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Real option analysis on offshore day-by-day contracts in the North Sea

Is there additional value for the charterer?

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Abstract

This thesis investigates the value of optionality in day-by-day spot contracts for PSV and AHTS in the OSV-spot market, where the charterer can replace the initial vessel with an alternative vessel in the spot market or renegotiate the contract, by using the real options embedded in the contracts. Even though there are usually replacement costs for the charterer when replacing the initially hired vessel prior to the contract's expiry, a significantly large spread between the initial contract rate and the current spot rate may still see the charterer benefit from terminating, or renegotiating, the initial contract. We perform an analysis based on a stochastic process, where parameter estimates are based on historic spot rates for various vessel types from 1996 to 2017. In addition to vessel type, we further distinguish between different exercise frequencies/options, where AHTS contracts allow for exercising the termination option every fifth day, and PSV contracts every day.

We find that the optionality may add value for the charterer for both PSV and AHTS, especially in markets with high demand, hence high rates. However, we find that the value of the optionality is significant more attractive in the AHTS, despite it lower assumed exercise frequency. This is mainly due to the higher volatility of AHTS spot rates, making thus AHTS has higher probability of exercise the option. We also perform a sensitivity analysis on the parameters, which suggest that replacement cost is the most sensitive parameter affecting the value of the optionality. Additionally, our thesis discusses how a possible renegotiation process might affect the valuation of the optionality.

Lastly, our conclusion provides an overview of the analyses performed, and sums up the valuation of the optionality for various contract specifications.

Preface

This master thesis is written as a concluding part of our Master of Science in Economics and Business Administration at Norwegian School of Economics (NHH). This thesis is written within the field of our major in Finance.

The maritime industry is a worldwide industry, with significant roots in Norway. The topic we chose was based on a discussion with our supervisor, Roar Aadland. For us, the optimal thesis combined both finance and maritime economics. Thus, real option valuation on OSV spot market contract was a perfect fit. Additionally, the literature on offshore freight rate, especially the spot market, is limited. That encouraged us even more.

We would like to thank the people who have provided guidance and expertise throughout this process. First, and foremost, supervisor Roar Aadland for professional feedback and guidance throughout the entire working period. Statoil, especially Per-Ove Sjåstad and Claus Wolff for helpful information and countless availability. Svein Leon Aure has provided intel for the renegotiation process, that has been very helpful. Finally, we are grateful to receive grants from The Norwegian Ship Owners` Associations' Fund at NHH. Hopefully, our work will be of relevance.

Bergen, December 2017

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1 Introduction

In this thesis we focus on the valuation of the optionality in the day-by-day Offshore Support Vessel (OSV) spot contract embedded with a daily optionality to terminate the contract. This market consists of a variety of different ships, all of which serve the same purpose – to assist offshore installations. Our valuation will investigate Platform Supply Vessel (PSV) and Anchor Handling Tug Supply (AHTS), respectively, as these vessel types are the most common once in the in the OSV market. PSV supports oil rigs with equipment, personnel, and necessities for operations, while AHTS supports rigs in transition. Vessels in this segment are equipped with a daily optionality, where the charterer can terminate the contract every day during the lifetime of the contract. This optionality is non-standardized and unique for the operational practice it concerns, which makes it a real option to investigate.

Chartering of vessels in these segments displays heterogeneous transactions at irregular intervals and low frequency, hence, extreme volatility. Every fixture is different from the last as the market continuously changes. Due to the extreme volatility in spot rates for OSV and the uncertainty in the maritime oil industry in general, most of the operational contracts have a daily optionality embedded. The extreme volatility suggests high fluctuation in rates, meaning that the spot rates will decrease often and rapidly providing possibilities for vessel replacements and thus additional value for the charterer.

Spot-contracts are considered to be a charter-party with a timeframe of less than 30 days (Shipbrokers, 2011), and correspond to the market spot rate at the inception of the contract.[1] Furthermore, the day-by-day contracts include optionality for charterer to terminate the contract during its lifetime. According to basic option theory, this day-by-day optionality is expected to create additional value for the charterer by replacing the vessel on contract with a vessel available in the spot market, if profitable. The length of the contract can vary from a couple of days, up to 30 days depending on the shipowner and charterers preferences and different market conditions. Market professionals state that the 14 days day-by-day contract is the standard contract length. However, contract length may depend on market conditions since the charterer, according to option theory, may have a natural interest in asking for as long a contract maturity as possible, in order to maximize the value of the optionality.

[1] The contract means that it is payable day-by-day, hence a 14 days contract is payable up to 14 days.

The relevance for such an option to replace derives from the spread of the initial contract and the spot market reaching a significant magnitude, and/or from the charterer not having an employment for the vessel, i.e. due to weather delay. In our analysis, we focus on the value of replacing the vessel, while only briefly commenting on the value of pure abandoning. When a termination is beneficial for the charterer, the market offers a better deal than the initial contract between the shipowner and the charterer, meaning that the remaining costs with the initial contract exceed the costs of a new contract in the spot market. Thus, the charterer will be better off going to the market to charter a new vessel. Even though the market offers a lower total net cost for the charterer, there are significant gross costs involving the replacement of vessels. These cost concerns, i.e. downtime, demobilization and tank-wash.[2] With all that considered, we suggest that the strike price of the initial contract resembles the rate where the contract can be exercised by the charterer. Note that the strike price will change during the lifetime of the contract period since the replacement cost is fixed and consequently independent of the remaining days of the contract. Thus, the strike price will decline for every day of the contract period, requiring larger rate falls.

Additionally, the charterer has the possibility to renegotiate the initial contract, and therefore avoid such replacement costs. This process will involve a counteraction between the shipowner and the charterer. Both, charterer, and shipowner will benefit from an agreement on a renegotiated rate that decreases the remaining aggregate cost for the charterer and increases the daily income for the shipowner in comparison with the spot rate the shipowner is offered in the spot market. In circumstances such as this, the information and rationality of the parties are crucial to negotiate the best deal. Thus, the costs of replacing the vessel are important for the shipowner in a negotiation process, since these costs determines the rate interval the charterer is willing to accept. Conversely, the shipowner will wish to have insight in the vessels operation I.e. additional costs concerning redelivery of equipment that will be ignored by a continuously cooperation, in order to gain leverage in negotiations with the charterers. The charterer again will have an incentive to hold back information on the forthcoming operational situation, knowledge of-which may benefit the shipowner in negotiations. The charterer will have an upper limit on the renegotiated rate where the total costs for replacing equals total costs for staying with the initial contract under a renegotiated rate. The shipowner, on the other hand, will have a lower boundary similar to the market spot rate. The renegotiation rate will therefore be expected to land somewhere in this range. However, the actual renegotiated rate is heavily

[2] Replacement cost will be explained further in section 4.

dependent on current market conditions. In low markets with many vessels available, the shipowner will have less bargaining power, and the renegotiated deal will tend to benefit the charterer, and vice versa. This situation occurs because the shipowner is willing to give up parts of his revenues in order to avoid offering his vessel in the over-supplied spot market.

The dynamics of the offshore market can often be fully explained by supply and demand dynamics alone (Alizadeh & Nomikos, 2011). The supply and demand are highly affected by the oil price, hence the world economy. Periods with high rates lead to increased demand for newbuilding, however the time-to-delivery in the newbuilding market is usually from two to four years. Consequently, the supply curve is convex, thus elastic at low rates and inelastic at high rates (Alizadeh & Nomikos, 2011). Shipowners tends to order too many new vessels in high markets, which eventually leads to an over-supply, hence a continuously period of low rates. Demand, on the other hand, is directly linked to oil production activity since their purpose is to support and provide oil rigs in operation or in transition. Oil production activity itself depends heavily on the oil price and the general state of the economy. In periods with high oil prices and robust economic growth, oil production will typically flourish. Consequently, the demand for OSV-vessels will increase in turn. A higher oil price again will lead to increased oil exploration, which means more demand for PSV-vessels, i.e. rig move and transport of drilling mud. However, there is no perfect correlation between oil prices and OSV rates, since oil production lags oil price. High oil prices will increase the demand for OSV-vessels, since oil projects take time to develop the long-term shift in demand for OSV-vessels changes gradually. In the spot market, the supply side changes slowly because regions fragment the OSV-market, meaning vessels seldom move between regions. Such regionality itself is affected by standardization of vessels and different specification, i.e. in the North Sea there may be harsh weather, meaning the vessel must be able to handle the rough sea. Weather is also an important factor for the short-term demand, where harsh weather forecasts could lead to a spike in demand prior to the event. Operational needs for the charterer are highly irregular and unpredictable, meaning demand is the main driver for the high short-term volatility.

The purpose of this thesis is to evaluate the additional value form optionality in spot market contracts to the charterer. By introducing a stochastic process for the rates of PSV and AHTS a simulation of daily rate will create the foundation of our valuation. Additionally, we will construct a real option valuation (ROV) based on the optionality to terminate the initial contract and consequently negotiate a better deal, this thesis will investigate the additional value for charter on the day-by-day contracts, with "*optionality without renegotiation"*, and contracts with *“optionality and renegotiation”* over contracts with no optionality for AHTS and PSV, respectively.

The remainder of this thesis is structured as follows: Section 2 gives a brief literature overview, section 3 presents data and overview over the data we use for our analysis, while section 4 discusses the underlying theories and methods for solving the valuation problems. Finally, section 5 shows the result of our empirical analysis, with section 6 summarizing and concluding our work.

2 Literature review

ROV on commodities and shipping freight rates is afield, which to a considerable extent, has been established. However, the literature on the offshore service market has been limited, but recently there have been a development of studies concerning OSVs. In the literature review, we will first focus on theoretical research on the valuation of optionality, followed by a discussion of methodology on the stochastic process.

The literature on ROV for commodities and shipping freight rates has mostly covered the optionality concerning switching between two or more states. Mossin (1968), introduce the entry-exit decision, where he attempted to set a lower critical value for when the vessel should be laid up, and an upper threshold for when it should be put back in operation. Brennan and Schwartz (1985) and Dixit (1989) developed Mossin`s (1968) work to a modern real option framework. Brennan and Schwartz's article evaluated the real option of opening and closing a mine, but the structure has a range of alternative use, such as Dixit's paper that developed the ROV framework concerning firm`s entry-exit decision further.

The literature mentioned above has created a framework for further study of optionality of the shipping- and OSV-market. However, the literature does mostly cover the conventional carrier market, such as dry bulk and tanker market. For instance, Tvedt (1997) valuated very large crude carriers (VLCC) under uncertainty, which incorporated three states; lay-up and scrapping as alternative to spot operation. Tvedt's paper ignored the possibility to operate on long-term time charter contract, hence a plausible state for VLCC-owners to operate where disregarded. Further, Sødal, Koekbakker and Adland (2008) investigated a real option model on flexibility for combination carrier to switch between dry bulk- and wet bulk-market.

The stochastic process has been a topic heavily discussed and researched. The studies assess the property of the dynamics of rates, and how to model the stochastic process. The freight rate markets in bulk shipping are often held as an example of perfectly competitive markets (Norman, 1979). Over time there has been developed two schools, first the classical school where authors model the rates based on supply and demand. See, for instance, (Norman, 1981). Secondly, the development led to modeling of rates directly in a stochastic process.

Further, Bjerksund and Ekern (1995) applied a mean reverting model, Ohrnstien-Uhlenbeck process (O-U process), in their research on evaluation problems involving mean-reverting cash flow in shipping. They developed a model to evaluate the options in time-charter contracts on the recent advances in financial economic tools. Dixit and Pindyck (1994) introduced a logarithmic model, the Geometric Mean Reversion process (GMR). Tvedt (1997) discussed the two alternative approaches, the O-U-process and the GMR, where he simulated rates for time charter equivalent spot rate in the VLCC. The result suggested by empirical findings that the GMR process is more appropriate than the O-U process. The O-U model could in some simulation present negative values. The GMR, on the other hand, contains a parameter that makes it downward restricted and thus avoids negative rates. Further, Insley and Robinson (2005) also presented a stochastic process, based on the GMR. Additionally, there has been developed other models, such as stochastic factor models, see for instance, (Javier, 2015) which included a seasonal variable.

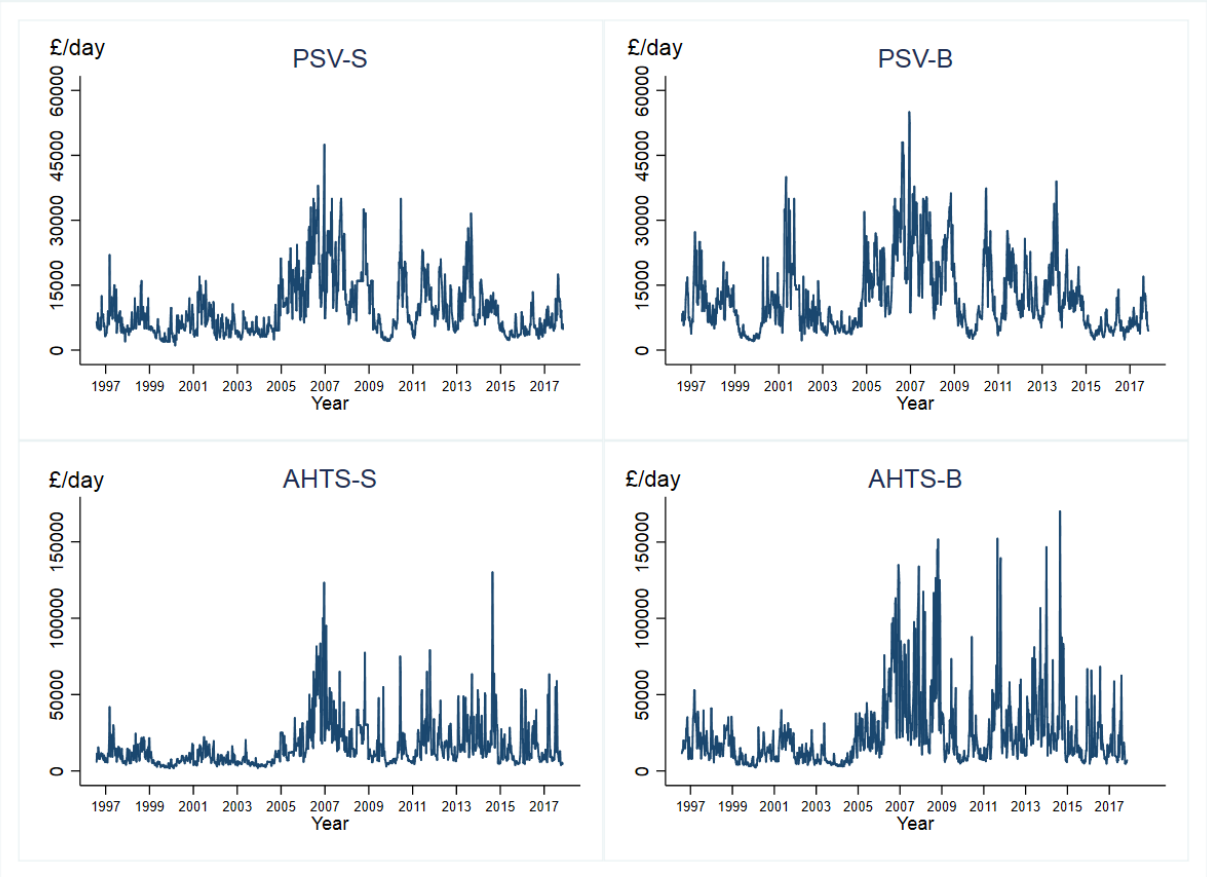
Literature on the stationarity of freight rates are published, see, for instance (Tvedt, 1997) and (Koekebakker, Adland, & Sødal, 2006). Tvedt applied the classic unit-root test presented by Dickey and Fuller (1981), the Augmented Dickey-Fuller (ADF) test to establish the rejection of unit root- hence a stationary process for the spot term-charter equivalent for the VLCC. Koekbakker et. all (2006) could not reject the unit-root for the bulk-markets by the ADF, however they proved stationarity by applying a test proposed by Kapetanois et. all (2004), (exponential smooth transition autoregressive model(ESTAR)).

Whereas all these studies consider conventional freight rates for bulk carriers mostly, research on offshore service markets has been limited but evolving. There have been presented thesis on the offshore service industry, see, (Bjørkelund, 2014) and (Dahle & Kvalsvik, 2016) and (Sværen, 2017). Bjørkelund (2014) present an extension to the geometric mean reversion model, by constructing a two-regime mean reversion of spot rates for PSV and AHTS. Dahle and Kvalsvik (2016) presented a thesis on microeconomic determinants of OSV-rates for PSV and AHTS. Sværen (2017) investigated the lay-up decision in the OSV-segment.

To our knowledge there has not been published ROV on optionality on day-by-day spot contracts in the offshore segment. Our thesis provides literature on the embedded optionality, on a topic where market participants have professional intuition on the value of the optionality.

3 Data

Clarksons Research Services Ltd. provide weekly spot rates for both PSV and AHTS for the period from August 6, 1996 through November 10, 2017 for different vessel specifications corresponding to 1111 observations. The data is plotted in figure 1, for each vessel and specification. The data set consist of two different vessel sizes for both PSV and AHTS. PSV is divided by deck-size -, one group of vessels between 500-899 m2 and one group above 900m2. AHTS has a similar separation, divided by Bullard Horse Power (BHP) which corresponds to the tug power of the vessel. AHTS has one group of vessels of above 20 000 BHP and one between 16,000 and 20,000 BHP.[3] We will denote the different sizes as PSV-S for PSV 500-899m2, PSV-B for PSV 900m2+, AHTS-S for AHTS 16-20,00 BHP and AHTS-B for AHTS 20,000+ BHP.



Source: Clarksons Reaserch Services

Figure 1. The spot rates for PSV and AHTS respectively.

[3] We will denote the different vessel sizes with adding small and big. I.e. PSV-big is vessel with free deck size above 900m2

The rates are provided by Clarksons Platou offshore brokers` estimate, which there are uncertainties for the outside observers on how this information is transformed into an indicator (Limited, 2015; Veenstra & Van Dalen, 2008). Indices are crucial for a transparent and efficient commodity markets. An efficient index relies upon many continuously transactions and homogenous equities (Adland, Cariou, & Wolff, 2017). The offshore market has heterogeneous equities and irregular transaction volume. Vessels have a range of different specs, different regional affiliation, operations have different requirements and contractual structure differ. However, different specifications for vessels have limited impact on spot rates (Dahle & Kvalsvik, 2016). Hence, we conclude that these rates are appropriate for our analysis.

Descriptive statistic for the series is given in table 1. The table illustrates evidence that AHTS has higher standard deviation than PSV. This was expected due to the nature of AHTS operation with highly unregular demand for rig moves. Additionally, it is evident that AHTS has higher rates, with maximum rates almost three times higher than the PSV. Further, the table shows that we can reject normal distribution in dataset by the Jargue-Bera test. However, this is expected since rates are downward restricted, hence the distribution are positively skewed and fat-tailed (Adland & Cullinane, 2006).

Table 1. Descriptive statistic from the data set.

	AHTS		PSV	
	16-20 BHB	20+ BHB	500-899m2	900m2+
Mean	15843	24590	9436	12300
Median	10778	17110	7319	9886
Maximum	130100	170165	47500	55000
Minimum	1800	2340	1100	2080
Std. Deviation	14888	23882	6824	8448
Skewness	2.774	2.482	1.665	1.406
Kurtosis	13.904	10.587	5.979	5.123
Jarque-Bera	6897.64	3788.23	919.98	572.10
Autocorrealation				
ρ(1)	0.690	0.743	0.851	0.866
ρ(2)	0.516	0.539	0.780	0.777
ρ(3)	0.461	0.484	0.725	0.725
ρ(4)	0.436	0.482	0.694	0.692

4 Theory and methods

In this section of the thesis, a presentation of theories and methods used to solve the research question will be introduced First, an introduction of financial theory, followed by an explanation of the uniqueness of our real options. Further, we will present the stochastic process and finally the model to value the real options.

Options in its simplest form are a contract on an underlying asset, where the owner has a *right* but not an obligation to *exercise* i.e. buy or sell. In option theory, real- and financial option has in many circumstances similar attributes. However, they distinguish in practice. Financial option is often used as a product for hedging or as a speculative instrument, whereas real option on the other hand concerns an important strategic decision for a real asset, and is the core strategic investing in investments under uncertainty (Smit, 2004). Myers (1977) introduced the term Real option; “Which are opportunities to purchase real assets on possibly favorable terms”.

The existence of real option is a consequence of imperfection in sectors, such as adjustment costs and market power. If markets would be perfectly competitive, such options would not exist (Myers, 1977). The offshore spot market is far from perfectly competitive, even though the transparency has increased. The day-by-day options on spot rates are a result of uncertainty in the OSV segment. Extreme volatility and weather permitted operations, creates an uncertainty for the charterer. Thus, the charterer favors contracts that are embedded with options to terminate the contract.

There are wide ranges of different option types, both for financial and real options. In this thesis, we will apply American and Bermudan option to price PSV and AHTS, respectively. Merton, Brennan and Schwartz (1977) presented a paper on valuation of American put options and on the question of optimal exercise strategy. These authors applied the Wiener process, which was first presented in option valuation by Black and Scholes (1973).[4] An American option can be described as an option that can be exercised at any point from inception date to expiration date. A Bermudan option on the other hand, outed in Schweizer, (2002) as: “A Bermudan option is an American-style option with a restricted set of possible exercise dates”.

We chose these options, due to the scope of the operations conducted by the vessels. A PSV-operation is short and uncomplex, with an average duration of approximately 48 hours. The operations concerns cargo runs, providing oil rigs with essential tools, liquid, and personnel

[4] We discuss the use of the Wiener process under section 4.1. The stochastic model.

both on deck and in tanks. PSV also support the rigs with disposal of contaminated chemicals, which must be disposed safely on land. Consequently, a replacement of a PSV is considered straightforward. However, before termination of the contract, charterer must clean the used tanks of the vessel. Even with the disadvantage of cleaning the tanks, we decide to value it as an American option of daily termination possibilities.

On the contrary, AHTS operations concern the tugging of oil rigs and anchor handling. These operations are heavily capital intensive and complex, with operations lasting for four days - all going well. Thus, we suggest that a termination will not be possible daily due to the significant costs of an operation break concerning rig transitions. Thus, we constructed a Bermudan option with termination every fifth day, which suggest that the vessel only can terminate the contract between operations. We are confident that this is a suited model to catch the real dynamics of the AHTS operations, hence to value the optionality.

In the real option literature, there are several types of options, such as options to abandon, switch and expand. Our real option model will be a combination of an option to switch and an option to abandon. Options to switch regards options to switch back and forth between two alternatives, while our option concerns switching to an alternative vessel without the possibility to switch back. Therefore, we suggest that the real options in this thesis is a combination of a switching option and an option to abandon, where the charterer has the right to abandon the contract, and consequently replace the initial vessel with an alternative on the spot market.

Additionally, we assume that the charterer will terminate the initial contract rate when the spot rate falls below the strike price. Even if the spot rates may decline more in the future, we suggest that the charterer will terminate instantly. As the charterer declines an exercise, he is dependent on a further decrease in spot the rates. These costs will be unaffected by the remaining days of the contract period as these are fixed costs concerning the replacement of a vessel. Consequently, the charterer will have fewer days to capitalize the cost of replacement in the future.

For the simplicity of the model we assume that the option to replace a vessel only is permitted once during the contract period. Several exercises on spot contracts will be very rare, see evidence in appendix (5). Thus, we concluded that the additional value from a multi-exercise model is limited.

4.1 Stochastic process

To determine the stochastic process, we must determine the nature of the dynamics affecting the freight rates of the OSV. There are many contributions to stochastic processes both in the field of shipping and others. Black & Scholes (1973) presented a method to calculate an European option using a Brownian motion as the stochastic parameter. Ornstein and Uhlenbeck (1930) presented a stochastic process which authors such as Bjerksund and Ekern (1995) applied to value freight rates for shipping. Dixit and Pindyck (1994) presented a logarithmic Ornstein-Uhlenbeck; the Geometric Mean Reversion(GMR). In this section we will discuss the best suited stochastic process for the thesis and provide the calculated estimations of the parameters.

Black & Scholes is the most common model to value options. Practitioners have used Black & Scholes since its origin, both for stocks and commodities, even though the model is preferable to value stocks. As Tvedt (1997) explained: Black and Scholes has been the most common model in lack of better methods to value options written on freight rates. Stocks are fundamentally different than commodities, hence valuing real options on freight rates based on Black & Scholes may not be appropriate. Additionally, the intention of the model is to value European options, not American options.

The stochastic equation is a Brownian Motion, given by:

$$dX_t = \mu X_t dt + \sigma X_t dZ_t \tag{1}$$

Where, μ is a constant variable, which describe the expected growth rate less the risk-free rate, hence the excess return. σ is the standard deviation of the relative change in the stock/freight rate. The last component, Z is a one-dimensional standard Brownian motion or wiener process. Z is an *independent increment*, *Markov process* and the changes are *normally distributed*. Independent increment means that the probability distribution for the process is independent of previous data. Markov process implies that only current information is useful for forecasting the future movement (A. K. Dixit, 1994). Z is normally distributed; $dZ_t \sim N[0, dt]$

The geometric Brownian motion is well suited to value financial stocks. However, it is evident from maritime economic theory that better stochastic specifications of the spot market may be more appropriate, such as assuming a mean reversion of rates (Sødal et al., 2008). As Tvedt (1997) explains; High rates will trigger new-buildings and rates will tend to revert to a mean reversion rate. For low market rates there are situations where it is not profitable for shipowner to operate the vessels and consequently might scrap or lay-up the vessel. Thus, the rates will tend to revert to a mean reversion rate as well.

Therefore, Bjerksund and Ekern (1995) suggested to apply an *Ornstein-Uhlenbeck process* (Ornstein & Uhlenbeck, 1930) to value a term charter contract in shipping. The model consists of a mean reversion property, which is reasonable to apply due to the nature of supply and demand forcing the rates to return to a long-run mean. This explanation of freight rates dynamics contradicts the *independent increment* since the rates are not independent of previous data.

The *Ornstein-Uhlenbeck* equation is stated as:

$$dX_t = k(\alpha - X_t)dt + \sigma dZ_t \tag{2}$$

The symbol α is the long-term mean and X_t is the current value of the stochastic process. If $X_t > \alpha$, the sign of the first term will be negative. The constant *k* is the speed of mean reversion, where a higher *k* will provide the stochastic process X_t to move back to α at a higher frequency and vice versa. The second term consist of volatility and the one-dimensional standard Brownian motion Z_t that is described above.

Shipowners have the option to lay up vessels if rates are lower than marginal costs. If operational cost is not covered, shipowners will choose to scrap or lay up their vessel, setting a floor to the rate on the low side (Tvedt, 1997). An O-U process can in some circumstances provide negative rates, which is an unrealistic scenario. O-U estimates negative values more frequently in high volatility markets than in low, because of the increased fluctuations in rates. As mentioned previously, the OSV-market is highly volatile, hence the O-U is more likely to simulate negative values. Therefore, it is appropriate to introduce a model that limits negative outputs. Dixit & Pindyck (1994) introduced a new version of O-U which effectively handles the issue with negative simulated rates. (Insley & Rollins, 2005) presented a GMR-model. The incremental process is given in the stochastic differential equation:

$$dX_t = k(\alpha - X_t)dt + \sigma X_t dZ_t \tag{3}$$

The first term of the GMR equation is the same as for the O-U. *k* is the tempo of the mean reversion and α is the mean. The second term differentiate GMR from O-U, where GMR includes the X_t, that denotes the rates at time *t*. The rest of the process is the same as for the O-U. Since the spot rate is included in the stochastic term, the equation is downward restricted. Thus, we suggest that the GMR given in equation (3) will be an appropriate model, since the rates often stay at moderate levels for a longer period, followed by a short period of high rates (Tvedt, 1997). The mean reversion parameter pulls the rates towards the long run equilibrium faster at high rates than low rates supporting the statement above, see figure 2[5]. Since this stochastic process is preferable compared to the processes discussed above we continue with GMR as the foundation of our analysis.

The autoregressive rate path is stated as the equation below (Insley & Rollins, 2005):

$$X_t - X_{t-1} = -k\Delta t X_{t-1} + k\alpha\Delta t + \sigma X_{t-1}\sqrt{\Delta_t \epsilon_t} \tag{4}$$

Where ϵ_t is N(0,1). We divide equation (4) by X_{t-1} to find the incremental changes and run an Ordinary Least square regression on equation (5):

$$\frac{X_t - X_{t-1}}{X_t} = \beta_0 + \beta_1 \frac{1}{X_{t-1}} + e_t \tag{5}$$

Where, e_t is the changes in the spot rate not explained by the mean reversion. The output from the regression is given in tables 2.

Table 2. Values obtained from the OLS regression.

PSV-B					
	Estimated coefficients	Standard Error	T-value		
Intercept	-0.0808	0.0154	-5.2448	Sample:	06aug1996 to 10nov2017
b1	642.27	101.96	6.30	Number of observation:	1111
				Standard error regression:	0.29
PSV-S			PSV 500-899		
	Estimated coefficients	Standard Error	T-value		
Intercept	-0.1265	0.0177	-7.134	Sample:	06aug1996 to 10nov2017
b1	780.33	92.37	8.45	Number of observation:	1111
				Standard error regression:	0.32
AHTS-B					
	Estimated coefficients	Standard Error	T-value		
Intercept	-0.1196	0.0216	-5.536	Sample:	06aug1996 to 10nov2017
b1	1504.62	210.67	7.14	Number of observation:	1111
				Standard error regression:	0.46
AHTS-S					
	Estimated coefficients	Standard Error	T-value		
Intercept	-0.1555	0.0234	-6.644	Sample:	06aug1996 to 10nov2017
b1	1402.84	171.70	8.17	Number of observation:	1111
				Standard error regression:	0.46

[5] Note that the real rates and our stochastic simulation has similar attributes.

β_0, β_1 and ϵ_t is stated in the equation below, which is explained by:

$$\beta_0 \equiv -k\Delta t \tag{6a}$$

$$\beta_1 \equiv k\Delta t\alpha \tag{6b}$$

$$e_t \equiv \sigma\sqrt{\Delta t\epsilon_t} \tag{6c}$$

From the values obtained in the OLS regression we estimate a value for mean reversion, mean reverting price and volatility by rearranging the equations (6a), (6b) and (6c).

$$k = -\beta_0 \tag{7a}$$

$$\alpha = \frac{\beta_1}{\beta_0} \tag{7b}$$

$$\sigma = SE \tag{7c}$$

For the analysis we will transform the weekly parameters to daily parameters since it is more appropriate to value day-by-day contracts on daily basis than weekly. We estimate the daily volatility by dividing the weekly volatility by the square root of 7.[6] To value the daily mean reversion, we divide the weekly mean reversion by 7. [7] The estimated values are stated in table 3. As expected the AHTS has higher standard deviation and equilibrium price. Additionally, AHTS seems to have a higher mean reversion parameter, indicating that rates tend to pull faster towards the long run equilibrium.

Table 3. Values obtained the OLS regression.

	AHTS-S	AHTS-B	PSV-S	PSV-B
Mean reversion	0.022	0.017	0.018	0.012
Equilibirum price	9022	12581	6167	7947
Standard deviation	0.17	0.17	0.12	0.11

To prove that the model is preferable for the stochastic dynamics of the spot market we conduct a Dickey-Fuller unit root test (Tvedt, 1997). Dickey-Fuller unit-root investigates the collected

[6] Vessels are often hired at weekdays. However, vessels may also be chartered during a weekend and are operational 24/7.

[7] To establish that it is justifiable to divide the mean reversion by 7 we conducted a simulation using the weekly- and the daily-parameters for up to 4 weeks. We established that the estimated value after week 2, 3 and 4 for both parameters gave similar expected values when applying a Monte-Carlo simulation. Hence, we concluded that it is appropriate and justifiable to divide the weekly parameter by 7 to use it as our daily parameter.

time-series of rates for stationarity, hence mean reverting (Insley & Rollins, 2005). From table 4, we conclude that the time-series is stationary at <1% significant level. Thus, we use a mean reversion model. Table 4, consist of a test for autocorrelation. The table illustrates high values from the Breusch-Goodfrey test suggest that we can reject the null hypothesis that there is no serial correlation. Consequently, it is evidence of autocorrelation in the residuals of the OLS regression, hence we fail to apply one of the Gaussian-Markov assumptions. Autocorrelation provide an inaccurate representation of the spot rates since changes in the variables tend to follow the previous changes in the same direction.[8] Benth and Koekebakker (2015) provided a model which incorporated the autocorrelations.

Table 4. Test on unit-root and autocorrelation.

Vessel type	Dickey-Fuller	Breusch-Goodfrey	R2
AHTS-B	-10.86	-3.95	0.06
AHTS-S	-9.80	-34.62	0.04
PSV-B	-8.57	-30.64	0.03
PSV-S	-7.44	-53.14	0.06

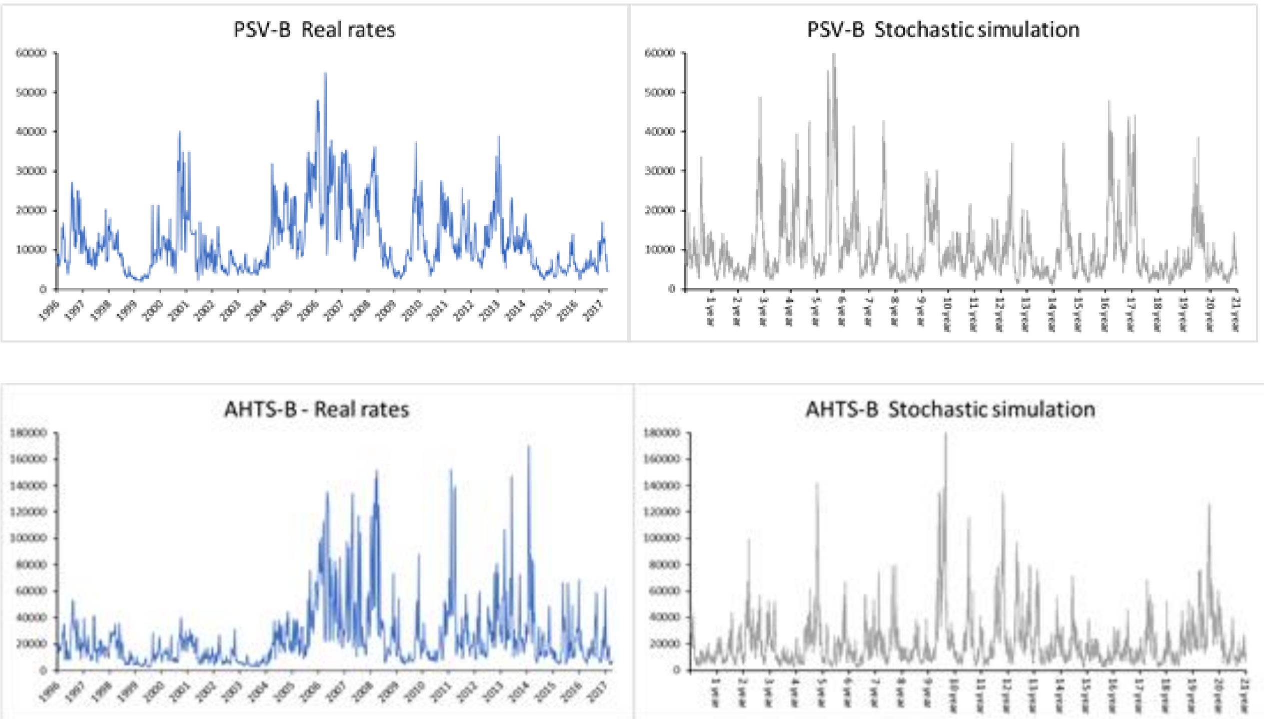


Figure 2: Real rates vs an example of a stochastic simulation for 21 years.

[8] Explained further in limitations

4.2 The model

In our valuation model we distinguish between PSV and ATHS regarding possible terminations as discussed in previously. Therefore, we present two different approaches to valuate PSV and AHTS separately. PSV do typically have a short operation cycle, where one supply run approximately takes 48 hours. Hence, we assume that replacing a vessel could be applicable continuously during the life span of the contract. Thus, the valuation of PSV will follow an American option pricing model with possible exercises $(X_t)_{0<t>T}$.

On the contrary, AHTS-vessels have a more complicated operational structure, since moving a rig is more time consumable and typically takes up to 4 days – all going well. Additionally, these operations are extremely costly for charterer to delay, since a delay in an operative rig could mean lost revenue and cost associated with crew. Consequently, in discussion with Statoil, we find an American option pricing somehow unrealistic. Therefore, we apply a model for valuing AHTS as a Bermudan option. Such an option is a denoted with (X, R) where $R \sqsubseteq [0, T]$ is the region of possible termination dates and $X = (X_t)_{0<t>T}$ is the Mean Reversion process concerning the vessel rates in the spot-market (Schweizer, 2002). In our valuation we assume that a replacement of a vessel is only applicable every fifth day, because of the 4-days operation period that is too capital intensive. Additionally, we assume that the charterer has continuously new operations during the contract period. All that considered, the R would be every fifth t value. Even though the limited exercise dates reduce the possible optionality, we find contract terminations that exceed these permitted dates as unrealistic. Further, the replacement cost will be significantly lower for the permitted dates than an American Option model would allow.

When the charterer hires a vessel from a shipowner, the rate is fixed during the contract period or until charterer terminates the contract. Consequently, X_0 is the daily cost of the initial contract. Let T be the contract length, and thus the aggregated cost of the charting if there is no optionality in the contract:

$$Aggregated\ cost_{no\ opt} = X_0 * T \tag{8}$$

Since the day-by-day contracts have optionality embedded we will construct an equation that represents the daily difference of the initial contract and the alternative spot market rate for each t from 0 to T.

$$P_t = X_0 - X_t \tag{9}$$

We assume that a chartering of a vessel on a contract period T, means that the charterer will have continuously operations in T days. This may not be a realistic assumption in all circumstances, since charterer in some situation might not have operations continuously during the contract period. Therefore, charterer will abandon the contract early and not replace the vessel. In our analysis we have not valuated the option to abandon the contract without rehiring a new vessel. Additionally, the total contract period will not change if the charterer decides to replace the initial vessel. I.e. if the charterer decides to replace the initial vessel after 5 days on a 14-day-by-day contract, the remaining days will be 9 days for the new vessel.

For this thesis, an analysis of the cost of replacement corresponding to the contract termination in the OSV spot-market is essential to solve the ROV. The replacement cost can be identified by the time and effort necessary to replace a vessel, risk of disrupting normal operation and costs occurred directly towards replacement. Consequently, the replacement cost need a benefit that surpasses the cost, hence generate a positive effect for the charterer.

To analyze the switching costs concerning the OSV market, we must discuss PSV and AHTS, respectively. The main costs concerning the replacement of a PSV follows:

- *Tank wash.* This cost will take the largest stake of the aggregated cost for replacement of PSV. The charterer is obliged to clean the vessel thoroughly before redelivery. Even though the charterer needs to wash the ship regardless, this will be an extra cost since the new vessel demands a wash at the end of the new contract period as well.
- *Down time* will occur if the ship is in operation when the replacement opportunity takes place. Consequently, this will add an extra cost due to the extra time the vessel needs to be in operation before redelivery. Additionally, when the charterer terminates the initial contract, the charterer will have an overlap of two vessels for a brief time until the first vessel is delivered back to the owner.
- *Bunker cost* is the cost regarding the additional fuel consumption relocating the terminated vessel to redelivery port and fuel consumption for transport of the new vessel.
- *Demobilization cost,* is cost concerning additional time of discharging the vessel before redelivery.

In discussion with Statoil, we suggest that the replacement cost for the PSV is £80,000 for the tank wash plus one initial contract rate for the additional down time of the vessel. That replacement cost will be the base case value of our model. In section 6, a thoroughly investigation of this value is presented.

Replacement cost regarding the AHTS will be quite similar for *Bunker and demobilization costs* for PSV. Despite the similar fuel and demobilization costs, AHTS will differentiate in other areas:

- *Tank wash* is not relevant for AHTS vessels, however washing cost occur for AHTS vessels as well, but these costs are limited compared to PSV, since the vessels do not carry chemicals.
- *Downtime,* is more significant for AHTS than PSV. The anchor handling and rig move operations are more complex and resource required activities. The operational costs will take the largest stake of the replacement costs for AHTS. The operations of tugging rigs are very expensive and thus it is not easy to replace vessels in such complex operations. However, by using a Bermudan option the downtime cost is reduced significantly due to fewer exercise opportunities which is only permitted between operations. Consequently, charterers can only replace vessels in operation under special operational situations or under a break in the operation.

For the AHTS, we will use a replacement cost of (0.5Initial contract rate + £7,000) *2 as the base case replacement cost. That value is the lowest possible replacement cost multiplied with two. This base case value will be further investigated in section 6.

The model for valuing the optionality of AHTS and PSV is similar, however, AHTS has limited possibilities to exercise the option. We introduce a dummy variable, which deduct whether the option is exercised or not. This dummy will have the value one, when the spot rate falls under the strike price, as discussed in the introduction. Let *D* denote a dummy variable with the following condition:

$$D = 1 \wedge (P_t * (T - t)) - \mathrm{R}C) > 0, and\ 0\ if\ not \quad (10)$$

This equation calculates P_t at every *t* value during the contract length, *T*, given the stochastic process. Further, the model multiplies the given P_t with the remainder of the contract length, $(T - t)$ to find the total additional value for the charterer. If this additional value surpasses the RC, the real option is exercised, thus the D will be given the value one. However, if RC exceeds the total additional value the dummy variable is given a value of zero, hence the option is not yet exercised. This process will continue until the D is 1 or the t = T, hence expired and no additional value.

Given the equation (10) above, let V_{opt} be the value of the optionality when the option is permitted to exercise once:

$$V_{opt} \sum_{t=1}^{T} X_t = D * (P_{\mathrm{t}}) * (T - t) - \mathrm{R}C \tag{11}$$

Equation (11) is based on a numerical approach where the valuation of the real option is calculated on an assumption of replacing the vessel when the market spot rate falls below the strike price. In the OSV spot-market segment, that approach is somehow imprecise. The charterer can contact the shipowner for a renegotiation process of the initial contract when the spot-rates fall below the strike price. Consequently, the charterer will attempt to renegotiate a new contract rate with the leverage of abandon the initial contract, using the daily optionality embedded.

We suggest that this bargain power will introduce an extra value to the optionality of the contracts:

$$V_{opt} \sum_{t=1}^{T} X_t = D * ((P_t) * (T - t) - \mathrm{R}C + (Y * \mathrm{R}C)) \tag{12}$$

Where the additional $Y * RC$ denotes the extra value a renegotiation on the initial contract, and consequently provide the charterer with the possibility to ignore the replacement cost.[9] This additional value will vary with a Y from 0 to 1 depending on the bargain power of the ship-owner and the charterer. In situations where the charterer renegotiates the initial contract to correspond to the market rate, the replacement cost will be totally ignored. Thus the Y will be 1 and:

$$V_{opt} \sum_{t=1}^{T} X_t = D * ((P_t) * (T - t) - \mathrm{R}C + (1 * \mathrm{R}C)) \tag{13}$$

Rearranged to:

[9] The renegotiation process will be thoroughly discussed in section 5.

$$V_{opt} \sum_{t=1}^{T} X_t = D * (P_t) * (T - t) \tag{14}$$

Note that equation 10 is unaffected by the new valuation in equation (12). Thus, the value of the *real option with renegotiation* will only have additional value when the D is 1. That suggests that the market spot rate falls below the strike price within the time frame of the contract length, providing the charterer the leverage to abandon the initial contract, and consequently find a new vessel in the spot market.

To investigate the value of optionality for PSV and AHTS in the spot market, we will perform simulations in Python. The simulations will run 100,000 replications of equation (10) and (11) to find the additional value the optionality provides the charterer for each simulation. Python will thereafter summarize the values and divide the aggregated value by the number of simulations (100,000) to find the average value for the specific real option.

5 Empirical analysis

In this section, an analysis of the base case parameters will be provided. The section will discuss vessel specifications and contract lengths for PSV and AHTS. Base case scenarios of various parameters for the most appropriate valuation of the optionality in the spot-market will be systematically investigated. Also, the renegotiation process will be thoroughly explained in the later part of the section. Finally, a valuation of the optionality for various contract specifications and vessel types will be provided as the summary of the empirical analysis.

Table 5 summarizes the base case values obtained from the stochastic modelling. The table provides the value of optionality with the most appropriate parameters. We decided to assume the long run equilibrium closer towards the median, since these estimates provide a more accurate replication of the historical rates. Hence, we suggest 1.25α as the long run equilibrium.

Table 5: Base case values for all vessel types[10]

	Base case value			
Parameter	PSV-S	PSV-B	AHTS-S	AHTS-B
α (£)	7,709	9,934	11,277	18,872
RC (£)	ICR + 80,000	ICR + 80,000	2RC	2RC
k	0.0222	0.0115	0.0181	0.0171
σ	0.12	0.11	0.17	0.17

As illustrated in table 6, the value of the optionality for S(small) and B(big) vessels will be, in general, quite similar, even though table 6 suggests a slightly higher valuation in smaller vessels. However, data provided by Ulstein Group suggest that the biggest vessels size for both PSV and AHST is the most common, see appendix (2) for further details. Thus, for the purpose of this analysis, we will use the PSV-B and AHTS-B to represent the PSV and AHTS segments, respectively. Table 6 shows that initial contract rates and value of the optionality are positively correlated. Additionally, contract lengths will increase the option value as well, as expected.

[10] ICR corresponds to initial contract rate

Table 6: Option value for various initial contract rates and vessel types

Initial contract rates(£)	*Option value(£)*	
PSV-S	*14-days contract*	*21-days contract*
5,000	-	-
10,000	-	80
15,000	14	2,116
20,000	334	6,730
25,000	1,401	12,464
30,000	3,282	18,944
PSV-B		
5,000	-	-
10,000	-	74
15,000	14	1,817
20,000	274	5,977
25,000	1,200	11,268
30,000	2,823	17,037
AHTS-S		
10,000	3,825	11,680
20,000	15,976	38,109
30,000	29,432	65,609
40,000	43,207	93,302
50,000	57,696	121,119
60,000	71,235	150,029
AHTS-B		
10,000	3,058	9,646
20,000	13,846	33,571
30,000	26,650	59,447
40,000	39,155	85,343
50,000	52,188	111,597
60,000	65,315	137,804

In table 7, the base case parameters are shown. For the PSV segment, we suggest a mean reversion equilibrium at 1.25α. Since the median of the PSV spot rates is 9,886, we find that the 1.25 multiple (9,934) will be the most appropriate parameter for PSV. Replacement cost is set to one initial contract plus tank wash, as suggested from Statoil for the best fit. The k and σ are parameters from the regression model. Y is an assumption that the two negotiating parts have equal bargaining power each, and thus split any profit from a plausible renegotiation process equally among them.

For AHTS, the base case value of the mean reversion equilibrium is 1.25α, as well. This value is closer to the median of 17,110 and thus, a more appropriate parameter for the model. The replacement cost in the case of AHTS is more complex. Since the RC $(X_0 + £7000)$ is the lowest possible replacement cost, we suggest that 2RC is more realistic for the model. The k, σ and Y are predicted in the same manner as for PSV.

Table 7: Base case parameters for PSV and AHTS

PSV-B

Parameter	Base case value	Range of variation
α(£)	9,934(α*1.25)	α - 5α
RC(£)	Initial contract + 80,000	Initial contract - 2Initial contract + 80,000
k	0,0115	-
σ	0.11	-
Y	0,5	0 - 1

AHTS-B

Parameter	Base case value	Range of variation
α(£)	18,872 (α*1.25)	α - 5α
RC(£)	2RC	RC - 2.5RC
k	0,0171	-
σ	0,17	-
Y	0,5	0 - 1

PSV

The scenario analysis that follows is built on the assumption that the charterer will terminate the initial contract instantly as soon as the alternative spot contract becomes more profitable or, in other words, that the spot rate falls below the strike price. Therefore, we find it relevant to test the plausible additional value of requiring an additional fall below the strike price. For the AHTS, this will not be realistic, as the possible termination dates occurs only every fifth day. Our model for PSV, in contrast, follows an American option, that can be exercised every day from contract inception to expiry. All charterers have different risk aversion that will affect their termination logic. Zakamouline (2003) stated that the more risk adverse investor, the earlier the investor would exercise an American option. It is not clear that the charterer will terminate the contract when the spot market offers a better deal. Even with possibility to secure profit, the charterer may consider waiting for a further drop in the rates. In table 8, the option value of various additional rate falls below the strike price is set as a barrier to exercise. By requiring an additional fall for the case of PSV, the table indicates the optimal additional fall in rates for a £30,000 contract is £1,250 while for a £20,000 contract, the optimal termination rate is £250 below the strike price.

Table 8 suggests further that for higher initial contract rates, it would be more profitable to wait for an even more pronounced decrease in rates before exercising. This could be explained by the mean reversion parameter in the stochastic model, where higher rates lead to stronger momentum towards the mean reversion rate.

Table 8: Shows how additional rate fall below the strike price affects the option value

Various termination barriers	*Initial contract rates(£)*					
	10,000	*(% terminations)*	20,000	*(% terminations)*	30,000	*(% terminations)*
-	-	-	273	3.2%	3,915	*22.8%*
250	-	-	287	2.6%	4,213	*21.4%*
500	-	-	275	1.7%	4,413	*19.8%*
750	-	-	251	1.4%	4,530	*18.3%*
1,000	-	-	248	1.3%	4,606	*17.0%*
1,250	-	-	219	1.0%	4,658	*15.7%*
1,500	-	-	183	0.8%	4,640	*14.5%*
1,750	-	-	153	0.6%	4,549	*13.2%*

However, we assume for the purpose of our scenario analysis that the charterer will terminate the contract immediately as soon as the spot rate falls below the strike price.

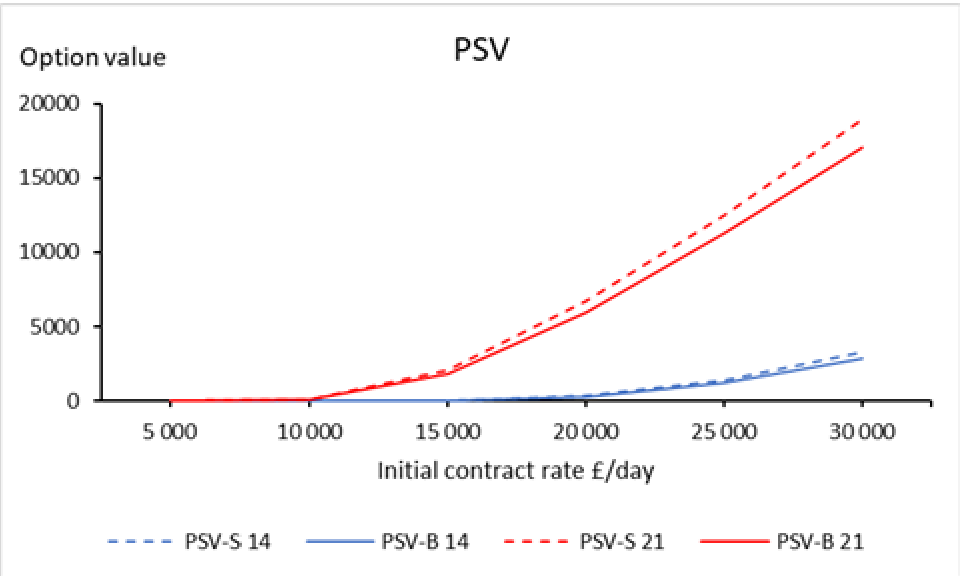


Figure 3: PSV-S and PSV-B comparison on contract lengths

Figure 3 is a graphic illustration of the option value for a set of initial contract rates and contract lengths with the base case parameters from table 7. The x-axis represents the initial contract rates for the spot market, while the y-axis represents the aggregated option value for the contract for each initial contract rate. As illustrated in the graphs above, the value of the optionality will be heavily dependent on the parameters in the x-axis and the contract length applied for the calculations. Since the replacement cost for PSV contains the initial contract rate, the replacement cost will also be higher for higher initial contract rates. However, the probability of termination prior to contract expiry increases significantly for increased initial contract rates as shown in figure 4 below. That occurs because of the greater possibility of spot rates to fall below the strike price, i.e. the initial contract will require a lower percentage fall in a £30,000 contract than for a £10,000 contract to cover the fixed cost of replacing the vessel in spot market. In figure 4, the probability of termination for contracts on both PSV-S and PSV-B are illustrated for 14 and 21- days contracts, denoted as PSV-S 14 for the PSV-S 14-days contract, PSV-B 21 for the PSV-B 21-days contract etc.

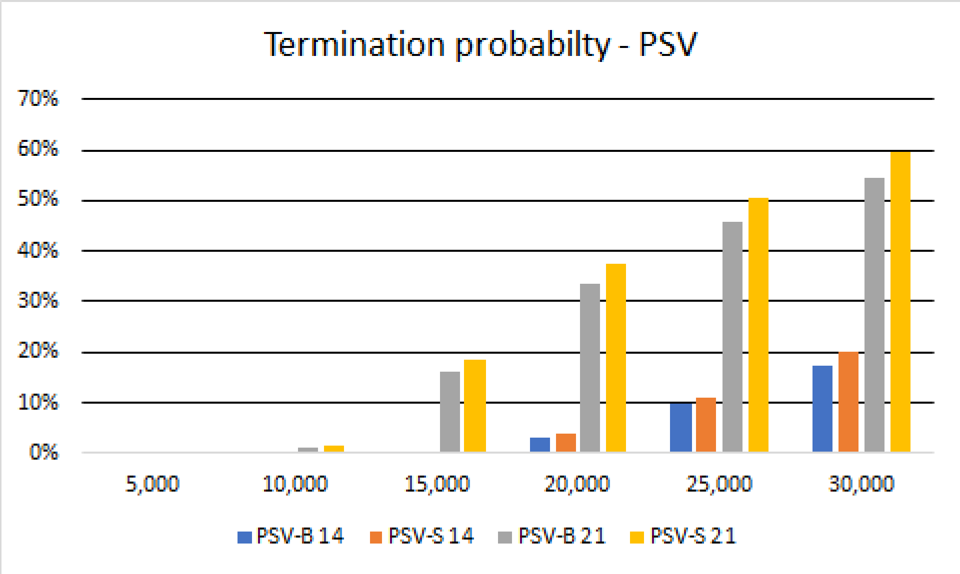


Figure 4: Probability of termination for PSV-S and B on 14-days and 21-days contracts

The initial contract rates represent the x-axis while the y-axis represent the probability of a termination of the contracts.

Longer contract periods imply higher probability of termination for the charterer. Intuitively, that is not surprising, as longer contract periods provide more options for terminating the contract than contracts of a shorter maturity. Additionally, longer contract periods lead to higher probability of a substantial change in spot rates.

Figure 3 has earlier illustrated very low values for the 14-days contracts, with zero option value for initial contract rates below approximately £15,000 for both PSV size categories. For 21-days contracts, the option value will be zero for contract rates below approximately £10,000. The value of a 14-days contract with an initial contract rate of £30,000 is £3,282, while the value for the same initial contract rate of 21-days is £18,944, or 578% higher. Table 9 illustrates the value of the optionality distributed over the total contract period. The daily implied discount is the (%) or (£) of the total cost of the contract the optionality provides to the charterer. The daily implied discount shows that the 21-days contract outperform the 14-days contract in both percentage (%) and cash (£) terms.

Table 9: Daily implied discount for initial contract rate of £30,000 PSV

	14-days	21-days
Totat cost of the contract(£)	420,000	630,000
Aggregated option value(£)	3,282	18,944
Daily implied discount(%)	0.056%	0.143%
Daily implied discount(£)	234	902

Even though the options are far more valuable for longer contract periods, these contracts are comparatively rare in reality. According to Statoil, the 14-days contracts are the most common contracts in the spot market. Consequently, our analysis will use 14 days as the base contract length.

AHTS

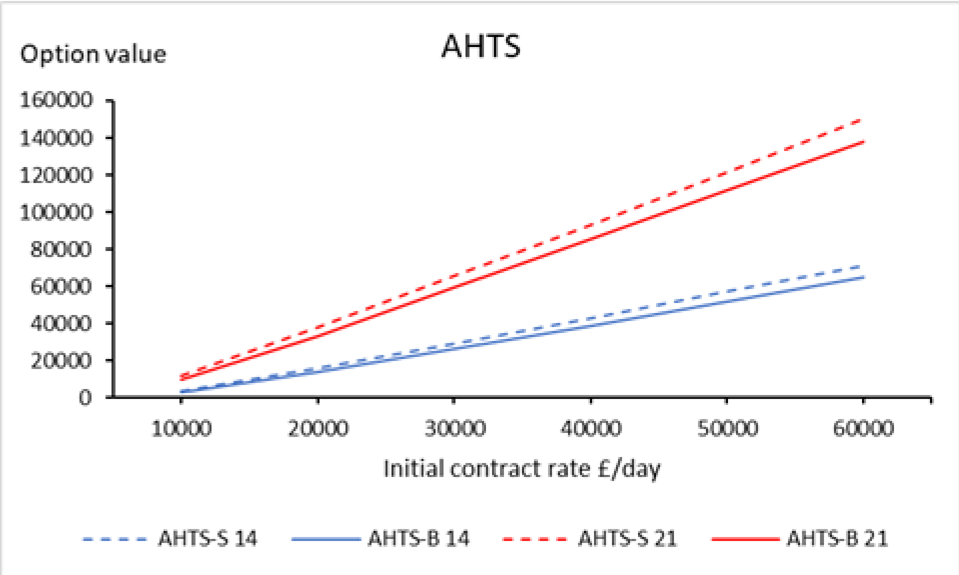


Figure 5: AHTS-S and AHTS-B comparison of contract lengths

Figure 5 represents the option value for an AHTS vessel in the same manner Figure 3 has earlier represented the respective values for PSV. As illustrated in table 6, AHTS have greater option value for all contract lengths when compared to PSV.

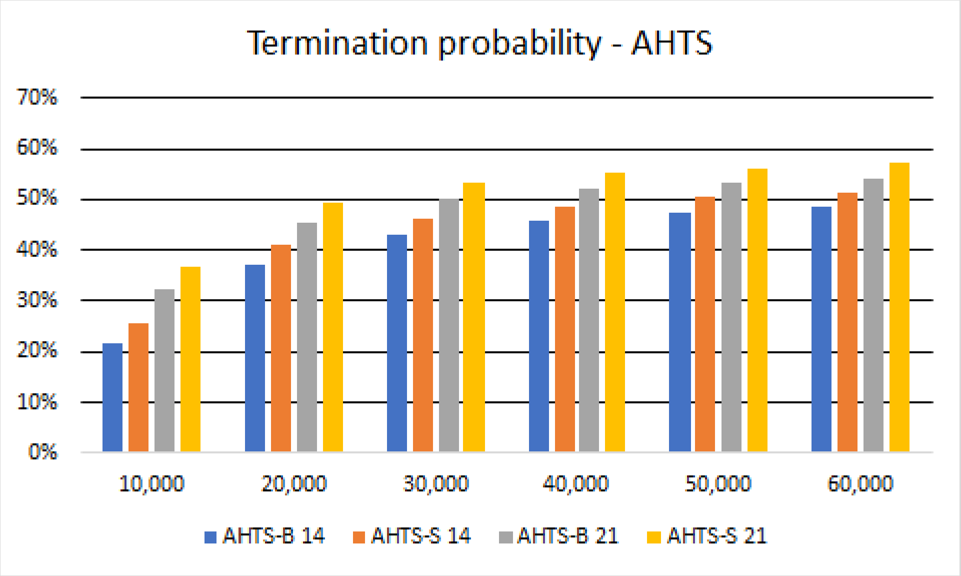


Figure 6: Contract termination for AHTS

Figure 5 has illustrated significant value for the charterer on all contract lengths. Consequently, Figure 6 shows that the termination probabilities are greater as well. The option value of a 14-days contract with £40,000 as the initial contract rate is £39,155, while the value for a 21-days contract with the same parameters is £85,343, or 218% higher.

Table 10 shows that the daily implied discount in percentage (%) terms is equal for both contract lengths. However, the daily discount in cash (£) terms suggests a higher value for the 21-days contract. This discount in cash (£) terms is greater for the longer contract period, but equal in percentage terms because the daily option value is compared to the total cost of the chartering, hence a higher aggregated cost for the 21-days contract than the 14-days contract applies.

Table 10: Daily implied discount for initial contract rate of 40,000 AHTS

	14-days	21-days
Totat cost of the contract(£)	560,000	840,000
Aggregated option value(£)	39,155	85,343
Daily implied discount(%)	0.499%	0.484%
Daily implied discount(£)	2,797	4,064

As expected, longer contract periods will provide more value to the charterer. However, from our discussions with Statoil, we find these contracts also rarely used in reality. The 14-days contracts are the most common in the AHTS. Hence, in the scenario analysis that follows, the evaluated contract periods will be assumed as 14 days for the AHTS.

Scenario analysis of base case values

The scenario analysis that follows will show that the model is sensitive to some variables. By using the range of variation on base case values presented in table 7, we will investigate the sensitivity of the parameters.

PSV

Table 11 sums up the value of optionality for the PSV-B on 14-days contracts for several replacement cost assumptions. Based on our discussion with Statoil, we suggest that the tank wash will cost £80,000. For the replacement of a vessel without required tank wash, this cost can be ignored. The impact of replacement cost on the option value is significant, and any reduction of the replacement cost will benefit the charterer substantially. As discussed previously, the replacement cost will include all additional cost for the charterer to replace the initial vessel with an alternative vessel in the spot market. However, if the charterer can avoid tank wash for instance, the replacement cost will be significantly reduced, and the option value will increase subsequently.

Table 11: PSV-B aggregated option value for various replacement costs

Replacement cost(£)	*Initial contract rates(£)*		
	10,000	20,000	30,000
Initial contract rate	6,054	13167	20159
2Initial contract rate	3,301	7431	11411
Initial contract rate + 80,000	-	273	2783
2Initial contract rate + 80,000	-	36	748

As the base case scenario takes into account the initial contract rate plus £80,000 for the tank wash, the value of the optionality requires initial contract rates of approximately £20,000 or above to provide additional value for the charterer. In Table 11, the contract length is 14 days. Consequently, the optionality values will be greater for a longer contract period, see Figure 3. However, as discussed previously, longer contract periods are very rare for this segment. If the charterer can manage to ignore the tank wash, the analysis suggests a significant additional value coming from the optionality. Without the tank wash, the option value for £20,000 contracts is £13,167.

Table 12 shows how the mean equilibrium rate affects the value of optionality. Since the stochastic model provides a mean equilibrium different from the median, we will present a scenario analysis of the PSV option value for this parameter.

Table 12: PSV-B option value for various mean reverting equilibriums.

Mean reverting equalibrium(£)		Initial contract rates(£)		
		10,000	20,000	30,000
6,358	*(α)*0.75*	-	308	4,111
7,947	*(α)*	-	298	4,069
9,934	*(α)*1.25*	-	272	3,945
11,921	*(α)*1.5*	-	263	3,860
13,907	*(α)*1.75*		251	3,791
15,894	(α)*2		222	3,669

As shown in the Table 12, the mean reverting equilibrium does not affect the option value severely. Thus, we suggest that the base case equilibrium is the α*1.25, which is closest to the median for PSV-B stated in Table 1.

Table 13 uses the α*1.25 as the mean reverting equilibrium, as well as the base case value of the replacement cost. By using these parameters as the most appropriate in our model, we can now plot initial contract rates in the table to investigate the value of optionality for PSV, and how the initial contract rate affects the option value.

Table 13: PSV-B option value for various initial contract rates

Initial contract rates(£)		*Value of optionality(£)*	*Termination probability*
6,167	*(α)*	-	0.0%
12,334	*(α*2)*	-	0.0%
18,501	*(α*3)*	121	1.7%
24,668	*(α*4)*	1,425	11.4%
30,835	(α*5)	4,475	24.8%

For the first row of results, the initial contract is the mean reverting equilibrium. This value gives no additional value for the charterer. The same can be stated for values up to(α*3). As expected, the initial contract rate must be quite high to add any value for the charterer. In situations where the charterer can ignore the replacement cost, there will be additional value as well. The termination probability in table T3 states therefore the obvious, i.e. that the higher the initial contract rate, the higher the probability of a contract termination during these 14 days.

AHTS

For AHTS, the replacement cost will be quite low, assuming the termination only can occur between operations, and thus, the capital-intensive operations will not affect the replacement. The replacement cost will vary for each contract, depending on where the vessel is located when the contract is terminated. We will conduct a scenario analysis on various replacement cost levels to examine the effect of the replacement cost for AHTS in the following table.

Table 14: PSV-B option value for various replacement costs

Replacement cost	*Initial contract rates(£)*		
	10,000	30,000	50,000
*(0.5*initial contract rate+£7000)*	6,439	36,709	68,677
*(0.5*initial contract rate+£7000)*1.5*	4,523	31,435	60,747
*(0.5*initial contract rate+£7000)*2*	3,039	26,416	52,645
*(0.5*initial contract rate+£7000)*2.5*	1,896	22,019	44,995

In Table 14, we used *0.5*initial contract rate + £7,000* as the lowest possible replacement cost. That rate considers the vessel to be off-operations and located at Mongstad port. Suggestions from Statoil show this cost level as the bottom possible replacement cost, assuming that the vessel is demobilized and ready for redelivery. However, according to Statoil this situation again is not very frequent. Thus, we need to analyze different replacement cost levels as well. From Table 14, the replacement cost scenarios indicate that the parameter is highly relevant for the value of the optionality. For instance, option values will vary from £68,677 to £44,995 by multiplying the replacement cost with 2.5. Actual cost from replacement will vary regarding location, specification and other parameters that will affect the plausible contract termination. By applying the Bermudan option, we will only investigate the off-operations situations. Thus, the vessel will be eligible for a replacement of an alternative in the spot market. Suggesting a replacement cost level of (*0.5*initial contract rate + £7,000) *2* we assume this cost level will represent the average cost of a vessel off-operation replacement and thus will be the base case for our valuation.

Table 15: AHTS-B option value for various mean reverting equilibriums

Mean reverting equalibrium(£)		Initial contract rates(£)		
		10,000	30,000	50,000
9,436	*(α*0.75)*	4,669	29,406	55,184
12,581	*(α)*	3,998	28,534	54,466
15,726	*(α*1.25)*	3,528	27,211	53,077
18,872	*(α*1.5)*	3,027	26,526	52,074
22,017	*(α*1.75)*	2,662	25,305	51,329
25,162	*(α*2)*	2,283	24,740	50,128

Table 15 presents the value of optionality for different mean reverting rates. Descriptive statistics in section 3 suggest a median of £17,110. The equilibrium rate α is lower than the median and thus we find it appropriate to investigate the influence this parameter has on the option value. In table 15 above, we see that the mean reverting equilibrium is not affecting the value significantly for initial contracts of £30,000 and £50,000. Even though initial contracts with value £10,000 will have a relative noteworthy impact when changing the mean reverting equilibrium with 25%, we suggest that the α*1.25 is the most appropriate parameter because it is closer to the median.

Table 16 sums up the option value for various initial contract rates with the base case parameters discussed previously.

Table 16: AHTS-B option value for various initial contract rates

Initial contract rates(£)		*Value of optionality(£)*	*Termination probability*
12,581	*(α)*	5,537	27.7%
25,162	*(α*2)*	20,338	40.7%
37,743	*(α*3)*	36,243	45.4%
50,324	*(α*4)*	53,017	47.6%
62,905	*(α*5)*	69,284	48.9%
75,486	*(α*6)*	85,910	50.0%

Table 16 illustrates the notion that higher initial contract rates provide higher value of the optionality. Even an initial contract rate at the long-run equilibrium level, α, provides the charterer with additional value. As stated in Table 16, the value will be approximately £5,537 and the termination probability is 27,7% for that parameter. For increasing initial contract rates, the value will increase significantly. Consequently, the termination probability increases as well. (α*6) will be the lowest initial contract rate that present a termination probability of over 50%, with the base case scenario of replacement cost and mean reverting. That initial contract rate will represent an additional value of £75,486.

Compared to the PSV, we find the AHTS optionality severely more valuable. As illustrated in table 13 and 16, the initial contract rate requires higher multiples of the α to generate a sound value for PSV than AHTS. That can mainly be explained by the 6%-point higher volatility for the letter vessel type. Additional, the replacement cost is more capital-intensive for the PSV, where the tank wash to clean the vessel after chemical delivery will be costly for the charterer. For AHTS we solved the capital-intensive operations by suggesting termination only to be possible between operations. Consequently, the low replacement cost and the higher volatility generate a solid value for the AHTS. However, if the charterer negotiates a contract longer than 14-days, the figure 3 indicates a substantial additional value for the PSV as well. Even though these contracts are rare, the charterer has the opportunity in some market conditions to negotiate contracts for longer periods than 14-days. In periods with low demand, the ship-owners are more willing to increase the contract lengths. Thus, the charterer has more bargain power to negotiate longer contract lengths and consequently increase the value of the contracts.

Renegotiation process of the initial contract rate

So far, our analysis is based on certain simulations where the valuation of the real options relies on the assumption that the vessel be replaced when the spread of the spot market and the initial contract reaches a certain level beneficial for the charterer. In the spot market, that approach lacks precision, because the charterer has the possibility of contacting the shipowner and consequently renegotiating the initial contract rate, thus rendering replacement cost irrelevant. The charterer will enter such renegotiations with the leverage/threat of abandoning the current contract, by using the optionality embedded in the spot market contracts.

The process of renegotiation starts when the charterer initiates a dialogue between the counterparties. Charterer and shipowner will have different degrees of leverage while negotiating, where the charterer uses the option to abandon the contract, while the shipowner will attempt to use the vessel's importance in the charterer's operations for leverage.

The leverage in the renegotiation will vary from situation to situation. The shipowner benefit from knowledge about the vessel's future operations before contract expiry. For instance, a PSV has many dirty tanks that need cleaning. Thus, the shipowner will use the leverage towards the charterer that it is costly and time consuming to replace the vessel. Consequently, the shipowner will require a rate higher than the actual spot rate. AHTS vessels could be stored with a lot of equipment for a rig move that needs to be delivered to another vessel. These costs are rendered irrelevant if the initial contract is renegotiated, rather than the contract terminated. In these circumstances, the shipowner will argue that the charterer must consider the cost of replacing the vessel in their calculations. Location of the vessel is also a factor where costs are associated with "sail-time" to delivery port. Therefore, communication between shipowners and the vessel captains is important for a successful renegotiation for the shipowners. The charterer on the other hand will intend to lower the initial contract by threatening the shipowner to abandon the contract, and therefore forcing the shipowner to re-offer the vessel in the spot market for a lower rate.

Remaining time to contract maturity is also crucial. If the contract is close to expiry, and the shipowner is aware of the replacement cost, the shipowner will have better cards to negotiate, hence the rate will only be slightly lower than the initial contract rate. However, if the contract duration is longer, the charterer has more leverage.

We suggest that the possibility of ignoring the replacement cost by threatening to abandon the contract will introduce an additional value of the optionality in spot contracts.

We assume both negotiating parties to act fully rationally, with all relevant information available to both parties. As shown in Table 17, both parties will benefit from a renegotiated rate between the market rate and the charterer's indifference rate (where the charterer is indifferent whether to replace the initial vessel or continue with the existing contract).

Table 17: Indifferent rate for 10 remaining days for a £20,000 PSV contract

		Contract rate(£)	*Total remaining cost(£)*		**Additional value of renegotiation(£)**		
					Charterer	*Ship-owner*	*Combined = RC*
(1)	*Initial contract rate*	20,000	**200,000**	*(20,000*10)*	**0**	**0**	**0**
(2)	*Indifferent rate(Y=0)*	18,000	**180,000**	*(18,000*10)*	**0**	**100,000** (18,000-8,000)*10	**100,000**
(3)	*Renegotiated rate(Y=0.3)*	15,000	**150,000**	*(15,000*10)*	**30,000** (180,000-15	**70,000** (15,000-8,000)*10	**100,000**
(4)	*Renegotiated rate(Y=0.5)*	13,000	**130,000**	*(13,000*10)*	**50,000** (180,000-13	**50,000** (13,000-8,000)*10	**100,000**
(5)	*Market spot rate*	8,000	**180,000**	*(8000*10 + RC)*	**0**	**0**	**0**

In Table 17, we constructed an example to show how the various indifference rates are calculated. The example suggests a contract with 10 remaining days of initial contract period, as well as a replacement cost of £100,000. For the initial contract rate (1), the total cost of the rest of the hiring period is £200,000 for the charterer. Additionally, if the spot market (5) offers £8,000/day for the same vessel type, this will result in a total cost of £180,000 including the replacement cost (£100,000) if the charterer choses to terminate the contract and hire a new vessel. As suggested in the numerical example, the charterer will terminate the initial contract, and consequently chose the alternative for a benefit of £20,000 (£200,000-£180,000). However, if the charterer can renegotiate the initial contract below the indifferent rate (2) of £18,000, and thus forego the replacement cost (RC), the value will be greater. The indifference rate resembles the renegotiated rate where the charterer is indifferent whether to terminate the contract. For example, if the charterer renegotiates a new rate for the initial contract of £15,000 (3), the new rate will save the charterer £50,000 ((£20,000-£15,000) *10). Additionally, the shipowner will have a greater return of £15,000/day than the alternative in the spot market (£8,000/a day).

Another look at Table 17 reveals that the spread between the indifference rate and the spot rate correspond to the replacement cost. £18,000 – £8,000 = £10,000. By multiplying that spread with the remaining days of the contract (10), we find that the spread resembles the replacement cost of £100,000. Consequently, the renegotiation process will add an additionally (Y*RC). As mentioned in chapter 4, the Y corresponds to the percentage of the replacement cost the charterer can save from a renegotiation of the initial rate.

In Table 17, the examples of Y=0 (2), Y=0.3 (3) and Y=0.5 (4) are presented as renegotiation results. The additional values of the given Y`s represent the percentage of the RC the charterer will gain from the renegotiation. As stated in the table, Y=0.3 provides the charterer with £30,000 (0.3*£100,000), hence the shipowner will profit the remaining part of the RC, which corresponds to £70,000. The additional value the shipowner gets from the deal can also be stated as the difference between the renegotiated rate of £15,000 and the spot rate the vessel would otherwise achieve on the spot market, multiplied by the remaining days of the contract. (£15,000 – £8,000) *10 = £70,000.

Table 18 shows how such renegotiation will increase the value for the charterer. In the table, (0% * RC) presents the value of the optionality for various initial contract rates when the charterer renegotiates the initial rate down to the indifference rate explained in Table 17. That rate provides the charterer the same total cost as the alternative in the market. Consequently, the (0% * RC) is the same as the numerical valuation in the model *without renegotiation* benefits.

Table 18: Value of optionality for AHTS-B and PSV-B for various Y.

AHTS	*Initial contract rates(£)*		
*Y*RC(%)*	10,000	30,000	50,000
*0% * RC*	*3,042*	26,292	52,313
*25% * RC*	*4,308*	31,385	59,822
*50% * RC*	*5,600*	35,743	67,740
*75% * RC*	*6,930*	40,500	75,224
*100% * RC*	8,171	45,343	82,503
PSV			
*Y*RC(%)*	10,000	20,000	30,000
*0% * RC*	-	274	3,943
*25% * RC*	-	1,061	9,729
*50% * RC*	-	1,960	15,407
*75% * RC*	-	2,724	21,295
*100% * RC*	-	3,364	27,151

The last scenario in Table 18 shows the situation where the charterer can renegotiate the initial contract down to the market rate. Thus, the replacement cost will be totally irrelevant, and the benefit from the negotiated rate will be the replacement cost if compared to the alternative vessel in the spot market where additional cost of replacement will occur. Note that the market rate presented in Table 18 resembles a replacement of the initial vessel, and therefore includes the replacement cost.

Exactly which Y the charterer will receive is hard to predict. For market conditions of low demand, the shipowner will be more willing to renegotiate the contract closer to the market rate. Shipowners under such market conditions can be assumed eager to keep their vessels in operations. The alternative to the renegotiated rate is a significant lower spot market rate or even off-contract, which is very costly for the shipowners. Thus, the charterer has the most bargaining power and, if acting rationally, can negotiate preferable deals. However, in market conditions with high demand, shipowners can be assumed to have more bargaining power. When the rates are relative high, charterers will for obvious reasons not have the same bargaining power as for the lower demand-markets. Thus, the renegotiation will be tougher.

We suggest that a Y of 50% is the most appropriate for our valuation. This rate is in the middle of the range between the indifference rate and the market rate. The rate provides 0.5RC to each counterparty. Consequently, the charterer and the ship-owner split the benefit of foregoing replacement costs equally.

Tables 19 and 20 present total cost of contracts without optionality, with optionality (without renegotiation) and with optionality (with renegotiation). In the tables, we suggest the base case parameters discussed throughout section 5 as the most appropriate for the 14-days contract valuation that follows.

Table 19: Total cost for PSV for various contract specifications

	Total cost					
	Initial contract rates(£)					
Contract specifications	10 000	*Change(%)*	20 000	*Change(%)*	30 000	*Change(%)*
Without optionality	140 000		280 000		420 000	
With optionality and without renegotiation	140 000	*0,0%*	279 729	*0,1%*	416 123	*0,9%*
With optionality and renegotiation	140 000	*0,0%*	278 080	*0,7%*	404 810	*3,6%*

The total cost of the 15-days contract for PSV are heavily dependent on the contract specifications. As illustrated in the table above, initial contracts for the PSV of £10,000 will have no additional value in the embedded optionality for the contracts. That is not surprising, considering that the termination probability of that initial contract rate is 0%, as stated in figure 4. However, for contracts of greater initial rates, there will be value. The additional value of optionality will be approximately 0,1% without the renegotiation possibility, while 0,7% with the renegotiation possibility for £20,000 as the initial contract rate. For initial contract rates of £30,000, the value will be significantly higher. Even without the renegotiation, the value surpasses all the £20,000 contract specifications. As illustrated in table 19, the additional value increases from 0,9% to 3,6% when adding the renegotiation feature, providing a solid additional value for the charterer.

Table 20: Total cost for AHTS for various contract specifications

	Total cost					
	Initial contract rates(£)					
Contract specifications	10 000	*Change(%)*	30 000	*Change(%)*	50 000	*Change(%)*
Without optionality	140 000		420 000		700 000	
With optionality and without renegotiation	136 946	*2,2%*	393 647	*6,3%*	647 357	*7,5%*
With optionality and renegotiation	134 396	*4,0%*	384 301	*8,5%*	633 085	*9,6%*

For AHTS, the additional value of the optionality will be solid for all initial contract rates. As illustrated in Table 20, the additional value with £10,000 as the initial contract rate is 2,9% without the renegotiation, while 5,3% with the renegotiation. Higher initial contract rates provide more additional value as evident in Table 20, with increasing additional value in percentage terms. For £50,000 as the initial contract the additional value of optionality without the renegotiation is 8,3%, while the renegotiation provides an additional 2,3% to the value.

The relative increase in the additional value for PSV is greater than for AHTS because of the significant replacement cost. These costs from for example tank wash can be totally ignored if the renegotiation process yields a new contractual rate. For AHTS, replacement costs are lower off-operations, leading to less additional value of a renegotiated rate.

6 Conclusion

The main conclusion of our thesis can be summarized as follows: *All optionality provides additional value to the charterer*. However, in markets with low demand, and consequently low spot rates, such additional value of a possible vessel replacement is limited. The PSV segment displays significantly higher replacement costs and lower volatility in spot rates than the AHTS segment. Additionally, rates in the PSV segment are on average lower than in the AHTS segment of the market. All this suggests a lower level of additional value to the embedded optionality in PSV spot contracts. Our empirical analysis confirms this notion that the value of the optionality is far more insignificant for the PSV segment.

Optionality in AHTS contracts proves to outperform optionality in PSV contracts under all types of market conditions. Even though the renegotiation process is, relatively speaking, more valuable in the PSV segment, we suggest that the total value of this optionality is still rather poor. Charting in the AHTS segment on the other hand, provides solid additional value for the charterer. However, it is only when the chartered vessel is off-operation that executing the termination option becomes valuable. For contracts of 14-days maturity, which we used as the base case contract length in our analysis, our assumption provides the charterer with only two dates for terminating the contract. Still, our analysis confirms excellent value of this optionality.

PSV requires initial contract of over approximately £15,000 while the AHTS will provide additional value of almost all initial contracts. In summary, we find the value of contract termination optionality to be far greater for the AHTS spot market. While the optionality of terminating PSV contracts is not completely void of value, such value does require generally stronger market conditions to become significant in the PSV segment, compared to the AHTS segment.

There are some limitations to this study that may affect the results. First, we have not considered the optionality to purely abandon the contract. The valuation is based on the possibility to replace the initial vessel, hence the value to abandon has not been included in the results. Furthermore, we assumed that the charterer provides operations for the vessels during the lifetime of the contract, hence the valuation of the replacement option may be overvalued. However, we believe that these limitations contradict each other providing a more realistic value for the entire optionality. Additionally, we found autocorrelation in the market spot rates. That suggests that the rates are not fully independent from previous rate movements. Hence, a fall in rates may anticipate a further decline. This contradicts our assumption of exercising at first profitable moment since charterer can expect further downfall, thus higher value.

Although this thesis has expanded the literature on OSV spot market optionality, there are several areas to be investigated further. For example, the real option in our thesis can be expanded to include the term market as an alternative to the spot market. Additionally, topics concerning whether there is a premium payment for the daily optionality in the spot market, or if the optionality is free for charterers.

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8 Appendices

Appendix 1. Augmented Dickey-Fuller

We performed a Dickey-Fuller test to investigate if the collected time-series sample of rates is stationary or non-stationary, hence mean reverting or a random walk (Brownian motion). The autoregressive equation is given by:

$$X_t = \alpha + \rho X_{t-1} + e_t$$

Where, ρ is the mean reversion, α is the level the rates move towards and e_t is the standard error. To explore whether the test is independent of previous rates or not, we investigate the incremental change of X_t.

$$X_t - X_{t-1} = \alpha + (1-\rho)X_{t-1} + e_t$$

Replace (1-ρ) with, θ.

$$X_t - X_{t-1} = \alpha + \theta X_{t-1} + e_t$$

We then define hypothesis, the H_0: $\theta = 0$ ➔ $\rho = 1$ and H_1: $\theta < 0$ ➔ $\rho < 1$. If $\theta = 0$ is statistically significant, then the series is a random walk, hence not stationary and mean reverting.

For a time-series with more than 500 observations the significance level at 5% and 1% are -2.86 and -3.43, respectively.

Appendix 2: Vessels in service North Sea

The dataset provided by Ulstein Group contains of 5002 vessels before cleansing. We choose vessels in service in the North Sea. Divide vessels by the specification in Clarkson´s spot rates: PSV- above 900m2 and between 500-899m2 and AHTS- above 20,000 BHP and between 16-20,000 BHP.

Vessels in service in the North Sea

	PSV		AHTS	
	All	2010	All	2010
Big	156	57	43	17
Small	134	27	34	12

Appendix 3: Jarque-Bera test

Jarque-Bera test (J-B) is a test to establish the if the residuals are normally distributed, with regards to skewness and kurtosis. The (J-B) value is given by:

$$JB = \frac{n-k}{6} * \left(S^2 + \frac{(C-3)^2}{4}\right)$$

S is the skewness, C is the kurtosis, n is the number of observations and k is the number of regressors.

Appendix 4: Python code to estimate option value

The python code below is constructed to value the PSV.

```
01. #/usr/bin/python
02.
03. import math
04. import sys
05. import numpy as np
06.
07.
08. def general_model(xt, contract_length, volatility, mean_reversion, median, replacement_cost, dummy_value, v_opt):
09.
10.
11.     global count
12.
13.     for j in range(dummy_value, contract_length):
14.
15.         tmp_x = (xt[0]) - xt[j]
16.         true_tmp_x = (xt[0] - xt[j])
17.         tmp_cont = contract_length - j
18.         tmp_v_opt = (tmp_x * tmp_cont) - (replacement_cost)
19.         true_tmp_v_opt = (true_tmp_x * tmp_cont) - (replacement_cost)
20.
21.         if tmp_v_opt > 0:
22.              count += 1
23.              return true_tmp_v_opt
24.
25.
26.
27.
28.     # Return if no dummy gives 1
29.     return 0
30.
31. def calc_xt(x0, contract_length, volatility, mean_reversion, median, replacement_cost):
32.     xt = [0] * contract_length
33.     xt[0] = x0
34.
35.     # variables for normal Gaussian distribution, mean and standard deviation
36.     mu, sigma = 0, 1
37.
38.     for i in range(1, contract_length):
39.         s = np.random.normal(mu, sigma)
40.
41.         xt[i] = xt[i-1] + (median - xt[i-1]) * mean_reversion + xt[i-1] * volatility * s
42.
43.     return xt
44.
45. if(len(sys.argv) != 7):
46.     print ("Run with: \n$ python general_model.py 'X0' 'T' 'Volatility' 'Mean reversion' 'Median' 'Switching Cost'")
47. else:
48.     x0 = int(sys.argv[1])
49.     contract_length = int(sys.argv[2]) + 1
50.     volatility = float(sys.argv[3])
51.     mean_reversion = float(sys.argv[4])
52.     median = int(sys.argv[5])
53.     replacement_cost = int(sys.argv[6])
54.
55.     v_opt = [0] * 100000
56.     count = 0
57.
58.     # run it 100 000 times to get a median
59.     for i in range(0, 100000):
60.
61.
62.         # Calculate x table, initial values
63.         xt = calc_xt(x0, contract_length, volatility, mean_reversion, median, replacement_cost)
64.
65.         # Run model
66.         v_opt[i] = general_model(xt, contract_length, volatility, mean_reversion, median, replacement_cost, 1, 0)
67.
68.
69.
70.
71.     # Average of all simulations
72.     avg = sum(v_opt) / len(v_opt)
73.
74.
75.
76.     print ("Average:", avg)
77.     print ("Number of times D happened:", (count/100000), "\n")
78.
```

For the AHTS, we must change code line 21. By adding (j==5), the model only accepts terminations every fifth day, as our Bermudan option proposes.

```
21         if tmp_v_opt > 0 and j==5:
22              count += 1
23              return true_tmp_v_opt
```

Appendix 5: Multiple exercises

```python
#/usr/bin/python

import math
import sys
import numpy as np


def general_model(xt, contract_length, volatility, mean_reversion, median, replacement_cost, dummy_value, v_opt, count=0):
    for j in range(dummy_value, contract_length):
        tmp_x = xt[0] - xt[j]
        tmp_cont = contract_length - j
        tmp_v_opt = (tmp_x * tmp_cont) - replacement_cost

        if tmp_v_opt > 0:
            x_new = xt
            x_new[0] = xt[j]

            # Will recurse for each new dummy found, and add v_opt to itself
            return v_opt + general_model(x_new, contract_length, volatility, mean_reversion, median, replacement_cost, j, ((tmp_x * tmp_cont) - replacement_cost), count+1)

    # For keeping track of how often each Dx happens
    global d1_counter
    global d2_counter
    global d3_counter
    global d4_counter

    if count > 0:
        d1_counter += 1
    if count > 1:
        print ("D reached:", count, "\n")
        d2_counter += 1
        print (xt, "\n")
    if count > 2:
        print ("D reached:", count, "\n")
        d3_counter += 1
        print (xt, "\n")
    if count > 3:
        print ("D reached:", count, "\n")
        d3_counter += 1
        print (xt, "\n")

    # Return if no dummy gives 1
    return v_opt

def calc_xt(x0, contract_length, volatility, mean_reversion, median, replacement_cost):
    xt = [0] * contract_length
    xt[0] = x0

    # variables for normal Gaussian distribution, mean and standard deviation
    mu, sigma = 0, 1

    for i in range(1, contract_length):
        s = np.random.normal(mu, sigma)

        xt[i] = xt[i-1] + (median - xt[i-1]) * mean_reversion + xt[i-1] * volatility * s

    return xt
```

```
if(len(sys.argv) != 7):
    print ("Run with: \n$ python general_model.py 'X0' 'T' 'Volatility' 'Mean reversion' 'Median' 'Replacement cost'")
else:
    x0 = int(sys.argv[1])
    contract_length = int(sys.argv[2]) + 1
    volatility = float(sys.argv[3])
    mean_reversion = float(sys.argv[4])
    median = int(sys.argv[5])
    replacement_cost = int(sys.argv[6])

    v_opt = [0] * 100000

    d_day = [0] * contract_length
    count = 0

    # Number of times Dx happened
    d1_counter = 0
    d2_counter = 0
    d3_counter = 0
    d4_counter = 0

    # run it 100 000 times to get a median
    for i in range(0, 100000):

        # Calculate x table, initial values
        xt = calc_xt(x0, contract_length, volatility, mean_reversion, median, replacement_cost)

        # Run model
        v_opt[i] = general_model(xt, contract_length, volatility, mean_reversion, median, replacement_cost, 1, 0)

        # Count up occurences of D for each day
        for j in range(1, contract_length):
            if(xt[0] == xt[j]):
                d_day[j] += 1
                count += 1

    print ("Number of times D happened:", count, "\n")

    print ("Times D1 happened:", d1_counter)
    print ("Times D2 happened:", d2_counter)
    print ("Times D3 happened:", d3_counter)
    print ("Times D4 happened:", d4_counter, "\n")

    print ("Times D happened on each day:\n", d_day, "\n")

    for i in range(0,contract_length):
        d_day[i] = float(d_day[i] / 100000.0)*100

        print ("Percentage of D on day", i, ":", d_day[i])

    # Average of all simulations
    avg = sum(v_opt) / len(v_opt)

    print ("\nAverage v_opt:", avg)
```

This code provides proof that the multiple exercises is extremely rare. We conducted an analysis for the PSV-B with £30,000 as the initial contract rate and replacement cost of £100,000 on a 14-days contract, with the following result:

```
Number of times D happened: 24053

Times D1 happened: 24053
Times D2 happened: 156
Times D3 happened: 0
Times D4 happened: 0
```

The illustration above shows how many of the 100,000 simulations that provides a termination. The D1 presents the termination as explained throughout the thesis, while the D2 presents the simulations where the termination occurs twice during the life-time of the contract.