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The Effect of Arbitrage Activity in Low Volatility Strategies

An Empirical Analysis of Return Comovements

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Acknowledgments

We would like to thank our supervisor, Francisco Santos, for providing invaluable guidance during the process of writing this thesis. Without his insights and advice, our goals would have been unattainable. In addition, we would like to thank the IT-department at NHH for giving us access to software which has been very helpful to us. Finally, we would also like to thank our parents for their unconditional love and support.

Bergen, June 2017

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Abstract

The goal of this thesis is to examine the effect arbitrageurs have on prices in the stock market. More specifically, we seek to investigate arbitrage activity in the low volatility anomaly by decomposing it into systematic- and firm-specific parts. Our main contribution is to create a measure of arbitrage activity for the idiosyncratic volatility strategy, which goes long stocks with low idiosyncratic- and short stocks with high idiosyncratic volatility. We fulfil this by mainly utilizing previous methodology of Ang et al. (2006), Lou and Polk (2013) and Huang et al. (2016).

First, for a proof that we are able to construct our own measure of arbitrage activity in low volatility strategies, we implement the methodology of Huang et al. (2016) and successfully replicate CoBAR, a measure of arbitrage activity in beta-strategies. We then proceed by creating our own measure of arbitrage activity in the idiosyncratic volatility strategy, which we dub CoIVOL. This proxy is used to identify periods of relatively low and high arbitrage activity and asses whether trading in the strategy is crowded. We use this to examine the implications and effects arbitrageurs have on prices. Our findings indicate that abnormal returns to the idiosyncratic volatility strategy, conditional on the arbitrage activity, are decreasing with time and activity. More specifically, we find that when activity is at its lowest, we achieve an average alpha of 1.71%/month for the first six months after portfolio formation. This alpha decreases monotonically with activity, and eventually becomes insignificant when arbitrage activity peaks. We conclude that arbitrageurs exploiting the idiosyncratic volatility anomaly has a stabilizing effect on prices.

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1   Introduction

The positive relationship between risk and return is one of the most widely accepted relations within the field of finance; an investor should be compensated for taking on risk, and the higher the risk, the higher the expected reward. One of the first models to explain this relationship was the Capital Asset Pricing Model (CAPM), originally documented by Sharpe (1964) and Lintner (1965). They also proposed that the only relevant measure of risk was a firm's sensitivity to the market as measured by beta, since market participants could remove other sources of risk by holding a diversified portfolio. However, later studies by Black, Jensen, and Scholes (1972) showed that the relationship may not be as positive as originally predicted by the CAPM. This, in turn, sparked interest for additional empirical studies examining the cross-sectional relationship between historical risk and return. One outcome of this research was the discovery of the low volatility puzzle[1], which is the phenomenon of low-volatility securities having higher risk-adjusted returns, on average, than their high-volatility counterparts. The puzzle has been studied as a whole as well as being decomposed into systematic- and idiosyncratic components of volatility, and empirical research has confirmed the anomaly in both.

The low volatility puzzle presents an opportunity to quasi-arbitrage[2] by exploiting the outperformance of low volatility stocks, an opportunity that should not exist according to the efficient-market hypothesis. There is no doubt that the role of those who try to exploit this, the arbitrageurs, in the financial marketplace is important. However, their impact on prices is hard to understand mainly because it is difficult to accurately measure the level of their activity at any given time. The lack or unavailability of accurate high-frequency information and other inputs such as the composition of arbitrageurs or capital under management, has made previous efforts of producing a good proxy fruitless. Lou and Polk (2013) proposed a new way to measure the activity of arbitrageurs in the financial markets by shifting their focus from the missing inputs to the actual outcome of the arbitrage process. More specifically, they measured the degree of abnormal return correlations among stocks that an arbitrageur

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[1]See Haugen and Heins (1975).

[2]Authors we refer to later use the term arbitrage, when they should be using quasi-arbitrage. In their spirit, the terms arbitrage and quasi-arbitrage will be used interchangeably for the remainder of this thesis.

would speculate on. In short, the approach captures the high-frequency return correlation that occurs when arbitrageurs long and short portfolios of stocks simultaneously. These return correlations can thus be used to measure the relative activity in that given trading strategy through time and assess whether it is high or low. Lou and Polk (2013) use this insight to shed new light on the actual impact on prices of arbitrageurs trading momentum-strategies.

In the wake of the low volatility anomaly publication and by building on the method proposed by Lou and Polk (2013), Huang, Lou, and Polk (2016) extend the analysis and inspect the excess comovement of stock returns in beta strategies, which exploits the low-beta anomaly proposed by Frazzini and Pedersen (2014). Their measure, unsurprisingly dubbed CoBAR, is constructed by sorting all stocks into deciles based on a pre-ranking market beta at the end of each month for the period 1970-2010 by using daily returns from the past 12 months. CoBAR is then computed as the average pairwise partial return correlation in the lowest beta-decile measured in the ranking period while controlling for the Fama and French (1992) three factor model (hereafter FF-3). Their results indicate that prices are not corrected as one would expect from the consequence of arbitrageurs exploiting the low-beta anomaly.

In this paper, we investigate how the methodology of Lou and Polk (2013) and Huang et al. (2016) can be applied to other arbitrage strategies to improve market timing and help understand the role of arbitrageurs in the market. Our main contribution is to develop a measure of the arbitrage activity in idiosyncratic volatility[3] (hereafter referred to as IVOL) strategies and then examine the performance of IVOL-sorted portfolios under this measure. To our knowledge, this is something that has never been done before. Because we also seek to decompose the low volatility puzzle and compare our new measure to the existing one, our focus in this thesis will be twofold.

First, for a proof of methodology, we replicate CoBAR, a measure of arbitrage activity in beta-strategies proposed by Huang et al. (2016), following the approach of Lou and Polk (2013). We choose to concentrate on the methodology of CoBAR because we aspire to

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[3]The terms idiosyncratic volatility, idiosyncratic risk, and firm-specific risk are used interchangeably throughout the text.

construct a measure related to the arbitrage activity in idiosyncratic volatility strategies. Therefore, a measure that has already been constructed for a low volatility strategy serves as an optimal starting point for what we want to achieve.

We find that our replication is near identical to the original measure in terms of time-series characteristics. Based on our 492 monthly values of CoBAR from 1970 to 2010, we find a mean of 0.105, standard deviation of 0.026, and a maximum value equivalent to 0.203. These deviate by just 0.001 compared to the corresponding values of the original measure. Our minimum value of 0.041 deviates by a mere 0.007 compared to the minimum value found by Huang et al. (2016). Based on the time-series we conclude that our replication has been very successful. However, we also need to confirm that the performance of the long-short beta-sorted portfolios shows the same trend as in the original paper. The original results suggests that when arbitrage activity in beta-strategies is low, abnormal returns are not realized immediately. When beta arbitrage activity is high, positive abnormal returns to the beta-strategy materialize within the first few months after forming portfolios before they revert and crash. Our portfolio results shows the same tendency for the abnormal returns and we are able to identify what Huang et al. (2016) refer to as "booms and busts in beta arbitrage". The results from the first part of our study suggests that we have been successful in replicating both CoBAR and the complimentary beta-portfolios.

In the second part of our thesis we move over to our contribution, namely constructing a measure of arbitrage activity in idiosyncratic volatility strategies, which we label CoIVOL. Our measure uses a combination of two methodologies. First, we sort stocks by idiosyncratic volatility using the methodology proposed by Ang, Hodric, Xing, and Zhang (2006), instead of beta that we used for CoBAR. To find the average excess comovement between stocks utilized in IVOL-strategies, we try to stay true to the methods used during the construction of CoBAR. Further, we investigate how our portfolios perform under various levels of CoIVOL for two reasons. One, we want to have comparable results to the beta-strategy and two, we want to see how arbitrageurs affect prices and thus if timing the market when using IVOL-strategies can be of use to investors.

Our results show that market-timing in the IVOL-strategy is not important during periods of low arbitrage activity. Monthly alphas, controlling for the Carhart (1997) four-factor

model, equals 1.71% on average for the first six months after portfolio formation when arbitrage activity is at its lowest. This is the most significant abnormal return we find for the holding periods we examine. From their peak, alphas decline both as time passes and activity increases before eventually diminishing when arbitrage activity in the strategy is at its highest. We conclude that this time- and activity-decaying pattern provides evidence that IVOL-arbitrageurs are indeed stabilizing on the stock prices. We also study the performance of our portfolios on the interaction between CoBAR and CoIVOL. That is, we look at how the beta- and IVOL-sorted portfolios perform when the arbitrage activity in the two strategies diverge. Our results indicate that when CoBAR is high compared to CoIVOL, the characteristics of the abnormal returns to the beta-strategy change for the first year compared to the original measure, while the long-run effects are the same. When CoIVOL is high compared to CoBAR, we find that the abnormal returns to the IVOL-strategy changes slightly, however, they still show the same tendency as in the original measure. To make sure our conclusions are correct, we also conduct what we believe are the most important tests of robustness. We look at the abnormal returns while using different asset-pricing models, two subsample tests, and controlling for general macro proxies, only to find that our initial results hold.

The rest of the paper is structured as follows. Chapter 2 contains a literature review of the topics that are discussed in this paper. In Chapter 3 we construct CoBAR, form beta-sorted portfolios, and test the performance of said portfolios under five levels of arbitrage activity in the beta-strategy. Chapter 4 outlines the process of computing CoIVOL, a measure of arbitrage activity in idiosyncratic volatility strategies, as well as the procedure for generating IVOL-sorted portfolios. Chapter 5 is dedicated to the analysis of our main results from the IVOL-strategy, including an examination of the CoIVOL time-series, the performance of the IVOL-sorted portfolios, the interaction between CoBAR and CoIVOL, and robustness tests. To give more depth to our results, we provide a discussion on risk-based investing by comparing the results of beta and IVOL arbitrage in Chapter 6. Finally, Chapter 7 marks the conclusion of this study.

2 Literature Review

In the following, we present literature that is closely related to the goal of this paper and try to include our own results where we deem it appropriate. First, we present the cross-sectional relationship between risk and return, and then look at the low volatility puzzle separated into the beta- and idiosyncratic anomaly. We then try to give some explanations on the persistence of the anomaly and relate these to the results we have obtained. Finally, we identify literature that concentrates on return comovements in order to support our findings on the CoBAR and CoIVOL measures.

2.1 The Cross-Sectional Relationship Between Risk and Return

In this section, we introduce literature that breaks with the traditional view of a positive relation between a stocks inherent risk and expected return. Our results, as will be shown later in this thesis, confirms the low volatility anomaly, both when analyzing the systematic- and firm-specific risk. In the two upcoming sections we decompose research on the subject into a systematic- and an idiosyncratic part, by first taking a look at the beta anomaly and then the idiosyncratic volatility anomaly.

2.1.1 The Beta Puzzle

The beta anomaly is the first of two strategies in which we attempt to construct a market-timing proxy for, based on Huang et al. (2016). It is therefore essential to highlight the literature of the strategy for further understanding our measure of CoBAR later in the thesis.

The beta puzzle is an anomaly in which stocks that have low systematic risk, as measured by beta in the CAPM equation, tend to outperform stocks with high systematic risk. The systematic risk strategy, more commonly known as the beta-strategy, was first published by Haugen and Heins (1975) and later updated by Frazzini and Pedersen (2014). Frazzini and Pedersen (2014) show that they can quasi-arbitrage by forming a zero-cost portfolio consisting of a short position in high beta stocks, a long position in low beta stocks, and rebalancing this portfolio on a monthly basis. Blitz, Pang, and Vliet (2013) supports the findings of Frazzini and Pedersen (2014). They empirically examine the relation between risk

and return in emerging equity markets and find that the relation is flat, or even negative. In Chapter 3, we confirm that the beta puzzle exists for the sample period we examine by proving that investors will earn significant positive alphas when shorting high beta stocks and longing low beta stocks.

2.1.2 The Idiosyncratic Volatility Puzzle

Our main contribution in this thesis is the arbitrage activity measure in idiosyncratic volatility strategies. Hence, we find it appropriate to have a more extensive review of the literature related to the IVOL anomaly. In terms of empirical research, we have found articles supporting a negative, positive and no relation between idiosyncratic risk and return. We therefore find it appropriate to briefly summarize all aspects of the anomaly to support our conclusion of this thesis.

The idiosyncratic volatility puzzle is an anomaly in which stocks with high IVOL tend to produce low risk-adjusted returns relative to their low IVOL counterparts. In the classical asset pricing models, like the CAPM, it is assumed that investors are diversified such that the IVOL disappears. Thus, according to said models, IVOL should not be related to stock returns. As this thesis is focused on our finding that stocks with high IVOL offers lower risk-adjusted returns than low IVOL stocks, we will start by discussing the literature supporting this.

Ang et al. (2006) are some of the researchers who finds that stocks with high IVOL perform worse than stocks with low IVOL. They define IVOL as the standard deviation of the residual term from the FF-3 model, which is the same method we will exercise when computing IVOL. In their first paper, Ang et al. (2006) found that the difference in alphas controlled for the FF-3 between high and low IVOL stocks in the period January 1980 to December 2003, is -1.31% on average per month and the results are highly significant. Our results show the same tendency regarding the alphas, but differ slightly in magnitude. For the same time period, we find significant differences in alphas, controlled for the FF-3, of -2.55% on average per month between the high- and low IVOL portfolios. This disparity can be attributed to the difference in portfolio size used, where we form decile buckets in order to get comparable results to the beta-strategy instead of quintiles as suggested in the original

paper. Ang et al. (2006) also found their results to be robust when controlling for size, book-to-market, leverage, liquidity, volume, turnover, bid-ask spreads, coskewness, dispersion in analysts' forecasts, aggregate volatility, and momentum effects. They also test the results in different subsamples, in NBER expansions and recessions, in volatile and stable periods of the market, and for different formation and holding periods, finding that the effect still holds for all these tests of robustness. Although we do not extend our analysis to all of these robustness tests, we still confirm that our results hold for a wide range of specifications and use of different models. In their follow-up paper, Ang et al. (2009) also confirm that their results hold for international markets.

In contrast to what we presented above, some researchers have found that there is a positive relationship between idiosyncratic volatility and return. Levy (1978) and Merton (1987) found that firms with larger firm-specific risk have larger alphas, inducing a positive relationship between firm-specific risk and return, which stand in contrast to what we find in this thesis. Merton further states that his results can be confirmed by Friend, Westerfield, and Granito (1978) who finds that expected return seems to depend on both market risk and total variance. Another interesting paper is by Stambaugh, Yu, and Yuan (2015) who finds that the idiosyncratic volatility effect is negative among overpriced stocks but positive among underpriced stocks where they use the argument of arbitrage asymmetry[4]. Our results supports the anomaly that stocks with low idiosyncratic volatility outperform stocks with high idiosyncratic volatility.

Next, we will look at some of the suggested explanations for the low volatility anomaly in order to give some depth to our results that we present later in the paper.

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[4]Stambaugh et al. (2015) argue that buying is easier than shorting for many investors, and the negative relationship between overpriced stocks is stronger, especially for stocks that are less easily shorted.

2.2   Reasons Behind the Low Volatility Anomaly

In the previous section we reviewed literature that ratified our results in this thesis. However, for the interpretation of our results we find it meaningful to mention some of the possible explanations of the low volatility anomaly. In the following we present a selection of the most relevant research on the subject, grouped by rational- and behavioral rationalizations[5].

We start by discussing the rational reasons for the existence of the low volatility anomaly. One such explanation relates the underperformance of high volatility securities to leverage-constrained investors. Black (1972), along with Frazzini and Pedersen (2014), points out that most investors are constrained in terms of the amount of leverage they can acquire. They claim that said investors tend to invest in stocks with high systematic risk in order to have higher expected returns to compensate for the lack of leverage. This in turn makes these stocks appreciate in value before eventually ending up as being overpriced, as calculated by the CAPM. With funding constraints also comes the benchmarking hypothesis by Baker, Bradley, and Wurgler (2011). They argue that parts of the anomaly can be explained by institutional investor's tendency to invest in high volatility stocks to compensate for the lack of access to leverage when aiming to beat a fixed benchmark. Due to time- and data constraints, we have not been able to check whether these in fact do explain our results. When examining the performance of our IVOL-portfolio conditional on the arbitrage activity in the strategy, our results are very hard to interpret as they are not intuitive nor are they explained by any of the rational theories we have mentioned. Our results are non-intuitive in the way that two different risk-measures gives two very different answers. We therefore look at some of the behavioral explanations for the low volatility anomaly.

Among the behavioral explanations, we find the lottery-preferences bias and the over-confidence bias. The former, argued by Baker et al. (2011), shows that individual investors who have a preference for lotteries have a tendency to overpay for highly volatile stocks for a chance of very high returns. Their demand could in turn make high volatility stocks overpriced and the consequence would be low average returns. Kumar (2009) side with this rationale and find that individual investors, on average, overweight stocks with high idiosyn-

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[5]Baker, Bradley, and Taliaferro (2014) decompose the low beta anomaly into micro and macro effects and offer an extensive collection of academical publications on a variety of explanations for the anomaly.

cratic volatility, higher skewness and lower prices. We try to control for this and find that our results still hold when excluding firms with the lowest 1% stock price in the portfolio formation period. Cornell (2009) argue the same case for overconfident investors who appear to be attracted to highly volatile stocks because they overestimate their own ability to forecast returns and are thus biased. Ang et al. (2006) propose that a reason for the strong relative performance of low IVOL stocks could be that higher idiosyncratic volatility earns higher returns over longer horizons than one month, and that short term overreaction forces returns to be low in the first month after forming the portfolio. Our findings support this, showing that the average monthly raw returns of the high IVOL portfolio actually reverts from being negative at -0.54% when looking at the first month, to becoming positive after six and twelve months at 0.20%/month and 0.54%/month, respectively, in our sample period. It should be noted that they are still lower than those of the low IVOL portfolio on average.

Based on our research, we can not find a common explanation for the low volatility anomaly in the literature. Rather, it seems as though there are multiple underlying factors that can explain the puzzle. Hou and Loh (2016) propose a simple methodology to evaluate a large number of potential causes and conclude that existing explanations account for 29-54% of the puzzle in individual stocks and 78-84% of the puzzle in idiosyncratic volatility-sorted portfolios.

2.3   Return Comovements

This section is devoted to literature on return comovements. We see this as a necessity to include because to achieve our main objective in this paper, we need to develop a measure of arbitrage activity by exploiting the comovement in stock returns.

The methodology we use to construct a measure of arbitrage activity was originally published by Lou and Polk (2013) for use in the momentum-strategy, and later adopted by Huang et al. (2016) for use in beta-strategies. Both of their papers are tied to the idea of comovement in stock returns. The traditional theory from economies without frictions and with rational investors, states that comovements in prices should reflect comovements in fundamental values (Barberis et al. (2005)). However, the preceding statement only holds for a frictionless economy with rational investors, whereas in economies with irrational investors,

frictions and limits to arbitrage, the comovement in prices will be tied to other factors than fundamentals as well. Lou and Polk (2013) apply this to measure arbitrage activity, but instead of measuring the process of arbitrage, which previous research concluded was near impossible, they measure the outcome of the arbitrage process. Specifically, they measure the ex-ante abnormal return correlations between a group of stocks in which an arbitrageur would perform the given arbitrage strategy on. They argue that this can be done because arbitrageurs follow a distinct strategy where they buy and sell portfolios of stocks simultaneously, and returns should therefore comove assuming that arbitrageurs influence stock prices. In the following, we use this to construct measures of the arbitrage activity in strategies based on the low volatility anomaly.

3 CoBAR – Activity, Portfolio Formation, Performance

In this chapter we attempt to replicate the arbitrage activity measure, CoBAR, proposed by Huang et al. (2016). We do this in order to prove that we are capable of constructing our own measure of arbitrage activity later. The first thing we do is to outline the process of constructing CoBAR as closely to the original paper as possible. We then form portfolios and eventually test the beta-strategy conditional on various levels of arbitrage activity. We compare our results to the ones found in the original paper as we go along, and in the end we draw conclusions on whether the replication was successful or not.

3.1 Data, Methodology, and Construction of CoBAR

In this first section of the chapter we utilize the methodology originally developed by Lou and Polk (2013) and later repurposed by Huang et al. (2016), to reconstruct their measure of arbitrage activity in systematic risk strategies, CoBAR. The proxy is a measure of arbitrage activity in the beta-strategy which goes long the value-weight lowest beta decile of stocks, and short the value-weight highest beta decile of stocks. The main rationale is that stocks that are targets of an arbitrage strategy should have comoving excess returns because arbitrageurs buy and sell portfolios simultaneously. This enables us to measure the activity in the strategy by looking at the outcome of the arbitrage process, which is the impact on stock prices.

The first step in the procedure is to prepare the required variables and clean the datasets we will be using during our computations. In the original construction, the authors analyze the sample period from January 1970 to December 2010, and use stock return data from the Center for Research in Security Prices (CRSP). They clean the dataset by only including common stocks traded on either the NYSE, NASDAQ, or Amex. Naturally, the first thing we do is to download daily stock returns from CRSP for the period December 1968 to December 2015. We will need the additional data later when we run regressions and generate the lagged excess market return. Following Huang et al. (2016), we also exclude all shares that are not classified as common shares (Share code 10 or 11) as well as stocks that are not traded on either NYSE, NASDAQ, or Amex (Exchange code 1, 2 and 3). After removing the aforementioned share classes, as well as missing values, we end up with a dataset containing

59,736,389 observations. Although not specified in the original paper, we also incorporate the delisting returns by adding them to the last observable stock return.

In addition to the raw returns of stocks, we also import the daily risk-free rate and the market risk-premium. Following the methodology of Huang et al. (2016), we create five lags of the market premium to account for illiquidity and non-synchronous trading in the regression described later. We do this in a separate data file in order to correctly join the lags on each daily stock return. Next, we calculate each stocks return in excess of the risk-free rate and merge them with the lags we generated earlier. The dataset obviously has the same amount of observations as before (59,736,389).

Following Huang et al. (2016), we are now ready to sort stocks into deciles at the end of each month based on their pre-ranking market betas. To obtain the pre-ranking beta, the authors run OLS regressions using the daily excess return of each stock for the past twelve months as the dependent variable, and five lags of the excess market return, in addition to the contemporaneous excess market return as independent variables. The pre-ranking beta is the sum of the six coefficients on the right-hand side after running the regression. To do this, we use a regression function where we set the window-length to twelve months. Specifically, we create a dataset containing date-intervals of one year for all stocks in the dataset, this dataset has 2,637,933 observations. The end date (formation date) is set to the end of each month and the beginning date equals this date minus twelve months. We then join the dataset containing the time-intervals where we want to run the regressions with the original return data (59,736,389 observations). We do this by joining where the date in the return dataset is larger than the beginning date of the regression and smaller than or equal to the end-date of the regression. Because we gave all observations an end-date, we can run the regression by this variable and the specific share identification numbers (PERMNO).

The outputs are the beta coefficients for all firms at the end of each month. The regression we run is the following:

$$Exret_{it} = \alpha_{it} + \beta_1 mktrf + \beta_2 mktrf_1 + \beta_3 mktrf_2 + \beta_4 mktrf_3 + \beta_5 mktrf_4 + \beta_6 mktrf_5 + \varepsilon_{it},$$

where $mktrf$ is the excess market return, $mktrf_{1-5}$ are the lagged excess market returns,

β_1 is the beta of the securities on the contemporaneous excess market return, and β_{2-6} are the betas of the securities on the lagged market excess return. ε_{it} is the residual and can be interpreted as the part of the excess stock return that is not explained by the model. In line with the original paper, we run regressions on stocks that have at least 200 observations in the 12-month interval in order to get valid regression coefficients. Because of the limitations of our computing power, we are forced to break down the dataset into smaller chunks[6]. When all of these subperiods are computed, we merge them together before we move on to the next step. As stated in Huang et al. (2016), the pre-ranking market beta is the sum of the six coefficients retrieved from the rolling-window OLS regression, as illustrated by the following equation:

$$Preranking \; beta = \widehat{\beta_1} mktrf + \widehat{\beta_2} mktrf_1 + \widehat{\beta_3} mktrf_2 + \widehat{\beta_4} mktrf_3 + \widehat{\beta_5} mktrf_4 + \widehat{\beta_6} mktrf_5$$

After these computations we are left with 2,408,375 observations in our dataset, which is the pre-ranking betas for each PERMNO computed at the respective formation dates. Following the original paper, we then sort the pre-ranking betas into deciles by the formation dates. We do this and then delete all observations that are not in decile 1, decile 5, and decile 10. We withhold decile 5 in order to test this against the two extreme deciles later. Our expectation is that the arbitrage activity in the extreme deciles should be uncorrelated with activity in decile 5 as this is the beta-neutral portfolio. We only need the lowest decile for the computation of CoBAR, however, we keep the highest decile for creating the long-short portfolios in the next section. The number of observations in the highest- and lowest decile are 240,661 and 240,614, respectively.

The next step in the original paper of Huang et al. (2016) is to compute the partial pairwise correlations using the past 52 weekly returns for all stocks in the lowest decile, while controlling for the FF-3. To get the weekly returns we use the daily returns that we downloaded earlier and scale them accordingly. We also import the FF-3 from Ken Frenchs' website and incorporate it into our weekly returns. Next, we join this data with the PERMNOs in the lowest decile where the week date variable is larger than 52 weeks before

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[6]We run six-year periods at a time.

the formation date. The resulting dataset has 12,502,872 observations after controlling for missing values. Further, we calculate the weekly excess return for all stocks and generate $retf_{-i}^{L}$ which is the equal-weight weekly return of each portfolio, excluding stock i. This variable will be used to calculate the partial correlations in the next step of the procedure. The calculation is as follows:

$$retf_{-i}^{l} = \frac{(\sum_{i=1}^{N} Exret_i) - Exret_i}{N - 1},$$

where $Exret_i$ is the weekly excess return of stock i and N is the number of stocks in the lowest decile for the given formation period. Our working dataset now contains the required variables to compute the average partial correlations for the stocks in the lowest decile. CoBAR is then, according to Huang et al. (2016), computed using the following formula:

$$CoBAR = \frac{1}{N} \sum_{i=1}^{N} partialCorr(retrf_{i}^{L}, retrf_{-i}^{L}|mktrf, smb, hml), \tag{1}$$

where $retf_{i}^{L}$ is the weekly return of stock i in the (L)owest beta decile, and $retf_{-i}^{L}$ is the same as before. As in the original paper by Huang et al. (2016), we end up with 492 monthly values of CoBAR based on the lowest beta decile, calculated over the period January 1970 to December 2010.

In the next section we will use the data we found in the procedure above to form a combined portfolio of the highest and lowest beta decile buckets.

3.2 Portfolio Formation in the Beta-strategy

We now describe the process of forming portfolios on the beta-strategy as first proposed by Frazzini and Pedersen (2014) and later used by Huang et al. (2016). The portfolio goes long the value-weight portfolio of stocks in the lowest beta decile and short the value-weight portfolio of stocks in the highest beta decile.

When creating our portfolios, we start by importing the necessary datasets generated in the CoBAR-construction. The datasets needed for creating the portfolios are the lowest- and highest deciles and, of course, stock return data. We import the latter from CRSP, including information on share code, exchange code, daily returns, share price, shares outstanding and delisting returns. We incorporate the delisting returns, and therefore also generate monthly returns instead of importing them. The monthly returns will later be used to track the performance of our portfolios. After cleaning the dataset for the incorrect share classes, we calculate the end-of-month market capitalization of each stock by multiplying the price with shares outstanding. We are now ready to form the zero-cost long-short portfolios of Huang et al. (2016) by combining the lowest- and highest beta deciles. The process of doing so is the same for both deciles, and we will thus only explain it once. "Decile" in the following explanation can therefore be understood as both the lowest- and highest decile.

We start by joining the PERMNOs in the decile with their corresponding monthly returns. We use the formation date of each portfolio in the decile as our starting date and create a variable set to 36 months ahead as our end date, to later measure the portfolios performance over longer holding periods. This enables us to join the monthly returns on each portfolio where the month is bigger than the portfolio formation date and smaller than or equal to the end date, in line with the original paper. We also incorporate the lagged market capitalization of the different firms for the computation of value-weighted returns. We do this because the portfolio is rebalanced one month before returns are realized. The value-weights are created within a month and for the specific portfolio by dividing each stocks market cap by the sum of the total market cap in that particular month for the particular portfolio. Next, we compute the value-weighted returns by multiplying the posterior value-weight of a given firm in one month with the returns that are realized in the consecutive month. To compute the portfolio return, we simply sum the value-weighted returns by month and portfolio.

We now have the value-weighted portfolio returns of the two extreme deciles and are ready to combine them into our long-short portfolio. The deciles each have 17,712 observations, equivalent to 36 monthly returns for each of the 492 formation dates between January 1970 and December 2010. To get the combined portfolio returns we join the value-weighted returns of the lowest- and highest deciles, before we subtract the returns of the highest decile from the lowest decile. Huang et al. (2016) evaluate the performance of the beta-strategy under five levels of arbitrage activity while separately controlling for the FF-3 and Carhart four-factor asset pricing models. Therefore, after creating the long-short returns, we download the monthly FF-3 and Carhart four-factor data from the WRDS database and incorporate it into our working dataset. Next, we import and sort the CoBAR-estimates into quintiles, following the methodology of Huang et al. (2016). We then use our portfolio formation dates including the corresponding 36 months of returns and join them on the end-of-month dates for the CoBAR estimates. This gives us a dataset containing the long-short value-weighted returns of the beta-strategy connected to their respective quintile for all months in our sample. We run the following regression for each of the five quintiles:

$$r_i - r_f = \alpha_i + \beta_i[r_m - r_f] + s_i SMB_t + h_i HML + \varepsilon_i, \tag{2}$$

where r_i is the expected return of portfolio i, r_f is the risk-free rate, α_i is the alpha of the portfolio, and r_m is the return on the value-weighted market portfolio. SMB_t is the excess return of a portfolio consisting of small stocks relative to a portfolio consisting of big stocks, and HML_t is the excess return of a portfolio consisting of high book-to-market ratio stocks relative to low book-to-market stocks. We also run regressions using the Carhart four-factor model:

$$r_i - r_f = \alpha_i + \beta_i[r_m - r_f] + s_i SMB_t + h_i HML + u_i UMD + \varepsilon_i \tag{3}$$

UMD, the momentum factor, is the addition to the FF-3 and is the return of a portfolio consisting of stocks with high past returns relative to a portfolio consisting of stocks with low past returns.

When running their regressions, Huang et al. (2016) control for auto-correlation and heteroskedasticity in the error term by using Newey-West standard errors. We do this in

SAS by the use of the `kernel=(bart, L+1, 0)` statement which corresponds to Newey-West standard errors with `L` lags. Because we look at the average monthly abnormal returns, as measured by alpha, for six and twelve month holding periods, we use `L+1=7` and `L+1=13` respectively. Identical to the original paper, we track the average abnormal returns in months 1 through 36 after portfolio formation. The results of the abnormal return analysis can be found in the upcoming section.

3.3 CoBAR and Beta-portfolio Results

In this section we present the results from our replication of the CoBAR measure and the performance of the beta-sorted portfolios conditional on five levels of CoBAR for four different time horizons. We compare our results to those obtained by Huang et al. (2016), and the conclusions on whether we have successfully replicated their paper or not will be made as we go along.

Figure 1: The Time-Series of CoBAR

The figure portrays the time series of the estimated CoBAR measure, plotted at the end of each December from 1970 to 2010. Panel A shows our estimation, while Panel B is the original time-series copied from Huang et al. (2016). At the end of each month, all stocks are sorted into deciles based on their pre-ranking market beta calculated using daily returns in the past 12 months, while controlling for illiquidity and non-synchronous trading. CoBAR is computed as the average pairwise partial weekly return correlation in the lowest-beta decile over the past 12 months. Like the authors of the original CoBAR measure, we begin measuring the arbitrage activity in 1969 (for being able to predict returns in January 1970), because that was the year when the low-beta anomaly was first acknowledged by academics. Summary statistics for CoBAR can be found in Table 1.

Panel A: The estimated time-series of CoBAR
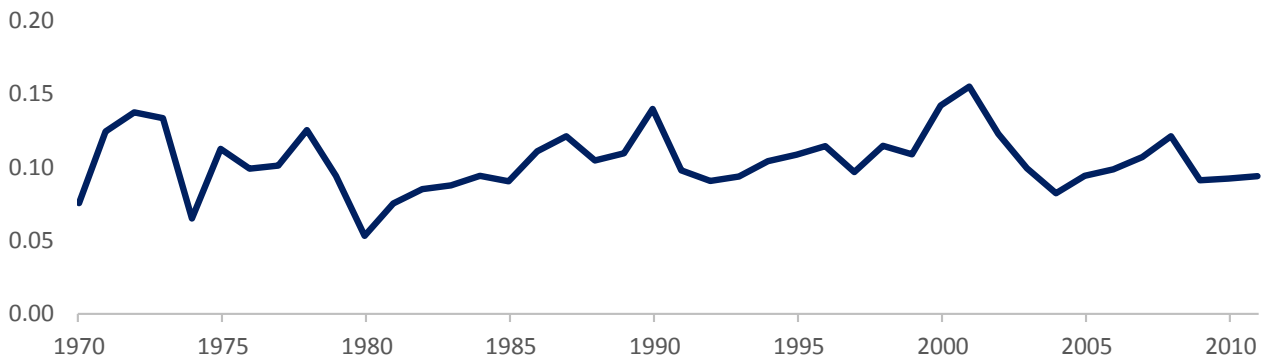
Panel B: The original time-series of CoBAR
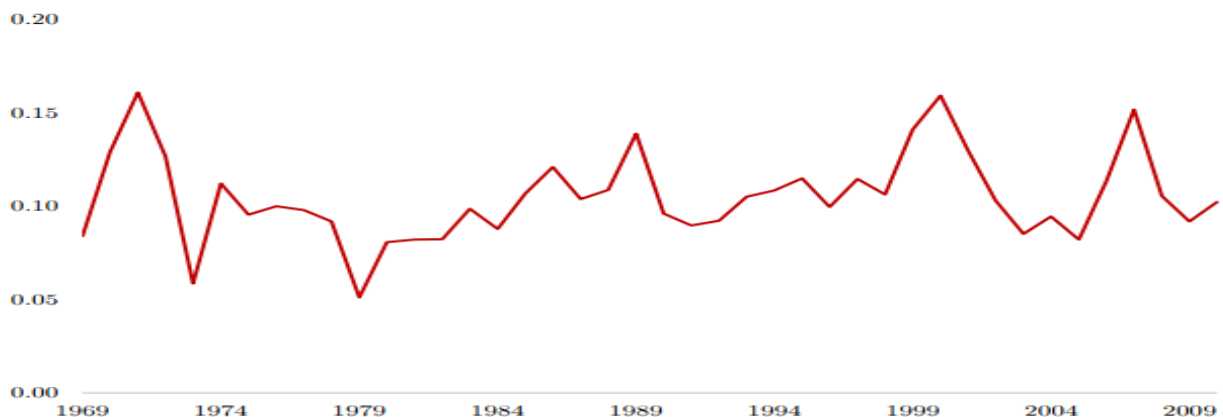
Huang et al. (2016) argue that stocks with the highest betas are susceptible to issues related to asynchronous trading and measurement noise and are thus not very reliable. For this reason we focus on CoBAR constructed on the lowest beta decile and present this in Figure 1 Panel A. Panel B in the same figure is a copy of the CoBAR time-series from the original paper. At first glance, the time-series of CoBAR, does not seem to indicate a clear trend regarding the arbitrage activity in the beta-strategy. However, it is easy to see that the average pairwise correlation fluctuates considerably over our 41 year sample period. From Table 1, the summary statistics show us that the mean of our CoBAR estimate is 0.105 with a standard deviation of 0.026, a minimum of 0.041, and a maximum of 0.203. Compared to the original results we deviate by 0.001 in terms of mean, standard deviation, and maximum value. The minimum values deviates by just 0.007.

Table 1: Summary statistics of the original and the estimated CoBAR

The table shows the summary statistics of the original CoBAR measure by Huang et al. (2016) and our estimates of the same measure, respectively. Reported are the number of observations, mean of the whole time-series, standard deviation, minimum- and maximum values.

Summary statistics of CoBAR					
Variable	Obs.	Mean	Std. Dev.	Min	Max
Original CoBAR	492	0.106	0.027	0.034	0.202
Estimated CoBAR	492	0.105	0.026	0.041	0.203

We also find that the correlation, reported in Table 2, between CoBAR as measured by the lowest beta decile is almost uncorrelated with both decile 5 and decile 10. Although it is surprising that the lowest and highest deciles are uncorrelated, the low correlation between decile 1 and decile 5 is exactly what we expected as trades in the extreme deciles follows a distinct strategy and should not be related to activity in the beta-neutral portfolio. The summary statistics as well as the correlation results are good indicators in confirming that we have managed to replicate CoBAR with very high precision and accuracy as they are almost identical to those found in the original paper. However, we still need to make sure that the beta-portfolios show similar results as Huang et al. (2016). That is, when CoBAR is low, investors have to wait longer, on average, to realize abnormal returns than when CoBAR is high. In contrast, when beta arbitrage activity is high, positive abnormal returns to the

beta-strategy occur relatively quickly before reverting and eventually crashing. The authors show that the long-run reversal of beta-arbitrage returns varies predictably through time and call these booms and busts in beta-arbitrage.

Table 2: The correlation between different bucket assignments for CoBAR

The table shows the correlation between CoBAR as measured by three different bucket assignments. Decile 1 contain stocks with the lowest beta values, decile 5 contain stocks with betas around 1, and decile 10 contain stocks with the highest beta values. Significance levels: * p <0.10, ** p <0.05, *** p <0.01.

	CoBAR correlation		
	Decile 1	Decile 5	Decile 10
Decile 1	1		
Decile 5	0.14***	1	
Decile 10	0.07	0.52***	1

In the original paper, Huang et al. (2016) find that the three-factor abnormal returns are statistically insignificant until year two after the initial trade was made in the lowest quintile of CoBAR. However, when using the four-factor model, abnormal returns does not occur until year three. In quintiles two through four, abnormal returns are insignificant with the exception of year two in the second quintile. This result disappears when adjusting for the momentum effect. In the highest quintile, the average abnormal return for the first six months following the trade has a significant positive alpha of 1.19% and the positive alpha continues to hold for the first twelve months, but diminishes when adjusting for the momentum-effect. The abnormal returns of the highest quintile become statistically insignificant in the preceding two years, before resulting in a significant negative alpha of -0.74% and -1.37% for the three- and four-factor models, respectively, in year three. Now that we have established what we aspire to replicate, we are ready to present our results.

In Table 3 on the next page we show our forecasts of abnormal returns to the beta-strategy under five levels of arbitrage activity as measured by CoBAR and indicated by the rank column. A rank equal to one represents the 20% of the sample with the lowest relative activity in the period between 1970 and 2010. The average abnormal return per month during the first six months as well as the first, second, and third year, after making the arbitrage trade are also displayed.

The FF-3 and Carhart four-factor model results, reported in Panel A and B of Table 3,

Table 3: Forecasting Beta-arbitrage Returns with CoBAR

The following tables reports returns to the beta arbitrage strategy as a function of lagged CoBAR. At the end of each month, all stocks are sorted into deciles based on their market beta, calculated using daily returns in the past 12 months, while controlling for illiquidity and non-synchronous trading. We sort CoBAR, the average pairwise partial weekly return correlation in the lowest-beta decile over the past 12 months, into quintiles. Reported below are the returns to the beta arbitrage strategy (i.e., going long the value-weighted low beta decile and short the value-weighted high beta decile) in each of the three years after portfolio formation during 1970 to 2010, following low to high values of CoBAR. Panels A and B report the average monthly three-factor alpha and Carhart four-factor alpha to the beta arbitrage strategy, respectively. "5-1" is the difference in monthly returns to the long-short strategy following high and low CoBAR. The t-statistics, which are shown in parentheses, are computed based on Bartlett kernel standard errors corrected for serial-dependence with 6 or 12 lags, depending on the number of overlapping observations. Statistically significant (5%) observations are highlighted in bold.

Panel A: Fama-French Adjusted Beta-arbitrage Returns

Rank	Obs.	Months 1-6 Est.	Months 1-6 t-stat	Year 1 Est.	Year 1 t-stat	Year 2 Est.	Year 2 t-stat	Year 3 Est.	Year 3 t-stat
1	98	0.43%	(1.77)	**0.58%**	**(3.51)**	**0.74%**	**(5.28)**	**0.88%**	**(4.90)**
2	99	0.06%	(0.24)	**0.49%**	**(2.71)**	**0.62%**	**(3.55)**	0.25%	(1.37)
3	98	-0.21%	(-0.80)	0.02%	(0.10)	**0.39%**	**(2.57)**	0.24%	(1.37)
4	99	-0.28%	(-0.96)	-0.05%	(-0.26)	-0.26%	(-1.19)	-0.01%	(-0.06)
5	98	**1.08%**	**(3.87)**	**0.57%**	**(3.15)**	-0.14%	(-0.61)	**-0.74%**	**(-3.90)**
5-1		0.65%	1.68	-0.01%	-0.12	**-0.88%**	**-3.36**	**-1.62%**	**-6.17**

Panel B: Four-Factor Adjusted Beta-arbitrage Returns

Rank	Obs.	Months 1-6 Est.	Months 1-6 t-stat	Year 1 Est.	Year 1 t-stat	Year 2 Est.	Year 2 t-stat	Year 3 Est.	Year 3 t-stat
1	98	0.28%	(1.06)	**0.46%**	**(2.76)**	**0.69%**	**(5.17)**	**0.75%**	**(4.25)**
2	99	-0.22%	(-0.89)	0.26%	(1.35)	**0.44%**	**(2.54)**	0.03%	(0.16)
3	98	-0.34%	(-1.18)	-0.07%	(-0.39)	**0.35%**	**(2.07)**	0.06%	(0.37)
4	99	**-0.81%**	**(-2.41)**	**-0.43%**	**(-2.13)**	**-0.48%**	**(-2.04)**	-0.15%	(-0.70)
5	98	**0.75%**	**(2.49)**	0.17%	(0.84)	**-0.57%**	**(-2.72)**	**-1.37%**	**(-6.73)**
5-1		0.47%	1.14	-0.29%	-1.14	**-1.27%**	**-5.15**	**-2.12%**	**-7.85**

respectively, shows the same tendency as those of Huang et al. (2016) in terms of abnormal returns. When arbitrage activity is low, we observe significantly positive abnormal returns in all time-periods, except for the first six months. The abnormal returns appear to be pretty consistent across all three years, with the highest ones occurring in year three ($\alpha = 0.75\%$ and t-stat = 4.25) using the four-factor model. When arbitrage activity is high, rank equals five, we observe a very different pattern. When looking at the FF-3, abnormal returns to the beta-strategy are 1.08% on average per month in the first six months with a t-stat of 3.87. In the first year the average alpha equals 0.57%/month with a corresponding t-stat of 3.15,

before becoming statistically insignificant in the second year and crashing in the third (α = -0.74% and t-stat = -3.90). Applying the four-factor model yields similar results, except for the year one alpha now being insignificant and crashing in year two, one year before the FF3-model in quintile five. To illustrate these results, we also calculate the cumulative abnormal returns to the portfolio formed in periods of low and high CoBAR, and present these in Figure 2. This graphical representation verify our results that abnormal returns to the beta-strategy in periods of low CoBAR are delayed. In the opposite case, when CoBAR is high, abnormal returns to the beta-strategy materialize relatively quickly before overshooting and crashing within the first 16 months. These results confirm the "booms and busts" found in the original paper by Huang et al. (2016).

Figure 2: Cumulative Four-Factor Alpha to the Beta-Strategy

The figure below displays the cumulative abnormal returns to the beta-strategy as a function of lagged CoBAR. At the end of each month, all stocks are sorted into deciles based on their pre-ranking market beta calculated using daily returns in the past 12 months, while controlling for illiquidity and non-synchronous trading. CoBAR is then computed as the average pairwise weekly three-factor residual correlation within the lowest-beta decile over the previous 12 months. We sort CoBAR into quintiles and join the beta-portfolios on each CoBAR computation date. The red curve shows the cumulative Carhart four-factor alpha to the beta arbitrage strategy formed when CoBAR is high, while the blue curve shows the cumulative four-factor alpha to the beta arbitrage strategy when CoBAR is low.
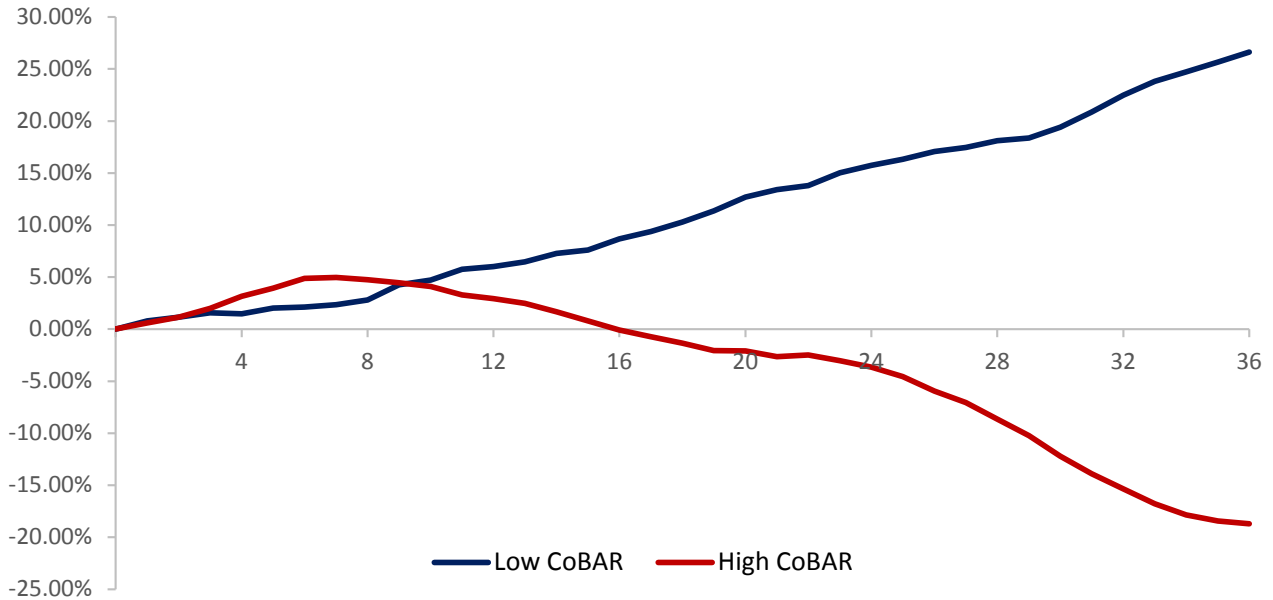
So far we have replicated the measure of arbitrage activity in beta-strategies as proposed by Huang et al. (2016) and the affiliated portfolios. We conclude that this has been successful and we will thus be able to construct our own arbitrage activity measure. In the next chapter, we will employ the methods we have used for the construction of CoBAR and the beta-portfolios on the construction of a measure to capture arbitrage activity in an idiosyncratic volatility strategy. Later, we will also look at how this strategy performs under five levels of activity in the respective arbitrage strategy between 1970 and 2010.

4 CoIVOL – Activity and Portfolio Formation

We devote this chapter to the main contribution of our paper, namely the measure of arbitrage activity in idiosyncratic volatility strategies, which we call CoIVOL. First, we will describe the steps we take to compute CoIVOL. Following that, we form portfolios on the IVOL-strategy while combining the methodologies of Ang et al. (2006) and Huang et al. (2016) to the best of our abilities. Because these procedures generally repeats what we outlined for CoBAR and the beta-portfolios, we will try to keep it brief.

4.1 Data, Methodology, and Construction of CoIVOL

In the following, we combine the methodologies of Huang et al. (2016) and Ang et al. (2006) to create a measure of the arbitrage activity in idiosyncratic risk strategies.

Before we start, we want to mention that we recognize other ways of measuring the idiosyncratic volatility of stocks and that they all have their pros and cons[7]. Some of them might give better indicators of arbitrage activity than the measure we use, but since we follow the methodology of Ang et al. (2006), we calculate IVOL as the standard deviation of the error term, ε_t^i, from a regression of daily stock returns on the FF-3 by month:

$$r_t^i = \alpha^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \varepsilon_t^i, \tag{4}$$

where r_t^i is the daily excess return of stock i at time t, MKT_t is the excess return of the market portfolio, SMB_t is the excess return of a portfolio consisting of small stocks relative to big stocks, and HML_t is the excess return of a portfolio consisting of high book-to-market ratio stocks relative to low book-to-market ratio stocks. Our measure of firm-specific risk is thus the volatility in stock returns that is not explained by the most common risk factors. We use the measure of IVOL as a ranking variable for each individual stock in each month throughout our sample. To find the arbitrage activity in IVOL-strategies, we examine the average pairwise excess weekly return correlation in the lowest IVOL decile over the past twelve months.

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[7]For example the residual of various asset-pricing models, forecasting models such as GARCH, and volatility implied by options, to mention a few.

We use the same starting dataset as we used for CoBAR which is already cleaned and have most of the necessary variables. The only thing we need to incorporate is the daily FF-3 that we will later use to find our ranking metric. In contrast to how we calculate CoBAR, we do not need to calculate the partial correlations in chunks. This is because IVOL, according to Ang et al. (2006), is calculated using daily return observations per month and not per year like the pre-ranking betas calculated by Huang et al. (2016).

Our dataset contains PERMNOs, daily excess returns for each stock and for the market, as well as the daily SMB and HML factors in addition to an end-of-the-month identifier. To stay consistent with previous literature, we require at least 17 daily return observations per month to include a stock in our regressions[8]. We use this dataset to run regression equation 4. IVOL is then calculated as the standard deviation of the monthly residuals from the regression on each stock, and we use this to rank each stock in every month. Ang et al. (2006) sort their IVOL observations into quintiles, however, to get comparable results to Huang et al. (2016) we choose to assign the monthly IVOLs into decile buckets. We also explore quintile buckets only to find that the results are very similar to those yielded by the deciles.

Following the sorting procedure, we keep decile 1 and decile 10, two portfolios that contain the stocks with the lowest and highest individual idiosyncratic risk, respectively, for a given month throughout our entire sample period. Going forward, we use stocks in decile 1 for the construction of CoIVOL for the same reasons as we use decile 1 for CoBAR – we expect the stocks to be larger and more liquid, making comovements more reliable.

Our decile 1 contains 257,428 observations on shares sorted by IVOL and end-of-month dates, going from 1970 to 2010. We merge the stocks in this file with their past 52 weeks of weekly returns and the corresponding weekly FF-3. From this file, we calculate excess weekly returns for stock i and excess returns for the entire portfolio excluding stock i, identically to how we did it for CoBAR. The file we end up with contains all the data we need to compute the partial correlations between the returns, it has a total of 12,955,075 observations.

The three-factor residuals used for CoIVOL are then calculated for stock i and for the

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[8]The exception is September 2001, where our dataset only report 15 trading days. We therefore set this as our minimum requirement for trading days instead of 17 for this particular month.

equal-weight portfolio that excludes stock i, in line with the methodology of Huang et al. (2016). To find the excess comovement of stocks involved in idiosyncratic volatility arbitrage, we compute the average correlation between the three-factor residuals for the portfolio for each month in the lowest IVOL decile as the following:

$$CoIVOL = \frac{1}{N} \sum_{i=1}^{N} partialCorr(retrf_i^L, retrf_{-i}^L | mktrf, smb, hml),$$

where $retrf_i^L$ is the weekly excess return of stock i in the (L)owest IVOL decile, $retrf_{-i}^L$ is the weekly return of the equal-weight lowest IVOL decile excluding stock i, and N is the number of stocks in the lowest IVOL decile in each formation month. We use the Pearson correlation coefficient and consequently end up with 492 monthly values of CoIVOL for the period from January 1970 to December 2010. We repeat the procedure using decile 5 and 10 as our basis for computing excess return comovement to later compare the activity in the three buckets. However, we want to emphasize that for comparable results we will use CoIVOL based on decile 1 going forward.

In the next section, we form a combined portfolio of the highest and lowest IVOL-decile buckets before we look at how this portfolio performs under different levels of CoIVOL. We do this to examine arbitrageurs effect on prices and to see if we can use our insights on arbitrage activity to time the market in this strategy.

4.2 Portfolio Formation in the IVOL-strategy

We will first outline the process of forming portfolios in the IVOL-strategy and try to keep it as close and consistent with the process outlined for the beta-strategy, before we evaluate their performance under CoIVOL. We want to emphasize that the portfolio formation procedure of Huang et al. (2016) deviates slightly from that of Ang et al. (2006), but the differences are negligible.

We start off by importing the lowest- and highest deciles that we obtained when constructing CoIVOL. They contain 257,428 and 257,484 observations, respectively. Our imported data from CRSP holds information on each stocks share code, exchange code, daily return, alternate share price[9] (not to be confused with adjusted share price), shares outstanding and prospective delisting return. Next, we need to compute the value-weights of each stock in every month, by first determining the market capitalizations using share price and shares outstanding. We calculate the market capitalization of each stock at the end of each month by multiplying the absolute value of the last observed alternate price with the corresponding outstanding shares for each month. These operations leaves us with a dataset containing PERMNOs, monthly identifiers, monthly returns and market caps for a total of 2,744,032 observations with data going from January 1970 up until and including December 2013. We include data for 2013 because we are interested in checking holding periods up to 36 months after portfolio formation. We use this dataset to construct the zero-cost long-short portfolio. The following procedure is the same for both the lowest- and highest deciles, and we will thus only explain it once. Referring to "decile" in the following can therefore be understood as both the lowest- and highest decile of IVOL stocks.

The first thing we do is join the PERMNOs in the decile with their corresponding monthly returns from the file we just created. We do this by using the formation date identifier that already exists in the decile dataset, and include the returns for 36 months ahead. We incorporate the market capitalization of each firm separately because we need the market cap of one month to match the returns in the subsequent month. Next, we compute the value-weighted returns by multiplying the value-weight of a given firm with the monthly

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[9]The alternate share price contains the last non-missing price in a given month, offering us more observations and allowing us to calculate the value-weights more accurately than we would have by using regular share prices.

return of the same firm in each month. To compute the portfolio return, we sum the value-weighted returns by months and portfolios. This leaves us with one dataset for each decile containing portfolio formation dates and monthly portfolio returns for a value-weighted, monthly rebalanced investment in stocks with the highest- and lowest idiosyncratic volatility. These datasets each contain 17,712 observations, which is equivalent to 36 monthly returns for each of the 492 formation dates between January 1970 and December 2010.

Lastly, we can create the long-short zero-cost portfolio. This is done by joining the portfolio returns of the lowest- and highest deciles and then subtracting the returns of the highest decile from the lowest decile. After creating the long-short returns, we import the monthly FF-3 and Carhart four-factor data from the WRDS database and merge it with our working dataset. We also sort the CoIVOL-estimates into quintiles, following the methodology of Huang et al. (2016). We then join formation dates and their respective 36 monthly returns from the portfolio dataset on the end-of-month dates of the CoIVOL dataset. After merging all the data together, we have a dataset containing the long-short portfolio returns assigned to their respective quintiles for all months in our sample. We then run regressions 2 and 3 by quintile while controlling for auto-correlation and heteroskedasticity in the error terms. The outcome of the regressions are average monthly alphas for our prespecified holding periods for each quintile of arbitrage activity. We now turn our attention to the main results of our study, specifically, the time-series of our arbitrage activity measure and the conditional performance of the respective IVOL-portfolio.

5 Main Results

In this chapter we present the results of our study on arbitrageurs effect on prices and market timing in idiosyncratic volatility strategies. We start off by investigating the time-series of CoIVOL, our measure of arbitrage activity in IVOL-strategies, before we move over to the performance of our IVOL-sorted portfolios. We have examined portfolio performance under various levels of arbitrage activity, as measured by the average pairwise partial weekly return correlation between stocks of lowest past IVOL. We also investigate how the interaction between CoBAR and CoIVOL can be used to forecast abnormal returns. The final section of this chapter includes various tests of robustness.
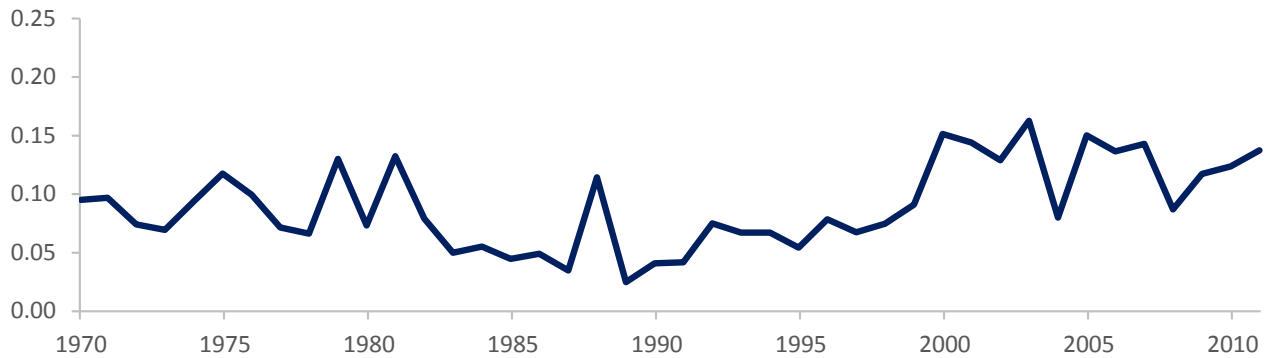
5.1 CoIVOL Time-Series

This section is devoted to the study of CoIVOL. Our monthly estimations of arbitrage activity in the IVOL-strategy are depicted in Figure 3. We will look at descriptive statistics, compare the arbitrage activity in IVOL-strategy with the activity in beta-strategy, and analyze when activity is high and low.

When inspecting the time-series of the CoIVOL estimates, we find quite surprising results. From the beginning of our sample period and up until 1990, there seems to be a negative trend in activity as measured by the lowest decile. After 1990, this trend reverts and the arbitrage activity increases on average for the remaining years of our sample period. There also seems to be an obvious relationship between market events and arbitrage activity in the IVOL-strategy as we see hikes followed by declines close to positive and negative macroeconomic events. For example, we detect an abrupt decline in arbitrage activity following the late 1980s and into the early 1990s, and the same for the period following 2006. We find the same pattern for the beginning of the 2000s with the dotcom-bubble. The period of lowest activity occurs in the late 1980s which is then followed by a steady increase in activity for the rest of the sample with a few exceptions. A global theme of the time-series is that the highest peaks are followed by drastic declines in activity which could potentially be related to crowded trading, but more on this later.

Figure 3: The time-series of CoIVOL

The figure portrays the time series of CoIVOL. The blue line in Panel A shows CoIVOL formed on the lowest decile of IVOL, while the red line in Panel B shows CoIVOL formed on the highest decile of IVOL. At the end of each month, all stocks are sorted into deciles based on their idiosyncratic volatility calculated using daily returns in the past month, requiring at least 17 observations (15 for September 2001). CoIVOL is computed as the average pairwise partial weekly return correlation in the lowest IVOL decile over the past 12 months. In order to compare the arbitrage activity in IVOL- and beta-strategies, we begin measuring CoIVOL in December 1969 so that our first prediction on returns can be made for January 1970. Summary statistics for CoIVOL can be found in Table 5.

Panel A: The time-series of CoIVOL formed on decile 1

Panel B: The time-series of CoIVOL formed on decile 10
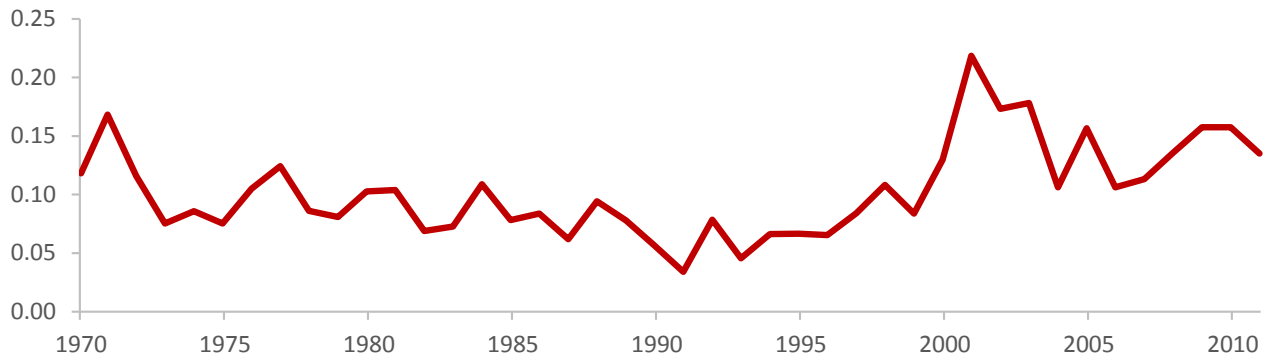
Table 4 shows that the correlation between CoIVOL as measured by the two extreme deciles equals 0.53 and is highly significant, compared to the much lower and insignificant correlation of 0.07 that we found for CoBAR. Although decile 1 and decile 10 appear to be somewhat correlated and of the same magnitude, we are worried that a portfolio of stocks with high individual IVOL may be prone to measurement noise and therefore give unreliable answers regarding the arbitrage activity. We also notice that decile 1 and decile 5 have a significant positive correlation of 0.38, which is a lot higher than the correlation of 0.14 found for the equivalent deciles in CoBAR. The IVOL-neutral portfolio, representing the average

activity, thus appear to have a closer connection to the activity in low IVOL stocks than the relation between low beta stocks and the beta-neutral portfolio.

Table 4: The correlation between different bucket assignments for CoIVOL

The table shows the correlation between CoIVOL as measured by three different bucket assignments. Decile 1 contain stocks with the lowest IVOL values, decile 5 contain stocks with median IVOL values, and decile 10 contain stocks with the highest IVOL values. Significance levels: * $p <0.10$, ** $p <0.05$, *** $p <0.01$.

	CoIVOL correlation		
	Decile 1	Decile 5	Decile 10
Decile 1	1		
Decile 5	0.38***	1	
Decile 10	0.53***	0.55***	1

As with CoBAR, we find that the average pairwise correlation vary a lot in our sample with a low of 0.005 and a high of 0.176 as can be seen in Table 5. The mean is slightly lower for CoIVOL than for CoBAR at 0.089, while the standard deviation is a little higher at 0.034. These results point to a higher frequency in the time-series, which could cause overlapping time periods of high and low activity when holding the IVOL-portfolio for longer periods.

Table 5: Summary statistics of the arbitrage activity measures

The table shows the summary statistics of the original and estimated CoBAR measure in addition to the statistics we find for CoIVOL. Reported are the number of observations, the mean of the whole time-series, standard deviation, minimum- and maximum values.

	Summary statistics				
Variable	Obs.	Mean	Std. Dev.	Min	Max
Original CoBAR	492	0.106	0.027	0.034	0.202
Estimated CoBAR	492	0.105	0.026	0.041	0.203
CoIVOL	492	0.089	0.034	0.005	0.176

In the next section we use CoIVOL to forecast abnormal returns to the IVOL-strategy in an attempt to determine whether arbitrageurs and their crowded trading affects the performance of the portfolio.

5.2 Forecasting IVOL-arbitrage Returns with CoIVOL

At last we present the performance of our portfolio formed on IVOL, conditional on the arbitrage activity induced by the strategy as measured by CoIVOL. Similar to how we analyze the beta-portfolio, we are interested in abnormal returns to the long-short IVOL-portfolio for up to three years from portfolio formation under five levels of arbitrage activity. We do this by examining six- and twelve month samples. This will give us insights on the effects of crowded trading on abnormal returns in said strategy. We do this by utilizing our previously constructed CoIVOL measure, from the methodology of Huang et al. (2016) and Ang et al. (2006). In order to stay consistent with our replication of CoBAR, we report results based on the Carhart four-factor model unless otherwise specified.

The FF-3 and Carhart four-factor model, reported in Panel A and B of Table 6 respectively, show the same trend in terms of average abnormal returns. While the pattern itself stands in stark contrast to what we found for the beta-strategy conditional on CoBAR, the most fascinating observations are the size and significance of the alphas compared to those obtained for the previous portfolio. When arbitrage activity in the IVOL-strategy is at its lowest, we find large significant positive alphas through all holding periods. The abnormal returns are largest in the first six months with an average alpha of 1.71%/month and a t-stat of 9.34. From there, alpha decreases as time passes and ends up at 0.40%/month with a t-stat of 3.29 in year three. Rank two through four shows the same tendency as rank one, with the exception that alphas are higher in year three than in year two. When activity is at its highest, we observe that the average abnormal returns in the FF-3 are positive and statistically significant in all holding periods. What separates this rank from the rest is that the abnormal returns do not follow the same time-decreasing pattern as we saw for the other levels of CoIVOL. When applying the four-factor model, we find that the significance of the highest rank disappears for all time-periods with the exception of year two, where the alpha is significant at 0.68%/month. This confirms that the momentum factor indeed deflates alphas of low volatility strategies when it is introduced into the regression, similar to what we saw for the beta-portfolio. To sum up, the theme appears to be that the alphas are at their highest in the first six months before decreasing with time and activity, for both models. To illustrate this, we graph the cumulative abnormal returns during periods of high and low

Table 6: Forecasting IVOL-arbitrage Returns with CoIVOL

The following table reports returns to the IVOL arbitrage strategy as a function of lagged CoIVOL. At the end of each month, all stocks are sorted into deciles based on their idiosyncratic risk calculated using daily returns in the past month, requiring at least 17 observations (15 for September 2001). We sort CoIVOL, the average pairwise partial weekly return correlation in the lowest-IVOL decile over the past 12 months, into quintiles. Reported below are the returns to the IVOL arbitrage strategy (i.e., going long the value-weighted low IVOL decile and short the value-weighted high IVOL decile) in each of the three years after portfolio formation during 1970 to 2010, following low to high values of CoIVOL. Panels A and B report, respectively, the average monthly three-factor alpha and Carhart four-factor alpha of the IVOL arbitrage strategy. "5-1" is the difference in monthly returns to the long-short strategy following high and low CoIVOL. The t-statistics, which are shown in parentheses, are computed based on Bartlett kernel standard errors corrected for serial-dependence with 6 or 12 lags, depending on the number of overlapping observations. Statistically significant (5%) observations are highlighted in bold.

Panel A: Fama-French Adjusted IVOL-arbitrage Returns

Rank	Obs.	Months 1-6 Est.	t-stat	Year 1 Est.	t-stat	Year 2 Est.	t-stat	Year 3 Est.	t-stat
1	98	**1.84%**	**(10.75)**	**1.41%**	**(11.84)**	**0.48%**	**(3.26)**	**0.29%**	**(2.19)**
2	99	**1.79%**	**(10.47)**	**1.44 %**	**(10.28)**	0.29%	(1.47)	**0.41%**	**(2.60)**
3	98	**1.61%**	**(7.14)**	**1.30%**	**(7.62)**	**0.50%**	**(2.56)**	**0.69%**	**(4.24)**
4	99	**0.94%**	**(3.43)**	**0.76%**	**(3.76)**	**0.47%**	**(2.54)**	**0.39%**	**(2.88)**
5	98	**0.72%**	**(2.20)**	**0.48%**	**(2.41)**	**0.81%**	**(5.56)**	**0.23%**	**(2.05)**
5-1		**-1.12%**	**-2.96**	**-0.91%**	**-3.97**	0.33%	1.66	-0.06%	-0.33

Panel B: Four-Factor Adjusted IVOL-arbitrage Returns

Rank	Obs.	Months 1-6 Est.	t-stat	Year 1 Est.	t-stat	Year 2 Est.	t-stat	Year 3 Est.	t-stat
1	98	**1.71%**	**(9.34)**	**1.34%**	**(11.13)**	**0.54%**	**(3.71)**	**0.40%**	**(3.29)**
2	99	**1.70%**	**(9.02)**	**1.39%**	**(9.82)**	0.36%	(1.97)	**0.65%**	**(3.93)**
3	98	**1.37%**	**(6.21)**	**1.11%**	**(6.73)**	**0.43%**	**(2.31)**	**0.68%**	**(4.79)**
4	99	**0.72%**	**(2.56)**	**0.58%**	**(2.66)**	**0.51%**	**(2.88)**	**0.37%**	**(2.67)**
5	98	0.47%	(1.37)	0.35%	(1.71)	**0.68%**	**(4.81)**	0.21%	(1.84)
5-1		**-1.24%**	**-3.10**	**-0.99%**	**-4.12**	0.14%	0.77	-0.19%	-1.16

CoIVOL in Figure 4.

When looking at the long and short legs of the IVOL-portfolio separately, we notice that around 82% of our abnormal return comes from the long leg, while the short leg explains roughly 18%. We find that abnormal returns from the long portfolio of low IVOL stocks gradually decrease from the first year until the third year when activity is low, while they are relatively stable when activity is high. From the short portfolio of high IVOL stocks, we see that average abnormal returns are even higher for the first six months during times of low arbitrage activity, and they are therefore the biggest contributor to the high and positive

alphas for that period of time in the combined portfolio. These returns, however, are not present in either the second or third year. When activity is high, we see that the high IVOL portfolio performs about the same for all holding periods.

Figure 4: Cumulative Four-Factor Alpha to the IVOL-strategy

The figure below displays the cumulative abnormal returns to the IVOL-strategy as a function of lagged CoIVOL. At the end of each month, all stocks are sorted into deciles based on their level of IVOL not explained by the most common risk factors in the preceding month. CoIVOL is then computed as the average pairwise weekly three-factor residual correlation within the lowest-IVOL decile over the previous 12 months. We sort CoIVOL into quintiles and join the beta-portfolios on each CoIVOL computation date. The red curve shows the cumulative Carhart four-factor alpha to the IVOL arbitrage strategy formed when CoIVOL is high, while the blue curve shoes the cumulative four-factor alpha to the beta arbitrage strategy when CoIVOL is low.
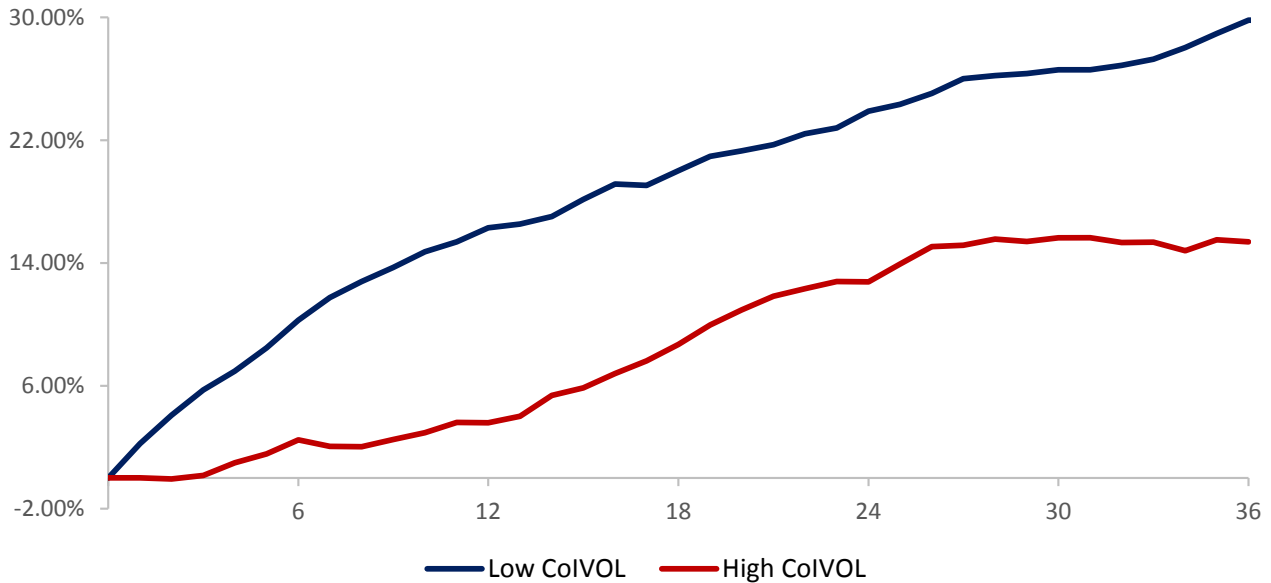
Our study indicates that while market timing in the beta-strategy appears to be important, efforts of timing the market in the IVOL-strategy is almost futile, at least in the lower quintiles. This is because positive and significant alphas are generated for most holding periods and for four out of five levels of activity when looking at the entire sample from 1970 to 2010. In the next section, we move over to examine the interaction between arbitrage activity in the systematic- and idiosyncratic risk strategies which we created measures for earlier in this thesis. We find this interesting as the decomposition of risk was what initially sparked our interest for examining the arbitrage activity in the IVOL-strategy.

5.3 The Interaction Between CoIVOL and CoBAR

In this thesis, we have focused on examining the arbitrage activity in the low volatility anomaly. Because we have decomposed the activity in this anomaly into a systematic- and an idiosyncratic component, we find it appropriate to examine the interaction between them. That is, we want to identify whether the activity in beta-strategies affect the performance of our IVOL-sorted portfolios, and vice versa. To do this we create two measures, one where we divide CoIVOL by CoBAR and another where we divide CoBAR by CoIVOL, before we assign them to quintile buckets. It is important to keep in mind that a high ratio of CoIVOL/CoBAR does not necessarily imply high activity in the IVOL-strategy, rather it indicates that the activity in the given strategy is high relative to the activity in the other strategy.

Table 7: Forecasting Abnormal Returns with CoBAR and CoIVOL Interactions

The following tables report abnormal returns to the beta and IVOL arbitrage strategy as a function of lagged CoBAR/CoIVOL and CoIVOL/CoBAR from 1970 to 2010, respectively. "5-1" is the difference in monthly returns to the long-short strategy following high and low CoBAR/CoIVOL (Panel A) and CoIVOL/CoBAR (Panel B). The t-statistics, which are shown in parentheses, are computed based on Bartlett kernel standard errors corrected for serial-dependence with 6 or 12 lags, depending on the number of overlapping observations. Statistically significant (5%) observations are highlighted in bold.

Panel A: Four-factor Adjusted Beta-returns on CoBAR/CoIVOL

Rank	Obs.	Months 1-6 Est.	t-stat	Year 1 Est.	t-stat	Year 2 Est.	t-stat	Year 3 Est.	t-stat
1	98	-0.01%	(-0.04)	0.26%	(1.36)	**0.76%**	**(4.33)**	**0.51%**	**(3.49)**
2	99	-0.51%	(-1.47)	-0.28%	(-1.32)	**0.58%**	**(3.39)**	**0.62%**	**(3.87)**
3	98	0.16%	(0.53)	0.26%	(1.41)	0.32%	(1.51)	**-0.56%**	**(-2.40)**
4	99	0.25%	(0.86)	0.15%	(0.91)	-0.27%	(-1.28)	**-0.56%**	**(-2.69)**
5	98	0.18%	(0.63)	-0.14%	(-0.87)	**-0.93%**	**(-6.00)**	**-0.77%**	**(-5.75)**
5-1		0.19%	(0.48)	-0.40%	(-1.60)	**-1.69%**	**(-7.23)**	**-1.28%**	**(-6.45)**

Panel B: Four-factor Adjusted IVOL-returns on CoIVOL/CoBAR

Rank	Obs.	Months 1-6 Est.	t-stat	Year 1 Est.	t-stat	Year 2 Est.	t-stat	Year 3 Est.	t-stat
1	98	**1.53%**	**(7.96)**	**1.03%**	**(8.40)**	0.14%	(0.86)	**0.49%**	**(4.30)**
2	99	**2.05%**	**(10.77)**	**1.61%**	**(12.10)**	0.17%	(0.87)	**0.32%**	**(2.02)**
3	98	**1.26%**	**(4.97)**	**1.17%**	**(6.84)**	**0.74%**	**(4.15)**	**0.38%**	**(2.39)**
4	99	0.61%	(1.71)	**0.53%**	**(2.30)**	**0.81%**	**(5.82)**	**0.70%**	**(5.72)**
5	98	0.45%	(1.57)	**0.42%**	**(2.20)**	**0.74%**	**(5.37)**	**0.38%**	**(3.24)**
5-1		**-1.08%**	**(-3.14)**	**-0.60%**	**(-2.64)**	**0.60%**	**(2.80)**	-0.11%	(-0.67)

We present our results in Table 7. Rank one shows abnormal returns to the strategy when the arbitrage activity is at its lowest relative to the activity in the other strategy. Contrary, rank five shows abnormal returns to the strategy when the activity is at its highest relative to the activity in the other strategy. Consequently, the two measures are the inverse of each other. In Table 7 Panel A, which reports the four-factor adjusted beta-returns conditional on CoBAR/CoIVOL, we find that there are no significant alphas within the first twelve months of forming portfolios. In year two, the alphas of the extreme quintiles becomes significant, yielding an alpha of 0.76% for the lowest quintile and an alpha of -0.93% in the highest quintile. This pattern continues in year three, where the lowest quintile has an alpha of 0.51% and the highest quintile has an alpha of -0.77%, both statistically significant. The results for year two and three aligns with our previously obtained results, where alphas in the portfolios constructed during low activity materialized slowly. However, the immediate abnormal returns in quintile five which was due to crowded-trading is now gone despite the strategy crashing in the long run.

In Panel B of Table 7, we report the results of forming IVOL-sorted portfolios conditional on CoIVOL/CoBAR. In the first six months, we find that when CoIVOL is low relative to CoBAR, the portfolios yield significant positive average alphas (1.53% for quintile one and 2.05% for quintile two), while they become insignificant when the discrepancy in activity levels out. When examining the first year as a whole we find the same pattern, except that all ranks now have significantly positive alphas which also shows a tendency of decreasing with relative activity. In year two, the lowest quintiles become insignificant while the abnormal returns of quintile three to five are now significantly positive. Alphas in the third year can be compared to those found in year one in terms of sign and significance.

The results of our study on the interaction between CoBAR and CoIVOL indicate that beta-portfolios do not earn significant abnormal returns at any levels of CoBAR/CoIVOL in the first year. When arbitrage activity in the beta-strategy is low relative to the IVOL-strategy we observe delayed abnormal returns, and we observe a crash in years two and three when the relative activity is reversed. When inspecting the abnormal returns to the IVOL-strategy conditional on CoIVOL/CoBAR, we find highly significant positive returns for most ranks and periods. The exception is for the highest quintiles in the first six months

and the lowest quintiles in year two. We argue that the interaction results serves as a good complement to our original measures. This is because it shows investors, when presented with the two strategies, where they should place their funds at any given point in time. The implications of these results will be discussed and compared to our original results later in Chapter 6.

We find our results so far to be very interesting but before we conclude, we want to perform robustness tests on the IVOL-sorted portfolios. In the next section we will therefore examine how changing specifications of our models can alter the outcome of our results and whether these give us additional insight on the effect of arbitrage activity.

5.4 Robustness Tests

In this section we test the robustness of our obtained results for various specifications in order to ensure that our result of time- and activity decaying abnormal returns in IVOL-arbitrage holds. We present the outcome of our tests in Table 8. The reported alphas are calculated as the average difference in returns to the IVOL-strategy between high and low CoIVOL in the first six months and in the third year after forming the portfolio, while controlling for the Carhart four-factor model. The t-statistics, which are shown in parentheses, are computed based on Bartlett kernel standard errors corrected for serial-dependence with 6 or 12 lags, depending on the number of overlapping observations and holding period. Statistically significant (5%) observations are highlighted in bold. The first row reports the main results that can also be found in Table 6 Panel B. Row two reports abnormal returns to the IVOL-strategy in our first subsample (1970-1989), while row three shows the same analysis conducted for our second subsample (1990-2010). On row four we exclude the tech bubble (2001) and row five shows abnormal returns to the strategy when excluding the financial crisis (2007-2009). In row six we control for a longer portfolio formation period. We control for small market cap companies in row seven and eight and exclude penny stocks in row nine. Row ten shows the difference in abnormal returns when sorting IVOL into quintile portfolios and row eleven reports average alphas when weighting individual stock returns equally in each portfolio. Due to the extensiveness of our robustness tests, we only report the most interesting findings. The rest of the results can be found in the aforementioned table.

5.4.1 Performance in different subsamples

Ang et al. (2006) argues that one explanation for the IVOL effect could be asymmetry of return distributions across business cycles. They study eight subsamples including every decade from 1963 to 2000, and their tests show that the low volatility effect holds in all of them. Earlier, we saw that the time-series of CoIVOL suggested a negative trend from 1970 to around 1989 and a similar positive trend from around 1990 to 2010. We therefore decide to split our sample into two subsamples for these periods and test whether our results hold.

From our test of subsamples we see that the reported difference in alphas in Table 8 are not significant. When examining the alphas individually, we find that the abnormal returns in subsample 1 are significant in the first six months and in the third year, both when arbitrage activity is high and low. Average alphas for the second subsample are significant within the first six months, both when activity is high and low, but are only significant in the third year after portfolio formation when activity is low. This is an indication that as the overall activity in the IVOL-strategy increases, abnormal returns are arbitraged away quicker when activity is at its highest. One explanation for the increased activity from the beginning of subsample 2, could be that the low volatility puzzle was gaining traction around the early 1990s with some influential papers being published[10]. We conclude that our original results hold in both subsamples despite the insignificant difference in alphas between high and low arbitrage activity

5.4.2 Excluding macro-economic events

When looking at the time-series of CoIVOL, we observe significant drops in arbitrage activity around times of negative macro-economic events. Huang et al. (2016) decide to exclude outlier years like the tech bubble crash (2001) and the financial crisis (2007-2009) when testing the beta-strategy, and find that their results remain robust. When we exclude portfolios formed in 2001 from our sample, we find that the results are almost identical to those obtained when looking at the full sample. Excluding 2007 to 2009 tells almost the same story, except that the difference in average alphas are also significant in the third year. This goes to show that negative macro-economic events does not change our results in any meaningful way.

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[10]See Campbell and Hentschel (1992) and Glosten, Jagannathan, and Runkle (1993)

5.4.3 Controlling for formation period

Ang et al. (2006) examines a twelve-month formation period, in order to make sure that the original one-month formation period is not capturing specific short-term corporate events, and find that the anomaly still holds. We change our portfolio formation period from one to twelve months to see how our results are affected. Our findings indicate that low IVOL stocks still outperform high IVOL stocks and that returns are higher during times of lower arbitrage activity, both in the short- and long-run. The reported difference in average alphas are not significant for the first six months, but they hold when inspecting them individually. Another interesting result from this test of robustness is that the year three alpha of the lowest quintile is 1.05% and highly significant, indicating that some of the time-decreasing returns to the strategy may be due to formation period and noise in the long run, although this is not reported in Table 8.

5.4.4 Excluding small cap companies and penny stocks

To make the IVOL-sorted portfolios more realistic, our next tests of robustness includes deleting the percentile and decile of stocks with the lowest market cap calculated at the end of each month. Ang et al. (2006) also control for market capitalization and although their abnormal returns are deflated, they conclude that it does not explain the low returns to high IVOL stocks. We conduct this test because we believe that stocks with the highest IVOL, the ones we take short positions on, have a tendency to be small cap companies which in turn have a very small number of shortable shares, if any at all. Our sample of stocks is thus reduced by 1% and 10% before we calculate IVOL and then sort into deciles.

On row 4 and 5 in Table 8, we see that our original results still hold both when deleting the lowest 1% and 10% market cap stocks each month. This result is very interesting because it goes to show that our combined portfolio is not dominated by the smallest traded companies in terms of market capitalization. Although this is interesting, it is not very surprising seeing as we previously found that 82% of the abnormal returns were generated by the lowest decile of IVOL stocks.

We also delete penny stocks, which we define as stocks with the lowest percentile share price in a given month. This test is conducted because stocks with a lower share price

generally translates to higher transaction costs due to bigger spreads. When removing penny stocks from our sample, we find much of the same as when controlling for small caps, the 5-1 alphas are statistically significant in the first six months before disappearing in year three.

5.4.5 Equal-weighted portfolios

To test if our results occur due to some stocks being weighted heavily, we give each individual stock in our portfolios equal weight before measuring their performance. We find that altering the weighting does not have any fundamental impact on our original results.

Table 8: Robustness Tests

In this table, we present a summary of the four-factor adjusted alphas of the IVOL-sorted portfolios conditional on CoIVOL, and controlled for various factors that might obscure our original results. The reported alphas are the difference in four-factor alpha to the IVOL arbitrage strategy between high CoIVOL and low CoIVOL periods. The first row reports the main results that can also be found in Table 6 Panel B. Row two reports abnormal returns to the IVOL-strategy in our first subsample (1970-1989), while row three shows the same analysis conducted for our second subsample (1990-2010). On row four we exclude the tech bubble (2001) and row five shows abnormal returns to the strategy when excluding the financial crisis (2007-2009). In row six we control for a longer portfolio formation period. We control for small market cap companies in row seven and eight and exclude penny stocks in row nine. Row ten shows the difference in abnormal returns when sorting IVOL into quintile portfolios and row eleven reports average alphas when weighting individual stock returns equally in each portfolio. The t-statistics, which are shown in parentheses, are computed based on Bartlett kernel standard errors corrected for serial-dependence with 6 or 12 lags, depending on the number of overlapping observations. Statistically significant (5%) observations are highlighted in bold.

	Months 1-6		Year 3	
	Estimate	t-stat	Estimate	t-stat
Subsamples				
Full sample	**-1.24%**	**-3.10**	-0.19%	-1.16
Subsample 1 (1970-1989)	-0.59%	-1.58	0.03%	0.15
Subsample 2 (1990-2010)	-0.62%	-1.21	-0.27%	-1.13
Tech bubble (2001)	**-1.22%**	**-3.11**	-0.24%	-1.49
Financial crisis (2007-2009)	**-1.16%**	**-2.85**	**-0.40%**	**-2.55**
Controlling for formation period				
Past 12 months	-0.34%	-0.69	**-0.75%**	**-4.15**
Other controls				
Excluding small cap (1%)	**-1.14%**	**-3.01**	-0.23%	-1.41
Excluding small cap (10%)	**-1.11%**	**-3.19**	-0.26%	-1.62
Excluding penny stocks (1%)	**-1.10%**	**-2.97**	-0.17%	-1.03
Quintile sorted portfolio	**-0.78%**	**-2.69**	-0.19%	-1.47
Equally weighted portfolio	**-1.00%**	**-2.16**	0.11%	0.51

6 Discussion

Before we conclude our thesis based on our findings on CoIVOL and the IVOL-sorted portfolios, we address a few topics that we think are worth discussing related to our results on risk-based investing. We start by discussing and interpreting our results obtained from the IVOL-portfolios. We concentrate on the CoIVOL-measure and its ability to forecast abnormal returns to the IVOL-strategy as this is the main contribution of this thesis. Additionally, we also discuss the differences in the two risk- and arbitrage activity measures, before we briefly debate the results from the interaction-measures between CoBAR and CoIVOL.

In our main results from Section 5.2, we found that the alphas were usually decreasing with time and arbitrage activity in the IVOL-strategy. We believe this is worth mentioning because it contradicts our conclusion on the beta-strategy made in Section 3.3. The beta-strategy yields the highest alphas in the first six months when arbitrage activity is at its highest, while the alphas increase with time when activity is at its lowest. The reason for the differences in the two strategies is that the beta-strategy is fundamentally anchored and will thus be the victim of positive-feedback trading. Huang et al. (2016) points out that when there is crowded arbitrage trading in the beta-strategy, prices of stocks with low betas will increase and prices of high-beta stocks will fall. The effect on levered firms in the portfolio will be even stronger because it deflates and inflates the low- and high-beta stocks respectively, ceteris paribus. It is obvious from the previously cited literature that the IVOL-strategy will not influence the fundamentals of a company and in basic portfolio theory, IVOL is neglected as it will disappear in a well-diversified portfolio.

An interesting result in the highest quintile of our IVOL-sorted portfolios is that alphas do not materialize until the second year after portfolio formation. Intuitively, it makes sense that there are no significant alphas in the first two years because the high activity in the strategy implies that many arbitrageurs are trading low IVOL stocks. This results in prices already being high for the low IVOL securities and low for the high IVOL securities. In Figure 3, we saw that periods of spikes in arbitrage activity were usually followed by a short period of sharp decline in activity. Thus, overlapping periods of arbitrage-crowdedness with lower quintiles of CoIVOL seem to be the reason for the delay in the realization of abnormal returns when

arbitrage activity is high. This is the exact opposite of what we find in the beta-strategy where alphas, on average, do not occur right after forming the portfolio when arbitrage activity is at its lowest. Instead, they require a longer holding period to become significant. When activity is at its highest, the beta-portfolio crashes after two years, again the exact opposite of what we find for the IVOL-strategy. However, the IVOL-alpha disappears in year three for the highest quintile of CoIVOL as the declines in activity tend to normalize. This aligns with the previously mentioned rationale that high CoIVOL does not result in a positive-feedback which is tied to the fundamentals of the respective securities.

During periods of low activity, on the other hand, alphas are positive and significant in all periods for the IVOL-strategy, however, they decrease as time passes. We ascribe this decaying behavior to the observation that periods of low activity are usually followed by a gradual surge in arbitrage activity. This tightens the spread between high- and low IVOL securities and makes the strategy less profitable over time. The latter is reinforced by the apparent dependency of the alphas on arbitrage activity. From the forecast of IVOL-arbitrage returns in Table 6 Panel B, we observe that the alphas are decreasing with arbitrage activity, an effect that is strongest in the first six months, however, they are more obscure in later periods. This shows that arbitrage activity in the IVOL-strategy is stabilizing, as higher activity results in alphas being arbitraged away.

In Section 5.3 we created two measures to test the interaction between CoBAR and CoIVOL. When examining the former relative to the latter we found that the initial abnormal returns to the beta-strategy diminished when CoBAR was relatively high compared to CoIVOL. This is interesting as we would expect that due to the positive-feedback effect in the beta-strategy, the initial returns would still be high in quintile five within the first six months. This could possibly be explained by the fact that when CoIVOL is low, the IVOL-sorted portfolios generate the highest abnormal returns.

In the second interaction measure we created, namely CoIVOL/CoBAR, we see that most of the abnormal returns to the IVOL-strategy still holds. We believe this can be attributed to the outstanding performance of the underlying strategy, both in times of high and low arbitrage activity. However, there are some difference that are worth mentioning. Specifically, we find the fact that the two highest quintiles in the first six months are insignificant

interesting. This shows that when CoIVOL is relatively high compared to CoBAR there are no short-run abnormal returns to be made from investing in the strategy. In the original CoIVOL measure the alphas were insignificant only in the highest quintile, thus it may seem like we have eliminated a possible fallacy of the strategy.

In the next chapter we summarize our findings on CoIVOL and the IVOL-sorted portfolios as well as the portfolios sorted on our measures of relative arbitrage activity in the respective strategies. We conclude our thesis on whether the effect of arbitrageurs is stabilizing on prices or not, and decide whether timing can be of use to investors exploiting the IVOL-strategy.

7 Conclusion

In this thesis we study the effect of arbitrage activity in two different arbitrage-strategies formed on the decomposition of the low volatility anomaly. Using the methodology of Lou and Polk (2013), we are able to successfully replicate the CoBAR measure of Huang et al. (2016), and confirm the booms and busts in the beta arbitrage strategy. Our results indicate that abnormal returns to the strategy materialize slowly when there is low arbitrage activity, while the portfolio generates high initial returns before crashing in the long run when formed during periods of high activity.

Our contribution has been to create an extension of CoBAR by constructing CoIVOL, a measure of arbitrage activity in idiosyncratic volatility strategies, using a combination of methodologies from Huang et al. (2016) and Ang et al. (2006). Following the original methodology of Lou and Polk (2013), we measure arbitrage activity in the IVOL-strategy by looking at the past degree of abnormal excess correlation within the group of stocks where the strategy would be traded. We document that the abnormal returns, on average, are at their highest immediately after forming portfolios and when arbitrage activity in the IVOL-strategy is low. Further, we find that the alphas are decreasing both with time and activity, indicating that arbitrageurs stabilize prices for this particular strategy. Our results also hold when controlling for a variety of specifications and tests of robustness. When studying the interaction between arbitrage activity in the beta- and IVOL-strategy, we find that when CoBAR is relatively high compared to CoIVOL, the abnormal returns materialize slowly and crash when the relationship inverts. The IVOL-portfolios conditional on CoIVOL relative to CoBAR shows that most of the abnormal returns are still positive and significant, indicating that the IVOL-strategy performs well regardless of activity in the beta-strategy.

We determine that there are few advantages of timing the market in the IVOL-strategy during periods of lower activity, while it can be valuable to investors when activity peaks. We conclude that this is a result of prices already being stabilized by arbitrageurs.

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