Discussion paper

Insider trading with non-fiduciary market makers

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Abstract

The single auction equilibrium of Kyle's (1985) is studied, in which market makers are not fiduciaries. They have some market power which they utilize to set the price to their advantage, resulting in positive expected profits. This has several implications for the equilibrium, the most important being that by setting a relatively modest "fee", the market maker is able to obtain a profit of the order of magnitude, and even better than, a perfectly informed insider. Our model indicates why speculative prices are more volatile than predicted by fundamentals. Noise traders may be uninformed, or partially informed. We analyze a situation where the market maker has private information as well as being non-fiduciary. In our model this leads to a more efficient market where the insider trades less and the market maker's profit increases.

KEYWORDS: Insider trading, asymmetric information, strategic trade, correlated trade, price distortion, partially informed market maker

1   Introduction

In his seminal paper on insider trading, Albert Kyle (1985) asks several questions: How valuable is private information to an insider? How does

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noise trading affect the volatility of prices? What determines the liquidity of a speculative market? He provides answers to these and other questions by modeling rigorously the trading strategy of an insider in a model of efficient price information.

Kyle focuses on a single auction model in which a risky asset is exchanged for a riskless asset among three kinds of traders. A single insider has access to perfect, private observation of the ex post liquidation value of the risky asset. Less informed noise traders trade randomly. Market makers set prices and clear the markets after observing the quantities traded by others.

In the Kyle model the noise traders can be considered as less than fully rational, since they expect to suffer losses equal to the insiders' gains. The market makers set the prices equal to the expected value of the risky asset conditional on the order flow; they are making zero profits. The market makers cannot distinguish the trading of the insider from the trading of the noise traders, who in effect provide camouflage, which enables the insider to make profits on their expense.

The market maker in the standard model has substantial market power, yet does not exploit this to his own advantage when setting the price; the market maker is assumed to be a fiduciary acting in the best interest of market participants.

One may ask how realistic this assumption is. In the testimony before the Financial Crisis Inquiry Commission, Goldman CEO Lloyd Blankfein laid out the Goldman Sachs perspective on the firm's role in CDO deals related to the 2008 financial crisis. From his answer it seems clear that he does not consider a market maker as a fiduciary agent:

In our market-making function, we are a principal. We represent the other side of what people want to do. We are not a fiduciary. We are not an agent. Of course, we have an obligation to fully disclose what an instrument is and to be honest in our dealings, but we are not managing sombody else's money.

Caveat emptor seems to be Mr. Blankfein's message, and this was also the basis of Goldman's defense against the SEC suit re the Abacus transactions. The case of Goldman Sachs is, we believe, not unique. Investment banks and other financial intermediaries are known to accumulate large fortunes, which should be difficult, or even impossible, if they were just disinterested auctioneers.

In this paper, we investigate the consequences of relaxing the assumption that market makers are fiduciaries. In our model, market makers are

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economic agents allowed to make a profit. Market makers generate profits by adding a margin to the conditional expected value of the risky asset when they are going short. Similarly they are subtracting a margin when taking long positions. Formally, the margin is a random variable, which is correlated with aggregate demand. Thus, market makers are not just adding or subtracting a fee; the size of the fee depends on trading volume. As in the standard model, informed traders realize what market makers are up to, and take their behavior into account when deciding their own trades. Noise traders just trade, but we allow them to have partial information. Despite of this, market makers may make unbounded profits taking advantage of noise traders, which would not make sense. To avoid this outcome a regulator is introduced. Alternatively, the market maker may be assumed to practice restraint in order to keep markets open.

Perhaps surprisingly, our analysis shows that for only a moderate correlation with the aggregate demand, the profits of the market maker may exceed that of the perfectly informed insider. This could serve as one explanation of why so much wealth tends to end up in the hands of financial intermediaries, a timely question that has been asked many times over after the 2008-financial crisis.

Another implication of our model is that the market maker's actions lead to more volatile prices than would be the case if dealings were fair. We also demonstrate that the volatility of prices increase with inside information. This may throw some light on the observation made by Campbell and Schiller (1988), that stock market prices display much more volatility than implied by dividends alone. The more recent approach to this problem is to consider the variations in the stochastic discount factor, see e.g., Campbell (1999). We investigate whether the actions of market makers may be part of the explanation.

To limit the distortion of prices, a regulatory authority (the SEC) imposes an upper bound on price volatility. In our model this limits the market maker's freedom to set prices. This implies the existence of an equilibrium in which the insider maximizes profits and the market maker trades "fees". We also extend the model by allowing market makers to obtain private information. The insider has more information about the value of the asset, but does not know what the market maker believes. The market makers do not use information strategically, but set prices equal to the asset's expected value conditional on private information and order flows; then they add the fee. This extension has several interesting implications. First of all the market

becomes more efficient, and the volatility of prices increases. As a secondary effect the increased basic volatility may allow the market maker to charge higher fees without disturbing the regulator. Informed market makers, and in particular privately informed market makers, is a problem for insiders. Opportunities for profit naturally decline, and if they do not know the information of the market makers, they reduce trading considerably. It follows that market makers will have incentives to share any private information they happen to receive with other market participants to increase trading and at the same time make markets more efficient. Thus, for market makers information is in general a mixed blessing. Less trading means less fee income. On the other hand, higher efficiency and increased price volatility may allow market makers to charge higher fees per trade.

This also throws some new light on one 'positive' side of insider trading (aside from the obvious negative aspects which we do not dwell on here); all information arriving to the market generally has a positive effect on efficiency.

Our model allows us to extend the analysis in another direction. By using the technique of Aase et. al. (2012a), we can let the noise traders be partially informed, as mentioned. Since this is not enough to bound the profits of the market maker, from Section 2.2 on we let the noise traders be uninformed.

Strategic market makers have been treated in the market microstructure literature by several authors, for example in Biais et al. (1998), where market makers are risk averse, but in their model information is symmetric, and the objective is to compare different market structures. In contrast, our market maker is not really strategic, is risk neutral, but exercises a certain degree of monopoly, as explained above. A regulator is introduced to mitigate this, or alternatively, the market maker may be assumed to practice restraint in order to keep markets open. After all, it is the market maker's responsibility that the markets function.

Dutta and Madhavan (1997) study collusion among market makers, and show that dealers who adopt noncooperative pricing strategies may set the bid-ask spreads above competitive levels, on organized stock exchanges. This form of implicit collusion differs from explicit collusion, where dealers cooperate to fix prices. In contrast, our market makers have a degree of "local" monopoly, where the customers may be thought of as having some level of ties to the dealers, and trade is thought to be over-the-counter. Also, given the order flow, there is no bid-ask spread in our model, but a unique price.

Other models of strategic trading behavior of market makers in security markets under adverse selection include Glosten (1989), who studies proper-

ties of a monopolistic specialist system as opposed to a competitive specialist system. Dennert (1993), Bernhardt and Hughson (1997) and Biais et al. (2000) analyze price competition among market makers when informed and uninformed traders are allowed to split their orders between markets. Bondarenko (2001) derives an equilibrium, in which each market maker behaves as a monopolist facing a residual demand curve resulting from maximizing behavior of the informed trader and the price schedules offered by the competitors.

As this literature shows, consistent models exist where market makers may make positive profits in equilibrium, even when there is competition between them.

There is a vast literature on insider trading, and in particular the question of who the noise traders are. It is the hope that our paper will shed some further light on this latter question. Regarding the topic of rationality, Spiegel and Subrahmanyam (1992) replaced Kyle's uninformed liquidity traders with strategic utility-maximizing agents trading for hedging purposes. Diamond and Verrecchia (1981) suggest adding a noise term to agents' risk exposures. Risk-averse agents will then have an insurance motive for trading. De Marzo and Duffie (1999) propose a model where different traders have different discount rates. These papers solve the problem of finding a logically consistent model that can be used for e.g., welfare statements, of markets with imperfect information revelation.

DeLong, Shleifer, Summers and Waldman (1990), Dow and Gorton (1994), and Shleifer and Vishny (1997) propose limits to arbitrage in order to explain that noise traders are not eliminated by informed traders. The view that noise traders are less than rational is discussed in Shiller (1984), Schleifer and Summers (1990) and Barberis and Thaler (2003). Admati and Pfleiderer (1988) introduce two types of liquidity traders, discretionary and non-discretionary. Dow and Gorton (2006) present a broad review of various aspects of noise traders, and conclude that the their identities, motivations and ability to persist remain topics of research.

The paper is organized as follows: In Section 2 we find an equilibrium in a one period model with a non-fiduciary market maker and partly informed noise traders, in Section 3 we assume that also the market maker has private information in addition to being non-fiduciary. Section 4 concludes.

2   A single auction equilibrium

In this section we find an equilibrium in the one period model with a non-fiduciary market maker. The structure of the model is the following: The risky asset's liquidation value $\tilde{v}$ is normally distributed with mean p_0 and variance σ_v^2, in short, $\tilde{v} \sim \mathcal{N}(p_0, \sigma_v^2)$. We assume a discount rate of zero between time 0 and time T. The quantity demanded by the noise traders is $\tilde{z}$, where $\tilde{z} \sim \mathcal{N}(0, \sigma_z^2)$. The quantity demanded by the informed trader is $\tilde{x}$, the price is denoted $\tilde{p}$.

Unlike Kyle (1985), we assume that the market maker does not set a fair price, explained shortly, and he can also have private information.

Regarding the noise traders, while Kyle (1985) assumes $\tilde{z}$ and $\tilde{v}$ to be independent, we allow $(\tilde{z}, \tilde{v})$ to be jointly normally distributed with correlation coefficient ρ, meaning Pearson's product-moment correlation. The interpretation is that when $\rho > 0$, the noise traders are more rational than in the standard model. This may be because they have information, or because whatever drives their demand is positively correlated with value. If $\rho < 0$ on the other hand, we interpret this as manipulation, i.e., the noise traders have been manipulated by the informed trader.

Trading is structured in two steps as follows: In step one, the exogenous values of $\tilde{v}$ and $\tilde{z}$ are realized and the informed trader chooses the quantity $\tilde{x}$ he demands. In doing so, he observes $\tilde{v}^{obs} = v$, but not $\tilde{z}$. The informed trader's trading strategy is given by some real, measurable function $x : R \to R$, i.e., $\tilde{x} = x(\tilde{v})$.

In step two the market maker determines the price $\tilde{p}$ at which he trades the quantity necessary to clear the market. In doing so he observes $\tilde{y} = (\tilde{x} + \tilde{z})$ but not $\tilde{x}$ or $\tilde{z}$ separately. He sets the price $\tilde{p}$ as follows:

(2.1) $$\tilde{p} = p(\tilde{y}, k) = E\{\tilde{v} + \tilde{u} | \tilde{y}\}$$

where the decision variable $\tilde{u} = k\tilde{y}$, k is a real non-negative parameter. The average price set by the market maker is $E\{p(\tilde{y}, k)\} = p_0$, and thus correct in expectation.

The market maker calculates the conditional expected value of the asset given the order flow. Then he adds a fee if he is selling, and subtracts a fee if he is buying. The fee is a positive linear function of market demand.

It would, perhaps, seem simpler with a fixed fee instead of a random one, but this typically leads to discontinuities which would make the model hard

to generalize to a continuous-time setting[1].

The pricing rule is thus determined by a real, measurable function p : $R \times R \to R$ so that $\tilde{p} = p(\tilde{x} + \tilde{z}, k)$. The informed trader's profit is denoted by $\tilde{\pi}_I$, where $\tilde{\pi}_I = (\tilde{v} - \tilde{p})\tilde{x}$. The market makers profit is $\tilde{\pi}_M$, where $\tilde{\pi}_M = (\tilde{p} - \tilde{v})(\tilde{x} + \tilde{z})$. Since $\tilde{\pi}_I$, $\tilde{\pi}_M$ and $\tilde{p}$ depend upon the real functions $x(\cdot)$ and $p(\cdot)$, we may write $\tilde{\pi}_I = \tilde{\pi}(x, p)$ and $\tilde{\pi}_M = \tilde{\pi}_M(x, p)$. We assume risk neutrality.

In order to limit the distortion of prices, we assume that a regulatory authority sets an upper bound on the price volatility $SD(\tilde{p})$ of the firm: $SD(\tilde{p}) \leq B$. When the volatility exceeds this boundary, the stock of the firm is suspended.

Instead of this measure, we can base the analysis on the conditional expected price deviation $E(\tilde{y}|\tilde{y} > 0)$, which is equal to $-E(\tilde{y}|\tilde{y} < 0)$ by symmetry of the normal distribution. We return to this possibility in Section 2.2.

If k is not bounded, market makers may exploit noise traders with impunity. This is not reasonable. The regulator is introduced to limit exploitation. In a repeated game setting, the incentive to keep markets open serve the same purpose.

Equilibrium. An equilibrium is a pair of functions (x, p) such that the following three conditions hold:

(1) Profit maximization (insider): For any other trading strategy x' and for any v

$$E\{\tilde{\pi}(x, p)|\tilde{v} = v\} \geq E\{\tilde{\pi}(x', p)|\tilde{v} = v\}.$$

(2) Price determination: The market price $\tilde{p}$ satisfying the regulatory constraint is given by

$$\tilde{p}(y, k, p) = E\{\tilde{v} + \tilde{u}|\tilde{x} + \tilde{z}\}.$$

(3) The market maker's profit is bounded in that there exists a maximal value of k, called k^*, such that $SD(\tilde{p}) \leq B$.

As mentioned in Kyle (1985), his model is not purely game theoretic, because the noise traders do not explicitly maximize any particular objective. In our model they may act more rationally, see Aase et.al. (2012) for details relating the noise traders. In addition the market maker obtains a positive profit on average, and 'optimizes' profits subject to the regulatory constraint.

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[1]In the present formulation the aggregate demand at time t, y_t, becomes a well defined diffusion process consisting of an 'innovation' term added to a mean reverting part, thus making the model appropriate for a continuous-time extension.

The profit depends on the parameter k, which is typically set equal to its largest possible value under this constraint. The market maker's strategy is ad hoc and very intuitive, but not a direct result of optimization. However, we believe it reflects what goes on in real markets. Our first result is the following:

Theorem 2.1. *In the situation described above, there exists an equilibrium in which x and p are linear functions. Defining*

$$(2.2) \quad \beta = \frac{1}{2\,\lambda} - \frac{\rho}{2}\frac{\sigma_z}{\sigma_v} \quad and \quad \lambda = \frac{\beta\sigma_v^2 + k^*(\beta^2\sigma_v^2 + \sigma_z^2) + (2k^*\beta + 1)\rho\sigma_v\sigma_z}{\beta^2\sigma_v^2 + \sigma_z^2 + 2\beta\rho\sigma_v\sigma_z}$$

where k^* is the value of k determined by the regulatory constraint, the equilibrium is given by

$$x(\tilde{v}) = \beta(\tilde{v} - p_0), \quad p(\tilde{x} + \tilde{z}) = p_0 + \lambda(\tilde{x} + \tilde{z}).$$

The parameter β is a solution to the cubic equation

$$(2.3) \quad 2k^*\sigma_v^2\beta^3 + (\sigma_v^2 + 5\rho k^*\sigma_v\sigma_z)\beta^2 + (\rho\sigma_z\sigma_v + 2k^*\rho^2\sigma_z^2 + 2k^*\sigma_z^2)\beta$$
$$+ \rho^2\sigma_z^2 + \frac{k^*\rho\sigma_z^3}{\sigma_v} - \sigma_z^2 = 0,$$

which also determines the constant λ. The expected profits are as follows:
The insider: $E(\tilde{\pi}_I) = \sigma_v^2\lambda\beta^2$.
The market maker: $E(\tilde{\pi}_M) = \lambda\sigma_z^2(1 - \frac{3}{4}\rho^2) - \frac{1}{2}\sigma_v^2\beta - \frac{1}{4}\rho\sigma_v\sigma_z$.
The noise traders: $E(\tilde{\pi}_N) = \frac{1}{2}\rho\sigma_v\sigma_z + \frac{1}{2}\rho^2\lambda\sigma_z^2 - \lambda\sigma_z^2$.

Proof. By the joint normality assumption between $\tilde{v} + \tilde{u}$ and $\tilde{y}$, the price is a linear function of $\tilde{y}$, or

$$p(y) = p_0 + \lambda y, \quad (y = x + z),$$

for some constant λ, since $E(\tilde{y}) = 0$. The insider, realizing how the market maker determines prices, computes his conditional expected profits, given his private information, as follows

$$\pi_I(v) = E\{[\tilde{v} - p(\tilde{x} + \tilde{z})]x|\tilde{v} = v\} = [v - p_0 - \lambda x - \lambda\rho\frac{\sigma_z}{\sigma_v}(v - p_0)]x,$$

since by our assumption that $(\tilde{z}, \tilde{v})$ is binormal,

$$E\{\tilde{z}|\tilde{v} = v\} = \rho\frac{\sigma_z}{\sigma_v}(v - p_0)$$

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by the 'projection' theorem. Profit maximization of this quadratic objective requires that x solves

$$v - p_0 - \lambda \rho \frac{\sigma_z}{\sigma_v}(v - p_0) - 2\lambda x = 0$$

or

$$x = \left(\frac{1}{2\lambda} - \frac{1}{2}\rho \frac{\sigma_z}{\sigma_v}\right)(v - p_0) = \beta(v - p_0).$$

In other words, $x(v)$ is linear in v, and β is as in the first part of (2.2). We must determine the price sensitivity parameter λ. By assumptions, $\tilde{v} + \tilde{u}$ is normally distributed with mean p_0 and

$$\mathrm{var}(\tilde{v} + \tilde{u}) = (1 + k\beta)^2 \sigma_v^2 + k^2 \sigma_z^2 + 2k\rho\sigma_v\sigma_z(1 + \beta k).$$

Also

$$\mathrm{var}(\tilde{y}) = \beta^2 \sigma_v^2 + \sigma_z^2 + 2\beta\rho\sigma_v\sigma_z,$$

and

$$\mathrm{cov}(\tilde{v} + \tilde{u}, \tilde{y}) = \mathrm{cov}(\tilde{v}, \tilde{y}) + k\,\mathrm{var}(\tilde{y}) =$$
$$\beta\sigma_v^2 + \rho\sigma_v\sigma_z + k\beta^2\sigma_v^2 + k\sigma_z^2 + 2k\beta\rho\sigma_v\sigma_z.$$

By the projection theorem

$$E\{\tilde{v} + \tilde{u}|\tilde{y} = y\} = p_0 + \rho_{\tilde{v}+\tilde{u},\tilde{y}} \frac{\sqrt{\mathrm{var}(\tilde{v} + \tilde{u})}}{\sqrt{\mathrm{var}(\tilde{y})}} \cdot y,$$

where

$$\rho_{\tilde{v}+\tilde{u},\tilde{y}} = \frac{\mathrm{cov}(\tilde{v} + \tilde{u}, \tilde{y})}{\sqrt{\mathrm{var}(\tilde{v} + \tilde{u})}\sqrt{\mathrm{var}(\tilde{y})}}.$$

We notice that the term $\sqrt{\mathrm{var}(\tilde{v} + \tilde{u})}$ cancels. We have found that

$$p = p(y) = p_0 + \frac{\beta\sigma_v^2 + k\beta^2\sigma_v^2 + (2k\beta + 1)\rho\sigma_v\sigma_z + k\sigma_z^2}{\beta^2\sigma_v^2 + \sigma_z^2 + 2\beta\rho\sigma_v\sigma_z} \cdot y.$$

This determines λ as a function of β as in the second part of (2.2). Since (2.2) is a system of two equations in the two unknowns λ and β, eliminating λ yields the cubic equation (2.3) in β.

This equation we have solved; it has at least one real root, one of which is the solution of the problem. The solution $\beta := \beta(k, \rho)$ is decreasing in k for each value of ρ. The expected profits of the market maker is

$$E(\tilde{\pi}_M) = E((\tilde{p} - \tilde{v})\tilde{y}) = \lambda\mathrm{var}(\tilde{y}) - E(\tilde{v}\tilde{y}).$$

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Using that $\tilde{y} = \tilde{x} + \tilde{z} = \beta(\tilde{v} - p_0) + \tilde{z}$, the relation between β and λ in (2.2) as well as the correlation between $\tilde{v}$ and $\tilde{z}$, we get

$$E(\tilde{\pi}_M) = \lambda\sigma_z^2(1 - \frac{3}{4}\rho^2) - \frac{1}{2}\sigma_v^2\beta - \frac{1}{4}\rho\sigma_v\sigma_z.$$

Since $\beta(k)$ decreases with k, by (2.2) $\lambda(k)$ must then increase with k. Hence the market maker's profit increases with k. Some algebra shows that the standard deviation of the price $\tilde{p}$, $SD(p)(k)$, also increases with k. Thus, the regulatory constraint limits the value of k to some maximum number, say k^*.

It remains to compute the expected profits of the informed trader and the noise traders: Starting with the insider, from (2.2) and the above analysis of the insider's profits it follows that

$$\pi_I(v) = E\{\tilde{v} - \tilde{p})\tilde{x}|\tilde{v} = v\} =$$

$$E\{(v - p_0 - \lambda\tilde{y})(\frac{1}{2\lambda}[v - p_0 - \lambda\rho\frac{\sigma_z}{\sigma_v}(v - p_0)])|\tilde{v} = v\}.$$

Since $E(\tilde{y}|\tilde{v} = v) = E\{\beta(\tilde{v} - p_0) + \tilde{z}|\tilde{v} = v\} = (v - p_0)(\beta + \rho\frac{\sigma_z}{\sigma_v})$, this gives

$$\pi_I(v) = (v - p_0)^2\left(1 - \lambda\beta - \lambda\rho\frac{\sigma_z}{\sigma_v}\right)\left(\frac{1}{2\lambda}(1 - \lambda\rho\frac{\sigma_z}{\sigma_v})\right).$$

By (2.2) $1 - \lambda\beta - \lambda\rho\frac{\sigma_z}{\sigma_v} = \lambda\beta$ and $1 - \lambda\rho\frac{\sigma_z}{\sigma_v} = 2\lambda\beta$. Taking expectations with respect to $\tilde{v}$, the insider's expected profits can be written

$$E(\tilde{\pi}_I) = \sigma_v^2\lambda\beta^2$$

as claimed. The noise traders' expected profits are

$$E(\tilde{v} - \tilde{p})\tilde{z}) = E\{\left((\tilde{v} - p_0) - \lambda(\beta(\tilde{v} - p_0) + \tilde{z})\right)\tilde{z}\} = (1 - \lambda\beta)\rho\sigma_v\sigma_z - \lambda\sigma_z^2,$$

and this can be written as claimed by using that $(1 - \lambda\beta) = \frac{1}{2} + \frac{\rho}{2}\lambda\frac{\sigma_z}{\sigma_v}$, which follows from the first part of (2.2).

Since we here have a pure exchange economy, it must be the case that these profits sum to zero, which they do. This completes the proof of the theorem. This completes the proof of the theorem. □

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2.1 Some Properties of the equilibrium

The first observation we make is that when $\rho = 0$ and $k = 0$, then $\beta = \sigma_z/\sigma_v$ and $\lambda = \sigma_v/2\sigma_z$ which are the same expressions as given by Kyle (1985). When this is the case, our model is the same as the standard one. When $k = 0$ but ρ is an arbitrary correlation coefficient, the cubic equation (2.3) becomes quadratic in β with solution

$$\beta = \frac{1}{2}\frac{\sigma_z}{\sigma_v}\left(\sqrt{4 - 3\rho^2} - \rho\right) \quad \text{and} \quad \lambda = \frac{\sigma_v}{\sigma_z\sqrt{4 - 3\rho^2}},$$

which are the same results as in Aase et. al. (2012), in which case the model has partly informed noise traders and fair price setting.

Figure 1 illustrates the parameter $\beta(k, \rho)$ as a function of k and ρ, while Figure 2 presents the projection of the resulting surface onto the (β, k)-plane when $\rho = .5$, and Figure 3 illustrates the projection onto the (β, ρ)-plane when $k = .5$.
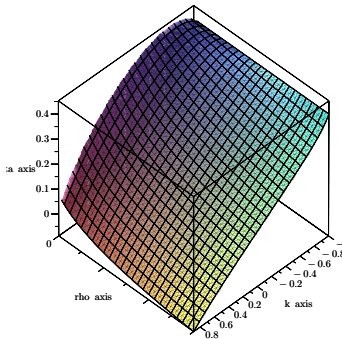
Fig. 1: The surface $\beta(k, \rho)$ as a function of k and ρ ($\sigma_v = 1.0$, $\sigma_z = .4$).

The parameter β can be negative for values of k larger than some k_0 when ρ is strictly positive, and β can be negative for values of ρ larger than some ρ_0 when k is strictly positive (Figure 3), both for the same values of σ_z and σ_v as above. In the remaining figures in this section we use the parameter values $\sigma_v = 1.0$, and $\sigma_z = .4$. For negative values of ρ is β non-negative as a function of k, and for $k = 0$ is β non-negative as a function of ρ. These features can also be seen from Fig. 1.
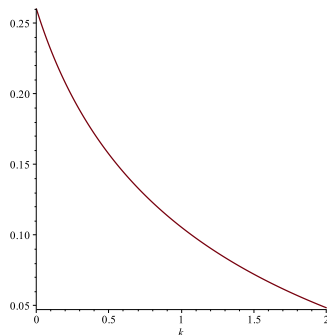
Fig. 2: The parameter $\beta(k, \rho)$ as a function of k when $\rho = .5$.
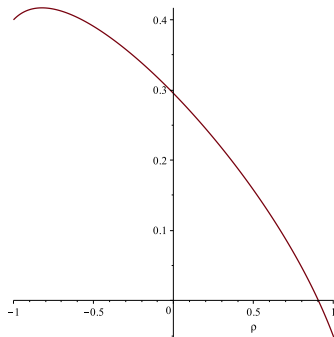

Fig. 3: The parameter $\beta(k, \rho)$ as a function of ρ when $k = .5$.

When the true value $v > p_0$ the insider would normally buy a positive quantum $x = \beta(v - p_0)$, but when $\beta < 0$ he then goes short. This only happens when k and ρ are both positive and large enough, and is not very realistic. A short discussion in nevertheless in order: In Figure 2 where $\rho = .5$ this happens when $k \geq 3.8$. In Figure 3 this occurs for $k = .5$ when $\rho \geq .9$. In this situation the market maker gets a high quality signal from the noise traders, and adjust the value accordingly. By adding a fee on top of that the stock becomes very expensive on average. The insider, acting strategically, may then obtain a positive profit $(v - p)x$ by going short, and thus reduces the profit of the market maker. Short-selling by the informed trader will move the price towards the correct value. Similar reasoning is valid when $v < p_0$.

A main finding in this paper is that the market maker may very well make a profit larger than that of the insider, by setting prices as assumed in relationship (2.1). Figure 4 illustrates this.
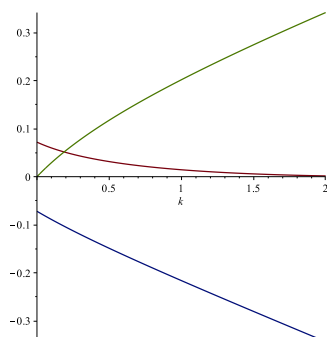
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Fig. 4: The three profits as function of k, when $\rho = .5$.

It shows the three profits as functions of k. The positive, decreasing curve is the insider's profit, the increasing curve is the market maker's. For $k > .2$, the *market maker's profit is the largest*. Thus, by modifying the price by a fairly modest amount[2], the uninformed market maker is able to obtain a strictly positive profit, which exceeds the well informed insider's profits.

To indicate the effect of the correlation coefficient ρ on the various profits, consider the profits as functions of ρ for a given k. Figure 5 illustrates this relationship for $k = .5$.
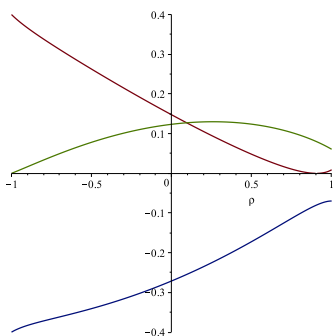

Fig. 5: The three profits as function of ρ, when $k = .5$.

Unlike the situation of a fiduciary market maker when $k = 0$, the noise traders do not achieve zero profits for $\rho = 1$. The fee charged by the market maker is a transfer from noise traders (and insider) to the market maker. The market maker's profit exceeds that of the insider for all values of $\rho > .1$. When the noise traders are mislead, both the other parties make positive profits, the insider the most. This is because the informed trader hides behind

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[2]Since k enters as a multiple of $\tilde{y}$, the effect on the price is not readily apparent. We will return to this issue below.

the noise traders. When the value is high, the noise traders tend to sell, and the informed trader will buy. The order stream is low and market makers earn a small profit. In the limit, when $\rho = -1$, the informed trader reads the behavior of the noise traders perfectly and takes the opposite position, eliminating market makers.

In the situation illustrated $\sigma_v / \sigma_z = 2.5 > 1$, in which case one would believe that the informed parties have an advantage, in particular the insider, since he has very precise information about the true value, which is rather uncertain to the market maker. For example, when $\sigma_v = \sigma_z = 1$, the market maker has the highest profit already from $\rho > -.4$ for $k = .5$ (Figure 5), and in Figure 4 the corresponding crossing point of the two profit curves occurs already at $k = .08$ for $\rho = .5$ (compared to $k = .17$ in Figure 4).

Figure 5 illustrates that when the noise traders have information, this limits the profit of the market maker as ρ increases, for any given value of k. However, Figure 4 indicates that this profit increases without bounds as a function of k, for any given value of ρ. Therefore, in the next sections we explore the simpler situation where $\rho = 0$, i.e., the noise traders are uninformed.

2.2 Non-fiduciary market makers, uninformed noise traders.

When noise trader demand is not correlated with the true value ($\rho = 0$), we have the following corollary to Theorem 1:

Corollary 1. *When $\rho = 0$ there exists an equilibrium in which x and p are linear functions. Defining*

$$(2.4) \qquad \beta = \frac{1}{2\lambda} \quad and \quad \lambda = \frac{\beta \sigma_v^2}{\beta^2 \sigma_v^2 + \sigma_z^2} + k^*$$

where k^* is the value of k determined by the regulatory constraint, the equilibrium is given by

$$x(\tilde{v}) = \beta(\tilde{v} - p_0), \quad p(\tilde{x} + \tilde{z}) = p_0 + \lambda(\tilde{x} + \tilde{z}).$$

The parameter β is a solution to the cubic equation

$$(2.5) \qquad 2k^* \sigma_v^2 \beta^3 + \sigma_v^2 \beta^2 + 2k^* \sigma_z^2 \beta - \sigma_z^2 = 0,$$

which also determines the constant λ. The expected profits are as follows:
The insider: $E(\tilde{\pi}_I) = \sigma_v^2 \lambda \beta^2 = \frac{1}{2}\sigma_v^2 \beta.$
The market maker: $E(\tilde{\pi}_M) = \lambda \sigma_z^2 - \frac{1}{2}\sigma_v^2 \beta = k(\beta(k)^2 \sigma_v^2 + \sigma_z^2).$
The noise traders: $E(\tilde{\pi}_N) = -\lambda \sigma_z^2.$

The last expression for the profit of the market maker is derived below. It says that this profit has a lower bound $k\sigma_z^2$, attained when the insider does not trade ($\beta = 0$).

Denoting the trading intensity $\beta := \beta(k, \sigma_v, \sigma_z)$, it is well known from Kyle (1985) that $\beta(0, \sigma_v, \sigma_z) = \frac{\sigma_z}{\sigma_v}$. It is also easy to see that $\beta(k, \sigma_v, \sigma_z) \leq \frac{\sigma_z}{\sigma_v}$ for $k \geq 0$. When $k \to \infty$ the equation approaches $\beta(\beta^2 - \sigma_z^2/\sigma_v^2) = 0$ which has the roots $\beta_1 = 0$, $\beta_2 = \sigma_z/\sigma_v$ and $\beta_3 = -\sigma_z/\sigma_v$, of which only the first makes sense. Kyle's solution cannot be valid here, since all trade will stop with this large price distortion. If the insider buys, he will on average be charged a very high price and suffers a loss. If he sells he will on average receive a very low price. With such a large "bid-ask spread", the market closes down.

It can be seen that $\beta = 0$ cannot solve (2.5) for finite k. Hence, for $k > 0$, $0 < \beta \leq \frac{\sigma_z}{\sigma_v}$. Why does the insider always trade regardless of k? The reason is that if he does not trade, he will loose since the expected return to trading is always strictly positive. However, his trading intensity decreases with k; implicit differentiation of (2.5) gives that

$$\frac{d\beta(k)}{dk} \leq 0, \quad \text{for all } k \geq 0.$$

A graph of $\beta(k)$ is shown in Figure 7 when $\sigma_v = \sigma_z = 1$.
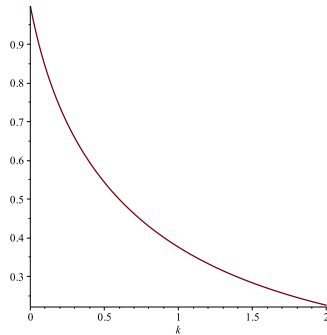

Fig. 7: The intensity $\beta(k)$ as a function of k ($\sigma_v = \sigma_z = 1$).

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The price sensitivity parameter $\lambda(k) = 1/(2\beta(k))$ thus increases with k, and the market depth $1/\lambda(k)$ decreases with k. This latter quantity measures the trade, i.e., order flow, necessary to change the price by one unit of account. Figure 8 shows a graph of the parameter λ as a function of k, of interest for later comparisons.
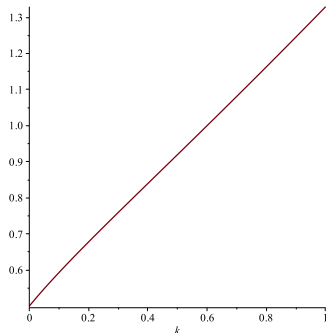

Fig. 8: The parameter $\lambda(k)$ as a function of k ($\sigma_v = \sigma_z = 1$).

To obtain further insight into market maker behavior, notice from (2.4) that the price sensitivity parameter contains two terms:

$$\lambda(k) = \frac{\beta(k)\sigma_v^2}{\beta^2(k)\sigma_v^2 + \sigma_z^2} + k.$$

The first term on the right hand side is the indirect effect on this parameter of a price perturbation. This term is decreasing in k. Ignoring k for the moment, interpreting the term

$$\tilde{\lambda} = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_z^2}$$

as a function of β, the function $\tilde{\lambda} := \tilde{\lambda}(\beta)$ has maximum for $\beta = \beta^*$, where

$$\beta^* = \frac{\sigma_z}{\sigma_v},$$

which interestingly is the optimal solution in the Kyle model. In this model β is a function of the σ's and will not vary independent of these.

The intuition is as follows. Outside equilibrium, if β is very high, informed traders overtrade and market makers must adjust prices downwards. On the other hand, if β is below β^*, market makers do not adjust prices upwards. Rather the un-informativeness of the order flow in this case makes the market makers reduce their responsiveness to orders.

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When $k > 0$ is introduced, β varies with k given the σ's. Indeed β is smaller, and so is the term $\frac{\beta(k)\sigma_v^2}{\beta^2(k)\sigma_v^2+\sigma_z^2}$, which is why this term is always smaller than $\frac{\sigma_v}{2\sigma_z}$ in our model, the value of λ in the Kyle model. In other words, the function $\tilde{\lambda}(\beta)$ is increasing in β for $\beta < \frac{\sigma_z}{\sigma_v}$.

Intuitively, increasing k has a positive direct effect on the price sensitivity. However, there is an indirect, negative effect on price sensitivity as well. As high k makes informed traders trade more softly, the order flow has a smaller effect on the indirect part of the conditional expected asset value.

Turning to the profits, for the insider the expected profit in Corollary 1 is

$$E(\tilde{\pi}_I) = E\{E\{(\tilde{v} - \tilde{p})\tilde{x}|\tilde{v}\}\} = \frac{1}{2}\beta(k)\,\sigma_v^2,$$

which follows from Theorem 1 since $\lambda(k)\beta(k) = 1/2$ when $\rho = 0$. The expected profit of the market makers can be computed as follows:

$$E(\tilde{\pi}_M) = E\{E\{(\tilde{p} - \tilde{v})(\tilde{x} + \tilde{z})|\tilde{y}\}.$$

This can written

$$E(\tilde{\pi}_M) = E\{E\{E(\tilde{v} + k\tilde{y}|\tilde{y})\tilde{y} - \tilde{v}\tilde{y}\}|\tilde{y}\} =$$

$$E\{E\{E(\tilde{v}\tilde{y}|\tilde{y}) + E(k\tilde{y}^2) - \tilde{v}\tilde{y}\}|\tilde{y}\} = k\,\mathrm{var}(\tilde{y}) = k(\beta(k)^2\sigma_v^2 + \sigma_z^2),$$

which proves the latter formula for this profit given in Corollary 1. Using Theorem 1, this quantity can also be written

$$E(\tilde{\pi}_M) = \frac{1}{2}\Big(\frac{\sigma_z^2}{\beta(k)} - \beta(k)\sigma_v^2\Big).$$

From this latter expression it is clear that $E(\tilde{\pi}_M)$ increases with k (since $\beta(k)$ is a decreasing function of k). Also note that equating these two expressions for the market maker's profits, leads directly to the cubic equation of β in (2.5).

Figure 9 shows these two profits as functions of the parameter k, when $\sigma_v = \sigma_z = 1$. The increasing curve is the market maker's profit.
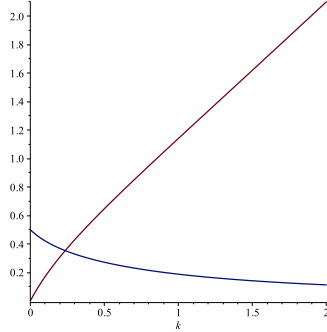
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Fig. 9: The two profits as a function of k ($\sigma_v = \sigma_z = 1$).

For $k \geq .23$ the market maker's profit exceeds that of the insider. The same situation was encountered in Figure 4, but then with $\rho = .5$. See also Figure 5.

One can get a measure of the price distortion at this level of k by calculating the standard deviation of the price with distortion in units of the standard deviation of the price of a fiduciary market maker. We then consider

$$SD(\tilde{p})(k) := \sqrt{\mathrm{var}(\tilde{p})(k)} = \lambda(k)\sqrt{\beta(k)^2\sigma_v^2 + \sigma_z^2}.$$

Note, when there is no inside information, $\tilde{p} = p_0$ and $SD(\tilde{p}) = 0$. Thus inside information increases the variance of $\tilde{p}$.

The relative price distortion is measured by

$$d(k) := SD(\tilde{p})(k)/SD(\tilde{p})(0).$$

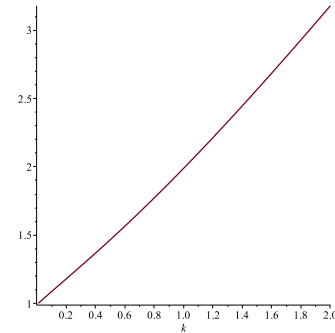A graph of $d(k)$ is shown in Figure 10.

Fig. 10: The relative price distortion $d(k)$ as a function of k.

At the point at which profits are equal $d(.23) = 1.22$, meaning that the relative price distortion has resulted in a standard deviation which is 22 per

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cent larger than with fair pricing. At this level price distortions by market makers can have a significant effect on price volatility.

If, for example, the regulatory authority sets a B corresponding to maximally 30 per cent increase in the volatility, this amounts to setting $k^* = .32$. In this case the market maker would have a profit about 1.4 times that of the well informed insider.

As mentioned in Section 2 we could alternatively consider the conditional expected price deviation $\lambda(k)E(\tilde{y}|\tilde{y} > 0)$, as measured in units of $\lambda(0)E(\tilde{y}|\tilde{y} > 0)$ when $k = 0$. Using truncation of the normal variable it follows that

$$E(\tilde{y}|\tilde{y} > 0) = \frac{2}{\sqrt{2\pi}}\sqrt{\beta(k)^2\sigma_v^2 + \sigma_z^2}$$

Thus the conditional expected price deviation so defined can be written

$$(2.6) \qquad pd(k) := \frac{\lambda(k)\,E(\tilde{y}|\tilde{y} > 0)}{\lambda(0)\,E(\tilde{y}|\tilde{y} > 0)} = \frac{\lambda(k)\sqrt{\beta(k)^2\sigma_v^2 + \sigma_z^2}}{\lambda(0)\sqrt{2}\sigma_z}.$$

Interestingly, this measure is seen to be the same in this model as the one we have applied above, i.e., $d(k) = pd(k)$ for all $k \geq 0$, which lends support to our former choice.

Finally we consider the informativeness in prices when price is distorted. A simple measure of *informativeness* of prices is defined by

$$\iota := 1 - \frac{\mathrm{var}(\tilde{v}|\tilde{p})}{\mathrm{var}(\tilde{v})}.$$

When the price carries no private information about the true value of the asset, the conditional variance equals the unconditional variance, and $\iota = 0$. When the price equals the value of the asset, the conditional variance equals zero and $\iota = 1$, in which case all private information is reflected in the price. Thus $0 \leq \iota \leq 1$.

In order to compute this measure, recall that

$$\mathrm{var}(\tilde{p}) = \lambda(k)^2(\beta(k)^2\sigma_v^2 + \sigma_z^2),$$

$$\mathrm{cov}(\tilde{v}, \tilde{p}) = \lambda(k)\beta(k)\sigma_v^2 = \frac{1}{2}\sigma_v^2,$$

$$\rho_{\tilde{v},\tilde{p}} = \frac{\frac{1}{2}\sigma_v^2}{\sigma_v\sqrt{\mathrm{var}(\tilde{p})}} = \frac{\beta(k)\sigma_v}{\sqrt{\beta(k)^2\sigma_v^2 + \sigma_z^2}},$$

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so that
$$\text{var}(\tilde{v}|\tilde{p}) = \text{var}(\tilde{v})(1 - \rho_{\tilde{v},\tilde{p}}^2) = \sigma_v^2 \left(1 - \frac{\beta(k)^2 \sigma_v^2}{\beta(k)^2 \sigma_v^2 + \sigma_z^2}\right).$$

From this it follows that
$$\iota(k) = \frac{\beta(k)^2 \sigma_v^2}{\beta(k)^2 \sigma_v^2 + \sigma_z^2},$$

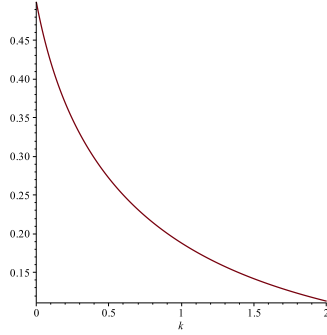or, $\iota(k) = \tilde{\lambda}\beta(k)$. In Figure 11 we present a graph of the informativeness as a function of k.

Fig. 11: Informativeness $\iota(k)$ as a function of k.

It shows that informativeness in prices decreases convexly as a function of the distortion level k, as one would, perhaps, think. Distorting prices is not informative to the other market participants.

As a summary of the model in Corollary 1, and for later comparisons, in Table 1 we give the connection between k^* , $d(k^*)$, $\iota(k^*)$, as well at the profits $E(\tilde{\pi}_M)$ and $E(\tilde{\pi}_I)$ for reasonable values of the price distortion d. As can be seen, at a price distortion of 10 per cent, the market maker's profit is 41.8 per cent of the perfectly informed insider. The informativeness (market efficiency) at the corresponding levels of k^* should be compared to $\iota(0) = 0.50$.

The insider has the highest profits for the values of $d < 1.25$, and this profit is decreasing with k^*. For $d = 1.25$ and above, the market maker's profit is the largest of the two.

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Model: Corollary 1					
$d(k^*)$	1.01	1.02	1.05	1.10	1.25
k^*	.010	.020	.051	.103	.263
$E(\tilde{\pi}_M)$	.020	.039	.093	.176	.387
$E(\tilde{\pi}_I)$	.490	.481	.455	.420	.343
$\iota(k^*)$	.49	.48	.45	.41	.32

Table 1: The connection between k^*, $d(k^*)$ and other quantities.

3 The market maker has privileged information as well

In the model of this last section the situation is as in Section 2.2 concerning the insider and the noise traders ($\rho = 0$), but now also the market maker has privileged information. We want to explore what effect this has on the equilibrium, when the market maker is not a fiduciary.

We think that this situation is not uncommon in the real world; that market makers of a certain size and importance may be 'more than well informed'. The Chinese walls may not be as soundproof as advertised!

Initially we assume that the market maker has private information. Later this situation will be compared with the simpler case when the insider knows the information the market maker actually possesses. The distribution of profits will be of interest, as well as market efficiency and price distortion.

Our assumption is that the market maker, but not the insider, receives an independent signal $\tilde{m}$ of the form

$$\tilde{m} = \tilde{v} + \tilde{\epsilon}$$

where $\tilde{\epsilon} \sim \mathcal{N}(0, \sigma_\epsilon^2)$. Also $E(\tilde{\epsilon}\,\tilde{v}) = E(\tilde{\epsilon}\,\tilde{y}) = 0$. As in the rest of the paper, joint normality is assumed.

The pricing mechanism is then

(3.1) $$\tilde{p} = p(\tilde{y}, \tilde{m}) = E\{\tilde{v} + \tilde{u}|\tilde{m}, \tilde{y}\},$$

where, as in Section 2, $\tilde{u} = k\tilde{y}$ for a non-negative parameter k.

Since we are in a 'normal universe', we know that the conditional expected value is linear in $\tilde{m}$ and $\tilde{y}$, i.e., there exists two constants λ and μ such that

(3.2) $$\tilde{p} = p(\tilde{y}, \tilde{m}) = p_0 + \lambda\tilde{y} + \mu(\tilde{m} - p_0).$$

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Equilibrium is defined as before. We then have the following result:

Theorem 3.1. *When the market maker has privileged information, there exists an equilibrium in which x and p are linear functions. Defining*

$$(3.3) \quad \beta = \frac{1}{2\lambda}(1-\mu), \quad \lambda = \frac{k^*\sigma_z^2\sigma_v^2 + \sigma_\epsilon^2(\beta\sigma_v^2 + k^*\beta^2\sigma_v^2 + k^*\sigma_z^2)}{\beta^2\sigma_v^2\sigma_\epsilon^2 + \sigma_z^2(\sigma_v^2 + \sigma_\epsilon^2)} \quad and$$

$$\mu = \frac{\sigma_v^2\sigma_z^2}{\beta^2\sigma_v^2\sigma_\epsilon^2 + \sigma_z^2(\sigma_v^2 + \sigma_\epsilon^2)}.$$

where k^* is the value of k determined by the regulatory constraint, the equilibrium is given by

$$x(\tilde{v}) = \beta(\tilde{v} - p_0), \quad p(\tilde{y}, \tilde{m}) = p_0 + \lambda\tilde{y} + \mu(\tilde{m} - p_0).$$

The parameter β is a solution to the cubic equation

$$(3.4) \quad 2k^*\sigma_v^2\sigma_\epsilon^2\beta^3 + (\sigma_v^2\sigma_\epsilon^2)\beta^2 + 2k^*\sigma_z^2(\sigma_\epsilon^2 + \sigma_v^2)\beta - \sigma_z^2\sigma_\epsilon^2 = 0,$$

which also determines the constants λ and μ. The expected profits are as follows:
The insider: $E(\tilde{\pi}_I) = \sigma_v^2\lambda\beta^2$.
The market maker: $E(\tilde{\pi}_M) = \lambda(\sigma_z^2 - \sigma_v^2\beta^2) = k(\beta^2\sigma_v^2 + \sigma_z^2)$.
The noise traders: $E(\tilde{\pi}_N) = -\lambda\sigma_z^2$.

    <u>Proof:</u> By the joint normality assumption between $\tilde{v} + \tilde{u}$ and $(\tilde{y}, \tilde{m})$, the price is a linear function of $\tilde{y}$ and $\tilde{m}$, or

$$p(y, m) = p_0 + \lambda y + \mu(m - p_0), \quad (y = x + z),$$

for some constants λ and μ, $(E(\tilde{y}) = 0)$. The insider, realizing how the market maker determines prices and that he has inside information, computes his conditional expected profits, given his private information, as follows

$$\pi_I(v) = E\{[\tilde{v} - p(\tilde{y}, \tilde{m})]x|\tilde{v} = v\} = E\{[v - p_0 - \lambda\tilde{y} - \mu(\tilde{m} - p_0)]x|\tilde{v} = v\} =$$

$$E\{[v - p_0 - \lambda(x + \tilde{z}) - \mu(v + \tilde{\epsilon} - p_0)]x|\tilde{v} = v\} =$$

$$(v - p_0)x - \lambda x^2 - \lambda x E\{\tilde{z}|\tilde{v} = v\} - \mu(v - p_0)x - \mu x E\{\tilde{\epsilon}|\tilde{v} = v\} =$$

$$(v - p_0)(1 - \mu)x - \lambda x^2.$$

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by the projection theorem. Profit maximization of this quadratic objective requires that x solves

$$(v - p_0)(1 - \mu) - 2\lambda x = 0$$

or

$$x = \frac{1}{2\lambda}(v - p_0)(1 - \mu) := \beta(v - p_0),$$

so that the trading intensity β is given by

$$\beta = \frac{1}{2\lambda}(1 - \mu).$$

In other words, $x(v)$ is linear in v, and given as claimed in the theorem.

We must determine the price sensitivity parameters λ and μ, and are then led to study the properties of the random vector $(\tilde{v} + k\tilde{y}, \tilde{y}, \tilde{m})$ which is jointly normal with mean vector $(p_0, 0, p_0)$ and covariance matrix M given by

$$\mathbf{M} = \begin{pmatrix} (1 + k\beta)^2\sigma_v^2 + k^2\sigma_z^2, & \beta\sigma_v^2(1 + k\beta) + k\sigma_z^2, & \sigma_v^2(1 + k\beta) \\ \beta\sigma_v^2(1 + k\beta) + k\sigma_z^2, & \beta^2\sigma_v^2 + \sigma_z^2, & \beta\sigma_v^2 \\ \sigma_v^2(1 + k\beta), & \beta\sigma_v^2, & \sigma_v^2 + \sigma_\epsilon^2 \end{pmatrix}$$

This is shown as follows: First $\text{var}(\tilde{y}) = \beta^2\sigma_v^2 + \sigma_z^2$ since $\tilde{v}$ and $\tilde{z}$ are uncorrelated (independent). It follows that

$$\text{var}(\tilde{v} + k\tilde{y}) = \text{var}\big(\tilde{v} + k(\beta(\tilde{v} - p_0) + \tilde{z})\big) =$$

$$\sigma_v^2 + k^2(\beta^2\sigma_v^2 + \sigma_z^2) + 2k\beta\sigma_v^2 = (1 + k\beta)^2\sigma_v^2 + k^2\sigma_z^2,$$

which accounts for the first two terms on the diagonal of the matrix $\mathbf{M}$. The last term on the diagonal is $\text{var}(\tilde{m}) = \text{var}(\tilde{v} + \tilde{\epsilon}) = \sigma_v^2 + \sigma_\epsilon^2$. Next

$$\text{cov}(\tilde{v} + k\tilde{y}, \tilde{y}) = \text{cov}(\tilde{v}, \beta(\tilde{v} - p_0) + \tilde{z}) + k\,\text{var}(\tilde{y}) =$$

$$\beta\sigma_v^2 + k(\beta^2\sigma_v^2 + \sigma_z^2) = \beta\sigma_v^2(1 + k\beta) + k\sigma_z^2,$$

which accounts for the first covariance in $\mathbf{M}$. The next covariance is

$$\text{cov}(\tilde{v} + k\tilde{y}, \tilde{m}) = \text{cov}(\tilde{v}, \tilde{m}) + k\,\text{cov}(\tilde{y}, \tilde{m}) =$$

$$\text{cov}(\tilde{v}, \tilde{v} + \tilde{\epsilon}) + k\,\text{cov}(\beta(\tilde{v} - p_0) + \tilde{z}, \tilde{v} + \tilde{\epsilon}) = \sigma_v^2(1 + k\beta).$$

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The last covariance is $\text{cov}(\tilde{y}, \tilde{m}) = \beta\sigma_v^2$, which completes the determination of the matrix $\mathbf{M}$.

We now partition the matrix $\mathbf{M}$ into four parts as follows: Let $M_{11} := (1 + k\beta)^2\sigma_v^2 + k^2\sigma_z^2$ (a scalar), define the vector

$$M_{12} := (\beta\sigma_v^2(1 + k\beta) + k\sigma_z^2, \sigma_v^2(1 + k\beta))$$

and finally the matrix

$$\mathbf{M}_{22} := \begin{pmatrix} \beta^2\sigma_v^2 + \sigma_z^2, & \beta\sigma_v^2 \\ \beta\sigma_v^2, & \sigma_v^2 + \sigma_\epsilon^2 \end{pmatrix}.$$

It follows from results in multivariable normal analysis[3] that

$$E\{\tilde{v} + k\tilde{y}|\tilde{y} = y, \tilde{m} = m\} = p_0 + M_{12}\mathbf{M}_{22}^{-1}\begin{pmatrix} y \\ m - p_0 \end{pmatrix}.$$

Also

$$\text{var}(\tilde{v} + k\tilde{y}|\tilde{y} = y, \tilde{m} = m) = M_{11} - M_{12}\mathbf{M}_{22}^{-1}M_{21},$$

where M_{21} is the transpose of M_{12} (T. W. Anderson (1958, Theorem 2.5.1, p29). It remains to compute these quantities: The inverse of the matrix $\mathbf{M}_{22}$ is given by

$$\mathbf{M}_{22}^{-1} = \frac{1}{\det(\mathbf{M}_{22})}\begin{pmatrix} \sigma_v^2 + \sigma_\epsilon^2, & -\beta\sigma_v^2 \\ -\beta\sigma_v^2, & \beta^2\sigma_v^2 + \sigma_z^2 \end{pmatrix},$$

where the determinant of $\mathbf{M}_{22}$ is

$$\det(\mathbf{M}_{22}) = \beta^2\sigma_v^2\sigma_\epsilon^2 + \sigma_z^2(\sigma_v^2 + \sigma_\epsilon^2).$$

The conditional expectation in question is computed in two steps: (i) First we find the vector

$$M_{12}\mathbf{M}_{22}^{-1} = \left(\frac{(\beta\sigma_v^2(1 + k\beta) + k\sigma_z^2)(\sigma_v^2 + \sigma_\epsilon^2) - \sigma_v^4(1 + k\beta)}{\beta^2\sigma_v^2\sigma_\epsilon^2 + \sigma_z^2(\sigma_v^2 + \sigma_\epsilon^2)}, \right.$$

$$\left. \frac{-(\beta\sigma_v^2(1 + k\beta) + k\sigma_z^2)\beta\sigma_v^2 + \sigma_v^2(1 + k\beta)(\beta^2\sigma_v^2 + \sigma_z^2)}{\beta^2\sigma_v^2\sigma_\epsilon^2 + \sigma_z^2(\sigma_v^2 + \sigma_\epsilon^2)} \right).$$

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[3]In several period models one has to use filtering theory, which at this point is not sufficiently developed to cope with the generality of this situation.

(ii) Finally we compute the scalar product

$$M_{12}\mathbf{M}_{22}^{-1}\begin{pmatrix} y \\ m - p_0 \end{pmatrix} = \frac{k\sigma_z^2\sigma_v^2 + \sigma_\epsilon^2(\beta\sigma_v^2(1 + k\beta) + k\sigma_z^2)}{\beta^2\sigma_v^2\sigma_\epsilon^2 + \sigma_z^2(\sigma_v^2 + \sigma_\epsilon^2)} y +$$

$$\frac{\sigma_v^2\sigma_z^2}{\beta^2\sigma_v^2\sigma_\epsilon^2 + \sigma_z^2(\sigma_v^2 + \sigma_\epsilon^2)}(m - p_0).$$

This shows that

$$p(y, m) = p_0 + \lambda y + \mu (m - p_0)$$

where

$$\lambda = \frac{k\sigma_z^2\sigma_v^2 + \sigma_\epsilon^2(\beta\sigma_v^2(1 + k\beta) + k\sigma_z^2)}{\beta^2\sigma_v^2\sigma_\epsilon^2 + \sigma_z^2(\sigma_v^2 + \sigma_\epsilon^2)}$$

and

$$\mu = \frac{\sigma_v^2\sigma_z^2}{\beta^2\sigma_v^2\sigma_\epsilon^2 + \sigma_z^2(\sigma_v^2 + \sigma_\epsilon^2)}$$

as claimed.

From the above connection between β, λ and μ, $2\beta\lambda = (1 - \mu)$, we can now derive the cubic equation for β, and the result is as given in Theorem 3.1. The monotonicity property in k of the various parameters can be verified, much as in Section 2.2, i.e., $\beta(k)$ is non-increasing in k, $\lambda(k)$ and $\mu(k)$ are both non-decreasing in k.

It remains to compute the two profit functions: The insider's expected profits are

$$E(\tilde{\pi}_I) = E\{(\tilde{v} - \tilde{p})\tilde{x}\} = E\{\beta(\tilde{v} - p_0)^2 - \lambda\tilde{y}\beta(\tilde{v} - p_0) - \mu(\tilde{m} - p_0)\beta(\tilde{v} - p_0)\} =$$

$$\beta\sigma_v^2 - \lambda\beta^2\sigma_v^2 - \mu\beta\sigma_v^2 = \sigma_v^2\beta(1 - \lambda\beta - \mu) = \lambda\beta^2\sigma_v^2,$$

since $2\lambda\beta = (1 - \mu)$.

The expected profits of the market maker is

$$E(\tilde{\pi}_M) = E((\tilde{p} - \tilde{v})\tilde{y}) = E\{(p_0 + \lambda\tilde{y} + \mu(\tilde{m} - p_0))\tilde{y}\} - E\{\tilde{v}\tilde{y}\} =$$

$$p_0 E(\tilde{y}) + \lambda E(\tilde{y}^2) + \mu E\{(\tilde{m} - p_0)\tilde{y}\} - E\{\tilde{v}(\beta(\tilde{v} - p_0) + \tilde{z})\} =$$

$$\lambda(\beta^2\sigma_v^2 + \sigma_z^2) + \beta\sigma_v^2(\mu - 1) = \lambda(\sigma_z^2 - \beta^2\sigma_v^2)$$

where we have used that $(\mu - 1) = -2\beta\lambda$. As in Section 2.2 this can also be computed as

$$E(\tilde{\pi}_M) = E\{E\{E(\tilde{v} + k\tilde{y}|\tilde{y})\tilde{y} - \tilde{v}\tilde{y}\}|\tilde{y}\} =$$

$$E\{E\{E(\tilde{v}\tilde{y}|\tilde{y}) + E(k\tilde{y}^2) - \tilde{v}\tilde{y}\}|\tilde{y}\} = k\operatorname{var}(\tilde{y}) = k(\beta(k)^2\sigma_v^2 + \sigma_z^2).$$

By equating these two expressions for the profit, the cubic equation for β again results. This completes the proof of the theorem. $\square$

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3.1 Some comments to the theorem

It can be seen that when $\sigma_\epsilon \to \infty$ the above results converge to the corresponding ones of Section 2, in particular $\mu \to 0$. In this case the market maker has no privileged information.

When $\sigma_\epsilon \to 0$, on the other hand, it follows that $\beta \to 0$ so in the limit the insider does not trade. He understands how the market maker sets prices equal to the expected true value plus fee, that the market maker has inside information, and how much. In this case $\lambda = k^*$, $\mu = 1$ and $p(\tilde{y}, \tilde{m}) = k^*\tilde{y} + \tilde{v}$. The market maker's expected profit is then $E(\pi_M) = k^*\sigma_z^2$ which is also the expected loss of the noise traders, while the insider has zero profits. The more noise trading, the higher the profit of the well informed market maker. These are polar cases.

When $k^* = 0$, then $\beta = \frac{\sigma_z}{\sigma_v}$, the Kyle solution. We then have the following corollary:

Corollary 2. *When $k = 0$ and the market maker has private information, there exists an equilibrium in which x and p are linear functions. Defining*

$$(3.5) \qquad \beta = \frac{1}{2\lambda}(1 - \mu), \quad \lambda = \frac{\sigma_v \sigma_\epsilon^2}{2\sigma_z \sigma_\epsilon^2 + \sigma_z \sigma_v^2} \quad and$$

$$\mu = \frac{\sigma_v^2}{2\sigma_\epsilon^2 + \sigma_v^2}.$$

the equilibrium is given by

$$x(\tilde{v}) = \beta(\tilde{v} - p_0), \quad p(\tilde{y}, \tilde{m}) = p_0 + \lambda\tilde{y} + \mu(\tilde{m} - p_0).$$

The parameter β is a solution to a quadratic equation, and given by

$$(3.6) \qquad \beta = \frac{\sigma_z}{\sigma_v}.$$

The expected profits are as follows:
The insider: $E(\tilde{\pi}_I) = \sigma_v^2 \lambda \beta^2$.
The market maker: $E(\tilde{\pi}_M) = 0$.
The noise traders: $E(\tilde{\pi}_N) = -\lambda\sigma_z^2$.

Note that λ is proportional to $(1 - \mu)$ since β is constant. The better informed the market maker is (larger μ), the smaller price adjustments will be made for given orders. Noise trader losses decrease in μ.

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We notice that $\lambda \to 0$ and $\mu \to 1$ as $\sigma_\epsilon \to 0$, in which case the price becomes the true value of the asset in the limit. The insider still trades, but obtains zero profits, and the noise traders do not loose. (When $k = 0$ the market maker makes no profit.) In the next section we explore an intermediate situation where $\mu \in (0, 1)$ and $k > 0$, that can otherwise be compared to the analysis in Section 2.2.

Notice that the market maker does not act strategically with respect to his private information. Instead he lets the market alone benefit from this signal, improving efficiency. As it turns out, this benevolent act may still be to his advantage, as we shall see below.

3.2 Properties of the equilibrium with a privately informed market maker

In order to compare with the equilibrium of Section 2, we consider the following set of parameters: $\sigma_v = \sigma_z = 1$, $\sigma_\epsilon = .1$. Here the market maker is fairly well informed, since σ_ϵ is relatively small compared to σ_v.

A graph of $\beta(k)$ as a function of the parameter k is shown in Figure 12 for the parameter values indicated above.
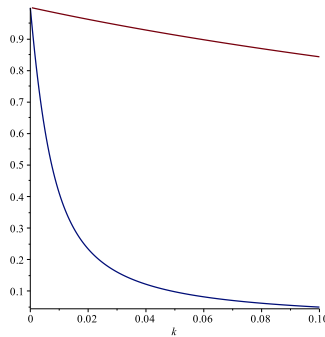

Fig. 12: The trading intensity $\beta(k)$ as a function of k.

In the same figure we also show the corresponding trading intensity for the model of Corollary 1 (Figure 7). Compared to this, the values of β are now smaller with the exception of $k = 0$, as we have just seen. As we shall see below, the insider trades more softly when he does not know the signal received by the market maker. Increasing the volatility σ_ϵ, moderately increases the values of $\beta(k)$. Also note that β is always positive.

Next, we turn to the price sensitive parameter $\lambda(k)$ as a function of k.

In Figure 13 we provide a graph using the same parameter values as above, where we also show the corresponding function in Figure 8 in Section 2.2 as the upper curve.
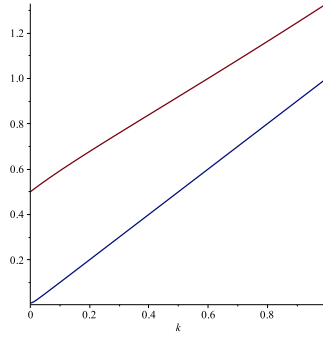

Fig. 13: The price sensitivity parameter $\lambda(k)$ as a function of k.

Comparing to Figure 8, the values of this parameter are smaller, which is natural since the market maker has an additional component from which to determine the price. In the relationship $\lambda(k) = \frac{1}{2\beta(k)}(1 - \mu(k))$, the factor $\frac{1}{2\beta(k)}$ is larger than the one in Section 2.2, but it is multiplied by the term $(1 - \mu(k))$ which is smaller than 1, here more than compensating for the increase in the first factor. Increasing the volatility σ_ϵ increases the values of $\lambda(k)$. As we know from Corollary 2, λ is not 0 when $k = 0$ (but close to 0 here as σ_ϵ is small).

The new quantity of interest, μ, influences the price due to the market maker's inside information. A graph of this parameter is shown in Figure 14, where the volatilities are the same as above.
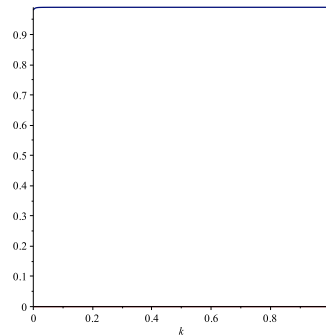

Fig. 14: The informed price sensitivity parameter $\mu(k)$ as a function of k.

As can be seen from this figure, most of the value in $\mu(k)$ is attributed

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to $\mu(0) = .98$. The subsequent increase of .01 takes place for small values of k. For k larger than about .02, this variable does not increase much; the parameter μ has to do with the market maker's private information, and only depends indirectly on k via $\beta(k)$. This information the market maker possesses independent of the values of k. Increasing the volatility σ_ϵ decreases this function in the leftmost end.

Our next issue is the profit of the market maker, shown in Figure 15 for the same values of the parameters as above.
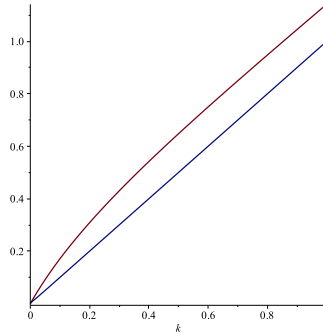

Fig. 15: The market maker's profits as as a function of k.

Comparing to Figure 9 in Section 2.2, also presented in the above figure as the upper curve, this profit is seen to be lower than the profit when the market maker has no inside information. When $k > 0$, the insider trades less for any given k, and the market maker takes in less in fees. It is, perhaps, surprising that the market maker do not use the private information to his/her advantage when it comes to profits. The reason is that he does not act strategically by assumption. Rather he lets the information be reflected in the price.

The market maker's information influences profits for a given k. However, this is only part of the story. The relative price distortion is a function of the information the market maker possesses as well as k. The market maker may choose a higher k when he is more informed, increasing profits. After comparing the profits of insiders and market makers, we shall address these issues in more detail.
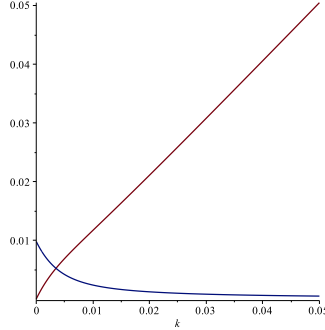
Fig. 16: The two profits π_M and π_I as functions of k.

Figure 16 shows the profits of the insider and the market maker of the present model, for the same parameter values as above, where the decreasing curve is the profits of the insider. As can be seen, for a small value of k onwards, here about 0.0035, the market maker's profit exceeds that of the insider. The reason is that the insider's profit has decreased much more than the market maker's for any given value of k. As we shall see below, this point corresponds to a relatively modest price change in terms of relative volatility. The corresponding point in Figure 9 is .25 , or about 71 times larger on the k-axis. The crossing point on the k-axis increases when the volatility σ_ϵ increases.

More fundamental to our analysis is the following, leading to Table 2 below. The percentwise price distortion measured by the function $d(k)$, defined in Section 2, is illustrated next. For this we need the standard deviation $SD(\tilde{p})$, which is the square root of the following expression

$$\text{var}(\tilde{p}) = \text{var}\big(p_0 + \lambda\tilde{y} + \mu(\tilde{m} - p_0)\big) =$$

$$\lambda^2\text{var}(\tilde{y}) + \mu^2\text{var}(\tilde{m}) + 2\lambda\mu\text{cov}(\tilde{y}, \tilde{m} - p_0) =$$
$$\lambda^2(\beta^2\sigma_v^2 + \sigma_z^2) + \mu^2(\sigma_v^2 + \sigma_\epsilon^2) + 2\lambda\mu\beta\sigma_v^2.$$

From this we can compute the change in volatility *relative* to fair pricing

$$d(k) = \frac{SD(\tilde{p}(k))}{SD(\tilde{p}(0))},$$

which is illustrated as the lower for the two curves in Figure 17a, while the upper curve is the same as in Figure 10. The parameters are still the same.
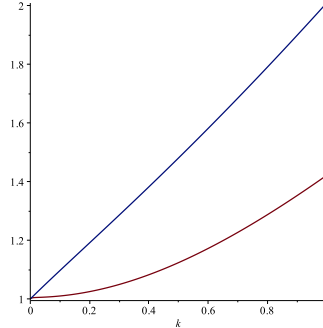
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Fig. 17a: The relative price distortion $d(k)$ as a function of k.

Suppose the regulator maximally accepts a price distortion of 1 per cent ($d = 1.01$). The corresponding value of k is then $k^* = .110$ in the present model. In the model with uninformed market maker the corresponding value of k is .010 (see Table 1). For a given d, the informed market maker makes a much higher profit than the uninformed. The reason is simply that the informed market maker may choose a higher level of k without attracting the attention of the regulator. The fact that profits are lower for a given k, is of less importance.
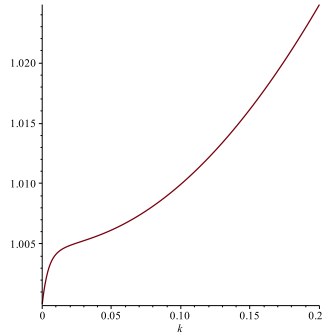

Fig. 17b: The relative price distortion $d(k)$ for the present model.

From the expression for the relative price distortion we notice that it is a rather complicated function of k, since it depends on $\beta(k), \mu(k)$, and $\lambda(k)$ and powers and products of these functions, which are themselves rather complicated functions of k.

$$d(k) = \sqrt{\frac{\lambda^2(k)(\beta^2(k)\sigma_v^2 + \sigma_z^2) + \mu^2(k)(\sigma_v^2 + \sigma_\epsilon^2) + 2\lambda(k)\mu(k)\beta(k)\sigma_v^2}{\lambda^2(0)(\beta^2(0)\sigma_v^2 + \sigma_z^2) + \mu^2(0)(\sigma_v^2 + \sigma_\epsilon^2) + 2\lambda(0)\mu(0)\beta(0)\sigma_v^2}}.$$

In Figure 17b we illustrate the peculiar shape of d for small values of k, not apparent in Fig. 17a. This will be further illustrated in the next section.

At the point where the profits cross in Figure 16, the relative price distortion is about .25 percent (a quarter of one percent). The corresponding distortion in Section 2.2 was about 22 percent. If as above, the regulator sets B corresponding to maximally 1 per cent relative price distortion, this amounts to setting $k^* = .110$, at which point the market maker has a profit about 400 times that of the perfectly informed insider. This ratio is much higher than in the model of Section 2.2.

When the volatility of $\tilde{\epsilon}$ increases tenfold to $\sigma_\epsilon = 1.0$, at the level of k where the profit curves cross, the relative price distortion increases to around 10 per cent, still far lower than with uninformed market maker.

It is also of interest to study the standard deviation of the price $\tilde{p}$. In Figure 18 we demonstrate this for the two models under consideration, the highest curve corresponds to the standard deviation $\mathrm{SD}(\tilde{p})(k)$ of the present model.
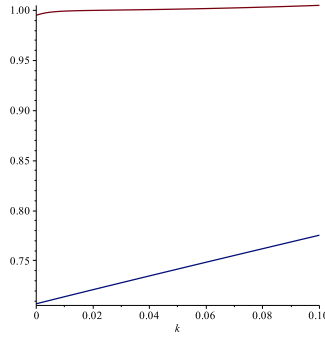

Fig. 18: The standard deviation of the price $\tilde{p}(k)$.

As we see, the price in the present model is more volatile than the price in the model where the market maker is uninformed. Comparing to the model of Corollary 1, this demonstrates how the volatility of prices increases with the information reflected in the price.

Finally we explore the informativeness in prices. For this we need to compute the variance of $\tilde{p}$ and the conditional variance of $\tilde{y}$ given the price $\tilde{p}$. The variance $\mathrm{var}(\tilde{p})$ was derived just above. The covariance between $\tilde{v}$ and $\tilde{p}$ is

$$\mathrm{cov}(\tilde{v}, \tilde{p}) = \mathrm{cov}\big(\tilde{v}, p_0 + \lambda\tilde{y} + \mu(\tilde{m} - p_0)\big) =$$
$$\mathrm{cov}\big(\tilde{v}, \lambda(\beta(\tilde{v} - p_0) + \tilde{z}) + \mathrm{cov}\big(\tilde{v}, \mu(\tilde{v} + \tilde{\epsilon} - p_0)\big) =$$
$$\lambda\beta\sigma_v^2 + \mu\sigma_v^2 = \sigma_v^2(\lambda\beta + \mu).$$

The correlation coefficient between $\tilde{v}$ and $\tilde{p}$ is

$$\rho_{\tilde{v},\tilde{p}} = \frac{\text{cov}(\tilde{v},\tilde{p})}{\sigma_v\sqrt{\text{var}(\tilde{p})}} = \frac{\left(\sigma_v^2(\lambda\beta+\mu)\right)}{\sigma_v\sqrt{\left(\lambda^2(\beta^2\sigma_v^2+\sigma_z^2)+\mu^2(\sigma_v^2+\sigma_\epsilon^2)+2\lambda\mu\beta\sigma_v^2\right)}}$$

and the quantity $\text{var}(\tilde{v}|\tilde{p}) = \sigma_v^2(1-\rho_{\tilde{v},\tilde{p}}^2)$ becomes

$$\text{var}(\tilde{v}|\tilde{p}) = \sigma_v^2\left(1 - \frac{\left(\sigma_v^2(\lambda\beta+\mu)\right)^2}{\sigma_v^2\left(\lambda^2(\beta^2\sigma_v^2+\sigma_z^2)+\mu^2(\sigma_v^2+\sigma_\epsilon^2)+2\lambda\mu\beta\sigma_v^2\right)}\right).$$

From the formula for the informativeness ι in Section 2.2 we get

$$\iota = \frac{\sigma_v^2(\lambda\beta+\mu)^2}{\lambda^2(\beta^2\sigma_v^2+\sigma_z^2)+\mu^2(\sigma_v^2+\sigma_\epsilon^2)+2\lambda\mu\beta\sigma_v^2}.$$

When $\mu = 0$ this expression for ι is seen to reduce to the corresponding one in Section 2.2.

A graph of $\iota(k)$ as a function of k is presented in Figure 19, where we also present informativeness in Corollary 1. The volatilities are as above.
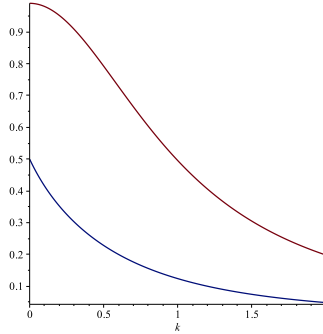
Fig. 19: Informativeness $\iota(k)$ as a function of k.

Comparing to Figure 11, the informativeness has increased, and has another shape as it starts out concave, and then becomes more convex at higher values of k. With fair pricing ($k = 0$), prices with informed market maker are almost perfectly informative, and close to twice as informative as without this information. This difference will naturally diminish as the volatility σ_ϵ increases. The improved market efficiency reflects the fact that the market maker acts as a fiduciary with respect to his own private information. The insider, on the other hand, trades more softly so as to withhold information from the market.

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We conclude that the model with an informed market maker, who treats his privileged information as laid out in Theorem 3.1, has several distinguishing features from the situation described in Section 2.2: (i) even if the profit function $\pi_M(k)$ has decreased, the market maker obtains a significantly higher profit for any given value of the maximal price disturbance allowed by the regulator; (ii) the perfectly informed insider obtains a significantly lower profit; (iii) the market becomes more efficient, in the sense of the price being more informative; (iv) the level of price distortion parameter k at which the market maker outperforms the insider is considerably lower.

The market maker can accordingly utilize his private information in an efficient manner, and still obtain a higher profit than without this information. This is illustrated below.

As a summary of the model in Theorem 3.1, in Table 2 we give the relationship between k^*, $d(k^*)$, $\iota(k^*)$, as well at the profits $E(\tilde{\pi}_M)$ and $E(\tilde{\pi}_I)$ for the same values of the price distortion d as in Table 1. As can be seen, the profits of the market maker have increased, while the profits of the insider have decreased compared to Table 1.

A trading fee related to the order flow reduces market efficiency since the order flow is weakly correlated with the value of the asset. If the regulator demands a certain level of efficiency, she is more inclined to accept a fee when markets are very efficient to begin with i.e. when market makers are well informed. Thus whether regulators focus on price distortion or efficiency, the argument is the same. Informed market makers may get away with charging higher fees to traders.

At a price distortion of 1 per cent, the market maker's profit has increased 5.5 times from the model in Corollary 1, while the insider's profit has decreased from .490 to .0002, which is dramatic. The informativeness (market efficiency) has increased at the corresponding levels of k^*.

Model: Theorem 3.1				
$d(k^*)$	1.01	1.02	1.05	1.10
k^*	.110	.179	.306	.448
$E(\tilde{\pi}_M)$	.1102	.1791	.3060	.4480
$E(\tilde{\pi}_I)$	.0002	.0001	.0001	.0001
$\iota(k^*)$	.98	.96	.90	.82

Table 2: The relationship between k^*, $d(k^*)$ and other quantities.

Compared to Table 1, higher values of k^* now correspond to the various price distortions. This difference is so substantial, that even if the profit function of the market maker is now lower as a function of k, the end result is significantly higher profits for him, and corresponding lower profits for the insider.

3.3 The insider knows the market maker's signal as well

There are two reasons why the insider obtains a smaller profit in the model of the last section (for a given k). One is that the market maker has more information than before, the other is that this information is private. In order to separate these two effects and see which one is the most important, we imagine that the insider also observes the signal $\tilde{m}$. This situation can be seen to reduce to the model analyzed in Section 2, with the difference that $\text{var}(\tilde{v})$ is replaced by $\text{var}(\tilde{v}|\tilde{m})$. Since $\tilde{m} = \tilde{v} + \tilde{\epsilon}$ where $E(\tilde{v}\tilde{\epsilon}) = 0$, it follows by joint normality that

$$\text{var}(\tilde{v}|\tilde{m}) = \sigma_v^2(1 - \rho_{m,v}^2), \quad \text{and} \quad \sigma_m^2 = \sigma_v^2 + \sigma_\epsilon^2$$

where

$$\rho_{m,v} = \frac{\text{cov}(\tilde{v}, \tilde{m})}{\sigma_v \sigma_m}$$

Accordingly

$$\text{var}(\tilde{v}|\tilde{m}) = \frac{\sigma_v^2 \sigma_\epsilon^2}{\sigma_v^2 + \sigma_\epsilon^2}.$$

We notice that when $\sigma_\epsilon \to \infty$ then $\text{var}(\tilde{v}|\tilde{m}) \to \sigma_v^2$, the situation in Section 2, since the information content in the signal $\tilde{m}$ gets more and more blurred as the volatility of $\tilde{\epsilon}$ increases.

By examining the model in Section 2, the situation where both the insider and the market maker observes $\tilde{m}$ (and the insider observes $\tilde{v}$ as before) can be analyzed by replacing σ_v^2 by $\frac{\sigma_v^2 \sigma_\epsilon^2}{\sigma_v^2 + \sigma_\epsilon^2}$ in the expressions for the insider's trading intensity β, and in the corresponding expression for the price sensitivity parameter λ. Clearly this amounts to reducing the variance relative to the model of Section 2, since $\sigma_v^2 > \sigma_v^2 \cdot \frac{\sigma_\epsilon^2}{\sigma_v^2 + \sigma_\epsilon^2}$. We then have the following

Corollary 3. *When the signal $\tilde{m}$ is public, there exists an equilibrium in which x and p are linear functions. Defining*

(3.7) $$\beta = \frac{1}{2\lambda} \quad and \quad \lambda = \frac{\beta\sigma_v^2\sigma_\epsilon^2}{\beta^2\sigma_v^2\sigma_\epsilon^2 + \sigma_z^2(\sigma_v^2 + \sigma_\epsilon^2)} + k^*$$

where k^* is the value of k determined by the regulatory constraint, the equilibrium is given by

$$x(\tilde{v}) = \beta(\tilde{v} - p_0), \quad p(\tilde{x} + \tilde{z}) = p_0 + \frac{1}{2}\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2}(\tilde{m} - p_0) + \frac{1}{2}(\tilde{v} - p_0) + \lambda\tilde{z}.$$

The parameter β is a solution to the cubic equation

(3.8) $$2k^*\frac{\sigma_v^2\sigma_\epsilon^2}{\sigma_v^2 + \sigma_\epsilon^2}\beta^3 + \frac{\sigma_v^2\sigma_\epsilon^2}{\sigma_v^2 + \sigma_\epsilon^2}\beta^2 + 2k^*\sigma_z^2\beta - \sigma_z^2 = 0,$$

which also determines the constant λ. The expected profits are as follows:
The insider: $E(\tilde{\pi}_I) = \frac{\sigma_v^2\sigma_\epsilon^2}{\sigma_v^2 + \sigma_\epsilon^2}\lambda\beta^2 = \frac{1}{2}\frac{\sigma_v^2\sigma_\epsilon^2}{\sigma_v^2 + \sigma_\epsilon^2}\beta.$
The market maker: $E(\tilde{\pi}_M) = \lambda\sigma_z^2 - \frac{1}{2}\frac{\sigma_v^2\sigma_\epsilon^2}{\sigma_v^2 + \sigma_\epsilon^2}\beta = k(\beta(k)^2\frac{\sigma_v^2\sigma_\epsilon^2}{\sigma_v^2 + \sigma_\epsilon^2} + \sigma_z^2).$
The noise traders: $E(\tilde{\pi}_N) = -\lambda\sigma_z^2.$

Proof: The formula for the updating of the price can be deduced as follows:
By the projection theorem we get

$$E(\tilde{v}|\tilde{m}) = p_0 + \rho_{v,m}\sigma_v(\tilde{m} - p_0)/\sqrt{\sigma_v^2 + \sigma_\epsilon^2}.$$

Since $\text{cov}(\tilde{m}, \tilde{v}) = \sigma_v^2$, it follows that

$$\rho_{v,m} = \frac{\sigma_v}{\sqrt{\sigma_v^2 + \sigma_\epsilon^2}}$$

which means that

$$E(\tilde{v}|\tilde{m}) = p_0 + \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2}(\tilde{m} - p_0).$$

Accordingly, the updated price $\tilde{p}$ can be written

$$\tilde{p} = p_0 + \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2}(\tilde{m} - p_0) + \lambda\left[\beta\left(\tilde{v} - p_0 - \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2}(\tilde{m} - p_0)\right) + \tilde{z}\right]$$

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$$= p_0(1 - \lambda\beta) + \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2}(\tilde{m} - p_0)(1 - \lambda\beta) + \lambda\beta\tilde{v} + \lambda\tilde{z}.$$

Recalling that $\lambda\beta = \frac{1}{2}$, this gives

$$(3.9) \qquad \tilde{p} = p_0 + \frac{1}{2}\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2}(\tilde{m} - p_0) + \frac{1}{2}(\tilde{v} - p_0) + \lambda\tilde{z}.$$

The rest of the corollary then follows. $\qquad \square$

In Table 3 we show the insider's profit in the various models of the paper when $k = 0$. Corollary 1 then refers to the Kyle solution, Corollary 2 to the situation where the market maker has private information, and Corollary 3 to the situation where this information is also known to the insider, the situation discussed in the present section. The parameter values are $\sigma_v = \sigma_z = 1, \sigma_\epsilon = 0.1$. As can be seen, for these parameter values the change

Model:	Corollary 1	Corollary 3	Corollary 2
$E(\tilde{\pi}_I)$	0.50	0.05	0.01

Table 3: $E(\tilde{\pi}_I)$ in the various models when $k = 0$;

in the insider's profit is largest when we move from the situation where the market maker does not have any information, to the situation where he has information which is also public. This leads to a reduction in the variance of the price, and as a consequence the profit is reduced by a factor of $1/10$. When this information is also private, i.e., there is differential information between these two parties, the insider's profit is further reduced by a factor of $1/5$, a total reduction by a factor of $1/50$.

The intuitive explanation is the following: Comparing the corollaries 1 and 3, the variance of profit per unit is reduced by a factor of 100 in the latter model. On the other hand the trading intensity β increases by a factor of 10. Knowledge of the market maker's signal enables the insider to take advantage of any mispricing much more confidently. The net effect is a tenfold reduction in profits. Comparing the models in corollaries 1 and 2, we have seen that the trading intensities are the same ($\beta = 1$). The insider, not knowing what the market maker has observed, trades more softly. However in Corollary 2 β is multiplied by $(1/2)\sigma_v(1 - \mu) = (1/2)(1/50) = 1/100$.

In the same vein, Table 4 shows the profits of the insider and the market maker when $k = 0.03$ for the same parameter values for the various models

in Corollary 1, Corollary 3 and Theorem 3.1. For this relatively modest value of k the market maker has a profit closer to that of the insider when his information is also public. When the information is private, however, he makes about 39 times as much as the insider, and the difference between these two profits is largest to his advantage. Also, with no information the market maker has the highest profit of these three scenarios, but then the insider makes the most. The largest single effect of increased private information thus seems to bring down the insider's profit.

Model:	Corollary 1	Corollary 3	Theorem 3.1	k
$E(\tilde{\pi}_I)$	0.472	0.033	0.0008	0.03
$E(\tilde{\pi}_M)$	0.057	0.043	0.031	0.03
$d(k)$	1.029	1.0015	1.0044	0.03
$\iota(k)$	0.47	0.99	0.99	0.03

Table 4: The two profits in the various models when $k = 0.03$;

Note that in the model of Theorem 3.1 there is a lower limit on the profit of the market maker equal to k times σ_z^2 which is attained when the insider stops trading. From Table 4 we can see that this lower bound is almost reached for the parameter values considered.

Table 4 may indicate that the market maker is worse off in terms of profit the more information he has. Apparently he would prefer to be uninformed, and if he happens to be informed he may benefit from sharing this information with the rest of the market. By assumption the market maker does not take advantage of his information. The market maker profits from trading fees. Ceteris paribus he makes more money, the more insiders trade (noise traders trade exogenously). Thus for a fixed k he prefers to be uninformed, and if informed he is better off sharing his information with the insider.

However, this conclusion is somewhat premature, since we consider profits for a fixed value of k. Again we must consider k a parameter indirectly determined by the regulator. To illustrate, suppose the regulator sets the limit to 1 per cent price distortion. Table 4 shows that the models in Theorem 3.1 and Corollary 3 both pass the test, but not the model in Corollary 1. When the market maker has private information, he can increase profits by increasing k to .11 in which case $\pi_M(.11) = .110$ in Theorem 3.1. When this information is also public, he can increase to $k = .12$ in Corollary 3 in which case $\pi_M(.12) = 0.13$. In this situation the market maker obtains the highest

profits when his information is public as in Corollary 3. The profit of the insider reduces to .00022 when information is private, and to 0.017 when the information is public. When information is private to the market maker, the market maker and, in particular, the insider suffer the most.

This illustrates that when price distortions are taken into account, the information is valuable for the market maker. He will profit from sharing his private information with the insider. The same conclusion follows a fortiori as the parameter σ_ϵ increases, i.e., as the market maker's information becomes less precise.

The effects of other values of k in Corollary 3 can be read out in more detail from the following figures. In each figure to follow, privately informed (Th. 3.1) is compared to publicly informed market maker (Cor. 3).
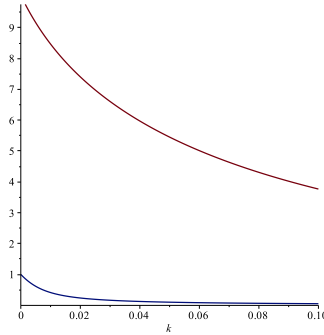

Fig. 20: $\beta(k)$ as a function of k.

In Figure 20 we present the trading intensity of the insider, and in the same graph also the corresponding trading intensity in Theorem 3.1. As already noted, for $k = 0$ β is 10 times higher when the market maker's information is public. This difference persists for positive values of k. The trading intensity in the model of Corollary 3 is also significantly higher than the corresponding intensity when the market maker is uninformed (see Fig. 7).

Figure 21 demonstrates the price sensitivity parameter $\lambda(k)$ as a function of k, in the same graph as the corresponding sensitivity of Theorem 3.1, the latter is the lowest curve.
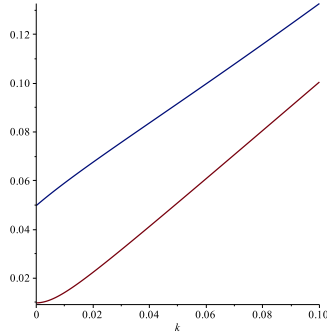
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Fig. 21: $\lambda(k)$ as a function of k.

As can be seen, $\lambda(k)$ is higher when the market maker has public information. In Corollary 3 (as well as Corollary 1), $\lambda(k) = 1/(2\beta(k))$. Also $\lambda_{Th3.1} = (1/(2\beta_{Th3.1}))(1 - \mu)$. So even if $\beta_{Coroll\,3} > \beta_{Th3.1}$, the price sensitivity parameter μ explains this "reversal" of the inequality. Again it is the private information that accounts for the relatively low value of $\lambda_{Th3.1}$. On the other hand λ with publicly informed market maker is lower than λ with uninformed market maker. This follows directly from the relationship between the corresponding β-parameters.

In Figure 22 we present a graph of the profit functions of the market maker for the two models under consideration, the highest curve being the one of the present model. As can be seen, they are fairly close but the latter dominates uniformly in k. The difference is larger for moderately positive values of k when insiders trade significant amounts. It first increases with σ_ϵ to a maximum, then falls. This is an important observation in explaining the ranking of the profits between these two models.
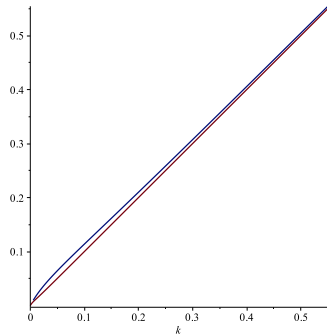

Fig. 22: Profits of the market maker as a function of k.

In Figure 23 we also include the two profit functions of the insider, the lowest falling curve corresponds to Theorem 3.1. The profit function of the

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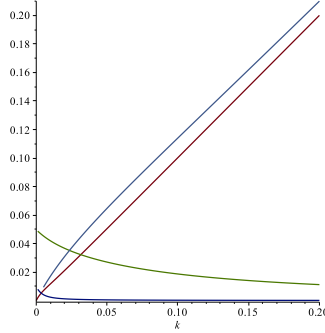insider is now larger, compared to the model of Theorem 3.1. This confirms the results from tables 3 and 4.

Fig. 23: The four profit functions as functions of k.

The value of k where the profit functions of the insider and the market maker are equal, is smallest when the market maker's information is private, and largest when the market maker has no information. This is due to the increasing profit of the insider the less private information the market maker possesses. Both pairs are seen to cross at a relatively low value of k (less than .025) in Fig. 23, indicating that also in this situation the market maker will quickly outperform the well informed insider as the parameter k increases. When the information is private, this crossing point is as low as $k = .0035$.

Next we consider the price distortion $d(k)$. In order to compute this quantity in the present context, the market maker needs to update his price conditional on the signal $\tilde{m}$, in other words p_0 must be updated. The result of this is given in Corollary 3. In order to calculate the price distortion, we need the variance of $\tilde{p}$ given in (3.9). It is

$$\text{var}(\tilde{p}) = \frac{3}{4} \frac{\sigma_v^4}{\sigma_v^2 + \sigma_\epsilon^2} + \frac{1}{4}\sigma_v^2 + \lambda^2 \sigma_z^2.$$

Notice the role that the information of the market maker (and the insider) plays: As this information becomes more precise ($\sigma_\epsilon \downarrow 0$), the variance of the security increases.

The relative price distortion is given by the formula

$$(3.10) \qquad d(k) = \sqrt{\frac{\frac{3}{4} \frac{\sigma_v^4}{\sigma_v^2 + \sigma_\epsilon^2} + \frac{1}{4}\sigma_v^2 + \lambda(k)^2 \sigma_z^2}{\frac{3}{4} \frac{\sigma_v^4}{\sigma_v^2 + \sigma_\epsilon^2} + \frac{1}{4}\sigma_v^2 + \lambda(0)^2 \sigma_z^2}}.$$

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A graph of $d(k)$ is presented in Fig. 24 together with the corresponding graph of Theorem 3.1.
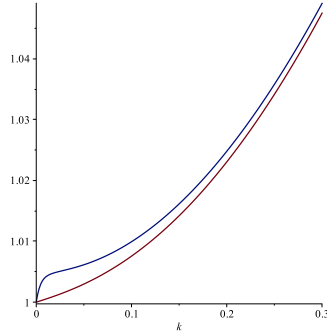

Fig. 24: Price distortions $d(k)$ as functions of k.

The price distortion is much lower than in the model where the market maker's has no information (recall Figure 17b), and also lower (uniformly in k) than the corresponding graph of Theorem 3.1. When σ_ϵ increases, the difference increases to a maximum, and then decreases to zero.

This is an important observation. From Figure 22 we know that market makers earn higher profits when information is public. If the regulator is concerned with price distortion, Figure 24 implies that publicly informed market makers may also increase k relative to privately informed market makers and increase profits even more.

The standard deviation of the price is larger for small values of k for the present model compared to Theorem 3.1, but this does not hold uniformly in k, as demonstrated in Figure 25a. When σ_ϵ increases, the relationship is basically unchanged.

Prices reflect noise as well as asset value. There are two sources of noise: noise trading and noise in the market maker's information. An increase in the variance of price may increase efficiency if the variance derives from variations in value. The opposite is the case if the increased variance is due to the effects of pure noise trading. Noise trading is translated into price via λ. Price reflects value via the informed market maker (μ) and via insider trading (λ and β). When the market maker is publicly informed, the information channels are separable. The market maker uses Bayes law to update price. Insiders influences price via trading. When the market maker has private information, information channels are intertwined. Theorem 3 implies that insider trading makes the market maker adjust prices less than Bayesian updating would imply. The adjustment depends on insider trading intensity.

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This reduces the variance of the market price. On the other hand, when the information of the market maker is public, the insider adjusts his/her demand. Hence, the noise in the information has less influence on the price reducing its variance. Comparing the variance across models thus depends on how these opposing effects play out. In particular, when fees strangle insider trading, the noise effect dominates making the price more volatile when information is private.
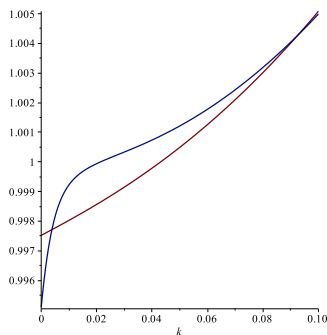

Fig. 25a: The standard deviations of the price as functions of k.

When k grows large, the standard deviations converge, which is according to intuition. This is demonstrated in Figure 25b.
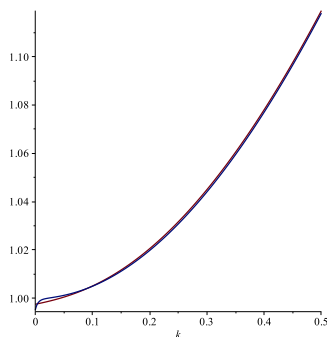

Fig. 25b: The standard deviations of the price for larger values of k.

Finally we consider market efficiency. The informativeness in the present model can be computed once we know

$$\mathrm{cov}(\tilde{v}, \tilde{p}) = \frac{1}{2}\sigma_v^2\Big(\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2} + 1\Big),$$

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and

$$\rho_{v,p} = \frac{\frac{1}{2}\sigma_v^2\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_\epsilon^2}+1\right)}{\sigma_v\sqrt{\frac{3}{4}\frac{\sigma_v^4}{\sigma_v^2+\sigma_\epsilon^2}+\frac{1}{4}\sigma_v^2+\lambda(k)^2\sigma_z^2}}.$$

From this we find the informativeness $\iota(k) = \rho_{v,p}^2(k)$ as follows

$$(3.11) \qquad \iota(k) = \frac{\frac{1}{4}\sigma_v^2\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_\epsilon^2}+1\right)^2}{\frac{3}{4}\frac{\sigma_v^4}{\sigma_v^2+\sigma_\epsilon^2}+\frac{1}{4}\sigma_v^2+\lambda(k)^2\sigma_z^2}.$$

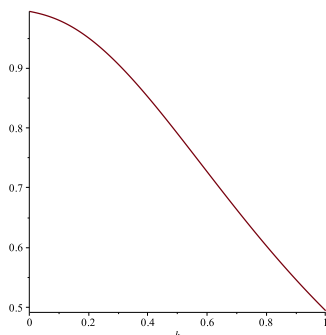A graph of $\iota(k)$ is presented in Fig. 26.

Fig. 26: The informativeness $\iota(k)$ as a function of k.

The informativeness in prices is identical to the one of Theorem 3.1. This is also the case when σ_ϵ increases. Market efficiency does not suffer when this information is shared with the insider.

As a summary of the model in Corollary 3, in Table 5 we present the connection between k^*, $d(k^*)$, $\iota(k^*)$, as well at the profits $E(\tilde{\pi}_M)$ and $E(\tilde{\pi}_I)$ for the same values of the price distortion d as in tables 1 and 2.

The informativeness (market efficiency) at the corresponding levels of k^* is still seen to be high and at the level of Table 2 for the value of $\sigma_\epsilon = .10$ that we consider. The corresponding values of k^* are higher than the ones in Table 2 for Theorem 3.1. The discrepancy is not nearly as large as between Table 1 and Table 2.

A major difference from Table 2 is that the profits of the insider have increased substantially. The profits of the market maker have also increased relative to the profits of Theorem 3.1. As pointed out above, this follows from the ranking of the price distortions, demonstrated in Figure 24, together with the ranking of the profits of the market maker, demonstrated in Figure 22.

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For a given value of the price distortion set by the regulator, we obtain a higher value of k^* for the model of the present section, which in its turn leads to a higher profit due to the profit ranking noted above.

Also in this model the market maker outperforms the insider in terms of profits. For a price distortion of 5 per cent the market maker now makes 3.3 per cent more than in the model of Theorem 3.1, while the insider makes 76 times as much.

Model: Corollary 3				
$d(k^*)$	1.01	1.02	1.05	1.10
k^*	.120	.184	.309	.450
$E(\tilde{\pi}_M)$	.1330	.1947	.3162	.4552
$E(\tilde{\pi}_I)$	.0165	.0120	.0076	.0054
$\iota(k^*)$	.98	.96	.90	.82

Table 5: The connection between k^*, $d(k^*)$ and other quantities.

It may seem surprising that the market maker is better off in terms of profits by sharing his information with the insider. In both models the market maker shares his private information with the market, but in the latter model the insider is in addition informed about the precise content of the market maker's signal. This situation turns out to improve the profits for both the informed parties, at the noise traders' expense.

To see why this may be, assume that the market maker has received a signal that happens to be far off the true value. If the insider sees this, he may take a large position to exploit the mispricing. This will move the price closer to the true value and increase the profits of market makers charging higher fees.

In conclusion, when the market maker has private information, and sets prices as explained, relatively low values of the price distortion d allows him to obtain high profits by trading fees, higher than the perfectly informed insider, since low values of d correspond to a high values of k^*. The difference in profits is most dramatic when the market maker's information is private, but when this information is public both the informed parties make the most, at the noise traders' expense.

We have not studied what happens when the noise traders have partial information in the above situation. From Figure 5 we know that the profits of the two other parties are reduced, but which one is reduced the most may

not be clear. However, using our model and the results in Theorem 2.1, the reader may be able to check this out without any major difficulties.

4 Summary

The single auction model of Kyle (1985) is studied, allowing market makers to maximize profit within regulatory limits by charging fees. This has several implications for the equilibrium, the most important being that by perturbing the price by a relatively modest amount, the market maker is able to obtain a profit of the order of magnitude, and even higher than, a perfectly informed insider. Noise traders may be uninformed, or partially informed.

We also analyze a situation where the market maker has private information as well as being non-fiduciary. Compared to the model where the market maker has no privileged information, we obtain several new insights. As expected the well informed insider's profit diminishes since his informational advantage has shrunk. This is because the market maker acts non-strategically with respect to his private information, and allows this information to be reflected into the price. Interestingly the informed market maker may then increase his fees without being detected by the regulator, which in turn increases his profits.

Our analysis indicates why speculative prices are more volatile than predicted by fundamentals. In particular, price volatility is shown to increase with informed market maker. This is an important aspect of the effect of privileged information on security prices, which may explain the price/dividend puzzle mentioned in the introduction.

Finally we analyze the situation where the market maker's information is public, which can be used to determine the value of differential information. One major difference is that the insider's profits is not nearly as low as when the market maker's information is private. In terms of market efficiency this situation is similar to the one with private information. Price distortions are now smaller, and profits larger as functions of the price perturbation parameter k. As a result of this, perhaps surprisingly, the market maker's profit has now increased, i.e., it pays for him to share his private information with the insider, but the noise traders are then even worse off.

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