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by

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Abstract: Helicopters are often used for transportation of the workers to and from off-shore installations. Flying helicopter can be a risky business and is often considered as one of the main risk factors in this industry. By applying different routing policies for these services, the expected number of fatalities can be influenced. The present paper is dealing with different routing policies for minimizing the expected number of fatalities where the transportation is performed in a hub and spoke fashion from a land based heliport to a set of off-shore installations either using the heliport as a hub or using one or more off-shore installations as hubs. Some theoretical results are offered as well as an exact model when one operates with more than one off-shore hub. In addition several naïve heuristics are offered for finding good solutions for the multi-hub situation.

1. Introduction

In the off-shore oil and gas industry oil companies most often use helicopters in order to bring the crews that are working on the off-shore installations out to the oil and gas platforms and back again. This is a fairly quick and efficient way to serve these installations both with the people who operate them on a daily base, but also bringing maintenance people and experts to platforms for shorter or longer times, performing special tasks. However, this type of transportation is often perceived by the workers to be uncomfortable and unpleasant due to the noise of the helicopter and turbulence that can be violent when the weather is bad. In addition, flying helicopter is a fairly risky activity and much more risky than flying an ordinary plane or driving a car. For more details about this see Qian et al (2011, 2012a and b)and Qian (2012) The risk during landings and take-offs are considered to be substantially larger than during cruising between the heliport (HP) and the installations or between the installations. This is especially true when it comes to landings or take-offs on the platforms, where the helicopter deck is narrow with a lot of equipment like cranes and containers create big challenges for the pilots. In the scenarios treated by Qian et al risks connected with both take offs/ landings as well as cruising risks are considered simultaneously and different routing policies are considered in order to minimize the expected number of fatalities. In this paper the same is done, but in contrast to Qian et al, we will consider other routing policies.

An accident is called fatal when one or more people die or is seriously injured in an accident. Routing of helicopters can neither decrease nor increase the probability that an unwanted fatality takes place. But – as described in the articles mentioned so far – by applying special routing policies the expected number of fatalities can be reduced. However, this will often lead to increased costs / increased number of flying hours. In Qian et al several different routing policies are considered. Basically all of them are variations of classical vehicle routing problems with pick-up and delivery demands under capacity restrictions on the helicopters. All helicopters start from a HP serving a subset of the off shore installations and return to the HP with the people that have been picked up from the installations out in the sea. In many cases the helicopters perform a flight from the heliport to a single customer, deliver whatever number of people and picks up those who are going back on shore and returns to the HP. In other cases the routes can be small cycles visiting two to three installations before returning to the HP. All the installations in such routes are usually visited exactly once, that is, the pick-up and delivery services are performed simultaneously. In some cases an installations may be visited twice. In such a case the out-bound passengers are delivered on the first visit and the home-bound passengers are picked up on the second visit.

In the present working paper we will consider some different options. The first option is to perform the required service in a hub and spoke fashion from the HP. That is, each and every installation (customer node) is served exclusively by a helicopter. This solution can be very expensive and time consuming, but it is proved in Qian et al that this solution outperforms any other routing policies treated in the traditional VRPPD way, whether the service is simultaneously or non-simultaneously performed given that the objective is to minimize the expected number of fatalities.

The new and different approach is to select one or more of the off-shore installations as hubs. The out-bound passengers are taken from the HP to one of these off shore hubs and then brought to their final destinations from this hub by exclusive flights to each of the spoke nodes that are assigned to this off-shore hub. The home-bound passengers are brought back to the off shore hub, and will wait there until all the spoke nodes are visited. Then finally all the home-bound passengers will be brought back together to the HP.

The rest of this paper is organized as follows: In section 2 using the HP as a hub is compared to a situation where a single installation is chosen as an off-shore hub. In section 3, two or more off-shore installations are chosen as a hub and heuristics for making these choices and how to assign the remaining installations as spoke nodes to one of the chosen off-shore hubs are offered. In section 4 small examples are offered in order to illustrate the different heuristics.

2. Two different single hub and spoke solutions

In Qian et al it is shown that using a single helicopter performing the services in a hub and spoke fashion outperformance the results compared to a Hamiltonian cycle. In this paper we will compare two hub and spoke situations. The first will be the situation where the HP is used as the hub, that is, all the installations will be served directly from the HP. In the second situation one of the installations will be chosen as a hub. All passengers will be brought from the HP to this installation and from this node be brought by single flights to their final destinations where the pick-up passengers are collected and brought back to the hub. When all installations have been visited, all the pick-up passengers will be brought back to the HP with the helicopter. See figure 1a and b below. The rectangle symbolizes the HP and the circles the customer nodes/ installations.

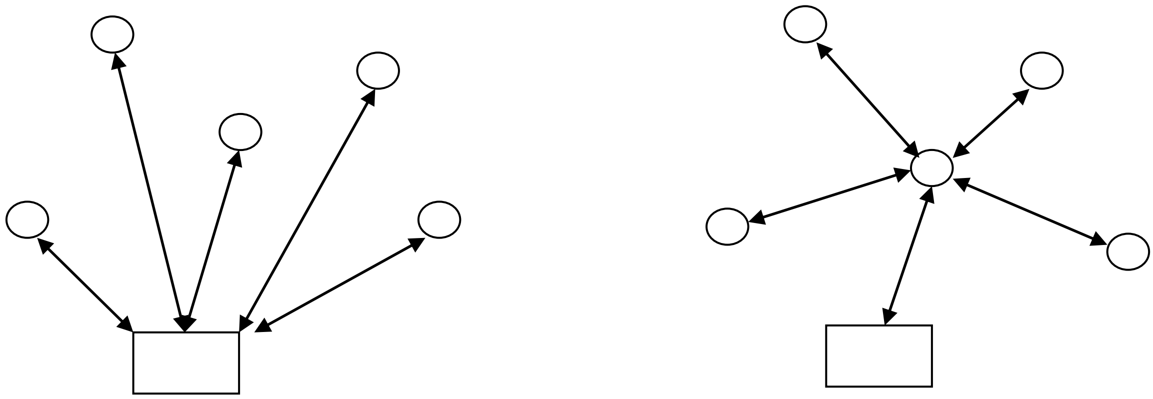

Fig 1 a and b. The two single hub solutions. Figure 1a to the left and 1b to the right.

Notation: We will denote the nodes by $i = 0,1,2,....,n$ where 0 denotes the HP and the other are the customers nodes. Each node has a certain delivery demand d_i, and a certain pick up demand p_i, such that $d_i + p_i > 0$. We will assume that each of the nodes that are not a hub will be visited only once, hence, for customer nodes not chosen to be a hub, the service will be simultaneous. We will let $\sum_{i=1}^{n} d_i = D$ and $\sum_{i=1}^{n} p_i = P$. Further, we let Q denote the capacity of the helicopter and assume that $D \leq Q$ and $P \leq Q$.

As mentioned in the introduction, choosing different routings for the helicopter(s) can only influence the expected number of fatalities, not the probabilities that some accident happens due to bad weather, mistakes made by the pilots or problems based on some technology breaking down. Until further notice, we will not consider the probabilities for fatal accidents neither during cruising time nor during landings or take offs. These probabilities will be different. It may even happen that there are different probabilities for landing and take offs on different installations due to the space on the platform deck and other equipment close to the helicopter deck. There will also be different probabilities concerning the installations and the land based HP. Hence there will be only three concepts that will be of interest in this working paper.

Let C=$[c_{ij}]_{(n+1)x(n+1)}$ the cost matrix of travelling from node i to node j and $c_{ii} = 0, \forall i, i = 0,1,2...,n$.The costs can be given as a distance, a certain amount of money or time, but in order to avoid confusion we will interpret these costs as distances. Further, we let $R_i(C) = \sum_{j=0}^{n} c_{i,j}$, that is the row sums in the cost matrix C, and $R_i(C \setminus \{0\}) = \sum_{j=1}^{n} c_{ij}$ which is the row sums for the matrix C restricted to the customer nodes.

Costs

C_{HP} is the cost of using helicopter(s) when the services are performed from the HP in a hub and spoke fashion (fig.1a)

$C_i, i = 1,2,...,n$ is the cost of using helicopter(s) when the services are performed from installation *i*, bringing all out-bond passengers from the HP to installation *i*, *i* acting as a hub for the other installations, collecting all the pick-up demands here before bringing them back to the HP via the hub (fig.1b).

Number of passengers taking part in landings and take offs.

PL_{HP} is the number of passengers taking part in landings when the HP is used as a hub.

PL_i is the number of passengers taking part in landings when installation *i* is used as a hub.

Number of passengers on board the helicopter multiplied with the flying distance they stay on board.

PC_{HP} is the number of passengers multiplied with the flying distance they stay on board when the HP is used as a hub.

PC_i is the number of passengers multiplied with the flying time they stay on board when installation *i* is used as a hub.

Note that these two last definitions can be interpreted as the transportation work performed by the helicopter and will be referred to as transportation work below.

From the definitions above the following expressions are straightforward, starting with the situation that the HP is the hub.

(1) $C_{HP} = 2R_0$

(2) $PL_{HP} = \sum_{i=1}^{n} \left(d_i + p_i\right) = D + P$

(3) $PC_{HP} = \sum_{j=1}^{n} c_{0j} \left(d_j + p_j\right)$

From (3) we can derive some simple upper and lower bounds on the transportation work performed by the helicopter:

(4) $\min_j \{d_j + p_j\} R_0 \le PC_{HP} \le \max_j \{d_j + p_j\} R_0$

Combining (1) and (4), we get the following inequalities:

(5) $\frac{1}{2}\min_j\{d_j + p_j\} \le \frac{PC_{HP}}{C_{HP}} \le \frac{1}{2}\max_j\{d_j + p_j\}$

(5) gives an upper and lower bound for the fraction of the transportation work relative to the flied distance.

Again from (3) we can obtain the inequalities in (6)

(6) $\min_j\{c_{0j}\}(D+P) \le PC_{HP} \le \max_j\{c_{0j}\}(D+P)$

which in combination with (2) can be transformed to

(7) $\min_j\{c_{0j}\} \le \frac{PC_{HP}}{PL_{HP}} \le \max_j\{c_{0j}\}$

(7) gives an upper and lower bound for the fraction of the transportation work relative to the number of passengers taking part in landings and take-offs.

Combining (5) and (6) we get an upper and lower bound for the number of passenger landings relatively to the distance the helicopter has to fly.

(8) $\frac{1}{2}\frac{\min_j\{d_j + p_j\}}{\max_j\{c_{0j}\}} \le \frac{PL_{HP}}{C_{HP}} \le \frac{1}{2}\frac{\max_j\{d_j + p_j\}}{\min_j\{c_{0j}\}}$

The expressions (9) to (11) below give similar formulas for the three concepts defined in (1) – (3) when an off-shore installation *i** is chosen as a hub.

(9) $C_{i*} = 2c_{0,i*} + 2R_{i*}(C \setminus \{0\}) = 2R_{i*}$

(10) $PL_{i*} = (D+P) + (D+P) - (d_{i*} + p_{i*}) = 2(D+P) - (d_{i*} + p_{i*})$

(11) $PC_{i*} = (D+P)c_{0,i*} + \sum_{j=1}^{n}(d_j + p_j)c_{i*,j}$

(11) can easily be reformulated into (12) giving upper and lower bounds for the transportation work performed from the chosen hub *i** to the other installations. Further, (12) shows that if one wants to have as small upper bound as possible, the hub should be chosen such that the sum of the distances from the hub to the other off-shore installations should be as small as possible. A reasonable interpretation of this will be to choose the hub among the installations located in the middle of all the off-shore installations. On the other hand, this upper bound will increase if the chosen hub is far from the HP.

(12) $(D+P)c_{0,i*} + \min_j\{d_j + p_j\}R_{i*}(C \setminus \{0\}) \le PC_{i*} \le (D+P)c_{0,i*} + \max_j\{d_j + p_j\}R_{i*}(C \setminus \{0\})$

Combining (12) with (9) we get (13)

$$(13)\qquad \min_j\{d_j+p_j\}\le \frac{PC_{i*}-(D+P)c_{0,i*}}{\frac{1}{2}C_{i*}-c_{0,i*}}\le \max_j\{d_j+p_j\}$$

Further, (11) can also easily be reformulated into (14).

$$(14)\qquad \min_j\{c_{i*,j}\}\le \frac{PC_{i*}}{D+P}-c_{0,i*}\le \max_j\{c_{i*,j}\}$$

Note that if the maximal and minimal distances from the chosen hub to the other off-shore installations are not very different, the upper and lower bounds given by (14) could be fairly close.

Lemma 1

If one chooses a customer node as a hub, the number of passenger landings will be minimized by choosing a node *k* such that $d_k+p_k=\max_j\{d_j+p_j\}$.

Proof:

The result follows directly from (10).

Hence, it is straight forward to find the best off-shore hub if we restrict our selves only to look at the number of passenger landings.

Choosing different off-shore installations as the hub will of course give different transportation work. This can be calculated by (11). Since we have only *n* off-shore installations the one generating the smallest transportation work can be easily found. However, we cannot á priory find which one that is the best as we could for the passenger landing. Lemma 2 below gives an upper bound on the difference between two choices for the transportation work.

Let $(c_0)_{\max}=\max_j\{c_{0.j}\}$ and $(c_0)_{\min}=\min_j\{c_{0.j}\}$. Further let $c_{\max}=\max_{\substack{i,j\\ i,j\neq 0}}\{c_{i.j}\}$ and $c_{\min}=\min_{\substack{i,j\\ i,j\neq 0}}\{c_{ij}\}$

Lemma 2

Let *i* and *k* be two off-shore nodes. Then

$$\Delta PC_{ik}=|PC_i-PC_k|\le 2c_{ik}(D+P)$$

Proof:

It is easily shown, using (11), that

$$PC_i - PC_k = (c_{0,i} - c_{0,k})(D+P) + \sum_{j=1}^{n} (c_{ij} - c_{kj})(d_j + p_j)$$

By the triangle inequality we have that $c_{0,i} \geq c_{0,k} - c_{ik}$ and $c_{ij} \geq c_{kj} - c_{ik}$. Substituting these into the expression for the difference between the two hub choices we get that

$$\Delta PC_{ik} \leq |c_{0,i} - c_{0,k}|(D+P) + \sum_{j=}^{n} |c_{ij} - c_{kj}|(d_j + p_j) \leq |-c_{ik}|(D+P) + \sum_{j=1}^{n} |-c_{ik}|(d_j + p_j) =$$
$$c_{ik}(D+P) + c_{ik} \sum_{j=1}^{n} (d_j + p_j) = 2c_{ik}(D+P)$$

The lemma shows that if two potential choices for a hub are close to each other, the difference between the transportation works for the two choices will be small.

Above we have looked at the two situations – HP as a hub and an off-shore installation as a hub – separately. Below two lemmas are given saying something about the relationship between the two choices.

Lemma 3

If the triangle inequality holds for the cost matrix *C*, the following inequality holds for any choice *k* as a hub among the customer nodes.

$$PC_k \geq PC_{HP} \text{ and } PL_i > PL_{HP}$$

Proof:

By (10) we have

$$PC_k = (D+P)c_{0,k} + \sum_{j=1}^{n} (d_j + p_j)c_{k,j} = \sum_{k=1}^{n} (d_k + p_k)c_{0.k} + \sum_{j=1}^{n} (d_j + p_j)c_{kj} =$$
$$\sum_{j=1}^{n} (d_j + p_j)(c_{0k} + c_{kj}) \geq \sum_{j=1}^{n} (d_j + p_j)c_{0k} = PC_{HP}$$

The second inequality follows immediately by comparing (2) and (10)

Hence, as a consequence, choosing the heliport as a hub will always outperform choosing a customer node as a hub in terms of number of passenger landings and transportation work.

Lemma 3

The following inequalities hold:

$$(c_0)_{\min}(D+P) \leq PC_{HP} \leq (c_0)_{\max}(D+P)$$

$$((c_0)_{\min} + c_{\min})(D+P) \leq PC_i \leq ((c_0)_{\max} + c_{\max})(D+P), \forall i, i = 1,2,\ldots,n$$

Corollary 1

The following inequality holds for all $i = 1,2,\ldots,n$:

$$\frac{(c_0)_{\min} + c_{\min}}{(c_0)_{\max}} \leq \frac{PC_i}{PC_{HP}} \leq \frac{(c_0)_{\max} + c_{\max}}{(c_0)_{\min}}, \forall i, i = 1,2,...,n$$

Above we have discussed the two of the three concepts. The third concept is the distance the helicopter has two fly or the time it has to be in the air. Comparing the flight hours when the HP is used as a hub with the flight hours used when a customer node is used as a hub is difficult and will rely heavily on how the customer nodes and the HP is located relatively to each other. In off-shore oil and gas activity the HP is often located close to the coast with all the installations out in the sea - broadly speaking – in the same direction. Hence, in this case the flight hours needed to serve the installations using the HP as hub often will lead to a large number of flying hours and much bigger than if some centrally located installation is used as a hub. Actually the node with the smallest row sum in the cost matrix *C* will be the optimal location for the hub if one wants to minimize the number of flying hours. This will not necessarily be the HP but could often be an installation out in the sea.

Algorithm for finding the best off-shore hub –single hub case.

Let π_L and π_C be the probabilities for a fatal accident during landings and cruising, respectively. We want to minimize the following objective, the total probability of having a fatal accident, TP.

$$(14) \quad TP = \pi_L PL_i + \pi_C PC_i = \pi_L \big(2(D+P) - (d_i + p_i)\big) + \pi_C \left((D+P)c_{0,i} + \sum_{j=1}^{n} (d_j + p_j) c_{i,j} \right)$$

Since we have *n* off-shore installations, we can calculate (14) for each of them and choose the one with the smallest probability. Hence, the algorithm is polynomial with complexity $O(n)$. The optimal solution will of course depend partly on the demand pattern. As a consequence the off-shore hub can change form one time horizon to the next. It may also happen that some of the off-shore installations are not suited as a hub due to different reasons like special activities going on making hindrances for frequent landing and take offs, that the number of people on board the platform is too big compared to the capacity of the life boats or that a platform does not have equipment for re-fueling the helicopter. Of course such platforms have to be excluded from the calculations above, even if they otherwise are good candidates for a temporarily hub.

A single helicopter performing all the visits may run out of time during the given time horizon. If this is the case we need two or more helicopters to perform the services. This situation will be treated in the next section.

3. Multiple helicopters and multiple hubs among the customer nodes

In this section we will assume that we have two or more helicopters performing the pick-up and delivery service. We will assume that the service is performed simultaneously, that is, every node is visited one and only one time by a helicopter except the chosen hubs that of course have to be visited several times.

There will be three basically different situations:

1. All the helicopters use the HP as a hub, visiting disjunctive sub-sets of the customers.
2. Each helicopter has different off-shore installation as its hub, starting from the HP with the delivery demands that belong to their specific disjunctive sub-sets of the customers in a hub and spoke fashion and bringing all the pick-up demands back to the HP.
3. A subset of the helicopters is using the HP as a hub and the remaining subset of the helicopters is using specific customer nodes as their hubs.

All the helicopters are using the HP as a hub

In this case the flying distance, the number of passenger landings and the transportation work will be the same as in the single helicopter case, that is, (1), (2), and (3) are still valid. The flying distance or the time used to serve the off-shore installation may become large, but using the HP as a hub having several helicopters will still outperform case 2 and 3 when it comes to passenger landings and transportation work. Using more than one helicopter one tacitly assumes that there is some restriction on time (or distance) that each helicopter can use to perform its service. If this time restriction is small, then one may have to use *n* helicopters, one for each of the off-shore installations. If it is somewhat bigger a helicopter can serve more than one installation during the given time horizon. It will be a separate problem to find which installations to combine such that the number of necessary helicopters will be minimized.

Using several helicopters each choosing a customer node as a hub

Assume that we have $k=1,2,\ldots,m$ helicopters. Let i_k be the customer node chosen as the hub for helicopter number *k*. Let N_k be a subset of $N \setminus \{0\}$ such that $N_k \cap N_l = \emptyset, k \neq l$ and $N_1 \cup N_2 \cup \ldots \cup N_m = N \setminus \{0\}$. N_k denotes the customer nodes that is served by helicopter *k*, starting from the HP with all the passengers that are going to the nodes in N_k, then using node *k* as a hub bringing all the delivery demands from *k* to the other nodes in N_k and bringing the pick-up demands back to *k* and bringing all pick up demands back to the HP.

The different concepts for this situation are given in the expressions (15) – (17) below.

$$(15)\qquad C = \sum_{k=1}^{m} C_{i_k} = 2\sum_{k=1}^{m} c_{0,i_k} + 2\sum_{k=1}^{m}\sum_{j\in N_k} c_{i_k,j}$$

(16) $$PL = \sum_{k=1}^{m} PL_{i_k} = \sum_{k=1}^{m} \sum_{s \in N_k} (d_s + p_s) + \sum_{k=1}^{m} \sum_{\substack{j \in N_k \\ j \neq i_k}} (d_j + p_j) =$$

$$D + P + (D + P) - \sum_{k=1}^{m} (d_{i_k} + p_{i_k}) = 2(D + P) - \sum_{k=1}^{m} (d_{i_k} + p_{i_k})$$

(17) $$PC = \sum_{k=1}^{m} \sum_{s \in N_k} (d_s + p_s) c_{0,i_k} + \sum_{k=1}^{m} \sum_{\substack{s \in N_k \\ s \neq i_k}} (d_s + p_s) c_{i_k,s}$$

Corollary 2

In order to minimize the number of passenger landings, the hubs chosen among the customer nodes should be done by choosing the nodes with the *m* biggest demands. If the number of helicopters is increased to *n*, the number of passenger landings becomes *D*+*P*, which is the minimal number of passenger landings that is possible.

Proof

The two statements follow directly from (16)

Note: The second part of the corollary shows that choosing each and every customer node as a hub, the hub concept degenerates and becomes identical with using the HP as a hub. In a way, using the HP as a hub will be identical with using all customer nodes as hubs.

Now, assuming that one wants two or more off-shore hubs, the following two decisions have to be made under the restriction of not violating the helicopters capacities:

1. Which installations to choose as off-shore hubs
2. Which installations should be assigned to which hub

We will assume that the demands during the time horizon are given as *D* and *P*, but are so big that they cannot be handled by a single helicopter. Further it is assumed that the number of helicopters is the smallest integer number *m*, such that $mQ \geq \max\{D, P\}$, where *Q* is the capacity for a single helicopter, all helicopters are uniform.

Lemma 5

For a given choice of *m* off-shore installations, the number of passenger landings will be independent of the way the spoke nodes are assigned to the off-shore hubs.

Proof:

Follows directly by (16).

An exact model for deciding which off-shore installation that should be chosen as hubs, when the number of helicopters, that is the number of off-shore hubs, is given.

In addition to the notation already given the following notation will be used:

m is a parameter denoting the number of helicopters.

Variables:

PL the number of passenger landings

PC the transportation work

$U_i = 1$ if node *i* is chosen as an off-shore hub and 0 otherwise, $i = 1,2,\ldots,n$

$W_{ik} = 1$ if node *k* is served from the off-shore hub *i*, and 0 otherwise, $i, k = 1,2,\ldots,n$

(19) $$\min(\pi_L PL + \pi_C PC)$$

Subject to

(20) $$PL = 2(D+P) - \sum_{i=1}^{n}(d_i + p_i)U_i$$

(21) $$PC = \sum_{i=1}^{n}\sum_{k=1}^{n} c_{0,i}(d_i + p_i)W_{ik} + \sum_{i=1}^{n}\sum_{k=1}^{n} c_{ik}(d_i + p_i)W_{ik}$$

(22) $$\sum_{i=1}^{n} U_i = m$$

(23) $$W_{ik} \le U_i, \forall i,k = 1,2,\ldots,n$$

(24) $$\sum_{\substack{k=1 \\ k\neq i}}^{n} d_k W_{ik} + d_i U_i \le Q, \forall i = 1,2,\ldots n$$

(25) $$\sum_{\substack{k=1 \\ k\neq i}}^{n} p_k W_{ik} + p_i U_i \le Q, \forall i = 1,2,\ldots n$$

(26) $$\sum_{i=1}^{n}\sum_{\substack{k=1 \\ k\neq i}}^{n} d_k W_{ik} + d_i U_i = D$$

(27) $$\sum_{i=1}^{n}\sum_{\substack{k=1 \\ k\neq i}}^{n} p_k W_{ik} + p_i U_i = P$$

The objective (19) minimizes the expected number of fatalities depending on the number of passenger landings and cruising distance multiplied with the number of passenger on board. (20) and (21) give the number of passenger landings and transportation work, respectively. (22) specifies the number of helicopters. (23) says that if an off-shore installation is not selected as a hub, no transportation can take place from this hub to any of the other off-shore installations. (24) and (25) ensure that the capacities of the helicopters are not violated, neither for the out-bound nor the homebound demands, respectively. Finally, (26) and (27) ensure that all the necessary transportation take place, both for the out-bound and the home bound traffic, respectively.

The above model is an integer program. As the number of installations increases, the number of 0/1 variables will also increase and solving the model to optimality may become difficult or take too long time, even if the model is treated as a tactical or strategically tool. If the model is used on a day to day basis where the hubs can be changed from one day to the next, time for solving the model can be limited, and a quicker way to find feasible solutions may be necessary. It is then necessary to rely on some heuristics to find feasible solutions. In the next section some heuristics are offered.

4. Heuristics for the multiple hub situations

Below several heuristics will be offered. They will all be of a naïve kind and based on the results above and/or inspired by heuristics from vehicle routing problems (VRP). Whether using an exact model or a heuristic two decisions have to be made in our context. As mentioned in section 3 these two decisions can be summarized as:

Assuming that one wants two or more off-shore hubs, the following two decisions have to be made under the restriction of not violating the helicopters capacities:

1. Which installations to choose as off-shore hubs
2. Which installations should be assigned to which hub

This opens up for two different approaches:

1. First one decides which installations that should be chosen as the off-shore hubs and then one decides which installations that should be assigned to which hub under the restriction of not violating the helicopters capacity, or alternativly
2. First one decides which off-shore installation that should belong to the same off-shore hub, given that the capacity of the helicopter is not violated and then one decides which node among the chosen ones that should serve as the off-shore hub.

Below both approaches are used.

Heuristic 1

This heuristic is based on the fact that the probability of an unwanted incident is much larger when landing on a platform than when the helicopter is cruising between the nodes. As a consequence the number of passenger landings should be minimized in order to minimize the number of expected fatalities.

Based on corollary 2 we then pick out off-shore installations with large demands.

Step 1: Choose the off shore installation with largest demands as a hub

Step 2: From this hub, assign the k nearest off-shore installations to the hub not assigned to any other hub, such that the demands between the HP and the hub is not violating the helicopters capacity, but is as big as possible. Go to step 3, unless all nodes are assigned to a hub. If so, stop.

Step 3: From the off-shore installations not used as a hub or assigned to a hub, choose as the next hub the remaining off-shore installation with the largest demand. Return to step 2.

Variety of heuristic 1: Instead of choosing the hubs one by one and for each of the hubs assigning the spoke nodes, one can instead choose the m nodes with the largest demands as the hubs á priory and then assigning the spoke nodes to the chosen hubs in one sequence or another, of course avoiding the nodes that are already chosen as hubs. Sub-variations of the assignment approach will then be how to assign spoke nodes to the chosen hubs. Several different approaches are possible.

Remark: According to lemma 5 and (16) the number of passenger landings will remain constant as soon as the off-shore hubs are fixed. The number of passenger landings could change if another choice of off-shore hubs is made.

Heuristic 2

This heuristic is inspired by the well known sweep heuristic in VRP. In addition we use the observation that usually in the off-shore industry the HPs are based on land with all the oil and gas installation out in the sea within in a certain sector that often will cover less than 180 degrees as is the case in figure 1 in section 4. For ease of exposition we will assume that all the installations are north of the HP and distributed from east to west out in the sea.

The heuristic will consist of two steps:

- First a clustering step, clustering the nodes such that the helicopters capacity is not violated.
- Second, choosing a hub for each of the clusters and assigning the other nodes belonging to the given cluster to this hub.

More formally:

Step 1: Start the sweep with a line starting from the HP and being east of all the off-shore installations.

Step2: Sweep clockwise towards west collecting nodes until the helicopters capacity is violated. This will be the first cluster. Repeat step 2 by continuing westward until all nodes are belonging to a cluster.

Step 3: In each cluster, choose the node with the biggest demand as the hub. Assign the other nodes in the cluster as spoke nodes to the chosen hub.

Of course one may start the sweep from west and sweeping counterclockwise creating.

Heuristic 3

This heuristic is inspired by the well known Fisher-Jaikumars (FJ) heuristic (Fisher and Jaikumar, 1981) for the classical VRP. This heuristic introduces so-called seeds nodes, one for each vehicle that is needed to perform either a delivery service or a pick-up service from a central depot to a set of customers. With the help of the chosen *m* seed nodes, the set of customers is clustered into *m* disjunctive subsets such that the vehicles capacities are not violated. The depot is then added to each of the clusters and *m* different Travelling Salesman Problems (TSPs) are solved for each of the subsets. Note that the clustering process has circumvented the capacity problem and as a consequence it is not necessary to solve VRPs, but only TSPs.

The present heuristic will first choose the seed nodes. These can be seen as the off-shore hubs and then a clustering will be performed in the FJ-fashion. In our case however, we have both delivery demands and pick-up demands. In their original paper Fischer and Jaikumar have several ways of finding good and suitable seed nodes. In our case we use the simple result of corollary 2 in order to keep the number of passenger landings as small as possible. Also note that as soon as the hubs are fixed by step 1 below, the number of passenger landings is given and will be independent on the assignment of the other off-shore installations to the hubs, see lemma 5.

Step 1: Choose the *m* off-shore nodes with the largest demands taken together

The clustering part can be performed in three different ways in our case. First we define the decision variables W_{is} = 1 if off-shore node *i* should be served from hub *s* and 0 otherwise, $i \neq s$.

FJ-heuristic, clustering part for variant 1

The first case is a direct extension of the original FJ-heuristic. For each of the off-shore nodes, not being chosen as a hub, we calculate the following cost elements. These cost elements are given by the formula below and is identical with the so-called savings for the cost matrix based on the HP. We assume a symmetric cost matrix. For further information about the saving values, see Clark and Wright, 1964.

$$(28) \quad \Delta_{is} = c_{0s} + c_{i0} - c_{si} \qquad \forall i, s, i = 1,2,...,n, s = 1,2,...,m, i \neq s$$

We then solve the following IP model:

$$(29) \quad \max \sum_{i=1}^{n} \sum_{s=1}^{m} \Delta_{is} W_{is}$$

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$$(30) \quad \sum_{\substack{s=1 \\ s \neq i}}^{m} W_{is} = 1 \quad \forall i, i = 1,2,...,n$$

$$(31) \quad \sum_{\substack{i=1 \\ i \neq s}}^{n} d_i W_{is} \leq Q \quad \forall s, s = 1,2,....,m$$

$$(32) \quad \sum_{\substack{i=1 \\ i \neq s}}^{n} p_i W_{is} \leq Q \quad \forall s, s = 1,2,....,m$$

The objective (29) tries to maximize the savings which is another way of minimizing the costs. (30) says a spoke node should be assigned to exactly one seed node / hub. (31) and (32) ensure that the helicopters' capacities are not violated. Solving the model (29) – (32) will create a cluster where the two types of demands will not exceed the helicopters capacity and each of the off-shore installations not chosen as a hub will be assigned to exactly one off-shore hub. In our case it is not necessary to solve TSPs since now sequencing is necessary in a hub and spoke situation.

FJ-heuristic, clustering part for variant 2

In variant 1 the saving costs given by (28) is clearly associated with a VRP-context. In our situation we have a hub and spoke situation where we want to minimize the expected number of fatalities. Since the number of passenger landings is decided in step1, we can only influence the transportation work through the clustering process. Hence it seems to be reasonable to replace the cost elements in (28) with cost elements that directly have something to do with the transportation work that has to be performed. We therefore propose the following cost elements:

$$(33) \quad \Theta_{is} = c_{is}(d_i + p_i)$$

We then replace the parameters in the objective (29) in the previous model and minimize instead of maximizing with (34) and keep the restriction (30) – (32).

$$(34) \quad \min \sum_{i=1}^{n} \sum_{s=1}^{m} \Theta_{is} W_{is}$$

FJ-heuristic, clustering part for variant 3

In heuristic 1 the hubs are chosen in the same way as it is done for the two variants for the FJ-heuristics. However, in heuristic 1 the clustering part is done in a sequential way by for example starting with one of the chosen hubs and the assign other off-shore installations to this hub if they are close to the hub. This assignment process can be done in many different ways and can be regarded as fairly naïve. To ensure that all possibilities are considered we can use the FJ-models above replacing the parameters in (34) with c_{is}. Actually one could use (33) in heuristic 1 and have another variation of this heuristic.

4 Examples

Below some examples are offered in order to illustrate the theoretical approaches treated in section 2 and 3. The examples are based on a small graph consisting of 7 nodes, 6 off-shores and one HP. The off-shore installations are all located "north" of the HP, see figure 2 for a map of the relative positions and the given distance matrix in table 1.

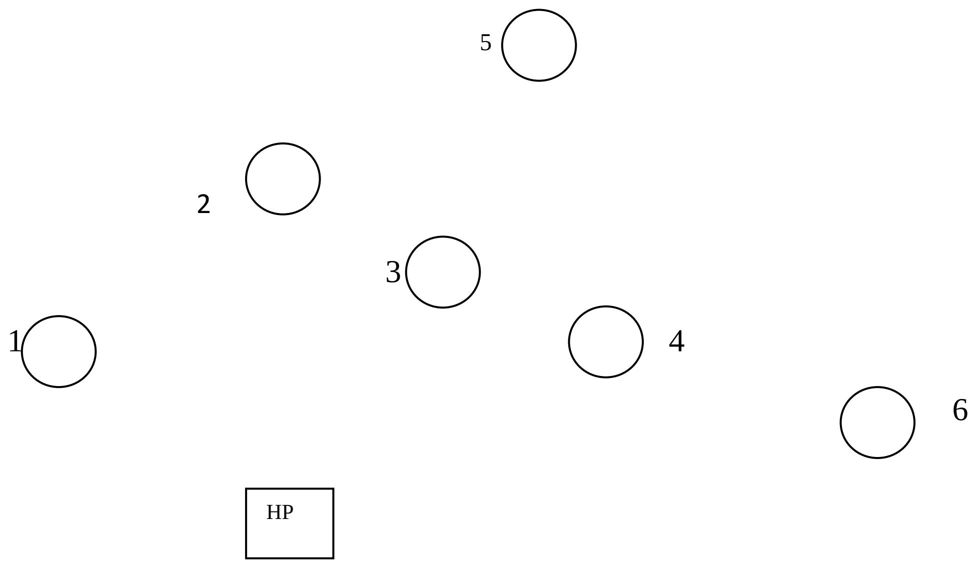

Figure 2: The map of the HP and the six off-shore installations

Table 1: Cost matrix for the graph in figure 2

	0	1	2	3	4	5	6	Row sum
0	-	28	40	36	45	67	71	287
1	28	-	28	41	60	64	91	312
2	40	28	-	22	45	36	76	247
3	36	41	22	-	22	32	54	207
4	45	60	45	22	-	41	32	245
5	67	64	36	32	41	-	64	304
6	71	91	76	54	32	64	-	388

Example 1

This example refers to the situation in section 2, where either the HP or one of the off-shore installations is chosen as a single hub for the helicopters. The demands are given in table 2 below and the results in table 3.

Table 2 Pick-up and delivery demands for example1

Node	Delivery demand	Pick-up demand	Sum
HP (0)	-	-	-
1	3	4	7
2	1	3	4
3	2	2	4
4	6	5	11
5	4	1	5
6	4	5	9
Sum	20	20	40

Table 3: The costs, number of passenger landings and transportation work

Node chosen as hub	Distance / costs	Number of passenger landings	Transportation work
HP	574	**40**	**1969**
1	624	73	3195
2	494	76	3243
3	**414**	76	2703
4	490	69	2981
5	608	75	4475
6	776	71	4669

As can be seen from table 3, the HP outperforms – as expected – all the off-shore installations as hubs apart from the travelling distances. This is in accordance with the theory given above. Choosing node 3 as a hub will minimize the travelling distance and will give the smallest transportation work among the off-shore installations, and choosing 4 as the off-shore hub will give the minimum number of passenger landings among the these potential hubs. Note that it is especially the passenger landings that will increase substantially using an off-shore installation as a hub compared to using the HP as a hub.

Examples for multiple hubs

Below several examples are given illustrating cases when one assumes multiple off-shore hubs based on the heuristics given in section 4. In all the examples the demands given in table 4 are used. As can be seen from this table, the demands can basically be handled by two helicopters, each with a capacity of 20 passengers.

Table 4 Pick-up and delivery demands for example 2 – 5.3

Node	Delivery demand	Pick-up demand	Sum
HP (0)	-	-	-
1	2	5	7
2	9	8	17
3	8	7	15
4	8	6	14
5	8	5	13
6	2	7	9
Sum	37	38	75

Example 2 based on heuristic 1 – basic version

Following the algorithm for heuristic 1, evidently node 2 will be the chosen hub by step 1. Then assigning node 1 and 3 as spoke nodes to node 2, the capacity is not violated, but there is not possible to assign any other node to this hub without violating the helicopter's capacity. Hence, node 2 is the first hub and node 1 and 3 are its spoke nodes.

Retreating to step 2 in the heuristic node 4 will be the remaining node with the largest demands and will then be the second hub. Assigning node 5 and 6 as the spoke nodes for this hub will not violate the helicopter's capacity. The result of this is given in table 5.

Table 5 The results of applying heuristic 1 – basic version to the data.

Hub	Distance / costs	Number of passenger landings	Transportation work
2 (with spoke nodes 1 and 3)	180	61	2086
4 (with spoke nodes 5 and 6)	236	58	2441
Sum	**416**	119	4527
HP as hub for two helicopters	574	**75**	**3658**

Example 3 based on heuristic 1 – variant

In this heuristic we first decide which off-shore installations that should act as hubs giving priority to those with the largest demands. From table 4 we get that the two nodes with the largest demands are node 2 and 3. We now have several choices as far as assigning spoke nodes to the two hubs. Alternative 1 will be to assign spoke nodes to the chosen hubs in a decreasing order when it comes to the total demands handled by the helicopters. In our example that will be assigning spoke nodes to node 2 first and then node 3. If one has more than two hubs there will exists *m!* different ways of doing this. Alternative 2 will be to start with the node that is closest to any of the chosen hubs and assigning this to the hub, then searching for the next one etc, until the helicopters' capacities are violated.

We will use both approaches starting with alternative 1.

Example 3.1 based on heuristic 1 – variant with alternative 1, sequence 1

The two nodes serving as hubs will be node 2 and 3. Then, starting with assigning spoke nodes to node 2, the two nodes closest to node 2 not serving as hubs will be node 1 and 5. It will be sufficient capacity to serve these three nodes, but adding another spoke node will violate the helicopter's capacity. Hence, the second hub – node 3 – will be the hub for nodes 4 and 6. The capacity will not be violated in this case either. The results are summarized in table 6.

Table 6 The results of applying heuristic 1 – variant with alternative 1 starting with node 2 as a hub to the data.

Hub	Distance / costs	Number of passenger landings	Transportation work
2 (with spoke nodes 1 and 5)	208	57	2144
3 (with spoke nodes 4 and 6)	224	61	2162
Sum	**432**	118	4306
HP as hub for two helicopters	574	**75**	**3658**

As can be seen from table 6 using HP as a hub will still be better than using two off-shore hubs. Comparing the results from table 6 with the results in table 5 as far as using two off-shore hubs, the results show that the in table 6 is slightly larger than in table 5, the number of passenger landings is marginally better in the latter case and the transportation work is somewhat better. The latter will give a lower expected probability for the number of fatalities.

Example 3.2 based on heuristic 1 – variant with alternative 1, sequence 2

In this case we will start with the same nodes as the off-shore hubs, but we will start with assigning spoke nodes to node 3 first. As shown in section 3 the number of passenger landings will be the same as in table 6.

Table 7 The results of applying heuristic 1 – variant with alternative 1, starting with node 3 as a hub to the data.

Hub	Distance / costs	Number of passenger landings	Transportation work
3 (with spoke nodes 1 and 4)	198	57	1891
2 (with spoke nodes 5 and 6)	304	61	2712
Sum	**502**	118	4603
HP as hub for two helicopters	574	**75**	**3658**

Comparing table 6 and 7, we see that the results are worse – apart from the passenger landings – in the latter case.

Example 3.3 based on heuristic 1 – variant with alternative 1, no priority between the hubs.

In this case we assign the spoke nodes to the closest hub, not necessarily utilizing the capacity for a helicopter before we start on the next one.

From the cost matrix we find that node 4 is the closest to any of the hubs and is closest to node 3. The next spoke node will be node 1, which will be assigned to node 2- The next one will be node 5 which should be assigned to hub 3, but this violates the capacity of the helicopter and must then be moved to node 2 as a spoke node. Finally node 6 will be a spoke node to hub 3. The assignment will be identical with the assignment in table 6.

Example 4 – The sweep heuristic, heuristic 2

Starting the sweep in a clockwise fashion we get nodes 1, 2 and 5 as the first cluster. Trying to add node 3 to this cluster will violate the helicopters capacity. The next cluster will be node 3, 4 and 6. In the first cluster node 2 has the largest demand and will be the hub with node 1 and 5 as spoke nodes. In the second cluster node 3 has the largest demand and node 4 and 6 will be assigned to this hub. The result will be the same as in table 6 above.

Trying to sweep counter clockwise will give the same clusters and hence the same results.

Example 5 – The Fischer-Jaikumar heuristic, heuristic 3

In all the three variants of heuristic 3, step 1 is common. With the demands taken from table 4, node 2 and 3 will be the hubs or the seed-nodes.

Example 5.1 Heuristic 3, variant 1

In this variant we use (28) to find the parameters used in the objective (29). These parameters are given in table 8.

Table 8. The savings for the assignment model with objective (29)

	0	1	2	3	4	5	6
Seed node 2	0	40	80	54	40	71	35
Seed node 3	0	23	54	72	59	71	53

Note that the shaded elements cannot be used in the optimization / assignment problem, since node 2 and 3 are seed nodes and shall not be assigned to each other.

Solving the associated model gives that $W_{12} = W_{52} = W_{43} = W_{53} = 1$ and all other variables are zero. This will be the same result as in table 6.

Example 5.1 Heuristic 3, variant 2

In this case we calculate the parameters in the second model with the help of (33) and minimize the objective. The parameters are given in table 9.

Table 9. The savings for the assignment model with objective (33)

	0	1	2	3	4	5	6
Seed node 2	0	196	0	330	630	468	684
Seed node 3	0	287	374	0	308	416	486

Note that the shaded elements cannot be used in the optimization / assignment problem, since node 2 and 3 are seed nodes and shall not be assigned to each other.

Solving the associated model gives that $W_{12} = W_{52} = W_{43} = W_{53} = 1$ and all other variables are zero. This will be the same result as for variant 1 and the details are shown in table 6.

Example 5.2 Heuristic 3, variant 2

In this case we calculate the parameters in the second model with the help of (33) and minimize the objective. The parameters are given in table 9.

Table 9. The savings for the assignment model with objective (33)

	0	1	2	3	4	5	6
Seed node 2	0	196	0	330	630	468	684
Seed node 3	0	287	374	0	308	416	486

Table 9. The parameters for the assignment model with objective (33)

	0	1	2	3	4	5	6
Seed node 2	0	196	0	330	630	468	684
Seed node 3	0	287	374	0	308	416	486

Solving the associated model gives that $W_{12} = W_{52} = W_{43} = W_{53} = 1$ and all other variables are zero. This will be the same result as for variant 1 and the details are shown in table 6.

Example 5.3 Heuristic 3, variant 3

In this case we use the distances from the seed nodes as parameters in the assignment model and minimize the objective. The parameters are given in table 10.

Table 10. The parameters for the assignment model when distances from the seed nodes are used

	0	1	2	3	4	5	6
Seed node 2	40	26	0	22	45	36	76
Seed node 3	36	41	22	0	22	32	54

Solving the associated model gives that $W_{12} = W_{52} = W_{43} = W_{53} = 1$ and all other variables are zero. This will be the same result as for variant 1 and the details are shown in table 6.

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