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Stepwise Investment and Capacity Sizing under Uncertainty

BY

Michail Chronopoulos, Verena Hagspiel AND **Stein-Erik Fleten**

Norges Handelshøyskole

NORWEGIAN SCHOOL OF ECONOMICS

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Michail Chronopoulos * Verena Hagspiel † Stein–Erik Fleten ‡

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Abstract

The relationship between uncertainty and managerial flexibility is particularly crucial in addressing capital projects. We consider a firm that can invest in a project in either a single (lumpy investment) or multiple stages (stepwise investment) under price uncertainty and has discretion over not only the time of investment but also the size of the project. We confirm that, if the capacity of a project is fixed, then lumpy investment becomes more valuable than a stepwise investment strategy under high price uncertainty. By contrast, if a firm has discretion over capacity, then we show that the stepwise investment strategy always dominates that of lumpy investment. In addition, we show that the total amount of installed capacity under a stepwise investment strategy is always greater than that under lumpy investment.

Keywords: investment analysis, capacity sizing, flexibility, real options

*Department of Business and Management Science, Norwegian School of Economics, Bergen, Norway
†Norwegian University of Science and Technology, Department of Industrial Economics, Trondheim, Norway
‡Norwegian University of Science and Technology, Department of Industrial Economics, Trondheim, Norway

1 Introduction

In irreversible investment, firms tend to split the projects in distinct phases. We explain this behavior in a setup where there is uncertainty, discretion over timing, and over the choice of project scale. According to standard economic literature (Arrow and Fisher, 1974; and Henry, 1974), investment decisions are influenced by three main factors, namely, uncertainty, irreversibility, and flexibility. The first refers to the uncertainty in the cash–flows that a project generates, the second to the inability to recover the investment cost after investment has taken place, and the third to the discretion over the timing of investment. The latter allows for uncertainty in underlying parameters to resolve before making an irreversible investment decision. Consequently, the ability to optimise the investment timing raises the expected value of the investment opportunity, which, in, turn, implies that investment is delayed relative to the traditional net present value (NPV) approach due to the opportunity cost of killing the timing option. In fact, this hesitation is prolonged as uncertainty increases, since the value of waiting increases. Interestingly, however, this result does not extend analogously to other types of flexibilities, and, in spite of the extensive literature that challenges the traditional views of how uncertainty and irreversibility explain investment behaviour (Alvarez and Stenbacka, 2004; Abel and Eberly, 1996), the interaction between uncertainty and different types of flexibilities has not been thoroughly examined yet. By developing an analytical framework for investment under uncertainty, we explore how discretion over project scale impacts a firms incentive to invest in stages.

Indeed, one crucial type of managerial discretion is the flexibility to either invest in an entire project at a single point in time (lumpy investment) or divide it into smaller, modular projects and then invest in each one at distinct points in time (stepwise investment). Empirical evidence indicates that modularity can have crucial implications for the value of a project. For example, Baldwin and Clark (2002) illustrate how the market value of the computer industry increased from the 1980s until the 1990s as a result of its transition from a highly concentrated market, in which IBM played a dominant role, to a large modular cluster of firms. By contrast, at the firm level and within the context of sequential capacity expansion, Kort *et al.* (2010) show how uncertainty reduces the value of modularity. More specifically, they show that, if the modularisation of a project is costly and the size of each module is fixed, then, high price uncertainty lowers the value of modularity in favour of lumpy investment. This happens because high levels of uncertainty lower the incentive to make costly switches between stages, and, as a result, lumpy investment dominates the stepwise investment strategy. A limitation of this work is that it studies a particular type of flexibility in isolation, when, in reality, firms can typically combine different types of managerial flexibilities. Consequently, how various types of managerial discretion, e.g., discretion over capacity, option to abandon, etc., interact to affect the value of modularity under increasing uncertainty remains an open question.

We address this disconnect by analysing how the flexibility to choose between a lumpy and a stepwise investment strategy interacts with discretion over capacity under price uncertainty. This situation is relevant to various industries, e.g., renewable energy (RE) power plants. Indeed, in the case of both on– and off–shore wind farms an area can, and often is, developed in stages. Additionally, for capital intensive projects, discretion over capacity is particularly crucial, since the installation of a large project increases a firm's exposure to downside risk in the case of a potential downturn, whereas the installation of a small project limits a firm's upside potential if market conditions suddenly become favourable. Thus, we contribute to the existing literature by developing an analytical framework in order to explore how discretion over capacity interacts with the flexibility to choose between lumpy and stepwise investment under price uncertainty. Additionally, we derive analytical results regarding the impact of uncertainty on the optimal investment threshold, the optimal capacity, and the choice of investment strategy.

We proceed in Section 2 by discussing some related literature and introduce assumptions and notation in Section 3. In Section 4, we formulate the problem and derive analytical expressions for the value of the option to invest, the optimal investment threshold, and the corresponding optimal capacity under lumpy and stepwise investment. In addition, we present analytical results regarding the impact of uncertainty on the choice of investment strategy. We present numerical results in Section 5 and conclude in Section 6.

2 Related Work

The seminal work of Majd and Pindyck (1987) and Dixit and Pindyck (1994) has spawned a substantial literature in the area of sequential investment. The former show how traditional valuation methods understate the value of a project by ignoring the flexibility embedded in the time to build, while the latter develop a sequential investment framework with infinite investment options assuming that the project value depreciates exponentially. The value of modularity is emphsasised in Gollier *et al.* (2005), who allow for electricity price uncertainty and compare a sequence of small nuclear power plants with a single nuclear power plant of large capacity. Intriguingly, their results indicate that the option value of modularity may trigger investment in the initial module at a level below the now–or–never NPV. In the same line of work, Malchow–Møller and Thorsen (2005) illustrate how the investment policy resembles the simple NPV rule when investing sequentially in subsequent upgrades of a technology. They find that the expected value of subsequent upgrades reduces the value of waiting to invest in the current version significantly, while the investment rule is less sensitive to changes in uncertainty. The advantages of modularity have also been stressed within the context of investment in distributed generation capacity. For example, Siddiqui and Maribu (2009) analyse how sequential investment may reduce the exposure of a microgrid to natural gas price volatility, and find that a direct (sequential) investment strategy is more preferable for low (high) levels of volatility. By contrast, Kort *et al.* (2010) show that, if stepwise investment is more costly than lumpy investment, then high price uncertainty promotes the latter strategy by reducing the incentive to make costly switches between stages. Siddiqui and Takashima (2013) combine strategic interactions with sequential capacity expansion in order to explore how sequential decision making offsets the effect of competition. They find that the loss in the value of a firm due to competition is reduced when the firm invests in stages and specify the conditions under which sequential capacity expansion is more valuable for a duopolist firm than for a monopolist.

From a more empirical standpoint, Rodrigues and Armada (2007) present a real options approach to the valuation of modular projects, and show that modularisation can increase the value of a project depending on the relative values, costs, and risk of each modular configuration. Gamba and Fusari (2009) develop a valuation approach based on real options theory in order to address the issues that a modularisation process poses in terms of financial valuation for capital budgeting. More specifically, they first create a stochastic optimal control framework for the six modular operators proposed by Baldwin and Clark (2000), and then adopt the least–squares Monte Carlo method of Longstaff and Schwartz (2001) in order to cope with the dynamic programming feature of the valuation problems. While the aforementioned literature offers a thorough analytical and empirical treatment of the value of modularity and of sequential investment under uncertainty, it ignores the potential implications from allowing for other types of managerial discretion that firms typically take into account when designing an optimal investment policy, e.g., discretion over capacity, suspension and resumption options, etc.

Indeed, apart from discretion over the investment strategy, e.g., lumpy versus stepwise, firms typically also have discretion over the size of a project, in the form of installed capacity. Dangl (1999) addresses the problem of a firm that invests in a project with continuously scalable capacity under demand uncertainty, and shows that, even when demand is high, low uncertainty makes waiting for further information the optimal strategy. A similar approach is adopted by Bøckman *et al.* (2008) for valuing small hydropower projects under electricity price uncertainty, however, unlike to Dangl (1999), they assume a cost function that is convex in capacity, and, therefore, their model is more pertinent to the energy sector. Huisman and Kort (2009) introduce game–theoretic considerations and show how, in a duopolistic competition, a leader can use discretion over capacity strategically in order to deter a follower's entry temporarily. A policy–oriented model for investment and capacity sizing is presented by Boomsma *et al.* (2012), who analyse the impact of uncertainty stemming from different types of policy mechanisms on investment and capacity sizing decisions. The impact of risk aversion on such decisions when a firm has operational flexibility is addressed in Chronopoulos *et al.* (2012), who find that higher risk aversion facilitates investment by decreasing the optimal capacity of a project.

In the area of discrete capacity sizing, Dixit (1993) analyses the choice among mutually exclusive projects of various capacities under uncertainty. The proposed decision rule requires that the projects are first ranked by capacity and then analysed separately in order to determine the corresponding investment thresholds. The optimal project is the largest one for which the optimal threshold is greater than the current price. A limitation in the approach of Dixit (1993) is identified by Décamps *et al.* (2006), who determine an intermediate waiting region around the indifference point between the NPVs of two projects. Consequently, it may be better to select the smaller project should the price drop sufficiently rather than wait for the price to hit the upper threshold in order to select the larger project. Additionally, they allow for the option to switch to a larger capacity after having made an initial investment in the smaller project. Fleten *et al.* (2007) adopt the framework of Décamps *et al.* (2006) in order to model investment in wind turbines taking the perspective of an investor who must choose among discrete alternatives and has discretion over both the time of investment and the size of the project.

Apart from analysing the value of discretion over capacity in isolation, a strand of literature combines it with various types of operational flexibilities. For example, He and Pindyck (1992) allow for demand uncertainty and examine the technology and capacity choice problem of a firm that can install either output–specific or flexible capital, which may be used to produce different outputs. They formulate the capacity choice problem as a stochastic control problem, and show that the value of the firm equals the value of its installed capital plus the expected value of its options to add capacity in the future. Hagspiel *et al.* (2014) allow for production flexibility and compare a flexible scenario, in which a firm can adjust production over time with the capacity level as the upper bound, to the inflexible scenario, in which a firm fixes production at capacity level from the moment of investment onward. Among other results, they find that the flexible firm invests in higher capacity than the inflexible firm and that the capacity difference increases with uncertainty. Considering the choice between two types of technologies, Takashima *et al.* (2012) find that price uncertainty induces investors to maximise expected profits by building larger plants, while the consideration of mutually exclusive projects increases the option value of the entire investment opportunity.

We extend the existing literature by developing and analytical framework that combines two important types of managerial discretion, i.e., the flexibility to invest in either a single or multiple stages with discretion over capacity. Although increasing uncertainty favours a lumpy over a more flexible, yet more costly, stepwise investment strategy when the capacity of a project is fixed, the implications from allowing for discretion over capacity are not thoroughly examined yet. For this reason, we assume that the capacity of the project is continuously scalable, and, in line with Dangl (1999), the firm has the option to fix the capacity of the project at investment. We first confirm the results of Kort *et al.* (2010) and then show that, although the relative value of the two strategies decreases with greater uncertainty, the stepwise investment strategy always dominates that of lumpy investment. This seemingly counter–intuitive result happens because the firm can optimise the size of the project in response to an increase in the cost of the stepwise investment strategy relative to that of lumpy investment. Intuitively, the extra flexibility to optimise the size of the project mitigates the loss in project value due to the higher cost associated with the flexibility to proceed in stages, thereby offsetting the benefit of a lower investment cost via lumpy investment.

3 Assumptions and Notation

We consider a price–taking firm that holds an option to invest in a project of infinite lifetime that may be completed in either a single or a sequence of i discrete stages with $i \in \mathbb{N}$. Also, the firm can either exercise an investment option immediately or delay investment in the light of price uncertainty. We assume that there is no variable production cost and that the output price at time t, P_t, where $t \geq 0$ is continuous and denotes time, follows a geometric Brownian motion (GBM) that is described in (1)

$$dP_t = \mu P_t dt + \sigma P_t dZ_t, \quad P_0 \equiv P > 0 \tag{1}$$

where μ is the annual growth rate, σ is the annual volatility, and dZ_t is the increment of the standard Brownian motion. Also, $\rho > \mu$ is the subjective discount rate. The capacity of the project is denoted by K_j when the firm has discretion over investment timing and by $\overline{K}_j$ when the firm exercises a now–or–never investment opportunity. Additionally, $\overline{F}_j(\cdot)$ is the expected value of the now–or–never investment opportunity, where $j \in \{\ell, s_i\}$ (denoting lumpy and staged investment respectively), while $\overline{k}_j$ is the corresponding optimal capacity. For example, $\overline{F}_\ell(\cdot)$ denotes the expected value of the now–or–never investment opportunity under lumpy investment and $\overline{k}_\ell$ is the corresponding optimal capacity. If the option to defer investment is available, then $F_j(\cdot)$ denotes the maximised option value, while τ_j, p_j, and k_j denote the time of investment, the optimal investment threshold, and the corresponding optimal capacity, respectively. Note that we use upper–case letters to denote state variables for capacity and output price and lower–case letter for the corresponding optimal thresholds. The investment cost, $I\left(K_j\right)$, is indicated in (2)

$$I\left(K_j\right) = a_j K_j + b_j K_j^{\gamma_j}, \quad a_j, b_j, \quad \text{and} \quad \gamma_j > 1 \tag{2}$$

where $\gamma_j > 1$ implies that this model is more suitable for describing projects that exhibit diseconomies of scale. In the energy sector, this is the case with RE power plants, while more general examples where the use of a convex cost function can be realistic include a monopsonistic environment in which a firm contemplates investment facing increasing prices due to increasing demand. As it becomes clear in Section 4, the assumption $\gamma_j > 1$ should be considered as an implication of the model itself, and, therefore, is not restricting the analytical results. In fact, as we discuss in Section 4, the results of the paper are more general as they are expected to hold also under economies of scale, i.e., $\gamma_j < 1$. For the purpose of comparing a lumpy investment to a strategy that entails a series of modular investments, we assume that each individual stage of the stepwise investment strategy is less costly than the entire project. However, in line with Kort *et al.* (2010), we assume that the flexibility to proceed in stages is costly, and, therefore, requires the firm to incur a premium. Consequently, the total investment cost under a stepwise investment strategy is greater than that under lumpy investment, as indicated in (3).

$$I\left(K_{s_i}\right) < I\left(K_\ell\right), \forall i \in \mathbb{N} \qquad \text{and} \qquad \sum_i I\left(K_{s_i}\right) > I\left(K_\ell\right) \tag{3}$$

Notice that, although a firm may have the flexibility to respond to low prices by producing at a level below the installed capacity, in this paper, we assume that a firm does not have production flexibility. This is often referred to as the *clearance* assumption and is widely used in the literature (Chod and Rudi, 2005; Anand and Girotra, 2007). For example, in the energy sector this assumption is relevant to baseload and RE power plants. Additionally, fixed costs, e.g., commitments to suppliers and production ramp–up, make it too costly to produce below the capacity level (Goyal and Netessine, 2007). In the car industry, firms often prefer to reduce prices in order to maintain production at full capacity, instead of producing below capacity (Mackintosh, 2003).

4 Model

The firm's optimisation objective under each investment strategy, i.e., lumpy and stepwise investment, is summarised in (4). The outer maximisation corresponds to the general decision on whether to invest immediately or delay investment. If the firm decides to wait for an infinitesimal time interval dt, then, according to the Bellman principle, the value that the firm holds is the discounted expected value of the capital appreciation of the option to invest. This is represented by the first argument of the maximisation on the right–hand side of (4). By contrast, the second argument of the outer maximisation represents the value that the firm receives if it decides to exercise a now–or–never investment opportunity. More specifically, the inner maximisation indicates that when the firm decides to invest it will chose the capacity of the project in such a way that maximises its expected NPV.

$$F_j(P) = \max\left\{(1-\rho dt)\mathbb{E}_P\left[F_j(P+dP)\right], \max_{\overline{K}_j}\left[\overline{F}_j\left(P, \overline{K}_j\right)\right]\right\}, \quad j = \ell, s_i \text{ and } i = 1, 2 \tag{4}$$

We begin by assuming that the firm adopts a lumpy investment strategy. In this case, the firm can delay investment until τ_ℓ, at which point it must fix the capacity of the entire project, K_ℓ, as shown in Figure 1. Consequently, K_ℓ is a function of the output price, P_ℓ, at time τ_ℓ.

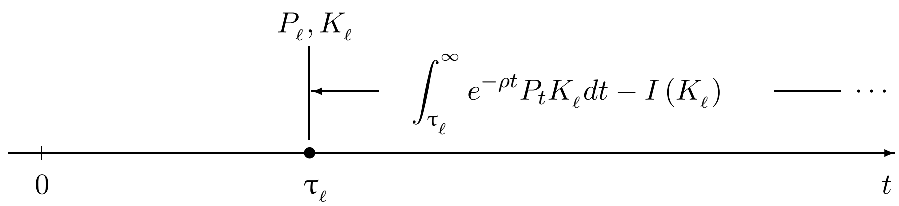

Figure 1: Lumpy investment

Initially, we assume that the firm ignores the option to wait for more information and invests in the project immediately. Hence, we first address the inner maximisation in (4). The expected value of the now–or–never investment opportunity is indicated in (5).

$$\overline{F}_\ell\left(P, \overline{K}_\ell\right) = \frac{P\overline{K}_\ell}{\rho - \mu} - I\left(\overline{K}_\ell\right) \tag{5}$$

Consequently, at investment, the output price, P, is known, and, therefore, the firm needs to determine only the corresponding optimal capacity, $\overline{k}_\ell$, by maximising the value of the now–or–never investment opportunity, as indicated in (6).

$$\max_{\overline{K}_\ell} \overline{F}_\ell\left(P, \overline{K}_\ell\right) \quad \Rightarrow \quad \overline{k}_\ell(P) = \left[\frac{1}{b_\ell \gamma_\ell}\left(\frac{P}{\rho - \mu} - a_\ell\right)\right]^{\frac{1}{\gamma_\ell - 1}} \tag{6}$$

We proceed by considering the outer maximisation in (4). If the firm can defer investment, then the value of the option to invest is described in (7), where $\mathcal{S}$ denotes the set of stopping times of the filtration generated by the price process and $\mathbb{E}_P$ is the expectation operator, which is conditional on the initial value, P, of the price process.

$$F_\ell(P) \quad = \quad \sup_{\tau_\ell \in \mathcal{S}} \mathbb{E}_P\left[\int_{\tau_\ell}^{\infty} e^{-\rho t} P_t K_\ell dt - I(K_\ell)\right] \tag{7}$$

Using the law of iterated expectations and the strong Markov property of the GBM, which states that price values after time τ_ℓ are independent of the values before τ_ℓ and depend only on the value of the process at τ_ℓ, we can rewrite (7) as in (8). The stochastic discount factor $\mathbb{E}_P\left[e^{-\rho\tau_\ell}\right] = \left(\frac{P}{P_\ell}\right)^{\beta_1}$ (Dixit and Pindyck, 1994, p.315), where and $\beta_1 > 1, \beta_2 < 0$ are the roots of $\frac{1}{2}\sigma^2\beta(\beta-1)+\mu\beta-\rho = 0$.

$$F_\ell(P) = \sup_{\tau_\ell \in \mathcal{S}} \mathbb{E}_P\left[e^{-\rho\tau_\ell}\right] \mathbb{E}_{P_\ell}\left[\int_0^\infty e^{-\rho t} P_t K_\ell dt - I\left(K_\ell\right)\right] = \max_{P_\ell \geq P}\left(\frac{P}{P_\ell}\right)^{\beta_1}\left[\frac{P_\ell K_\ell}{\rho-\mu} - I\left(K_\ell\right)\right] \qquad (8)$$

Solving the unconstrained maximisation problem (8), we can express the maximised option value, $F_\ell(P)$, as in (9). The endogenous constant, A_ℓ, the optimal investment threshold, p_ℓ, and the corresponding optimal capacity, k_ℓ, are determined via value–matching and smooth–pasting conditions between the two branches of (9) together with the condition for optimal capacity choice at investment (6) and are indicated in (A–7), (A–8), and (A–9), respectively for $j = \ell$ (all proofs can be found in the appendix).

$$F_\ell(P) = \begin{cases} A_\ell P^{\beta_1} & , \text{ for } P < p_\ell \\ \frac{P k_\ell}{\rho-\mu} - I\left(k_\ell\right) & , \text{ for } P \geq p_\ell \end{cases} \qquad (9)$$

Next, we assume that the firm adopts a stepwise investment strategy, and, without loss of generality, we assume that a stepwise investment comprises of two stages, i.e., $i \leq 2$. As indicated in Figure 2, the firm has the option to delay investment in the first stage until τ_{s_1}, at which point it must fix the corresponding capacity, K_{s_1}. The firm receives the revenues of the first stage until τ_{s_2}, at which point it fixes the capacity of the second stage, K_{s_2}. After the firm invests in the second stage, it incurs the corresponding cost and receives the revenues from both stages.

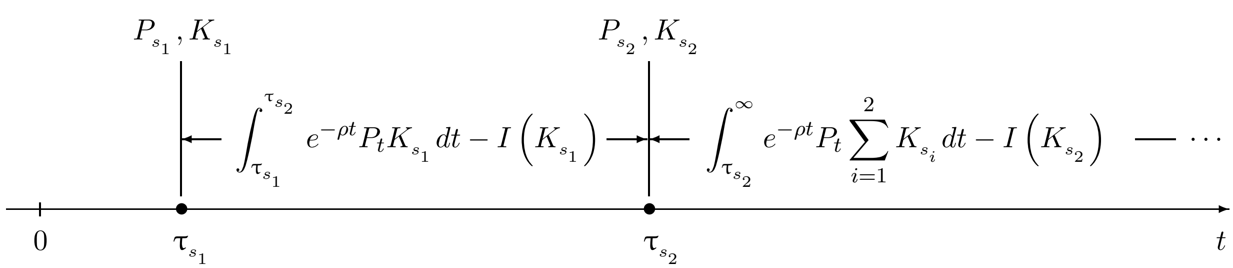

Figure 2: Stepwise investment

The optimal capacity at each stage of the project when the firm invests immediately is obtained by maximising the value of the now–or–never investment opportunity. Following the same approach as in the case of lumpy investment, the optimal capacity for each stage is indicated in (10). Notice that the value of the now–or–never investment opportunity is the sum of the maximised NPVs from each stage, i.e., $\overline{F}_s(P) = \sum_i \overline{F}_{s_i}\left(P, \overline{k}_{s_i}\right)$.

$$\max_{\overline{K}_{s_i}} \overline{F}_{s_i}\left(P, \overline{K}_{s_i}\right) \quad \Rightarrow \quad \overline{k}_{s_i}(P) = \left[\frac{1}{b_{s_i}\gamma_{s_i}}\left(\frac{P}{\rho-\mu} - a_{s_i}\right)\right]^{\frac{1}{\gamma_{s_i}-1}} \tag{10}$$

If the option to delay investment is available, then the optimisation objective is described in (11). Notice that by completing the first stage, the firm receives the option to proceed to the second. As a result, the option to invest in the first stage may be seen as a compound option.

$$\begin{aligned} F_s(P) \quad = \quad & \sup_{\tau_{s_1} \in \mathcal{S}} \mathbb{E}_P \left[\sup_{\tau_{s_2} \geq \tau_{s_1}} \mathbb{E}_P \left[\int_{\tau_{s_1}}^{\tau_{s_2}} e^{-\rho t} P_t K_{s_1} dt - I\left(K_{s_1}\right) \right.\right. \\ & \left.\left. + \int_{\tau_{s_2}}^{\infty} e^{-\rho t} P_t \sum_{i=1}^{2} K_{s_i} dt - I\left(K_{s_2}\right) \right]\right] \end{aligned} \tag{11}$$

By decomposing the first integral on the right–hand side of (11), we can express the original problem as two separate optimal stopping–time problems, as in (12)

$$\begin{aligned} F_s(P) \quad = \quad & \sup_{\tau_{s_1} \in \mathcal{S}} \mathbb{E}_P \left[\int_{\tau_{s_1}}^{\infty} e^{-\rho t} P_t K_{s_1} dt - I\left(K_{s_1}\right) \right] \\ & + \sup_{\tau_{s_2} \geq \tau_{s_1}} \mathbb{E}_P \left[\int_{\tau_{s_2}}^{\infty} e^{-\rho t} P_t K_{s_2} dt - I\left(K_{s_2}\right) \right] \end{aligned} \tag{12}$$

and the solution of each of the two optimal stopping–time problems is expressed in (13), where A_{s_i}, p_{s_i}, and k_{s_i} are indicated in (A–7), (A–8), and (A–9), respectively. Notice that the value of the option to invest is the sum of the respective option values of each stage, i.e., $F_s(P) = \sum_i F_{s_i}(P)$.

$$F_{s_i}(P) \quad = \quad \begin{cases} A_{s_i} P^{\beta_1} & , \text{ for } P < p_{s_i} \\ \frac{P k_{s_i}}{\rho - \mu} - I\left(k_{s_i}\right) & , \text{ for } P \geq p_{s_i} \end{cases} \tag{13}$$

Proposition 4.1 *The optimal investment threshold and the corresponding optimal capacity under* *lumpy and stepwise investment are:*

$$p_j\left(k_j\right) = \frac{I\left(k_j\right)}{k_j}\frac{\beta_1(\rho-\mu)}{\beta_1-1} \quad and \quad k_j = \left[\frac{a_j}{b_j}\frac{1}{\gamma_j(\beta_1-1)-\beta_1}\right]^{\frac{1}{\gamma_j-1}}, \quad \gamma_j(\beta_1-1)-\beta_1>0 \tag{14}$$

Assuming that $\tau_{s_2} \geq \tau_{s_1}$, Proposition 4.2 indicates that the decision to invest in the first stage is independent of the presence of the second. Intuitively, this result is a consequence of the optimality of myopic behavior based on which a firm disregards subsequent investment decisions when evaluating the current one. Within the context of capacity expansion, this property implies that an investment in new capacity is evaluated assuming that it is the last one in the horizon. The optimality of myopic behavior is not generally true but holds under certain assumptions. For example, Bertola (1989) and Pindyck (1988, 1993) show that it holds in case of a monopoly as considered in this paper. [1]

Proposition 4.2 *p_{s_1} is independent of p_{s_2}.*

In line with the standard real options intuition, Proposition 4.3 indicates that greater uncertainty raises both the optimal capacity of the project and the optimal investment threshold. This happens because greater uncertainty increases the opportunity cost of an irreversible investment decision, thereby raising the value of waiting. Furthermore, from (6) we know that the optimal capacity of the project is a monotonic function of the output price. Consequently, an increase in the optimal investment threshold results in the installation of a bigger project.

Proposition 4.3 $\frac{\partial k_j}{\partial \sigma} > 0$ *and* $\frac{\partial p_j}{\partial \sigma} > 0$.

Interestingly, as Proposition 4.4 indicates, if the firm has discretion over capacity, then the value of the option to proceed in stages is always greater than that under lumpy investment. This is in contrast to Kort *et al.* (2010) who show that, under relatively large uncertainty, the single stage investment is more attractive relative to a more flexible, yet more costly, stepwise investment strategy. This seemingly counter–intuitive result is based on the endogenous relationship between the price at investment and the capacity of the project. Notice that, if a firm has discretion over capacity, then, according to (14), the optimal capacity, k_j is non–negative if $\gamma_j(\beta_1-1)-\beta_1>0$. However, while greater uncertainty lowers the relative value of the two strategies, it also decreases

[1] Optimality of myopia also holds within a context of strategic interactions provided that the profit is additively separable if more that one technology is considered (Baldurson and Karatzas, 1997).

β_1. According to Proposition 4.4, the relative value of the two strategies does not decrease below one for non–negative values of k_j. Intuitively, although the value of the stepwise investment strategy is reduced due to the cost that a firm incurs for the flexibility to proceed in stages, the extra flexibility to scale the capacity of the project allows the firm to offset the reduction in the value of the stepwise investment strategy completely. Indeed, if the capacity of the project was fixed, then greater uncertainty would delay investment but the amount of installed capacity would remain unaffected. In fact, according to Kort *et al.* (2010), the stepwise investment strategy dominates under low levels of uncertainty even if it entails the installation of the same capacity size at a greater cost than lumpy investment. By contrast, discretion over capacity allows a firm to respond to an increase in the investment cost by optimising the endogenous relationship between the size of the project and the time of investment.

Proposition 4.4 *If a firm has discretion over capacity, then* $F_s(P) > F_\ell(P)$.

Moreover, from (14) we see that the existence of an optimal solution to the investment problem under each strategy requires that the cost function is strictly convex, i.e., $\gamma_j(\beta_1 - 1) - \beta_1 > 0 \Leftrightarrow \gamma_j > \frac{\beta_1}{\beta_1 - 1} > 1$. Therefore, the convexity of the cost function is not an assumption, as indicated in (2), but rather a property implied by the analytical framework itself. More specifically, convexity ensures that the optimal capacity of the project is finite. Indeed, if $\gamma > \frac{\beta_1}{\beta_1 - 1}$, then $0 < k_j < \infty$, whereas if $\gamma \to \frac{\beta_1}{\beta_1 - 1}$, then $k_j \to \infty$. Consequently, the result of Proposition 4.4 is in line with the more general intuition that a firm is typically induced to adjust its capital stock more slowly due to diseconomies of scale associated with more rapid changes in the investment cost. Hence, a convex investment cost implies that it is more expensive to perform adjustments, e.g., expand capacity, at a greater than at a lower rate (Jøhansen and Kort, 1993). Additionally, note that $\gamma > 1$ is a consequence of the exogenous price, while the result of Proposition 4.4 depends upon the endogenous relationship between the price at investment and the capacity of the project, as this is described in (6). Allowing for the price to depend on the amount of quantity produced via an inverse demand function will result in a concave cost function (Dangl, 1999), yet the qualitative relationship between the price and capacity will remain the same. Indeed, Dangl (1999) illustrates how the optimal capacity increases monotonically with the output price under economies of scale, i.e., $\gamma < 1$. Since the endogenous relationship between the price at investment and the capacity of the project remains unaffected, Proposition 4.4 should hold under both diseconomies and economies of scale, and, therefore, we omit the analysis of the latter case.

Another consequence of the endogenous relationship between the output price at investment and the size of the project, is that the amount of installed capacity under lumpy investment is always lower than the total amount of capacity installed under a stepwise investment strategy, as shown in Proposition 4.5. Indeed, as the investment cost associated with the stepwise investment strategy increases, it raises both the optimal investment threshold and the amount of installed capacity. Consequently, the firm compensates for the extra cost it incurs for the flexibility to proceed in stages by adjusting the size of the project so that it offsets the reduction in the value of the investment opportunity. As a result, the stepwise investment strategy leads to the installation of a bigger project than that under lumpy investment. This is in contrast to Kort *et al.* (2010), where a firm may delay investment due to an increase in the investment cost, yet it is restricted in terms of the amount of capacity that it can install.

Proposition 4.5 $k_\ell < \sum_{i=1}^{n} k_{s_i}$.

In order to obtain a deeper intuition of the underlying dynamics that determine the optimal investment policy, we analyse the impact of uncertainty on the marginal benefit (MB) and the marginal cost (MC) of delaying investment under each investment strategy assuming that the capacity of the project is either fixed or scalable. Therefore, we first express the firm's maximised option value as in (15)

$$F_j(P) = A_j P^{\beta_1}, \quad \text{where} \quad A_j = \frac{1}{p_j^{\beta_1}} \left[\frac{p_j k_j}{\rho - \mu} - I\left(k_j\right) \right] \tag{15}$$

and then describe the optimal investment rule by equating the MB of delaying investment to the MC, as in (16). The first term on the left–hand side of (16) is positive and represents the incremental project value created by a marginal increase in the output price. Notice that this term is a decreasing function of the output price, since waiting longer enables the project to start at a higher initial price, yet the rate at which this benefit accrues diminishes due to the effect of discounting. The second term is also positive and represents the reduction in the MC of waiting to invest due to saved investment cost. Together, these two terms constitute the MB of delaying investment. The right–hand side of (16) represents the MC of delaying investment. This term is positive and reflects the opportunity cost of forgone cash flows. As shown in Corollary 4.1, when the output price is low it is worthwhile to postpone investment since the MB is greater than the

MC.

$$\text{MB} = \text{MC} \quad \Leftrightarrow \quad \frac{\beta_1 I\left(k_j\right)}{p_j} + \frac{k_j}{\rho - \mu} = \frac{\beta_1 k_j}{\rho - \mu} \tag{16}$$

Corollary 4.1 *The MB is steeper than the MC.*

As Proposition 4.6 indicates, if the capacity of the project is fixed, then greater uncertainty decreases both the MB and the MC of delaying investment, however, the impact of uncertainty on the MC is more pronounced than that on the MB. By contrast, the opposite is true if the firm has discretion over capacity. In fact, although in both cases greater uncertainty postpones investment, the incentive to delay investment is greater when the firm has the flexibility to scale the capacity of the project.

Proposition 4.6 *If k_j is fixed, then $\frac{\partial}{\partial \sigma} MB < 0$, $\frac{\partial}{\partial \sigma} MC < 0$, and $\left|\frac{\partial}{\partial \sigma} MB\right| < \left|\frac{\partial}{\partial \sigma} MC\right|$, whereas if k_j is scalable, then $\frac{\partial}{\partial \sigma} MB > 0$, $\frac{\partial}{\partial \sigma} MC > 0$, and $\frac{\partial}{\partial \sigma} MB > \frac{\partial}{\partial \sigma} MC$.*

Notice that, if the capacity of the project, k_j, is fixed, then from (16) we see that both the MB and MC of delaying investment decrease with greater uncertainty, since $\frac{\partial \beta_1}{\partial \sigma} < 0$. In addition, from (A–12) we have $\frac{I(k_j)}{p_j} < \frac{k_j}{\rho - \mu}$, and, therefore, greater uncertainty lowers the MC by more than the MB. As a result, the marginal value of delaying investment increases, thereby raising the incentive to postpone investment. Intuitively, although the extra benefit from allowing the project to start at a higher output price is fixed, the extra benefit from saving on the investment cost and the extra cost of the forgone cash flows decrease due to the effect of discounting. In fact, the latter becomes more pronounced as both the output price and the volatility increase. By contrast, if the capacity of the project is scalable, then the increase in the optimal capacity of the project with greater uncertainty presents an opposing force, which mitigates the reduction in the value of β_1. As proposition 4.6 indicates, in the latter case both MB and MC of delaying investment increase with greater uncertainty, and, unlike the case of fixed capacity, the MB increases by more than the MC, thus increasing the incentive to delay investment. Consequently, discretion over capacity allows the firm to manage price uncertainty more efficiently by adjusting the size of the project in response to an increase in the investment cost.

5 Numerical Examples

For the numerical examples we assume that $\mu = 0.01$, $\rho = 0.1$, and $\sigma \in [0, 0.4]$. Also, the cost parameters for the lumpy investment are $a_\ell = 30$, $b_\ell = 0.5$, and $\gamma_\ell = 3$, while for the stepwise investment these are $a_{s_1} = 15$, $a_{s_2} = 25$, $b_{s_1} = b_{s_2} = 0.5$, and $\gamma_{s_1} = \gamma_{s_2} = 3$. Under fixed capacity, the investment cost is $I_\ell = 1000$, $I_{s_1} = 500$, and $I_{s_2} = 900$ for lumpy and stepwise investment respectively, while the corresponding capacity levels are $K_\ell = 10$, $K_{s_1} = 3.4$, and $K_{s_2} = 6.6$. Figure 3 illustrates the impact of uncertainty on the optimal investment threshold under scalable capacity, as well as on the optimal capacity of the project. According to the left panel, $p_{s_1} < p_{s_2}$, and, therefore, the numerical assumptions satisfy the condition $\frac{I\left(k_{s_1}\right)}{I\left(k_{s_2}\right)} < \frac{k_{s_1}}{k_{s_2}}$. Additionally, as the right panel illustrates, with the flexibility to scale the size of the project the total capacity when proceeding in stages exceeds that of the lumpy investment, as shown in Proposition 4.5. In fact, the wedge between $k_{s_1} + k_{s_2}$ and k_ℓ reflects the extra value that the firm has due its discretion over capacity. Notice that, since $k_{s_1} + k_{s_2} > k_\ell$ and $a_{s_1} + a_{s_2} > a_\ell$, the condition that stepwise investment is more costly than lumpy investment, as indicated in (3), is also satisfied. Consequently, apart from discretion over capacity, the remaining assumptions are the same as the ones underlying the model of Kort *et al.* (2010).

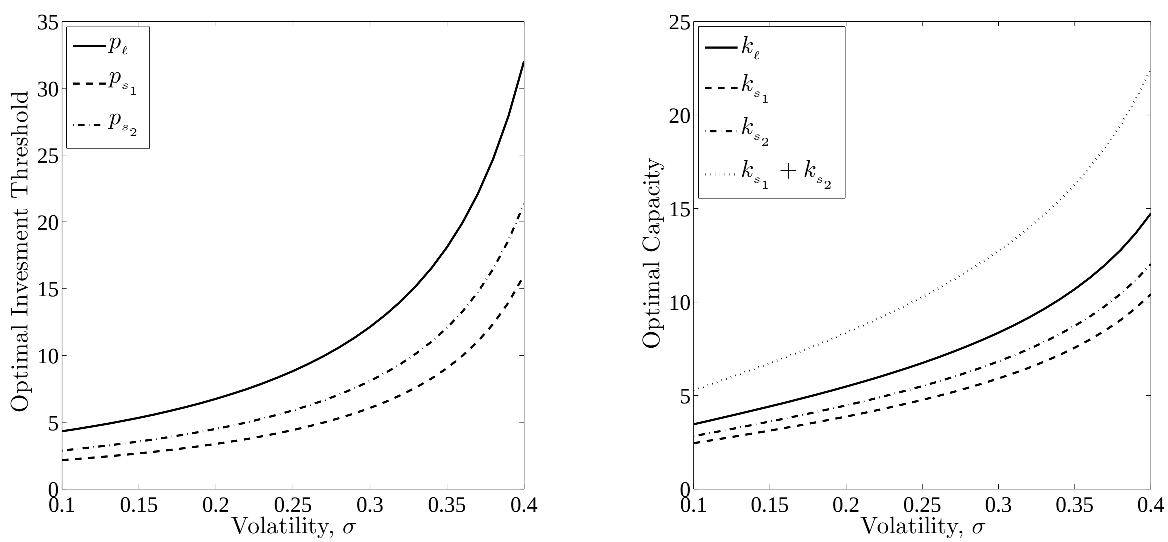

Figure 3: Optimal investment threshold (left) and optimal capacity (right) versus σ

Figure 5 illustrates the impact of uncertainty on the relative value of the two strategies, i.e., $\frac{F_s(P)}{F_\ell(P)}$, under fixed (left panel) and scalable capacity (right panel). In line with Kort *et al.* (2010), if the capacity of the project is fixed, then the relative value of the two strategies is greater than one for low levels of uncertainty, yet drops below one as uncertainty increases (left panel). Hence, lumpy investment becomes more attractive than stepwise investment with greater uncertainty. This happens because greater uncertainty increases inertia and raises the incentive to avoid costly switches between stages, thereby promoting a lumpy investment strategy. By contrast, if a firm has discretion over capacity, then the stepwise investment strategy always dominates that of lumpy investment, as shown in Proposition 4.4. Intuitively, the flexibility to scale the capacity of the project offsets the reduction in the value of the stepwise investment strategy due to the cost that a firm must incur for the flexibility to proceed in stages. Additionally, as the right panel illustrates, the relative value of the two strategies is not only strictly greater than one, but swifts upwards as the investment cost becomes more convex, i.e., as γ_j increases. This implies that a more pronounced increase in the marginal cost of investment creates an extra incentive to adopt a stepwise investment strategy.

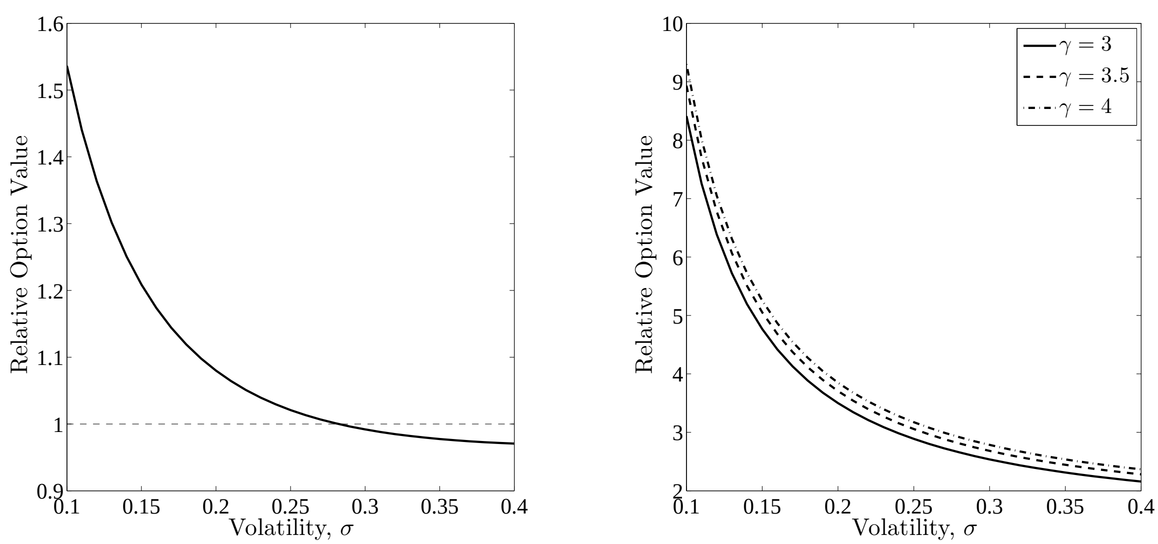

Figure 4: Relative value of the two investment strategies, i.e., lumpy and stepwise, versus σ under fixed capacity (left) and scalable capacity (right)

The left panel in Figure 5 illustrates the MB and MC of delaying investment under scalable capacity for each stage of the project. Notice that for low price levels the MB exceeds the MC, and, as a result, the firm has an incentive to postpone investment. Furthermore, the MB decreases as the output price increases due to the effect of discounting, while the MC is constant. The right panel illustrates the impact of uncertainty on the total MB and MC of delaying investment in each stage of the project under fixed and scalable capacity. In the former case, the MB and MC decrease with greater uncertainty, while, in the latter case, the MB and MC increase, as shown in Proposition 4.6. Intuitively, the incentive to delay investment is greater when the capacity of the project is scalable because a modular investment enables flexibility, thereby making it possible to adapt to uncertain market conditions.

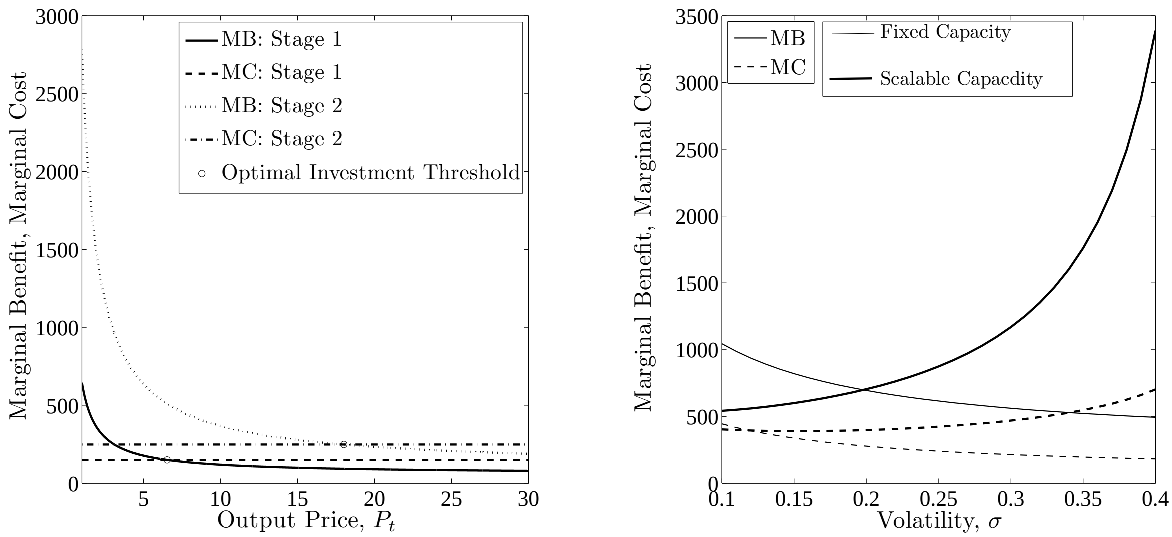

Figure 5: MB and MC of delaying investment for stages $i = 1, 2$ and $\sigma = 0.2$ under scalable capacity (left) and total MB and MC under stepwise investment (right)

6 Concluding remarks

Managerial flexibility is crucial for addressing the valuation and tradeoffs involved in capital projects, that are typically more complex than simple now–or–never investments. In this paper, we extend the results of Kort *et al.* (2010) by assuming that a firm does not only have the flexibility to choose the investment strategy, in terms of lumpy versus stepwise investment, but also has discretion over both the investment timing and the size of the project. Thus, we determine not only the optimal investment threshold and the corresponding optimal capacity under lumpy and stepwise investment, but also the impact of price uncertainty on the relative value of the two investment strategies.

While Kort *et al.* (2010) show that the flexibility to proceed in stages becomes less valuable than lumpy investment with greater uncertainty, which is in contrast to the traditional real options intuition that emphasises on the positive relationship between flexibility and uncertainty, implications from including different types of managerial flexibilities have not been examined thoroughly yet. We confirm the results of Kort *et al.* (2010), however, in addition we show that, if a firm has discretion over capacity, then the stepwise investment strategy always dominates that of lumpy investment. This result emphasises that the relationship between flexibility and uncertainty requires further investigation. Indeed, not only is the positive relationship between the value of flexibility and uncertainty case specific, but, more importantly, the impact of uncertainty on an isolated type of managerial discretion may be completely mitigated if the latter is combined with another type of flexibility. In this paper, we show that, although the flexibility to proceed in stages becomes less valuable than lumpy investment with greater uncertainty when a project has a fixed capacity, allowing for discretion over capacity mitigates this effect completely. More specifically, the reduction in the value of the stepwise investment strategy due to the cost that a firm incurs in order to have the flexibility to proceed in stages is completely offset by the extra value from the flexibility to scale the capacity of the project. Additionally, we show that the amount of installed capacity under stepwise investment is always greater than that under lumpy investment.

The assumption that investment decisions do not affect future prices can be relaxed by linking the output price with the amount of installed capacity via an inverse demand function. Although this in not expected to influence the main result of the paper, it would still be interesting to investigate any quantitative difference due to the implications of installing a very large project. In order to obtain further insights on the robustness of the results regarding the relationship between uncertainty and various combinations of different types of flexibilities, we may also allow for production flexibility in the context of Hagspiel *et al.* (2014), operational flexibility in the form of options to suspend and resume operations, or an alternative stochastic process, e.g., arithmetic Brownian motion or mean–reverting process. Finally, in line with Siddiqui and Takashima (2012), this setup allows for exploration of game–theoretic considerations, e.g., how the presence of a rival impacts the decision to invest and the relative value of the two investment strategies under duopolistic competition.

Acknowledgements The authors would like to express their gratitude to Peter Kort for his valuable comments that helped improve the paper.

APPENDIX

Proposition 4.1: *The optimal investment threshold and the corresponding optimal capacity under lumpy and stepwise investment are:*

$$p_j\left(k_j\right) = \frac{I\left(k_j\right)}{k_j}\frac{\beta_1(\rho-\mu)}{\beta_1-1} \quad and \quad k_j = \left[\frac{a_j}{b_j}\frac{1}{\gamma_j(\beta_1-1)-\beta_1}\right]^{\frac{1}{\gamma_j-1}}, \quad \gamma_j(\beta_1-1)-\beta_1>0 \tag{A–1}$$

Proof: By maximising the value of the now–or–never investment opportunity, we obtain the expression for the optimal capacity, $\overline{k}_j$, corresponding to the current output price P, as indicated in (A–2) for $j = \ell, s_i$.

$$\max_{\overline{K}_j}\overline{F}_j\left(P,\overline{K}_j\right) \quad \Rightarrow \quad \overline{k}_j(P) = \left[\frac{1}{b_j\gamma_j}\left(\frac{P}{\rho-\mu}-a_j\right)\right]^{\frac{1}{\gamma_j-1}} \tag{A–2}$$

Next, the value of the option to invest is described in (A–3).

$$F_j(P) = \begin{cases}(1-\rho dt)\mathbb{E}_P\left[F_j(P+dP)\right] & ,P<p_j\\ \frac{Pk_j}{\rho-\mu}-I\left(k_j\right) & ,P\geq p_j\end{cases} \tag{A–3}$$

By expanding the first branch on the right–hand side of (A–3) using Itô's lemma, we obtain the differential equation (A–4)

$$\frac{1}{2}\sigma^2P^2F_j''(P)+\mu PF_j'(P)-\rho F_j(P)=0 \tag{A–4}$$

which, for $P<p_j$, has the general solution that is indicated in (A–5).

$$F_j(P) = A_jP^{\beta_1}+B_jP^{\beta_2} \tag{A–5}$$

Notice that since $\beta_2<0$ we have $P\to 0\Rightarrow B_jP^{\beta_2}\to\infty$. Consequently, we must have $B_j=0$, and, thus, we finally obtain (A–6).

$$F_j(E) = \begin{cases}A_jP^{\beta_1} & ,P<p_j\\ \frac{Pk_j}{\rho-\mu}-I\left(k_j\right) & ,P\geq p_j\end{cases} \tag{A–6}$$

By applying value–matching and smooth–pasting conditions between the two branches of (A–6) we obtain the expression for the endogenous constant and the optimal investment threshold, that are indicated in (A–7) and (A–8), respectively.

$$A_j = \frac{1}{p_j^{\beta_1}} \left[\frac{p_j k_j}{\rho - \mu} - I\left(k_j\right) \right] \tag{A–7}$$

$$p_j\left(k_j\right) = \frac{I\left(k_j\right)}{k_j} \frac{\beta_1(\rho-\mu)}{\beta_1 - 1} \tag{A–8}$$

Finally, by inserting (A–8) into (A–2), we obtain the expression for the optimal capacity.

$$k_j = \left[\frac{a_j}{b_j} \frac{1}{\gamma_j(\beta_1 - 1) - \beta_1} \right]^{\frac{1}{\gamma_j - 1}}, \quad \gamma_j(\beta_1 - 1) - \beta_1 > 0 \tag{A–9}$$

■

Proposition 4.2: p_{s_1} *is independent of* p_{s_2}.

Proof: If we assume that $\tau_{s_2} \geq \tau_{s_1}$, then $p_{s_1} \leq p_{s_2}$ and the maximised option value in the case of staged investment is indicated in (A–10).

$$F_s(P) = \left(\frac{P}{p_{s_1}}\right)^{\beta_1} \left[\frac{p_{s_1} k_{s_1}}{\rho - \mu} - I\left(k_{s_1}\right) + \left(\frac{p_{s_1}}{p_{s_2}}\right)^{\beta_1} \left[\frac{p_{s_2} k_{s_2}}{\rho - \mu} - I\left(k_{s_2}\right) \right] \right] \tag{A–10}$$

Hence, p_{s_1} satisfies that FONC (A–11)

$$\beta_1 \left(-\frac{1}{p_{s_1}} \right) \left[\frac{p_{s_1} k_{s_1}}{\rho - \mu} - I\left(k_{s_1}\right) \right] + \frac{k_{s_1}}{\rho - \mu} = 0 \tag{A–11}$$

from which we have:

$$p_{s_1} = \frac{I\left(k_{s_1}\right)}{k_{s_1}} \frac{\beta_1(\rho-\mu)}{\beta_1 - 1} \tag{A–12}$$

Consequently, p_{s_1} is independent of p_{s_2}, i.e., the presence of the second stage does not affect the decision to invest in the first one. Note that the assumption $p_{s_1} < p_{s_2}$ can be expressed as in

(A–13).

$$\frac{I\left(k_{s_1}\right)}{I\left(k_{s_2}\right)} < \frac{k_{s_1}}{k_{s_2}} \tag{A–13}$$

■

Proposition 4.3: $\frac{\partial k_j}{\partial \sigma} > 0$ *and* $\frac{\partial p_j}{\partial \sigma} > 0$.

Proof: By differentiating the expression of the optimal capacity in (A–9) we have:

$$\frac{\partial k_j}{\partial \sigma} = k_j \left[\frac{-\frac{\partial}{\partial \sigma}\beta_1}{\gamma_j(\beta_1 - 1) - \beta_1}\right], \quad \gamma_j(\beta_1 - 1) - \beta_1 > 0 \tag{A–14}$$

Since $\frac{\partial}{\partial \sigma}\beta_1 < 0$, we have $\frac{\partial k_j}{\partial \sigma} > 0$. Additionally, the expression of the optimal investment threshold is:

$$\begin{aligned} p_j\left(k_j\right) &= \frac{I\left(k_j\right)}{k_j}\frac{\beta_1(\rho-\mu)}{\beta_1 - 1} \\ &= \frac{a_j k_j + b_j k_j^{\gamma_j}}{k_j}\frac{\beta_1(\rho-\mu)}{\beta_1 - 1} \\ &= \left(a_j + b_j k_j^{\gamma_j - 1}\right)\frac{\beta_1(\rho-\mu)}{\beta_1 - 1} \end{aligned} \tag{A–15}$$

Since $\frac{\partial}{\partial \sigma}\frac{\beta_1}{\beta_1 - 1} > 0$ and $\frac{\partial}{\partial \sigma}k_j > 0$ we have $\frac{\partial}{\partial \sigma}p_j > 0$. ■

Proposition 4.4: *If a firm has discretion over capacity, then* $F_s(P) > F_\ell(P)$.

Proof: The relative value of the two strategies is indicated in (A–16).

$$\frac{F_s(P)}{F_\ell(P)} = \sum_i \frac{F_{s_i}(P)}{F_\ell(P)} \tag{A–16}$$

In order to show that $F_s(P) > F_\ell(P)$, we will show that each term on the right–hand side of (A–16) is greater than one, i.e:

$$\frac{F_{s_i}(P)}{F_\ell(P)} = \frac{\left(\frac{P}{p_{s_i}}\right)^{\beta_1}\left[\frac{p_{s_i}k_{s_i}}{\rho-\mu} - I\left(k_{s_i}\right)\right]}{\left(\frac{P}{p_\ell}\right)^{\beta_1}\left[\frac{p_\ell k_\ell}{\rho-\mu} - I\left(k_\ell\right)\right]} = \left(\frac{k_{s_i}}{k_\ell}\right)^{\beta_1}\frac{I\left(k_\ell\right)^{\beta_1 - 1}}{I\left(k_{s_i}\right)^{\beta_1 - 1}} > 1, \quad i = 1, 2 \tag{A–17}$$

By manipulating the expression of the relative value in (A–17) we obtain (A–18).

$$\left(\frac{k_{s_i}}{k_\ell}\right)^{\beta_1}\frac{I\left(k_\ell\right)^{\beta_1-1}}{I\left(k_{s_i}\right)^{\beta_1-1}} > 1 \Leftrightarrow \frac{\frac{I\left(k_\ell\right)^{\beta_1-1}}{k_\ell^{\beta_1}}}{\frac{I\left(k_{s_i}\right)^{\beta_1-1}}{k_{s_i}^{\beta_1}}} > 1 \Leftrightarrow \frac{k_{s_i}}{k_\ell}\frac{\frac{I\left(k_\ell\right)^{\beta_1-1}}{k_\ell^{\beta_1-1}}}{\frac{I\left(k_{s_i}\right)^{\beta_1-1}}{k_{s_i}^{\beta_1-1}}} > 1 \tag{A–18}$$

Without loss of generality, we assume that $\gamma_\ell = \gamma_{s_i} = \gamma$ and $b_\ell = b_{s_i} = b$. Consequently, any difference between the investment cost of each stage i and that of lumpy investment is expressed through a_j. Notice that this maintains the convexity of the investment cost without violating condition (3). The implication of this assumption is indicated in (A–19).

$$\frac{k_{s_i}}{k_\ell} = \left(\frac{a_{s_i}}{a_\ell}\right)^{\frac{1}{\gamma-1}} \tag{A–19}$$

By substituting the expression for $\frac{k_{s_i}}{k_\ell}$ into (A–18) and by inserting the expression for the optimal capacity from (A–9) into (A–18), we finally obtain (A–20).

$$\frac{k_{s_i}}{k_\ell}\frac{\frac{I\left(k_\ell\right)^{\beta_1-1}}{k_\ell^{\beta_1-1}}}{\frac{I\left(k_{s_i}\right)^{\beta_1-1}}{k_{s_i}^{\beta_1-1}}} = \left(\frac{a_{s_i}}{a_\ell}\right)^{\frac{1}{\gamma-1}}\left(\frac{a_\ell}{a_{s_i}}\right)^{\beta_1-1} = \left(\frac{a_\ell}{a_{s_i}}\right)^{\beta_1-\frac{\gamma}{\gamma-1}} \tag{A–20}$$

Notice that $(\beta_1 - 1)(\gamma - 1) > 1 \Leftrightarrow \beta_1 > \frac{\gamma}{\gamma-1}$, which is the required condition so that $k_j \in \mathbb{R}^+$. Additionally, by differentiating (A–17) with respect to σ as in (A–21), we can determine the relationship between uncertainty and the relative value of the two strategies.

$$\frac{\partial}{\partial\sigma}\frac{F_{s_i}(P)}{F_\ell(P)} = \frac{\partial}{\partial\sigma}\left(\frac{a_\ell}{a_{s_i}}\right)^{\beta_1-\frac{\gamma}{\gamma-1}} = \left(\frac{a_\ell}{a_{s_i}}\right)^{\beta_1-\frac{\gamma}{\gamma-1}}\ln\left(\frac{a_\ell}{a_{s_i}}\right)\frac{\partial\beta_1}{\partial\sigma} \tag{A–21}$$

Notice that if $a_{s_i} < a_\ell$, then greater uncertainty decreases the relative value of the stepwise investment strategy. Consequently, if $a_{s_i} < a_\ell\ \forall i \in \mathbb{N}$, then $\sigma \nearrow \Rightarrow \frac{F_s(P)}{F_\ell(P)} \searrow$. In addition, from (A–20) we conclude that the stepwise investment strategy is always more valuable than lumpy investment.

$$\frac{F_s(P)}{F_\ell(P)} = \sum_i \frac{F_{s_i}(P)}{F_\ell(P)} = \sum_i \left(\frac{a_\ell}{a_{s_i}}\right)^{\beta_1-\frac{\gamma}{\gamma-1}} > 0, \quad \text{if} \quad a_{s_i} < a_\ell,\ \forall i \in \mathbb{N} \tag{A–22}$$

■

Proposition 4.5: $k_\ell < \sum_{i=1}^{n} k_{s_i}$.

Proof: The optimal capacity of the project under lumpy and stepwise investment is described in (A–23).

$$k_j = \left[\frac{a_j}{b_j}\frac{1}{\gamma_j(\beta_1 - 1) - \beta_1}\right]^{\frac{1}{\gamma_j - 1}} \tag{A–23}$$

Setting $\gamma_\ell = \gamma_{s_i} = \gamma$ and $b_\ell = b_{s_i} = b$, the assumption $I(k_\ell) < \sum_i I\left(k_{s_i}\right)$ is equivalent to (A–24).

$$a_\ell < a_{s_1} + a_{s_2} \tag{A–24}$$

Thus, by setting $\xi = \frac{1}{b}\frac{1}{\gamma(\beta_1-1)-\beta_1}$, we obtain from (A–24):

$$a_\ell \xi < a_{s_1}\xi + a_{s_2}\xi \quad \Rightarrow \quad k_\ell^{\frac{1}{\gamma-1}} < k_{s_1}^{\frac{1}{\gamma-1}} + k_{s_2}^{\frac{1}{\gamma-1}} \tag{A–25}$$

Since (A–25) is true $\forall \gamma : \gamma > \frac{\beta_1}{\beta_1 - 1} > 1$, it is also true for $\gamma = 2$, and, thus, we finally have:

$$k_\ell < k_{s_1} + k_{s_2} \tag{A–26}$$

■

Corollary 4.1: *The MB is steeper than the MC.*

Proof: The result follows from differentiating the MB and MC of delaying investment with respect to the output price. Notice that the MC is positive and independent of the output price, while $\frac{\partial}{\partial P}MB < 0$. ■

Proposition 4.6: *If k_j is fixed, then $\frac{\partial}{\partial\sigma}MB < 0$, $\frac{\partial}{\partial\sigma}MC < 0$, and $\left|\frac{\partial}{\partial\sigma}MB\right| < \left|\frac{\partial}{\partial\sigma}MC\right|$, whereas if k_j is scalable, then $\frac{\partial}{\partial\sigma}MB > 0$, $\frac{\partial}{\partial\sigma}MC > 0$, and $\frac{\partial}{\partial\sigma}MB > \frac{\partial}{\partial\sigma}MC$.*

Proof: The MB and MC of delaying investment is indicated in (A–27).

$$\text{MB} = \text{MC} \quad \Leftrightarrow \quad \frac{\beta_1 I\left(k_j\right)}{p_j} + \frac{k_j}{\rho - \mu} = \frac{\beta_1 k_j}{\rho - \mu} \tag{A–27}$$

Notice that if the capacity of the project is fixed, then an increase in σ lowers both the MB and the MC as indicated in (A–28).

$$\frac{\partial MB}{\partial \sigma} = \frac{I\left(k_j\right)\frac{\partial}{\partial\sigma}\beta_1}{p_j} < 0 \text{ and } \frac{\partial MC}{\partial \sigma} = \frac{k_j\frac{\partial}{\partial\sigma}\beta_1}{\rho-\mu} < 0 \tag{A–28}$$

However, from (A–12) we know that $\frac{I(k_j)}{p_j} < \frac{k_j}{\rho-\mu}$, and, therefore, the MC of delaying investment decreases by more than the MB.

$$\left|\frac{\partial MC}{\partial \sigma}\right| > \left|\frac{\partial MB}{\partial \sigma}\right| \tag{A–29}$$

By contrast, if the capacity of the project is scalable, then the MB and MC of delaying investment increase with greater price uncertainty. Indeed, for $P < p_j$ we have:

$$\frac{\partial MB}{\partial \sigma} = \frac{I\left(k_j\right)\frac{\partial}{\partial\sigma}\beta_1 + \beta_1\frac{\partial}{\partial\sigma}I\left(k_j\right)}{p_j} + \frac{\frac{\partial}{\partial\sigma}k_j}{\rho-\mu} > 0 \tag{A–30}$$

The second term on the right–hand side of (A–30) is positive since $\frac{\partial}{\partial\sigma}k_j > 0$. Notice also that, even though $\frac{\partial}{\partial\sigma}\beta_1 < 0$, β_1 is bounded from below since $\beta_1 > 1$. By contrast, since the capacity of the project is not bounded and $\frac{\partial}{\partial\sigma}I\left(k_j\right) > 0$, the decrease in β_1 is mitigated by the increase in the investment cost. The impact of σ on the MC is indicated in (A–31).

$$\frac{\partial MC}{\partial \sigma} = \frac{k_j\frac{\partial}{\partial\sigma}\beta_1 + \beta_1\frac{\partial}{\partial\sigma}k_j}{\rho-\mu} \tag{A–31}$$

Notice that reduction in β_1 makes the impact of σ on k_j less pronounced, and, therefore, the second term on the right–hand side of (A–30) is greater than right–hand side of (A–31). Since the first term on the right–hand side of (A–30) is positive, we have:

$$\frac{\partial MB}{\partial \sigma} > \frac{\partial MC}{\partial \sigma} \tag{A–32}$$

Finally, notice that the second term on the left–hand side of (A–27) is constant when the capacity is fixed and increasing when the capacity is scalable. Similarly, the reduction of first term due to the decrease in β_1 with greater uncertainty is offset by the increase in k_j. Consequently, the impact of σ on the MB of delaying investment is not only reversed when the firm has discretion over capacity but it is also more pronounced. ■

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