation beta estimation technique												
	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}
Panel B: Percentage eliminated												
El %	45%	45%	45%	45%	35%	45%	70%	70%	70%	60%	50%	60%
Panel C: β estimates												
β_{vw}	-0.428	-0.434	-0.431	-0.422	-0.403	-0.416	-0.739	-0.706	-0.716	-0.548	-0.519	-0.530
t	[-4.35]	[-4.30]	[-4.28]	[-7.09]	[-6.29]	[-6.66]	[-11.78]	[-10.98]	[-10.97]	[-9.15]	[-8.47]	[-8.55]
β_{el}	-0.049	0.001	-0.079	-0.040	0.006	-0.074	-0.180	-0.119	-0.348	-0.087	-0.053	-0.104
t	[-0.51]	[0.02]	[-0.81]	[-0.83]	[-0.12]	[-1.10]	[-3.00]	[-2.18]	[-5.60]	[-1.56]	[-0.93]	[-1.78]
Δ_β	-88.63%	-100.31%	-81.62%	-89.62%	-101.48%	-82.10%	-75.63%	-83.13%	-51.41%	-84.20%	-89.79%	-80.33%
Panel D: α estimates												
α_{vw}	0.797	0.605	0.652	0.484	0.439	0.465	0.931	0.786	0.843	0.870	0.663	0.723
t	[4.78]	[3.68]	[3.90]	[4.65]	[4.11]	[4.45]	[4.53]	[3.81]	[4.06]	[4.36]	[3.23]	[3.51]
α_{el}	0.581	0.207	0.233	0.284	0.144	0.271	0.459	0.245	0.321	0.578	0.249	0.273
t	[3.25]	[1.24]	[1.21]	[2.72]	[1.36]	[2.47]	[2.39]	[1.15]	[1.46]	[3.08]	[1.17]	[1.36]
Δ_α	-27.15%	-65.82%	-65.86%	-41.43%	-67.21%	-41.69%	-50.69%	-68.80%	-61.90%	-33.57%	-62.41%	-62.30%
Panel E: Information ratios												
IR_{vw}	0.464	0.357	0.382	0.528	0.475	0.505	0.507	0.422	0.445	0.465	0.349	0.377
t	[4.39]	[3.34]	[3.58]	[4.98]	[4.45]	[4.73]	[4.81]	[3.96]	[4.18]	[4.41]	[3.27]	[3.54]
IR_{el}	0.324	0.122	0.126	0.314	0.160	0.273	0.251	0.122	0.162	0.328	0.128	0.142
t	[3.06]	[1.14]	[1.18]	[2.96]	[1.50]	[2.56]	[2.39]	[1.14]	[1.52]	[3.11]	[1.20]	[1.33]
Δ_{ir}	-30.24%	-65.94%	-67.10%	-40.52%	-66.30%	-45.97%	-50.39%	-71.07%	-63.68%	-29.54%	-63.28%	-62.32%

technique β^{BAB}. In these cases, all four anomaly strategies experience insignificant abnormal returns. The reductions in abnormal returns are smaller when stocks are eliminated on the basis of β^{SR}, as abnormal returns still remain significant. The magnitude of reductions is somewhere in between when β^{FF} is used.

Presented in Panel E are the corresponding information ratios[7] of the anomaly portfolios before and after the elimination procedure is implemented. This panel displays reductions in information ratios that closely resemble the reductions in CAPM alpha estimates from Panel D. This provides evidence that the portfolios with eliminated stocks experience similar residual risk compared to the value-weighted portfolios. As a result, residual risk cannot explain the reductions in abnormal returns, which therefore seem to be driven by the reduced exposure to systematic risk.

Findings in Table 3 present evidence that an elimination of portfolio constituents that cause the beta imbalance is associated with significant reductions in abnormal returns. Furthermore, removing stocks based on estimates from the pre-formation beta estimation technique β^{BAB} is the most effective. This method also results in the most substantial reductions in abnormal returns.

The main advantage of the elimination technique is that it preserves the value-weighting scheme from the original anomaly portfolio constructions. The disadvantage is the process of elimination itself because the sample is reduced and therefore the composition of the portfolios change. A part of the reductions in abnormal returns may therefore be attributed to the reduction in the sample, and not to the reduction in beta exposure itself. In order to complement the disadvantages of the elimination method otherwise, we continue following the methodology of Liu (2018) and provide alternative methods that alter portfolio constituents' weights.

[7] The annualized portfolio information ratios are defined as: $IR = \sqrt{12} \cdot \frac{\alpha}{RMSE}$. The first factor seeks to annualize the information ratio, which can be interpreted as the Sharpe ratio of the portfolio after the market risk is removed (Goodwin, 1998).

4.2 Correcting the Beta Imbalance: Modification of Weights

Instead of thinking of the beta imbalance as an outnumbering of stocks with a certain characteristic, we can view them as overweighted in the respective portfolio legs. In that regard, the long leg of each anomaly strategy has an overweight of low-beta stocks, while the short legs have an overweight of high-beta stocks. Modification of individual stocks' weights within each portfolio leg can be tailor-made such that the realized beta of each leg changes in the desired direction. The goal is to change the weights of stocks within the long leg of each anomaly strategy such that the realized beta increases. Similarly, we want the realized beta of the short leg to decrease. As a result, the anomaly strategies will become more market neutral. We provide three methods that manipulate individual stocks' weights in order to mitigate the beta imbalance. The first two are replications of the techniques proposed in Liu (2018) and prior drafts. The third method is a contribution where we modify the second method in order to improve beta mitigation efficiency. We do also contribute with separate analyses of the anomaly portfolios for each beta estimation technique.

4.2.1 Weighting by Beta Ranks

We borrow the beta-rank weighting method in its entirety from Liu (2018). Because each pre-formation beta serves as an estimate for the future systematic risk of a stock, the weighted average of pre-formation betas may be interpreted as a proxy for the future realized beta of a portfolio leg. This is exploited in the beta-rank weighting method. Changing the weight of stocks based on the ranking of their beta estimates in each portfolio leg serves as a way for correcting the beta imbalance. More specifically, the weights of high-beta stocks in the long leg is increased relative to the low-beta stocks. In the short leg of each anomaly portfolio, the weights of low-beta stocks are increased relative to the high-beta stocks.

Each month t, stocks are ranked and assigned to deciles based on their pre-formation beta estimate. Stocks in the long leg are ranked in ascending order. Therefore, low-beta stocks receive low ranks relative to the high-beta stocks. In the short leg, stocks are ranked in descending order such that low-beta stocks receive higher ranks relative to high-beta stocks. Subsequently, the sum of all the stocks' ranks in each portfolio leg is calculated on a monthly basis. The return contributed by every stock to the long and short leg of the portfolio is calculated in the equation below.

$$r_{i,t}^{br} = (r_{i,t} - r_{f,t}) \frac{rank_{i,t}}{\sum_i^n rank_{i,t}}, \quad rank_{i,t} \in [1, 10] \tag{15}$$

In specification (15), $r_{i,t}$ is the return to stock i, $r_{f,t}$ is the risk-free rate and $rank_{i,t}$ is the rank of stock i in month t. Thus, $r_{i,t}^{br}$ is the return contributed to the portfolio by every stock after the weighting scheme has been applied.

In Panel B of Table 4, we observe that the exposure to beta is reduced for the anomaly portfolios based on momentum. However, the changes in realized beta for the idiosyncratic volatility strategies are marginal for instance. The corresponding reductions in beta for strategies formed on composite equity issuance and return volatility are also modest. Thus, the effectiveness of the beta-rank weighting method is rather low. It is therefore peculiar that reductions in abnormal returns are as large as Panel C suggests. For idiosyncratic volatility, these range from 74% to 105%. Based on these findings we can hardly conclude that the reductions in abnormal returns are attributed to the mitigation of exposure to the beta anomaly.

Having demonstrated the ineffectiveness of this method, we claim that this beta mitigation technique is inferior to alternative approaches. The method of weighting by beta ranks is proposed in the working paper edition of Liu (2018) and omitted in the final draft. We argue that the weighting scheme applied in this technique allows individual stock weights that are too extreme in many cases. For instance, microcap stocks may receive ten times the relative weight of much larger firms, based solely on their beta estimates. In this sense, we argue that this method runs the risk of applying an inverse value-weighting scheme in some instances that make too large of an impact to be ignored. It is therefore too far of a stretch to compare the returns to the value-weighted anomaly portfolios with the beta-ranked weighted portfolios.

The method of weighting by beta ranks does, however, provide two advantages. First, it preserves the original portfolios' constituents from the value-weighted portfolios (Liu, 2018), as opposed to the elimination method. The second advantage that is presented in the working paper version of Liu (2018) is that the beta-rank weighting method only considers the

Table 4: CAPM estimates for beta-rank weighted long-short anomaly portfolios

Reported in this table are the CAPM estimates of long-short anomaly portfolios and the corresponding t-statistics. The sample period is 1927 to 2016. In each month, value-weighted anomaly portfolios are formed from univariate sorts into quintiles of all NYSE, AMEX and NASDAQ stocks. The monthly anomaly portfolio returns are defined as the difference between value-weighted returns of extreme quintiles. Alpha estimates are denoted in percent. α_{vw} (β_{vw}) is the CAPM alpha (beta) estimate of the value-weighted long-short portfolios. α_{br} (β_{br}) is the CAPM alpha (beta) estimate of the beta-rank weighted long-short portfolios. Δ_α (Δ_β) is the difference between α_{vw} (β_{vw}) and α_{br} (β_{br}) in percent. The t-statistics are corrected for heteroscedasticity using Newey and West (1987) standard errors.

	MOM			CEI			VOL			IVOL		
Panel A: Pre-formation beta estimation technique												
	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}
Panel B: β estimates												
β_{vw}	-0.428	-0.434	-0.431	-0.422	-0.403	-0.416	-0.739	-0.706	-0.716	-0.548	-0.519	-0.530
t	[-4.35]	[-4.30]	[-4.28]	[-7.09]	[-6.29]	[-6.66]	[-11.78]	[-10.98]	[-10.97]	[-9.15]	[-8.47]	[-8.55]
β_{br}	-0.177	-0.141	-0.198	-0.354	-0.320	-0.404	-0.662	-0.611	-0.711	-0.506	-0.457	-0.561
t	[-1.71]	[-1.46]	[-2.32]	[-4.40]	[-4.19]	[-6.03]	[-7.92]	[-7.43]	[-9.10]	[-6.20]	[-5.59]	[-7.05]
Δ_β	-58.64%	-67.59%	-53.93%	-16.20%	-20.67%	-2.80%	-10.46%	-13.48%	-0.67%	-7.71%	-11.94%	5.79%
Panel C: α estimates												
α_{vw}	0.797	0.605	0.652	0.484	0.439	0.465	0.931	0.786	0.843	0.870	0.663	0.723
t	[4.78]	[3.68]	[3.90]	[4.65]	[4.11]	[4.45]	[4.53]	[3.81]	[4.06]	[4.36]	[3.23]	[3.51]
α_{br}	0.438	0.175	0.271	0.325	0.156	0.202	0.330	0.072	0.104	0.223	-0.033	0.0155
t	[2.73]	[1.10]	[1.78]	[2.56]	[1.15]	[1.68]	[1.71]	[0.35]	[0.54]	[1.15]	[-0.16]	[0.08]
Δ_α	-45.05%	-71.12%	-58.40%	-32.95%	-64.43%	-56.54%	-64.56%	-90.89%	-87.70%	-74.32%	-104.98%	-97.86%
Panel D: Information ratios												
IR_{vw}	0.464	0.357	0.382	0.528	0.475	0.505	0.507	0.422	0.445	0.465	0.349	0.377
t	[4.39]	[3.34]	[3.58]	[4.98]	[4.45]	[4.73]	[4.81]	[3.96]	[4.18]	[4.41]	[3.27]	[3.54]
IR_{br}	0.243	0.097	0.165	0.272	0.121	0.190	0.170	0.035	0.055	0.113	-0.016	0.008
t	[2.30]	[0.91]	[1.55]	[2.57]	[1.13]	[1.78]	[1.61]	[0.33]	[0.52]	[1.08]	[-0.15]	[0.08]
Δ_{ir}	-47.74%	-72.81%	-56.80%	-48.51%	-74.50%	-62.30%	-66.52%	-91.73%	-87.55%	-75.61%	-104.47%	-97.87%

information in the relative ranking of betas, rather than relying on specific values. However, we argue that noisy beta estimates are less of an issue in this thesis because we consider three different pre-formation beta estimation techniques.

4.2.2 The Method of Shifting Weights

Due to flaws in the method of weighting by beta ranks, Liu (2018) provides a different technique for correcting the beta imbalance through the weight-shifting method. The advantages of the beta-rank weighting method are conserved in this third beta mitigation technique. Instead of weighting stocks by their beta-ranks, the weight-shifting method subtracts weight from low-beta stocks in the long leg and distributes it onto the high-beta stocks in the same portfolio leg. Symmetrically, weight is shifted from high-beta stocks to low-beta stocks in the short leg. We contribute with different estimates for each of the pre-formation beta estimation techniques.

Following Liu (2018), we assign all stocks in the long leg of each anomaly strategy with a pre-formation beta estimate below 1 the subscript i. However, this is conditional on pre-formation beta estimation techniques β^{SR} or β^{BAB} is being used. When portfolios are to be modified on the basis of β^{FF}, we enforce the condition that the pre-formation beta estimate must be below 1.4 in order for a stock to be assigned subscript i. The reason is that a breakpoint of 1 proxy for an average beta estimate for all securities. The simple average of beta estimates provided by β^{FF} is 1.4 because it does not take into account the number of stocks in each size-beta portfolio. Thus, we adjust the proxy for average beta in this case to avoid skewed results. The original value-weight of low-beta stocks vw_i is the weight of the stock determined by its market capitalization. If a stock is not assigned subscript i, then it is given the subscript j. The original value-weight of high-beta stock j is therefore vw_j, determined by its market capitalization. Subsequently, low-beta stocks in the long leg of each anomaly portfolio are assigned the new weight wt_i given by

$$wt_{i,t} = vw_{i,t} - vw_{i,t} \cdot w, \quad w \in [0, 0.7], \tag{16}$$

where w is determined such that the realized beta of the long-short anomaly portfolio is close to zero. Following Liu (2018), we proceed by distributing the weight that is subtracted from each low-beta stock i ($vw_{i,t} \cdot w$) equally among the high-beta stocks in the long leg. The new weight for each high-beta stock in the long leg is therefore given in the following expression

$$wt_{j,t} = vw_{j,t} + \frac{1}{N} \sum_{i} vw_{i,t} \cdot w. \tag{17}$$

In a symmetrical, and otherwise same procedure, weight is subtracted from high-beta stocks and distributed equally among low-beta stocks in the short leg of each anomaly portfolio.

Panel B of Table 5 displays the amount of weight that is subtracted in the various anomalies across all beta estimation methods. The upper-bound of subtracted weight for each stock is set to 70% in order to keep some of the original weight that is acquired through value-weighting. Results in Panel C indicate that the weight-shifting method successfully neutralizes the realized betas for anomaly strategies formed on momentum. This is also true for composite equity issuance, except for when stocks' weights are modified based on pre-formation beta estimation technique β^{FF}. The method is less effective for anomaly portfolios formed on the volatility characteristics. We can, however, observe that market risk is effectively neutralized when stocks are removed based on β^{BAB} for the idiosyncratic volatility strategy. Overall, the weight-shifting method appears to be the most effective when stocks' weights are altered on the basis of β^{BAB}.

Panel D shows that the mitigation of low-beta exposure is related to substantial reductions in the anomaly strategies' abnormal returns. Shifting weights on the basis of beta estimation method β^{BAB} is associated with the largest reductions in abnormal returns for the momentum and composite equity issuance strategies. For the volatility related strategies, however, reductions are largest when weights are shifted on the basis of beta estimates from β^{FF}. Nonetheless, the anomaly strategies do also experience significant reductions when weights are modified on the basis of β^{SR}. Panel E displays reductions in information ratios that closely resemble the corresponding reductions in Panel D. This suggests similar residual risk in the weight-shifted portfolios compared to the original value-weighted strategies.

The advantages of the weight-shifting method are similar to those in the method of weighting by beta ranks. The original portfolio constituents are kept intact (Liu, 2018). Fur-

Table 5: CAPM estimates for long-short anomaly portfolios after shifting weights

Reported in this table are the CAPM estimates of long-short anomaly portfolios and the corresponding t-statistics. The sample period is 1927 to 2016. In each month, value-weighted anomaly portfolios are formed from univariate sorts into quintiles of all NYSE, AMEX and NASDAQ stocks. The monthly anomaly portfolio returns are defined as the difference between value-weighted returns of extreme quintiles. Alpha estimates are denoted in percent. α_{vw} (β_{vw}) is the CAPM alpha (beta) estimate of the value-weighted long-short portfolios. α_{ws} (β_{ws}) is the CAPM alpha (beta) estimate of the weight-shifted long-short portfolios. Δ_α (Δ_β) is the difference between α_{vw} (β_{vw}) and α_{ws} (β_{ws}) in percent. The t-statistics are corrected for heteroscedasticity using Newey and West (1987) standard errors.

	MOM			CEI			VOL			IVOL		
Panel A: Pre-formation beta estimation technique												
	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}
Panel B: Percentage of weight shifted												
w%	70%	70%	70%	65%	55%	70%	70%	70%	70%	70%	70%	70%
Panel C: β estimates												
β_{vw}	-0.428	-0.434	-0.431	-0.422	-0.403	-0.416	-0.739	-0.706	-0.716	-0.548	-0.519	-0.530
t	[-4.35]	[-4.30]	[-4.28]	[-7.09]	[-6.29]	[-6.66]	[-11.78]	[-10.98]	[-10.97]	[-9.15]	[-8.47]	[-8.55]
β_{ws}	-0.079	0.019	-0.043	-0.091	-0.053	-0.109	-0.424	-0.243	-0.409	-0.258	-0.074	-0.173
t	[-0.78]	[0.23]	[-0.59]	[-1.12]	[-0.95]	[-2.15]	[-6.03]	[-4.90]	[-7.26]	[-3.82]	[-1.50]	[-3.05]
Δ_β	-81.44%	-104.48%	-90.02%	-78.51%	-86.75%	-73.84%	-42.65%	-65.65%	-42.88%	-52.98%	-85.79%	-67.44%
Panel D: α estimates												
α_{vw}	0.797	0.605	0.652	0.484	0.439	0.465	0.931	0.786	0.843	0.870	0.663	0.723
t	[4.78]	[3.68]	[3.90]	[4.65]	[4.11]	[4.45]	[4.53]	[3.81]	[4.06]	[4.36]	[3.23]	[3.51]
α_{ws}	0.415	0.070	0.326	0.259	0.154	0.281	0.400	0.219	0.189	0.308	0.094	0.083
t	[2.57]	[0.47]	[2.21]	[2.55]	[1.51]	[2.68]	[2.17]	[1.21]	[1.03]	[1.68]	[0.53]	[0.45]
Δ_α	-48.00%	-88.45%	-49.93%	-46.64%	-64.82%	-39.72%	-57.09%	-72.18%	-77.63%	-64.61%	-85.85%	-88.56%
Panel E: Information ratios												
IR_{vw}	0.464	0.357	0.382	0.528	0.475	0.505	0.507	0.422	0.445	0.465	0.349	0.377
t	[4.39]	[3.34]	[3.58]	[4.98]	[4.45]	[4.73]	[4.81]	[3.96]	[4.18]	[4.41]	[3.27]	[3.54]
IR_{ws}	0.238	0.046	0.219	0.255	0.169	0.323	0.224	0.130	0.114	0.172	0.056	0.049
t	[2.25]	[0.43]	[2.06]	[2.40]	[1.58]	[3.03]	[2.13]	[1.22]	[1.07]	[1.63]	[0.52]	[0.46]
Δ_{ir}	-48.67%	-87.12%	-42.52%	-51.81%	-64.52%	-36.04%	-55.79%	-69.12%	-74.35%	-63.08%	-83.95%	-87.02%

thermore, Liu (2018) argues that this method only considers the binary outcome of whether a stock's beta estimate is above or below the cross-sectional average, rather than relying on the specific values, which can be rather noisy. However, we show that the choice of pre-formation beta estimation technique impacts the effectiveness of the weight-shifting method. It is also shown that this leads to varying reductions in abnormal returns.

The weight-shifting method suggests a weighting scheme that is not nearly as extreme as in the method of weighting by beta ranks. Liu (2018) claims that the weight shifting method preserves stock weights that more closely resemble the original value-weights in the original portfolios. We agree with this argument. However, the weighting scheme of Liu (2018) includes an equal distribution of the subtracted weight onto the high-beta stocks in the long leg, and onto the low-beta stocks in the short leg. Thus, we argue that the weight-shifting scheme would resemble the original value-weighting scheme more closely if the individual market capitalization of each stock was taken into account in the distribution of the subtracted weight. On the basis of this argument, we contribute with a modification of the weight-shifting method.

4.2.3 A Modification of the Weight-Shifting Method

In order to make a more precise comparison with the original value-weighted anomaly portfolios, we contribute to the findings of Liu (2018) with a modification of the weight-shifting method. This modification ensures that the weighting of stocks more closely resembles a value-weighting scheme after the method of shifting weights has been applied. As a result, a comparison with the original value-weighted portfolios is more accurate.

In order to make this modification, we follow the method of Liu (2018) until we distribute the weight that has been subtracted onto the high-beta stocks in the long leg, and onto the low-beta stocks in the short leg. Subsequently, we modify equation (17) such that the new weight of high-beta stocks $wt_{j,t}$ in the long leg is given by

$$wt_{j,t} = vw_{j,t} + \frac{ME_{j,t}}{\sum_j ME_{j,t}} \cdot \sum_i vw_{i,t} \cdot w. \tag{18}$$

In the above equation, $wt_{j,t}$ is the new weight of high beta stock j, $ME_{j,t}$ is the market capitalization of the high-beta stock j and $\sum_j vw_{j,t} \cdot w$ is the total weight that is subtracted from all low-beta stocks in month t. As weight is shifted, this ensures that stocks are distributed weight that is proportional to their size in the cross-section in every month t. Symmetrically, the same distribution of subtracted weight is made in the short leg, except that weight is subtracted from high beta stocks and distributed to low-beta stocks.

In order to compare the original method of weight-shifting with this modified version, we keep the reduction parameters in Panel B of Table 6 equal to the corresponding values in Table 5. Results in Panel C of Table 6 show larger reductions in realized beta for all anomalies across beta estimation methods with only one exception compared to corresponding findings in Table 5. The exception is when the momentum strategy is constructed on the basis of β^{BAB}. The difference in reduction in beta between Table 5 and Table 6 in this particular case is, however, a marginal 7%. Overall, the modified weight-shifting method appears to be more successful in neutralizing portfolio betas.

Based on the reductions in realized beta estimates from the modified weight-shifting method, the corresponding reductions in abnormal returns in Panel D of Table 6 are surprising. Nearly all long-short anomaly strategies experience lower reductions in abnormal returns compared to Table 5. The exceptions are the two composite equity issuance strategies, where weights are shifted on the basis of β^{SR} and β^{FF}. Due to the consistency in reductions in information ratios compared to abnormal returns, we cannot conclude that this results from changes in residual risk.

When we compare the modified weight-shifting method with the original that is proposed in Liu (2018), there are two advantages. First, the weighting scheme more closely resembles value-weighting because the subtracted weight is distributed according to stocks' size in the cross-section. Second, the modified weight-shifting technique more effectively mitigates the realized betas of the anomaly strategies. Therefore, we argue that a comparison of the original value-weighted portfolios and the portfolios acquired from the modified weight-shifting technique is more precise than using the original weight-shifting method. Thus, the reductions in abnormal returns in Panel D of Table 6 reflect more accurate estimates for the explanatory power of low-beta compared to the unmodified weight-shifting method and corresponding results in Table 5. It is also interesting that the reductions in abnormal returns

Table 6: CAPM estimates for long-short anomaly portfolios after modifying the weight-shifting method

Reported in this table are the CAPM estimates of long-short anomaly portfolios and the corresponding t-statistics. The sample period is 1927 to 2016. In each month, value-weighted anomaly portfolios are formed from univariate sorts into quintiles of all NYSE, AMEX and NASDAQ stocks. The monthly anomaly portfolio returns are defined as the difference between value-weighted returns of extreme quintiles. Alpha estimates are denoted in percent. α_{vw} (β_{vw}) is the CAPM alpha (beta) estimate of the value-weighted long-short portfolios. α^*_{ws} (β^*_{ws}) is the CAPM alpha (beta) estimate of the modified weight-shifted long-short portfolios. Δ_α (Δ_β) is the difference between α_{vw} (β_{vw}) and α^*_{ws} (β^*_{ws}) in percent. The t-statistics are corrected for heteroscedasticity using Newey and West (1987) standard errors.

	MOM			CEI			VOL			IVOL		
Panel A: Pre-formation beta estimation technique												
	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}
Panel B: Percentage of weight shifted												
w%	70%	70%	70%	65%	55%	70%	70%	70%	70%	70%	70%	70%
Panel C: β estimates												
β_{vw}	-0.428	-0.434	-0.431	-0.422	-0.403	-0.416	-0.739	-0.706	-0.716	-0.548	-0.519	-0.530
t	[-4.35]	[-4.30]	[-4.28]	[-7.09]	[-6.29]	[-6.66]	[-11.78]	[-10.98]	[-10.97]	[-9.15]	[-8.47]	[-8.55]
β^*_{ws}	-0.033	-0.011	-0.024	0.034	0.010	0.007	-0.288	-0.212	-0.309	-0.141	-0.043	-0.027
t	[-0.44]	[-0.12]	[-0.26]	[0.66]	[0.23]	[0.14]	[-5.48]	[-3.44]	[-4.48]	[-2.65]	[-0.68]	[-0.41]
Δ_β	-92.28%	-97.49%	-94.45%	-107.95%	-102.59%	-101.70%	-61.04%	-69.99%	-56.89%	-74.35%	-91.80%	-94.82%
Panel D: α estimates												
α_{vw}	0.797	0.605	0.652	0.484	0.439	0.465	0.931	0.786	0.843	0.870	0.663	0.723
t	[4.78]	[3.68]	[3.90]	[4.65]	[4.11]	[4.45]	[4.53]	[3.81]	[4.06]	[4.36]	[3.23]	[3.51]
α^*_{ws}	0.554	0.189	0.342	0.231	0.168	0.271	0.602	0.435	0.414	0.515	0.262	0.288
t	[3.17]	[1.11]	[2.10]	[2.40]	[1.78]	[2.40]	[3.21]	[2.06]	[1.96]	[2.78]	[1.29]	[1.42]
Δ_α	-30.50%	-68.73%	-47.48%	-52.37%	-61.81%	-41.76%	-35.33%	-44.71%	-50.87%	-40.82%	-60.54%	-60.10%
Panel E: Information ratios												
IR_{vw}	0.464	0.357	0.382	0.528	0.475	0.505	0.507	0.422	0.445	0.465	0.349	0.377
t	[4.39]	[3.34]	[3.58]	[4.98]	[4.45]	[4.73]	[4.81]	[3.96]	[4.18]	[4.41]	[3.27]	[3.54]
IR^*_{ws}	0.331	0.115	0.214	0.266	0.192	0.284	0.344	0.219	0.211	0.302	0.138	0.147
t	[3.13]	[1.07]	[2.01]	[2.51]	[1.80]	[2.67]	[3.26]	[2.06]	[1.98]	[2.87]	[1.30]	[1.38]
Δ_{ir}	-28.80%	-67.82%	-43.93%	-49.66%	-59.55%	-43.70%	-32.14%	-48.02%	-52.66%	-34.95%	-60.37%	-61.14%

from the modified weight-shifting method harmonize with the results from the elimination method in Table 3.

4.3 Correcting the Beta Imbalance: Application of Leverage

Liu (2018) discusses how the use of leverage, in the spirit of Frazzini and Pedersen (2014), to the individual anomaly portfolio legs may serve as an alternative approach for correcting the beta imbalance. He concludes that the use of leverage is unsuited for this purpose. Liu (2018) states that the application of leverage does not change the fact that the long leg of the anomaly portfolios on average hold low-beta stocks compared to the short leg. While we do not dispute such an argument, we disagree on the conclusion. Even though the portfolio legs are overrepresented or overweighted by stocks with a certain characteristic, we argue that the use of leverage is a viable alternative. The reason is that we are concerned with the effect of neutralizing beta when we aggregate returns to the portfolio level.

Based on the preceding argument, we contribute to the results of Liu (2018) through an application of the leveraging technique proposed by Frazzini and Pedersen (2014). Again, we do also contribute with separate estimations on each anomaly portfolio across beta estimation techniques. In order to perform this technique, we apply leverage to the long leg of the original value-weighted anomaly strategies. Simultaneously, we de-lever the short leg and fund the difference by borrowing at the risk-free rate. The returns to each of the new anomaly portfolio strategies are defined as

$$r_{p,t} = \frac{1}{\beta^L_{t-1}}(r^L_{p,t} - r_{f,t}) - \frac{1}{\beta^S_{t-1}}(r^S_{p,t} - r_{f,t}). \tag{19}$$

In equation (19), β^L is the realized CAPM beta of the long leg in the original value-weighted portfolio, while β^S is the corresponding beta of the short leg. $r^L_{p,t}$ and $r^S_{p,t}$ are the respective time series returns of the long and short leg in each original value-weighted portfolio.

Panel B in Table 7 shows that the realized beta is perfectly neutralized for all anomaly portfolios across beta estimation methods when the leverage technique is applied. Panel C displays the corresponding reductions in abnormal returns to each strategy, which provides contradictory evidence to the impact of the choice of beta estimation technique compared

Table 7: CAPM estimates for leveraged long-short anomaly portfolios

Reported in this table are the CAPM estimates of long-short anomaly portfolios and the corresponding t-statistics. The sample period is 1927 to 2016. In each month, value-weighted anomaly portfolios are formed from univariate sorts into quintiles of all NYSE, AMEX and NASDAQ stocks. The monthly anomaly portfolio returns are defined as the difference between value-weighted returns of extreme quintiles. Alpha estimates are denoted in percent. α_{vw} (β_{vw}) is the CAPM alpha (beta) estimate of the value-weighted long-short portfolios. α_l (β_l) is the CAPM alpha (beta) estimate of the leveraged long-short portfolios. Δ_α (Δ_β) is the difference between α_{vw} (β_{vw}) and α_l (β_l) in percent. The t-statistics are corrected for heteroscedasticity using Newey and West (1987) standard errors.

	MOM			CEI			VOL			IVOL		
Panel A: Beta measure												
	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}
Panel B: β estimates												
β_{vw}	-0.428	-0.434	-0.431	-0.422	-0.403	-0.416	-0.739	-0.706	-0.716	-0.548	-0.519	-0.530
t	[-4.35]	[-4.30]	[-4.28]	[-7.09]	[-6.29]	[-6.66]	[-11.78]	[-10.98]	[-10.97]	[-9.15]	[-8.47]	[-8.55]
β_l	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.001	0.001	0.001
t	[0.01]	[0.01]	[0.01]	[0.02]	[0.02]	[0.02]	[0.04]	[0.04]	[0.04]	[0.03]	[0.03]	[0.03]
Δ_β	-100.22%	-100.24%	-100.23%	-100.26%	-100.27%	-100.26%	-100.27%	-100.29%	-100.28%	-100.26%	-100.27%	-100.27%
Panel C: α estimates												
α_{vw}	0.797	0.605	0.652	0.484	0.439	0.465	0.931	0.786	0.843	0.870	0.663	0.723
t	[4.78]	[3.68]	[3.90]	[4.65]	[4.11]	[4.45]	[4.53]	[3.81]	[4.06]	[4.36]	[3.23]	[3.51]
α_l	0.557	0.417	0.451	0.346	0.311	0.333	0.557	0.470	0.505	0.550	0.415	0.455
t	[4.05]	[3.02]	[3.23]	[3.61]	[3.18]	[3.47]	[3.58]	[2.97]	[3.20]	[3.60]	[2.59]	[2.85]
Δ_α	-30.10%	-31.01%	-30.78%	-28.55%	-29.05%	-28.48%	-40.17%	-40.23%	-40.03%	-36.82%	-37.43%	-37.09%
Panel D: Information ratios												
IR_{vw}	0.464	0.357	0.382	0.528	0.475	0.505	0.507	0.422	0.445	0.465	0.349	0.377
t	[4.39]	[3.34]	[3.58]	[4.98]	[4.45]	[4.73]	[4.81]	[3.96]	[4.18]	[4.41]	[3.27]	[3.54]
IR_l	0.392	0.293	0.316	0.418	0.375	0.401	0.398	0.326	0.348	0.383	0.279	0.306
t	[3.71]	[2.74]	[2.97]	[3.94]	[3.51]	[3.76]	[3.75]	[3.05]	[3.27]	[3.63]	[2.61]	[2.87]
Δ_{ir}	-15.50%	-17.81%	-17.15%	-20.86%	-21.06%	-20.53%	-21.47%	-22.70%	-21.82%	-17.62%	-19.95%	-18.91%

to prior tables. There is minimal variation among reductions in abnormal returns across beta estimation methods for each anomaly strategy. Furthermore, the reductions in abnormal returns seem to be driven by the realized portfolio beta of the original value-weighted portfolios. For instance, because strategies formed on return volatility have the most negative portfolio beta in the original value-weighted portfolios, these anomalies experience the largest reductions in abnormal returns once the realized betas are neutralized. This finding is consistent across all anomalies. Results in Panel D of Table 7 show that the reductions in information ratios only amount to about half of the reduction in abnormal returns for each strategy, but sometimes more. Nonetheless, the reductions in information ratios for the levered anomaly strategies are lower than the reduction in abnormal returns. The reason is that the residual risk for each strategy decreases once leverage is applied.

The advantages of the leverage technique are that the relative weight within each portfolio is maintained and that it is highly effective in neutralizing realized beta. However, as Liu (2018) argues, the fact that the portfolio is leveraged does not change the fact that the portfolios still on average hold low-beta stocks in the long leg relative to the short leg. An additional disadvantage of the leverage technique is reflected in the change in information ratios relative to the change in abnormal returns. Because the residual risk is significantly lower for the leveraged portfolios, one could argue that they are fundamentally different from the original anomaly portfolios. This contaminates a potential comparison of these strategies.

4.4 Double Sorts on Beta and Anomaly Characteristic

A double sort is a common technique used to study the change in one variable while holding another constant. We follow the methodology of Fama and French (1992) and create double sorts on anomaly characteristic and beta in order to study the abnormal returns of each strategy. Liu (2018) presents a challenge concerning the use of double sorts for this particular problem. Even though we sort on beta and the anomaly characteristic, there is still significant variation in beta within each beta quintile. This variation makes it harder to decipher the results in the double sorts.

We contribute with a potential solution to the problem proposed in Liu (2018). In order to do that, we plot the abnormal return and realized beta from each combination of the anomaly strategies in a diagram. This allows us to study the overall trend that comes about from the neutralization of portfolio betas. Additionally, we make the usual contribution that involves a presentation of our findings for each beta estimation technique.

In particular, we assign the entire sample of stocks to quintiles based on an ascending sort on pre-formation betas in every month. Subsequently, we assign each stock within each beta quintile into new quintiles based on an anomaly characteristic. Thus, the intersections form 25 portfolios. We then proceed to estimate the value-weighted return to each portfolio, in order to perform a CAPM regression for each of the 25 portfolios. Panel (a) in Table 8 presents the time series average of pre-ranking beta estimates for a double sort on momentum that is based on beta estimation technique β^{SR}. In order to create double sorts on realized beta and abnormal returns, we estimate a CAPM regression where the time series return to each portfolio is regressed on the market. Panel (b) of Table 8 displays the realized beta estimate for each of the 25 portfolios, while Panel (c) shows the abnormal returns to the corresponding portfolios.

Consistent with the findings of Liu (2018), the estimates in Panel (b) show significant variation within each quintile of realized beta. This illustrates the challenges relating to a direct interpretation of the findings in the double sorts. However, if realized beta had no impact on a long-short momentum strategy, then the abnormal returns from buying low-beta winners and selling high-beta losers should yield about the same abnormal returns as buying high-beta winners and selling low-beta losers. Based on Panel (c), these strategies would yield monthly abnormal returns of 1.558% and 0.276%, respectively. The realized portfolio betas for the same strategies based on Panel (b) would be -1.313 and 0.663, respectively. In order to illustrate this more precisely, we plot the abnormal returns and realized betas for all 25 possible combinations of momentum strategies in Figure (d) in Table 8.

In order to construct Figure (d), we define A as the 5×5 matrix containing the abnormal returns from the double sort on momentum and beta. Furthermore, let C be the corresponding 5×5 matrix containing the post-formation beta estimates from the same double sort. Also, simply define $k = 1$ and $p = 1$.

Table 8: Double sort on momentum and beta constructed with β^{SR}

Reported in the panels below are double sorts on momentum and beta. The sample period is 1927 to 2016 Each month, stocks are sorted into quintiles based on their pre-formation beta estimate. Subsequently, stocks within each beta quintile are assigned into quintiles based on their prior six-month cumulative return with a one-month gap between the end of the measurement period and the portfolio formation date. Intersections form 25 portfolios for every month. The monthly return is defined as the value-weighted return of each portfolio. Panel (a) displays the time-series average of pre-formation beta estimates for each of the 25 portfolios. Panel (b) reports the time-series estimate of post-formation beta for the same 25 portfolios. Panel (c) displays the corresponding time series abnormal return of the 25 portfolios in percent. Figure (d) displays a plot where each observation represents a combination of long-short momentum strategy from extreme momentum quintiles. Thus, each observation represents a strategy that is long a winner quintile (5) and short a loser quintile (1). This returns a total of 25 combinations of momentum strategies. The realized beta of each long-short strategy is reflected on the x-axis, while the corresponding abnormal return for the same strategy is reflected on the y-axis.

(a) Pre-formation Beta

Quintiles	β^1	β^2	β^3	β^4	β^5
1 (Losers)	0.291	0.714	1.041	1.409	2.072
2	0.322	0.712	1.034	1.397	1.984
3	0.334	0.711	1.031	1.389	1.946
4	0.338	0.713	1.031	1.389	1.948
5 (Winners)	0.310	0.715	1.038	1.403	2.026

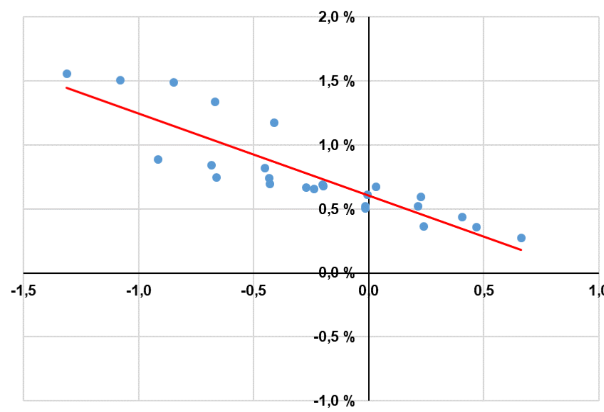
(b) Realized Beta

Quintiles	β^1	β^2	β^3	β^4	β^5
1 (Losers)	0.848	1.042	1.271	1.525	1.923
2	0.742	0.907	1.129	1.425	1.773
3	0.605	0.829	1.084	1.355	1.695
4	0.568	0.827	1.020	1.338	1.601
5 (Winners)	0.610	0.842	1.074	1.255	1.511

(c) Abnormal Returns

Quintiles	β^1	β^2	β^3	β^4	β^5
1 (Losers)	-0.276	-0.359	-0.362	-0.507	-1.175
2	0.067	0.085	-0.058	-0.295	-0.577
3	0.131	0.151	0.174	-0.018	-0.292
4	0.177	0.105	0.209	-0.078	-0.263
5 (Winners)	0.383	0.333	0.316	0.160	0.000

(d) Combinations of Strategies



$$A = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{13} & \alpha_{15} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \alpha_{25} \\ \dots & \dots & \dots & \dots & \dots \\ \alpha_{51} & \alpha_{52} & \alpha_{53} & \alpha_{54} & \alpha_{55} \end{bmatrix} \quad (20) \qquad C = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{13} & \beta_{15} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} & \beta_{25} \\ \dots & \dots & \dots & \dots & \dots \\ \beta_{51} & \beta_{52} & \beta_{53} & \beta_{54} & \beta_{55} \end{bmatrix} \quad (21)$$

Subsequently, let each possible anomaly strategy from buying past winners and selling past losers be defined as

$$for\ i \in [1,5]\ and\ j \in [1,5] \qquad \begin{cases} B(k) = A(5,i) - A(1,j), & k = k+1 \\ D(p) = C(5,i) - C(1,j), & p = p+1 \end{cases} \tag{22}$$

Then, $B(k)$ and $D(p)$ are 1×25 matrices that include the corresponding abnormal returns and post-formation beta estimates for all 25 long-short momentum strategies. Lastly, we define Q as a 2×25 matrix consisting of matrices $B(k)$ and $D(p)$, where each column refers to a coordinate in the two-dimensional system in Figure (d). Realized portfolio betas are reflected along the x-axis, while abnormal returns are reflected along the y-axis.

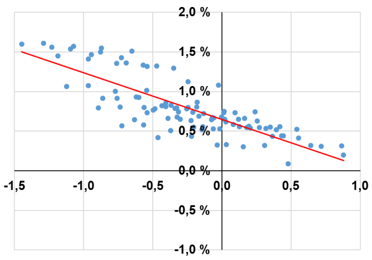
Figure (d) displays a clear trend in long-short momentum strategy abnormal returns as the realized portfolio beta changes. The diagram predicts abnormal returns in the magnitude of about 0.6% per month once the portfolio beta is neutralized. The trend is very similar for the other asset pricing anomalies we study. In order to present these results, we plot the corresponding diagrams for all four anomalies across all beta estimation techniques in Figure 2. In an attempt to increase sample frequency in the diagrams, we extend the double sorts to include deciles on beta instead of quintiles. All else equal, we therefore construct 5×10 double sorts and otherwise plot the abnormal returns against realized beta estimates in the same fashion as in Figure (d) in Table 8.

In Figure 2, rows of diagrams display individual anomalies, while columns represent beta estimation methods. All of the twelve diagrams present evidence that abnormal returns are reduced as the realized portfolio betas are neutralized. The diagrams also present some additional interesting findings. First, the intercept on the y-axis is consistently lowest for strategies formed on β^{BAB}. However, abnormal returns for these strategies are also the lowest

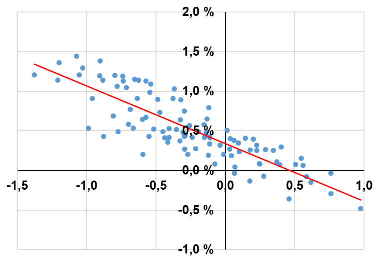
Figure 2: Double sorts on anomaly characteristic and realized portfolio

Each figure displays the realized beta and abnormal return from being long-short extreme anomaly quintiles. The sample period is 1927 to 2016. Each month, stocks are sorted into deciles based on their pre-formation beta estimate. Subsequently, stocks within each beta decile are sorted into quintiles based on an anomaly characteristic. Intersections form 50 portfolios for every month. The monthly return is defined as the value-weighted return for each portfolio. These are used to estimate CAPM abnormal returns and realized beta for each of the 50 portfolios. Each observation represents a strategy that is long the desired anomaly quintile and short a corresponding undesired quintile. This returns a total of 100 combinations for each anomaly strategy. The realized beta of each long-short strategy is reflected on the x-axis, while the corresponding abnormal return for the same strategy is reflected on the y-axis. Columns of figures display the pre-formation beta estimation technique that is used in the construction of portfolios while rows show the individual anomalies.

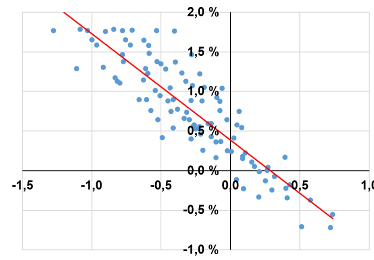
(a) MOM (β^{SR})



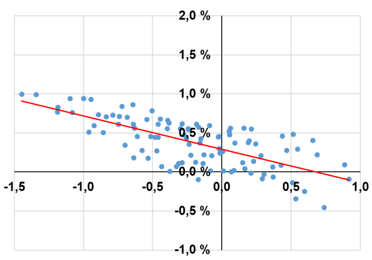
(b) MOM (β^{BAB})



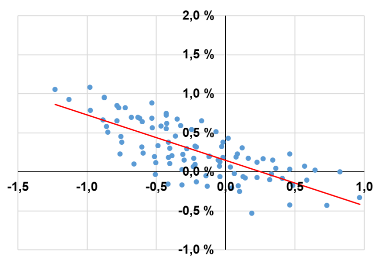
(c) MOM (β^{FF})



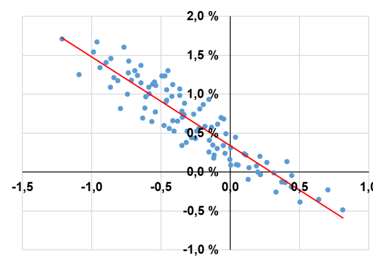
(d) CEI (β^{SR})



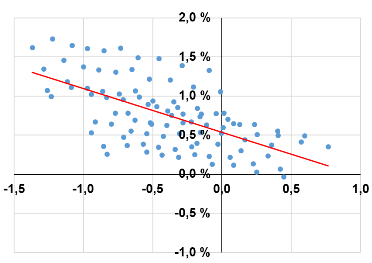
(e) CEI (β^{BAB})



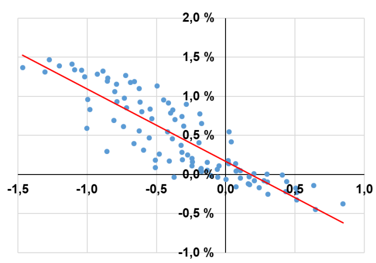
(f) CEI (β^{FF})



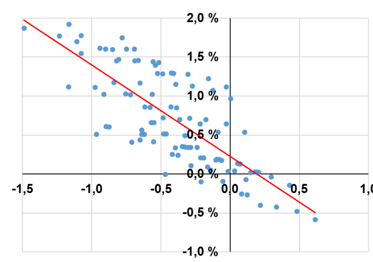
(g) VOL (β^{SR})



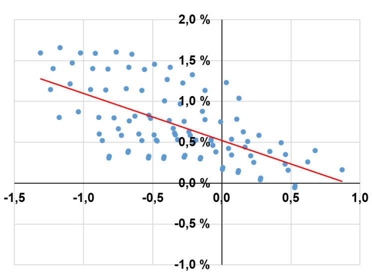
(h) VOL (β^{BAB})



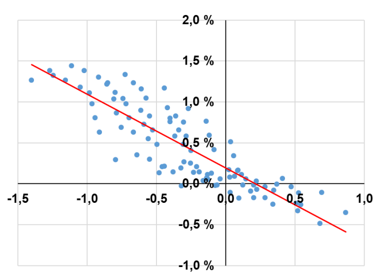
(i) VOL (β^{FF})



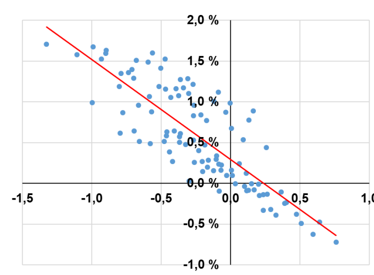
(j) IVOL (β^{SR})



(k) IVOL (β^{BAB})



(l) IVOL (β^{FF})



initially. We do therefore turn our attention to the slopes. These are consistently the steepest for β^{FF}, and the flattest for strategies formed on β^{SR}. Coupled with the large errors for the diagrams on VOL and IVOL based on β^{SR}, findings in Figure 2 provide evidence that β^{SR} is less efficient in explaining abnormal returns compared to the alternative beta estimation techniques.

The prevalent advantage of the double sort technique in general is that it also considers the construction of portfolios based on quintiles of characteristics. Thus, the portfolios that are created in the double sorts are essentially the same as the original portfolios, except that they are decomposed into portfolios consisting of fewer stocks based on beta estimates. The double sort method is therefore in harmony with our preceding analysis and presents evidence that is consistent with our prior findings. This is related to the choice of beta estimation technique, and the impact it has on the explanatory power of low-beta with regards to other cross-sectional anomalies. Figure 2 provides evidence that β^{BAB} and β^{FF} most efficiently explain this effect. An additional and important advantage of the double sort technique is that it maintains the value-weighting scheme that is utilized in the original long-short anomaly portfolios.

4.5 Regression Tests: Controlling for the Beta Anomaly

Our final contribution to the findings of Liu (2018) includes regression tests of the anomaly strategies. We compare the abnormal return predicted by the CAPM against the abnormal returns of the same model once we also control for the beta anomaly. Liu (2018) argues two reasons that such tests are unsuited in this particular context. First, a CAPM specification where a low-beta strategy that is long-short extreme quintiles of beta is used as a proxy for the beta anomaly is shown to suffer from multicollinearity. Furthermore, he argues that using the beta anomaly as an explanatory variable implies that it proxies for a systematic risk factor. In Liu (2018) beta is considered as stock characteristics, while the relevant regression specification is not particularly well suited for adjustments in characteristics.

We confirm the concern of Liu (2018) by finding the exact same correlation coefficient of -0.77 between his proxy for the beta anomaly and the market portfolio. In order to address this problem, we use the BAB-factor as a proxy for the beta anomaly instead of a strategy

Table 9: CAPM estimates for value-weighted long-short anomaly portfolios while controlling for the BAB-factor

Reported in this table are regression estimates and the corresponding t-statistics for each of the original value-weighted long-short anomaly portfolios. The sample period is 1927 to 2016. Each panel displays which pre-formation beta estimation method has been used in the formation of the long-short anomaly portfolios. The leftmost column displays the relevant model that is estimated in addition to the difference in abnormal returns between the two models (Δ_α). Alpha estimates are shown in percent. The remaining columns represent the individual long-short anomaly portfolios. CAPM is estimated in the specification $r_{i,t} - r_{f,t} = \hat{\alpha}_i + \hat{\beta}(r_{m,t} - r_{f,t}) + \hat{\epsilon}_{i,t}$, while CAPM + BAB is estimated as $r_{i,t} - r_{f,t} = \hat{\alpha}_i + \hat{\beta}(r_{m,t} - r_{f,t}) + BAB \cdot r_{b,t} + \hat{\epsilon}_{i,t}$. In the preceding specifications, $r_{i,t}$ is the return to long-short anomaly portfolio i, $r_{f,t}$ is the risk-free rate, $r_{m,t}$ is the return to the market portfolio and $r_{b,t}$ is the return to the replicated betting against beta portfolio in month t. The t-statistics are adjusted for heteroscedasticity using Newey and West (1987) standard errors.

	MOM			CEI			VOL			IVOL		
Panel A: Coefficient	α	β	BAB	α	β	BAB	α	β	BAB	α	β	BAB
Panel B: β^{SR}												
CAPM	0.797	-0.428		0.484	-0.422		0.931	-0.739		0.870	-0.548	
t	[4.78]	[-4.35]		[4.65]	[-7.09]		[4.53]	[-11.78]		[4.36]	[-9.15]	
CAPM + BAB	0.423	-0.397	0.430	0.264	-0.388	0.313	0.484	-0.676	0.606	0.501	-0.500	0.495
t	[1.85]	[-5.67]	[2.69]	[2.52]	[-8.45]	[6.66]	[2.41]	[-9.45]	[5.84]	[2.30]	[-7.36]	[3.73]
Δ_α	-46.95%			-45.48%			-48.03%			-42.44%		
Panel C: β^{BAB}												
CAPM	0.605	-0.434		0.439	-0.403		0.786	-0.706		0.663	-0.519	
t	[3.68]	[-4.30]		[4.11]	[-6.29]		[3.81]	[-10.98]	[3.23]	[-8.47]		
CAPM + BAB	0.312	-0.388	0.416	0.230	-0.371	0.297	0.401	-0.646	0.548	0.367	-0.473	0.420
t	[1.41]	[-5.29]	[2.76]	[2.16]	[-7.42]	[6.26]	[1.99]	[-9.09]	[5.59]	[1.66]	[-7.07]	[3.45]
Δ_α	-48.40%			-47.55%			-49.09%			-44.59%		
Panel D: β^{FF}												
CAPM	0.652	-0.431		0.465	-0.416		0.843	-0.716		0.723	-0.530	
t	[3.90]	[-4.28]		[4.45]	[-6.66]		[4.06]	[-10.97]		[3.51]	[-8.55]	
CAPM + BAB	0.328	-0.390	0.420	0.248	-0.381	0.308	0.420	-0.655	0.580	0.392	-0.484	0.448
t	[1.46]	[-5.36]	[2.74]	[2.37]	[-7.94]	[6.50]	[2.04]	[-8.89]	[5.49]	[1.75]	[-7.05]	[3.48]
Δ_α	-49.62%			-46.74%			-50.15%			-45.71%		

that is long-short extreme quintiles of beta. We find that the correlation coefficient between the BAB-factor and the market portfolio return only amounts to -0.15. Additionally, a regression of the BAB-factor onto the market return results in an R^2 of only 2%, such that the variance inflation factor is close to 1. Therefore, we find no evidence of multicollinearity.

The first row in each panel of Table 9 displays the coefficients from ordinary CAPM regressions of the time-series of returns to the original value-weighted portfolios on the excess market return. Each panel indicates which beta estimation technique that has been used in the formation of the anomaly strategies. The second to bottom row in each panel display regression coefficients for the same specification when we include the BAB-factor as an explanatory variable. Our findings suggest, unconditional on anomaly or beta estimation technique, that all strategies are loading significantly in the BAB-factor. This finding is robust to alternative asset pricing models, including the Fama and French 3-factor and 5-factor models. Furthermore, the inclusion of the BAB-factor is related to significant reductions in abnormal returns in the range of 42% to 50%. These reductions display little variation across beta estimation methods for individual anomalies. This provides evidence to the notion that the choice of pre-formation beta estimation technique has little effect on the explanatory power of low-beta on other cross-sectional anomalies.

Regression tests have a general advantage in that they can be tailor-made to analyze very specific problems. For instance, we can extend these regressions to show how the factor-loadings change after beta mitigation techniques have been applied to the original value-weighted portfolios. This exercise seeks to address the second concern of Liu (2018), that our regression specification is not well-suited for adjustments in characteristics. In order to do this, we estimate separate CAPM regressions after the elimination method and weight-shifting methods have been applied to the original anomaly strategies for all beta estimation techniques. Our findings suggest that none of the strategies are loading in the BAB-factor once any of the three methods have been successful in mitigating beta exposure. Furthermore, the abnormal returns to the modified portfolios display little variation when we add the BAB-factor as an explanatory variable, conditional on beta being neutralized. These results are enclosed in appendix A.

4.6 Robustness Tests: Common Sample of Stocks

One particular concern relating to the use of alternative pre-formation beta estimation techniques may be the different sample sizes that are acquired. We require available estimates on both beta and anomaly characteristics in order for a stock in a given month to be included in our sample. Because β^{SR} consistently results in the largest sample, this serves as an explanation for why strategies formed on this technique always experience the largest abnormal returns. Studying the changes in percent naturally serves as a way to mitigate the potential bias that occurs from different sample sizes. In order to increase the validity of our findings, we include a robustness test where we use a common sample of stocks.

Thus far, we have demonstrated that the method of elimination and the modified weight-shifting method are superior to the beta-rank weighting technique and the original weight-shifting method. Both of the superior methods indicate that the explanatory power of the beta anomaly varies across pre-formation beta estimation techniques. This finding is also true for the double sorts. In order to test the validity of these findings, we replicate the elimination method, modified weight-shifting technique and double sorts where we use a common sample of stocks. Thus, we require that a stock must have an available estimate of beta from each pre-formation beta estimation technique in a given month, in order for it to be included in the sample. All robustness tests indicate that the explanatory power of the beta anomaly still varies across beta estimation techniques. Furthermore, the tests confirm our prior findings. The overall trend is that reductions in abnormal returns are the largest when β^{BAB} is used. These results are enclosed in appendix B.

5 Conclusion

In this thesis, we apply the techniques that are proposed in Liu (2018) in order to mitigate long-short anomaly portfolios' exposure to the beta anomaly. These include the elimination method, the beta-rank weighting technique and the method of weight-shifting. Furthermore, we provide contributions that we argue to improve and add to the methodology of Liu (2018). These include the modification of the weight-shifting method, application of leverage, double sorts and regression tests where we control for the beta anomaly. We exploit all of these techniques with the aim of investigating if the low-beta anomaly can explain the abnormal returns to other documented asset pricing anomalies. In addition, we contribute by testing if alternative pre-formation beta estimation techniques impact the explanatory power of the beta anomaly.

Consistent with the findings of Liu (2018), we find that all methods mitigate the long-short anomaly portfolios' exposure to the beta anomaly to some extent. The degree to which realized beta is neutralized, and the corresponding reductions in abnormal returns, do however vary across beta mitigation methods. While we show that the beta-rank weighting method embeds flaws and thus displays mixed results, the other techniques are quite successful. Realized portfolio betas are non-different from zero after the elimination, and both weight-shifting methods are applied for most anomaly strategies. They do however struggle in mitigating the beta exposure for the return volatility portfolios. The elimination and modified weight-shifting techniques maintain the value-weighting scheme from the original anomaly strategies. Their advantages complement each other as one eliminates stocks and the other changes the weights of the original portfolio constituents. Both techniques result in similar reductions in abnormal returns. The leveraging technique is the most efficient for neutralizing portfolio betas and show reductions in abnormal returns that are consistent with the magnitude of which beta exposure is reduced. The double sorts and regression tests also present compelling evidence that the beta anomaly can explain part of the abnormal returns to other asset pricing anomalies.

The inclusion of three alternative pre-formation beta estimation methods allows us to present how the choice such impacts the preceding results. The methods of elimination, modified weight-shifting as well as double sorts show that mitigation of beta exposure and corresponding reductions in abnormal returns vary across beta estimation techniques. Overall, these beta mitigation methods attribute the highest explanatory power to the beta anomaly when beta exposure is mitigated on the basis of β^{BAB}. This is due to the overall superior reductions in realized betas. The leveraging technique and regression test do however present evidence that choice of pre-formation beta estimation technique has little to no impact on the explanatory of the beta anomaly.

We find compelling evidence that part of the abnormal returns to long-short anomaly strategies on momentum, composite equity issuance, return volatility and idiosyncratic volatility result from their exposure to the low-beta anomaly. We conclude that the magnitude of this explanatory power varies both on the basis of pre-formation beta estimation technique and beta mitigation method.

References

Ang, A., Hodrick, R. J., Xing, Y., & Zhang, X. (2006). The cross-section of volatility and expected returns. *The Journal of Finance*, *61*(1), 259–299.

Daniel, K., & Titman, S. (2006). Market reactions to tangible and intangible information. *The Journal of Finance*, *61*(4), 1605–1643.

Dimson, E. (1979). Risk measurement when shares are subject to infrequent trading. *Journal of Financial Economics*, *7*(2), 197–226.

Fama, E. F., & French, K. R. (1992). The cross-section of expected stock returns. *the Journal of Finance*, *47*(2), 427–465.

Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial economics*, *33*(1), 3–56.

Fama, E. F., & French, K. R. (2016). Dissecting anomalies with a five-factor model. *The Review of Financial Studies*, *29*(1), 69–103.

Frazzini, A., & Pedersen, L. H. (2014). Betting against beta. *Journal of Financial Economics*, *111*(1), 1–25.

Goodwin, T. H. (1998). The information ratio. *Financial Analysts Journal*, 34–43.

Jegadeesh, N., & Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of finance*, *48*(1), 65–91.

Jensen, M. C., Black, F., & Scholes, M. S. (1972). The capital asset pricing model: Some empirical tests.

Lintner, J. (1965). Security prices, risk, and maximal gains from diversification. *The journal of finance*, *20*(4), 587–615.

Liu, R. (2018). Asset pricing anomalies and the low-risk puzzle. *Available at SSRN 3258015*.

Markowitz, H. (1952). Portfolio selection. *The journal of finance*, *7*(1), 77–91.

Merton, R. C. (1980). On estimating the expected return on the market: An exploratory investigation. *Journal of financial economics*, *8*(4), 323–361.

Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica: Journal of the econometric society*, 768–783.

Newey, W. K., & West, K. D. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, *55*(3), 703–708.

Roll, R. (1977). A critique of the asset pricing theory's tests part i: On past and potential testability of the theory. *Journal of financial economics*, *4*(2), 129–176.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, *19*(3), 425–442.

Vasicek, O. A. (1973). A note on using cross-sectional information in bayesian estimation of security betas. *The Journal of Finance*, *28*(5), 1233–1239.

Appendices

A CAPM Regressions Including the BAB-factor

Table 10: CAPM estimates for long-short anomaly portfolios based on β^{SR} while controlling for the BAB-factor

Reported in this table are regression estimates and the corresponding t-statistics for long-short anomaly portfolios constructed on pre-formation beta estimation technique β^{SR}. The sample period is 1927 to 2016. Panel B through E reports which long-short portfolios' excess return is used as the dependent variable. The leftmost column displays the relevant model that is estimated. The remaining columns represent the coefficients that are estimated for the various anomaly portfolios. Alpha estimates are shown in percent. CAPM is estimated in the specification $r_{i,t} - r_{f,t} = \hat{\alpha}_i + \hat{\beta}(r_{m,t} - r_{f,t}) + \hat{\epsilon}_{i,t}$, while CAPM + BAB is estimated as $r_{i,t} - r_{f,t} = \hat{\alpha}_i + \hat{\beta}(r_{m,t} - r_{f,t}) + BAB \cdot r_{b,t} + \hat{\epsilon}_{i,t}$. In the preceding specifications, $r_{i,t}$ is the return to long-short anomaly portfolio i, $r_{f,t}$ is the risk-free rate, $r_{m,t}$ is the return to the market portfolio and $r_{b,t}$ is the return to the replicated betting against beta portfolio in month t. The t-statistics are adjusted for heteroscedasticity using Newey and West (1987) standard errors.

	MOM			CEI			VOL			IVOL		
Panel A: Coefficient	α	β	BAB	α	β	BAB	α	β	BAB	α	β	BAB
Panel B: Value-weighted												
CAPM	0.797	-0.428		0.484	-0.422		0.931	-0.739		0.870	-0.548	
t	[4.78]	[-4.35]		[4.65]	[-7.09]		[4.53]	[-11.78]		[4.36]	[-9.15]	
CAPM + BAB	0.423	-0.397	0.430	0.264	-0.388	0.313	0.484	-0.676	0.606	0.501	-0.500	0.495
t	[1.85]	[-5.67]	[2.69]	[2.52]	[-8.45]	[6.66]	[2.41]	[-9.45]	[5.84]	[2.30]	[-7.36]	[3.73]
Panel C: Eliminated												
CAPM	0.581	-0.049		0.284	-0.044		0.459	-0.180		0.578	-0.087	
t	[3.25]	[-0.51]		[2.72]	[-0.83]		[2.39]	[-3.00]		[3.08]	[-1.56]	
CAPM + BAB	0.429	-0.046	0.153	0.283	-0.039	0.023	0.332	-0.164	0.179	0.513	-0.081	0.075
t	[1.73]	[-0.58]	[0.89]	[2.62]	[-0.76]	[0.55]	[1.55]	[-2.74]	[1.78]	[2.37]	[-1.48]	[0.64]
Panel D: Weight-shifted (original)												
CAPM	0.415	-0.079		0.259	-0.091		0.400	-0.424		0.308	-0.258	
t	[2.57]	[-0.78]		[2.55]	[-0.091]		[2.17]	[-6.03]		[1.68]	[-3.82]	
CAPM + BAB	0.267	-0.078	0.130	0.265	-0.089	0.004	0.088	-0.380	0.428	0.109	-0.232	0.265
t	[1.19]	[-0.87]	[0.130]	[2.24]	[-1.13]	[0.07]	[0.43]	[-6.13]	[3.81]	[0.51]	[-3.69]	[2.08]
Panel E: Weight-shifted (modified)												
CAPM	0.554	-0.330		0.231	0.034		0.602	-0.288		0.515	-0.141	
t	[3.17]	[-0.44]		[2.40]	[0.66]		[3.21]	[-5.48]		[2.78]	[-2.65]	
CAPM + BAB	0.458	-0.042	0.053	0.293	0.030	-0.068	0.370	-0.254	0.329	0.428	-0.131	0.114
t	[2.05]	[-0.62]	[0.39]	[2.97]	[0.59]	[-1.72]	[1.77]	[-4.96]	[3.31]	[2.04]	[-2.48]	[1.02]

Table 11: CAPM estimates for long-short anomaly portfolios based on β^{BAB} while controlling for the BAB-factor

Reported in this table are regression estimates and the corresponding t-statistics for long-short anomaly portfolios constructed on pre-formation beta estimation technique β^{BAB}. The sample period is 1927 to 2016. Panel B through E reports which long-short portfolios' excess return is used as the dependent variable. The leftmost column displays the relevant model that is estimated. The remaining columns represent the coefficients that are estimated for the various anomaly portfolios. Alpha estimates are shown in percent. CAPM is estimated in the specification $r_{i,t} - r_{f,t} = \hat{\alpha}_i + \hat{\beta}(r_{m,t} - r_{f,t}) + \hat{\epsilon}_{i,t}$, while CAPM + BAB is estimated as $r_{i,t} - r_{f,t} = \hat{\alpha}_i + \hat{\beta}(r_{m,t} - r_{f,t}) + BAB \cdot r_{b,t} + \hat{\epsilon}_{i,t}$. In the preceding specifications, $r_{i,t}$ is the return to long-short anomaly portfolio i, $r_{f,t}$ is the risk-free rate, $r_{m,t}$ is the return to the market portfolio and $r_{b,t}$ is the return to the replicated betting against beta portfolio in month t. The t-statistics are adjusted for heteroscedasticity using Newey and West (1987) standard errors.

	MOM			CEI			VOL			IVOL		
Panel A: Coefficient	α	β	BAB	α	β	BAB	α	β	BAB	α	β	BAB
Panel B: Value-weighted												
CAPM	0.605	-0.434		0.439	-0.403		0.786	-0.706		0.663	-0.519	
t	[3.68]	[-4.30]		[4.11]	[-6.29]		[3.81]	[-10.98]		[3.23]	[-8.47]	
CAPM + BAB	0.312	-0.388	0.416	0.230	-0.371	0.297	0.400	-0.646	0.548	0.367	-0.473	0.420
t	[1.41]	[-5.29]	[2.76]	[2.16]	[-7.42]	[6.26]	[1.99]	[-9.09]	[5.59]	[1.66]	[-7.07]	[3.45]
Panel C: Eliminated												
CAPM	0.207	0.001		0.144	0.006		0.245	-0.119		0.249	-0.053	
t	[1.24]	[0.02]		[1.36]	[0.12]		[1.15]	[-2.18]		[1.17]	[-0.93]	
CAPM + BAB	0.151	0.010	0.080	0.186	-0.001	-0.059	0.215	-0.114	0.043	0.268	-0.056	-0.026
t	[0.69]	[0.13]	[0.52]	[1.76]	[-0.01]	[-1.72]	[0.88]	[-2.11]	[0.36]	[1.09]	[-0.99]	[-0.19]
Panel D: Weight-shifted (original)												
CAPM	0.070	0.019		0.154	-0.053		0.219	-0.243		0.094	-0.074	
t	[0.47]	[0.23]		[1.51]	[-0.95]		[1.21]	[-4.90]		[0.53]	[-1.50]	
CAPM + BAB	0.089	0.016	-0.027	0.180	-0.057	-0.037	0.042	-0.215	0.252	0.025	-0.063	0.099
t	[0.49]	[0.21]	[-0.24]	[1.64]	[-1.05]	[-0.75]	[0.21]	[-4.26]	[2.12]	[0.12]	[-1.26]	[0.71]
Panel E: Weight-shifted (modified)												
CAPM	0.189	-0.011		0.168	0.010		0.435	-0.212		0.262	-0.043	
t	[1.11]	[-0.12]		[1.78]	[0.23]		[2.06]	[-3.44]		[1.29]	[-0.68]	
CAPM + BAB	0.175	-0.009	0.020	0.198	0.006	-0.043	0.277	-0.187	0.224	0.187	-0.031	0.106
t	[0.82]	[-0.10]	[0.15]	[1.99]	[0.13]	[-1.08]	[1.14]	[-3.31]	[1.80]	[0.76]	[-0.53]	[0.73]

Table 12: CAPM estimates for long-short anomaly portfolios based on β^{FF} while controlling for the BAB-factor

Reported in this table are regression estimates and the corresponding t-statistics for long-short anomaly portfolios constructed on pre-formation beta estimation technique β^{FF}. The sample period is 1927 to 2016. Panel B through E reports which long-short portfolios' excess return is used as the dependent variable. The leftmost column displays the relevant model that is estimated. The remaining columns represent the coefficients that are estimated for the various anomaly portfolios. Alpha estimates are shown in percent. CAPM is estimated in the specification $r_{i,t} - r_{f,t} = \hat{\alpha}_i + \hat{\beta}(r_{m,t} - r_{f,t}) + \hat{\epsilon}_{i,t}$, while CAPM + BAB is estimated as $r_{i,t} - r_{f,t} = \hat{\alpha}_i + \hat{\beta}(r_{m,t} - r_{f,t}) + BAB \cdot r_{b,t} + \hat{\epsilon}_{i,t}$. In the preceding specifications, $r_{i,t}$ is the return to long-short anomaly portfolio i, $r_{f,t}$ is the risk-free rate, $r_{m,t}$ is the return to the market portfolio and $r_{b,t}$ is the return to the replicated betting against beta portfolio in month t. The t-statistics are adjusted for heteroscedasticity using Newey and West (1987) standard errors.

		MOM			CEI			VOL			IVOL	
Panel A: Coefficient	α	β	BAB	α	β	BAB	α	β	BAB	α	β	BAB
Panel B: Value-weighted												
CAPM	0.652	-0.431		0.465	-0.416		0.843	-0.716		0.723	-0.530	
t	[3.90]	[-4.28]		[4.45]	[-6.66]		[4.06]	[-10.97]		[3.51]	[-8.55]	
CAPM + BAB	0.328	-0.390	0.420	0.248	-0.381	0.308	0.420	-0.655	0.580	0.392	-0.484	0.448
t	[1.46]	[-5.36]	[2.74]	[2.37]	[-7.94]	[6.50]	[2.04]	[-8.89]	[5.49]	[1.75]	[-7.05]	[3.48]
Panel C: Eliminated												
CAPM	0.223	-0.079		0.271	-0.074		0.594	-0.477		0.273	-0.104	
t	[1.21]	[-0.81]		[2.47]	[-1.10]		[2.82]	[-7.75]		[1.36]	[-1.78]	
CAPM + BAB	0.071	-0.062	0.177	0.198	-0.059	0.115	0.185	-0.415	0.562	0.079	-0.079	0.249
t	[0.29]	[-0.76]	[1.01]	[1.62]	[-1.00]	[1.76]	[0.88]	[-6.11]	[5.57]	[0.35]	[-1.29]	[2.00]
Panel D: Weight-shifted (original)												
CAPM	0.326	-0.043		0.282	-0.109		0.189	-0.409		0.089	-0.173	
t	[2.21]	[-0.05]		[2.68]	[-2.15]		[1.03]	[-7.26]		[0.45]	[-3.05]	
CAPM + BAB	0.186	-0.028	0.172	0.190	-0.091	0.137	-0.116	-0.364	0.418	-0.087	-0.152	0.223
t	[0.95]	[-0.47]	[1.19]	[1.68]	[-2.12]	[2.83]	[-0.62]	[-6.03]	[4.62]	[-0.43]	[-2.55]	[1.93]
Panel E: Weight-shifted (modified)												
CAPM	0.342	-0.024		0.271	0.007		0.414	-0.309		0.288	-0.027	
t	[2.10]	[-0.26]		[2.40]	[0.14]		[1.96]	[-4.48]		[1.42]	[-0.41]	
CAPM + BAB	0.229	-0.012	0.126	0.257	0.015	0.032	0.109	-0.263	0.414	0.196	-0.019	0.105
t	[1.06]	[-0.16]	[0.83]	[2.07]	[0.32]	[0.58]	[0.51]	[-3.55]	[4.15]	[0.87]	[-0.28]	[0.84]

B Robustness Tests: Common Sample of Stocks

Table 13: CAPM estimates for long-short anomaly portfolios after elimination with a common sample of stocks

Reported in this table are the CAPM estimates of long-short anomaly portfolios and the corresponding t-statistics. The sample period is 1927 to 2016. All stocks included in the sample have a beta estimate from each pre-formation beta estimation technique in any given month. In each month, value-weighted anomaly portfolios are formed from univariate sorts into quintiles of all NYSE, AMEX and NASDAQ stocks. The monthly anomaly portfolio returns are defined as the difference between value-weighted returns of extreme quintiles. Alpha estimates are denoted in percent. α_{vw} (β_{vw}) is the CAPM alpha (beta) estimate of the value-weighted long-short portfolios. α_{el} (β_{el}) is the CAPM alpha (beta) estimate of the long-short portfolios where stocks are eliminated. Δ_α (Δ_β) is the difference between α_{vw} (β_{vw}) and α_{el} (β_{el}) in percent. The t-statistics are corrected for heteroscedasticity using Newey and West (1987) standard errors.

	MOM			CEI			VOL			IVOL		
Panel A: Beta measure												
	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}
Panel B: Percentage eliminated												
El%	45%	45%	45%	45%	35%	45%	70%	70%	70%	60%	50%	60%
Panel C: β estimates												
β_{vw}	-0.422	-0.422	-0.422	-0.398	-0.398	-0.398	-0.717	-0.717	-0.717	-0.528	-0.528	-0.528
t	[-4.23]	[-4.23]	[-4.23]	[-6.22]	[-6.22]	[-6.22]	[-11.42]	[-11.42]	[-11.42]	[-8.83]	[-8.83]	[-8.83]
β_{el}	-0.046	0.003	-0.080	-0.022	0.007	-0.063	-0.170	-0.145	-0.361	-0.063	-0.074	-0.125
t	[-0.48]	[0.03]	[-0.83]	[-0.39]	[0.13]	[-0.94]	[-2.77]	[-2.62]	[-6.02]	[-1.09]	[-1.34]	[-2.19]
Δ_β	-89.10%	-100.71%	-81.04%	-94.47%	-101.76%	-84.17%	-76.29%	-79.78%	-49.65%	-88.07%	-85.98%	-76.33%
Panel D: α estimates												
α_{vw}	0.568	0.568	0.568	0.451	0.451	0.451	0.734	0.734	0.734	0.598	0.598	0.598
t	[3.50]	[3.50]	[3.50]	[4.28]	[4.28]	[4.28]	[3.67]	[3.67]	[3.67]	[3.02]	[3.02]	[3.02]
α_{el}	0.398	0.182	0.187	0.282	0.155	0.277	0.258	0.127	0.238	0.261	0.128	0.239
t	[2.25]	[1.10]	[1.05]	[2.64]	[1.50]	[2.48]	[1.33]	[0.60]	[1.13]	[1.30]	[0.62]	[1.25]
Δ_α	-29.83%	-67.93%	-67.05%	-37.40%	-65.59%	-38.51%	-64.85%	-82.70%	-67.57%	-56.35%	-78.60%	-60.03%
Panel E: Information ratios												
IR$_{vw}$	0.339	0.339	0.339	0.490	0.490	0.490	0.407	0.407	0.407	0.327	0.327	0.327
t	[3.18]	[3.18]	[3.18]	[4.59]	[4.59]	[4.59]	[3.81]	[3.81]	[3.81]	[3.06]	[3.06]	[3.06]
IR$_{el}$	0.224	0.108	0.108	0.307	0.172	0.277	0.139	0.065	0.124	0.142	0.068	0.130
t	[2.10]	[1.01]	[1.01]	[2.88]	[1.62]	[2.60]	[1.30]	[0.61]	[1.16]	[1.33]	[0.63]	[1.22]
Δ_{ir}	-33.92%	-68.14%	-68.14%	-37.35%	-64.90%	-43.47%	-65.85%	-84.03%	-69.53%	-56.57%	-79.20%	-60.24%

Table 14: CAPM estimates for long-short anomaly portfolios after modifying the weight-shifting method with a common sample of stocks

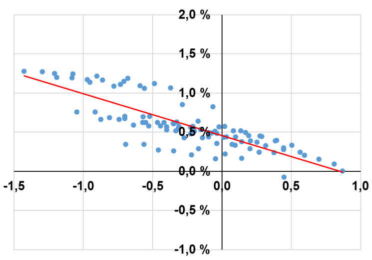
Reported in this table are the CAPM estimates of long-short anomaly portfolios and the corresponding t-statistics. The sample period is 1927 to 2016. All stocks included in the sample have a beta estimate from each pre-formation beta estimation technique in any given month. In each month, value-weighted anomaly portfolios are formed from univariate sorts into quintiles of all NYSE, AMEX and NASDAQ stocks. The monthly anomaly portfolio returns are defined as the difference between value-weighted returns of extreme quintiles. Alpha estimates are denoted in percent. α_{vw} (β_{vw}) is the CAPM alpha (beta) estimate of the value-weighted long-short portfolios. α^*_{ws} (β^*_{ws}) is the CAPM alpha (beta) estimate of the modified weight-shifted long-short portfolios. Δ_α (Δ_β) is the difference between α_{vw} (β_{vw}) and α^*_{ws} (β^*_{ws}) in percent. The t-statistics are corrected for heteroscedasticity using Newey and West (1987) standard errors.

	MOM			CEI			VOL			IVOL		
Panel A: Beta measure												
	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}	β^{SR}	β^{BAB}	β^{FF}
Panel B: Percentage of weight shifted												
w%	70%	70%	70%	65%	55%	70%	70%	70%	70%	70%	70%	70%
Panel C: β estimates												
β_{vw}	-0.422	-0.422	-0.422	-0.398	-0.398	-0.398	-0.717	-0.717	-0.717	-0.528	-0.528	-0.528
t	[-4.23]	[-4.23]	[-4.23]	[-6.22]	[-6.22]	[-6.22]	[-11.42]	[-11.42]	[-11.42]	[-8.83]	[-8.83]	[-8.83]
β^*_{ws}	-0.030	-0.004	-0.026	0.047	0.013	0.023	-0.278	-0.220	-0.318	-0.125	-0.073	-0.035
t	[-0.38]	[-0.04]	[-0.28]	[0.89]	[0.28]	[0.43]	[-5.32]	[-3.44]	[-4.75]	[-2.31]	[-1.21]	[-0.54]
Δ_β	-92.89%	-99.05%	-93.84%	-111.73%	-103.27%	-105.78%	-61.23%	-69.32%	-55.65%	-76.33%	-86.17%	-93.37%
Panel D: α estimates												
α_{vw}	0.568	0.568	0.568	0.451	0.451	0.451	0.734	0.734	0.734	0.598	0.598	0.598
t	[3.50]	[3.50]	[3.50]	[4.28]	[4.28]	[4.28]	[3.67]	[3.67]	[3.67]	[3.02]	[3.02]	[3.02]
α^*_{ws}	0.369	0.175	0.228	0.242	0.180	0.295	0.414	0.278	0.248	0.236	0.089	0.233
t	[2.14]	[1.04]	[1.42]	[2.51]	[1.93]	[2.54]	[2.17]	[1.28]	[1.19]	[1.25]	[0.43]	[1.20]
Δ_α	-34.98%	-69.16%	-59.75%	-46.28%	-60.04%	-34.52%	-43.60%	-62.13%	-66.21%	-60.54%	-85.12%	-61.04%
Panel E: Information ratios												
IR_{vw}	0.339	0.339	0.339	0.490	0.490	0.490	0.407	0.407	0.407	0.327	0.327	0.327
t	[3.18]	[3.18]	[3.18]	[4.59]	[4.59]	[4.59]	[3.81]	[3.81]	[3.81]	[3.06]	[3.06]	[3.06]
IR^*_{ws}	0.222	0.108	0.144	0.276	0.206	0.307	0.231	0.142	0.128	0.136	0.048	0.122
t	[2.08]	[1.01]	[1.35]	[2.59]	[1.93]	[2.88]	[2.17]	[1.33]	[1.20]	[1.27]	[0.45]	[1.15]
Δ_{ir}	-34.51%	-68.14%	-57.52%	-43.67%	-57.96%	-37.35%	-43.24%	-65.11%	-68.55%	-58.41%	-85.32%	-62.69%

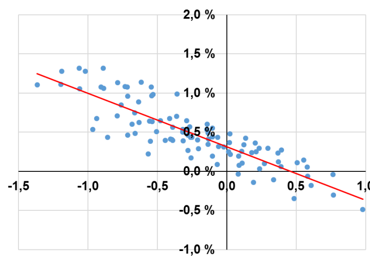
Figure 3: Double sorts on anomaly characteristic and realized portfolio beta with a common sample of stocks

Each figure displays the realized beta and abnormal return from being long-short extreme anomaly quintiles. The sample period is 1927 to 2016. All stocks included in the sample have a beta estimate from each pre-formation beta estimation technique in any given month. Each month, stocks are sorted into deciles based on their pre-formation beta estimate. Subsequently, stocks within each beta decile are sorted into quintiles based on an anomaly characteristic. Intersections form 50 portfolios for every month. The monthly return is defined as the value-weighted return for each portfolio. These are used to estimate CAPM abnormal returns and realized beta for each of the 50 portfolios. Each observation represents a strategy that is long a desired anomaly quintile and short a corresponding undesired quintile. This returns a total of 100 combinations for each anomaly strategy. The realized beta of each long-short strategy is reflected on the x-axis, while the corresponding abnormal return for the same strategy is reflected on the y-axis.

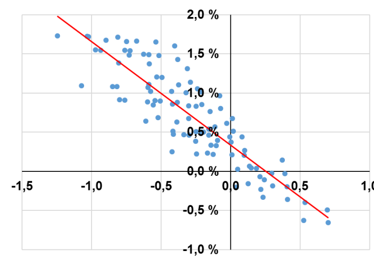
(a) MOM (β^{SR})



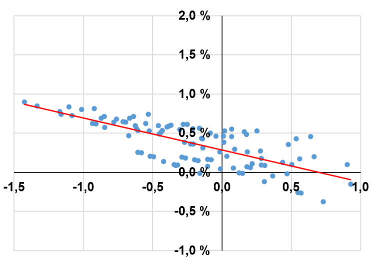
(b) MOM (β^{BAB})



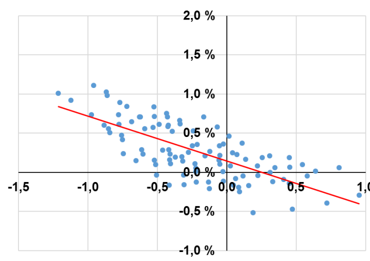
(c) MOM (β^{FF})



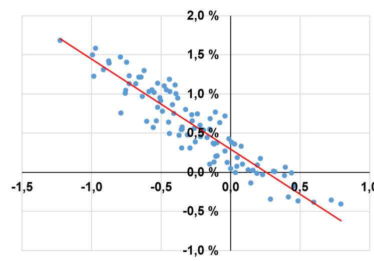
(d) CEI (β^{SR})



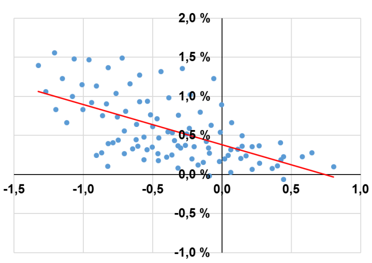
(e) CEI (β^{BAB})



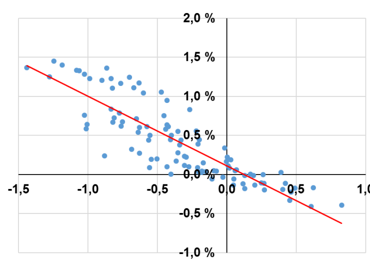
(f) CEI (β^{FF})



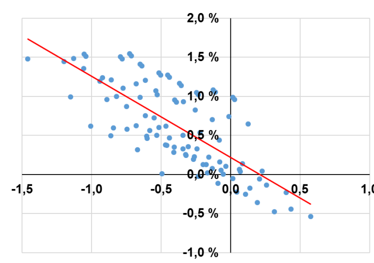
(g) VOL (β^{SR})



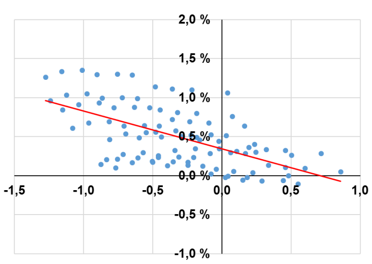
(h) VOL (β^{BAB})



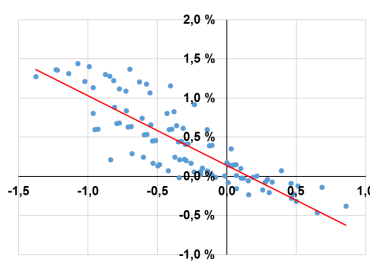
(i) VOL (β^{FF})



(j) IVOL (β^{SR})



(k) IVOL (β^{BAB})



(l) IVOL (β^{FF})