A SUSTAINABLE MODEL FOR OPTIMAL DYNAMIC ALLOCATION OF PATROL TUGS TO OIL TANKERS

Brice Assimizele*
Faculty of Maritime Technology and Operations
Aalesund University College
P O Box 1517
NO-6025 Ålesund
Norway

Johan Oppen
Planning, Optimization and Decision Support Group
Molde University College
P O Box 2110
NO-6402 Molde
Norway

Robin T. Bye
Faculty of Engineering and Natural Sciences
Aalesund University College
P O Box 1517
NO-6025 Ålesund
Norway

* Corresponding author. Email: bras@hials.no

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ABSTRACT

Oil tanker traffic constitutes a vital part of the maritime operations in the High North and is associated with considerable risk to the environment. As a consequence, the Norwegian Coastal Administration (NCA) administers a number of vessel traffic services (VTS) centers along the Norwegian coast, one of which is located in the town of Vardø, in the extreme northeast part of Norway. The task of the operators at the VTS center in Vardø is to command a fleet of tug vessels patrolling the northern Norwegian coastline such that the risk of oil tanker drifting accidents is reduced. Currently, these operators do not use computer algorithms or mathematical models to solve this dynamic resource allocation problem but rely on their own knowledge and experience when faced with constantly changing weather and traffic conditions. We therefore propose a novel sustainable model called the *receding horizon mixed integer programming (RHMIP)* model for optimal dynamic allocation of patrol vessels to oil tankers. The model combines features from model predictive control and linear programming. Simulations run with real-world parameters highlight the performance and quality of our method. The developed *RHMIP* model can be implemented as an operational decision support tool to the NCA.

1. INTRODUCTION

Maritime shipping is an important channel of international trade. More than seven billion tons of goods are carried by ships every year (Acar et al. 2009). In Norway, several hundred oil tankers transit each year along its northern coastline (Bye 2012). This traffic is associated with potential grounding accidents due to oil tankers losing control of steering or propulsion, a problem that is highly underreported and likely occurs almost every day[1]. Such accidents can have severe environmental consequences from oil spill and may even lead to loss of lives.

The VTS center in Vardø, located in the extreme northeast part of Norway, controls a fleet of tugs patrolling the coastline. By means of the automatic identification system (AIS) used by ships and VTS centers all over the world, the VTS center in Vardø obtains static (e.g., identity, dimensions, cargo, destination) and dynamic (e.g., position, speed, course, rate of turn) information about vessels along the coast. In addition to the AIS information, weather forecasts and dynamic models of wind, wave heights, and ocean currents can be used to predict possible drift trajectories and grounding locations of tankers that lose maneuverability. The aim of the patrolling tug vessels is to move along the coastline in a collectively intelligent manner such that potential drift trajectories can be intercepted. The closest tug vessel will then intercept the drifting oil tanker before it runs aground (Eide et al. 2007).

The number of oil tanker transits off the northern coastline of Norway is predicted to increase rapidly in the coming years (Institute of Maritime Research 2010). In addition, the number of patrolling tug vessels may increase as a response to the increase in oil tanker traffic. Consequently, the VTS operators' task of manually commanding the fleet of patrol tugs is becoming unmanageable without the aid of a decision support tool. Addressing this need, Bye (2012) and Bye, van Albada, and Yndestad (2010) used a heuristic and suboptimal *receding horizon genetic algorithm (RHGA)* to dynamically allocate patrolling tug vessels to oil tankers along the northern coastline of Norway. Our aim here is to present a *receding horizon mixed integer programming (RHMIP)* model to *optimally* solve the same fleet optimization problem.

The remainder of this paper is organized as follows: Section 2 explicitly describes the problem whereas Section 3 presents a methodology for the solution. Section 4 reports some computational experiments.

[1] Information provided by the NCA.

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Finally, discussions and propositions for future research are made in Section 5.

2. PROBLEM DESCRIPTION

Oil tankers move, by law, along piecewise-linear corridors well defined in advance and approximately parallel to the coastline. We adopt the problem description used in Bye (2012) and Bye et al. (2010). Accordingly, we assume a set C of oil tankers moving in one dimension along a line of motion *z*. Moreover, we assume a set P of tug vessels moving along a line of motion *y* parallel to *z* and close to shore. An illustration of the problem is presented in Figure 1, where patrol tug vessels are represented as black circles, oil tankers as white circles, predicted oil tanker positions as dashed circles, and circles with a cross represent points where the predicted drift trajectories cross the patrol line *y*. We refer to these points as *cross points*.

Based on real-time information of oil tankers from the AIS and on a set of forecasting models developed previously by the NCA and partners, we assume that it is possible to predict future oil tankers positions as well as the corresponding potential drift trajectories.

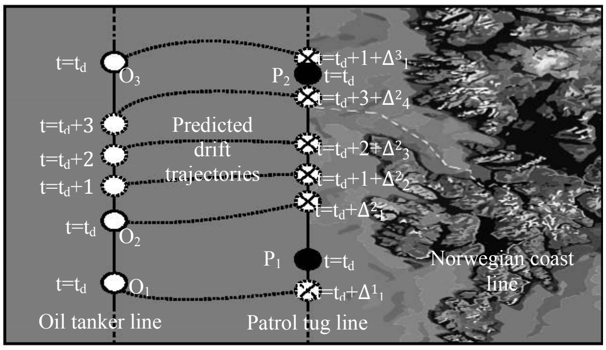

Figure 1: Problem illustration

Specifically, for each current position of a given oil tanker $z(t)$, there is a corresponding predicted drift trajectory which crosses the line y at a cross point $y(t')$ where $t'-t=\Delta$ represents the estimated drift time. Thus, the main goal is to make sure there is always a tug vessel at a position $y'(t')$ close enough to any potential cross point $y(t')$ to rescue the drifting oil tanker. That is, what is the optimal positioning of tug vessels along the coastline for a minimum rescue time of potentially drifting oil tankers?

3. METHODOLOGY

Task allocation in real-time systems in order to meet certain deadlines is known to be an NP-hard problem (Gertphol and Prasanna 2005). In addition, the highly uncertain weather conditions and the dynamic environment add to the complexity of the problem. To overcome these challenges, we propose using a combination of different methods that complement each other. An iterative solution approach for different types of problems that integrate optimization and simulation methodologies have been developed by several researchers in the literature (Acar et al. 2009). Here, we make use of the receding horizon control principle together with a linear optimization approach to develop our novel *RHMIP* model. Whilst this approach can be used to solve the specific problem presented in this paper, our model can likely be extended to solve other problems such as dynamic fleet optimization of platform supply vessels (PSVs) or other resource allocation problems both offshore and on land.

Model predictive control *(MPC)* or receding horizon control *(RHC)* is a class of control algorithms that uses explicit process models to predict the future response of a system and guide a system to a desired output using optimization as an intermediate step (Park et al. 2009). Receding horizon optimization is widely recognized as a highly practical approach with high performance (Zheng et al. 2011). It has become a very successful strategy in real-time control problems (Goodwin et al. 2006). Morari and Lee (1999) showed that many important practical and theoretical problems can be formulated in the RHC framework. The RHC algorithm consists of two main steps: (1) prediction of future system behavior on the basis of current measurements and a system model and (2) solution of an optimization problem for determining future values of the manipulated variables, subject to constraints (Wang et al. 2007). For a given planning time horizon T, with step $k \geq 0$ corresponding to the time instant $t_k = k\lambda$ with λ the sampling time, the future control sequence $\mu(k), \mu(k+1), \ldots, \mu(k+T-1)$ is computed by solving a discrete-time optimization problem over the period $[t_k, t_{k+T\lambda})$ in a way that a performance index defined over the considered period is optimized subject to some operational constraints. For our problem, tug vessels are constrained to move no faster than their maximal speed, which leads to a limitation on the number of oil tankers allocated to a given tug vessel. Once the optimal control sequence is computed, only the first control sample is implemented, and then the horizon is shifted. Subsequently, the new state of the system is estimated, and a new optimization problem at time t_{k+1} is solved using this new information (Tarău et al. 2011). In effect, the *RHC* principle introduces feedback control, and thus robustness to changes in the environment.

Mixed-integer linear programming (*MIP*) problems are optimization problems with a linear objective function, subject to linear equality and inequality constraints and where some variables are constrained to be integers. The advantage of using this approach is the availability of efficient solvers that can compute the global optimal solution within reasonable time (Tarău et al. 2011).

Despite the high uncertainty related to weather, wave heights and ocean currents, we have decided to develop a deterministic *MIP* model. This decision is justified by the fact that the model is run dynamically and parameters are updated at every time step, thus implicitly handling the stochasticity of the problem.

3.1 *RHMIP* model

Previous work done by Bye (2012) and Bye et al. (2010) aimed to reduce the distances between all cross points and the nearest patrol points in the planning horizon (which is equivalent to minimizing rescue time if all patrol tugs have the same maximal speed). Indeed, this is a logical choice that tries to maximize the number of oil tankers that can be rescued at any time.

The following cost function was used as a minimization objective: $\sum_{t=t_d}^{t_d+T}\sum_{c=1}^{N_o}\min_{p\in P}\left|y_t^c - y_t^p\right|$

Here, y_t^c represents the cross point of the drift trajectory of oil tanker c at time t, y_t^p is the position of patrol tug p at time t; N_o is the number of oil tankers, P is the set of patrol tugs, and T is the planning horizon. The above cost function can be rewritten as a linear cost function by adding some extra variables and can therefore be solved optimally using linear *MIP*.

In this study, we implement two variants of our model, one using static tug vessels *(Static MIP)* and the other with dynamic tug vessels *(RHMIP)*. The definition of sets, parameters and variables are as follows.

Sets

P set of tug vessels

C set of oil tankers

Parameters

$v_{\max}^p$ maximal speed of tug vessel p

$v_{\max}^c$ maximal speed of oil tanker c

y_t^c cross point of the cth oil tanker's predicted drift trajectory at time t. Note that t represent the time at which the drifting oil tanker crosses the patrol line.

y_0^p initial position of tug vessel p

y_0^c initial position of oil tanker c

$\Delta_{\min}$ drift time of oil tankers

λ length of each time period

T_{RHMIP} number of simulation steps

T length of the planning horizon of the *MIP* optimization model

M a large number[2]

Decision variables

Y_t^p position of tug vessel p in period t

I_t^p, J_t^p direction of tug vessel p in period t. If $I_t^p - J_t^p > 0$ the tug vessel will move forward and backward if $I_t^p - J_t^p < 0$, otherwise it will remain static.

X_t^{cp}, Z_t^{cp} distance between potentially drifting oil tanker c and tug vessel p in period t. One of the variables will contain the distance and the other variable will be equal to zero.

Ψ_t^{cp} distance from potentially drifting oil tanker c to its allocated tug vessel p in period t. Specifically, t represent the time period where the potentially drifting oil tanker c cross the patrol line.

$$W_t^{cp}\begin{cases}1 \text{ if tug vessel } p \text{ is allocated to oil tanker } c\\ \text{ in period } t\\ 0 \text{ otherwise}\end{cases}$$

3.2 Algorithm

Below is an algorithm implementing the RHMIP model.

Step 1:

a- *Let* $t := 0$; *let* $y_t^p :=$ *initial value* $\forall p \in P$;

b- Compute predicted drift trajectories, cross points of oil tankers and the predicted maximal speed of oil tankers and tug vessels.

c- Run *MIP* model to obtain the optimal position and allocation of patrolling tug vessels over the planning horizon $[t, t+1, ..., T]$.

d- Implement the first period of the *MIP* solution

Step 2:

a- Let t:=t+1;

Let $y_0^p := y_0^p + v_{\max}^p \cdot t \cdot \left(I_{\Delta_{\min}}^p - J_{\Delta_{\min}}^p\right) \forall \mathrm{p} \in \mathrm{P}$

(Obtain current position of tug vessels)

b- Update the predicted drift trajectories, cross points and the predicted new maximal speed of oil tankers and tug vessels. In addition, update the current number of oil tankers moving along the coastline as well as the available number of tug vessels.

c- Run *MIP* model to obtain the optimal position and allocation of patrolling tug vessels over the planning horizon $[t, t+1, ..., T+t]$.

d- Implement the first period of the new *MIP* solution

Step 3: Go back to Step 2 or stop if *t:=* T_{RHMIP} *+1*

The basic idea in Step 2 is that the maximal speed of oil tankers and tug vessels may vary over time due to changing weather conditions such as ocean currents, wave heights, and wind, or change in cargo weight after loading or unloading. In addition, some tug vessels may be unavailable due to maintenance or change of crew. Finally, an oil tanker leaving the defined protection zone should be removed from the set for the next

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[2] See constraints (4) and (8).

planning period, whereas other oil tankers may enter the zone and should be included in the next planning period.

The *MIP* model is used as an optimization phase in the algorithm:

Minimize $\displaystyle\sum_{c\in C,p\in P,t\in\{\Delta_{min}..T\}} \Psi_t^{cp}$

Subject to

$$Y_t^p = Y_{t-1}^p + v_{max}^p \cdot \lambda \cdot \left(I_t^p - J_t^p\right), \quad \forall p \in P, \forall t \in \left\{\left(\Delta_{min}+1\right)..T\right\} \tag{1}$$

$$I_t^p + J_t^p \le 1, \ \forall p \in P, \forall t \in \left\{\Delta_{min}..T\right\} \tag{2}$$

$$X_t^{cp} - Z_t^{cp} = y_t^c - Y_t^p, \ \forall c \in C, \forall p \in P, \forall t \in \left\{\Delta_{min}..T\right\} \tag{3}$$

$$M \cdot \left(1 - W_t^{cp}\right) + \Psi_t^{cp} \ge X_t^{cp} + Z_t^{cp}, \quad \forall c \in C, \forall p \in P, \forall t \in \left\{\Delta_{min}..T\right\} \tag{4}$$

$$\sum_{p\in P} W_t^{cp} = 1, \forall c \in C, \forall t \in \left\{\Delta_{min}..T\right\} \tag{5}$$

$$Y_{\Delta_{min}}^p = y_0^p + v_{max}^p \cdot \Delta_{min} \cdot \left(I_{\Delta_{min}}^p - J_{\Delta_{min}}^p\right), \ \forall p \in P \tag{6}$$

$$Y_t^p \ge 0, \forall p \in P, \forall t \in \left\{\Delta_{min}..T\right\}$$

$$I_t^p, J_t^p \in [0,1], \ \forall p \in P, \forall t \in \left\{\Delta_{min}..T\right\} \tag{7}$$

$$W_t^{cp} \in \{0,1\}, \forall c \in C, \forall p \in P, \forall t \in \left\{\Delta_{min}..T\right\}$$

$$X_t^{cp} \ge 0, Z_t^{cp} \ge 0, \Psi_t^{cp} \ge 0, \forall c \in C, \forall p \in P, \forall t \in \left\{\Delta_{min}..T\right\}$$

$$M \ge \max_{p\in P,c\in C} \left| y_0^p - \left(y_0^c + T \cdot v_{max}^c\right)\right| \tag{8}$$

Constraints (1), (2), and (6) determine the optimal speed and direction of each tug vessel at every time period. Because there are no cross points for $t \in [0, \Delta_{min} - 1]$, the model determines, in constraint (6), the speeds and directions of tug vessels at these time periods for an optimal allocation in period Δ_{min}. Constraints (3) through (5) optimally allocate each oil tanker to one tug vessel, while constraint (7) define bounds on the decision variables.

For each time period of length λ in the planning horizon *T*, there is a predicted drift trajectory for each oil tanker which is expected to cross the patrol line after Δ_{min} periods ahead in time. Thus, there will be cross points at every time period starting from $\Delta_{\min}$. In case an oil tanker starts drifting in period *t*, the model gives direction and speed to its allocated tug vessel at each time period such that their distance, after $t+\Delta_{min}$ time period, is minimized. A tug vessel in period *t* will be allocated to cross point(s) in period $t+\Delta_{min}$ and possibly different cross point(s) at the next time period. The result is that the tug vessels will proactively move to make sure there is enough time to rescue any drifting oil tanker.

3.3 *Static MIP* model

The variant of the model with static tug vessels is obtained by replacing the variable Y_t^p by y_0^p in constraints (3) of the *MIP* model. In addition, only constraints (3), (4), (5) and (8) are kept and the rest are removed. The model is then run once for each time period in the planning horizon. This variant simply gives optimal allocation of static tug vessels to oil tankers and is only used for comparison.

4. COMPUTATIONAL SIMULATION STUDY

In this section, we present the simulation settings and results of the computational experiment used to evaluate the performance of our solution method to the tug vessels allocation problem**.**

4.1 Simulation settings

The mathematical models and algorithms for simulations were coded in AMPL and the *MIP* models were optimally solved with CPLEX 9.0. All the experiments were executed on a personal computer with an Intel Pentium IV 3.0 GHz CPU and 4.0 GB of RAM, with the operating system Microsoft Windows 7.

Notwithstanding the expected increase in oil tanker traffic along the northern coast of Norway, we have decided to use 6 oil tankers and 3 tug vessels for the simulation. These realistic numbers were provided by the NCA in 2010 and are reasonable choices for comparison with previous work done by Bye (2012).

The typical maximum speed of tug vessels in this region is about 28 km/h and the normal operating speed of oil tankers is about 18 to 26 km/h (Bye 2012). Based on this information, we conservatively chose to use a random speed of each oil tanker generated in the interval $\pm[20,30]$ (km/h), whereas a maximum speed of ±30 km/h was used for the tug vessels. In both cases, a positive speed denotes a northbound movement and a negative speed a southbound movement. A drift time of only 10 hours is considered fast drift, while slow drift means most tankers will not run ashore before having drifted for 20 to 30 hours (Eide et al. 2007). For this reason, Bye (2012) used a conservative estimate in the interval $[8,12]$ (hours). Note that this interval represents the possible values of Δ presented in section 2. To be even more risk averse, we decided to use a constant value of $\Delta_{\min} = 8$ hours for each oil tanker in this study.

To introduce nonlinearity of drift trajectories, such as that caused by wave heights, wind, ocean currents, and oil tanker size and shape, we used the same simple formula as Bye (2012), which has no physical relation to real drift trajectories but was merely chosen for its nonlinearity:

$y(t') = z(t) + v\sin\left(\frac{2\pi}{T}\Delta_{\min}\right)$. Hence, any oil tanker will follow an eastbound sinusoidal trajectory with period equal to *T* scaled by its velocity *v*. The initial positions of oil tankers on the z line were randomly chosen in the range $[-750, 750](km)$. For simplicity in the implementation, this interval was translated to $[1000, 2500](km)$ to obtain only positive values. In order to compare the dynamic *RHMIP* model with the *static MIP* model, we decided to divide the above interval into 3 equal subintervals and place one tug vessel, at a "tug base", at the center of each segment for the static model.

The *RHMIP* and *static MIP* models were simulated for T_{RHMIP} = 26 hours, a duration picked somewhat randomly, although we emphasize that the models should be simulated for at least a duration long enough to allow the tug vessels to move from initially bad to good positions and thenceforth remain in good positions, where "good" and "bad" positions refer to how well the tugs collectively optimize the cost function presented above.

At every step of λ =1 hour, the associated *MIP* model was run for a planning period of *T*=24 hours, but only the solution for the first hour was implemented. A total number of 30 scenarios were simulated. Details on the simulation settings are presented in Table 1.

Table 1: Simulation settings

Number of oil tankers	6
Random initial position (km)	$[1000, 2500]$
Random velocity (km/h)	$\pm[20, 30]$
Minimal drift time $\Delta_{\min}$ (hours)	8
Number of tug vessels	3
Initial tug positions	$\{1250, 1750, 2250\}$
Maximal velocity (km/h)	±30
Planning horizon T (hours)	24
Simulation step length λ (hours)	1
Simulation steps T_{RHMIP} (hours)	26
Number of scenarios	30

4.2 Results

The static tug vessel policy resulted in an average total distance of 22234, with a high standard deviation of 5880. The best case scenario had a total minimal distance of 12915 while the worst case scenario had a maximal distance of 27325. Unsurprisingly, using the static policy, a considerable number of potentially drifting oil tankers will not be rescued even at a maximal speed of the nearest tug vessel. However, the developed static *MIP* model can at least provide an optimal allocation of tug vessels to oil tankers, which cannot easily be achieved manually or using heuristic methods. The average running time for the *MIP* of this model was 10 sec for each time step.

The average total distance of the *RHMIP* model was 7702 with a standard deviation of 2912. The best case scenario had a minimal total distance of 2989, whereas the worst case scenario had a maximal total distance of 10609. The average performance improvement in terms of mean total distance of the *RHMIP* solution compared to the static policy was 66%. This dynamic variant of the model had a *MIP* average running time of 20 min at each time step, which is in the order of two magnitudes greater than the static *MIP* but is still acceptable for real-time implementation, since the calculation only needs to finish before the beginning of the next hourly receding horizon control step. The results are summarized in Table 2.

The parameters of our simulations were the same as those used in Bye (2012) except for the length of the drift trajectories, where our model was implemented as a worst case analysis with the minimum of 8 hours instead of random drift times in the interval $[8,12]$ (hours). As a consequence, a few of our simulated scenarios will have a slightly higher number of cross points. Nevertheless, the results from the two studies are still comparable, since the scenarios were randomly drawn from the same population but with different random samples. Moreover, the main comparison is based on the performance improvement from the static tug vessel policy. Comparison of RHGA vs. RHMIP simulation results are presented in Table 3.

Table 2: Simulation results

	Static MIP	*RHMIP*	Reduction by *RHMIP*
Mean	22234	7702	66%
STD	5880.7	2911.6	50.5%
Min	12915	2989	-
Max	27325	10609	-
Step time	10 sec	20 min	

The *RHGA* and *RHMIP* approaches used the same planning horizon of 24 hours at the optimization step. Compared with a static policy, the *RHMIP* showed a 66% improvement, whereas the *RHGA* showed 57.5%, thus the *RHMIP* outperformed the *RHGA* by 8.5%.

Table 3: Improvement from static policy

	RHGA	*RHMIP*
Improvement	57.5%	66%

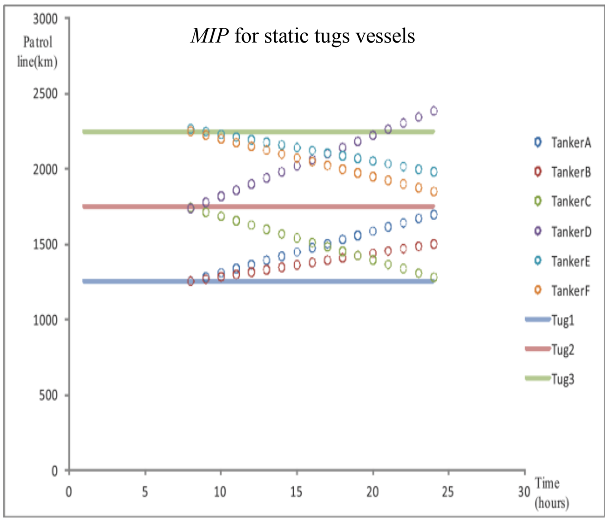
Figure 2 highlights the difference between the *MIP* models. The two models were run once, with the same parameters, for *T=24* hour time periods. The straight lines and piecewise linear functions represent the dynamic allocation of patrolling tug vessels over the planning horizon. The cross points, starting in period eight, are represented by circles in the figure. Compared to the *MIP* for static tug vessels, our dynamic *MIP* model cleverly and optimally allocates and tracks the potential drifting oil tankers for the given fixed planning horizon.

We recall that only the first period of the *MIP* model solution is implemented at each step in the simulation of the *RHMIP* model. At every step, the parameters (current numbers of oil tankers, maximal speeds, cross points and tug vessel positions) are updated. This allows tackling the weather uncertainty at each simulation step and coping with the variation of the parameters.

Another advantage of using the receding horizon approach is that of better tug vessels allocation at each time period. In fact, if we assume a situation where the weather is stable or accurately predicted for *T=24* hours planning horizon, one may be tempted to implement the entire solution planned from a single *MIP* optimization, which will not allow better allocation of tug vessels at each period. The *RHMIP* model, run in the same conditions for T_{RHMIP} = 24 hours, will give better allocation because the planning at each period will not be influenced by that of the previous, which is not the case in the *MIP* model. This is illustrated in Table 4, where the letters A to F represent the oil tankers and columns for distances represent the sum of the distances between a tug vessel and its allocated oil tankers. The total distances demonstrate the advantage of using the *RHMIP* model although the weather is accurately forecasted and the parameters constant.

4.3 Conclusions

The combined features of receding horizon control and mixed integer programming allow our model to optimally control tug vessels and allocate them to oil tankers in a dynamic and highly uncertain environment.



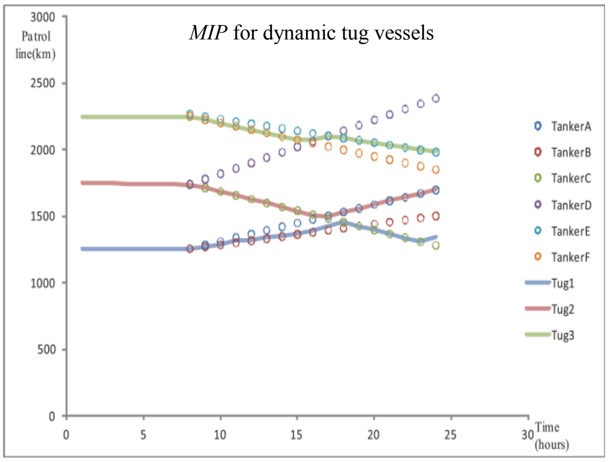


Figure 2: An illustration of employing static (top) and dynamic (bottom) tug vessels for a planning horizon of *T*=24 hours.

Table 4: Tug vessels allocations

		MIP									*RHMIP*						
		A	B	C	D	E	F	Distance			A	B	C	D	E	F	Distance
	Tug1							1		Tug1							1
t=8	Tug2							7	t=8	Tug2							7
	Tug3							16		Tug3							16
	Tug1							13		Tug1							13
t=9	Tug2							63	t=9	Tug2							63
	Tug3									Tug3							23
	Tug1							25		Tug1							25
t=10	Tug2							132	t=10	Tug2							132
	Tug3							31		Tug3							31
…	…	…	…	…	…	…	…		…	…	…	…	…	…	…	…	
	Tug1							134		Tug1							305
t=22	Tug2							0	t=22	Tug2							225
	Tug3							404		Tug3							0
	Tug1							0		Tug1							361
t=23	Tug2							387	t=23	Tug2							14
	Tug3							346		Tug3							0
	Tug1							223		Tug1							223
t=24	Tug2							0	t=24	Tug2							283
	Tug3							536		Tug3							0
Total distance								5365									4567

5. DISCUSSION AND FUTURE RESEARCH

Combining features from model predictive control and linear programming, this paper presents a novel sustainable model called the *receding horizon mixed integer programming (RHMIP)* model for optimal dynamic allocation of patrol vessels to oil tankers. Compared with previous work (Bye, 2012; Bye et al., 2010) that used a genetic algorithm to suboptimally minimize the proposed cost function, our model provides an exact (optimal) solution at every receding horizon time step. At the expense of slower computational evaluation, our optimal model outperforms the suboptimal heuristic method as well as providing a benchmark for future models.

5.1 Sustainability

International communities and government bodies such as NCA are expressing concern about the environmental impacts from shipping related activities. In fact, international shipping accounts for 2.7% of worldwide CO_2 emissions (Psaraftis and Kontovas 2013). One of the measures used, at a tactical or operational level, to address this issue is the speed reduction of ships. Accordingly, the *RHMIP* is a sustainable model as it explicitly reduces the speed of tugs vessels from the parameters settings and implicitly inside the model as well. Noticeably, the average operational speed of tug vessels was equal to 5 km/h for all scenarios. This is considerable slow-steaming compared to the 30 km/h maximum speed. In addition, a constraint on the maximal daily fuel consumption of tug vessels could be easily included in the model. However, limiting fuel consumption would cause a trade-off to be made between the short term CO_2 emissions reduction plan with long term potential environmental impact caused by drifting oil tankers that could not be rescued on time.

5.2 Robustness

For each time period of one hour in the simulation, the initial speed of each tug vessel is determined by the *MIP* model and a tug vessel is supposed to move with the same speed through the whole time period. However, the wave heights, ocean currents and other factors may also affect the speed of the related tug vessel, thus causing deviations from the predicted future position of the tugs. This problem is overcome by the receding horizon control strategy, which at every planning interval will take into consideration the very latest current information about tug and tanker positions as well as updated weather forecasts. In addition, some tug vessels may not be available for some time periods due to maintenance or other possible reasons. It will be interesting to run the model with a variable number of tug vessels in the planning horizon.

The consequences of accidents will likely depend on the type and characteristics of oil tankers as well as the place or zone of accident in the coastline. Identifying the high risk zone and weighing the oil tankers will be of great benefit and can be easily included in the model.

Simulations with very large test instance size may highly increase the computational time. But one way of handling this issue is to subdivide the problem into small reasonable sizes. That is, the coastline can be divided into a few numbers of zones and each group of tug vessels will then patrol along its allocated coastline zone.

5.3 Future research

Although oil tankers are required by law to sail along predetermined piecewise linear corridors parallel to the coastline, more research can be done on a 2D dimensions.

This paper aimed to minimize the distance between potential drifting oil tankers and their respective allocated tug vessels. Future research may be focused on other optimizations objective. For instance, one could decide to reduce the probability of an oil tanker running ashore. This can be achieved with probabilistic models or robust optimizations.

The development of oil and gas fields in the Barents Sea will considerably increase the number of oil tankers transits along the coastline in the next 10-15 years (Bye, 2012). Further research could be conducted to determine the optimal number of required tug vessels as well as deciding whether the vessels should be homogeneous or heterogeneous.

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