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A model for Cash Management:

An Aquaculture case study

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Abstract

The problem of maximizing interest earned on cash surplus gained from a firm's operation can involve considerable complexity, especially when there are seasonal factors and uncertainty involved. The network flow model with gains and losses for use in cash management was first presented in 1979 by Golden and Libertore. Their model is deterministic, but in this thesis, stochastic techniques are implemented to the model, as well as introducing different asset classes.

Value at risk (VaR) is commonly used in the financial industry to quantify risk in asset portfolios. Cash-flow-at-Risk (CFaR) has been considered the VaR alternative for non-financial firms, by quantifying the potential loss in earnings from operations. In this thesis, the CFaR is implemented to the network flow model to determine the minimum level of cash in the operation of an aquaculture company. For this strategy to be successfully implemented it is necessary to include a rolling planning horizon to achieve the optimum investment strategy.

There is considerable number of risk factors involved in the operation of an aquaculture company. The price of salmon has historically been very volatile and difficult to predict. The operations of an aquaculture company involves cycles and periods of low and high margin. In such cases, effective cash management is important. By retaining earning from periods of high earning to meet the potential lower margin periods is a step toward ensuring that the company has enough cash resources to meet its future obligations.

Preface

This thesis is the final product of my master´s degree in Engineering Management from Reykjavik University. A degree that I combined with an exchange program from University of Stavanger.

In my previous professional work, managing finance. I realized that there were often at times sums of cash lying on cash accounts bearing little to no interest. Cash resources obtained from operations that was to be used or allocated at later times. When I started my master degree studies in Engineering Management, I decided in the start of my study, that my master´s project would be a model for optimizing the allocation process of cash resources.

I would like to thank my instructor and supervisor Dr. Roy Endré Dahl, for his guidance and motivation in the thesis. His positive feedback motivated me greatly in the process of producing this thesis. I would also like to thank Dr. Páll Jensson at Reykjavik University for his mentoring in management science and Dr. Frank Asche at UIS for his valuable inputs on the Aquaculture industry. I would also like to mention that a management science paper from my friend Þorlákur H. Hilmarsson, MSc and MCF, inspired me to research network flow modelling in cash management.

June 4, 2014

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Erlendur Ingi Jónsson

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1. Introduction

The problem in financial decision making, related to the short term and future short term cash management planning is considerably complicated, and is faced by companies, investors and consumers alike. These kind of problem where one of the first problems solved in mathematical programming and operation research. [51]

The importance of efficient cash management is great when inflation, cost of capital is high and the access external financing is not easily accessible. There are some aspect to consider in the efficient management of cash. An important and perhaps the most covered literature in treasury management textbooks is the need for efficient working capital management. Such as, speeding up collections and credit management, accounts receivables and payable management. Yet another important factor is the short term financing and effective investment plans.

By maximizing the interest earned on net cash generated from operations and minimizing the cost of short term financing, is a step toward limiting the influence of inflation and increase the net present value of cash available to distribute the company´s shareholders.

In this thesis, a model for optimizing cash resources will be introduced. Based on network flow algorithm with gains and losses. The goal of the model is to maximize the interest earned on cash resources obtained from operations. This problem increases in complexity when there are cycles and seasonality in costs and revenues. Where surplus in cash from operations have to be kept in reserves to meet a period of lower margin.

The research method chosen for the thesis is a case study within aquaculture. Good access of information and the interesting characteristics of the industry such as predictability in production and harvesting, seasonality in costs and revenues, makes this research choice fascinating. The case study will look at a 12 month cash flow planning horizon for the fictitious Norwegian aquaculture company AquaSalmon AS.

The objectives of this thesis can be summarized as follows.

1. Locate the key variables in costs and revenues in the Norwegian aquaculture industry and construct a case study for an average size salmon company.

2. Implement Cash-Flow-a-Risk to the Network flow with gains and losses model.
3. Determine if Cash-flow-at-Risk can be used to evaluate uncertainty and in decision making of setting a minimum cash account, for a 12 month period.
4. Implement different asset classes to the Network-flow with gains and losses algorithm.

The research question of the thesis can be summarized as:

Can the network flow algorithm with gain and losses be effectively used in the medium term decision making in cash management?

1.1 Scope of the thesis

The thesis will address the determinants of risk in the net cash flow, and how the associated risk can be treated in order to make efficient investment planning. This is done be using available public data from the Norwegian Directorate of fisheries to determine the characteristics of a medium size salmon farming company. Taxes, legal and accounting regulations will be disregarded in the study.

1.2 Background and purpose

The aim of the study is to introduce a cash management model that maximizes the interest earned on the net cash flow, by allocating the capital resources into higher interest bearing assets. The optimization method is based on Network flow with gain and losses algorithm presented by Golden B. and Liberatore M. (1979). In their paper they noted that "future research should investigate the implications of the bounding results further and try to apply stochastic techniques to deal with data sets were all supplies and demands are not known with certainty". They also noted that the model should be tested on a company´s cash flow and compared to current cash management planning techniques. Pacheco and Morabito (2011) tested the model with an agriculture company for short-term decision making in investment and financing. In the model, they disregarded stochastic properties as the planning horizon is short-term. Their solution method is a deterministic one, but recommended the use of a rolling planning horizon.

In this thesis, there will be an attempt to improve the model to deal the underlying uncertainty associated with the cash flow, for a planning horizon of 1 year. Many researches and papers have been published on cash management and involve; short-term borrowing, short term investing, cash positions and cash flow forecasting. In this thesis, these topics will be addressed by taking into account the underlying uncertainty in the projected cash flow.

Value-at-Risk (VaR) has been considered the leading tool in in managing risk for financial firms. Cash-Flow-at-Risk (CFaR) measure has been implemented as the VaR equalent for non-financial firms. With the purpose of estimating the uncertainty associated with different operation performance metrics of a company. In this study, the stochastic properties of the CFaR are implemented to the Network flow with gains and losses model.

1.3 Structure of the thesis

Chapter 2 will be an introduction to cash management, the principles related to efficient cash management and literature review on the subject. Chapter 3 will cover cash flow forecasting, introduction to Cash-flow-at-risk and a literature review. Chapter 4 is the summary of the method and model of the study. Chapter 5 is an introduction chapter on the Norwegian Aquaculture industry and the determinants that influence the cash flow of an aquaculture company. Chapter 6 covers the results of the case study and chapter 7 the conclusions and discussions, followed by discussions for further study.

2. Cash management

Cash management and treasury management are sometimes used interchangeably and are subjected to the same goal. The principal goal of cash management is to allocate cash resources as efficiently as possible and in accordance with corporate strategy [12]. Graham and Harvey (2001) demonstrated in their survey that management value flexibility over other measures in their financial management when making decisions. The determinants of corporate cash management policies around the world where studied by Kusnadi and Wei (2009). They concluded that managers should acknowledge the importance of legal protection set by regulators before setting the optimal cash management policies. Early study of the costs and benefits of holding cash was studied by Keynes (1936), which suggested that firm´s cash management policy should depend on the access to external financing[1].

According to the Association of Financial Professionals (AAFP) the major objectives of treasury management are to:

(1) Maintain liquidity
(2) Optimize cash resources
(3) Manage risk
(4) Maintain access to short term financing
(5) Maintain investment
(6) Maintain access to medium and long term financing and to support investment in capital assets
(7) Coordinate financial functions and share financial information.
(8) Enhance global and cross border focus.

In this thesis (1), (2), (4) and (5) will be addressed and a model for optimizing the allocation of cash into low risk securities is presented. Number (3) gets special attention when risk in revenues and expenditure are evaluated.

[1] Are the funds that is obtained from outside the company

2.1 Liquidity management

According the AAFP “The primary objective of liquidity management is to maintain a cash position that allows a company to meet daily obligations without incurring the opportunity costs that arise from holding excess cash of from not having enough cash on hand to meet those obligations”. Liquidity is sometimes referred to as the ability to meet short-term obligations such as paying for supplies, salaries, tax and to pay back creditors. For a company that does not meet these obligations will be insolvent quickly as it does has not access to necessary cash to meet its commitments. A company can minimize this risk by effectively manage its liquidity. In efficient liquidity management it is therefore a necessary to keep some level of liquid cash assets. There are four reasons for an organization to maintain liquidity. [12]

(1) Transaction requirement
(2) Precautionary requirements
(3) Opportunistic requirements
(4) Regulatory or covenant requirements

Reasons (1), (2) and (3) are addressed in the thesis. Transaction requirements are the constant in and outflow of cash that most organizations have to meet.

Financial institutions sometimes require compensation for their loans and services. That is, require that a minimum cash balance is held at the bank to cover the costs of providing these services. These cash deposits are sometimes called compensating balances. But these kind of requirement have declined over the years. 84,7% of all responding companies in a survey from in 1979 (see Lawrence et al 1979) were required to hold a minimum cash balance at the financial institution, decreasing to 28% in 1996 (see Maxwel et al 1996). Bank institutions have increasingly focused on charging service fees for their services instead of the compensating balances. [36]

2.1.1 Target cash balance

The target cash balance is the amount of cash that a company wants to keep in reserve for a period of time. The complication is that holding too much cash, can result in a lost investment or opportunity costs. But on the opposite of holding too little, the risk is not being able to honor the company's obligations. The risk of holding too little cash can bring about that the company has to make a necessary and undesirable sales of profitable investments, delay or cancel expansion of the operation [36]. Companies with large fluctuation in their cash flows, are the ones likely to have a need for hold high levels of cash. And on the contrary the larger firms with stable cash flow and high credit rating have a less need for holding large cash positions, as they are likely to have better access to capital market financing [36]. One of the major advantages of stocking a large amounts of cash is that it will help to fund investments in the future, especially when the large spread between internal and external cost of financing is high [6]. This can be referred to as an opportunistic balance of cash.

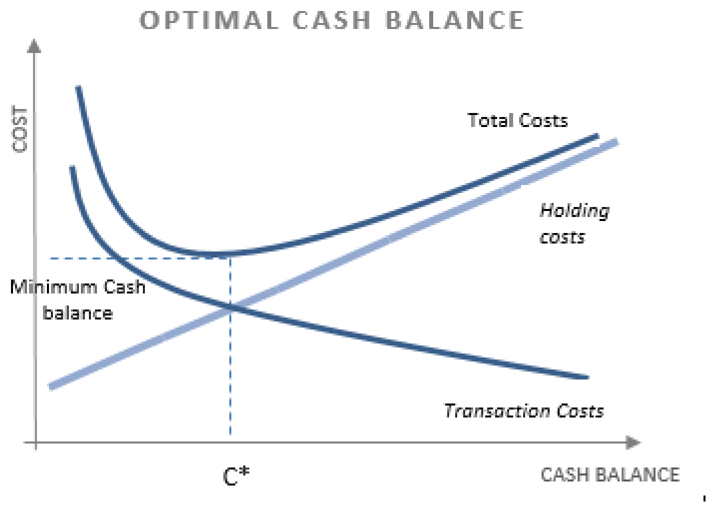


Figure 1. The optimal cash balance.

Different methods and models have been presented to determine the optimum cash balance. In the following the two most common one will be reviewed.

Baumol (1952) presented a model[2] which can be used to determine the optimal level of cash. His model can be used equally as an inventory and cash management model. The model is deterministic and makes the assumption that the firm is able to forecast its cash requirements with certainty and that cash is received at known intervals, that is, at a steady rate of cash. It also assumes that the associated transaction cost and holding costs are fixed over the planned period. The model has a starting cash balance of c, and that the cost is withdrawn with an

[2] The classical model by Baumol (1952) can be reviewed in multiple academic financial management books. Figure 1 is obtained from [17].

equal amount of cash every day, until the balance equals zero. Then the firm sells its marketable securities to achieve the target balance of C* (See figure 2).

The optimal cash balance is presented as C* and can be formulated as:

$$C^* = \sqrt{\frac{2 \cdot b \cdot t}{r}} \qquad \text{(Eq. 1).}$$

Where *b* is the demand for cash over the period, *t* is the transaction cost. The average cash balance can then be calculated as:

$$C = \frac{C^*}{2} \qquad \text{(Eq. 2)}.$$

And the total cost (*TC*) is calculated as:

$$TC = TrC + HC \qquad \text{(Eq. 3).}$$

Where *TrC* is the associated transaction cost and *HC* the holding cost of cash. The model expects that all revenues received are converted into interest bearing assets and that the company has to liquidate them to get cash. As a precautionary requirement, a minimal acceptable cash balance (Min) can be introduced (see figure 2). While the precautionary requirements are the unplanned in and outflow, and therefore requires that the company should hold a precautionary amount of cash to meet these unexpected outflows of cash.

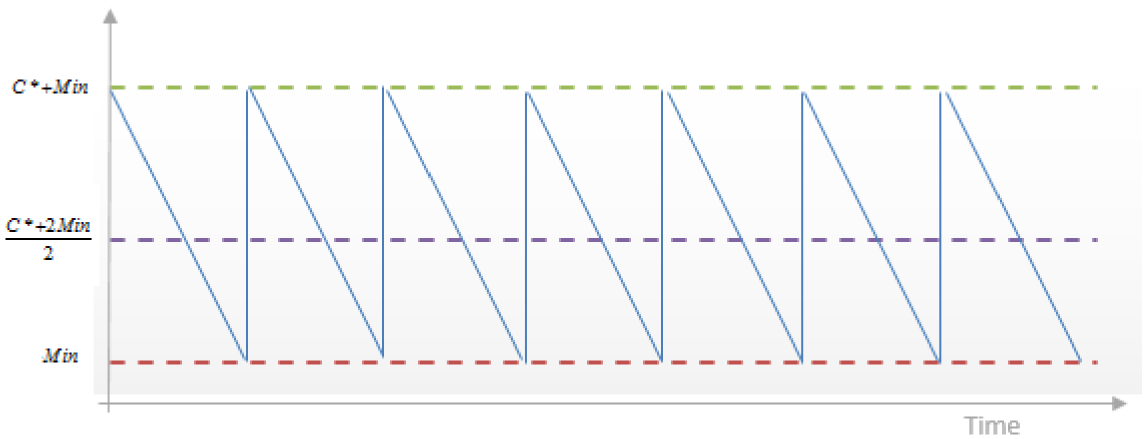


Figure 2. Baumol model with minimum cash balance

The model ignores seasonal and cyclical cash flows. This makes it rather unpractical as most companies have cash flows that fluctuate at some level, and the possibility of short term financing or an overdraft is not considered. But the model provides good insight and a foundation for more sophisticated models on how to maintain an optimal cash balance

Miller and Orr (1966) presented a model that treats the optimum cash balance from a probability aspect. Then it is necessary to obtain the variance of the expected cash flow. The model takes into account the uncertainty of the cash flow by assuming that the cash flow fluctuates randomly from day to day (see figure 3). Similar to the Baumol (1952) model, the goal of the Miller-Orr model is to minimize the loss of possible interest earned by holding cash balances while taking into account the risk of having insufficient balance of cash.

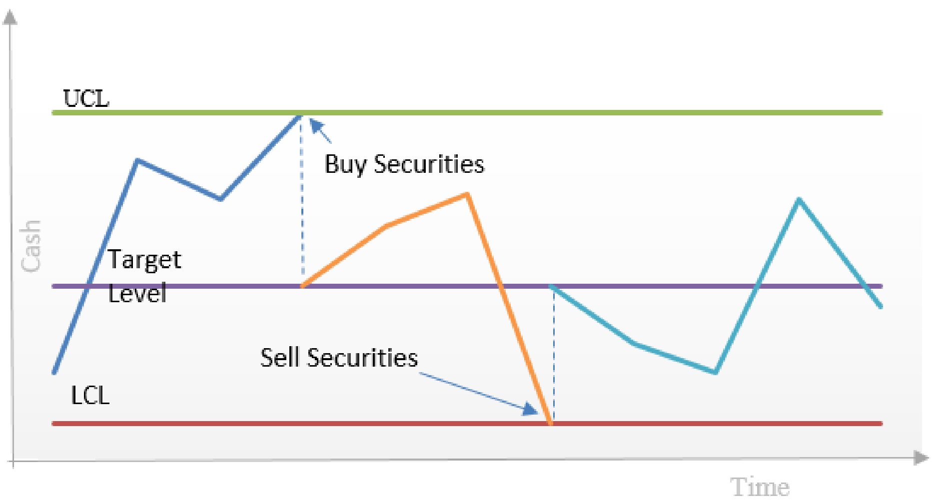


Figure 3. Demonstrating the Miller-Orr model.

The target level (C*) of the model can be found by determining the spread Z:

$$Z = \left(\frac{3 \cdot t \cdot \sigma^2}{4r} \right) \qquad \text{(Eq. 4).}$$

Where *t* is the cost per transaction, and r is the interest rate per period from market securities and σ^2 is the variance[3] in the cash flow. The first step of the model is to determine the minimum level of cash (LCL), this can be set by management or calculated from the variance from the cash flow. The upper control limit (UCL) is set 3 Z´s above the LCL, then the target

[3] Explanation of variance can be seen in appendix B

level C* is set a level of Z above the LCL. When the cash level reached either LCL or UCL, market securities are either bought or sold to reach the target level C*, which can be computed as:

$$C^* = LCL + \frac{4}{3}(Z) \qquad \text{(Eq. 5).}$$

It has been pointed out by Golden and Liberatore (1979) that the assumption of random in and outflow of cash are unrealistic, since some cash flows are quite predictable. Gormley et al (2007) presented a stochastic cash management model based on the model of Baumol (1952) and Miller and Orr (1966) and proposed an approach that uses forecasted cash flows and the associated uncertainty.

Opler et al (1999) defined the transaction cost of converting cash to interest bearing asset as liquidity premium, and pointed out that the premium cannot be compared to risk premium. And if the holding asset bears a lower interest because different risk characteristics, it should not be included as a cost.

2.2 Managing risk

In the overall risk management there are several factors that can pose a risk to an organization, such as[4]:

(1) Market Risk
(2) Credit Risk
(3) Liquidity risk
(4) Operational Risk
(5) Legal and regulatory risk

Market risk is the risk of loss due to unexpected price movement in financial securities and include price fluctuation in equities, commodities, interest rate and exchange rates. Credit risk is the risk of loss due to counterparty default, and liquidity risk is the inability to meet the required obligations. These risks are usually referred to as financial risk. [12]

[4] There are many types of risk that can be measured and evaluated, such as buisness risk, strategic risk, reputational risk, outsourcing risk. (1)-(4) are often refered to in financial management.

Financial risk is the exposure to the unexpected events that affect profitability of an institution and could in the worst scenarios lead to bankruptcy. It can include; failure of financial systems, regulatory or compliance issues. Also; bad debt, adverse changes in exchange rates, overdependence on a single supplier, loss of a key customer, loss of investments. Many kinds of risk factors can in fact be hedged, but poor hedging decisions can also become a financial risk. [55] Measuring financial risk can be done in many ways, such as with sensitivity analysis, scenario analysis, simulation and Value-at-Risk[5].

Insurance, or sometimes called insurance risk management[6] is the process of identifying potential risk exposures, or accidental losses, that affect the company´s operations. By using insurance contracts, the potential financial loss from operational disruption can be limited. The goal of insurance risk management is to limit risk against disastrous loss, decide what and when to insure and at what price. [12]

2.3 Cash flow from operations

Cash flow analysis is perhaps the most useful tool available in assessing a company strength [34]. Free cash flow (FCF) represents the operating cash flow and then subtracting the capital expenditures as:

$$\text{FCF} = \text{Net income} + (\text{Depreciation and Amortization}) - \text{Change in Working Capital} - \text{Capital Expenditures} \quad \text{(Eq. 6).}$$

FCF is the cash that a company is able to produce after the expenditures required to maintain or expand the operation. And can be defined as the cash available for distribution to investors. [17] This measurement is often used in the valuation of companies, as is considered to be a better representation of the value of a company, due to the fact that it is telling us how much operating cash profit is generated in the business. FCF disregards the possibility of creative accounting, as the “creation” of cash is almost impossible. [34]

A negative cash flow does not have to be bad by itself, as it could be a indication that a company has made a large investment or is growing very quickly. For this to hold true, it would have to be supported by a potential payoff in the future. [12]

[5] Value-at-Risk is covered in chapter 3.3
[6] According to AFP

Cash Flow From Operation	
Net Income	$
Adjustment to net Income	
Depreciation	$
Increase in Account Receivables	($)
Increase in Inventory	($)
Increase in Account Payable	$
Net Cash provided from Operations	$

Table 1 Cash flow from operations

For the yearly statement of cash flow the changes in investment activities and financing activities are also included, in addition to table 1.

The net cash flow (CF) from operation is the cash produced from regular operations and can also be defined as:

$$CF = \text{Operating Cash Flow} - \text{Capital Expenditures} \quad \text{(Eq. 7).}$$

Equation 7 is the one that is used in the case study of chapter 5, as data on a monthly basis are used in the study.

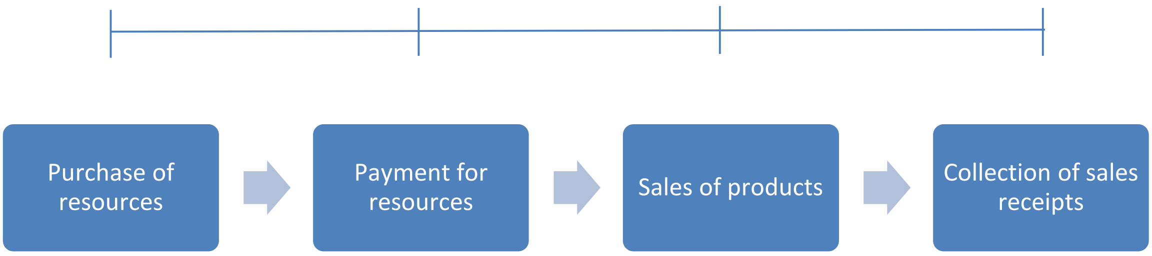


Figure 4. The cash flow timeline

The operating cash cycle involves the purchasing of the resources required and sales of the produced goods or services. The timing between the payments of resources and the collection of the sales receipt is one of the key elements of effective cash management. This procedure is called the cash conversion cycle (CCC) in working capital management.

2.4 Cash management performance metrics

Working capital management has an important role in cash management. As working capital measure how well an institution manages its liquidity. In daily operations, evaluation of working capital is important so that the company has an overview of in and outflow of cash.

Working capital is defined as:

Current Assets – Current Liabilities (Eq. 8)

Current assets are assets which can be converted to cash within a year and consist of cash, account receivables and marketable securities inventory and pre-paid expenses. Current liabilities are the liabilities that have to be paid within a year, that is, account payable and accrued liabilities. [12]

There are in principal three ways of acquiring working capital. First, is by raising cash from owners and investors, secondly by financing from financial institutions and by issuing bonds. And the third option by retained earnings, which for a newly founded company is not an option. In reality, a mixture of these strategies are likely to be applied. [18]

A high Current ratio (Current Assets / Current liabilities) tells the story of how strong the company´s liquidity is. But it is important to realize that current assets generally deliver lower rates of return than the interest paid on current liabilities [12]. High current ratio could then be a liability to the overall performance in cash management, and therefor necessary to have an efficient working capital management where optimal liquidity is determined.

3. Forecasting cash flows and risk

This chapter covers forecasting of cash flows and the associated risk.

3.1 Forecasting process

Forecasting is considered the most important factor in cash management. There are a number of motivations for using forecasting in cash management, such as, credit worthiness, loan structuring and security perfections; where specific shortfalls can be identified and the potential credit risk can be managed. [34]

The time horizon for forecasting is important to consider, as short term forecasting has a different purpose than long term forecasting and vice versa. The time intervals can be divided into long-term, medium term and short-term. Short-term usually refers to a period of one day, a week or a month. Long-term usually refer to time horizons exceeding one year. The medium-term forecasting, ranging from 1-12 months is the time horizon of interest in the thesis. It covers the expected inflow (income) and outflow (costs) of cash, and by this, the company´s need for short term financing and short term investing can be determined. [12]

When forecasting future cash flow the financial manager starts by estimating the future in and outflow. This can be done in multiple ways, such as, based on previous experience or using forecasting techniques. By performing variance analysis, a company can compare the actual cash flows with forecasted cash flows. By this, it is possible to estimate the future uncertainty associated with the cash flow.

Cash flow needs to be divided into key components to deliver the most reliable forecast. This is normally done by dividing the flow into operating, investing, and financial activities. To increase the degree of certainty, key components are split up to: Certain flows, such as interest and principal repayment. Predictable flows, like the level or pattern of sales (quantity and price). Less predictable cash flow, such as future legal settlements or sales of a new product. [12]

3.1.1 Exponential smoothing

In the case study, the Holt-Winter´s procedure[7] for exponential smoothing (HW) is used for forecasting production, harvesting and price of salmon. Followed by a variance analysis that is performed by testing the forecast on historical data. By this, there is an opportunity to evaluate the uncertainty in the projected cash flow. The motivation for using the exponential smoothing method is the ease of use and the capabilities of the method to deal with seasonal data.

The exponential smoothing is one of the most widely used forecasting method in business with ability to forecast trends in sales and costs where there are historical patterns and seasonality in the data. [46] There are essentially nine common trends[8] in econometrics that can be forecasted by the exponential smoothing. All the method of exponential smoothing involve the same process, data averaging. For data with no trend or seasonal patterns, a simple exponential smoothing is appropriate. In the thesis, the linear trend with multiplicative seasonal factors is applied, or the three parameter exponential model.

Originally Holt (1957) formulated a trend model using double exponential smoothing and one smoothing constant and a smoothing constant on the slope. Later, Winter´s (1960) extended this model using different smoothing constants. [9] The Holt-Winter´s procedure is based on three smoothing equations, one for level, one for trend and one for the seasonality. [46] The common way of measuring the accuracy of the forecast can be done by:

Percentage mean error (ME)

$$ME = \sum\left(\frac{Actual}{Forecast}\right)/n \qquad \text{(Eq. 9).}$$

And the Mean Absolute Percentage Error (MAPE) can be found by:

$$MAPE = \sum\left|\left(\frac{Actual}{Forecast}\right)/n\right| \qquad \text{(Eq. 10).}$$

[7] See appendix C

[8] These trends of constants level, linear trend, dampended trend involve the combination of non-seasonal, additive seasonality and multiplicative seasonality (See Hirchey, 2008, 218 „Manageral Economics“)

The use of MAPE as an accurate measurement is very popular, but at the same time very controversial due to the fact that no forecasting method gives an unmistakable indication of forecasting performance [30]. See Goodwin and Lawton (1999), for review of modified MAPE. Makridakis (1993) points out that from a theoretical point, it is difficult to single out the best performing forecasting method from an accuracy point of view. It is then necessary to review the selected method for the purpose and the nature of the forecasting.

3.1.2 On confidence intervals and risk

In the case study in chapter 5, the cash flow of a salmon company is predicted using a forecasting technique. But for the MAPE as an accuracy measurement for forecasting, it cannot fully describe the uncertainty associated with the future. The measure does not give us a posterior probability of the how the underlying variables will behave in the future, only the *h*-step ahead of the model forecast.

Aven (2010) concluded in his book on the misconceptions of risk "History data provides insight into risk. Assuming that the future will be as history shows, we may obtain good predictions about the future. However, there is in principle a huge step from history to risk as any assumption transforming the data to the future may be challenging. To fully express risk we need to look beyond historical based data."

In the thesis, the prediction is compared to the historical data, which gives an estimate of how well the forecasting method has performed in the past. By this notation, the historical accuracy error is treated as a standard deviation. Due to the complication associated with prediction intervals and the risk associated with the cash flow, the probabilities are described as knowledge based probabilities or subjective probabilities.

3.2 Value at Risk

VaR was first used by JP Morgan in 1993 [25], although the first paper on it can be traced back to 1923 (see Hardy, 1923). It has since then, been integrated to The Basel II as a mandatory risk assessment protocol for regulated financial institutions. The method has become considerably popular with financial professionals and academics.

Value at Risk (VaR) determines how changes in financial variables affect the company value over a certain time period. It tells us how much we can lose on specific time period with a given level of certainty. It is usually referred to with a time horizon as 1-day and therefore answer the question “How much can I lose tomorrow given a certain confidence level?”

VaR is defined as the 100p% quantile from the distribution of the potential loss X and is described as:

$$P(X \le x_p) = p \qquad \text{(Eq. 11).}$$

Where p is usually 1% (99% confidence) or 5% (95% confidence). By VaR calculation the output is one number that is expected to represent the total risk in a portfolio. This measurement is widely used in the financial sector, and often used as the basis for risk management models within financial institutions. The standard approach for estimating VaR for a financial institution would be to use the “bottom up” method [27]. That would be to identify risk exposed assets, such as loans, derivatives and securities, which are then quantified. Then, the overall risk exposure is calculated with a level of confidence.

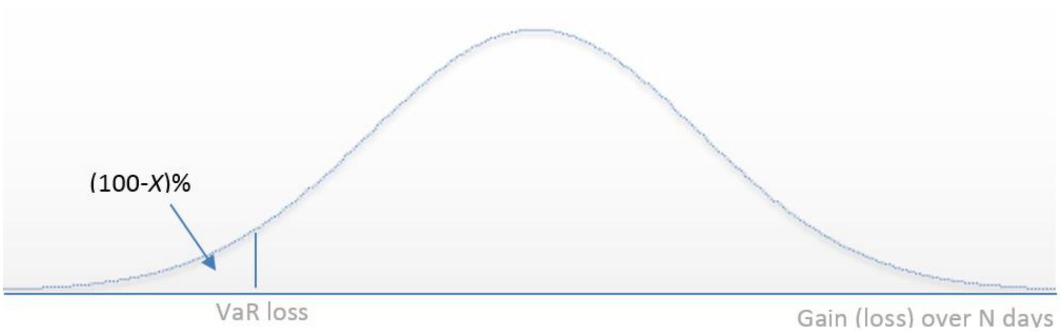


Figure 5. Value-at-Risk.

The two main approaches of quantifying the VaR are the model-building approach approach and the historical simulation. [8]

In the historical simulation, the historical data is used to describe the future. The first step is to locate the market variables that affect the portfolio, such as, different interest rates, commodity prices and equity price. The data are collected and changes in the variables over a certain period of time quantified, typically over a 500 day period. By this, there are 500 possible scenarios for what can happen over one day period.

The model building approach is considered the main alternative to the historical simulation [8] and will be the method of choice in the thesis.

The 1-day VaR can be calculated as:

$$(1\text{-}day\ 99\%\ VaR) = 2{,}33\sigma \qquad \text{(Eq. 12).}$$

The time horizon of model building VaR can be altered by the assumption that:

$$N - day\ \ VaR = 1 - day\ VaR \times \sqrt{N} \qquad \text{(Eq. 13).}$$

Koheler (1990) concluded that the assumption is only appropriate for a random walk, and not theoretically correct as *h*-step ahead forecast errors.

3.2.1 Cash Flow at Risk

Cash Flow at Risk (CFaR) was first introduced by the firm RiskMetric. CFaR is considered to be an extension of the VaR, and requires an estimate of the distribution of the cash flow of a company at a certain time in the future. It can be used to estimate the risk in the EBITA, EBIT or different performance metrics of a firm. The CFaR could be used for example to answer the question: "How much can the company expect loose for a specific period in the future if we experience a 5% tail event?"

This method was presented for non-financial firms by Stein et al (2001), introducing a "top down" method as an alternative to the "bottom-up" of classical VaR. Stein et al (2001) pointed out that the lack of operational data from companies makes the "bottom up" approach difficult, as there are usually only quarterly data available, at best. In the modelthey measured the operating cash flow or EBITA[9], and also mentioned that using EBIT[10] would provide virtually identical results.

The " top down" model uses quarterly data from multiple companies is the same industry, for a 5 year period. The first step of the model is to have a forecast of the expected cash flow of the companies. Stein et al (2001) uses a simple autoregressive forecast to estimate the future variable and to get an estimation of a prediction error. Then a dummy variable is included to adjust the data to the possible seasonality in the data. The data is then collected and the

[9] EBITA is earnings before interest taxes and amortization.
[10] EBIT is earning before interest and taxes.

companies are categorized by their relevant characteristics. The output is then a 5% CFaR tail in a bucket of the categorized firms by industries, which is then used to estimate the potential tail of a specific firm.

Although CFaR can be described in the similar manner as the VaR, or the potential loss over some given period of the time, given a level of confidence. It is a bit more difficult to get a reliable CFaR estimate. Since CFaR focuses on operational cash flow and a quarterly or yearly estimation horizon while VaR focuses on assets, with a time horizon typically measured in days.

Andrén, Jankensgård and Oxelheim concluded (2005) that CFaR is the cash flow equivalent of VaR It is pointed out that VaR is an effective tool for managing risk in financial firms, and noted that the standard VaR with "bottom-up" approach is not suitable for non-financial firms. As the likelihood of leaving some important sources of risk behind is high. In the paper an exposure based CFaR model is presented, where a set of exposure coefficients provide information about how various microeconomic and market variables are expected to affect the cash flow of the company. Although VaR is effectively used by financial firms, when utilized for non-financial firm it only describes a part of the firms overall risk, this is due to the fact that the VaR ignores the firm´s cash flow from regular operations. Andrén et al (2005) concluded that to get the full description of the CFaR it is necessary to calculate the overall variance of the firm.

Both Stein et al (2001) and Andrén et al (2005) look at the data from an external analyst point of view, and only take into account the market information, yearly and quarterly statements. And do not take into account that the treasury or cash manager of the company could have much more accurate information about the cash flow of the firm, and provide significantly better estimations with a "bottom up" approach.

3.2.2 Conditional Value at Risk

Although VaR is widely used in risk management, it has its drawbacks. VaR gives us a valuable insight on how much we can potentially loose at a certain point in time it, but does not tell us the size of potential loss exceeding that specific loss. That proceedure is called C-VaR or the conditional Value-at-Risk. Therefor it is necessary to review and evaluate the calculated VaR value.

Figure 6. Conditional Value-at-Risk.

The CVaR gives the probability that *f*(*x*,*y*) does not exceed the threshold α is given by

$$\Psi(x,\alpha) = \int_{f(x,y)\le\alpha} p(y)dy \qquad \text{(Eq. 14).}$$

Where α is a function of x, Ψ is the cumulative distribution of the potential loss associated with x. *f*(x, y) is assumed continuous from the right, but not necessarily from the left due to the possibility of a risk lump [28], as can be seen in figure 5. In this thesis, the CFaR value is be treated as a right side intergral, and the parameter used to set the cash account in the optimization model. This approach will also be used when presenting figures that associate forecasting uncertainty.

3.3 Optimizing Cash Resources and short term financing

As pointed out in chapter 2.1, cash securities can be held for two main reasons; secure future liquidity and to finance future long term investments. It is typical that short term marketable securities and financial products bear lower interest then operating assets. Most cash and checking account bears little to no interest rate. This motivates firms to allocate part of their cash resources to marketable resources [36].

Orgler (1969) introduced an optimization model that deals with the cash management problem as multi period linear programing model. Orgler (1969) takes into account four types of decision variables, that is; short term financing, payment schedules, investment transactions and the target cash balance. The objective of the model is to minimize the net cost of the cash budget over the planning period. Orgler´s formulation is deterministic, and therefore it disregards the uncertainty associated with the cash flow, but multiple sensitivity and scenario analysis can be done by changing the parameter involved.

A transhipment model for cash management was introduced by Srinivasan (1974). The model's goal is to optimize payment schedules for certain periods and how to invest in a securities portfolio at the beginning of a planning period. And to determine the need for short term financing over the period. Srinivasan (1974) applied the model to minimize the total cost of allocating cash from operations to different assets, with the option of transferring cash between asset classes. But as Golden and Liberatore (1979) pointed out, that the serious disadvantage of the model was that re-investment and interest compounding is not taken into account.

The network flow model with gain and losses algorithm has been used in power electricity distribution networks and severing networks in cities, both network produce flows with gain and losses. Golden and Liberatore (1979) proposed that the network flow algorithm with gains and losses that can be used for optimization in cash management. Where the losses and gains are the exchange rates between assets classes, cost of financing and the interests gained from investments. Kornbluth and Salkin (1987) published some numerical examples of the model by allocating resources to different asset classes. Mulvey and Vladimirou (1992) presented and discussed a network flow model that deals with the stochastic distribution of resources to different classes for different periods.

The network flow model used in the thesis is based on the Golden and Liberatore (1979) model of gains and losses. In the paper the parameters of the model are defined as:

S_t	Supply (Accounts receivables) of asset x in period t (t=1,…,n)
D_t	Demand (Accounts payable) of asset x in period t (t=1,…,n)
x_t	The balance of asset x at time t
y_t	The balance of asset y at time t
α	The interest rate per period of asset x
β	The interest rate per period of asset y
c_{xy}	The per unit cost of conversion from asset x to y
c_{yx}	The per unit cost of conversion from asset y to x

Table 2. Parameter of Golden and Liberatore model.

The model can be shown in a network graph G = (N, A, W) with N = (S, T, Z, 1, 2, 3,…, n, $\underline{1}$, $\underline{2}$, …$\underline{n}$), see figure 7. And consists of the node set N, and an arc set A. The arc weights or multipliers are given by the matrix W = [$w_{i,j}$], which is the matrix of weight multipliers of the corresponding arc A (i,j). Node S is a supply node, node T (Terminal) is the demand node. Node Z is the goal node and the objective function is to maximize the value into Z. [14]

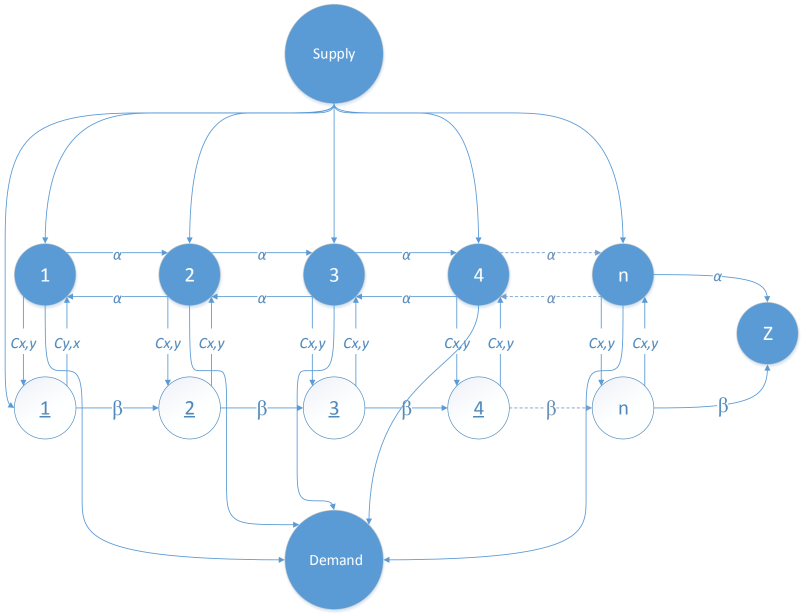


Figure 7. The Network Flow model, with gains and losses.

Golden et al (1979) defined the input flow *f*(*i*,*j*), and *g*(*i*,*j*) as output flow for each arc (*i*,*j*). The *f*(*i*,*j*) and *g*(*i*,*j*) are related by the multiplier *w*(*i*,*j*) as:

$$g(i,j) = w(i,j)\,f(i,j) \qquad \text{(Eq. 15).}$$

Where the objective function can be described as:

$$Max\; g(n,Z) + g(\overline{n},Z) \qquad \text{(Eq. 16).}$$

Subject to

$$\sum_{j\in N^-} f(i,j) - \sum_{j\in N^-} g(i,j) = \begin{cases} S(i) - D(i)\; for\; i = 1,2,\ldots,n, \\ y_0 \;\; for\; i = \overline{1} \\ 0 \;\; for\; i = \overline{2},\overline{3},\ldots,\overline{n}, \end{cases} \qquad \text{(Eq. 17).}$$

$$f(i,j) \geq 0 \ for\ i \in N^-, and\ (i,j) \in A,$$

Where $N^- = N - S - T$

Then, if the $g(i,j)$ is substituted from the equation (3), (4) and (5) then there is a problem in the decision variables $f(i,j)$ only. Only $w(i,j)$ remains, which is represent the gains and losses in transaction costs or interest charges and gains.

$$w_{i,j} = \begin{cases} i+\alpha & for\ (i,j) = (t,t+1)\ or\ (n,Z) \\ i+\beta & for\ (i,j) = (\underline{t},\underline{t}+1)\ or\ (\underline{n},Z) \\ 1-c_{xy} & for\ (i,j) = (t+1,1) = (\mathrm{t},\underline{\mathrm{t}}) \\ 1-c_{yx} & for\ (i,j) = (t+1,1) = (\underline{\mathrm{t}},\mathrm{t}) \\ 1 & otherwise \end{cases} \qquad \text{(Eq. 18).}$$

When dealing with seasonal or fluctuating cash flow it is often a possibility of negative flow of cash for a certain periods of the years, the model can be extended to deal with a borrowing capacity by constructing reverse arcs (t+1, t), that have the multipliers:

$$w(t+1,\mathrm{t}) = \frac{1}{1+\gamma} \qquad \text{(Eq. 19).}$$

And an upper limit bound u is placed on the amount borrowed given by:

$$g_{t+1,\mathrm{t}} = \frac{1}{1+\gamma} f_{t+1,t} \leq u \ \ for\ t = 1,\ldots,n-1 \qquad \text{(Eq. 20).}$$

And the lower bound of the cash account can set as:

$$f(t,t+1) \geq \mathrm{m} \qquad \text{(Eq. 21).}$$

Golden et al. (1979) pointed out that when more uncertainty in the inflows and outflows of cash this option becomes quite valuable, as will be discussed later in the thesis.

The constraints can then be mathematically formulated as:

$$\left.\begin{array}{l} f(t,t+1)\geq \mathrm{m} \\ \frac{1}{1+\gamma}\,f(t+1,t)\leq u \end{array}\right\}\ \mathit{for}\ t=1,2,\ldots,n-1$$

$$f(n,Z)\geq m. \qquad \text{(Eq. 22).}$$

Golden et al (1979) pointed out that the accuracy of the forecast of the supply and demand and interest rates is the most important estimate when evaluating the effectiveness of the model.

4. Method

In this chapter, the method of the Aquaculture Case study will be summarized, and the model for the cash allocation optimization explained. This can be broken into the following steps:

(1) Locate the operational and market variables and determine the planning horizon.
(2) Forecast the operational and market variables and evaluate the accuracy of the forecast.
(3) Line up the operating cash flow model with associated costs and revenues.
(4) Run a simulation to estimate the uncertainty in the net free cash flow.
(5) Determine the level of the minimum cash account.
(6) Use gains-and loss algorithm to allocate the cash surplus from operations to higher interest bearing assets of different classes.

The objective is to find the maximal value of cash Z at time T:

$$Z = f\,[(0, T); Z] \qquad \text{(Eq. 23).}$$

Where Z is the total amount of capital at the end node.

In the following chapters the components of method used in the study will be introduce. Starting by the planning horizon, abide by, cash flow forecast, how the minimum cash account is determined and explain the solution method of the cash allocation model.

4.1 Planning horizon

The planning horizon is the horizon when the company plans its cash transactions. In the case study in chapter 5, the cash allocation optimization is performed from a 12 month operating forecast. The concept of a rolling horizon of often applied in the practice of managing cash flows [13]. That is, when there is a planning horizon of many periods, the planning is updated after the first period has ended. The previous month is removed and a new one is added at the end. The desired inputs are then forecasted and evaluated again, and the optimization model is applied. This is then repeated each time, as a rolling planning horizon.

4.2 Determining the minimum cash balance

In this section the method for determining the minimum cash account is explained, and a corresponding probability constraint on the cash account is introduced. The first step is to predict the future cash flow and to evaluate how well the forecast has performed in the past.

4.2.1 Cash flow forecasting

Like Stein et al (2001) points out in his CFaR paper, the forecasting technique is not of the most importance. But rather, how well the forecasting method fits the data and how well it has performed in the past. An experienced treasury manager or a seasoned aquaculture farmer might have a different opinion and knowledge on how to forecast the operational and market variables. As most, if not all medium to large firms have forecasts of their expected cash flow, there should be an opportunity to evaluate how accurate they are. A previous empirical prediction error could provide as good information as from sophisticated forecasting methods. In this study, Stat-tools from Palisade will be used for forecasting the cash flow from operations, and use the built in exponential smoothing forecasting tools. The motivation for using this method is the simplicity of the calculations and the seasonal smoothing constants, which gives interesting insight on the seasonal effect the expected price, sales and costs.

4.2.2 Setting the minimum cash balance

The minimum cash balance is determined by the uncertainty of the underlying operational and market variables. Opler et al (1999) pointed out that “Uncertainty lead to situations in which, at times, the firm has more outlays that expected. Therefore, one would expect firms with greater cash flow uncertainty to hold more cash.”

In this model, the CFaR method to set the minimum cash balance, based on the model’s presented in chapter 3.3. That is to estimate the potential loss at certain period of time in the expected cash flow. Opposed to the Baumol´s model and the Miller-Orr model, where the objective is to acquire a target cash account to determine the projected minimum cash account. The forecasting gives an uncertainty interval on the projected cash flow, which can be used to determine the minimum cash balance with a level of confidence.

The solution can be split into two, a minimum cash for precautionary measure that enables the firm to deal with the potential loss from regular operations and the opportunistic measure that is kept in reserves for possible unexpected opportunities. Different approaches can be used when setting this minimum limit, but in the following there one example will be introduced, that will be used in the case study of chapter 5.

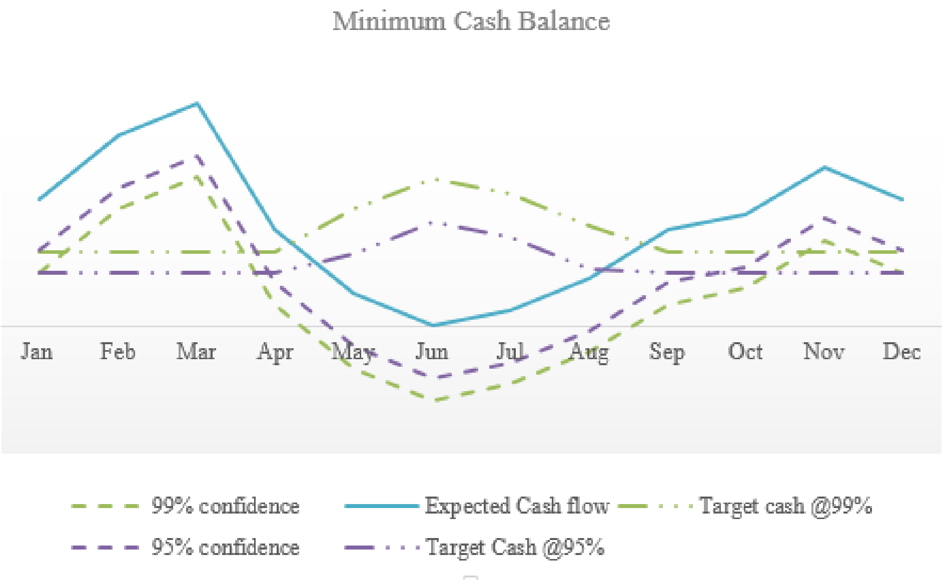


Figure 8. Demonstrating CFaR to set a minimum cash balance.

The 99% and 95% confidence lines in figure 8 represent the level of confidence of the forecasted cash flow. To explain, according to prediction there are 95% certainty that the net cash flow will be higher than this 95% CFaR value at time *t*.

Say that a company wants to determine by some level of confidence, that it has sufficient cash to meet operational shortfalls and keep in reserves for potential business opportunities. Assuming normal distribution. The minimum cash account can then be set for time *t* as:

$$\text{Minimum Cash Account} = \max\left(Z - \text{CfaR}(\alpha)_t, Z\right)_t \quad \text{(Eq. 24).}$$

Where Z is the opportunistic cash minimum set by management. The CFaR value at certain confidence level $\alpha\sigma_t$ at time *t* can be calculated as:

$$\text{CFaR}(X>x_\alpha)_t = E(\text{cash flow})_t - \alpha\,\sigma_t \quad \text{(Eq. 25).}$$

The standard deviation at time *t* is calculated from the historical accuracy of the forecasting method. Different techniques of forecasting can be used, but so that the level of confidence can set with a meaning the method of forecasting has to be validated, for example by comparing to historical accuracy.

As stated in chapter 1 the minimum balance of is thought of as a requirement of precautionary, opportunistic or regulatory requirements. The problem with this approach is that it could be considered unlikely that a company would like to hold an opportunistic cash requirement when the net cash flow for a specific period is expected to be 0 or negative. But justification could be made if other periods of the planning period would compensate the loss. These kind of decisions call for a management review[11].

These requirements can of course vary from company to company and could in theory be 0, which in practical circumstances should not be the case. If the cash flow is insufficient to meet these minimum cash account requirements, then the company has to consider the possibility of a line of credit, overdraft or other kind of short term financing. After these considerations, constraint on the cash account can be set and a minimum cash account parameter for the optimization.

4.2.3 Probability constraint

As the minimum cash account has been determined, a constraint in the optimization model can to be introduced. This is described as:

$$CFaR(X>x_p)_t = m_t \qquad \text{(Eq. 26).}$$

Where *m* is the lower limit of the cash account at time *t*. There is no need to set up an upper limit on the cash balance, as will be demonstrated in chapter 6.

4.3 Allocating the cash resources

Pacheco and Morabito (2011) tested the network flow model with gains and losses with numerical results on different kind of cash flows, where they utilized it for tactical back payment of loans, operation solution, treasury solution with cash account and an investment option. Their treasury solution with a bank account and an additional investment asset is of particular interest, as it can be used within a week (short-term) forecasting period.

A typical investment option is introduced, fixed deposit bank accounts. By intuition and observation from the websites of various banking institutions it can be observed that fixed

[11] Review and judgement by the decision-maker.

bank accounts carry a higher interest rate than standard checking accounts. Liquidity theory of the term structure says that an investor prefers short-term fixed income investments over long-term investments. The justification is that investors prefer their cash resources to be more liquid rather than tied down [11]. After the level of cash on the minimum cash balance[12] has been evaluated and chosen, it is now possible to plan the allocation of some of the cash on higher interest bearing securities or accounts. Now new asset classes are introduced to the model, the fixes deposits, represented by the arcs $1+\varepsilon_t$. Where *t* is the fixed periods of 3, 6 or 9 months.

The parameters of the capital allocation model can be summarized in table 3.

b_k	The net cash flow
x_t	The balance of asset x
y_t	The balance of asset y
ε_t	The interest rate for fixed account *t* (t=3,6,9)
α	The interest rate per period of asset x
β	The interest rate per period of asset y
c_{xy}	The per unit cost of conversion from asset x to y
c_{yx}	The per unit cost of conversion from asset y to x

Table 3. The parameters of implied model.

The network flow with gains and losses model is then utilized where a given constant $a_{i,j}$ presents units of flow exiting arc (*i*,*j*) at note *j* for every 1 unit of flow entering at *i*, by:

[12] Chapter 3.2.2

$$a_{i,j} = \begin{cases} i+\alpha & for\ (i,j)=(t,t+1)\ or\ (n,Z) \\ i+\beta & for\ (i,j)=(t,t+1)\ or\ (n,Z) \\ i+\varepsilon_t & for\ (i,j)=(t,t+3,t+6,t+9)\ or\ (n,Z) \\ \frac{1}{1+\gamma} & for\ (i,j)=(t,t+1)\ or\ (n,Z) \\ 1-c_{xy} & for\ (i,j)=(t+1,1)=(\mathrm{t},\overline{\mathrm{t}}) \\ 1-c_{yx} & for\ (i,j)=(t+1,1)=(\overline{\mathrm{t}},\mathrm{t}) \\ 1 & otherwise \end{cases}$$ (Eq. 28).

Then the value $a_{i,j} > 1$ indicates a gain, $a_{i,j} < 1$ a loss, and $a_{i,j} = 1$ is an ordinary network flow without loss or a gain. This can be shown in figure 7. In this model application, c_{xy} is the transaction costs between the cash account and bonds. There are no transaction costs associated with transferring cash from the bank account to fixed account.

The nodes $a_{i,j}$ are adjusted for the effective rate in according to the period length they are fixed for according the formula:

$$r_{\text{effective rate}} = \left(1+\frac{r}{m}\right)^{T\cdot m} - 1$$ (Eq 29).

Where $r_{\text{effective rate}}$ is the effective rate. r is the nominal interest rate. T is the time in years and m is number of periods per year.

Increase in cash available in each period is dependent on the net cash received from operations in each period. The problem is solved as a set of matrixes and a linear optimization technique. The objective function is described as:

$$Z = \max \sum\sum c_{i,j} \cdot x_{i,j}$$ (Eq. 30).

Where $c_{i,j}$ determines the cash available and the end node. The variable $x_{i,j}$ is the decision variable and tells the amount of cash is transferred from time i to j. There is no constraint on the upper limit of the cash available in the beginning or end of each year. In the model the net cash flow is forecasted at a certain level of confidence and the net cash at time t (where t = 1,2…12) is set as the difference in supply, $S(i)$ is the cash from operations and the demand, $D(i)$ represent the operating expenditures. Where:

$S(i) - D(i) = b_k$ where $i = k$ (Eq. 31).

b_k becomes the difference of the in- and outflow on time k. And therefore can be solved[13] as:

$$\sum_{i,\mathrm{k}\in A} a_{i,\mathrm{k}} \cdot x_{i,\mathrm{k}} - \sum_{k,j\in A} d_{i,\mathrm{k}} \cdot x_{i,j} = b_k \ for\, k = 1, 2, ..., \mathrm{n} \qquad \text{(Eq. 32).}$$

Where k is a synonym of i and j, $a_{i,k}$ is the cash borrowed, conversion rate and interest on time i to k. $d_{i,k}$ is the decision arc multiplier, that determines the direction of flow, from and to nodes.

The upper limit of short term financing is set as:

$$0 \leq x_{i,j} \leq u_{i,j} \qquad \text{(Eq. 33).}$$

Where $u_{i,j}$ is an upper limit of loan on time i to j. Which is deterministic, and it can be assumed that a maximum loan amount is set by management as a rule or limited by a financial institution. Then probability constraint from Eq. 32 is introduced as:

$$\mathrm{m}i, \mathrm{j} \leq \ xij \qquad \text{(Eq. 34).}$$

Where $m_{i.j}$ is the lower limit of the cash account. Where the lower limit is decided from the minimum cash balance in section 3. There is no need to set an upper limit of the cash account as the model is expected to allocate the cash inflow to a highest interest bearing security according to the optimum strategy.

The one of the most important feature of the model is the visual information that is received from the model. This makes it an attractive decision tool in practical cash management and increases the likelihood of managers being willing to implement the model in practise. [14] See figure 8.

The starting position at every node can be set by b_k, or the expected cash flow at time k. Then for example, for an initial cash position of 500.000 NOK at node 49 then b_{49} = 500.000. The model is then imported to the mathematical programming language MPL (or Ilog) and solved by a Cplex300 solver. Where the decision variable $x_{i,j}$ is iterated by the solver to the maximum value of the end node Z (node 55 in figure 8.).

[13] Solution obtained from Rardin (1998)

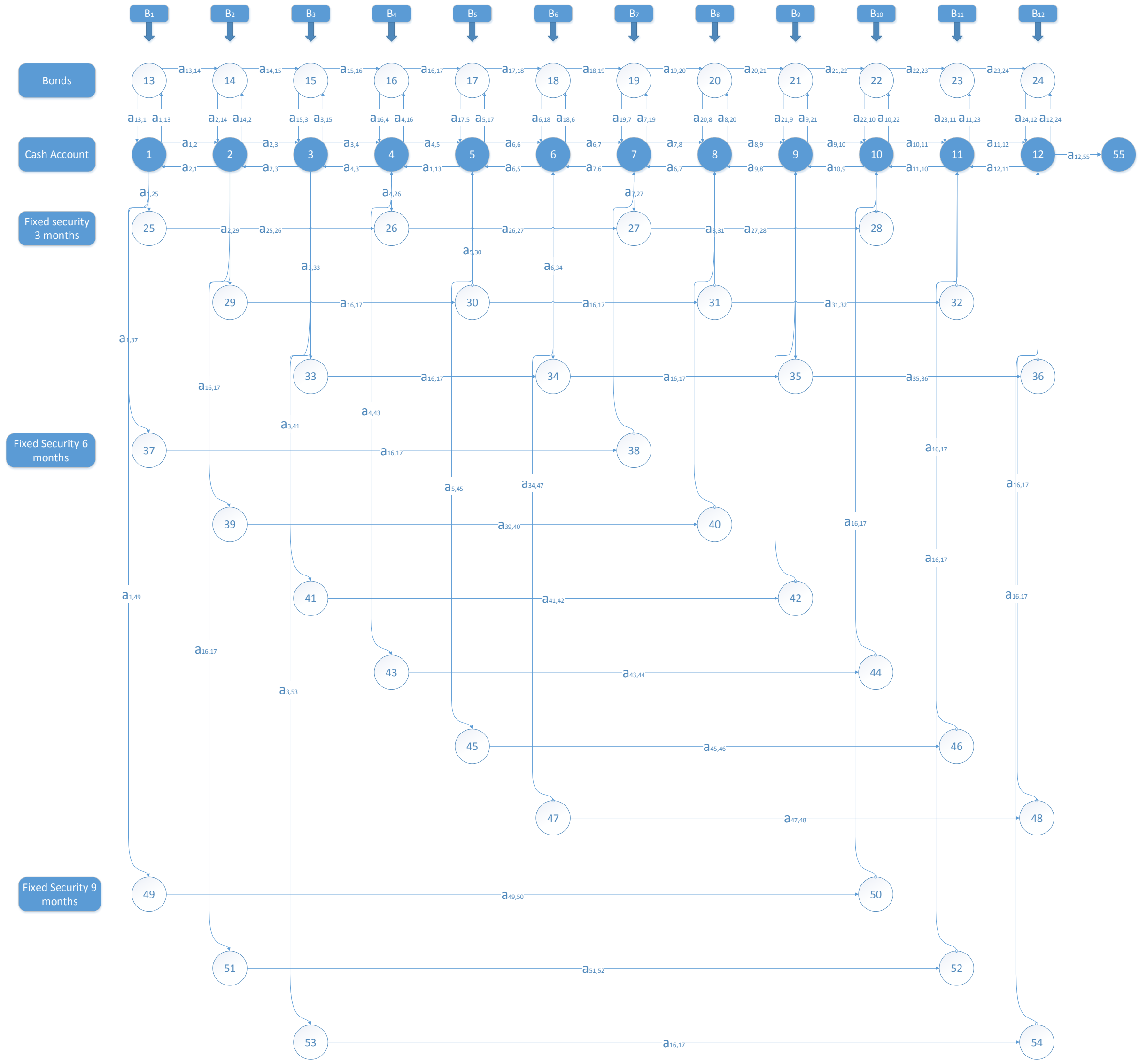


Figure 9. Arc network flow with gains and losses.

5. Case study

In this chapter, the key characteristics that influence the cash flow of an aquaculture company will be identified. The cash flow data of the study are acquired from the Norwegian Directorate of fisheries.

5.1 Aquaculture

The study is viewed from the viewpoint of the person responsible for the cash management of an Aquaculture company, here known as AquaSalmon AS. The uncertainty in the cash flow is estimated by forecasting accuracy and the basic market and operational variables affecting the cash flow of AquaSalmon AS. Variables included in the study:

(1) Atlantic Salmon price (3-6 kg)
(2) Harvesting and sales quantity
(3) Feed costs
(4) Other fixed costs

In the following chapter the, characteristics of the industry and the associated uncertainty is evaluated and measured. It is estimated that the expected value of sales quantity of salmon is 6.700 tonnes for the planning horizon and the following assumptions:

- Data is on a monthly basis, and cash is received at the end of the month.
- Forecasting period is 01.2014-12.2014.
- Historical prices are based on figures from www.fishpool.eu (2006-2014).
- Production and harvesting profiles are based on figures from the Norwegian Directorate of fisheries (2005-2013).
- Output and usage uncertainty from harvesting and feed is fixed for the forecasting period.
- Fixed costs are the same over the whole period, with an assigned standard deviation.
- The feed is purchased on a monthly basis and average inventory is one month.

- The cash conversion cycle (CCC)[14] is 30 days, from harvesting to payment.

Given the fact that good access to information are available for the industry, both costs and revenues on a monthly basis, makes it is a good choice to do the case study. But as Stein et al (2001) pointed out the main drawback of the CFaR method is the general lack of access to information.

5.2 Introduction to aquaculture

Norway became the first country to start salmon farming, where the systematic breeding of salmon started in 1972. Environmental conditions such as long fjords, islands and inlets and relatively stable water temperatures are considered favourable for the production of the Norwegian salmon [18]. The conditions have contributed to make Norway the largest producer of salmon in the world, with an estimated total sales of 1.241.482 tonnes in 2012 with a value of 28.570 million NOK. The Atlantic salmon is a commodity traded globally and is it still experiencing an immense growth in both supply and demand [18].

The biological process in salmon aquaculture can be described in the following steps[15]. [18]

(1) Production of broodstock and roe
(2) Production of fry
(3) Production of smolts
(4) Productions of farmed fish

The objective of a salmon company is to maximize the net present value of its operation. Aquaculture farming involves a considerable amount of risk, therefore the objective is to maximize the expected utility of net present value if the aquaculture farmer is risk averse. [20]

5.2.1 Salmon price and profits

The historical Salmon prices and profits have been characterized by cyclical fluctuation. Andersen et al (2008) concluded that an aquaculture farmer has limited possibilities to respond to price changes in the short run. This can be explained by the fact that, salmon farms have limited flexibility in harvesting of salmon [20]. These limitations, such as, constrained

[14] It usually refers to the time period when the cash is layed out for resources, here we will view it as the time we pay for harvesting.
[15] See Asche and Bjørndal (2011) for more details.

transportation capacity and limited time window of harvesting, influence the high fluctuation of salmon spot prices and can result in a delayed price responsiveness. Oglend and Sikveland (2008) pointed out that volatility increases when the prices are in high period trend. By that notation, it can be observed that if a farmer is responding to high prices of salmon by adding production, he is expected to enter a period of higher price risk. And therefore, higher volatility in the net cash flow. On the contrast, if the production is reduced due to low salmon prices is can be expected that the company will receive less margin in a later time when prices go up again. Due to the delayed price responsiveness of the industry. These cycles can be dealt with by retaining earning from the price tops and reserving cash for the bottom of the cycle [18]. To estimate how much it is necessary keep in reserves, it is important to have active cash management procedures and policies within the company.

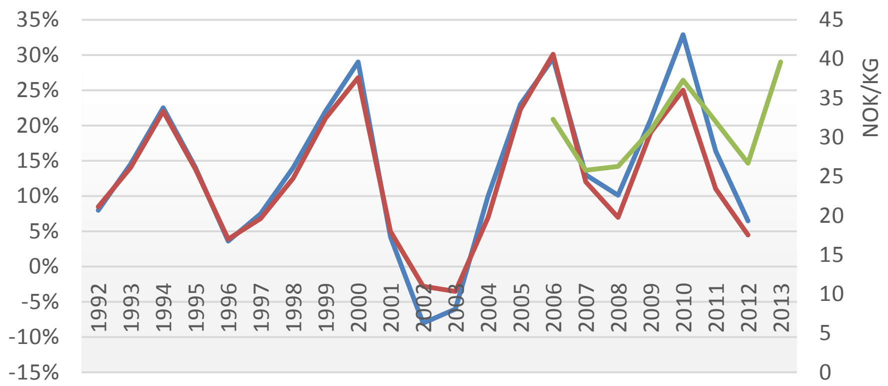


Figure 10. Operating profit and Return on Total assets for the industry as whole. Source Norwegian Directorate of fisheries. Average salmon prices 2006-2013.Source fishpool.eu.

Figure 10 demonstrates the cycles and fluctuation in the operating margin of the industry. As would be expected the operating margin follows the price movement of salmon. These movements are not perfectly synchronized [18], and can partially be explained by the constraints in production that delay the response to price changes

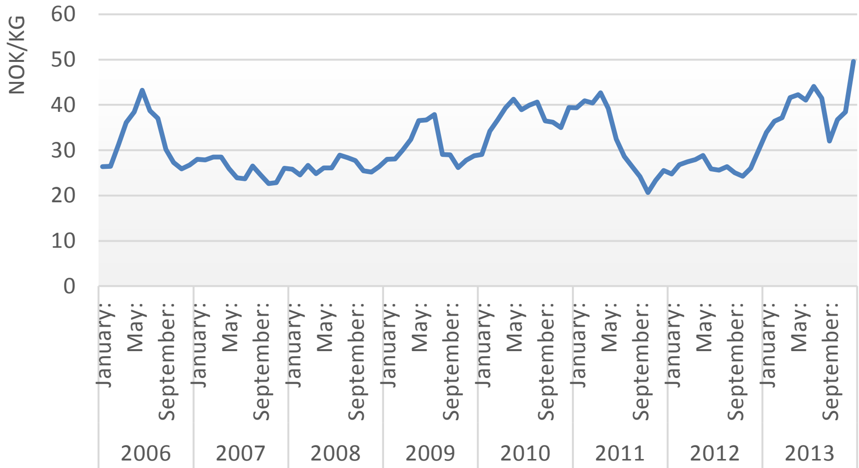


Figure 11. Norwegian Atlantic salmon Spot Prices, 2006-2013.Source www.fishpool.eu

5.3 Seasonality in price and harvesting

There is considerable seasonality in the price and the harvesting of salmon within the year, as can be seen by the seasonality index (see figure 15). The two variables have a correlation coefficient of -0,64. Simply put, when there is an increase in sales quantities (harvesting) the price goes down. The sales revenues from salmon are smoothed out for the whole year, but with a top in December.

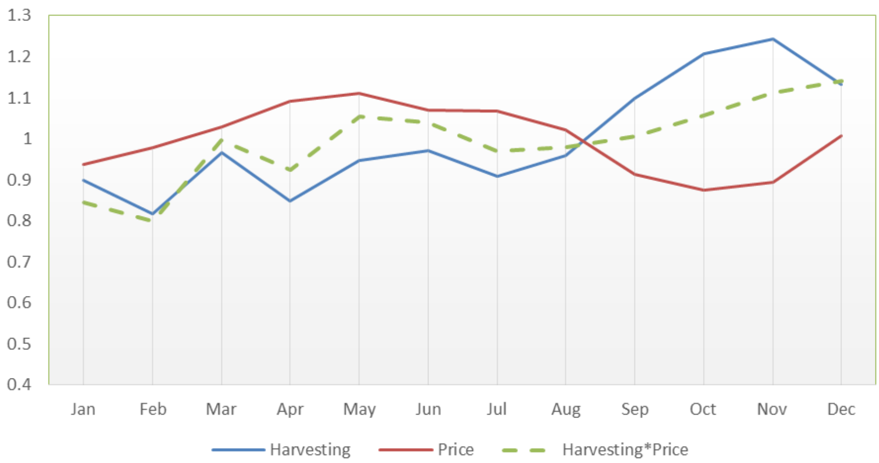


Figure 12. Total sales by months.

5.3.1 Aquaculture risk management

It could be argued that the general aquaculture farmers is a risk-seeker. As price and profits in the industry has historically been very volatile, and by entering the industry you are willing to take on risk. The different kind of risk management strategies and plans have to be considered in the industry. Bergfjord (2009) published as survey where farmer rated the importance of different risk strategies in their companies. According to the study, producing at the lowest possible cost is considered to be of highest importance. Abide by, preventing deceases and fish escape, prioritize solidity (low debt/equity ratio) and to prioritize liquidity.

The median number of employees on an aquaculture farm are 6 employees, while the largest one employs approximately 3000. It can be assumed, by the sheer size difference that there it must be different amounts of risk appetite within the industry. The larger companies have better ability to diversify their overall risk, for example by; producing in different locations, have a wider range of products and expected to have a more diversified customer base. While the smaller farmer has limited ability to diversify, do to his smaller production. It could then be assumed that a smaller farmers are more risk averse than the larger ones and therefore less willing to reserve their cash resources provided from operations in risky assets. A farmer with larger operations would also have better ability to invest their capital resources in a diversified portfolio. For example, diversify by investing in a negatively correlated portfolio to salmon prices or in multiple investment portfolios. Due to the high fluctuation in the price and profits in the industry, it is important to keep liquid cash resources for reserves for the period of lower prices.

Bergfjord (2009) demonstrated in a su, that farmers consider the future price of salmon the biggest individual source of risk. Guttormsen (1999) presented a paper with the purpose of finding good forecasting methods for salmon prices[16]. The Vector Auto Regression (VAR) method proved to produce the least MAPE for most weight and age categories. While the Classical Additive Decomposition (CAD) produced the best ratio of an accurate forecast. The Holt Winter (exponential smoothing) method underperformed other methods in the study, by consistently overestimating the prices. Oglend et al (2008) found that an AR(5)-GARCH(1,1) process describes the price process best. But as Guttormsen (1999) points out, a simple forecasting techniques might be a better tool in practice for risk management, as they are easier to understand and implement. And require limited knowledge of statistics and econometrics.

[16] Guttormsen (1999) tested The Classical Additive Decomposition (CAD), Holt-Winters Exponential Smoothing (HW) Auto Regressive Moving Average (ARMA), Vector Auto Regression and two versions of naïve methods. Based on weekly prices from 1992-97.

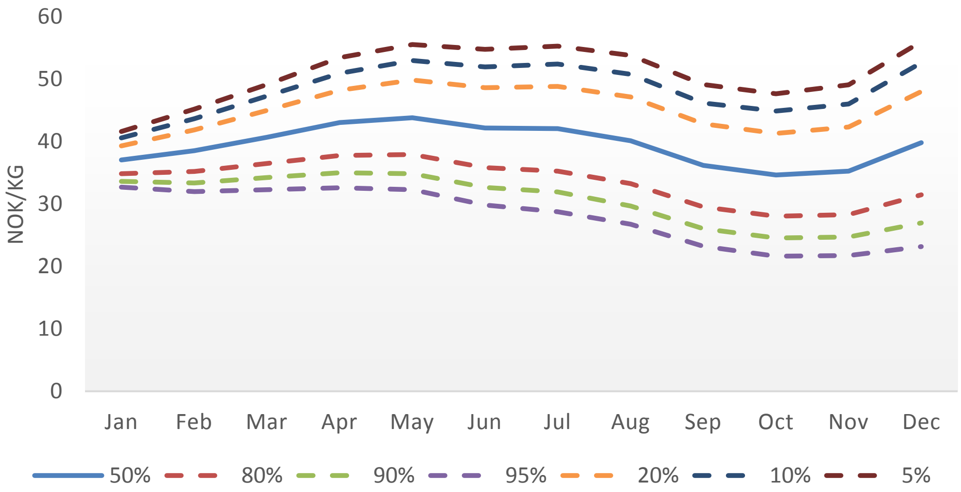


Figure 13. Random walk with seasonal dummy variables for salmon spot prices for 2014.

Figure 13 is the random walk for salmon, with a forecasting period based on historical fluctuation and with the drift factor treated as seasonal dummy variables[17]. Under the assumption that the standard deviation increases by $\sqrt{t}\ \sigma$, with t being 1 month. The probability lines in figure 13, are the cumulative probabilities at each month. Where the level of confidence is approached from the right side of the distribution. That is, for the 90% line at time t; that the prediction expects that there it is a 90 % probability that the price will be higher than that specific value at time t.

Ogeland et al (2008) noted that the market is chronically missing stability and predictability, and that forecast cannot be expected to be accurate. The salmon price risk can in fact be minimized by the use of future contracts as some companies have entered with buyers. The motivation for selling the salmon by future contracts is to minimize the associated risk of selling spot, as salmon prices have been historically very volatile. By trading with futures, it would then decrease the deviation in the forecasted net cash flow, but on the other hand, risking selling at a lower or even higher prices than could be received by selling spot.

There is an active derivatives market for the Atlantic salmon. Salmon Fish pool ASA is a regulated market for derivative with fish and seafood at a global level. It started exchanging in May 2006 and is located in Bergen, Norway. With the objective to become the leading global

[17] The seasonal dummy variable is calcualted by summing up the average price in each month and then dividing by the global average price.

exchange of fish products. The first product offered is financial salmon contract – forwards, futures and options. The prices reflect the expected future price for fresh, gutted salmon 3-6 kg, superior quality (in 1 ton lots) delivered free carrier (FCA) Oslo, Norway. There is a membership access to through Norsk Opjon Sentral (NOS) Clearing ASA, where approved members can actively trade on the exchange. [5]

5.3.2 Salmon price forecast

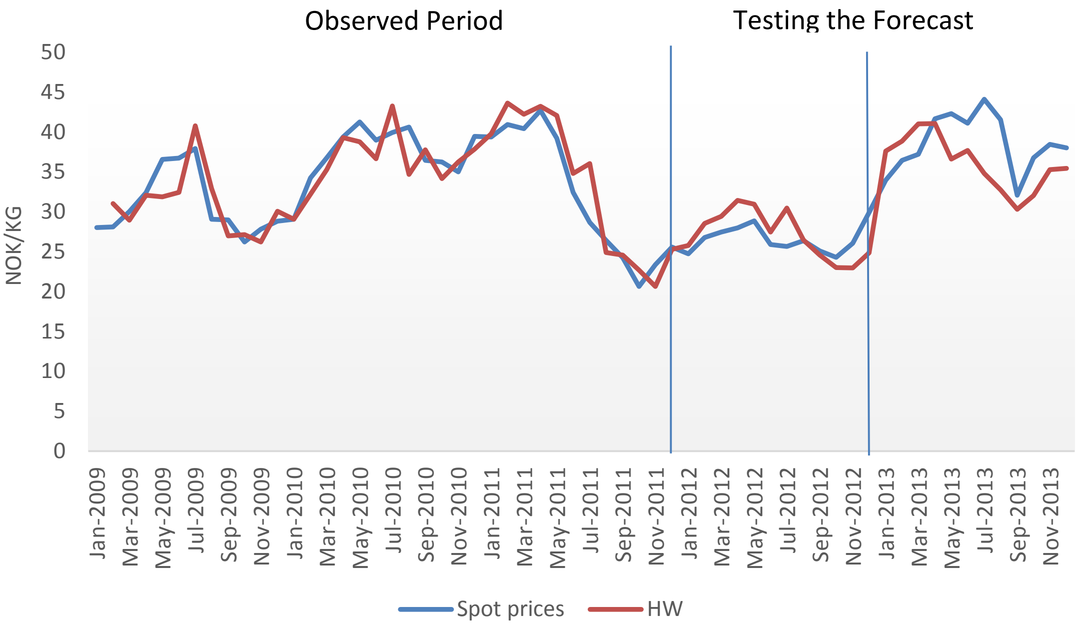


Figure 14. Back testing the price forecast (HW). Price source Fishpool.eu 2006-2013.

The accuracy of the market prediction (forward prices) was tested and compared to the spot prices. This was done by looking at the forward prices in December of the previous year and then the forward prices treated as a forecast to the spot prices. It is interesting to note that the market prediction based on this specific period had MAPE of 21,3% (average for both 1 year periods). The HW outperformed the market prediction in accuracy for this period. The error of the HW method was calculated as average 5,70% over the 2 year period (see figure 14) by forecasting for 2012 and then 2013 and compared with actual spot prices.

When using the exponential smoothing (HW) forecasting, resulted in the average price of 40,5 NOK/kg for 2014 (figure 15), which is considerably over the 7 year average of 31,5 NOK/kg. The forecast is based on price figures from January 2006 to August 2013.

Forecasting prices with data up to December 2013 were also tested, but it resulted in an average price over 52 NOK/kg, which would be by a long way over the 7 years average.

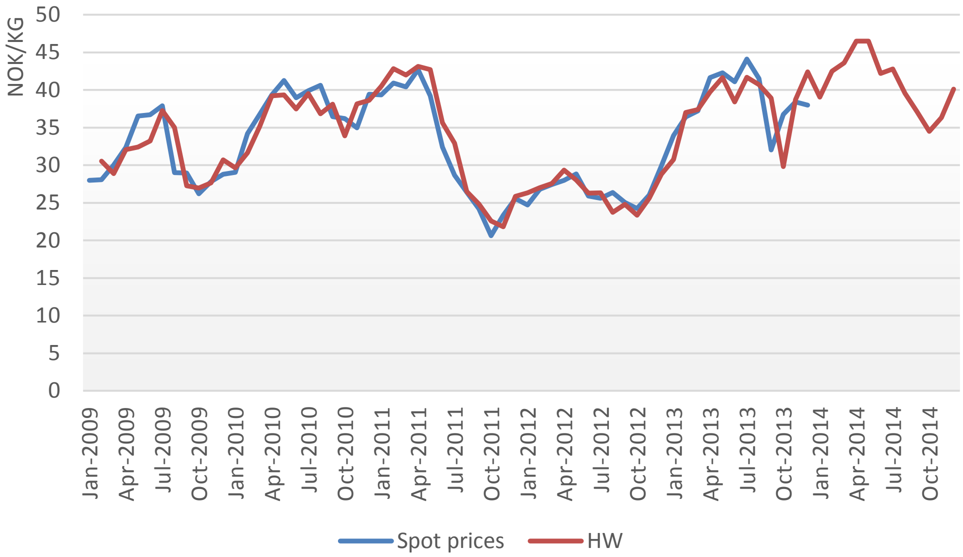


Figure 15. Forecasted prices for 2014 with exponential smoothing (HW).

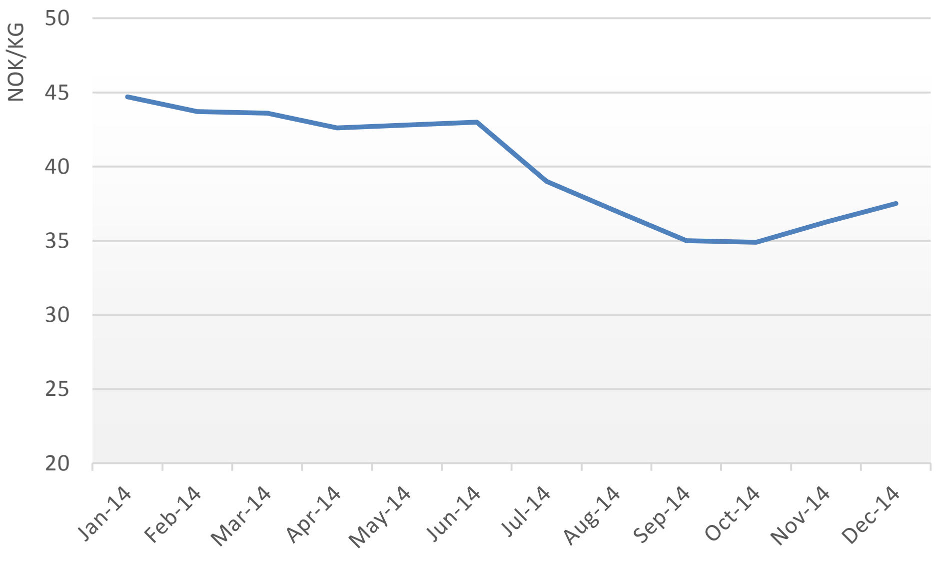


Figure 16. Forward prices as published 16.12.2013, source [www.fishpool.no.](http://www.fishpool.no)

5.4 Biomass production and slaughtering

Guttormsen (2008) concluded that a well developed production plan can be the difference between a loss and profit in the operation of an aquaculture farm. And that the timing of harvesting and the distribution of limited resources to production costs has a significant impact on the cash flow.

Asheim, Dahl, Kumbhakar, Oglend and Tveteras (2011) published a paper with the goal to answer the question whether the price of farmed salmon is driven by the biology of the industry or the short term supply of salmon. They concluded that the price of salmon or the feed has no significant effect on the short term harvest supply. Loland, Aldrin, Steinbakk and Huseby (2011) introduced a statistical model to predict the production and slaughtering of Norwegian farmed Salmon. They pointed out that cycles for producing salmon are rather predetermined and that the production cycle involves systematic fluctuation in biomass and the quantity of slaughtered fish. They also pointed out that the slaughtered biomass have been historically more variable than the biomass production quantities. Their model is a deterministic one, where the cycles are determined by a pair of cosine and sine function, but the underlying variables are stochastic. Asheim et al (2011) concluded that the biomass of live salmon and seasonal factors have a statistically significant influence on the harvest quantity.

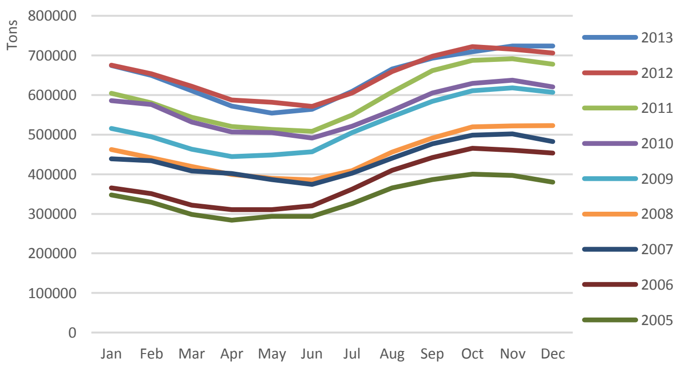


Figure 17. Total Biomass production. Source Norwegian Directorate of fisheries.

The fish is usually between 1,5 to 3 years old when it is slaughtered, and that gives a rather predictable growth horizon, since the growth is largely systematic and seasonal (see figure 14). Asheim et al (2011) also concluded that sea temperature is the most influential factor in the production of live biomass, this can be explained by the fact that the salmon feeds best at warmer sea temperatures. [18]

Asheim et al (2011) concluded that the biomass of live salmon and seasonal factors have a statistically significant influence on the harvest quantity. When calculating the seasonality index for slaughtering quantities, it is shown that there is considerable seasonality in the data, with a yearly top in November. It can be assumed that this is due to the fact that there is a potential increase in demand for salmon in December and that the salmon is at a desirable weight for harvesting in the period.

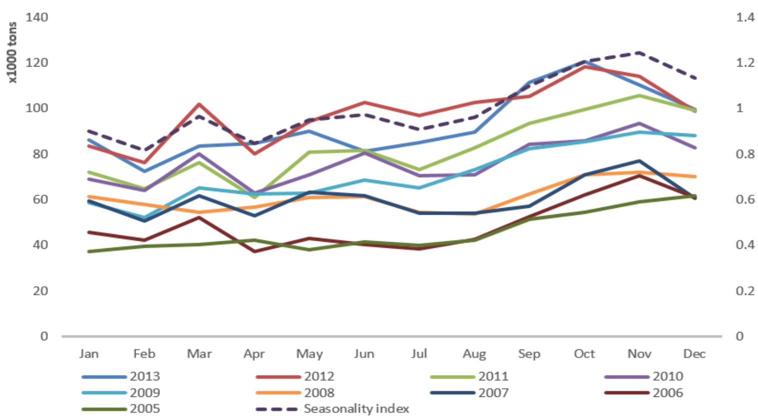


Figure 18. Harvesting of live biomass. Source Norwegian Directorate of fisheries.

5.4.1 Harvesting forecast

AquaSalmon AS has an estimation of total slaughtering output of 6.700 tonnes a year. With a monthly error of 5,9%, which are based on the accuracy of the forecasting method that is used to forecast the biomass slaughtering in the total market.

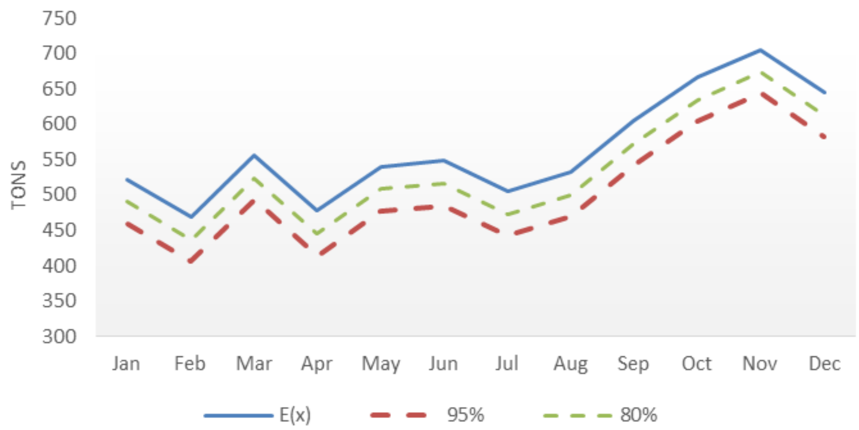


Figure 19. Estimated sales quantity.

It is assumed that prediction interval stays the same over the prediction period. This is can be explained by the fact that the slaughtering is estimated to be a fixed quantity over the whole year and related to the underlying seasonal short-time demand and selective harvesting. The forecast is then the expected value in each month, normally distributed, and with a standard deviation of 38,7 tons. In practical circumstances, it could be justified that the harvesting is really dependent on the harvesting of the prior month. This could be in the case of deceases and lower survival rate in the biomass production, such properties are neglected in the study. Potential business disruption in operations, production risks or crop risk, can be insured to some point.

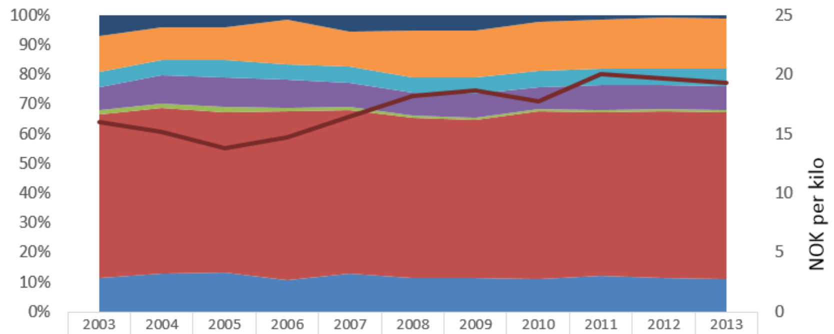
Sales revenues from slaughtered biomass are described as:

$$V_t = p(w)(0{,}99)\,\mathrm{B}_{t+1} \qquad \text{(Eq. 35).}$$

Where t = 0,1,2…12 and B_{t+1} is the biomass weight in month t+1 and $p(w)$ is the price per kilogram and 0,99 is the empirical constant that is used to correct the biomass weight for sales weight. Which is due to the fact that the fish loses its weight and is starved in the slaughtering process. [18]

5.4.2 Production costs

The costs of production (see figure 20) have been increasing again in nominal price since 2008, after many years of cost reduction[18]. With feed being the largest individual production cost.



	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Financial Cost	1.12	0.63	0.55	0.23	0.93	0.93	0.95	0.39	0.29	0.19	0.22
Other operating costs	1.94	1.68	1.52	2.23	1.91	2.85	2.94	2.94	3.30	3.37	3.26
Depreciation	0.81	0.76	0.83	0.74	0.89	0.95	1.08	1.01	1.16	1.09	1.15
Wages	1.23	1.42	1.38	1.43	1.38	1.43	1.46	1.31	1.69	1.61	1.55
Insurance	0.26	0.25	0.22	0.16	0.15	0.15	0.15	0.14	0.15	0.14	0.12
Feed Costs	8.81	8.47	7.46	8.36	9.07	9.76	9.95	10.01	10.97	11.02	10.85
Smolt costs	1.85	1.94	1.85	1.58	2.13	2.09	2.13	1.97	2.45	2.28	2.16
Production Cost	16.02	15.15	13.81	14.73	16.46	18.16	18.66	17.76	20.00	19.70	19.31

Figure 20. Production cost on a yearly basis. Source Norwegian Directorate of fisheries.

[18] For more information see fiskeridir.no

During the years 2008 to 2013 the feed cost represented about 50 to 60 percent of the total production costs. In this case study, the feed costs are estimated to be 53 percent of the total production costs of 20 NOK/kg or 10,6 NOK/kg fixed for the entire year and with a standard deviation of 5,5%. Which is the accuracy of our method of the prediction method. The feed costs are distributed monthly according the seasonal smoothing constants that are obtained from the exponential smoothing forecasting.

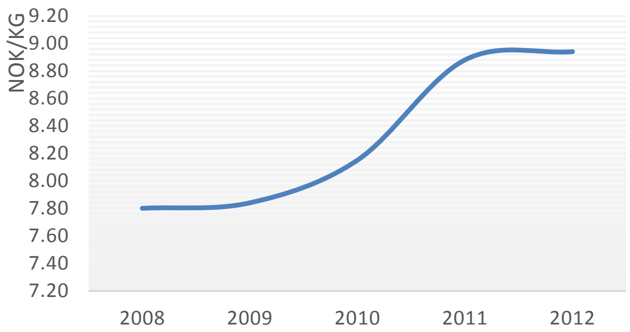


Figure 21. The price of feed 2008-2012. Source Norwegian Directorate of fisheries.

The harvesting cost is estimated to be fixed at 2,5 NOK/kg[19] and transportation cost 3,0NOK/kg. The feed is not a tradable commodity, but purchased from a feed producer at a fixed or negotiated price[20]. It is then expected that the feed is purchased at fixed price on a monthly basis, with limited levels of fluctuation.

The salmon feeds best at hotter sea temperatures and some research indicate that it does not feed at all during the winter season. And if the sea temperature is lower over the year than expected, then the fish will feed less over the year. [20] It is known that feeding influences the growth rate of the salmon. [18]

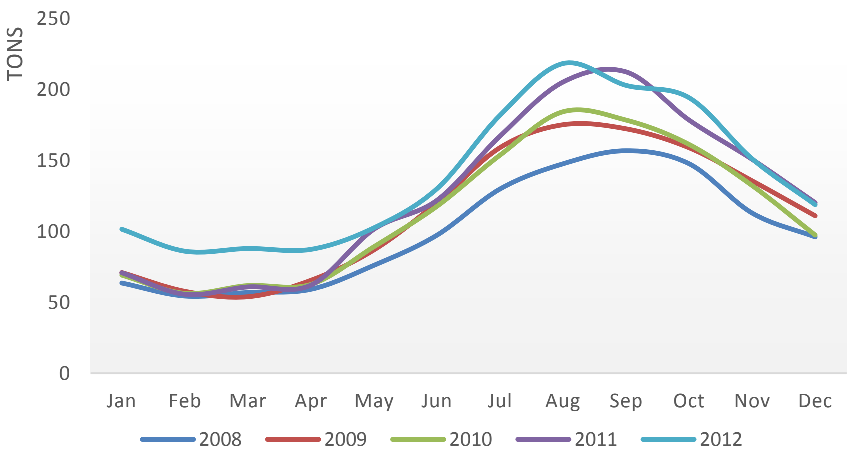


Figure 22. The use of feed, 2008-2012. Source Norwegian Directorate of fisheries.

[19] Authors judgement, based on average figures in the industry 2008-2012
[20] This information was received from feed production specialists

Guttormsen (2002) pointed out that that feed costs can be treated as the only variable factor in the short run. But the feed quantities used in the production also depend on the sea temperature and potential production disruption, deceases and the survival rate in the production [47]. The conversion ratio associated with feed costs can be described by the function:

$$f_t = \frac{F(t)}{w'(t)} \qquad \text{(Eq. 36).}$$

Where $F(t)$ is the feed quantity and $w'(t)$ is the weight change of the salmon. This ration is the relation between feed quantity and the growth of the salmon. [18]

5.5 Case study parameters

When all the factors are taken together, it can be observed that aquaculture farming is characterized by cycles and fluctuation and the need for effective cash management is considerable. The data indicated that the potential short term high margin is followed by a period of lower margin.

In the case study of Aquasalmon AS the uncertainty parameters are summarized in table 4.

AquaSalmon AS	Assigned standard deviation
Salmon price	5,7%
Sales quantity	5.7%
Feed costs	5.5%
Harvesting costs	4.6%
Fixed costs	4,0%

Table 4. Case study uncertainty parameters

AquaSalmon AS assumes that they can predict their sales and costs by some level of confidence (See table 4). It is also assumed that Aquasalmon AS sells part of their harvest by contracts with an estimated average price of 39.5 NOK/kg (see figure 16), and the rest is sold spot with an average price of 40.5 NOK/kg (see figure 15). The yearly harvesting quantity of 6.700 tonnes of salmon is expected to be distributed over the planning period according to the exponential smoothing function. Transportation and harvesting costs are estimated to be a fixed function of harvesting. Feed costs of follows the seasonal components according to the seasonal exponential smoothing function (HW) and are based on the expected biomass production. Other production costs are expected to be fixed on a monthly basis with the assigned standard deviation of 4%. It is estimated that Aquasalmon AS has 2 million a month in other cash payments, such as, taxes, fees, dividends, legal costs and loan back payments. The cash flow is simulated with 100.000 iterations in @Risk from Palisade. This can be seen in figure 23.

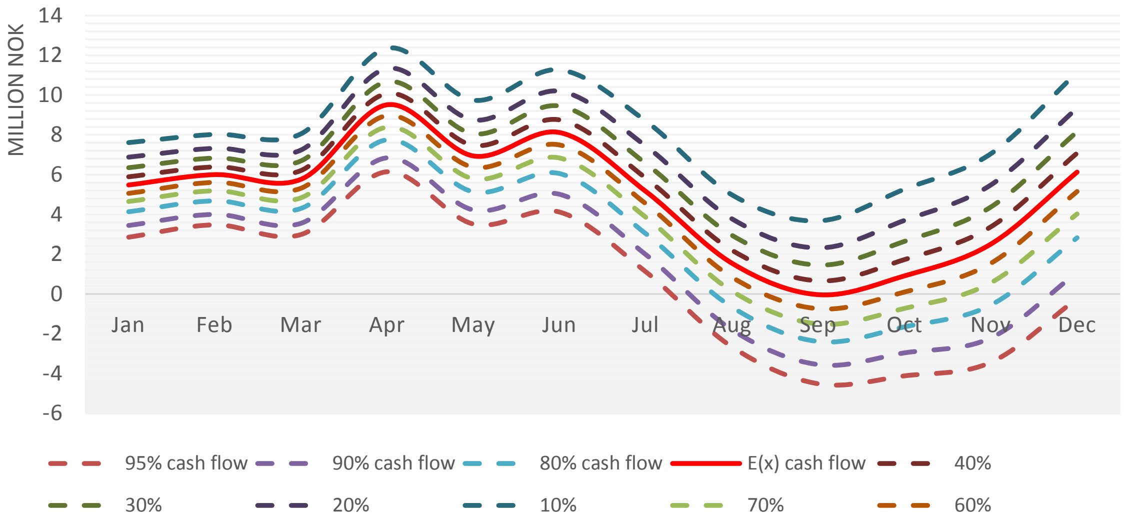


Figure 23. The expected cash flow and the associated level of confidence.

In figure 23, it can be shown how the uncertainty in the cash flow increases with time. This can be explained by the fact that the uncertainty in the Salmon prices increases with time. Figure 23, illustrates the importance of a rolling planning horizon[21]. As it is known that the level of uncertainty decreases when the time in question comes closer.

Tail	Jan	Feb	Mar	Apr	Mai	Jun	Jul	Aug	Sept	Okt	Nov	Dec
95%<	2.854	3.475	2.988	6.146	3.512	4.115	1.104	-2.677	-4.526	-4.124	-3.442	-236
90%<	3.443	3.998	3.553	6.844	4.217	5.006	1.968	-1.815	-3.558	-2.964	-2.211	1.077
80%<	4.134	4.583	4.303	7.744	5.132	6.052	3.020	-674	-2.398	-1.658	-560	2.824
E[X]	5.475	5.995	5.763	9.511	6.934	8.112	5.143	1.529	-46	904	2500	6.122
Forecast	5.362	6.025	5.791	9.524	7.008	8.112	5.279	1.576	-18	1.003	2.453	6.122
Prob FCF<0	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,7%	28,0%	51.3%	38,5%	24,9%	5,0%

Table 5. Table of the expected cash flow of AquaSalmon AS and probabilities of negative cash flow

[21] See chapter 3.1

The minimum cash account is set by equation 24. Then, by longer planning horizons the model sets a higher minimum cash account due to the higher level of uncertainty later in the planning horizon. This also explained by the higher probability of negative cash flow (Table 5). Therefore, accurate investment decision cannot effectively be made for the future by medium term planning. In figure 24, the bars represent different minimum cash accounts given level of confidence.

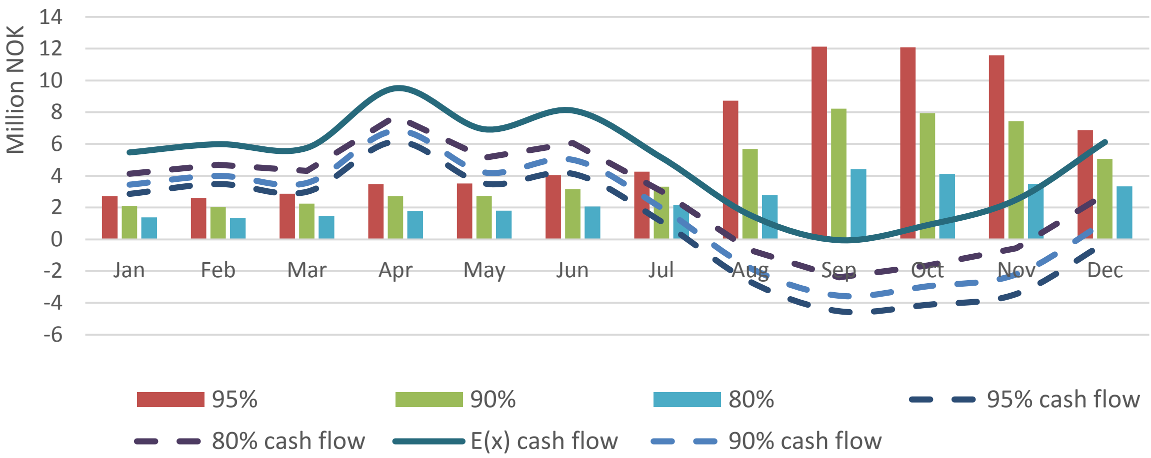


Figure 24. Minimum Cash account of Aquasalmon AS.

The minimum cash account can now be decided as a probability boundary. It can be seen that the minimum cash account covers the precautionary minimum up to a certain level of confidence, plus the opportunistic requirement. The opportunistic requirement should be set by management or could be calculated as a function of the uncertainty in the cash flow. By this, AquaSalmon AS has the resources to deal with the potential shortfalls in the cash flow and the potential opportunities that could present themselves over the planning horizon.

In this case (fig. 24), the opportunistic requirement is set as $\alpha\sigma$[22] of the net cash flow volatility at each time and the same level of confidence as the precautionary requirement. But as stated earlier, the minimum cash account is not expected to be fixed over the entire planning period, as the potential deviation decreases when the time in question is approached. Now it can be determined, how much cash Aquasalmon AS cannot afford to invest at future dates. As the level of cash on the minimum cash account has been set for the planning period.

[22] Where α is the chosen level of confidence

In the study the expected values of the investment options are used, it can be argued by the strong law of large numbers, together with the central limit theorem. This would suggest that over a large period the distribution of price change for any product and any market should converge to the expected value. The return of the fixed accounts is usually negotiated with the financial institution or quoted at fixed yield for some period of time. Bonds with a longer maturity than the planning horizon would have a higher duration and therefore involve interest rate risk to the model. Therefore, bonds with maturity less than 1 year are included. In table 6, is the initial cash position of AquaSalmon AS and the investment options available to the company can be shown. The company has an initial cash position of 5,5 million NOK and a bond balance of 1,5 million.

	Type	Effective	Nominal	Arc value	Initial position
Bank account					
1 month		0.5%	0.499%	1.00042	5.500.000 NOK
1 month	Financing	10.5%	8.186%	0.99133	
Fixed account					
3 months		3.30%	3.251%	1.00825	0 NOK
6 months		3.40%	3.348%	1.01700	0 NOK
9 months		3.50%	3.445%	1.02625	0 NOK
Security					
1 month bonds		1,40%	1.39%	1.00117	1.500.000 NOK
Transaction cost		0.10%		0.99900	

Table 6. Account and investment parameters of the model. Source www.financeportalen.no, www.OsloBors.no.

Aquasalmon AS is risk neutral and does not want to increase their total CFaR by investing in risky assets. But deposits its cash surplus on a fixed period bank accounts and government zero coupon bonds, with maturity within a year. By this, there is no increase in counterparty or financial risk and the overall CFaR does not increase with investments. The government bonds are expected to have lower return than the bank account as the probability of default by the government is lower than the banks. Also, the fact that they are more liquid that the fixed accounts. The maximum short term borrowing capacity is set at 10.000 million NOK, $u_{i,j} <$ 10.000.

6. Results

The optimization is solved in MPL, a mathematical programming software from Maximal Software and solved with Cplex300 solver. The computing time for the model is about 1-1.5 minutes, where the solution takes only about 2 seconds to calculate. This is computed on a Quad Core Intel i5 processor that runs at 1,8 GHz. The rest of the time required is the time it takes to read and write the data between the Excel database and the programming software. The planning horizon is 1 year on a monthly basis and each node represents the parallel month, that is, node 1 is January and node 2 February and so on. The arc´s between the nodes are the cash resources that are transferred between the nodes. When optimizing for 80% confidence on the minimum cash account, and the expected cash flow the resulting arc set is obtained in figure 25. As the model is maximizing the cash at the end node. The top bars represent the net cash flow at each period

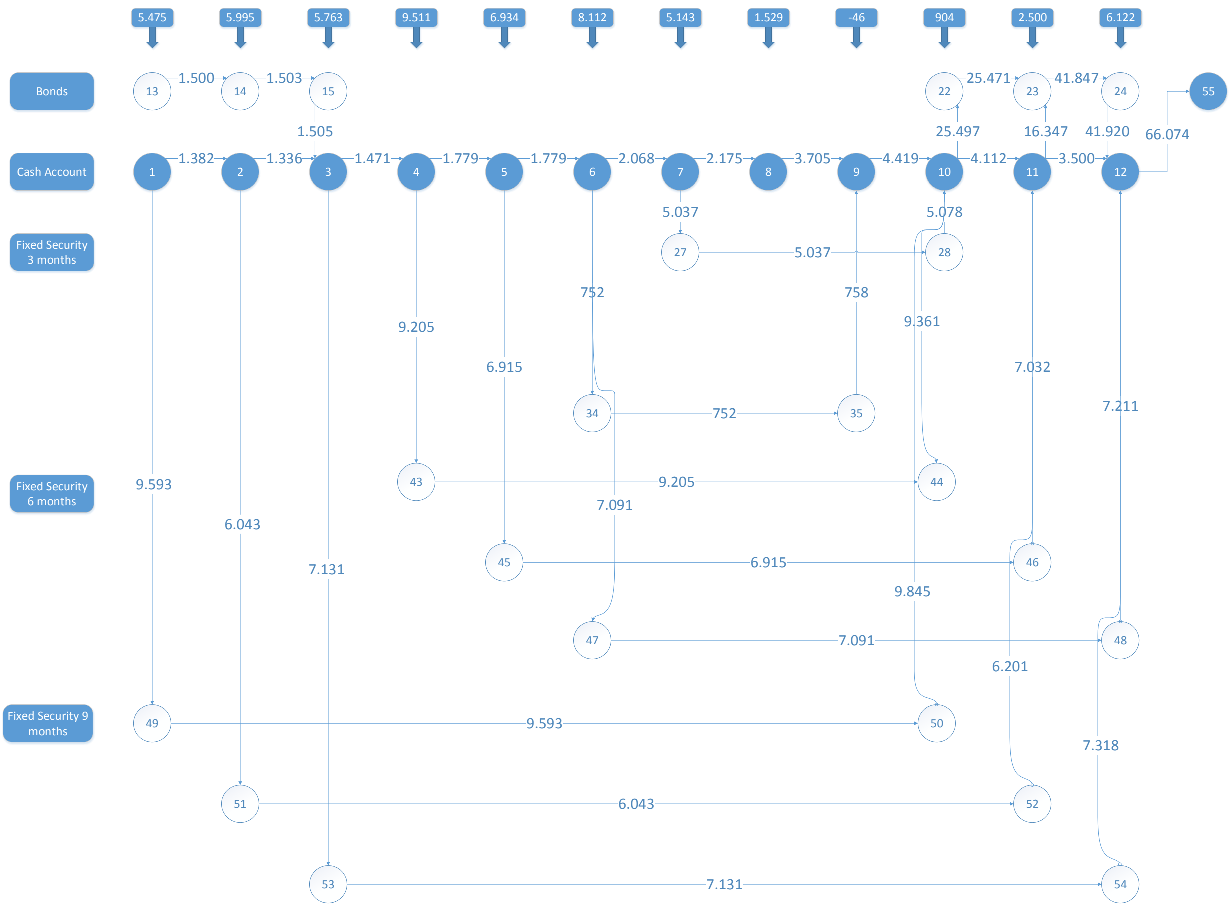


Figure 25. The optimized model with 80% level of confidence on the minimum cash account and the expected cash flow.

For the net cash flow of 64,942 million NOK, the model expects a net cash balance 66,074 million NOK compared to 65,134 million NOK without any optimization, where the cash lies in the cash for the entire planning periods. This results in 1,76% increase in net cash at the end node.

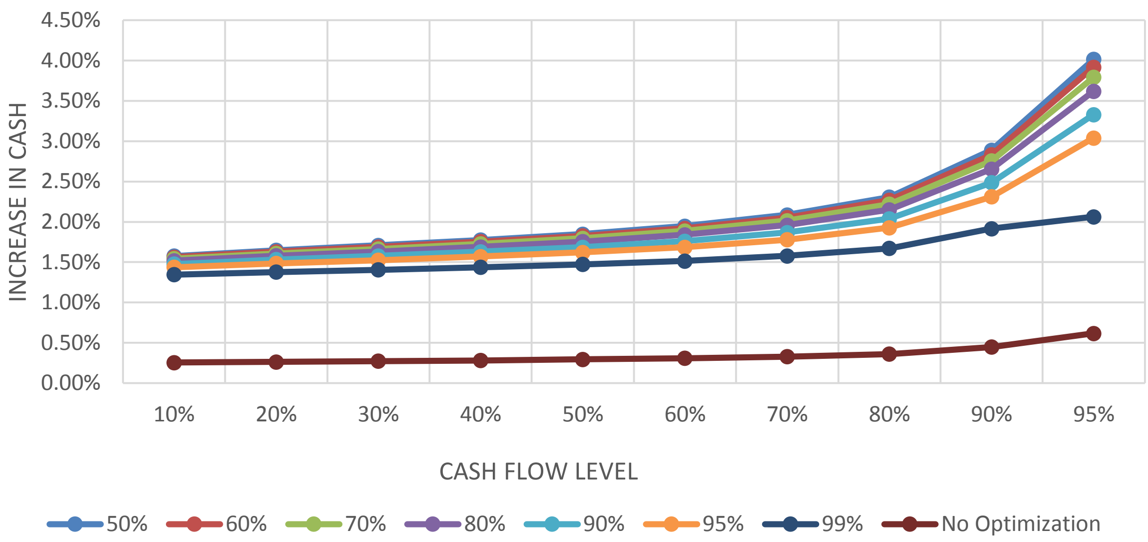


Figure 26. Changes in the cash flow and cash account

The lines (colored lines, 50-99%) in figure 26 represent the return associated with the chosen level of confidence in the minimum cash account. It is interesting to note that when the cash flow is lower, the return increases and even outperforms the highest interest bearing asset class. This can be explained by the fact, that the cash flow is positive for the first months of the year, then when the cash flow turns negative for the second half of the year the previous cash surplus bears interest.

		Minimum cash account							
Δr=0	50%	60%	70%	80%	90%	95%	99%	No Opt.	CF
10%	106.533	106.518	106.499	106.472	106.429	106.387	106.291	105.147	104.879
20%	92.263	92.248	92.228	92.202	92.159	92.116	92.019	91.008	90.768
30%	82.508	82.492	82.472	82.446	82.402	82.359	82.262	81.344	81.122
40%	73.847	73.932	73.812	73.785	73.741	73.699	73.602	72.764	72.559
Cashflow E(X)	66.143	66.128	66.109	**66.074**[23]	66.037	65.995	65.898	65.134	64.942
60%	58.341	58.325	58.306	58.279	58.234	58.191	58.093	57.403	57.226
70%	50.020	50.004	49.985	49.958	49.913	49.870	49.771	49.159	48.998
80%	40.525	40.510	40.490	40.463	40.419	40.375	40.273	39,754	39.611
90%	27.324	27.309	27.289	26.262	27.217	27.171	27.066	26.676	26.557
95%	16.838	16.822	16.802	16.774	16.727	16.680	16.522	16.288	16.188
99%	**-2.604**[24]	Inf	Inf	Inf	Inf	Inf	Inf	-2.958	-3.021

Table 7. Table of changes in the minimum cash account and the cash flow. In million NOK

For the cash flow at the 10%[25] probability the gain from optimizing the cash flow is 106,533 million NOK associated with no minimum cash on the cash account, compared to the net cash flow of 104,879 million NOK. The expected gain in percentages is higher for the lower cash flow, but measured in NOK the capital gains increases with more cash flow.

It is important to note that the amount that the model expects to hold on the cash account decreases when the time in question comes closer. By this, it can be known that model is always underestimating the expected value in the final node. As the cash account returns the lowest yield. This can be considered the main limitation of the model as the uncertainty of the model increases with time, due to the price uncertainty of salmon.

[23] Arc set in figure 25
[24] Arc set in figure 28
[25] See figure 20

6.1 Shadow prices

Figure 23 shows how much can be gained from 1 extra unit of cash (1 NOK) in each period (month) with respect to the final value. The shadow prices converge to zero when the end of the planning horizon comes closer, this is explained by the fact that the cash has less time to earn interest.

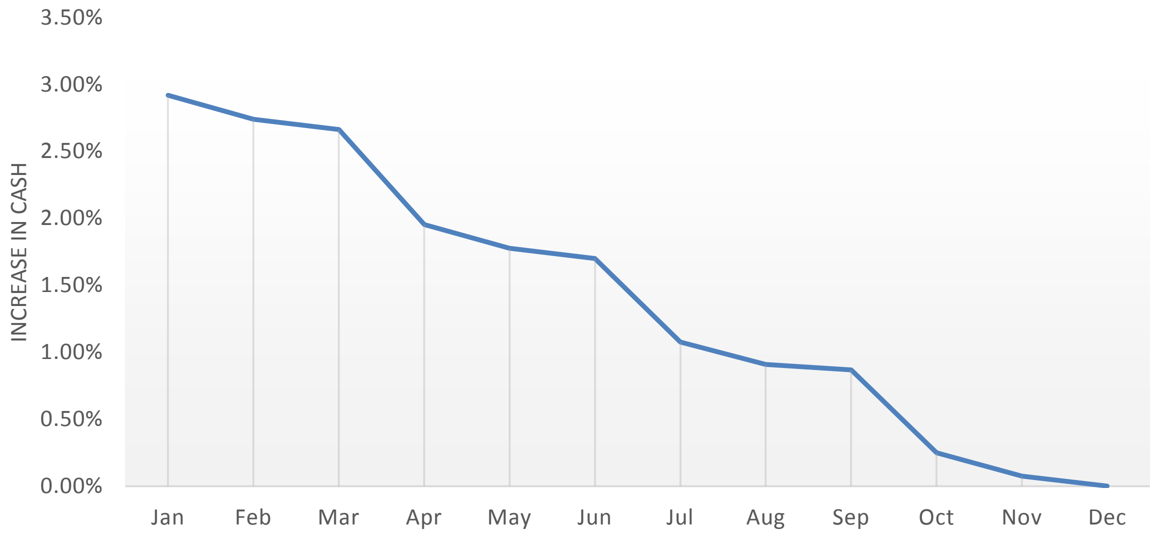


Figure 27. Shadow prices, expected cash flow and 80% confidence on the minimum cash account.

By this it can be concluded that all extra cash above the expected value for the cash flow will be directed to the highest interest bearing asset available. The model has four asset classes, bank account (C1), bonds (C2), fixed deposit for 3 months (C3) fixed deposit for 6 months (C4) and fixed deposit for 9 months (C5).

As C1<C2<C3<C4<C5, and since $CFaR_t > 0$, for t = 1, 2…12. Then, all extra cash at time *t* will allocated to the highest interest bearing asset available. This holds true in the case when the net cash flow is positive for the whole period. Then, by this notation, it can be seen that it would be sensible to assume a lower cash flow than the expected value, when planning. For example at the 80% level or 90% level[26].

In the scenario when there is the probability of a negative net cash flow, the cash account can be used to compensate for the loss from operations.

[26] From figure 20.

6.2 Changes in the cash flow

Multiple sensitivity analysis can be performed on the model. The short computing time enables the possibility of extensive sensitivity analysis. When optimizing the cash flow at the 99% level where the net cash flow of the year is expected to be negative, some interesting characteristics are revealed.

For the cash flow at the 99% confidence[27] level the model delivers a 13.91% better return on the net cash flow of the year, -2.604.000 NOK compared to the -3.021.420 NOK net cash flow of the year. This is an interesting characteristics as the highest interest bearing asset (9 months fixed account) returns only 3,5% interest per year. The model expects an overdraft at period 12, which is borrowed from next year. Where the loan interests are paid in the next year.

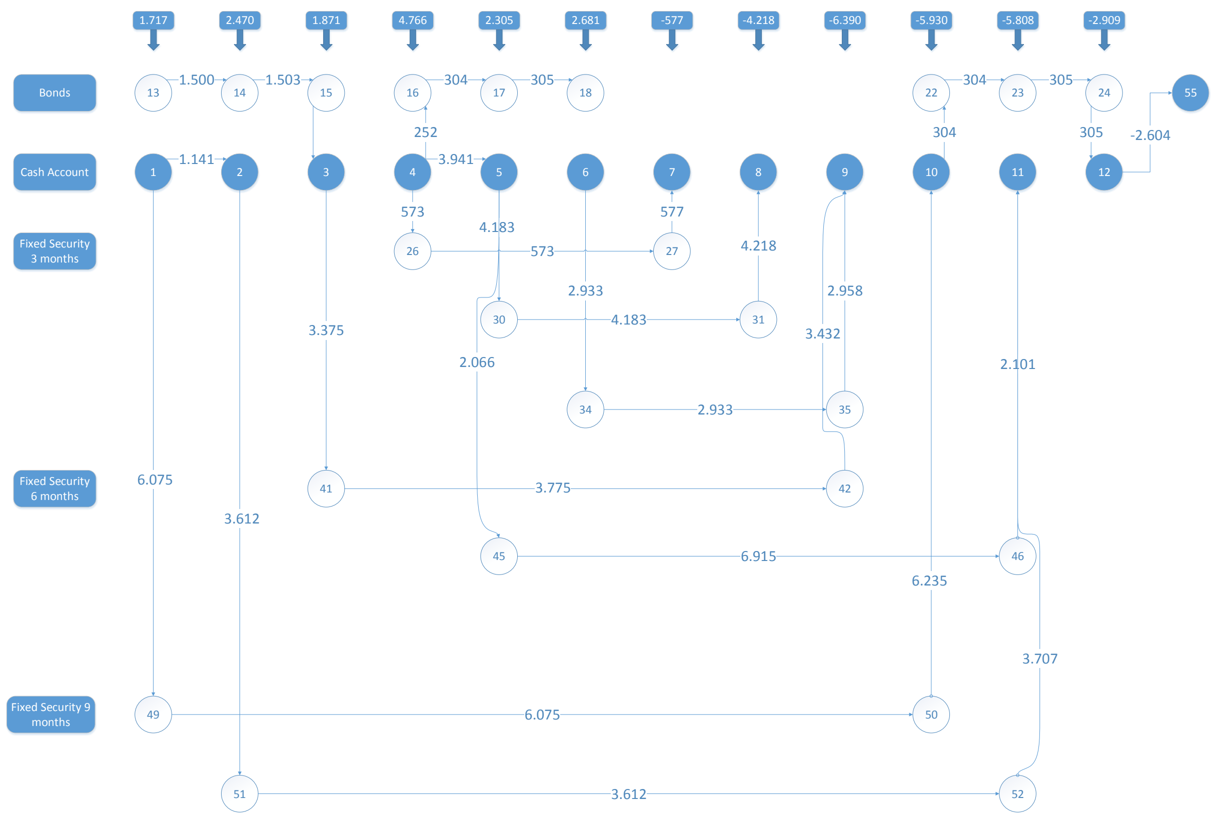


Figure 28. The network flow with cash flow at the 99% level and no minimum cash account

[27] See figure 20.

In real life circumstances, this might be an unlikely planning scenario. Or at least if the company is expecting a loss, then some measures would likely be taken to turn things around. But it gives a good insight on how much is really gained by optimizing the investment allocation process.

To demonstrate the capabilities of model and how it handles short term financing. If there is added a onetime payment of 15 million NOK in node 2, with a net cash flow expected to be -9 million NOK in period 2 and a starting cash balance of 0 in the cash account. This could be a tax payment, a loan payment, dividends or a long term investment. Then, the capital allocation planning changes completely and the model opens the line of credit, by borrowing from the next month 3,392 million NOK at the arc (3,2). Then, all constraints of the model are satisfied.

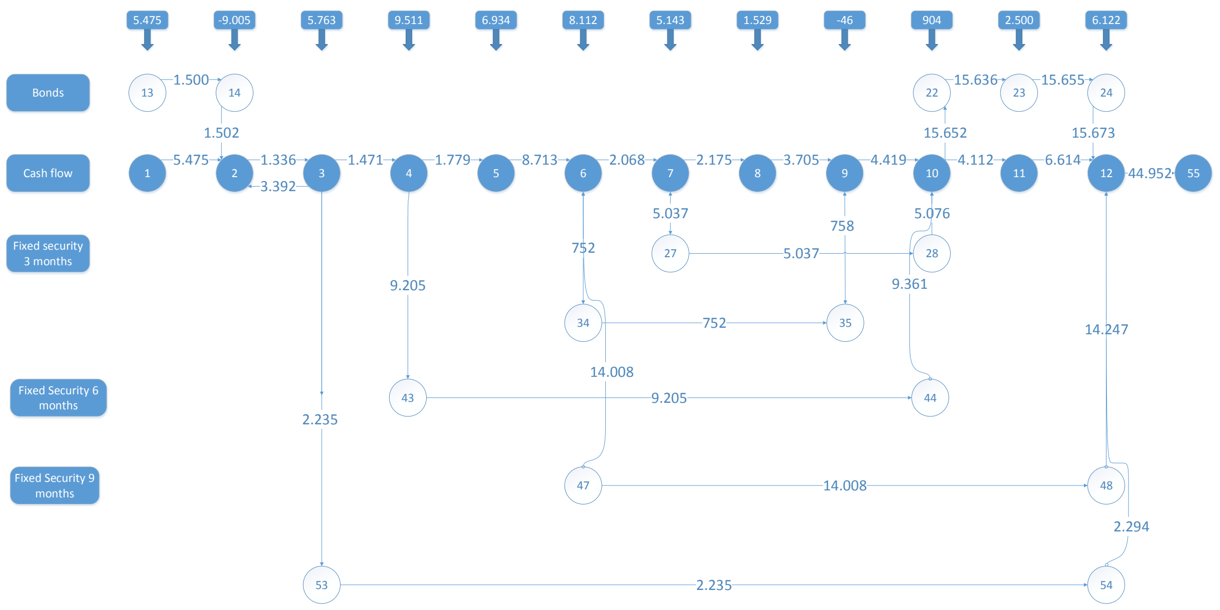


Figure 29. Arc set with short term financing. In thousand NOK

The net cash flow of the year in this case is expected to be 44,442 million NOK, 44,952 or 1,15% better result.

As the problem is a planning and scheduling model it is very important that all expenditures are included in the planning process. One change in a period of the planning horizon would alter the entire investment schedule.

6.2.1 Changes in the short term financing

When the short term financing rate decreases to 9% the model starts to open a line of credit and allocate the borrowed cash to the highest interest bearing asset. This, without any major expectations of a negative cash flow in any period in the planning horizon. For the scenario of 8% borrowing rates and lower the model starts to maximize the credit line of 10.000 million NOK at some point.

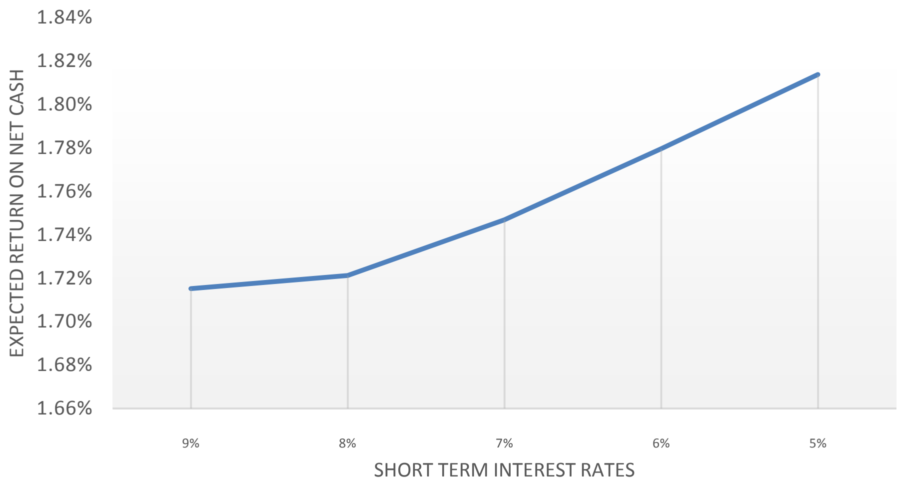


Figure 30. Changes in the expected return. Expected cash flow and minimum cash account at 80% level.

Under the assumption that the model can borrow at 7.5% (7,3% nominal) on a yearly basis for one month and invest at a 3,5% (3,4% nominal) yearly rate. By first glance it does not sound rational that borrowing money and deposit them in a fixed account is an economical strategy. But at node 3 in figure 31, the model maximizes the line of credit and pays back the loan in one month time by the cash flow of next month, paying 66.225 NOK in interest in node 4. The model deposits that cash in a fixed account for 9 months and receives interest payment of 262.500 NOKS in node 12. This strategy is then repeated in nodes 6 and 10, as can be seen in figure 31.

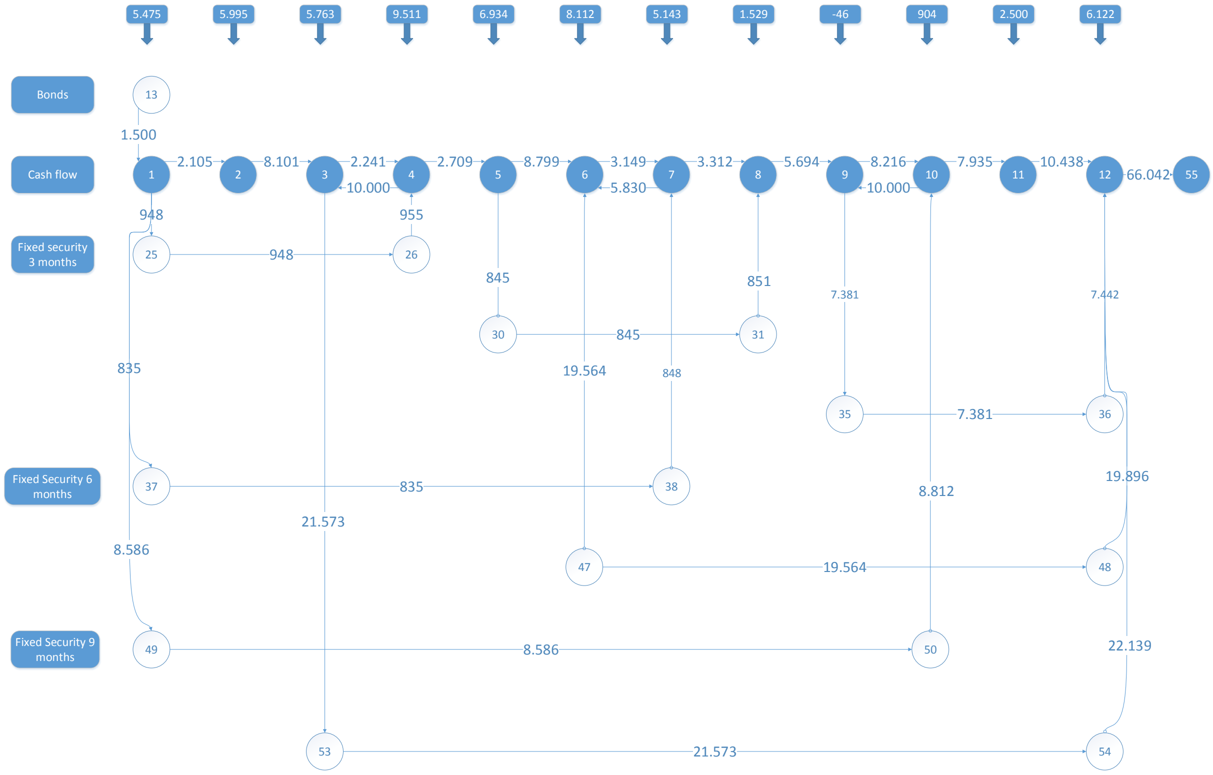


Figure 31. The arc set with 7.5% borrowing rates and the expected cash flow and minimum cash account of 80% confidence level.

6.2.2 Changes in the short term deposit rates and bond rates

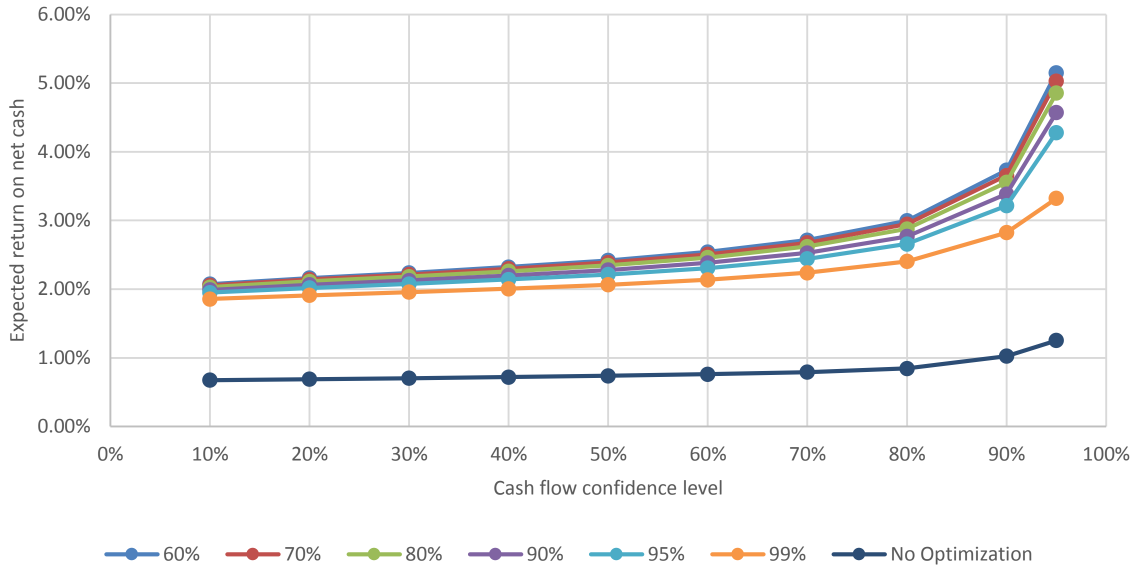


Figure 32. With a linear increase of 1% of interest return and loan rates

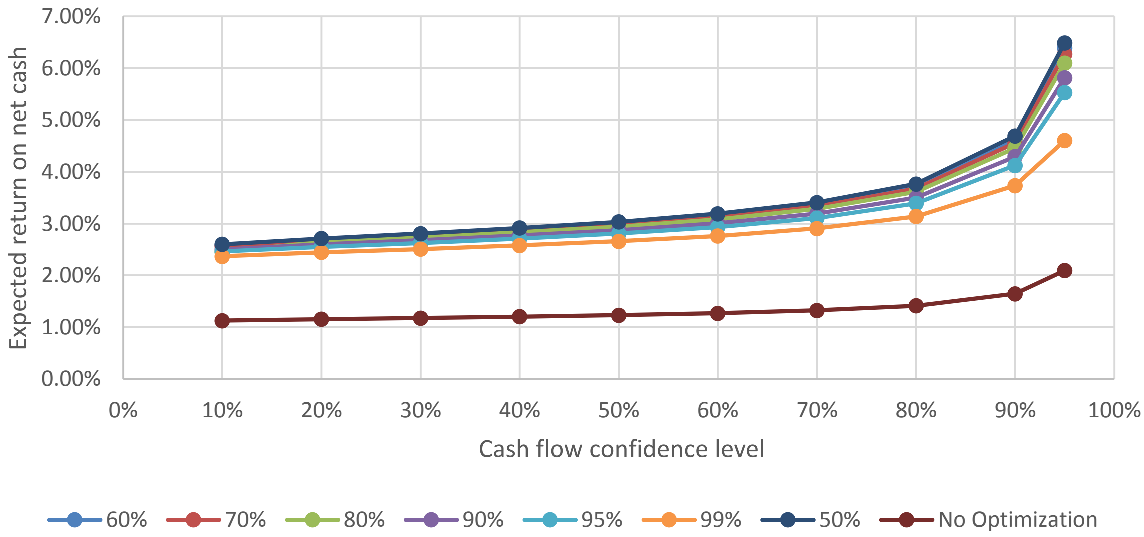


Figure 33. With a linear increase of 2% of interest return and loan rates

As expected, the return on net cash surplus increases proportionally with an increase in interest rates. It can be observed from figures 32 and 33, that the importance of cash management increases when there is large deviation between the rates of the minimum cash account and other deposits and investment options.

6.2.3 Different scenario

It can be considered unlikely that interest rates increase linearly. In this chapter the model is tested according to the Icelandic market environment, where historical inflation has been considerably higher than in Norway. The market information are obtained from Islandsbanki, www.islandsbanki.is, one of the largest banks in Iceland, and the yield of short term bonds published by the government in Iceland, from www.bonds.is.

	Type	Effective	Nominal	Arc value	Initial position
Bank account					
1 month	Balance	1.0%	0.995%	1.00042	5.500.000 NOK
1 month	Overdraft	12.80%	12.11%	0.99133	
Fixed account					
3 months		4.91%	4.803%	1.00825	0 NOK
6 months		5.06%	4.946%	1.01700	0 NOK
9 months		5.38%	5.252%	1.02993	0 NOK
Asset					
1 year bonds		4.20%	4.12%	1.00150	1.500.000 NOK
Transaction cost		0.10%		0.99900	

Table 8.Icelandic interest yield enviroment.

It can be seen from figure 34, by comparing to figure 27, the structure of the investment does not change much, although interest rates increase. But it illustrates the increased importance of how important active short term investment management in cash management is. 66,714 million NOK in cash at the end node and 2,73% increase is cash compared to the net cash flow of 64,942 million NOK. (See Appendix B for sensitivity analysis)

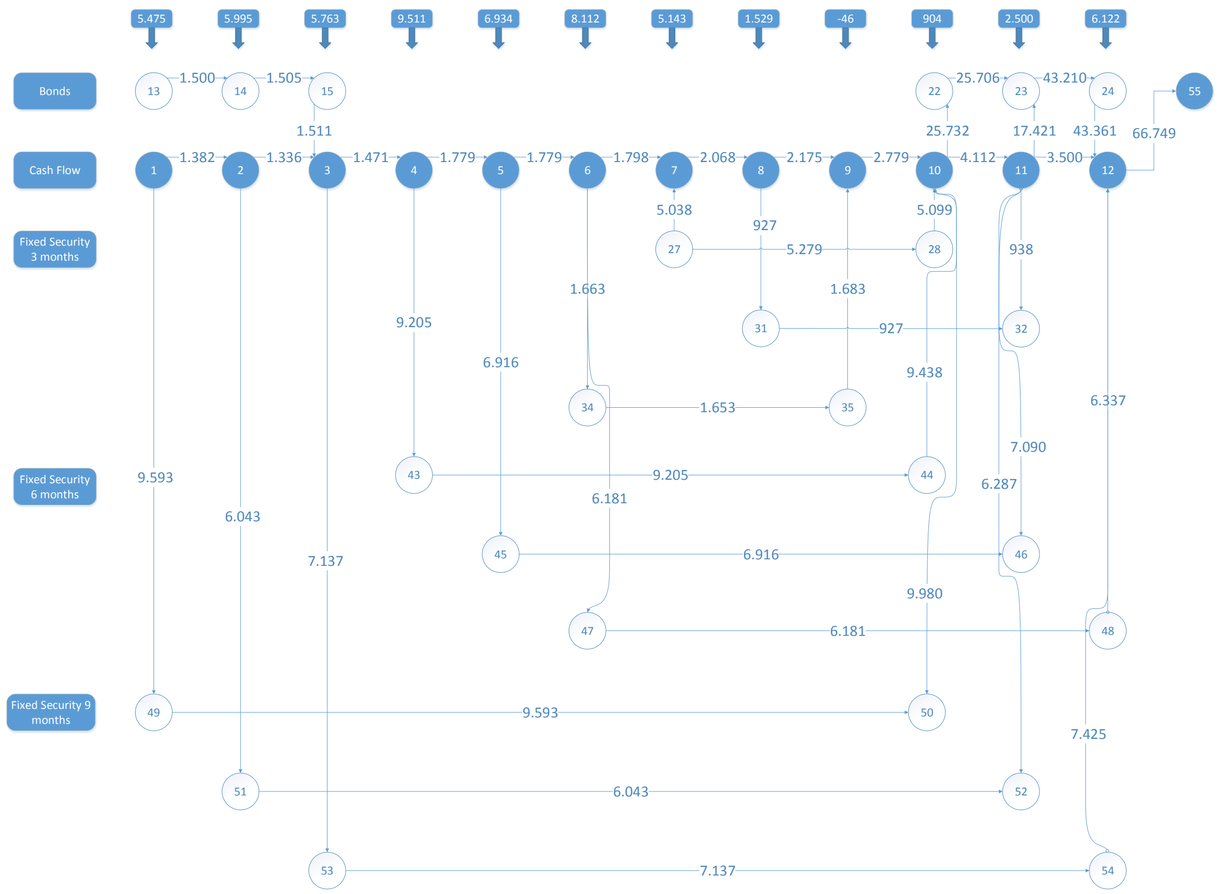


Figure 34. Arc set for the Icelandic environment.

7. Summary and Conclusion

In the thesis, it was demonstrated that network flow model with gains and losses is a flexible and effective tool in cash management.

The objectives of this thesis were to:

1. Locate the key variables in costs and revenues in the Norwegian aquaculture industry and construct a case study for an average size salmon company.
2. Implement Cash-Flow-at-Risk to the Network flow with gains and losses model.
3. Determine if Cash-flow-at-Risk can be used to evaluate uncertainty and in decision making of setting a minimum cash account for a 12 month period.
4. Implement different asset classes to the Network-flow with gains and losses algorithm.

(1) Chapter 5 covered the determinants in the harvesting and sales in the Norwegian salmon aquaculture.

(2) Cash-flow-at-risk was implemented to the Network flow model in chapter 4. The conclusion is that Cash flow at risk can be used in for medium term planning (1-12 months) in the Norwegian aquaculture industry. The problem is however, how well the farmer is able to predict its prices of salmon, as the uncertainty associated with spot prices increases with time. This risk can be limited by the use of future contracts or purchase agreements with customers.

(3) It can be concluded that Cash-flow-at-risk can be used to set the minimum cash account. But so it has any meaning, it is necessary to include a rolling planning horizon, that is, repeat the forecast and planning after each period ends. As further is looked into the future the more uncertainty there is, and according to the proposed model the increased level of cash is required to be kept on the cash account. How the level of confidence set on the cash account should be set by a management review as different companies has different needs and long term strategies. The minimum cash account has also an important part to play when determining how much can be invested at future dates.

(4) The different asset classed was introduced in chapter 4.

In the introduction the research question was to answer the question if the model can be used in practical cash management, the simple answer is yes, but with its limitations. Under the assumption that it is associated with a rolling planning horizon strategy. As the plan loses its validity after the first period in the planning horizon has ended. The main limitation is that using CFaR to set the minimum cash account, will underestimate the expected return of the model, as the uncertainty in the salmon price increases with longer planning horizons. Then the model assumes that there will be more cash in the minimum cash account than will actually be necessary to keep, when the time period in question is reached. When this is taken into account, the importance of a rolling planning horizon can be noticed. The continuous use of the model in each period is necessary to achieve the optimum investment strategy.

In the case of aquaculture companies, the effectiveness of the model depends on the how well a company is able to predict its cash flow. The cost of production is rather predetermined, but the variable in question is the price of salmon, which has historically been very volatile. The risk in the operation of salmon farming is difficult to generalize, as all companies have different sources of risk associated with their cash flow. Such as, different counterparty risk, environmental and risk associated with the location of the operation. The risk of leaving some important risk factor behind is quite high, then to be used in practice, the risk modelling has to be tailor made for each company.

As demonstrated in the previous chapter the model is flexible and when it has been implemented, continuous use should be relatively straightforward. The model could be improved with empirical information from the company´s operations. Sensitivity analysis and scenario analysis on the cash flow and capital allocation model can be performed to identify potential vulnerabilities in the cash flow. In the case of aquaculture the cycles of profit and price fluctuations span over the 1 year forecasting period of the study. When making cash flow investment decisions it is necessary to include a long term forecast, exceeding the 1 year planning horizon to support the medium term decision making.

Another important factor is that the company has a visual oversight of their capital allocation process. That is, the company might have an opportunity to invest in an ad-hoc unexpected projects. By using the model, the company has a visual oversight how long it can expect to be able to tie down capital for such projects.

7.1 Further research

In the study there was an attempt to simulate the cash flow of an aquaculture company by a “bottom up” CFaR approach. But in the thesis, only part of the associated risk affecting the cash flow of an aquaculture company is taken into account, from the available market information.

By using information from a company in the salmon production industry, one could expect to get much more detailed information of the overall variance of a specific firm.

It should be relatively straightforward to add details to the model by adding different kind of investment options. An interesting feature would be to construct a negatively correlated investment portfolio to the aquaculture industry, and introduce a probability constraint to the model similar to the one used on the minimum cash account. The model could be enhanced by adding a potential deviation in each node and therefore the overall CFaR value of the firm could be calculated, if risky assets would be associated with the model.

In the case of corporations, who have even number of international subsidiaries or possibly number of sources of income, the synchronization of the underlying cash flows should be an interesting problem to solve and optimize. By adding cash accounts, associated with exchange rates and different economic conditions.

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9. Appendix

9.1 Appendix A

Changes in interest rate by +1%

r=+1%	50%	60%	70%	80%	90%	95%	99%	No Opt.	CF
10%	107.069	107.054	107.035	107.008	106.965	106.923	106.827	105.586	104.879
20%	92.745	92.73	92.711	92.685	92.641	92.599	92.502	91.394	90.768
30%	82.953	82.937	82.918	82.892	82.848	82.805	82.709	81.693	81.122
40%	74.26	74.245	74.225	74.199	74.155	74.112	74.015	73.081	72.559
E(X)	66.528	66.512	66.493	66.466	66.422	66.379	66.283	65.421	64.942
60%	58.696	58.68	58.66	58.634	58.59	58.546	58.449	57.661	57.226
70%	50.342	50.328	50.309	50.282	50.237	50.194	50.095	49.386	48.998
80%	40.813	40.798	40.778	40.751	40.707	40.664	40.563	39.946	39.611
90%	27.563	27.548	27.528	27.501	27.456	27.411	27.307	26.829	26.557
95%	17.038	17.022	17.002	16.974	16.928	16.881	16.726	16.391	16.188
99%	-2.472	Inf	Inf	Inf	Inf	Inf	Inf	-2.928	-3.021

+1%	50%	60%	70%	80%	90%	95%	99%	No Optimization
10%	2.09%	2.07%	2.06%	2.03%	1.99%	1.95%	1.86%	0.67%
20%	2.18%	2.16%	2.14%	2.11%	2.06%	2.02%	1.91%	0.69%
30%	2.26%	2.24%	2.21%	2.18%	2.13%	2.07%	1.96%	0.70%
40%	2.34%	2.32%	2.30%	2.26%	2.20%	2.14%	2.01%	0.72%
50%	2.44%	2.42%	2.39%	2.35%	2.28%	2.21%	2.06%	0.74%
60%	2.57%	2.54%	2.51%	2.46%	2.38%	2.31%	2.14%	0.76%
70%	2.74%	2.71%	2.68%	2.62%	2.53%	2.44%	2.24%	0.79%
80%	3.03%	3.00%	2.95%	2.88%	2.77%	2.66%	2.40%	0.85%
90%	3.79%	3.73%	3.66%	3.55%	3.39%	3.22%	2.82%	1.02%
95%	5.25%	5.15%	5.03%	4.86%	4.57%	4.28%	3.32%	1.25%
99%	18.17%							

Changes in interest rate by +2%

+2%	50%	60%	70%	80%	90%	95%	99%	No Optim	CF
10%	106.533	106.518	106.499	106.472	106.429	106.387	106.291	105.147	104.879
20%	92.263	92.248	92.228	92.202	92.159	92.116	92.019	91.008	90.768
30%	82.508	82.492	82.472	82.446	82.402	82.359	82.262	81.344	81.122
40%	73.847	73.832	73.812	73.785	73.741	73.699	73.602	72.764	72.559
50%	66.143	66.128	66.109	66.082	66.037	65.995	65.898	65.134	64.942
60%	58.341	58.325	58.306	58.279	58.234	58.191	58.093	57.403	57.226
70%	50.020	50.004	49.985	49.958	49.913	49.870	49.771	49.159	48.998
80%	40.525	40.510	40.490	40.463	40.419	40.375	40.273	39.754	39.611
90%	27.324	27.309	27.289	27.262	27.217	27.171	27.066	26.676	26.557
95%	16.838	16.822	16.802	16.774	16.727	16.680	16.522	16.288	16.188
99%	-2.600	inf	inf	inf	inf	inf	inf	-2.958	-3.021

	50%	60%	70%	80%	90%	95%	99%	No Opt	CF
10%	1.58%	1.56%	1.54%	1.52%	1.48%	1.44%	1.35%	0.26%	104.879
20%	1.65%	1.63%	1.61%	1.58%	1.53%	1.49%	1.38%	0.26%	90.768
30%	1.71%	1.69%	1.66%	1.63%	1.58%	1.52%	1.41%	0.27%	81.122
40%	1.78%	1.75%	1.73%	1.69%	1.63%	1.57%	1.44%	0.28%	72.559
50%	1.85%	1.83%	1.80%	1.76%	1.69%	1.62%	1.47%	0.30%	64.942
60%	1.95%	1.92%	1.89%	1.84%	1.76%	1.69%	1.52%	0.31%	57.226
70%	2.09%	2.05%	2.01%	1.96%	1.87%	1.78%	1.58%	0.33%	48.998
80%	2.31%	2.27%	2.22%	2.15%	2.04%	1.93%	1.67%	0.36%	39.611
90%	2.89%	2.83%	2.76%	2.65%	2.49%	2.31%	1.92%	0.45%	26.557
95%	4.02%	3.92%	3.79%	3.62%	3.33%	3.04%	2.06%	0.62%	16.188
99%	13.94%	Inf	Inf	Inf	Inf	Inf	Inf	-118.27%	-3.021

9.2 Appendix B

Icelandic scenario

	Minimum Cash Account						
	50%	80%	90%	95%	99%	No Opt	CF
20%	93.136	93.046	92.981	92.916	92.767	91.185	90.768
Increase in return	2.61%	2.51%	2.44%	2.37%	2.20%	0.46%	
e(x)	66.839	66.748	66.682	66.616	66.468	65.261	64.942
Increase in return	2.92%	2.78%	2.68%	2.58%	2.35%	0.49%	
80%	41.047	40.956	40.889	40.824	40.666	39.834	39.611
Increase in return	3.63%	3.40%	3.23%	3.06%	2.66%	0.56%	

9.3 Appendix C

The mathematical programming code in MPL for solving network flow and losses algorithm. The basic network flow algorithm was obtained from Dr. Páll Jensson at Reykjavik University.

```
Title
                              Cash_Flow_Planning;
Options
                              ExcelWorkbook =
"C:\Users\erlenduri\Desktop\Optimization.xlsx"
                              ExcelSheetName = "12months"
Index
                              i := 1..54 ;
                              j := 1..55 ;
                              k := i ;
Data
                              C[i,j]          = Excelrange(Cij);
                              A[i,j]          = Excelrange(Aij);
                              D[i,j]          = Excelrange(Dij);
                              B[k]            = Excelrange(Bk);
```

```
                              U[i,j]           = Excelrange(Uij);
                              M[i,j]           = Excelrange(Mij);
Variables
                              X[i,j] export to
excelrange("Optimization.xlsx","Xij");
Model
                              Max Final = SUM(i,j: C*X);
Subject to
                              Constr[k] EXPORT ShadowPrice TO
EXCELRANGE("ShadowPrice") :
                              sum(i,j=k: A*X) - SUM(i=k,j: D*X) = B[k];
                                          !
Bounds
                              X < U ;
                              X > M ;
End
```

9.4 Appendix D

For a random variable X with mean μ has a variance defined as

$$Var(X) = E\left[(x-\mu)^2\right] = E[X^2] - (E[X])^2$$

That is the variance is the expected value of the square of X minus the squared expected value of the mean. Then the standard deviation is defined as:

$$\sigma = \sqrt{Var(X)}$$

Covariance is the measurement how much two random variables change together. If the value of variable X changes in one direction and value of Y changes in a similar manner than their covariance is positive. In the opposite, the covariance would be negative.

$$Cov(X,Y) = E((X-\mu_X)(Y-\mu_Y)) = \sum_{i=1}^{N} \frac{(x_i-\bar{x})(y_i-\bar{y})}{N}$$

Although covariance is generally an expression of correlation, the correlation coefficient is in some way easier to describe. Like covariance, it tells us about how two variables behave interactively. Correlation can take the value between -1 and 1. Where 1 means perfect correlation, 0 no dependence and -1 that they move in opposite directions.

$$corr(X,Y) = \rho_{x,y} = \frac{Cov(X,Y)}{\sigma_{x,y}} = \frac{E((X-\mu_X)(Y-\mu_X)}{\sigma_x \sigma_y}$$

9.5 Appendix E

In the following we will describe the Holt-Winter methods, as described in Madsen (2008).

$$Y_{N+j} = T_{N+j} + S_{N+j} + \varepsilon_{N+j}$$

Where the trend at time *N+j* equals the mean Y at *N* and the slope at *N+j*

$$T_{N+j} = \mu_{N+j} = \mu_N + \beta_{N+j} \, .$$

The seasonal components are described by

$$S_t = \sum_{i=1}^{s} S_i \delta_{ti}$$

An estimate of the mean of mean at the time origin *N*+1 is found by

$$\hat{u}_{N+1} = \alpha_1 (Y_{N+1} - \hat{S}_{N+1-s}) + (1-\alpha_1)(\hat{\mu}_N + \hat{\beta}_N)$$

Then the new estimate is a weighted average of the new observation Y_{N+1} adjusted for the seasonal component S_{N+1-s} and the previous estimates for the level and the slope, $\hat{\mu}_N$ and $\hat{\beta}$. Then $(\hat{\mu}_N + \hat{\beta}_N)$ is the estimate for the level at time origin *N*+1 based on the time *N* observation. The estimate of the slope at time origin *N*+1 is found by

$$\hat{\beta}_{N+1} = \alpha_2 (\hat{\mu}_{N+1} - \hat{\mu}_N) + (1-\alpha_2)\hat{\beta}_N$$

That is, the weighted average between the new observation of the slope and the previous estimate $\hat{\beta}_N$. Then the seasonal component is updated using

$$\hat{S}_{N+1} = \alpha_3 (Y_{N+1} - \hat{\mu}_{N+1}) + (1-\alpha_3)\hat{S}_{N+1-s} \, .$$

That is the weight between the new observation of the seasonal component $(\hat{Y}_{N+q} - \hat{\mu}_{N+1})$ and the previous estimate $\hat{S}_{N+1-s}$ .Using these estimates of slope, level and seasonal component, we can predict by

$$(Y_{N+1|\mathrm{N}} - \mu_{N+1}) = \hat{\mu} + \beta_N \ell + S_{N+\ell-s}, \ \ell = 1, 2, ..., s$$

When setting the parameters α_1, α_2 and α_3 to minimize the sum square error are chosen