

# Modeling Freight Markets for Coal

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## Abstract

In this paper we study bulk shipping of coal between the central regions in the world. We compare the performance of cost-minimizing models with a gravity model approach. The main finding in the paper is that cost minimizing models provide relative poor fits to data. A simple one parameter gravity model, however, provides very satisfactory fits to observed behaviour.

Keywords: Bulk freight, cost efficiency, gravity modeling

Jel codes: F10, F17, R41

## 1 Introduction

In this paper we wish to examine yearly trade matrices for coal between the central regions in the world. The basic point of view in the paper is that distance is an important factor determining trades. Assuming that there is sufficient supply, and all else being equal, it seems reasonable to assume that import will take place from the closest exporting region. The observable import-export trade matrix for a given commodity is a reflection of both supply and demand factors, however, and the interaction of the two. When there is excess demand from a region, clearly import must be made from a more distant alternative. The final trade matrix is hence a compromise between several forces acting together in a rather complex manner. Nevertheless it seems reasonable to assume that the overall system should be cost efficient in the sense that the overall transportation cost is as low as possible.

To study whether or not cost efficiency is reflected in the actual trades, we have used data for total seaborne trades of coal in the period 1967-2006. Our data sets are described in detail in Section 2. For every year we have an observed trade matrix, and the marginal totals of this matrix defines the total demand and supply for coal in each region. Once the marginal totals are known, we can solve a cost minimization problem, i.e., compute mathematically how trades must be carried out to minimize the total transportation costs in the system. Once we have computed the mathematical solution to this problem, the solution can be compared with the observed trades.

In the period 1967 to 1979, we find that there are quite strong signs of cost efficiency in the sense that there is a good match between the cost efficient solution and the actual trades. From 1980 and onwards, however, this relationship fails completely. In the latter time period the observed trades are very different from what one would expect in a cost efficient system. It is hence of interest to consider more refined models that can explain the observed trades.

Gravity models have been an important tool to study trades deterred by distance for a considerable length of time. The first uses dates back to the early 60's with the works by Isard (1960), Tinbergen (1961), Pöyhönen (1963), Leontief and Strout (1963) and Linneman (1966). Since then a long series of scientific papers have been devoted to the subject. A detailed list of references to these papers would be too long to include here. We refer instead to the introductory chapter in Andersson (2007) where a considerable number of references are discussed. Gravity models can be specified and derived in many different ways. The references to the various versions and derivations are again too many to be included here. We refer instead to the seminal textbook of Sen and Smith (1995) and the references therein. In this paper we will only make use of a particularly simple version, including only one parameter. As the model fit using this simple version turns out to be very satisfactory, it seems like a one parameter version is sufficient to capture the main effects of distance. More parameters can of course be included into the model, and there are standard ways of doing this. As these more sophisticated extensions are concerned with secondary effects, they will not be discussed in this paper.

Our paper is organized as follows: As already mentioned our data sets are described in detail in Section 2. In Section 3 we describe the basic models used in the empirical study of our data sets.

In Section 4 we consider minimum cost models and compute how they fit the yearly observations. As the fit becomes really poor from 1980 and onwards, we compute the corresponding model fits using a one parameter standard gravity model. It turns out that this model provides very satisfactory fits to the observed behavior in the whole time period from 1967-1967. Finally in Section 5 we offer some concluding remarks and suggestions for future work.

## 2 The data sets

Our data sets have been extracted from Fearnley Reviews, and have been manually converted to the format we apply in the paper. As most imports of coal to Europe are shipped via Hampton Roads, the import data to different parts of Europe have been aggregated in our data sets. Exports of coal from North America have been split into data for USA and Canada. The total annual export from North America is reported by Fearnley Reviews, and we have used data from the IEA/OECD Coal statistics to carry out the split. The data for the first year (1967) is shown in Table 1.

Table 1: Coal total seaborne trade 1967

To From	Europe	South America	Japan
Europe	15.02	0.25	3.06
USA	17.62	2.34	11.1
Canada	0	0	0
Australia	0	0	8.96

(figures in million tonnes)

The format in Table 1 only applies in the period 1967-1979. New regions are included in the data sets from the year they start to appear in the reports from Fearnley Reviews: in 1980 exports from South Africa are included in the figures, in 1984 imports to S/E Asia are included, in 1990 exports from South America Carracas and China are included, and in 2000 exports from Indonesia are included as well. The final format which applies for the period 2000-2006 is shown in Table 2.

Table 2: Coal total seaborne trade 2006

To From	Europe	South America	Japan	S/E Asia
Europe	71.91	0.62	9.2	6.15
USA	24.68	7.42	0.3	0.52
Canada	0	0	8.6	6.46
Australia	28.36	14.32	103.22	40.75
South Africa	54.43	1.23	0	0.02
South Am. Car.	25.85	3.73	0.03	0
China	2.59	0.23	20.7	20.84
Indonesia	21.48	1.67	31.55	43.21

(figures in million tonnes)

Table 3: Distances in miles between the regions

To From	Europe	South America	Japan	S/E Asia
Europe	2000	6000	15000	14500
USA	3500	4700	9500	9700
Canada	8900	8300	4500	4700
Australia	11700	7400	4200	4000
South Africa	6900	3000	7500	7350
South Am. Car.	4500	3700	8080	8470
China	13700	11300	1049	505
Indonesia	9600	9500	2500	1300

### 3 Models of freight markets

In this section we will consider models for freight of a commodity. In general we assume that there are  $M$  origins from which the commodity is exported and  $N$  destinations importing the commodity. We let  $\mathbf{T} = \{T_{ij}\}_{i,j=1}^{M,N}$  denote the trade matrix, i.e.

$$T_{ij} = \text{total volume of trade from origin } i \text{ to destination } j \quad (1)$$

The basic point of view in the paper is that trade is deterred by distance, i.e., that freight over long distances are costly, so that import from nearby origins are preferred over more distant alternatives. We let  $\mathbf{d} = \{d_{ij}\}_{i,j=1}^{M,N}$  denote the distance matrix, i.e., the geographical distance from origin  $i$  to destination  $j$ . We assume that each origin  $i$  has a given supply  $S_i$  and that every destination  $j$  has a given demand  $D_j$ ,  $i = 1, \dots, M$ ,  $j = 1, \dots, N$ . That leaves us with

$M + N$  marginal constraints on the system, i.e.

$$\sum_{i=1}^M T_{ij} = D_j \quad \sum_{j=1}^N T_{ij} = S_i \quad i = 1, \dots, M, j = 1, \dots, N \quad (2)$$

We will always assume that supply equals demand for the system as a whole, i.e., that

$$\sum_{i=1}^M S_i = \sum_{j=1}^N D_j \quad (3)$$

One reasonable hypothesis on trade is that trade is subject to cost minimizing, i.e., that trade is carried out in a way such that the total traveling expenses in the system are as low a possible. A model based on this principle we call a cost minimizing model, and it is the solution of of the cost minimizing problem: Given the marginal constraints  $S_i, D_j, i = 1, \dots, M, j = 1, \dots, N$ , find a trade matrix  $\mathbf{T}^{\text{cost min}}$  such that the total traveling cost  $\sum_{i,j=1}^{M,N} T_{ij}d_{ij}$  is as small as possible.

In many circumstances, however, the approach above is too simple. In trade matrices of the above type one often observes zero entries. Sometimes such observations can happen by coincidence, but is more likely explained by an absence of demand for trade between the regions in question. This point of view makes it more natural to search for a trade matrix that is a solution of a modified problem: Given the marginal constraints  $S_i, D_j, i = 1, \dots, M, j = 1, \dots, N$  and an observed trade matrix  $\mathbf{T}^{\text{observed}}$ , find a trade matrix  $\mathbf{T}^{\text{cost min}}$  with zero entries where observed trades are zero and such that the total traveling cost  $\sum_{i,j=1}^{M,N} T_{ij}d_{ij}$  is as small as possible. When we discuss cost minimizing models in the empirical parts of the paper, these are always the solution of this modified problem.

While the cost minimizing model is the most cost efficient way of carrying out the trades, this model does not provide very good fits to the data we consider in this paper. Hence it is natural to consider alternative models which provides better fits to data. As we already have mentioned in the introduction, gravity models have been used to study export/import problems for a considerable length of time. These models can be formulated/derived in many different ways, all leading to more or less the same type of model. To mention a few main lines of approach, they can be derived from monopolistic competition, see, e.g., Harrigan (2003) and Fenstra (2004),

entropy-maximizing principles, see, e.g., Wilson (1967) or Batten and Boyce (1986), and random utility theory, see, e.g. Anas (1983). To link the gravity models to cost minimizing models, we include a brief discussion on how to derive a gravity model from probabilistic cost efficiency, see, e.g., Erlander and Smith (1990) or Jörnsten and Ubøe (2006).

Probabilistic cost efficiency is a weaker concept than the cost minimum principle we have discussed so far. To fit this machinery to the framework above, we need to assume that there is a discrete number of units being shipped from each region. Trades consistent with the marginal constraints then can be carried out in an extremely large number of different ways. The resulting trade matrices will, however, not be equally probable. The basic idea of probabilistic cost efficiency is to search for a probability measure with the property that trade matrices with lower total transportation costs are always more probable than trade matrices with higher transportation costs. Apparently this kind of structure would lead to a minimum cost matrix, but that is not true. Other solutions than a minimum cost matrix are indeed possible, and one can prove that the trades consistent with probabilistic cost efficiency forms a 1-parameter family of trade matrices given by the expression

$$T_{ij} = A_i B_j \exp[-\beta d_{ij}] \quad (4)$$

In this expression  $\beta \geq 0$  is a given parameter, and  $A_i$  and  $B_j$  are balancing factors that can be computed numerically by the classical Bregman balancing algorithm, Bregman (1967), using the marginal constraints

$$\sum_{i=1}^M T_{ij} = D_j \quad \sum_{j=1}^N T_{ij} = S_i \quad i = 1, \dots, M, j = 1, \dots, N \quad (5)$$

This family of trade matrices contains the cost minimizing model as a limiting case when  $\beta \rightarrow \infty$ . The model can easily be modified to force zero entries in positions where we have observed zero flows. Formally this is done setting the geographical distance to infinity in such cases, i.e.

$$T_{ij} = \begin{cases} A_i B_j \exp[-\beta d_{ij}] & \text{if } d_{ij} \neq \infty \\ 0 & \text{if } d_{ij} = \infty \end{cases} \quad (6)$$

The calculations carried out in this paper make use of the formula in (6) where the convention of

infinite distance is applied whenever the observed trade matrix has a zero entry. To calibrate this model against an observed matrix, we compute a value for  $\beta$  such that we obtain a best possible fit in the sense of loglikelihood. The likelihood values are, however, unsuitable to compare performance of different kinds of models across different sized data sets. In the next section we will hence consider more appropriate ways of describing the goodness-of-fit.

## 4 Model performance of cost minimum and gravity models

Model performance is often measured in terms of likelihood values, and variations thereof, like AIC. While these goodness of fit measures are very suitable to determine whether or not a model extension is statistically significant, there are cases where other measures provide more relevant information. In particular this happens when the difference in performance is very large, when different models are compared or when data sets differ in dimension. Knudsen and Fotheringham (1986) argue that the standardized root mean square error, SRMSE, is a more suitable tool in such cases. In this paper we wish to consider model performance in cases where our observations are  $M \times N$  matrices. In this case the SRMSE is defined as follows:

$$\text{SRMSE} = \frac{\sqrt{\frac{\sum_{i,j=1}^{M,N} (T_{ij}^{\text{observation}} - T_{ij}^{\text{model}})^2}{M \cdot N}}}{\frac{\sum_{i,j=1}^{M,N} T_{ij}^{\text{observation}}}{M \cdot N}} \quad (7)$$

Verbally one computes the root of the average squared error, and divides this by the average size of the observations. The advantage of this approach is that it produces a normalized value that can be interpreted regardless of the size of the observed matrix. Some rough rules of thumb is provided in Table 4 below:

SRMSE value	Interpretation
$\geq 1$	a poor fit to data, several entries out of scale
$\approx 0.75$	a fair fit capturing major trends in the data
$\leq 0.5$	a very good fit, large entries to scale, small entries small

To illustrate the guidelines in Table 4 we compute model fits using trade data from 2006, see Table 2. We first compare the observed trade volumes by a cost minimizing model, i.e., an

model where the total freight cost is as small as possible. The results are shown below.

$$T^{\text{obs}} = \begin{pmatrix} 71.91 & 0.62 & 9.2 & 6.15 \\ 24.68 & 7.42 & 0.3 & 0.52 \\ 0 & 0 & 8.6 & 6.46 \\ 28.36 & 14.32 & 103.22 & 40.75 \\ 54.43 & 1.23 & 0 & 0.02 \\ 25.85 & 3.73 & 0.03 & 0 \\ 2.59 & 0.23 & 20.7 & 20.84 \\ 21.48 & 1.67 & 31.55 & 43.21 \end{pmatrix} \quad T^{\text{cost min}} = \begin{pmatrix} 87.89 & 0 & 0 & 0 \\ 32.93 & 0 & 0 & 0 \\ 0 & 0 & 15.06 & 0 \\ 23.21 & 29.23 & 134.22 & 0 \\ 55.68 & 0 & 0 & 0 \\ 29.61 & 0 & 0 & 0 \\ 0 & 0 & 24.34 & 20.02 \\ 0 & 0 & 0 & 97.92 \end{pmatrix}$$

Observed values and values produced by a cost minimizing model (2006 data).

While some entries are quite similar, the overall performance of this approach is rather poor. We particularly note some very large differences in  $T_{44}, T_{81}, T_{83}$ . Several more entries are predicted to be zero by the model but observed values are not very small. In summary a cost minimizing model does not seem to offer a very good explanation of the observed trades. This lack of performance is reflected in the SRMSE value which is 0.92 in this case.

As a next step we calibrate a one parameter standard gravity model to the same data. The results are shown below.

$$T^{\text{obs}} = \begin{pmatrix} 71.91 & 0.62 & 9.2 & 6.15 \\ 24.68 & 7.42 & 0.3 & 0.52 \\ 0 & 0 & 8.6 & 6.46 \\ 28.36 & 14.32 & 103.22 & 40.75 \\ 54.43 & 1.23 & 0 & 0.02 \\ 25.85 & 3.73 & 0.03 & 0 \\ 2.59 & 0.23 & 20.7 & 20.84 \\ 21.48 & 1.67 & 31.55 & 43.21 \end{pmatrix} \quad T^{\text{gravity model}} = \begin{pmatrix} 79.66 & 3.64 & 2.79 & 1.8 \\ 25.26 & 2.03 & 3.62 & 2.02 \\ 0 & 0 & 9.66 & 5.4 \\ 39.55 & 9.6 & 85.64 & 51.87 \\ 37.65 & 8.44 & 0 & 9.58 \\ 21.88 & 2.63 & 5.1 & 0 \\ 3.95 & 0.65 & 24.11 & 15.65 \\ 21.37 & 2.23 & 42.7 & 31.62 \end{pmatrix}$$

Observed values and values produced by a standard gravity model (2006 data).

By inspection of the matrices above, we see that model performance is considerably better in



this case. In particular we note that the large differences in  $T_{44}, T_{81}, T_{83}$  has disappeared. The improved fit is reflected in the SRMSE value which is 0.40 in this case. Taking into account that our model only has one degree of freedom, the model performance is better than one could reasonably expect. This suggests that gravity models might be the right tool to study trades of this type. Clearly one cannot draw any definite conclusions based on only one such data set. To pursue this question in more detail, we have carried out the above comparison for trade data for every year in the period 1967 to 2006. It then turns out that our results using 2006 data are typical. The SRMSE values for the two models are shown in Figure 1:

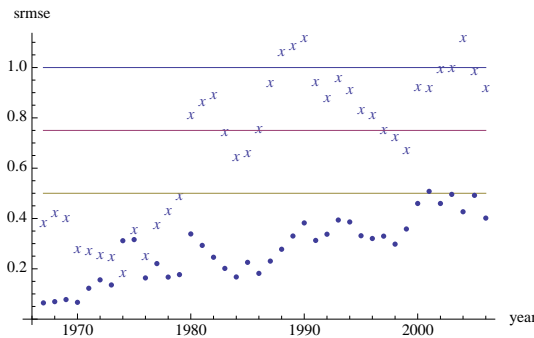


Figure 1

In Figure 1 “x” denotes the SRMSE value for the cost minimizing model, while “.” denotes the corresponding fit for a one parameter gravity model. From the figure we see that the gravity model provides a very good fit to data in the whole period from 1967 to 2006. The worst fit is observed in 2001, SRMSE = 0.51, which is still very good. The cost minimizing model performs reasonably good up to 1979. In particular we note the year 1974, where the observed trades are very close to a cost minimum. In the remaining time period, however, the performance of the cost minimizing model is rather poor.

One possible hypothesis explaining the results above would be that freight cost are so small compared to other expenses, that freight distance has very little impact on trade. To examine this question in more detail we carried out the same calculations for a gravity model where the impact of distance is set to zero. A model of this type distributes trades directly in proportion to the size of demand and supply. The results for the 2006 data are shown below:

$$T^{\text{obs}} = \begin{pmatrix} 71.91 & 0.62 & 9.2 & 6.15 \\ 24.68 & 7.42 & 0.3 & 0.52 \\ 0 & 0 & 8.6 & 6.46 \\ 28.36 & 14.32 & 103.22 & 40.75 \\ 54.43 & 1.23 & 0 & 0.02 \\ 25.85 & 3.73 & 0.03 & 0 \\ 2.59 & 0.23 & 20.7 & 20.84 \\ 21.48 & 1.67 & 31.55 & 43.21 \end{pmatrix} \quad T^{\text{proportional model}} = \begin{pmatrix} 35.28 & 4.5 & 29.64 & 18.47 \\ 13.22 & 1.68 & 11.1 & 6.92 \\ 0 & 0 & 9.28 & 5.78 \\ 74.93 & 9.55 & 62.95 & 39.22 \\ 33.72 & 4.3 & 0 & 17.65 \\ 15.05 & 1.92 & 12.64 & 0 \\ 17.81 & 2.27 & 14.96 & 9.32 \\ 39.31 & 5.01 & 33.03 & 20.58 \end{pmatrix}$$

Observed values and values produced by a proportional trade model (2006 data).

As we can see by inspection of the values, the overall performance is very poor. There are hardly any similarities between the two matrices. The very bad fit is reflected in the SRMSE value which is 0.94 in this case. The pattern we observe in 2006 turns out to be typical and is what we observe every year in the period 1967-2006. The model fits for the whole time period are shown in Figure 2:

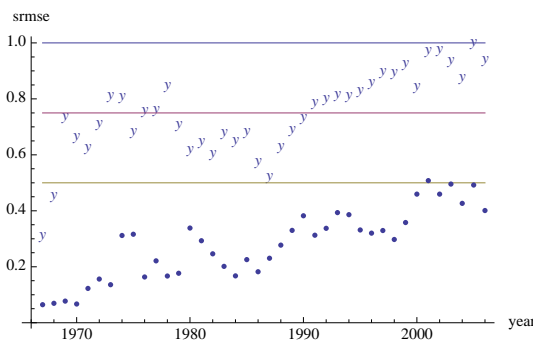


Figure 2

In Figure 2 “y” denotes the proportional trade model, while “.” denotes the corresponding fit for a one parameter gravity model. The conclusion we can draw from this model is very clear. Distances do matter, and a model ignoring distances and distributing trades directly in proportion to the size of demand and supply is unsuitable to explain the observed trades.

## 5 Concluding remarks

In this paper we have studied export-import data for coal in the time period 1967-2006. The main finding in the paper is that a minimum cost model offers a reasonably good fit to the observed data in the period 1967-1979. In the period 1980-2006, a minimum cost model has relatively poor fits to data. A 1-parameter gravity model, however, provides very good fits to data in the whole time period from 1967-2006.

An interesting next step is to see how these gravity models can be used to predict trade flows. Such predictions can be made in several different ways, and we will here only outline two possible lines of approach. The gravity model takes the total supply and demand in each region as input. To predict flows in the future, one could consider time series of the supply and demand in each region together with the time series for the deterrence parameter in the gravity model, i.e., consider for  $t = 1, \dots, T$

$$L_i^{(t)} \quad i = 1, \dots, M \quad E_j^{(t)} \quad j = 1, \dots, N \quad \beta_t \quad (8)$$

From the time series in (8), one could then try to predict the level of demand and supply and the value for the deterrence parameter in the next time period, i.e.

$$L_i^{(T+1)} \quad i = 1, \dots, M \quad E_j^{(T+1)} \quad j = 1, \dots, N \quad \beta_{T+1} \quad (9)$$

These predicted values could then be used as input to the gravity model, and the output would offer a prediction for the trade flows at time  $T + 1$ . One complicating matter is that the total demand must equal the total supply in the gravity model. If this is not satisfied, a solution cannot be found. The prediction of supply and demand must take this into account, and it is not clear how this can be done. Many lines of approach are possible but these are left for future research. Another line of approach would be to consider time series for the balancing factors in the gravity model, i.e., to consider

$$T_{ij}^{(t)} = A_i^{(t)} B_j^{(t)} \exp[-\beta_t d_{ij}] \quad (10)$$

where  $A_i^{(t)}$ ,  $B_j^{(t)}$  and  $\beta_t$  are considered as latent time series processes. This can be formulated as

a state space model and estimated by means of the Kalman filter or computer intensive methods. One could then try to predict

$$A_i^{(T+1)} \quad i = 1, \dots, M \quad B_j^{(T+1)} \quad j = 1, \dots, N \quad \beta_{T+1} \quad (11)$$

and compute a prediction using the new balancing factors in the model. The latter approach has the advantage that it avoids the matching of demand and supply in the first approach. A further analysis of these issues is beyond the scope of the present paper, however, and is left for future research.

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